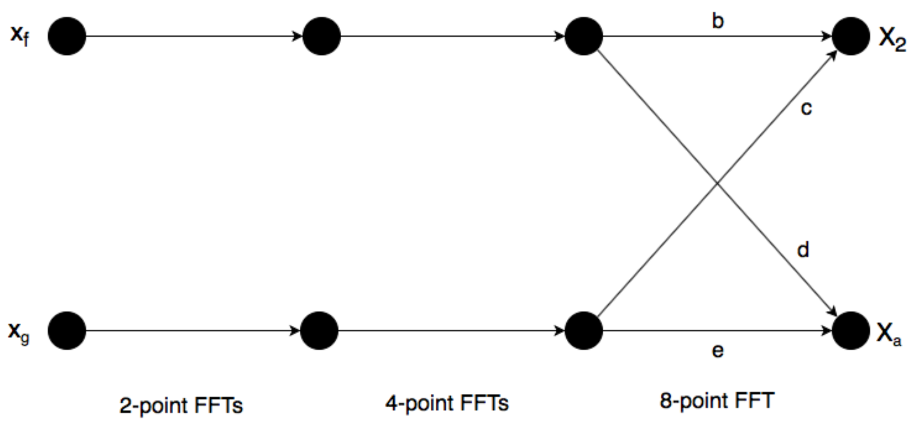
FFT Butterfly Structures

1. Suppose you're given the partially-completed 8-point radix-2 FFT butterfly structure shown below.



   (a) Is the FFT implemented using decimation-in-time or decimation-in-frequency?

   (b) Find a, b, c, d, e, f, and g.

   (c) Assuming no multiplications are trivial, how many complex multiplications and additions would be required to implement the full 8-point FFT?

2. Draw the 4-point decimation-in-time FFT butterfly structure. How would you have to modify it to create the 4-point decimation-in-frequency butterfly? Will the computational complexity of the FFT change upon doing so?

System Block Diagrams

1. Consider the causal LSI system described by the following difference equation:

$$y[n] = x[n] + \frac{1}{16}y[n-4] - x[n-4]$$

   (a) Draw the Direct Form II implementation of the system. Show the system transfer function. Is the system BIBO stable?

   (b) Consider a cascade implementation of the system using second-order subsections with real coefficients. What are the transfer functions of the sub-systems? Implement the cascade using Direct Form I subsections.

2. Suppose you're given the following transfer function:

$$H(z) = \frac{(1-2z^{-1})}{(1-0.5z^{-1})(1-1.5z^{-1})}, |z| > 1.5$$

   (a) Is the system BIBO stable?

   (b) Draw the Direct Form I implementation of the system.

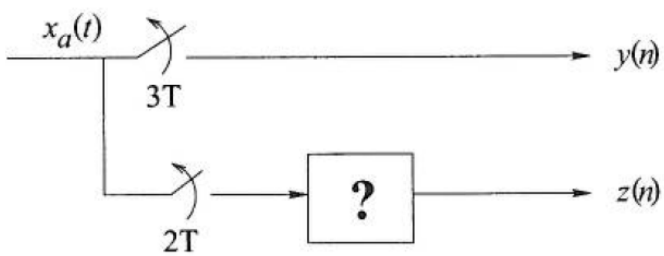
   (c) Draw the Transpose Form I implementation of the system.

   (d) Implement the transfer function in parallel form using two first-order sections, both in Direct Form II.

Rate Conversion Systems

1. For the following system, determine the smallest integer values for L, M, and n_0, respectively and the corresponding $H_d(\omega)$ such that $Y_d(\omega) = X_d(\omega)e^{-j4.5\omega}$.

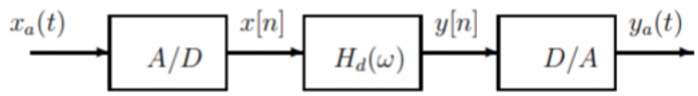
2. Consider the following system consisting of two synchronized ideal A/D converters. Assume that the input analog signal $x_a(t)$ is bandlimited to $\Omega_c = \frac{\pi}{3T}$. Design a digital rate conversion subsystem marked with "?" using down-sampler(s), up-sampler(s), and digital filter(s) as necessary such that $y[n] = z[n]$. Draw a block diagram and determine all the essential parameters of the subsystem. Is your choice of subsystem unique?



Digital Processing of CT Signals

1. After completing ECE 310, Jimmy and Johns decided to start up a new company specialized in DSP. Their first contract was to develop a lowpass system that would filter out all frequencies above 3 kHz in speech signals, which are assumed to be bandlimited to 5 kHz.

   (a) They first started with an ideal design using the following DSP system:

   where $x_a(t)$ is the input analog speech signal, $y_a(t)$ is the output analog speech signal, A/D is an analog-to-digital converter with sampling interval T, D/A is an ideal digital-to-analog converter with the same interpolating interval T, and $H_d(\omega)$ is a digital filter.

   Determine T for the Nyquist sampling frequency and sketch the desired frequency response of the digital filter $H_d(\omega)$ for this system.

   (b) Suppose that an input speech signal has the following Fourier transform

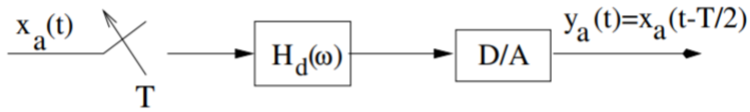
   $$X_a(\Omega) = \begin{cases} 1 - \frac{|\Omega|}{10^4\pi}, & \text{if } |\Omega| \leq 10^4\pi \\ 0 & \text{else} \end{cases}$$

   Sketch $X_d(\omega)$, $Y_d(\omega)$, and $Y_a(\Omega)$ with the T and $H_d(\omega)$ found in part (a).

   (c) Jimmy and Johns then realized that instead of an ideal D/A, they have only a zero-order-hold (ZOH) D/A. Sketch the magnitude of the new output $Y_a(\Omega)$ of part (b) when the ideal D/A is replaced by a ZOH D/A with the same T.

   (d) To obtain the desired output, Jimmy and Johns add a compensated analog filter $F_a(\Omega)$ after the ZOH D/A. Sketch the magnitude response of this filter $F_a(\Omega)$ and specify its transition bandwidth.

2. Consider the following system:



where the D/A convertor is an ideal D/A. Assume that $x_a(t)$ is bandlimited to $\Omega_{\max}$ (rad/sec), T is chosen to be $T < \frac{\pi}{\Omega_{\max}}$ and the impulse response of overall system is $h(t) = \delta(t - T/2)$ (or $H_a(\Omega) = e^{-j\Omega T/2}$).

(a) Determine the frequency response $H_d(\omega)$ of the **desired** digital filter.

(b) Determine the unit pulse response $h[n]$ of the **desired** digital filter.

(c) Determine a length-2 FIR filter $g[n]$ that approximates the above desired filter $h[n]$ using a rectangular window design. Is this **designed** FIR filter $g[n]$ LP or GLP?

D/A Conversion, ZOH, and Compensation Filters

1. Let x[n] be the input to a D/A converter with $T = 5$ms. Sketch the output signal $x_a(t)$ for the following cases. **Label your axis tick marks and units clearly.**

   (a) The D/A converter is a ZOH and $x[n] = 2\delta[n] + 3\delta[n-7]$.

   (b) The D/A converter is an "ideal" D/A and $x[n] = 3\delta[n-7]$.

2. Suppose you're given the following system:

$$y_a(t) \rightarrow \boxed{A/D} \xrightarrow{y[n]} \boxed{\uparrow L} \rightarrow \boxed{G_d(\omega)} \xrightarrow{\tilde{y}[n]} \boxed{ZOH} \rightarrow \boxed{F_a(\Omega)} \rightarrow y_a(t)$$

   where $Y_a(\Omega)$ is bandlimited to $\Omega_0 < \frac{\pi}{T}$, $G_d(\omega)$ is an ideal low-pass filter with a cutoff frequency of $\frac{\pi}{L}$, and the ZOH operates at $\hat{T} = \frac{T}{L}$. We wish to design an analog compensation filter $F_a(\Omega)$, such that from input $y[n]$ to output $y_a(t)$, the system acts as an *ideal* D/A. For any arbitrary L, *derive* an expression for the allowable transition region width of the analog filter.

Convolution

1. The linear convolution $\{x_n\}_{n=0}^{8} * \{h_n\}_{n=0}^{10}$ is to be evaluated using the DFT method. Namely, $\text{DFT}^{-1}\{\text{DFT}\{x_n\} \cdot \text{DFT}\{h_n\}\}$.

   (a) Determine the minimum number of zeros that should be padded to $\{x_n\}$ and $\{h_n\}$, respectively, before the DFTs are applied.

   (b) If the DFTs are to be calculated with a radix-2 FFT algorithm, how many zeros should now be padded to $\{x_n\}$ and $\{h_n\}$, respectively?

   (c) In (a), can the zeros be padded at the beginning (instead of the end) of the sequences? If so, how do you obtain $\{x_n\}_{n=0}^{8} * \{h_n\}_{n=0}^{10}$ from $\text{DFT}^{-1}\{\text{DFT}\{x_n\} \cdot \text{DFT}\{h_n\}\}$?

2. Recall that the convolution of two discrete signals $\{x[n]\}$ and $\{h[n]\}$ is denoted as:

$$(x * h)[h] = \sum_{k=-\infty}^{\infty} x[k]h[n-k].$$

   Prove that $((x * h_1) * h_2)[n] = (x * (h_1 * h_2))[n]$ for all n. Draw equivalent system block diagrams for the left side and right side of the equation. **Note:** In order to use z-transforms, you must prove that the z-transform of $x[n] * h[n]$ is $X(z)H(z)$.

Filter Design

1. (a) Design a GLP length-30 low-pass filter with a cutoff frequency of $\omega_c = \frac{\pi}{4}$, using the window design method with a rectangular window. Give a closed-form expression for the filter coefficients, $h_{LPF}[n]$, as your answer.

   (b) Describe how to modify $h_{LPF}[n]$ to create a high-pass filter with a cutoff frequency of $\omega_c = \frac{3\pi}{4}$.

   (c) Describe how to modify $h_{LPF}[n]$ to create a band-pass filter with cutoff frequencies of $\frac{\pi}{4}$ and $\frac{3\pi}{4}$.

   (d) Describe how to use $h_{LPF}[n]$ to create a band-stop filter, that removes all frequencies between $\frac{\pi}{4}$ and $\frac{3\pi}{4}$.

2. True or False:

   (a) IIR filters can exhibit GLP.

   (b) IIR filters are always stable.

   (c) If an FIR filter is used to approximate $H_d(\omega)$, an IIR filter can provide a similar approximation using a smaller order.

   (d) The Butterworth filter has ripples in the passband, but not in the stopband.

   (e) The Elliptical filter has equiripple in both the passband and stopband.

3. Sketch the magnitude response of the Elliptical, Butterworth, Chebyshev Type I, and Chebyshev Type II low-pass filters. Describe how to identify them, and explain the trade-off between ripple and transition region width.

4. For each of the following impulse responses, determine whether the system is a GLP filter. If so, determine the type and whether it is also a strictly LP filter.

   (a) $\{h[n]\}_{n=0}^{1} = \{1, 1\}$

   (b) $\{h[n]\}_{n=0}^{2} = \{2, 0, -2\}$

   (c) $\{h[n]\}_{n=0}^{2} = \{2, -1, -2\}$