University of Illinois
ECE 310 **Profs. Do and Liang**

Midterm Exam

7:00-8:30pm, Thursday, March 1, 2018

Name: ____________________

Section: 9:00 AM 12:00 PM 3:00 PM

NetID: ____________

Score: ____________

Problem	Pts.	Score
1	10	
2	12	
3	8	
4	10	
5	10	
6	5	
7	7	
8	15	
9	15	
10	8	
Total	100	

Instructions

- You may not use any books, calculators, or notes other than two handwritten two-sided sheets of 8.5" x 11" paper.
- Show all your work to receive full credit for your answers.
- When you are asked to "calculate", "determine", or "find", this means providing closed-form expressions (i.e., without summation or integration signs).
- Neatness counts. If we are unable to read your work, we cannot grade it.
- Turn in your entire booklet once you are finished. No extra booklet or papers will be considered.

(10 Pts.)

1. Answer **True** or **False** to each of the following statements:

   (a) Let $\sum_{n=-\infty}^{\infty} x[n]\, \delta[\sin(\pi n)] = 2$. Then $\sum_{n=-\infty}^{\infty} x[n] = 2$. **True/False**

   (b) Suppose that $x[n]$ has z-transform $X(z)$. The DTFT of $x[n]$ can always be expressed as: $X_d(\omega) = X(z)|_{z=e^{i\omega}}$. **True/False**

   (c) The DTFT of the sequence $x[n] = \cos\left(\frac{\pi}{2}n\right)$, $-\infty < n < \infty$ is $X_d(\omega) = \pi\left[\delta(\omega - \frac{\pi}{2}) + \delta(\omega + \frac{\pi}{2})\right]$, for $-\infty < \omega < \infty$. **True/False**

   (d) The BIBO stability of ***any*** system is completely determined by the system's unit pulse response. **True/False**

   (e) The output $y[n]$ of a system for an arbitrary input $x[n]$ is given by $y[n] = x[n] * h[n]$, where $h[n]$ is the unit pulse response of the system. The system must be linear and shift-invariant. **True/False**

(12 Pts.)

2. For each of the systems with input $x[n]$ and output $y[n]$ shown in the table, indicate by "**yes**" or "**no**" whether the properties indicated apply to the system.

	Linear	Shift-Invariant	Causal	Stable
$y[n] = x[n-1] + x[n] + x[n+1]$				
$y[n] = y[n-1] + \frac{n-1}{n}x[n], \quad n = 0, 1, 2, \ldots$				
$y[n] = \frac{\sin(x[n])}{x[1]}, \quad n = 0, 1, 2, \ldots$				

(8 Pts.)

3. Calculate and plot the results of the following convolution: $\{\underset{\uparrow}{1}, 2, 3, 2, 1\} * \{\underset{\uparrow}{1}, -1\}$

(10 Pts.)

4. Calculate the z-transform (if it exists) and the corresponding region of convergence for each of the following signals. Simplify your expressions.

   (a) $x[n] = 3\delta[n+1] - \delta[n-100]$

   (b) $x[n] = 3^n u[n] + 2^n u[n+3]$

(10 Pts.)

5. The z-transform of $x[n]$ is given below:

$$X(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{3}z^{-1}}$$

(a) Determine all valid ROCs for $X(z)$.

(b) Assuming that $x[n]$ is a right-sided sequence, determine $x[n]$.

(5 Pts.)

6. Determine the sequence $x[n]$ whose discrete-time Fourier transform is:

$$X_d(\omega) = \frac{1}{1 - \frac{1}{5}e^{-j\omega}}$$

(**Hint:** *Consider the relationship between DTFT and z-transform.*)

(7 Pts.)

7. Consider a causal LTI system with the following linear constant coefficient difference equation (LC-CDE):

$$y[n] = \frac{1}{2}y[n-1] + x[n] - 5x[n-4], \quad n = 0, 1, 2, \ldots$$

Determine the transfer function $H(z)$ of the system and sketch its zero-pole plot.

(15 Pts.)

8. Consider the following cascaded system:

$$x \rightarrow \boxed{S_1} \rightarrow \boxed{S_2} \rightarrow y$$

Assume that the transfer function of overall system is $H(z) = \frac{1-3z^{-10}}{1-(1/2)z^{-1}}$ with ROC $|z| > 1/2$, and S_1 is implemented by an LCCDE:

$$y[n] = \frac{1}{2}y[n-1] + x[n], \quad n = 0, 1, 2, \ldots.$$

(a) Determine the transfer function of S_2: $H_2(z)$

(b) Determine the unit pulse response of S_2: $h_2[n]$

(c) Express S_2 in the form of an LCCDE.

(15 Pts.)

9. The difference equation of a causal LTI system is given by

$$y[n] - \frac{1}{2}e^{j\frac{\pi}{2}}y[n-1] = x[n], \qquad -\infty < n < \infty.$$

Determine $y[n]$ for input $x[n] = 1 + 2\cos(\frac{\pi}{4}n), \quad -\infty < n < \infty$.

(8 Pts.)

10. The transfer function of a causal LSI system is: $H(z) = \frac{z-1}{z+j}$. Find a **bounded, real-valued** input to the system which will produce an **unbounded** output $y[n]$. Give an expression for the input in the z-domain, $X(z)$.