PuMaS for Multiple Response Pk/Pd

Chris Rackauckas

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1 Introduction

This is an introduction to PuMaS, a software for pharmacometric modeling and simulation. In this tutorial we will show how to simulate from a multiple response PK/PD model defined by ordinary differential equations, and how to extract information about the solution.

1.1 Installation

Because PuMaS is still unregistered, you will need to give the Git repository in order to add the package. To do this, use the command <code>ladd https://github.com/UMCTM/PuMaS.jl</code>. Doing it this way, PuMaS and its dependencies will install automatically. If one cannot authenticate for this command (since the repository is currently private!), then first clone the repository how you please, use <code>ladd path/to/package</code>, then then do <code>lbuild PuMaS</code>. Using the build command will download and install the dependencies.

1.2 Getting Started

To load the package, use using PuMaS

1.3 Our Example Model

1.4 Using the @model Macro

Now let's define a model. A model is defined in an Qmodel block. Inside of this block we have a few subsections. The first of which is Qparam. In here we define what kind of parameters we have. For this model we will define a vector parameter θ of size 12:

```
\begin{array}{l} {\tt Oparam \ begin} \\ \theta \in {\tt VectorDomain(12)} \\ {\tt end} \end{array}
```

Next we define our random effects. The random effects are defined by a distribution from Distributions.jl. For more information on defining distributions, please see the Distributions.jl

documentation. For this tutorial, we wish to have a multivariate normal of 11 uncorrelated random effects, so we utilize the syntax:

Notice that here we imported I from LinearAlgebra and and said that our Normal distribution's covariance is said I, the identity matrix.

Now we define our pre-processing step in **@pre**. This is where we choose how the parameters, random effects, and the covariates collate. We define the values and give them a name as follows:

```
Opre begin
                  = \theta[1]
     Ka1
     CL
                  = \theta[2] * \exp(\eta[1])
      Vс
                  = \theta[3] * \exp(\eta[2])
                  = \theta[4] * \exp(\eta[3])
      Vр
                 = \theta[5] * \exp(\eta[4])
                 = \theta[6] * \exp(\eta[5])
     Kin
     Kout
                 = \theta[7] * \exp(\eta[6])
      IC50
                  = \theta[8] * \exp(\eta[7])
      XAMI
                 = \theta[9] * \exp(\eta[8])
                  = \theta[10] * \exp(\eta[9])
                 = \theta[11] * \exp(\eta[10])
      Vmax
                  = \theta[12] * \exp(\eta[11])
end
```

Next we define the **@init** block which gives the inital values for our differential equations. Any variable not mentioned in this block is assumed to have a zero for its starting value. We wish to only set the starting value for **Resp**, and thus we use:

```
Cinit begin

Resp = \theta[6]/\theta[7]

end
```

Now we define our dynamics. We do this via the **@dynamics** block. Differential variables are declared by having a line defining their derivative. For our model, we use:

Lastly we utilize the **@derived** macro to define our post-processing. We can output values using the following:

```
\begin{array}{rcl} \textbf{@derived begin} \\ & \text{ev1} & = \text{Ev1} \\ & \text{cp} & = \text{Cent} \ / \ \theta \text{[3]} \\ & \text{periph} & = \text{Periph} \\ & \text{resp} & = \text{Resp} \\ & \text{end} \end{array}
```

The Qmodel block is all of these together, giving us the following model:

```
using LinearAlgebra
model = @model begin
     @param begin
       \theta \in \texttt{VectorDomain}(12)
     end
     @random begin
       \eta \sim MvNormal(Matrix{Float64}(I, 11, 11))
     Opre begin
          Ka1
                    = \theta[1]
          CL
                    = \theta[2]*exp(\eta[1])
                    = \theta[3] * \exp(\eta[2])
          Vс
          Q
                    = \theta[4] * \exp(\eta[3])
                   = \theta[5] * \exp(\eta[4])
          ďγ
                   = \theta[6] * \exp(\eta[5])
          Kin
          Kout
                   = \theta[7] * \exp(\eta[6])
          IC50
                   = \theta[8] * \exp(\eta[7])
                 = \theta[9] * \exp(\eta[8])
          IMAX
                   = \theta[10] * \exp(\eta[9])
                 = \theta[11] * \exp(\eta[10])
          Vmax
                   = \theta[12] * \exp(\eta[11])
     end
     @init begin
          Resp = \theta[6]/\theta[7]
     end
     @dynamics begin
          Ev1'
                    = -Ka1*Ev1
                   = Ka1*Ev1 - (CL+Vmax/(Km+(Cent/Vc))+Q)*(Cent/Vc) + Q*(Periph/Vp)
          Cent'
          Periph' = Q*(Cent/Vc) - Q*(Periph/Vp)
                  = Kin*(1-(IMAX*(Cent/Vc)^{\gamma}/(IC50^{\gamma}+(Cent/Vc)^{\gamma}))) - Kout*Resp
     end
     @derived begin
          ev1
                  = Ev1
                  = Cent / \theta[3]
          periph = Periph
          resp
                  = Resp
     end
end
```

1.5 Grabbing Data

In this tutorial we will utilize an example dataset in NMTRAN form. To bring in data we utilize the process_nmtran function. The first argument is the path to the dataset. The path to our example data is example_nmtran_data("event_data/data23"). Next we give it the name of the covariate from the dataset. Our example data has no covariates, so we note this as []. Lastly, we declare the names of the columns which correspond to the observation variables. In this dataset, we have observations for all of [:ev1,:cp,:periph,:resp]. Together, this gives us the command:

```
pop = process_nmtran(example_nmtran_data("event_data/data23"),[],
        [:ev1,:cp,:periph,:resp])
```

That gives us a whole population. If we wish to grab a subject out of the population, we simply index the population. Let's grab the first subject:

```
subject = pop[1]
```

1.6 Running a Simulation

The main function for running a simulation is simobs. simobs on a population simulates all of the population (in parallel), while simobs on a subject simulates just that subject. If we wish to change the parameters from the initialized values, then we pass them in. Let's simulate subject 1 with a set of chosen parameters:

```
x0 = (θ = [

1, # Ka1 Absorption rate constant 1 (1/time)
1, # CL Clearance (volume/time)
20, # Vc Central volume (volume)
2, # Q Inter-compartmental clearance (volume/time)
10, # Vp Peripheral volume of distribution (volume)
10, # Kin Response in rate constant (1/time)
2, # Kout Response out rate constant (1/time)
2, # IC50 Concentration for 50% of max inhibition (mass/volume)
1, # IMAX Maximum inhibition
1, # γ Emax model sigmoidicity
0, # Vmax Maximum reaction velocity (mass/time)
2 # Km Michaelis constant (mass/volume)
],)
```

sim = simobs(model, subject, x0)

Notice that in our model we said that there was a single parameter θ so our input parameter is a named tuple with just the name θ . When we only give the parameters, the random effects are automatically sampled from their distributions. If we wish to prescribe a value for the random effects, we pass initial values similarly:

```
y0 = (\eta = zeros(11),)
sim = simobs(model, subject, x0, y0)
```

The points which are saved automatically match the observations from the dataset. If you wish to change the saving time points, or did not have observations in your dataset, pass the keyword argument obstimes. For example, let's save at every 0.1 hours and run the simulation for 19 hours:

```
sim = simobs(model, subject, x0, y0, obstimes = 0:0.1:19)
```

1.7 Handling the SimulatedObservations

The resulting SimulatedObservations type has two fields. sim.times is an array of time points for which the data was saved. sim.derived is the result of the derived variables. From there, the derived variables are accessed by name. For example,

```
sim.derived.cp

191-element Array{Float64,1}:
    0.0
    0.47221914014817434
```

```
0.892563879689724
```

- 1.2661629995273587
- 1.5976533425197088
- 1.8912202652117394
- 2.1506525123265425
- 2.379365921095179
- 2.5804446918538138
- 2.7566848994999047

:

- 1.7208030796955243
- 1.7151514165326058
- 1.709525592732175
- 1.7039252684684807
- 1.698350113429305
- 1.6927998068159575
- 1.6872740373432795
- 1.6817725009423412
- 1.6762949074483278

is the array of cp values at the associated time points. We can turn this into a DataFrame via using the DataFrame command:

DataFrame(sim.derived)

| 1 0.0 0.0 5.0 2 90.4837 0.472219 0.0478097 4.90282 3 81.8731 0.892564 0.182936 4.68846 4 74.0818 1.26616 0.393897 4.42606 5 67.032 1.59765 0.6704 4.15103 6 60.6531 1.89122 1.00324 3.88186 7 54.8811 2.15065 1.38416 3.62842 8 49.6585 2.37937 1.80581 3.3952 9 44.9329 2.58044 2.26161 3.18391 10 40.657 2.75668 2.74568 2.99469 11 36.7879 2.91061 3.2528 2.82656 12 33.287 3.04448 3.77834 2.67809 13 30.1194 3.16035 4.31818 2.54768 14 27.2532 3.26008 4.86862 2.43359 15 24.6596 3.34535 5.42643 2.33411 < | | ev1 | cn. | periph | resp |
|---|----|---------|-----------|---------|-----------------------|
| 2 90.4837 0.472219 0.0478097 4.90282 3 81.8731 0.892564 0.182936 4.68846 4 74.0818 1.26616 0.393897 4.42606 5 67.032 1.59765 0.6704 4.15103 6 60.6531 1.89122 1.00324 3.88186 7 54.8811 2.15065 1.38416 3.62842 8 49.6585 2.37937 1.80581 3.3952 9 44.9329 2.58044 2.26161 3.18391 10 40.657 2.75668 2.74568 2.99469 11 36.7879 2.91061 3.2528 2.82656 12 33.287 3.04448 3.77834 2.67809 13 30.1194 3.16035 4.31818 2.54768 14 27.2532 3.26008 4.86862 2.43359 15 24.6596 3.34535 5.42643 2.33411 16 22.3129 3.41765 5.98878 | 1 | | <u>cp</u> | | |
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| 19 16.5299 3.56976 7.67934 2.0527 20 14.9567 3.60258 8.2375 2.00499 21 13.5333 3.62802 8.79038 1.96429 22 12.2456 3.6469 9.33671 1.92974 23 11.0804 3.65996 9.87538 1.90057 24 10.026 3.66786 10.4054 1.87608 25 9.07168 3.67119 10.926 1.85568 26 8.20821 3.67048 11.4366 1.8389 27 7.42708 3.6662 11.9366 1.82534 28 6.72043 3.65877 12.4255 1.81459 29 6.08107 3.64859 12.903 1.80631 30 5.50248 3.63602 13.3688 1.80018 31 4.97873 3.62137 13.8226 1.7959 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.54703 15.5165 1.7 | | | | | |
| 20 14.9567 3.60258 8.2375 2.00499 21 13.5333 3.62802 8.79038 1.96429 22 12.2456 3.6469 9.33671 1.92974 23 11.0804 3.65996 9.87538 1.90057 24 10.026 3.66786 10.4054 1.87608 25 9.07168 3.67119 10.926 1.85568 26 8.20821 3.67048 11.4366 1.8389 27 7.42708 3.6662 11.9366 1.82534 28 6.72043 3.65877 12.4255 1.81459 29 6.08107 3.64859 12.903 1.80631 30 5.50248 3.63602 13.3688 1.80018 31 4.97873 3.62137 13.8226 1.7959 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1. | | | | | |
| 21 13.5333 3.62802 8.79038 1.96429 22 12.2456 3.6469 9.33671 1.92974 23 11.0804 3.65996 9.87538 1.90057 24 10.026 3.66786 10.4054 1.87608 25 9.07168 3.67119 10.926 1.85568 26 8.20821 3.67048 11.4366 1.8389 27 7.42708 3.6662 11.9366 1.82534 28 6.72043 3.65877 12.4255 1.81459 29 6.08107 3.64859 12.903 1.80631 30 5.50248 3.63602 13.3688 1.80018 31 4.97873 3.62137 13.8226 1.7959 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 | | | | | |
| 22 12.2456 3.6469 9.33671 1.92974 23 11.0804 3.65996 9.87538 1.90057 24 10.026 3.66786 10.4054 1.87608 25 9.07168 3.67119 10.926 1.85568 26 8.20821 3.67048 11.4366 1.8389 27 7.42708 3.6662 11.9366 1.82534 28 6.72043 3.65877 12.4255 1.81459 29 6.08107 3.64859 12.903 1.80631 30 5.50248 3.63602 13.3688 1.80018 31 4.97873 3.62137 13.8226 1.7959 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.56754 15.1113 1.79231 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 | | | | | |
| 23 11.0804 3.65996 9.87538 1.90057 24 10.026 3.66786 10.4054 1.87608 25 9.07168 3.67119 10.926 1.85568 26 8.20821 3.67048 11.4366 1.8389 27 7.42708 3.6662 11.9366 1.82534 28 6.72043 3.65877 12.4255 1.81459 29 6.08107 3.64859 12.903 1.80631 30 5.50248 3.63602 13.3688 1.80018 31 4.97873 3.62137 13.8226 1.7959 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.56754 15.1113 1.79231 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 | | 13.5333 | 3.62802 | 8.79038 | 1.96429 |
| 24 10.026 3.66786 10.4054 1.87608 25 9.07168 3.67119 10.926 1.85568 26 8.20821 3.67048 11.4366 1.8389 27 7.42708 3.6662 11.9366 1.82534 28 6.72043 3.65877 12.4255 1.81459 29 6.08107 3.64859 12.903 1.80631 30 5.50248 3.63602 13.3688 1.80018 31 4.97873 3.62137 13.8226 1.7959 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.56754 15.1113 1.79231 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1. | 22 | 12.2456 | 3.6469 | 9.33671 | 1.92974 |
| 25 9.07168 3.67119 10.926 1.85568 26 8.20821 3.67048 11.4366 1.8389 27 7.42708 3.6662 11.9366 1.82534 28 6.72043 3.65877 12.4255 1.81459 29 6.08107 3.64859 12.903 1.80631 30 5.50248 3.63602 13.3688 1.80018 31 4.97873 3.62137 13.8226 1.7959 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.56754 15.1113 1.79231 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.4377 17.0162 | 23 | 11.0804 | 3.65996 | 9.87538 | 1.90057 |
| 26 8.20821 3.67048 11.4366 1.8389 27 7.42708 3.6662 11.9366 1.82534 28 6.72043 3.65877 12.4255 1.81459 29 6.08107 3.64859 12.903 1.80631 30 5.50248 3.63602 13.3688 1.80018 31 4.97873 3.62137 13.8226 1.7959 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.56754 15.1113 1.79231 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1. | 24 | 10.026 | 3.66786 | 10.4054 | 1.87608 |
| 27 7.42708 3.6662 11.9366 1.82534 28 6.72043 3.65877 12.4255 1.81459 29 6.08107 3.64859 12.903 1.80631 30 5.50248 3.63602 13.3688 1.80018 31 4.97873 3.62137 13.8226 1.7959 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.56754 15.1113 1.79231 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1 | 25 | 9.07168 | 3.67119 | 10.926 | 1.85568 |
| 28 6.72043 3.65877 12.4255 1.81459 29 6.08107 3.64859 12.903 1.80631 30 5.50248 3.63602 13.3688 1.80018 31 4.97873 3.62137 13.8226 1.7959 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.56754 15.1113 1.79231 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 26 | 8.20821 | 3.67048 | 11.4366 | 1.8389 |
| 29 6.08107 3.64859 12.903 1.80631 30 5.50248 3.63602 13.3688 1.80018 31 4.97873 3.62137 13.8226 1.7959 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.56754 15.1113 1.79231 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 27 | 7.42708 | 3.6662 | 11.9366 | 1.82534 |
| 30 5.50248 3.63602 13.3688 1.80018 31 4.97873 3.62137 13.8226 1.7959 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.56754 15.1113 1.79231 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 28 | 6.72043 | 3.65877 | 12.4255 | 1.81459 |
| 31 4.97873 3.62137 13.8226 1.7959 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.56754 15.1113 1.79231 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 29 | 6.08107 | 3.64859 | 12.903 | 1.80631 |
| 32 4.50471 3.60492 14.2643 1.79328 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.56754 15.1113 1.79231 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 30 | 5.50248 | 3.63602 | 13.3688 | 1.80018 |
| 33 4.07588 3.5869 14.6939 1.79214 34 3.68801 3.56754 15.1113 1.79231 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 31 | 4.97873 | 3.62137 | 13.8226 | 1.7959 |
| 34 3.68801 3.56754 15.1113 1.79231 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 32 | 4.50471 | 3.60492 | 14.2643 | 1.79328 |
| 35 3.33719 3.54703 15.5165 1.79363 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 33 | 4.07588 | 3.5869 | 14.6939 | 1.79214 |
| 36 3.01982 3.52557 15.9095 1.79595 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 34 | 3.68801 | 3.56754 | 15.1113 | 1.79231 |
| 37 2.73259 3.50331 16.2904 1.79915 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 35 | 3.33719 | 3.54703 | 15.5165 | 1.79363 |
| 38 2.47251 3.48041 16.6593 1.80311 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 36 | 3.01982 | 3.52557 | 15.9095 | 1.79595 |
| 39 2.23701 3.457 17.0162 1.80774 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 37 | 2.73259 | 3.50331 | 16.2904 | 1.79915 |
| 40 2.02392 3.43317 17.3614 1.81297 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 38 | 2.47251 | 3.48041 | 16.6593 | 1.80311 |
| 41 1.8312 3.40902 17.695 1.81873 42 1.65695 3.38463 18.0173 1.82496 | 39 | 2.23701 | 3.457 | 17.0162 | 1.80774 |
| 42 1.65695 3.38463 18.0173 1.82496 | 40 | 2.02392 | 3.43317 | 17.3614 | 1.81297 |
| 42 1.65695 3.38463 18.0173 1.82496 | | 1.8312 | 3.40902 | 17.695 | 1.81873 |
| | 42 | 1.65695 | 3.38463 | 18.0173 | 1.82496 |
| 43 1.49941 3.3601 18.3283 1.83159 | 43 | 1.49941 | 3.3601 | 18.3283 | 1.83159 |
| 44 1.35692 3.33548 18.6283 1.83857 | | | | | |
| 45 1.22797 3.31086 18.9175 1.84586 | | | | | |
| 46 1.11117 3.28627 19.1961 1.8534 | | | | | |
| 47 1.00531 3.26179 19.4642 1.86116 | | | | | |
| 48 0.909441 3.23743 19.7222 1.8691 | | | | | |
| 49 0.822713 3.21324 19.9703 1.87721 | | | | | |
| 50 0.744316 3.18923 20.2087 1.88546 | | | | | |
| 51 0.673482 3.16544 20.4377 1.689383 | | | | | |
| 52 0.609488 3.14188 20.6575 1.9023 | | | | | |
| 53 0.551654 3.11858 20.8683 1.91085 | | | | | |

From there, any Julia tools can be used to analyze these arrays and DataFrames. For example, if we wish the plot the result of ev1 over time, we'd use the following:

using Plots
plot(sim.times,sim.derived.ev1)

