4.6 Theorem. Let $a, n \in \mathbb{N}$ with $(a, n) = 1$. Then there exists a natural number k such that $a^k \equiv 1 \pmod{n}$.

Proof. Let $a, n \in \mathbb{N}$ with $(a, n) = 1$ be given. By Theorem 4.2, $(a^k, n) = 1$ for any $k \in \mathbb{N}$. By Theorem 2.32 and letting $n = a^k - 1$, we find that $(a^k, a^k - 1) = 1$. Now observing $n(1) = a^k - 1$, we find $a^k \equiv 1 \pmod{n}$. $\square$