University of Illinois

# ECE 310: Problem Set 8

**Due:** 5pm, Friday October 26, 2018

1. For each of the following finite length sequences, determine the DFT $X[k]$. Sketch the magnitude and phase for parts (a) and (b).
   (a) $x[n] = \delta[n-5]$, $0 \le n \le 5$
   (b) $x[n] = \begin{cases} 0, & 0 \le n \le 2 \\ 1, & 3 \le n \le 5 \end{cases}$
   (c) $x[n] = \cos\left(\frac{\pi n}{4}\right)$, $0 \le n \le 7$
   (d) $x[n] = \begin{cases} 1, & n \text{ even}, 0 \le n \le 6 \\ 0, & n \text{ odd}, 0 \le n \le 6 \end{cases}$

2. Let $X[k] = \left\{1, e^{-j\frac{\pi}{2}}, 0, e^{j\frac{\pi}{2}}\right\}$ be the DFT of $\{x[n]\}_{n=0}^{3}$.
   (a) Without computing the inverse DFT of $X[k]$, show that $x[n]$ is real-valued.
   (b) Compute the inverse DFT of $X[k]$ to find $x[n]$.
   (c) Use DFT properties to find the inverse DFT of the sequence $Y[k] = \{1, -1, 0, -1\}$, using your answer to part (a).

3. Let $X[k]$ be the 8-point DFT of the sequence $x[n] = [1, -1, 2, 3, -3, 0, 0, 0]$. Let $y[n]$ be a finite length sequence whose DFT is $Y[k] = X[k]e^{-j\frac{2\pi}{6}kn_0}$, where $n_0 = 3$. Determine the sequence $y[n]$.

4. Consider the real finite length sequence $\{x[n]\}_{n=0}^{4} = \{0, 1, 2, 3, 4\}$, and its DFT $\{X[k]\}_{k=0}^{4}$.
   (a) Let $y[n]$ be a finite length sequence whose DFT is $Y[k] = e^{j4\pi k/5}X[k]$. Sketch $y[n]$.
   (b) Let $w[n]$ be a finite length sequence whose DFT is $W[k] = \mathrm{Im}\{X[k]\}$. Sketch $w[n]$.

5. A real continuous-time signal $x_c(t)$ is bandlimited to frequencies below 5kHz, i.e., $X_c(\Omega) = 0$ for $|\Omega| \ge 2\pi(5000)$. The signal $x_c(t)$ is sampled with a sampling rate of 10,000 Hz to produce a sequence $x[n] = x_c(nT)$. Let $X[k]$ be the 1000-point DFT of $x[n]$. To what continuous-time frequency do the indices $k = 150$ and $k = 800$ correspond?

6. Let $X[k]$ denote the 48-point DFT of $x[n], 0 \le n \le 47$. The sequence $y[n]$ is obtained by zero-padding $x[n]$ to length 256. Determine $k_0$ such that $Y[48] = X[k_0]$.

7. Let $X[k], (0 \le k \le 20)$ and $X_d(\omega)$ respectively be the 21-point DFT and DTFT of a *real-valued* sequence $\{x_n\}_{n=0}^{7}$ that is zero-padded to length 21. Determine all the **correct** relationships and justify your answer.
   (a) $X[19] = X_d(-\frac{4\pi}{21})$.
   (b) $X[2] = X_d^*(-\frac{4\pi}{21})$
   (c) $X[12] = X_d(-\frac{4\pi}{21})$
   (d) $X[4] = X_d^*(-\frac{4\pi}{21})$