Plotting Magnitude and Phase

Problem 1

Determine the magnitude and phase of the complex function $X(\omega) = e^{-j3\omega} - e^{-j7\omega}$. Sketch the magnitude and phase from $\omega = -\pi$ to $\omega = \pi$.

Problem 2

Determine the magnitude and phase of the complex function $X(\omega) = 1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega}$. Sketch the magnitude and phase from $\omega = -\pi$ to $\omega = \pi$.

System Properties

Problem 1

Determine whether the following systems are: (a) linear or non-linear, (b) causal or non-causal, and (c) shift-invariant or shift-varying. Justify your answers.

(a) $y^2[n-1] + 2y[n] = x[n]$

(b) $y[n] = \sum_{m=-\infty}^{n+1} x[m]$

(c) $y[n] = x[1]x[n]$

(d) $y[n] = \frac{n}{x[n+1]}$

(e) $y[n] = x[n]\sin^2(x[n])$

(f) $y[n] + y[n-1] = x[n] + 2x[n-2]$

Impulse Response and Convolution

Problem 1

Consider an LTI system with the following input-output relationships. **Note:** Bold indicates the time origin ($n = 0$).

$$x_1[n] = \{\mathbf{1}, -1, -2\} \rightarrow y_1[n] = \{\mathbf{1}, 1, -1, -7, -6\}$$
$$x_2[n] = \{-\mathbf{2}, 3, 6\} \rightarrow y_2[n] = \{-\mathbf{2}, -1, 6, 21, 18\}$$

Find the output of the system when the input is $x[n] = \delta[n-1]$.

Problem 2

Compute the convolution $y[n] = x[n] * h[n]$ *using the definition*, where $x[n] = \left(\frac{1}{2}\right)^n u[n]$ and $h[n] = a^n u[n-5]$, where $|a| < 1$.

Problem 3

Compute the convolution $y[n] = x[n] * h[n]$, where $x[n] = \cos(\frac{\pi}{2}n)$ and $h[n] = n(u[n] - u[n-4])$.

Z Transforms

Problem 1

The relationship between input $x[n]$ and output $y[n]$ for a causal system is given by

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n] - \frac{1}{2}x[n-1]$$

(a) Determine the transfer function $H(z)$ of the system.

(b) Draw a block diagram representation of the system.

(c) Determine the output for the input $x[n] = (-3)^{-n}u[n-1]$.

BIBO Stability

Problem 1

Determine whether or not each of the following systems with input $x[n]$ and output $y[n]$ is BIBO stable. **Justify your answer.**

(a) A system described by the equation $y[n] = e^{jn^2} x^3[n^2]$.

(b) A system described by the equation: $y[n] = x[n] * z[n]$ where $z[n] = \frac{u[n]}{n+1}$.

(c) A causal system described by the equation $y[n] = 0.5y[n-1] + 0.5y[n-2] + 0.1x[n]$.

(d) A causal LSI system for which the z-transforms of the input and output are related by

$$Y(z) = \frac{z + 0.5}{z^5 + j} X(z)$$

Problem 2

Consider an LSI system with transfer function $H(z) = \frac{z^2}{z^2+1}$. Determine which of the following inputs will produce an **unbounded** output. **(-1 pt for wrong answers)**.

(a) $x[n] = \delta[n]$

(b) $x[n] = \sin(\frac{\pi n}{2})u[n]$

(c) $x[n] = u[n]$

(d) $x[n] = \cos(\pi n)u[n]$

(e) $x[n] = 5^n u[n]$

Problem 3

The transfer function of a causal LSI system is: $H(z) = \frac{z-1}{z+j}$. Find a **bounded, real-valued** input to the system which will produce an **unbounded** output $y[n]$. Given an expression for the input in the z-domain, $X(z)$.

The DTFT

Problem 1

The DTFT of the sequence $x[n] = na^n u[n]$ for $|a| < 1$ is

$$X_d(\omega) = \frac{ae^{-j\omega}}{(1 - ae^{-j\omega})^2}$$

Derive a closed-form expression (no sums) for the DTFT $V_d(\omega)$ of the sequence

$$v[n] = n \left(\frac{1}{3}\right)^n \cos(n) u[n-7]$$

Problem 2

The DTFT of a real signal $x[n]$ is given for $\pi \leq \omega \leq 2\pi$, by

$$X_d(\omega) = \begin{cases} 2 & \pi \leq \omega \leq 1.5\pi \\ 1 & 1.5\pi \leq \omega \leq \frac{11\pi}{6} \\ 2 & \frac{11\pi}{6} < \omega \leq 2\pi \end{cases}$$

Determine a closed-form expression for the signal $x[n]$. Your answer should not contain imaginary numbers.

Eigensequences

Problem 1

A causal system is described by the equation $y[n] = \frac{j}{2}y[n-1] + x[n]$. Find the output of this system if the input is $x[n] = \cos\left(\frac{\pi n}{3} + 30°\right)$.

Problem 2

The frequency response $H_d(\omega)$ of a digital filter is defined as below:

$$|H_d(\omega)| = \begin{cases} \frac{1}{2}, & |\omega| \leq \frac{\pi}{2} \\ \frac{3}{4}, & \frac{\pi}{2} < |\omega| < \frac{3\pi}{4} \\ 1, & \frac{3\pi}{4} \leq |\omega| < \pi \end{cases} \qquad \angle H_d(\omega) = \begin{cases} 2\omega, & \omega \geq 0 \\ 0, & \omega < 0 \end{cases}$$

Determine the system's output $y[n]$ for input

$$x[n] = 2 + \cos\left(\frac{\pi}{8}n + 34°\right)$$

Concept Checks - True/False and Multiple Choice

Problem 1

Answer **True** or **False** to each of the following statements:

(a) The causal LSI system $H(z) = \frac{1+z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-1})}$ is stable.

(b) An FIR filter is always BIBO stable.

(c) Knowing the ROC of the transfer function of a causal system is sufficient to determine if the system is BIBO stable.

(d) If the impulse response of a system is $h[n] = \delta[n] + \delta[n-1] + (-1-j)^n$ then the system is unstable.

Problem 2

A system's transfer function is $H(z)$ and its input is $\cos(\omega_0 n)$. Under what conditions does it hold that $y[n] = |H_d(\omega_0)| \cos(\omega_0 n + \angle H_d(\omega_0))$, where $H_d(\omega) = H(z)|_{z=e^{j\omega}}$? Check all that apply. **(-1 pt for each wrong answer)**

(a) The system must be causal.

(b) The system must be BIBO stable.

(c) The impulse response of the system must be real.

(d) The impulse response of the system must be of finite length.

(e) It never holds.

Problem 3

If the response of a discrete LTI system to input $x_1[n] = \sin(\frac{\pi}{4}n)$ is $y_1[n] = e^{j(\frac{\pi}{4}n+\pi)}$, then its response to $x_2[n] = \cos(\frac{\pi}{4}n + 1)$ is

(a) $y_2[n] = j\cos(\frac{\pi}{4}n + 1 + \pi)$

(b) $y_2[n] = -je^{j(\frac{\pi}{4}n+1)}$

(c) $y_2[n] = \sin(\frac{\pi}{4}n + 1 - \pi)$

(d) There is not enough information to determine the correct answer.

Problem 4

Given that the z-transform of $x[n] = 5^n u[n]$ is $X(z) = \frac{z}{z-5}$, the DTFT of $x[n]$ is given by

1. $\frac{e^{j\omega}}{e^{j\omega}-5}$
2. $\frac{e^{-j\omega}}{e^{-j\omega}-5}$
3. $\frac{1}{1-5e^{-j\omega}}$
4. Does not exist.

Problem 5

(a) The transfer function of a discrete-time LTI system has the form $H(z) = \frac{z^4}{(z-0.5)^3}$. Is the system causal or noncausal?

(b) What is the ROC for the z-transform of $x[n] = n^5 2^{-n} u[n] + n^{33} e^{-n} u[n-1]$?

(c) Is the system $y[n] = 2^{x[3n]}$ time-invariant or time-varying?