ECE 310: Quiz #7 Solution (Section E) Fall 2018

October 31, 2018

1. (6pts) Let $x[n] = \{1, 0, 0, 1\}$,
   (a) Compute the DFT $X[k]$ of $x[n]$.
   (b) Sketch the magnitude of $X[k]$.

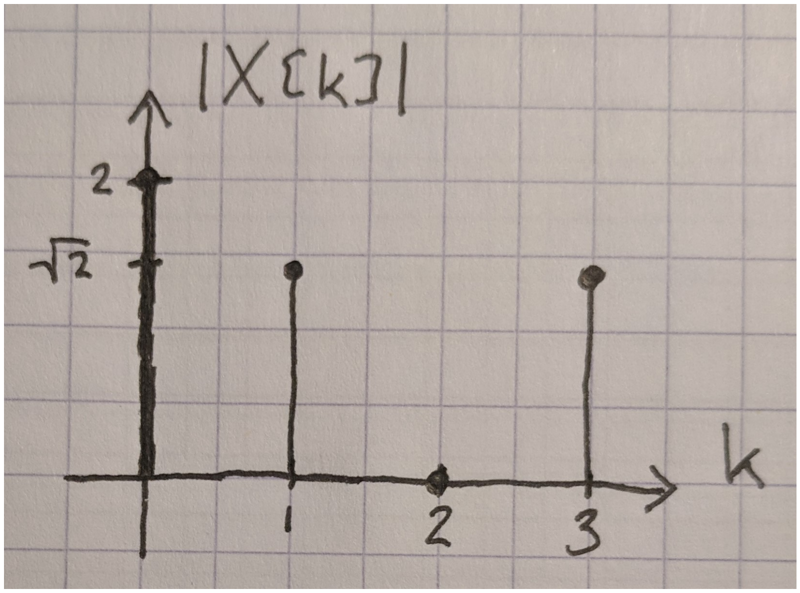
Solution

(a)

$$\begin{aligned} X[k] &= \sum_{n=0}^{3} x[n]\, e^{-j\frac{2\pi}{4}nk} \\ &= \sum_{n=0}^{3} x[n]\, e^{-j\frac{\pi}{2}nk} \\ &= 1 \cdot e^{-j\frac{\pi}{2}(0)k} + 0 \cdot e^{-j\frac{\pi}{2}(1)k} + 0 \cdot e^{-j\frac{\pi}{2}(2)k} + 1 \cdot e^{-j\frac{\pi}{2}(3)k} \\ &= 1 + e^{-j\frac{3\pi}{2}k} \end{aligned}$$

(b)

$$\begin{aligned} |X[0]| &= \left|1 + e^{-j\frac{3\pi}{2}(0)}\right| = |1+1| = 2 \\ |X[1]| &= \left|1 + e^{-j\frac{3\pi}{2}(1)}\right| = |1+j| = \sqrt{2} \\ |X[2]| &= \left|1 + e^{-j\frac{3\pi}{2}(2)}\right| = |1-1| = 0 \\ |X[3]| &= \left|1 + e^{-j\frac{3\pi}{2}(3)}\right| = |1-j| = \sqrt{2} \end{aligned}$$



2. (4 pts) Let $X[k]$ be the DFT of $x[n] = \{0, 3, 2, 1\}$. Determine the sequence $y[n]$ whose DFT is given by $Y[k] = 2(-1)^k X[k]$.

Solution

$$
\begin{aligned}
Y[k] &= 2(-1)^k X[k] \\
&= 2e^{-j\pi k} X[k] \\
&= 2e^{-j\frac{2\pi}{4}2k} X[k] \\
&= 2W_N^{2k} X[k]
\end{aligned}
$$

Apply the linearity and time shifting properties of the DFT:

$$
\begin{aligned}
y[n] &= 2x[\langle n-2 \rangle_4] \\
&= \{4, 2, 0, 6\}
\end{aligned}
$$