

ECE 310 Quiz 3 Section E Fall 2018 Solutions

1. (5 pts)

$$x[n] = \frac{1}{2} \left(\frac{1}{2} \right)^{n-1} u[n-1] - \delta[n-1]$$

Recall the following:

$$\begin{aligned} a^n u[n] &\xleftrightarrow{z} \frac{z}{z-a} \quad \text{ROC : } |z| > a \\ \delta[n] &\xleftrightarrow{z} 1 \quad \text{ROC : } \forall z \\ x[n-k] &\xleftrightarrow{z} z^{-k} X(z) \end{aligned}$$

Use the above to transform $x[n]$ into the z-domain:

$$\begin{aligned} X(z) &= \frac{1}{2} z^{-1} \left(\frac{z}{z - \frac{1}{2}} \right) - z^{-1} \\ &= \frac{1}{2} z^{-1} \left(\frac{z}{z - \frac{1}{2}} - 2 \right) \\ &= \frac{1}{2} z^{-1} \left(\frac{z}{z - \frac{1}{2}} - 2 \frac{z - \frac{1}{2}}{z - \frac{1}{2}} \right) \\ &= \frac{1}{2} z^{-1} \left(\frac{-z + 1}{z - \frac{1}{2}} \right) \\ X(z) &= \boxed{-\frac{1}{2} \left(\frac{z - 1}{z^2 - \frac{1}{2}z} \right) \quad \text{ROC : } |z| > \frac{1}{2}} \end{aligned}$$

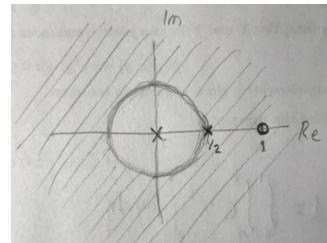
$$\boxed{\text{zeros : } z = 1 \quad \text{poles : } z = 0, \frac{1}{2}}$$

Grading:

+3 for $X(z)$

+1 for ROC

+1 for poles/zeros



2. (5 pts)

$$y[n] = x[n] * (3^{n-2}x[n-2]) \quad X(z) = \frac{1}{1 - \frac{1}{6}z^{-1}} \quad \text{ROC : } |z| > \frac{1}{6}$$

Recall that:

$$a^n u[n] \quad \xleftrightarrow{z} \quad \frac{1}{1 - az^{-1}} \quad \text{ROC : } |z| > a$$

Therefore, $x[n] = (\frac{1}{6})^n u[n]$.

$$\begin{aligned} \text{Let } h[n] &= 3^{n-2}x[n-2] \\ &= 3^{n-2} \left(\frac{1}{6}\right)^{n-2} u[n-2] \\ &= \left(\frac{1}{2}\right)^{n-2} u[n-2] \end{aligned}$$

Then:

$$H(z) = z^{-2} \frac{1}{1 - \frac{1}{2}z^{-1}} \quad \text{ROC : } |z| > \frac{1}{2}$$

Recall $y[n] = x[n] * h[n] \quad \xleftrightarrow{z} \quad Y(z) = X(z)H(z)$

$$Y(z) = \left(\frac{1}{1 - \frac{1}{6}z^{-1}}\right) \left(\frac{z^{-2}}{1 - \frac{1}{2}z^{-1}}\right) \quad \text{ROC : } |z| > \frac{1}{2}$$

Alternatively we can find $H(z)$ by recalling the scaling property:

$$\begin{aligned} a^n x[n] &\quad \xleftrightarrow{z} \quad X(a^{-1}z) \quad \text{ROC : } |a|R_x \\ H(z) &= z^{-2} X(3^{-1}z) = \frac{z^{-2}}{1 - \frac{1}{6}(3z^{-1})} = \frac{z^{-2}}{1 - \frac{1}{2}z^{-1}} \quad \text{ROC : } |z| > \frac{1}{2} \end{aligned}$$

Grading:

- +2 for $H(z)$
- +2 for $Y(z) = X(z)H(z)$
- +1 for ROC

Note that both $Y(z)$ and the ROC are needed for the z-transform of $y[n]$.

The book's definition of the z-transform: The z-transform of a sequence consists of an algebraic formula and its associated ROC. Thus, to uniquely specify a sequence $x[n]$, we need both $X(z)$ and its ROC.