

ECE210 / ECE211 - Homework 12

Due : Wednesday, November 28 at 6 p.m.

1. Consider a poor quality LPF with frequency response $H(\omega) = \frac{1}{1+j\omega}$:
 - (a) Plot or sketch $|H(\omega)|$.
 - (b) Plot the response to $\cos(t/2)$ on the same plot as $|H(\omega)|$.
 - (c) Plot the response to $\cos(2t)$ on the same plot as $|H(\omega)|$.
 - (d) In what sense are the responses in parts (b) and (c), samples of $H(\omega)$?
 - (e) Is $H(\omega)$ a reconstruction from samples? Why?
 - (f) How is $H(\omega)$ related to both the Fourier transform and the Fourier series of the output?
2. A system is described by an impulse response $h(t) = \delta(t - \frac{1}{2}) - \delta(t + \frac{1}{2})$. Sketch the system response $y(t) = h(t) * f(t)$ to the following inputs:
 - (a) $f(t) = u(t)$
 - (b) $f(t) = \text{rect}(t)$
3. Determine the Fourier transform of the following signals-simplify the results as much as you can. For parts (a), (b) and (c), sketch the magnitude and phase of the result.
 - (a) $f(t) = 5 \cos(3t) + 2 \sin(10t)$.
 - (b) $x(t) = 2 \cos^2(2t)$.
 - (c) $y(t) = e^{-t}u(t) * \cos(4t)$.
 - (d) $z(t) = (1 + \cos(2t))e^{-t}u(t)$
4. Determine the inverse Fourier transforms of the following:
 - (a) $F(\omega) = j[\delta(\omega - 2) - \delta(\omega + 2)] + 8\delta(\omega - 4)$.
 - (b) $A(\omega) = 3\pi \cos(2\omega)$.
 - (c) $B(\omega) = \sum_{n=-\infty}^{\infty} \frac{3}{1+n^2} \delta(\omega - n)$
 - (d) $C(\omega) = \frac{8}{2+j\omega} + 4\pi\delta(\omega)$
5. Consider the following zero-state input-output relations for a variety of systems. In each case, determine whether the system is zero-state linear, time invariant, and causal. You must show work to prove each property.
 - (a) $y(t) = f(t - 1) + f(-t - 1)$
 - (b) $y(t) = f(2t)$.
 - (c) $y(t) = 5f(t) * u(t)$.
 - (d) $y(t) = \delta(t - 4) * f(t) - \int_{-\infty}^{t+2} f^2(\tau) d\tau$.
 - (e) $y(t) = \int_{-\infty}^{t-2} f(\tau) d\tau$. (Hint: is it possible to write this as a convolution?)
6. Find the impulse responses $h(t)$ of the LTI systems having the following unit-step responses.
 - (a) $g(t) = 5u(2t - 5)$.
 - (b) $g(t) = t^3u(t)$.
 - (c) $g(t) = (2 - e^{-t})u(t - 5)$.
7. Determine the minimum sampling frequencies ω_s needed to sample the following analog signals without causing aliasing error.
 - (a) Arbitrary signal $f(t)$ with bandwidth 20 kHz.
 - (b) $f_1(t) = \text{sinc}(4000\pi t)$.
 - (c) $f_2(t) = \text{sinc}^2(4000\pi t)$. Compare this sampling frequency to the one in part (b).
 - (d) $f_3(t) = \text{sinc}(4000\pi t) \cos(12000\pi t)$. Compare this sampling frequency to the one in part (b).