University of Illinois Fall 2018

ECE 310: Problem Set 3

Due: 5 pm, September 21, 2018

1. Determine the z-transform and sketch the pole-zero plot with the ROC for each of the following sequences:

   (a) $x[n] = \frac{1}{2}\delta[n-4] - 2\delta[n]$

   (b) $x[n] = \begin{cases} [1,-1,0,4,2], & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$

   (c) $x[n] = \left(\frac{1}{4}\right)^n nu[n] + \left(\frac{1}{3}\right)^n u[n]$

2. Given the z-transform pair $x[n] \leftrightarrow X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$ with ROC: $|z| > 1/2$, use the z-transform properties to determine the z-transform of the following sequences:

   (a) $y[n] = x[n-2]$
   (b) $y[n] = x[n] * x[n-1]$
   (c) $y[n] = nx[n]$
   (d) $y[n] = \left(\frac{3}{2}\right)^n x[n]$

3. Given the z-transform pair $x[n] \leftrightarrow X(z) = 1/(1-0.6z^{-1})$ with ROC: $|z| > 0.6$, use the z-transform properties to determine the sequences corresponding to the following transforms:

   (a) $Y(z) = X^*(3z^*)$
   (b) $Y(z) = X^2(z)$
   (c) $Y(z) = -dX(z)/dz$

4. Find the z-transform (if it exists) and the corresponding region of convergence for each of the following signals. To the extent possible, use the properties of the z-transform to enable the re-use of standard results and reduce calculations. Simplify your expressions. (Recall that for real-valued signals, the transform should only have real-valued coefficients.)

   (a) $x[n] = (\frac{1}{4})^{(n-1)} \sin(\frac{n\pi}{4} + \frac{\pi}{8})u[n-1]$
   (b) $x[n] = n^2 \left(\frac{1}{2}\right)^n u[n]$

5. Find the inverses of the following z-transforms:

   (a) $X(z) = 2 + 3z^{-2} + z^{-4}, \quad 0 < |z| < \infty$

   (b) $X(z) = \dfrac{1}{(1-\frac{1}{2}z^{-1})} + \dfrac{2}{(1-\frac{1}{3}z^{-1})}, \quad |z| > \frac{1}{2}$

   (c) $X(z) = \dfrac{1}{(1-z^{-1})(1-\frac{1}{2}z^{-1})^2}, \quad |z| > 1$