

1.10 Theorem. Let $a, b, n \in \mathbb{Z}$ with $n > 0$. If $a \equiv b \pmod{n}$, then $b \equiv a \pmod{n}$.

Proof. Let $a, b, n \in \mathbb{Z}$ with $n > 0$ be given such that $a \equiv b \pmod{n}$. Then by definition, $n \mid (a - b)$. We may choose $k \in \mathbb{Z}$ such that $a - b = nk$. Multiplying both sides by -1 ,

$$\begin{aligned} -(a - b) &= -(nk), \\ b - a &= -kn. \end{aligned}$$

By CPI, we may choose $t \in \mathbb{Z}$ such that $-k = t$. Therefore, $b - a = tn$, and by definition of divisibility, $n \mid (b - a)$. Lastly, by definition of congruence of modulo, $b \equiv a \pmod{n}$. \square