1. (a)

$$\begin{aligned} S &\Rightarrow X \\ &\Rightarrow bX \\ &\Rightarrow bbX \\ &\Rightarrow bbbX \\ &\Rightarrow bbb \end{aligned}$$

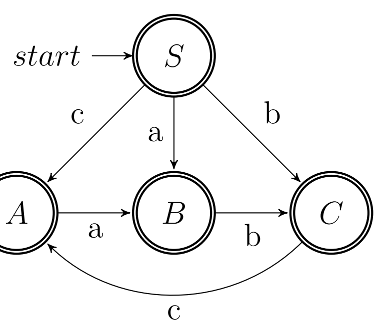
(b)

$$\begin{aligned} S &\Rightarrow aS \\ &\Rightarrow aaS \\ &\Rightarrow aaX \\ &\Rightarrow aabX \\ &\Rightarrow aabbX \\ &\Rightarrow aabb \end{aligned}$$

(c)

$$\begin{aligned} S &\Rightarrow X \\ &\Rightarrow \epsilon \end{aligned}$$

2.



3. (a)

$$\begin{aligned} S &\rightarrow aX \mid \epsilon \\ X &\rightarrow bS \end{aligned}$$

(b)

$$S \to aX \mid X$$
$$X \to bS \mid \epsilon$$

(c)

$$S \to aX \mid X$$
$$X \to bS \mid \epsilon$$

(d)

$$S \to aX \mid X$$
$$X \to bS \mid \epsilon$$

4. Prove $\{a^n b^n c^n\}$ is not regular.
   Proof. Suppose not. That is, let $L_1 = \{a^n b^n c^n\}$ be regular such that the Pumping Lemma gives us k. Choose some $x = a^k, y = b^k, z = c^k$ such that $xyz = a^k b^k c^k \in L_1$ and $|y| \geq k$. By the Pumping Lemma, let $y = uvw$ with $|v| > 0$ such that $xuv^iwz \in L_1$ for all $i \geq 0$. Since v is a sequence of one or more bs, uv^2w has more bs than uvw, and xuv^2wz has more bs than as and cs. This contradicts our initial assumption that $|a| = |b| = |c|$. Thus, $\{a^n b^n c^n\}$ is not regular. □

5. (a)

$$S \to 0A \mid 1A \mid \epsilon$$
$$A \to 0B \mid 1B$$
$$B \to 0S \mid 1S$$

(b)

$$S \to aaSbb \mid \epsilon$$

(c)

$$S \to aSb \mid A \mid \epsilon$$
$$A \to bAa \mid S \mid \epsilon$$

(d)

$$S \to aSa \mid bSb \mid \epsilon$$

6. Two different parse trees for the string ( ) [ ].

<s>

$\downarrow$

<round><square>

$\downarrow$

( ) [ ]

Second parse tree for the same string.

<s>

$\downarrow$

<outer>

$\downarrow$

(<inner>]

$\downarrow$

( ) [ ]