Lemma TP. Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $b|c$, then $a|c$.

Proof. Let $a, b, c \in \mathbb{Z}$ be given such that $a|b$ and $b|c$. By definition, $b = ax$ and $c = by$ for some $x, y \in \mathbb{Z}$. Substituting into c,

$$\begin{aligned} c &= (ax)y \\ &= a(xy). \end{aligned}$$

By CPI, $xy = t$ for $t \in \mathbb{Z}$. Thus, $a|c$. $\square$

2.1 Theorem. If n is a natural number greater than 1, and that there exists a prime p such that $p|n$.

Proof. By contradiction, let $n \in \mathbb{N}$ with $n > 1$ be given, and for all primes p, $p \not| n$. Let $\mathbb{S} = \{\text{the numbers greater than 1 that divide } n\}$. Since any number divides itself, the set is nonempty. By the WOANN, there exists a smallest number $p_0 \in \mathbb{S}$. Since p_0 is a composite number, $p_0 = xy$ for some $x, y \in \mathbb{Z}$ and both greater than 1. Since $x|p_0$ and $p_0|n$, by Lemma TP, $x|n$.

Notice that both x and p_0 divide n. Since $1 < x < p_0$, this contradicts the WOANN where p_0 is the smallest number that divides n. Thus, there exists a prime p such that $p|n$. $\square$