ECE 310: Quiz #6 (10am Section G) Fall 2018 Solutions

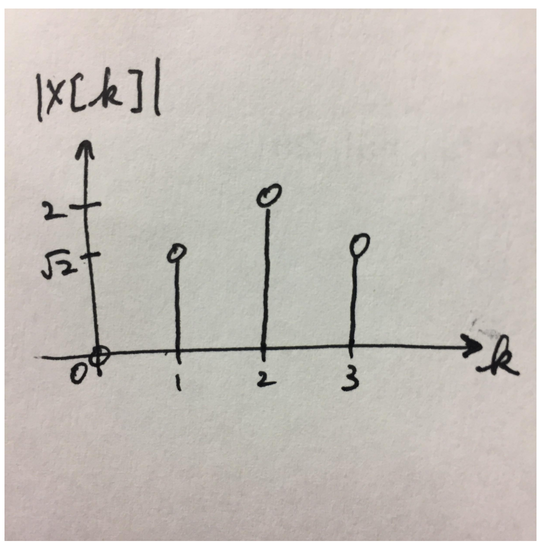
1. Let $x[n] = \{-1, 1, 0, 0\}$.

   (a) Compute the DFT $X[k]$ of $x[n]$ (4pts)

$$
\begin{aligned}
X[k] &= \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi}{4}kn} \\
&= -1e^{-jk\cdot 0} + 1e^{-j\frac{\pi}{2}k} \\
&= -e^{-j\frac{\pi}{4}k}\left(e^{j\frac{\pi}{4}k} - e^{-j\frac{\pi}{4}k}\right) \\
&= \boxed{-2j\sin(\frac{\pi}{4}k)e^{-j\frac{\pi}{4}k}} \\
&= \{0, 1-j, -2, -1+j\}
\end{aligned}
$$

   (b) Sketch the magnitude of $X[k]$ (2 pts)

$$|X[k]| = |-2j\sin(\frac{\pi}{4}k)| = \left\{0, \sqrt{2}, 2, \sqrt{2}\right\}$$



2. Let $X[k]$ be the DFT of $x[n] = \{1, 2, 3, 4\}$. Determine the sequence $y[n]$ whose DFT is given by $Y[k] = 3e^{-j\pi(k+1)/2}X[k]$. (4pts)

$$
\begin{aligned}
Y[k] &= 3e^{-j\frac{\pi}{2}}e^{-j\frac{\pi}{2}k}X[k] \\
&= 3e^{-j\frac{\pi}{2}}W_N^{1k}X[k]
\end{aligned}
$$

The first coefficient in front of $X[k]$ is simply a scalar, $-j3$, so we can apply the property of homogeneity for the linear DFT. The second coefficient corresponds to a right circular shift by 1.

$$\boxed{y[n] = -j3x[\langle n-1\rangle_N] = \{-j12, -j3, -j6, -j9\}}$$