1.23 Theorem. If the sum of the digits of a natural number expressed in base 10 is divisible by 3, then the number is divisible by 3 as well.

Proof. Let the sum of the digits of a natural number expressed in base 10 be divisible by 3 be given such that $a, a' \in \mathbb{N}$ and the number a is represented as $a = a_k a_{k-1} \cdots a_1 a_0$ and the sum of its digits is represented as

$$a' = \sum_{i=0}^{k} a_i 10^i$$

$$\begin{aligned}
a' &= a_k 10^k + a_{k-1} 10^{k-1} + \cdots a_1 10^1 + a_0 10^0 \\
&= a_k(10^k + 1 - 1) + a_{k-1}(10^{k-1} + 1 - 1) + \cdots a_1(10 + 1 - 1) + a_0(1 + 1 - 1) \\
&= a_k(10^k + 1 - 1) + a_{k-1}(10^{k-1} + 1 - 1) + \cdots a_1(10 + 1 - 1) + a_0(1 + 1 - 1) \\
&= a_k(10^k - 1) + a_k + a_{k-1}(10^{k-1} - 1) + a_{k-1} + \cdots a_1(9) + a_1 + a_0 \\
&= a_k(10^k - 1) + a_{k-1}(10^{k-1} - 1) + \cdots 9a_1 + a_k + a_{k-1} + \cdots + a_1 + a_0 \\
&= (a_k(10^k - 1) + a_{k-1}(10^{k-1} - 1) + \cdots 9a_1) + (a_k + a_{k-1} + \cdots + a_1 + a_0).
\end{aligned}$$

Observe each of the terms in the first group contains a factor of 3 and therefore is divisible by 3. Since the second term is also divisble by 3, the number itself is divisible by 3. $\square$