1.50 Exercise. Find solution(s) to $31x + 21y = 1770$.

Finding (31,21),

$$\begin{aligned} 31 &= 21(1) + 10, \\ 21 &= 10(2) + 1. \end{aligned}$$

Substituting,

$$\begin{aligned} 1 &= 21 - 2(10) \\ &= 21 - 2(31 - 21) \\ &= 21 - 2(31) + 2(21) \\ &= -2(31) + 3(21). \end{aligned}$$

Let $-2 = s, 3 = t$ for some $s, t \in \mathbb{Z}$. Now, consider $31x+21y = 1$. Multiplying both sides by 1770,

$$31(1770s) + 21(1770t) = 1770.$$

Substitute in for s, t and the equation is true when $x = -3540$ and $y = 5310$. But I can't figure out a general form.