BOOK : LADR , SEAN AXLER

F^n

Higher dimensional analogues of R^2 and R^3
are defined with F^n

F^n is the set of all lists of length n of elements of F.
$F^n = \{ (x_1, \ldots, x_n) : x_k \in F \text{ for } k = 1, \ldots, n \}$.

For $(x_1, \ldots, x_n) \in F^n$ and $k \in \{1, \ldots, n\}$
x_k is the kth coordinate of $(x_1, \ldots, x_n)$

Eg. C^4 is the set of all lists of four complex numbers
$C^4 = \{ (z_1, z_2, z_3, z_4) : z_1, z_2, z_3, z_4 \in C \}$

At $n \geq 4$ we cannot visualize R^n as a physical object

But algebraic manipulations can still be performed
as easily

Elements of a vector space are called vectors or points
R^n is a vector space over R, C^n is a vector space over C

A vector space over R, is called a <u>real vector space</u>
A vector space over C is called a <u>complex vector space</u>

F^∞ is defined to be the set of all sequences of elements
F, $F^\infty = \{(x_1, x_2, \dots) : x_k \in F \text{ for } k = 1, 2, \dots\}$

Addition of F^∞: $(x_1, x_2, \dots) + (y_1, y_2, \dots) = (x_1 + y_1, x_2 + y_2, \dots)$
$\lambda (x_1, x_2, \dots) = (\lambda x_1, \lambda x_2, \dots)$

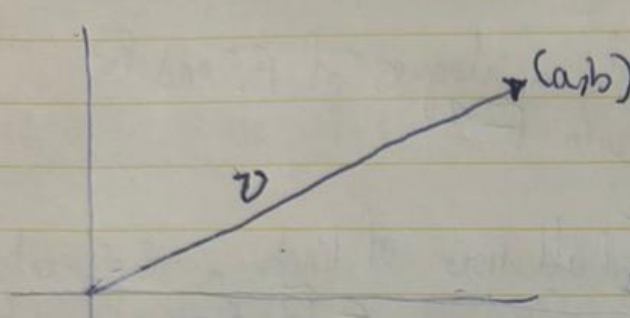NB: 0 is the simplest vector space

- Vector space involving a set of functions
If S is a set, then F^S denotes the set of functions from
S to F

for $f, g \in F^S$, the sum $f + g \in F^S$ is the function defined
by $(f + g)(x) = f(x) + g(x)$

For $\lambda \in F$ and $f \in F^S$, the product $\lambda f \in F^S$
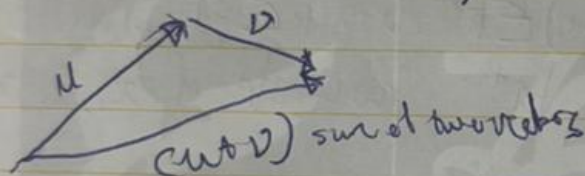is the function defined by $(\lambda f)(x) = \lambda f(x)$

Addition   In higher   dimensional order F^n

$$(x_1, \dots, x_n) + (y_1, \dots, y_n) = (x_1 + y_1, \dots, x_n + y_n)$$

(a,b) A typical element of
R^2 is a point $v = (a,b)$
which when thought of
as an arrow, becomes a vector.

$$R^5 = (2, -3, 17, \pi, \sqrt{2})$$

$(u + v)$ sum of two vectors

Scalar multiplication in F^n

$$\lambda(x_1, \dots x_n) = (\lambda x_1, \dots, \lambda x_n),$$
here $\lambda \in F$ and $(x_1, \dots, x_n) \in F^n$

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Scalar multiplication in R^2

If $\lambda > 0$ and $x \in R^2 =$
The scalar points in same direction as x,
and gives a resultant vector λ times the length of x

A vector space is a set V along with an addition on V and a scalar multiplication on V such that the following properties hold.

Commutativity

$u + v = v + u$ for all $u, v \in V$

associativity

$(u + v) + w = u + (v + w)$ and $(ab)v = a(bv)$
for all $u, v, w \in V$ and for all $a, b \in F$

additive identity

An element $0 \in V$ such that $v + 0 = v$ for all $v \in V$

additive inverse

for every $v \in V$, there exists $w \in V$ such that
$v + w = 0$.

multiplicative inverse

$1v = v$ for all $v \in V$

distributive vector property

$a(u + v) = au + av$ and $(a + b)u = au + bu$
for all $a, b \in F$ and all $u, v \in V$

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