ORIE 4820: Spreadsheet-Based Modeling and Data Analysis
Portfolio Optimization Lab Exercise
Spring 2013

During this lab exercise, you will build a simple portfolio optimization tool that determines an optimal portfolio allocation from among five user-selected securities. Two different portfolio optimization models will be implemented. The primary purpose is to give you practice in ***formulating decision problems that have nonlinear components and solving these problems using Excel's Solver.*** We will also review referencing and matrix manipulation functions.

The template file for this exercise is ***portfolio-optimization.xlsm***. Download a copy of the file from the course Blackboard site and *save it on your computer*.

Topics/Tools we will cover:
- ***Computing pairwise stock return correlations*** using historical data
- ***Determining the risk and return of a selected portfolio allocation*** given the risk and return parameters of the individual securities and the correlation matrix
- ***Formulating the portfolio optimization problem*** using two different nonlinear programming models and ***solving these NLPs*** using Excel's Solver
- ***Recording macros that call Excel's Solver*** to find optimal solutions
- ***Interpreting*** the optimal solutions

If you have problems or questions at any point during the session, please raise your hand.

Background:

The *Portfolio Selection* worksheet contains a ***security selection area***, an ***optimization area***, and several supporting tables and graphs:

- The ***security selection area*** enables the user to select, via dropdown lists, five securities from among the 21 securities listed in the ***Ticker Symbol Table*** (see cell K32) and to specify an investment allocation. The 21 securities in the Ticker Symbol Table include 18 publicly traded stocks, two market indices (GSPC = the S&P 500 and RUT = the Russell 2000), and a fixed income security (ticker symbol FIXED) that yields a constant annual return of 2%. Once completed, the *security selection area* will display the expected return, standard deviation, and variance of each of the selected securities.

- The ***optimization area*** will display the expected return and risk associated with the portfolio allocation specified in the *security selection area*. The user will also have the ability to execute two different portfolio optimization models. Specifically, the user will be able to:

  - Determine the ***lowest-risk portfolio*** comprised of the five selected securities ***whose expected return exceeds a specified target return***; and/or

  - Determine the ***highest-expected-return portfolio*** comprised of the five selected securities ***whose standard deviation does not exceed a target risk percentage***.

- The data source for the above areas is the ***Return Table*** (see cell N32). The Return Table contains ten years' worth of annual return data for the securities listed in the Ticker Symbol Table (2003-2012), as well as summary statistics. These data will be used in populating the *security selection area*, as well as the Correlation and Covariance Matrices.

- The rightmost two columns of the ***Portfolio Combination Table*** (see cell C31) are populated by the macro `chart_portfolios`. (The button labeled "Chart Portfolio Footprint" runs this macro, but ***it will not work properly until you have completed Section 2*** of this lab exercise.) You do not need to modify this table or the macro. Both exist solely for the purpose of updating the two risk-return graphs located directly below the optimization area:

  - The first graph depicts the risk-return "footprint" of all possible portfolio allocations comprised of the five user-selected securities. Since there are an infinite number of allocation combinations, this graph contains a representative set only, but you should be able to discern the general "umbrella" shape. Note that the northwest-most points on this graph describe the ***efficient frontier*** of portfolios comprised of the five selected securities.

  - The second graph depicts the risk-return curve of ***portfolios comprised of the first and second selected securities only***.

Section 1: Extracting Individual Security Parameters

Note that the calculated rows below the "blue data" in the Return Table (rows 45 and 46) depict the averages and standard deviations of the annual returns for each security over the past ten years. Although these data are limited and imperfect, in our models we will use these sample data as surrogates for the parameters of the one-year return random variables R_j , j = 1, …, 5. Therefore, these statistics can be pulled directly into the *security selection area*:

(1) Populate column D of the *security selection area* by using the **match** function on the Return Table headers in conjunction with the **index** function on the row 45 entries.

(2) Populate column E of the *security selection area* by using the **match** function on the Return Table headers in conjunction with the **index** function on the row 46 entries.

(3) Populate column F of the *security selection area* by squaring column E.

Section 2: Analyzing a Portfolio Allocation

Based on the investment percentages that the user enters in cells G5:G9 for the five selected securities (i.e., w_j, j = 1, …, 5, which should total 100%), the designated cells in the *optimization area* of the worksheet should ***calculate the expected return of the associated portfolio, E[R_P], as well as the portfolio variance, Var[R_P], and standard deviation, Std Dev[R_P]***. Recall that:

$$\mu_P = E[R_P] = E[\sum_{j=1}^{N} w_j R_j] = \sum_{j=1}^{N} w_j E[R_j] = \sum_{j=1}^{N} w_j \mu_j$$

and

$$\sigma_P{}^2 = Var[R_P] = Var[\sum_{j=1}^{N} w_j R_j] = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j Cov(R_i, R_j) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \rho_{ij} \sigma_i \sigma_j$$

The computation of E[R_P] is straightforward given the data in the *security selection area* (simply use the **sumproduct** function). To compute Var[R_P], we will compute the pieces of the above expression from right to left:

(1) **Matrix of Correlations** (ρ_{ij}): Note that each "blue column" of data in the Return Table has been ***named*** after its ticker symbol (e.g., the cell range Q34:Q43 is named AMZN). This fact makes populating the Correlation Matrix in cells C15:G19 relatively easy. Recall that the **indirect** function in Excel allows the user to refer to a named cell range using the text of the name, so that if cell C14 contains the text "AMZN", then the formula =**average**(**indirect**(C14)) will yield the average annual return of Amazon stock over the past 12 years. Use the **indirect** function in conjunction with the **correl** function to compute the ***pairwise correlations*** of the selected security returns in cells C15:G19.

(2) **Matrix of Covariances**: To determine the ***matrix of covariances*** in cells C23:G27, use the fact that:

$$Cov(R_i, R_j) = \rho_{ij}\ \sigma_i\ \sigma_j$$

Given the layout of the data in the *security selection area*, you can easily retrieve the appropriate standard deviation values using the **vlookup** function. (Note: You could also use the Excel **covariance.s** function here.)

(3) **Portfolio Variance**: Var[R_P] is just the sum of all 25 weighted covariance values:

$$w_i\ w_j\ Cov(R_i, R_j)$$

Instead of computing the individual entries and summing them, however, we can use nested ***matrix multiplication***, entered as an ***array function***:

MMULT(MMULT(TRANSPOSE(w_j),Covar),w_j)

(4) **Portfolio Standard Deviation (i.e., "Risk")**: Take the square root of Var[R_P].

Section 3: Lowest-Risk Portfolio Meeting a Target Return

Now that you have completed the mechanism to compute portfolio risk and expected return, you are ready to determine "optimal" portfolios using Excel's Solver. First, we want to ***build an optimization model that finds the lowest-risk portfolio that meets a user-specified target average rate of return*** (TARR, in cell J7):

$$\text{Minimize } \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \rho_{ij}\sigma_i\sigma_j} \quad \text{(i.e., } \sigma_{\text{P}}\text{)}$$

$$\text{subject to:}$$

$$\sum_{j=1}^{N} w_j \mu_j \geq TARR$$

$$\sum_{j=1}^{N} w_j = 1$$

$$w_j \geq 0 \quad \text{for all } j = 1,\ldots N.$$

(1) Enter the above ***nonlinear program*** into Solver (Data->Analysis->Solver from the ribbon) and solve. Under "Select a Solving Method", be sure to select the "GRG Nonlinear" option.

(2) ***Save*** the model in the designated area of column R named Min_Risk_Model. (If you need more cells than are specified, redefine the cell range Min_Risk_Model.)

(3) ***Record a macro*** to re-solve the model at the click of a button:

   a. From the ribbon, select Developer->Code->Record Macro.
   b. Type in a descriptive name for the macro, like "solve_min_risk", and press "OK".
   c. Do exactly (and only) the steps you want to repeat:
      i. From the ribbon, select Data->Analysis->Solver.
      ii. On the Solver form, select Load/Save, then specify the cell range where you saved the model, and press Load.
      iii. If asked, replace the current model, then press Solve.
      iv. Once Solver completes, press OK to accept the solution.
   d. When you have finished, select Developer->Code->Stop Recording.
   e. Attach a button to the macro using Developer->Controls->Insert. Press the button icon under Form Controls, and create a box on the spreadsheet by holding down and dragging the left mouse button.
   f. Select the macro you just created, and press "OK".
   g. Press Alt-F11 to enter the VB Editor. ***Eliminate all hard-coded cell addresses***.

Before running your macro, be sure that Visual Basic recognizes the calls to the Solver tool:

   h. From the Visual Basic Editor main menu, select Tools->References…
   i. Make sure the SOLVER box is checked, and uncheck any "MISSING" references to Solver. Press OK.

Section 4: Highest-Return Portfolio Not Exceeding a Target Risk

Now we want to ***build an optimization model that finds the highest-expected-return portfolio that does not exceed a user-specified target risk*** (TRISK, in cell O7):

$$\text{Maximize } \sum_{j=1}^{N} w_j \mu_j \quad \text{(i.e., } \mu_P\text{)}$$

$$\text{subject to:}$$

$$\sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \rho_{ij} \sigma_i \sigma_j} \le TRISK$$

$$\sum_{j=1}^{N} w_j = 1$$

$$w_j \ge 0 \quad \text{for all } j = 1,\ldots N.$$

(1) Enter the above nonlinear program into Solver and solve.

(2) ***Save*** the model in the designated area of column T named Max_Return_Model. (If you need more cells than are specified, redefine the cell range Max_Return_Model.)

(3) ***Record a macro*** to re-solve the model at the click of a button:

a. From the ribbon, select Developer->Code->Record Macro.

b. Type in a descriptive name for the macro, like “solve_min_risk”, and press “OK”.

c. Do exactly (and only) the steps you want to repeat:

   i. From the ribbon, select Data->Analysis->Solver.

   ii. On the Solver form, select Load/Save, then specify the cell range where you saved the model, and press Load.

   iii. If asked, replace the current model, then press Solve.

   iv. Once Solver completes, press OK to accept the solution.

d. When you have finished, select Developer->Code->Stop Recording.

e. Attach a button to the macro using Developer->Controls->Insert. Press the button icon under Form Controls, and create a box on the spreadsheet by holding down and dragging the left mouse button.

f. Select the macro you just created, and press “OK”.

g. Press Alt-F11 to enter the VB Editor. ***Eliminate all hard-coded cell addresses***.

Questions to Consider:

Suppose you select the following securities: AMZN, GE, IBM, PG, WMT

(A) ***What is the lowest risk portfolio allocation that <u>returns at least 25</u>%? PG has a higher expected return and lower risk than WMT, so why is WMT included in the optimal mix?***

(B) ***What is the lowest risk portfolio allocation that <u>returns at least 25%</u> AND <u>allocates no more than 30% to any one security</u>? Explain the Solver results.***

(C) ***What is the highest return portfolio allocation that has a <u>standard deviation of no more than 8%</u>? Each of the individual securities in the optimal mix has a standard deviation higher than 8% -- how is this possible?***

(D) ***Re-solve part (A) <u>without enforcing nonnegativity constraints</u> on the decision variables. How does the optimal allocation differ from part (A)? How does the achieved risk differ from part (A)? Can you explain the results? What do you think is the interpretation of "negative" investment allocation in a security?***

(E) ***Replace WMT with the FIXED security and run the chart_portfolios macro. How does the efficient frontier of portfolios change? Explain the change in the shape of the "footprint".***

