Numerical Analysis HW 8

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May 1, 2017

Problem 1. For this problem and using Dr. Glunt's code, I had to change a couple lines of code. First thing I changed was setting "iknowtheactualsolution" to "false", and second change was to set function f equal to $16\pi^2(x^2-x)\sin(4\pi y)-2\sin(4\pi y)$. Here is the code:

```
implicit none
double precision, allocatable, dimension(:,:) :: u, unew, rhs
double precision :: delta, f, relerr, abserr, relgot, absgot
double precision :: actual
logical :: iknowtheactualsolution
integer :: iterate, n, i, j, itmax

n = 50
itmax = 1000000

delta = 1.0d0/dble(n+1)
relerr = 1.0d-7
abserr = 1.0d-7

iknowtheactualsolution = .false.

allocate(u(0:n+1, 0:n+1), unew(0:n+1, 0:n+1), rhs(0:n+1, 0:n+1))
u = 0.0d0
unew = u
print*, 'Setting right hand sides'

call setrhs(n, delta, rhs)

open(unit = 9, file = 'Greport', status = 'replace')
print*, 'Main loop for GS is starting'
do iterate = 1, itmax
call gssweep(n, u, unew, rhs, absgot, relgot)
u = unew
write(9,*) iterate, absgot, relgot
```

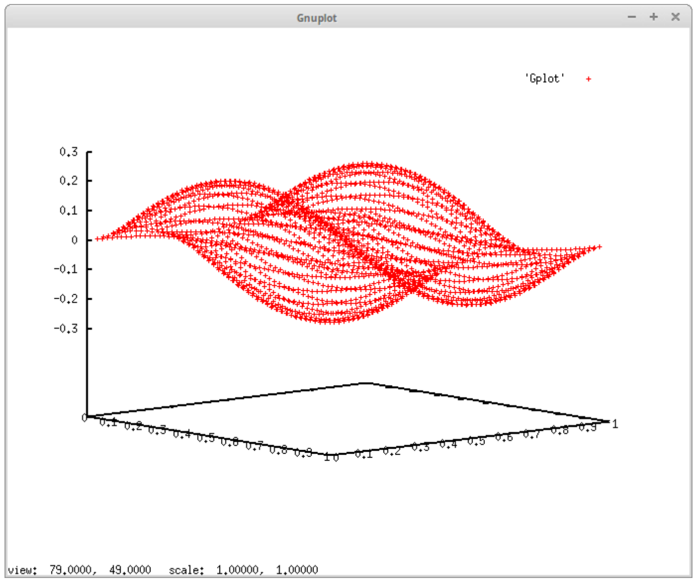
```
if(absgot < abserr .and. relgot < relerr) then
print*, 'Ha! Victory is mine at step:', iterate
open(unit = 7, file = 'Gplot', status = 'replace')

do i = 1, n

do j = 1, n
write(7,*) delta*dble(i), delta*dble(j), u(i,j)
end do

write(7,*)
end do

if(iknowtheactualsolution) then
call compare(n,u)
end if

close(7)
close(9)
print*, 'Halt'
stop
end if
end do
print*, 'Convergence not got', absgot, relgot, 'writing Gplot anyway'

open(unit = 7, file = 'Gplot', status = 'replace')
do i = 1, n

do j = 1, n
write(7,*) delta*dble(i), delta*dble(j), u(i,j)
end do

write(7,*)
end do

if(iknowtheactualsolution) then
call compare(n, u)
end if

close(7)
close(9)

deallocate(u, unew, rhs)
stop
end
```

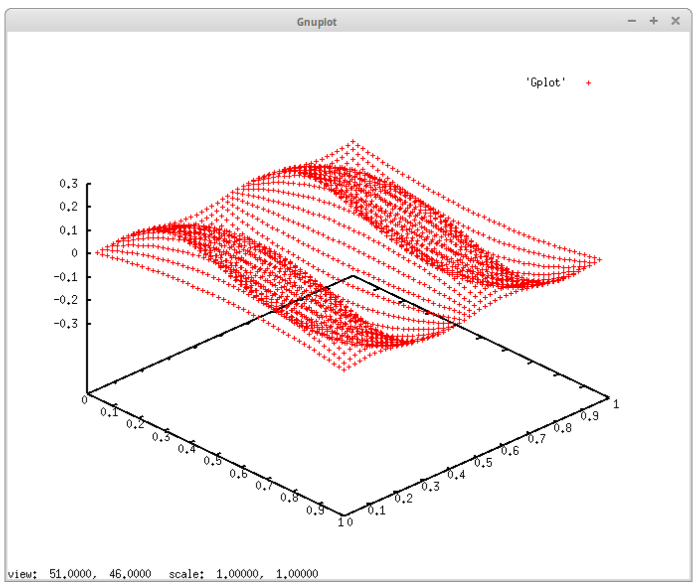
```
!--------subroutines and functions ---------------

double precision function f(x, y)
implicit none
double precision :: x, y, pi

pi = 3.14159265359d0

f = 16.0d0*pi**2 *(x**2 -x) *sin(4.0d0*pi*y)-2.0d0*sin(4.0d0*pi*y)

return
end

subroutine setrhs(n, delta, rhs)
implicit none
integer :: n, i, j
double precision :: delta, f, rhs(0:n+1, 0:n+1)

do i = 1, n
do j = 1, n
rhs(i,j) = delta**2 * f(delta*dble(i), delta*dble(j))
end do
end do

return
end

subroutine gssweep(n, u, unew, rhs, absgot, relgot)
implicit none
integer :: n, i, j
double precision :: absgot, relgot, diffa, diffr
double precision :: u(0:n+1, 0:n+1), unew(0:n+1, 0:n+1), rhs(0:n+1, 0:n+1)
double precision :: bot

absgot = 0.0d0
relgot = 0.0d0

do i = 1, n
do j = 1, n
unew(i,j) = (rhs(i,j) -u(i-1,j) -u(i+1,j) -u(i,j-1) -u(i,j+1)) /(-4.0d0)

diffa = abs(u(i,j) - unew(i,j))
if(diffa > absgot) then
absgot = diffa
end if
```

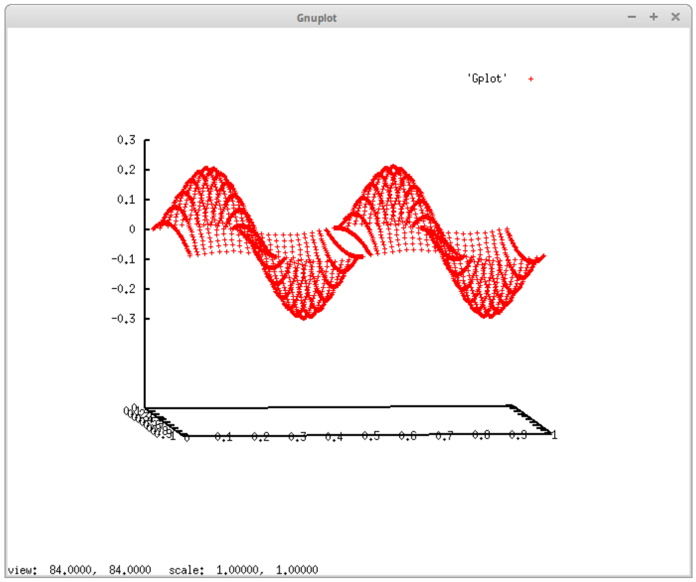
```
bot = abs(u(i,j))
if(bot == 0.0d0) then
bot = 1.0d0
end if

diffr = diffa/bot
if(diffr > relgot) then
relgot = diffr
end if

u(i,j) = unew(i,j)
end do
end do

return
end

double precision function actual(x,y)
implicit none
double precision :: x, y

actual = x*y * (1.0d0 - x) * (1.0d0 - y)

return
end

subroutine compare(n,u)
implicit none
integer :: n, i, j
double precision :: u(0:n+1, 0:n+1), x, y, delta, actual
double precision :: diff, worstabs, worstrel, bot

delta = 1.0d0/dble(n+1)
worstabs = 0.0d0
worstrel = 0.0d0

print*, '-----------------------------------------------'
write(9,*) '-----------------------------------------------'
do i = 1,n
do j = 1,n
x = dble(i)*delta
y = dble(j)*delta
diff = abs(u(i,j) - actual(x,y))
if(diff > worstabs) then
worstabs = diff
end if
```

```
bot = abs(u(i,j))
if(bot == 0.0d0) then
bot = 1.0d0
end if

diff = diff/bot
if(diff > worstrel) then
worstrel = diff
end if

end do
end do

write(9,*) 'Max Uij-actual(xi,yj)=', worstabs
write(9,*) 'Max relerror=', worstrel
write(*,*) 'Max Uij-actual(xi,yj)=', worstabs
write(*,*) 'Max relerror=', worstrel

print*, '--------------------------------------------------'
write(9,*) '----------------------------------------------------'

return
end
```

I then ran the output file Gplot in gnuplot and here are a few shots of the graph:


Gnuplot
'Gplot'
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
view: 79.0000, 49.0000 scale: 1.00000, 1.00000


Gnuplot
'Gplot'
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
view: 51.0000, 46.0000 scale: 1.00000, 1.00000


Gnuplot
'Gplot'
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
view: 84.0000, 84.0000 scale: 1.00000, 1.00000

Problem 2. To solve the problem $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u + f(x, y)$, I did some basic algebra to isolate $f(x, y)$, and then using the descretionization of the second derivatives as described in our lectures, we have:

$$\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{\Delta^2} + \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{\Delta^2} - u_{i,j} = f_{i,j}$$

$$u_{i-1,j} - 2u_{i,j} + u_{i+1,j} + u_{i,j-1} - 2ui, j + u_{i,j+1} - \Delta^2 u_{i,j} = \Delta^2 f_{i,j}$$

$$(-4 - \Delta^2)u_{i,j} = \Delta^2 f_{i,j} - u_{i-1,j} - u_{i+1,j} - u_{i,j-1} - u_{i,j+1}$$

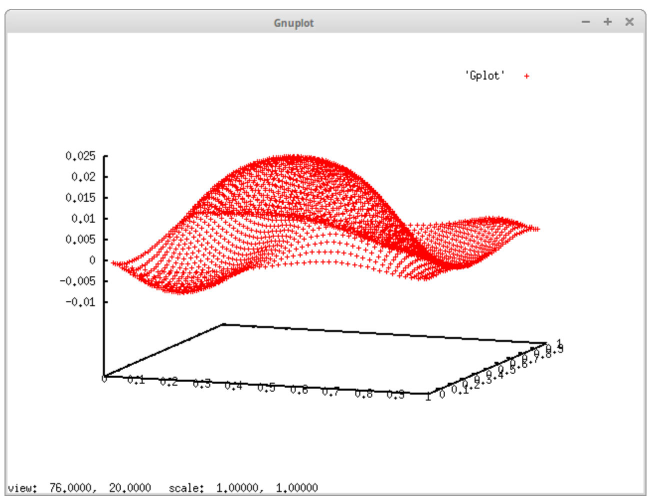
$$u_{i,j} = \frac{\Delta^2 f_{i,j} - u_{i-1,j} - u_{i+1,j} - u_{i,j-1} - u_{i,j+1}}{-4 - \Delta^2}$$

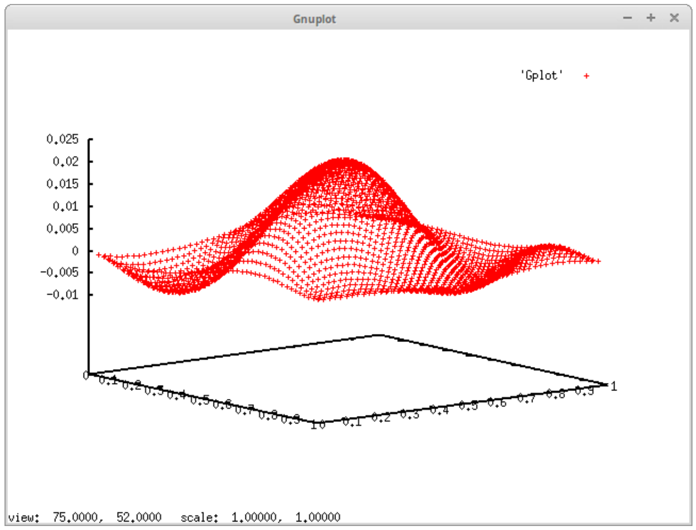
Using our new $u_{i,j}$, I would then change the code in the program so that $unew_{i,j} = \frac{\Delta^2 f_{i,j} - u_{i-1,j} - u_{i+1,j} - u_{i,j-1} - u_{i,j+1}}{-4-\Delta^2}$. Here are the snips of sections of code I changed from the previous problem's code:

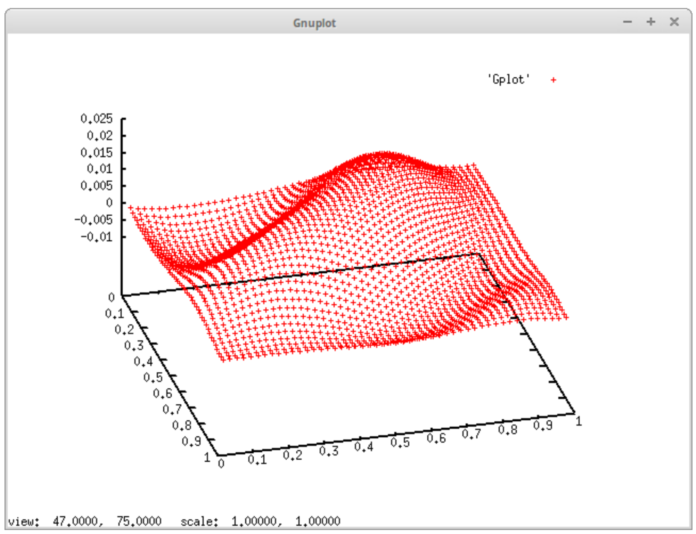
```
double precision function f(x, y)
implicit none
double precision :: x, y

f = cos(6.0d0*(x**2 + y**2))

return
end

subroutine gssweep(n, u, unew, rhs, absgot, relgot)
implicit none
integer :: n, i, j
double precision :: absgot, relgot, diffa, diffr
double precision :: u(0:n+1, 0:n+1), unew(0:n+1, 0:n+1), rhs(0:n+1, 0:n+1)
double precision :: bot

absgot = 0.0d0
relgot = 0.0d0

do i = 1, n
do j = 1, n
unew(i,j) = (rhs(i,j) -u(i-1,j) -u(i+1,j) -u(i,j-1) -u(i,j+1)) /(-4.0d0-(1.0d0/dble(n+1))**2

diffa = abs(u(i,j) - unew(i,j))
if(diffa > absgot) then
absgot = diffa
```

```
end if

bot = abs(u(i,j))
if(bot == 0.0d0) then
bot = 1.0d0
end if

diffr = diffa/bot
if(diffr > relgot) then
relgot = diffr
end if

u(i,j) = unew(i,j)
end do
end do

return
end
```

I then ran the output file Gplot through gnuplot, and here are a few pictures of the graph:




Gnuplot
'Gplot'
view: 75.0000, 52.0000 scale: 1.00000, 1.00000


Gnuplot
'Gplot'
view: 47.0000, 75.0000 scale: 1.00000, 1.00000