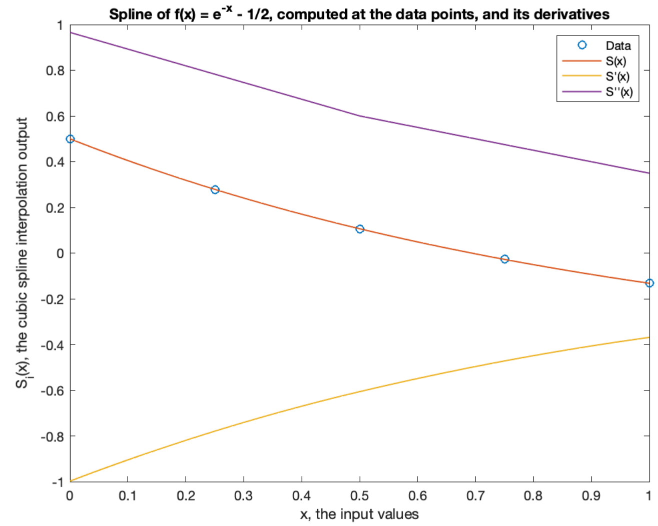
Part A

The function $f(x) = e^{-x} - \frac{1}{2}$, evaluated at $x = 0, 0.25, 0.5, 0.75, 1$, is given by the circular data points. From this data, a cubic spline with not-a-knot end conditions was constructed, and its first and second derivatives were computed. Each is shown for reference.

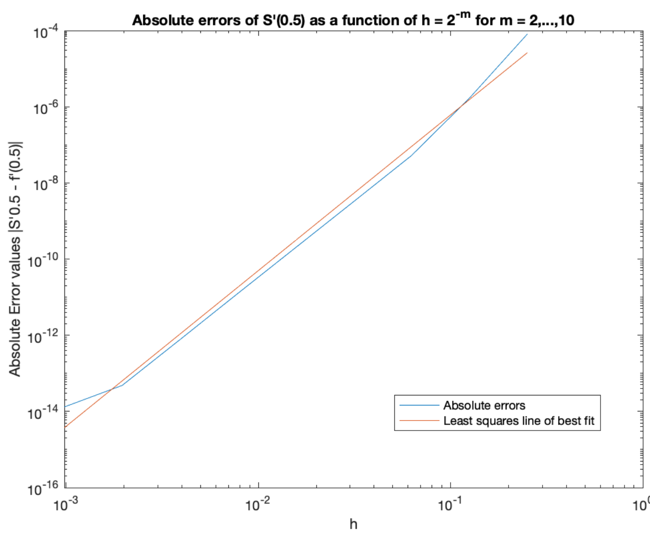


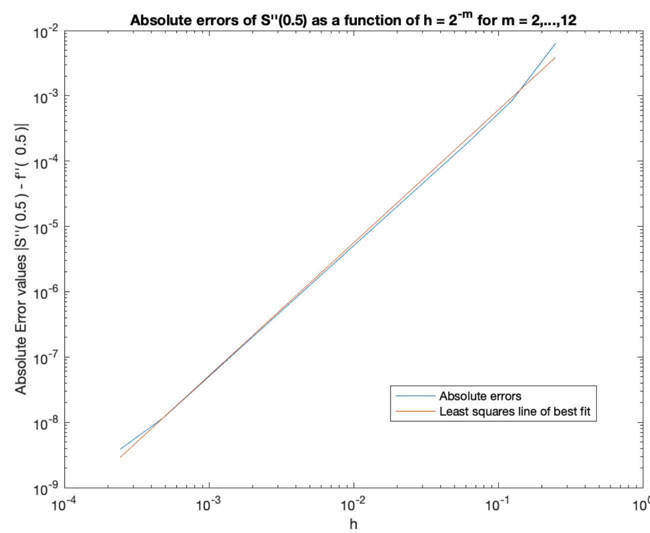
Since the spline is cubic, it has the form $S_i(x) = a_i + b_i x + c_i x^2 + d_i x^3$. The coefficients for the spline S are as follows:

-0.1218	0.4828	-0.9979	0.5000
-0.1218	0.3914	-0.7793	0.2788
-0.0835	0.3001	-0.6065	0.1065
-0.0835	0.2374	-0.4721	-0.0276

From which the derivatives are simple to construct. The approximations for $f'(0.5)$ and $f''(0.5)$ are $S'(0.5) \approx -0.6065$, $S''(0.5) \approx 0.6001$, and their absolute errors are $|S'(0.5) - f'(0.5)| \approx$ 7.9565e-05 and $|S''(0.5) - f''(0.5)| \approx 0.0064$.

Part B





We repeat Part A over the interval [0,1] with equal node spacings $h = 2^{-m}, m = 2, 3, ..., 10$ for the LHS plot and $m = 2, 3, ..., 12$ for the RHS plot. We limit m since higher values introduce erratic behaviour. Since $error = c \cdot h^p$, solving for p will allow us to describe the error in our approximations in O notation. We linearize the data by plotting $\log(error) = \log(ch^p)$, for each $h = 2^{-m}$, for each of $S'(0.5)$ and $S''(0.5)$. We then approximate these lines by applying a least squares regression. Recall that these lines have the form $\log(error) = \log(c \cdot h^p) \Rightarrow \log(error) = \log(c) + p\log(h)$. Solving for the slope, p, allows us to describe the error, as desired:

The error in the approximations for $f'(0.5) \approx S'(0.5)$ as a function of h are $O(h^{4.087})$, thus $p \approx 4.087$.
The error in the approximations for $f''(0.5) \approx S''(0.5)$ as a function of h are $O(h^{2.035})$, thus $p \approx 2.035$.