Defining and Simulating Populations

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using PuMaS, DataFrames, LinearAlgebra, Plots

1 Introduction

In this tutorial, we will cover the fundamentals of generating populations to simulate with PuMaS. We will demonstrate how to specify dosage regimens and covariates, and then how to piece these together to form a population to simulate.

1.1 The model

Below is a PuMaS model that specifies a 1-compartment oral absorption system with betweensubject variability on all the parameters. Details of the model specification are provided in the introduction tutorial.

```
model = @model begin
  @param begin
     \theta \in {\tt VectorDomain}(4)
     \Omega \in \mathtt{PSDDomain}(3)
     \sigma_{\mathtt{prop}} \in \mathtt{RealDomain}(\mathtt{init=0.1})
  @random begin
     \eta \sim MvNormal(\Omega)
  @covariates isPM Wt
  Opre begin
     \mathsf{TVCL} = \mathsf{isPM} == 1 \ ? \ \theta[1] \ : \ \theta[4]
     CL = \theta[1]*(Wt/70)^0.75*exp(\eta[1])
     V = \theta[2]*(Wt/70)^0.75*exp(\eta[2])
     Ka = \theta[3] * \exp(\eta[3])
  Odynamics begin
     Depot' = -Ka*Depot
     Central' = Ka*Depot - Central*CL/V
```

```
end

Ovars begin

conc = Central/V

end

Oderived begin

dv ~ O.Normal(conc, sqrt(conc^2*\sigma_prop+ eps()))

end

PKPDModel

Parameters: \theta, \Omega, \sigma_prop

Random effects: \eta

Covariates: isPM, Wt

Dynamical variables: Depot, Central

Derived: conc, dv
```

1.2 Setting up parameters

Next we provide the initial estimates of the parameters to simulate from. The fixed effects are provided in the θ vector (CL, V, Ka) and the between-subject variability parameteres are provided in the Ω vector as variances. So, 0.04 variance on Ω 11 suggests a 20% coefficient of variation. Similarly, σ prop has a 20% proportional residual error.

```
\begin{array}{l} \mathbf{p} = (\\ \theta = [0.4,20,1.1,2],\\ \Omega = \mathbf{PDMat}(\mathbf{diagm}(0 \Rightarrow [0.04,0.04,0.04])),\\ \sigma_{\mathbf{prop}} = 0.04 \\ ) \\ \\ (\theta = [0.4,\ 20.0,\ 1.1,\ 2.0],\ \Omega = \mathbf{PDMats}.\mathbf{PDMat}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},2\}\}\{3,\ [0.04\ 0.0\ 0.0;\ 0.0\ 0.0\ 0.0\ 0.04],\ \mathbf{LinearAlgebra}.\mathbf{Cholesky}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\{\mathbf{Float64},\mathbf{Array}\}\}\}\}\}\}\}\}
```

1.3 Single dose example

DosageRegimen() is the function that lets you construct a dosing regimen. The first argument of the DosageRegimen is amt and is not a named argument. All subsequent arguments need to be named. Lets try a simple example where you provide a 100 mg dose at time=0.

```
ev = DosageRegimen(100, time=0)
first(ev.data)
```

	$_{ m time}$	cmt	amt	evid	ii	addl	rate	ss
	Float64	Int64	Float64	Int8	Float64	Int64	Float64	Int8
_1	0.0	1	100.0	1	0.0	0	0.0	0

As you can see above, we provided a single 100 mg dose. DosageRegimen provides some defaults when it creates the dataset, time=0, evid=1, cmt=1, rate=0, ii=0 & addl=0. We can also provide units to the amt and any other variable that is derived from amt, e.g. rate, will have associated units. Handling of units will be covered in a different tutorial.

Note that ev is of type DosageRegimen. Specified like above, DosageRegimen is one of the four fundamental building block of a Subject (more on Subject below).

1.3.1 Building Subjects

Let's create a single subject

```
s1 = Subject(id=1,evs=ev,cvs=(isPM=0, Wt=70))
for fn in fieldnames(Subject)
           x = getproperty(s1, fn)
           if !isa(x, Nothing)
               println(fn)
               println(x)
           end
end
id
observations
NamedTuple{(),Tuple{}}[]
covariates
(isPM = 0, Wt = 70)
PuMaS.Event{Float64,Float64,Float64,Float64,Float64,Float64}[Dose event
 dose amount = 100.0
 dose time = 0.0
 compartment = 1
  instantaneous
  interdose interval = 0.0
  infusion start time = 0.0
]
time
Int64[]
```

Note that each Subject is an individual composed of:

- id: an unique identifier
- obs: observations, represented by PuMaS.Observation[]
- cvs: covariates
- evs: events, represented by PuMaS.Event[]

In the example above, we only provided the id, evs, and the cvs. Since obs were not provided, they are represented by an empty array. Lets take a closer at the events for this subject 1.

```
1-element Array{PuMaS.Event{Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float
```

The events are presented by basic information such as the dose of drug and associated units if specified, the time of dose administration, the compartment number for administration and whether the dose is an instantaneous input or an infusion.

Below is how the covariates are represented

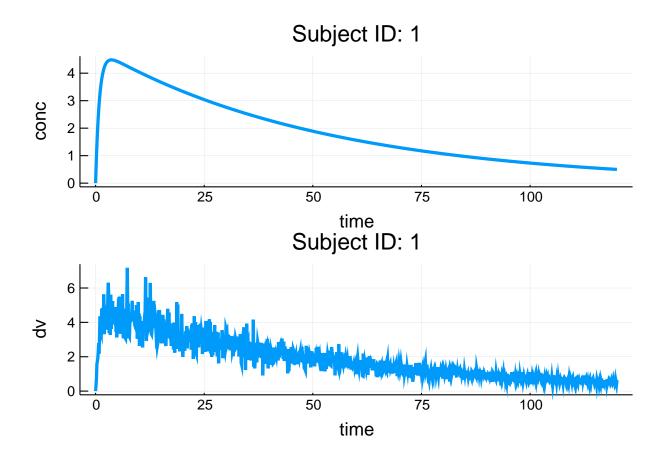
s1.covariates

```
(isPM = 0, Wt = 70)
```

(Note: defining distributions for covariates will be discussed in detail later.)

Using this one subject, s1, let us simulate a simple concentration time profile using the model above:

```
obs = simobs(model,s1,p,obstimes=0:0.1:120)
plot(obs)
```



1.3.2 Building Populations

Now, lets create one more subject, s2.

```
s2 = Subject(id=2,evs=ev,cvs=(isPM=1,Wt=70))
```

```
Subject
ID: 2
Events: 1
Observations: 0
Covariates: isPM, Wt
```

twosubjs = Population([s1,s2])

If we want to simulate both s1 and s2 together, we need to bring these subjects together to form a Population. A Population is essentially a collection of subjects.

```
2-element PuMaS.Population{PuMaS.Subject{StructArrays.StructArray{NamedTupl
e{(),Tuple{}},1,NamedTuple{(),Tuple{}}},NamedTuple{(:isPM, :Wt),Tuple{Int64},Int64}},Array{PuMaS.Event{Float64,Float64,Float64,Float64,Float64,Float64},
1},Int64}}:
Subject
ID: 1
Events: 1
Observations: 0
```

Covariates: isPM, Wt

Subject
ID: 2
Events: 1
Observations: 0
Covariates: isPM, Wt

Let's see the details of the first and the second subject

twosubjs.subjects[1]

Subject
ID: 1
Events: 1
Observations: 0

Covariates: isPM, Wt

twosubjs.subjects[2]

Subject
ID: 2
Events: 1
Observations: 0
Covariates: isPM, Wt

Now, we can simulate this Population of 2 subjects as below

obs = simobs(model,twosubjs,p,obstimes=0:0.1:120)

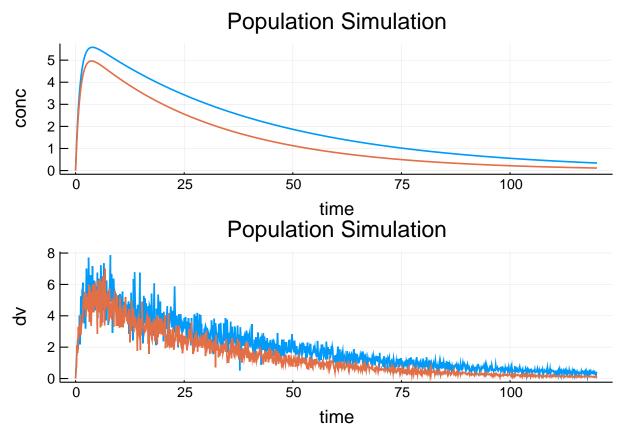
PuMaS.SimulatedPopulation{Array{PuMaS.SimulatedObservations{PuMaS.Subject{S tructArrays.StructArray{NamedTuple{(),Tuple{}},1,NamedTuple{(),Tuple{}}},Na medTuple{(:isPM, :Wt),Tuple{Int64,Int64}},Array{PuMaS.Event{Float64,Float64 ,Float64,Float64,Float64,float64},1},Int64},StepRangeLen{Float64,Base.Twice Precision{Float64},Base.TwicePrecision{Float64}},NamedTuple{(:conc, :dv),Tu ple{Array{Float64,1},Array{Float64,1}}},1}}(PuMaS.SimulatedObservations{Pu MaS.Subject{StructArrays.StructArray{NamedTuple{(),Tuple{}},1,NamedTuple{()} ,Tuple{}}},NamedTuple{(:isPM, :Wt),Tuple{Int64,Int64}},Array{PuMaS.Event{Fl oat64,Float64,Float64,Float64,Float64,Float64,1},1},Int64},StepRangeLen{Float 64, Base. TwicePrecision {Float64}, Base. TwicePrecision {Float64}}, NamedTuple {(: conc, :dv),Tuple{Array{Float64,1},Array{Float64,1}}} [SimulatedObservations {Subject{StructArray{NamedTuple{(), Tuple{}}, 1, NamedTuple{(), Tuple{}}}}, Named Tuple{(:isPM, :Wt),Tuple{Int64,Int64}},Array{Event{Float64,Float64,Float64, Float64,Float64,Float64},1},Int64},StepRangeLen{Float64,TwicePrecision{Floa t64},TwicePrecision{Float64}},NamedTuple{(:conc, :dv),Tuple{Array{Float64,1 },Array{Float64,1}}})(Subject

ID: 1 Events: 1 Observations: 0 Covariates: isPM, Wt , 0.0:0.1:120.0, (conc = [0.0, 0.572321, 1.08975, 1.55744, 1.98002, 2.36172, 2.70637, 3.01743, 3.29804, 3.55107 ... 0.349222, 0.348375, 0.347531, 0.34688, 0.345848, 0.34501, 0.344174, 0.343339, 0.342507, 0.341677], $dv = [-2.57008e-9, 0.515089, 1.16012, 1.53255, 2.07712, 2.5435, 1.84729, 3.1113, 2.67327, 4.01581 ... 0.311728, 0.417526, 0.350573, 0.318168, 0.406706, 0.319189, 0.316772, 0.206393, 0.39207, 0.405278])), SimulatedObservations{Subject{StructArray{NamedTuple{(),Tuple{}},1,NamedTuple{(),Tuple{}}},NamedTuple{(:isPM,:Wt),Tuple{Int64,Int64}},Array{Event{Float64,Float64,Float64,Float64,Float64},TwicePrecision{Float64},1,Int64},StepRangeLen{Float64,TwicePrecision{Float64},TwicePrecision{Float64},NamedTuple{(:conc,:dv),Tuple{Array{Float64,1}},Array{Float64,1}}}$

ID: 2
Events: 1
Observations: 0
Covariates: isPM, Wt
, 0.0:0.1:120.0, (conc = [0.0, 0.505564, 0.963718, 1.37876, 1.75459, 2.0947
7, 2.40253, 2.68081, 2.93228, 3.15938 ... 0.11715, 0.116767, 0.116385, 0.11
6004, 0.115625, 0.115247, 0.11487, 0.114494, 0.11412, 0.113747], dv = [-1.8
3559e-8, 0.674143, 0.69721, 2.12128, 1.79477, 1.63338, 3.27304, 2.45614, 3.
29477, 3.27392 ... 0.108601, 0.137456, 0.117079, 0.0796513, 0.110989, 0.099
8515, 0.160552, 0.152971, 0.101073, 0.122878]))])

When using simobs on more than one subject, i.e., on a Population, the simulation is automatically parallelized across the subejets.

plot(obs)



Similarly, we can build a population of any number of subjects. But before we do that, let's dive into covariate generation.

1.3.3 Covariates

As was discussed earlier, a Subject can also be provided details regarding covariates. In the model above, there are two covariates, isPM which stands for is the subject a poor metabolizer and takes a boolean of yes and no. The second covariate is a continuous covariate where body weight Wt impacts both CL and V. Let us now specify covariates to a population of 10 subjects.

choose_covariates (generic function with 1 method)

choose_covariates will randomly choose a isPM and an Wt between 55-80 kgs.

We can make a list with covariates for ten subjects through a list comprehension

cvs = [choose_covariates() for i in 1:10]
DataFrame(cvs)

	isPM	Wt
	Int64	Int64
1	0	61
2	1	55
3	1	58
4	1	55
5	0	66
6	1	58
7	0	61
8	1	69
9	1	78
10	1	73

Now, we add these covariates to the population as below. The map(f,xs) will return the result of f on each element of xs. Let's map a function that build's a subject with the randomly chosen covariates in order to build a population:

```
Subject
 ID: 2
 Events: 1
 Observations: 0
 Covariates: isPM, Wt
 Subject
  ID: 3
 Events: 1
 Observations: 0
 Covariates: isPM, Wt
 Subject
 ID: 4
 Events: 1
 Observations: 0
 Covariates: isPM, Wt
 Subject
 ID: 5
 Events: 1
 Observations: 0
 Covariates: isPM, Wt
 Subject
 ID: 6
 Events: 1
 Observations: 0
 Covariates: isPM, Wt
 Subject
 ID: 7
 Events: 1
 Observations: 0
 Covariates: isPM, Wt
Subject
 ID: 8
 Events: 1
 Observations: 0
 Covariates: isPM, Wt
 Subject
 ID: 9
 Events: 1
 Observations: 0
 Covariates: isPM, Wt
 Subject
 ID: 10
 Events: 1
 Observations: 0
 Covariates: isPM, Wt
Simulate into the population
```

obs = simobs(model,pop_with_covariates,p,obstimes=0:0.1:120)

 ${\tt PuMaS.SimulatedPopulation \{Array \{PuMaS.Simulated 0bservations \{PuMaS.Subject \{Snumber 1 \} \} \}} \\$ tructArrays.StructArray{NamedTuple{(),Tuple{}},1,NamedTuple{(),Tuple{}}},Na medTuple{(:isPM, :Wt),Tuple{Int64,Int64}},Array{PuMaS.Event{Float64,Float64 ,Float64,Float64,Float64,Float64},1},Int64},StepRangeLen{Float64,Base.Twice Precision{Float64},Base.TwicePrecision{Float64}},NamedTuple{(:conc, :dv),Tu ple{Array{Float64,1},Array{Float64,1}}},1}}(PuMaS.SimulatedObservations{Pu MaS.Subject{StructArrays.StructArray{NamedTuple{(),Tuple{}},1,NamedTuple{()} ,Tuple{}}},NamedTuple{(:isPM, :Wt),Tuple{Int64,Int64}},Array{PuMaS.Event{Fl oat64,Float64,Float64,Float64,Float64,Float64},1},Int64},StepRangeLen{Float 64, Base. TwicePrecision {Float64}, Base. TwicePrecision {Float64}}, NamedTuple {(: conc, :dv),Tuple{Array{Float64,1},Array{Float64,1}}}}[SimulatedObservations {Subject{StructArray{NamedTuple{(), Tuple{}}},1,NamedTuple{(),Tuple{}}},Named Tuple{(:isPM, :Wt),Tuple{Int64,Int64}},Array{Event{Float64,Float64,Float64, Float64,Float64,Float64},1},Int64},StepRangeLen{Float64,TwicePrecision{Floa t64},TwicePrecision{Float64}},NamedTuple{(:conc, :dv),Tuple{Array{Float64,1 },Array{Float64,1}}})(Subject

ID: 1
Events: 1
Observations: 0
Covariates: isPM, Wt

, 0.0:0.1:120.0, (conc = [0.0, 0.517143, 0.990667, 1.42415, 1.82087, 2.1838
4, 2.51583, 2.81938, 3.09681, 3.35028 ... 0.345319, 0.344468, 0.34362, 0.34
2774, 0.34193, 0.341087, 0.340247, 0.339409, 0.338574, 0.33774], dv = [8.45
652e-10, 0.564901, 1.24412, 1.08586, 1.88453, 1.45852, 1.70473, 2.196, 3.22
758, 2.10355 ... 0.399099, 0.426837, 0.251284, 0.367955, 0.301023, 0.317181
, 0.330994, 0.312297, 0.361814, 0.386246])), SimulatedObservations{Subject{
StructArray{NamedTuple{(),Tuple{}},1,NamedTuple{(),Tuple{}}},NamedTuple{(:isPM, :Wt),Tuple{Int64,Int64}},Array{Event{Float64,Float64,Float64,Float64,Float64,Float64,Float64},1},Int64},StepRangeLen{Float64,TwicePrecision{Float64},TwicePrecision{Float64},NamedTuple{(:conc, :dv),Tuple{Array{Float64,1},Array{Float64,1}}}}

ID: 2
Events: 1
Observations: 0
Covariates: isPM, Wt

, 0.0:0.1:120.0, (conc = [0.0, 0.55092, 1.05323, 1.51113, 1.92846, 2.30872,
 2.65512, 2.97058, 3.25778, 3.51915 ... 0.702689, 0.701376, 0.700065, 0.698
757, 0.697451, 0.696148, 0.694847, 0.693549, 0.692253, 0.690959], dv = [-2.
78785e-8, 0.594679, 1.09298, 2.12482, 2.00969, 2.48297, 2.4688, 3.04511, 3.
36634, 2.94427 ... 0.712652, 0.86545, 0.710122, 0.649028, 0.735146, 0.94173
2, 0.589043, 0.606787, 0.795027, 0.791767])), SimulatedObservations{Subject {StructArray{NamedTuple{(),Tuple{}},1,NamedTuple{(),Tuple{}}},NamedTuple{(:isPM,:Wt),Tuple{Int64,Int64}},Array{Event{Float64,Float64,Float64,Float64,Float64,Float64,Float64},
Float64,Float64},1},Int64},StepRangeLen{Float64,TwicePrecision{Float64},TwicePrecision{Float64},TwicePrecision{Float64},TwicePrecision{Float64},NamedTuple{(:conc,:dv),Tuple{Array{Float64,1},Array{Float64,1}}}}

ID: 3
Events: 1
Observations: 0
Covariates: isPM, Wt

, 0.0:0.1:120.0, (conc = [0.0, 0.707116, 1.34036, 1.90731, 2.41477, 2.86886
, 3.27505, 3.63825, 3.96289, 4.25292 ... 0.770005, 0.768578, 0.767153, 0.76
5731, 0.764311, 0.762895, 0.76148, 0.760069, 0.75866, 0.757254], dv = [2.80
606e-9, 0.737379, 1.34516, 1.07859, 2.00251, 2.9659, 4.02289, 4.16401, 4.61
925, 3.32301 ... 0.604963, 0.937181, 0.61327, 0.578587, 0.7679, 0.831049, 0
.849429, 1.00733, 0.637914, 1.03514])), SimulatedObservations{Subject{StructArray{NamedTuple{(),Tuple{}}},1,NamedTuple{(),Tuple{}}},NamedTuple{(:isPM,:Wt),Tuple{Int64,Int64}},Array{Event{Float64,Fl

```
4,Float64,1},Int64},StepRangeLen{Float64,TwicePrecision{Float64},TwicePrec
ision{Float64}},NamedTuple{(:conc, :dv),Tuple{Array{Float64,1},Array{Float6
4,1}}}}(Subject
    ID: 4
    Events: 1
    Observations: 0
    Covariates: isPM, Wt
, 0.0:0.1:120.0, (conc = [0.0, 0.514403, 0.974842, 1.38684, 1.75535, 2.0848
2, 2.37925, 2.64222, 2.87695, 3.08634 ... 0.212658, 0.212089, 0.211521, 0.2
1.50148e-8, 0.406951, 0.989509, 1.52345, 1.78076, 1.87723, 3.20431, 2.73657
, 2.31311, 4.56895 ... 0.169895, 0.261722, 0.28784, 0.210316, 0.265663, 0.1
86258, 0.297109, 0.225643, 0.178878, 0.192208])), SimulatedObservations{Sub
ject{StructArray{NamedTuple{(),Tuple{}}},1,NamedTuple{(),Tuple{}}}},NamedTupl
e{(:isPM, :Wt),Tuple{Int64,Int64}},Array{Event{Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64
t64,Float64,Float64},1},Int64},StepRangeLen{Float64,TwicePrecision{Float64}
,TwicePrecision{Float64}},NamedTuple{(:conc, :dv),Tuple{Array{Float64,1},Ar
ray{Float64,1}}}(Subject
    ID: 5
    Events: 1
    Observations: 0
    Covariates: isPM, Wt
, 0.0:0.1:120.0, (conc = [0.0, 0.710231, 1.34245, 1.90505, 2.40553, 2.85058
, 3.24617, 3.59763, 3.90972, 4.18666 ... 0.481999, 0.480933, 0.479869, 0.47
8808, 0.477748, 0.476692, 0.475637, 0.474585, 0.473535, 0.472488], dv = [-1]
.03659e-8, 0.698639, 1.52847, 1.71862, 1.58437, 2.4865, 3.73867, 3.78431, 3
.34784, 4.3906 ... 0.519496, 0.590827, 0.37275, 0.415483, 0.431961, 0.39677
, 0.544815, 0.370722, 0.657458, 0.332047])), SimulatedObservations{Subject{
StructArray{NamedTuple{(),Tuple{}},1,NamedTuple{(),Tuple{}}},NamedTuple{(:i
sPM, :Wt),Tuple{Int64,Int64}},Array{Event{Float64,Float64,Float64,Float64,F
loat64, Float64}, 1}, Int64}, StepRangeLen {Float64, TwicePrecision {Float64}, TwicePrecision {F
ePrecision{Float64}},NamedTuple{(:conc, :dv),Tuple{Array{Float64,1},Array{F
loat64,1}}}{(Subject
    ID: 6
    Events: 1
    Observations: 0
    Covariates: isPM, Wt
, 0.0:0.1:120.0, (conc = [0.0, 0.596983, 1.1149, 1.56412, 1.95366, 2.29135,
 2.584, 2.83752, 3.05702, 3.247 ... 1.03683, 1.03553, 1.03424, 1.03295, 1.0
3166, 1.03037, 1.02908, 1.0278, 1.02651, 1.02523], dv = [1.102e-9, 0.569917]
, 0.882307, 2.40268, 2.36094, 2.79859, 2.81885, 2.47526, 2.44672, 4.55466
... 0.975169, 1.14241, 1.17355, 0.764812, 0.978039, 1.11853, 1.09721, 0.8796
71, 0.758454, 0.996563])), SimulatedObservations{Subject{StructArray{NamedT
uple{(),Tuple{}},1,NamedTuple{(),Tuple{}}},NamedTuple{(:isPM, :Wt),Tuple{In
t64, Int64}}, Array{Event{Float64, Float64, Float64, Float64, Float64, Float64, Float64, Float64}, 1}
,Int64},StepRangeLen{Float64,TwicePrecision{Float64},TwicePrecision{Float64
}},NamedTuple{(:conc, :dv),Tuple{Array{Float64,1},Array{Float64,1}}}}(Subje
ct
    ID: 7
    Events: 1
    Observations: 0
    Covariates: isPM, Wt
, 0.0:0.1:120.0, (conc = [0.0, 0.664789, 1.24553, 1.75271, 2.19552, 2.58199
 , 2.91915, 3.21316, 3.46939, 3.69258 ... 0.770639, 0.76938, 0.768124, 0.766
869, 0.765617, 0.764367, 0.763119, 0.761872, 0.760628, 0.759386, dv = [1.0]
1655e-8, 0.681652, 1.28973, 2.00735, 1.79631, 3.7435, 3.1855, 2.79707, 3.38
912, 3.10694 ... 0.784871, 0.922855, 0.910753, 0.59235, 0.764558, 0.644324,
 0.961122, 1.06492, 0.497474, 0.596659])), SimulatedObservations{Subject{St
ructArray{NamedTuple{(),Tuple{}},1,NamedTuple{(),Tuple{}}},NamedTuple{(:isP
```

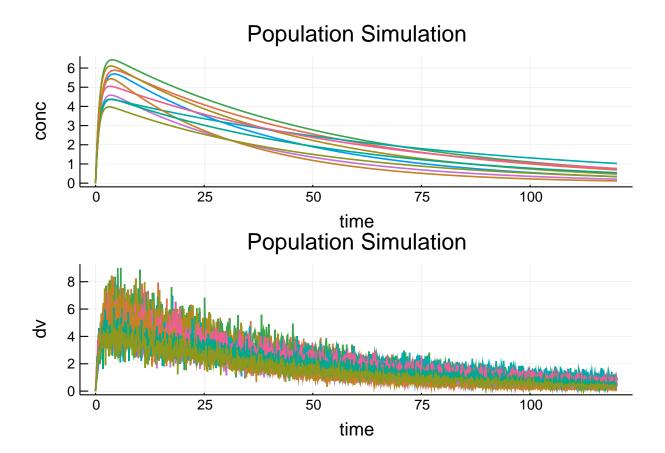
```
M, :Wt), Tuple{Int64,Int64}}, Array{Event{Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Float64,Floa
at64, Float64}, 1}, Int64}, StepRangeLen{Float64, TwicePrecision{Float64}, TwiceP
recision{Float64}},NamedTuple{(:conc, :dv),Tuple{Array{Float64,1},Array{Flo
at64,1}}}{(Subject
   ID: 8
   Events: 1
   Observations: 0
   Covariates: isPM, Wt
, 0.0:0.1:120.0, (conc = [0.0, 0.577877, 1.09917, 1.56924, 1.99293, 2.37464
, 2.71835, 3.02764, 3.3058, 3.55576 ... 0.116881, 0.11649, 0.1161, 0.115712
, 0.115324, 0.114938, 0.114554, 0.11417, 0.113788, 0.113407], dv = [1.7719e
-8, 0.289641, 0.857423, 1.3876, 1.7667, 2.72551, 2.12081, 2.21408, 3.69457,
 4.20912 ... 0.12588, 0.13358, 0.118749, 0.148105, 0.167703, 0.0616205, 0.0
942864, 0.115281, 0.0903927, 0.0918065])), SimulatedObservations{Subject{St
ructArray{NamedTuple{(),Tuple{}},1,NamedTuple{(),Tuple{}}},NamedTuple{(:isP
M, :Wt),Tuple{Int64,Int64}},Array{Event{Float64,Float64,Float64,Float64,Flo
at64, Float64}, 1}, Int64}, StepRangeLen{Float64, TwicePrecision{Float64}, TwiceP
recision{Float64}},NamedTuple{(:conc, :dv),Tuple{Array{Float64,1},Array{Flo
at64,1}}}{(Subject
   ID: 9
   Events: 1
   Observations: 0
   Covariates: isPM, Wt
, 0.0:0.1:120.0, (conc = [0.0, 0.51299, 0.96865, 1.37329, 1.73251, 2.05132,
 2.33416, 2.58498, 2.80731, 3.00427 ... 0.551495, 0.5505, 0.549506, 0.54851
4, 0.547524, 0.546536, 0.545549, 0.544564, 0.543581, 0.5426], dv = [1.69407
e-8, 0.514171, 0.755338, 0.898257, 1.72195, 2.0114, 2.53831, 2.46977, 2.347
25, 3.2279 ... 0.388992, 0.613899, 0.727955, 0.512591, 0.589046, 0.533358,
0.518497, 0.316814, 0.479108, 0.434677])), SimulatedObservations{Subject{St
ructArray{NamedTuple{(),Tuple{}},1,NamedTuple{(),Tuple{}}},NamedTuple{(:isP
M, :Wt),Tuple{Int64,Int64}},Array{Event{Float64,Float64,Float64,Float64,Flo
at64,Float64},1},Int64},StepRangeLen{Float64,TwicePrecision{Float64},TwiceP
recision{Float64}},NamedTuple{(:conc, :dv),Tuple{Array{Float64,1},Array{Flo
at64,1}}}{(Subject
   ID: 10
   Events: 1
   Observations: 0
   Covariates: isPM, Wt
0.0:0.1:120.0, (conc = [0.0, 0.53706, 1.00496, 1.41246, 1.76721, 2.0759,
```

2.34437, 2.57771, 2.78037, 2.95625 ... 0.348305, 0.34757, 0.346837, 0.34610 5, 0.345374, 0.344646, 0.343918, 0.343192, 0.342468, 0.341746], dv = [1.568 71e-8, 0.250012, 1.00921, 0.991236, 1.35162, 2.43761, 2.10681, 2.61234, 2.7 2883, 2.24335 ... 0.380086, 0.302562, 0.338812, 0.291212, 0.323308, 0.32634

and visualize the output

8, 0.272792, 0.40908, 0.325354, 0.309878]))])

plot(obs)



1.4 Multiple dose example

The additional dosage regimen controls of the NMTRAN format are available in DosageRegimen. For example, ii defines the "interdose interval", or the time distance between two doses, while add1 defines how many additional times to repeat a dose. Thus, let's define a dose of 100 that's repeated 7 times at 24 hour intervals:

```
md = DosageRegimen(100,ii=24,addl=6)
```

```
PuMaS.DosageRegimen(1×8 DataFrames.DataFrame
Row
     time
               cmt
                                evid
                                               addl
                                                      rate
      Float64
               Int64
                      Float64
                               Int8
                                      Float64
                                               Int64
                                                      Float64
                                                                Int8
      0.0
                      100.0
                                      24.0
                                                      0.0
                                                                0
 1
               1
                                               6
```

Let's create a new subject, s3 with this dosage regimen:

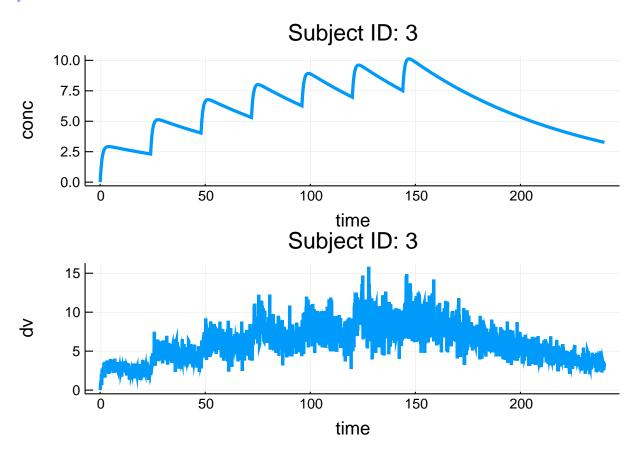
```
s3 = Subject(id=3,evs=md, cvs=(isPM=0,Wt=70))
```

Subject

ID: 3 Events: 7 Observations: 0 Covariates: isPM, Wt

and see the results:

obs = simobs(model, s3, p,obstimes=0:0.1:240)
plot(obs)



1.5 Combining dosage regimens

We can also combine dosage regimens to build a more complex regimen. Recall from the introduction that using arrays will build the element-wise combinations. Thus let's build a dose of 500 into compartment 1 at time 0, and 7 doses into compartment 1 of 100 spaced by 24 hours:

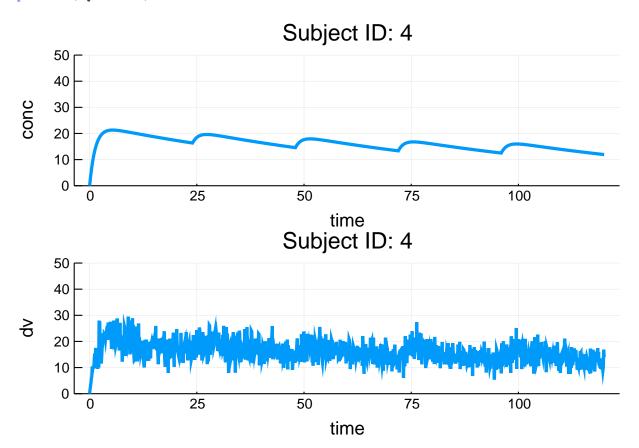
ldmd = DosageRegimen([500,100],cmt=1, time=[0,24], addl=[0,6],ii=[0,24])

PuMaS.DosageRegimen(2×8 DataFrames.DataFrame Row time cmt amt evid ii addl rate ss Float64 Int64 Float64 Int8 Float64 Int64 Float64 Int

```
1 0.0 1 500.0 1 0.0 0 0.0 0
2 24.0 1 100.0 1 24.0 6 0.0 0
```

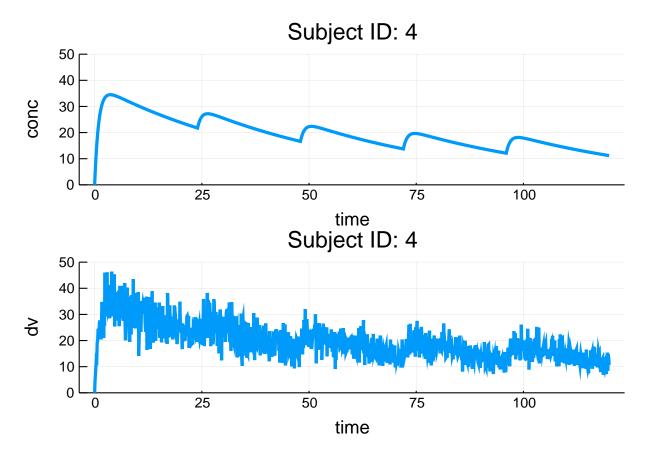
Let's see if this result matches our intuition:

```
s4 = Subject(id=4, evs=ldmd, cvs=(isPM=0,Wt=70))
obs = simobs(model, s4, p,obstimes=0:0.1:120)
plot(obs, ylims=(0,50))
```



Another way to build complex dosage regiments is to combine previously constructed regimens into a single regimen. For example:

```
e1 = DosageRegimen(500,cmt=1, time=0, addl=0,ii=0)
e2 = DosageRegimen(100,cmt=1, time=24, addl=6,ii=24)
evs = DosageRegimen(e1,e2)
obs = simobs(model, s4, p,obstimes=0:0.1:120)
plot(obs, ylims=(0,50))
```



is the same regimen as before.

Putting these ideas together, we can define a population where individuals with different covariates undergo different regimens, and simulate them all together with automatic parallelism:

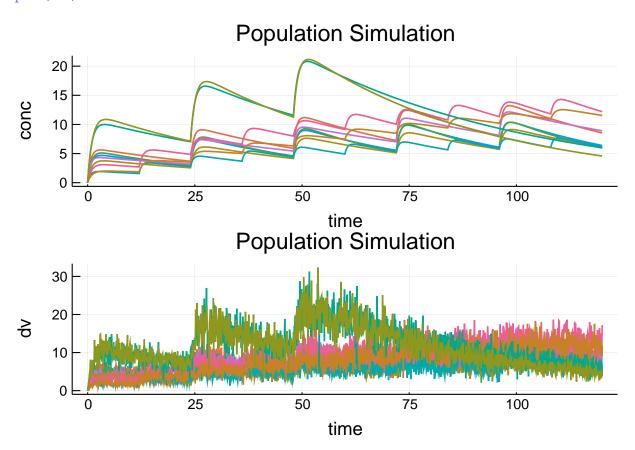
```
e1 = DosageRegimen(100, ii=24, addl=6)
e2 = DosageRegimen(50, ii=12, addl=13)
e3 = DosageRegimen(200, ii=24, addl=2)
PuMaS.DosageRegimen(1 \times 8 DataFrames.DataFrame
 Row time
               cmt
                      amt
                               evid ii
                                              addl
                                                     rate
                                                               SS
      Float64 Int64 Float64 Int8 Float64
                                             Int64
                                                    Float64
                                                              Int8
      0.0
                      200.0
                               1
                                     24.0
                                              2
                                                     0.0
                                                              0
 1
pop1 = Population(map(i -> Subject(id=i,evs=e1,cvs=choose_covariates()),1:5))
pop2 = Population(map(i -> Subject(id=i,evs=e2,cvs=choose_covariates()),6:8))
pop3 = Population(map(i -> Subject(id=i,evs=e3,cvs=choose_covariates()),9:10))
pop = Population(vcat(pop1,pop2,pop3))
```

 ${\tt 10-element~PuMaS.Population \{PuMaS.Subject \{StructArrays.StructArray \{NamedTupartarray \} \} \} } \\$

```
le{(),Tuple{}},1,NamedTuple{(),Tuple{}}},NamedTuple{(:isPM, :Wt),Tuple{Int6
4, Int64}}, Array{PuMaS.Event{Float64,Float64,Float64,Float64,Float64,Float64
},1},Int64}}:
Subject
 ID: 1
 Events: 7
 Observations: 0
 Covariates: isPM, Wt
 Subject
 ID: 2
 Events: 7
  Observations: 0
 Covariates: isPM, Wt
 Subject
 ID: 3
 Events: 7
 Observations: 0
 Covariates: isPM, Wt
Subject
 ID: 4
 Events: 7
  Observations: 0
 Covariates: isPM, Wt
 Subject
 ID: 5
 Events: 7
 Observations: 0
 Covariates: isPM, Wt
 Subject
 ID: 6
 Events: 14
 Observations: 0
 Covariates: isPM, Wt
 Subject
 ID: 7
 Events: 14
 Observations: 0
 Covariates: isPM, Wt
 Subject
 ID: 8
  Events: 14
 Observations: 0
 Covariates: isPM, Wt
 Subject
 ID: 9
 Events: 3
 Observations: 0
 Covariates: isPM, Wt
 Subject
  ID: 10
```

Events: 3
Observations: 0
Covariates: isPM, Wt

obs = simobs(model,pop,p,obstimes=0:0.1:120)
plot(obs)



1.6 Defining Infusions

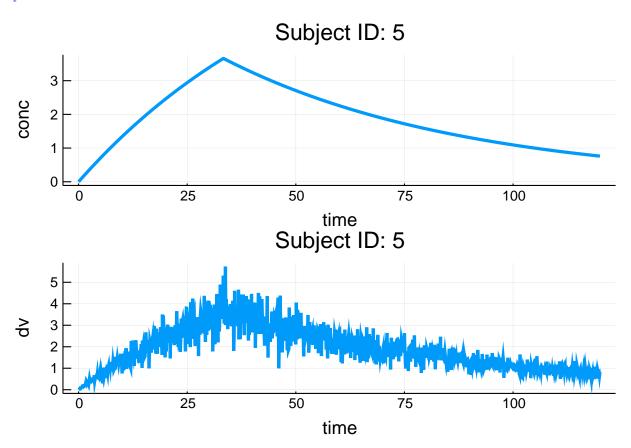
As specified in the NMTRAN format, an infusion is a dosage which is defined as having a non-zero positive rate at which the drug enters the system. Let's define a single infusion dose of total amount 100 with a rate of 3 into the second compartment:

inf = DosageRegimen(100, rate=3, cmt=2)

PuMaS.DosageRegimen(1×8 DataFrames.DataFrame Row time cmtamt evid ii addl rate SS Float64 Int64 Float64 Int8 Float64 Int64 0.0 100.0 0.0 3.0 1

Now let's simulate a subject undergoing this treatment strategy:

```
s5 = Subject(id=5, evs=inf, cvs=(isPM=0,Wt=70))
obs = simobs(model, s5, p, obstimes=0:0.1:120)
plot(obs)
```



1.7 Final Note on Julia Programming

Note that all of these functions are standard Julia functions, and thus standard Julia programming constructions can be utilized to simplify the construction of large populations. We already demonstrated the use of map and a comprehension, but we can also make use of constructs like for loops.

1.8 Conclusion

This tutorial shows the tools for generating populations of infinite complexity, defining covariates and dosage regimens on the fly and simulating the results of the model.