2.35 Theorem. There are infinitely many prime numbers.

Proof. Suppose not. That is, suppose there is only a finite number of primes. Let $S = \{p_1, p_2, ..., p_k\}$ be the set of all the primes. By FTA, let n be natural number with a prime factorization of all the elements in set S such that $n = p_1 p_2 ... p_k$. Now consider $n + 1$. By FTA, it too has a prime factorization such that one of its prime factors, by Theorem 2.1, divide $n+1$ (and this shows the existence of a prime factor). By Theorem 2.32, we know $(n, n + 1) = 1$ which means they do not share any common prime factors. This contradicts the set S containing all the primes. Thus, there are infinitely many prime numbers. $\square$