

1.24 Theorem. An integer is divisible by 5 if and only if its last digit is 0 or 5.

Proof. Let $n \in \mathbb{Z}$ be divisible by 5 such that $n = 5k$ for some $k \in \mathbb{Z}$. If k is even then $k = 2p$ for some $p \in \mathbb{Z}$.

$$\begin{aligned}n &= 5(2p) \\ &= 10p.\end{aligned}$$

Observe all multiples of 10 end in 0.

If k is odd, then $k = 2p + 1$.

$$\begin{aligned}n &= 5(2p + 1) \\ &= 10p + 5.\end{aligned}$$

Any multiple of 10 plus 5 ends in 5.

Suppose now we let n be a number whose last digit is 0 or 5. Any number whose last digit is 0 can be represented as $10q$ for some $q \in \mathbb{Z}$. Since 10 is divisible by 5, 5 divides any multiple of 10. Any number whose last digit is 5 can be represented as $10q + 5$. Since 5 divides any multiple of $10q$ and 5, 5 divides $10q + 5$. Thus, an integer is divisible by 5 if and only if its last digit is 0 or 5. \square