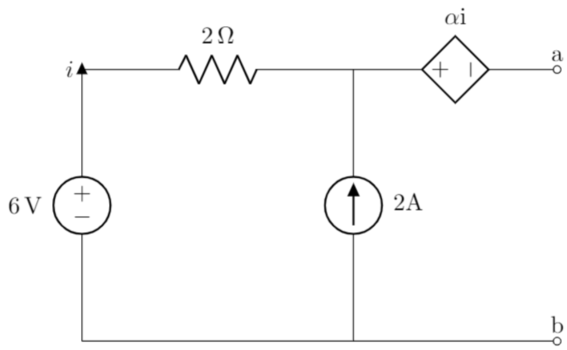
ECE210 / ECE211 - Homework 03

Solutions

1. Determine the Thevenin and Norton equivalent circuits of the following network between nodes a to b and then determin the available power of the network. **Note**: your answers can be left in terms of α.



Solution:

To get the open-circuit voltage we analyze the circuit as shown in the problem statement. Applying KCL at the upper central node, we obtain

$$i + 2 = 0$$

which gives

$$i = -2\text{A}.$$

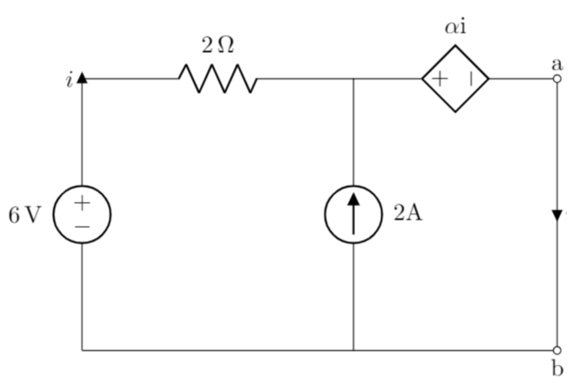
Then we can apply the KVL for the outer loop and the equation can be written as:

$$2i + \alpha i + v_{oc} = 6,$$

which gives

$$v_{oc} = 10 + 2\alpha\text{V}.$$

To get the short-circuit current we analyze the following figure:



By applying the KVL for the outer loop, we could get:

$$6 = 2i + \alpha i,$$

which gives

$$i = \frac{6}{2+\alpha}\text{A}.$$

Nodal equation could be written as:

$$i_N = i + 2,$$

which gives

$$i_N = \frac{10+2\alpha}{2+\alpha}\text{A}.$$

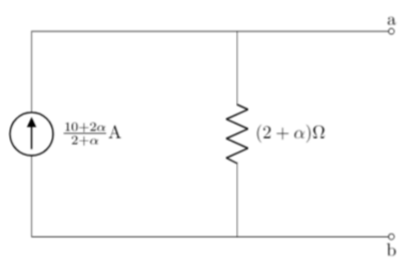
The equivalent resistance can be calculated as

$$R_{eq} = \frac{v_{oc}}{i_{sc}} = \frac{10+2\alpha}{\frac{10+2\alpha}{2+\alpha}} = (2+\alpha)\Omega,$$

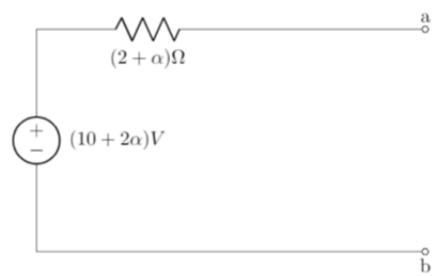
The available power is given as follow:

$$P_a = \frac{v_T^2}{4R_T} = \frac{(10+2\alpha)^2\text{V}}{4\times(2+\alpha)A} = \frac{(10+2\alpha)^2}{4(2+\alpha)}\text{W}.$$

As a result, the Norton and Thevenin equivalent circuits can be drawn as shown below:

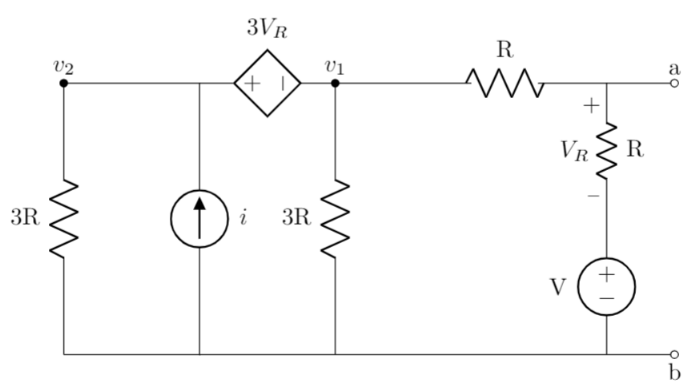


(a) Norton equivalent circuit



(b) Thevenin equivalent circuit

2. Determine the Thevenin equivalent of the following network between nodes a and b, and then determine the available power of the network. **Note**: your answers can be left in terms of V, R, i.



Solution:

To get the open-circuit voltage we analyze the circuit as shown in the problem statement. Let v_1 denote the voltage across the right $3R$ resistor. Then the voltage across the left $3R$ resistor v_2 is $v_1 + 3V_R$. By applying the KCL at node v_1,

$$\frac{v_1 + 3V_R}{3R} - i + \frac{v_1}{3R} + \frac{v_1 - v_{oc}}{R} = 0,$$

Also, for the independent voltage source, we have

$$V_R = v_{oc} - V,$$

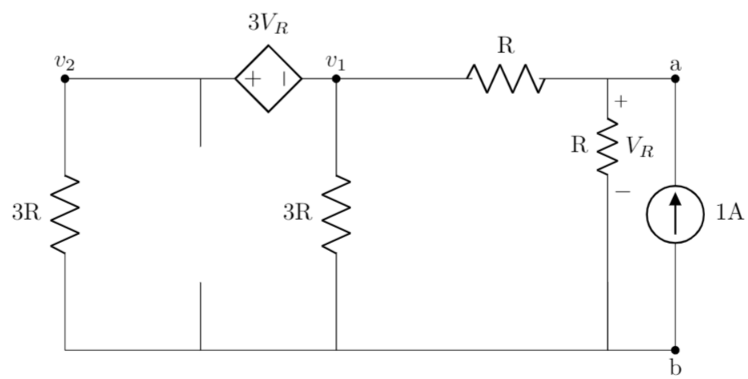
By applying the KCL at the node which connects the two R resistor, we have

$$\frac{v_1 - v_{oc}}{R} = \frac{V_R}{R},$$

The three equations give us:

$$v_{oc} = \frac{3R \times i + 8V}{10}.$$

In order to find the equivalent resistance, we add a 1A current to the right of the circuit and suppress the independent sources:



The voltage across the test current source is $v_{ab} = V_R$. The voltage across the left $3R$ is $3V_R + v_1$. By applying KCL at node v_1, we have:

$$\frac{v_1 + 3V_R}{3R} + \frac{v_1}{3R} + \frac{v_1 - V_R}{R} = 0,$$

also, by applying the KCL at node a, we have:

$$1 + \frac{v_1 - V_R}{R} = \frac{V_R}{R}.$$

Since $v = V_R$, by combining the two equations, we have:

$$v = \frac{1}{2}R.$$

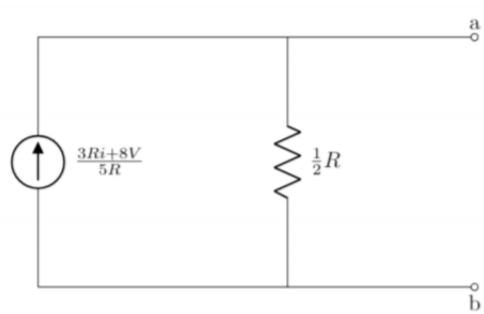
Thus,

$$R_{eq} = \frac{v}{1\text{A}} = \frac{1}{2}R.$$

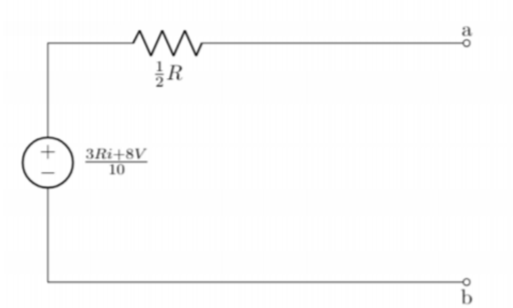
and Norton current is:

$$i_N = \frac{v_{oc}}{R_{eq}} = \frac{3Ri + 8V}{5R}.$$

The Thevenin equivalent circuit is:



(a) Norton equivalent circuit

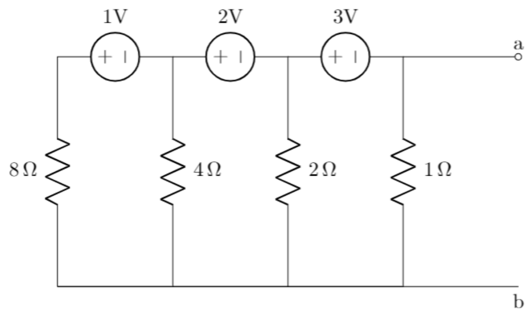


(b) Thevenin equivalent circuit

As a result, the available power is:

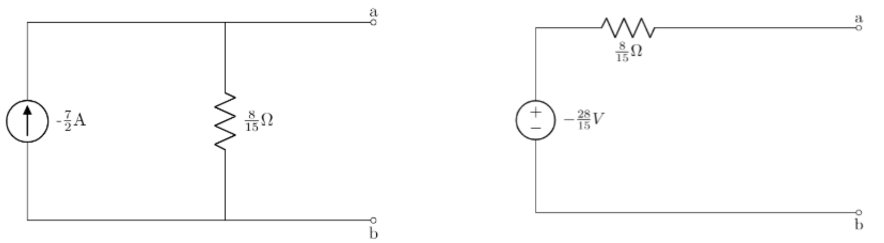
$$p_{\text{av}} = \frac{v_{oc}^2}{4R_{TH}} = \frac{(3R \times i + 8V)^2}{200R}.$$

3. Determine the Thevenin and Norton equivalent circuits of the following network between nodes a to b and then determin the available power of the network.



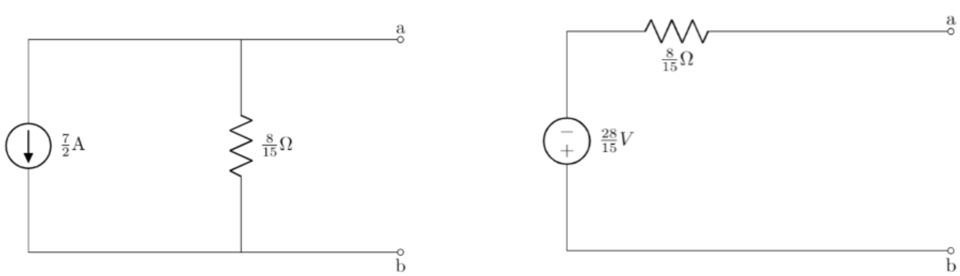
Solution:

In this problem, we apply the source transformation multiple times to simplify the circuit above. The equivalent circuit will be:



(a) Norton equivalent circuit (b) Thevenin equivalent circuit

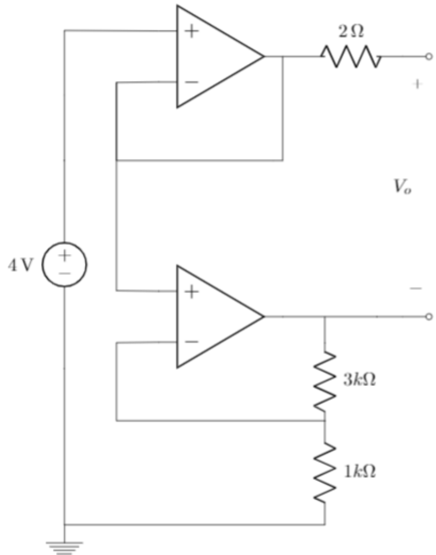
Then, we could simply get the Thevenin voltage and Norton current by just flipping the signs of the power sources:



The available power is:

$$P_a = \frac{V_T^2}{4 \times R_T} = \frac{49}{30}\text{W}.$$

4. Determine the voltage V_o in the following circuit, assuming linear operation.



Solution:

Considering the ideal op-amp assumptions, we have $v^+ = v^- = 4\text{V}$ for both op-amps. At the bottom of the circuit we identify a non-inverting amplifier with a gain of

$$G = 1 + \frac{3\text{k}\Omega}{1\text{k}\Omega} = 4,$$

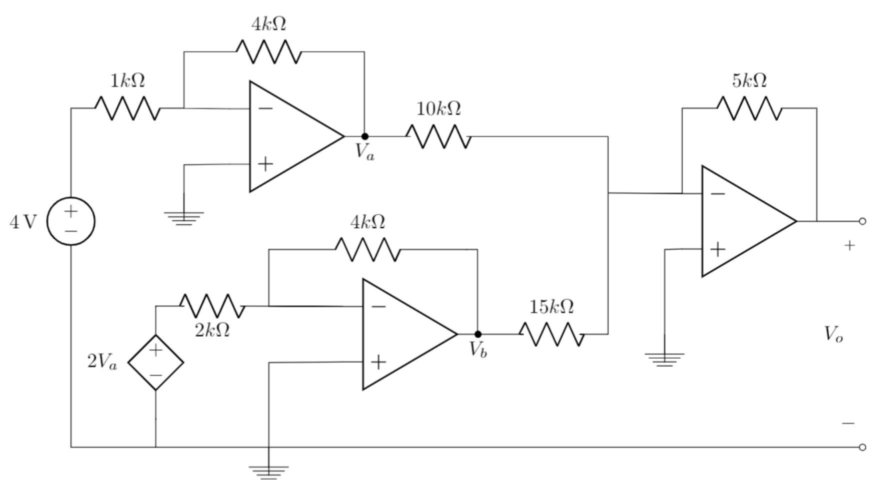
with the output

$$v_{bottom} = G \times 4\text{V} = 16\text{V}.$$

At the top of the circuit we have a voltage follower and no current flowing through the resistor. Consequently, the output v_{top} is 4V. Finally we obtain

$$v_o = v_{top} - v_{bottom} = 4V - 16\text{V} = -12\text{V}.$$

5. In the op-amp circuit shown below, determine the voltage V_a , V_b, V_o. Assume the circuit behaves linearly and make use of the ideal op-amp approximation.



Solution:

Similarly, we have $v^+ = v^- = 0$ and $i^+ = i^- = 0$ for all op-amps. For the upper left op-amp, the current through the $1k\Omega$ resister and $4k\Omega$ resistor is the same, hence

$$\frac{4 - v^-}{1} = \frac{v^- - V_a}{4}.$$

which gives

$$V_a = -16\text{V}.$$

Now, let's look at the lower left op-amp. The current through the $2k\Omega$ and $4k\Omega$ resistors are the same. Hence,

$$\frac{2V_a - v^-}{2} = \frac{v^- - V_b}{4},$$

which gives

$$V_b = 64V.$$

Since we know V_a and V_b, we can calculate the current through $10k\Omega$ resistor and the $15k\Omega$ resistor,

$$i_{10} = \frac{V_a}{10k\Omega} = -\frac{8}{5}m\text{A},$$

and

$$i_{15} = \frac{V_b}{15k\Omega} = \frac{64}{15}\text{mA}.$$

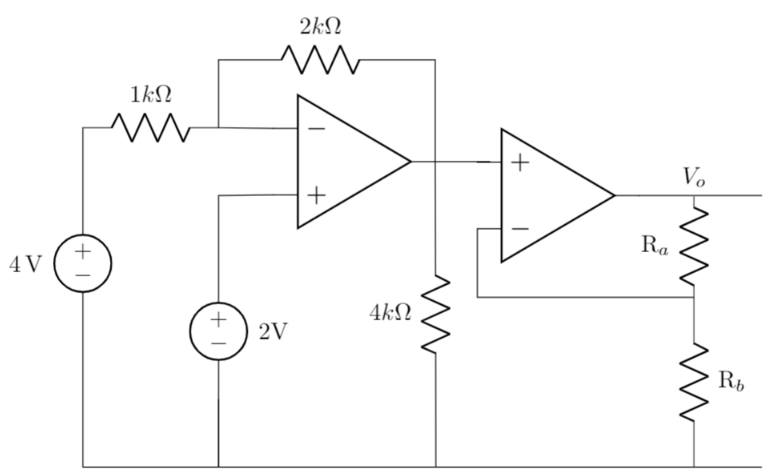
Applying the KCL, we get the current through the $5k\Omega$ resistor is:

$$i_{15} + i_{10} = \frac{v^- - V_o}{5} = \frac{8}{3}\text{mA},$$

which gives

$$V_o = -\frac{40}{3}\text{V}.$$

6. Determine the node voltage V_o using the ideal op-amp approximation.
   (a) Assume that $R_a = R_b = 2k\Omega$.
   (b) Assume that $R_a = 0$, and $R_b = \infty$.



Solution:

(a) For the left op-amp, we have $v_1^+ = v_1^- = 2\text{V}$. Let v_2^+ and v_2^- denote the voltage for the right op-amp. By applying KVL, we have:

$$\frac{4 - v_1^-}{1} = \frac{v_1^- - v_2^+}{2},$$

which gives

$$v_2^+ = v_2^- = -2\text{V}.$$

The current through R_a and R_b is the same, then we have:

$$\frac{V_o - v_2^-}{R_a} = \frac{v_2^-}{R_b},$$

which gives

$$V_o = -4\text{V}.$$

(b) For $R_a = 0$, and $R_b = \infty$, we can still apply the KVL and get the voltage of the right op-amp

$$v_2^+ = v_2^- = -2\text{V}.$$

Since V_o and v_2^- are connected, $V_o = v_2^- = -2$V. Thus,

$$V_o = -2\text{V}.$$