1.32 Theorem. Let $a, n, b, r, k \in \mathbb{Z}$. If $k \mid a$, $k \mid b$, and $a = nb + r$, then $k \mid r$.

Proof. Let $a, n, b, r, k \in \mathbb{Z}$ be given such that $k \mid a$, $k \mid b$, and $a = nb+r$. Let $x, y \in \mathbb{Z}$ such that, by definition, $a = kx$ and $b = ky$. Substituting $a = nb+r$ into $a = kx$

$$\begin{aligned} a &= kx, \\ nb + r &= kx. \end{aligned}$$

Substituting for $b = ky$

$$\begin{aligned} n(ky) + r &= kx, \\ r &= kx - kny, \\ &= k(x - ny). \end{aligned}$$

By CPI, we can see $(x - ny) \in \mathbb{Z}$. Thus $k \mid r$. $\square$