A.10 Theorem. Let n be a natural number. Then $1+2+3+\cdots+n = \frac{n(n+1)}{2}$.

Proof. Let $P(n)$ be the statement $1+2+3+\cdots+n = \frac{n(n+1)}{2}$ and $n \in \mathbb{N}$. We consider the base case where $n = 1$.

For $P(1)$,

$$\begin{aligned} 1 &= \frac{1(1+1)}{2} \\ &= \frac{2}{2} \\ &= 1. \end{aligned}$$

Since the base case is true, we will prove by induction. Suppose now $1+2+3+\cdots+k = \frac{k(k+1)}{2}$ for some $k \in \mathbb{N}$. We want to show $1+2+3+\cdots+k+(k+1) = \frac{(k+1)(k+2)}{2}$. It follows,

$$\begin{aligned} 1+2+3+\cdots+k+(k+1) &= \frac{k(k+1)}{2} + (k+1) \\ &= \frac{k(k+1)}{2} + \frac{2(k+1)}{2} \\ &= \frac{k(k+1)+2(k+1)}{2} \\ &= \frac{(k+1)(k+2)}{2}. \end{aligned}$$

Thus, by induction, $1+2+3+\cdots+n = \frac{n(n+1)}{2}$. $\square$