University of Illinois

ECE 310: Problem Set 2

Due: 5pm, Friday September 14, 2018

1. Consider the discrete-time system given by

$$y[n] = 5y[n-1] + x[n]$$

with initial condition $y[-1] = 0$.

(a) Determine the impulse response $h[n]$ of the system.

(b) Use the impulse response $h[n]$ to determine whether the system is stable.

2. Consider the systems specified by the following input-output relations, where $x[n]$ is the input and $y[n]$ the output:

(a) $y[n] = y[n-3] + x[n] - 2x[n-2]$

(b) $y[n-2] + 2y[n] = e^{j\pi n}x[n]$

(c) $y[n] = 5x[n] + 1$

(d) $y[n] = x[0] \cdot x[n]$

For each system, determine if it is: (i) linear or non-linear, (ii) time-invariant or time-varying. **Justify your answers with proofs or counter-examples**.

3. Suppose we have an LTI system with step response $g[n]$ (i.e., the response of the system to a unit step $u[n]$ as input). Express the output of the system in terms of $g[n]$ and the input $x[n]$.

Hint: Represent $\delta[n]$ in terms of shifted versions of $u[n]$ and compute the impulse response.

4. Assume that the response of an LTI system to input $x[n] = 3^{(-n)}u[n-3]$ is $y[n] = 2^{(-n)}u[n-5]$. Use the system's properties (linearity and shift-invariance) to find $h[n]$, the system's impulse response.

5. Compute the convolution $x[n] * h[n]$ for the $x[n]$ and $h[n]$ given below. **Note**: The arrow indicates $n = 0$.

(a) $x[n] = \{1, \underset{\uparrow}{2}, 3\}, h[n] = \{\underset{\uparrow}{1}, 0, 3, -1\}$

(b) $x[n] = 3^{(-n)}u[n], h[n] = \{\underset{\uparrow}{-1}, 2, 3\}$

(c) $x[n] = (0.1)^n u[n], h[n] = n(u[n] - u[n-3])$

(d) $x[n] = (-1)^{(-n)}u[n], h[n] = e^{(-n)}u[n-2]$

6. Suppose $x[n] = n(u[n-1] - u[n-10]) + 0.5^n u[n-30]$, $h[n] = 0.3^n(u[n] - u[n-7])$ and let $y[n] = x[n] * h[n]$. For what values of n is $y[n]$ non-zero?