1.12 Theorem. Let $a, b, c, d, n \in \mathbb{Z}$ with $n > 0$. If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $a + c \equiv b + d \pmod{n}$.

Proof. Let $a, b, c, d, n \in \mathbb{Z}$ with $n > 0$ be given such that $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$. By definition, $n \mid (a - b)$ and $n \mid (c - d)$. We may choose $t, u \in \mathbb{Z}$ such that $a - b = nt$ and $c - d = nu$. Adding both equations

$$\begin{aligned} a - b + c - d &= nt + nu \\ &= n(t + u). \end{aligned}$$

By CPI, we may choose $z \in \mathbb{Z}$ such that $t + u = z$. Using algebra, $(a + c) - (b + d) = nz$. By definition, $n \mid [(a + c) - (b + d)]$. Therefore, $a + c \equiv b + d \pmod{n}$. $\square$