1 Part A

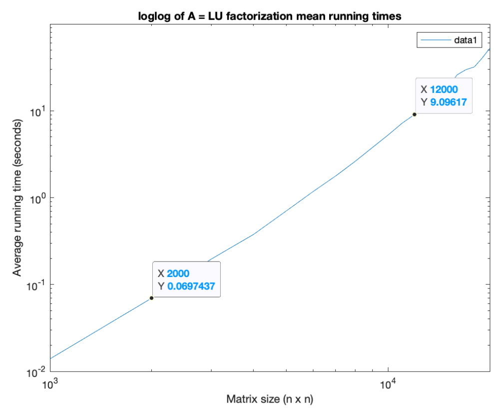
The following graph shows the logarithmically linearized relationship between the average computing time that Matlab's LU factorization algorithm requires to factor a matrix of size n into L and U. Values of n range from $[10^3, 2 \times 10^4]$ in increments of 10^3. Average run times are averaged over 10 $A = LU$ factorizations. The cost of creating the matrices was not considered.

The relationship is exponential, so taking the log of the relationship has the effect of forming a line for easier analysis. Solving for m will give us an appropriate upper bound on the growth rate of the average time since

$$\log(avg\ time) = m\log(n) + C$$

$$\Rightarrow \quad 2^{\log(avg\ time)} = 2^{\log(n^m)+C}$$

$$\Rightarrow \quad avg\ time = n^m 2^c$$



So, $slope = m = \frac{\triangle y}{\triangle x} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\log(12\times10^3)-\log(2\times10^3)}{\log(9.09617)-\log(6.97437\times10^{-2})} \approx 2.7184$

Hence, the growth of the average run time for $A = LU$ factorization grows in the order of $O(n^{2.7184})$.

2 Part B

Here we see the value of utilizing tridiagonal matrices for LU factorization. Values of n range from $[10^3, 10^5]$ in increments of 10^3 and the number of factorizations averaged over remains 10. Growth in average computing time is directly proportional to n as n increases. Hence, the growth of the average run time for $A = LU$ factorization when utilizing tridiagonal matrices in Matlab is $O(n)$. A line of best fit gives us a coefficient for the slope, $m = 1.879e - 07$.