1.38 Theorem. Let $a, b \in \mathbb{Z}$. If $(a, b) = 1$, then there exist $x, y \in \mathbb{Z}$ such that $ax + by = 1$.

Proof. Let $a, b \in \mathbb{Z}$ be given such that $(a, b) = 1$. We want to show there exist $x, y \in \mathbb{Z}$ such that $ax + by = 1$. By contradiction, suppose for all $x, y \in \mathbb{Z}$ that $ax + by \neq 1$, provided $(a, b) = 1$.