ECE 310: Quiz #5 (3pm Section E) Fall 2018 Solution

Given: October 10, 2018

1. (5 pts) Compute the DTFT of $x[n] = \left(\frac{j}{4}\right)^n e^{-j\frac{\pi}{4}n} u[n]$.

   Express j as $e^{j\frac{\pi}{2}}$.

$$\begin{aligned} x[n] &= \left(\frac{e^{j\frac{\pi}{2}}}{4}\right)^n e^{-j\frac{\pi}{4}n} u[n] \\ &= \left(\frac{e^{j\frac{\pi}{4}}}{4}\right)^n u[n] \end{aligned}$$

   Take the z-transform using the transform pair $a^n u[n] \leftrightarrow \frac{1}{1-az^{-1}}$, ROC: $|z| > |a|$.

$$X(z) = \frac{1}{1 - \left(\frac{e^{j\frac{\pi}{4}}}{4}\right) z^{-1}}$$

   The signal is right-sided (causal) and the magnitude of the pole is less than one. Therefore the ROC will include the unit circle and the DTFT can be obtained by substituting $z = e^{j\omega}$.

$$\begin{aligned} X_d(\omega) &= \frac{1}{1 - \left(\frac{e^{j\frac{\pi}{4}}}{4}\right) e^{-j\omega}} \\ &= \frac{1}{1 - \frac{1}{4} e^{-j\left(\omega - \frac{\pi}{4}\right)}} \end{aligned}$$

   Alternatively, recall the frequency shift property.

$$e^{j\omega_c n} x[n] \overset{DTFT}{\longleftrightarrow} X_d(\omega - \omega_c)$$

   Take the z-transform of $\left(\frac{j}{4}\right)^n u[n]$ using the transform pair $a^n u[n] \leftrightarrow \frac{1}{1-az^{-1}}$.

$$\frac{1}{1 - \left(\frac{j}{4}\right) z^{-1}}$$

The signal is right-sided (causal) and the magnitude of the pole is less than one. Therefore the ROC will include the unit circle and the DTFT can be obtained by substituting $z = e^{j\omega}$.

$$\frac{1}{1 - \left(\frac{j}{4}\right) e^{-j\omega}}$$

Apply the frequency shift property with $\omega_c = -\frac{\pi}{4}$.

$$X_d(\omega) = \frac{1}{1 - \left(\frac{j}{4}\right) e^{-j\left(\omega + \frac{\pi}{4}\right)}}$$

2. (5 pts) Let $x[n]$ be a signal with DTFT $X_d(\omega)$.

   (a) Find an expression for the DTFT of $y[n] = x[n] \cos\left(\frac{\pi}{2} n\right)$ in terms of $X_d(\omega)$.

   (b) Suppose $X_d(\omega)$ is as shown below. Sketch the DTFT of $y[n]$. Label the axes and "important points" on your sketch.

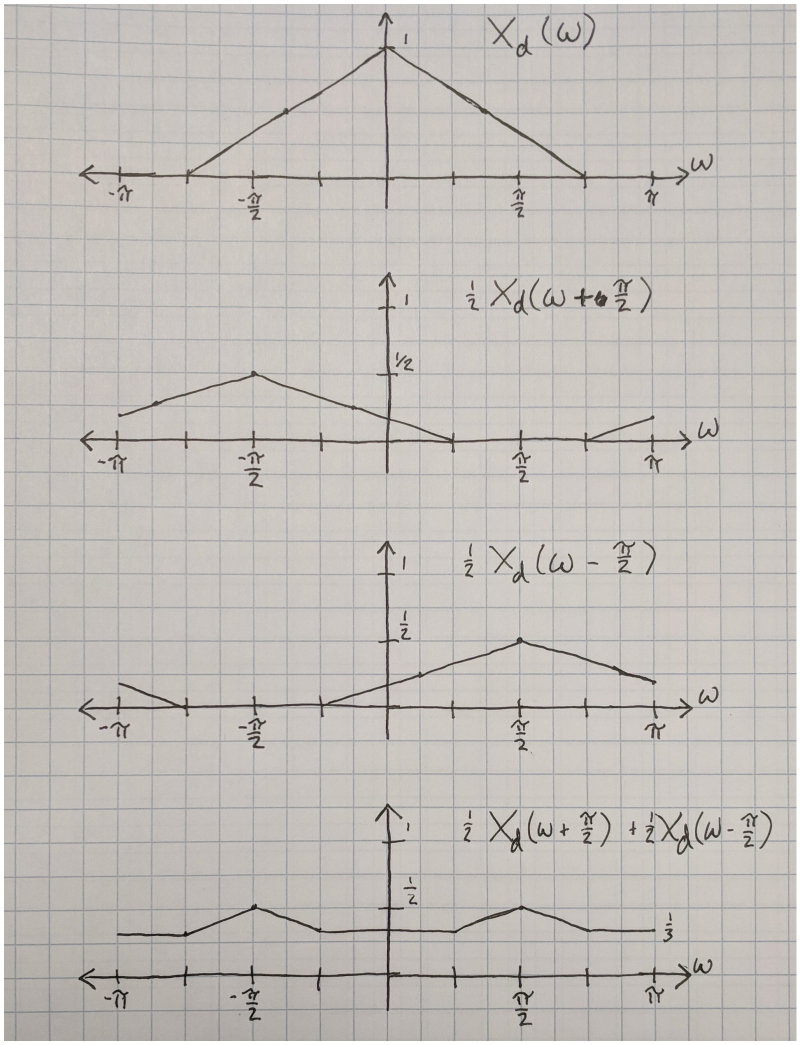
(a) Recall the modulation property.

$$x[n] \cos(\omega_c n) \overset{DTFT}{\longleftrightarrow} \frac{1}{2} X_d(\omega + \omega_c) + \frac{1}{2} X_d(\omega - \omega_c)$$

Apply the property with $\omega_c = \frac{\pi}{2}$.

$$Y_d(\omega) = \frac{1}{2} X_d\left(\omega + \frac{\pi}{2}\right) + \frac{1}{2} X_d\left(\omega - \frac{\pi}{2}\right)$$

(b)


$X_d(\omega)$
$\frac{1}{2}X_d(\omega+\frac{\pi}{2})$
$\frac{1}{2}X_d(\omega-\frac{\pi}{2})$
$\frac{1}{2}X_d(\omega+\frac{\pi}{2})+\frac{1}{2}X_d(\omega-\frac{\pi}{2})$
ω
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
1
1/2
$\frac{1}{2}$
$\frac{1}{3}$