1. Evaluate the following integrals.

(7) a) $\int x^2 e^{-x}\, dx$

(8) b) $\int \frac{\sqrt{x^2-1}}{x}\, dx$ (Hint: trig. substitution)

(8) c) $\int \frac{x^3-2x+1}{x^3-x^2-2x}\, dx$ (Hint: partial fractions)

(8) d) $\int \sec^5 x \tan^5 x \ dx$

(8) e) $\int e^{-x} \cos 2x \ dx$

(8) 2. Solve the initial value problem $(1+x^2)\dfrac{dy}{dx} - e^y = 2xe^y\,,\ y(0) = -2\,$, for $y\,$.

(5) 3a) Find the length of the curve

$$y = 2x^{3/2} + 1, \qquad 0 \le x \le 1.$$

(4) b) Set up **but do not evaluate** an integral representing the area of the surface obtained by revolving the curve in (a) about the (i) x-axis, (ii) $x = -1$.

(5) c) Set up **but do not evaluate** an integral representing the volume of the solid obtained by revolving the region under the curve in (a) and above the x-axis about the line (i) $x = -1$, (ii) $y = 3$.

4. Evaluate the following limits. Show all work.

(7) a) $\displaystyle\lim_{x\to 0} \frac{\ln(1+3x) - 3x}{x^3 + 3x^2}$

(7) b) $\displaystyle\lim_{x\to\infty} (x + e^x)^{3/x}$

5. Decide whether the series converges or diverges. For convergent series find the sum.

(6) a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n2^n}$

(7) b) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{2n-1}}{2^n 5^{n+1}}$

(4) c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2^{3n-2}}{e^{n-3}}$

6. Determine whether the series diverges, converges conditionally or converges absolutely.

(7) a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n n e^{-n}$

(7) b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{\ln n}}$

7. Determine whether the following series converge or diverge. State which test you are using and implement the test as clearly as you can (5 of the 7 points are for the work!).

(7) a) $\sum_{n=0}^{\infty}(-1)^n \frac{\sqrt{n^2+5}}{n^4+1}$

(7) b) $\sum_{n=0}^{\infty} e^{1/n}$

(7) c) $\sum_{n=3}^{\infty} \frac{(n!)^2}{(2n)!}$

(7) d) $\sum_{n=1}^{\infty}(-1)^n \frac{n^{2n}}{2^n(n+1)^n}$

(12) 8. Find the interval of convergence for the power series. Make clear the status of any end points.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{\sqrt{n}3^n}$.

(ii) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x-1)^n}{n!}$.

(iii) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x+1)^n}{n^2}$.

(10) 9. Find the first **four** non-zero terms of the Taylor series for $f(x) = 4\sqrt{x}$ centered at $a = 4$.

(8) 10. a) Find the Maclaurin series for $\dfrac{1}{1+4x^2} - \dfrac{3}{1-2x^2}$

b) Evaluate the integral as an infinite series: $\displaystyle\int_0^{\frac{\pi}{4}} \cos(x^2)\,dx$.

(10) 11. a) Use known series (binomial series, geometric series, series given on the cover sheet etc) to find the first four nonzero terms of the Maclaurin series for

(i) $f(x) = e^{-x}\ln(1+x)$.

(ii) $g(x) = \dfrac{\sqrt[3]{1+3x}}{2-e^x}$.

(5) 12. a) Sketch the curve with parametric equations $x = 2 + \sin^2 t$, $y = \cos t$ for $0 \leq t \leq \pi$ (indicate the direction with an arrow).

(5) b) Set up (but do not evaluate) an integral representing the area of the surface generated revolving this curve about the y-axis.

(5) 13. a) Sketch the polar curve $r = 2\cos\theta - 1$, $0 \leq \theta \leq 2\pi$.

(6) b) Find the tangent to the curve at the point $\theta = \pi/2$.

(5) c) Set up but do not evaluate an integral representing the area of the inner loop.