DIAGNOSTIC TEST AB

Directions: Solve the following problems, using available space for scratchwork. After examining the form of the choices, decide which one is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

Note: For the actual test, no scrap paper is provided.

In this test:

(1) The domain of a function f is the set of all real numbers x for which $f(x)$ is a real number, unless otherwise specified.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. Evaluate $\lim\limits_{h\to\frac{1}{2}} \dfrac{e^h - \sqrt{e}}{h - \frac{1}{2}}$.

   (A) e^2
   (B) $e^2 - 1$
   (C) $\sqrt{e}$
   (D) e
   (E) The limit does not exist.

2. $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\cos x \; dx =$

   (A) 0
   (B) 1
   (C) $\sqrt{\pi}$
   (D) $\dfrac{2}{3}$
   (E) $\dfrac{2\pi}{3}$

3. The graph of $y = x^3 + 21x^2 - x + 1$ is concave down for

   (A) $x < -7$
   (B) $-7 < x < 7$
   (C) all x
   (D) $x > 7$
   (E) $x < -7$ and $x > 7$

4. If $F(x) = \dfrac{x^3 + x^5}{\ln(x^2)}$, then $F'(\sqrt{e}) =$

   (A) $e^2(3e + 1)$
   (B) $e(3e + 1)$
   (C) $2e$
   (D) $\sqrt{e}(e + 1)$
   (E) $3e^2 - 1$

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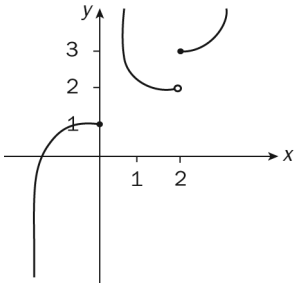
5. What are the values for which the function $f(x)=\frac{2}{3}x^3-x^2-4x+3$ is increasing?
   (A) $-1<x<2$
   (B) $x<-1$
   (C) $x<-1$ and $x>2$
   (D) $0<x<2$
   (E) $x>-1$

6. If $f(x)=\cos^2(\sin(2x))$, then $f'\left(\frac{\pi}{8}\right)=$
   (A) $1+\sqrt{2}\cos\left(\frac{\sqrt{2}}{2}\right)$
   (B) $\sqrt{2}\sin\left(\frac{\sqrt{2}}{2}\right)$
   (C) $\cos\left(\sin\left(\frac{\sqrt{2}}{2}\right)\right)$
   (D) $-2\sqrt{2}\cos\left(\frac{\sqrt{2}}{2}\right)\sin\left(\frac{\sqrt{2}}{2}\right)$
   (E) $-\sqrt{2}\cos\left(\frac{\sqrt{2}}{2}\right)$

7. The graph of f is given below. Which of the following statements is true about f?



   (A) $\lim_{x\to 0} f(x)=1$
   (B) $\lim_{x\to 0^-} f(x)=2$
   (C) $\lim_{x\to 2^+} f(x)=3$
   (D) $\lim_{x\to 2} f(x)=2$
   (E) $\lim_{x\to 2^-} f(x)=$ undefined

8. Evaluate $\lim_{x\to 0}\frac{\sin^2(4x)}{x^2}$.
   (A) $\frac{1}{4}$
   (B) 0
   (C) 16
   (D) 4
   (E) The limit does not exist.

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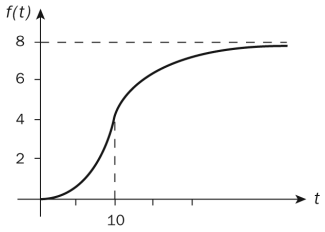
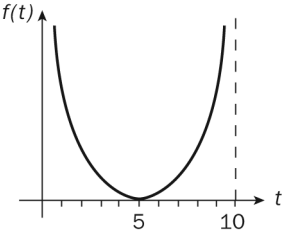


9. The graph of f is given above. Which graph below could represent the graph of f'?

(A)


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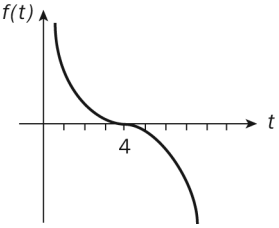
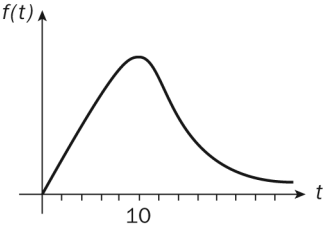
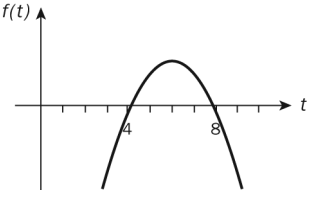
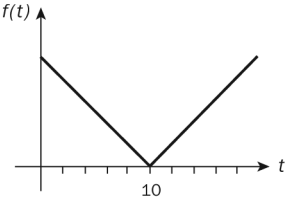


(C)


(D)


(E)


10. Evaluate $\lim_{x\to\infty} \dfrac{x^3+3x^5+x}{2x^5+3x^2+5}$.

(A) 0
(B) $-\dfrac{4}{3}$
(C) $\dfrac{2}{3}$
(D) $\dfrac{3}{2}$
(E) The limit does not exist.

11. Evaluate $\lim_{x\to 1} \dfrac{2x^2+x-3}{1-x^2}$.

(A) $-\dfrac{5}{2}$
(B) 0
(C) $\dfrac{4}{3}$
(D) $-\dfrac{3}{2}$
(E) The limit does not exist.

12. The solution to the differential equation $\dfrac{dy}{dx}=\dfrac{x^2}{y^3}$, where $y(3)=3$ is

(A) $y=\sqrt[4]{\frac{3}{4}x^3-45}$
(B) $y=\sqrt[4]{\frac{4}{3}x^3+45}$
(C) $y=\sqrt[4]{\frac{4}{3}x^4+5}$
(D) $y=\sqrt[4]{\frac{4}{3}x^4}+\sqrt[4]{45}$
(E) $y=\sqrt[4]{\frac{4}{3}x^4+45}$

13. The slope of the tangent to the curve $2x^3y^2-5x^2y=18$ at the point $(1,2)$ is

(A) $-\dfrac{4}{5}$
(B) $-\dfrac{3}{2}$
(C) 0
(D) $-\dfrac{4}{3}$
(E) -1

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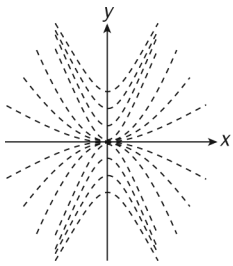
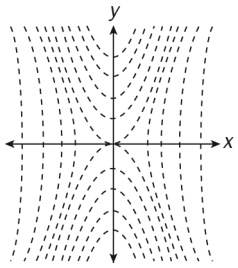
14. Which of the following is a slope field for the differential equation $\frac{dy}{dx}=\frac{2x^3}{y}$?

(A)


(B)


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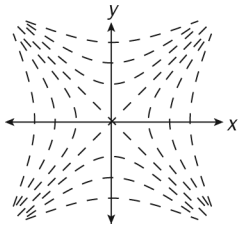
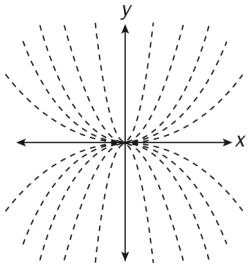
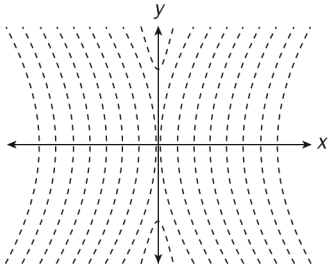


(D)


(E)


15. The area of the region enclosed by the graph of $y=2x^2+1$ and the line $y=2x+5$ is

(A) 5
(B) 3
(C) $\frac{7}{2}$
(D) 9
(E) 1

16. At what point on the graph of $y=2x^{\frac{3}{2}}$ is the tangent line perpendicular to the line $2x+3y=6$?

(A) (0, 0)
(B) $\left(\frac{1}{16},\frac{1}{32}\right)$
(C) $\left(\frac{3}{2},\frac{3\sqrt{6}}{2}\right)$
(D) $\left(\frac{1}{4},\frac{1}{4}\right)$
(E) $\left(\frac{3}{16},\frac{3\sqrt{3}}{32}\right)$

17. If the region enclosed by the x-axis, the line $x=1$, the line $x=3$, and the curve $y=x^{\frac{3}{2}}$ is revolved around the x-axis, the volume of the solid is

(A) 20π
(B) $\frac{25}{2}\pi$
(C) 21π
(D) 4ω
(E) $\frac{100}{3}\pi$

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18. An equation of the line tangent to the graph of $y = 2\sin\left(2x + \frac{3\pi}{4}\right)$ at $x = \frac{\pi}{8}$ is

(A) $y - \frac{\pi}{2} = -4x$

(B) $y + \frac{\pi}{4} = \frac{3}{4}x$

(C) $y = 4x + \frac{\pi}{2}$

(D) $y + \frac{\pi}{2} = 4x$

(E) $2y = -4x + \pi$

19. Find the derivative of $f(x) = \int_0^{x^2} \cos(t^2)\,dt$.

(A) $\cos(x^4)$

(B) $\cos(x^2)$

(C) $2x\cos(x^2)$

(D) $x\cos(x^4)$

(E) $2x\cos(x^4)$

20. Suppose that f is a continuous function and differentiable everywhere. Suppose also that $f(0) = 1$, $f(5) = -4$, $f(-5) = -3$. Which of the following statements must be true about f?

I. f has exactly two zeros.

II. f has at least two zeros.

III. f must have a zero between 0 and −5.

IV. There is not enough information to determine anything about the zeros of f.

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) IV

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Free-Response Question

Directions: Solve the following problem, using available space for scratchwork. Show how you arrived at your answer.

- You should write out all your work for each part. On the actual test, you will do this in the space provided in the test booklet. Be sure to write clearly and legibly. If you make a mistake, you can save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. Clearly label any functions, graphs, tables, or other objects that you use. On the actual exam, you will be graded on the correctness and completeness of your methods as well as your answers. Answers without any supporting work may not receive credit.
- Justifications (i.e., the request that you "justify your answer") require that you give mathematical (non-calculator) reasons.
- Work must be expressed in standard mathematical notation, not calculator syntax.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in calculations, the readers of the actual exam will grade you on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of function f is the set of all real numbers x for which $f(x)$ is a real number.

21. A particle moves along the x-axis so that its velocity at any time $t > 0$ is given by $v(t) = 5t^2 - 4t + 7$. The position of the particle, $x(t)$, is 8 for $t = 3$.
    (a) Write a polynomial for the position of the particle at any time $t \geq 0$.
    (b) Find the total distance traveled by the particle from time $t = 0$ until time $t = 2$.
    (c) Does the particle achieve a minimum velocity? And if so what is the position of the particle at this time?

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IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT TURN TO ANY OTHER SECTION IN THE TEST.

STOP