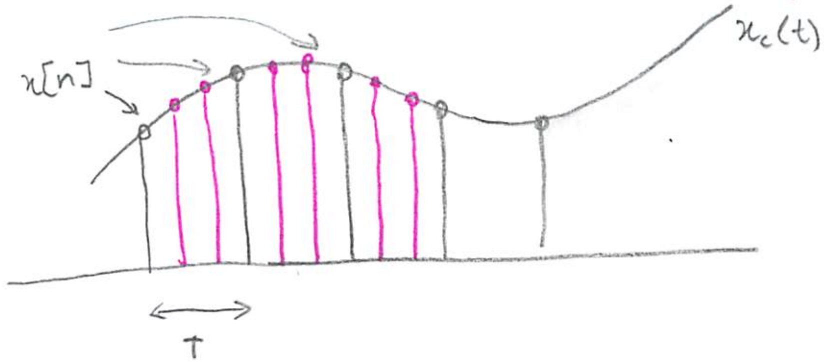
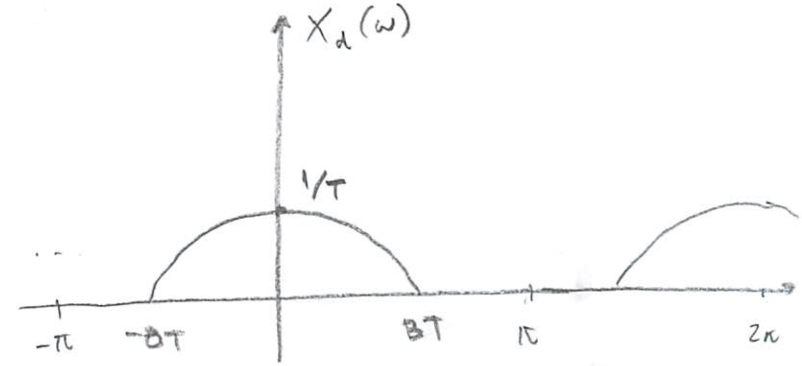
Lecture 25 (Multirate DSP)

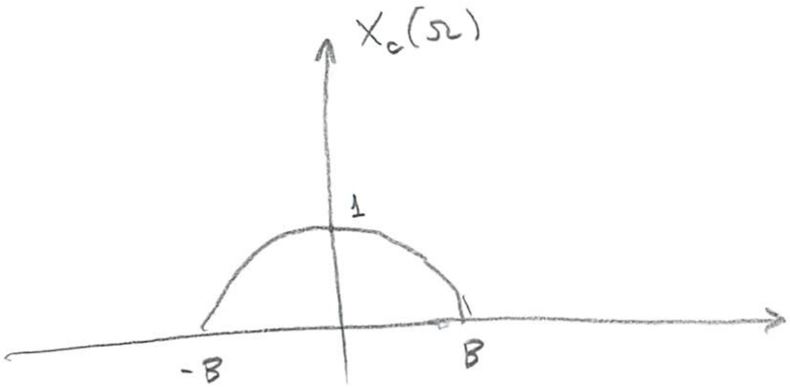
Upsampling (by integer U)

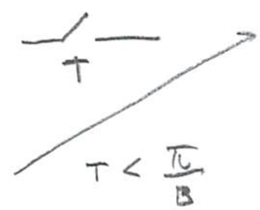


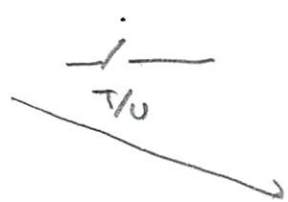
(e.g. U = 3)

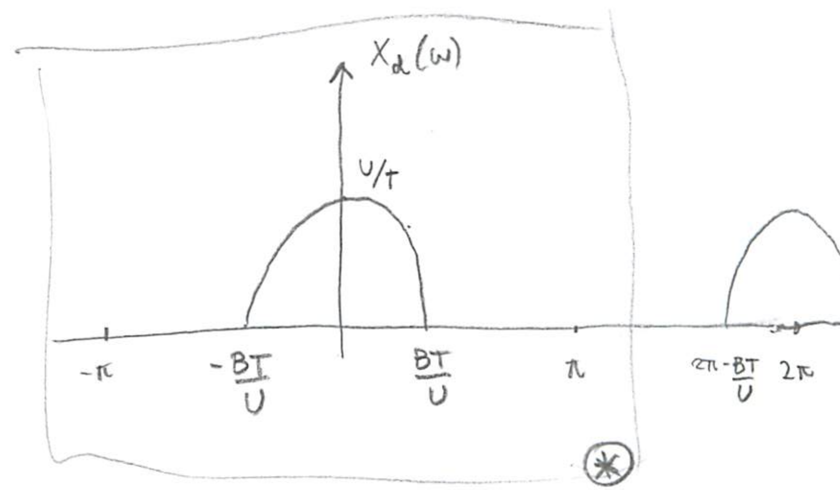


How are the DTFTs related?







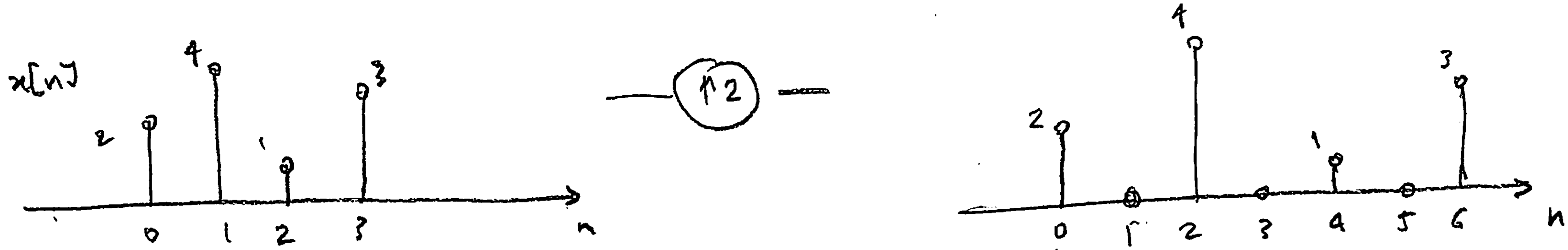


this is our goal

Let's define the upsampler operator as follows:

$$x[n] \longrightarrow \boxed{\uparrow U} \longrightarrow y[n] = \begin{cases} x[n/U] & \text{if } n \text{ is a multiple of } U \\ 0 & \text{otherwise} \end{cases}$$

Ex: $U = 2$



i.e., upsampler places $U-1$ zeros between consecutive samples of $x[n]$

In frequency domain:

$$x[n] \xleftrightarrow{\text{DTFT}} X_d(w) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jwn}$$

$$y[n] = \begin{cases} x[n/U] & \text{if } n = U \cdot \ell \\ 0 & \text{otherwise} \end{cases}$$

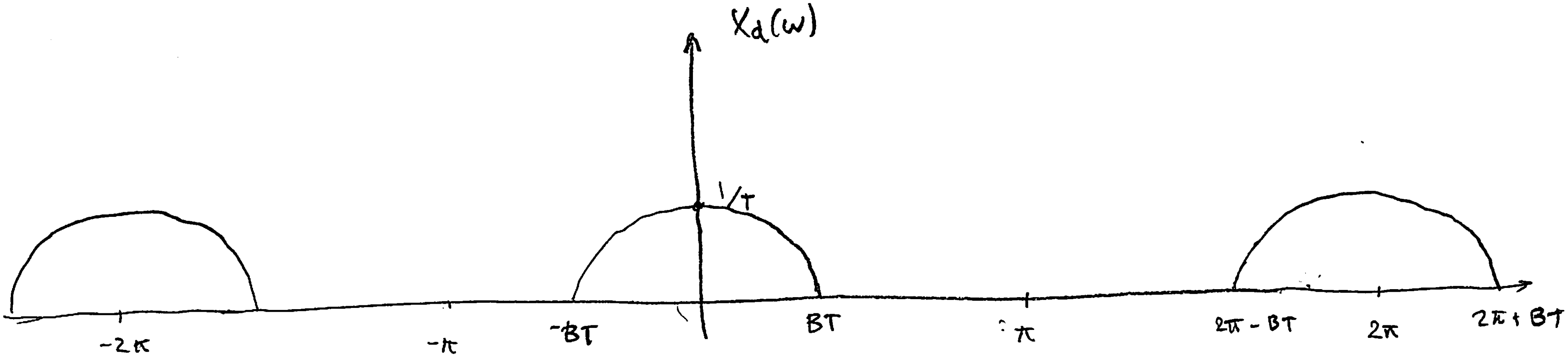
$$Y_d(w) = \sum_{n=-\infty}^{\infty} y[n]\, e^{-jwn} = \sum_{\ell=-\infty}^{\infty} y[U\cdot \ell]\, e^{-jw(U\cdot \ell)}$$

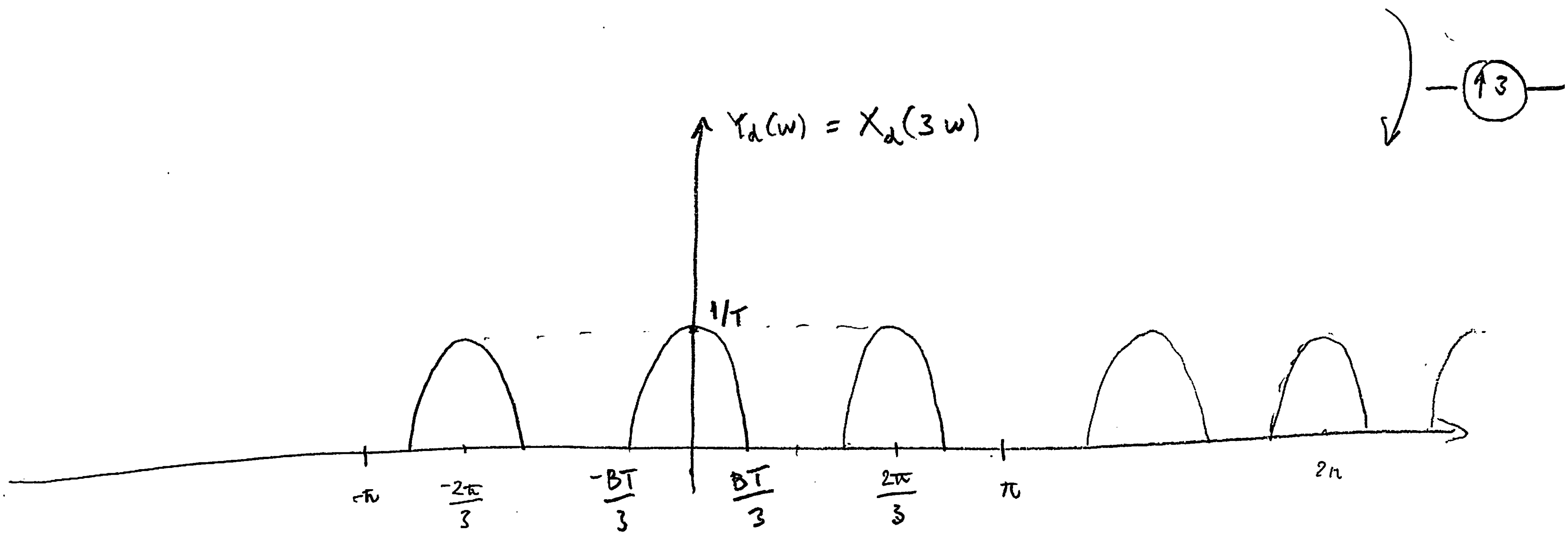
$$= \sum_{\ell=-\infty}^{\infty} x[\ell]\, e^{-j(w\cdot U)\cdot \ell} = X_d(U\cdot w)$$

Upsampler operator (places $U-1$ zeros between samples)

shrinks $X_d(w)$ along the w-axis by factor of U

Ex: $U = 3$.



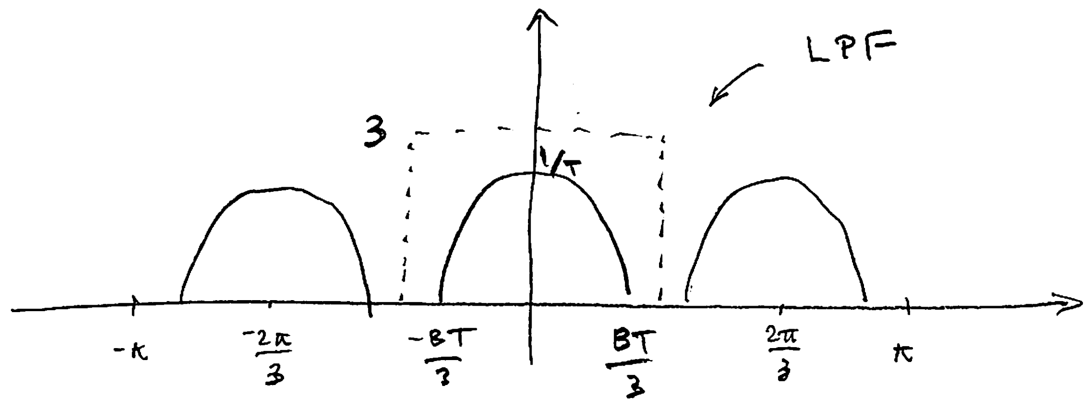


This is almost what we wanted in ⊛.

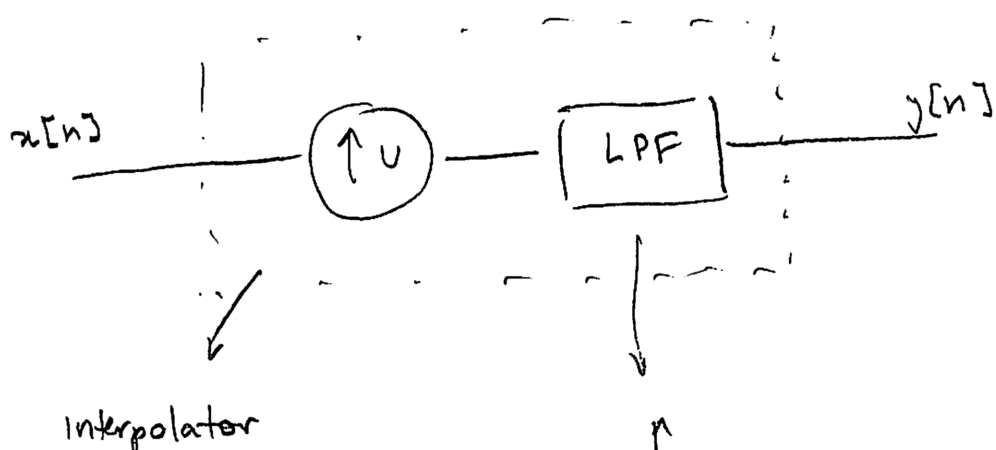
Difference: • 3 copies between $-\pi$ and π (instead of 1)

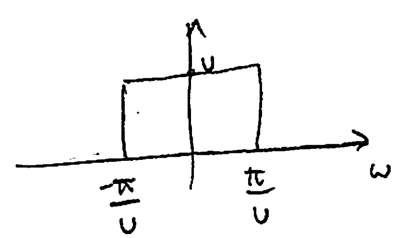
• height is $1/T$ instead of $3/T$

Solution:



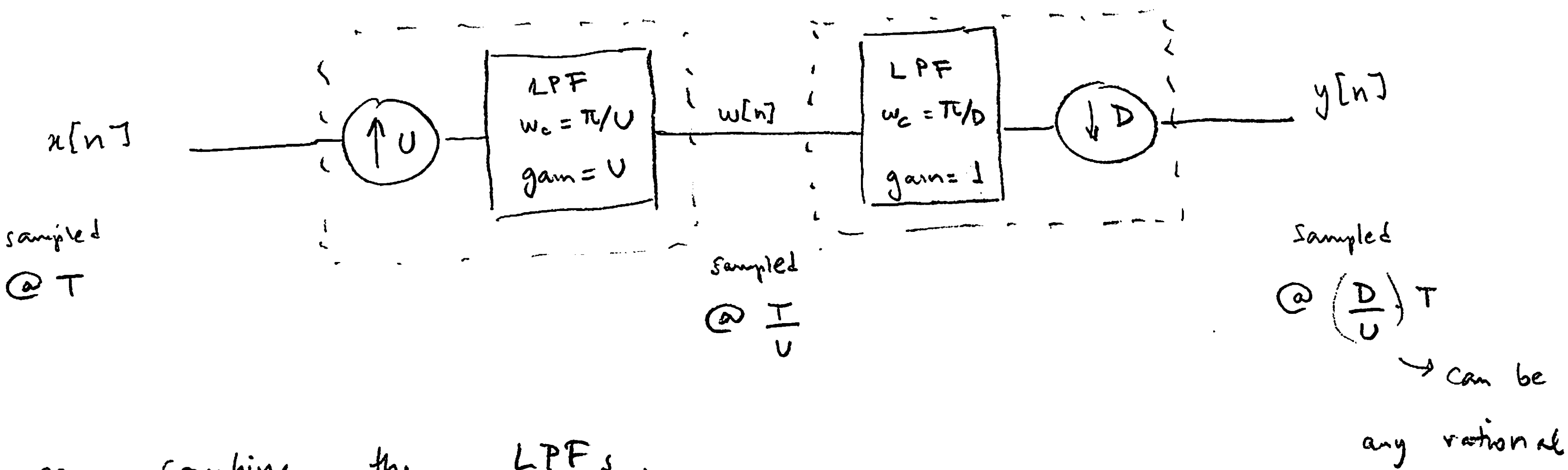
In general, we add a LPF with cutoff $w_c = \frac{\pi}{U}$ and gain $= U$ after the upsampler



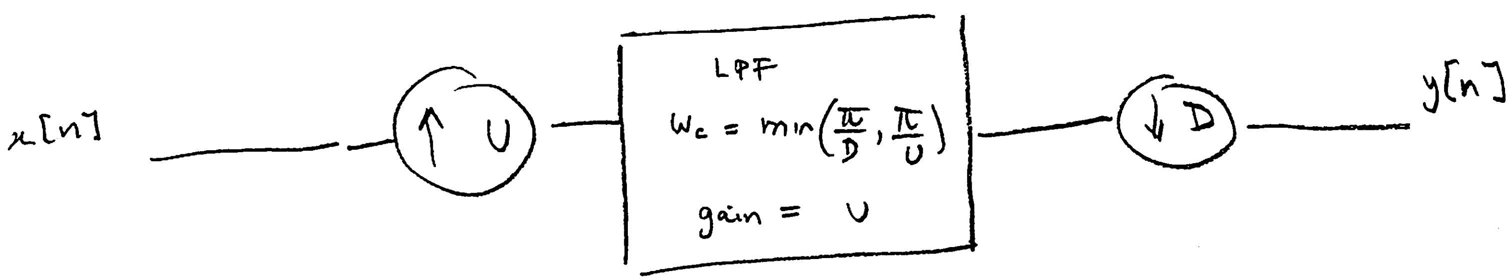


How about non-integer sampling rate conversion?

We combine the interpolator and the decimator



can combine the LPFs:



Can we switch the order of the blocks? No, because upsampler and downsampler are not LTI