University of Illinois **Fall 2018**

ECE210 / ECE211 - Homework 08 - Updated

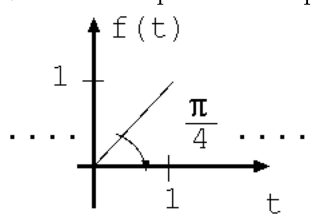
due: Wednesday, October 24, 2017 at 6:00 p.m.

1. Consider the function $f(t) = Re\left\{2e^{j3t} + 3e^{-j2t}\right\}$,where $Re\left\{x\right\}$ denotes the real part of x. Find its period, T, its fundamental frequency, ω_0, and plot it over at least two periods.

2. For each one of the following functions of t, indicate whether they are periodic or not. If periodic, indicate its period, and if not periodic, indicate why. Assume n is a positive integer.
   (a) $\sin t - \sin(t/2) - \sin(t/4)$
   (b) $\sin(2\pi t) + \cos\sqrt{2}t$
   (c) $\sin\left(\frac{\pi t}{2}\right) + \cos(2nt) + \sin\left(\frac{3\pi t}{5}\right) + \sin\left(\frac{et}{2}\right)$
   (d) $\cos(2nt)$
   (e) $\cos(\pi t) - \cos\left(\frac{\pi}{n}t\right)$
   (f) $\sin(nt)\cos(2nt)$

3. What property of RLC circuits allows the use of Fourier series for analysis? Show the property by means of a diagram.

4. Consider the periodic ramp shown in the diagram below, where the angle of the line is $\frac{\pi}{4}$, and the period $T \geq 1$.

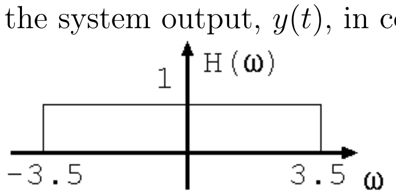


   (a) Determine its exponential Fourier series coefficients, F_n, for all n.
   (b) Determine its fundamental frequency, ω_0.
   (c) Determine $\Delta\omega = \omega_n - \omega_{n-1}$, where ω_n is the frequency of the n-th harmonic, and explain what happens as $\Delta\omega \to 0$.

5. Consider the following periodic signal $f(t)$ with period $T = 2\pi$ s and exponential Fourier series coefficients F_n :

$$f(t) = \begin{cases} \cos(2t) & 0 \leq t < \pi, \\ 0, & \pi \leq t < 2\pi. \end{cases} \qquad F_n = \begin{cases} \frac{jn}{\pi(4-n^2)} & n \text{ odd}, \\ \frac{1}{4} & n = \pm 2, \\ 0, & \text{else}. \end{cases}$$

   (a) Express $f(t)$ in exponential Fourier series form.
   (b) Obtain the compact Fourier series coefficients c_n and θ_n for all n and express $f(t)$ in its compact Fourier series form.
   (c) Let $f(t)$ be the input to an LTI system with frequency response $H(\omega)$ plotted below. Find the system output, $y(t)$, in compact form.



(d) Obtain the exponential Fourier series coefficients of the periodic signal $g(t)$, with period $T = 2\pi$ s, in terms of the exponential Fourier series coefficients F_n. (Hint: sketch both functions and find their relationship.)

$$g(t) = \begin{cases} \sin\left(2\left(t + \frac{\pi}{4}\right)\right) & -\frac{\pi}{2} \le t < \frac{\pi}{2}, \\ 0, & \frac{\pi}{2} \le t < \frac{3\pi}{2}. \end{cases}$$

(e) Express $g(t)$ in exponential Fourier series form.

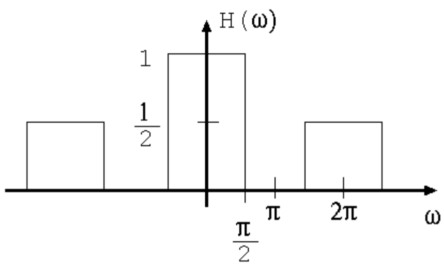
6. Consider a periodic signal $f(t)$ with period $T = 2$s, given by:

$$f(t) = \begin{cases} e^{t} & 0 \le t < 1 \text{ s} \\ e^{2-t} & 1 \le t < 2 \text{ s}. \end{cases}$$

(a) The function $f(t)$ can be expressed as a Fourier series with exponential coefficients

$$F_n = \frac{e^{1-jn\pi} - 1}{1 + n^2\pi^2}.$$

Let $f(t)$ be the input to an LTI system with frequency response $H(\omega)$, given in the plot below. Determine the steady-state output, $y_{ss}(t)$ in compact form.



(b) Consider the periodic signal $g(t)$ with period $T = 2$s, given by:

$$g(t) = \begin{cases} e^{t} & -1 \le t < 0 \text{ s} \\ e^{-t} & 0 \le t < 1 \text{ s}. \end{cases}$$

Determine its exponential Fourier coefficients G_n in terms of the exponential Fourier coefficients F_n. (Hint: sketch both functions and find their relationship.)

(c) Express $g(t)$ in exponential Fourier series form.