

### Midterm Exam

7:00-8:30pm, Thursday, March 1, 2018

Name: \_\_\_\_\_

Section: 9:00 AM      12:00 PM      3:00 PM

NetID: \_\_\_\_\_

Score: \_\_\_\_\_

Problem	Pts.	Score
1	10	
2	12	
3	8	
4	10	
5	10	
6	5	
7	7	
8	15	
9	15	
10	8	
Total	100	

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### Instructions

- You may not use any books, calculators, or notes other than two handwritten two-sided sheets of 8.5" x 11" paper.
  - Show all your work to receive full credit for your answers.
  - When you are asked to "calculate", "determine", or "find", this means providing closed-form expressions (i.e., without summation or integration signs).
  - Neatness counts. If we are unable to read your work, we cannot grade it.
  - Turn in your entire booklet once you are finished. No extra booklet or papers will be considered.
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(10 Pts.)

1. Answer **True** or **False** to each of the following statements:

- (a) Let  $\sum_{n=-\infty}^{\infty} x[n] \delta[\sin(\pi n)] = 2$ . Then  $\sum_{n=-\infty}^{\infty} x[n] = 2$ . **True/False**
- (b) Suppose that  $x[n]$  has  $z$ -transform  $X(z)$ . The DTFT of  $x[n]$  can always be expressed as:  
 $X_d(\omega) = X(z)|_{z=e^{j\omega}}$ . **True/False**
- (c) The DTFT of the sequence  $x[n] = \cos\left(\frac{\pi}{2}n\right)$ ,  $-\infty < n < \infty$  is  $X_d(\omega) = \pi \left[ \delta(\omega - \frac{\pi}{2}) + \delta(\omega + \frac{\pi}{2}) \right]$ ,  
for  $-\infty < \omega < \infty$ . **True/False**
- (d) The BIBO stability of **any** system is completely determined by the system's unit pulse response. **True/False**
- (e) The output  $y[n]$  of a system for an arbitrary input  $x[n]$  is given by  $y[n] = x[n] * h[n]$ , where  $h[n]$  is the unit pulse response of the system. The system must be linear and shift-invariant. **True/False**

(12 Pts.)

2. For each of the systems with input  $x[n]$  and output  $y[n]$  shown in the table, indicate by “yes” or “no” whether the properties indicated apply to the system.

	Linear	Shift-Invariant	Causal	Stable
$y[n] = x[n - 1] + x[n] + x[n + 1]$				
$y[n] = y[n - 1] + \frac{n-1}{n}x[n], \quad n = 0, 1, 2, \dots$				
$y[n] = \frac{\sin(x[n])}{x[1]}, \quad n = 0, 1, 2, \dots$				

**(8 Pts.)**

3. Calculate and plot the results of the following convolution:  $\{1, 2, 3, 2, 1\} * \{1, -1\}$

**(10 Pts.)**

4. Calculate the  $z$ -transform (if it exists) and the corresponding region of convergence for each of the following signals. Simplify your expressions.

- (a)  $x[n] = 3\delta[n + 1] - \delta[n - 100]$   
(b)  $x[n] = 3^n u[n] + 2^n u[n + 3]$

**(10 Pts.)**

5. The  $z$ -transform of  $x[n]$  is given below:

$$X(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{3}z^{-1}}$$

(a) Determine all valid ROCs for  $X(z)$ .

(b) Assuming that  $x[n]$  is a right-sided sequence, determine  $x[n]$ .

(5 Pts.)

6. Determine the sequence  $x[n]$  whose discrete-time Fourier transform is:

$$X_d(\omega) = \frac{1}{1 - \frac{1}{5}e^{-j\omega}}$$

(**Hint:** Consider the relationship between DTFT and z-transform.)

(7 Pts.)

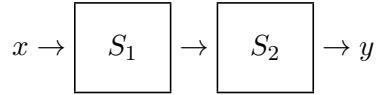
7. Consider a causal LTI system with the following linear constant coefficient difference equation (LC-CDE):

$$y[n] = \frac{1}{2}y[n-1] + x[n] - 5x[n-4], \quad n = 0, 1, 2, \dots$$

Determine the transfer function  $H(z)$  of the system and sketch its zero-pole plot.

(15 Pts.)

8. Consider the following cascaded system:



Assume that the transfer function of overall system is  $H(z) = \frac{1-3z^{-10}}{1-(1/2)z^{-1}}$  with ROC  $|z| > 1/2$ , and  $S_1$  is implemented by an LCCDE:

$$y[n] = \frac{1}{2}y[n-1] + x[n], \quad n = 0, 1, 2, \dots$$

- (a) Determine the transfer function of  $S_2$ :  $H_2(z)$
- (b) Determine the unit pulse response of  $S_2$ :  $h_2[n]$
- (c) Express  $S_2$  in the form of an LCCDE.

(15 Pts.)

9. The difference equation of a causal LTI system is given by

$$y[n] - \frac{1}{2}e^{j\frac{\pi}{2}}y[n-1] = x[n], \quad -\infty < n < \infty.$$

Determine  $y[n]$  for input  $x[n] = 1 + 2\cos(\frac{\pi}{4}n)$ ,  $-\infty < n < \infty$ .

(8 Pts.)

10. The transfer function of a causal LSI system is:  $H(z) = \frac{z-1}{z+j}$ . Find a **bounded, real-valued** input to the system which will produce an **unbounded** output  $y[n]$ . Give an expression for the input in the  $z$ -domain,  $X(z)$ .