1.21 Theorem. Let a natural number n be expressed in base 10 as

$$n = a_k a_{k-1} \cdots a_1 a_0$$

(Note that what we mean by this notation is that each a_i is a digit of a regular base 10 number, not that the a_i's are being multiplied together.) If $m = a_k + a_{k-1} + \cdots a_1 + a_0$, then $n \equiv m \pmod 3$.

Proof. Let $n \in \mathbb{N}$ expressed in base 10 where $n = a_k a_{k-1} \cdots a_1 a_0$ be given, and let $m = a_k + a_{k-1} + \cdots a_1 + a_0$. Observe that $10 \equiv 1 \pmod 3$, and by Theorem 1.18 $10^i \equiv 1^i \pmod 3$, or simply $10^i \equiv 1 \pmod 3$ for some $i \in \mathbb{N}$.

$$\begin{aligned}
n &\equiv (a_k 10^k + a_{k-1} 10^{k-1} + \cdots a_1 10^1 + a_0 10^0) \pmod 3 \\
&\equiv (a_k 1^k + a_{k-1} 1^{k-1} + \cdots a_1 1^1 + a_0 1^0) \pmod 3 \\
&\equiv (a_k 1 + a_{k-1} 1 + \cdots a_1 1 + a_0) \pmod 3 \\
&\equiv (a_k + a_{k-1} + \cdots a_1 + a_0) \pmod 3 \\
&\equiv m \pmod 3.
\end{aligned}$$

Thus, $n \equiv m \pmod 3$. $\square$