ECE 310: Quiz #10 Solution (3PM Section E)
Fall 2018

December 5th, 2018

1. (5 pts) A video signal $x_c(t)$ is assumed to be bandlimited to 10 MHz. It is desired to filter this signal with a **bandstop** filter that will block the frequencies between 2 MHz and 3 MHz and pass the other, by using a digital filter with frequency response $H_d(\omega)$ sandwiched between an ideal A/D and an ideal D/A.

   (a) Determine the Nyquist sampling rate for the input signal, and specify the frequency response $H_d(\omega)$ for the necessary discrete-time filter, when sampling at the Nyquist rate.

   ### Solution

   The Nyquist rate is twice the maximum frequency of the input. Therefore the Nyquist rate is $2 \times 10\text{MHz} = 20\text{MHz}$.
   Use the relationships $\Omega = 2\pi F$ and $\omega = \Omega T$ to find the cutoff frequencies off the bandpass filter.

   $$\omega_{c1} = 2\pi \times 2\text{MHz} \times 50\text{ns} = 0.2\pi$$
   $$\omega_{c2} = 2\pi \times 3\text{MHz} \times 50\text{ns} = 0.3\pi$$

   The frequency response is therefore:

   $$H_d(\omega) = \begin{cases} 1, & |\omega| \leq 0.2\pi \\ 0, & 0.2\pi < |\omega| < 0.3\pi \\ 1, & 0.3\pi \leq |\omega| \leq \pi \end{cases}$$

   (b) Smart Alec claims that the system can perform the desired filtering function even when the sampling rate is lower than the Nyquist rate. Is this true? Justify your answer.

   ### Solution

   This system cannot be implemented with a lower sampling frequency. In the case where sampling is performed below the Nyquist rate, frequencies just above one half of the sampling rate will alias into the upper passband of the filter.

2. (5 pts) A system for processing analog signals $x_c(t)$ is composed of the following parts, connected in cascade: (i) an ideal analog LPF with cutoff frequency F_c; followed by (ii) a causal digital system whose input $x[n]$ and output $y[n]$ are related as $y[n] = -0.3y[n-2] + x[n]$, which is sandwiched between an ideal A/D and an ideal D/A operating at a sampling rate of 20 kHz. The output of this entire system is denoted by $y_c(t)$.

   (a) What is the largest value of F_c for which the entire system will act as an analog LTI system, from input $x_c(t)$ to output $y_c(t)$. Justify your answer.

   ### Solution

   In order to behave as an LTI system, the analog filter must prevent aliasing in the A/D conversion because the digital system defined by $y[n] = -0.3y[n-2] + x[n]$ will pass through all frequencies. For this to be the case, the cutoff frequency of the analog filter must be at most half of the sampling frequency of the A/D. Therefore the filter must have a cutoff F_c of 10kHz.

   (b) For the F_c determined in (a), determine the analog frequency response $H_c(\Omega)$ of the entire system.

   ### Solution

   Since the A/D and D/A are ideal and sampling at the Nyquist frequency, the system response is determined by the response of the analog filter and the digital system. Multiply the analog equivalent of the digital system by the frequency response of the filter.

$$
\begin{aligned}
`y[n] &= -0.3y[n-2] + x[n] \\
Y_d(\omega) &= -0.3e^{-j2\omega} Y_d(\omega) + X_d(\omega) \\
\frac{Y_d(\omega)}{X_d(\omega)} &= \frac{1}{1 + 0.3e^{-j2\omega}}
\end{aligned}
$$

   Use the relationship $\Omega = \frac{\omega}{T}$ to determine the analog equivalent of the digital system

$$
\begin{aligned}
H_c(\Omega) &= H_d(\Omega T) \\
&= \begin{cases} \frac{1}{1+0.3e^{-j2\Omega \times 50\mu s}}, & |\Omega| \leq \frac{\pi}{T} \\ 0, & |\Omega| > \frac{\pi}{T} \end{cases}
\end{aligned}
$$

Multiply by the response of the filter for the complete system response

$$
\begin{aligned}
H_{\text{total}}(\Omega) &= G(\Omega)\, H_c(\Omega) \\
&= \begin{cases} \frac{1}{1+0.3e^{-j2\Omega \times 50\mu\text{s}}}, & |\Omega| < 2\pi F_c \\ 0, & |\Omega| > 2\pi F_c \end{cases}
\end{aligned}
$$