2.26 Theorem. Let p be a prime number and let a be an integer. Then p does not divide a if and only if $(a, p) = 1$.

Proof. Suppose $p \not| a$, then $(a, p) = 1$. By its contrapositive, suppose $(a, p) > 1$, then $p|a$. Since p is prime, a is an integer, and its gcd is greater than 1, $(a, p) = p$. Thus, $p|a$. Since the contrapositive is true, Then p does not divide a if $(a, p) = 1$.

Now suppose $(a, p) = 1$, then $p \not| a$. Since a, p are co-primes, they do not have any prime factors. Thus, $p \nmid a$.

Since both directions of the biconditional statement are true, p does not divide a if and only if $(a, p) = 1$. $\square$