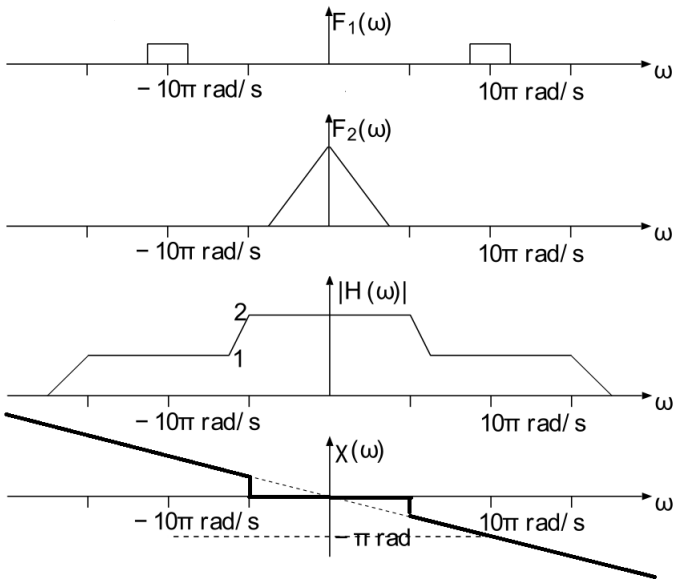
**ECE210 / ECE211 -** Homework 10

**Due** : Wednesday, November 7 at 6 p.m.

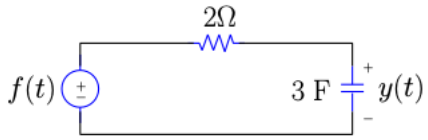
1. Let $f(t) = f_1(t) + f_2(t)$ such that $f_1(t) \leftrightarrow F_1(\omega)$ and $f_2(t) \leftrightarrow F_2(\omega)$, and let $H(\omega) = |H(\omega)|e^{j\chi(\omega)}$. The functions $F_1(\omega)$, $F_2(\omega)$, $H(\omega)$ and $\chi(\omega)$ are given graphically below. The signal $f(t)$ is the input to an LTI system with a frequency response $H(\omega)$. Express the output $y(t)$ of the system as a superposition of scaled and/or shifted versions of $f_1(t)$ and $f_2(t)$.




2. Consider the circuit shown below.

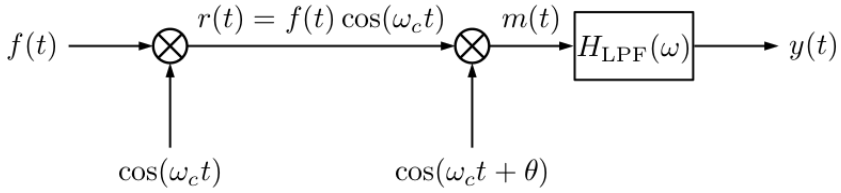
   (a) Consider an arbitrary input $f(t)$ and determine the response, $y(t)$, in the form of an inverse Fourier transform.

   (b) Evaluate $y(t)$ for the case $f(t) = e^{-\frac{t}{6}}u(t)\,\mathrm{V}$.




3. Given that $f(t)e^{\pm j\omega_0 t} \leftrightarrow F(\omega \pm \omega_0)$, determine the Fourier transform of $g(t) = f(t)\sin(\omega_o t)$ in terms of scaled and/or shifted versions of $F(\omega)$.

4. A signal $f(t)$ is bandlimited to the interval $\omega \in [-\Omega, \Omega]$ and modulated by a cosine carrier of frequency $\omega_c > \Omega$ . The resulting modulated signal $r(t)$ is then coherently demodulated with a mismatched carrier signal $\cos(\omega_c t + \theta)$, and filtered with an ideal low pass filter $H_{\mathrm{LPF}}(\omega) = \mathrm{rect}\left(\frac{\omega}{2\Omega}\right)$ as shown in the figure below.




   (a) Find an expression for $y(t)$ in terms of $f(t)$ and $\theta$.

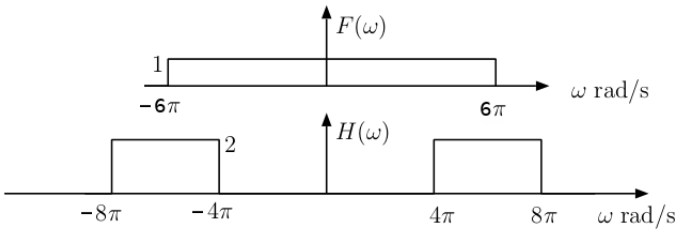
   (b) For what values of $\theta$ is the amplitude of $y(t)$ smallest and largest?

   (c) Consider what would happen when $\theta$ is slowly time varying (e.g. $\theta = \Delta\omega t$, where $0 < \Delta\omega \ll \Omega$). If you were to play $y(t)$ on a loudspeaker, what qualitative effect with this have on the signal you hear?

5. A linear system with frequency response $H(\omega)$ is excited with an input

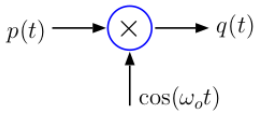
$$f(t) \leftrightarrow F(\omega).$$

$H(\omega)$ and $F(\omega)$ are plotted below:




(a) Sketch the Fourier transform $Y(\omega)$ of the system output $y(t)$ and calculate the energy $W_y$ of $y(t)$.

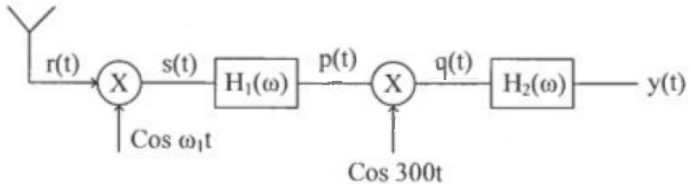
(b) It is observed that output $q(t)$ of the following system equals $y(t)$ determined in part (a).

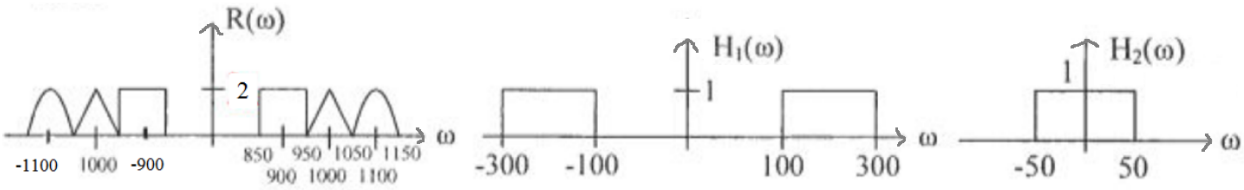



Sketch $P(\omega)$ and determine $\omega_o$.

(c) Express $y(t)$ in terms of $f(t)$.

6. Consider the system below, where $\omega_1 = 1,200$rad/s, $R(\omega), H_1(\omega)$, and $H(\omega)$ are plotted below the system figure.




(a) Sketch the spectra $S(\omega), P(\omega), Q(\omega)$,and $Y(\omega)$.

(b) Calculate the energy of $y(t)$.