This Week and Next

Monday, April 1

- TVOM Fundamentals

Wednesday, April 3

- Liquair-Pro Project Overview

Monday, April 8: ***NO CLASS***

Wednesday, April 10

- Project Cash Flow Analysis
- Liquair-Pro Cash Flows

Topics

- TVOM Terminology
- Economic Equivalence
- Converting Cash Flows in Time

A Problem Solving Framework

1. *Define the Problem*
2. *Collect and Organize Data*
3. *Characterize Uncertainty and Data Relationships*
4. *Build an Evaluation Model*
5. *Formulate a Solution Approach*
6. *Evaluate Potential Solutions*
7. *Recommend a Course of Action*

The Time Value of Money

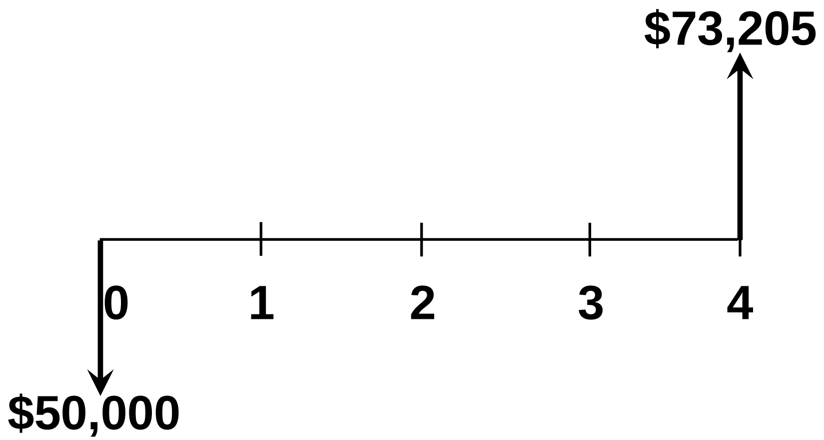
- The economic value of an amount of money depends upon ***when the money is received***.
- A dollar you have today is worth more than a dollar promised to you in the future:
  - The dollar you have today is yours for ***certain***.
  - The dollar has ***earning power*** - you can invest or lend the dollar and receive ***interest***.
- Interest is the ***cost of having, or the benefit of making, money available for use***.

Interest

- ***Interest*** is the cost of having, or the benefit of making, money available for use.
- Interest is expressed as a ***percentage rate per period*** (i) for a specified ***number of periods*** (N).
- Interest is earned or accrued over time. For most cash flow analyses, interest is assumed to ***compound*** each period on ***the entire outstanding balance***.
- By convention, the beginning of the evaluation timeframe is defined to be ***the end of period 0***.

Cash Flow Diagram

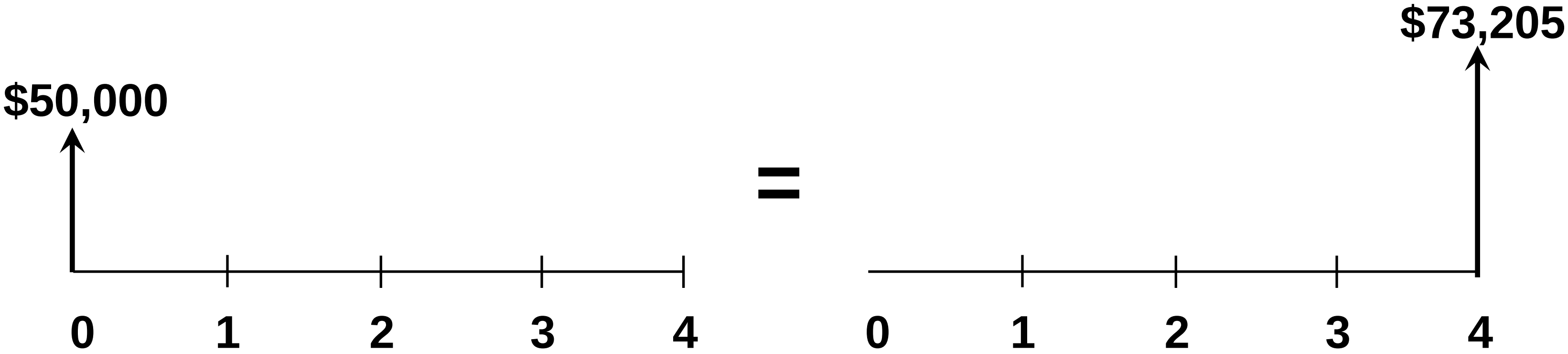
- A ***cash flow diagram*** is a graphical representation of a series of periodic cash flows (usually monthly or annual). Cash outflows point down and cash inflows point up (depending on your point of view).
- Example: You loan \$50,000 to a friend now and he pays you \$73,205 in four years. So, from your point of view:



- End-of-Period Convention: All cash flow transactions that occur during a period are assumed to occur ***at the end of the period***.

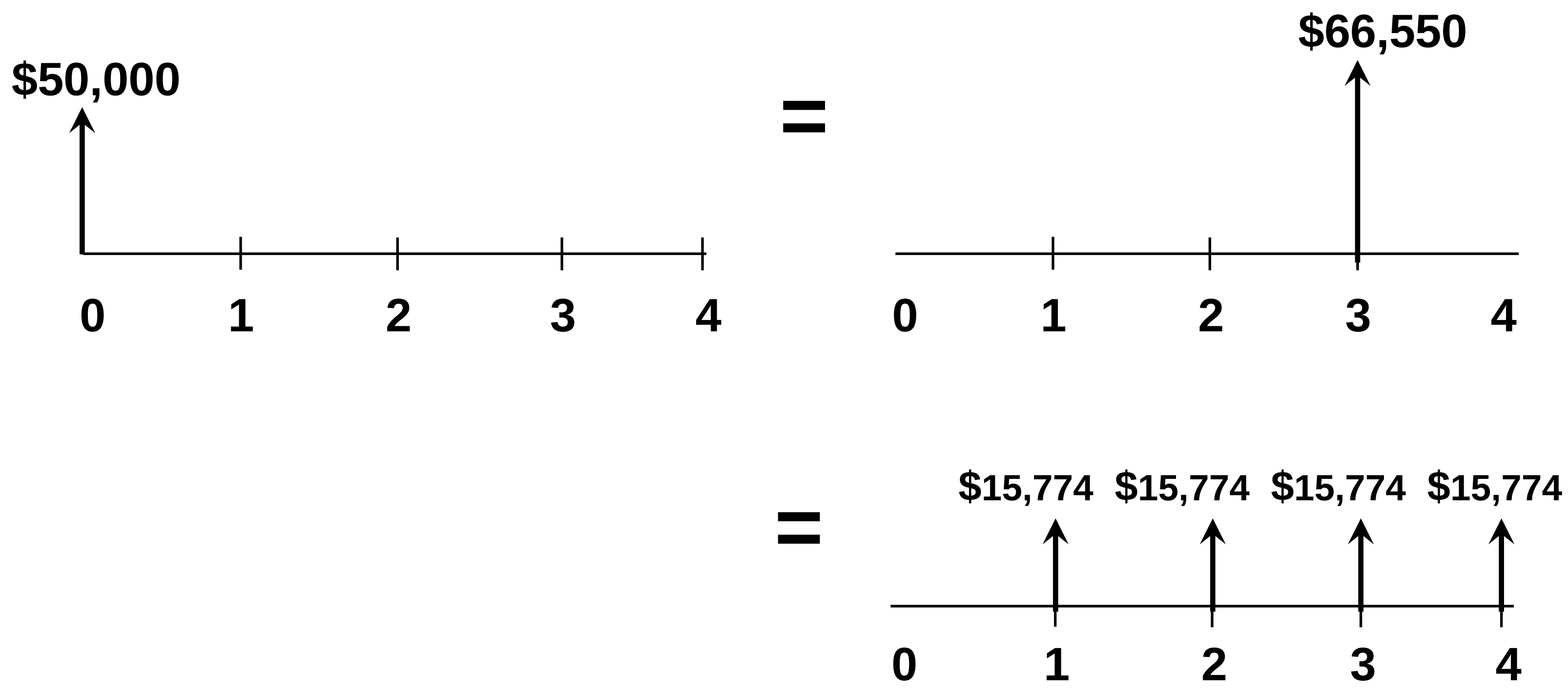
Economic Equivalence

- Cash flows are ***economically equivalent with respect to a given interest rate*** if they have the same economic value when evaluated at the same point in time using that rate.
- From an economic standpoint, ***equivalent cash flows can be substituted***, or traded, for one another.
- Example: At i = 10%, \$50,000 received today is economically equivalent to \$73,205 received four years from now, since \$50,000 x $(1.1)^4$ = \$73,205:



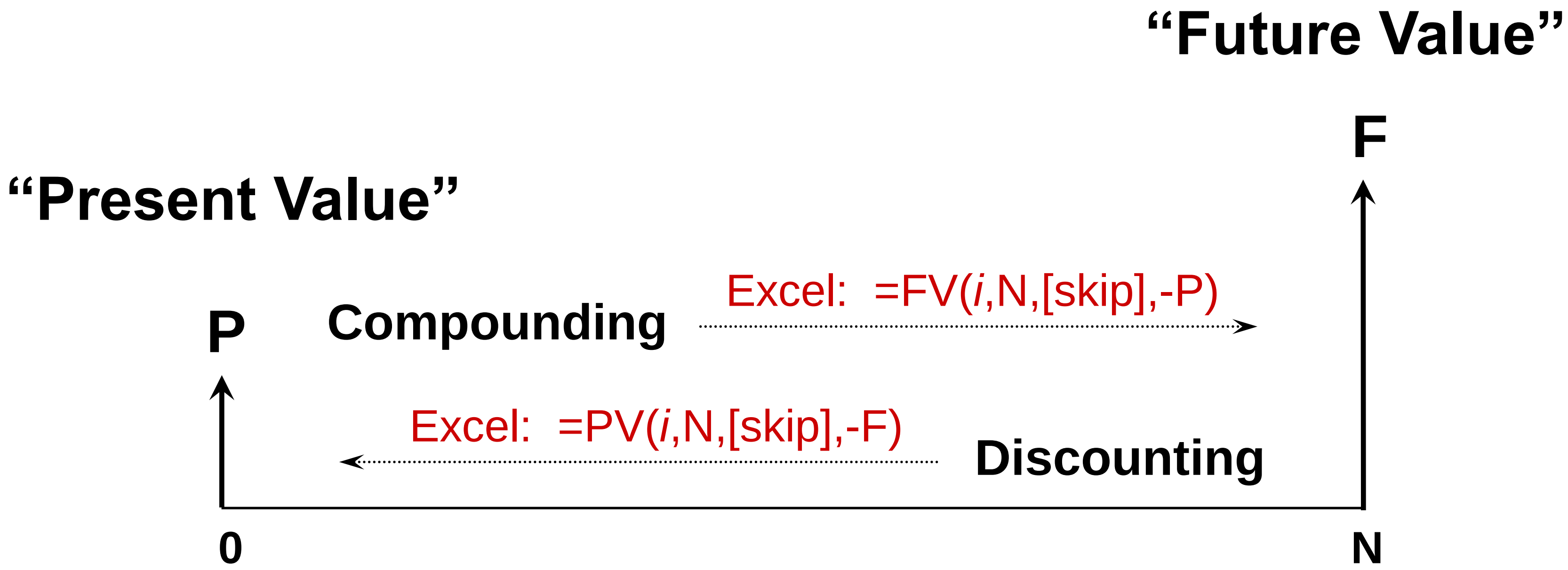
Economic Equivalence

- At *i* = 10%, the following cash flows are economically equivalent:



- Determining economic equivalence often requires the ***conversion of multiple cash flows to a single cash flow***, or vice-versa.

Converting Single Amounts



$$P = F * (1 + i)^{-N} \text{ or}$$

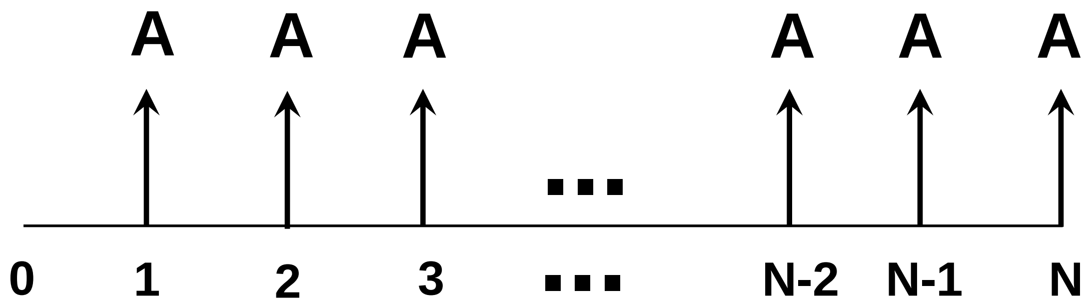
$$P = F * \underbrace{(P/F, i, N)}_{\text{"Discounting Factor" or "Present Worth Factor"}}$$

$$F = P * (1 + i)^{N} \text{ or}$$

$$F = P * \underbrace{(F/P, i, N)}_{\text{"Compound Amount Factor"}}$$

Annuities

- An ***annuity*** is a series of equal cash flows (payments or receipts) spaced equally in time (usually annually).



N = Number of payments

A = Amount of each payment

Standard Assumption: First payment is made at the END of period 1

- Converting a single cash flow, or a series of unequal cash flows, to an annuity equivalent provides a comparative ***N-year annual equivalent worth*** (AEW).

Converting Annuities

$$(F/A,i,N) = \left[\frac{(1+i)^N - 1}{i}\right]$$

"Uniform Series Compound Factor"

Excel: =FV(*i*,N,-A)

$$(A/F,i,N) = \left[\frac{i}{(1+i)^N - 1}\right]$$

"Sinking Fund Factor"

Excel: =PMT(*i*,N,[skip],-F)

$$(P/A,i,N) = \left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

"Uniform Series Present Worth Factor"

Excel: =PV(*i*,N,-A)

$$(A/P,i,N) = \left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

"Capital Recovery Factor"

Excel: =PMT(*i*,N,-P)

Excel: =RATE(N,A,-P) to get *i*, given P and A

Annuity Examples

- Starting in the year 2021, suppose you invest \$10,000 per year for 35 years. You can earn *i* =10% interest annually. How much will you have by the end of 2055?

*Want F = A * (F/A, i = 10%, N=35) = \$10,000 * [(1.1)^{35} – 1]/0.1*
or =FV(10%, 35, -\$10,000)

- If you want to have \$5,000,000 in 2055, how much will you need to invest each year?

*Want A = F * (A/F, i = 10%, N=35) = \$5,000,000 * 0.1/[(1.1)^{35} – 1]*
or =PMT(10%, 35, [skip], -\$5,000,000)

- If you start taking equal annual distributions in 2056 and plan to take 25 such distributions, what is the maximum amount you can withdraw?

*Want A = P * (A/P, i = 10%, N=25) = \$5,000,000 * 0.1 *(1.1)^{25} /[(1.1)^{25} – 1]*
or =PMT(10%, 25, -\$5,000,000)

Perpetuities

- A ***perpetuity*** is an annuity that continues forever.

$$\lim_{N\to\infty}(P/A,i,N) = \lim_{N\to\infty}\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = \frac{1}{i}$$

"Capitalization Factor"

Excel: = A / *i*

- <u>Example</u>: A university wants to establish a scholarship fund that will give 100 students a $10,000 stipend each year. How much do they need today to establish the fund if interest can be earned at *i* = 5% per year?

Want $P = A_{\infty} * (P/A_{\infty}, i = 10\%) = \$1{,}000{,}000 / 0.05 = \$20{,}000{,}000$

TVOM: Key Ideas

- Different series of cash flows can be compared by evaluating them at ***the same point in time***.
- Cash flows are ***equivalent with respect to a particular interest rate*** if they have the same economic value at any point in time.
- Given an interest rate and any combination of single cash flows, annuities, and/or perpetuities, we can easily calculate the equivalent:
  - Present Value
  - Future Value
  - Annuity Value
- These ideas are commonly used in ***evaluating proposed business project alternatives***.