University of Illinois

ECE 310: Problem Set 5

Due: 5pm, Friday October 5, 2018

1. Evaluate the following integrals:

   (a) $\int_{-\infty}^{\infty}(t^2 + t - 9)\delta(t + 2)dt$

   (b) $\int_{3}^{\infty}(t^2 + t - 9)\delta(t - 2)dt$

   (c) $\int_{-\infty}^{1}(t^2 + t - 9)\delta(3t - 2)dt$

   (d) $[\cos(\omega t)u(t)] * \delta(3t - 2)$, where $u(t)$ is a unit step function.

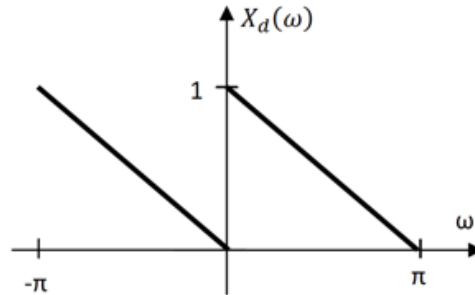2. Determine the Fourier transform of the following functions:

   (a) $\delta(3t - 2)$
   (b) $\sin(2\Omega_0 t + \phi_0)$, where Ω_0 and ϕ_0 are known real numbers.
   (c) $e^{-\frac{1}{2}|t|}$
   (d) $(u(t - 1) - u(t - 3))\, e^{j2\pi t}$

3. Let $x[n]$ be a signal with DTFT as shown in the following figure. Determine and sketch the DTFT of $y[n] = x[n]\cos(\pi n/3)$.

4. Derive closed-form expressions for the DTFT of the following sequences. Sketch the magnitude and phase for parts (a) and (b).
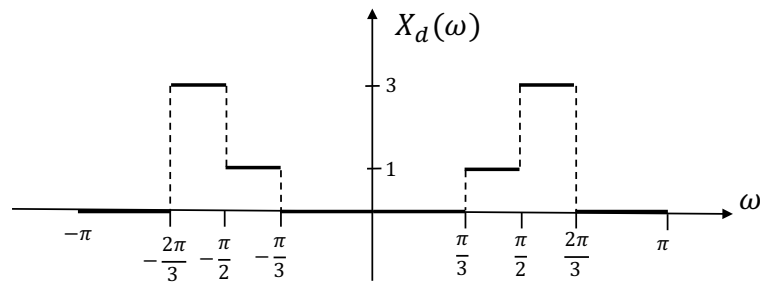
   (a) $x[n] = \delta[n + 3] + \delta[n - 3]$
   (b) $x[n] = u[n] - u[n - 7]$
   (c) $x[n] = \left(\frac{1}{4}\right)^n u[n - 3]$
   (d) $x[n] = \left(\frac{1}{4}\right)^n e^{j\pi n/3}u[n - 5]$
   (e) $x[n] = n\left(\frac{1}{4}\right)^n u[n - 3]$

5. The DTFT of $x[n]$ is as shown below. Determine $x[n]$.

6. Let $X_d(\omega)$ denote the DTFT of the complex valued signal $x[n]$, where the real and imaginary parts of $x[n]$ are given below. Perform the following calculations **without** explicitly evaluating $X_d(\omega)$.

   (a) Evaluate $X_d(0)$

   (b) Evaluate $X_d(\pi)$

   (c) Evaluate $\displaystyle\int_{-\pi}^{\pi} X_d(\omega)d\omega$

   (d) Determine and sketch the signal whose DTFT is $X_d^*(-\omega)$