ECE310: Quiz#3 (6pm Section CSS) Fall 2018 Solutions

1. (5 pts) Determine the z-transform and sketch the pole-zero plot with the ROC for the signal

$$x[n] = \left[\left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n\right] u[n]$$

Solution: Use the z-transform pair:

$$a^n u[n] \leftrightarrow \frac{1}{a - z^{-1}}, |z| > a$$

This gives

$$\boxed{X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{1}{1 - \frac{1}{3}z^{-1}}, |z| > \frac{1}{2}}$$

The ROC is $|z| > \frac{1}{2}$ because the individual z-transforms will have ROCs of $|z| > \frac{1}{3}$ and $|z| > \frac{1}{2}$; we must take the stricter of the two. To find the poles and zeros, we simplify the expression into a ratio of two polynomials in z:

$$\begin{aligned} X(z) &= \frac{(1 - \frac{1}{3}z^{-1}) + (1 - \frac{1}{2}z^{-1})}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z^{-1})} \\ &= \frac{2 - \frac{5}{6}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z^{-1})} \\ &= \frac{2z^2 - \frac{5}{6}z}{(z - \frac{1}{2})(z - \frac{1}{3})} \end{aligned}$$

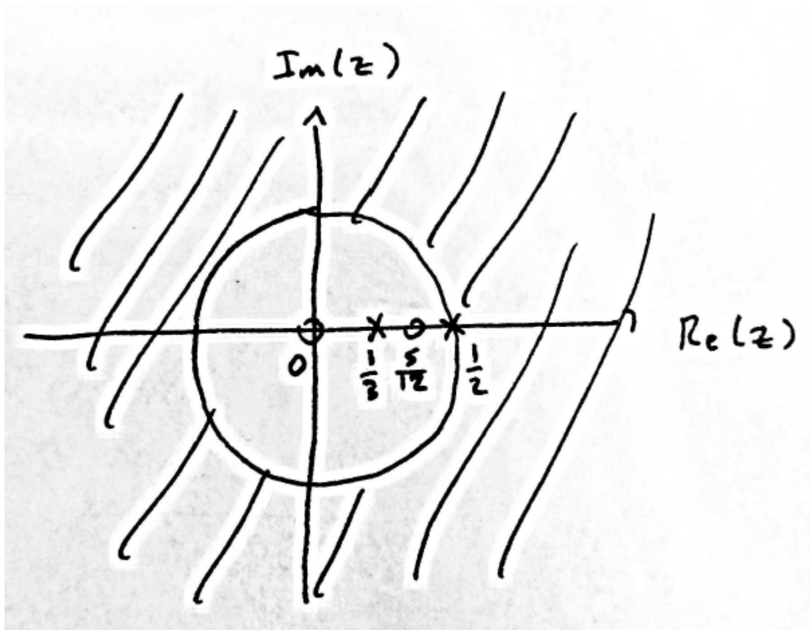
We can now solve for the zeros by setting the numerator equal to zero:

$$2z^2 - \frac{5}{6}z = 0 \rightarrow \boxed{z = 0, z = \frac{5}{12}}$$

Similarly, we can solve for the poles by setting the denominator equal to zero:

$$\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right) = 0 \rightarrow \boxed{z = \frac{1}{2}, z = \frac{1}{3}}$$

This leads to the pole-zero plot given below.



Im(z)
Re(z)
0
1
3
5
12
1
2

2. (5 pts) Given the z-transform pair $x[n] \leftrightarrow X(z) = \frac{1}{1-\frac{1}{5}z^{-1}}$ with ROC: $|z| > 1/5$, use the z-transform properties to determine the z-transform of the signal $y[n] = (n-3)x[n-3]$.

Solution, Approach 1: Apply the shifting and differentiation properties directly to the z-transform. If we let $w[n] = nx[n]$, then

$$W(z) = -z\frac{dX(z)}{dz} = -z\left[\frac{(1-\frac{1}{5}z^{-1})(0)-(1)(\frac{1}{5}z^{-2})}{(1-\frac{1}{5}z^{-1})^2}\right] = \frac{\frac{1}{5}z^{-1}}{(1-\frac{1}{5}z^{-1})^2}$$

Then, noting that $y[n] = w[n-3]$, we get

$$Y(z) = z^{-3}W(z) = \boxed{\frac{\frac{1}{5}z^{-4}}{(1-\frac{1}{5}z^{-1})^2}, |z| > \frac{1}{5}}$$

Solution, Approach 2: Taking the inverse z-transform gives

$$x[n] = \left(\frac{1}{5}\right)^n u[n]$$

So,

$$y[n] = (n-3)\left(\frac{1}{5}\right)^{n-3} u[n-3]$$

Using the transform pair

$$na^n u[n] \leftrightarrow \frac{az^{-1}}{(1-az^{-1})^2}, |z| > a$$

and the shifting property, we get

$$Y(z) = z^{-3}\left(\frac{\frac{1}{5}z^{-1}}{(1-\frac{1}{5}z^{-1})^2}\right) = \boxed{\frac{\frac{1}{5}z^{-4}}{(1-\frac{1}{5}z^{-1})^2}, |z| > \frac{1}{5}}$$