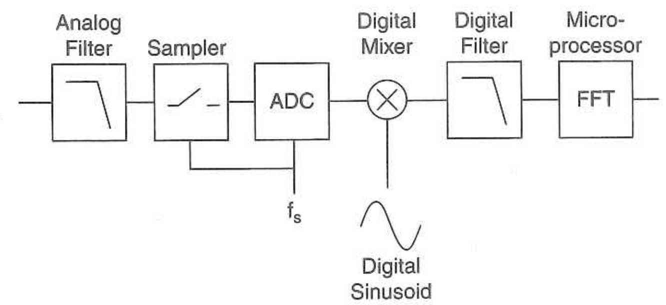

Figure 4-13 A digital mixer provides band selectable analysis in an FFT analyzer.

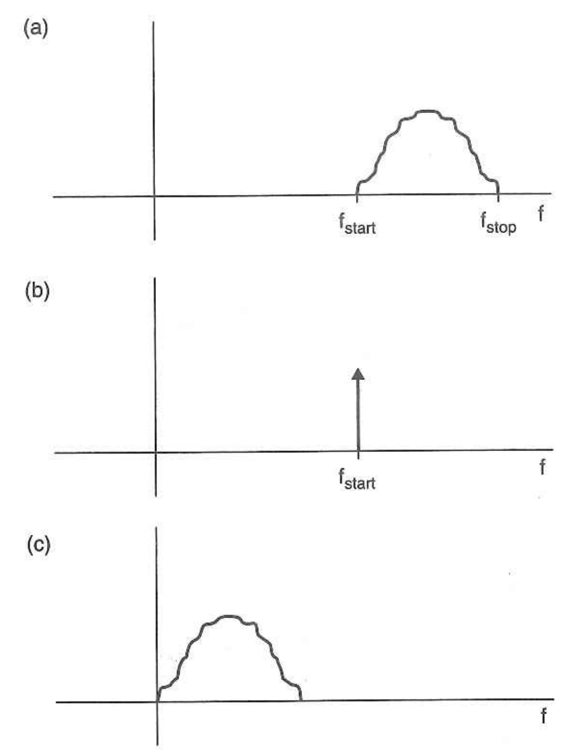

Figure 4-14 (a) The spectrum of the signal to be measured. (b) The spectrum of the digital oscillator. (c) The frequency translated version of the original spectrum.

superheterodyne technique used in radio receivers and swept spectrum analyzers. The frequency span of interest (Figure 4-14) is mixed with a complex sinusoid at the center frequency, which causes that frequency span to be mixed down to baseband. The digital filter is configured for the proper span by using the appropriate decimation factor. The FFT is used to obtain the frequency spectrum from the output of the digital filter. The bandwidth of the digital filter can be narrowed significantly, producing frequency spans as narrow as 1 Hz.

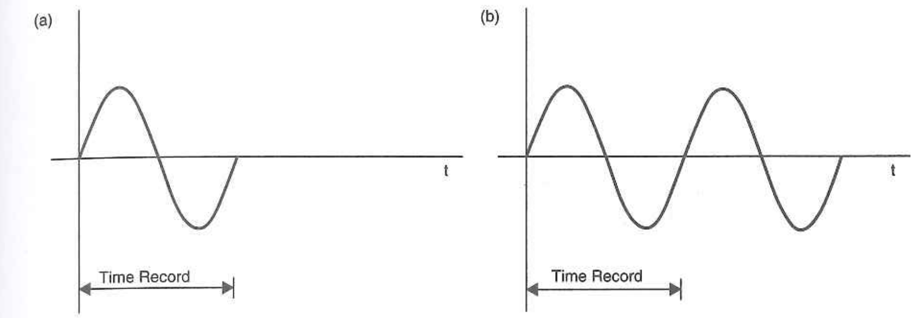

Figure 4-15 (a) A waveform that exactly fits one time record. (b) When replicated, no transients are introduced.

4.9 Leakage

The FFT operates on a finite length time record in an attempt to approximate the Fourier transform, which integrates over all time. The mathematics of the FFT operate on the finite length time record but have the effect of replicating the finite length time record over all time (Figure 4-15).[7] With the waveform shown in Figure 4-15b, the finite length time record represents the actual waveform quite well, so the FFT result will approximate the Fourier integral very well.

However, the shape and phase of a waveform may be such that a transient is introduced when the waveform is replicated for all time, as shown in Figure 4-16. In this case, the FFT spectrum is not a good approximation for the integral form of the Fourier transform. Since the instrument user often does not have control over how the waveform fits into the time record, in general, it must be assumed that a discontinuity may exist. This effect, known as *leakage*, is very apparent in the frequency domain. Instead of the spectral line appearing thin and slender, it spreads out over a wide frequency range (Figure 4-17).

The usual solution to the problem of leakage is to force the waveform to zero at the ends of the time record; then they will always be the same, and no transient will exist when the time record is replicated. This is accomplished by multiplying the time record by a *window* function. Of course, the shape of the window is important because it will affect the data; it also must not introduce a transient of its own. Many different window functions have been developed for particular digital signal processing applications. The ones common to spectrum analyzers will be examined here.

4.10 Hanning Window

Also known as the *Hann window*, the *Hanning window* is one of the most common windows in digital signal processing. The time record samples are weighted by

$$w_n = \frac{1}{2}\{1 - \cos[2\pi n/(N-1)]\} \tag{4-13}$$

[7] The FFT has the effect of replicating the time record. This is a consequence of the mathematics, and there is no need for the algorithm to actually produce the replicated time record.

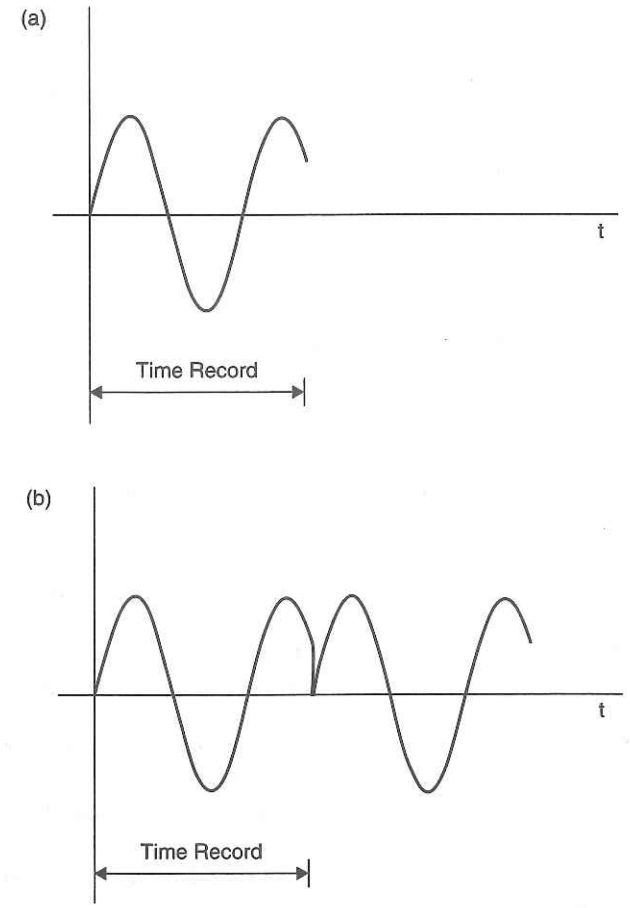

Figure 4-16 (a) A waveform that does not exactly fit into one time record. (b) When replicated, severe transients are introduced, causing leakage in the frequency domain.

where

n = bin number
N = number of bins

The Hanning window provides a smooth transition to zero as either end of the time record is approached (Figure 4-18). Therefore, the windowed time record will not produce a transient when replicated by the FFT algorithm. Clearly, the original time record has been modified and the effect in the frequency domain must be considered. The shape of the Hanning window in the frequency domain is the Fourier transform of the window function.

The frequency domain response of the window function determines the passband shape of the individual filters that the FFT produces mathematically. Figure 4-19a shows the overlapped response of several frequency bins using a Hanning window. The filter shape is rounded off, and the net response of the analyzer drops off somewhat between bins. Therefore, a spectral line falling where the two filters meet will be measured with an error

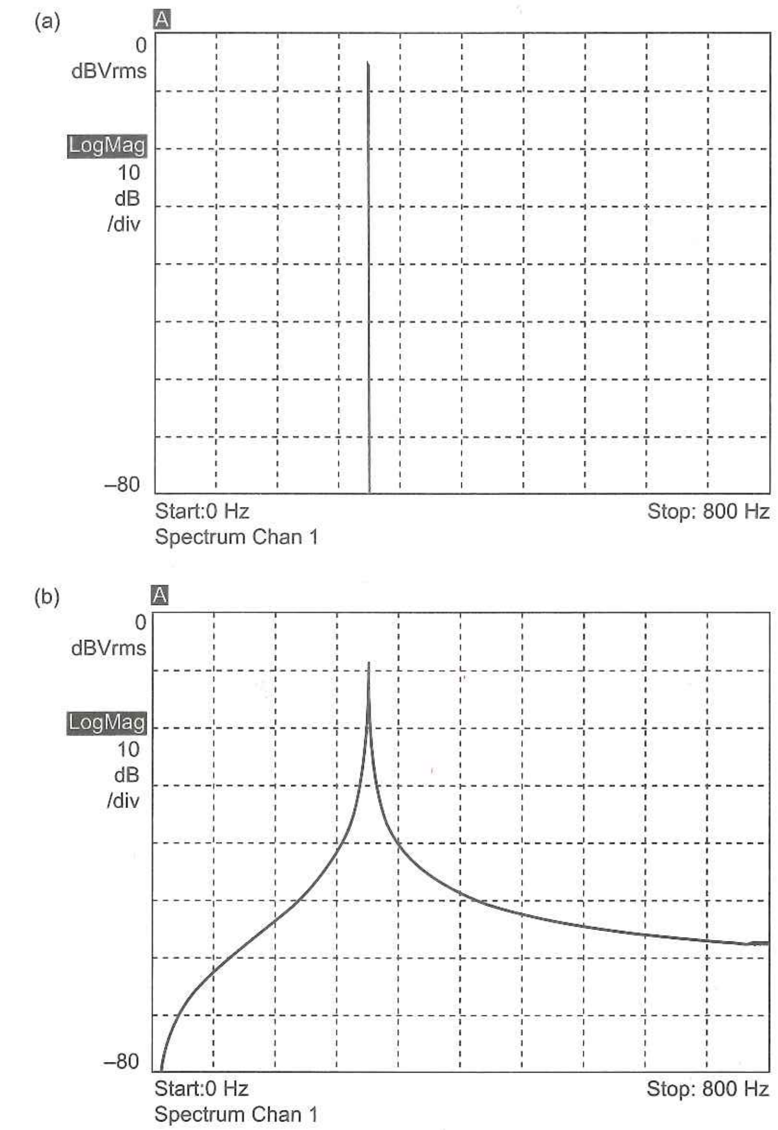

Figure 4-17 (a) Measurement of a spectral line with no leakage. (b) Measurement of a spectral line with leakage.

determined by the shape of the filter. The Hanning window introduces a maximum amplitude error of 1.5 dB (16%), which may be a significant error in some applications. The shape of a window is always a compromise between amplitude accuracy (which depends on the flatness of the filter passband) and frequency resolution (which depends on the width of the filter). The Hanning window, compared with other common windows, provides good frequency resolution at the expense of somewhat less amplitude accuracy. Figure 4-20a shows the spectrum of a sine wave measured using the Hanning window.

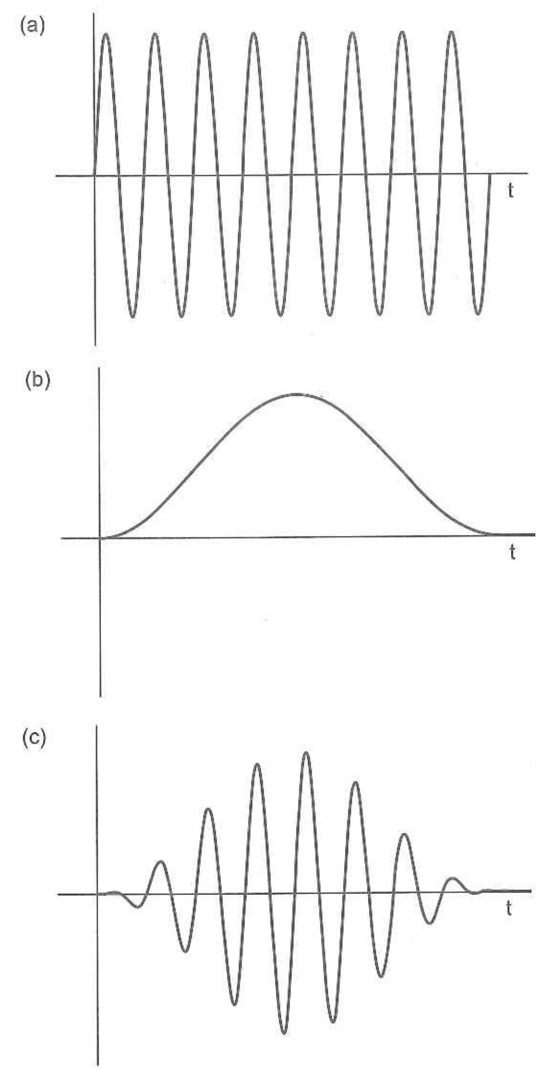

Figure 4-18 (a) The original time record. (b) The Hanning window. (c) The windowed time record.

4.11 Flattop Window

A window that has a flat passband reduces the size of the amplitude dips between bins and minimizes the amplitude error. A spectral line that falls halfway between the centers of two bins will be attenuated by a much smaller amount. The *flattop* window has such a characteristic and is shown in Figure 4-19b. Since the response of each bin overlaps considerably more than with the Hanning window, the disadvantage of the flattop window is reduced frequency resolution due to its wider profile. The spectral line will appear wider on the spectrum analyzer display, limiting the ability to resolve two closely spaced spectral lines.

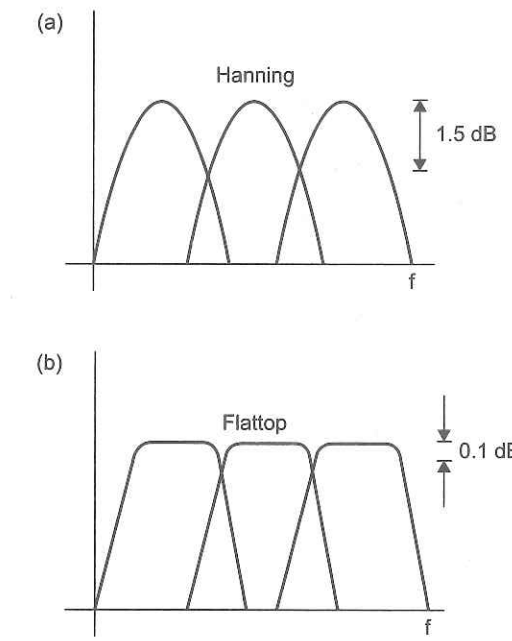

Figure 4-19 (a) The Hanning window introduces a maximum amplitude error of 1.5 dB. (b) The flattop window introduces a maximum amplitude error of up to 0.1 dB.

The flattop window is considered a high-amplitude accuracy window, having a maximum amplitude error of 0.1 dB or less, depending on the implementation. Figure 4-20b shows the spectrum of a sine wave as measured using the flattop window.

4.12 Uniform Window

The uniform window is really no window at all; all the samples are left unchanged. Although the uniform window has the potential for severe leakage problems, in some cases the waveform in the time record has the same value at both ends of the record, thereby eliminating the transient introduced by the FFT. Such waveforms are called *self-windowing.* Waveforms such as *pseudorandom noise* (PRN),[8] sine bursts, impulses, and decaying sinusoids can all be self-windowing.

The uniform window is appropriate for making network measurements when the internal noise source of the analyzer is used. The noise source is usually a PRN generator that produces a noise waveform that is periodic within the time record of the instrument. Since the noise source and the time record are synchronized, no transients occur at the ends of the time record and leakage in the frequency domain is avoided.

[8] PRN is not truly random noise but instead repeats at some interval.

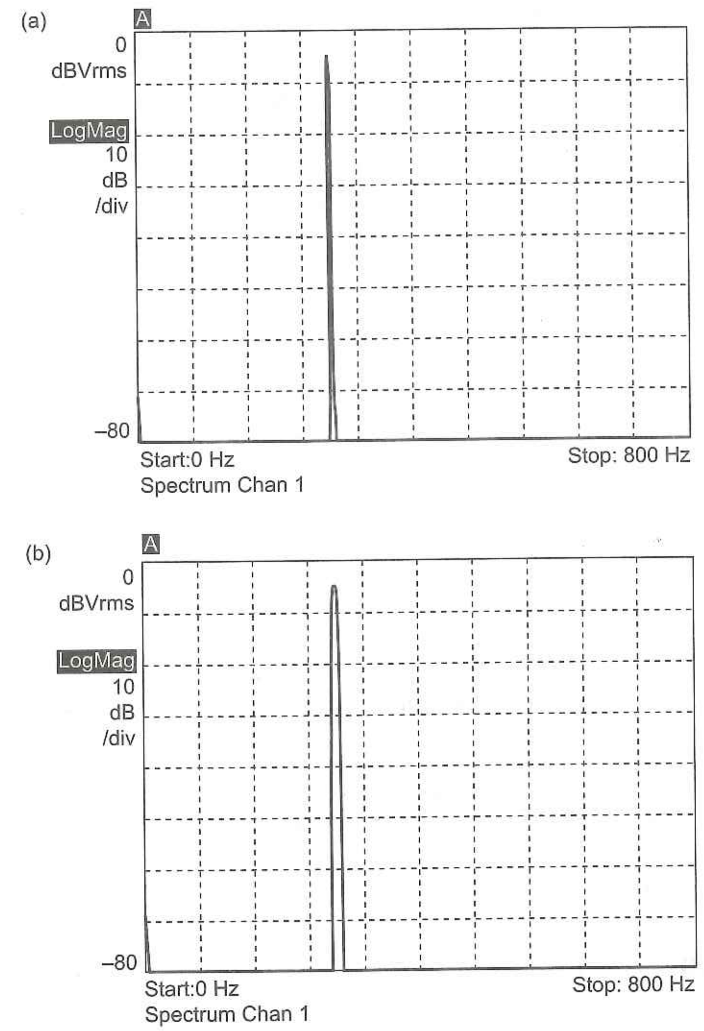

Figure 4-20 (a) Sine wave spectrum using the Hanning window. (b) Sine wave spectrum using the flattop window.

4.13 Exponential Window

One of the advantages of an FFT analyzer is that it can be used to measure the frequency content of a fast transient. (This is not usually possible in the more conventional swept analyzer since it may miss some of the transient as it is sweeping through its frequency span.) Such a transient might be the step or impulse response of an electrical network or mechanical system.

A typical transient response is shown in Figure 4-21a. As shown, the waveform is self-windowing because it dies out within the length of the time record, reducing the leakage. If the waveform does not dissipate within the time record (as shown in Figure 4-21b), then a

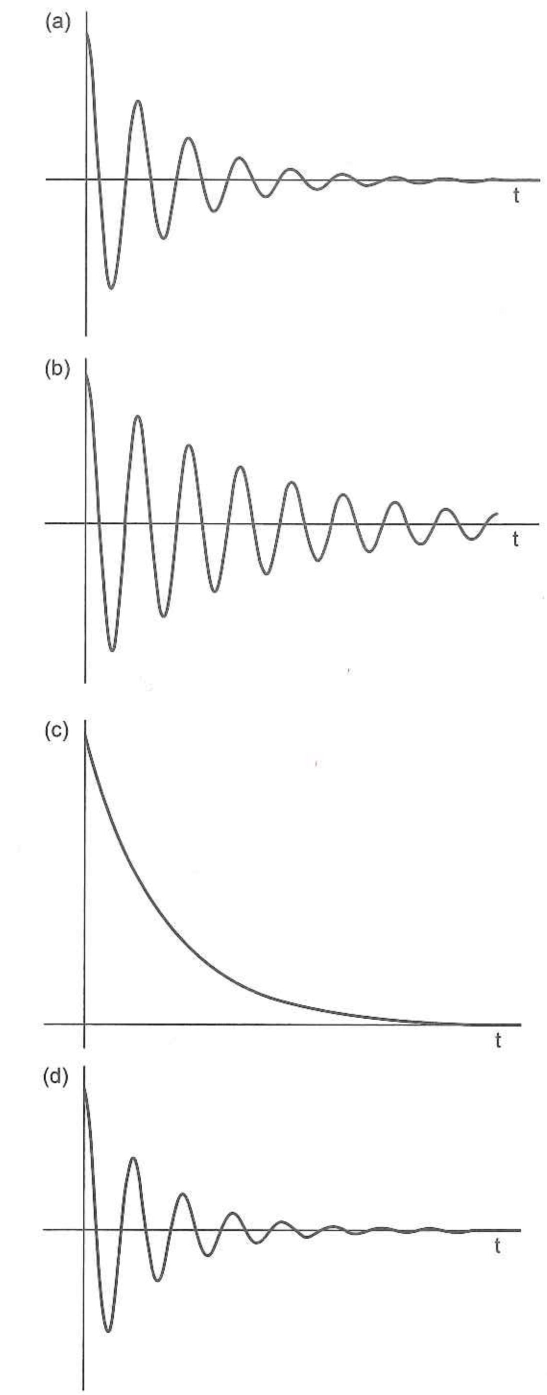

Figure 4-21 (a) A transient response that is self-windowing. (b) A transient response that requires windowing. (c) The exponential window. (d) The windowed transient response.