

## ECE 310 Quiz 3 Section E Fall 2018 Solutions

1. (5 pts)

$$x[n] = \frac{1}{2} \left( \frac{1}{2} \right)^{n-1} u[n-1] - \delta[n-1]$$

Recall the following:

$$\begin{aligned} a^n u[n] &\xleftrightarrow{\mathcal{Z}} \frac{z}{z-a} \quad \text{ROC: } |z| > a \\ \delta[n] &\xleftrightarrow{\mathcal{Z}} 1 \quad \text{ROC: } \forall z \\ x[n-k] &\xleftrightarrow{\mathcal{Z}} z^{-k} X(z) \end{aligned}$$

Use the above to transform  $x[n]$  into the z-domain:

$$\begin{aligned} X(z) &= \frac{1}{2} z^{-1} \left( \frac{z}{z - \frac{1}{2}} \right) - z^{-1} \\ &= \frac{1}{2} z^{-1} \left( \frac{z}{z - \frac{1}{2}} - 2 \right) \\ &= \frac{1}{2} z^{-1} \left( \frac{z}{z - \frac{1}{2}} - 2 \frac{z - \frac{1}{2}}{z - \frac{1}{2}} \right) \\ &= \frac{1}{2} z^{-1} \left( \frac{-z + 1}{z - \frac{1}{2}} \right) \\ X(z) &= \boxed{-\frac{1}{2} \left( \frac{z-1}{z^2 - \frac{1}{2}z} \right) \quad \text{ROC: } |z| > \frac{1}{2}} \end{aligned}$$

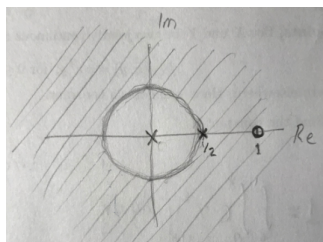
$$\boxed{\text{zeros: } z = 1 \quad \text{poles: } z = 0, \frac{1}{2}}$$

**Grading:**

+3 for  $X(z)$

+1 for ROC

+1 for poles/zeros



2. (5 pts)

$$y[n] = x[n] * (3^{n-2}x[n-2]) \quad X(z) = \frac{1}{1 - \frac{1}{6}z^{-1}} \quad \text{ROC} : |z| > \frac{1}{6}$$

Recall that:

$$a^n u[n] \xleftrightarrow{\mathcal{Z}} \frac{1}{1 - az^{-1}} \quad \text{ROC} : |z| > a$$

Therefore,  $x[n] = \left(\frac{1}{6}\right)^n u[n]$ .

$$\begin{aligned} \text{Let } h[n] &= 3^{n-2}x[n-2] \\ &= 3^{n-2}\left(\frac{1}{6}\right)^{n-2}u[n-2] \\ &= \left(\frac{1}{2}\right)^{n-2}u[n-2] \end{aligned}$$

Then:

$$H(z) = z^{-2} \frac{1}{1 - \frac{1}{2}z^{-1}} \quad \text{ROC} : |z| > \frac{1}{2}$$

Recall  $y[n] = x[n] * h[n] \xleftrightarrow{\mathcal{Z}} Y(z) = X(z)H(z)$

$$\boxed{Y(z) = \left(\frac{1}{1 - \frac{1}{6}z^{-1}}\right)\left(\frac{z^{-2}}{1 - \frac{1}{2}z^{-1}}\right) \quad \text{ROC} : |z| > \frac{1}{2}}$$

Alternatively we can find  $H(z)$  by recalling the scaling property:

$$\begin{aligned} a^n x[n] &\xleftrightarrow{\mathcal{Z}} X(a^{-1}z) \quad \text{ROC} : |a|R_x \\ H(z) &= z^{-2}X(3^{-1}z) = \frac{z^{-2}}{1 - \frac{1}{6}(3z^{-1})} = \frac{z^{-2}}{1 - \frac{1}{2}z^{-1}} \quad \text{ROC} : |z| > \frac{1}{2} \end{aligned}$$

**Grading:**

+2 for  $H(z)$

+2 for  $Y(z) = X(z)H(z)$

+1 for ROC

Note that both  $Y(z)$  and the ROC are needed for the z-transform of  $y[n]$ .

The book's definition of the z-transform: The z-transform of a sequence consists of an algebraic formula and its associated ROC. Thus, to uniquely specify a sequence  $x[n]$ , we need both  $X(z)$  and its ROC.