Работа №10
Задача о рюкзаке
Имеется рюкзак грузоподъемностью W. $weight_i$ – вес одного предмета i-ого типа, $cost_i$ – стоимость (ценность) одного предмета i-ого типа, x_i – число предметов i-ого типа, которые будут загружаться на транспортировочное средство. Требуется заполнить его грузом, состоящим из предметов N различных типов таким образом, чтобы стоимость (ценность) всего груза была максимальной.

$$W(x) = \sum_{i=1}^{N} x_i \cdot cost_i \to max$$

$$\sum_{i=1}^{N} x_i \cdot weight_i \leq W, \quad x_i \in \{0\} \cup \mathbb{N}$$

Решение задачи разбивается на N этапов. На каждом i-ом этапе определяется максимальная стоимость груза, состоящего из предметов типа $k = \overline{1, i}$

Рекуррентное уравнение Беллмана для задачи о рюкзаке
$W_i(weight)$ - максимальная стоимость груза, состоящего из предметов типа $k = \overline{1, i}$ с общим весом не более $weight$.

$$\forall \, weight \colon \; weight \in \overline{0, W}$$

$$W_i(weight) = \max_{x_i \in 0, \left[\frac{weight}{weight_i}\right]} \{x_i \cdot cost_i + W_{i-1}(weight - x_i \cdot weight_i)\}$$

$$\forall \, weight \colon weight \in \overline{0, W} \quad W_0(weight) = 0$$

Количество типов предметов 6, грузоподъемность: 20

5	9	8	7	10	13

Веса

28	20	13	6	21	18

Стоимости

Решение
Шаг №1
$$W_1(0) = \max_{x_i \in 0, \left[\frac{20}{5}\right]} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_1 = 0$$

$$W_1(1) = \max_{x_i \in 0, \left[\frac{20}{5}\right]} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_1 = 0$$

$$W_1(2) = \max_{x_i \in 0, \left[\frac{20}{5}\right]} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_1 = 0$$

$$W_1(3) = \max_{x_i \in 0, \left[\frac{20}{5}\right]} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_1 = 0$$

$$W_1(4) = \max_{x_i \in 0, \left[\frac{20}{5}\right]} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_1 = 0$$

$$W_1(5) = \max_{x_i \in 0, \left[\frac{20}{5}\right]} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28\} = 28, \quad x_1 = 1$$

$$W_1(6) = \max_{x_i \in 0, \left[\frac{20}{5}\right]} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28\} = 28, \quad x_1 = 1$$

$$W_1(7) = \max_{x_i \in 0, \left[\frac{20}{5}\right]} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28\} = 28, \quad x_1 = 1$$

$$W_1(8) = \max_{x_i \in 0, \left[\frac{20}{5}\right]} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28\} = 28, \quad x_1 = 1$$

$$W_1(9) = \max_{x_i \in 0, \left[\frac{20}{5}\right]} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28\} = 28, \quad x_1 = 1$$

$$W_1(10) = \max_{x_i \in 0, \left[\frac{20}{5}\right]} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28, 56\} = 56, \quad x_1 = 2$$

$$W_1(11) = \max_{x_i \in 0, \left[\frac{20}{5}\right]} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28, 56\} = 56, \quad x_1 = 2$$

$$W_1(12) = \max_{x_i \in \overline{0, \left[\frac{20}{5}\right]}} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28, 56\} = 56, \quad x_1 = 2$$

$$W_1(13) = \max_{x_i \in \overline{0, \left[\frac{20}{5}\right]}} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28, 56\} = 56, \quad x_1 = 2$$

$$W_1(14) = \max_{x_i \in \overline{0, \left[\frac{20}{5}\right]}} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28, 56\} = 56, \quad x_1 = 2$$

$$W_1(15) = \max_{x_i \in \overline{0, \left[\frac{20}{5}\right]}} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28, 56, 84\} = 84, \quad x_1 = 3$$

$$W_1(16) = \max_{x_i \in \overline{0, \left[\frac{20}{5}\right]}} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28, 56, 84\} = 84, \quad x_1 = 3$$

$$W_1(17) = \max_{x_i \in \overline{0, \left[\frac{20}{5}\right]}} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28, 56, 84\} = 84, \quad x_1 = 3$$

$$W_1(18) = \max_{x_i \in \overline{0, \left[\frac{20}{5}\right]}} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28, 56, 84\} = 84, \quad x_1 = 3$$

$$W_1(19) = \max_{x_i \in \overline{0, \left[\frac{20}{5}\right]}} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28, 56, 84\} = 84, \quad x_1 = 3$$

$$W_1(20) = \max_{x_i \in \overline{0, \left[\frac{20}{5}\right]}} \{x_i \cdot cost_i + W_{1-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0, 28, 56, 84, 112\} = 112, \quad x_1 = 4$$

Шаг №2

$$W_2(0) = \max_{x_i \in \overline{0, \left[\frac{20}{9}\right]}} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_2 = 0$$

$$W_2(1) = \max_{x_i \in \overline{0, \left[\frac{20}{9}\right]}} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_2 = 0$$

$$W_2(2) = \max_{x_i \in 0, \left[\frac{20}{9}\right]} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_2 = 0$$

$$W_2(3) = \max_{x_i \in 0, \left[\frac{20}{9}\right]} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_2 = 0$$

$$W_2(4) = \max_{x_i \in 0, \left[\frac{20}{9}\right]} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_2 = 0$$

$$W_2(5) = \max_{x_i \in 0, \left[\frac{20}{9}\right]} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_2 = 0$$

$$W_2(6) = \max_{x_i \in 0, \left[\frac{20}{9}\right]} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_2 = 0$$

$$W_2(7) = \max_{x_i \in 0, \left[\frac{20}{9}\right]} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_2 = 0$$

$$W_2(8) = \max_{x_i \in 0, \left[\frac{20}{9}\right]} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_2 = 0$$

$$W_2(9) = \max_{x_i \in 0, \left[\frac{20}{9}\right]} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28, 20\} = 28, \quad x_2 = 0$$

$$W_2(10) = \max_{x_i \in 0, \left[\frac{20}{9}\right]} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 20\} = 56, \quad x_2 = 0$$

$$W_2(11) = \max_{x_i \in 0, \left[\frac{20}{9}\right]} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 20\} = 56, \quad x_2 = 0$$

$$W_2(12) = \max_{x_i \in 0, \left[\frac{20}{9}\right]} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 20\} = 56, \quad x_2 = 0$$

$$W_2(13) = \max_{x_i \in \overline{0,\left[\frac{20}{9}\right]}} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 20\} = 56, \quad x_2 = 0$$

$$W_2(14) = \max_{x_i \in \overline{0,\left[\frac{20}{9}\right]}} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 48\} = 56, \quad x_2 = 0$$

$$W_2(15) = \max_{x_i \in \overline{0,\left[\frac{20}{9}\right]}} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 48\} = 84, \quad x_2 = 0$$

$$W_2(16) = \max_{x_i \in \overline{0,\left[\frac{20}{9}\right]}} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 48\} = 84, \quad x_2 = 0$$

$$W_2(17) = \max_{x_i \in \overline{0,\left[\frac{20}{9}\right]}} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 48\} = 84, \quad x_2 = 0$$

$$W_2(18) = \max_{x_i \in \overline{0,\left[\frac{20}{9}\right]}} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 48, 40\} = 84, \quad x_2 = 0$$

$$W_2(19) = \max_{x_i \in \overline{0,\left[\frac{20}{9}\right]}} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 76, 40\} = 84, \quad x_2 = 0$$

$$W_2(20) = \max_{x_i \in \overline{0,\left[\frac{20}{9}\right]}} \{x_i \cdot cost_i + W_{2-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{112, 76, 40\} = 112, \quad x_2 = 0$$

Шаг №3

$$W_3(0) = \max_{x_i \in \overline{0,\left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_3 = 0$$

$$W_3(1) = \max_{x_i \in \overline{0,\left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_3 = 0$$

$$W_3(2) = \max_{x_i \in \overline{0,\left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_3 = 0$$

$$W_3(3) = \max_{x_i \in \overline{0, \left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_3 = 0$$

$$W_3(4) = \max_{x_i \in \overline{0, \left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_3 = 0$$

$$W_3(5) = \max_{x_i \in \overline{0, \left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_3 = 0$$

$$W_3(6) = \max_{x_i \in \overline{0, \left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_3 = 0$$

$$W_3(7) = \max_{x_i \in \overline{0, \left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_3 = 0$$

$$W_3(8) = \max_{x_i \in \overline{0, \left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28, 13\} = 28, \quad x_3 = 0$$

$$W_3(9) = \max_{x_i \in \overline{0, \left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28, 13\} = 28, \quad x_3 = 0$$

$$W_3(10) = \max_{x_i \in \overline{0, \left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 13\} = 56, \quad x_3 = 0$$

$$W_3(11) = \max_{x_i \in \overline{0, \left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 13\} = 56, \quad x_3 = 0$$

$$W_3(12) = \max_{x_i \in \overline{0, \left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 13\} = 56, \quad x_3 = 0$$

$$W_3(13) = \max_{x_i \in \overline{0, \left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 41\} = 56, \quad x_3 = 0$$

$$W_3(14) = \max_{x_i \in \overline{0,\left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 41\} = 56, \quad x_3 = 0$$

$$W_3(15) = \max_{x_i \in \overline{0,\left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 41\} = 84, \quad x_3 = 0$$

$$W_3(16) = \max_{x_i \in \overline{0,\left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 41, 26\} = 84, \quad x_3 = 0$$

$$W_3(17) = \max_{x_i \in \overline{0,\left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 41, 26\} = 84, \quad x_3 = 0$$

$$W_3(18) = \max_{x_i \in \overline{0,\left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 69, 26\} = 84, \quad x_3 = 0$$

$$W_3(19) = \max_{x_i \in \overline{0,\left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 69, 26\} = 84, \quad x_3 = 0$$

$$W_3(20) = \max_{x_i \in \overline{0,\left[\frac{20}{8}\right]}} \{x_i \cdot cost_i + W_{3-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{112, 69, 26\} = 112, \quad x_3 = 0$$

Шаг №4

$$W_4(0) = \max_{x_i \in \overline{0,\left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_4 = 0$$

$$W_4(1) = \max_{x_i \in \overline{0,\left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_4 = 0$$

$$W_4(2) = \max_{x_i \in \overline{0,\left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_4 = 0$$

$$W_4(3) = \max_{x_i \in \overline{0,\left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_4 = 0$$

$$W_4(4) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_4 = 0$$

$$W_4(5) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_4 = 0$$

$$W_4(6) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_4 = 0$$

$$W_4(7) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28, 6\} = 28, \quad x_4 = 0$$

$$W_4(8) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28, 6\} = 28, \quad x_4 = 0$$

$$W_4(9) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28, 6\} = 28, \quad x_4 = 0$$

$$W_4(10) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 6\} = 56, \quad x_4 = 0$$

$$W_4(11) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 6\} = 56, \quad x_4 = 0$$

$$W_4(12) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 34\} = 56, \quad x_4 = 0$$

$$W_4(13) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 34\} = 56, \quad x_4 = 0$$

$$W_4(14) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 34, 12\} = 56, \quad x_4 = 0$$

$$W_4(15) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 34, 12\} = 84, \quad x_4 = 0$$

$$W_4(16) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 34, 12\} = 84, \quad x_4 = 0$$

$$W_4(17) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 62, 12\} = 84, \quad x_4 = 0$$

$$W_4(18) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 62, 12\} = 84, \quad x_4 = 0$$

$$W_4(19) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 62, 40\} = 84, \quad x_4 = 0$$

$$W_4(20) = \max_{x_i \in \overline{0, \left[\frac{20}{7}\right]}} \{x_i \cdot cost_i + W_{4-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{112, 62, 40\} = 112, \quad x_4 = 0$$

Шаг №5

$$W_5(0) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_5 = 0$$

$$W_5(1) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_5 = 0$$

$$W_5(2) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_5 = 0$$

$$W_5(3) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_5 = 0$$

$$W_5(4) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_5 = 0$$

$$W_5(5) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_5 = 0$$

$$W_5(6) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_5 = 0$$

$$W_5(7) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_5 = 0$$

$$W_5(8) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_5 = 0$$

$$W_5(9) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_5 = 0$$

$$W_5(10) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 21\} = 56, \quad x_5 = 0$$

$$W_5(11) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 21\} = 56, \quad x_5 = 0$$

$$W_5(12) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 21\} = 56, \quad x_5 = 0$$

$$W_5(13) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 21\} = 56, \quad x_5 = 0$$

$$W_5(14) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 21\} = 56, \quad x_5 = 0$$

$$W_5(15) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 49\} = 84, \quad x_5 = 0$$

$$W_5(16) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 49\} = 84, \quad x_5 = 0$$

$$W_5(17) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 49\} = 84, \quad x_5 = 0$$

$$W_5(18) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 49\} = 84, \quad x_5 = 0$$

$$W_5(19) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 49\} = 84, \quad x_5 = 0$$

$$W_5(20) = \max_{x_i \in \overline{0, \left[\frac{20}{10}\right]}} \{x_i \cdot cost_i + W_{5-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{112, 77, 42\} = 112, \quad x_5 = 0$$

Шаг №6

$$W_6(0) = \max_{x_i \in \overline{0, \left[\frac{20}{13}\right]}} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_6 = 0$$

$$W_6(1) = \max_{x_i \in \overline{0, \left[\frac{20}{13}\right]}} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_6 = 0$$

$$W_6(2) = \max_{x_i \in \overline{0, \left[\frac{20}{13}\right]}} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_6 = 0$$

$$W_6(3) = \max_{x_i \in \overline{0, \left[\frac{20}{13}\right]}} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_6 = 0$$

$$W_6(4) = \max_{x_i \in \overline{0, \left[\frac{20}{13}\right]}} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{0\} = 0, \quad x_6 = 0$$

$$W_6(5) = \max_{x_i \in \overline{0, \left[\frac{20}{13}\right]}} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_6 = 0$$

$$W_6(6) = \max_{x_i \in 0,\left[\frac{20}{13}\right]} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_6 = 0$$

$$W_6(7) = \max_{x_i \in 0,\left[\frac{20}{13}\right]} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_6 = 0$$

$$W_6(8) = \max_{x_i \in 0,\left[\frac{20}{13}\right]} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_6 = 0$$

$$W_6(9) = \max_{x_i \in 0,\left[\frac{20}{13}\right]} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{28\} = 28, \quad x_6 = 0$$

$$W_6(10) = \max_{x_i \in 0,\left[\frac{20}{13}\right]} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56\} = 56, \quad x_6 = 0$$

$$W_6(11) = \max_{x_i \in 0,\left[\frac{20}{13}\right]} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56\} = 56, \quad x_6 = 0$$

$$W_6(12) = \max_{x_i \in 0,\left[\frac{20}{13}\right]} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56\} = 56, \quad x_6 = 0$$

$$W_6(13) = \max_{x_i \in 0,\left[\frac{20}{13}\right]} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 18\} = 56, \quad x_6 = 0$$

$$W_6(14) = \max_{x_i \in 0,\left[\frac{20}{13}\right]} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{56, 18\} = 56, \quad x_6 = 0$$

$$W_6(15) = \max_{x_i \in 0,\left[\frac{20}{13}\right]} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 18\} = 84, \quad x_6 = 0$$

$$W_6(16) = \max_{x_i \in 0,\left[\frac{20}{13}\right]} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 18\} = 84, \quad x_6 = 0$$

$$W_6(17) = \max_{x_i \in \overline{0,\left[\frac{20}{13}\right]}} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 18\} = 84, \quad x_6 = 0$$

$$W_6(18) = \max_{x_i \in \overline{0,\left[\frac{20}{13}\right]}} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 46\} = 84, \quad x_6 = 0$$

$$W_6(19) = \max_{x_i \in \overline{0,\left[\frac{20}{13}\right]}} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{84, 46\} = 84, \quad x_6 = 0$$

$$W_6(20) = \max_{x_i \in \overline{0,\left[\frac{20}{13}\right]}} \{x_i \cdot cost_i + W_{6-1}(20 - x_i \cdot weight_i)\} =$$

$$= \max\{112, 46\} = 112, \quad x_6 = 0$$

Ответ

Максимальная стоимость: 112

$$W_6(20) \Rightarrow x_6^o = 0$$

$$W_5(20 - 0 * 13) = 112 \text{ при } x_5^o = 0$$

$$W_4(20 - 0 * 10) = 112 \text{ при } x_4^o = 0$$

$$W_3(20 - 0 * 7) = 112 \text{ при } x_3^o = 0$$

$$W_2(20 - 0 * 8) = 112 \text{ при } x_2^o = 0$$

$$W_1(20 - 0 * 9) = 112 \text{ при } x_1^o = 4$$

Оптимальное решение: (4, 0, 0, 0, 0, 0)