1.45 Theorem. Let $a, b, c, n \in \mathbb{Z}$ with $n > 0$. If $ac \equiv bc \pmod{n}$ and $(c, n) = 1$, then $a \equiv b \pmod{n}$.

Proof. Let $a, b, c, n \in \mathbb{Z}$ with $n > 0$ be given such that $ac \equiv bc \pmod{n}$ and $(c, n) = 1$. By definition, $n \mid (ac - bc)$. Factoring c,

$$n \mid c(a - b).$$

By Theorem 1.41, since $(c, n) = 1$, $n \mid (a - b)$, and by definition, $a \equiv b \pmod{n}$. $\square$