2.32 Theorem. For all natural numbers n, $(n, n+1) = 1$.

Proof. Let $n \in \mathbb{N}$ be given. Let $(n, n+1) = d$ with $d \geq 1$ such that $d|n$ and $d|(n+1)$. By Theorem 1.2, $d|[n-(n+1)]$. By definition, $[n-(n+1)] = dt$ for $t \in \mathbb{Z}$. Thus,

$$\begin{aligned} dt &= n - n + 1 \\ &= 1. \end{aligned}$$

Since $d \geq 1$, we could assume $d = 1$. But suppose not. That is, given $dt = 1$, suppose $d > 1$ such that $1 \leq t < dt$. It follows that $1 < dt$. This is a contradiction to our given $dt = 1$. Thus, $d = 1 = (n, n+1)$. $\square$