Section 1 – Multivariate Statistics

Question 1

- Please refer to the coding section at GitHub

Question 2

- Please refer to the coding section at GitHub
- After loading input data for SP500 and FX time series, I combined/merged them based on the common column 'Date', and then sorted by ascending order.
- Then my script calculates the mean and standard deviation for both FX and SP500 values.
- Then normalize FX and SP500 to N(0,1) distribution.
- Create a new time series, Z_norm, to with correlation of 0.5. To be specific, Z is a linear combination of the normalized SP500 and FX data, then scaled by 0.5 to achieve the desired correlation.
- Rescaling to original scale, Z. Lastly, I scaled the Z_norm back to the original data scale by multiplying std_z and adding mean_z and.
- Now the time series Z, will have correlation of 0.5 with the FX series.

Question 3

- Please refer to the coding section at GitHub

Section 2 – Investment /Total Fund

Question 4:

- **Execution**. Below are the various approaches that we can allocate the S&P500 exposure
  1) Invest in **equity index futures**, i.e. long S&P500 Index future with notional amount of about $USD 726.1 million, [CAD 1billion/1.3772, where 1.3772 is the USD/CAD FX rate as of April 12]
  2) Invest in **ETFs (Exchange-Traded-Funds**). We could purchase S&P500 ETFs with value worth of CAD $1billion
  3) Invest in **pooled funds**. We could allocate the $1billion capital to our equity External managers which can help invest S&P500.
  4) **Other derivatives such as equity options and equity total return swaps** can also be the approach, but those exposure won't be linear products and the risk is typically higher than above three approaches.

- Implication to total fund besides equity asset class.
  1) **Asset class exposure/weight** will be changed. OTPP is a global investor and has exposure across various asset classes, and from total fund allocation perspective, each asset class will have an target weight. Through adding the $1billion on S&P500, the equity asset class weight will for sure increase, **we will need to make sure this won't breach the equity exposure limit**.

2) **Impact on risk**
    a. **Total fund level asset risk**. The S&P allocation will **impact the correlation with other assets classes** including the fixed income and alternative assets (Real Estate, Infrastructure, Private Equity). As a result, the total fund asset risk will change and as Total Risk team, we need to make sure this new allocation will not breach any Total Fund risk limit.
    b. **Liquidity risk**. Above four approaches have different impact to liquidity risk.
        i. The first approach **with S&P Futures is the ideal solution to me from the liquidity perspective**. As we only need to post the margin requirement rather than the full capital of 1 billion, and this can free up some liquidity for the fund.
        ii. ETFs and Pooled fund are both cash products which have higher liquidity requirement.
    c. **Stress test results**. We should also evaluate the impact to the stress test such as the 2008 GFC scenario to make sure the Fund performance is still financially acceptable.

Section 3 – Data

Question 5:

Please refer to the coding part for answer. For solving the missing values, I demonstrated both forward fill and linear interpolation method.

Question 6

I would use linear regression to estimate the missing DV01 Convexity, through modelling the relationship between [DV01 Convexity] and [MV, and DV01].

To validate my method, I also predicted the DV01 Convexity for those four dates with good DV01 data, and it confirmed that my method fit well with the data.

Please refer to the coding section to see the result.

Question 7

Data normalization in database design is a method used to organize data in a database efficiently. This method reduces redundancy (i.e., repetition of data) and avoids undesirable features like data anomalies (unexpected errors that occur when adding, updating, or deleting data).

Normalization is useful because:

- It reduces data duplication, which saves storage and makes data easier to manage.
- It helps avoid data errors particularly when you update, add, or delete data.
- It makes the database more organized by clearly defining relationships between different parts of the data.

When designing a new database, data normalization can set up database efficiently. When updating an existing database, data normalization improves its structure and how it handles data.

Employee Data Table before Normalization:

EmployeeID	EmployeeName	Title	DepartmentID	AnnualSalary	StartDate
1001	James Charles	Analyst	Risk	80000	2023-01-02
1002	Tom Steven	Analyst	Finance	100000	2024-04-01

Table after Normalization:

The normalization process firstly eliminates redundancy by separating department data into its own table; additionally, enhances data integrity by isolating employee-specific data from department data.

Employee Table

EmployeeID	EmployeeName	Title	DepartmentID	AnnualSalary	StartDate
1001	James Charles	Analyst	D001	80000	2023-01-02

1002	Tom Steven	Analyst	D002	100000	2024-04-01

Department Table:

DepartmentID	DepartmentName
D001	Risk
D002	Finance

Section 4

Question 8

- As a Canadian investor, any non-CAD asset/exposure will be considered as currency exposure. In the example of this portfolio, other than S&P/TSX and CAD 10Y, allocation on all the rest assets including S&P500, S&P GSCI Index, Gold, and US 10Y Treasuries will be included in the currency exposure.
- At any day, the summation of 1-day PnL of all holdings in the portfolio will serve as the portfolio level PnL. Repetitively doing this throughout the history, we will get the portfolio level 1-day PnL. Then find the X% percentile value from the time series, that value is the X% confident level 1-Day VaR fo the portfolio
- I wasn't able to find the data for US 10Y Treasuries and CAD 10Y Treasuries given the time constraint, but the same theory applies even using the rest of assets.
- Please refer to tab 'Q8 rtn' in the Q8 Data excel file attached for the calculation.

Question 9

- There are two ways I can think of to calculate the 10-day VaR
    - One is the extend the return horizon from 1-day to 10-day and then redo the calculation to get the 10-day VaR
    - Second approach is through linear scaling method through square root of time. To be more specific

$$VaR_{10day} = VaR_{1day} * sqrt(10)$$

Question 10

- Since this is an estimate, the linear scaling method is more straightforward

$$VaR_{1year} = VaR_{3month} * sqrt\left(\frac{12}{3}\right) = 3.6 * 2 = \$7.2\ billion$$

- The 1-year VaR estimate is at $7.2 Billion

Question 11

The value of a call option for a non-dividend-paying underlying stock in terms of the Black–Scholes parameters is:

$$C(S_t, t) = N(d_+)S_t - N(d_-)Ke^{-r(T-t)}$$
$$d_+ = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right]$$
$$d_- = d_+ - \sigma\sqrt{T-t}$$

Based on above Black-Schole option pricing formula, when Vol goes to infinite, d+ also goes to infinite and d- goes to negative infinite. Thus, N(d+) goes to 1 and N(d-) goes to 0.

Therefore, a Call option is equal to S_t when with infinity volatility.

Question 12

Please refer to code for the process. Basically to first calculate the SP500 daily return, then get the daily volatility from the daily return, lastly annualize the daily vol through sqrt(252) to get annual vol

Question 13

CDS Spread=PD×(1−Recovery Rate)

Thus PD = 0.0050/（1-0.5）=0.01

Question 14

- If the price of SPY increases, the absolute value of the put's delta will decrease.
- Put option on SPY can be used as a downside risk insurance, particularly during periods of high market volatility or bear market conditions.
- Use Calendar Spreads, selling a short-term put and buying a longer-term put at the same strike price, the premium from the short-term put offsets part of the cost of the long-term put. This strategy is useful when expecting volatility to increase in the future but not immediately.

Question 15

I think the 5.8B risk off is overestimated. Because net MV impact of the two transaction is zero on day 1. Then normally equity would be more volatile than credit, thus a lower risk estimation seems incorrect to me.

I will rerun the simulation based on the new allocation (total fund, less credit bond and more equity) to validate to see if the impact is true or not.