from random import randint import numpy as np from scipy.optimize import linear sum assignment def hungarian\_algorithm(cost\_matrix): cost matrix = np.array(cost matrix) # Check if the matrix is rectangular and non-empty if len(cost\_matrix.shape) != 2 or cost\_matrix.shape[0] == 0 or cost\_matrix.shape[ raise ValueError("Cost matrix must be a non-empty 2D array.") row indices, col indices = linear sum assignment(cost matrix) return row\_indices.tolist(), col\_indices.tolist() def main(): try: n = int(input())should\_be\_generated = input("Write something to generate table") if should be generated:  $cost_matrix = [[randint(1, 9) for j in range(n)] for _ in range(n)]$ for i in cost matrix: print(\*i) else: cost\_matrix = [list(map(int, input().split())) for \_ in range(n)] row\_ind, col\_ind = hungarian\_algorithm(cost\_matrix) print("Optimal assignment:") optimal\_assignment = [[0 for j in range(n)] for i in range(n)]for r, c in zip(row\_ind, col\_ind):  $optimal_assignment[r][c] = 1$ 

```
for i in optimal assignment:
       print(*i)
     total_cost = sum(cost_matrix[r][c] for r, c in zip(row_ind, col_ind))
     print(f"Total cost: {total cost}")
  except Exception as e:
     print(f"Error: {e}")
if name == " main ":
  main()
  Ввод-вывод:
  10
  1655828468
  9916324899
  2526882397
  8886795222
  5275747942
  1377287147
  5692368583
  8967592373
  2229882151
  2684867957
  Optimal assignment:
  0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0
  0\; 0\; 1\; 0\; 0\; 0\; 0\; 0\; 0\; 0
  0\; 0\; 0\; 0\; 0\; 0\; 0\; 1\; 0\; 0
  0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0
  0\; 1\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0
  0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0
  0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0
  0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0
  0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1
  10000000000
  Total cost: 19
```

## $N_{\overline{2}}1$

1.

,

```
5 4 4 6 0 7
               3
8
     5
  0
       1
                    8
          0
       5
         5
3
     4
            0
               1
                    5
  0
                 7
    4 4 6
            3
               0
6
  6
                    0
     3 4
            5
  5
         1
0
                    0
         6
     6 0
2
  6
            6
               0
                    6
       0
         3
               3
    0
            6
     5
       2
         6
               1
  4
            0
     8
       6
         6
            1
1
  1
               0
                 4
                    0
4 6 2 5 3 5
               7 3 5
```

2.

$$(1, 6).$$
 1 6 .  $(2; 6), (1; 1).$ 

10 (

7),

$$5,$$
  $3,$   $8,$   $5,$   $4,$   $1,$   $7,$   $9.$ 

```
6
            0
                 3
  5
    4
       4
               7
                    5
  8
       5
          1
               3
     0
            0
                 7
                    8
                       8
8
  3
       4
          5
            5
               0
                 1
     0
                    7
                       5
0
  6
       4 4
            6
               3
                 0
     6
                    0
                       0
6
3
  0
     5
       3 4
            1
               5
                 7
                    2
                      0
       6 0 6
              6 0
  2
                    3
    6
                      6
            3 6
                 3
  4
     7 0
         0
                    6
                       1
       5
          2
            6
                 1
                    5
  7
     4
               0
                       1
       8 6
     1
            6
               1
                 0
                    4
  1
                       0
       2 5 3 5 7 3 5
  4 6
```

 $(\min(2, 6, 6, 3, 6, 4, 7, 3, 6, 1, 7, 4, 6, 5, 1, 4, 6, 3,$ 

(3, 5) = 1

**5**  0 **8 4**  5  $\mathbf{2}$ 

$$3 \ 3 \ 6 \ 0 \ 0 \ 2 \ 6 \ 3 \ 5 \ 0$$

$$6 \ 6 \ 3 \ 5 \ 2 \ 5 \ 0 \ 1 \ 4 \ 0$$

$$\mathbf{2} \ 1 \ 1 \ \mathbf{9} \ \mathbf{7} \ 6 \ \mathbf{2} \ \mathbf{1} \ 4 \ 0$$

$$0\ \ 3\ \ 5\ \ 2\ \ 5\ \ 2\ \ 5\ \ 7\ \ 3\ \ 5$$

```
1.
```

```
[0]
                                            8 4 5
                         1 5
                               4
                                   5 7
                              [0]
                                   6
                                     2
                                        [0]
                                            4 8 8
                         9
                            8
                                                   8
                            3
                               [0]
                                   5 6
                                        5
                                            1 2 7 5
                         1
                                   5 5
                                        6
                                            4 1 0 0
                           6
                               6
                                  4 5
                                        1
                                            6 8 2 0
                         4
                               5
                           0
                                        5
                           1
                               5
                                  6 0
                                            6 0 2 5
                         0
                                        2
                         3 3
                              6
                                  0 0
                                            6 3 5 0
                                        5
                                  5 2
                           6
                              3
                         6
                                             1 4 0
                                            0
                                  9 7
                                        6
                                              1
                         2
                           1
                                            2
                                                4 0
                              1
                         0 3
                                   2 5
                                        2
                                            5 7 3 4
                               5
                                                             10
            2),
3.
                                                                 6,
                                                                        2,
                                                         10,
7,
                      1,
(
                              3,
                                   4,
                                             7.
               2,
       1,
                           1 5 4
                                  5
                                     7 0
                                          8
                                               5
                                            4
                                                  7
                             8
                               0
                                  6
                                     2
                                          4
                                       0
                                             8
                                               8
                                                  8
                             3 0
                                  5
                                    6
                                       5
                                            2
                                          1
                                               7 5
                                  5
                                     5
                                            1
                             6 6
                                       6
                                          4
                                               0 0
                            0 5
                                  4 5
                                       1 6 8
                                               2 0
                             1
                               5
                                  6
                                       5
                                          6 0
                                               2 5
                                     0
                                       2
                             3 6
                                  0
                                     0
                                          6 3
                                               5 0
                             6 3
                                     2
                                       5
                                            1
                                  5
                                         0
                                               4 0
                                  9
                                     7
                                       6
                                          2
                                            1
                                               4 0
                             1
                               1
                          0 3 5 2 5 2 5 7 2 4
                              (\min(5, 6, 5, 2, 7, 4, 5, 1, 8, 2, 5, 2, 5, 1, 4, 9, 7, 6,
1, 4, 2, 5, 2, 7, 2) = 1
```

,

 $N_{\overline{2}}3$ 

1.

```
10 (
```

1),

3.

: 5, 8, 7,

6, 8, 1, 3, 5, 4, 10.

2 **6 5 5** 7 0 **9** 4 **5** 8

10 **9 1 6** 2 0 **5** 8 **8** 9

 $1 \quad 3 \quad 0 \quad 4 \quad 5 \quad 4 \quad 1 \quad 1 \quad 6 \quad 5$ 

8 7 7 5 5 6 5 1 0 1

 $4 \quad 0 \quad 5 \quad 3 \quad 4 \quad 1 \quad 6 \quad 8 \quad 1 \quad 0$ 

1 **2 6 6** 0 **5 7** 0 **2** 6

 $4 \quad 4 \quad 7 \quad 0 \quad 0 \quad 2 \quad 7 \quad 3 \quad 5 \quad 1$ 

7 7 4 **5 2 5** 0 **1** 4 1

2 **1 1 9** 6 **6** 2 **1** 4 0

0 **3 5 2** 5 **2** 5 **7** 3 4

 $(\min(6, 5, 5, 9, 5, 9, 1, 6, 5, 8, 2, 6, 6, 7, 2, 1, 1, 8,$ 

2, 3, 3, 5, 1, 5, 1) = 1

2 **5 4 4** 7 0 **8** 4 **4** 8

10 **8 0 5** 2 0 **4** 8 **7** 9

1 3 0 4 5 4 1 1 6 5

 $8 \ \ 7 \ \ 7 \ \ 5 \ \ 5 \ \ 6 \ \ 5 \ \ 1 \ \ 0 \ \ 1$ 

 $4 \ \ 0 \ \ 5 \ \ 3 \ \ 4 \ \ 1 \ \ 6 \ \ 8 \ \ 1 \ \ 0$ 

1 **1 5 5** 0 **5 6** 0 **1** 6

 $4 \quad 4 \quad 7 \quad 0 \quad 0 \quad 2 \quad 7 \quad 3 \quad 5 \quad 1$ 

7 7 4 **4 1 4** 0 **0 3** 1

2 **0 0 8** 6 **5** 2 **0** 4 0

0 **2 4 1** 4 **1** 5 **6 0** 4

,

```
2 \ \ 5 \ \ 4 \ \ 4 \ \ 7 \ \ 0 \ \ 8 \ \ 4
      0 5
                      8
             2
                   4
   8
                0
                             9
         4
            6
               5 1
      0
                      2 6 6
   7 7 5 6
               7 5
                      2
                         0 2
      5
         3 5
                1 6 8
                         1 1
         5 0
                5 6
                      0
   4 7 0 1 3 7 4 5
   7 4 5 2 5 0
                      1 4
   0 0 8 6 5 1
                      0 4 0
0 \quad 2 \quad 4 \quad 1 \quad 4 \quad 1 \quad 5 \quad 6 \quad 1 \quad 4
```

**№**4

1.

•

2.

(1, 6). 1 6 (2, 6). (2, 3). 2

3 . (3; 3), (9; 3).

:

2	5	4	4	7	[0]	8	4	4	8
10	8	[0]	5	2	[-0-]	4	8	7	9
2	3	[-0-]	4	6	5	1	2	6	6
9	7	7	5	6	7	5	2	0	2
5	0	5	3	5	1	6	8	1	1
1	1	5	5	0	5	6	0	1	6
5	4	7	0	1	3	7	4	5	2
7	6	3	4	2	5	0	1	3	1
2	0	[-0-]	7	6	5	1	0	2	0
0	2	4	0	4	1	4	6	0	4

10

2),

**3.**9, 10, 3, 6, 6, 2, 4, 4, 8.

5	4	4	7	0	8	4	4	8
8	0	5	2	0	4	8	7	9
3	0	4	6	5	1	2	6	6
7	7	5	6	7	5	2	0	2
0	5	3	5	1	6	8	1	1
1	5	5	0	5	6	0	1	6
4	7	0	1	3	7	4	5	2
6	3	4	2	5	0	1	3	1
0	0	7	6	5	1	0	2	0
2	4	0	4	1	4	6	0	4
	8 3 7 0 1 4 6 0	8 0 3 0 7 7 0 5 1 5 4 7 6 3 0 0	8 0 5 3 0 4 7 7 5 0 5 3 1 5 5 4 7 0 6 3 4 0 0 7	8         0         5         2           3         0         4         6           7         7         5         6           0         5         3         5           1         5         5         0           4         7         0         1           6         3         4         2           0         0         7         6        0         0         7         6	8         0         5         2         0           3         0         4         6         5           7         7         5         6         7           0         5         3         5         1           1         5         5         0         5           4         7         0         1         3           6         3         4         2         5           0         0         7         6         5           0         0         7         6         5	8       0       5       2       0       4         3       0       4       6       5       1         7       7       5       6       7       5         0       5       3       5       1       6         1       5       5       0       5       6         4       7       0       1       3       7         6       3       4       2       5       0         0       0       7       6       5       1	8       0       5       2       0       4       8         3       0       4       6       5       1       2         7       7       5       6       7       5       2         0       5       3       5       1       6       8         1       5       5       0       5       6       0         4       7       0       1       3       7       4         6       3       4       2       5       0       1         0       0       7       6       5       1       0	8       0       5       2       0       4       8       7         3       0       4       6       5       1       2       6         7       7       5       6       7       5       2       0         0       5       3       5       1       6       8       1         1       5       5       0       5       6       0       1         4       7       0       1       3       7       4       5         6       3       4       2       5       0       1       3         0       0       7       6       5       1       0       2

1)

1	5	4	4	6	0	7	3	3	7
9	8	0	5	1	0	3	7	6	8
1	3	0	4	5	5	0	1	5	5
9	7	7	5	6	7	5	2	0	2
4	0	5	3	4	1	5	7	0	0
1	1	5	5	0	5	6	0	1	6
4	4	7	0	0	3	6	3	4	1
7	6	3	4	2	5	0	1	3	1
2	0	0	7	6	5	1	0	2	0
0	2	4	0	4	1	4	6	0	4

,

1	5	4	4	6	0	7	3	3	7
9	8	0	5	1	0	3	7	6	8
1	3	0	4	5	5	0	1	5	5
9	8	8	6	6	8	5	2	0	2
4	0	5	3	4	1	5	7	0	0
1	2	6	6	0	6	6	0	1	6
4	4	7	0	0	3	6	3	4	1
7	7	4	5	2	6	0	1	3	1
2	1	1	8	6	6	1	0	2	0
0	3	5	1	4	2	4	6	0	4

**№**5

1.

2.

1 6 (2, 3). (3; 3). (1, 6).(2; 6).2

3

1	5	4	4	6	[0]	7	3	3	7
9	8	[0]	5	1	[-0-]	3	7	6	8
1	3	[-0-]	4	5	5	[0]	1	5	5
9	8	8	6	6	8	5	2	[0]	2
4	[-0-]	5	3	4	1	5	7	[-0-]	[0]
1	2	6	6	[-0-]	6	6	[0]	1	6
4	4	7	[-0-]	[0]	3	6	3	4	1
7	7	4	5	2	6	[-0-]	1	3	1
2	1	1	8	6	6	1	[-0-]	2	[-0-]
0	3	5	1	4	2	4	6	[_0_]	4

10

**3.** 5, 3, 6, 6, 7, 7, 9, 9, 10. ( ):

1	5	4	4	6	0	7	3	3	7
9	8	0	5	1	0	3	7	6	8
1	3	0	4	5	5	0	1	5	5
9	8	8	6	6	8	5	2	0	2
4	0	5	3	4	1	5	7	0	0
1	2	6	6	0	6	6	0	1	6
4	4	7	0	0	3	6	3	4	1
7	7	4	5	2	6	0	1	3	1
2	1	1	8	6	6	1	0	2	0
0	3	5	1	4	2	4	6	0	4

1)

,

0	4	4	3	5	0	7	2	3	6
8	7	0	4	0	0	3	6	6	7
0	2	0	3	4	5	0	0	5	4
8	7	8	5	5	8	5	1	0	1
4	0	6	3	4	2	6	7	1	0
1	2	7	6	0	7	7	0	2	6
4	4	8	0	0	4	7	3	5	1
6	6	4	4	1	6	0	0	3	0
2	1	2	8	6	7	2	0	3	0
0	3	6	1	4	3	5	6	1	4

**№**6

1.

.

2.

(1, 6). 1 6 (2; 6), (1; 1).

,,,

$\boxed{[-0-]}$	4	4	3	5	[0]	7	2	3	6
8	7	[0]	4	[-0-]	[-0-]	3	6	6	7
$\boxed{[-0-]}$	2	[-0-]	3	4	5	[0]	[-0-]	5	4
8	7	8	5	5	8	5	1	[0]	1
4	[0]	6	3	4	2	6	7	1	[-0-]
1	2	7	6	[0]	7	7	[-0-]	2	6
4	4	8	[0]	[-0-]	4	7	3	5	1
6	6	4	4	1	6	[0]	[-0-]	3	[-0-]
2	1	2	8	6	7	2	[-0-]	3	[0]
[0]	3	6	1	4	3	5	6	1	4

k = 10.

 $C_e$ :

0	4	4	3	5	0	7	2	3	6
8	7	0	4	0	0	3	6	6	7
0	2	0	3	4	5	0	0	5	4
8	7	8	5	5	8	5	1	0	1
4	0	6	3	4	2	6	7	1	0
1	2	7	6	0	7	7	0	2	6
4	4	8	0	0	4	7	3	5	1
6	6	4	4	1	6	0	0	3	0
2	1	2	8	6	7	2	0	3	0
0	3	6	1	4	3	5	6	1	4

4. X,

$\boxed{[-0-]}$	4	4	3	5	[0]	7	2	3	6
8	7	[0]	4	[-0-]	[-0-]	3	6	6	7
$\boxed{[-0-]}$	2	[-0-]	3	4	5	[-0-]	[0]	5	4
8	7	8	5	5	8	5	1	[0]	1
4	[0]	6	3	4	2	6	7	1	[-0-]
1	2	7	6	[0]	7	7	[-0-]	2	6
4	4	8	[0]	[-0-]	4	7	3	5	1
6	6	4	4	1	6	[0]	[-0-]	3	[-0-]
2	1	2	8	6	7	2	[-0-]	3	[0]
[0]	3	6	1	4	3	5	6	1	4

:

$$C_{\min} = 2 + 2 + 2 + 2 + 2 + 2 + 1 + 2 + 2 + 1 + 3 = 19$$

(4;9), (5;2), (7;4), (1;6), (10;1), (2;3), (6;5), (8;7), (9;10), (3;8) (9;10), (10;1), (1;6), (6;5), (5;2), (2;3), (3;8), (8;7), (7;4)

 $X_0 = (1, 2); (2, 3); (3, 4); (4, 5); (5, 6); (6, 7); (7, 8); (8, 9); (9, 10); (10, 1)$  $F(X_0) = 34 + 56 + 91 + 83 + 51 + 77 + 26 + 44 + 87 + 58 = 607.$ 

D

$$d_i = \min_j d_{ij}$$

i j	1	2	3	4	5	6	7	8	9	10	$\mathbf{d}_i$
1	M	34	68	18	63	80	12	44	58	87	12
2	56	M	56	94	62	65	18	38	67	22	18
3	34	53	M	91	13	73	70	51	13	37	13
4	28	19	14	M	83	89	25	9	89	22	9
5	67	13	1	19	M	51	7	13	31	4	1
6	3	78	24	90	14	M	77	6	35	69	3
7	96	32	100	4	8	19	M	26	37	36	4
8	79	2	48	25	63	99	17	M	44	45	2
9	97	49	33	74	23	72	23	73	M	87	23
10	58	83	24	39	17	76	64	78	100	M	17

 $d_i$ 

i j	1	2	3	4	5	6	7	8	9	10
1	M	22	56	6	51	68	0	32	46	75
2	38	M	38	76	44	47	0	20	49	4
3	21	40	M	78	0	60	57	38	0	24
4	19	10	5	M	74	80	16	0	80	13
5	66	12	0	18	M	50	6	12	30	3
6	0	75	21	87	11	M	74	3	32	66
7	92	28	96	0	4	15	M	22	33	32
8	77	0	46	23	61	97	15	M	42	43
9	74	26	10	51	0	49	0	50	M	64
10	41	66	7	22	0	59	47	61	83	M

$$d_j = \min_i d_{ij}$$

i j	1	2	3	4	5	6	7	8	9	10
1	M	22	56	6	51	68	0	32	46	75
2	38	M	38	76	44	47	0	20	49	4
3	21	40	M	78	0	60	57	38	0	24
4	19	10	5	M	74	80	16	0	80	13
5	66	12	0	18	M	50	6	12	30	3
6	0	75	21	87	11	M	74	3	32	66
7	92	28	96	0	4	15	M	22	33	32
8	77	0	46	23	61	97	15	M	42	43
9	74	26	10	51	0	49	0	50	M	64
10	41	66	7	22	0	59	47	61	83	M
$\mathbf{d}_{j}$	0	0	0	0	0	15	0	0	0	3

 $d_i d_j$ 

i j	1	2	3	4	5	6	7	8	9	10
1	M	22	56	6	51	53	0	32	46	72
2	38	M	38	76	44	32	0	20	49	1
3	21	40	M	78	0	45	57	38	0	21
4	19	10	5	M	74	65	16	0	80	10
5	66	12	0	18	M	35	6	12	30	0
6	0	75	21	87	11	M	74	3	32	63
7	92	28	96	0	4	0	M	22	33	29
8	77	0	46	23	61	82	15	M	42	40
9	74	26	10	51	0	34	0	50	M	61
10	41	66	7	22	0	44	47	61	83	M

H:

$$H = \sum d_i + \sum d_j$$

**№**1.

$$\begin{array}{ccc} & (i,j) & (i^*,j^*). \\ M \ ( & & ) \end{array}$$

i j	1	2	3	4	5	6	7	8	9	10	$ \mathbf{d}_i $
1	M	22	56	6	51	53	0(6)	32	46	72	6
2	38	M	38	76	44	32	0(1)	20	49	1	1
3	21	40	M	78	0(0)	45	57	38	0(30)	21	0
4	19	10	5	M	74	65	16	0(8)	80	10	5
5	66	12	0(5)	18	M	35	6	12	30	0(1)	0
6	0(22)	75	21	87	11	M	74	3	32	63	3
7	92	28	96	0(6)	4	0(32)	M	22	33	29	0
8	77	0(25)	46	23	61	82	15	M	42	40	15
9	74	26	10	51	0(0)	34	0(0)	50	M	61	0
10	41	66	7	22	0(7)	44	47	61	83	M	7
$\mathbf{d}_{j}$	19	10	5	6	0	32	0	3	30	1	0

$$d(1,7) = 6+0 = 6;$$
  $d(2,7) = 1+0 = 1;$   $d(3,5) = 0+0 = 0;$   $d(3,9) = 0+30 = 30;$   $d(4,8) = 5+3 = 8;$   $d(5,3) = 0+5 = 5;$   $d(5,10) = 0+1 = 1;$   $d(6,1) = 3+19 = 30;$   $d(7,4) = 0+6 = 6;$   $d(7,6) = 0+32 = 32;$   $d(8,2) = 15+10 = 25;$   $d(9,5) = 0+0;$   $d(9,7) = 0+0 = 0;$   $d(10,5) = 7+0 = 7;$   $(0+32) = 32$   $(7,6),$   $(7,6)$   $(7^*,6^*).$ 

$$d_{76} = 0 M, (7^*, 6^*),$$

i j	1	2	3	4	5	6	7	8	9	<b>10</b>	$\mid \mathbf{d}_i \mid$
1	M	22	56	6	51	53	0	32	46	72	0
2	38	M	38	76	44	32	0	20	49	1	0
3	21	40	M	78	0	45	57	38	0	21	0
4	19	10	5	M	74	65	16	0	80	10	0
5	66	12	0	18	M	35	6	12	30	0	0
6	0	75	21	87	11	M	74	3	32	63	0
7	92	28	96	0	4	M	M	22	33	29	0
8	77	0	46	23	61	82	15	M	42	40	0
9	74	26	10	51	0	34	0	50	M	61	0
10	41	66	7	22	0	44	47	61	83	М	0
$\mathbf{d}_{j}$	0	0	0	0	0	32	0	0	0	0	32

$$H(7^*, 6^*) = 120 + 32 = 152$$

(7,6) 7-

 $d_{67}$  M,  $(9 \times 9)$ ,

i j  $\mathbf{5}$  $\mathbf{d}_i$ M M $\mathbf{2}$ М M M  $\mathbf{5}$ M M M M  $\overline{\mathbf{d}_{i}}$ 

:

$$\sum d_i + \sum d_j = 6$$

(7,6) :

$$H(7,6) = 120 + 6 = 126 \le 152$$

(7,6) ,  $(7^*,6^*)$ , (7,6)

H = 126.