1.37 Exercise. Find integers x and y such that $162x + 31y = 1$.

$$\begin{aligned} 162 &= 31(5) + 7 \\ 31 &= 7(4) + 3 \\ 7 &= 3(2) + 1 \end{aligned}$$

Doing some fancy isolation of the various equations

$$\begin{aligned} 1 &= 7 - 3(2) \\ &= 7 - [31 - 7(4)](2) \\ &= [162 - 31(5)] - 2\{31 - [162 - 31(5)]4\} \\ &= 162 - 5(31) - 2\{31 - [4(162) - 20(31)]\} \\ &= 162 - 5(31) - 2(31) + 8(162) - 40(31) \\ &= 9(162) + (-47)(31). \end{aligned}$$

We find $x = 9$ and $y = -47$.