ECE310: Quiz#5 (10am Section G) Fall 2018 Solutions

1. (5 pts) Compute the DTFT of $x[n] = \left(\frac{1}{3}\right)^n e^{j\frac{\pi}{3}n} u[n-1]$.

Solution

The easiest approach is to use z-transforms and intermediate signals. Letting $v[n] = \left(\frac{1}{3}\right)^n u[n-1]$, we can find $V(z)$:

$$v[n] = \frac{1}{3}\left(\frac{1}{3}\right)^{n-1} u[n-1]$$

$$V(z) = \frac{\frac{1}{3}z^{-1}}{1 - \frac{1}{3}z^{-1}}, \ |z| > \frac{1}{3}$$

Because the ROC contains the unit circle, we can find the DTFT of $v[n]$ by substituting $z = e^{j\omega}$. Doing so gives

$$V_d(\omega) = \frac{\frac{1}{3}e^{-j\omega}}{1 - \frac{1}{3}e^{-j\omega}}$$

Then, noting that $x[n] = v[n]e^{j\frac{\pi}{3}n}$, we can use the modulation property of the DTFT to get the result:

$$\boxed{X_d(\omega) = V_d(\omega - \frac{\pi}{3}) = \frac{1}{3}e^{-j(\omega - \frac{\pi}{3})} \frac{1}{1 - \frac{1}{3}e^{-j(\omega - \frac{\pi}{3})}}}$$

Grading:

- 1 point for separating $\left(\frac{1}{3}\right)^n = \frac{1}{3}\left(\frac{1}{3}\right)^{n-1}$.
- 2 points for calculating the DTFT of $\frac{1}{3}\left(\frac{1}{3}\right)^{n-1}u[n-1]$.
- 1 point for applying the modulation property.
- 1 point for the final answer.

2. (5 pts) Let $x[n]$ be a signal with DTFT $X_d(\omega)$.

    (a) Find an expression for the DTFT of $y[n] = \cos\left(\frac{\pi}{4}n\right)$ in terms of $X_d(\omega)$.

    (b) Suppose $X_d(\omega)$ is as shown below. Sketch the DTFT of $y[n]$. Label the axes and "important points" on your sketch,.
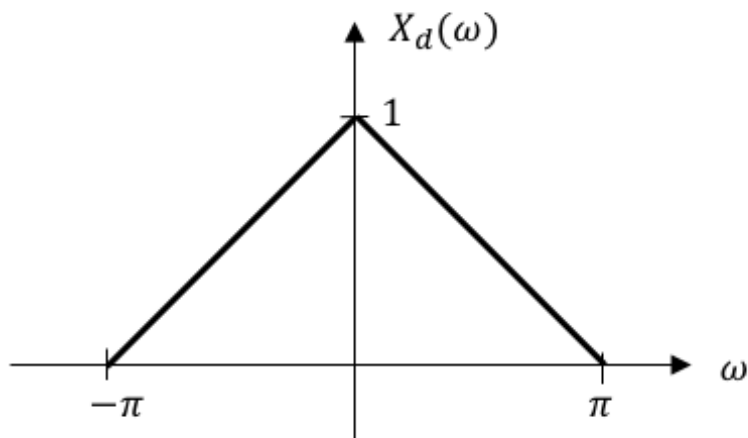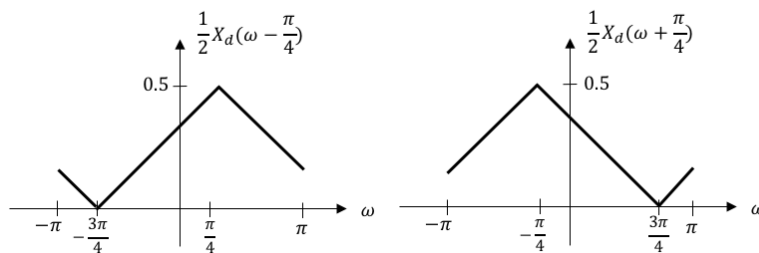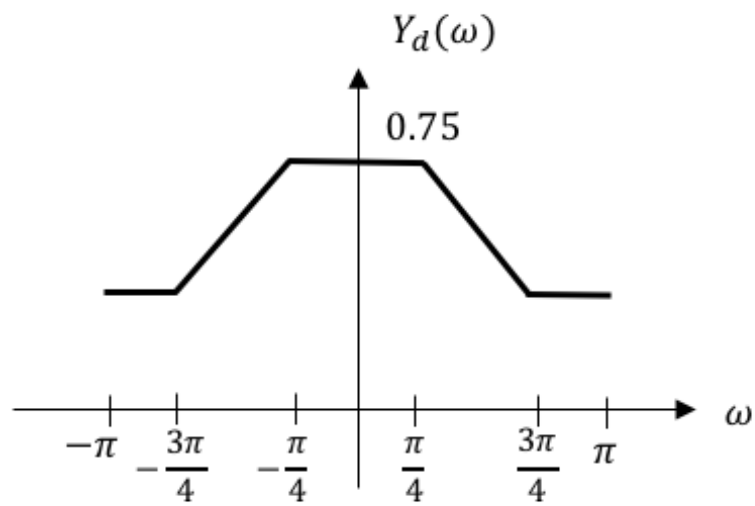

Solution

(a) Recall the modulation property of the DTFT; if $y[n] = x[n]e^{j\omega_0 n}$, then $Y_d(\omega) = X_d(\omega - \omega_0)$. Therefore,

$$y[n] = \frac{1}{2}e^{j\frac{\pi}{4}n}x[n] + \frac{1}{2}e^{-j\frac{\pi}{4}n}x[n] \rightarrow \boxed{Y_d(\omega) = \frac{1}{2}X_d(\omega - \frac{\pi}{4}) + \frac{1}{2}X_d(\omega + \frac{\pi}{4})}$$

(b) It's easiest to perform the addition graphically, sketching the two copies separately. Recalling that any DTFT must be 2π-periodic, the two copies are shown below:

Performing the addition gives the result below.

$$Y_d(\omega)$$

0.75

$-\pi \quad -\dfrac{3\pi}{4} \quad -\dfrac{\pi}{4} \quad \dfrac{\pi}{4} \quad \dfrac{3\pi}{4} \quad \pi \qquad \omega$

Grading:

- 2 points for (a).
- 1 point for each shifted graph.
- 1 point for the final answer.