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Bayesian persuasion with costly messages ☆

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Abstract

We study a model of Bayesian persuasion in which the Sender commits to a signal structure, *privately* observes the signal realization, and then sends a message to the Receiver at a cost that depends on both the signal realized and the message sent. Our setup weakens the Sender's commitment to truthfully reveal information in Bayesian persuasion. We provide sufficient conditions for full communication by the Sender in the Sender-preferred equilibrium, and these conditions are satisfied under many commonly studied communication games. Under these conditions, the Sender's (lack of) commitment in the persuasion problem is quantified as a communication cost to induce a belief distribution for the Receiver. We apply this approach to study test design and information provision by lobbyists.

JEL classification: D82; D83; D72

Keywords: Bayesian persuasion; Partial commitment; Full communication; Costly messages; Information misrepresentation

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1. Introduction

Many economic situations involve an agent wanting to influence the action of a decision maker. When monetary transfers are not possible, the agent can instead strategically control the decision maker's information to influence her beliefs and thus affect the actions that she takes. Kamenica and Gentzkow (2011) (hereafter KG) model this as a "Bayesian persuasion" problem in which the agent (Sender, he) designs an information structure that generates information about the underlying state to the decision maker (Receiver, she).

In this paper, we study a variant of a Bayesian persuasion model with the innovation that new information is transmitted to the Receiver by the Sender through potentially costly messages. The Sender first commits to a signal structure that generates a signal about an unknown state. Upon *privately* observing the realized signal, the Sender sends the Receiver a message at a cost that depends on both the message and the signal realization. The Receiver then updates her belief and takes an action that affects the utility of both players.

Our model differs from a canonical Bayesian persuasion model in two substantial ways. First, when information misrepresentation by the Sender is possible but costly, our setup weakens the Sender's commitment to truthfully revealing new information to the Receiver, which is a key assumption in the Bayesian persuasion literature. This setup is of particular relevance to persuasion activities that require expert interpretation or the preparation of new information. For example, a drug company can commit to a type of scientific research on its drug, but the results require expert interpretation, which is susceptible to misrepresentation.

Second, the "belief-based" approach reduces a typical Bayesian persuasion problem to choosing a distribution of posteriors for the Receiver subject to only the posteriors averaging back to the prior ("Bayes plausibility"). Therefore, the signals used to encode the respective posteriors are fully interchangeable in Bayesian persuasion. In contrast, because the Sender's communication cost depends on the identity of the realized signal, our model also captures the Sender's choice of signal to use to encode each posterior belief in the signal structure.

To illustrate the implication of the second feature, consider a situation in which a drug company wants to persuade the FDA to approve a new drug by first announcing a type of news to search for regarding the drug. Let p_G (p_B) be the probability that the announced news is found when the drug is good (bad). Say that it is a "positive news test" if $p_G > p_B$, and it is a "negative news test" if $p_G < p_B$. News has a different effect in the two types of tests. In a positive news test, the arrival of news improves the belief about the drug, whereas the lack of news worsens the belief; in contrast, in a negative news test, the arrival of news worsens the belief, whereas the lack of news improves it.

If the drug company's problem is modeled as a Bayesian persuasion problem, then the drug company is assumed to be committed to truthfully revealing the test outcome to the FDA, or (equivalently) the FDA can directly observe the outcome. In this case, the choice between a positive or a negative news test is irrelevant because the belief-based approach reduces the problem to choosing the distribution of two posteriors, and any distribution of two posteriors can be induced by a set of (p_G, p_B) that has the property of $p_G > p_B$ (i.e., positive news test) or $p_G < p_B$ (i.e., negative news test).[1]

Suppose now that the FDA relies on the drug company to report its test outcome (i.e., the setup in this paper), and it is infinitely costly for the drug company to fabricate news, whether

[1] Consider any test, (p_G, p_B), with $p_G > p_B$ (i.e., positive news test). The test with probabilities $p'_G = 1 - p_G$ and $p'_B = 1 - p_B$ is a negative news test that generates the same distribution of posteriors as (p_G, p_B).

positive or negative, but concealing news is always costless. In this case, the type of test becomes important. In a negative news test, the bad posterior is attached to a readily manipulated signal (because any news can be costlessly hidden). Since the drug company always wants to generate the good posterior, its “no news” message in a negative news test is never credible to the FDA. However, this credibility problem is absent from a positive news test because the bad posterior is attached to a signal that the drug company is committed to truthfully report (because it cannot fabricate news when none is found). Therefore, the solution to the persuasion problem now involves not only pinning down the distribution of posterior beliefs (as in Bayesian persuasion) but also the property that new information for persuasion must be generated by a positive news test.

More generally, in this paper, we define a *message technology* as the triple of (1) the signal space for generating new information; (2) the message space for communicating the new information to the Receiver; and (3) the message costs associated with each signal realization. We consider how the persuasion activity is also affected by the message technology as opposed to only the players’ preferences over actions, which has been the focus of the Bayesian persuasion literature. In the example above, the signals and messages available in the message technology are “news” and “no news.” A positive (negative) news test is represented by a signal structure that attaches a favorable belief to the “news” (“no news”) signal, and the messaging cost after each signal realization renders the use of a negative test infeasible. Our main model considers more general forms of message technologies that allow for abstract signal and message spaces and cover a wide range of communication games.

In our model, the Sender faces two interacting problems: what information to generate and what information to reveal to the Receiver. Because the information design/acquisition problem is followed by strategic communication, any subgame-perfect equilibrium involves characterizing two interconnected belief distributions: the Sender’s belief distribution generated by his chosen signal structure and the Receiver’s belief distribution in the equilibrium of the communication subgame. With communication taking place through costly messages, the two players’ beliefs can differ in equilibrium, thus complicating the problem.

Our first contribution is providing a set of sufficient conditions (Condition 1) on the message technology for the Sender-preferred equilibrium to be supportable by full communication between the Sender and the Receiver (Proposition 1). These conditions are satisfied under many natural communication games and do not put any restrictions on the preferences of the Sender or the Receiver. Armed with Proposition 1, our second contribution is showing that the persuasion problem reduces to finding *only* the (Sender-)optimal belief distribution held by the Receiver ex post while obeying a constraint for full communication in the equilibrium of the communication subgame (Proposition 2). This constraint manifests as a cost for the Sender to induce each belief distribution for the Receiver and is derived from a cost-minimization problem of choosing signals and messages to encode beliefs such that it is incentive-compatible for the Sender to truthfully reveal information. Therefore, the solution characterization consists of a two-step process that is reminiscent of the solution to a standard discrete-action moral hazard problem[2]: first, determine the cost for each belief distribution; second, optimize over the belief distributions while accounting for its cost.

The costs to sustain various belief distributions can be interpreted as the Sender’s commitment power (or lack of it) under the message technology. If the cost of a belief distribution is infinite,

[2] In a discrete-action moral hazard problem, we often first solve for the optimal contract to induce any given effort by the agent; then, we optimize over the effort to induce. See, for example, Laffont and Martimort (2002) and Bolton and Dewatripont (2005).

then it implies that the message technology does not allow the Sender to credibly transmit such information to the Receiver. In Proposition 3, we provide a broad class of message technologies with the feature that the commitment power afforded to the Sender is simply the set of the feasible signal structures — i.e., the cost of a belief distribution is always either zero or infinite. Therefore, under such message technologies, the problem is equivalent to doing "KG persuasion" within a constrained set of signal structures. We illustrate its applications using some examples.

The remainder of the paper proceeds as follows. We discuss the related literature in the next subsection. Then, in Section 2, we introduce our model. In Section 3, we study issues related to full communication. Subsequently, in Section 4, we show how the problem reduces to finding the optimal belief distribution with a cost attached to each distribution and provide some properties of this cost function. In Section 5, we consider two examples, and in Section 6, we provide an extension with an information misrepresentation cost. Finally, in Section 7, we conclude. All omitted proofs are found in Appendices A and B.

1.1. Related literature

Our paper contributes to the literature on weakening the Sender's commitment to truth-telling in Bayesian persuasion problems. Both the online appendix of KG and Gentzkow and Kamenica (2017) consider models in which the Sender privately observes the signal realization and then sends a message to the Receiver. In contrast to our model, in the online appendix of KG, the Sender chooses the message costs; and in Gentzkow and Kamenica (2017), the communication stage is always a verifiable disclosure game (à la Grossman, 1981; Milgrom, 1981). Next, Lipnowski and Ravid (2020) consider a model in which the Sender's message is cheap talk, and Min (2017), Fréchette et al. (2018) and Lipnowski et al. (2018) study "intermediate" models in which the Sender can costlessly lie about the signal realization with some probability but must report it truthfully with the complement probability. These intermediate models are equivalent to having a stochastic communication cost in our model, which we do not consider.[3] Finally, there is also a set of papers that relax the Sender's commitment power in dynamic settings (Best and Quigley, 2017; Che et al., 2019; Mathevet et al., 2019, and Pei, 2020). In these papers, the dynamic incentives play a crucial role in the Sender's choice of information-generating process and/or reporting strategy, which contrasts with our static setting, in which the related incentives hinge on the message technology.

Next, our paper is also related to the literature on strategic communication with endogenous information for the Sender. Ivanov (2010) studies a cheap talk game in which the Receiver controls the Sender's information structure, and Perez-Richet and Skreta (2018) study test design with signal falsification. Both of these papers focus on Receiver-optimality, whereas we focus on Sender-optimality. Pei (2015) and Argenziano et al. (2016) study models in which the Sender *covertly* chooses a signal structure at a cost, privately observes the realized signal, and then sends a cheap talk message to the Receiver.[4] Both papers have a full-communication property in equilibrium as in our paper, but the driving force for the result is very different from ours.

[3] However, we allow for more general players' preferences, whereas the Sender's utility in these papers is either state-independent or, in Min (2017), of a quadratic-loss form. Li (2020) also studies a model with a similar setup to ours but under substantial restrictions on the state space and the Sender's preference, and the message cost structure is also quite different from ours.

[4] Argenziano et al. (2016) allow for both covert and overt information acquisition but have a different cost structure for the signal structure from Pei (2015).

Other related models include Guo and Shmaya (2020) and Libgober (2020): in Guo and Shmaya (2020), the Sender privately chooses a signal structure, privately observes the signal realization and then reports the resulting belief at a cost of information misrepresentation, which is similar to the cost $\psi(\rho|\sigma)$ in Section 6 in our paper; in Libgober (2020), the Sender publicly chooses some dimensions of the signal structure and privately chooses the other dimensions, but information misrepresentation is not possible.

Finally, our paper is also related to the literature on strategic communication with costly lying. Gneezy (2005), Sánchez-Pagés and Vorsatz (2007, 2009), Vanberg (2008), Hurkens and Kartik (2009), Fischbacher and Föllmi-Heusi (2013) and Gneezy et al. (2018) are some examples of experimental evidence of individuals exhibiting aversion to lying or a preference for promise-keeping. Kartik et al. (2007) and Kartik (2009) incorporated lying costs into communication games. In their models, the Sender incurs a cost of lying whenever the literal meaning of his message does not match his private information, even if this "lie" is anticipated by the Receiver and does not result in misinformation. The message cost $c(m|s)$ in our baseline model follows this "non-consequentialist" view on lying. Sobel (2020) provides a comprehensive perspective on this issue, distinguishing between (non-consequentialist) lying and "deception," which arises when the Receiver is induced to hold a wrong belief. Our information misrepresentation cost $\psi(\rho|\sigma)$ in Section 6 is consistent with a cost of deception.

2. A model of persuasion with costly messages

2.1. Setup

Let Ω, S, M and A be finite sets, denoting the state space, signal space, message space and action space, respectively.[5] We assume that $|S|, |M| \geq |\Omega| \geq 2$.

There is an unknown state $\omega \in \Omega$, and a Sender (he) and a Receiver (she) share a common prior $\beta^o \in \Delta\Omega$. The game has two stages. In stage 1, the Sender publicly chooses a signal structure $\pi : \Omega \to \Delta S$, where $\pi(s|\omega)$ is the conditional probability of signal $s \in S$ under state ω. In stage 2, the Sender *privately* observes the realized signal and then reports a message $m \in M$ to the Receiver. The Receiver then takes an action $a \in A$.

The Receiver and the Sender's utility functions are $u(a, \omega)$ and $v(a, \omega)$, respectively. Both players are expected utility maximizers; therefore, we allow the arguments of u and v to also be probability distributions on the respective arguments. The Receiver maximizes u, whereas the Sender maximizes his net payoff of

$$v(a, \omega) - c(m|s),$$

where $c : M \times S \longrightarrow \mathbb{R}^+ \cup \{\infty\}$ is his *message cost* of sending message m after (privately) observing signal s. The triple $\{S, M, c\}$ is jointly called the *message technology.*

2.2. Discussion and interpretation of the message technology

A signal s is *not* a literal description of the state because the meaning that each signal holds about the state is derived from the signal structure that generates it. Rather, each signal should be

[5] The finiteness assumptions simplify our exposition, but all of our results extend to any S, M, and A that are compact metric spaces. Throughout, ΔX denotes the set of probability distributions on X.

interpreted as only a *label* (e.g., a statement or an event) that, by itself, is neutral about the state. For most of the paper, we leave these signals as abstract labels but note that, when these labels have meanings by themselves, they can carry over to the information-gathering process that the signal structure represents. We provide an example of this in Section 5.1.

Next, a message m is a descriptive statement of the signal. We do not need to restrict M to be S, which means that language limitations are possible or that the Sender might be allowed to remain silent. Because each message has an objective meaning relative to the signals, the natural interpretation of the message cost c is a broadly defined "lying cost," which can be motivated by individuals' intrinsic aversion to lying (see examples of experimental evidence discussed in Section 1.1), the expected cost of being caught lying ex post (e.g., reputational costs and punishment by the law), and the cost of falsifying signals (e.g., the effort and resources expended to doctoring documents).[6]

Interpreting it as a lying cost, c depends on only the literal meanings of the signal and message but not their informational content about the state, which depends on the signal structure chosen. In Section 6, we extend this baseline model to include a cost that punishes the Sender when his message leads to a misrepresentation of information about the state. For expositional clarity, we defer its introduction and discussion to that section.

2.3. Strategies, beliefs and equilibrium

The Sender's strategy is a choice of a signal structure π in stage 1, followed by a message rule $\mu : S \to \Delta M$ in stage 2, where $\mu(m|s)$ denotes the probability of sending message m after the Sender observes signal s. To differentiate the players' beliefs, we use "σ" to denote the Sender's belief and "ρ" to denote the Receiver's belief. Specifically, under π, $\sigma_\pi(\cdot|s) \in \Delta\Omega$ denotes the Sender's posterior after observing signal s, and, together with μ, $\rho_{\pi,\mu}(\cdot|m) \in \Delta\Omega$ denotes the Receiver's posterior upon receiving message m, where

$$\sigma_\pi(\omega|s) = \frac{\pi(s|\omega)\,\beta^o(\omega)}{\sum_{\omega'\in\Omega}\pi(s|\omega')\,\beta^o(\omega')} \tag{1}$$

$$\rho_{\pi,\mu}(\omega|m) = \frac{\sum_{s\in S}\mu(m|s)\,\pi(s|\omega)\,\beta^o(\omega)}{\sum_{\omega'\in\Omega}\sum_{s'\in S}\mu(m|s')\,\pi(s'|\omega')\,\beta^o(\omega')} \tag{2}$$

For convenience, we define the Receiver's strategy, denoted by α, as a mapping from her belief (instead of the message that she received) to her action — i.e., $\alpha : \Delta\Omega \to \Delta A$, where $\alpha(a|\rho)$ is the probability of the Receiver taking action a when her belief is ρ. Let $\bar{A}(\rho)$ be the set of the Receiver's optimal actions under belief ρ — i.e., $\bar{A}(\rho) := \arg\max_{a\in A} \sum_{\omega\in\Omega}\rho(\omega)\,u(a,\omega)$. Let

$$\mathcal{A}^* := \left\{ \alpha \;\middle|\; \text{for any } \rho \in \Delta\Omega,\ \alpha(a|\rho) > 0 \implies a \in \bar{A}(\rho) \right\}$$

$\mathcal{A}^*$ is the Receiver's set of best-response strategies — i.e., strategies in which if a is played with a strictly positive probability under belief ρ, then a must be an optimal action for the Receiver with belief ρ.

Our equilibrium concept is perfect Bayesian equilibrium.

Definition 1. The triple $(\pi, \mu; \alpha)$ is an equilibrium if

[6] Lacker and Weinberg (1989) and Maggi and Rodriguez-Clare (1995) are examples of some early works that study agency problems that incorporate costly signal distortion.

1. Upon receiving message $m \in M$, the Receiver forms a posterior $\rho_{\pi,\mu}(\cdot|m) \in \Delta\Omega$ using Bayes' rule whenever applicable.
2. $\alpha \in \mathcal{A}^*$.
3. For any s in the support of π and $m \in M$ such that $\mu(m|s) > 0$,

$$c\left(m'|s\right) - c(m|s) \geq v\Big(\alpha\left(\cdot|\rho_{\pi,\mu}\left(\cdot|m'\right)\right), \sigma_\pi(\cdot|s)\Big) - v\Big(\alpha\left(\cdot|\rho_{\pi,\mu}(\cdot|m)\right), \sigma_\pi(\cdot|s)\Big) \quad \forall m' \neq m. \tag{S-IC}$$

Furthermore, if μ is also a pure and fully separating strategy,[7] then $(\pi, \mu; \alpha)$ is a *full-communication equilibrium.*

Let

$$V(\mu;\alpha|\pi) := \sum_{\omega\in\Omega} \beta^o(\omega) \sum_{s\in S} \pi(s|\omega) \sum_{m\in M} \mu(m|s)\, v\Big(\alpha\left(\cdot|\rho_{\pi,\mu}(\cdot|m)\right), \sigma_\pi(\cdot|s)\Big), \tag{3}$$

$$C(\mu|\pi) := \sum_{\omega\in\Omega} \beta^o(\omega) \sum_{s\in S} \pi(s|\omega) \sum_{m\in M} \mu(m|s)\, c(m|s), \tag{4}$$

$$W(\mu;\alpha|\pi) := V(\mu;\alpha|\pi) - C(\mu|\pi). \tag{5}$$

Under $(\pi, \mu; \alpha)$, V and C are the Sender's expected utility and message cost, respectively, and W is his expected net payoff. We are interested in the *Sender-preferred equilibria*. If $(\pi^*, \mu^*; \alpha^*)$ is a Sender-preferred equilibrium, then we say that π^* is an *optimal signal structure*, and $W(\mu^*; \alpha^*|\pi^*)$ is the Sender's *value of persuasion*.

Remark 1. (On Sender-preferred equilibria.) First, note that in a Sender-preferred equilibrium, the Receiver need not always break a tie within her best responses in favor of the Sender. This is because the Sender's equilibrium constraint (S-IC) depends on α; therefore, an $\alpha \in \mathcal{A}^*$ that does not always break a tie in favor of the Sender might allow the Sender to play certain strategies that result in an equilibrium with a more favorable ex ante distribution of the Receiver's posteriors. Section 5.1 provides an illustration of this property. Second, because we do not bound the Sender's message costs, the Sender can potentially be trapped in a costly communication game equilibrium in which he would not have participated in the first place. To simplify the exposition, we implicitly assume that the Sender's value of persuasion $W(\mu^*; \alpha^*|\pi^*)$ is always sufficiently high such that it satisfies the Sender's (unmodeled) participation constraint.

Remark 2. (On the Sender's commitment.) In Bayesian persuasion, the Sender can commit to the signal structure and also truthfully report the realized signal. Although our setup is framed as relaxing only the latter commitment, it has an equivalence to a model that relaxes the first commitment instead. We provide details of this equivalence in the Online Appendix. However, our model is different from one that studies the initial information design when the Sender can commit to truthfully revealing the signals but only through costly messages.[8]

7 μ is pure if $\mu(m|s)$ is 1 or 0 for all m, s; μ is fully separating if, for any two signals $s \neq s'$, there does not exist any m such that $\mu(m|s), \mu\left(m|s'\right) > 0$.

8 This is an interesting question that relates to the literature on costly disclosure (e.g., Verrecchia, 1983; Hedlund, 2015) but endogenizing the Sender's information.

3. Full communication in the communication subgame

In this section, we consider when the Sender-preferred equilibrium is supportable with full communication. Note that the revelation principle in Myerson (1986) — that considering the Sender truthfully revealing his private information is without loss of generality — relies on the Sender's messages being costless; thus, it does not apply here. Moreover, the literature on mechanism design with costly signal falsification has established that non-truthful mechanisms can implement outcomes that are not implementable via truthful mechanisms.[9] In Section 3.1, we provide an example to illustrate this intuition in the context of our setup. Our main results are in Section 3.2, where we provide sufficient conditions on the message technology that rule out such situations.

3.1. Example of optimality without full communication

Let $\Omega = \{0, 1\}$ and the prior be uniform, and let $S = \{x, y, z\}$ and $M = \left\{m_x, m_y\right\}$. To shorten notations, we denote a belief by the probability of $\omega = 1$. The Sender's utility is $v\left(a, \omega\right) = a\omega$, and the message costs are as follows:

$$
\begin{aligned}
c\left(m_x|x\right) = 0 \;\; &; \;\; c\left(m_y|x\right) = 0.1 \\
c\left(m_x|y\right) = 0.1 \;\; &; \;\; c\left(m_y|y\right) = 0 \\
c\left(m_x|z\right) = \infty \;\; &; \;\; c\left(m_y|z\right) = 0.1
\end{aligned}
$$

Next, the set of the Receiver's optimal actions is summarized as follows[10]:

$$
\begin{aligned}
\bar{A}\left(0.2\right) &= \{1\} \\
\bar{A}\left(0.8\right) &= \{2\} \\
a \leq 0 \; \forall a \in \bar{A}\left(\rho\right) &\quad \text{in which } \rho \neq 0.2, 0.8
\end{aligned} \tag{6}
$$

It is readily observed that, in any Sender-preferred equilibrium, the support of the Receiver's belief distribution must consist of only beliefs 0.2 and 0.8. By Bayes theorem, the expectation of the posterior belief is the prior, which means that the Receiver must then hold these two beliefs in equilibrium with a probability of $\frac{1}{2}$ each. Let τ denote this belief distribution.

The best full-communication equilibrium for the Sender. In a full-communication equilibrium, the Sender chooses a π that generates the belief distribution τ. To minimize the message costs, the Sender wants to use only signals x and y in π and then send m_x after $s = x$ and m_y after $s = y$. However, this is impossible because, when the Sender's belief is $\sigma = 0.2$, his utility gain from inducing the Receiver to hold belief $\rho = 0.8$ (instead of $\rho = 0.2$) is 0.2, which is higher than any message cost after signals x and y. Therefore, belief 0.2 must be generated by $s = z$, with the Sender sending m_y after $s = z$ in equilibrium, as illustrated in the top part of Fig. 1. The Sender's equilibrium expected message cost is $\frac{1}{2}c\left(m_y|z\right)$.

[9] For example, Green and Laffont (1986) illustrates this point under partially verifiable information disclosure, and Lacker and Weinberg (1989) illustrates this point under a falsification cost that is smooth and increasing in the magnitude of falsification.

[10] The Receiver's utility and action set that rationalize these optimal actions are found in the Online Appendix. For ease of exposition, we have removed some irrelevant elements from the original sets $\bar{A}\left(0.2\right)$ and $\bar{A}\left(0.8\right)$, which results in $\bar{A}\left(\rho\right)$ in (6) not having the upper-hemicontinuity property possessed by any best-response correspondence of an expected utility function.

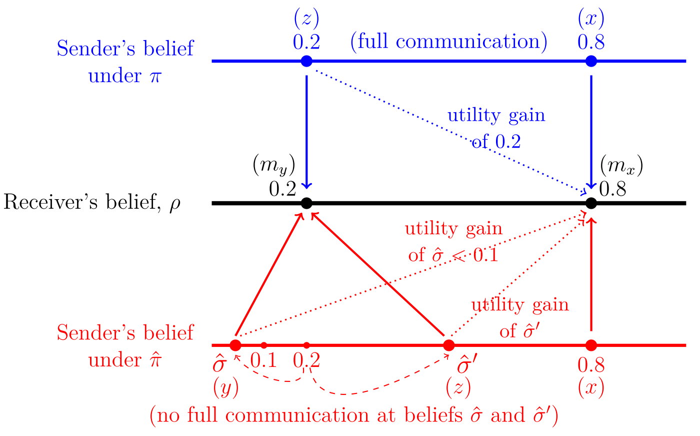

Fig. 1. Optimality without full communication.

The Sender-preferred equilibrium. Consider signal structure $\hat{\pi}$, illustrated in the bottom part of Fig. 1. Relative to belief distribution τ, $\hat{\pi}$ also generates belief 0.8 (using $s = x$) but splits belief 0.2 in a mean-preserving manner into two beliefs, $\hat{\sigma}$ (with $s = y$) and $\hat{\sigma}'$ (with $s = z$), where $\hat{\sigma} < 0.1$. Under the message strategy depicted by the solid arrows, the Receiver's belief distribution is still τ, which leaves his expected utility unchanged from before.[11] However, this message strategy constitutes an equilibrium with a lower expected message cost — in particular, under belief $\hat{\sigma}$ (i.e., $s = y$), the Sender's utility gain from inducing the Receiver to hold belief $\rho = 0.8$ (instead of $\rho = 0.2$) is only $\hat{\sigma} < 0.1$, which is less than $c\,(m_x|y)$. Because a message cost is incurred only after $s = z$, which occurs with a probability of less than $\frac{1}{2}$, the Sender's equilibrium expected message cost here is lower than before.[12] Therefore, this is a better equilibrium for the Sender.

Intuitively, the decrease in the expected message cost arises because, under $\hat{\pi}$, the information used by the Sender to induce the Receiver to hold belief $\rho = 0.2$ is now partially generated by signal y, which incurs no cost to send message m_y. In contrast, under π, the same information must be completely generated by signal z, from which it is costly to send a message. Therefore, the optimal signal structure minimizes the probability of using signal z under the constraint that $\hat{\sigma} \leq 0.1$, which implies choosing $\hat{\sigma} = 0.1$ and $\hat{\sigma}' = 1$. (Further details are provided in the Online Appendix.) Finally, note that not only is the Sender-preferred equilibrium not a full-communication equilibrium here, but the size of the support of the optimal signal structure is also larger than the size of the state space.

3.2. *Conditions for full communication*

Condition 1. For any subset of messages $\hat{M} \subseteq M$ such that $\left|\hat{M}\right| = |\Omega|$, there exists a subset of $|\Omega|$ signals in S, which is denoted by $\{\bar{s}_m\}_{m\in\hat{M}}$, such that for every $m \in \hat{M}$,

[11] Note that expected utilities are linear in beliefs; thus, a mean-preserving spread in the Sender's belief does not change his expected utility when the Receiver's action is unchanged.

[12] Because the belief distribution of $\hat{\pi}$ is formed by subjecting τ to a mean-preserving spread at belief 0.2 (which occurs with a probability of $\frac{1}{2}$ under τ), the total probability of beliefs $\hat{\sigma}$ and $\hat{\sigma}'$ is $\frac{1}{2}$.

1. $c(m|\bar{s}_m)$ is finite, and $c(m|\bar{s}_m) \leq c\left(m|s'\right)$ for all $s' \in S$.
2. for any $m' \in M$, $c\left(m'|\bar{s}_m\right) - c(m|\bar{s}_m) \geq c\left(m'|s'\right) - c\left(m|s'\right)$ for all $s' \in S$.[13]

Fixing $\hat{M}$, we call $\bar{s}_m$ the *efficient signal of message* m. Condition 1 states two properties that make it efficient for any message m to be sent only after its efficient signal $\bar{s}_m$ in an equilibrium. The first requirement is that the message cost of m is the lowest after $\bar{s}_m$. The second concerns sustaining an equilibrium — if the Sender is supposed to send m in equilibrium, he must face a "deviation cost" of sending $m' \neq m$, and this deviation cost must be highest after $\bar{s}_m$.

Proposition 1. *Suppose that $(\pi, \mu; \alpha)$ is a Sender-preferred equilibrium. If the message technology satisfies Condition 1, then there always exists an equilibrium $(\bar{\pi}, \bar{\mu}; \alpha)$ such that $W(\bar{\mu}; \alpha|\bar{\pi}) = W(\mu; \alpha|\pi)$, and*

1. *$\bar{\mu}$ is a pure and separating message strategy, and*
2. *the number of signals in the support of $\bar{\pi}$ is at most $|\Omega|$.*

The key to the first point of Proposition 1 is showing that if Condition 1 is satisfied, a non-separating message strategy always leads to a (weakly) suboptimal equilibrium for the Sender.[14] Intuitively, this is because if there is pooling to a message $\hat{m}$ from two different signals — say, s_1 and s_2 — then we can construct another equilibrium with a signal structure that replaces both s_1 and s_2 by $\bar{s}_{\hat{m}}$ (the efficient signal of $\hat{m}$) and let $\hat{m}$ be sent only after $\bar{s}_{\hat{m}}$. Due to the second property in Condition 1, if the constraint (S-IC) to report $\hat{m}$ after both s_1 and s_2 is satisfied, then the constraint (S-IC) to report $\hat{m}$ after $\bar{s}_{\hat{m}}$ also holds. Next, by the first property in Condition 1, the new equilibrium also has a lower message cost.

To provide a heuristic illustration of this argument, suppose that $(\pi, \mu; \alpha)$ is an equilibrium with $\mu\left(\hat{m}|s_1\right), \mu\left(\hat{m}|s_2\right) = 1$. For simplicity, suppose that $\bar{s}_{\hat{m}}$ is not in the support of π and $\mu\left(\hat{m}|s\right) = 0$ for all other $s \neq s_1, s_2$. To shorten the notation, let the unconditional probabilities of s_1 and s_2 being realized under π be p_1 and p_2, respectively; let the Sender's beliefs upon observing s_1 and s_2 be σ_1 and σ_2, respectively; and let the Receiver's belief after receiving $\hat{m}$ be $\hat{\rho}$. Next, let signal structure $\bar{\pi}$ be the modification of π, in which, conditional on each state, all of the probabilities of both s_1 and s_2 are shifted away to signal $\bar{s}_{\hat{m}}$ — this implies that the unconditional probability of $\bar{s}_{\hat{m}}$ under $\bar{\pi}$ is $p_1 + p_2$. Finally, let $\bar{\mu}$ be the modification of μ in which $\bar{\mu}\left(\hat{m}|\bar{s}_{\hat{m}}\right) = 1$.

We first check that $(\bar{\pi}, \bar{\mu}; \alpha)$ is also an equilibrium, which only requires verifying the constraint (S-IC) that the Sender sends $\hat{m}$ after observing $\bar{s}_{\hat{m}}$. First, note that, under $\bar{\pi}$, the Sender's belief after observing $\bar{s}_{\hat{m}}$ is $\hat{\rho}$. Suppose that the Sender deviates to sending $m' \neq \hat{m}$, which leads to the Receiver having belief ρ'. We have

$$v\Big(\alpha\left(\cdot|\rho'\right), \hat{\rho}\Big) - v\Big(\alpha\left(\cdot|\hat{\rho}\right), \hat{\rho}\Big) = \sum_{i=1,2} \frac{p_i}{p_1 + p_2}\Big[v\Big(\alpha\left(\cdot|\rho'\right), \sigma_i\Big) - v\Big(\alpha\left(\cdot|\hat{\rho}\right), \sigma_i\Big)\Big] \tag{7}$$

[13] We adopt the convention that "$\infty - \infty = 0$," and that the inequality also holds if the left- and right-hand sides are both simultaneously ∞.

[14] If the equilibrium message strategy has only randomization but is still separating, then in equilibrium, the Receiver still perfectly learns the Sender's privately observed signal. In turn, because the two players' beliefs always coincide in equilibrium, the Sender's expected utility cannot be greater than that in a full-communication equilibrium.

$$\leq \sum_{i=1,2} \frac{p_i}{p_1+p_2} \left[c\left(m'|s_i\right) - c\left(\hat{m}|s_i\right) \right] \tag{8}$$

$$\leq \sum_{i=1,2} \frac{p_i}{p_1+p_2} \left[c\left(m'|\bar{s}_{\hat{m}}\right) - c\left(\hat{m}|\bar{s}_{\hat{m}}\right) \right] = c\left(m'|\bar{s}_{\hat{m}}\right) - c\left(\hat{m}|\bar{s}_{\hat{m}}\right) \tag{9}$$

In line (7), the equality follows from Bayes theorem that $p_1\sigma_1 + p_2\sigma_2 = (p_1 + p_2)\,\hat{\rho}$ and the linearity of expected utilities in beliefs. In line (8), the inequality follows from constraint (S-IC) holding under $(\pi, \mu; \alpha)$ after signals s_1 and s_2. In line (9), the inequality follows from the second part of Condition 1. Lines (7) to (9) establish that $(\bar{\pi}, \bar{\mu}; \alpha)$ is an equilibrium. Moreover, the linearity of expected utilities in beliefs implies that $V(\bar{\mu}; \alpha|\bar{\pi}) = V(\mu; \alpha|\pi)$, whereas $c\left(\hat{m}|\bar{s}_{\hat{m}}\right) \leq c\left(\hat{m}|s_1\right), c\left(\hat{m}|s_2\right)$ (i.e., the first part of Condition 1) implies that $C(\bar{\mu}|\bar{\pi}) \leq C(\mu|\pi)$. Jointly, this implies that $W(\bar{\mu}; \alpha|\bar{\pi}) \geq W(\mu; \alpha|\pi)$.

The second point of Proposition 1 (on the size of the support of $\bar{\pi}$) is related to the result that, in the canonical Bayesian persuasion model, only $|\Omega|$ posteriors are needed in the optimal signal structure (see proposition 4 in the online appendix of KG). However, there are two issues specific to our model that create some complications in showing this result. The first issue concerns quantifying the Sender's payoff — in KG, the Sender's payoff at every belief is well-defined by the primitives (see the "$\hat{v}$" function in KG), whereas in our model, the Sender's net payoff at each belief includes both his utility *and* his message cost, and the message cost is an equilibrium object. The second issue is that after reducing the support of the signal structure, we must also ensure that the communication equilibrium in stage 2 is unaffected. Further details on this are provided in Appendix A.[15]

Lemma 1. *The following message technologies satisfy Condition 1:*

- *(Discrete-lying cost.) $M = S$, with $c(m|s) = 0$ if $m = s$, and $c(m|s) = k \geq 0$ if $m \neq s$.*
- *(Distance-lying cost.) $M = S$, with $c(m|s) = d(m, s)$, where d is any metric on S.*
- *(Partial verifiability.) $S = \mathcal{V} \cup \mathcal{N}$, where $\mathcal{V}$ and $\mathcal{N}$ are disjointed and nonempty sets, and $M = S \cup \{\phi\}$, with the following:*
  - *$c(\phi|s) = 0$ for all $s \in S$.*
  - *for any $m \in \mathcal{N}$, $c(m|s) = 0$ for all $s \in S$.*
  - *for any $m \in \mathcal{V}$, $c(m|s) = 0$ if $s = m$, and $c(m|s) = k > 0$ if $s \neq m$.*

These message technologies are examples of "lying costs." The discrete-lying cost treats all forms of lying as equal, and this encompasses cheap talk communication (Crawford and Sobel, 1982) when $k = 0$. The distance-lying cost provides an intuitive measure of the degree of lying using a distance function. Under partial verifiability, message ϕ represents the Sender remaining silent, which is always costless. $\mathcal{N}$ is a set of nonverifiable signals that the Sender can always costlessly claim to have received, whereas $\mathcal{V}$ is a set of verifiable signals in which falsifying such a signal incurs a fixed cost k. When k is infinite, signals in $\mathcal{V}$ are hard evidence that is impossible to falsify, similar to the private signals in verifiable disclosure games studied in Dye (1985) and Jung and Kwon (1988).

[15] See, in particular, Lemma 6.

4. Value of persuasion

In this section, we apply Proposition 1 to compute the Sender's value of persuasion.

4.1. From costly messages to costly belief distributions

Let $\mathcal{D}(\beta^o)$ be the set of belief distributions supported on $|\Omega|$ beliefs and averaging back to β^o. Each belief distribution $\tau \in \mathcal{D}(\beta^o)$ consists of two $|\Omega|$-dimensional vectors: $\tau = \left\{\vec{\beta}; \vec{\delta}\right\} = \left\{(\beta_j)_{j=1,\dots,|\Omega|}; (\delta_j)_{j=1,\dots,|\Omega|}\right\}$, in which each $\beta_j \in \Delta\Omega$ is a belief, and $\vec{\delta}$ is a probability vector, with $\delta_j = \Pr_\tau\left[\beta_j\right]$ and $\sum_{j=1}^{|\Omega|} \delta_j \beta_j = \beta^o$.

Let $V^*(\tau, \alpha)$ denote the Sender's expected utility when both the Sender and the Receiver have the belief distribution τ, and the Receiver's strategy is α — i.e.,

$$V^*\left(\left\{\vec{\beta}; \vec{\delta}\right\}, \alpha\right) := \sum_{j=1}^{|\Omega|} \delta_j v\left(\alpha\left(\cdot|\beta_j\right), \beta_j\right).$$

Next, define program $(\mathcal{C})$ as follows: for any $\left\{\vec{\beta}; \vec{\delta}\right\} \in \mathcal{D}(\beta^o)$ and $\alpha \in \mathcal{A}^*$,

$$C^*\left(\left\{\vec{\beta}; \vec{\delta}\right\}, \alpha\right) := \min_{\substack{s_1,\dots,s_{|\Omega|} \in S \\ m_1,\dots,m_{|\Omega|} \in M}} \sum_{j=1}^{|\Omega|} \delta_j c\left(m_j|s_j\right) \tag{10}$$

$$\text{s.t. } c\left(m_{j'}|s_j\right) - c\left(m_j|s_j\right) \geq v\Big(\alpha\left(\cdot|\beta_{j'}\right), \beta_j\Big) - v\Big(\alpha\left(\cdot|\beta_j\right), \beta_j\Big), \forall j, j' = 1, \dots, |\Omega|, \tag{11}$$

with the convention that $C^*\left(\left\{\vec{\beta}; \vec{\delta}\right\}, \alpha\right) = \infty$ if the feasible set in (11) is empty.

To explain program $(\mathcal{C})$, fix a belief distribution $\left\{\vec{\beta}; \vec{\delta}\right\}$ and a Receiver's strategy α. Let s_j denote the signal chosen to encode belief β_j for the Sender. If the Receiver conjectures that the Sender sends m_j after s_j, she will also hold belief β_j after receiving m_j. Thus, constraint (11) is the Sender's incentive-compatibility constraint to report m_j (instead of $m_{j'}$) after observing s_j, which then leads to full communication between the Sender and the Receiver. The objective in (10) is the Sender's expected message cost of sending m_j after s_j for every $j = 1, \dots, |\Omega|$. Therefore, given a belief distribution $\left\{\vec{\beta}; \vec{\delta}\right\}$ and a Receiver's strategy α, program $(\mathcal{C})$ considers the most cost-efficient way of choosing the signals and messages to encode the beliefs in $\vec{\beta}$ for the Sender and the Receiver, respectively, while ensuring full communication in the communication subgame.

Proposition 2. *Suppose that the message technology satisfies Condition 1, and there exist $\underline{\rho} \in \Delta\Omega$ and $\underline{a} \in \bar{A}\left(\underline{\rho}\right)$ such that $v\left(\underline{a}, \sigma\right) = -\infty$ for all $\sigma \in \Delta\Omega$. A Sender-preferred equilibrium exists if and only if there exists a solution to the following program:*

$$\max_{\tau \in \mathcal{D}(\beta^o),\ \alpha \in \mathcal{A}^*} V^*(\tau, \alpha) - C^*(\tau, \alpha). \tag{$\mathcal{P}$}$$

Moreover, if W^* is the value of program ($\mathcal{P}$), then the Sender's value of persuasion is W^*. A solution to program ($\mathcal{P}$) always exists if there exists $(\tau, \alpha) \in \mathcal{D}(\beta^o) \times \mathcal{A}^*$ such that $C^*(\tau, \alpha) < \infty$.[16]

If $(\pi, \mu; \alpha)$ is a full-communication equilibrium and τ is the belief distribution generated by π, then the Sender's expected net payoff in this equilibrium is no greater than $V^*(\tau, \alpha) - C^*(\tau, \alpha)$ — this follows from program ($\mathcal{C}$) being the problem of sustaining τ in a full-communication equilibrium with the lowest expected message cost, as previously explained.

Conversely, if there exist a belief distribution $\left\{\vec{\beta}, \vec{\delta}\right\}$ and α such that $V^*\left(\left\{\vec{\beta}, \vec{\delta}\right\}, \alpha\right) - C^*\left(\left\{\vec{\beta}, \vec{\delta}\right\}, \alpha\right) = \hat{W}$, then there exists a full-communication equilibrium that gives the Sender's an expected net payoff of $\hat{W}$. To illustrate the equilibrium construction, let $\chi = \left\{s_1^*, \ldots, s_{|\Omega|}^*; m_1^*, \ldots, m_{|\Omega|}^*\right\}$ denote the solution of program ($\mathcal{C}$) that obtains $C^*\left(\left\{\vec{\beta}, \vec{\delta}\right\}, \alpha\right)$. Set the Sender's equilibrium strategy $\hat{\pi}$ and $\hat{\mu}$ to be $\hat{\pi}\left(s_j^*|\omega\right) = \frac{\beta_j(\omega)\delta_j}{\beta^o(\omega)}$ and $\mu^*\left(m_j^*|s_j^*\right) = 1$ for all j. From equation (1), the Sender's belief after observing signal s_j^* is β_j. Because χ satisfies constraint (11), μ^* satisfies the Sender's equilibrium constraint (S-IC) against a deviation to any on-equilibrium-path messages m_1^* to $m_{|\Omega|}^*$. If the Sender also does not have an incentive to deviate to sending an off-equilibrium-path message, then $\left(\hat{\pi}; \hat{\mu}; \alpha\right)$ is an equilibrium with $V\left(\hat{\mu}; \alpha|\hat{\pi}\right) - C\left(\hat{\mu}|\hat{\pi}\right) = V^*\left(\left\{\vec{\beta}, \vec{\delta}\right\}, \alpha\right) - \sum_{j=1}^{|\Omega|} \delta_j c\left(m_j^*|s_j^*\right) = \hat{W}$. To simplify our analysis, we have assumed the existence of a Sender's least preferred action $\underline{a}$ that is rationalizable by the Receiver, and we let the Receiver choose $\underline{a}$ after every off-equilibrium-path message.[17] However, with a more explicit specification of the utility functions u and v and the message cost function c, specifying the Receiver's actions after receiving an off-equilibrium-path message to support an equilibrium should usually not be complicated in our model.

Therefore, program ($\mathcal{P}$) solves for the Sender-preferred full-communication equilibrium, which, by Proposition 1, is also a Sender-preferred equilibrium. As in KG, the program involves choosing a belief distribution for the Receiver that affects the Sender's expected utility V^*. The difference here is that each belief distribution incurs a cost of C^* that is endogenously determined via program ($\mathcal{C}$). Thus, the solution characterization consists of a two-step process reminiscent of the classic discrete-action moral hazard problem: first, determine the cost C^* for each belief distribution; then, optimize over the belief distributions while considering the cost. Program ($\mathcal{C}$) is a finite $|\Omega|$-dimensional linear program that is solvable by standard techniques, and the existence of a solution for program ($\mathcal{P}$) is guaranteed under a very mild assumption.

Program ($\mathcal{P}$) is reminiscent of Gentzkow and Kamenica (2014) who use the Shannon (1948) entropy information measure to quantify the cost of a belief distribution. Because of the posterior separability property of the entropy cost, the concavification method in KG can still be applied. In contrast, C^* in program ($\mathcal{P}$) is a cost to sustain a belief distribution in the stage-2 communication game and generally does not have the posterior separability property. Thus, the problem has a different structure from that in Gentzkow and Kamenica (2014). However, as we show next, the

[16] As noted in footnote 5, our results extend to the case in which S, M, and A are any compact metric spaces. In this case, the sufficient condition for the existence of a solution to program ($\mathcal{P}$) also requires that c is continuous and $\mathcal{A}^*$ is compact — these two conditions always hold when S, M and A are finite sets.

[17] Action $\underline{a}$ can be motivated as, for example, the Receiver breaking away from the relationship with the Sender who has a low outside option.

C^*-functions for the message technologies in Lemma 1 exhibit a property that still allows us to exploit the concavification argument for the solution.

Proposition 3. *Under the message technologies in Lemma 1, $C^*(\tau,\alpha)=0$ or $C^*(\tau,\alpha)=\infty$ for all $\tau\in\mathcal{D}(\beta^o)$ and $\alpha\in\mathcal{A}^*$. Moreover, program ($\mathcal{P}$) is equivalent to program ($\mathcal{P}_{NL}$):*

$$\max_{\left\{\vec{\beta};\vec{\delta}\right\}\in\mathcal{D}(\beta^o),\ \alpha\in\mathcal{A}^*} V^*\left(\left\{\vec{\beta};\vec{\delta}\right\},\alpha\right) \tag{$\mathcal{P}_{NL}$}$$

subject to the existence of $s_1,\ldots,s_{|\Omega|}\in S$ such that

$$c\left(s_{j'}|s_j\right)\ \geq\ v\Big(\alpha\left(\cdot|\beta_{j'}\right),\beta_j\Big)-v\Big(\alpha\left(\cdot|\beta_j\right),\beta_j\Big)\quad\forall j,j'. \tag{NL}$$

Under the message technologies in Lemma 1, program ($\mathcal{P}$) becomes program ($\mathcal{P}_{NL}$). Qualitatively, constraint (NL) is a *no-lying constraint*; therefore, a signal structure is feasible if and only if there is a no-lying equilibrium in the subsequent communication subgame. Thus, the problem reduces to doing "KG persuasion" within a constrained set of (feasible) signal structures. The form of program ($\mathcal{P}_{NL}$) also makes it possible to provide some properties of the persuasion solution.

Corollary 1. *Under the discrete-lying cost in Lemma 1, the Sender's value of persuasion is continuous and nondecreasing in k. Therefore, a small change in the discrete-lying cost never leads to a large change in the Sender's value of persuasion.*[18]

4.2. *Attaining full commitment value without full commitment*

Let the Sender's "full commitment value" denote his value of persuasion in the KG setup with the players' utility functions u and v.[19] Let $\mathcal{D}^{KG}(\beta^o)\subset\mathcal{D}(\beta^o)$ be the set of belief distributions generated by the optimal signal structures when the Sender has full commitment power à la KG, and let $\alpha^{KG}\in\mathcal{A}^*$ be the Receiver's strategy in which she always chooses the Sender-preferred action among her best responses (as assumed in KG). The following is a corollary of Proposition 2:

Corollary 2. *The Sender's value of persuasion is his full commitment value if and only if there exists $\tau^{KG}\in\mathcal{D}^{KG}(\beta^o)$ such that $C^*\left(\tau^{KG},\alpha^{KG}\right)=0$.*

We emphasize the "only if" part of Corollary 2 — in our model, if the Sender can achieve his full commitment value with π^*, then π^* must also be an optimal signal structure in the associated KG setup. This property contrasts with how the Sender might obtain his full commitment value in other persuasion models with a weakened Sender's commitment discussed in Section 1.1, in which the full commitment value (if attainable) is necessarily obtained from a signal structure that is not KG-optimal.

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[18] It is less obvious how a change in the distance-lying cost should be defined. One possible way is to let $c(m|s)=k\times d(m,s)$, where d is a metric and k is a positive constant. In this case, the Sender's value of persuasion is also continuous and nondecreasing in k.

[19] This refers to the Sender's expected utility from the optimal signal structure in the KG model, which assumes that the Receiver can directly observe the signal realization and, thus, there is no message cost for the Sender.

As an example of the use of Corollary 2, suppose that the players have the commonly used quadratic-loss utility functions — i.e., $u(a,\omega) = -(a-\omega)^2$ and $v(a,\omega) = -(a-\omega-b)^2$, where $b \in (0,1)$. Let Ω be a finite subset of $[0,1]$ and $A = M = S = [0,1]$. Therefore,

$$v(a,\beta) = E_\beta\left[-(a-\omega-b)^2\right] = -Var_\beta[\omega] - \left(a-b-E_\beta[\omega]\right)^2. \tag{12}$$

Thus, the Receiver's optimal action under belief β is uniquely $a = E_\beta[\omega]$, and the no-lying constraint (NL) is (after some rearrangement)

$$\left(E_{\beta_{j'}}[\omega] - E_{\beta_j}[\omega] - b\right)^2 \geq b^2 - c\left(s_{j'}|s_j\right) \quad \forall j, j' = 1, \ldots, |\Omega|. \tag{13}$$

Constraint (13) is always satisfied if there exist $|\Omega|$ signals such that the right-hand side of (13) is always negative. The reader can verify that τ^{KG} here is uniquely the fully informative signal structure, which sets the variance to zero, and the full commitment value is $-b^2$. Therefore, by Corollary 2 and some rearrangement of (13), the full commitment value is attainable if and only if there exist $s_1, \ldots, s_{|\Omega|}$ such that $c\left(s_{j'}|s_j\right) \geq b^2 - \min_{\omega',\omega''\in\Omega}\left(\omega''-\omega'-b\right)^2$ for all $j, j' = 1, \ldots, |\Omega|$. For example, under a message technology with a discrete-lying cost k, this condition is equivalent to $k \geq b^2 - \min_{\omega',\omega''\in\Omega}\left(\omega''-\omega'-b\right)^2$.

5. Examples

In this section, we complement our abstract analysis with two examples.

5.1. *Test design under partial verifiability*

Environment. A drug company (Sender) wants to persuade the FDA (Receiver) to approve a new drug. There are two states of the world: the drug is good ($\omega = G$) or the drug is bad ($\omega = B$). For convenience, we denote the belief as the probability that $\omega = G$. The common prior is $\beta^o = 0.3$, and the FDA has two actions: $A = \{approve, reject\}$.

Regardless of the state, the drug company receives a utility of $\bar{v} > 0$ if the FDA approves his drug and a utility of 0 otherwise. In contrast, the FDA's utility is 1 when she makes the right decision and 0 otherwise — i.e., $u(approve, G) = u(reject, B) = 1$, and $u(approve, B) = u(reject, G) = 0$. Therefore,

$$\bar{A}(\beta) = \begin{cases} \{reject\} & \text{, if } \beta < 0.5 \\ \{approve, reject\} & \text{, if } \beta = 0.5 \\ \{approve\} & \text{, if } \beta > 0.5 \end{cases}$$

Let α_z^* be the FDA's strategy under which she rejects when $\beta < 0.5$, approves when $\beta > 0.5$, and approves with a probability of z when $\beta = 0.5$. Therefore, $\mathcal{A}^* = \left\{\alpha_z^*\right\}_{z\in[0,1]}$.

This setting mirrors the leading prosecutor-judge example in KG. The optimal signal structure there is unique and generates beliefs $\beta = 0$ and $\beta = 0.5$ with probabilities 0.4 and 0.6, respectively. KG assumes that the FDA always breaks a tie in favor of the drug company (i.e., plays α_1^*); therefore, the drug company's full commitment value is $0.6\bar{v}$.

Next, let $S = \{\checkmark, \times\}$ and $M = \{\checkmark, \times, \phi\}$, where "✓" is short for "news found," and "×" is short for "no news found,", and

$$c(\checkmark|\times) = k, \quad c(\times|\checkmark) = c(\times|\times) = c(\checkmark|\checkmark) = 0, \quad c(\phi|s) = 0\ \forall s.$$

This is a partial verifiability message technology in Lemma 1, with $\mathcal{V} = \{\checkmark\}$ and $\mathcal{N} = \{\times\}$. It models the signal structure as a search for news about the drug. Hiding news is costless, whereas fabricating news incurs a cost of k. A belief distribution here is denoted by $\tau = \{(\beta_\checkmark, \beta_\times); (\delta_\checkmark, \delta_\times)\}$, in which β_s is the belief attached to signal s and δ_s is the probability that signal s is realized.

Negative news tests. A belief distribution represents a *"negative news test"* if $\beta_\checkmark < \beta^0 < \beta_\times$ — i.e., the discovery of news (i.e., $s = \checkmark$) worsens the belief about the drug, whereas the lack of news (i.e., $s = \times$) improves the belief. Since the FDA simply rejects the drug all the time when $\beta_\times < 0.5$, we only have to consider negative news tests with $\beta_\times \geq 0.5$.

Lemma 2. *For any negative news test τ_- in which $\beta_\times \geq 0.5$, $C^*\left(\tau_-, \alpha_z^*\right) = \infty \; \forall z \in (0, 1]$.*

Proof. When $\beta_x > 0.5$ or $\beta_\times = 0.5$ and $z > 0$, constraint (NL) for $s = \checkmark$ is always violated because $c\,(\times|\checkmark) = 0$. By Proposition 3, C^* is infinite. □

Positive news tests. A belief distribution represents a *"positive news test"* if $\beta_\checkmark > \beta^0 > \beta_\times$ — i.e., the discovery (lack) of news improves (worsens) the belief about the drug. Analogous to before, we only have to consider positive news tests with $\beta_\checkmark \geq 0.5$.

Lemma 3. *Let $\tau_+ \in \mathcal{D}\,(\beta^o)$ be a positive news test with $\beta_\checkmark \geq 0.5$.*

1. *If $k \geq \bar{v}$, $C^*\left(\tau_+, \alpha_z^*\right) = 0$ for all $z \in [0, 1]$.*
2. *If $k < \bar{v}$ and $\beta_\checkmark > 0.5$, $C^*\left(\tau_+, \alpha_z^*\right) = \infty$ for all $z \in [0, 1]$.*
3. *If $k < \bar{v}$ and $\beta_\checkmark = 0.5$, $C^*\left(\tau_+, \alpha_z^*\right) = 0$ if $z \leq k$, and $C^*\left(\tau_+, \alpha_z^*\right) = \infty$ if $z > k$.*

To show Lemma 3, we must verify that the truthful message strategy satisfies (violates) constraint (NL) to show that $C^* = 0$ ($C^* = \infty$). We relegate the details to Appendix B.5. Intuitively, when $k \geq \bar{v}$, the lying cost exceeds any gain from lying; thus, constraint (NL) is always satisfied. In contrast, when $k < \bar{v}$, if lying induces the FDA to always approve, which is the case when $\beta_\checkmark > 0.5$, then constraint (NL) is never satisfied under truth-telling. However, if $\beta_\checkmark = 0.5$, the FDA is indifferent between approving and rejecting. In this case, if the FDA approves only some of the time, the drug company's gain from lying decreases, which helps satisfy (NL) under a low lying cost. In turn, this allows the drug company to still generate and transmit information to the FDA. Therefore, the Sender-preferred equilibrium here entails the Receiver *not* always breaking a tie in favor of the Sender,[20] thus illustrating Remark 1. The following summarizes our discussion:

Proposition 4. *The drug company's optimal test is uniquely the "KG positive news test" of $(\beta_\checkmark, \beta_\times) = (0.5, 0)$ and $(\delta_\checkmark, \delta_\times) = (0.6, 0.4)$.*

1. *When $k \geq \bar{v}$ (i.e., high commitment level), the drug company's value of persuasion is his full commitment value of $0.6\bar{v}$.*

[20] This feature in which the Receiver has to randomize over her actions is reminiscent of the literature on mechanism design under limited commitment (e.g., Khalil, 1997; Bester and Strausz, 2001), in which some noncompliance by the agent is necessary in equilibrium. Analogously, the Sender's lack of commitment to truthfully reveal information here means that his influence over the Receiver's action is also limited.

2. *When $k < \bar{v}$ (i.e., low commitment level), the drug company's value of persuasion is $0.6k < 0.6\bar{v}$. The Sender-preferred equilibrium involves the FDA approving with a probability of $\frac{k}{\bar{v}} < 1$ when she is indifferent.*

A negative news test can never be used.

This example captures the phenomenon that positive news is more likely to be published than negative news in clinical research, as documented in Riveros et al. (2013), who also note that reporting is often delayed or incomplete when a trial yields negative results. For instance, despite a 2008 federal law mandating that all results from any clinical trial of at least 20 human subjects be reported to http://www.ClinicalTrials.gov, an investigation by STAT reveals that many institutions have neither compiled with this law nor faced any punishment, thus raising concerns about the validity of clinical trials and the use of their results.[21]

5.2. *Lobbying, information provision and lying cost*

Environment. Consider a policymaker (Receiver) choosing a policy's intensity parametrized by $a \in [0, 1]$. The state space is $\Omega = \{0, 1\}$, and the policy is suitable if $\omega = 1$ and unsuitable if $\omega = 0$. An expert lobbyist (Sender) provides information about ω to the policymaker.[22] The lobbyist represents a special interest group that benefits from a more intensive policy. The lobbyist's utility function is $v(a, \omega) = \nu(a)$, where ν is strictly increasing and convex, and $\nu(0)$ is normalized to zero. In contrast, the policymaker cares about choosing the right policy. Her marginal utility from choosing a higher intensity under $\omega = 1$ is normalized to 1 — such gains can include the policymaker's intrinsic desire to improve social welfare or obtain re-election after a good performance. However, when $\omega = 0$, the policymaker's marginal loss from implementing a higher a is $L > 0$, which is interpreted as her *cost of mistake*. In addition, the policymaker incurs a cost of $\frac{a^2}{2}$ for choosing a, which is interpreted as her effort cost. Jointly, the policymaker's utility function is $u(a, \omega) = [\omega - L(1-\omega)]\, a - \frac{a^2}{2}$. We assume that the lobbyist can commit to the type of research (i.e., signal structure) but then lie about its result at a cost. We model this as a message technology with a discrete-lying cost (Lemma 1) with $k > 0$ and $S = M = \{0, 1\}$.

Optimal information provision. Denote a belief β as the probability of $\omega = 1$ and assume that the common prior is $\beta^o < \frac{L}{1+L}$. Let a^*_β denote the Receiver's optimal action at belief β. It is readily observed that $a^*_\beta = 0$ if $\beta \le \frac{L}{1+L}$ and $a^*_\beta = \beta(1+L) - L$ if $\beta > \frac{L}{1+L}$.

Proposition 5. *The optimal signal structure is as follows:*

1. *If $k \ge \nu(1)$, the lobbyist chooses a fully informative signal structure and obtains his full commitment value.*
2. *If $k < \nu(1)$, the lobbyist's value of persuasion is $\frac{k\beta^o(1+L)}{\nu^{-1}(k)+L}$, and the optimal signal structure generates beliefs 0 and $\bar{\beta} = \frac{\nu^{-1}(k)+L}{1+L} < 1$ with probabilities $1 - \frac{\beta^o(1+L)}{\nu^{-1}(k)+L}$ and $\frac{\beta^o(1+L)}{\nu^{-1}(k)+L}$,*

[21] See "Failure to report: A STAT investigation of clinical trials reporting," published on December 13, 2015, which is available at https://www.statnews.com/2015/12/13/clinical-trials-investigation/.

[22] Situations in which an expert lobbyist with special interests provides information to a policymaker to influence her action have received much attention in the literature. See, for example, Austen-Smith and Wright (1992) and Grossman and Helpman (2001).

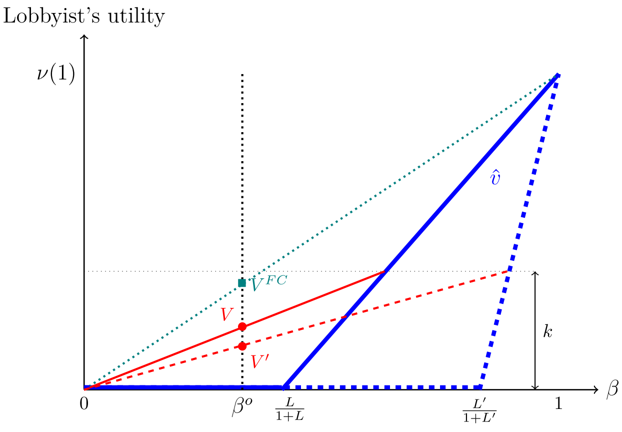

Fig. 2. Optimal signal structure provided by the lobbyist. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

respectively. $\bar{\beta}$ *increases (i.e., the informativeness of the signal structure increases) with both* k *and* L*. The lobbyist's value of persuasion increases with* k *but decreases with* L*.*

In the current example, because the lobbyist's preference is state-independent, a belief distribution $\left\{\vec{\beta};\vec{\delta}\right\} \in \mathcal{D}(\beta^o)$ satisfies constraint (NL) if and only if

$$\max_j \hat{v}\left(\beta_j\right) - \min_j \hat{v}\left(\beta_j\right) \leq k, \tag{14}$$

where $\hat{v}(\beta) := \nu\left(a^*_\beta\right)$ is the lobbyist's expected utility when the policymaker's belief is β. This simplification holds more generally when the Sender has a state-independent preference and, in the context here, allows for a useful geometric representation of constraint (NL).

Fig. 2 illustrates the case for $\nu(a) = a$, which implies that $\hat{v}(\beta) = a^*_\beta$. The solid graph is the $\hat{v}$-function. A belief distribution is represented by a line, and its intersection points with $\hat{v}$ are the beliefs generated. Constraint (14) requires that the vertical distance between the two intersection points be less than k. The dotted line plots the fully informative signal structure, which is the optimal signal structure under full commitment. The lobbyist's full commitment value is V^{FC}. If $k \geq \nu(1)$, the fully informative signal structure satisfies constraint (14); therefore, the lobbyist attains his full commitment value.

Suppose that $k < \nu(1)$. The optimal signal structure now is the line that crosses the dotted vertical line from β^o at the highest possible point while satisfying constraint (14). This is the solid line in the figure, and the lobbyist's persuasion value is marked by V. It is readily observed that, if k decreases, the optimal signal becomes a flatter line from the origin, representing a less informative signal structure and resulting in a lower value of persuasion. Intuitively, when the lobbyist's lying cost decreases, his ability to generate a larger spread in the policymaker's posterior beliefs diminishes because such a larger spread increases the lobbyist's gains from lying. Because the lobbyist benefits from spreading the policymaker's beliefs here, the inability to do so decreases his value of persuasion.

Next, when the policymaker's cost of mistake increases from L to L', the $\hat{v}$ graph shifts right, as represented by the dashed graph. Under full commitment, the lobbyist's optimal signal structure and the full commitment value remain unchanged. However, when $k < \nu(1)$, the optimal signal structure is now the dashed line in the figure. This signal structure generates a larger spread in the policymaker's posteriors (i.e., a more informative signal structure), but the lobbyist's value of persuasion drops to V'. Intuitively, under any interior belief, the policymaker takes a lower action when her cost of mistake L increases. Without full commitment, the lobbyist cannot credibly let the policymaker perfectly learn the lobbyist's preferred state (i.e., $\omega = 1$); therefore, the policymaker's conservativeness against a higher action can adversely affect the lobbyist's value of persuasion, which was not the case if the lobbyist had the commitment power to fully reveal information about ω.

6. Information misrepresentation cost

In this section, we extend the Sender's communication cost to include a cost that directly depends on the information transmitted about the state. Let $\psi : \Delta\Omega \times \Delta\Omega \to \mathbb{R}^+$, where $\psi(\rho|\sigma)$ is the Sender's *information misrepresentation cost* when the Receiver's and the Sender's *ex post* beliefs are ρ and σ, respectively. Accordingly, if the state is ω, the Sender had observed signal s from signal structure π and sent message m, and the Receiver's ex post belief is ρ and takes action a, then the Sender's ex post net payoff is

$$v(a,\omega) - c(m|s) - \psi(\rho|\sigma_\pi(\cdot|s)).$$

As actual information about the state is essentially a belief, it is natural to model the Sender's information misrepresentation cost as the discrepancy between his and the Receiver's beliefs. However, one potential drawback of our formulation is that the Receiver's posterior belief is endogenous to the equilibrium, which might then affect the interpretation of ψ as the Sender's lying cost. In particular, because the Receiver's posterior belief depends on how she interprets the Sender's messages, ψ could be penalizing the Sender for the Receiver's own misinterpretation. Nevertheless, we believe that this "consequentialist" view is still a reasonable formulation of an information misrepresentation cost because the consequence of the Receiver's belief (and hence action) is often the basis for the judgment of the Sender's conduct in practice. For example, Gneezy (2005) provided experimental evidence that individuals specifically care about the harm caused by their lies, and criminal sentencing considers not only the crime but also the damages caused. Overall, our formulation of the communication costs is also consistent with the distinction between "lying" (penalized by c) and "deception" (penalized by ψ) in Sobel (2020).

We now consider the analysis of our extended model with the additional term ψ. First, the definition of an equilibrium in Definition 1 remains unchanged except that the Sender's best response condition (S-IC) is now replaced by (S-IC$_\psi$):

$$\begin{aligned}&\Big[c\left(m'|s\right) - c\left(m|s\right)\Big] + \Big[\psi\left(\rho_{\pi,\mu}\left(\cdot|m'\right)|\sigma_\pi\left(\cdot|s\right)\right) - \psi\left(\rho_{\pi,\mu}\left(\cdot|m\right)|\sigma_\pi\left(\cdot|s\right)\right)\Big]\\ \geq\ & v\Big(\alpha\left(\cdot|\rho_{\pi,\mu}\left(\cdot|m'\right)\right), \sigma_\pi\left(\cdot|s\right)\Big) - v\Big(\alpha\left(\cdot|\rho_{\pi,\mu}\left(\cdot|m\right)\right), \sigma_\pi\left(\cdot|s\right)\Big) \quad \forall m' \neq m. \qquad \text{(S-IC}_\psi\text{)}\end{aligned}$$

Let

$$\Psi(\mu;\alpha|\pi) := \sum_{\omega\in\Omega} \beta^o(\omega) \sum_{s\in S} \pi(s|\omega) \sum_{m\in M} \mu(m|s)\, \psi\Big(\rho_{\pi,\mu}(\cdot|m)\,|\sigma_\pi(\cdot|s)\Big) \tag{15}$$

denote the Sender's expected information misrepresentation cost. We abuse the notation for W in (5); henceforth, let

$$W(\mu;\alpha|\pi) := V(\mu;\alpha|\pi) - C(\mu|\pi) - \Psi(\mu;\alpha|\pi). \tag{16}$$

Our goal here is to provide a condition on ψ under which all of our previous results follow through. Condition 2 below provides the condition, Lemma 4 provides examples of ψ-functions that satisfy Condition 2, and Proposition 6 provides the main result of this section.

Condition 2. ψ satisfies the following:

1. $\psi(\rho|\sigma) = 0$ if $\rho = \sigma$.
2. For any $\sigma \in \Delta\Omega$ and $\tau \in \Delta\Delta\Omega$ such that $\int \sigma' d\tau(\sigma') = \sigma$,

$$\psi(\rho|\sigma) \geq \int \psi(\rho|\sigma') - \psi(\sigma|\sigma') \, d\tau(\sigma') \quad \text{for any } \rho \in \Delta\Omega. \tag{17}$$

The first property of Condition 2 states that the information misrepresentation cost is zero if the Receiver's belief is the same as the Sender's belief. The second property states that misrepresenting information is less costly when the Sender has less precise information. To see this, first note that the left-hand side of equation (17) can be written as $\psi(\rho|\sigma) - \psi(\sigma|\sigma)$ because $\psi(\sigma|\sigma) = 0$; therefore, the left-hand side is the Sender's cost of changing the Receiver's belief from σ to ρ when his private information is a belief of σ. On the right-hand side of equation (17), τ is a distribution of beliefs that averages out to σ; therefore, the right-hand side is also the Sender's (expected) cost of changing the Receiver's belief from σ to ρ but under less precise private information — instead of (privately) knowing that the belief is precisely σ, the Sender knows only that the belief is σ *in expectation*.

Lemma 4. *The following parameterizations of ψ satisfy Condition 2:*

- *$\psi(\rho|\sigma) = d(\rho, \sigma)$, where $d : \Delta\Omega \times \Delta\Omega \longrightarrow \mathbb{R}^+$ is any metric on $\Delta\Omega$ (e.g., Euclidean distance, total variation distance, and the Hellinger distance).*
- *Kullback-Leibler divergence: $\psi(\rho|\sigma) = \sum_{\omega\in\Omega} \sigma(\omega) \log \frac{\sigma(\omega)}{\rho(\omega)}$.*
- *Squared Euclidean distance: $\psi(\rho|\sigma) = \sum_{\omega\in\Omega} (\sigma(\omega) - \rho(\omega))^2$.*

Proposition 6. *Suppose that the Sender also has an information misrepresentation cost ψ that satisfies Condition 2. Proposition 1 holds. In turn, Proposition 2 also holds, with constraint (11) replaced by*

$$\begin{aligned} \forall j, j' = 1, \ldots, |\Omega|, \quad & c(m_{j'}|s_j) - c(m_j|s_j) + \psi(\beta_{j'}|\beta_j) \\ & \geq \; v\Big(\alpha(\cdot|\beta_{j'}), \beta_j\Big) - v\Big(\alpha(\cdot|\beta_j), \beta_j\Big) \end{aligned} \tag{18}$$

The proof of Proposition 6 is subsumed in the proofs of Proposition 1 and Proposition 2. As before, the key is to establish that message pooling leads to a suboptimal equilibrium for the Sender. To see the role of Condition 2, consider the scenario in the earlier explanation for Proposition 1, in which the Sender's belief after signals s_1 and s_2 are σ_1 and σ_2, respectively, and the Sender reports message $\hat{m}$ after both of these signals, which leads to the Receiver holding belief $\hat{\rho}$. There, we explained how having a signal structure that shifts all of the conditional probabilities of s_1 and s_2 to the efficient signal $\bar{s}_{\hat{m}}$ and having the Sender reporting $\hat{m}$ after this signal lead to a better equilibrium for the Sender.

Given the first property of Condition 2, such a change also always leads to a lower expected ψ-cost because the Sender's belief at signal $\bar{s}_{\hat{m}}$ is also $\hat{\rho}$; therefore, reporting $\hat{m}$ after signal $\bar{s}_{\hat{m}}$ leads to $\psi = 0$. Next, the second property of Condition 2 ensures that this new construction is still an equilibrium. Intuitively, this is because constraint (S-IC$_\psi$) at $\bar{s}_{\hat{m}}$ is a convex combination of (S-IC$_\psi$) at s_1 and s_2 with weights $\frac{p_1}{p_1+p_2}$ and $\frac{p_2}{p_1+p_2}$, respectively. Because Condition 2 implies that $\psi\left(\rho'|\hat{\rho}\right) \geq \frac{p_1}{p_1+p_2}\psi\left(\rho'|\sigma_1\right) + \frac{p_2}{p_1+p_2}\psi\left(\rho'|\sigma_2\right)$, then, for any ρ', the additional ψ-cost of deviating to reporting a message m' that induces a belief $\rho' \neq \hat{\rho}$ is higher when the Sender's belief is $\hat{\rho}$ than when his belief was a convex combination of σ_1 and σ_2 that averages out to $\hat{\rho}$.

Finally, we note that, because a zero function satisfies Condition 1, our results hold if ψ is the only communication cost in the model. In fact, ψ tightens the full communication property in Sender-preferred equilibria:

Corollary 3. *Suppose that the message technology satisfies Condition 1 and ψ satisfies Condition 2. If, in addition, $\psi\left(\rho|\sigma\right) = 0$ only if $\rho = \sigma$, then there is full communication in every Sender-preferred equilibrium. This additional condition is satisfied by all of the examples in Lemma 4.*

Finally, because Proposition 1 holds, Proposition 2 also extends with constraint (11) becoming constraint (18), in which the ψ-cost is zero on the equilibrium path because the two players' ex post beliefs always coincide in a full-communication equilibrium. In turn, Proposition 3 also extends.

Corollary 4. *Suppose that the Sender's information misrepresentation cost ψ satisfies Condition 2. Proposition 3 holds with the no-lying constraint in (NL) replaced by*

there exist $s_1, \ldots, s_{|\Omega|} \in S$ such that

$$c\left(s_{j'}|s_j\right) + \psi\left(\beta_{j'}, \beta_j\right) \geq v\left(\alpha\left(\cdot|\beta_{j'}\right), \beta_j\right) - v\left(\alpha\left(\cdot|\beta_j\right), \beta_j\right) \quad \forall j, j'. \tag{NL$_\psi$}$$

7. Conclusion

In the strategic communication literature, which includes games of cheap talk, communication with costly lying and verifiable disclosure, the Sender's information is usually given, and the focus is on understanding the possible communication outcomes. In contrast, in the Bayesian persuasion literature, the communication issues are put aside, and the focus is on the design of information acquisition. In this paper, we provide a framework that spans both strains of literature, which then allows us to study the interaction between the two sets of issues. Within our framework, we derive sufficient conditions on the communication costs such that the Sender-preferred equilibrium is always supportable with full communication, and these conditions are satisfied by many communication games that have been widely studied in the literature. We view this result as having two substantial implications. First, this result allows for a simple and useful characterization of the persuasion solution under limited commitment. Second, it connects the issues in these strategic communication games with Bayesian persuasion — for an information design to be ex ante optimal for the Sender, the constraints that it creates for the subsequent (strategic) communication stage should not be stringent enough to prohibit full communication. Using examples, we show how this connection helps explain some test design features and why less information might be acquired even if full information is mutually beneficial a priori.

Appendix A. Proof of Proposition 1

To streamline the exposition, we consider the extended setup with the information misrepresentation cost ψ introduced in Section 6. Readers who are interested in only the baseline setup can simply take ψ to be the zero function whenever it arises.

Notations. We drop the finiteness assumption on S, M, and A, and let them be any compact metric spaces. This requires us to redefine some notations. Henceforth, for any compact metric space X, let ΔX denote the set of all Borel probability measures on X, endowed with the weak* topology. A signal structure is a measurable map $\pi : \Omega \to \Delta S$, where $\pi(\cdot|\omega) \in \Delta S$ denotes the probability measure on S in state ω. The Sender's stage-2 message rule is a measurable map $\mu : S \to \Delta M$, where $\mu(\cdot|s) \in \Delta M$ is the probability measure on M after the Sender observes signal s. The Receiver's strategy is a measurable map $\alpha : \Delta\Omega \to \Delta A$, where $\alpha(\cdot|\rho) \in \Delta A$ is the probability measure on A when the Receiver holds belief ρ. We further assume that $\bar{A}(\rho)$ is nonempty for all $\rho \in \Delta\Omega$ and v is bounded above by $\bar{v} < \infty$; these two assumptions hold trivially when A is finite.

The players' beliefs are still denoted by σ and ρ. Fixing $\omega \in \Omega$, for any Borel $\hat{S} \subseteq S$ in the support of π and any Borel $\hat{M} \subseteq M$ in the support of π and μ,

$$\pi\left(\hat{S}|\omega\right)\beta^o(\omega) = \sum_{\omega' \in \Omega} \beta^o\left(\omega'\right) \int_{s \in \hat{S}} \sigma_\pi(\omega|s)\, d\pi\left(s|\omega'\right), \tag{19}$$

$$\int_{s\in S} \mu\left(\hat{M}|s\right) d\pi(s|\omega)\,\beta^o(\omega) = \sum_{\omega' \in \Omega} \beta^o\left(\omega'\right) \int_{s\in S}\int_{m\in\hat{M}} \rho_{\pi,\mu}(\omega|m)\, d\mu(m|s)\, d\pi\left(s|\omega'\right). \tag{20}$$

Because S and M are complete and separable, the regular conditional probabilities (i.e., posterior beliefs) when conditioned on s and m, respectively, exist — see Shiryaev (1996), chapter 7. By the Radon-Nikodym theorem, σ_π and $\rho_{\pi,\mu}$ in equations (19) and (20) define the players' respective posteriors almost everywhere.

Outline of the proof. We first define an auxiliary problem in Subsection A.1. Next, in Subsection A.2, we first prove Lemmas 5 and 6, which are essentially the statement of Proposition 1 but for the auxiliary problem. Finally, in Subsection A.3, we prove Proposition 1 by mapping the auxiliary problem to the original problem using Lemma 7, which connects the results in Lemmas 5 and 6 to the original problem.

A.1. Defining an auxiliary problem

Call the problem in the main text the *original problem*. Its *auxiliary problem* (to be described) is an analogous problem with the same message set but a richer signal set. As a convention for notations, every symbol in the auxiliary problem has an additional superscript "$\mathbb{A}$" to its counterpart in the original problem. The signal set in the auxiliary problem is $S^{\mathbb{A}} = S \times \mathbb{R}$. A signal $s \in S^{\mathbb{A}}$ is a double $s = (x, y)$, in which the first dimension $x \in S$ is the *base dimension*. The message cost in the auxiliary problem is $c^{\mathbb{A}} : M \times S^{\mathbb{A}} \to \mathbb{R}^+ \cup \{\infty\}$, in which the base dimension of the signal fully determines its message cost according to the message cost function c in the original problem — i.e.,

$$c^{\mathbb{A}}(m|(x, y)) = c(m|x) \quad \forall m \in M \text{ and } (x, y) \in S^{\mathbb{A}}.$$

Next, a signal structure in the auxiliary problem is $\pi^{\mathbb{A}} : \Omega \to \Delta S^{\mathbb{A}}$, and a Sender's message strategy is $\mu^{\mathbb{A}} : S^{\mathbb{A}} \to \Delta M$. The players' posteriors are as expressed in (19) and (20) but with the signal set S replaced by $S^{\mathbb{A}}$. A Receiver's strategy in the auxiliary problem is still $\alpha : M \to \Delta A$, as in the original problem. The functions $V^{\mathbb{A}}$, $C^{\mathbb{A}}$, $\Psi^{\mathbb{A}}$ and $W^{\mathbb{A}}$ are the natural extensions of (3), (4), (15) and (16), respectively:

$$
\begin{aligned}
& V^{\mathbb{A}}\left(\mu^{\mathbb{A}};\alpha|\pi^{\mathbb{A}}\right) \\
& \quad := \sum_{\omega\in\Omega}\beta^{o}(\omega)\int_{s\in S^{\mathbb{A}}}\int_{m\in M} v\Big(\alpha\left(\cdot|\rho_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}(\cdot|m)\right),\sigma_{\pi^{\mathbb{A}}}(\cdot|s)\Big)d\mu^{\mathbb{A}}(m|s)\,d\pi^{\mathbb{A}}(s|\omega) \\
& C^{\mathbb{A}}\left(\mu^{\mathbb{A}}|\pi^{\mathbb{A}}\right) \\
& \quad := \sum_{\omega\in\Omega}\beta^{o}(\omega)\int_{s\in S^{\mathbb{A}}}\int_{m\in M} c^{\mathbb{A}}(m|s)\,d\mu^{\mathbb{A}}(m|s)\,d\pi^{\mathbb{A}}(s|\omega) \\
& \Psi^{\mathbb{A}}\left(\mu^{\mathbb{A}};\alpha|\pi^{\mathbb{A}}\right) \\
& \quad := \sum_{\omega\in\Omega}\beta^{o}(\omega)\int_{s\in S^{\mathbb{A}}}\int_{m\in M} \psi\Big(\rho_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}(\cdot|m)\,|\sigma_{\pi^{\mathbb{A}}}(\cdot|s)\Big)d\mu^{\mathbb{A}}(m|s)\,d\pi^{\mathbb{A}}(s|\omega) \\
& W^{\mathbb{A}}\left(\mu^{\mathbb{A}};\alpha|\pi^{\mathbb{A}}\right) := V\left(\mu^{\mathbb{A}};\alpha|\pi^{\mathbb{A}}\right) - C\left(\mu^{\mathbb{A}}|\pi^{\mathbb{A}}\right) - \Psi\left(\mu^{\mathbb{A}};\alpha|\pi^{\mathbb{A}}\right)
\end{aligned}
$$

The definition of an equilibrium is the same as Definition 1 but with the appropriate replacement of the signal set and the message cost function in constraint (S-IC$_\psi$) to the following: for any $s \in S^{\mathbb{A}}$ and $m \in M$ in the support of $\mu^{\mathbb{A}}(\cdot|s)$,

$$
\begin{aligned}
& \left[c^{\mathbb{A}}\left(m'|s\right) - c^{\mathbb{A}}(m|s)\right] \\
& \quad + \left[\psi\left(\rho_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}\left(\cdot|m'\right)|\sigma_{\pi^{\mathbb{A}}}(\cdot|s)\right) - \psi\left(\rho_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}(\cdot|m)\,|\sigma_{\pi^{\mathbb{A}}}(\cdot|s)\right)\right] \\
& \geq\ v\Big(\alpha\left(\cdot|\rho_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}\left(\cdot|m'\right)\right),\sigma_{\pi^{\mathbb{A}}}(\cdot|s)\Big) - v\Big(\alpha\left(\cdot|\rho_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}(\cdot|m)\right),\sigma_{\pi^{\mathbb{A}}}(\cdot|s)\Big) \quad \forall m' \neq m.
\end{aligned}
\tag{S-IC$^{\mathbb{A}}$}
$$

Similarly, $\left(\pi^{\mathbb{A}},\mu^{\mathbb{A}};\alpha\right)$ is a full-communication equilibrium if $\mu^{\mathbb{A}}$ is also pure and separating.

A.2. *Full communication in the auxiliary problem*

Definition 2. (Direct signal mapping.) Fix a signal structure $\pi^{\mathbb{A}}$ and a message strategy $\mu^{\mathbb{A}}$. Let $\bar{M}$ be the set of messages supported by $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$, and let $\Lambda(\cdot|m) \in \Delta S^{\mathbb{A}}$ be the regular conditional probability measure over $S^{\mathbb{A}}$ when conditioned on the Receiver receiving message $m \in \bar{M}$ under $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$.[23] Say $\gamma : \bar{M} \to S^{\mathbb{A}}$ is a *"direct signal mapping* of $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$" if for all $m \in \bar{M}$,

[23] I.e., for any Borel set $\hat{M} \subseteq \bar{M}$ and $\hat{S}^{\mathbb{A}} \subseteq S^{\mathbb{A}}$,

$$
\sum_{\omega\in\Omega}\beta^{o}(\omega)\int_{s'\in S^{\mathbb{A}}}\int_{m\in\hat{M}}\Lambda\left(\hat{S}^{\mathbb{A}}|m\right)d\mu^{\mathbb{A}}\left(m|s'\right)d\pi^{\mathbb{A}}\left(s'|\omega\right) = \sum_{\omega\in\Omega}\beta^{o}(\omega)\int_{s\in\hat{S}^{\mathbb{A}}}\int_{m\in\hat{M}} d\mu^{\mathbb{A}}(m|s)\,d\pi^{\mathbb{A}}(s|\omega).
$$

.

1. $c^{\mathbb{A}}(m|\gamma(m)) \le \int_{s\in S^{\mathbb{A}}} c^{\mathbb{A}}(m|s)\, d\Lambda(s|m)$.
2. for any $m' \in M$ such that $\int_{s\in S^{\mathbb{A}}} c^{\mathbb{A}}\left(m'|s\right) d\Lambda(s|m) < \infty$, $c^{\mathbb{A}}(m|\gamma(m)) - c^{\mathbb{A}}\left(m'|\gamma(m)\right) \le \int_{s\in S^{\mathbb{A}}} c^{\mathbb{A}}(m|s)\, d\Lambda(s|m) - \int_{s\in S^{\mathbb{A}}} c^{\mathbb{A}}\left(m'|s\right) d\Lambda(s|m)$.
3. for any $m' \in M$ such that $\int_{s\in S^{\mathbb{A}}} c^{\mathbb{A}}\left(m'|s\right) d\Lambda(s|m) = \infty$, $c^{\mathbb{A}}\left(m'|\gamma(m)\right) = \infty$.
4. $\gamma\left(m'\right) = \gamma\left(m''\right)$ if and only if $m' = m''$.

Lemma 5. *Suppose that ψ satisfies Condition 2, $\left(\pi^{\mathbb{A}}, \mu^{\mathbb{A}}; \alpha\right)$ is an equilibrium in the auxiliary problem, and γ is a direct signal mapping of $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$ (as defined in Definition 2). Let $\bar{M} \subseteq M$ denote the set of messages supported by $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$. There exists an equilibrium $\left(\bar{\pi}^{\mathbb{A}}, \bar{\mu}^{\mathbb{A}}; \alpha\right)$ such that*

1. *The support of $\bar{\pi}^{\mathbb{A}}$ is $\bar{S}^{\mathbb{A}} := \left\{\gamma(m)\,|m \in \bar{M}\right\}$*
2. *For all $m \in \bar{M}$, $\bar{\mu}^{\mathbb{A}}(\cdot|\gamma(m))$ is a Dirac measure on m (i.e., $\bar{\mu}^{\mathbb{A}}$ is pure and separating).*
3. *$V^{\mathbb{A}}\left(\bar{\mu}^{\mathbb{A}}; \alpha|\bar{\pi}^{\mathbb{A}}\right) = V^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha|\pi^{\mathbb{A}}\right)$, $C^{\mathbb{A}}\left(\bar{\mu}^{\mathbb{A}}|\bar{\pi}^{\mathbb{A}}\right) \le C^{\mathbb{A}}\left(\mu^{\mathbb{A}}|\pi^{\mathbb{A}}\right)$, and $\Psi^{\mathbb{A}}\left(\bar{\mu}^{\mathbb{A}}; \alpha|\bar{\pi}^{\mathbb{A}}\right) = 0$.*

Proof. As in Definition 2, let $\Lambda(\cdot|m) \in \Delta S^{\mathbb{A}}$ be the regular conditional probability measure over $S^{\mathbb{A}}$ when conditioned on $m \in \bar{M}$ under $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$. Let $g : \bar{S}^{\mathbb{A}} \longrightarrow \bar{M}$ be the function in which $g(\gamma(m)) = m\ \forall m \in \bar{M}$.[24] Let $\bar{\pi}^{\mathbb{A}} : \Omega \longrightarrow \Delta S^{\mathbb{A}}$ be the following mapping: for any Borel set $\hat{S}^{\mathbb{A}} \subseteq S^{\mathbb{A}}$,

$$\bar{\pi}^{\mathbb{A}}\left(\hat{S}^{\mathbb{A}}|\omega\right) := \int\limits_{s\in S^{\mathbb{A}}} \mu^{\mathbb{A}}\left(\left\{g\left(s'\right)|s' \in \hat{S}^{\mathbb{A}} \cap \bar{S}^{\mathbb{A}}\right\}|s\right) d\pi^{\mathbb{A}}(s|\omega) \quad \forall \omega \in \Omega. \tag{21}$$

Notice that $\left\{g\left(s'\right)|s' \in S^{\mathbb{A}} \cap \bar{S}^{\mathbb{A}}\right\} = \bar{M}$. This implies that $\bar{\pi}^{\mathbb{A}}\left(S^{\mathbb{A}}|\omega\right) = 1\ \forall \omega \in \Omega$ and, hence, $\bar{\pi}^{\mathbb{A}}$ is a valid signal structure that is supported on $\bar{S}^{\mathbb{A}}$.

We first verify that $\left(\bar{\pi}^{\mathbb{A}}, \bar{\mu}^{\mathbb{A}}; \alpha\right)$ is an equilibrium. Because $\left(\pi^{\mathbb{A}}, \mu^{\mathbb{A}}; \alpha\right)$ is an equilibrium, $\alpha \in \mathcal{A}^*$. Therefore, we only have to verify the Sender's incentive-compatibility condition, which is (S-IC$^{\mathbb{A}}$) for the auxiliary problem. First, note that the messages supported under $\bar{\pi}^{\mathbb{A}}$ and $\bar{\mu}^{\mathbb{A}}$ is $\bar{M}$ (i.e., same as that of under $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$). For any Borel $\hat{M} \subseteq \bar{M}$, let $P_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}\left(\hat{M}|\omega\right)$ denote the probability measure of $\hat{M}$ conditional on ω under $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$. Because

$$\begin{aligned} P_{\bar{\pi}^{\mathbb{A}},\bar{\mu}^{\mathbb{A}}}\left(\hat{M}|\omega\right) &= \int\limits_{s\in S^{\mathbb{A}}} \bar{\mu}^{\mathbb{A}}\left(\hat{M}|s\right) d\bar{\pi}^{\mathbb{A}}(s|\omega) = \int\limits_{s\in\left\{\gamma(m)|m\in\hat{M}\right\}} d\bar{\pi}^{\mathbb{A}}(s|\omega) \\ &= \int\limits_{s'\in S^{\mathbb{A}}} \int\limits_{m\in\hat{M}} d\mu^{\mathbb{A}}\left(m|s'\right) d\pi^{\mathbb{A}}\left(s'|\omega\right) = P_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}\left(\hat{M}|\omega\right), \end{aligned} \tag{22}$$

we have

[24] γ is injective but not necessarily surjective because its range is $\bar{S}^{\mathbb{A}} \subseteq S^{\mathbb{A}}$. If we define its codomain to be $\bar{S}^{\mathbb{A}}$, then γ will be bijective, and g is simply the inverse of γ.

$$\rho_{\bar{\pi}^{\mathbb{A}},\bar{\mu}^{\mathbb{A}}}(\cdot|m) = \rho_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}(\cdot|m) \quad \forall m \in \bar{M}. \tag{23}$$

Therefore,

$$\sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m)) = \rho_{\bar{\pi}^{\mathbb{A}},\bar{\mu}^{\mathbb{A}}}(\cdot|m) = \rho_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}(\cdot|m) = \int\limits_{s'\in S^{\mathbb{A}}} \sigma_{\pi^{\mathbb{A}}}\left(\cdot|s'\right) d\Lambda\left(s'|m\right) \quad \forall m \in \bar{M}, \tag{24}$$

where the first equality follows from $\bar{\mu}^{\mathbb{A}}(\cdot|\gamma(m))$ being Dirac on m $\forall m \in \bar{M}$, the second equality is (23), and the third equality follows from the martingale property of Bayesian posteriors. Next, due to (23), we can ease notation by letting $\rho_m = \rho_{\bar{\pi}^{\mathbb{A}},\bar{\mu}^{\mathbb{A}}}(\cdot|m) = \rho_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}(\cdot|m)$ and $\alpha_m(\cdot) = \alpha(\cdot|\rho_m)$ $\forall m \in \bar{M}$. To verify (S-IC$^{\mathbb{A}}$) for $\left(\bar{\pi}^{\mathbb{A}}, \bar{\mu}^{\mathbb{A}}; \alpha^{\mathbb{A}}\right)$, we have to show that $\forall m \in \bar{M}$ and $m' \neq m$,

$$\begin{aligned} & v\Big(\alpha_m, \sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))\Big) - v\Big(\alpha_{m'}, \sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))\Big) \\ \geq\ & \Big[c^{\mathbb{A}}(m|\gamma(m)) - c^{\mathbb{A}}\left(m'|\gamma(m)\right)\Big] \\ & + \left[\psi\left(\rho_m|\sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))\right) - \psi\left(\rho_{m'}|\sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))\right)\right]. \end{aligned} \tag{25}$$

$c^{\mathbb{A}}(m|\gamma(m))$ must be finite for all $m \in \bar{M}$ because $\left(\pi^{\mathbb{A}}, \mu^{\mathbb{A}}; \alpha\right)$ is an equilibrium and m is in the support of $\mu^{\mathbb{A}}(\cdot|\gamma(m))$. Moreover, the left-hand-side of the inequality in (25) is finite by assumption. If m' is such that $\int_{s'\in S^{\mathbb{A}}} c^{\mathbb{A}}\left(m'|s'\right) d\Lambda\left(s'|m\right) = \infty$, property 3 of γ implies that $c^{\mathbb{A}}\left(m'|\gamma(m)\right) = \infty$, which implies that (25) is trivially satisfied. Therefore, we consider $\int_{s'\in S^{\mathbb{A}}} c^{\mathbb{A}}\left(m'|s'\right) d\Lambda\left(s'|m\right) < \infty$. By property 2 of γ,

$$\int\limits_{s\in S^{\mathbb{A}}} c^{\mathbb{A}}(m|s) - c^{\mathbb{A}}\left(m'|s\right) d\Lambda(s|m) \geq c^{\mathbb{A}}(m|\gamma(m)) - c^{\mathbb{A}}\left(m'|\gamma(m)\right). \tag{26}$$

From (24), $\int_{s'\in S^{\mathbb{A}}} \sigma_{\pi^{\mathbb{A}}}\left(\cdot|s'\right) d\Lambda\left(s'|m\right) = \sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))$ and $\rho_m = \sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))$, which implies that $\psi\left(\rho_m|\sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))\right) = 0$. Next, the second part of Condition 2 implies that for all m', $\psi\left(\rho_{m'}|\sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))\right) \geq \int_{s'\in S^{\mathbb{A}}} \psi\left(\rho_{m'}|\sigma_{\pi^{\mathbb{A}}}\left(\cdot|s'\right)\right) - \psi\left(\sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))|\sigma_{\pi^{\mathbb{A}}}\left(\cdot|s'\right)\right) d\Lambda\left(s'|m\right)$; thus,

$$\begin{aligned} & \int\limits_{s'\in S^{\mathbb{A}}} \psi\left(\rho_m|\sigma_{\pi^{\mathbb{A}}}\left(\cdot|s'\right)\right) - \psi\left(\rho_{m'}|\sigma_{\pi^{\mathbb{A}}}\left(\cdot|s'\right)\right) d\Lambda\left(s'|m\right) \\ & \quad \geq \psi\left(\rho_m|\sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))\right) - \psi\left(\rho_{m'}|\sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))\right) \end{aligned} \tag{27}$$

From (24), for any $\alpha \in \Delta A$ and $m \in \bar{M}$,

$$\begin{aligned} v\left(\alpha, \sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))\right) &= \sum_{\omega\in\Omega} \sigma_{\bar{\pi}^{\mathbb{A}}}(\omega|\gamma(m))\, v(\alpha, \omega) \\ &= \sum_{\omega\in\Omega} \int\limits_{s'\in S^{\mathbb{A}}} \sigma_{\pi^{\mathbb{A}}}\left(\omega|s'\right) d\Lambda\left(s'|m\right) v(\alpha, \omega) \\ &= \int\limits_{s'\in S^{\mathbb{A}}} v\left(\alpha, \sigma_{\pi^{\mathbb{A}}}\left(\cdot|s'\right)\right) d\Lambda\left(s'|m\right). \end{aligned} \tag{28}$$

Putting everything together, we have,

$$
\begin{aligned}
& v\Big(\alpha_m, \sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))\Big) - v\Big(\alpha_{m'}, \sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))\Big) \\
&= \int\limits_{s'\in S^{\mathbb{A}}} \left[v\left(\alpha_m, \sigma_{\pi^{\mathbb{A}}}\left(\cdot|s'\right)\right) - v\left(\alpha_{m'}, \sigma_{\pi^{\mathbb{A}}}\left(\cdot|s'\right)\right)\right] d\Lambda\left(s'|m\right) && (29) \\
&\geq \int\limits_{s'\in S^{\mathbb{A}}} \left[c^{\mathbb{A}}\left(m|s'\right) - c^{\mathbb{A}}\left(m'|s'\right)\right] + \left[\psi\left(\rho_m|\sigma_{\pi^{\mathbb{A}}}\left(\cdot|s'\right)\right) - \psi\left(\rho_{m'}|\sigma_{\pi^{\mathbb{A}}}\left(\cdot|s'\right)\right)\right] d\Lambda\left(s'|m\right) && (30) \\
&\geq \left[c^{\mathbb{A}}\left(m|\gamma(m)\right) - c^{\mathbb{A}}\left(m'|\gamma(m)\right)\right] + \left[\psi\left(\rho_m|\sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))\right) - \psi\left(\rho_{m'}|\sigma_{\bar{\pi}^{\mathbb{A}}}(\cdot|\gamma(m))\right)\right], && (31)
\end{aligned}
$$

which is (25) — the equality in (29) follows from (28); the inequality in (30) follows from (S-IC$^{\mathbb{A}}$) for $\left(\pi^{\mathbb{A}}, \mu^{\mathbb{A}}; \alpha^{\mathbb{A}}\right)$, and the inequality in (31) follows from (26) and (27). Thus, $\left(\bar{\pi}^{\mathbb{A}}, \bar{\mu}^{\mathbb{A}}; \alpha\right)$ satisfies (S-IC$^{\mathbb{A}}$); therefore, it is an equilibrium, and

$$
\begin{aligned}
V^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha|\pi^{\mathbb{A}}\right) &= \sum_{\omega'\in\Omega} \beta^o\left(\omega'\right) \int\limits_{s\in S^{\mathbb{A}}} \int\limits_{m\in\bar{M}} v\left(\alpha_m, \sigma_{\pi^{\mathbb{A}}}(\cdot|s)\right) d\mu^{\mathbb{A}}(m|s)\, d\pi^{\mathbb{A}}\left(s|\omega'\right) \\
&= \sum_{\omega\in\Omega}\sum_{\omega'\in\Omega} \beta^o\left(\omega'\right) \int\limits_{s\in S^{\mathbb{A}}} \sigma_{\pi^{\mathbb{A}}}(\omega|s) \int\limits_{m\in\bar{M}} v\left(\alpha_m, \omega\right) d\mu^{\mathbb{A}}(m|s)\, d\pi^{\mathbb{A}}\left(s|\omega'\right) \\
&= \sum_{\omega\in\Omega} \beta^o(\omega) \int\limits_{s\in S^{\mathbb{A}}} \int\limits_{m\in\bar{M}} v\left(\alpha_m, \omega\right) d\mu^{\mathbb{A}}(m|s)\, d\pi^{\mathbb{A}}(s|\omega) && (32) \\
&= \sum_{\omega\in\Omega} \beta^o(\omega) \int\limits_{m\in\bar{M}} v\left(\alpha_m, \omega\right) dP_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}(m|\omega). && (33)
\end{aligned}
$$

By the same argument, $V^{\mathbb{A}}\left(\bar{\mu}^{\mathbb{A}}; \alpha|\bar{\pi}^{\mathbb{A}}\right) = \sum_{\omega\in\Omega} \beta^o(\omega) \int_{m\in\bar{M}} v\left(\alpha_m, \omega\right) dP_{\bar{\pi}^{\mathbb{A}},\bar{\mu}^{\mathbb{A}}}(m|\omega)$. Since $P_{\bar{\pi}^{\mathbb{A}},\bar{\mu}^{\mathbb{A}}}(\cdot|\omega) = P_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}(\cdot|\omega)\, \forall\omega$ from (22), $V^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha|\pi^{\mathbb{A}}\right) = V\left(\bar{\mu}^{\mathbb{A}}; \alpha|\bar{\pi}^{\mathbb{A}}\right)$. Next,

$$
\begin{aligned}
& C^{\mathbb{A}}\left(\bar{\mu}^{\mathbb{A}}|\bar{\pi}^{\mathbb{A}}\right) \\
&= \sum_{\omega'\in\Omega} \beta^o\left(\omega'\right) \int\limits_{m\in\bar{M}} \left[c^{\mathbb{A}}\left(m|\gamma(m)\right)\right] dP_{\bar{\pi}^{\mathbb{A}},\bar{\mu}^{\mathbb{A}}}(m|\omega) \\
&\leq \sum_{\omega'\in\Omega} \beta^o\left(\omega'\right) \int\limits_{m\in\bar{M}} \left[\int\limits_{s\in S^{\mathbb{A}}} c^{\mathbb{A}}(m|s)\, d\Lambda(s|m)\right] dP_{\bar{\pi}^{\mathbb{A}},\bar{\mu}^{\mathbb{A}}}(m|\omega) && (34) \\
&= \sum_{\omega'\in\Omega} \beta^o\left(\omega'\right) \int\limits_{m\in\bar{M}} \left[\int\limits_{s\in S^{\mathbb{A}}} c^{\mathbb{A}}(m|s)\, d\Lambda(s|m)\right] dP_{\pi^{\mathbb{A}},\mu^{\mathbb{A}}}(m|\omega) && (35)
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{\omega' \in \Omega} \beta^{o}\left(\omega'\right) \int\limits_{s' \in S^{\mathbb{A}}} \int\limits_{m \in \bar{M}} \left[\int\limits_{s \in S^{\mathbb{A}}} c^{\mathbb{A}}\left(m|s\right) d\Lambda\left(s|m\right) \right] d\mu^{\mathbb{A}}\left(m|s'\right) d\pi^{\mathbb{A}}\left(s'|\omega'\right) \\
&= \sum_{\omega \in \Omega} \beta^{o}\left(\omega\right) \int\limits_{s' \in S^{\mathbb{A}}} \int\limits_{m \in \bar{M}} c^{\mathbb{A}}\left(m|s'\right) d\mu^{\mathbb{A}}\left(m|s'\right) d\pi^{\mathbb{A}}\left(s'|\omega\right) \qquad (36) \\
&= C^{\mathbb{A}}\left(\mu^{\mathbb{A}}|\pi^{\mathbb{A}}\right). \qquad (37)
\end{aligned}
$$

The inequality in (34) follows from the first property of γ, the equality in (35) follows from (22), and the equality in (36) follows from noting that the integral $\int_{s \in S^{\mathbb{A}}} c^{\mathbb{A}}\left(m|s\right) d\Lambda\left(s|m\right)$ integrates over only signals s in which m is in the support of $\mu\left(\cdot|s\right)$. Finally, it is immediate that $\Psi^{\mathbb{A}}\left(\bar{\mu}^{\mathbb{A}}; \alpha|\bar{\pi}^{\mathbb{A}}\right) = 0 \leq \Psi^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha|\pi^{\mathbb{A}}\right)$. Therefore,

$$
W^{\mathbb{A}}\left(\bar{\mu}^{\mathbb{A}}; \alpha|\bar{\pi}^{\mathbb{A}}\right) \geq W^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha|\pi^{\mathbb{A}}\right). \quad \square
$$

Lemma 6. *Suppose that $\left(\pi^{\mathbb{A}}, \mu^{\mathbb{A}}; \alpha\right)$ is a Sender-preferred equilibrium in the auxiliary problem and $\mu^{\mathbb{A}}$ is pure and separating. There exists a signal structure $\bar{\pi}^{\mathbb{A}}$ such that $\left(\bar{\pi}^{\mathbb{A}}, \mu^{\mathbb{A}}; \alpha\right)$ is also a Sender-preferred equilibrium and $\left|\bar{S}^{\mathbb{A}}\right| = \left|\bar{M}\right| \leq |\Omega|$, where $\bar{S}^{\mathbb{A}}$ is the support of $\bar{\pi}^{\mathbb{A}}$ and $\bar{M}$ is the set of messages supported by $\bar{\pi}^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$.*

Proof. We first introduce a few notations. Let $\tilde{S}^{\mathbb{A}} \subseteq S^{\mathbb{A}}$ be the support of $\pi^{\mathbb{A}}$, $\tilde{M} \subseteq M$ be the set of messages supported by $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$, and $Q \in \Delta\tilde{M}$ be the probability measure over the equilibrium messages under $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$.[25] Assume that $\left|\tilde{M}\right| > |\Omega|$ (if not, we are done). Since $\mu^{\mathbb{A}}$ is pure and separating, for each $m \in \tilde{M}$, let s_m denote the signal in which $\mu^{\mathbb{A}}\left(\{m\}\,|s_m\right) = 1$. To shorten the notations, let $c^{\mathbb{A}}\left(m|s_m\right)$ be c_m, $\alpha\left(\cdot|m\right)$ be α_m, $\sigma_{\pi^{\mathbb{A}}}\left(\cdot|s_m\right)$ be σ_m, $v\left(\alpha_m, \sigma_m\right)$ be v_m, and $v_m - c_m = w_m$. Finally, let $\mathcal{B} = \{\sigma_m\}_{m \in \tilde{M}} \subset \Delta\Omega$ and $\chi = \{\sigma_m, w_m\}_{m \in \mathcal{M}} \subset \mathbb{R}^{|\Omega|}$.

We first note the following observation: for any finite subset of messages $\hat{M} \subset \tilde{M}$, if there exists a probability vector $\left(p_m\right)_{m \in \hat{M}}$ such that $\sum_{m \in \hat{M}} p_m \sigma_m = \beta^o$, then there exists a signal structure $\hat{\pi}^{\mathbb{A}}$ that generates a belief distribution supported on only beliefs $\{\sigma_m\}_{m \in \hat{M}}$, $\left(\hat{\pi}^{\mathbb{A}}, \mu^{\mathbb{A}}; \alpha\right)$ is also an equilibrium, and $W^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha|\hat{\pi}^{\mathbb{A}}\right) = \sum_{m \in \hat{M}} p_m w_m$. This is because from KG, since $\sum_{m \in \hat{M}} p_m \sigma_m = \beta^o$, there exists a signal structure $\hat{\pi}^{\mathbb{A}}$ that generates a belief distribution supported on only beliefs $\{\sigma_m\}_{m \in \hat{M}}$ with σ_m occurring with a probability of p_m. Let belief σ_m be attached to signal s_m in $\hat{\pi}^{\mathbb{A}}$. Notice that under $\hat{\pi}^{\mathbb{A}}$ and the stage-2 strategy profile $\left(\mu^{\mathbb{A}}; \alpha\right)$, the set of constraints (S-IC$^{\mathbb{A}}$) is a subset of that under the original $\pi^{\mathbb{A}}$. Therefore, $\left(\hat{\pi}^{\mathbb{A}}, \mu^{\mathbb{A}}; \alpha\right)$ is also an equilibrium, and $W^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha|\hat{\pi}^{\mathbb{A}}\right) = \sum_{m \in \hat{M}} p_m w_m$.

Next, for any set X, let $co\left(X\right)$ denotes its convex hull. Let $\xi = \left(\inf_{m \in \tilde{M}} w_m\right) - \varepsilon$, where $\varepsilon > 0$, and let $Y : co\left(\mathcal{B}\right) \to \mathbb{R}$ be a function defined by

[25] i.e., for any Borel set $\hat{M} \subseteq M$, $Q\left(\hat{M}\right) = \sum_{\omega \in \Omega} \beta^o\left(\omega\right) \int_{s \in \tilde{S}^{\mathbb{A}}} \mu^{\mathbb{A}}\left(\hat{M}|s\right) d\pi^{\mathbb{A}}\left(s|\omega\right)$.

$$Y(\beta) = \begin{cases} w_m & \text{if } \beta \in \mathcal{B} \text{ and } \beta = \sigma_m \\ \xi & \text{if } \beta \notin \mathcal{B} \end{cases}$$

Let $hyp(Y)$ be the hypograph of Y — i.e., $hyp(Y) = \{(\beta, y) \,|\, y \leq Y(\beta)\} \subset \mathbb{R}^{|\Omega|}$. Note that since v is bounded and ξ must be finite, $hyp(Y)$ is path-connected and hence connected. As $\beta^o = \int_{m\in\tilde{M}} \sigma_m dQ(m)$ and $W\left(\mu^{\mathbb{A}}; \alpha|\pi^{\mathbb{A}}\right) = \int_{m\in\tilde{M}} w_m dQ(m)$, this implies that $\left\{\beta^o, W\left(\mu^{\mathbb{A}}; \alpha|\pi^{\mathbb{A}}\right)\right\} \in co(\chi) \subset co(hyp(Y))$. By the Fenchel-Bunt theorem (see Hiriart-Urruty and Lemaréchal (2012), p. 30, Proposition 1.3.7), there exist $\mathcal{Z} = \{z_1, \ldots, z_N\} \in hyp(Y)$, with $N \leq |\Omega|$, and a probability vector $\delta = (\delta_1, \ldots, \delta_N)$ such that $\sum_{n=1}^N \delta_n z_n = \left(\beta^o, W\left(\mu^{\mathbb{A}}; \alpha|\pi^{\mathbb{A}}\right)\right)$. By the observation in the previous paragraph, if we can show that $\mathcal{Z} \subset \chi$, then we would have proven the Lemma. We show this next.

Let $\mathcal{Z} = \{(\beta_1, y_1), \ldots, (\beta_N, y_N)\}$, where, for each n, $\beta_n \in co(\mathcal{B})$ and $y_n \in hyp(Y(\beta_n))$. Therefore, $\sum_{n=1}^N \delta_n \beta_n = \beta^o$ and $\sum_{n=1}^N \delta_n y_n = W\left(\mu^{\mathbb{A}}; \alpha|\pi^{\mathbb{A}}\right)$. Suppose, for a contradiction, that $\mathcal{Z} \not\subset \chi$. For ease of exposition, suppose that only $z_1 \notin \chi$ but $z_2, \ldots, z_N \in \chi$, and that $\left|\tilde{M}\right|$ is finite — the argument is readily extended. Let $\tilde{M} = \{m_1, m_2, \ldots, m_J\}$, where $J > |\Omega| \geq N$, and without loss of generality, let $\beta_2 = \sigma_{m_2}, \ldots, \beta_N = \sigma_{m_N}$. If $z_1 \notin \chi$, then it implies that either (i) $\beta_1 \in \mathcal{B}$ but $y_1 < Y(\beta_1)$, or (ii) $\beta_1 \notin \mathcal{B}$.

Consider Case (i) first. Without loss of generality, let $\beta_1 = \sigma_{m_1}$. By the observation above, there exists an equilibrium in which the Sender's expected net payoff is $\sum_{n=1}^N \delta_n w_{m_n} = \delta_1 Y\left(\sigma_{m_1}\right) + \sum_{n=2}^N \delta_n y_n > \delta_1 y_1 + \sum_{n=2}^N \delta_n y_n = W^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha|\pi^{\mathbb{A}}\right)$, which then contradicts the assumption that the Sender's value of persuasion is $W^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha|\pi^{\mathbb{A}}\right)$.

Next, consider Case (ii). Since $\beta_1 \in co(\mathcal{B})$, by the Carathéodory theorem, there exists a probability vector $\left(q_j\right)_{j=1,\ldots,J}$ such that $\beta_1 = \sum_{j=1}^J q_j \sigma_{m_i}$ (note that some q_j's are possibly zero). Consider the probability vector $\left(\hat{\delta}_1, \ldots, \hat{\delta}_J\right)$ where $\hat{\delta}_j = q_j \delta_1$ if $j \neq 2, \ldots, N$, and $\hat{\delta}_j = \delta_j + q_j \delta_1$ if $j = 2, \ldots, N$.[26] Notice that

$$\sum_{j=1}^J \hat{\delta}_j \sigma_{m_j} = \sum_{j=1}^J q_j \delta_1 \sigma_{m_j} + \sum_{j=2}^N \delta_j \sigma_{m_j} = \delta_1 \beta_1 + \sum_{n=2}^N \delta_n \beta_n = \beta^o.$$

Therefore, by the observation above, there exists an equilibrium in which the Sender's net expected payoff is $\sum_{j=1}^J \hat{\delta}_j w_{m_j}$. However,

$$\begin{aligned} \sum_{j=1}^J \hat{\delta}_j w_{m_j} &= \sum_{j=1}^J q_j \delta_1 w_{m_j} + \sum_{j=2}^N \delta_j w_{m_j} \\ &= \delta_1 w_1 + \sum_{n=2}^N \delta_n w_n > \delta_1 \xi + \sum_{n=2}^N \delta_n y_n = W^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha|\pi^{\mathbb{A}}\right), \end{aligned}$$

where the inequality is because $\beta_1 \notin \mathcal{B}$ implies that $y_1 = \xi$. Again, this contradicts the assumption that the Sender's value of persuasion is $W^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha|\pi^{\mathbb{A}}\right)$.

[26] $\sum_{j=1}^J \hat{\delta}_j = \sum_{j=1}^J q_j \delta_1 + \sum_{j=2}^N \delta_j = \sum_{n=1}^N \delta_n = 1$.

Jointly, this implies that $\mathcal{Z} \subset \chi$. $\square$

Lemma 7. *If Condition 1 is satisfied in the original problem, then for any signal structure $\pi^{\mathbb{A}}$ and message strategy $\mu^{\mathbb{A}}$ in the auxiliary problem, there exists a direct signal mapping of $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$ (as defined in Definition 2). Moreover, if $\bar{M}$, the set of messages supported by $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$, is such that $\left|\bar{M}\right| \leq |\Omega|$, then there exists a direct signal mapping γ such that for all $m \in \bar{M}$, $\gamma(m) = (x, y)$ with $y = 0$.*

Proof. Let $\bar{s}_m \in S$ be the efficient signal of m in set $\bar{M}$ as defined in Condition 1. Condition 1 implies that setting $\gamma(m) = (\bar{s}_m, y) \in S^{\mathbb{A}}$ satisfies the first three properties in Definition 2 for any $y \in \mathbb{R}$. The last property is then trivially satisfied given the freedom in the choice of y. Since $\bar{s}_m = \bar{s}_{m'}$ if and only if $m = m'$, we have $\gamma(m) = \gamma\left(m'\right)$ if and only if $m = m'$. The last part of the lemma is immediate. $\square$

A.3. Proof of Proposition 1

Proof. Let $(\pi, \mu; \alpha)$ be an equilibrium in the original problem, and let $\tilde{M}$ denote the set of messages supported in this equilibrium. For any Borel $\hat{S}^{\mathbb{A}} \subset S^{\mathbb{A}}$ (in the auxiliary problem), define $\mathcal{X}\left(\hat{S}^{\mathbb{A}}\right) := \left\{(x, y) \in \hat{S}^{\mathbb{A}} | y = 0\right\} \subset S^{\mathbb{A}}$ — i.e., the subset of signals in $\hat{S}^{\mathbb{A}}$ in which the second dimension is zero — and let $\mathcal{B}\left(\hat{S}^{\mathbb{A}}\right) = \left\{x | (x, y) \in \hat{S}^{\mathbb{A}}\right\} \subseteq S$ — i.e., the collection of base signals of $\hat{S}^{\mathbb{A}}$. For all $\omega \in \Omega$ and Borel $\hat{S}^{\mathbb{A}} \subset S^{\mathbb{A}}$, let $\pi^{\mathbb{A}}\left(\hat{S}^{\mathbb{A}} | \omega\right) = \pi\left(\mathcal{B}\left(\mathcal{X}\left(\hat{S}^{\mathbb{A}}\right)\right) | \omega\right)$. Next, for any $x \in S$, let $\mu^{\mathbb{A}}(\cdot | (x, 0)) = \mu(\cdot | x)$; the mapping $\mu^{\mathbb{A}}(\cdot | (x, y))$ for any $y \neq 0$ can arbitrarily specified because such a signal is not supported by $\pi^{\mathbb{A}}$. The following two observations are immediate: (i) the set of messages supported by $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$ is also $\tilde{M}$, and (ii) $\sigma_\pi(\cdot | x) = \sigma_{\pi^{\mathbb{A}}}(\cdot | (x, 0))$ for any $x \in S$ in the support of π, $\rho_{\pi, \mu}(\cdot | m) = \rho_{\pi^{\mathbb{A}}, \mu^{\mathbb{A}}}(\cdot | m)$ $\forall m \in \tilde{M}$. These two observations imply that if $(\pi, \mu; \alpha)$ satisfy (S-IC$_\psi$), then $\left(\pi^{\mathbb{A}}, \mu^{\mathbb{A}}; \alpha\right)$ satisfy (S-IC$^{\mathbb{A}}$). Therefore, $\left(\pi^{\mathbb{A}}, \mu^{\mathbb{A}}; \alpha\right)$ is an equilibrium in the auxiliary problem, and $W^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha | \pi^{\mathbb{A}}\right) = W(\mu; \alpha | \pi)$. By Lemma 7, there exists a direct signal mapping of $\pi^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$, which we denote by γ. By Lemmas 5 and 6, there exist $\bar{\pi}^{\mathbb{A}}$ and $\bar{\mu}^{\mathbb{A}}$ such that $\left(\bar{\pi}^{\mathbb{A}}, \bar{\mu}^{\mathbb{A}}; \alpha\right)$ is also an equilibrium and

1. $\left|\bar{S}^{\mathbb{A}}\right| = \left|\bar{M}\right| \leq |\Omega|$, where $\bar{S}^{\mathbb{A}}$ is the support of $\bar{\pi}^{\mathbb{A}}$ and $\bar{M}$ is the set of messages supported by $\bar{\pi}^{\mathbb{A}}$ and $\mu^{\mathbb{A}}$.
2. $\bar{S}^{\mathbb{A}} := \left\{\gamma(m) | m \in \bar{M}\right\}$
3. For all $m \in \bar{M}$, $\bar{\mu}^{\mathbb{A}}(\cdot | \gamma(m))$ is a Dirac measure on m (i.e., $\bar{\mu}^{\mathbb{A}}$ is pure and separating).
4. $W^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha | \bar{\pi}^{\mathbb{A}}\right) \geq W^{\mathbb{A}}\left(\mu^{\mathbb{A}}; \alpha | \pi^{\mathbb{A}}\right)$.

By Lemma 7, since $\left|\bar{M}\right| \leq |\Omega|$, we can set γ to be such that for all $m \in \bar{M}$, $\gamma(m) = (\bar{s}_m, 0)$. Therefore, $\bar{\pi}^{\mathbb{A}}$ is a signal structure supported on $\left|\bar{M}\right|$ signals in which the second dimension of every supported signal is always zero. Next, define a signal structure $\bar{\pi}$ in the original problem as follows: the support of $\bar{\pi}$ is $\{\bar{s}_m\}_{m \in \bar{M}}$, and $\bar{\pi}(\{\bar{s}_m\} | \omega) = \bar{\pi}^{\mathbb{A}}(\{(\bar{s}_m, 0)\} | \omega)$ $\forall \omega$. Define a message strategy $\bar{\mu}$ in the original problem as follows: for each $m \in \bar{M}$, $\bar{\mu}(\{m\} | \bar{s}_m) = 1 =$

$\bar{\mu}^{\mathbb{A}}(\{m\} \mid (\bar{s}_m, 0))$. By a similar argument as above, because $\left(\bar{\pi}^{\mathbb{A}}, \bar{\mu}^{\mathbb{A}}; \alpha\right)$ is an equilibrium in the auxiliary problem, $(\bar{\pi}, \bar{\mu}; \alpha)$ is an equilibrium in the original problem, and $W^{\mathbb{A}}\left(\bar{\mu}^{\mathbb{A}}; \alpha|\bar{\pi}^{\mathbb{A}}\right) = W(\bar{\mu}; \alpha|\bar{\pi})$. We have thus proved Proposition 1. □

Remark 3. In our construction of the full-communication equilibrium, when $\bar{M}$ is the set of messages supported in the equilibrium, the set of signals supported by the signal structure is the set of efficient signals $\{\bar{s}_m\}_{m\in\bar{M}}$ (as defined in Condition 1).

Appendix B. Proofs

This section provides the proofs for all the results except for Proposition 1, whose proof is found in Appendix A. As in Appendix A, we also streamline the exposition by considering the extended setup with the information misrepresentation cost ψ introduced in Section 6.

B.1. Proof of Lemma 1

Proof. Consider the distance-lying cost first. For any $\hat{M} \subseteq M$, set $\bar{s}_m = m\ \forall m \in \hat{M}$. As $c(m|\bar{s}_m) = 0$, the first property must hold. For the second property, for any $m \in \hat{M}$, $m' \neq m$ and $s \in S$, $c\left(m'|\bar{s}_m\right) - c(m|\bar{s}_m) = d\left(m', m\right) \geq d\left(m', s\right) - d(m, s) = c\left(m'|s\right) - c(m|s)$, where the inequality follows from the triangle inequality property of d. Next, the discrete-lying cost is the discrete metric under the distance-lying cost.

Finally, consider partial verifiability. If $\phi \notin \hat{M}$, set $\bar{s}_m = m\ \forall m \in \hat{M}$; if $\phi \in \hat{M}$, then set $\bar{s}_\phi$ to be any $s \notin \hat{M}$ and $\bar{s}_m = m$ for the other $m \neq \phi$. As $c(m|\bar{s}_m) = 0$, the first property must hold. For the second property, if $m' = \phi$ or $m' \in \mathcal{N}$, the left-hand side is $-c(m|s)$, whereas the right-hand side is 0 because $c(m|\bar{s}_m) = 0$; therefore, the second property holds if $m' \notin \mathcal{V}$. If $m' \in \mathcal{V}$, the right-hand side is $c\left(m'|\bar{s}_m\right) = k$, whereas the left-hand side is weakly lower than k; therefore, the second property also holds if $m' \in \mathcal{V}$. □

B.2. Proof of Proposition 2

Proof. Let $\Phi^*(\tau, \alpha) := V^*(\tau, \alpha) - C^*(\tau, \alpha)$. Call $(\pi, \mu; \alpha)$ a $|\Omega|$-full-communication equilibrium if it is a full-communication equilibrium and π is support on $|\Omega|$ or less signals. We first prove the following two statements:

1. For any $\tau = \left\{\vec{\beta}, \vec{\delta}\right\} \in \mathcal{D}(\beta^o)$ and $\alpha \in \mathcal{A}^*$, there exists a $|\Omega|$-full-communication equilibrium $\left(\hat{\pi}, \hat{\mu}; \hat{\alpha}\right)$ such that $W\left(\hat{\mu}; \hat{\alpha}|\hat{\pi}\right) = \Phi(\tau, \alpha)$.
2. If $(\pi, \mu; \alpha)$ is a $|\Omega|$-full-communication equilibrium and τ is the belief distribution generated by π, then $V^*(\tau, \alpha) = V(\mu; \alpha|\pi)$ and $C^*(\tau, \alpha) \leq C(\mu|\pi)$.

For statement 1, let $\chi = \left\{s_1^*, \ldots, s_{|\Omega|}^*; m_1^*, \ldots, m_{|\Omega|}^*\right\}$ denote the solution that obtains $C^*\left(\left\{\vec{\beta}, \vec{\delta}\right\}, \alpha\right)$. For $j = 1$ to $|\Omega|$, let $\hat{\pi}\left(s_j^*|\omega\right) = \frac{\beta_j(\omega)\delta_j}{\beta^o(\omega)}$ for each $\omega \in \Omega$ and $\mu^*\left(m_j^*|s_j^*\right) = 1$.

Let $\hat{\alpha}$ be such that $\hat{\alpha}\left(\cdot|\beta_j\right)=\alpha\left(\cdot|\beta_j\right)$ for $j=1$ to $|\Omega|$, and $\hat{\alpha}\left(\left\{\underline{a}\right\}|\underline{\rho}\right)=1$; note that $\hat{\alpha}\in\mathcal{A}^*$.[27] The Sender's belief after observing signal s_j^* is β_j. χ satisfying constraint (11) (resp., (18)) implies that μ^* satisfies constraint (S-IC) (resp., (S-IC$_\psi$)). Therefore, $\left(\hat{\pi},\hat{\mu};\hat{\alpha}\right)$ is a $|\Omega|$-full-communication equilibrium, and $V\left(\hat{\mu};\hat{\alpha}|\hat{\pi}\right)=V^*\left(\tau,\alpha\right)$, and $C\left(\hat{\mu}|\hat{\pi}\right)=C^*\left(\tau,\alpha\right)$.

For statement 2, the equality between V^* and V is trivial. For the second part, let s_1 to $s_{|\Omega|}$ be the signals supported by π, and let m_j be the message such that $\mu\left(\left\{m_j\right\}|s_j\right)=1$. Since $(\pi,\mu;\alpha)$ satisfies constraint (S-IC) (resp., (S-IC$_\psi$)), $\left\{s_1,\ldots,s_{|\Omega|};m_1,\ldots,m_{|\Omega|}\right\}$ satisfies constraint (11) (resp., (18)) and, hence, is a feasible solution for program ($\mathcal{C}$) for (τ,α). Therefore, $C^*\left(\tau,\alpha\right)\le\sum_{j=1}^{|\Omega|}\delta_j c\left(m_j|s_j\right)=C\left(\mu|\pi\right)$.

Next, fix any $|\Omega|$-full-communication equilibrium $(\pi,\mu;\alpha)$. If program ($\mathcal{P}$) does not have a solution, then by statement 2, there exist $\hat{\tau}\in\mathcal{D}\left(\beta^o\right)$ and $\hat{\alpha}\in\mathcal{A}^*$ such that $\Phi^*\left(\hat{\tau},\hat{\alpha}\right)>W\left(\mu;\alpha|\pi\right)$. In turn, by statement 1, there exists a $|\Omega|$-full-communication equilibrium $\left(\hat{\pi},\hat{\mu};\hat{\alpha}\right)$ such that $W\left(\hat{\mu};\hat{\alpha}|\hat{\pi}\right)>W\left(\mu;\alpha|\pi\right)$. Therefore, if program ($\mathcal{P}$) does not have a solution, there does not exist a Sender-preferred $|\Omega|$-full-communication equilibrium.

Conversely, suppose that program ($\mathcal{P}$) has a solution (τ^*,α^*). By statement 1, there exists a $|\Omega|$-full-communication equilibrium $(\pi^*,\mu^*;\alpha^*)$ such that $W\left(\mu^*;\alpha^*|\pi^*\right)=\Phi\left(\tau^*,\alpha^*\right)$. Consider any $|\Omega|$-full-communication equilibrium $(\pi,\mu;\alpha)$. By statement 2, $W\left(\mu;\alpha|\pi\right)\le\Phi\left(\tau^*,\alpha^*\right)$. Therefore, $(\pi^*,\mu^*;\alpha^*)$ is a Sender-preferred $|\Omega|$-full-communication equilibrium.

Jointly, the two preceding paragraphs imply that a Sender-preferred $|\Omega|$-full-communication equilibrium exists if and only if a solution to program ($\mathcal{P}$) exists. Moreover, if $(\pi^*,\mu^*;\alpha^*)$ is a Sender-preferred equilibrium, then $W\left(\mu^*;\alpha^*|\pi^*\right)=\Phi\left(\tau^*,\alpha^*\right)$, where (τ^*,α^*) is a solution to program ($\mathcal{P}$). The first part of Proposition 2 then follows from Proposition 1 (and the first part of Proposition 6), where the Sender-preferred $|\Omega|$-full-communication equilibrium is also a Sender-preferred equilibrium.

Finally, we show that program ($\mathcal{P}$) always has a solution. Per footnote 16, we assume that c is continuous and $\mathcal{A}^*$ is compact. For any $(\tau,\alpha)\in\mathcal{D}\left(\beta^o\right)\times\mathcal{A}^*$, let

$$\begin{aligned}\chi\left(\tau,\alpha\right)=\Big\{\vec{s}=\left(s_1,\ldots,s_{|\Omega|}\right);\\ \vec{m}=\left(m_1,\ldots,m_{|\Omega|}\right)\ |\ \vec{s},\vec{m}\text{ jointly satisfy (11) under }(\tau,\alpha)\Big\}.\end{aligned}$$

Next, let $\bar{\mathcal{D}}=\{\tau\in\mathcal{D}\left(\beta^o\right)|\chi\left(\tau,\alpha\right)$ is non-empty for some $\alpha\in\mathcal{A}^*\}$ be the subset of $\mathcal{D}\left(\beta^o\right)$ such that the sets of signals and messages satisfying constraint (11) are non-empty. Given that there exists $(\tau,\alpha)\in\mathcal{D}\left(\beta^o\right)\times\mathcal{A}^*$ such that $C^*\left(\tau,\alpha\right)<\infty$, $\bar{\mathcal{D}}$ is non-empty, and it is without loss of generality to search for the optimal signal structure over $\bar{\mathcal{D}}$. Since constraint (11) (resp., (18)) is a finite set of (N^2-N) weak inequalities and $\mathcal{A}^*$ is compact, $\bar{\mathcal{D}}$ is a closed set, which implies that it is also compact. Next, as v is an expected utility function, v is continuous in α for $\alpha\in\mathcal{A}^*$; c is continuous by assumption. By noting that the right-hand side of the inequality (as written in (11)) (resp., (18)) is a scalar, it is readily verified that $\chi\left(\tau,\alpha\right)$ is both upper and lower hemicontinuous for all $(\tau,\alpha)\in\bar{\mathcal{D}}\times\mathcal{A}^*$. By the maximum theorem, $C^*\left(\tau,\alpha\right)$ is continuous in $\bar{\mathcal{D}}\times\mathcal{A}^*$. It is

[27] There is some tension if one of the β_j's is $\underline{\rho}$. This problem is created because we defined a strategy as a mapping from the Receiver's belief instead of from the message she receives. To be precise, let $\hat{\alpha}:M\to\Delta A$ be a Receiver's strategy that is a mapping from the message that she receives such that $\hat{\alpha}\left(\cdot|m_j^*\right)=\alpha\left(\cdot|\beta_j\right)\ \forall j$, and let $\hat{\alpha}\left(\left\{\underline{a}\right\}|m\right)=1$ for all other m. The Receiver's belief after receiving any (off-equilibrium) message $m\notin\left\{m_1^*,\ldots,m_{|\Omega|}^*\right\}$ is $\underline{\rho}$; therefore, $\hat{\alpha}$ is a Receiver's best-response against μ^*.

immediate that $V^*(\tau, \alpha^*)$ is also continuous; therefore, $V^* - C^*$ is continuous in $\bar{\mathcal{D}} \times \mathcal{A}^*$, which is compact. Thus, $V^* - C^*$ must attain a maximum in $\bar{\mathcal{D}} \times \mathcal{A}^*$. $\square$

B.3. Proof of Proposition 3

Proof. As noted, we consider the extended model with ψ; therefore, constraint (11) in the baseline model is replaced by constraint (18).

Fix any $\left\{\vec{\beta}; \vec{\delta}\right\}$ and α and suppose that $C^*\left(\left\{\vec{\beta}; \vec{\delta}\right\}, \alpha\right) \neq \infty$. This implies that the feasible set for constraint (18) is nonempty. Let messages $m_1, \ldots m_{|\Omega|}$ and signals $s_1, \ldots s_{|\Omega|}$ be a set of messages and signals that satisfy constraint (18). If it is the partial verifiability message technology, assume for the moment that none of these messages are ϕ. Set $\hat{s}_j = m_j$ for all j. From the proof of Lemma 1, for each j, $\hat{s}_j$ is an efficient signal (as defined in Condition 1) of message m_j given the set of messages $\left\{m_1, \ldots m_{|\Omega|}\right\}$. By the second property of Condition 1, for any j and j',

$$\begin{aligned} c\left(m_{j'}|\hat{s}_j\right) - c\left(m_j|\hat{s}_j\right) &\geq c\left(m_{j'}|s_j\right) - c\left(m_j|s_j\right) \\ &\geq v\Big(\alpha\left(\cdot|\beta_{j'}\right), \beta_j\Big) - v\Big(\alpha\left(\cdot|\beta_j\right), \beta_j\Big) - \psi\left(\beta_{j'}|\beta_j\right). \end{aligned}$$

Thus, $\left\{m_1, \ldots m_{|\Omega|}\right\}$ and $\left\{\hat{s}_1, \ldots \hat{s}_{|\Omega|}\right\}$ satisfy constraint (18), and $\sum_{j=1}^{|\Omega|} \delta_j c\left(m_j|\hat{s}_j\right) = 0$. Therefore, $C^*\left(\left\{\vec{\beta}; \vec{\delta}\right\}, \alpha\right) = 0$. Given this solution, program ($\mathcal{P}_{NL}$) is immediate.

Next, suppose that one of the messages in the feasible set for constraint (18) is ϕ under partial verifiability. Let messages $\phi, m_2, \ldots m_{|\Omega|}$ and signals $s_1, \ldots s_{|\Omega|}$ be a set of messages and signals that satisfy constraint (18). Consider the set of messages $m_1, m_2, \ldots m_{|\Omega|}$, where m_1 is any message that is not m_2 to $m_{|\Omega|}$. Consider signals $\hat{s}_1, \ldots, \hat{s}_{|\Omega|}$ such that $\hat{s}_j = m_j$ for all j. By the argument above, constraint (18) at $\hat{s}_j$ for $j \geq 2$ is satisfied. For $\hat{s}_1$, notice that $c\left(m_1|\hat{s}_1\right) = 0 = c\left(\phi|s_1\right)$, and for any $m_{j'} \neq m_1$, since $m_1 = \hat{s}_1$, $c\left(m_{j'}|\hat{s}_1\right) \geq c\left(m_{j'}|s_1\right)$.

$$\begin{aligned} \implies \quad c\left(m_{j'}|\hat{s}_j\right) - c\left(m_j|\hat{s}_j\right) &\geq c\left(m_{j'}|s_j\right) - c\left(\phi|s_j\right) \\ &\geq v\Big(\alpha\left(\cdot|\beta_{j'}\right), \beta_j\Big) - v\Big(\alpha\left(\cdot|\beta_j\right), \beta_j\Big) - \psi\left(\beta_{j'}|\beta_j\right). \end{aligned}$$

Therefore, constraint (18) for $\hat{s}_1$ also holds. $\square$

B.4. Proof of Corollary 1

Proof. Let

$$\begin{aligned} &\Gamma : [0, \infty) \rightrightarrows \mathcal{D}\left(\beta^o\right) \times \mathcal{A}^*, \\ &\Gamma(k) := \left\{(\tau, \alpha) \,|\, k \geq v\Big(\alpha\left(\cdot|\beta_{j'}\right), \beta_j\Big) - v\Big(\alpha\left(\cdot|\beta_j\right), \beta_j\Big)\ \forall j, j'\right\}. \end{aligned}$$

Γ is both upper and lower hemicontinuous. By the maximum theorem, the value function is continuous in k. $\square$

B.5. Proof of Lemma 3

Proof. As mentioned in the main text, we need to verify that the truthful message strategy satisfies (violates) constraint (NL) to show that $C^* = 0$ ($C^* = \infty$). At $s = \checkmark$, constraint (NL) holds trivially. Consider (NL) at $s = \times$. The LHS of (NL) is always k. The RHS is at most $\bar{v}$, which thus establishes the first point. If $\beta_{\checkmark} > 0.5$, the RHS is $\bar{v}$, which establishes the second point. If $\beta_{\checkmark} = 0.5$, the RHS is $z\bar{v}$, which establishes the third point. □

B.6. Proof of Proposition 5

Proof. By Proposition 2, we only have to consider Bayes plausible distributions of beliefs supported on two beliefs. Let $\underline{\beta}$ and $\bar{\beta}$ denote the two beliefs under the optimal signal structure and, without loss of generality, assume that $\underline{\beta} \leq \beta^o \leq \bar{\beta}$. The probabilities of beliefs $\underline{\beta}$ and $\bar{\beta}$ are $\underline{\tau} = \frac{\bar{\beta}-\beta^o}{\bar{\beta}-\underline{\beta}}$ and $\bar{\tau} = \frac{\beta^o-\underline{\beta}}{\bar{\beta}-\underline{\beta}}$, respectively. As $\underline{\beta} \leq \beta^o \leq \frac{L}{1+L}$, $\hat{v}\left(\underline{\beta}\right) = 0$. Therefore, the Sender's value of persuasion is $\hat{V}\left(\underline{\beta}, \bar{\beta}\right) = \tau_0 \hat{v}\left(\underline{\beta}\right) + \tau_1 \hat{v}\left(\bar{\beta}\right) = \frac{\beta^o-\underline{\beta}}{\bar{\beta}-\underline{\beta}} \hat{v}\left(\bar{\beta}\right)$, and constraint (14) requires that $\hat{v}\left(\bar{\beta}\right) \leq k$. As $\bar{\beta} \geq \beta^o$, $\hat{V}\left(\underline{\beta}, \bar{\beta}\right)$ decreases with $\underline{\beta}$; thus $\underline{\beta} = 0$ and $\hat{V}\left(0, \bar{\beta}\right) = \frac{\beta^o}{\bar{\beta}} \hat{v}\left(\bar{\beta}\right)$. If $\bar{\beta} \leq \frac{L}{1+L}$, $\hat{V}\left(0, \bar{\beta}\right) = 0$; if $\bar{\beta} > \frac{L}{1+L}$, $\hat{V}\left(0, \bar{\beta}\right) = \frac{\beta^o}{\bar{\beta}} \nu\left(\bar{\beta} - L\left(1 - \bar{\beta}\right)\right) > 0$. Therefore, $\bar{\beta} > \frac{L}{1+L}$. As ν is convex, it is differentiable almost everywhere, and

$$
\begin{aligned}
\frac{d}{d\bar{\beta}} \hat{V}\left(0, \bar{\beta}\right) &\propto \nu'\left(-L + \bar{\beta}(1+L)\right) - \frac{\nu\left(-L + \bar{\beta}(1+L)\right)}{\bar{\beta}(1+L)} \\
&> \nu'\left(-L + \bar{\beta}(1+L)\right) - \frac{\nu\left(-L + \bar{\beta}(1+L)\right)}{-L + \bar{\beta}(1+L)} \geq 0.
\end{aligned}
$$

The last inequality follows from $\nu'(x) \geq \frac{\nu(x)}{x}$ $\forall x > 0$, implied by the convexity of ν. Since $\hat{V}\left(0, \bar{\beta}\right)$ increases with $\bar{\beta}$, the optimal $\bar{\beta}$ is to set such that $\hat{v}\left(\bar{\beta}\right) = k$, thus implying that $\bar{\beta} = \frac{\nu^{-1}(k)+L}{1+L}$ and the Sender's value of persuasion is $\frac{(k)\beta^o(1+L)}{\nu^{-1}(k)+L}$. $\frac{d}{dk}\left(\frac{k\beta^o(1+L)}{\nu^{-1}(k)+L}\right) = \frac{\beta^o(1+L)}{\left(\nu^{-1}(k)+L\right)^2}\left(L + \nu^{-1}(k) - k\frac{d\nu^{-1}(k)}{dk}\right) > 0$ because $\frac{\nu^{-1}(z)}{z} \geq \frac{d\nu^{-1}(z)}{dz}$ for any z, which is due to ν^{-1} being concave. Therefore, the Sender's value of persuasion increases with k. Finally, $\nu^{-1}(k) < 1$ implies that the Sender's value of persuasion decreases with L. □

B.7. Proof of Lemma 4

Proof. The first property is immediate. We prove the second property. Fix any $\sigma \in \Delta\Omega$, $\tau \in \Delta\Delta\Omega$ such that $\int \sigma' d\tau\left(\sigma'\right) = \sigma$, and consider any $\rho \in \Delta\Omega$.

For any metric d, the triangle inequality property implies that

$$
\begin{aligned}
\int \psi\left(\rho|\sigma'\right) - \psi\left(\sigma|\sigma'\right) d\tau\left(\sigma'\right) &= \int d\left(\rho, \sigma'\right) - d\left(\sigma, \sigma'\right) d\tau\left(\sigma'\right) \\
&\leq \int d(\rho, \sigma) d\tau\left(\sigma'\right) = d(\rho, \sigma) = \psi(\rho|\sigma).
\end{aligned}
$$

For the Kullback-Leiber divergence, note that for any σ',

$$
\begin{aligned}
\psi\left(\rho|\sigma'\right) - \psi\left(\sigma|\sigma'\right) &= \sum_{\omega\in\Omega} \sigma'(\omega) \log \frac{\sigma'(\omega)}{\rho(\omega)} - \sum_{\omega\in\Omega} \sigma'(\omega) \log \frac{\sigma'(\omega)}{\sigma(\omega)} \\
&= \sum_{\omega\in\Omega} \sigma'(\omega) \log \frac{\sigma(\omega)}{\rho(\omega)},
\end{aligned}
$$

$$\Longrightarrow \quad \int \psi\left(\rho|\sigma'\right) - \psi\left(\sigma|\sigma'\right) d\tau\left(\sigma'\right) = \int \sum_{\omega\in\Omega} \sigma'(\omega) \log \frac{\sigma(\omega)}{\rho(\omega)} d\tau\left(\sigma'\right)$$
$$= \sum_{\omega\in\Omega} \sigma(\omega) \log \frac{\sigma(\omega)}{\rho(\omega)} = \psi(\rho|\sigma).$$

For the squared Euclidean distance, note that for any σ',

$$\psi\left(\rho|\sigma'\right) - \psi\left(\sigma|\sigma'\right) = \sum_{\omega\in\Omega} \left(\sigma'(\omega) - \rho(\omega)\right)^2 - \sum_{\omega\in\Omega} \left(\sigma'(\omega) - \sigma(\omega)\right)^2$$
$$= \sum_{\omega\in\Omega} \rho(\omega)^2 - \sigma(\omega)^2 - 2\sigma'(\omega)\left[\rho(\omega) - \sigma(\omega)\right],$$
$$\Longrightarrow \quad \int \psi\left(\rho|\sigma'\right) - \psi\left(\sigma|\sigma'\right) d\tau\left(\sigma'\right)$$
$$= \sum_{\omega\in\Omega} \rho(\omega)^2 - \sigma(\omega)^2 - 2\int \sigma'(\omega)\, d\tau\left(\sigma'\right)\left[\rho(\omega) - \sigma(\omega)\right]$$
$$= \sum_{\omega\in\Omega} \rho(\omega)^2 - \sigma(\omega)^2 - 2\sigma(\omega)\left[\rho(\omega) - \sigma(\omega)\right]$$
$$= \sum_{\omega\in\Omega} \left[\sigma(\omega) - \rho(\omega)\right]^2 = \psi(\rho|\sigma). \quad \square$$

Appendix C. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2021.105212.

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