4.16 Theorem. (Fermat's Little Theorem, Version II) If p is a prime and a is any integer, then $a^p \equiv a \pmod{p}$.

Proof. We will examine by cases.

Case 1. Suppose $(a, p) = 1$. By Theorem 4.15, $a^{p-1} \equiv 1 \pmod{p}$. Multiplying a to both sides,

$$a^{p-1}a \equiv 1(a) \pmod{p},$$
$$a^p \equiv a \pmod{p}.$$

Thus, letting $(a, p) = 1$ satisfies the theorem.

Case 2. Suppose $(a, p) \neq 1$. This implies $(a, p) = p$. Since $p|a$, $a \equiv 0 \pmod{p}$. Also, p divides any integer multiple of a let's us know $a^p \equiv 0 \pmod{p}$. Thus, $a^p \equiv a \pmod{p}$ by transitivity and reflexivity.

Since both cases are true, if p is a prime and a is any integer, then $a^p \equiv a \pmod{p}$. □