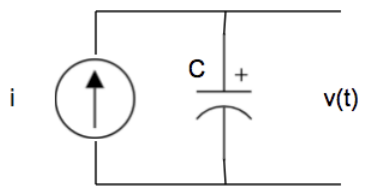
ECE210 / ECE211 - Homework 04
Solutions
Due date: Wednesday, Sep. 26, 2018

1. In the circuit shown below, let $C = 1\text{F}$ and $v(0^-) = 1\text{V}$. Determine $v(t)$ for $t > 0$ and sketch it.



Solution:

For capacitor C we have the following relation:

$$i(t) = C \times \frac{dV}{dt} \tag{1}$$

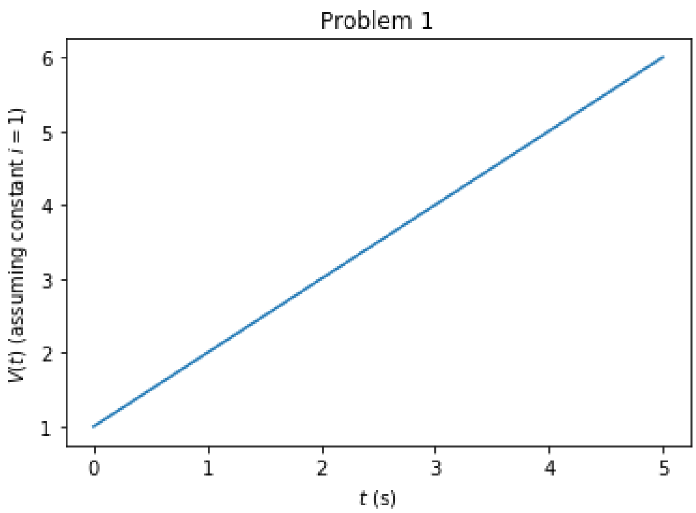
Given $C = 1\text{F}$ and and initial condition $v(0^-) = 1\text{V}$,

$$\frac{dV}{dt} = \frac{1}{C} \times i(t) \tag{2}$$

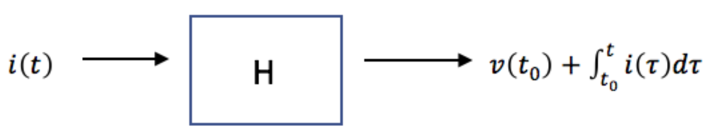
$$\int_0^t \frac{dV}{d\tau} d\tau = \frac{1}{C} \int_0^t i(\tau) d\tau \tag{3}$$

$$\Longrightarrow V(t) = 1 + \int_0^t i(\tau) d\tau \tag{4}$$

For constant current i in this question, $V(t) = 1 + it$, and the plot is shown below.



2. Suppose we have the system below. Show that this system is zero-state linear.



Solution: For zero-state, $v(t_0) = 0$, so $v(t) = \int_{t_0}^{t} i(\tau)d\tau$.

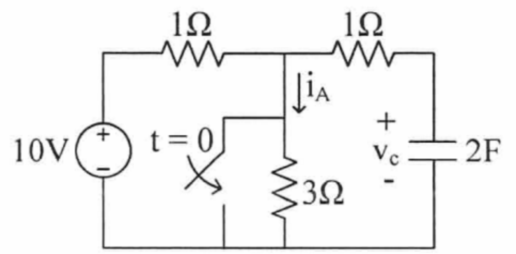
$$i_1(t) \rightarrow v_1(t) = \int_{t_0}^{t} i_1(\tau)d\tau \tag{5}$$

$$i_2(t) \rightarrow v_2(t) = \int_{t_0}^{t} i_2(\tau)d\tau \tag{6}$$

$$i_3(t) = ai_1(t) + bi_2(t) \rightarrow v_3(t) = \int_{t_0}^{t} i_3(\tau)d\tau = \int_{t_0}^{t} [ai_1(t) + bi_2(t)]\, d\tau = a\int_{t_0}^{t} i_1(t)d\tau + b\int_{t_0}^{t} i_2(t)d\tau = av_1(t) + bv_2(t) \tag{7}$$

Therefore the system is zero-state linear.

3. The circuit shown below has been in DC steady state before the switch closes at $t = 0$s.
   (a) Obtain $i_A(0^-)$, $i_A(0^+)$, $v_c(0^-)$ and $v_c(0^+)$.
   (b) Determine $i_A(t)$ for $t > 0$, and sketch it for $t > -1$.
   (c) Determine $v_c(t)$ for $t > 0$ and sketch it for $t > -1$.



Solution: (a)

$$i_A(0^-) = \frac{10\ V}{1\Omega + 3\Omega} = 2.5A \tag{8}$$

$$i_A(0^+) = \frac{10\ V}{1\Omega} + \frac{7.5\ V}{1\Omega} = 17.5A \tag{9}$$

$$v(0^-) = v(0^+) = 10V \cdot \frac{3\Omega}{1\Omega + 3\Omega} = 7.5V \tag{10}$$

(b) The general solution of $i_A(t)$ should take the following form

$$i_A(t) = A + Be^{-t/\tau} \tag{11}$$

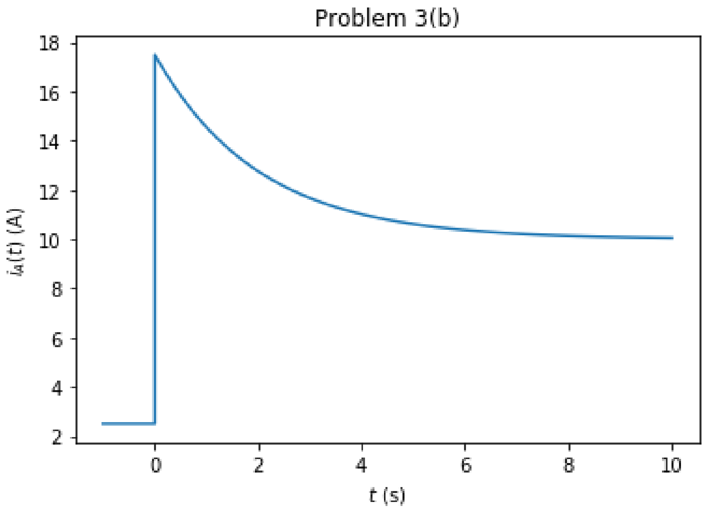
where $\tau = R_{eq} \cdot C$ is the time constant of this RC circuit. First we find equivalent resistance $R_{eq} = 1\Omega$ by removing the voltage source, resulting $\tau = 2$.

Furthermore this general solution needs to satisfy the initial condition ($t = 0$) and the steady state condition ($t \rightarrow \infty$). plugging in those two numbers, we find

$$i_A(0^+) = A + B = 17.5 \tag{12}$$

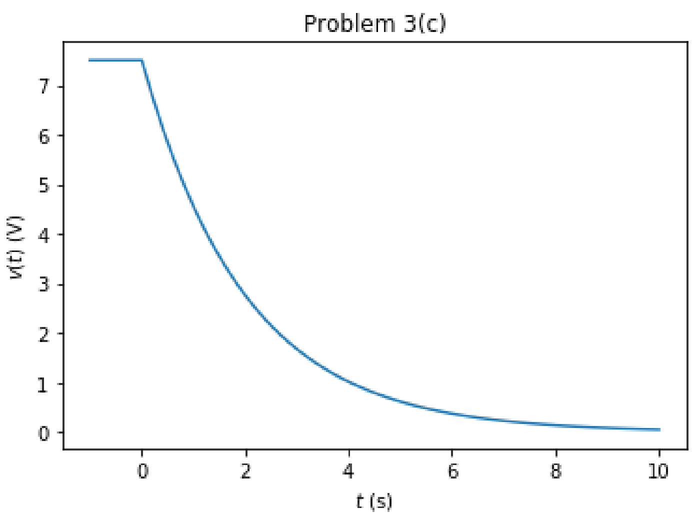
$$i_A(\infty) = A = 10 \tag{13}$$

$$\Longrightarrow i_A(t) = 10 + 7.5e^{-t/2} \tag{14}$$



(c) Similar to the procedures in (b), we first write down the general solution of $v(t)$ and solve for two unknowns:

$$v(t) = A + Be^{-t/2} \tag{15}$$
$$v(0^+) = A + B = 7.5 \tag{16}$$
$$v(\infty) = A = 0 \tag{17}$$
$$\implies v(t) = 7.5e^{-t/2} \tag{18}$$

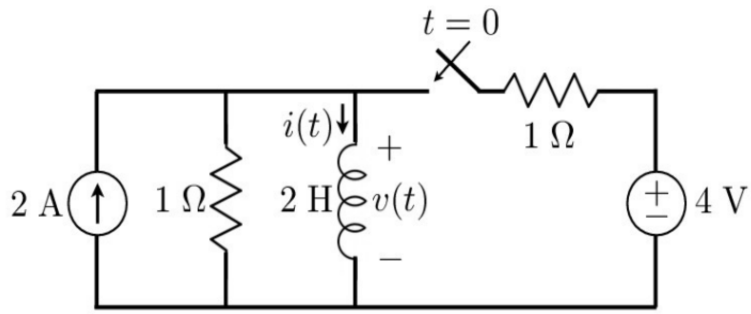


4. The circuit shown below has been in DC steady state before the switch closes at $t = 0$.

   (a) Obtain $i(0^-)$, $i(0^+)$, $v(0^-)$ and $v(0^+)$.

   (b) Determine $i(t)$ for $t > 0$, and sketch it for $t > -1$.

   (c) Determine $v(t)$ for $t > 0$ and sketch it for $t > -1$.

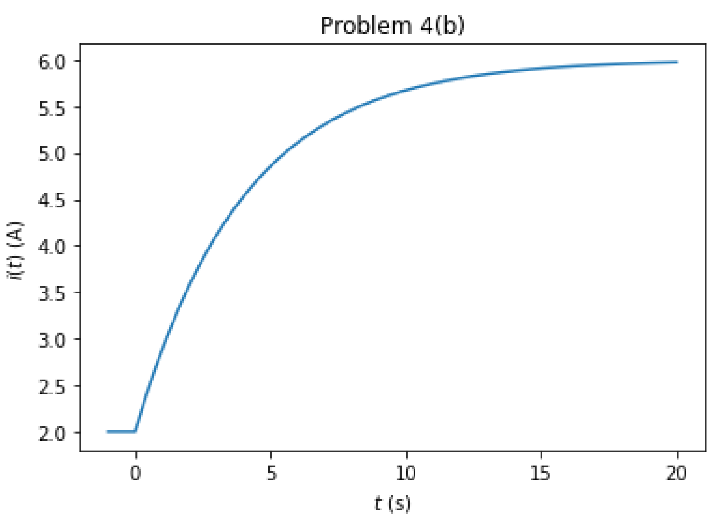


Solution: (a)

$$i_A(0^-) = i_A(0^+) = 2A \tag{19}$$
$$v(0^-) = 0V \tag{20}$$
$$v(0^+) = 2V \tag{21}$$

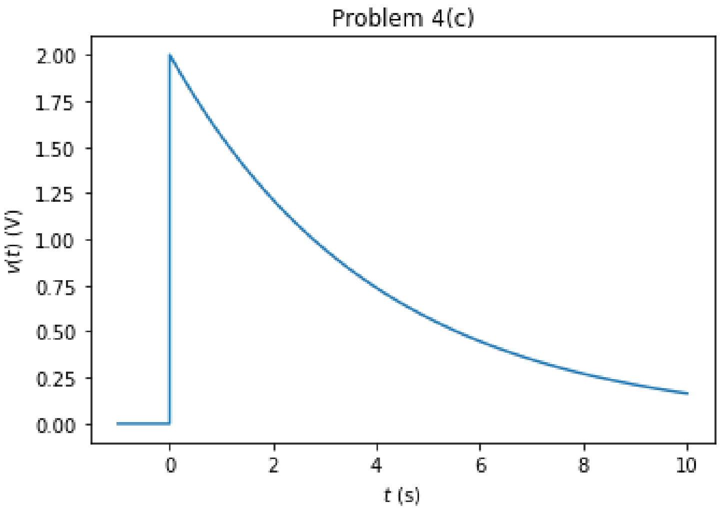
(b) The time constant for a RL circuit is $\tau = L/R_{eq} = 2H/\frac{1}{2}\Omega = 4s$, following the same procedures as in Problem 3, we find

$$i(t) = A + Be^{-t/4} \tag{22}$$
$$i(0^+) = A + B = 2 \tag{23}$$
$$i(\infty) = A = 2 + 4 = 6 \tag{24}$$
$$\implies i(t) = 6 - 4e^{-t/4} \tag{25}$$



(c) Similar to the procedures in (b), we first write down the general solution of $v(t)$ and solve for two unknowns:

$$v(t) = A + Be^{-t/4} \tag{26}$$
$$v(0^+) = A + B = 2 \tag{27}$$
$$v(\infty) = A = 0 \tag{28}$$
$$\implies v(t) = 2e^{-t/4} \tag{29}$$



Problem 4(c)
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
v(t) (V)
0
2
4
6
8
10
t (s)