1.11 Theorem. Let $a, b, c, n \in \mathbb{Z}$ with $n > 0$. If $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then $a \equiv c \pmod{n}$.

Proof. Let $a, b, c, n \in \mathbb{Z}$ with $n > 0$ be given such that $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$. Then by definition, $n \mid (a-b)$ and $n \mid (b-c)$. We may choose $t, u \in \mathbb{Z}$ such that $a - b = nt$ and $b - c = nu$, by definition of divisibility. Using algebra, $b = nu + c$, and by substitution,

$$\begin{aligned} a - (nu + c) &= nt, \\ a - nu - c &= nt, \\ a - c &= nt + nu \\ &= n(t + u). \end{aligned}$$

By CPI, we may choose $k \in \mathbb{Z}$ such that $t + u = k$. Therefore, $a - c = nk$, and by definition of divisibility, $n \mid (a - c)$. Lastly, by definition of congruence of modulo, $a \equiv c \pmod{n}$. $\square$