ECE 310: Problem Set 1

Due:5pm, Friday September 7, 2018

Problem 1 (20pts)

(a) $10\angle 135° + 6\angle -120°$

In Cartesian form (3pts)

$$10\angle 135° = -10\cos 45° + j10\sin 45° = -5\sqrt{2} + j5\sqrt{2}$$
$$6\angle -120° = -6\cos 60° - j6\sin 60° = -3 - j3\sqrt{3}$$
$$\Rightarrow 10\angle 135° + 6\angle -120° = \boxed{-5\sqrt{2} - 3 + j(5\sqrt{2} - 3\sqrt{3})} \approx -10.071 + j1.875$$

In polar form (3pts, final approximation is accepted)

$$\text{Magnitude} = \sqrt{(5\sqrt{2}+3)^2 + (5\sqrt{2} - 3\sqrt{3})^2} \approx \sqrt{10.071^2 + 1.875^2} = 10.244$$
$$\text{Angle} = \tan^{-1}\frac{5\sqrt{2} - 3\sqrt{3}}{-5\sqrt{2} - 3} \approx \tan^{-1}\frac{1.875}{-10.071} = -10.55° + 180° = 169.45°$$
$$\Rightarrow \boxed{10.244\angle 169.45°}$$

(b) $\frac{(1-j)^2}{1+j}$

In Cartesian form: (3pts)

$$\frac{(1-j)^2}{1+j} = \frac{(1-j)^3}{(1+j)(1-j)} = \frac{1}{2}(1-j)^3 = \frac{1}{2}(1 - j3 - 3 - j) = \boxed{-1-j}$$

In polar form: (3pts)

$$\text{Magnitude} = \sqrt{1+1} = \sqrt{2}$$
$$\text{Angle} = 180° + \tan^{-1} 1 = 225° = -135°$$
$$\Rightarrow \boxed{\sqrt{2}\angle -135°}$$

(c) In Cartesian form: (6pts)

$$\left(\frac{-1+j3}{1-j} + \frac{3+j}{1+j2}\right)^n = \left(\frac{(-1+j3)(1+j2) + (3+j)(1-j)}{(1-j)(1+j2)}\right)^n = \left(\frac{-3-j}{3+j}\right)^n = (-1)^n$$
$$= \boxed{\begin{cases} 1, & n \text{ even} \\ -1, & n \text{ odd} \end{cases}}$$

In polar form: (2pts, $e^{-j\pi n}$ is not required for full credits)

$$\begin{cases} 1, & n \text{ even} \\ -1, & \text{odd} \end{cases} = \boxed{e^{-j\pi n}}$$

Problem 2 (20 pts)

Recall Euler's formula: $e^{j\omega} = \cos\omega + j\sin\omega$.

$$\begin{aligned} G(\omega) &= 1 - e^{-2j\omega} = e^{-j\omega}(e^{j\omega} - e^{-j\omega}) \\ &= e^{-j\omega}(\cos\omega + j\sin\omega - \cos(-\omega) - j\sin(-\omega)) \\ &= e^{-j\omega}(\cos\omega + j\sin\omega - \cos\omega + j\sin\omega) \\ &= e^{-j\omega} j2\sin\omega \end{aligned}$$

Note that we can write j as $e^{j\frac{\pi}{2}}$

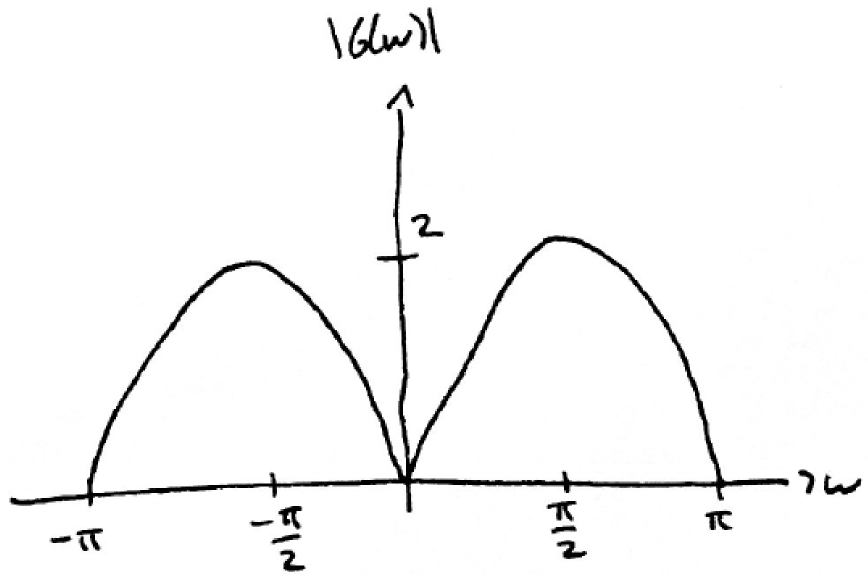
$$\begin{aligned} G(\omega) &= e^{-j\omega}e^{j\frac{\pi}{2}}2\sin\omega \\ &= e^{j(\frac{\pi}{2}-\omega)}2\sin\omega \end{aligned}$$

Therefore, for $\omega \in [-\pi, \pi]$

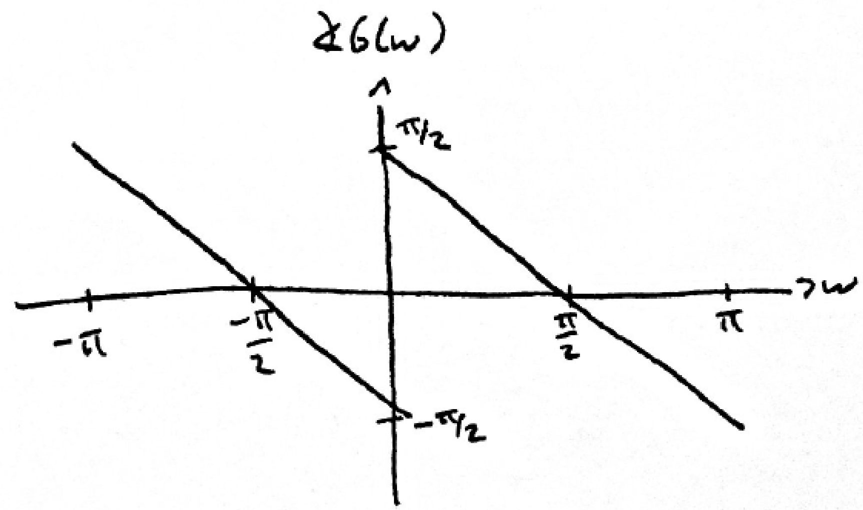
$$\begin{aligned} |G(\omega)| &= \boxed{2|\sin\omega|} \\ \angle G(\omega) &= \boxed{\begin{cases} \frac{\pi}{2} - \omega - \pi & -\pi < \omega < 0 \\ \frac{\pi}{2} - \omega & 0 < \omega < \pi \end{cases}} \end{aligned}$$

(10pts, equivalent answers that can produce correct plots are accepted)

Plot of $|G(\omega)|$, for $\omega \in [-\pi, \pi]$: (5pts)



Plot of $\angle G(\omega)$, for $\omega \in [-\pi, \pi]$: (5pts)



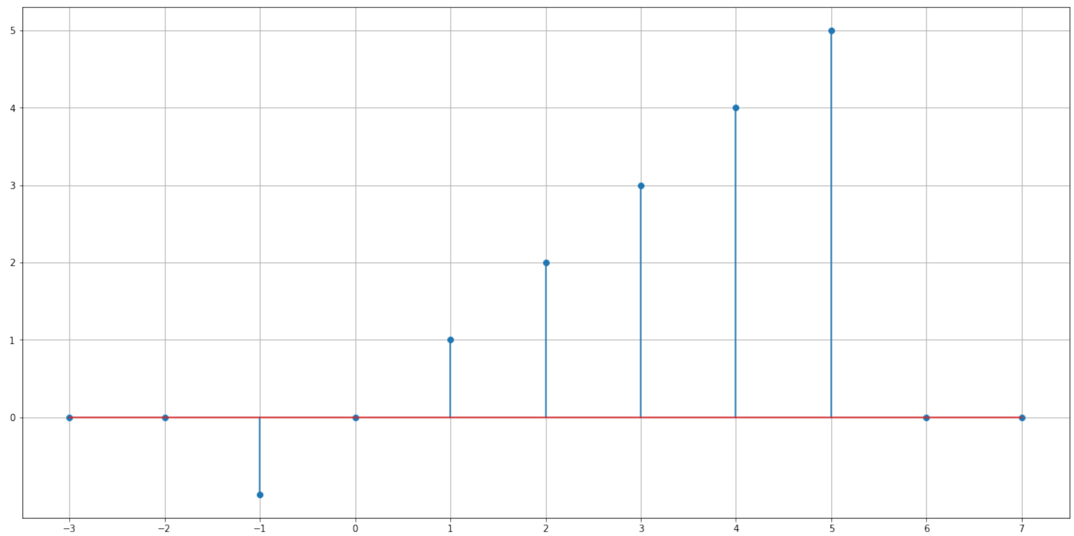
Problem 3 (20 pts)

(a) Recall that $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$.

So, $u[n+1] = \begin{cases} 1 & n \geq -1 \\ 0 & \text{otherwise} \end{cases}$ and $u[n-6] = \begin{cases} 1 & n \geq 6 \\ 0 & \text{otherwise} \end{cases}$.

Therefore, $u[n+1] - u[n-6] = \begin{cases} 1 & -1 \leq n \leq 5 \\ 0 & \text{otherwise} \end{cases}$. We multiply with n to produce $n(u[n+1] - u[n-6])$.

(5pts for derivation, 5pts for plot)

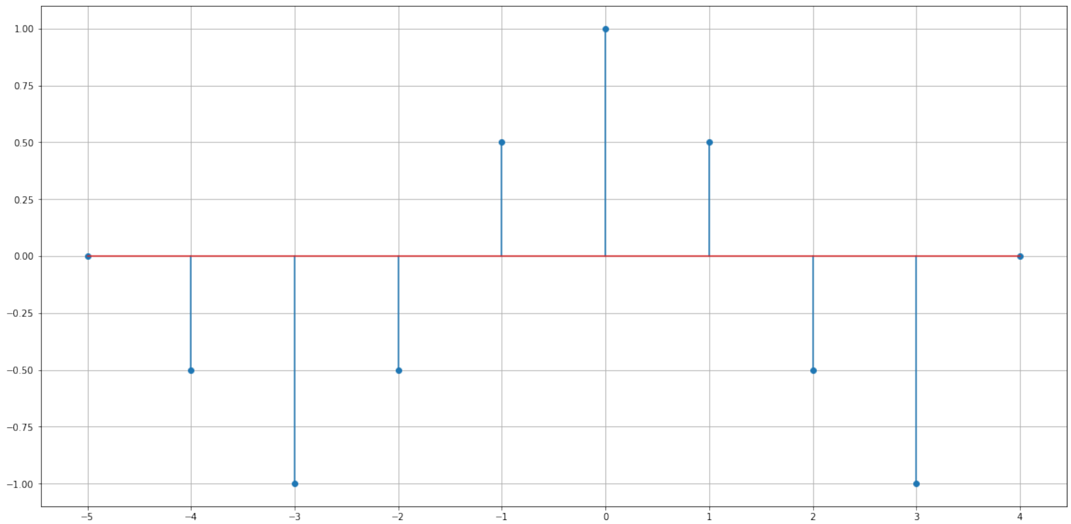


(b) $u[n+4] = \begin{cases} 1 & n \geq -4 \\ 0 & \text{otherwise} \end{cases}$ and $u[-n+3] = \begin{cases} 1 & n \leq 3 \\ 0 & \text{otherwise} \end{cases}$.

Therefore, $u[n+4]u[-n+3] = \begin{cases} 1 & -4 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$.

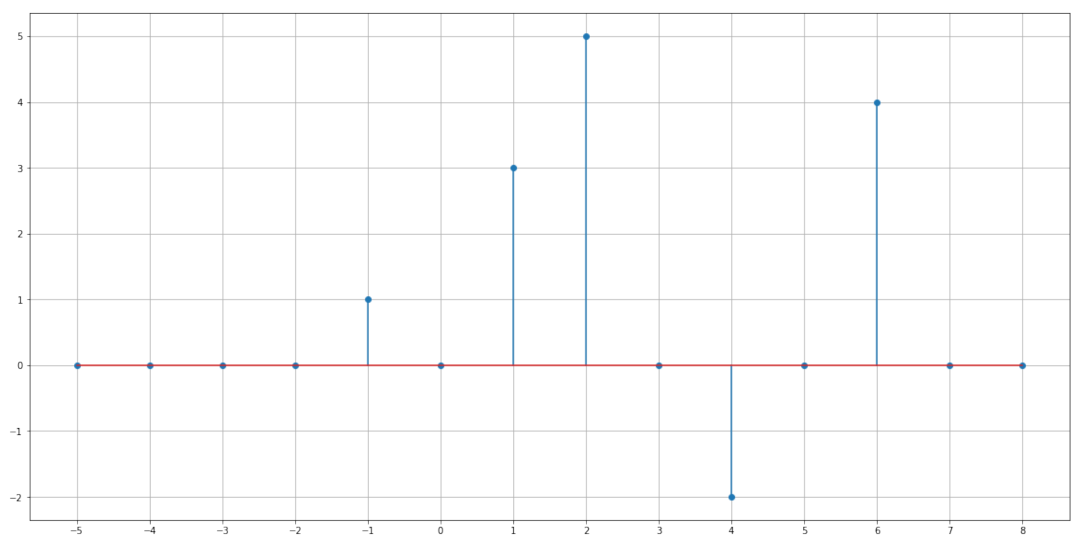
$\cos(\frac{n\pi}{3})u[n+4]u[-n+3] = \cos(\frac{n\pi}{3})$, for $-4 \leq n \leq 3$.

(5pts for derivation, 5pts for plot)

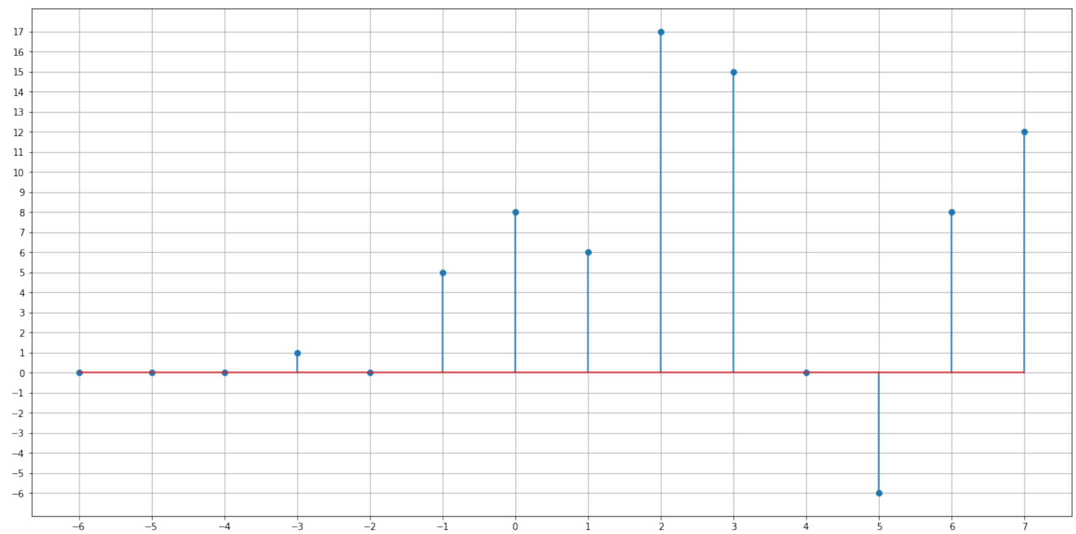


Problem 4 (20pts)

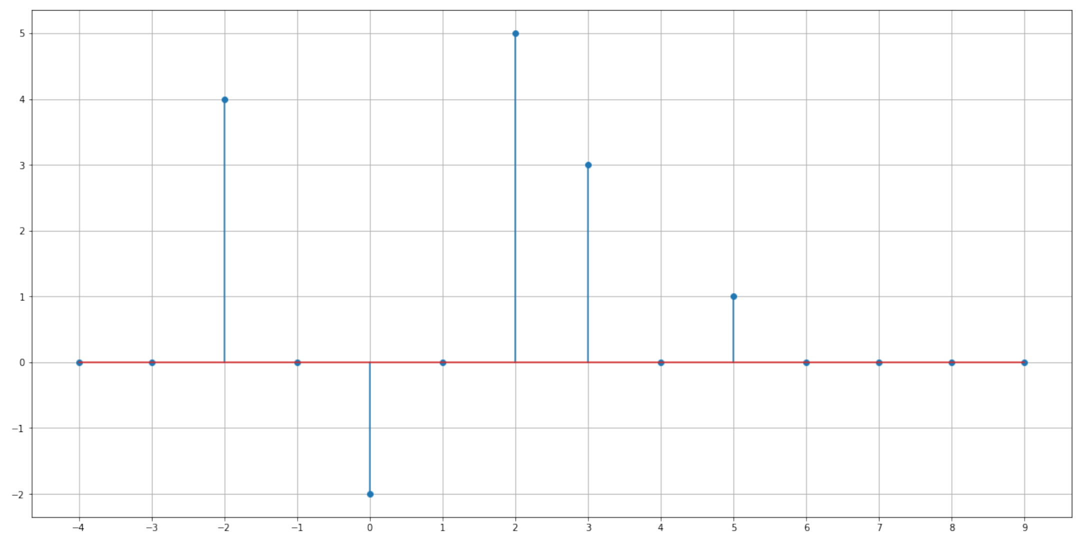
(a) $y[n] = x[n-2]$ (5pts)



(b) $w[n] = x[n] + 2x[n-2] + 3x[n-3]$ (5pts)



(c) $v[n] = x[-n+2]$ (5pts)



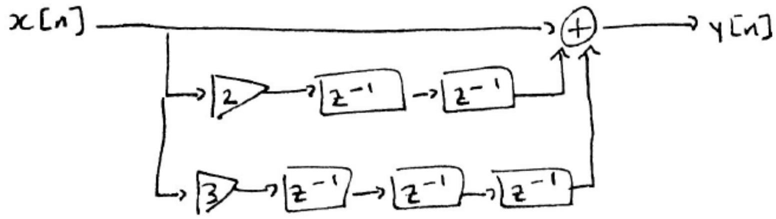
(d) $x[n] = \boxed{\delta[n+3] + 3\delta[n+1] + 5\delta[n] - 2\delta[n-2] + 4\delta[n-4]}$ (5pts)

Problem 5 (20pts)

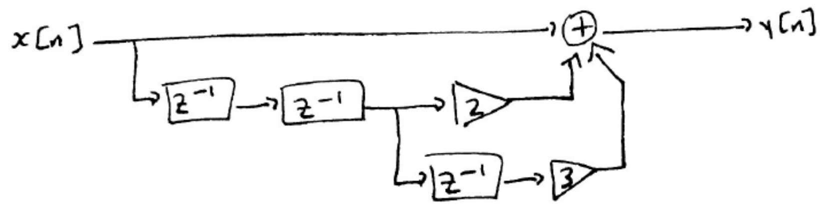
(a) $y[n] = x[n] + 2x[n-2] + 3x[n-3]$ (10pts)

(Blocks of z^{-k} are accepted here, but it is suggested to use multiple blocks of z^{-1}.)

Method 1:



Method 2:



(b) $y[n] = -0.5y[n-1] + 2x[n]$ (10pts)