1.28 Theorem. Let $a, b, n \in \mathbb{Z}$ with $n > 0$. Then $a \equiv b \pmod{n}$ if and only if a and b have the same remainder when divided by n. Equivalently, $a \equiv b \pmod{n}$ if and only if when $a = nq + r$ $(0 \leq r < n)$ and $b = nq' + r'$ $(0 \leq r' < n)$, then $r = r'$.

Proof. Let $a, b, n \in \mathbb{Z}$ with $n > 0$ be given, and let $a \equiv b \pmod{n}$. By definition, $a - b = nk$ for some $k \in \mathbb{Z}$. Thus,

$$\begin{aligned} a &= nk + b \\ &= nk + nq' + r' \\ &= n(k + q') + r'. \end{aligned}$$

Examining $a = nq + r$ and $a = n(k + q') + r'$, by uniqueness of TDA, $r = r'$.

Let $r = r', a = nq + r$, and $b = nq' + r'$ be given. Then,

$$\begin{aligned} a - nq &= b - nq', \text{ and} \\ a - b &= nq - nq' \\ &= n(q - q'). \end{aligned}$$

Thus, $a - b = nt$ where $q - q' = t$ for some $t \in \mathbb{Z}$, and $a \equiv b \pmod{n}$. $\square$