1.14 Theorem. Let $a, b, c, d, n \in \mathbb{Z}$ with $n > 0$. If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $ac \equiv bd \pmod{n}$.

Proof. Let $a, b, c, d, n \in \mathbb{Z}$ with $n > 0$ be given such that $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$. By definition, $n \mid (a - b)$ and $n \mid (c - d)$. We may choose $t, u \in \mathbb{Z}$ such that $a - b = nt$ and $c - d = nu$. Using algebra, $a = nt + b$ and $c = nu + d$. Multiplying both equations

$$\begin{aligned} ac &= (nt + b)(nu + d) \\ &= ntnu + ntd + bnu + bd \\ &= n(tnu + td + bu) + bd. \end{aligned}$$

By CPI, we may choose $z \in \mathbb{Z}$ such that $tnu + td + bu = z$. Using algebra, $ac = nz + bd$, and consequently $ac - bd = nz$. By definition, $n \mid [(ac) - (bd)]$. Therefore, $ac \equiv bd \pmod{n}$. $\square$