1.54 Exercise. Find all integer solution to the equation $24x + 9y = 33$.

Work. $(24, 9) = 3, x_0 = 1, y_0 = 1$

$$\begin{aligned} c &= a\left[x_0 + \left(\frac{bk}{(a,b)}\right)\right] + b\left[y_0 - \left(\frac{ak}{(a,b)}\right)\right] \\ 33 &= 24\left[1 + \left(\frac{9k}{3}\right)\right] + 9\left[1 - \left(\frac{24k}{3}\right)\right] \text{ for all } k \in \mathbb{Z} \\ 33 &= 24\left[1 + 3k\right] + 9\left[1 - 8k\right] \end{aligned}$$

Thus, $x = 1 + 3k$ and $y = 1 - 8k$ for all $k \in \mathbb{Z}$.