

1.6 Theorem. Let $a, b, c \in \mathbb{Z}$. If $a \mid b$, then $a \mid bc$.

Proof. Let $a, b, c \in \mathbb{Z}$ be given such that $a \mid b$. We may choose $k \in \mathbb{Z}$ such that $b = ka$. Multiplying both sides by c ,

$$\begin{aligned} b(c) &= ka(c) \\ &= a(kc). \end{aligned}$$

By CPI, we may choose $m \in \mathbb{Z}$ such that $kc = m$. Therefore, $bc = am$, and by definition, $a \mid (bc)$. \square