1.26 Theorem. (The Division Algorithm) Let $n, m \in \mathbb{N}$. Then (*existence part*) there exist integers q (for quotient) and r (for remainder) such that

$$m = nq + r$$

and $0 \leq r < n$...

Proof. Let $m, n \in \mathbb{N}$ be given. Consider $S = \{m - nq \mid m - nq \in \mathbb{N}$ and $0 \leq m - nq\}$.

Note: Taking $q = 0$, establishes S as a non-empty set since $m \geq 0$.

By WOANN, S contains a smallest element, $m - nq \in S$. Since $m - nq$ is minimal, $m - n(q + 1) < 0$. Therefore, $m - nq < n$. Letting $r = m - nq$, $0 \leq r < n$. $\square$