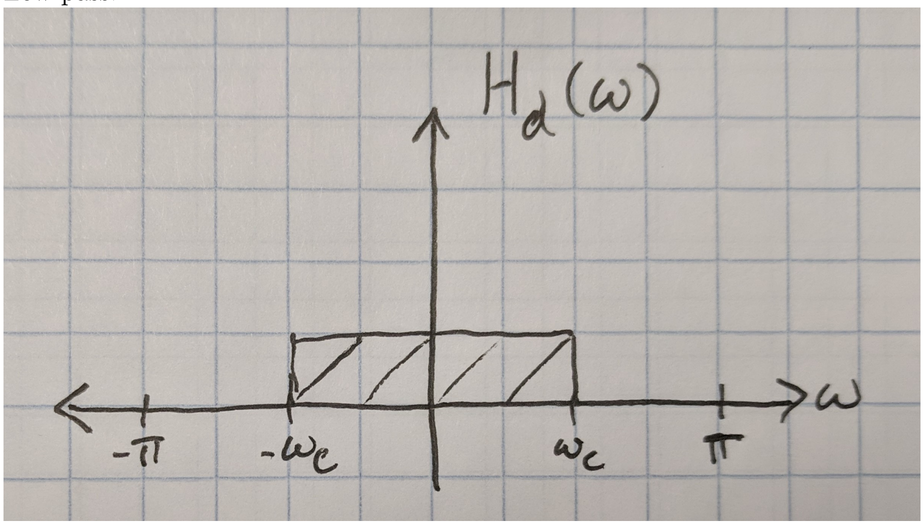
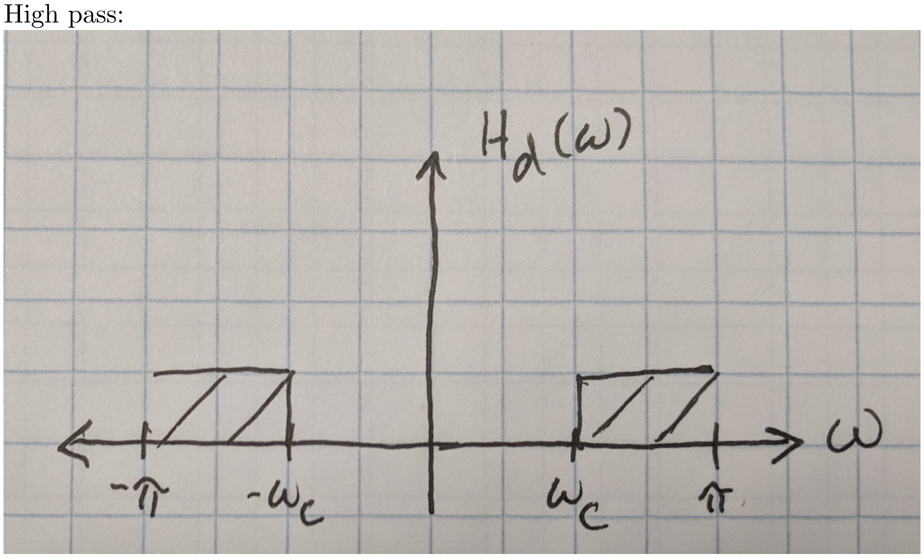
ECE 310: Recitation #10 Fall 2018

FIR Filter Design

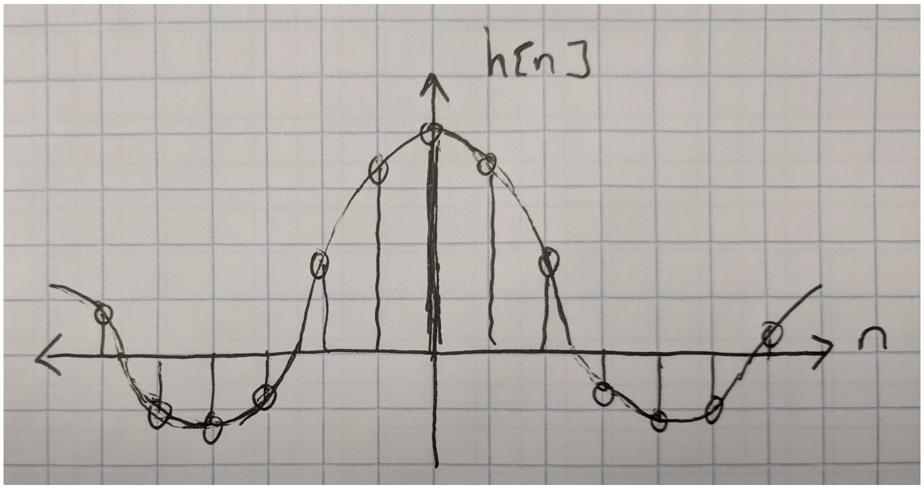
To design a filter, start by considering the DTFT of an ideal filter's impulse response:
Low pass:



High pass:



These will correspond to impulse responses in the shape of a sinc function: $h[n] = \frac{\sin(\omega_c n)}{\pi n}$. This is determined through the IDTFT, defined by the integral $\int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} d\omega$:



If the goal is to perform real-time processing, there are 2 issues with this impulse response:

1. It is non-causal. The output must change before the input does.
2. It is infinitely long. Processing a single output requires performing an infinite number of computations.

To address these issues, apply a window and time delay to the impulse response. The window function is used to eliminate non-causal portions of the impulse response and limit it to a finite length. Shifting preserves the symmetry of the impulse response after windowing, which is necessary for generalized linear phase. For a low pass filter, time shifting leads to the responses:

$$h\left[n\right]=\frac{\sin\left(\omega_c\left(n-\alpha\right)\right)}{\pi\left(n-\alpha\right)}$$

$$H_d\left(\omega\right)=\begin{cases}e^{-j\alpha\omega}, & |\omega|<\omega_c\\ 0, & \omega_c<|\omega|<\pi\end{cases}$$

The value α is chosen so that the sinc function is centered in the window in order to keep GLP. Thus it is chosen $\alpha=\frac{M}{2}=\frac{N-1}{2}$. The type of symmetry present in the impulse response and the length of the window determine which of the rows of the following table govern its properties.

Table 10.1 Properties of impulse response sequence $h[n]$ and frequency response function $H(e^{j\omega}) = A(e^{j\omega})e^{j\Psi(e^{j\omega})}$ of FIR filters with linear phase.

Type	$h[k]$	M	$A(e^{j\omega})$	$A(e^{j\omega})$	$\Psi(e^{j\omega})$
I	even	even	$\sum_{k=0}^{M/2} a[k]\cos\omega k$	even–no restriction	$-\frac{\omega M}{2}$
II	even	odd	$\sum_{k=1}^{\frac{M+1}{2}} b[k]\cos\left[\omega\left(k-\frac{1}{2}\right)\right]$	even $A(e^{j\pi})=0$	$-\frac{\omega M}{2}$
III	odd	even	$\sum_{k=1}^{M/2} c[k]\sin\omega k$	odd $A(e^{j0})=0$ $A(e^{j\pi})=0$	$\frac{\pi}{2}-\frac{\omega M}{2}$
IV	odd	odd	$\sum_{k=1}^{\frac{M+1}{2}} d[k]\sin\left[\omega\left(k-\frac{1}{2}\right)\right]$	odd $A(e^{j0})=0$	$\frac{\pi}{2}-\frac{\omega M}{2}$

The windowing is done by multiplication in the time domain, corresponding to convolution in the frequency domain. This is what leads to the ripple effects in the passband and stopband, and causes the transition to be non-ideal. The effects caused by convolution with various common window functions are summarized in the following table.

Table 10.3 Properties of commonly used windows ($L = M + 1$).

Window name	Side lobe level (dB)	Approx. $\Delta\omega$	Exact $\Delta\omega$	$\delta_p \approx \delta_s$	A_p (dB)	A_s (dB)
Rectangular	-13	$4\pi/L$	$1.8\pi/L$	0.09	0.75	21
Bartlett	-25	$8\pi/L$	$6.1\pi/L$	0.05	0.45	26
Hann	-31	$8\pi/L$	$6.2\pi/L$	0.0063	0.055	44
Hamming	-41	$8\pi/L$	$6.6\pi/L$	0.0022	0.019	53
Blackman	-57	$12\pi/L$	$11\pi/L$	0.0002	0.0017	74