

2.37 Theorem. If r_1, r_2, \dots, r_m are natural numbers and each one is congruent to 1 modulo 4, then the product $r_1 r_2 \dots r_m$ is also congruent to 1 modulo 4.

Proof. Let $r_1, r_2, \dots, r_n \in \mathbb{N}$ such that any $r_i \equiv 1 \pmod{4}$. Let $m = 2$ be our base case. By Theorem 1.14, $r_1 r_2 \equiv 1(1) \pmod{4}$. Thus, $r_1 r_2 \equiv 1 \pmod{4}$ satisfies the theorem. Suppose any m is true where $1 \leq m \leq k$. By induction, we want to show $r_1 r_2 \dots r_k r_{k+1} \equiv 1 \pmod{4}$. From our induction hypothesis, we know up to k such that $r_1 r_2 \dots r_k \equiv 1 \pmod{4}$. By Theorem 1.14, $r_1 r_2 \dots r_k r_{k+1} \equiv 1(1) \pmod{4}$. Thus, for each natural number congruent 1 modulo 4, the product of those natural numbers are also congruent 1 modulo 4. \square