University of Illinois Fall 2018

ECE 310: Problem Set 4

Due: 5pm, Friday September 28, 2018

1. Use the (unilateral) z-transform to determine the convolution of the signals $x[n] = 2^{-n}u[n-2]$ and $v[n] = (1 + 3^{-n})u[n]$

2. A causal system is described by the following relationship between input $x[n]$ and output $y[n]$:
$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n]$$
   (a) Use the z-transform to find the impulse response, h[n], of this system.
   (b) Use the z-transform to find the output of this system for the input $x[n] = 3^{-n}u[n]$ when the system is initially at rest, i.e. zero initial conditions.

3. Determine whether or not the following systems, with input $x[n]$ and output $y[n]$, are bounded input, bounded output (BIBO) stable.
   (a) $y[n] = x^3[n] + n^2 e^{-0.01n}$
   (b) $y[n] = \tan(x[n])$
   (c) $y[n] = n\cos(x[n])$
   (d) $y[n] = h[n] * x[n]$ where $h[n] = \begin{cases} 0 & \text{if } n < 0 \\ 100^{100} & 0 \le n \le 10^{10} \\ n(0.99)^n & 10^{10} < n < \infty \end{cases}$

4. Determine whether or not each of the following system functions represents that of a BIBO stable system. Assume that each of these system functions is the one-sided z-transform of the impulse response for a causal LTI system.
   (a) $H(z) = \frac{z+100}{z^2+1/2}$
   (b) $H(z) = \frac{z+100}{z^2-1.8z+0.8}$
   (c) $H(z) = \frac{z+100}{z-4}$
   (d) $H(z) = \frac{z-0.5}{z^3+j}$

   For each case above in which the system is determined to be BIBO unstable, find a bounded real-valued input that produces an unbounded output.

5. A causal LTI system is observed to produce the output $y[n] = 2^{-n}u[n]$ when its input is $x[n] = 3^{-n}(0.5u[n] - u[n-1])$.
   (a) Find the impulse response $h[n]$ of the system. Is the impulse response unique?
   (b) Determine whether or not the system is BIBO stable. Where are the poles of the transfer function located?
   (c) Find the difference equation that characterizes this system.