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Machine Learning
HW 2

1. Design Matrix $X = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ target vector $= \begin{bmatrix} 1 \\ 4 \\ 3 \\ 7 \end{bmatrix}$

Closed form solution $\rightarrow w = (X^T X)^{-1} X^T y$

$$w = \left(\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 3/4 \\ 5/2 \\ 7/2 \end{bmatrix} \rightarrow \begin{matrix} w_0 = 3/4 \\ w_1 = 5/2 \\ w_2 = 7/2 \end{matrix}$$

$$RSS = \sum_{i=1}^n (y^{(i)} - \hat{y}^{(i)})^2 = (1 - 3/4)^2 + (4 - 4 1/4)^2 + (3 - 3 1/4)^2 + (7 - 6 3/4)^2 = 1/4$$

$$TSS = \sum_{i=1}^n (y^{(i)} - \bar{y})^2 = (1 - 3 3/4)^2 + (4 - 3 3/4)^2 + (3 - 3 3/4)^2 + (7 - 3 3/4)^2 = 18 3/4$$

$$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{1/4}{18 3/4} = 74/75$$

74/75 of the variance in y is explained by x .

predict the value of $x = (0.5, 0.5)$

$$y = \frac{3}{4} + \frac{5}{2}(0.5) + \frac{7}{2}(0.5)$$

$$y = \frac{3}{4} + \frac{5}{4} + \frac{7}{4}$$

$$y = \frac{15}{4} = 3 \frac{3}{4}$$

2. Code is in jupyter notebook

$$RSS = 14517.552$$

$$TSS = 42716.295$$

$$R^2 = 0.660$$

0.66 of the variance in y is explained by x .

3. You can use the four features as parameters in x . So, the amount of rainfall, amount of fertilizer, average temp, and the number of sunny days will all be in a design matrix with an additional column of ones on the left. The target vector is the crop yields.

4. Code is in jupyter notebook

f_1

steps	w_1	w_2
1	7	17
2	-85	-222
3	1202	3106
4	-16725	-43265

f_2

step	w_1	w_2
1	15	20
2	-95	-460
3	2045	10060
4	-44315	-219420