University of Illinois Fall 2018

ECE 310: Problem Set 10

Due: 5pm, Friday November 9, 2018

1. The nonzero part of the impulse response of a particular filter is given by $\{h[n]\}_{n=0}^{2} = \{0.5; -1; 0.5\}$. Your goal is to convolve $h[n]$ with the following signal:

$$\{x[n]\}_{n=0}^{27} = \{1; 2; 3; 4; 5; 6; 7; 8; -8; -7; -6; -5; -4; -3; -2; -1; 1; 2; 3; 4; 5; 6; 7; 8; 7; 6; 5; 4\}$$

You decide to use the overlap-add method with a length-8 FFT, dividing the input into frames $x_m[n]$, where m is the frame index, and each frame $x_m[n]$ is a signal of length $K = 8$. (A frame $x_m[n]$ may include zero-padding - but if so, use the minimum possible amount of zero-padding.)

   (a) Specify the contents of the first two frames, $\{x_0[n]\}_{n=0}^{7}$ and $\{x_1[n]\}_{n=0}^{7}$.

   (b) Specify the contents of the first two intermediate output frames, $\{y_0[n]\}$ and $\{y_1[n]\}$.

   (c) Let $y[n] = h[n] * x[n]$. Recall that in the overlap-add method, the values of the output $y[n]$ are formed by a combination of the values in the intermediate output frames. Give expressions for each of the outputs $\{y[n]\}_{n=4}^{9}$ in terms of the entries of the intermediate output frames, $\{y_0[n]\}$ and $\{y_1[n]\}$.

2. We wish to digitally lowpass filter a real-valued analog signal. Although this signal has an infinite number of samples, we have already determined that truncating the signal to keep only 25,000 consecutive time samples is adequate to meet our error requirements in this application. We wish to filter this signal with a real-valued length-240 FIR low-pass filter.

   (a) Determine the number of real multiplications and additions required to implement this linear convolution directly.

   (b) Determine the number of real multiplications and additions required to implement this linear convolution using "fast convolution," for which two complex-valued radix-2 FFT and one inverse FFT (with transform length chosen as a power of two) are to be used. Assume in this sub-problem and the next one reduced butterflies (with one complex multiply per butterfly), and that 4 real multiplies and 2 real additions are needed to implement each complex multiply. Do not try to exclude from your count multiplies by "trivial" twiddle factors. By what factor is fast convolution faster compared to direct convolution?

   (c) Since the data vector is much longer than the filter length, the overlap-add algorithm yields a more efficient solution. Determine the size of the power-of-two length FFT that minimizes the total number of multiplies per output sample using the overlap-add method of fast convolution. Find the total computational cost (number of real multiplies and additions) using this algorithm. Assume that the appropriate DFT of the filter has been precomputed and stored. Compare to the costs of direct convolution, and to the (single block) fast convolution that you found in (a) and (b), respectively.

3. The transfer functions of three causal LTI systems are given below. For each system, determine whether it is an FIR or an IIR filter.

   (a) $\frac{z^2+3z+2}{2z^2+3z-1}$

   (b) $\frac{1+0.8z^{-1}}{1-0.64z^{-2}}$

   (c) $3 + z^{-1} - 4z^{-2}$

4. Problem 28, Ch. 9 in the textbook.

5. Consider the following causal LSI system with the following transfer function:

$$H(z) = \frac{(1 - 3z^{-1})}{(1 - 1.2z^{-1} + 0.4z^{-2})(1 - 0.5z^{-1})}$$

   Draw the filter structures for implementations of the system in each of the following forms:

   (a) Direct form I.

   (b) Direct form II.

   (c) Transpose form I.

   (d) Transpose form II.

   (e) Cascade form using first- and second-order direct form II sections.

   (f) Parallel form using first- and second-order direct form II sections.

   (g) Cascade form using first- and second-order transpose form II sections.

6. The frequency response of a generalized linear phase (GLP) filter can be expressed as $H_d(\omega) = R(\omega)e^{j(\alpha-\beta\omega)}$ where $R(\omega)$ is a real function and α and β are constant. For each of the following LTI systems, described by their impulse response, transfer function, or difference equation, determine whether it is a GLP filter. If it is, determine $R(\omega)$, α and β , and indicate whether it is also a strictly linear phase filter.

   (a) $y[n] = x[n] - x[n-2]$

   (b) $y[n] = 0.5y[n-1] + 0.5y[n-2] + x[n]$

   (c) $y[n] = 0.5x[n] + x[n-1] - x[n-2] - 0.5x[n-3]$

   (d) $\{h[n]\}_{n=0}^{2} = \{3, 7, 3\}$

   (e) $\{h[n]\}_{n=0}^{2} = \{3, 4, 3\}$

   (f) $H(z) = 1 - 3z^{-1} - z^{-2}$