University of Illinois **Fall 2018**

ECE210 / ECE211 - Homework 14

Due: Wednesday, December 12 @ 6pm

1. For each of the following Laplace transforms $\hat{F}(s)$, determine the inverse Laplace transform $f(t)$.

   (a) $\hat{F}(s) = \frac{s+3}{(s+2)(s+4)}$

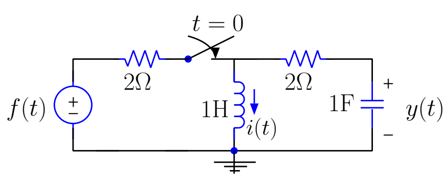
   (b) $\hat{F}(s) = \frac{1}{s(s-5)^2}$

   (c) $\hat{F}(s) = \frac{s^2+2s+1}{(s+1)(s+2)}$

2. Determine the transfer functions $\hat{H}(s)$ and the zero-state response for the LTIC system described by the following ODE:
$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = f(t),$$
where $f(t) = e^{3t}u(t)$.

3. Take the Laplace transform of the following ODE to determine $\hat{Y}(s)$ assuming $f(t) = u(t)$, $y(0^-) = 1$, and $y'(0^-) = 0$. Determine $y(t)$ for $t > 0$ by taking the inverse Laplace transform of $\hat{Y}(s)$.
$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y(t) = \frac{df}{dt} + 2f(t).$$

4. Consider the circuit:

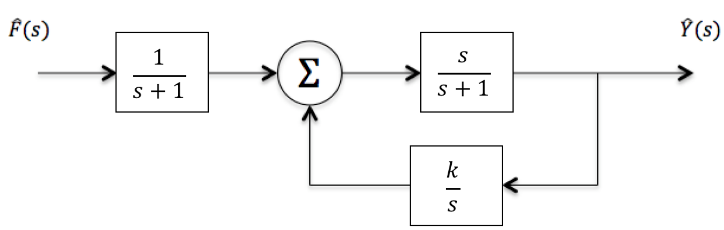


   (a) Show that the transfer function of the circuit for $t > 0$ is $\hat{H}(s) = \frac{\hat{Y}(s)}{\hat{F}(s)} = \frac{s}{4s^2+5s+2}$.

   (b) What are the characteristic modes of the circuit?

   (c) Determine $y(t)$ for $t > 0$ if $f(t) = 1$ V, $y(0^-) = 1$ V, and $i(0^-) = 0$.

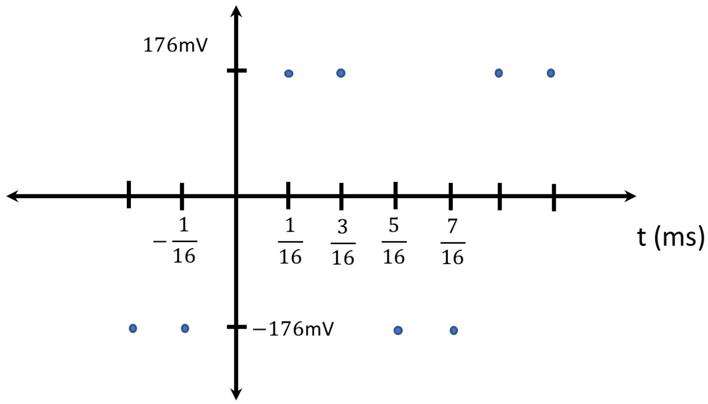
5. (a) Determine the transfer function $\hat{H}(s)$ of the system shown below.



   (b) When $k = 2$, determine whether the system is BIBO stable or not.

   (c) Which values of k can you have so that the system is BIBO stable?

6. Lab 5 is about signal sampling and reconstruction. Let's review a part about what we did there in this question: Part 2.1.6, we had a 2kHz sinusoid wave with amplitude 250 mV. Let's take 4 samples each period, thus our sampling frequency is 8kHz (4 samples per period), what you have in your computer hard drive is shown in the following graph.



(a) Do you think it's possible to get your original 2kHz sine wave back from these dots, as shown in the graph?

(b) Please explain step-by-step how you can reconstruct your original signal from these 4 dots per period by drawing the resulting signal. (those 4 points per period is all you have).

(c) According to Nyquist's sampling principle, as long as you have more than two dots per period, you should be able to get your original sine wave back. If you didn't get a perfect sine wave in part (b), try again.