

№8

Ввод-вывод:

10

1 6 5 5 8 2 8 4 6 8

9 9 1 6 3 2 4 8 9 9

2 5 2 6 8 8 2 3 9 7

8 8 8 6 7 9 5 2 2 2

5 2 7 5 7 4 7 9 4 2

1 3 7 7 2 8 7 1 4 7

5 6 9 2 3 6 8 5 8 3

8 9 6 7 5 9 2 3 7 3

2 2 2 9 8 8 2 1 5 1

2 6 8 4 8 6 7 9 5 7

Optimal assignment:

0 0 0 0 0 1 0 0 0 0

0 0 1 0 0 0 0 0 0 0

0 0 0 0 0 0 0 1 0 0

0 0 0 0 0 0 0 0 1 0

0 1 0 0 0 0 0 0 0 0

0 0 0 0 1 0 0 0 0 0

0 0 0 1 0 0 0 0 0 0

0 0 0 0 0 0 1 0 0 0

0 0 0 0 0 0 0 0 0 1

1 0 0 0 0 0 0 0 0 0

Total cost: 19

:

1	6	5	5	8	2	8	4	6	8
9	9	1	6	3	2	4	8	9	9
2	5	2	6	8	8	2	3	9	7
8	8	8	6	7	9	5	2	2	2
5	2	7	5	7	4	7	9	4	2
1	3	7	7	2	8	7	1	4	7
5	6	9	2	3	6	8	5	8	3
8	9	6	7	5	9	2	3	7	3
2	2	2	9	8	8	2	1	5	1
2	6	8	4	8	6	7	9	5	7

№1

1.

0	5	4	4	7	1	7	3	5	7	1
8	8	0	5	2	1	3	7	8	8	1
0	3	0	4	6	6	0	1	7	5	2
6	6	6	4	5	7	3	0	0	0	2
3	0	5	3	5	2	5	7	2	0	2
0	2	6	6	1	7	6	0	3	6	1
3	4	7	0	1	4	6	3	6	1	2
6	7	4	5	3	7	0	1	5	1	2
1	1	1	8	7	7	1	0	4	0	1
0	4	6	2	6	4	5	7	3	5	2

0	5	4	4	6	0	7	3	5	7
8	8	0	5	1	0	3	7	8	8
0	3	0	4	5	5	0	1	7	5
6	6	6	4	4	6	3	0	0	0
3	0	5	3	4	1	5	7	2	0
0	2	6	6	0	6	6	0	3	6
3	4	7	0	0	3	6	3	6	1
6	7	4	5	2	6	0	1	5	1
1	1	1	8	6	6	1	0	4	0
0	4	6	2	5	3	5	7	3	5

2.

(1, 6). 1 6 .
(2; 6), (1; 1).

[0]	5	4	4	6	[0]	7	3	5	7
8	8	[0]	5	1	[0]	3	7	8	8
[0]	3	[0]	4	5	5	[0]	1	7	5
6	6	6	4	4	6	3	[0]	[0]	[0]
3	[0]	5	3	4	1	5	7	2	[0]
[0]	2	6	6	[0]	6	6	[0]	3	6
3	4	7	[0]	[0]	3	6	3	6	1
6	7	4	5	2	6	[0]	1	5	1
1	1	1	8	6	6	1	[0]	4	[0]
0	4	6	2	5	3	5	7	3	5

10 (7), .

3.

: 1, 4, 2,
5, 3, 8, 5, 4, 1, 7, 9.
():

0	5	4	4	6	0	7	3	5	7
8	8	0	5	1	0	3	7	8	8
0	3	0	4	5	5	0	1	7	5
6	6	6	4	4	6	3	0	0	0
3	0	5	3	4	1	5	7	2	0
0	2	6	6	0	6	6	0	3	6
3	4	7	0	0	3	6	3	6	1
6	7	4	5	2	6	0	1	5	1
1	1	1	8	6	6	1	0	4	0
0	4	6	2	5	3	5	7	3	5

(min(2, 6, 6, 3, 6, 4, 7, 3, 6, 1, 7, 4, 6, 5, 1, 4, 6, 3, 3, 5) = 1)
:

0	5	4	4	6	0	7	3	5	7
8	8	0	5	1	0	3	7	8	8
0	3	0	4	5	5	0	1	7	5
6	6	6	4	4	6	3	0	0	0
3	0	5	3	4	1	5	7	2	0
0	1	5	6	0	5	6	0	2	5
3	3	6	0	0	2	6	3	5	0
6	6	3	5	2	5	0	1	4	0
1	1	1	8	6	6	1	0	4	0
0	3	5	2	5	2	5	7	2	4

,

:

1	5	4	5	7	0	8	4	5	7
9	8	0	6	2	0	4	8	8	8
1	3	0	5	6	5	1	2	7	5
7	6	6	5	5	6	4	1	0	0
4	0	5	4	5	1	6	8	2	0
0	1	5	6	0	5	6	0	2	5
3	3	6	0	0	2	6	3	5	0
6	6	3	5	2	5	0	1	4	0
2	1	1	9	7	6	2	1	4	0
0	3	5	2	5	2	5	7	3	5

№2

1.

.

1	5	4	5	7	[0]	8	4	5	7
9	8	[0]	6	2	[0]	4	8	8	8
1	3	[0]	5	6	5	1	2	7	5
7	6	6	5	5	6	4	1	0	0
4	0	5	4	5	1	6	8	2	0
0	1	5	6	0	5	6	0	2	5
3	3	6	0	0	2	6	3	5	0
6	6	3	5	2	5	0	1	4	0
2	1	1	9	7	6	2	1	4	0
0	3	5	2	5	2	5	7	3	4

2), .

3.

7, 1, 2, 1, 3, 4, 7. : 10, 6, 2,
():

1	5	4	5	7	0	8	4	5	7
9	8	0	6	2	0	4	8	8	8
1	3	0	5	6	5	1	2	7	5
7	6	6	5	5	6	4	1	0	0
4	0	5	4	5	1	6	8	2	0
0	1	5	6	0	5	6	0	2	5
3	3	6	0	0	2	6	3	5	0
6	6	3	5	2	5	0	1	4	0
2	1	1	9	7	6	2	1	4	0
0	3	5	2	5	2	5	7	2	4

(min(5, 6, 5, 2, 7, 4, 5, 1, 8, 2, 5, 2, 5, 1, 4, 9, 7, 6,
1, 4, 2, 5, 2, 7, 2) = 1) :

1	5	4	5	7	0	8	4	5	7
9	8	0	6	2	0	4	8	8	8
1	3	0	4	5	4	1	1	6	5
7	6	6	5	5	6	4	1	0	0
4	0	5	3	4	0	6	7	1	0
0	1	5	6	0	5	6	0	2	5
3	3	6	0	0	2	6	3	5	0
6	6	3	4	1	4	0	0	3	0
2	1	1	8	6	5	2	0	3	0
0	3	5	1	4	1	5	6	1	4

, :

	2	6	5	5	7	0	9	4	5	8
10	9	1	6	2	0	5	8	8	9	
1	3	0	4	6	5	1	2	6	6	
8	7	7	5	5	6	5	1	0	1	
4	0	5	3	5	1	6	8	1	1	
1	2	6	6	0	5	7	0	2	6	
4	4	7	0	1	3	7	3	5	2	
7	7	4	5	2	5	0	1	4	1	
2	1	1	9	7	6	2	1	4	0	
0	3	5	2	5	2	5	7	3	4	

№3

1.

.

2	6	5	5	7	[0]	9	4	5	8
10	9	1	6	2	[0]	5	8	8	9
1	3	0	4	5	4	1	1	6	5
8	7	7	5	5	6	5	1	0	1
4	0	5	3	4	1	6	8	1	0
1	2	6	6	0	5	7	0	2	6
4	4	7	0	0	2	7	3	5	1
7	7	4	5	2	5	0	1	4	1
2	1	1	9	6	6	2	1	4	0
0	3	5	2	5	2	5	7	3	4

10 (

1), .

3.

6, 8, 1, 3, 5, 4, 10. : 5, 8, 7, ():

2	6	5	5	7	0	9	4	5	8
10	9	1	6	2	0	5	8	8	9
1	3	0	4	5	4	1	1	6	5
8	7	7	5	5	6	5	1	0	1
4	0	5	3	4	1	6	8	1	0
1	2	6	6	0	5	7	0	2	6
4	4	7	0	0	2	7	3	5	1
7	7	4	5	2	5	0	1	4	1
2	1	1	9	6	6	2	1	4	0
0	3	5	2	5	2	5	7	3	4

$$(\min(6, 5, 5, 9, 5, 9, 1, 6, 5, 8, 2, 6, 6, 7, 2, 1, 1, 8, 2, 3, 3, 5, 1, 5, 1) = 1) :$$

2	5	4	4	7	0	8	4	4	8
10	8	0	5	2	0	4	8	7	9
1	3	0	4	5	4	1	1	6	5
8	7	7	5	5	6	5	1	0	1
4	0	5	3	4	1	6	8	1	0
1	1	5	5	0	5	6	0	1	6
4	4	7	0	0	2	7	3	5	1
7	7	4	4	1	4	0	0	3	1
2	0	0	8	6	5	2	0	4	0
0	2	4	1	4	1	5	6	0	4

, :

2	5	4	4	7	0	8	4	4	8
10	8	0	5	2	0	4	8	7	9
2	3	0	4	6	5	1	2	6	6
9	7	7	5	6	7	5	2	0	2
5	0	5	3	5	1	6	8	1	1
1	1	5	5	0	5	6	0	1	6
5	4	7	0	1	3	7	4	5	2
8	7	4	5	2	5	0	1	4	1
2	0	0	8	6	5	1	0	4	0
0	2	4	1	4	1	5	6	1	4

1.

$$\begin{array}{ccccc} (1, 6). & 1 & 6 & . & \\ (2; 6). & & (2, 3). & & 2 \\ & & (3; 3), (9; 3). & & \end{array}$$

3

$$\vdots$$

2	5	4	4	7	[0]	8	4	4	8
10	8	[0]	5	2	[−0−]	4	8	7	9
2	3	[−0−]	4	6	5	1	2	6	6
9	7	7	5	6	7	5	2	0	2
5	0	5	3	5	1	6	8	1	1
1	1	5	5	0	5	6	0	1	6
5	4	7	0	1	3	7	4	5	2
7	6	3	4	2	5	0	1	3	1
2	0	[−0−]	7	6	5	1	0	2	0
0	2	4	0	4	1	4	6	0	4

10 (

3.

9, 10, 3, 6, 6, 2, 4, 4, 8.
():

2	5	4	4	7	0	8	4	4	8
10	8	0	5	2	0	4	8	7	9
2	3	0	4	6	5	1	2	6	6
9	7	7	5	6	7	5	2	0	2
5	0	5	3	5	1	6	8	1	1
1	1	5	5	0	5	6	0	1	6
5	4	7	0	1	3	7	4	5	2
7	6	3	4	2	5	0	1	3	1
2	0	0	7	6	5	1	0	2	0
0	2	4	0	4	1	4	6	0	4

(min(2, 7, 8, 4, 4, 8, 10, 2, 4, 8, 7, 9, 2, 6, 1, 2, 6, 6, 5, 5, 6,

1) :

1	5	4	4	6	0	7	3	3	7
9	8	0	5	1	0	3	7	6	8
1	3	0	4	5	5	0	1	5	5
9	7	7	5	6	7	5	2	0	2
4	0	5	3	4	1	5	7	0	0
1	1	5	5	0	5	6	0	1	6
4	4	7	0	0	3	6	3	4	1
7	6	3	4	2	5	0	1	3	1
2	0	0	7	6	5	1	0	2	0
0	2	4	0	4	1	4	6	0	4

, :

1	5	4	4	6	0	7	3	3	7
9	8	0	5	1	0	3	7	6	8
1	3	0	4	5	5	0	1	5	5
9	8	8	6	6	8	5	2	0	2
4	0	5	3	4	1	5	7	0	0
1	2	6	6	0	6	6	0	1	6
4	4	7	0	0	3	6	3	4	1
7	7	4	5	2	6	0	1	3	1
2	1	1	8	6	6	1	0	2	0
0	3	5	1	4	2	4	6	0	4

№5

1. .

2. , .

(1, 6). 1 6 .

(2; 6). (2, 3). 2

3 . (3; 3).

:

1	5	4	4	6	[0]	7	3	3	7
9	8	[0]	5	1	[−0−]	3	7	6	8
1	3	[−0−]	4	5	5	[0]	1	5	5
9	8	8	6	6	8	5	2	[0]	2
4	[−0−]	5	3	4	1	5	7	[−0−]	[0]
1	2	6	6	[−0−]	6	6	[0]	1	6
4	4	7	[−0−]	[0]	3	6	3	4	1
7	7	4	5	2	6	[−0−]	1	3	1
2	1	1	8	6	6	1	[−0−]	2	[−0−]
0	3	5	1	4	2	4	6	[−0−]	4

10 (

7), .

3. . :

5, 3, 6, 6, 7, 7, 9, 9, 10.
():

1	5	4	4	6	0	7	3	3	7
9	8	0	5	1	0	3	7	6	8
1	3	0	4	5	5	0	1	5	5
9	8	8	6	6	8	5	2	0	2
4	0	5	3	4	1	5	7	0	0
1	2	6	6	0	6	6	0	1	6
4	4	7	0	0	3	6	3	4	1
7	7	4	5	2	6	0	1	3	1
2	1	1	8	6	6	1	0	2	0
0	3	5	1	4	2	4	6	0	4

(min(1, 5, 4, 6, 3, 7, 9, 8, 5, 1, 7, 8, 1, 3, 4, 5, 1, 5, 9, 8, 6, 6

1) :

0	4	4	3	5	0	7	2	3	6
8	7	0	4	0	0	3	6	6	7
0	2	0	3	4	5	0	0	5	4
8	7	8	5	5	8	5	1	0	1
4	0	5	3	4	1	5	7	0	0
1	2	6	6	0	6	6	0	1	6
4	4	7	0	0	3	6	3	4	1
6	6	4	4	1	6	0	0	3	0
2	1	1	8	6	6	1	0	2	0
0	3	5	1	4	2	4	6	0	4

,

:

0	4	4	3	5	0	7	2	3	6
8	7	0	4	0	0	3	6	6	7
0	2	0	3	4	5	0	0	5	4
8	7	8	5	5	8	5	1	0	1
4	0	6	3	4	2	6	7	1	0
1	2	7	6	0	7	7	0	2	6
4	4	8	0	0	4	7	3	5	1
6	6	4	4	1	6	0	0	3	0
2	1	2	8	6	7	2	0	3	0
0	3	6	1	4	3	5	6	1	4

№6

1. .

. ,

. .

2. , 1 6 .

(1, 6).
(2; 6), (1; 1).
:

$[-0-]$	4	4	3	5	$[0]$	7	2	3	6
8	7	$[0]$	4	$[-0-]$	$[-0-]$	3	6	6	7
$[-0-]$	2	$[-0-]$	3	4	5	$[0]$	$[-0-]$	5	4
8	7	8	5	5	8	5	1	$[0]$	1
4	$[0]$	6	3	4	2	6	7	1	$[-0-]$
1	2	7	6	$[0]$	7	7	$[-0-]$	2	6
4	4	8	$[0]$	$[-0-]$	4	7	3	5	1
6	6	4	4	1	6	$[0]$	$[-0-]$	3	$[-0-]$
2	1	2	8	6	7	2	$[-0-]$	3	$[0]$
$[0]$	3	6	1	4	3	5	6	1	4

$k = 10.$

$C_e:$

0	4	4	3	5	0	7	2	3	6
8	7	0	4	0	0	3	6	6	7
0	2	0	3	4	5	0	0	5	4
8	7	8	5	5	8	5	1	0	1
4	0	6	3	4	2	6	7	1	0
1	2	7	6	0	7	7	0	2	6
4	4	8	0	0	4	7	3	5	1
6	6	4	4	1	6	0	0	3	0
2	1	2	8	6	7	2	0	3	0
0	3	6	1	4	3	5	6	1	4

4.

$X,$

()

.

$[-0-]$	4	4	3	5	$[0]$	7	2	3	6
8	7	$[0]$	4	$[-0-]$	$[-0-]$	3	6	6	7
$[-0-]$	2	$[-0-]$	3	4	5	$[-0-]$	$[0]$	5	4
8	7	8	5	5	8	5	1	$[0]$	1
4	$[0]$	6	3	4	2	6	7	1	$[-0-]$
1	2	7	6	$[0]$	7	7	$[-0-]$	2	6
4	4	8	$[0]$	$[-0-]$	4	7	3	5	1
6	6	4	4	1	6	$[0]$	$[-0-]$	3	$[-0-]$
2	1	2	8	6	7	2	$[-0-]$	3	$[0]$
$[0]$	3	6	1	4	3	5	6	1	4

:

$$C_{\min} = 2 + 2 + 2 + 2 + 2 + 1 + 2 + 2 + 1 + 3 = 19$$

: (4;9), (5;2), (7;4), (1;6), (10;1), (2;3), (6;5), (8;7), (9;10), (3;8) : (4;9),
 (9;10), (10;1), (1;6), (6;5), (5;2), (2;3), (3;8), (8;7), (7;4)

:

$$X_0 = (1, 2); (2, 3); (3, 4); (4, 5); (5, 6); (6, 7); (7, 8); (8, 9); (9, 10); (10, 1)$$

$$F(X_0) = 34 + 56 + 91 + 83 + 51 + 77 + 26 + 44 + 87 + 58 = 607.$$

,

D

$$d_i = \min_j d_{ij}$$

i j	1	2	3	4	5	6	7	8	9	10	d_i
1	M	34	68	18	63	80	12	44	58	87	12
2	56	M	56	94	62	65	18	38	67	22	18
3	34	53	M	91	13	73	70	51	13	37	13
4	28	19	14	M	83	89	25	9	89	22	9
5	67	13	1	19	M	51	7	13	31	4	1
6	3	78	24	90	14	M	77	6	35	69	3
7	96	32	100	4	8	19	M	26	37	36	4
8	79	2	48	25	63	99	17	M	44	45	2
9	97	49	33	74	23	72	23	73	M	87	23
10	58	83	24	39	17	76	64	78	100	M	17

d_i

i j	1	2	3	4	5	6	7	8	9	10
1	M	22	56	6	51	68	0	32	46	75
2	38	M	38	76	44	47	0	20	49	4
3	21	40	M	78	0	60	57	38	0	24
4	19	10	5	M	74	80	16	0	80	13
5	66	12	0	18	M	50	6	12	30	3
6	0	75	21	87	11	M	74	3	32	66
7	92	28	96	0	4	15	M	22	33	32
8	77	0	46	23	61	97	15	M	42	43
9	74	26	10	51	0	49	0	50	M	64
10	41	66	7	22	0	59	47	61	83	M

$$, \qquad \qquad \qquad :$$

$$d_j = \min_i d_{ij}$$

i j	1	2	3	4	5	6	7	8	9	10
1	M	22	56	6	51	68	0	32	46	75
2	38	M	38	76	44	47	0	20	49	4
3	21	40	M	78	0	60	57	38	0	24
4	19	10	5	M	74	80	16	0	80	13
5	66	12	0	18	M	50	6	12	30	3
6	0	75	21	87	11	M	74	3	32	66
7	92	28	96	0	4	15	M	22	33	32
8	77	0	46	23	61	97	15	M	42	43
9	74	26	10	51	0	49	0	50	M	64
10	41	66	7	22	0	59	47	61	83	M
d_j	0	0	0	0	0	15	0	0	0	3

$$, \qquad \qquad \qquad d_i \quad d_j$$

.

i j	1	2	3	4	5	6	7	8	9	10
1	M	22	56	6	51	53	0	32	46	72
2	38	M	38	76	44	32	0	20	49	1
3	21	40	M	78	0	45	57	38	0	21
4	19	10	5	M	74	65	16	0	80	10
5	66	12	0	18	M	35	6	12	30	0
6	0	75	21	87	11	M	74	3	32	63
7	92	28	96	0	4	0	M	22	33	29
8	77	0	46	23	61	82	15	M	42	40
9	74	26	10	51	0	34	0	50	M	61
10	41	66	7	22	0	44	47	61	83	M

$$H:$$

$$H = \sum d_i + \sum d_j$$

$$H = 12+18+13+9+1+3+4+2+23+17+0+0+0+0+0+0+15+0+0+0+3 = 120$$

№1.

$(i, j) \rightarrow (i^*, j^*)$.

$M = (m_{ij})$

,

.

i \ j	1	2	3	4	5	6	7	8	9	10	d_i
1	M	22	56	6	51	53	0(6)	32	46	72	6
2	38	M	38	76	44	32	0(1)	20	49	1	1
3	21	40	M	78	0(0)	45	57	38	0(30)	21	0
4	19	10	5	M	74	65	16	0(8)	80	10	5
5	66	12	0(5)	18	M	35	6	12	30	0(1)	0
6	0(22)	75	21	87	11	M	74	3	32	63	3
7	92	28	96	0(6)	4	0(32)	M	22	33	29	0
8	77	0(25)	46	23	61	82	15	M	42	40	15
9	74	26	10	51	0(0)	34	0(0)	50	M	61	0
10	41	66	7	22	0(7)	44	47	61	83	M	7
d_j	19	10	5	6	0	32	0	3	30	1	0

$$d(1, 7) = 6 + 0 = 6; \quad d(2, 7) = 1 + 0 = 1; \quad d(3, 5) = 0 + 0 = 0; \quad d(3, 9) = 0 + 30 = 30;$$

$$d(4, 8) = 5 + 3 = 8; \quad d(5, 3) = 0 + 5 = 5; \quad d(5, 10) = 0 + 1 = 1; \quad d(6, 1) = 3 + 19 = 22;$$

$$d(7, 4) = 0 + 6 = 6; \quad d(7, 6) = 0 + 32 = 32; \quad d(8, 2) = 15 + 10 = 25; \quad d(9, 5) = 0 + 0 = 0;$$

$$d(9, 7) = 0 + 0 = 0; \quad d(10, 5) = 7 + 0 = 7;$$

$$(0 + 32) = 32 \quad (7, 6), \quad ,$$

$$(7, 6) \rightarrow (7^*, 6^*).$$

$$(7, 6) \quad d_{76} = 0 \quad M, \\ (7^*, 6^*),$$

.

i j	1	2	3	4	5	6	7	8	9	10	d_i
1	M	22	56	6	51	53	0	32	46	72	0
2	38	M	38	76	44	32	0	20	49	1	0
3	21	40	M	78	0	45	57	38	0	21	0
4	19	10	5	M	74	65	16	0	80	10	0
5	66	12	0	18	M	35	6	12	30	0	0
6	0	75	21	87	11	M	74	3	32	63	0
7	92	28	96	0	4	M	M	22	33	29	0
8	77	0	46	23	61	82	15	M	42	40	0
9	74	26	10	51	0	34	0	50	M	61	0
10	41	66	7	22	0	44	47	61	83	M	0
d_j	0	0	0	0	0	32	0	0	0	0	32

:

$$H(7^*,6^*) = 120 + 32 = 152$$

$$(7,6) \qquad \qquad \qquad 7- \qquad \qquad \qquad 6- \qquad \qquad \qquad ,$$

$$d_{67} \qquad \qquad \qquad M, \qquad \qquad \qquad . \qquad \qquad \qquad : \\ (9 \times 9), \qquad \qquad \qquad . \qquad \qquad \qquad .$$

i j	1	2	3	4	5	7	8	9	10	d_i
1	M	22	56	6	51	0	32	46	72	0
2	38	M	38	76	44	0	20	49	1	0
3	21	40	M	78	0	57	38	0	21	0
4	19	10	5	M	74	16	0	80	10	0
5	66	12	0	18	M	6	12	30	0	0
6	0	75	21	87	11	M	3	32	63	0
8	77	0	46	23	61	15	M	42	40	0
9	74	26	10	51	0	0	50	M	61	0
10	41	66	7	22	0	47	61	83	M	0
d_j	0	0	0	6	0	0	0	0	0	6

:

$$\sum d_i + \sum d_j = 6$$

$$(7,6) \qquad \qquad \qquad :$$

$$H(7,6) = 120 + 6 = 126 \leq 152$$

$$\begin{array}{ccccc}
 & (7,6) & , & (7^*,6^*), & (7,6) \\
 H = 126. & & & &
 \end{array}$$