Notes on Ortoleva (2012)

Modeling the change of paradigm: Non-Bayesian reactions to unexpected news

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Abstract

If we attach zero probability to an event because we believe it cannot happen in any way, if there is an evidence that show it had happened, we cannot use Bayesian methods to updating our beliefs, so what should we use instead? Let's see how Ortoleva think about this issue (Ortoleva, 2012).

1 Bayes' rule

First, let us rethink about why we use the Bayesian methods?

It is induced by the definition of *conditional probability*, see Hogg et al. (2005) *section 1.4*.

But the *conditional probability* is defined on events which have positive probabilities, so we must realize that the response to the unexpected news *is not an actual theoretical issue*.

From this consideration, I doubt the logic foundations behind the assumptions of *uncommon support* (Galperti, 2015), and more aggressive, the foundations behind the assumptions of *heterogeneous priors* (Alonso and Câmara, 2016)[1].

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[1] There are some classical papers that have considered the origin of *heterogeneous priors* have been included in the literature review.

It has two limitations (*maybe only on the case of subjective probability[2]*):

1. It contains no prescriptions on zero probability (*subjective*) events or information.

2. There are psychology and behavioral evidence show that the existence of non-Bayesian reactions to "unexpected" news (small but positive probability event).

2   An Example

Consider the case of an investor who wants to forecast the future return of a certain stock:

- The investor forms a *subjective prior*[3]

- As she receives new information, she will update her beliefs

There may be three types of information and the investor responses in different ways:

1. The information is about "business as usual" situations $\Rightarrow$ she will revise her beliefs following Bayes' rule

2. The information is unexpected (the event is assigned probability *zero*)[4] $\Rightarrow$ she cannot use Bayes' rule, she will somehow *rationally* choose a new prior given the evidence

3. A possible but unlikely event happened [5] $\Rightarrow$ she may question her *subjective* prior instead of updating her beliefs by Bayes' rule [6]

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[2]??

[3]Since there is no objective known distribution about the likelihood of these returns.

[4]For example, the market enters a financial crisis

[5]Which is assigned a positive but small probability

[6]For instance, the investor could have used an economic model to form her prior over stock returns, but if she saw that this model assigned a really small probability to the information that was later revealed, she might question its validity and look for an alternative one

3 Overview of the Results

The procedure behind the above example is like a *Hypothesis Testing* process, but is it reasonable? If the answer is Yes, why is it reasonable? i.e. what is the logic behind it?

Ortoleva (2012) answered this question.

The procedure we described above can be formalized as follows:

Hypothesis Testing

The agent has a triple profile (u, ρ, ϵ)

- u: a utility function over consequences

- ρ: a prior over priors[7]

- ϵ: a threshold between 0 and 1

The agent react to the information by the following steps:

1. Forms the prior π according to ρ, that is, she chooses the prior which has *maximum likelihood*

2. As new information i revealed, the agent *test her prior* to verify whether she used a correct one:

    - If $\pi(i) > \epsilon$, the prior is not rejected, the agent simply updates it using Bayes' rule

    - If $\pi(i) \leq \epsilon$, then the prior is rejected, the agent goes back to ρ, and updates it by Bayes' rule using the new information, then chooses the new π' according to *the updating prior over priors*

A behavioral property:

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[7] A belief over which belief she should use

Dynamic Coherence

The following three facts contain the same information content for the agent:

- High unemployment

- Weak housing market

- Low inflation

So each of them leads her the same beliefs.

Equivalence

Dynamic Coherence + Other standard postulates

⇕

Hypothesis Testing procedure

4

References

Alonso, R. and Câmara, O. (2016). Bayesian persuasion with heterogeneous priors. *Journal of Economic Theory*, 165:672–706.

Galperti, S. (2015). Hide or surprise: Persuasion without common-support priors. Technical report, mimeo.

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