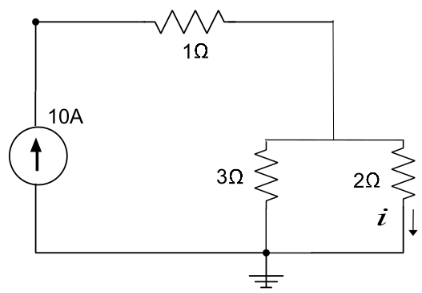
ECE210 / ECE211 - Homework 01

Due: Wednesday, September 5 @ 6pm.

Solution

1. Consider the following circuit. Obtain the current i and the power absorbed by the 2Ω resistor.



Solution:

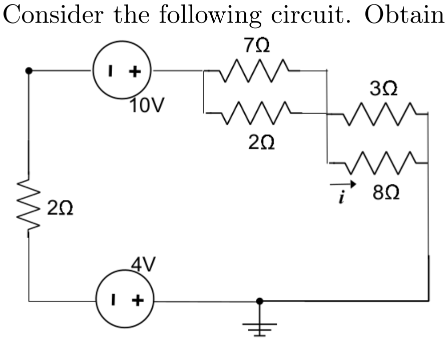
Using current divider ($2\Omega$and 3Ω are in parallel)

$$i = 10 \times \frac{3}{3+2} = 6A$$

Correspondingly, the power absorbed by 2Ω is

$$P = i^2 R_{2\Omega} = 6^2 \times 2 = 72W$$

2. Consider the following circuit. Obtain the current i.



Solution:

Since $7\Omega \,||\, 2\Omega$ and $3\Omega \,||\, 8\Omega$, calculate equivalent resistance these two parallel resistor pairs:

$$7\Omega \,||\, 2\Omega = \frac{7 \times 2}{7+2} = \frac{14}{9}\Omega,$$

$$3\Omega \,||\, 8\Omega = \frac{8 \times 3}{8+3} = \frac{24}{11}\Omega.$$

Thus, after parallel simplification, the resistors are in series. The overall resistance R

$$R \quad = \quad (3\Omega \,||\, 8\Omega) + (7\Omega \,||\, 2\Omega) + 2\Omega = \frac{568}{99}\Omega$$

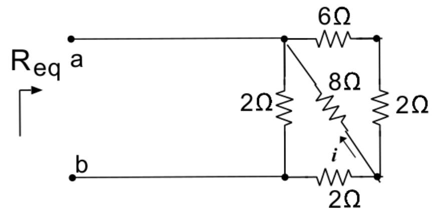
For the resistors which are in series, voltage divider is possible,

$$V_{3\Omega \,||\, 8\Omega} = (10-4) \times \frac{\frac{24}{11}}{\frac{14}{9} + \frac{24}{11} + 2} = 6 \times \frac{27}{71} \approx 2.28V$$

Therefore,

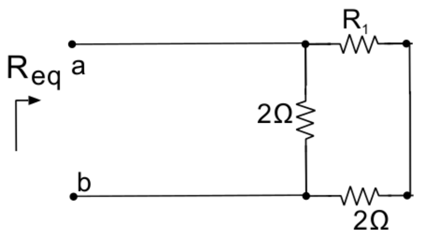
$$i = \frac{V_{3\Omega \,||\, 8\Omega}}{8\Omega} = 0.285A.$$

3. Consider the following circuit with $v_{ab} = 10$V. Obtain the current i and the equivalent resistance R_{eq}.



Solution:

Because $(6+2)\Omega \,||\, 8\Omega$, we define the equivalent resistor R_1 as follow,



The equivalent resistance can be calculated,

$$R_1 = R_{(6+2)\Omega\,||\,8\Omega} = \frac{8 \times 8}{8+8}\Omega = 4\Omega,$$

Therefore, the overall resistance is

$$R_{eq} = (R_1 + 2\Omega)\,||\,2\Omega = \frac{6 \times 2}{6+2}\Omega = \frac{3}{2}\Omega$$

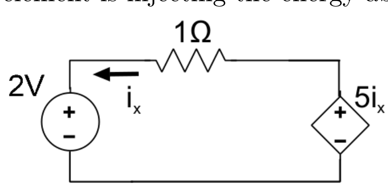
Given $v_{ab} = 10V$, we can find the voltage across the 8Ω (equivalently R_1) by voltage divider

$$V_{8\Omega} = V_{R_1} = 10 \times \frac{R_1}{R_1 + 2} = 10 \times \frac{2}{3}V = \frac{20}{3}V$$

Be careful about the direction. Since $V_{ab} = V_a - V_b = 10V$, current is then flowing downward which is opposite to direction of the label i. Therefore, the current is

$$i = -\frac{V_{8\Omega}}{8\Omega} = -\frac{5}{6}A \approx -0.83A.$$

4. Consider the circuit below. Determine i_x and calculate the absorbed power for each circuit element. Which element is injecting the energy absorbed in the circuit?.



Solution:

Apply KVL

$$5i_x - 1 \times i_x - 2 = 0,$$

Solve for i_x

$$i_x = \frac{1}{2}A$$

Calculate the absorbed power for independent voltage source

$$P_{2V} = Vi_x = (2V)(\frac{1}{2}A) = 1W \text{ (absorbed)}$$

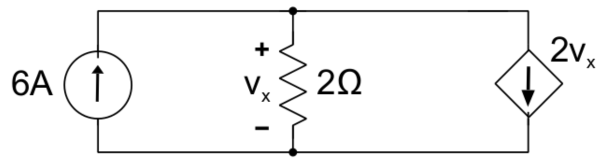
Calculate the absorbed power for resistor

$$P_{1\Omega} = i_x^2 R = (\frac{1}{2}A)^2 \times 1\Omega = \frac{1}{4}W \text{ (absorbed)}$$

Calculate the absorbed power for dependent voltage source

$$P_{5i_x} = (5i_x)(-i_x) = (\frac{5}{2}V)(-\frac{1}{2}A) = -\frac{5}{4}W \text{ (injecting)}$$

5. Consider the circuit below. Determine v_x and calculate the absorbed power for each circuit element. Which element is injecting the energy absorbed in the circuit?.



Solution:

Denote the top node as A, and the node voltage is V_x. Apply KCL,

$$6 - \frac{V_x}{2} - 2V_x = 0,$$

Solve for V_x

$$V_x = \frac{12}{5}V = 2.4V$$

Calculate the absorbed power for independent current source

$$P_{6A} = V_x \times (-6A) = -(2.4V)(6A) = -14.4W \text{ (injects energy)}$$

Calculate the absorbed power for resistor

$$P_{2\Omega} = \frac{V_x^2}{R} = \frac{2.4^2}{2} = 2.88W \text{ (absorbs energy)}$$

Calculate the absorbed power for dependent current source

$$P_{2V_x} = (2V_x)(V_x) = 2 \times 2.4^2 = 11.52W \text{ (absorbs energy)}$$

Therefore, the independent current source is injecting energy in the circuit, and the power is conserved.

6. This problem relates to complex numbers.
   (a) Let $A = 3 - j3$. Express A in exponential form.
   (b) Let $B = -1 - j$. Express B in exponential form.
   (c) Determine the magnitudes of $A + B$ and $A - B$.
   (d) Express AB and A/B in rectangular form.

Solution:

(a) Exponential form can be written as $A = Re^{j\theta}$, where $R = \sqrt{3^2 + (-3)^2} = 3\sqrt{2}$, and $\hat{\theta} = \tan^{-1}(\frac{-3}{3}) = -\frac{\pi}{4}$. Note that A is in the $4th$ quadrant, therefore $-\frac{\pi}{2} < \theta < 0$, $\theta = \hat{\theta}$:

$$A = Re^{j\theta} = 3\sqrt{2}e^{-j\frac{\pi}{4}}$$

(b) Similarly in (a), we can calculate $R = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$, $\hat{\theta} = \tan^{-1}(\frac{-1}{-1}) = \frac{\pi}{4}$. Since B is in $3rd$ quadrant, $-\pi < \theta < -\frac{\pi}{2}$, $\theta = \hat{\theta} - \pi = -\frac{3\pi}{4}$:

$$B = \sqrt{2}e^{-j\frac{3\pi}{4}}$$

(c) When calculating $A+B$ or $A-B$, it is more convenient to use rectangular form. Suppose $A = X_A + jY_A$, $B = X_B + jY_B$, $A \pm B = (X_A \pm X_B) + (Y_A \pm Y_B)j$.
Since $A + B = (3-1) + (-3-1)j = 2 - 4j$, we have

$$|A + B| = \sqrt{2^2 + (-4)^2} = 2\sqrt{5}$$

Similarly, $A - B = (3+1) + (-3+1)j = 4 - 2j$, thus

$$|A - B| = \sqrt{(-2)^2 + 4^2} = 2\sqrt{5}$$

(d) When calculating AB or A/B, it is more convenient to use exponential form. Suppose $A = R_A e^{j\theta_A}$ and $B = R_B e^{j\theta_B}$, $AB = (R_A \times R_B)e^{j(\theta_A + \theta_B)}$, $\frac{A}{B} = (R_A/R_B)e^{j(\theta_A - \theta_B)}$.
Since $AB = (3\sqrt{2} \times \sqrt{2})e^{j(-\frac{3\pi}{4} - \frac{\pi}{4})} = 6e^{-\pi j} = 6e^{\pi j}$, rectangular form is

$$AB = -6$$

Similarly, $A/B = \frac{3\sqrt{2}}{\sqrt{2}}e^{j(+\frac{3\pi}{4} - \frac{\pi}{4})} = 3e^{\frac{\pi}{2}j}$, rectangular form is

$$AB = 3j$$

7. This problem relates to complex numbers.
   (a) Determine the rectangular forms of e^{j0}, $e^{j\frac{\pi}{2}}$, $e^{-j\frac{\pi}{2}}$, $e^{j\pi}$, $e^{-j\pi}$, and $e^{2j\pi}$.
   (b) Simplify $P = e^{j\pi} + e^{-j\pi}$, $Q = e^{j\frac{\pi}{2}} + e^{-j\frac{\pi}{2}}$ and $R = 1 - e^{j\pi}$.

Solution:

(a) Convert exponential form $A = Re^{j\theta}$ to rectangular form $A = X + jY$: $X = R\cos(\theta)$, $Y = R\sin(\theta)$. Therefore, we have

$$\begin{aligned} e^{j0} &= 1 \\ e^{j\frac{\pi}{2}} &= j \\ e^{-j\frac{\pi}{2}} &= -j \\ e^{j\pi} &= -1 \\ e^{-j\pi} &= -1 \\ e^{2j\pi} &= 1 \end{aligned}$$

(b) Based on results in (a), we have

$$\begin{array}{rcccl} P &=& e^{j\pi} + e^{-j\pi} &=& -1 - 1 = -2 \\ Q &=& e^{j\frac{\pi}{2}} + e^{-j\frac{\pi}{2}} &=& j - j = 0 \\ R &=& 1 - e^{j\pi} &=& 1 - (-1) = 2 \end{array}$$