ECE310: Quiz#1 (10am Section G) Fall 2018 Solutions

1. (6 pts)

   (a) Derive closed-form expressions for the magnitude and phase of the function $G(\omega) = -1 - e^{j2\omega}$ of the real variable ω.

   (b) Sketch the phase over the interval $-\pi \leq \omega \leq \pi$. Label the axes in your plot, and mark values at the "interesting points."

Note that we can write $G(\omega)$ as $(-1)(1 + e^{j2\omega})$. Now, the easiest approach is to "split the phase":

$$\begin{aligned} G(\omega) &= (-1)(1 + e^{j2\omega}) \\ &= (-1)e^{j\omega}(e^{j\omega} + e^{-j\omega}) \\ &= (-1)e^{j\omega}(2\cos(\omega)) \end{aligned}$$

Noting that we can write -1 as $e^{-j\pi}$, this becomes

$$G(\omega) = e^{j(\omega-\pi)}(2\cos(\omega))$$

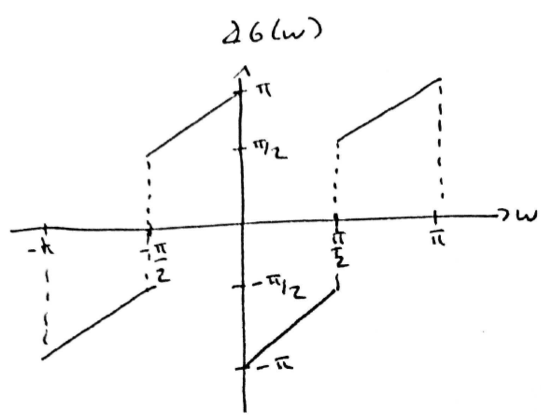
From here, we calculate the magnitude to be

$$\boxed{|G(\omega)| = 2|\cos(\omega)|}$$

and the phase to be

$$\boxed{\angle G(\omega) = \begin{cases} \omega - \pi, & \cos(\omega) \geq 0 \rightarrow \omega \in [-\frac{\pi}{2}, \frac{\pi}{2}] \\ \omega, & \cos(\omega) < 0 \rightarrow \omega \in [-\pi, -\frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi] \end{cases}}$$

The π offset in the phase comes from the fact that the magnitude must always be positive. So, whenever $2\cos(\omega) < 0$, we signify that $G(\omega)$ takes a negative value by adding/subtracting π from the phase. The plot of the phase can be seen below.



2. (4 pts) Draw a block diagram of a system with input $x[n]$ and output $y[n]$, defined by $y[n] = 2y[n-2] - 0.5x[n] + 3x[n-2]$.

An example block diagram can be seen below.