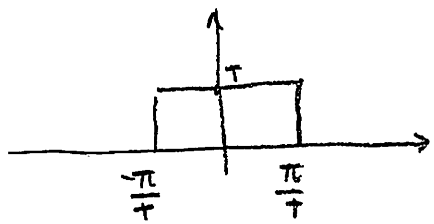
Lecture 26 - Practical D/A

$$y[n] \longrightarrow \boxed{D/A} \longrightarrow y_c(t)$$

$$\uparrow T$$

Ideal D/A

Ideal D/A : $Y_c(\Omega) = \boxed{G(\Omega)}\, Y_d(\Omega T)$ (lecture 14)



Ideal interpolating function.

$$g(t) = CTFT^{-1}\{G(\Omega)\} = \frac{\sin\left(\frac{\pi}{T} t\right)}{\frac{\pi}{T} t}$$

$$\Rightarrow y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \cdot g(t - nT)$$

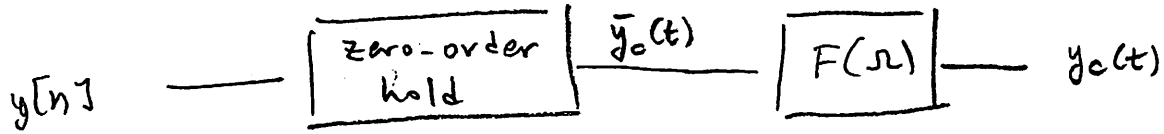
This approach is impractical:

$y_c(t)$ depends on all values of $y[n]$

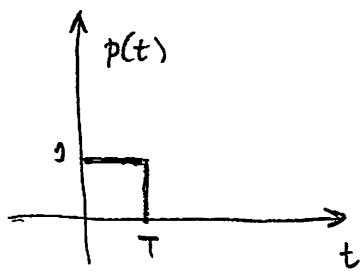
in the past and in the future

Practical D/A system:

compensator filter (analog)

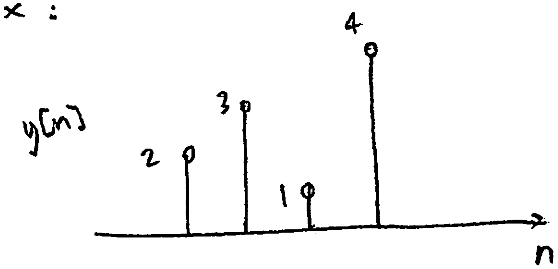


ZOH interpolates y[n] using a rectangular pulse:

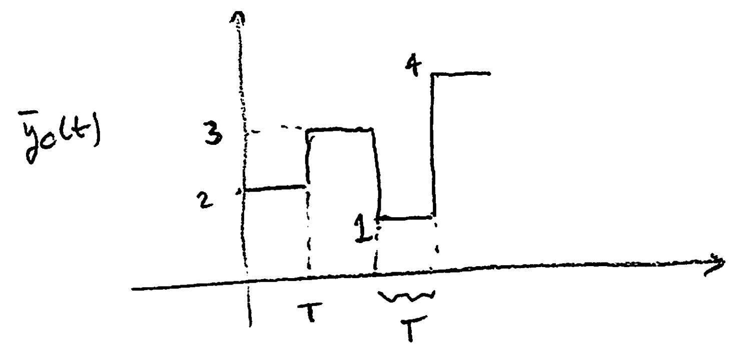


$$\bar{y}_c(t) = \sum_{n=-\infty}^{\infty} y[n] \cdot p(t - nT)$$

Ex:



$\xrightarrow{\text{ZOH}}$



ZOH can be implemented with an analog circuit

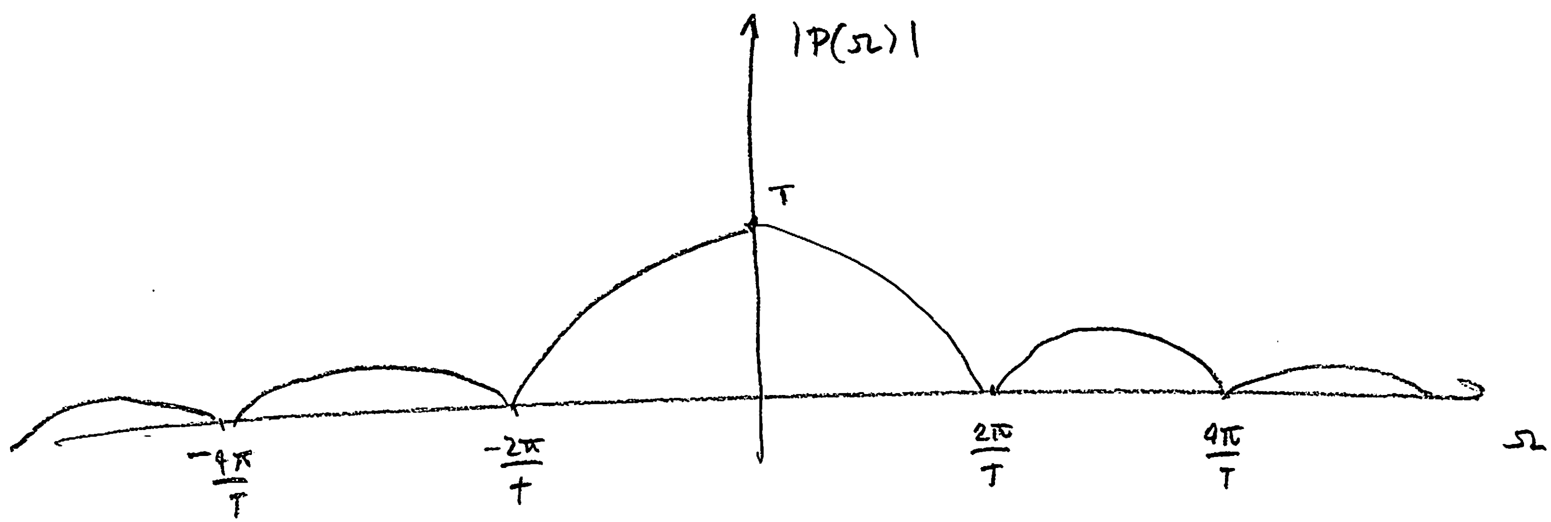
(resistor / capacitor ladder)

In frequency domain: $Y_c(\Omega) = P(\Omega)\, Y_d(\Omega T)$

$$P(\Omega) = \int_{-\infty}^{\infty} p(t)\, e^{-j\Omega t}\, dt = \int_0^T e^{-j\Omega t}\, dt = \left.\frac{e^{-j\Omega t}}{-j\Omega}\right|_0^T = \frac{e^{-j\Omega T} - 1}{-j\Omega}$$

$$= e^{-j\frac{\Omega}{2}T} \frac{\left(e^{j\frac{\Omega}{2}T} - e^{-j\frac{\Omega}{2}T}\right)}{j\Omega} = e^{-j\frac{\Omega}{2}T} \frac{2\sin\left(\frac{\Omega}{2}T\right)}{\Omega}$$

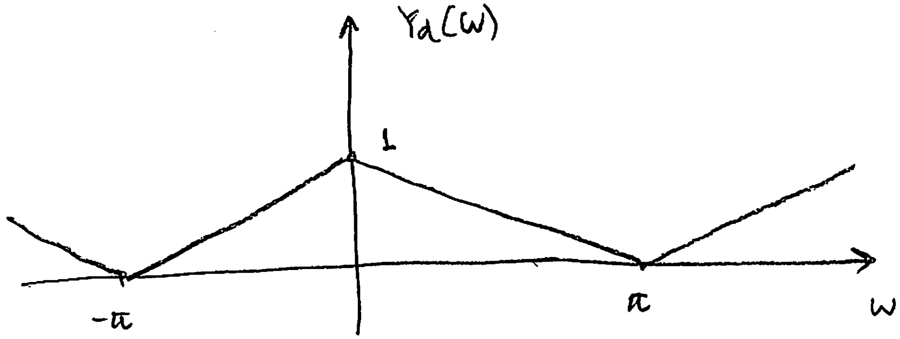
$$= e^{-j\frac{\Omega}{2}T} \cdot T \cdot \mathrm{sinc}\left(\frac{\Omega}{2}T\right)$$

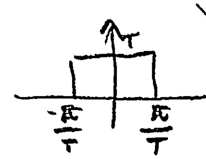


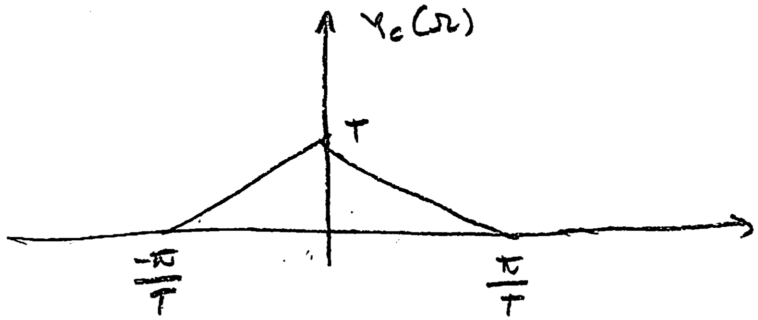
Very different from ideal $G(\Omega)$. Frequency components from $-\infty$ to $+\infty$

We will pick $F(\Omega)$ to compensate the non-ideality of $P(\Omega)$

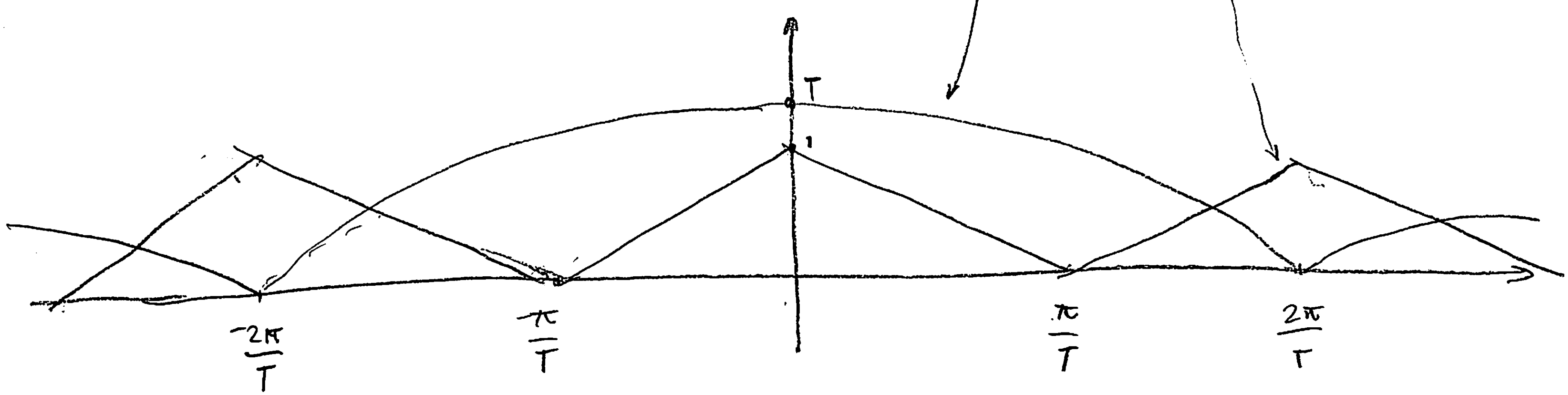
Suppose we have $y[n]$ with FT:

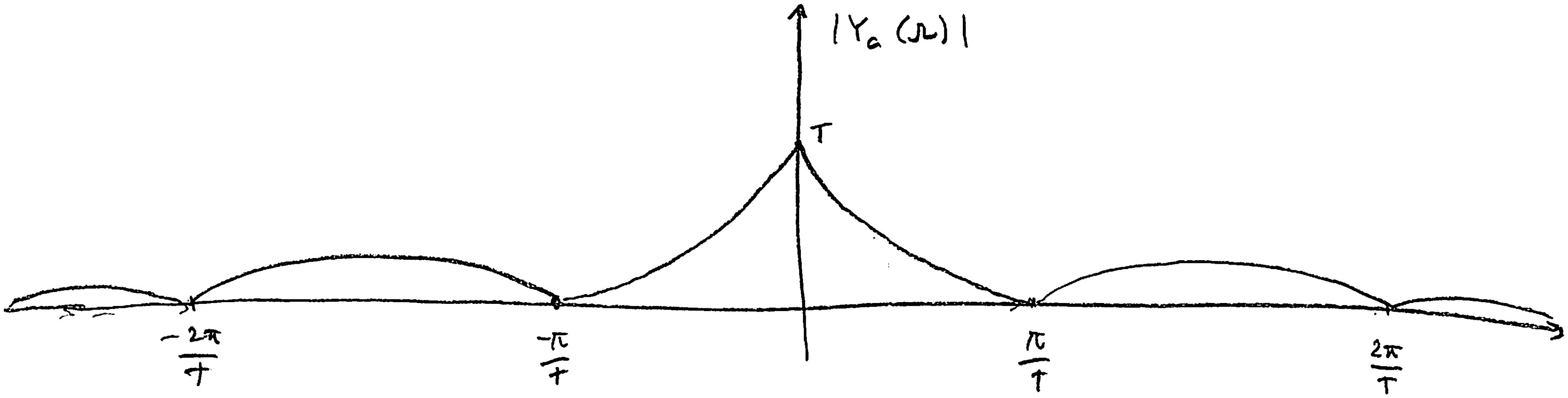


If we use ideal D/A (), the output would be:



For ZOH, let's plot $|\bar{Y}_c(\Omega)| = |P(\Omega)|\,|Y_d(\Omega T)|$





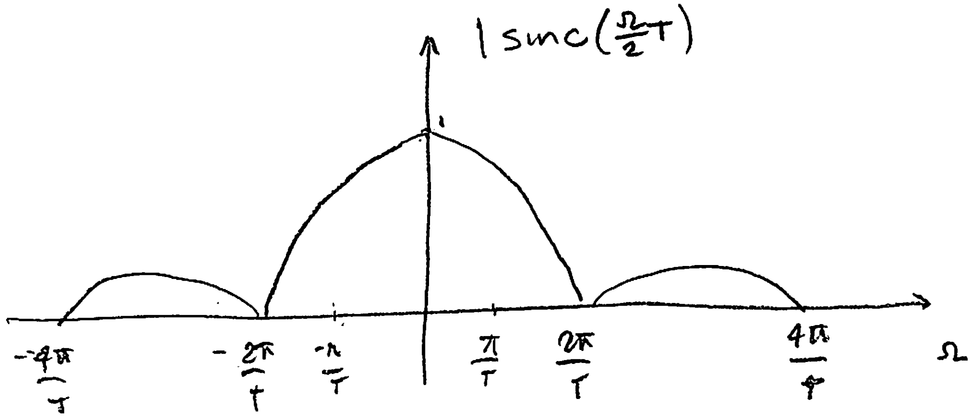
Unlike ideal D/A, $|Y_c(\Omega)|$ has frequencies from $-\infty$ to $+\infty$

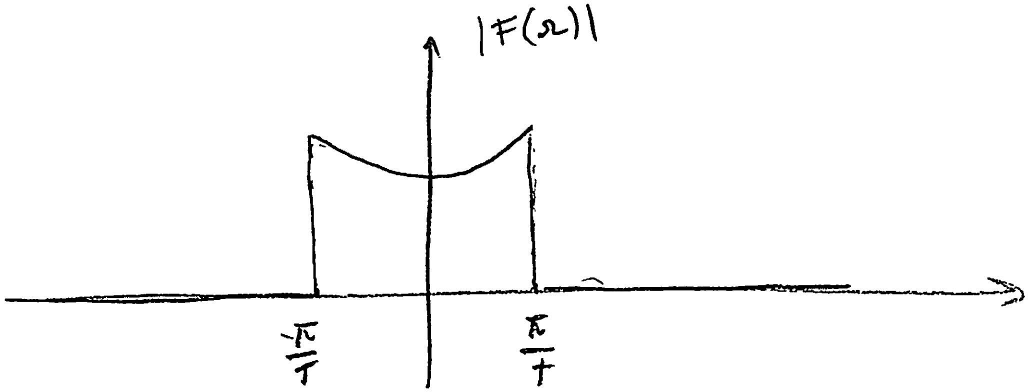
Let's use $F(\Omega)$ to "fix" $P(\Omega)$. We want:

$$F(\Omega)P(\Omega)Y_d(\Omega T) = \begin{cases} T\,Y_d(\Omega T) & \text{for } -\frac{\pi}{T} \le \Omega \le \frac{\pi}{T} \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow F(\Omega) = \begin{cases} \dfrac{T}{P(\Omega)} = \dfrac{e^{j\frac{\Omega}{2}T}}{\operatorname{sinc}\left(\frac{\Omega}{2}T\right)} & \text{for } -\frac{\pi}{T} \le \Omega \le \frac{\pi}{T} \\ 0 & |\Omega| > \pi/T \end{cases}$$

What does $F(\Omega)$ look like?





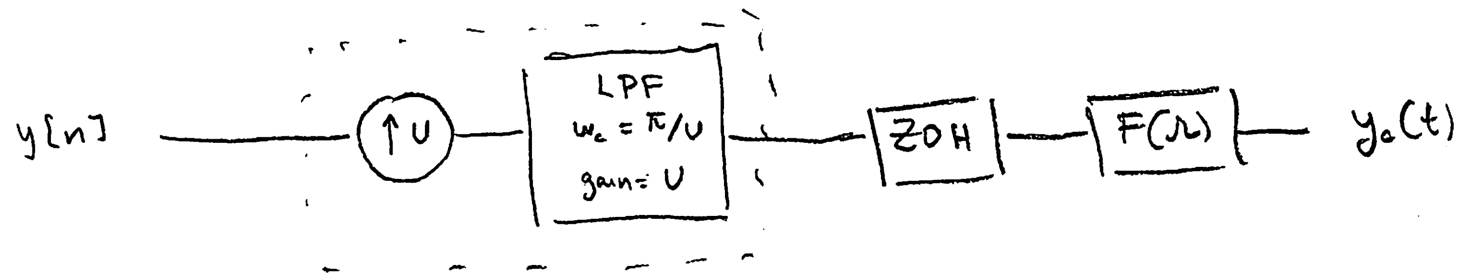
Using this ideal analog compensator, our ZOH-based D/A operates as an ideal D/A

Note 1: $F(\Omega)$ is still a non-causal filter, but it can be made causal via shift and truncate

Note 2: Some designs use a digital pre-compensator before the ZOH so that the analog $F(\Omega)$ can have flat response

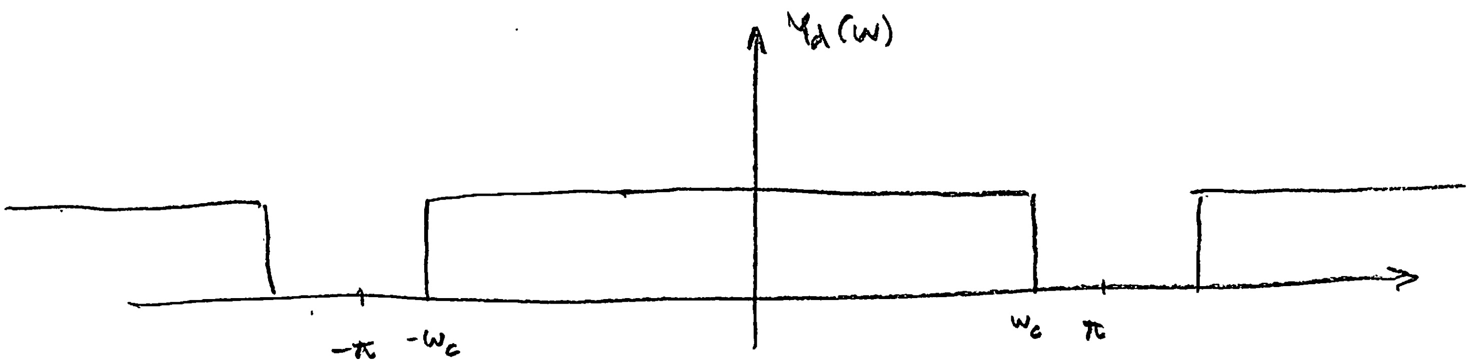
Overgampling D/A :

Idea : Relax the requirements on $F(\Omega)$ by using digital interpolation before ZOH



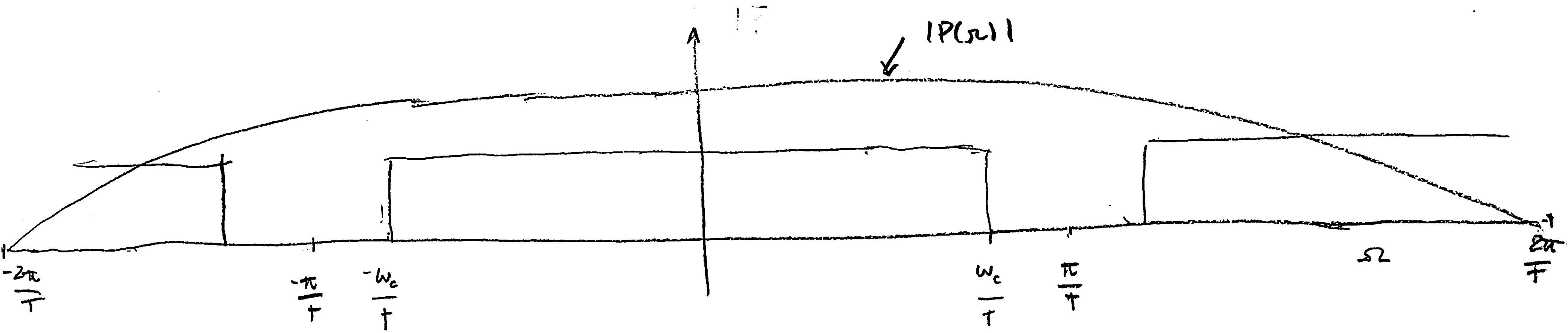
This will allow transition band of $F(\Omega)$ to be wider and the response in pass band to be flatter.

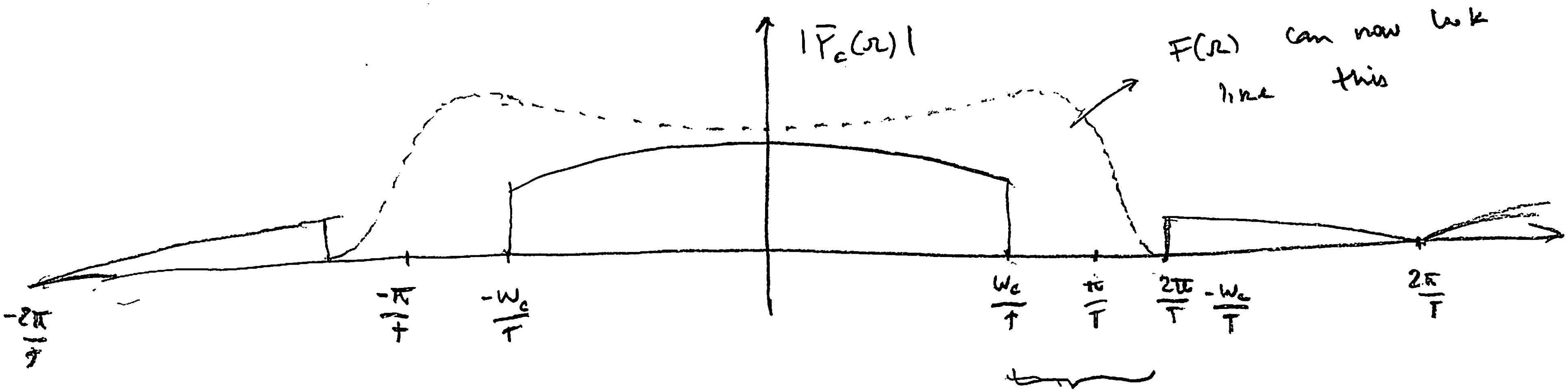
Suppose we have:



If we apply ZOH without oversampling first we get

$$\bar{Y}_c(\Omega) = P(\Omega) \cdot Y_d(\Omega T)$$





can be used for transition band of $F(\Omega)$

By oversampling D/A, we will increase the "room" for transition band from $\frac{2(\pi - w_c)}{T}$ to $\frac{2(U\pi - w_c)}{T}$