**3.19 Theorem.** Let $a, b$, and $c$ be integers with $n > 0$. Show that $ax \equiv b \pmod{n}$ has a solution if and only if there exist integers $x, y$ such that $ax + ny = b$.

**Proof.** Let $ax \equiv b \pmod{n}$ be given such that a solution exists. By definition, $ax - b = nk$ for $k \in \mathbb{Z}$. Rearranging, $ax + ny = b$ where $y = -k$. Thus, $ax + ny = b$ has a solution. Now let $x, y \in \mathbb{Z}$ be given such that $ax + ny = b$. Rearranging, $ax - b = -ny$. Substituting $-y = t$ for $t \in \mathbb{Z}$, $ax - b = nt$. By definition, $ax \equiv b \pmod{n}$. Thus, $ax \equiv b \pmod{n}$ has a solution if and only if there exist integers $x, y$ such that $ax + ny = b$. $\square$