ECE310: Quiz#6 (3pm Section E) Fall 2018 Solutions

1. (5 pts) The frequency response of an LTI system is

$$H_d(\omega) = (\omega^3 + \omega + 2)e^{j\omega \sin(3\omega)}, \quad \frac{\pi}{7} \le \omega \le \frac{4\pi}{5}$$

   (a) Is the system real?

   (b) Determine the output $y[n]$ for the input $x[n] = 2 + j^n + \cos(\frac{\pi}{9})\sin(\frac{7\pi n}{8})$.

Solution

There are two ways to approach the problem in this case. One approach would be to note that the support isn't symmetric; $H_d(\omega)$ is only nonzero over a subset of positive ω. Therefore, the system is **not real**.

Alternatively, you could show that $H_d(\omega) \neq H_d^*(-\omega)$:

$$\begin{aligned} H_d(\omega) &= (\omega^3 + \omega + 2)e^{j\omega \sin(3\omega)} \\ H_d^*(\omega) &= (\omega^3 + \omega + 2)e^{-j\omega \sin(3\omega)} \\ H_d^*(-\omega) &= ((-\omega)^3 - \omega + 2)e^{j\omega \sin(-3\omega)} \\ &= (-\omega^3 - \omega + 2)e^{-j\omega \sin(3\omega)} \\ &\neq H_d(\omega) \end{aligned}$$

Since $H_d(\omega) \neq H_d^*(-\omega)$, the system is not real.

(b) Since the system is not real, we need to break the input into its constituent complex exponentials in order to find the output. Then we can apply the eigensequence property:

$$x[n] = e^{j\omega_0 n} \rightarrow \boxed{H_d(\omega)} \rightarrow y[n] = H_d(\omega_0)e^{j\omega_0 n}$$

We can write the input as

$$x[n] = 2e^{j0} + e^{j\frac{\pi}{2}n} + \frac{1}{2j}\cos\left(\frac{\pi}{9}\right)e^{j\frac{7\pi}{8}n} - \frac{1}{2j}\cos\left(\frac{\pi}{9}\right)e^{-j\frac{7\pi}{8}n}$$

Therefore, the output is given as

$$y[n] = 2H_d(0)e^{j0} + H_d\left(\frac{\pi}{2}\right)e^{j\frac{\pi}{2}n} + \frac{1}{2j}\cos\left(\frac{\pi}{9}\right)H_d\left(\frac{7\pi}{8}\right)e^{j\frac{7\pi}{8}n} - \frac{1}{2j}\cos\left(\frac{\pi}{9}\right)H_d\left(-\frac{7\pi}{8}\right)e^{-j\frac{7\pi}{8}n}$$

Calculating $H_d(\omega)$ at the desired values gives

$$\begin{aligned}
H_d(0) &= 0 \text{ (outside the bounds)} \\
H_d\left(\frac{\pi}{2}\right) &= \left(\frac{\pi^3}{8} + \frac{\pi}{2} + 2\right) e^{j\frac{\pi}{2}\sin\left(\frac{3\pi}{2}\right)} = \left(\frac{\pi^3}{8} + \frac{\pi}{2} + 2\right) e^{-j\frac{\pi}{2}} \\
H_d\left(\frac{7\pi}{8}\right) &= 0 \text{ (outside the bounds)} \\
H_d\left(-\frac{7\pi}{8}\right) &= 0 \text{ (outside the bounds)}
\end{aligned}$$

So, the output is given as

$$\boxed{y[n] = \left(\frac{\pi^3}{8} + \frac{\pi}{2} + 2\right) e^{j\frac{\pi}{2}(n-1)}}$$

Grading:

- 2 points for (a).
- 1 point for application of the eigensequence property.
- 1 point for calculating the correct values of $H_d(\omega)$.
- 1 point for the final answer.

2. (5 pts) Consider the discrete-time signal $x[n] = \cos(\frac{\pi}{4})\cos(\frac{7\pi n}{11})$. Find two continuous-time signals $x_c(t)$ that will produce $x[n]$ when sampled at a rate of 440 samples per second.

Solution

Since cosine is a 2π-periodic even function, we can write

$$x[n] = \cos\left(\frac{\pi}{4}\right)\cos\left(\left(\frac{7\pi}{11} \pm 2\pi k\right) n\right), \quad k \in \mathbb{Z}$$

Therefore, since $x[n] = x_c(nT)$, and using the relationship $\omega = \Omega T$ with $T = \frac{1}{440}$ s, we find that

$$x_c(t) = \cos\left(\frac{\pi}{4}\right)\cos\left(\left(280\pi \pm 880\pi k\right) t\right), \quad k \in \mathbb{Z}$$

We get infinitely many possible continuous-time signals, depending on the value of k chosen. For example, when $k = 0$, we get

$$\boxed{x_c(t) = \cos\left(\frac{\pi}{4}\right)\cos(280\pi t)}$$

and when $k = 1$, we get

$$\boxed{x_c(t) = \cos\left(\frac{\pi}{4}\right)\cos(1160\pi t)}$$

Grading:

- 3 points for correct reasoning.
- 1 point for each continuous-time signal.