CMPT 440 – Spring 2019: Quantum Finite Automata

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Theoretical Background

Quantum finite automata (QFA) are divided into five main models. For the purposes of this paper I will be discussing the most basic models of QFAs, which are one-way quantum finite automata (1QFA). There are two main types of 1QFA which are measure-many and measure-one models. A measure-many one-way quantum finite automaton (MM-1QFA) is defined by a 6-tuple(Q, Sigma, delta, q0, Qacc, Qrej).

- Q is a finite set of symbols
- Sigma is an input alphabet
- delta is a transition function
- q0 is an initial state
- Qacc is a set of accepting sets
- Qrej is a set of rejecting sets

An Example

MM-1QFA example:
Sigma = [a]
Q = [Q0,Q1,Qacc,Qrej]
Va(Q0) = ($\frac{1}{2}$Q0)+($\frac{1}{2}$Q1) +($\frac{1}{2}$Qreq)
word = "aa"
1. The QFA starts in q0, then Va is used. The computation has a $\frac{1}{2}$ probability of entering a rejecting state and a $\frac{1}{2}$ probability of entering an accepting state.
2. Next ($\frac{1}{2}$Q0)+($\frac{1}{2}$Q1) is put in a non-halting state by mapping itself by Va
3. Input is empty and so it maps the superposition to ($\frac{1}{2}$Qrej)+($\frac{1}{2}$Qacc)
The total probability of accepting is 1/4, the probability of rejecting is 3/4

References

A. Ambainis and R. Freivalds. 1-way quantum finite automata: strengths, weaknesses andgeneralizations. In Proceedings 39th Annual Symposium on Foundations of Computer-Science (Cat. No. 98CB36280), pages 332-341. IEEE, 1998.