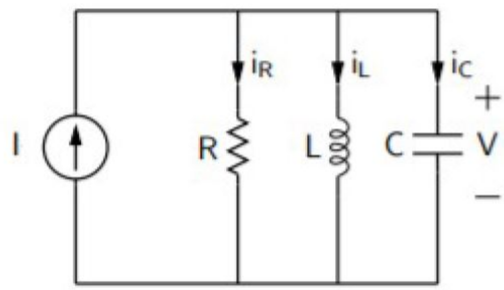
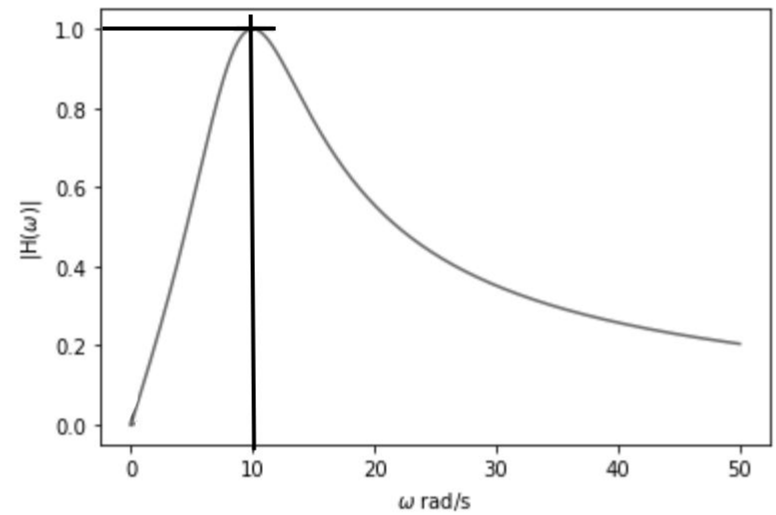
ECE210/ ECE211 - Homework 7 - Solutions

1. Consider the circuit drawn below:

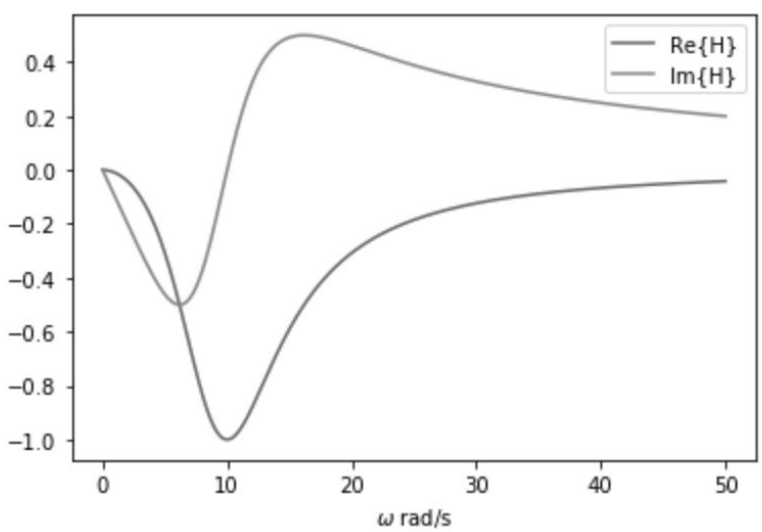


(a) $\omega = \frac{1}{\sqrt{LC}} = 10\frac{rad}{s}$

(b) $H(\omega) = [\frac{1}{R} + \frac{1}{j\omega L} + j\omega C]^{-1}$

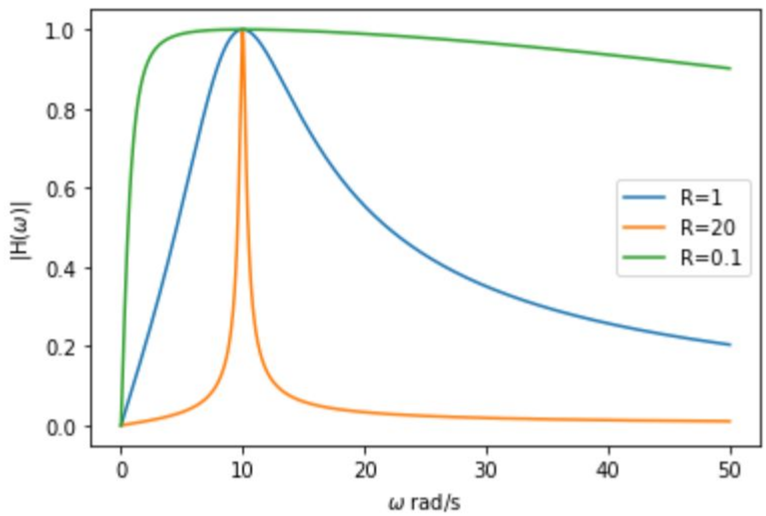


(c) See plots below:



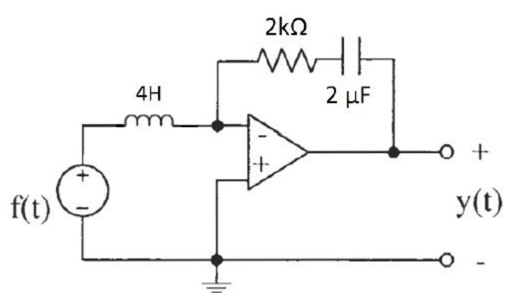
(d) The circuit acts as a band-pass filter because the the signal with frequency away from 10rad/s will be attenuated (it has a gain close to 1 only for a small band of frequencies).

(e) For both cases, we see that the resonant frequencies stay at 10rad/sec:



(f) From the plot above, we see that as the resistor value increase, the bandwidth decrease (narrower).

2. From the following circuit:



$H(\omega) = \frac{Y}{F} = -\frac{2000 - \frac{5E5j}{\omega}}{4j\omega}$

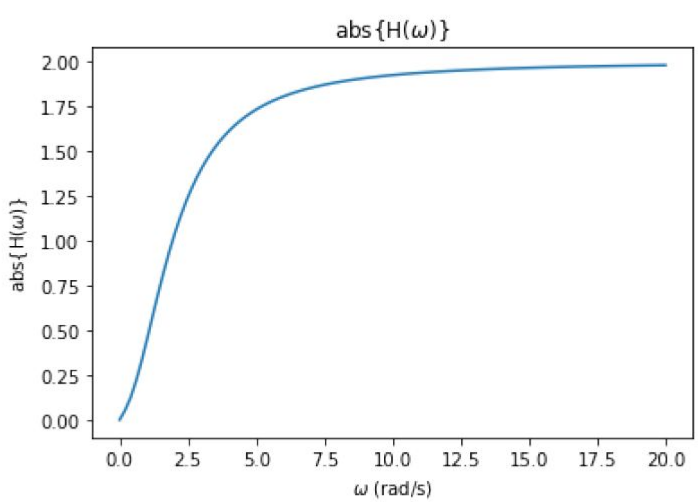
3. Given a linear system:

$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y(t) = \frac{df}{dt} + 2\frac{d^2f}{dt^2}$

(a) $(j\omega)^2 - 4(j\omega)Y + 4Y = (j\omega)F + (j\omega)^2 2F$

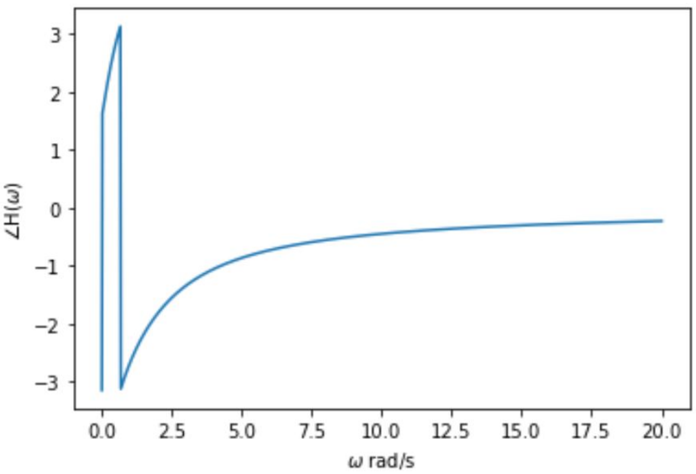
$H(\omega) = \frac{Y}{F} = \frac{j\omega - 2\varpi^2}{-4j\omega - \omega^2 + 4}$

(b) See plot below:



(c) High Pass Filter

(d) See plot below:



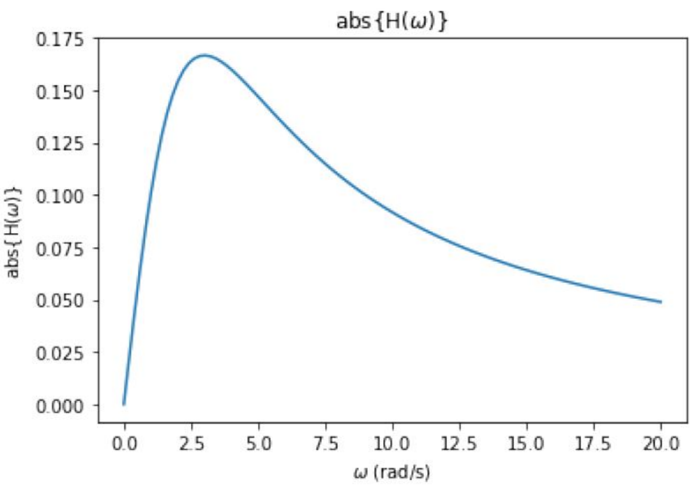
4. A linear system is given below:

   $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y(t) = \frac{df}{dt}$

   (a) $(j\omega)^2 Y - 6(j\omega)Y + 9Y = (j\omega)F$

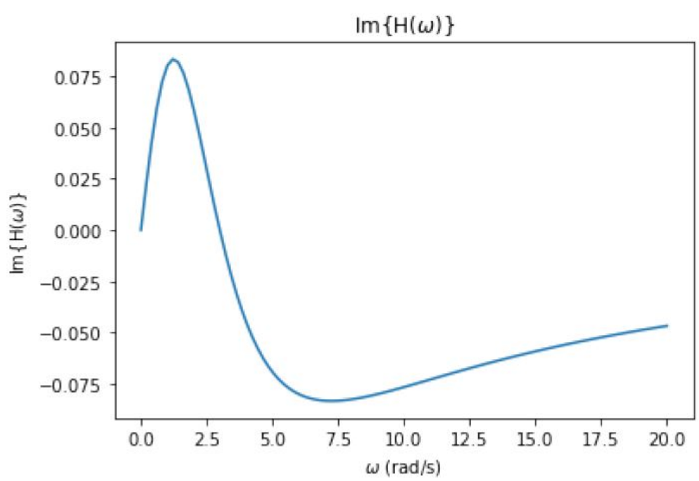
   $H(\omega) = \frac{Y}{F} = \frac{j\omega}{-6j\omega - \omega^2 + 9}$

   (b) See plot below:

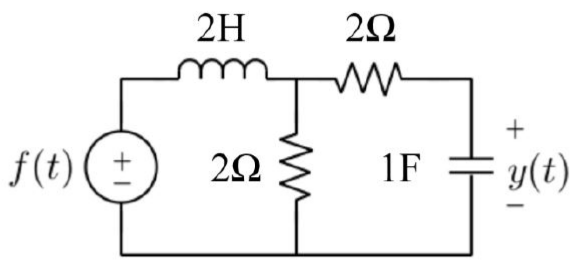


   (c) Bandpass filter

   (d) See plot below:

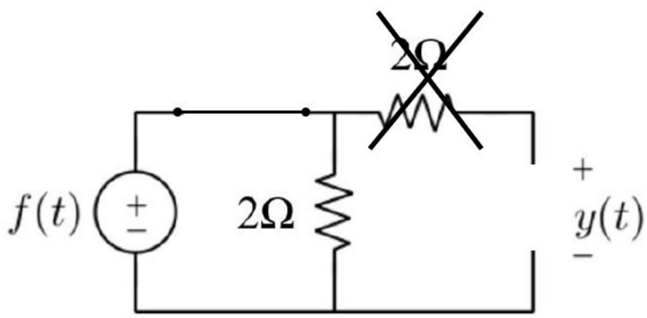


5. For circuit below:



Given $f(t) = 4 - cos(2t)$, we need to divide our analysis into DC and AC:

(a) DC Analysis: f(t)=4V
At stead state: Inductor will act like a short whereas the capacitor will act like an open:



Therefore, y(t) = 4

(b) AC Analysis: f(t)=-cos(2t)
Convert the whole circuit to a phasor and use KCL and VDR to find solution.
Using KCL:
$\frac{-1-Vo}{j4} = \frac{Vo-Y}{} + \frac{Vo}{2}$
$-1 = (1 + j4)Vo - j2Y$ (1)
Using VDR:
$Y = Vo(\frac{\frac{1}{j2}}{2+\frac{1}{j2}})$ (2)
Plugging (2) into (1) and we get:
$-1 = (j8 - 15)Y - j2Y$
$Y = \frac{1}{15-j6} = 0.06\angle 21.8°$

Combine DC and AC analysis and we get:

$y(t) = 4 + 0.06cos(2t + 21.8°)$

6. We can express f(t) as:
$f(t) = 4cos(2t) + 2sin(t) + 2cos(t) = 4cos(2t) + 2\sqrt{2}cos(t - \frac{\pi}{4})$
With $H(\omega) = \frac{1-j\omega}{1+j\omega}$
We get:
$|H(1)| = 1$; $\angle H(1) = -1.57$
$|H(2)| = 1; \angle H(2) = -2.21$
Therefore, we get:
$y(t) = 2\sqrt{2}cos(t - \frac{\pi}{4} - 1.57) + 4cos(2t - 2.21)$