ECE310: Quiz#6 (6pm Section CSS) Fall 2018 Solutions

1. (5 pts) The frequency response of an LTI system is

$$H_d(\omega) = (\omega^2 + 2\cos(2\omega))e^{j\omega\cos(4\omega)}, \quad \frac{\pi}{8} \leq |\omega| \leq \frac{6\pi}{7}$$

   (a) Is the system real?

   (b) Determine the output $y[n]$ for input $x[n] = 1 + \cos(\frac{\pi n}{4}) + \cos(\frac{\pi}{5})\sin(\frac{9\pi n}{10})$.

Solution

(a) Since the support is symmetric, to verify whether or not the system is real, we need to see if $X_d(\omega) = X_d^*(-\omega)$. Plug in:

$$\begin{aligned}
X_d(\omega) &= (\omega^2 + 2\cos(2\omega))e^{j\omega\cos(4\omega)} \\
X_d^*(\omega) &= (\omega^2 + 2\cos(2\omega))e^{-j\omega\cos(4\omega)} \\
X_d^*(-\omega) &= ((-\omega)^2 + 2\cos(-2\omega))e^{-j(-\omega)\cos(-4\omega)} \\
&= (\omega^2 + 2\cos(\omega))e^{j\omega\cos(4\omega)} \\
&= X_d(\omega)
\end{aligned}$$

using the fact that cosine is an even function. Since $X_d(\omega) = X_d^*(-\omega)$, **the system is real**.

(b) Since the system is real, we know that

$$x[n] = \cos(\omega_0 n + \phi) \rightarrow \boxed{H_d(\omega)} \rightarrow y[n] = |H_d(\omega_0)|\cos(\omega_0 n + \phi + \angle H_d(\omega_0))$$

Therefore, we can write the output as

$$y[n] = H_d(0) + \left|H_d\left(\frac{\pi}{4}\right)\right|\cos\left(\frac{\pi n}{4} + \angle H_d\left(\frac{\pi}{4}\right)\right) + \cos\left(\frac{\pi}{5}\right)\left|H_d\left(\frac{9\pi}{10}\right)\right|\sin\left(\frac{9\pi n}{10} + \angle H_d\left(\frac{9\pi}{10}\right)\right)$$

Calculating $H_d(\omega)$ at the desired values gives

$$\begin{aligned}
H_d(0) &= 0 \text{ (outside the bounds)} \\
H_d\left(\frac{\pi}{4}\right) &= \left(\frac{\pi^2}{16} + 2\cos\left(\frac{\pi}{2}\right)\right)e^{j\frac{\pi}{4}\cos(\pi)} = \frac{\pi^2}{16}e^{-j\frac{\pi}{4}} \\
H_d\left(\frac{9\pi}{10}\right) &= 0 \text{ (outside the bounds)}
\end{aligned}$$

Therefore, the output is given as

$$\boxed{y[n] = \frac{\pi^2}{16}\cos\left(\frac{\pi n}{4} - \frac{\pi}{4}\right)}$$

Grading:

- 2 points for (a).
- 1 point for application of the eigensequence property.
- 1 point for calculating $H_d(0), H_d(\frac{\pi}{4})$, and $H_d(\frac{9\pi}{10})$.
- 1 point for the final answer.

2. (5 pts) Consider the discrete-time signal $x[n] = 2\tan(\frac{\pi}{5})\cos(\frac{10\pi n}{13})$. Find two continuous-time signals $x_c(t)$ that will produce $x[n]$ when sampled at a rate of 260 samples per second.

Solution

Since cosine is a 2π-periodic even function, we can write

$$x[n] = 2\tan\left(\frac{\pi}{5}\right)\cos\left(\left(\frac{10\pi}{13} \pm 2\pi k\right)n\right), \quad k \in \mathbb{Z}$$

Therefore, since $x[n] = x_c(nT)$, and using the relationship $\omega = \Omega T$ with $T = \frac{1}{260}$ s, we find that

$$x_c(t) = 2\tan\left(\frac{\pi}{5}\right)\cos((200\pi \pm 520\pi k)t), \quad k \in \mathbb{Z}$$

We get infinitely many possible continuous-time signals, depending on the value of k chosen. For example, when $k = 0$, we get

$$\boxed{x_c(t) = 2\tan\left(\frac{\pi}{5}\right)\cos(200\pi t)}$$

and when $k = 1$, we get

$$\boxed{x_c(t) = 2\tan\left(\frac{\pi}{5}\right)\cos(720\pi t)}$$

Grading:

- 3 points for correct reasoning.
- 1 point for each continuous-time signal.