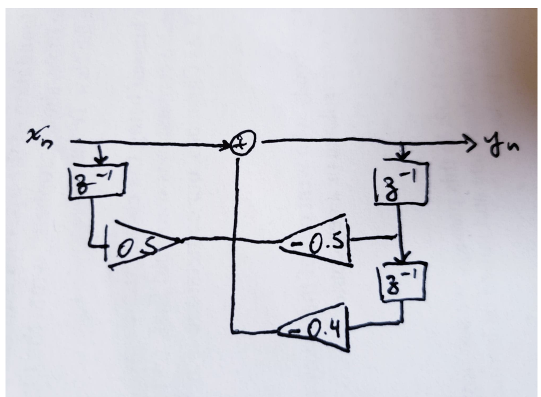
ECE 310: Quiz #9 (10am Section G) Fall 2018 Solutions

1. (5 pts) Draw the Direct Form I filter structure for a system with the following transfer function:

$$\frac{1+0.5z^{-1}}{1+0.5z^{-1}+0.4z^{-2}}$$

Solution
Convert the transfer function into a difference equation by taking the inverse z-transform and then solving in terms of $y[n]$.

$$\begin{aligned}
\frac{Y(z)}{X(z)} &= \frac{1+0.5z^{-1}}{1+0.5z^{-1}+0.4z^{-2}} \\
Y(z)(1+0.5z^{-1}+0.4z^{-2}) &= X(z)(1+0.5z^{-1}) \\
y[n]+0.5y[n-1]+0.4y[n-2] &= x[n]+0.5x[n-1] \\
y[n] &= -0.5y[n-1]-0.4y[n-2]+x[n]+0.5x[n-1]
\end{aligned}$$



2. (5 pts) Design a length-5 GLP FIR low pass filter with cutoff frequency $\omega_c = \frac{\pi}{4}$ radians. Use the window design method with a Hann window ($w[n] = 0.5 - 0.5\cos(2\pi n/M)$ for $n = 0, \ldots, M$). Give your answer in terms of a closed-form expression for the filter coefficients $\{h_n\}_{n=0}^{4}$.

Solution

Recall that a FIR LPF with GLP of order M has the following form.

$$h_{lp}[n] = \frac{\sin(\omega_c(n-M/2))}{\pi(n-M/2)}$$

Choosing our values of $\omega_c = \frac{\pi}{4}$ and $M = 4$, and then applying the Hann window, gives the following filter coefficients. (Here, $\text{sinc}(x) = \sin(x)/x$.)

$$\begin{aligned}
\{h_n\}_{n=0}^{4} &= (0.5 - 0.5\cos(\frac{\pi}{2}n))\frac{\sin(\frac{\pi}{4}(n-2))}{\pi(n-2)} \\
&= (0.5 - 0.5\cos(\frac{\pi}{2}n))\frac{1}{4}\text{sinc}(\frac{\pi}{4}(n-2))
\end{aligned}$$