

3.14 Theorem. Given any integer a and any natural number n , there exists a unique integer t in the set $\{0, 1, 2, \dots, n - 1\}$ such that $a \equiv t \pmod{n}$.

Proof. Let $a \in \mathbb{Z}$ be given. By TDA, there exists a unique quotient q and unique remainder t such that

$$\begin{aligned}a &= nq + t \text{ for } n \in \mathbb{Z}, \\a - t &= nq.\end{aligned}$$

Thus, by definition, $a \equiv t \pmod{n}$.

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