

Lemma 01. Let $a, b \in \mathbb{Z}$ and both not 0. If $(a, b) = d$, then for $a = da'$ and $b = db'$, $(a', b') = 1$.

Proof. Let $a, b \in \mathbb{Z}$ with both not 0 and $(a, b) = d$ be given such that $a = da'$ and $b = db'$. By contradiction, suppose $(a', b') \neq 1$. Thus, $(a', b') = k$ such that $a' = ka''$ and $b' = kb''$. Substituting in to a and b ,

$$a = dka'' \text{ and } b = dkb''$$

Letting $dk = t$ for $t \in \mathbb{Z}$,

$$a = ta'' \text{ and } b = tb''.$$

Notice that $t|a$ and $t|b$. Also observe that $t > d$. This contradicts $(a, b) = d$. Thus, $(a', b') = 1$. \square