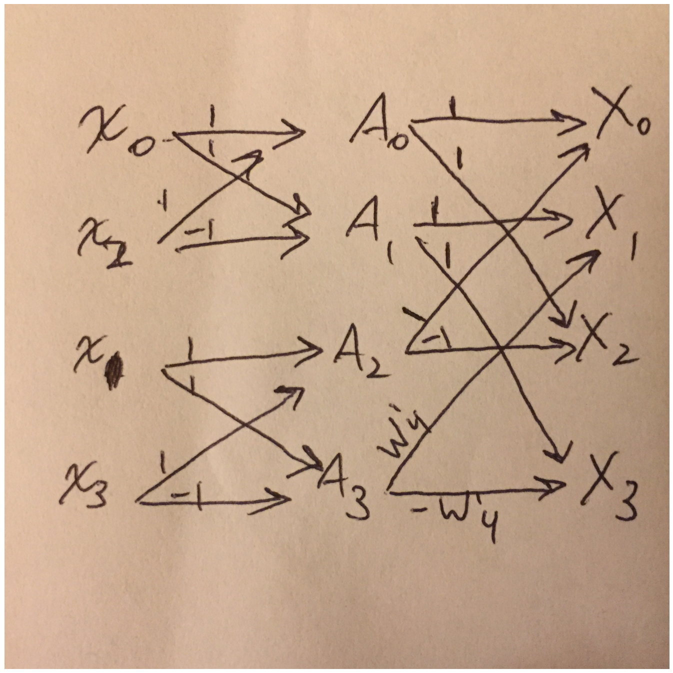
ECE 310: Quiz #8 (10am Section G) Fall 2018 Solutions

1. Consider the signal $\{x[n]\}_{n=0}^{3} = \{1, -1, 2, -2\}$. Use a decimation-in-time radix-2 FFT to compute the DFT of $x[n]$, i.e., calculate explicitly all the intermediate quantities that are computed in the FFT, and show how they are combined to produce the final output, which you should also give explicitly. (4pts)



First calculate the intermediate quantities from the flowchart.

$$\begin{aligned} A[0] &= x[0] + x[2] = 1 + 2 = 3 \\ A[1] &= x[0] - x[2] = 1 - 2 = -1 \\ A[2] &= x[1] + x[3] = -1 - 2 = -3 \\ A[3] &= x[1] - x[3] = -1 + 2 = -1 \end{aligned}$$

Now we can calculate $\{X[k]\}_{k=0}^{3}$.

$$\begin{aligned} X[0] &= A[0] + A[2] = 3 - 3 = 0 \\ X[1] &= A[1] + e^{-j\frac{2\pi}{4}} A[3] = -1 + j \\ X[2] &= A[0] - A[2] = 3 + 3 = 6 \\ X[3] &= A[1] - e^{-j\frac{2\pi}{4}} A[3] = -1 - j \end{aligned}$$

$$\boxed{\{X[k]\}_{k=0} = \{0, -1 + j, 6, -1 - j\}}$$

2. Given $\{x[n]\}_{n=0}^{N-1}$, with $x[n] = x_c(nT)$ and $T = 100\mu$sec, you compute a length-N FFT of $x[n]$ and plot the magnitude. Using this method, you wish to resolve analog sinusoidal signals that are

separated by as little as 5 Hz in frequency. Assume that the frequency resolution for windowed DTFT-based spectral analysis is equal to the main lobe of the DTFT of the window. Determine the minimum length $N = 2^\nu$ that will meet your resolution requirement. (**Hint**: The DTFT of the sequence $v[n] = u[n] - u[n-N]$ is $V_d(\omega) = e^{-j\omega(N-1)/2}\frac{\sin(N\omega/2)}{\sin(\omega/2)}$. (3pts)

We are given the desired analog frequency resolution, $\Delta\Omega$, which we need to convert to discrete time frequency, $\Delta\omega$.

$$\Delta\omega = \Delta\Omega T = 2\pi \times 5 \times 100 \times 10^{-6} = 1 \times 10^{-3}\pi \text{ rad/sec}$$

We can find $\Delta\omega$ as a function of N by setting it equal to the width of the mainlobe, which is the width between the first two zero crossings of the digital sinc function. Since digital sinc is an even function, the width is twice the value of the first zero crossing, ω_0.

$$\begin{aligned} \frac{N\omega_0}{2} &= \pi \\ \omega_0 &= \frac{2\pi}{N} \\ \Delta\omega &= \frac{4\pi}{N} \end{aligned}$$

Lastly, solve for N, then set it to be the closest power of 2.

$$\boxed{N = 2^{\lceil \log_2(4\pi/\Delta\omega) \rceil} = 4096}$$

3. Let $\{x[n]\}_{n=0}^{2} = \{2, 4, 5\}$ and $\{v[n]\}_{n=0}^{2} = \{1, 0, -1\}$. A new sequence $\{g[n]\}_{n=0}^{3}$ is generated as follows: $\{g[n]\}_{n=0}^{3} = \text{IFFT}\left(\{G[k]\}_{k=0}^{3}\right)$ where the IFFT is a 4-point inverse FFT, $G[k] = X[k]V[k], k = 0, 1, 2, 3$, and the sequences $X[k]$ and $V[k]$ are generated each by a 4-point FFT of the sequences $\{x[n]\}$ and $\{v[n]\}$, respectively, after zero padding them to length 4. Determine $g[1]$ and $g[3]$. (4pts)

   We know that multiplication in the discrete frequency domain is equivalent to circular convolution in the discrete time domain. Then $g[n] = x_{zp}[n] \circledast v_{zp}[n]$ where the 'zp' subscript denotes the zero-padded sequences. One method of solving this circular convolution is using the matrix method.

$$g[n] = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 3 \\ -4 \end{bmatrix}$$

   Which gives us our solution,

$$\boxed{g[1] = 4,\ g[3] = -4}$$