

3.17 Theorem. Let n be a natural number. Any set, $\{a_1, a_2, \dots, a_n\}$, of n integers for which no two are congruent modulo n is a complete residue system.

Proof. Let the set $S = \{a_1, a_2, \dots, a_n\}$ of n integers for which no two or more are congruent modulo n be given. This implies that when each of the elements are divided by n , there will be distinct remainders. More specifically, there will be n distinct remainders. Thus, fulfilling the CRS definition. Suppose there are elements from the CCRS that did not map to the one or more elements of S . By the Pigeonhole principle, this means that one of the elements from the CCRS maps to two or more elements in S , which is a contradiction. Thus, given a set of integers for which no two are congruent modulo n is a complete residue system. \square