ECE 310 Quiz 2 (Section CSS) Fall 2018 Solutions

1. (6 pts)

$$y[n] = x[n] \cdot x[n-2]$$

(i) linear or non-linear

Suppose the following:
$y_1[n] = x_1[n]x_1[n-2]$
$y_2[n] = x_2[n]x_2[n-2]$
$y_3[n] = x_3[n]x_3[n-2]$

Let $y_3'[n] = ay_1[n] + by_2[n]$ and $x_3[n] = ax_1[n] + bx_2[n]$

$$\begin{aligned} y_3[n] &= x_3[n]x_3[n-2] \\ &= (ax_1[n] + bx_2[n])(ax_1[n-2] + bx_2[n-2]) \\ &= a^2x_1[n]x_1[n-2] + abx_1[n]x_2[n-2] + abx_2[n]x_1[n-2] + b^2x_2[n]x_2[n-2] \end{aligned}$$

$$\begin{aligned} y_3'[n] &= ay_1[n] + by_2[n] \\ &= a(x_1[n]x_1[n-2]) + b(x_2[n]x_2[n-2]) \end{aligned}$$

Because $y_3[n] \neq y_3'[n]$, the system is **non-linear**

(ii) time-invariant or time-varying

$x[n-n_0] = x_1[n] \xrightarrow{\mathcal{H}} y_1[n] = x_1[n]x_1[n-2] = x[n-n_0]x[n-n_0-2]$
$y[n-n_0] = x[n-n_0]x[n-n_0-2]$

Because $y_1[n] = y[n-n_0]$, the system is **time-invariant**

2. (4 pts)

$$x[n] = u[n-2] - 0.5u[n-5]$$
$$h[n] = (n^3 - 2)(u[n] - u[n-2])$$

$$\begin{aligned} h[n] &= (n^3 - 2)(u[n] - u[n-2]) \\ &= (n^3 - 2)(\delta[n] + \delta[n-1]) \\ &= -2\delta[n] - \delta[n-1] \end{aligned}$$

$$\begin{aligned} x[n] * h[n] &= x[n] * (-2\delta[n] - \delta[n-1]) \\ &= x[n] * (-2\delta[n]) + x[n] * (-\delta[n-1]) \\ &= -2x[n] - x[n-1] \\ &= -2u[n-2] + u[n-5] - u[n-3] + 0.5u[n-6] \end{aligned}$$