1.

read	pop	push
a	S	S1
c	S	ϵ
b	1	ϵ

2. (a)

read	pop	push
0	q_0	q_1
1	q_0	q_0
0	q_1	q_2
1	q_1	q_1
0	q_2	q_0
1	q_2	q_2
0	q_0	ϵ
1	q_0	ϵ

(b)

read	pop	push
a	S	Sbb
b	S	Sb
b	S	ϵ
b	b	ϵ

(c)

read	pop	push
a	S	Sb
ϵ	S	ϵ
b	b	ϵ
b	S	Sa
a	S	Sa
b	S	Sb
b	S	ϵ
a	a	ϵ

(d)

read	pop	push
a	S	Sd
b	S	Sc
ϵ	S	ϵ
c	c	ϵ
d	d	ϵ

3.

read	pop	push
ϵ	S	S_1S_2
ϵ	S	S_3S_1
ϵ	S_1	aS_1b
ϵ	S_1	ϵ
ϵ	S_2	bS_2
ϵ	S_2	b
ϵ	S_3	aS_3
ϵ	S_3	a
a	a	ϵ
b	b	ϵ

4.

$$
\begin{aligned}
S &\to S_1S_2S_3 \\
S_1 &\to aS_1b \mid \epsilon \\
S_2 &\to cS_2 \mid \epsilon \\
S_3 &\to dS_3 \mid \epsilon
\end{aligned}
$$

5. **Prove.** $L = \{a^nb^mc^n \mid m \geq n\}$ is not context free.

Proof. Suppose not. That is, let $L = \{a^nb^mc^n \mid m \geq n\}$ be context free such that the pumping lemma holds for L. Let k be given by the pumping lemma. Choose $z = a^kb^kc^k$ such that $z \in L$ and $|z| \geq k$. Let u, v, w, x, y be given such that $z = uvwxy = a^kb^kc^k$, v, x are not both ϵ, $|vwx| \leq k$, and for all i, $uv^iwx^iy \in L$. Consider when $i = 2$, since v and x cannot contain more than one type of symbol each, it follows that $|vwx| \geq k$, which contradicts our assumption that $|vwx| \leq k$. Thus, L is not context free. $\square$