PBPK in PuMaS, A Model for ACAT

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1 Introduction

2 Code

```
using PuMaS, LinearAlgebra, DiffEqSensitivity, Distributions, Optim, QuadGK
pbpkmodel = @model begin
    @param begin
        GER ∈ ConstDomain(0.066)
        \rho \in \texttt{ConstDomain}(5e-6)
        r ∈ ConstDomain(1)
        T ∈ ConstDomain(3e-5)
        d ∈ ConstDomain(1e-4)
        SST ∈ ConstDomain(5.5)
        kilST ∈ ConstDomain(0.5)
        kaST \in ConstDomain(14040.00076)
        kaGU \in ConstDomain(14040.000063)
        kt ∈ ConstDomain(0.035)
        SGU1 ∈ ConstDomain(5.5)
        SGU2 

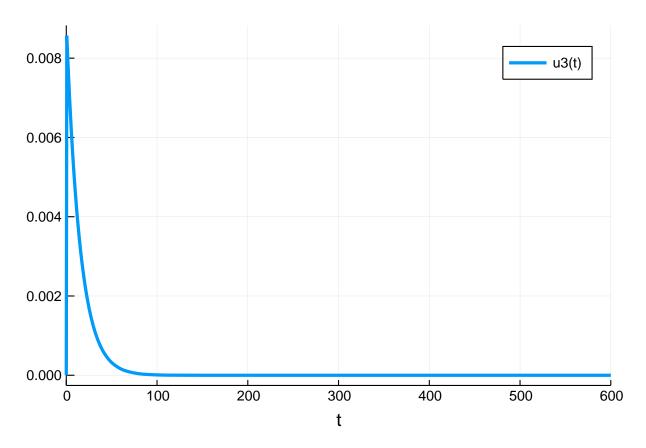
ConstDomain(5.5)
        SGU3 ∈ ConstDomain(5.5)
        SGU4 \in ConstDomain(5.5)
        SGU5 ∈ ConstDomain(5.5)
        SGU6 ∈ ConstDomain(5.5)
        SGU7 \in ConstDomain(5.5)
        kilGU1 ∈ ConstDomain(0.5)
        kilGU2 ∈ ConstDomain(0.5)
        kilGU3 ∈ ConstDomain(0.5)
        kilGU4 ∈ ConstDomain(0.5)
        kilGU5 \in ConstDomain(0.5)
        kilGU6 ∈ ConstDomain(0.5)
        kilGU7 ∈ ConstDomain(0.5)
        EHR ∈ ConstDomain(0 )
        kbil ∈ ConstDomain(0.0)
        VLI ∈ ConstDomain(1690)
        Kp \in ConstDomain(1.3)
        ktCO ∈ ConstDomain(0.0007)
        SCO ∈ ConstDomain(5.5)
        VCO ∈ ConstDomain(700)
        kilCO ∈ ConstDomain(0.0007)
        kaCO ∈ ConstDomain(14040.0000542)
        CP ∈ ConstDomain(0)
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QLU ∈ ConstDomain(5233)

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VLU ∈ ConstDomain(1172)
    VST1 ∈ ConstDomain(50)
    VST2 ∈ ConstDomain(154)
    VGU ∈ ConstDomain(1650)
    VAR ∈ ConstDomain(1698)
    AIR \in ConstDomain(0.0)
    VVE ∈ ConstDomain(3396)
    VIR \in ConstDomain(0.0)
    QBR \in ConstDomain(700)
    VBR ∈ ConstDomain(1450)
    QLI \in ConstDomain(1650)
    QKI \in ConstDomain(1100)
    QHR \in ConstDomain(150)
    VHR ∈ ConstDomain(310)
    QMU ∈ ConstDomain(750)
    VMU ∈ ConstDomain(35000)
    QAD \in ConstDomain(260)
    VAD ∈ ConstDomain(10000)
    QSK \in ConstDomain(300)
    VSK ∈ ConstDomain(7800)
    QBO \in ConstDomain(250)
    VBO ∈ ConstDomain(4579)
    QTH ∈ ConstDomain(80)
    VTH ∈ ConstDomain(29)
    QST \in ConstDomain(38)
    QGU \in ConstDomain(1100)
    Ker \in ConstDomain(0.0)
    QPA ∈ ConstDomain(133)
    VPA ∈ ConstDomain(77)
    QSP \in ConstDomain(77)
    VSP ∈ ConstDomain(192)
    CLint ∈ ConstDomain(0)
    QHA ∈ ConstDomain(302)
    VKI ∈ ConstDomain(280)
    R ∈ ConstDomain(1)
end
@random begin
    \eta \sim \text{MvNormal}(\text{Matrix}(1.0\text{I},2,2))
end
Odynamics begin
    #Absorption compartments
    #Stomach compartment
    AUNDST' = -GER * AUNDST - (((3*d)/(\rho*r*T)) * AUNDST* (SST - (ADIST/VST1)))
    ADIST' = -GER * ADIST + (((3*d)/(\rho*r*T)) *AUNDST*(SST - (ADIST/VST1))) - kilST*
ADIST -kaST *ADIST
    ADEGST' = -GER * ADEGST + kilST * ADIST
    AABSST' = kaST * ADIST
    #GU1 small intestinal compartment
    AUNDGU1' = GER * AUNDST - kt * AUNDGU1 - ((3*d)/(\rho*r*T)) * AUNDGU1 *(SGU1 -
(ADISGU1/VGU))
    ADISGU1' = GER * ADIST - kt * ADISGU1 + ((3*d)/(\rho*r*T)) * AUNDGU1 *(SGU1 -
(ADISGU1/VGU)) - kilGU1*ADISGU1 - kaGU*ADISGU1 + (EHR*kbil*CLI *VLI)/( Kp)
    ADEGGU1' = GER * ADEGST - kt * ADEGGU1 + kilGU1 * ADISGU1
    AABSGU1' = kaGU * ADISGU1
    # Other small intestinal compartments (GU2-GU7)
```

```
AUNDGU2' = kt * AUNDGU1 - kt * AUNDGU2 - ((3*d)/(\rho*r*T)) * AUNDGU2 * (SGU2 - (3*d)/(\rho*r*T)) * AUNDGU2 * (SGU2 - (3*d
(ADISGU2/VGU))
               ADISGU2' = kt * ADISGU1 - kt * ADISGU2 + ((3*d)/(\rho*r*T)) * AUNDGU2 * (SGU2 - P)
(ADISGU2/VGU)) - kilGU2*ADISGU2 - kaGU*ADISGU2
              ADEGGU2' = kt*ADEGGU1 - kt*ADEGGU2 + kilGU2 * ADISGU2
               AABSGU2' = kaGU * ADISGU2
              AUNDGU3' = kt * AUNDGU2 - kt * AUNDGU3 - ((3*d)/(\rho*r*T)) * AUNDGU3 * (SGU3 - P)
(ADISGU3/VGU))
               ADISGU3' = kt * ADISGU2 - kt * ADISGU3 +((3*d)/(\rho*r*T)) * AUNDGU3 *(SGU3 -
(ADISGU3/VGU)) - kilGU3*ADISGU3 - kaGU*ADISGU3
              ADEGGU3' = kt*ADEGGU2 - kt*ADEGGU3 + kilGU3 * ADISGU3
               AABSGU3' = kaGU * ADISGU3
               AUNDGU4' = kt * AUNDGU3 - kt * AUNDGU4 - ((3*d)/(\rho*r*T)) * AUNDGU4 * (SGU4 - (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) + (1) +
(ADISGU4/VGU))
               ADISGU4' = kt * ADISGU3 - kt * ADISGU4 + ((3*d)/(\rho*r*T)) * AUNDGU4 * (SGU4 - \rho*r*T)
(ADISGU4/VGU)) - kilGU4*ADISGU4 - kaGU*ADISGU4
              ADEGGU4' = kt*ADEGGU3 - kt*ADEGGU4 + kilGU4 * ADISGU4
               AABSGU4' = kaGU * ADISGU4
              AUNDGU5' = kt * AUNDGU4 - kt * AUNDGU5 - ((3*d)/(\rho*r*T)) * AUNDGU5 * (SGU5 - 1)
(ADISGU5/VGU))
               ADISGU5' = kt * ADISGU4 - kt * ADISGU5 + ((3*d)/(\rho*r*T)) * AUNDGU5 * (SGU5 - P)
(ADISGU5/VGU)) - kilGU5*ADISGU5 - kaGU*ADISGU5
               ADEGGU5' = kt*ADEGGU4 - kt*ADEGGU5 + kilGU5 * ADISGU5
               AABSGU5' = kaGU * ADISGU5
               AUNDGU6' = kt * AUNDGU5 - kt * AUNDGU6 - ((3*d)/(\rho*r*T)) * AUNDGU6 * (SGU6 - V)
(ADISGU6/VGU))
               ADISGU6' = kt * ADISGU5 - kt * ADISGU6 +((3*d)/(\rho*r*T)) * AUNDGU6 *(SGU6 -
(ADISGU6/VGU)) - kilGU6*ADISGU6 - kaGU*ADISGU6
               ADEGGU6' = kt*ADEGGU5 - kt*ADEGGU6 + kilGU6 * ADISGU6
               AABSGU6' = kaGU * ADISGU6
              AUNDGU7' = kt * AUNDGU6 - kt * AUNDGU7 - ((3*d)/(\rho*r*T)) * AUNDGU7 * (SGU7 - (3*d)/(\rho*r*T)) * AUNDGU7 * (SGU7 - (3*d
(ADISGU7/VGU))
              ADISGU7' = kt * ADISGU6 - kt * ADISGU7 + ((3*d)/(\rho*r*T)) * AUNDGU7 * (SGU7 - r)
(ADISGU7/VGU)) - kilGU7*ADISGU7 - kaGU*ADISGU7
               ADEGGU7' = kt*ADEGGU6 - kt*ADEGGU7 + kilGU7 * ADISGU7
               AABSGU7' = kaGU * ADISGU7
               # Colon compartment
               AUNDCO' = kt * AUNDGU7 - ktCO * AUNDCO - ((3*d)/(\rho*r*T)) * AUNDCO * (SCO - \rho*r*T)
(ADISCO/VCO))
               ADISCO' = kt * ADISGU7 - kt * ADISCO + ((3*d)/(\rho*r*T)) * AUNDCO * (SCO - P)
(ADISCO/VCO)) - kilCO*ADISCO - kaCO*ADISCO + (CP*CLI*VLI*kbil)/( Kp)
               ADEGCO' = kt * ADEGGU7 - ktC0 * ADEGGU7 + kilC0 * ADISCO
               AABSCO' = kaCO * ADISGU7
               #Total intestinal absorption (IA)
               AIA' = kaGU*ADISGU1 + kaGU*ADISGU2 + kaGU*ADISGU3 + kaGU*ADISGU4 + kaGU*ADISGU5
+ kaGU*ADISGU6 + kaGU*ADISGU7
               #Somatic Compartments
               # Lungs
              CLU' = (QLU/VLU) * (CVE - (CLU*R)/(Kp))
               #Arterial blood (AR)
```

```
CAR' = (1/VAR) * (QLU*(((CLU*R)/(Kp)) - CAR) + AIR)
                        # Venous blood (VE)
                       CVE' = (1/VVE) * (((QBR *CBR*R)/( Kp)) + ((QLI *CLI*R)/( Kp)) + ((QKI *CKI*R)/(
           *CSK*R)/( Kp)) + ((QBO *CBO*R)/( Kp)) +((QTH *CTH*R)/( Kp)) - QLU * CVE + VIR)
                        #Brain
                       CBR' = (QBR/VBR)* (CAR - (CBR*R)/(Kp))
                        #Heart
                       CHR' = (QHR/VHR)* (CAR - (CHR*R)/(Kp))
                       CMU' = (QMU/VMU)* (CAR - (CMU*R)/(Kp))
                       #Adinose
                       CAD' = (QAD/VAD)* (CAR - (CAD*R)/(Kp))
                        #Skin
                       CSK' = (QSK/VSK)* (CAR - (CSK*R)/(Kp))
                       #Bone
                       CBO' = (QBO/VBO)* (CAR - (CBO*R)/(Kp))
                       #Thymus
                       CTH' = (QTH/VTH)* (CAR - (CTH*R)/(Kp))
                       #Pancreas
                       CPA' = (QPA/VPA)* (CAR - (CPA*R)/(Kp))
                       #Spleen
                       CSP' = (QSP/VSP)* (CAR - (CSP*R)/(Kp))
                       #Stomach
                       CST' = (1/VST2) * (QST*(CAR - ((CST*R)/(Kp))) + AABSST)
                       CGU' = (1/VGU) * (QGU*(CAR - ((CGU*R)/(Kp))) + AIA)
                       #Kidney
                       CKI' = ((1/VKI) * QKI*(CAR - ((CKI*R)/(Kp)))) - ((CKI*Ker)/Kp)
                       #Liver
                       CLI' = (1/VLI)*(QHA*CAR + ((QGU *CGU*R)/(Kp)) + ((QPA *CPA*R)/(Kp)) + ((QSPA*R)/(Kp)) + ((QSPA*R)/(K
           *CSP*R)/( Kp)) + ((QST *CST*R)/( Kp)) - ((QLI *CLI*R)/( Kp)) - (CLI*CLint)/Kp )
            end
end
subject = Subject(evs = DosageRegimen(250,cmt=[1],time=[0.0]))
param = (GER = 0.066, \rho = 5e-6, r = 1, T = 3e-5, d = 1e-4, SST = 5.5, kilST = 0.5, kaST =
           14040.00076,kaGU = 14040.000063,kt = 0.035,SGU1 = 5.5,SGU2 = 5.5,SGU3 = 5.5,SGU4 =
           5.5, SGU5 = 5.5, SGU6 = 5.5, SGU7 = 5.5, kilGU1 = 0.0, kilGU2 = 0.0, kilGU3 = 0.0, kilGU4
           = 0.0, kilGU5 = 0.0, kilGU6 = 0.0, kilGU7 = 0.0, EHR = 0
                       kbil = 0.0, VLI = 1690, Kp = 1.3, ktCO = 0.0007, SCO = 5.5, VCO = 700, kilCO = 1.3, ktCO = 1.3, ktCO
           0.0007,kaC0 = 14040.0000542,CP = 0,QLU = 5233,VLU = 1172,VST1 = 50,VST2 = 154,VGU =
           1650, VAR = 1698, AIR = 0.0, VVE = 3396, VIR = 0.0, QBR = 700, VBR = 1450, QLI = 1650, QKI =
           1100,QHR = 150,VHR = 310,QMU = 750,VMU = 35000,QAD = 260,
                       VAD = 10000,QSK = 300,VSK = 7800,QBO = 250,VBO = 4579,QTH = 80,VTH = 29,QST =
           38,QGU = 1100,Ker = 10.0,QPA = 133,VPA = 77,QSP = 77,VSP = 192,CLint = 0.315,QHA =
           302,VKI = 280,R = 1)
y0 = (\eta = [0.0, 0.0])
sol_diffeq = solve(pbpkmodel,subject,param,y0,tspan=(0.0,600.0),progress=true)
using Plots
plot(sol_diffeq,vars=3)
```



3 Global Sensitivity Analysis

```
function sensivity_func(pars)
    y0 = (\eta = [0.0,0.0])
    sim = solve(pbpkmodel, subject, pars, y0, tspan=(0.0,600.0))
    f = t \rightarrow -sim(t;idxs=3)
    res = optimize(f,0.0,600.0,Brent())
    i,e = quadgk(f,0.0,600.0)
    [-Optim.minimum(res),-250/i]
end
a = []
for i in param
    if i != 0
        push!(a,[i-0.05*i,i+ 0.05*i])
    else
        push!(a,[0.0,1e-4])
    end
end
using Random
Random.seed!(5)
m = DiffEqSensitivity.morris_sensitivity(
                       sensivity_func,a,[10 for i in 1:70];
                       relative_scale= false,len_trajectory=75,
                       total_num_trajectory=50,num_trajectory=20)
```