

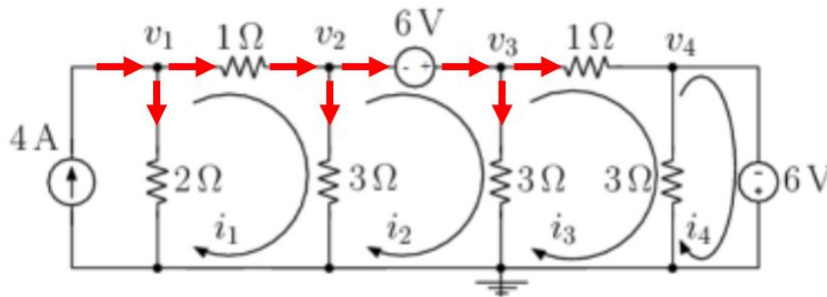
## ECE210 / ECE211 - Homework 02 Solution

Due: Wednesday, September 12 @ 6pm.

## Problems:

1. Consider the following circuit

- (a) Use the node-voltage method to obtain a set of equations, in terms of the node voltages **only**, that can be solved to obtain all node voltages using the bottom node as the reference node.



Using KCL:

$$\text{@ Node } v_1 : 4 = \frac{v_1}{2} + \frac{v_1 - v_2}{1}$$

$$\text{@ Node } v_2 \text{ and } v_3 : \frac{v_1 - v_2}{1} = \frac{v_2}{3} + \frac{v_3 - v_4}{1} + \frac{v_3}{3}$$

From the circuit you can see that :

$$v_4 = -6$$

$$v_3 - v_2 = 6$$

When we simplify the above equations, we get the equations:

$$3v_1 - 2v_2 = 8$$

$$-3v_1 + 8v_2 = -42$$

- (b) Use the loop-current method to obtain a set of equations, in terms of loop currents **only**, that can be solved to obtain all loop currents.

Using KVL:

$$\text{@ Loop } i_1 : -2(4 - i_1) + i_1 + 3(i_1 - i_2) = 0$$

$$\text{@ Loop } i_2 : -3(i_1 - i_2) - 6 + 3(i_2 - i_3) = 0$$

$$\text{@ Loop } i_3 : -3(i_2 - i_3) + i_3 + 3(i_3 - i_4) = 0$$

$$\text{@ Loop } i_4 : -3(i_3 - i_4) - 6 = 0$$

When we simplify the above equations, we get the equations:

$$6i_1 - 3i_2 = 8$$

$$-3i_1 + 6i_2 - 3i_3 = 6$$

$$-3i_2 + 7i_3 - 3i_4 = 0$$

$$-3i_3 + 3i_4 = 6$$

- (c) Obtain all node voltages. You can use software (Matlab, Mathematica, etc.) to solve the equations, but include your code.

Using equations obtained from part a, we get:

$$v_1 = \frac{-10}{9}V$$

$$v_2 = \frac{-17}{3}V$$

$$v_3 = \frac{1}{3}V$$

$$v_4 = -6V$$

- (d) Obtain all loop currents. You can use software (Matlab, Mathematica, etc.) to solve the equations, but include your code.

Using equations obtained from part b, we get:

$$i_1 = \frac{41}{9} A$$

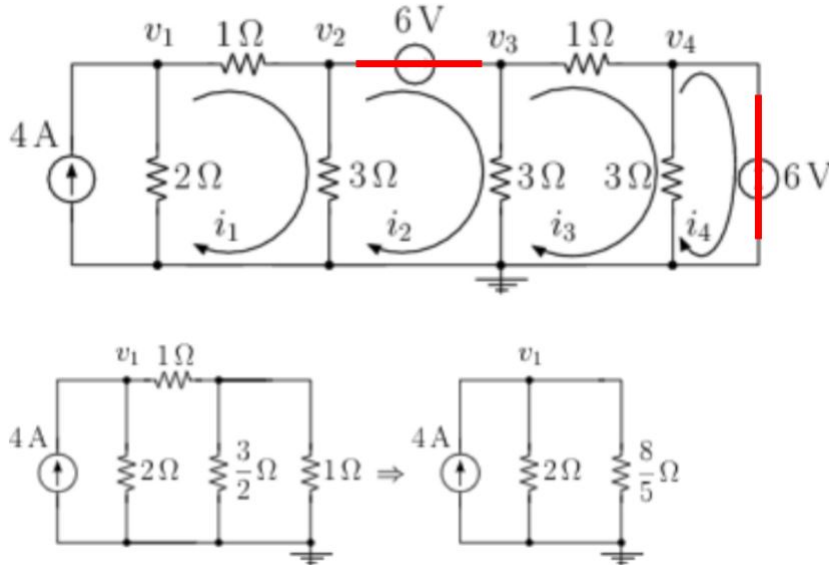
$$i_2 = \frac{58}{9} A$$

$$i_3 = \frac{19}{3} A$$

$$i_4 = \frac{25}{3} A$$

- (e) Use superposition to obtain the voltage across the terminals of the  $2\Omega$  resistor.

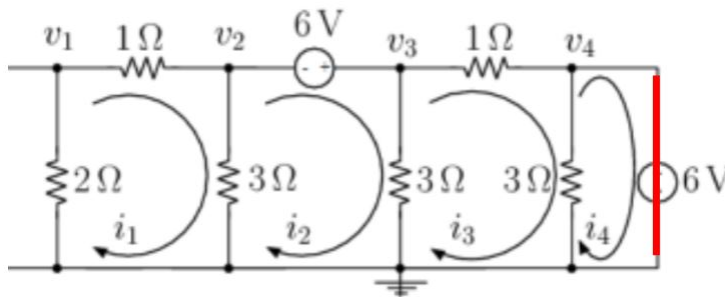
STEP 1: Suppress all sources except 4A current source:



From KCL:

$$4 = \frac{1}{2}v_1 + \frac{5}{8}v_1 \text{ thus } v_1 = \frac{32}{9} V$$

STEP 2: Suppress all sources except 6V on the top:



From KCL:

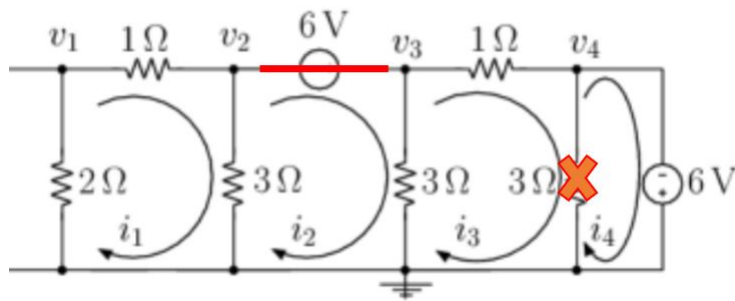
$$\frac{v_1}{2} + \frac{v_1 - v_2}{1} = 0$$

$$\frac{v_1 - v_2}{1} = \frac{v_2}{3} + \frac{v_2 + 6}{4}$$

After solving these two equations we get:

$$v_1 = -\frac{8}{3}$$

STEP3: Suppress all sources except 6V on the right:



Using VDR:

$$v_1 = -2V$$

LAST STEP: Sum it all up:

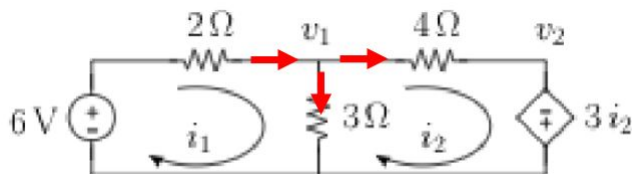
$$V_1 = -2 - \frac{8}{3} + \frac{32}{9} = \frac{-10}{9}V$$

- (f) Obtain the power absorbed by the  $2\Omega$  resistor

$$P = \frac{V^2}{R} = \frac{(10/9)^2}{2} = \frac{50}{81}W$$

2. Consider the following circuit.

- (a) Use the node-voltage method to obtain a set of equations, in terms of the node voltages **only**, that can be solved to obtain all node voltages using the bottom node as the reference node.



Using KCL:

$$\frac{6-v_1}{2} = \frac{v_1}{3} + \frac{v_1-v_2}{4}$$

From the circuit:

$$v_2 = -3i_2$$

From Ohm's law:

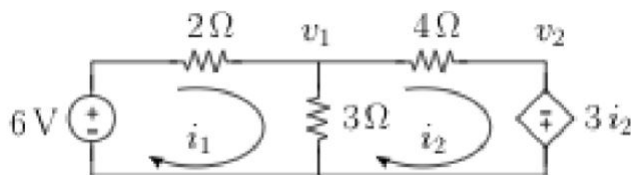
$$i_2 = \frac{v_1-v_2}{4}$$

From the above equations, we can get the equations:

$$13v_1 - 3v_2 = 36$$

$$3v_1 + v_2 = 0$$

- (b) Use the loop-current method to obtain a set of equations, in terms of loop currents **only**, that can be solved to obtain all loop currents.



From KVL:

$$6 - 2i_1 - 3(i_1 - i_2) = 0$$

$$3(i_1 - i_2) - 4i_2 + 3i_2 = 0$$

From the equations above, we get:

$$5i_1 - 3i_2 = 6$$

$$-3i_1 + 4i_2 = 0$$

- (c) Obtain all node voltages. You can use software (Matlab, Mathematica, etc.) to solve the equations, but include your code.

$$v_1 = \frac{18}{11}V$$

$$v_2 = \frac{-54}{11}V$$

- (d) Obtain all loop currents. You can use software (Matlab, Mathematica, etc.) to solve the equations, but include your code.

$$i_1 = \frac{24}{11}A$$

$$i_2 = \frac{18}{11}A$$

- (e) Use superposition to obtain the current through the dependent voltage source.

Because we only have one independent source no superposition needed:

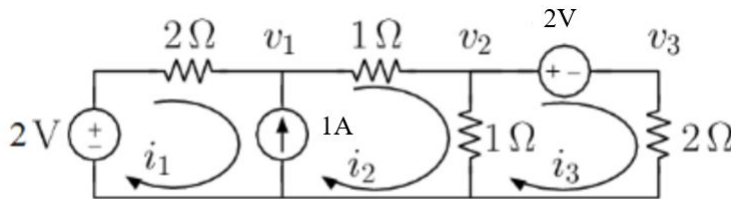
$$I = \frac{18}{11}A$$

- (f) Obtain the power absorbed by the dependent voltage source.

$$P = vi = (3i_2)(-i_2) = -\frac{972}{121} \approx -8.03W$$

3. Consider the following circuit

- (a) Use the node-voltage method to obtain a set of equations, in terms of the node voltages **only**, that can be solved to obtain all node voltages using the bottom node as the reference node.



From KCL:

$$\frac{v_1 - 2}{2} + \frac{v_1 - v_2}{1} = 1$$

$$\frac{v_2 - v_1}{1} + v_2 + \frac{v_2 - 2}{2} = 0$$

Simplifying above eqn. we get:

$$3v_1 - 2v_2 = 4$$

$$2v_1 - 5v_2 = -2$$

- (b) Use the loop-current method to obtain a set of equations, in terms of loop currents **only**, that can be solved to obtain all loop currents.

$$2i_1 + i_2 + (i_2 - i_3) = 2$$

$$2i_3 + (i_3 - i_2) + 2 = 0$$

$$i_2 = 1 + i_1$$

$$4i_2 - i_3 = 4$$

$$i_2 - 3i_3 = 2$$

Simplifying above eqn. we get:

$$4 = 4i_2 - i_3$$

$$-2 = 3i_3 - i_2$$

- (c) Obtain all node voltages. You can use software (Matlab, Mathematica, etc.) to solve the equations, but include your code.

$$v_1 = \frac{24}{11}V$$

$$v_2 = \frac{14}{11}V$$

$$v_3 = \frac{-8}{11}V$$

- (d) Obtain all loop currents. You can use software (Matlab, Mathematica, etc.) to solve the equations, but include your code.

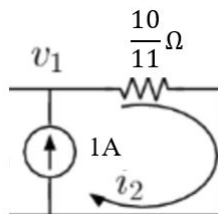
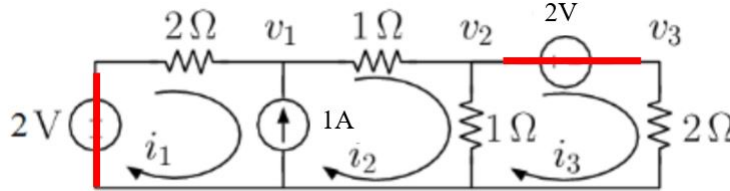
$$i_1 = \frac{-1}{11} A$$

$$i_2 = \frac{10}{11} A$$

$$i_3 = \frac{-4}{11} A$$

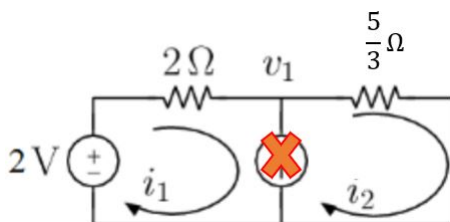
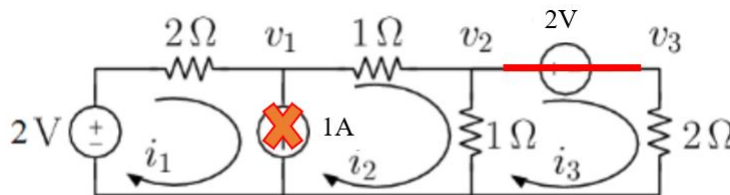
- (e) Use superposition to obtain the voltage across the terminals of the current source.

STEP1: Suppress all sources but 1A current source



$$v_1 = \frac{10}{11} V$$

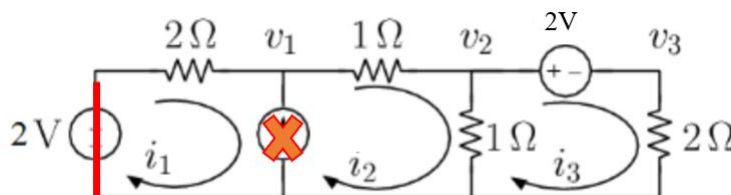
STEP2: Suppress all sources but 2V voltage source on the left



From VDR:

$$v_1 = \frac{10}{11} V$$

STEP3: Suppress all sources but 2V voltage source on the right



Using source conversion:

$$v_1 = \frac{4}{11} V$$

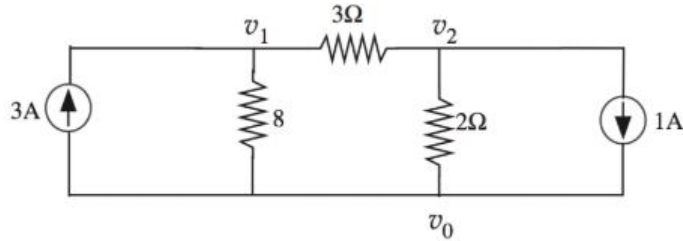
FINAL STEP: Sum all the contribution of the three sources:

$$V_1 = \frac{10}{11} + \frac{10}{11} + \frac{4}{11} = \frac{24}{11} V$$

- (f) Obtain the power absorbed by the current source

$$P = -v_1 * 1A = -\frac{24}{11}W$$

4. Consider the circuit below



- (a) Write nodal equations for the circuit below, assuming that  $v_0 = \text{ground}$ . Solve for  $v_1$  and  $v_2$ . You can use software (Matlab, Mathematica, etc.) to solve the equations, but include your code.

Using KCL:

$$\frac{v_1}{8} + \frac{v_1 - v_2}{3} = 3$$

$$\frac{v_2 - v_1}{3} + \frac{v_2}{2} + 1 = 0$$

Using the above two equations we solve:

$$v_1 = 8V$$

$$v_2 = 2V$$

- (b) Write nodal equations for the same circuit, but this time, assume that  $v_1 = \text{ground}$ . Solve for  $v_0$  and  $v_2$ . You can use software (Matlab, Mathematica, etc.) to solve the equations, but include your code.

Again, using KCL:

$$\frac{v_o}{8} + \frac{v_o - v_2}{2} + 3 = 1$$

$$\frac{v_2}{3} + \frac{v_2 - v_o}{2} + 1 = 0$$

Using the above two equations, we solve:

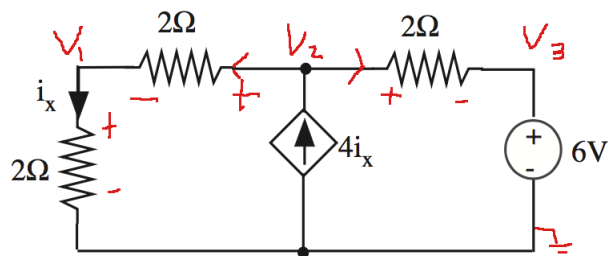
$$v_o = -8V$$

$$v_2 = -6V$$

- (c) How are your answers to parts (a) and (b) related?

Absolute potential changes but the potential difference remains the same.

5. Find  $i_x$  using nodal analysis. You **cannot** use software (Matlab, Mathematica, etc.) to solve this problem, you have to solve it by hand.



$$6 = V_3 - 0$$

$$\frac{V_1 - 0}{2} = \frac{V_2 - V_1}{2}$$

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{2} = 4i_x$$

$$i_x = \frac{V_1 - 0}{2}$$

Multiply the second equation by 2 on both sides and solve for  $V_2 = 2V_1$ .

Then substitute this and equations 1 and 4 into the third equation, multiply both sides by 2 and solve for  $V_1 = -6V$ .

Substituting back into the fourth equation we get

$$i_x = \frac{-6}{2} = -3A$$

6. Let  $Z = \frac{j(e^{j\frac{\pi}{6}} - e^{-j\frac{\pi}{6}})}{e^{-j\frac{\pi}{3}} + e^{j\frac{\pi}{3}}}$ . Write  $Z$  in rectangular and polar forms.

Recall that  $\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$  and  $\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$ .

Hence,  $Z = \frac{-\sin(\pi/6)}{\cos(\pi/3)} = \frac{-1/2}{1/2} = -1 = e^{j\pi}$ .