ECE 310: Quiz #6 (10am Section G) Fall 2018 Solutions

1. The frequency response of an LTI system is

$$H_d(\omega) = \left(\omega^4 + 2\omega^2\right) e^{j\omega\cos(2\omega)}, \; \frac{\pi}{6} \le |\omega| \le \frac{3\pi}{4}$$

(a) Is the system real? (2pts)

Yes. The system displays Hermitian symmetry.

$$\begin{aligned} H_d^*(-\omega) &= \left((-\omega)^4 + 2(-\omega)^2\right) e^{-j(-\omega)\cos(2(-\omega))}, \; \frac{\pi}{6} \le |\omega| \le \frac{3\pi}{4} \\ &= \left(\omega^4 + 2\omega^2\right) e^{j\omega\cos(2\omega)}, \; \frac{\pi}{6} \le |\omega| \le \frac{3\pi}{4} = H_d(\omega) \end{aligned}$$

We can also see from inspection that the magnitude displays even symmetry, as it is a sum of even-degree polynomials, and the phase displays odd symmetry, since it is the product of an even function $(\cos(2\omega))$ and an odd function (ω).

(b) Determine the output $y[n]$ for the input $x[n] = 1 + \cos(\frac{\pi}{2}n) + \cos(\frac{\pi}{3})\sin(\frac{6\pi}{7}n)$ (3 pts)

First, because $H_d(\omega)$ characterizes an LTI system, we can use the eigensequence property.

$$x[n] = Ae^{j\omega_0 n} \rightarrow \boxed{H_d(\omega)} \rightarrow y[n] = H_d(\omega_0)Ae^{j\omega_0 n} \tag{1}$$

Furthermore, because the system is *real*, we can use a special property of sinusoids:

$$x[n] = A\cos(\omega_0 n + \phi) \rightarrow \boxed{H_d(\omega)} \rightarrow y[n] = |H_d(\omega_0)|A\cos(\omega_0 n + \phi + \angle H_d(\omega_0)) \tag{2}$$

We can identify the frequencies of our three input components as 0, $\frac{\pi}{2}$, and $\frac{6\pi}{7}$.

$$\begin{aligned} H_d(0) &= 0 \\ H_d(\frac{\pi}{2}) &= \left((\frac{\pi}{2})^4 + 2(\frac{\pi}{2})^2\right)e^{-j\frac{\pi}{2}} \\ H_d(\frac{6\pi}{7}) &= 0 \end{aligned}$$

Applying (2) gives us the following:

$$y[n] = \left((\frac{\pi}{2})^4 + 2(\frac{\pi}{2})^2\right)\cos\left(\frac{\pi}{2}n - \frac{\pi}{2}\right)$$

2. Consider the discrete-time signal $x[n] = \cos\left(\frac{5\pi}{9}n\right)$. Find two continuous-time signals $x_c(t)$ that will produce $x[n]$ when sampled at a rate of 180 samples per second. (5pts)

First, consider the the case of no aliasing.

$$\Omega_0 = \frac{\omega_0}{T} = \frac{5\pi \cdot 180}{9} = 100\pi \text{ rads}$$

Then, consider any aliased signal whose frequencies align with either of the following conditions.

$$\begin{aligned} \omega_1 &= \omega_0 + 2\pi k \; \forall k \in \mathbb{Z} \\ \omega_1 &= -\omega_0 + 2\pi k \; \forall k \in \mathbb{Z} \end{aligned}$$

For example, we take the aliased signal with the lowest frequency.

$$\omega_1 = -\omega_0 + 2\pi = \frac{13\pi}{9}$$
$$\Omega_1 = \frac{\omega_1}{T} = \frac{13\pi \cdot 180}{9} = 260\pi \text{ rads}$$

This gives us the two following signals:

$$x_{c,0}(t) = \cos(100\pi t)$$
$$x_{c,1}(t) = \cos(260\pi t)$$