1.22 Theorem. If a natural number is divisible by 3, then, when expressed in base 10, the sum of its digits is divisible by 3.

Proof. Let a $n \in \mathbb{N}$ be divisible by 3 be given such that when expressed in base 10, we want to show $(a_k 10^k + a_{k-1}10^{k-1} + \cdots a_1 10^1 + a_0 10^0)$ is divisible by 3. Since 10 is not divisible by 3, we can do a little algebra to make these factors of 10 work to our benefit. $a_k 10^k + a_{k-1}10^{k-1} + \cdots a_1 10^1 + a_0 10^0$

$$
\begin{aligned}
&= a_k(10^k + 1 - 1) + a_{k-1}(10^{k-1} + 1 - 1) + \cdots a_1(10 + 1 - 1) + a_0(1 + 1 - 1) \\
&= a_k(10^k + 1 - 1) + a_{k-1}(10^{k-1} + 1 - 1) + \cdots a_1(10 + 1 - 1) + a_0(1 + 1 - 1) \\
&= a_k(10^k - 1) + a_k + a_{k-1}(10^{k-1} - 1) + a_{k-1} + \cdots a_1(9) + a_1 + a_0 \\
&= a_k(10^k - 1) + a_{k-1}(10^{k-1} - 1) + \cdots 9a_1 + a_k + a_{k-1} + \cdots + a_1 + a_0 \\
&= (a_k(10^k - 1) + a_{k-1}(10^{k-1} - 1) + \cdots 9a_1) + (a_k + a_{k-1} + \cdots + a_1 + a_0).
\end{aligned}
$$

Observe each of the terms in the first group contains a factor of 3. Since the number is divisible by 3, the sum of the second term, the sum of the digits, is also divisible by 3. $\square$