Hohmann Transfer Orbit
LEO to GEO

- most efficient orbit transfer
- tangential to initial & target orbital
- 2 impulsive engine burns
  - ↳ 1st transfer orbit
  - ↳ 2nd adjust to match target orbit

↳ lowest possible amt of impuls
↳ Δv = change in v = measure of impuls
  ↳ thrust per unit mass

thrust: $\boxed{T(t)} = v \frac{dm}{dt}$ ← expelled mass (fuel)

$\frac{m}{s} \frac{kg}{s} = N$

Delta v: $\Delta v = \int_{t_0}^{t_1} \frac{T(t)}{m(t)} dt$

$kg \frac{m}{s^2}$

impulse: $\Delta p = m \Delta v$

propellant flow rate to combustion chamber

thruster: $T = v_{exh} \dot{p}$

spacecraft

$T = m\dot{v} \Rightarrow \boxed{\dot{v} = \frac{T}{m} = v_{exh} \frac{\dot{p}}{m}}$

$\int_{t_0}^{t_1} T(t)/m \, dt$

$\dot{m} = -\dot{p}$

$\frac{dm}{dt}$

$\Delta v = -\int_{t_0}^{t_1} v_{exh} \frac{\dot{m}}{m} dt = -\int_{m_0}^{m_1} v_{exh} \frac{dm}{m}$

$= v_{exh} \ln\left(\frac{m_0}{m_1}\right)$ ← hohmann transfer Δv

kinetic $E = \frac{mv^2}{2} - \frac{GMm}{r} = \frac{-GMm}{2a}$ a = avg dist

then, $v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$

↳ GM → grav parameter

orbit speed $v = \sqrt{\frac{\mu}{r}}$ → $r = \frac{\mu}{v^2}$

semimajor axis $a_t = \frac{r_1 + r_2}{2}$ recall

$r_1 = r_{LEO}$, $r_2 = r_{GEO}$

v @ perigee (clossest point) of transfer ellipse

$$v_{t_1} = \sqrt{2\frac{\mu}{r_1} - \frac{\mu}{a_t}}$$

$r =$

@ apogee

$$v_{t_2} = \sqrt{2\frac{\mu}{r_2} - \frac{\mu}{a_t}}$$

$$a_t = \frac{r_1 + r_2}{2} = \frac{\mu}{v_1^2}\frac{1}{2} + r_2\frac{1}{2} \Rightarrow a_t - r_2\frac{1}{2} = \frac{\mu}{v_1^2}\frac{1}{2}$$

$$\Rightarrow v_1^2 = \frac{\mu}{2}\left(\frac{1}{a_t - r_2\frac{1}{2}}\right)\frac{(a_t + r_2\frac{1}{2})}{(a_t + r_2\frac{1}{2})}$$

$$= \frac{\mu}{2}\left(\frac{a_t + r_2\frac{1}{2}}{a_t^2 - a_t r_2\frac{1}{2} - r_2^2\frac{1}{4}}\right)$$

$\cancel{a_t = \frac{r_1+r_2}{2} \Rightarrow r_1 = 2a_t - r_2}$

$$\Rightarrow \frac{\mu}{v_1^2} = 2a_t - r_2$$

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$ vis-viva eqn

LEO | GEO

porogee | apogee

$$T = \pi\sqrt{\frac{a_t^3}{\mu}}$$

$\to v_{LEO} \to v_{GEO}$

$\to$ equivalent to $\sqrt{\frac{GM}{r}}$ for circular

$$\Delta v = I_{sp} \cdot g_0 \cdot \ln\left(\frac{m_0}{m_f}\right) \Rightarrow e^{\Delta v} = e^{I_{sp} g_0 \ln(m_0/m_f)}$$

$$m_f = \frac{m_0}{e^{\Delta v/(I_{sp} g_0)}}$$

Starship dry mass

initial fuel mass

I_{sp}

Position

$$m\frac{d^2\vec{r}}{dt^2} = \vec{F} = -G\frac{Mm}{r^3}\vec{r} = -\frac{\mu m}{r^3}\vec{r}$$

$$\Rightarrow \frac{d^2\vec{r}}{dt^2} = -\frac{\mu}{r^3}\vec{r}$$

$\vec{v} = \frac{d\vec{r}}{dt}$ $\quad \frac{d\vec{v}}{dt} = -\frac{\mu}{r^3}\vec{r}$, $\frac{dx}{dt} = v_x$, $\frac{dy}{dt} = v_y$

so, $\frac{dv_x}{dt} = -\frac{\mu}{r^3}x$ same for y

then, $\vec{v}(t) = v(t_0) + \int_{t_0}^{t} \frac{d\vec{v}}{d\tau}\,d\tau = v_0 - \int_{t_0}^{t} \frac{\mu}{r(\tau)^3}\vec{r}(\tau)$

dummy (every time)

$$\vec{r}(t) = \vec{r}(t_0) + \int_{t_0} \vec{v}(\tau)\,d\tau$$

solve_ivp(, ,

$\frac{dy}{dt} = f(t,y)$ $\quad (t0, tf)$ $\quad$ initial value @ t_0, y_0