A.30 Theorem. Every natural number can be written as the sum of distinct powers of 2.

Proof. Assume $P(k)$ for $n \leq k$ can be written as the sum of distinct powers of 2. We will prove $P(n)$ for all natural numbers n by strong induction.

Base Case. For $P(1)$,

$$\begin{aligned} 1 &= 2^0 \\ &= 1. \end{aligned}$$

The base case $P(1)$ is true.

Inductive Step. Suppose $P(n)$ is true for every natural number $n \leq k$. We want to show $P(k+1)$ is also true.

Case 1. If k is even, it is easy to see that the sum of distinct power of 2 does not contain 2^0. When we consider $k+1$, it is easy to see this is the previous sum plus $2^0 = 1$, a distinct power of 2.

Case 2. If k is odd, then $k = 2m - 1$ for some $m \in \mathbb{Z}$. Then $k + 1 = 2m$. Since $m \leq k$, m can be written as the sum of distinct powers of 2, the same way $k + 1$ is true in the previous case.

Since the base case of $P(1)$ and both cases of $P(k+1)$ are true, every natural number can be written as the sum of distinct powers of 2. $\square$