**1.41 Theorem.** Let $a, b, c \in \mathbb{Z}$. If $a \mid bc$ and $(a, b) = 1$, then $a \mid c$.

**Proof.** Let $a, b, c \in \mathbb{Z}$ be given such that $a \mid bc$ and $(a, b) = 1$. By Theorem 1.38, we know there exists $x, y \in \mathbb{Z}$ such that $ax + by = 1$. Multiplying this equation by $c$,

$$c(ax + by) = c(1),$$
$$acx + bcy = c.$$

Observing the left hand side of the equation, $a$ divides any integer multiple of $a$ and $a$ divides any integer multiple of $bc$. Since $a$ divides both terms, $a \mid c$. $\square$