ECE 310: Problem Set 12

Due: 5 pm, November 30, 2018

1. A speech signal $x_a(t)$ is assumed to be bandlimited to 10 kHz. It is desired to filter this signal with a bandpass filter that will pass the frequencies between 200 Hz and 4 kHz by using a digital filter $H_d(\omega)$ sandwiched between an ideal A/D and an ideal D/A.
   (a) Determine the Nyquist sampling rate for the input signal.
   (b) Sketch the frequency response $H_{d,1}(\omega)$ for the necessary discrete-time filter, when sampling at the Nyquist rate.
   (c) Find the largest sampling period T for which the A/D, digital filter response ($H_{d,2}(\omega)$), and D/A can perform the desired filtering function. (Hint: some amount of aliasing may be permissible during A/D conversion for this part.)
   (d) For the system using T from part (c), sketch the necessary $H_d(\omega)$.

2. You are given the task of implementing an analog double-echo generator using a system composed of a digital filter sandwiched between an ideal A/D and D/A, both operating at a common sampling period T. The echo generator is specified by the following system equation:

$$y_a(t) = x_a(t) + \alpha x_a(t - \tau) + \beta x_a(t - 2\tau),$$

   where $x_a(t)$ and $y_a(t)$ are the input and output of the analog system, respectively, and τ is the time delay constant. Assume $x_a(t)$ is bandlimited to 20 kHz.
   (a) Find the desired analog frequency response $H_a(\Omega)$. (Your answer will be in terms of τ, α, β, which are assumed to be known.)
   (b) Find the appropriate maximum sampling period $T = T_0$ making this design possible.
   (c) Find the digital filter response $H_d(\omega)$ that is needed to implement the analog system.
   (d) Assuming that $\tau = 100T_0$, find the transfer function $H(z)$ corresponding to $H_d(\omega)$, and draw a block diagram for its implementation.
   (e) Suppose the input to your system is $x_a(t) = \mathrm{sinc}(2000\pi t)$. Determine the output $y_a(t)$.
   (f) Suppose the input to your system is $x_a(t) = \cos(50 \cdot 10^3 \pi t)$. Determine the output $y_a(t)$.

3. Consider the system in Fig. 1. Suppose that $x_a(t)$ ($X_a(f)$ is given in Fig. 2) is bandlimited to $F_c = 20$ kHz and $H(\omega)$ is an ideal low-pass filter with cutoff frequency $\omega_c = \pi/2$. Sketch and label the Fourier Transform of $y_a(t)$ for each of the following cases:
   (a) $T_1 = T_2 = 25 \cdot 10^{-6}$
   (b) $T_1 = T_2 = 12.5 \cdot 10^{-6}$
   (c) $T_1 = 12.5 \cdot 10^{-6}, T_2 = 25 \cdot 10^{-6}$
   (d) $T_1 = 25 \cdot 10^{-6}, T_2 = 12.5 \cdot 10^{-6}$

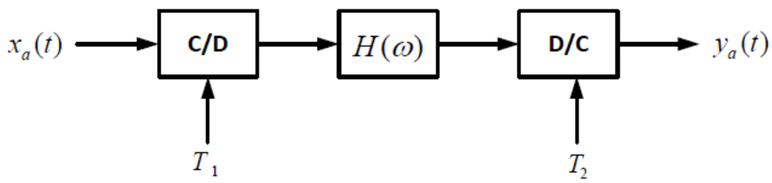

Figure 1: Discrete-Time Implementation of Analog Filtering. The C/D and D/C blocks correspond to ideal A/D and D/A converters, respectively.

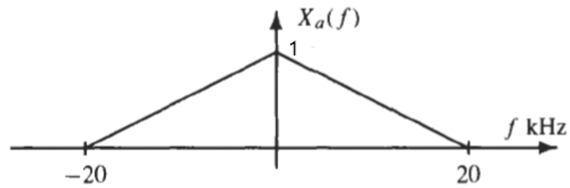

Figure 2: $X_a(f)$ for the input signal $x_a(t)$.

4. Suppose that an analog signal $x_a(t)$ is to be processed using an analog high-pass filter with a passband starting at 2.5 kHz, stopband being the interval $[0, 2\text{kHz}]$ and a stopband attenuation of -40 dB. This filter is to be implemented digitally, as illustrated in Fig. 1 with $T_1 = T_2 = T_s$.

   (a) The block $H(\omega)$ represents the frequency response of a digital FIR filter. Design this filter to have the shortest length possible using the windowing method with one of the standard windows (rectangular, triangular, Hanning, or Hamming) to meet the analog filter specifications. Assume sampling frequency $F_s = 10$ kHz. Your answer should be a closed-form expression for the designed impulse response $h[n]$.

   (b) Suppose that in your design procedure, you forgot to apply the window you have chosen for the design, and used instead a rectangular window of the same length. Sketch (by hand) the approximate analog magnitude frequency response $|H_a(\Omega)|$ for your entire system, over the range of frequencies for which the system functions as an LTI system. Discuss what happens for input with frequencies outside this range. Your sketch should show the approximate shapes of the passband and stopband magnitude responses, and should be labeled to show the height or ripples if any, and band edge frequencies. Do the sketch twice: once using a linear scale, and once using a dB scale for the vertical axis.