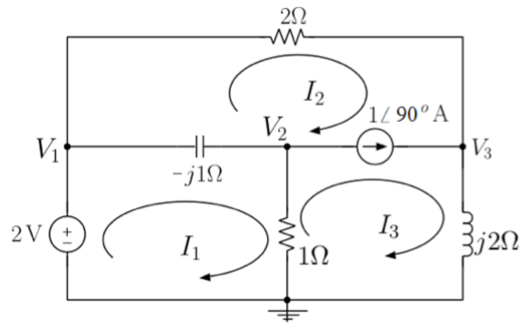
University of Illinois **Fall 2018**

ECE210 / ECE211 - Homework 6 Solution

1. In the following circuit determine the node-voltage phasors V_1, V_2, and V_3 and express them in polar form.



Solution:

$$\mathbf{V_1 = 2}\ V = \mathbf{2\angle 0}\ V$$

Solve for V_2 by solving the node equation at node V_2:

$$\frac{V_1 - V_2}{-j} = \frac{V_2}{1} + j$$

$$j(V_1 - 1) = V_2(1 + j)$$

$$j = V_2(1 + j)$$

$$V_2 = \frac{j}{1 + j}$$

$$\mathbf{V_2} = \frac{\mathbf{\sqrt{2}}}{\mathbf{2}}\mathbf{e^{j\frac{\pi}{4}}}\ V = \frac{\mathbf{\sqrt{2}}}{\mathbf{2}}\angle\frac{\pi}{\mathbf{4}}\ V$$

Solve for V_3 by solving the node equation at node V_3:

$$\frac{V_1 - V_3}{2} + j = \frac{V_3}{j2}$$

$$j(V_1 - V_3) - 2 = V_3$$

$$j2 - 2 = V_3(1 + j)$$

$$V_3 = \frac{j2 - 2}{1 + j} = \frac{j4}{2} = 2e^{j\frac{\pi}{2}}$$

$$\mathbf{V_3 = 2e^{j\frac{\pi}{2}}}\ V = \mathbf{2}\angle\frac{\pi}{\mathbf{2}}\ V$$

2. In the circuit shown for Problem 1, determine the loop-current phasors I_1, I_2, and I_3 and express them in polar form.

Solution:

Node loop equations:

(1) $j + I_2 = I_3$

(2) $2 - (-j)(I_1 - I_2) - 1(I_1 - I_3) = 0$

(3) $2 - 2I_2 - j2I_3 = 0$

Add 2*(1) to (3)

$$2 + j2 - I_3(2 + j2) = 0$$
$$\mathbf{I_3 = 1}\ A = \mathbf{1\angle 0}\ A$$

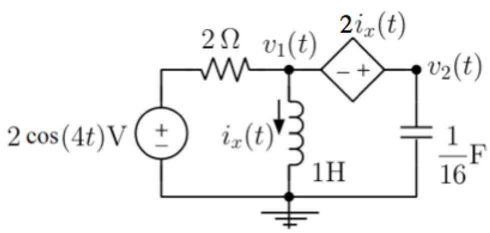
Plug I_3 into (1)

$$I_2 = 1 - j$$
$$\mathbf{I_2 = \sqrt{2}e^{-j\frac{\pi}{4}}}\ A = \mathbf{\sqrt{2}\angle\frac{-\pi}{4}}\ A$$

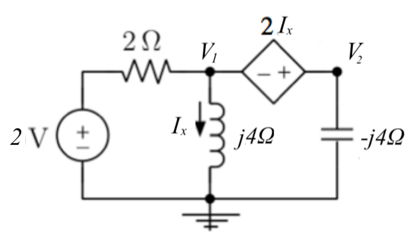
Plug I_2 and I_3 into (2)

$$2 + j(I_1 - 1 + j) - (I_1 - 1) = 0$$
$$2j + 1 = I_1(1 + j)$$
$$I_1 = \frac{2j + 1}{1 + j} = \frac{j + 3}{2}$$
$$\mathbf{I_1 = \frac{\sqrt{10}}{2}e^{j\tan^{-1}(1/3)}}\ A = \mathbf{\frac{\sqrt{10}}{2}\angle\tan^{-1}(1/3)}\ A$$

3. Use the phasor method to determine $v_1(t)$ in the following circuit:



Solution: In Phasor form:



Equations:

(1) $\frac{2-V_1}{2} = \frac{V_1}{j4} + \frac{V_2}{-j4}$

(2) $V_1 + 2I_x = V_2$

(3) $I_x = \frac{V_1}{j4}$

Plug (2) into (1):

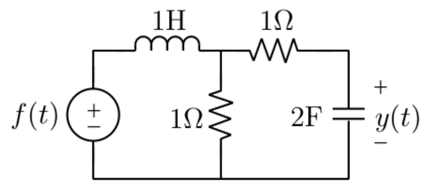
$$\frac{2 - V_1}{2} = \frac{V_1}{j4} + \frac{V_1 + 2I_x}{-j4}$$

$$j2(2 - V_1) = V_1 - V_1 - 2I_x$$

Plug in value of I_x from (3)

$$
\begin{aligned}
j4 - j2V_1 &= -2\frac{V_1}{j4} \\
-16 + 8V_1 &= -2V1 \\
\frac{8}{5} &= V_1 \\
\mathbf{v_1(t)} &= \mathbf{\frac{8}{5}cos(4t)}\ V
\end{aligned}
$$

4. In the following circuit, the input is $f(t) = 4 - \cos(2t)$. Determine the steady-state output $y(t)$ of the circuit.



Solution: The input has two frequencies, $\omega = 0$ rad/s and $\omega = 2$ rad/s, so we have to use linearity to get

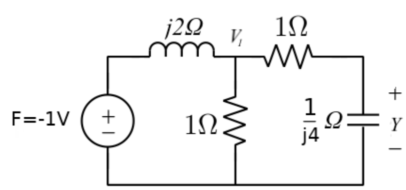
$$y(t) = y_0(t) + y_2(t)$$

For $\omega = 0$ rad/s, in steady-state the inductor acts like a short and the capacitor as an open circuit, so that

$$y_0(t) = 4\ V$$

.

For $\omega = 2$ rad/s, the circuit in phasor for becomes



(1) $Y = V_1 \frac{\frac{1}{j4}}{1+\frac{1}{j4}}$ (Voltage Divider)

(2) $\frac{-1-V_1}{j2} = \frac{V_1}{1} + \frac{V_1-Y}{1}$

Re-work (2):

$$
\begin{aligned}
\frac{-1-V_1}{j2} &= 2V_1 - Y \\
-1 - V_1 &= j4V_1 - j2Y \\
j2Y - 1 &= (j4+1)V_1
\end{aligned}
$$

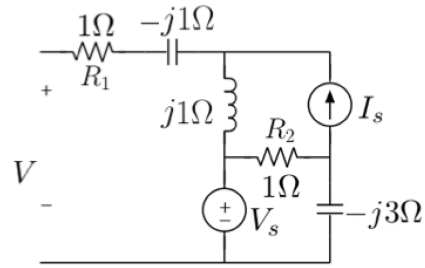
(1) can be re-written as: $Y(j4+1) = V_1$

Plug in (1):

$$
\begin{aligned}
j2Y - 1 &= (j4+1)(j4+1)Y \\
j2Y - 1 &= (-15+j8)Y \\
-1 &= (-15+j6)Y \\
Y &= \frac{1}{15-j6} \\
y_2(t) &= 0.06cos(2t+0.38)\ V
\end{aligned}
$$

$$\mathbf{y(t) = y_0(t) + y_2(t) = 4 + 0.06cos(2t+0.38)}\ V$$

5. Consider the network below:



(a) Determine the phasor V when $I_s = 0$.
Solution: $\mathbf{V} = \mathbf{V_s}\ V$

(b) Determine the phasor V when $Vs = 0$.
Solution: $\mathbf{V} = \mathbf{jI_s}\ V$

(c) Determine V when $Vs = 4V$ and $Is = 2A$, and calculate the average power absorbed in the resistors.
Solution: $\mathbf{V} = \mathbf{4 - j2}\ V$

$$V_{R1} = 0\ V$$
$$\mathbf{P_{R1}} = \mathbf{0}\ W$$

$$\frac{V_s - V_1}{1} = I_s + \frac{V_1}{-j3}$$
$$\frac{4 - V_1}{1} = 2 + \frac{V_1}{-j3}$$
$$V_1 = \frac{6}{3+j}$$

$$V_{R2} = V_S - V_1 = 4 - \frac{6}{3+j} = \frac{6+4j}{3+j}$$
$$P_{R2} = \frac{|V_{R2}|^2}{2R_2} = \frac{\frac{52}{10}}{2}$$
$$\mathbf{P_{R2}} = \frac{\mathbf{13}}{\mathbf{5}} = \mathbf{2.6}\ W$$

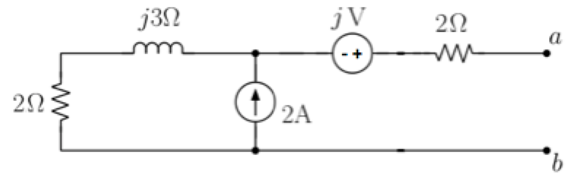
(d) What is the Thevenin equivalent and the available average power of the network when $Vs = 4V$ and $Is = 2A$?
Solution:

$$\mathbf{V_T} = \mathbf{4 - j2}\ V$$
$$\mathbf{Z_T} = \mathbf{1}\ \Omega$$
$$P_a = \frac{|V_T|^2}{8R_T} = \frac{20}{8}$$
$$\mathbf{P_a} = \mathbf{2.5}\ W$$

6. Determine the impedance Z_L of a load that is matched to the following network at terminals a and b, and determine the net power absorbed by the matched load.

Solution:

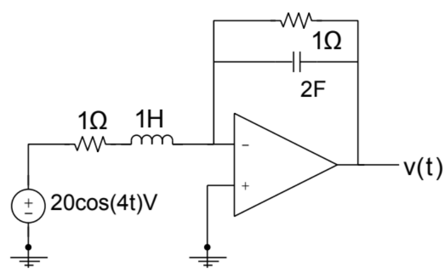
$$V_1 = 2(2 + j3) = 4 + j6\ V$$
$$V_T = V_1 + j = 4 + j7\ V$$
$$Z_T = 4 + j3\ \Omega$$



$$\mathbf{Z_L = Z_T^* = 4 - j3}\ \Omega$$

$$P_a = \frac{|V_T|^2}{8R_T} = \frac{65}{32}$$
$$\mathbf{P_a = 2.03}\ W$$

7. Use the phasor method to determine the steady-state voltage $v(t)$ in the following op-amp circuit. Please use ideal op-amp approximations.



Solution:

$$\frac{20}{1+j4} = \frac{-V}{(1+j8)^{-1}}$$
$$\frac{20}{1+j4} = -V(1+j8)$$
$$V = \frac{-20}{(1+j4)(1+j8)} = \frac{-20}{-31-j12}$$

$$\mathbf{v(t) = 0.6cos(4t + 0.37)}\ V$$