1.42 Theorem. Let $a, b, n \in \mathbb{Z}$. If $a \mid n$, $b \mid n$ and $(a, b) = 1$, then $ab \mid n$.

Proof. Let $a, b, n \in \mathbb{Z}$ be given such that $a \mid n$, $b \mid n$, and $(a, b) = 1$. By definition, $a \mid n$ and $b \mid n$ are equivalent to $n = as$ and $n = bt$, respectively, for some $s, t \in \mathbb{Z}$. By Theorem 1.38, since $(a, b) = 1$, there exists $x, y \in \mathbb{Z}$ such that $ax + by = 1$. Multiplying both sides by n,

$$\begin{aligned} n &= n(ax + by) \\ &= nax + nby. \end{aligned}$$

Replacing n in the right hand side,

$$\begin{aligned} n &= bt(ax) + as(by) \\ &= ab(xt) + ab(sy) \\ &= ab(xt + sy). \end{aligned}$$

By CPI, $xt + sy$ is an integer. Thus, $ab \mid n$. $\square$