1. (a)

$$\begin{aligned} 24 &= 0(18) + 6 \\ 18 &= 3(6) + 0 \\ (24, 8) &= 6 \end{aligned}$$

(b)

$$\begin{aligned} 7469 &= 3(2464) + 77 \\ 2469 &= 32(77) + 0 \\ (7469, 2464) &= 77 \end{aligned}$$

(c)

$$\begin{aligned} 243 &= 1(198) + 45 \\ 198 &= 4(45) + 18 \\ 45 &= 2(18) + 9 \\ 18 &= 2(9) + 0 \\ (198, 243) &= 9 \end{aligned}$$

2. (a)

i	q_i	r_i	s_i	t_i
0		24	1	0
1		18	0	1
2	1	6	1	-1
3	3	0	-3	4

(b)

i	q_i	r_i	s_i	t_i
0		7469	1	0
1		2464	0	1
2	7	77	1	-3
3	32	0	-32	97

(c)

i	q_i	r_i	s_i	t_i
0		243	1	0
1		198	0	1
2	1	45	1	-1
3	4	18	-4	5
4	2	9	9	-11
5	2	0	-22	27

3. (a) $a^{-1} = 15$

i	q_i	r_i	s_i	t_i
0		26		0
1		7		1
2	3	5		-3
3	1	2		4
4	2	1		-11

(b) $a^{-1} = 631$

i	q_i	r_i	s_i	t_i
0		999		0
1		19		1
2	52	11		-52
3	1	8		53
4	1	3		-105
5	2	2		263
6	1	1		-368

4. (a) $\Phi(8) = 8(1 - \frac{1}{2}) = 4$
   (b) $\Phi(15) = 15(1 - \frac{1}{3})(1 - \frac{1}{5}) = 8$
   (c) $\Phi(17) = 17(1 - \frac{1}{17}) = 16$

5. (a)

$$\begin{aligned} 4^6 &\equiv 1 \pmod 7 \\ 4(4^5) &\equiv 1 \pmod 7 \\ 4^{-1} &\equiv 4^5 \pmod 7 = 2 \end{aligned}$$

(b)

$$\begin{aligned} 5^4 &\equiv 1 \pmod{12} \\ 5(5^3) &\equiv 1 \pmod{12} \\ 5^{-1} &\equiv 5^3 \pmod{12} = 5 \end{aligned}$$

(c)

$$\begin{aligned} 6^{12} &\equiv 1 \pmod{13} \\ 6(6^{11}) &\equiv 1 \pmod{13} \\ 6^{-1} &\equiv 6^{11} \pmod{13} = 11 \end{aligned}$$

6. (a)

$$\begin{aligned} p &= 5 & n &= 55 \\ q &= 11 & \Phi(n) &= 40 \\ e &= 3 & d &= 27 \\ x &= 9 & y &= 14 \end{aligned}$$

(b)

$$\begin{aligned} p &= 7 & n &= 91 \\ q &= 13 & \Phi(n) &= 72 \\ e &= 5 & d &= 29 \\ x &= 2 & y &= 32 \end{aligned}$$