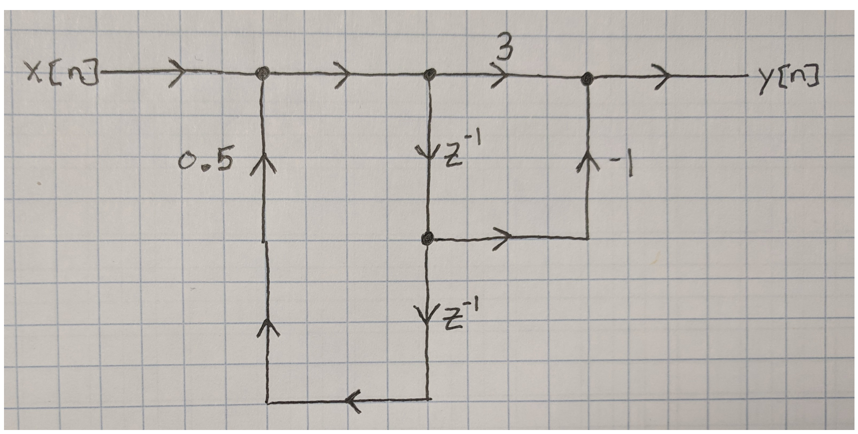
ECE 310: Quiz #9 Solution (3pm Section E) Fall 2018

November 28th, 2018

1. (5 pts) Draw the Direct Form II filter structure for a system with the following transfer function:

$$\frac{3 - z^{-1}}{1 - 0.5z^{-2}}$$

Solution



2. (5 pts) Design a length-5 GLP FIR **high-pass** filter with cutoff frequency $\omega_c = \frac{\pi}{5}$ radians. Use the window design method with a Hann window $\left(w\left[n\right] = 0.5 - 0.5\cos\left(\frac{2\pi n}{M}\right)\right.$ for $\left. n = 0, \ldots, M\right)$. Give your answer in terms of a closed-form expression for the filter coefficients $\{h_n\}_{n=0}^{4}$.

Solution

This problem can be solved either by direct design or by designing an appropriate low-pass filter and modulating by $e^{j\pi n}$ or $e^{-j\pi n}$. A high-pass filter can be implemented using either a Type I or a Type IV GLP FIR filter. In this case, the filter order is $M = L - 1 = 5 - 1 = 4$, so a Type I filter will be used.

Direct design:
The desired frequency response is

$$H_d\left(\omega\right) = \begin{cases} 1 & , \frac{\pi}{5} < |\omega| < \pi \\ 0 & , \text{otherwise} \end{cases}$$

Perform the inverse DTFT

$$\begin{aligned}
h_d\left[n\right] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d\left(\omega\right) e^{j\omega n} d\omega \\
&= \frac{1}{2\pi}\int_{-\pi}^{-\frac{\pi}{5}} 1 e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{\frac{\pi}{5}}^{\pi} 1 e^{j\omega n} d\omega \\
&= \frac{1}{2\pi j n}\left[e^{j\omega n}\right]_{-\pi}^{-\frac{\pi}{5}} + \frac{1}{2\pi j n}\left[e^{j\omega n}\right]_{\frac{\pi}{5}}^{\pi} \\
&= \frac{1}{2\pi j n}\left(e^{-j\frac{\pi}{5}n} - e^{-j\pi n}\right) + \frac{1}{2\pi j n}\left(e^{j\pi n} - e^{j\frac{\pi}{5}n}\right) \\
&= \frac{1}{2\pi j n}\left(e^{-j\frac{\pi}{5}n} - \left(-1\right)^n + \left(-1\right)^n - e^{j\frac{\pi}{5}n}\right) \\
&= \frac{1}{2\pi j n}\left(e^{-j\frac{\pi}{5}n} - e^{j\frac{\pi}{5}n}\right) \\
&= \frac{1}{2\pi j n} e^{-j\pi n}\left(e^{j\frac{4\pi}{5}n} - e^{j\frac{6\pi}{5}n}\right) \\
&= \frac{1}{2\pi j n} e^{-j\pi n}\left(e^{j\frac{4\pi}{5}n} - e^{-j\frac{4\pi}{5}n}\right) \\
&= e^{-j\pi n}\frac{\sin\left(\frac{4\pi}{5}n\right)}{\pi n}
\end{aligned}$$

Now shift by $\frac{M}{2}$ and then apply the window

$$
\begin{aligned}
h[n] &= h_d\left[n - \frac{M}{2}\right] w[n] \\
&= \begin{cases} (-1)^n \frac{\sin\left(\frac{4\pi}{5}(n-2)\right)}{\pi(n-2)} \left(0.5 - 0.5\cos\left(\frac{2\pi n}{4}\right)\right) & , 0 \leq n \leq 4 \\ 0 & , \text{otherwise} \end{cases}
\end{aligned}
$$

Modulating a low-pass filter:
Modulate a low-pass filter with cutoff frequency $\pi - \frac{\pi}{5} = \frac{4\pi}{5}$ by $e^{j\pi n}$ or $e^{-j\pi n}$ (here, the 2π-periodicity of the DTFT is used). The desired frequency response of the low-pass filter is

$$
H_{lp}(\omega) = \begin{cases} 1 & , |\omega| \leq \frac{4\pi}{5} \\ 0 & , \text{otherwise} \end{cases}
$$

This corresponds to

$$
h_{lp}[n] = \frac{\sin\left(\frac{4\pi}{5} n\right)}{\pi n}
$$

Obtain the desired impulse response of the high-pass filter by modulation

$$
h_d[n] = e^{j\pi n} h_{lp}[n] = e^{j\pi n} \frac{\sin\left(\frac{4\pi}{5} n\right)}{\pi n}
$$

Now shift by $\frac{M}{2}$ and then apply the window

$$
\begin{aligned}
h[n] &= h_d\left[n - \frac{M}{2}\right] w[n] \\
&= \begin{cases} (-1)^n \frac{\sin\left(\frac{4\pi}{5}(n-2)\right)}{\pi(n-2)} \left(0.5 - 0.5\cos\left(\frac{2\pi n}{4}\right)\right) & , 0 \leq n \leq 4 \\ 0 & , \text{otherwise} \end{cases}
\end{aligned}
$$

Note that both methods give the same result.