University of Illinois Fall 2018

ECE 310: Problem Set 13

Due: 5pm, Friday December 7, 2018

1. Given an input $x[n]$, let $v[n]$ be the output of a sample rate expander (upsampler, zero insertion) by an integer factor L, and $w[n]$ be the output of a sample rate compressor (downsampler) by an integer factor D. That is, $w[n] = x[nD]$, and

$$v[n] = \begin{cases} x[\ell], & n = L\ell \\ 0 & \text{otherwise} \end{cases}$$

Given that

$$X_d(\omega) = \begin{cases} 1 - \frac{2|\omega|}{\pi}, & \text{if } |\omega| \le \pi/2 \\ 0 & \text{if } \pi/2 < |\omega| \le \pi \end{cases}$$

   (a) Sketch $V_d(\omega)$ for $L = 2$.
   (b) Sketch $V_d(\omega)$ for $L = 3$.
   (c) Sketch $W_d(\omega)$ for $D = 2$.
   (d) Sketch $W_d(\omega)$ for $D = 3$.

2. Consider the system illustrated in Fig. 1. The block labeled as $H_d(\omega)$ is an ideal LPF with cutoff $\pi/3$ and gain of 1 in the passband. Find the output $y[n]$ for the following inputs:

   (a) $x[n] = \cos(0.25\pi n)$
   (b) $x[n] = \cos(0.75\pi n)$
   (c) $x[n] = \frac{\sin(\pi n/8)}{\pi n}$

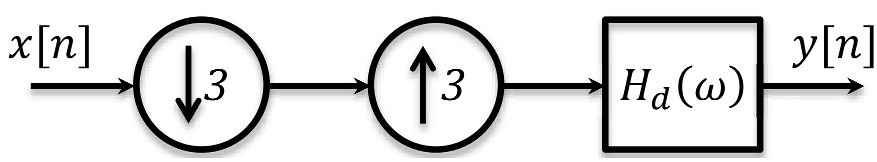
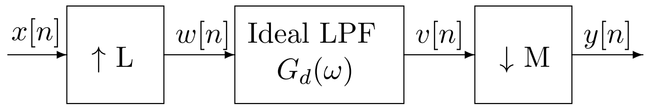


Figure 1 (Problem 2)

3. A speech signal $x_a(t)$ has been low-pass filtered (anti-aliasing) at 6 kHz and then sampled at a rate of 12 kHz to produce the sampled sequence $x[n] = x_a(nT_1)$. It is desired to convert $x[n]$ digitally to $y[n] = x_a(nT_2)$ where $y[n]$ is the sequence that would have been obtained by low-pass filtering $x_a(t)$ to 5 kHz and then sampling it at 10 kHz.

   Design a digital system that converts $x[n]$ to $y[n]$ using the block diagram shown below, i.e., specify L, M, and $G_d(\omega)$ that would achieve this design goal.



4. **Spectral Gymnastics.** Consider again the system in Problem 3. For each of the following cases, sketch the DTFTs $W_d(\omega), V_d(\omega)$, and $Y_d(\omega)$, for input $x[n]$ that has the DTFT $X_d(\omega) = |\omega|/\pi$ for $|\omega| \leq \pi$. Make sure you label both the axes and indicate all the "important" points. (If a sketch is identical to a previous one you don't need to repeat it; just indicate the corresponding identical sketch.)

   In each case, also determine (justify your answer) whether the entire system, for *any* input $x[n]$ to output $y[n]$, is shift invariant. When it is, sketch the magnitude of the frequency response $H_d(\omega)$ of the entire system from input to output. In all cases, the magnitude frequency response of the filter $G_d(\omega)$ in its pass band is equal to 1.

   (a) $L = 3$, $M = 2$, $G_d(\omega)$ is an ideal LPF with cutoff $\omega_c = \pi/3$.
   (b) $L = 3$, $M = 2$, $G_d(\omega)$ is an ideal LPF with cutoff $\omega_c = \pi/2$.
   (c) $L = 3$, $M = 2$, $G_d(\omega)$ is an ideal LPF with cutoff $\omega_c = 2\pi/3$.
   (d) $L = 3$, $M = 3$, $G_d(\omega)$ is an ideal LPF with cutoff $\omega_c = \pi/3$.
   (e) $L = 3$, $M = 3$, $G_d(\omega)$ is an ideal LPF with cutoff $\omega_c = 2\pi/3$.
   (f) $L = 3$, $M = 2$, $G_d(\omega)$ is an ideal HPF with cutoff $\omega_c = \pi/3$.
   (g) $L = 3$, $M = 2$, $G_d(\omega)$ is an ideal HPF with cutoff $\omega_c = \pi/2$.
   (h) $L = 3$, $M = 2$, $G_d(\omega)$ is an ideal HPF with cutoff $\omega_c = 2\pi/3$.
   (i) $L = 3$, $M = 3$, $G_d(\omega)$ is an ideal HPF with cutoff $\omega_c = \pi/3$.
   (j) $L = 2$, $M = 3$, $G_d(\omega)$ is an ideal LPF with cutoff $\omega_c = \pi/3$.
   (k) $L = 2$, $M = 3$, $G_d(\omega)$ is an ideal LPF with cutoff $\omega_c = \pi/2$.
   (l) $L = 2$, $M = 3$, $G_d(\omega)$ is an ideal LPF with cutoff $\omega_c = \pi$ (i.e, a "do-nothing" filter, a "wire").

5. Let the input to a ZOH operating with period $T = 1$ msec be $y[n] = \cos(n\pi/3)$. Sketch the output of the ZOH by hand for $0 \leq t < 8$ msec.

6. Sketch by hand the magnitude of the Fourier transform of the output of a ZOH operating at 8 Hz for an input $y[n] = \cos(n\pi/3)$. Do the sketch for $0 \leq |\Omega| \leq 32\pi$. Because it does not provide perfect bandlimiting like the ideal D/A, the ZOH will have at its output frequency components that do not appear at the output of an ideal D/A with the same input $y[n]$. Determine the magnitude and frequency of the largest (in the frequency domain) such spurious component at the output.

7. In a digital audio recorder, the recorded signal is bandlimited to 20 kHz, and the sampling rate is 44 kHz. The recorder uses a D/A with a zero-order hold to produce the analog output. To filter out spurious components at the output, the system uses an (imperfect) analog filter that has a non-zero transition band. Determine the maximum allowed width of this transition band for

   (a) a standard D/A using a zero-order hold.
   (b) an oversampling D/A using a zero-order hold with an oversampling factor of 5.