2.13 Theorem. If a and b are natural numbers and $a^2|b^2$, then $a|b$.

Proof. Let a and b be given natural numbers such that $a^2|b^2$. Let $p_1^{r_1}p_2^{r_2}...p_m^{r_m}$ be the unique prime factorization of a and let $q_1^{t_1}q_2^{t_2}...q_s^{t_s}$ be the unique prime factorization of b. By definition, $b^2 = a^2k$ for some $k \in \mathbb{Z}$ such that

$$
\begin{aligned}
(q_1^{t_1}q_2^{t_2}...q_s^{t_s})^2 &= (p_1^{r_1}p_2^{r_2}...p_m^{r_m})^2k, \\
q_1^{2t_1}q_2^{2t_2}...q_s^{2t_s} &= p_1^{2r_1}p_2^{2r_2}...p_m^{2r_m}k.
\end{aligned}
$$

By Theorem 2.12, since $a^2|b^2$, $p_i = q_j$ and $2r_i \leq 2t_j$. Thus, we can simplify the exponents to $r_i \leq t_j$. Notice we are left with

$$
\begin{aligned}
q_1^{t_1}q_2^{t_2}...q_s^{t_s} &= p_1^{r_1}p_2^{r_2}...p_m^{r_m}k, \\
b &= ak.
\end{aligned}
$$

Thus, $a|b$. $\square$