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ECE 310 DIGITAL SIGNAL PROCESSING

Notes on Filter Design

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Suppose we have a filter with **even** symmetry, and length N (which can be even or odd). The standard filter design approach works as follows:

(a) Let $D_d(\omega)$ be the magnitude of the desired frequency response.

(b) Define $G_d(\omega) = D_d(\omega)e^{-j\omega\frac{N-1}{2}}$.

(c) Compute $g[n] = \text{DTFT}^{-1}\{G_d(\omega)\}$.

(d) Set $h[n] = g[n]w[n]$, where $w[n]$ is the desired windowing function.

However, we can also use the shortcut:

(a) Define $c[n] = \text{DTFT}^{-1}\{D_d(\omega)\}$. Note that here $c[n]$ is defined in **functional form, not** as a sequence. For the purpose of this course, that means that $c[n]$ is defined for **all** $n \in \mathbb{R}$.

(b) Set $g[n] = c[n - \frac{N-1}{2}]$. Because $c[n]$ is defined for all n, we're not shifting a sequence by a non-integer amount; this operation is allowed. However, $g[n]$ is only defined for **integer** values of n.

(c) Set $h[n] = g[n]w[n]$, where $w[n]$ is the desired windowing function.

To prove that the two are identical, it suffices to take the inverse DTFT of the $G_d(\omega)$ obtained by the standard method:

$$g[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} D_d(\omega)e^{-j\omega\frac{N-1}{2}}e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} D_d(\omega)e^{j\omega(n-\frac{N-1}{2})}d\omega = c[n - \frac{N-1}{2}]$$

as we defined $c[n]$ to be the inverse DTFT of $D_d(\omega)$, and using the time-shift property of the DTFT.

Now, suppose we wish to design a filter with **odd** symmetry. We need to be slightly more careful in this case, due to extra factors of $e^{j\frac{\pi}{2}}$. Using the standard approach, since the GLP form of the filter is $R(\omega)e^{j(\frac{\pi}{2}-\frac{N-1}{2}\omega)}$, $R(\omega)$ must be odd. Therefore, we define

$$G_d(\omega) = \begin{cases} D_d(\omega)e^{j(\frac{\pi}{2}-\frac{N-1}{2}\omega)} & \omega \geq 0 \\ D_d(\omega)e^{j(\frac{\pi}{2}-\frac{N-1}{2}\omega)} & \omega < 0 \end{cases}$$

Then, to follow the standard procedure, we:

(a) Compute $g[n] = \text{DTFT}^{-1}\{G_d(\omega)\}$.

(b) Take $h[n] = g[n]w[n]$, where $w[n]$ is the desired windowing function.

If we want to use the shortcut approach, we need to realize that the frequency response corresponding to $c[n]$ will be **imaginary**. So, we need to set

$$C_d(\omega) = \begin{cases} jD_d(\omega) & \omega \geq 0 \\ -jD_d(\omega) & \omega < 0 \end{cases}$$

This also preserves the odd symmetry (recall that $D(\omega)$ is the magnitude of the ideal response). Then, we follow the same shortcut procedure:

(a) Determine $c[n] = \text{DTFT}^{-1}\{C_d(\omega)\}$.

(b) Compute $g[n] = c[n - \frac{N-1}{2}]$. Again, $c[n]$ is given in functional form, while $g[n]$ is a sequence.

(c) Set $h[n] = g[n]w[n]$.

To prove that the two are identical, it again suffices to show that $g[n] = c[n - \frac{N-1}{2}]$. We can write

$$\begin{aligned} c[n - \frac{N-1}{2}] &= \frac{1}{2\pi}\int_{-\pi}^{0} -jD_d(\omega)e^{j\omega(n-\frac{N-1}{2})}d\omega + \frac{1}{2\pi}\int_{0}^{\pi} jD_d(\omega)e^{j\omega(n-\frac{N-1}{2})}d\omega \\ &= -\frac{1}{2\pi}\int_{-\pi}^{0} e^{j(\frac{\pi}{2}-\frac{N-1}{2}\omega)}e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{0}^{\pi} D_d(\omega)e^{j(\frac{\pi}{2}-\frac{N-1}{2}\omega)}e^{j\omega n}d\omega \\ &= g[n] \end{aligned}$$

Therefore, the shortcut approach will produce identical results.