ECE310: Quiz#1 (6pm Section CSS) Fall 2018 Solutions

1. (6 pts)

   (a) Derive closed-form expressions for the magnitude and phase of the function $G(\omega) = (-j + je^{j2\omega})e^{\omega}$ of the real variable ω.

   (b) Sketch the phase over the interval $-\pi \leq \omega \leq \pi$. Label the axes in your plot, and mark values at the "interesting points."

It's easiest to "split the phase", and use the fact that $-1 = e^{-j\pi}$:

$$-j + je^{j2\omega} = j(e^{j2\omega} - 1)$$
$$= je^{j\omega}(e^{j\omega} - e^{-j\omega})$$
$$= -2\sin(\omega)e^{j\omega}$$
$$= 2\sin(\omega)e^{j(\omega - \pi)}$$

So, we have that

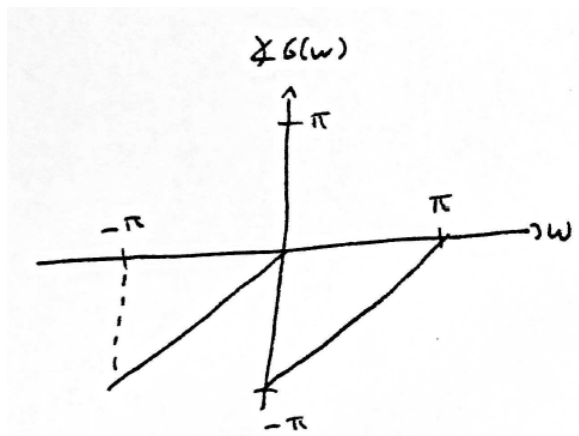$$G(\omega) = 2\sin(\omega)e^{j(\omega - \pi)}e^{\omega}$$

From here, we calculate the magnitude to be

$$\boxed{|G(\omega)| = 2e^{\omega}|\sin(\omega)|}$$

and the phase to be

$$\boxed{\angle G(\omega) = \begin{cases} \omega - \pi & \omega > 0 \\ \omega & \omega < 0 \end{cases}}$$

The π offset in the phase comes from the fact that the magnitude must always be positive. So, whenever $2e^{\omega}\sin(\omega) < 0$, or when $\sin(\omega) < 0$, we signify that $G(\omega)$ takes a negative value by adding/subtracting π from the phase. The plot of the phase can be seen below.

2. (4 pts) Draw a block diagram of a system with input $x[n]$ and output $y[n]$, defined by
$y[n] = 3y[n-2] - 2y[n-1] - 0.5x[n]$.

An example block diagram can be seen below.