2.33 Theorem. Let $k \in \mathbb{N}$. Then there exists a $n \in \mathbb{N}$ (which will be much larger than k) such that no natural number less than k and greater than 1 divides n.

Proof. Let $k \in \mathbb{N}$ be given. Let $n > k$ for $n \in \mathbb{N}$ and $1 < a < k$ for $a \in \mathbb{N}$ such that $a \nmid n$. By contradiction, suppose $a|n$. recalling a is any natural number less than k, it follows that $a|k!$ since $a = k-i$ and $k! = k(k-1)(k-2)...(2)(1)$ for all $1 \leq i < k$. Let $n = k! + 1$. We know that $k!$ and a are divisible, thus they have a common factor. By Theorem 2.32, $(k!, k!+1) = 1$. This means n does not have any common factors with $k!$ and a. Thus, there exists a $n \in \mathbb{N}$ (which will be much larger than k) such that no natural number less than k and greater than 1 divides n. $\square$