ECE310: Quiz#5 (6pm Section CSS) Fall 2018 Solutions

1. (5 pts) Compute the DTFT of $x[n] = n(-\frac{1}{3})^n u[n-2]$.

Solution

We first compute the z-transform, and verify that its ROC contains the unit circle. If it does, we obtain the DTFT by setting $z = e^{j\omega}$.

Note that the n indices are under different shifts, so we cannot apply the shifting property unless everything is in terms of $n-2$. We can accomplish this by writing $n = n + 2 - 2$:

$$\begin{aligned}
x[n] &= n\left(-\frac{1}{3}\right)^{n-2+2} u[n-2] \\
&= \frac{1}{9} n \left(-\frac{1}{3}\right)^{n-2} u[n-2] \\
&= \frac{1}{9}(n-2+2)\left(-\frac{1}{3}\right)^{n-2} u[n-2] \\
&= \frac{1}{9}(n-2)\left(-\frac{1}{3}\right)^{n-2} u[n-2] + \frac{2}{9}\left(-\frac{1}{3}\right)^{n-2} u[n-2]
\end{aligned}$$

Taking the z-transform gives

$$X(z) = \frac{1}{9} z^{-2}\left(\frac{-\frac{1}{3}z^{-1}}{(1+\frac{1}{3}z^{-1})^2}\right) + \frac{2}{9}z^{-2}\left(\frac{1}{1+\frac{1}{3}z^{-1}}\right), |z| > \frac{1}{3}$$

Because the ROC contains the unit circle, we can substitute $z = e^{j\omega}$, giving the final result as

$$\boxed{X_d(\omega) = \frac{-\frac{1}{27}e^{-j3\omega}}{(1+\frac{1}{3}e^{-j\omega})^2} + \frac{\frac{2}{9}e^{-j2\omega}}{1+\frac{1}{3}e^{-j\omega}}}$$

Alternatively, this solution could have been reached by applying the differentiation property on the z-transform of $\left(-\frac{1}{3}\right)^n u[n-2]$.

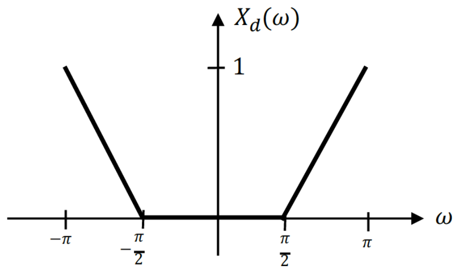
Grading:

- 1 point for applying the time-shift property.
- 1 point for changing the exponent of $\left(-\frac{1}{3}\right)^n$.
- 2 points for breaking $x[n]$ up into two parts, or for application of the differentiation property.
- 1 point for the final answer.

2. (5 pts) Let $x[n]$ be a signal with DTFT $X_d(\omega)$.

   (a) (2 pts) Find an expression for the DTFT of $y[n] = x[n]\cos(\frac{3\pi}{4}n)$ in terms of $X_d(\omega)$.

   (b) (3 pts) Suppose $X_d(\omega)$ is as shown below. Sketch the DTFT of $y[n]$. Label the axes and "important points" on your sketch.



Solution

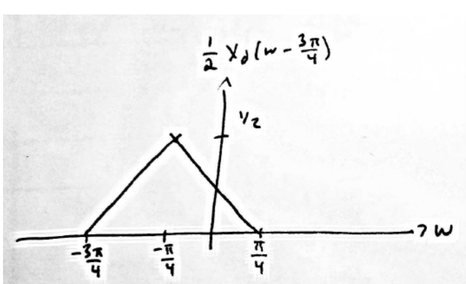
(a) Recalling the modulation property of the DTFT, we can write

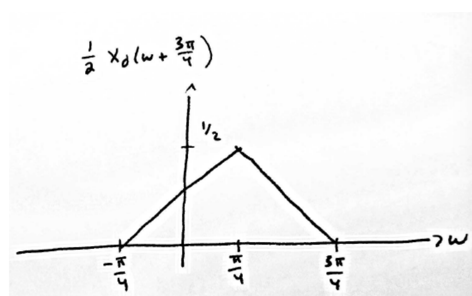
$$x[n]\cos\left(\frac{3\pi}{4}n\right) = x[n]\left(\frac{1}{2}e^{j\frac{3\pi}{4}n} + \frac{1}{2}e^{-j\frac{3\pi}{4}n}\right)$$

making it obvious that

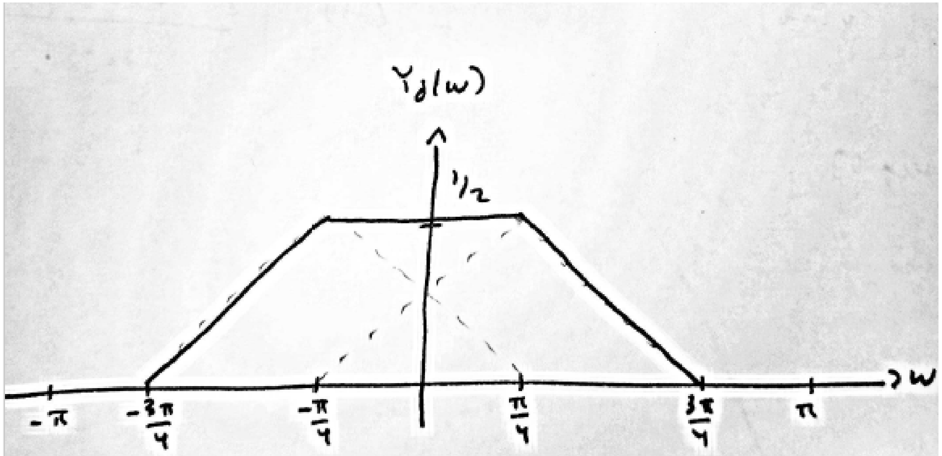
$$\boxed{Y_d(\omega) = \frac{1}{2}\left(X_d\left(\omega - \frac{3\pi}{4}\right) + X_d\left(\omega + \frac{3\pi}{4}\right)\right)}$$

(b) It's easiest to perform the addition graphically, recalling the 2π-periodicity of the DTFT. Sketching out $X_d(\omega + \frac{3\pi}{4})$ and $X_d(\omega - \frac{3\pi}{4})$ gives the following:





making it more obvious that the addition of the two gives



Grading:

- 1 point for graph of $X_d(\omega + \frac{3\pi}{4})$.
- 1 point for graph of $X_d(\omega - \frac{3\pi}{4})$.
- 1 point for the final answer.