# ECE 310: Quiz 10 Section CCS Solutions

1. (5 pts) A communications signal $x_c(t)$ is assumed to be bandlimited to 300 MHz. It is desired to filter this signal with a **lowpass** filter with cutoff of 100 MHz, by using a digital filter with frequency response $H_d(\omega)$ sandwiched between an ideal A/D and an ideal D/A.

(a) Determine the Nyquist sampling rate for the input signal, and specify the frequency response $H_d(\omega)$ for the necessary discrete-time filter, when sampling at the Nyquist rate.

**Solution:** The Nyquist rate requires sampling at twice the maximum frequency of input signals. $x_c(t)$ is bandlimited to 300 MHz. Therefore, the Nyquist rate is 600 MHz.

The lowpass filter has a cutoff of $f_c = 100$ MHz. As a digital filter, this will map to a frequency $\omega_c = \Omega_c T = 2\pi f_c T = \frac{\pi}{3}$ where $\frac{1}{T}$ corresponds to the Nyquist rate. So,

$$H_d(\omega) = \begin{cases} 1 & |\omega| \le \frac{\pi}{3} \\ 0 & \text{else} \end{cases}$$

**Grading**
+0.5 for correct Nyquist rate
+2.5 for $H_d(\omega)$

(b) Smart Alec claims that the system can perform the desired filtering function even when the sampling rate is lower than the Nyquist rate. Is this true? Justify your answer.

**Solution:** This is true. Aliased components from spectral copies in the digital frequency domain are okay as long as they do not leak into the interval of the lowpass filter. They will just be cut off. Refer to the solutions of HW 12 problem 1(c). For this problem, the lowest sampling rate that can perform the desired filtering function is 400 MHz.

**Grading**
+1 for correct answer
+1 for justification

2. (5 pts) A system for processing analog singals $x_c(t)$ is composed of the followign parts, connected in cascade: (i) an analog LPF with frequency response

$$G_c(\Omega) = \begin{cases} 1 - 0.5\frac{|\Omega|}{\Omega_c} & \text{for } |\Omega| \le \Omega_c \\ 0 & \text{for } |\Omega| > \Omega_c \end{cases}$$

followed by (ii) a causal digital system whose input $x[n]$ and output $y[n]$ are related as

$$y[n] + 0.5y[n-3] + x[n] = 0$$

which is sandwiched between an ideal A/D and an ideal D/A operating at a sampling rate of 50 kHz. The output of this entire system is denoted by $y_c(t)$.

(a) What is the largest value of $\Omega_c$ for which the entire system will act as an analog LTI system, from input $x_c(t)$ to output $y_c(t)$. Justify your answer.

**Solution:** Because the digital system is operating at a sampling rate of 50 kHz, any signal with frequencies greater than 25 kHz will alias. This is because the digital filter passes all frequencies in the sampled signals. This will break the LTI property of the system. So,

$$\Omega_c = 2\pi f_c = 2\pi(25000) = 50000\pi$$

**Grading**
+1 for correct $\Omega_c$
+1 for justification

(b) For the $\Omega_c$ determined in (a), determine the analog frequency response $H_c(\Omega)$ of the entire system from input $x_c$ to output $y_c$.

**Solution:** The analog frequency response of the entire system $H_c(\Omega)$ will be composed of the LPF $G_c(\Omega)$ and the analog frequency response of the A/D, the digital filter (which we will define as $C_d(\omega)$), and the D/A. Because these two components are cascaded, then

$$H_c(\Omega) = G_c(\Omega)C_c(\Omega)$$

where $C_c(\Omega) = \begin{cases} C_d(\Omega T) & |\Omega| \leq \frac{\pi}{T} \\ 0 & \text{else} \end{cases}$. We first find $C_d(\omega)$:

$$y[n] + 0.5y[n-3] + x[n] = 0 \implies Y_d(\omega) + 0.5Y_d(\omega)e^{-j\omega 3} + X_d(\omega) = 0$$

So then we get:

$$\frac{Y_d(\omega)}{X_d(\omega)} = C_d(\omega) = \frac{-1}{1 + 0.5e^{-j\omega 3}} \implies C_c(\Omega) = \frac{-1}{1 + 0.5e^{-j\Omega 3/50000}}$$

Finally, we get the frequency response:

$$H_c(\Omega) = \begin{cases} (1 - 0.5\frac{|\Omega|}{\Omega_c})(\frac{-1}{1+0.5e^{-j\Omega 3/50000}}) & |\Omega| \leq \Omega_c \\ 0 & \text{else} \end{cases}$$

**Grading**
+1 for $C_d(\omega)$
+1 for $C_c(\Omega)$
+1 for correct $H_c(\Omega)$