2.34 Theorem. Let k be a natural number. There exists a prime greater than k.

Proof. Suppose not. That is, let $k \in \mathbb{N}$ such that all primes are less than or equal k. Let the set $S = \{p_1, p_2, ...p_m\}$ be all the primes less than or equal to k. Let $n + 1 = p_1p_2...p_m + 1$ such that no p_i divides $n + 1$ since $(n, n + 1) = 1$. This allows us to conclude there is a prime in the prime factorization of n that is not only, greater than k, but also not in set S. Thus, we have reach a contradiction such that there exists a prime greater than a natural number k. $\square$