ECE310: Quiz#9 (6pm Section CSS) Fall 2018 Solutions

1. (5 pts) Draw a cascade structure with first-order sections, each in Direct Form II structure, for a system with the following transfer function:

$$\frac{0.5 - z^{-1}}{(1 - 0.25z^{-1})(1 + 0.5z^{-1})}$$

Solution

If we want to implement a cascade structure, we want to find $H(z) = H_1(z)H_2(z)$, and implement $H_1(z)$ and $H_2(z)$ separately: the system looks like

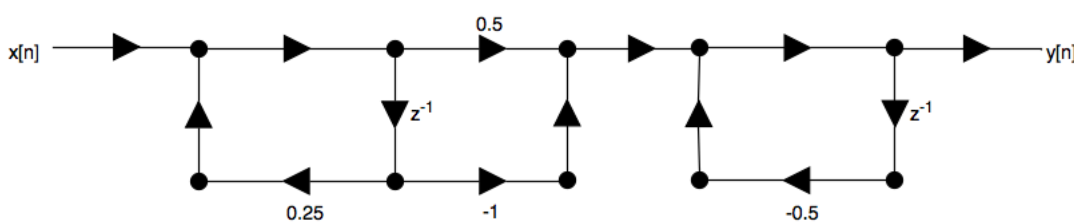
$$x[n] \rightarrow \boxed{H_1(z)} \rightarrow \boxed{H_2(z)} \rightarrow y[n]$$

Therefore, to draw the cascade structure, we draw $H_1(z)$ in Direct Form II, and connect its output to the input of the implementation of $H_2(z)$ in Direct Form II.

Note that the transfer function is already factored; we can write

$$H(z) = \underbrace{\left(\frac{0.5 - z^{-1}}{1 - 0.25z^{-1}}\right)}_{H_1(z)} \underbrace{\left(\frac{1}{1 + 0.5z^{-1}}\right)}_{H_2(z)}$$

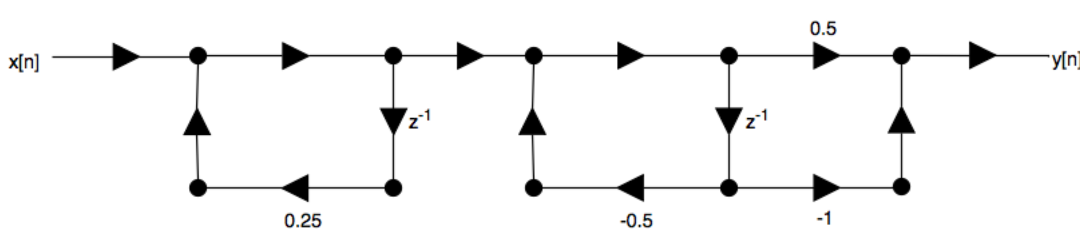
We can immediately write down the Direct Form II structure, using the form given on Page 492 of the textbook. This is given below.



Note that another possible implementation would have been to set

$$H(z) = \underbrace{\left(\frac{1}{1 - 0.25z^{-1}}\right)}_{H_1(z)} \underbrace{\left(\frac{0.5 - z^{-1}}{1 + 0.5z^{-1}}\right)}_{H_2(z)}$$

This leads to the block diagram implementation given below.



Grading:

- 1 point for setting $H(z) = H_1(z)H_2(z)$.
- 2 points for the Direct Form II implementation of $H_1(z)$.
- 2 points for the Direct Form II implementation of $H_2(z)$.
- -0.5 points for any sign errors in the gains.

Partial Credit:

- 2 points for giving a single correct Direct Form II implementation of $H(z)$.
- 2 points for performing a partial fraction expansion and giving a correct parallel structure.
- 2 points for performing a partial fraction expansion and giving an incorrect cascade structure.

2. (5 pts) Design a length-6 GLP FIR low-pass filter with cutoff frequency $\omega_c = \frac{\pi}{5}$ radians. Use the window design method with a Hamming window ($w[n] = 0.54 - 0.46\cos(2\pi n/M)$ for $n = 0, ..., M$). Give your answer in terms of a closed-form expression for the filter coefficients $\{h_n\}_{n=0}^{5}$.

Solution

Since we want to design a low-pass filter with even length, we must use even symmetry; GLP filters with even length and odd symmetry can never be low-pass. Going through the standard filter design approach, we have

$$D_d(\omega) = \begin{cases} 1 & |\omega| < \frac{\pi}{5} \\ 0 & \text{else} \end{cases}$$

Therefore, we obtain the desired GLP response, $G_d(\omega)$, by multiplying by $e^{j(\alpha - \beta\omega)}$. Here, $\alpha = 0$ since we're using even symmetry, and $\beta = \frac{N-1}{2} = 2.5$. So

$$G_d(\omega) = \begin{cases} e^{-j\omega 2.5} & |\omega| < \frac{\pi}{5} \\ 0 & \text{else} \end{cases}$$

We obtain $g[n]$ by taking the inverse DTFT:

$$g[n] = \frac{1}{2\pi}\int_{-\frac{\pi}{5}}^{\frac{\pi}{5}} e^{j\omega(n-2.5)}d\omega = \frac{2j\sin(\frac{\pi}{5}(n-2.5))}{2\pi j(n-2.5)} = \frac{1}{5}\text{sinc}(\frac{\pi}{5}(n-2.5))$$

Finally, we obtain the filter $h[n]$ by multiplying by the window; we have

$$\boxed{h[n] = g[n]w[n] = \begin{cases} \frac{1}{5}\text{sinc}(\frac{\pi}{5}(n-2.5))(0.54 - 0.46\cos(\frac{2\pi n}{5})) & 0 \le n \le 5 \\ 0 & \text{else} \end{cases}}$$

Grading:

- 1 point for using a filter with even symmetry.
- 1 point for the sinc form of $g[n]$.
- 1 point for shifting by 2.5.
- 1 point for using the correct window (no shift, and correct value of M).
- 1 point for restricting the result to $0 \le n \le 5$.