

ECE310 Fall 2018 Section E Quiz 1 Solutions

1 (6 pts)

- (a) Derive closed-form expressions for the magnitude and phase of the function
 $G(\omega) = -(1+j)\cos(\omega)e^{-j\omega}$

$$\begin{aligned}G(\omega) &= -(1+j)\cos(\omega)e^{-j\omega} \\&= (-1-j)\cos(\omega)e^{-j\omega} \\&= \left(\sqrt{2}e^{j\frac{5\pi}{4}}\right)\cos(\omega)e^{-j\omega} \\&= \sqrt{2}\cos(\omega)e^{j\left(\frac{5\pi}{4}-\omega\right)}\end{aligned}$$

$$|G(\omega)| = \sqrt{2}|\cos(\omega)|$$

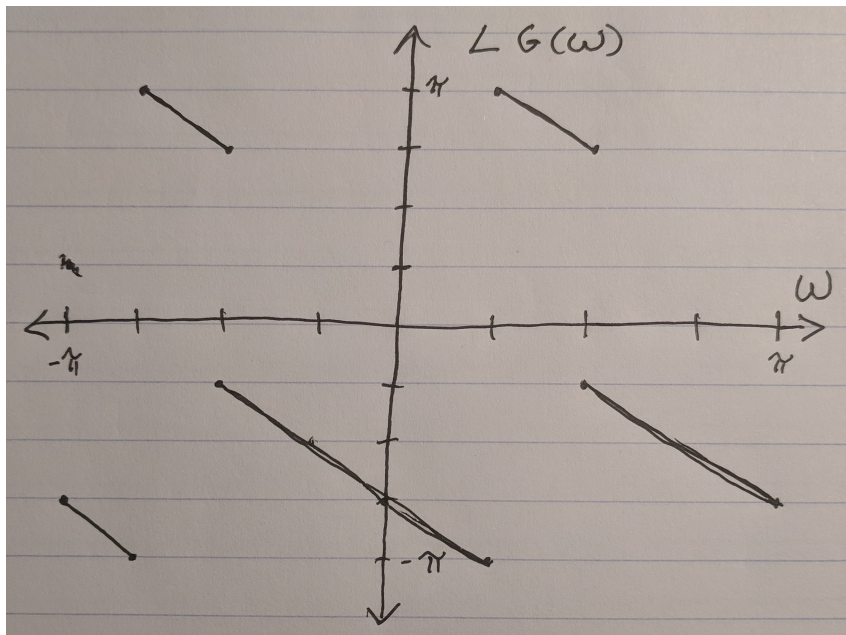
For $\omega \in [-\pi, \pi]$ $\angle G(\omega)$ is:

$$\angle G(\omega) = \begin{cases} \frac{\pi}{4} - \omega & -\pi \leq \omega < -\frac{\pi}{2} \\ \frac{5\pi}{4} - \omega & -\frac{\pi}{2} \leq \omega \leq \frac{\pi}{2} \\ \frac{\pi}{4} - \omega & \frac{\pi}{2} < \omega \leq \pi \end{cases}$$

- (b) Sketch the phase over the interval $-\pi \leq \omega \leq \pi$. Label the axes in your plot, and mark values at the "interesting points."

In order to keep $G(\omega) \in [-\pi, \pi]$ when graphing, a shift of 2π is added to sections which would go outside this range:

$$\angle G(\omega) = \begin{cases} \frac{-7\pi}{4} - \omega & -\pi \leq \omega \leq \frac{-3\pi}{4} \\ \frac{\pi}{4} - \omega & \frac{-3\pi}{4} \leq \omega < \frac{-\pi}{2} \\ \frac{-3\pi}{4} - \omega & \frac{-\pi}{2} \leq \omega \leq \frac{\pi}{4} \\ \frac{5\pi}{4} - \omega & \frac{\pi}{4} \leq \omega \leq \frac{\pi}{2} \\ \frac{\pi}{4} - \omega & \frac{\pi}{2} < \omega \leq \pi \end{cases}$$



2

(4 pts) Draw a block diagram of a system with input $x[n]$ and output $y[n]$, defined by $y[n] - 0.2y[n-1] - 0.5x[n-1] + x[n-2] = 0$.

$$y[n] = 0.2y[n-1] + 0.5x[n-1] - x[n-2]$$

