

Lemma TP. Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $b|c$, then $a|c$.

Proof. Let $a, b, c \in \mathbb{Z}$ be given such that $a|b$ and $b|c$. By definition, $b = ax$ and $c = by$ for some $x, y \in \mathbb{Z}$. Substituting into c ,

$$\begin{aligned} c &= (ax)y \\ &= a(xy). \end{aligned}$$

By CPI, $xy = t$ for $t \in \mathbb{Z}$. Thus, $a|c$. □