**Lemma IR** For all primes $p$, $\sqrt{p}$ is irrational.

**Proof.** Suppose not. That is let $\sqrt{p} \in \mathbb{Q}$. By definition, $\sqrt{p} = \frac{a}{b}$ for $a, b \in \mathbb{Z}$. Suppose $\frac{a}{b}$ is irreducible such that $(a, b) = 1$. Squaring both sides,

$$\begin{aligned} p &= \frac{a^2}{b^2}, \\ pb^2 &= a^2. \end{aligned}$$

Since $p|a^2$, $p$ must be a prime factor of $a$. Thus $a = pk$ for some $k \in \mathbb{Z}$. Substituting,

$$\begin{aligned} pb^2 &= (pk)^2, \\ b &= pk^2. \end{aligned}$$

Notice that both $a$ and $b$ share the prime factor $p$. Since primes are greater than 1, this contradictions the assumption that $(a, b) = 1$. Thus, for all primes $p$, $\sqrt{p}$ is irrational. $\square$