This Week

Monday

- Modeling and Solving Linear Optimization Problems

Wednesday

- Lab Exercise: Using Excel Solver

Topics

- **Modeling Decision Problems as Linear Programs (LPs)**
- **Solving Simple LPs Graphically**
- **Solving LPs using Excel Solver**

A Problem Solving Framework

1. *Define the Problem*
2. *Collect and Organize Data*
3. *Characterize Uncertainty and Data Relationships*
4. *Build an Evaluation Model*
5. *Formulate a Solution Approach*
6. *Evaluate Potential Solutions*
7. *Recommend a Course of Action*

Product Mix Decision Problem

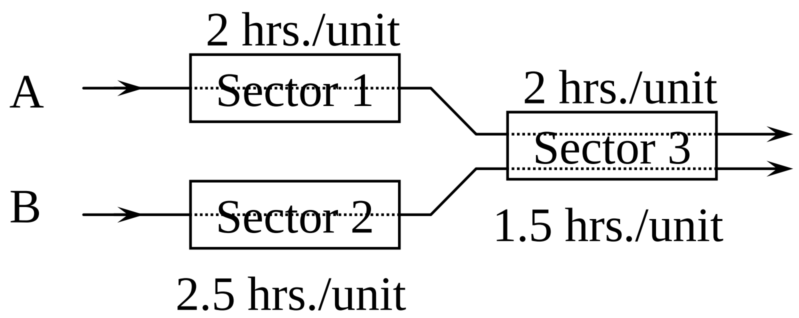
- A company manufactures two products: **A** and **B**.
- Product information and next month's labor availability by Sector are given in the tables below:

Product Information

	Unit Selling	Unit RM	Labor Hours Required			Unit Labor	Unit Gross	Marketing Forecast	
Product	Price	Cost	Sector 1	Sector 2	Sector 3	Cost	Margin	Min	Max
A	$600	$50	2	0	2	$100.00	$450.00	75	140
B	$600	$100	0	2.5	1.5	$80.00	$420.00	0	140

Resource Information

	Sector 1	Sector 2	Sector 3
Hourly Labor Cost:	$30	$20	$20
Labor Hours Available:	336	336	336



How much of each product should be produced to maximize next month's profit?

Organizing the Problem

- The ***decisions*** are to choose production levels for products A and B.
- The ***objective*** is to maximize next month's profit.
- In order for the decisions to be ***feasible***, they must simultaneously satisfy the following conditions, or ***constraints***:

Labor constraints
- In Sector 1, no more than 336 labor hours can be used.
- In Sector 2, no more than 336 labor hours can be used.
- In Sector 3, no more than 336 labor hours can be used.

Marketing constraints
- Between 75 and 140 units of A must be produced.
- Between 0 and 140 units of B must be produced.

Formulating the Problem

- Let the ***decision variables*** be denoted x_A and x_B, where:

  x_A = the number of units of product A to produce next month.

  x_B = the number of units of product B to produce next month.

- The contribution margin per unit of product A is $450, and the contribution margin per unit of product B is $420. Hence, the ***objective*** is to:

$$\text{Maximize } \underbrace{450\,x_A + 420\,x_B}_{\text{This is the } \textbf{\textit{objective function}}.}$$

Formulating the Problem (cont'd)

Labor Constraints

- Each unit of product A requires 2 labor hours in Sector 1, and product B does not require any labor in Sector 1. So, the ***Sector 1 labor constraint*** is:

$$2\,x_A \leq 336.$$

- Similarly, the labor constraints for Sectors 2 and 3 are:

Sector 2 $$2.5\,x_B \leq 336.$$

Sector 3 $$2\,x_A + 1.5\,x_B \leq 336.$$

Formulating the Problem (cont'd)

Marketing Constraints

- The marketing constraints for product A can be represented with the following inequalities:

$$x_A \geq 75.$$

$$x_A \leq 140.$$

- Similarly, marketing constraints for product B can be written as:

$$x_B \geq 0.$$

$$x_B \leq 140.$$

The Linear Program

Putting this all together, we formulate the problem as:

Maximize $450\, x_A + 420\, x_B$

subject to:

Labor constraints

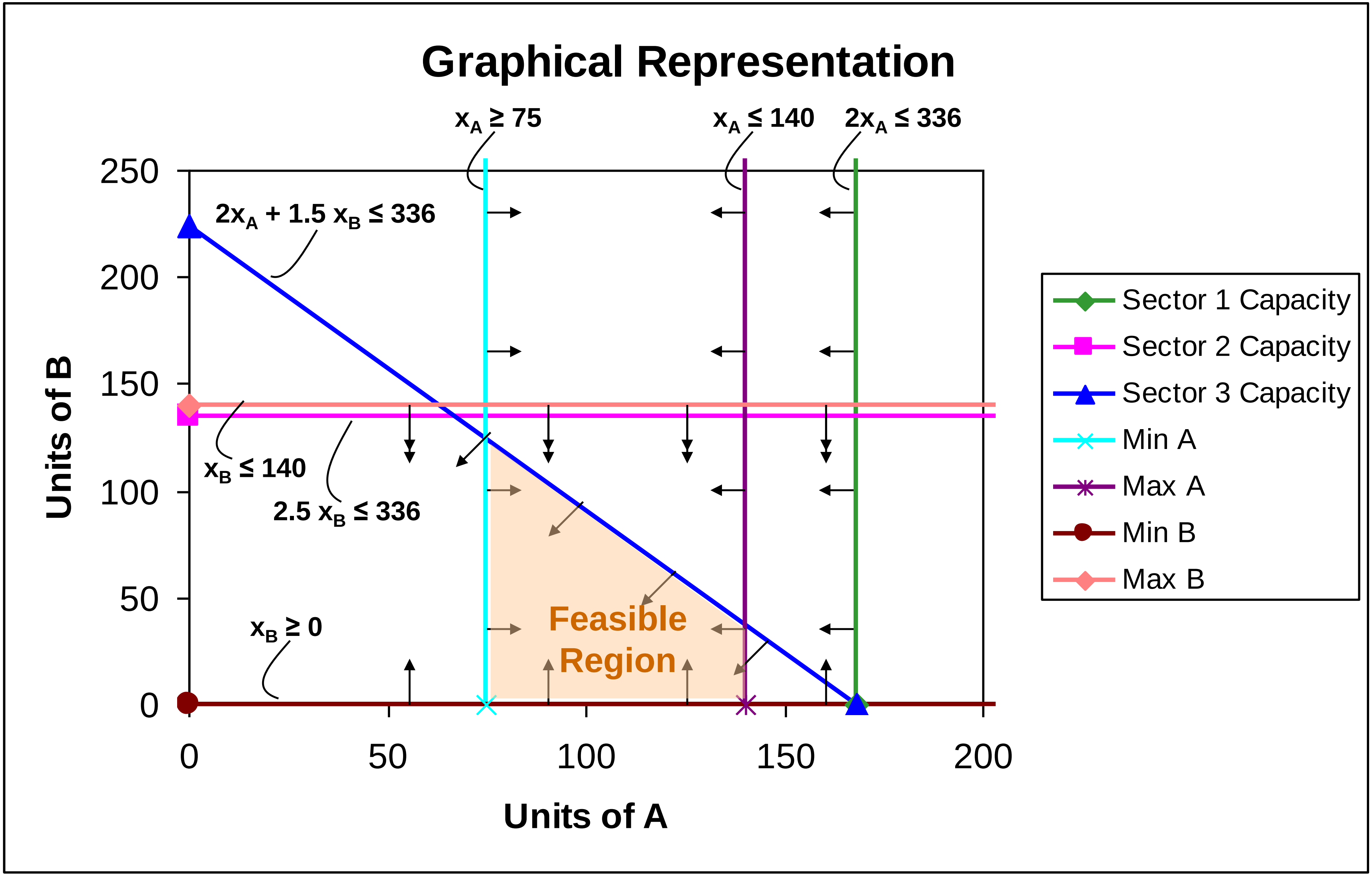
$$2\, x_A \leq 336.$$
$$2.5\, x_B \leq 336.$$
$$2\, x_A + 1.5\, x_B \leq 336.$$

Marketing constraints

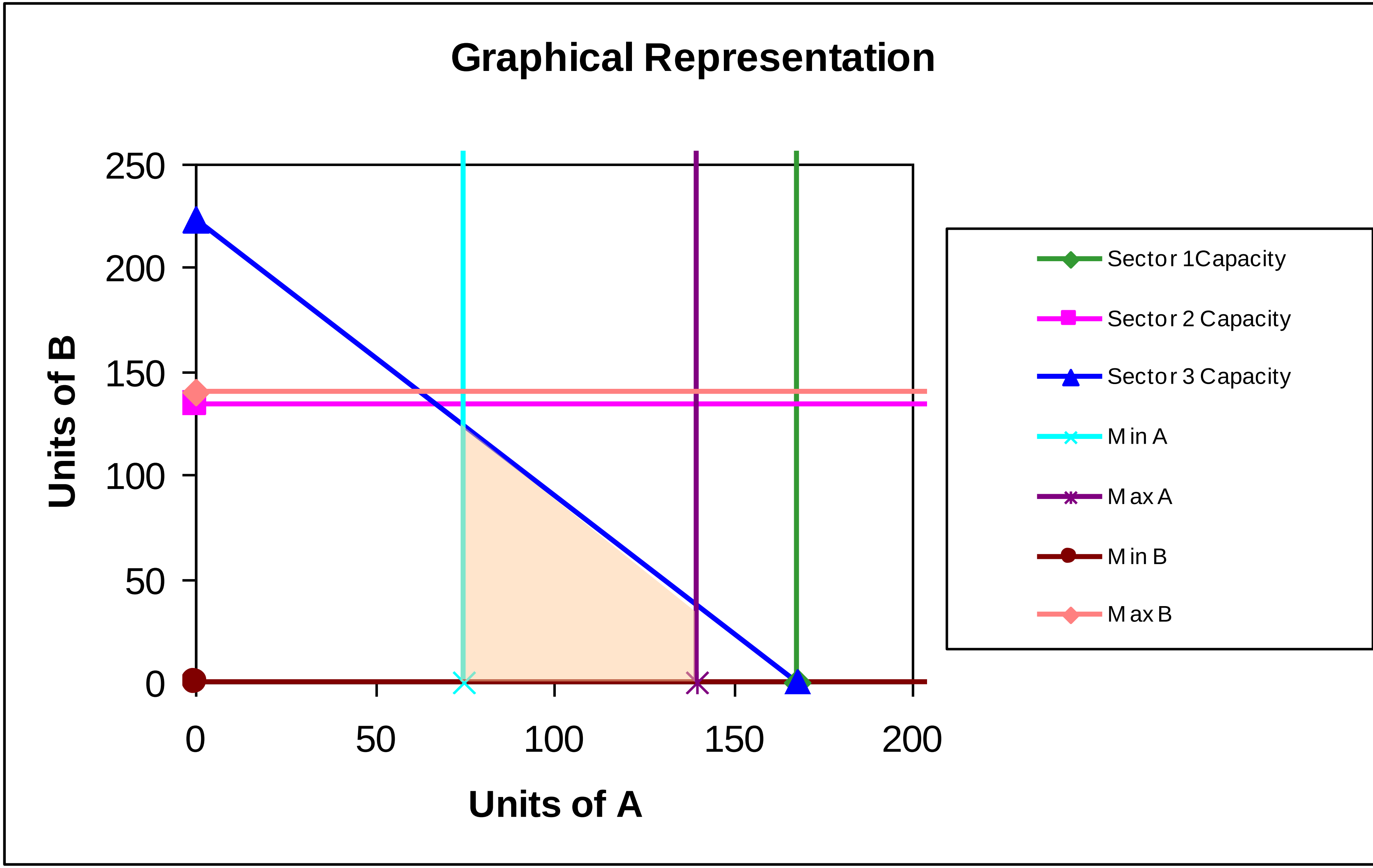
$$x_A \geq 75.$$
$$x_A \leq 140.$$
$$x_B \geq 0.$$
$$x_B \leq 140.$$

Solving Linear Programs

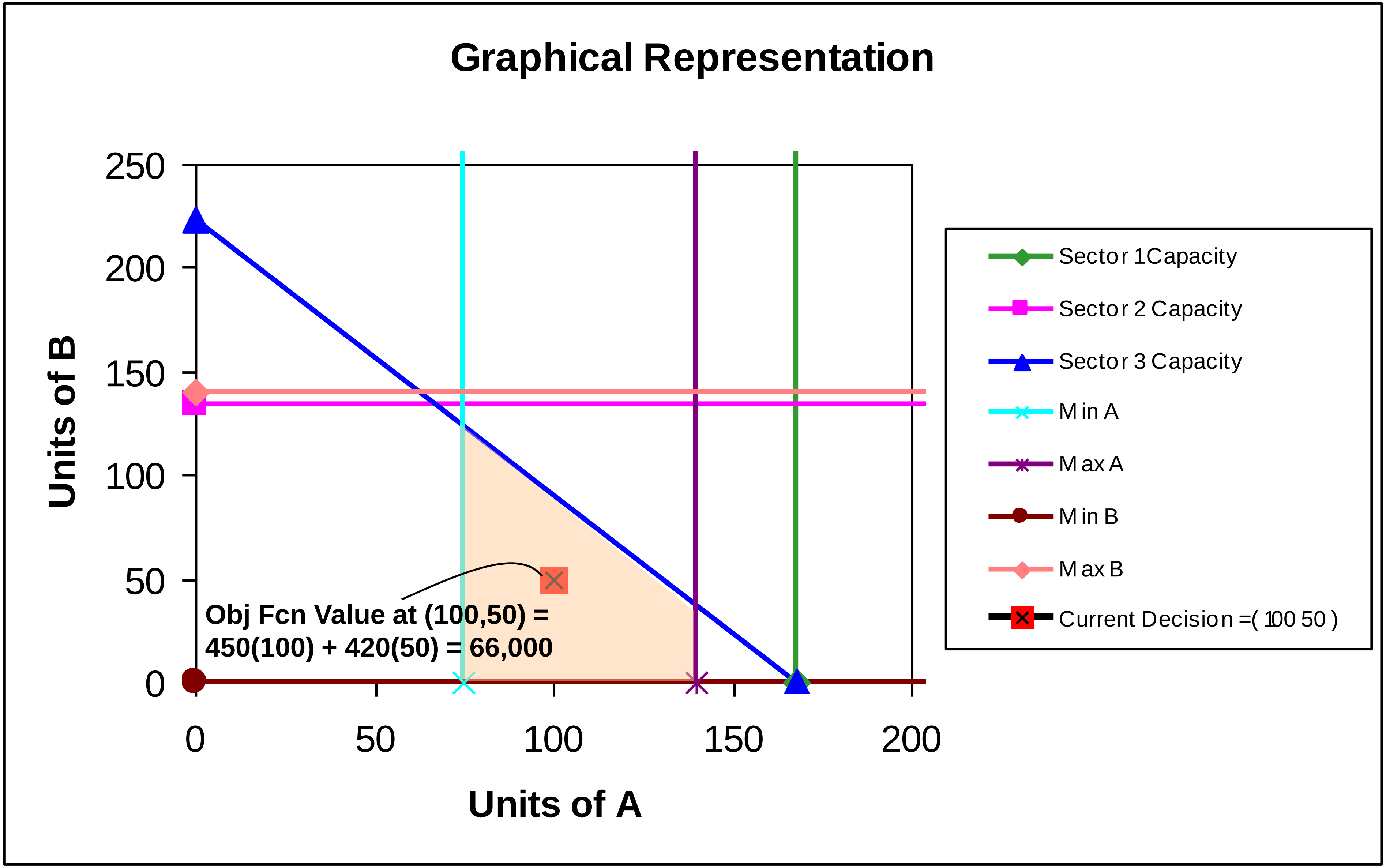
- Many different methods exist for solving LPs. Variations of the ***simplex method***, first developed by George Dantzig in 1947, are still the most widely used.
- Since our problem has only two variables, x_A and x_B, we can represent the problem on a two-dimensional graph, where each point (x_A, x_B) represents a possible solution to the LP.
- By plotting the constraints on the graph, we will be able to tell which points are feasible.
- The problem can then be solved using simple algebra.



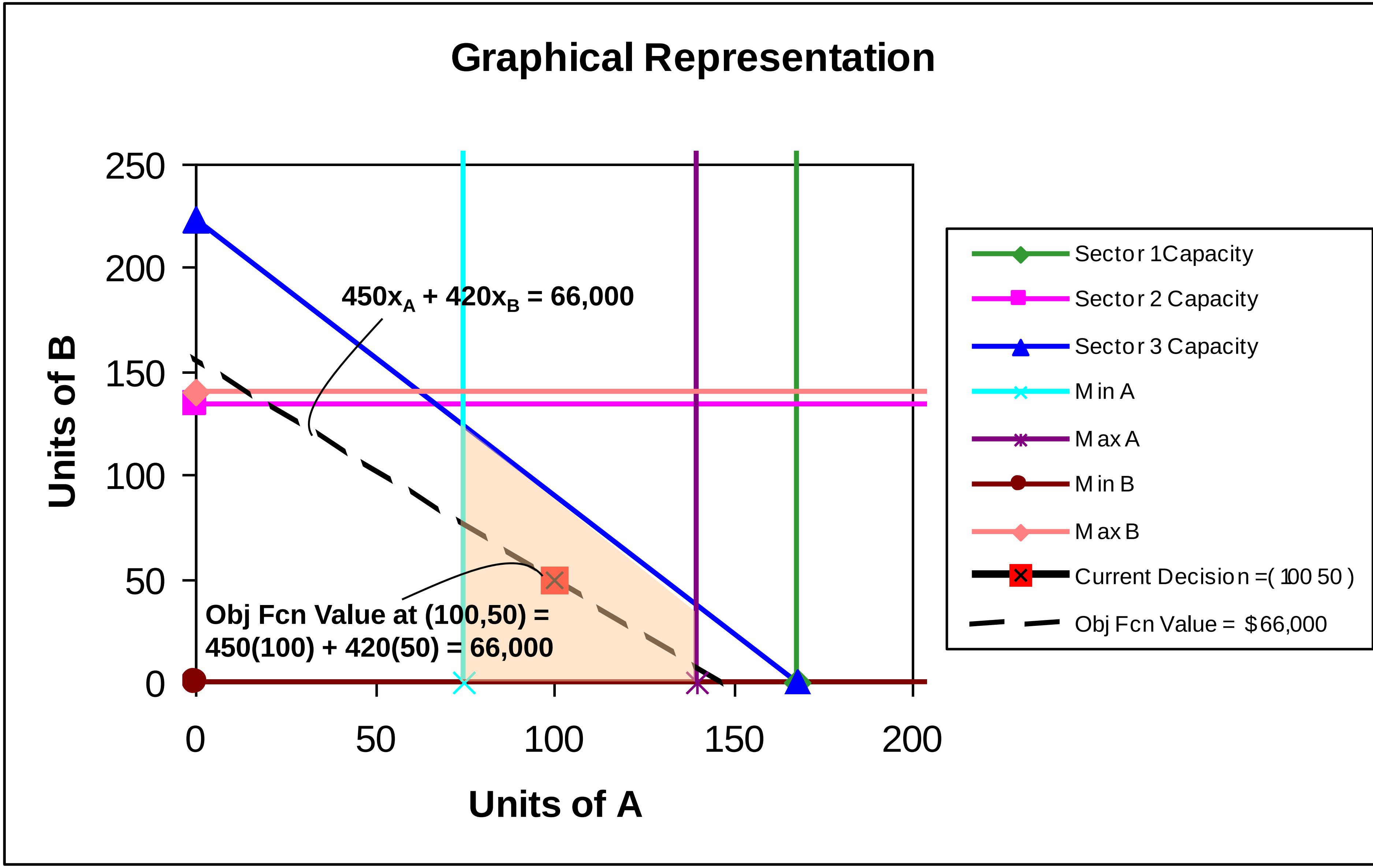
Graphical Representation
xA ≥ 75
xA ≤ 140
2xA ≤ 336
2xA + 1.5 xB ≤ 336
xB ≤ 140
2.5 xB ≤ 336
xB ≥ 0
Feasible Region
Units of B
Units of A
0
50
100
150
200
250
Sector 1 Capacity
Sector 2 Capacity
Sector 3 Capacity
Min A
Max A
Min B
Max B

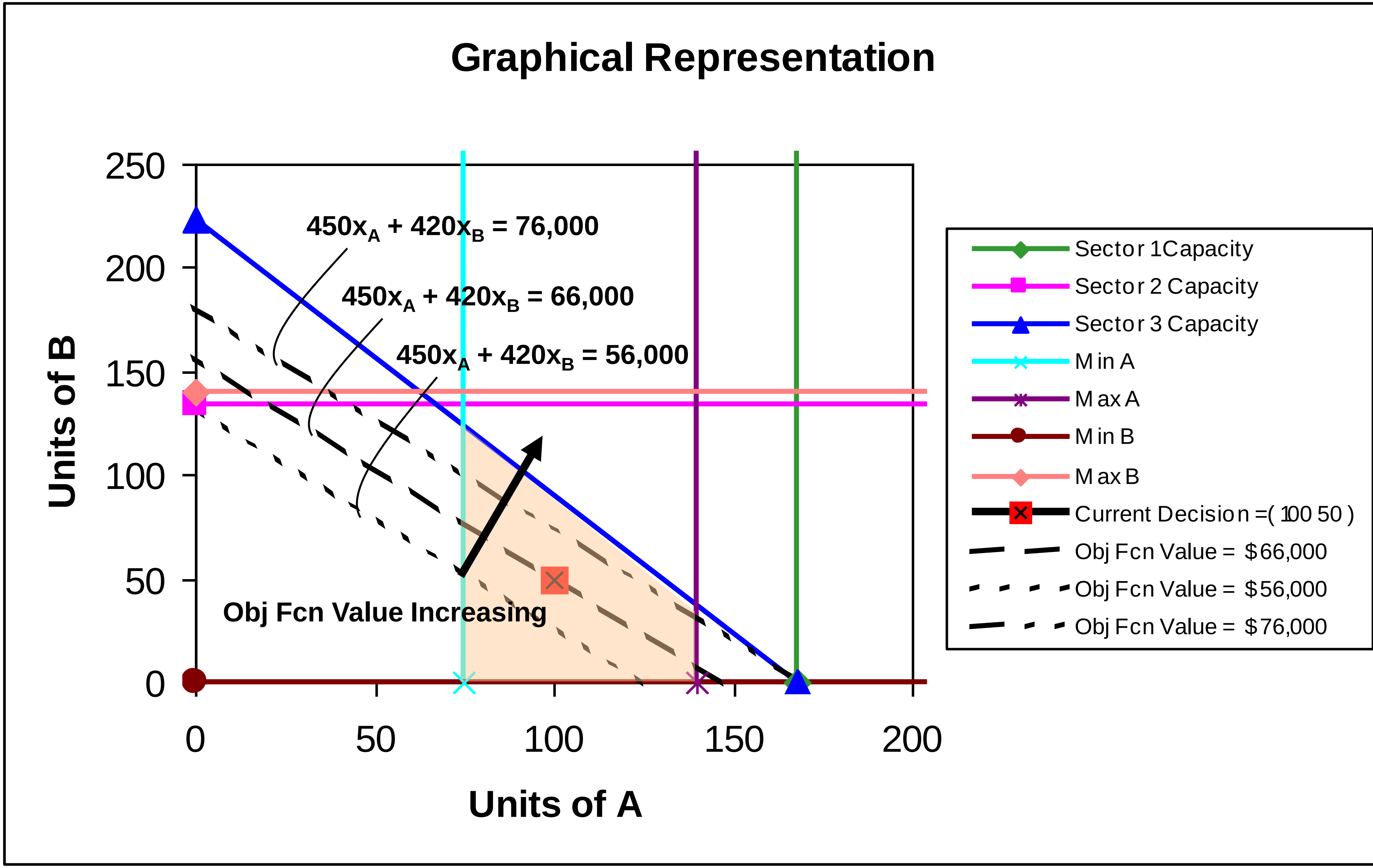


Graphical Representation
250
200
150
100
50
0
Units of B
0
50
100
150
200
Units of A
Sector 1Capacity
Sector 2 Capacity
Sector 3 Capacity
Min A
Max A
Min B
Max B

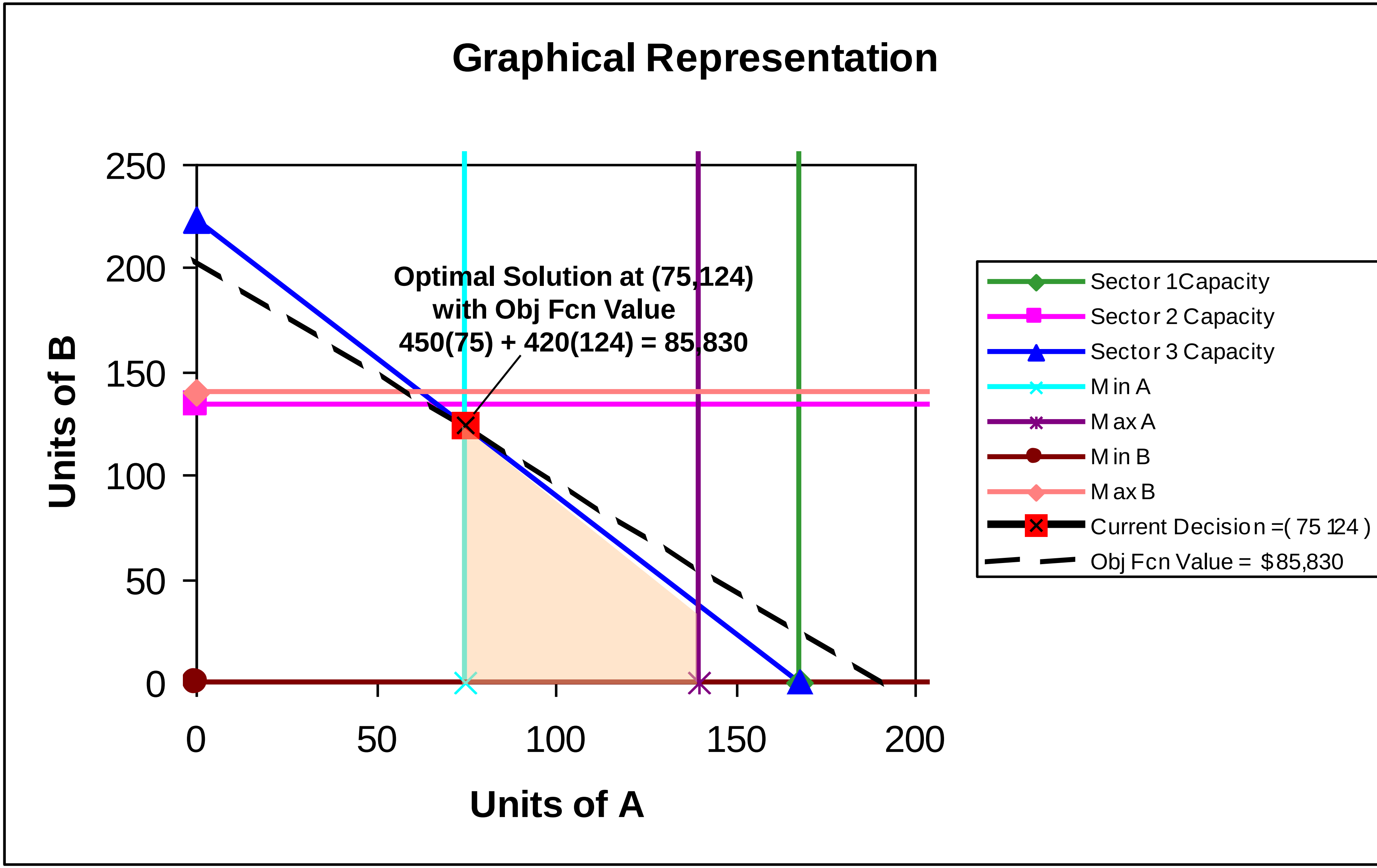


Graphical Representation
Units of B
250
200
150
100
50
0
0
50
100
150
200
Units of A
Obj Fcn Value at (100,50) =
450(100) + 420(50) = 66,000
Sector 1 Capacity
Sector 2 Capacity
Sector 3 Capacity
Min A
Max A
Min B
Max B
Current Decision = (100 50)


Graphical Representation
Units of B
Units of A
250
200
150
100
50
0
0
50
100
150
200
450x_A + 420x_B = 66,000
Obj Fcn Value at (100,50) =
450(100) + 420(50) = 66,000
Sector 1 Capacity
Sector 2 Capacity
Sector 3 Capacity
Min A
Max A
Min B
Max B
Current Decision =(100 50)
Obj Fcn Value = $66,000



Graphical Representation
250
200
150
100
50
0
0
50
100
150
200
Units of B
Units of A
450xA + 420xB = 76,000
450xA + 420xB = 66,000
450xA + 420xB = 56,000
Obj Fcn Value Increasing
Sector 1Capacity
Sector 2 Capacity
Sector 3 Capacity
Min A
Max A
Min B
Max B
Current Decision =(100 50)
Obj Fcn Value = $66,000
Obj Fcn Value = $56,000
Obj Fcn Value = $76,000



Which labor resource would you invest in to increase profit?

Setting Up LPs In Excel

Fact of life: Excel Solver is extremely fussy and does not always behave the way you think it should.

Before attempting a Solver implementation, make sure that you can ***write down your LP in standard form***:

$$\text{Max} \quad \boldsymbol{cx} \quad \text{(or Min)}$$

subject to:

$$\boldsymbol{Ax} \le \boldsymbol{b} \qquad (\ge \; =)$$

In particular, **the entries of *c*, *b*, and *A* should *NOT* depend upon the decision variable vector *x***.

Setting Up LPs In Excel

Next, organize the worksheet so that you have ***one cell representing each "piece" of the LP***:

Maximize $\boxed{450\ x_A + 420\ x_B}$ **Objective Function**

subject to:

Decision Variable Cells $\boxed{x_A}$ $\boxed{x_B}$

LHS Cells / **RHS Cells**

LHS Cells		RHS Cells
$2\ x_A$	$\leq$	336
$2.5\ x_B$	$\leq$	336
$2\ x_A + 1.5\ x_B$	$\leq$	336
x_A	$\geq$	75
x_A	$\leq$	140
x_B	$\geq$	0
x_B	$\leq$	140

Building the Model In Excel

1. ***Create cells for the decision variables*.** Designate one cell for each decision variable in the model. These cells should ***not*** contain formulas.

	Decision Area	
A:	**1**	
B:	**1**	

Building the Model In Excel

2. ***Create cells for the objective function coefficients*.** Designate one cell for each objective function coefficient in the model. In each cell, enter the appropriate formula or value for the coefficient.

$$\text{Maximize } \boxed{450}\, x_A + \boxed{420}\, x_B$$

	Unit Gross Margin
Selling Price – RM Cost – Labor Cost =	$450.00
	$420.00

Building the Model In Excel

3. ***Create the objective function cell*.**
   Designate one cell for the objective function value. In this cell, put the formula for the objective function by referencing the decision variable cells and the coefficient cells.

$$\text{Maximize } \boxed{450\ x_A + 420\ x_B}$$

Total Gross Margin:	**$870**

	Decision Area	
A:	**1**	
B:	**1**	

Unit Gross Margin
$450.00
$420.00

Building the Model In Excel

4. ***Create cells for the constraint coefficients*****.** Designate one cell for each constraint coefficient in the model. In each cell, enter the appropriate formula or value for the coefficient.

	Labor Hours Required		
Product	**Sector 1**	**Sector 2**	**Sector 3**
A	2	0	2
B	0	2.5	1.5

$$\boxed{2}\,x_A \qquad \boxed{} \qquad \leq 336$$
$$\boxed{} \qquad \boxed{2.5}\,x_B \leq 336$$
$$\boxed{2}\,x_A + \boxed{1.5}\,x_B \leq 336$$
$$\left\{\begin{array}{ll} x_A & \geq 75 \\ x_A & \leq 140 \\ x_B & \geq 0 \\ x_B & \leq 140 \end{array}\right.$$

Note: We do not designate separate cells for these simple constraints since we can describe the left-hand sides using the decision variable cells directly.

Building the Model In Excel

5. ***Create cells for the constraint left-hand sides*.** Designate one cell for each constraint LHS in the model. In each cell, enter the formula for the LHS by referencing the decision variable cells and the coefficient cells.

$$\boxed{2\,x_A \qquad\qquad} \leq 336$$

$$\boxed{\qquad\qquad 2.5\,x_B} \leq 336$$

$$\boxed{2\,x_A + 1.5\,x_B} \leq 336$$

	Sector 1	Sector 2	Sector 3
Labor Hours Used in Production Decision:	2.0	2.5	3.5

	Labor Hours Required		
Product	Sector 1	Sector 2	Sector 3
A	2	0	2
B	0	2.5	1.5

	Decision Area
A:	1
B:	1

Building the Model In Excel

6. ***Create cells for the constraint right-hand sides***. Designate one cell for each constraint RHS in the model. In each cell, enter the appropriate formula or value for the RHS.

			Sector 1	Sector 2	Sector 3
Labor Hours Available:			336	336	336

$$2\,x_A \leq \boxed{336}$$
$$2.5\,x_B \leq \boxed{336}$$
$$2\,x_A + 1.5\,x_B \leq \boxed{336}$$

	Marketing Forecast	
Product	Min	Max
A	75	140
B	0	140

$$x_A \geq \boxed{75}$$
$$x_A \leq \boxed{140}$$
$$x_B \geq \boxed{0}$$
$$x_B \leq \boxed{140}$$

Solving LPs With Excel Solver

We can now solve the LP using Data→Analysis→Solver

Maximize $\boxed{450\,x_A + 420\,x_B}$

subject to:

$\boxed{x_A}$ $\boxed{x_B}$

$$\boxed{2\,x_A} \leq \boxed{336}$$

$$\boxed{2.5\,x_B} \leq \boxed{336}$$

$$\boxed{2\,x_A + 1.5\,x_B} \leq \boxed{336}$$

$$\boxed{x_A} \geq \boxed{75}$$

$$\boxed{x_A} \leq \boxed{140}$$

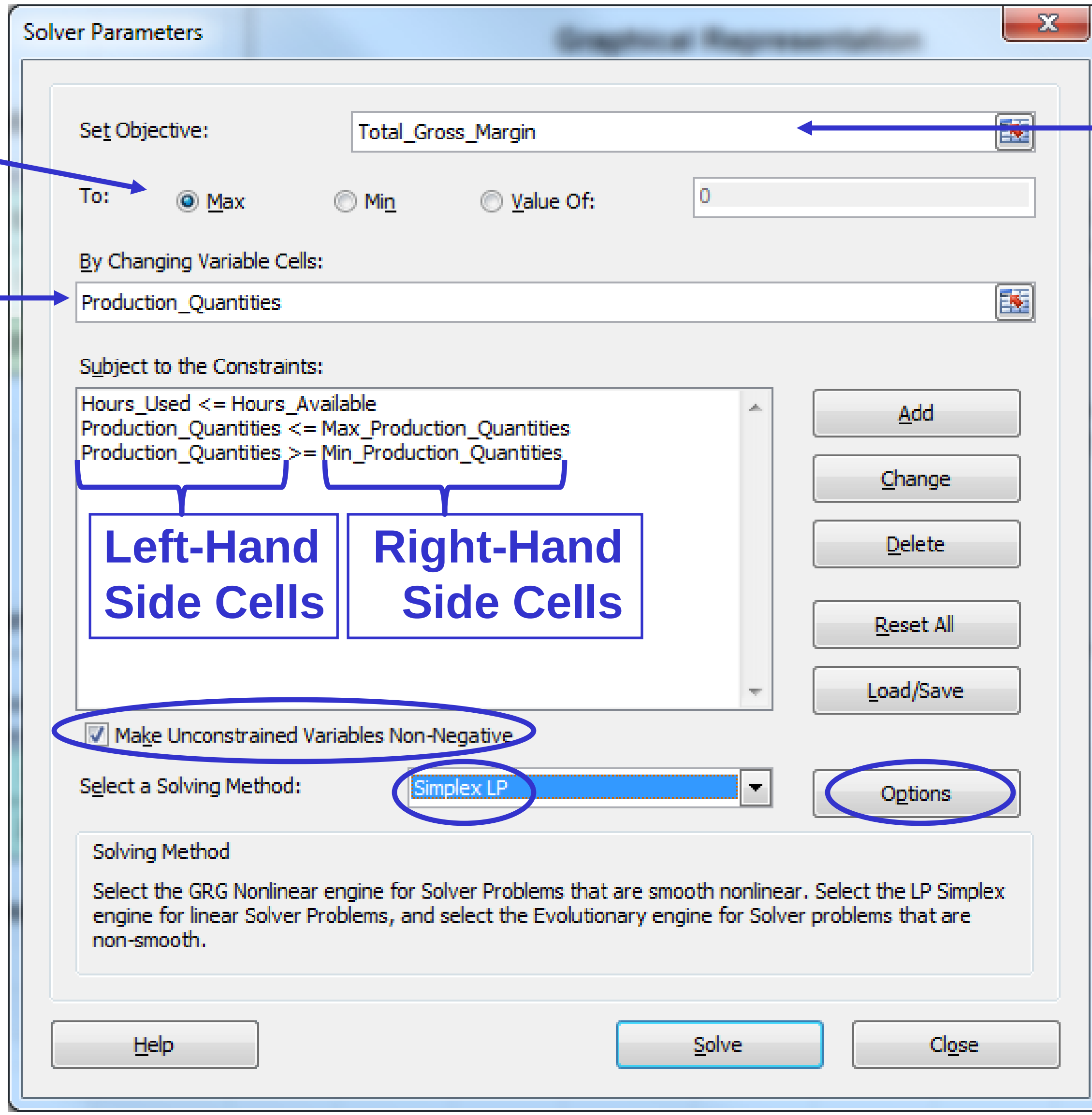
$$\boxed{x_B} \geq \boxed{0}$$

$$\boxed{x_B} \leq \boxed{140}$$

Solving LPs With Excel Solver

Objective Function Direction

Decision Variable Cells



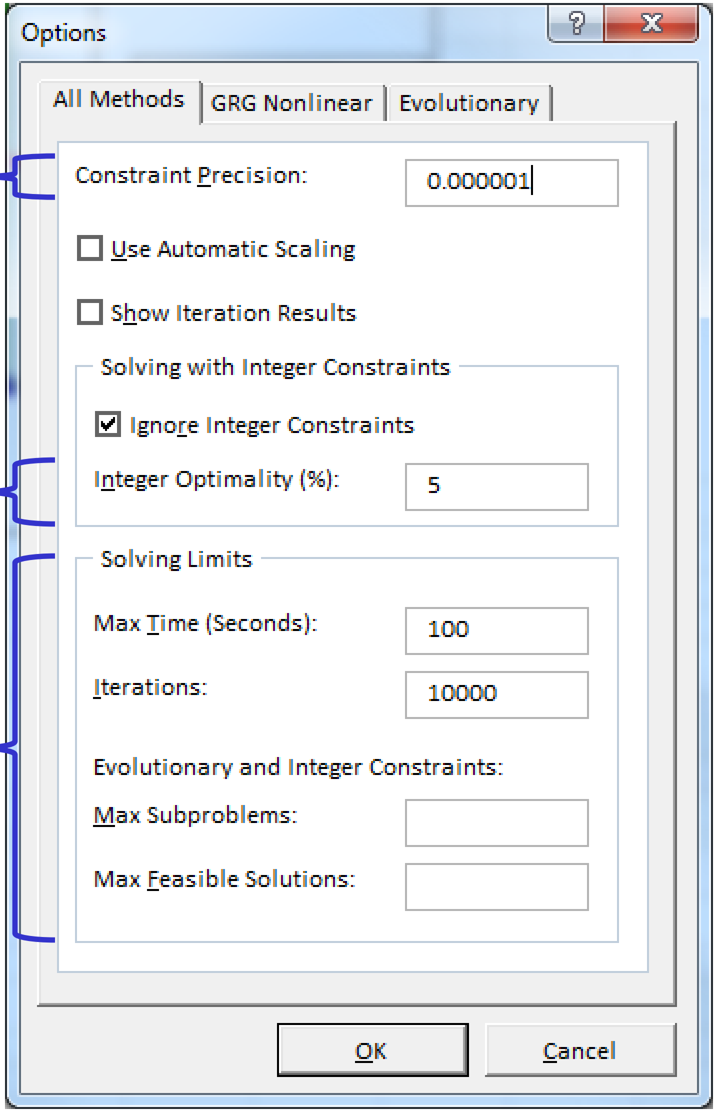
Objective Function Cell

Solver Options

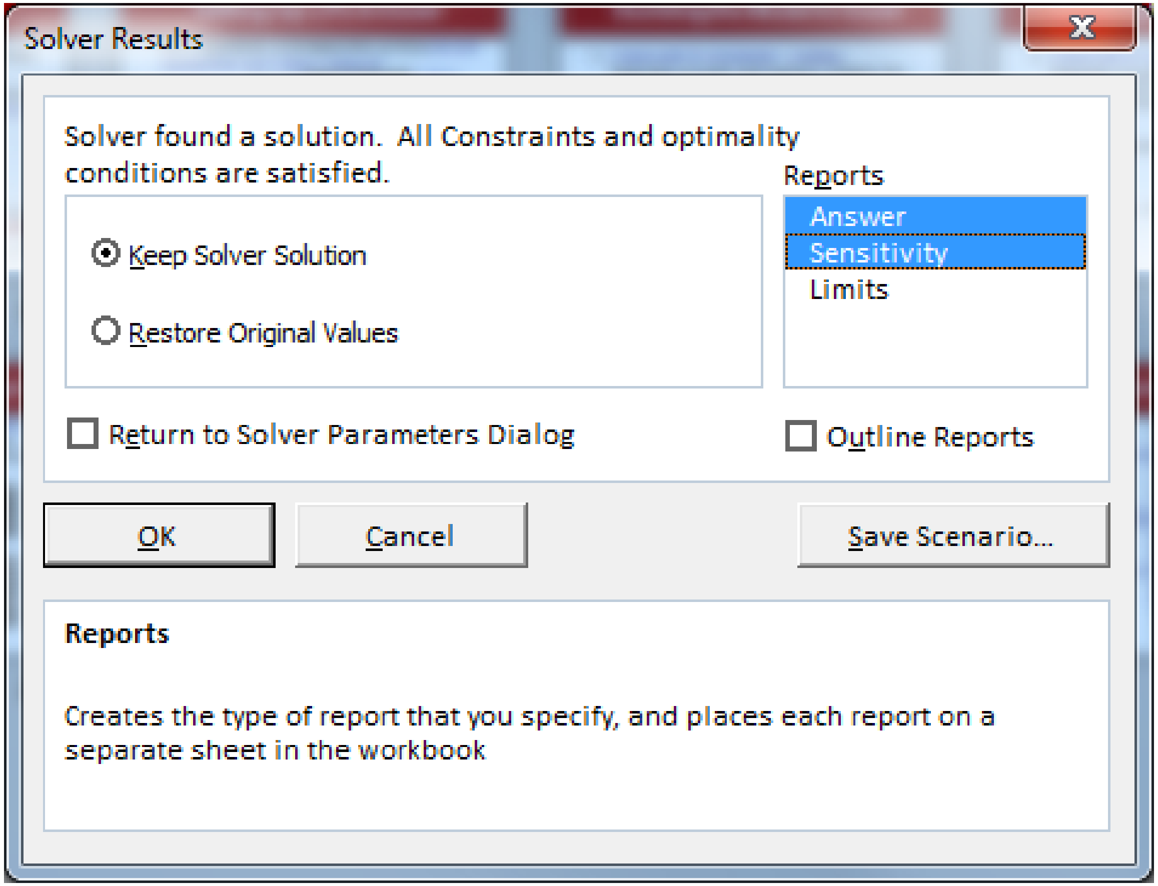
Constraints Satisfied?

Close enough to Optimal Obj Func Value (IP)?

Stop Search?



Solver Results and Reports



Solver Answer Report

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$E$25	Total_Gross_Margin	$66,000	$85,830

Optimal Objective Function Value

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$E$22	x_A	100	75	Contin
$E$23	x_B	50	124	Contin

Optimal Decision Variable Values

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$9	Sector_1_Hours_Used	150.00	$E$9<=$E$8	Not Binding	186
$F$9	Sector_2_Hours_Used	310.00	$F$9<=$F$8	Not Binding	26
$G$9	Sector_3_Hours_Used	336.00	$G$9<=$G$8	Binding	0
$E$22	x_A	75	$E$22<=$K$17	Not Binding	65
$E$23	x_B	124	$E$23<=$K$18	Not Binding	16
$E$22	x_A	75	$E$22>=$J$17	Binding	0
$E$23	x_B	124	$E$23>=$J$18	Not Binding	124

Optimal Left-Hand Side Values

Which constraints are binding at the optimal solution?

Solver Sensitivity Report

How will the objective function value change if we increase a decision variable value by one unit (while maintaining feasibility)?

How much can the Objective Function Coefficients change without affecting the optimal decision variable values?

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$E$22	x_A	75	-110	450	110	1E+30
$E$23	x_B	124	0	420	1E+30	82.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$9	Sector_1_Hours_Used	150	0	336	1E+30	186
$F$9	Sector_2_Hours_Used	310	0	336	1E+30	26
$G$9	Sector_3_Hours_Used	336	280	336	15.6	186

How will the optimal objective function value change if the R.H.S. increases by one unit?

How much can the Constraint R.H.S.s change without affecting the constraints that are binding at the optimal solution?