Literature Survey of Post-Quantum Cryptography

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Abstract

Current cryptography methods rely heavily on the how computationally infeasible it is to use brute force attacks on trapdoor functions. In particular, methods like RSA rely on the difficulty of doing prime factorization on incredibly large numbers. However, recent advances in quantum computing mean that algorithms like Shor's Algorithm, that take advantage of quantum mechanics to quickly and efficiently make accurate guesses on prime factors, are able to break our current encryption methods in a matter of seconds. To rectify this, newer approaches to encryption are being found and used in what is called "post-quantum cryptography". These post-quantum cryptographic algorithms use many different methods of encryption, including complex polynomial math, stateless hash based approaches, and more. These new forms of encryption can provide the standards of protection against non-quantum attacks that are necessary, while also protecting against quantum attacks.

Keywords: post-quantum, cryptography, encryption, Shor's Algorithm, Grover's Algorithm

Summary

1 Introduction

We currently use RSA (Rivest Shamir Adleman) and ECDSA (Elliptic Curve Public-Key Cryptography) for a lot of our encryption methods. These methods are generally what is called public-key cryptography. In a public-key system, there are two keys used, one is called the public-key which is a specific number or formula used for encrypting messages, and the other is called the private key, which is generally an incredibly large prime number. Public-key cryptography works incredibly well because normal computers are not good at factoring prime numbers, which means that obtaining the private key is pointless in terms of computational time, power, and overall resources. For example, on a normal computer, a password comprised of 3763863863761 would take approximately 4 minutes to crack for a given computer, however, as we increase the length we can see that the time that it takes to crack increases exponentially. So if we just use that number two times in a row to get 376386386376137638638637 61, then the new time to crack the password becomes 79 million years. Obviously, there is more complexity to passwords than that, but it gives a good idea of how large prime numbers are harder for computers to crack. However, in 1994 a mathematician at MIT named Peter Shor came up with an algorithm that uses a lot of mathematical cleverness to create better guesses for the factors of a prime number. The algorithm works best in quantum systems, especially quantum systems with a higher number of what is referred to as qubits. In a normal computer system, the most basic unit of information is a bit, which represents a logical state where a value is either 1 or 0. Bit is actually an amalgamation of binary integer, and can be thought of as representing either an on or off state, and then using binary code and increasing layers of complexity, we get the computers that we use today. In a quantum system the most basic unit of information is not a bit, but a qubit (short for quantum bit). These qubits do not represent either on or off like a regular bit, but rather both numbers at once. Essentially, Shor's algorithm takes advantage of how qubits function to create a system that efficiently and effectively outputs the highest probability prime factors of a number. What would normally take current computers say 79 million years to crack, with Shor's algorithm it would take quantum computers a few minutes. This is obviously an incredibly big issue, because we use public-key cryptography all of the time, and so much of the internet, hardware, and infrastructure rely on these methods of encryption that would be rendered almost useless. Luckily, we still have time before reaching that point, because according to several estimates, for Shor's algorithm to be effective enough, it would require a system that has around 1300 to 1600 qubits, and currently the most advanced systems are only beginning to near 1000 qubits. The point of this project is to help update hardware for the transition period when Shor's algorithm does become a concern to encryption, and to open avenues for future quantum encryption methods in our software and hardware systems.

1.1 Motivations and Contributions

2 Background Information

2.1 Asymmetric Cryptography

Asymmetric Cryptography is the backbone of our current encryption infrastructure and requires several pieces to function properly. When using asymmetric key cryptography, it can also be referred to as public-key cryptography, this is because the system uses a public key and a private key.

2.1.1 Public Key

The public key in a public-key system is generally a number generated by a cryptographic function for each user in a system. It is known by everyone, and is used to encrypt messages. For example, if Alice, wants to send a message to a different user, Bob, but doesn't want the information to be known to a malicious third party, then she can use Bob's public key, which is known to everyone, to encrypt the message. This does not compromise the security of the message, but means that it is now encrypted, and can in theory only be decrypted by Bob who has the private key that goes with his public key.

2.1.2 Private Key

The private key in a public-key system is generally a number, or set of different numbers used in tandem, that correspond to a user's public key. Private keys are what make public-key systems work, they give the user the knowledge needed to take a message encrypted with their public key and decrypt it. Without the private-key it is computationally infeasible to decrypt the message. This is what's called a trapdoor function or a one-way function.

2.1.3 Trapdoor Functions

Trapdoor functions provide a major part of the foundation of modern cryptography. These functions are called trapdoor functions or one-way functions, because they work to take a message, perform an operation on it, and then obscure it to the point where it can't be undone. However, in cryptography, these functions have clever mathematics that allows a user with the right knowledge to undo the operation and get back to the original message. It is similar to taking 100 different pages of paper and shredding them. If a third party tries to put them back together, it will take them an unreasonable amount of time, however, since what was on each page is known to us, it is much faster, but still not instant, for us to piece the pages back together.

2.1.4 Putting it Together

In a public-key cryptosystem, there are several necessary pieces. There is the system itself, which is needed to generate the public and private keys, to

encrypt and decrypt messages using trapdoor functions, and finally a signature scheme is necessary. Signature schemes are used to prove the authenticity of the message, and can be thought of like signatures on check. With all of these pieces, a system is capable of fully encrypting and decrypting messages and ensuring authenticity and message integrity.

2.2 Symmetric Cryptography

2.3 Quantum Cracking

2.4 Post-Quantum Algorithms

3 Results

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4 This is an example for first level head—section head

4.1 This is an example for second level head—subsection head

4.1.1 This is an example for third level head—subsubsection head

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5 Equations

Equations in LaTeX can either be inline or on-a-line by itself ("display equations"). For inline equations use the `$...$` commands. E.g.: The equation $H\psi = E\psi$ is written via the command `$H \psi = E \psi$`.

For display equations (with auto generated equation numbers) one can use the equation or align environments:

$$\|\tilde{X}(k)\|^2 \leq \frac{\sum_{i=1}^{p} \left\|\tilde{Y}_i(k)\right\|^2 + \sum_{j=1}^{q} \left\|\tilde{Z}_j(k)\right\|^2}{p+q}. \tag{1}$$

where,

$$\begin{aligned} D_\mu &= \partial_\mu - ig\frac{\lambda^a}{2}A^a_\mu \\ F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^a_\nu \end{aligned} \tag{2}$$

Notice the use of `\nonumber` in the align environment at the end of each line, except the last, so as not to produce equation numbers on lines where no equation numbers are required. The `\label{}` command should only be used at the last line of an align environment where `\nonumber` is not used.

$$Y_\infty = \left(\frac{m}{\text{GeV}}\right)^{-3}\left[1 + \frac{3\ln(m/\text{GeV})}{15} + \frac{\ln(c_2/5)}{15}\right] \tag{3}$$

The class file also supports the use of `\mathbb{}`, `\mathscr{}` and `\mathcal{}` commands. As such `\mathbb{R}`, `\mathscr{R}` and `\mathcal{R}` produces $\mathbb{R}$, $\mathscr{R}$ and $\mathcal{R}$ respectively (refer Subsubsection 4.1.1).

6 Tables

Tables can be inserted via the normal table and tabular environment. To put footnotes inside tables you should use `\footnotetext[]{...}` tag. The footnote appears just below the table itself (refer Tables 1 and 2). For the corresponding footnotemark use `\footnotemark[...]`

Table 1 Caption text

Column 1	Column 2	Column 3	Column 4
row 1	data 1	data 2	data 3
row 2	data 4	data 5[1]	data 6
row 3	data 7	data 8	data 9[2]

Source: This is an example of table footnote. This is an example of table footnote.

[1]Example for a first table footnote. This is an example of table footnote.

[2]Example for a second table footnote. This is an example of table footnote.

The input format for the above table is as follows:

```
\begin{table}[<placement-specifier>]
\begin{center}
\begin{minipage}{<preferred-table-width>}
\caption{<table-caption>}\label{<table-label>}%
\begin{tabular}{@{}llll@{}}
\toprule
Column 1 & Column 2 & Column 3 & Column 4\\
\midrule
row 1 & data 1 & data 2  & data 3 \\
row 2 & data 4 & data 5\footnotemark[1] & data 6 \\
row 3 & data 7 & data 8  & data 9\footnotemark[2]\\
```

```
\botrule
\end{tabular}
\footnotetext{Source: This is an example of table footnote.
This is an example of table footnote.}
\footnotetext[1]{Example for a first table footnote.
This is an example of table footnote.}
\footnotetext[2]{Example for a second table footnote.
This is an example of table footnote.}
\end{minipage}
\end{center}
\end{table}
```

Table 2 Example of a lengthy table which is set to full textwidth

	Element 1[1]			Element 2[2]		
Project	Energy	σ_{calc}	σ_{expt}	Energy	σ_{calc}	σ_{expt}
Element 3	990 A	1168	1547 ± 12	780 A	1166	1239 ± 100
Element 4	500 A	961	922 ± 10	900 A	1268	1092 ± 40

Note: This is an example of table footnote. This is an example of table footnote this is an example of table footnote this is an example of table footnote this is an example of table footnote.

[1]Example for a first table footnote.

[2]Example for a second table footnote.

In case of double column layout, tables which do not fit in single column width should be set to full text width. For this, you need to use `\begin{table*} ... \end{table*}` instead of `\begin{table} ... \end{table}` environment. Lengthy tables which do not fit in textwidth should be set as rotated table. For this, you need to use `\begin{sidewaystable} ... \end{sidewaystable}` instead of `\begin{table*} ... \end{table*}` environment. This environment puts tables rotated to single column width. For tables rotated to double column width, use `\begin{sidewaystable*} ... \end{sidewaystable*}`.

7 Figures

As per the LaTeX standards you need to use eps images for LaTeX compilation and `pdf/jpg/png` images for `PDFLaTeX` compilation. This is one of the major difference between LaTeX and `PDFLaTeX`. Each image should be from a single input .eps/vector image file. Avoid using subfigures. The command for inserting images for LaTeX and `PDFLaTeX` can be generalized. The package used to insert images in `LaTeX/PDFLaTeX` is the graphicx package. Figures can be inserted via the normal figure environment as shown in the below example:

Table 3 Tables which are too long to fit, should be written using the "sidewaystable" environment as shown here

	Element 1[1]			Element[2]		
Projectile	Energy	σ_{calc}	σ_{expt}	Energy	σ_{calc}	σ_{expt}
Element 3	990 A	1168	1547 ± 12	780 A	1166	1239 ± 100
Element 4	500 A	961	922 ± 10	900 A	1268	1092 ± 40
Element 5	990 A	1168	1547 ± 12	780 A	1166	1239 ± 100
Element 6	500 A	961	922 ± 10	900 A	1268	1092 ± 40

Note: This is an example of table footnote this is an example of table footnote this is an example of table footnote this is an example of table footnote this is an example of table footnote.

[1]This is an example of table footnote.

```
\begin{figure}[<placement-specifier>]
\centering
\includegraphics{<eps-file>}
\caption{<figure-caption>}\label{<figure-label>}
\end{figure}
```

Fig. 1 This is a widefig. This is an example of long caption this is an example of long caption this is an example of long caption this is an example of long caption

In case of double column layout, the above format puts figure captions/images to single column width. To get spanned images, we need to provide `\begin{figure*} ... \end{figure*}`.

For sample purpose, we have included the width of images in the optional argument of `\includegraphics` tag. Please ignore this.

8 Algorithms, Program codes and Listings

Packages `algorithm`, `algorithmicx` and `algpseudocode` are used for setting algorithms in LaTeX using the format:

```
\begin{algorithm}
\caption{<alg-caption>}\label{<alg-label>}
\begin{algorithmic}[1]
. . .
\end{algorithmic}
\end{algorithm}
```

You may refer above listed package documentations for more details before setting `algorithm` environment. For program codes, the "program" package is required and the command to be used is `\begin{program} ... \end{program}`. A fast exponentiation procedure:

```
begin
  for i := 1 to 10 step 1 do
      expt(2, i);
      newline() od                Comments will be set flush to the right margin
where
proc expt(x, n) ≡
  z := 1;
  do if n = 0 then exit fi;
```

```
    do if odd(n) then exit fi;
       comment: This is a comment statement;
       n := n/2; x := x * x od;
    {n > 0};
    n := n − 1; z := z * x od;
  print(z).
end
```

Algorithm 1 Calculate $y = x^n$

Require: $n \geq 0 \vee x \neq 0$
Ensure: $y = x^n$

```
y ⇐ 1
if n < 0 then
    X ⇐ 1/x
    N ⇐ −n
else
    X ⇐ x
    N ⇐ n
end if
while N ≠ 0 do
    if N is even then
        X ⇐ X × X
        N ⇐ N/2
    else[N is odd]
        y ⇐ y × X
        N ⇐ N − 1
    end if
end while
```

Similarly, for `listings`, use the `listings` package. `\begin{lstlisting} ... \end{lstlisting}` is used to set environments similar to `verbatim` environment. Refer to the `lstlisting` package documentation for more details.

```
for i:=maxint to 0 do
begin
{ do nothing }
end;
Write('Case insensitive ');
Write('Pascal keywords.');
```

9 Cross referencing

Environments such as figure, table, equation and align can have a label declared via the `\label{#label}` command. For figures and table environments use the `\label{}` command inside or just below the `\caption{}` command. You can then use the `\ref{#label}` command to cross-reference them. As an example, consider the label declared for Figure 1 which is `\label{fig1}`. To cross-reference it, use the command `Figure \ref{fig1}`, for which it comes up as "Figure 1".

To reference line numbers in an algorithm, consider the label declared for the line number 2 of Algorithm 1 is `\label{algln2}`. To cross-reference it, use the command `\ref{algln2}` for which it comes up as line 2 of Algorithm 1.

9.1 Details on reference citations

Standard LaTeX permits only numerical citations. To support both numerical and author-year citations this template uses `natbib` LaTeX package. For style guidance please refer to the template user manual.

Here is an example for `\cite{...}`: [1]. Another example for `\citep{...}`: [2]. For author-year citation mode, `\cite{...}` prints Jones et al. (1990) and `\citep{...}` prints (Jones et al., 1990).

All cited bib entries are printed at the end of this article: [3], [4], [5], [6], [7], [8], [9], [10], [11] and [12].

10 Examples for theorem like environments

For theorem like environments, we require `amsthm` package. There are three types of predefined theorem styles exists—`thmstyleone`, `thmstyletwo` and `thmstylethree`

`thmstyleone`	Numbered, theorem head in bold font and theorem text in italic style
`thmstyletwo`	Numbered, theorem head in roman font and theorem text in italic style
`thmstylethree`	Numbered, theorem head in bold font and theorem text in roman style

For mathematics journals, theorem styles can be included as shown in the following examples:

Theorem 1 (Theorem subhead) *Example theorem text. Example theorem text. Example theorem text. Example theorem text. Example theorem text. Example theorem text. Example theorem text. Example theorem text. Example theorem text. Example theorem text.*

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

Proposition 2 *Example proposition text. Example proposition text. Example proposition text. Example proposition text. Example proposition text. Example proposition text. Example proposition text. Example proposition text. Example proposition text. Example proposition text.*

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

Example 1 Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem.

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

Remark 1 Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem.

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

Definition 1 (Definition sub head) Example definition text. Example definition text. Example definition text. Example definition text. Example definition text. Example definition text. Example definition text. Example definition text.

Additionally a predefined "proof" environment is available: `\begin{proof} ... \end{proof}`. This prints a "Proof" head in italic font style and the "body text" in roman font style with an open square at the end of each proof environment.

Proof Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. □

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

Proof of Theorem 1 Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text. Example for proof text.

Example for proof text. Example for proof text. Example for proof text. Example for proof text. □

For a quote environment, use `\begin{quote}...\end{quote}`

> Quoted text example. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text (refer Figure 1). Sample body text. Sample body text. Sample body text (refer Table 3).

11 Methods

Topical subheadings are allowed. Authors must ensure that their Methods section includes adequate experimental and characterization data necessary for others in the field to reproduce their work. Authors are encouraged to include RIIDs where appropriate.

Ethical approval declarations (only required where applicable) Any article reporting experiment/s carried out on (i) live vertebrate (or higher invertebrates), (ii) humans or (iii) human samples must include an unambiguous statement within the methods section that meets the following requirements:

1. Approval: a statement which confirms that all experimental protocols were approved by a named institutional and/or licensing committee. Please identify the approving body in the methods section
2. Accordance: a statement explicitly saying that the methods were carried out in accordance with the relevant guidelines and regulations
3. Informed consent (for experiments involving humans or human tissue samples): include a statement confirming that informed consent was obtained from all participants and/or their legal guardian/s

If your manuscript includes potentially identifying patient/participant information, or if it describes human transplantation research, or if it reports results of a clinical trial then additional information will be required. Please visit (https://www.nature.com/nature-research/editorial-policies) for Nature Portfolio journals, (https://www.springer.com/gp/authors-editors/journal-author/journal-author-helpdesk/publishing-ethics/14214) for Springer Nature journals, or (https://www.biomedcentral.com/getpublished/editorial-policies#ethics+and+consent) for BMC.

12 Discussion

Discussions should be brief and focused. In some disciplines use of Discussion or 'Conclusion' is interchangeable. It is not mandatory to use both. Some journals

prefer a section 'Results and Discussion' followed by a section 'Conclusion'. Please refer to Journal-level guidance for any specific requirements.

13 Conclusion

Conclusions may be used to restate your hypothesis or research question, restate your major findings, explain the relevance and the added value of your work, highlight any limitations of your study, describe future directions for research and recommendations.

In some disciplines use of Discussion or 'Conclusion' is interchangeable. It is not mandatory to use both. Please refer to Journal-level guidance for any specific requirements.

Supplementary information. If your article has accompanying supplementary file/s please state so here.

Authors reporting data from electrophoretic gels and blots should supply the full unprocessed scans for key as part of their Supplementary information. This may be requested by the editorial team/s if it is missing.

Please refer to Journal-level guidance for any specific requirements.

Acknowledgments. Acknowledgments are not compulsory. Where included they should be brief. Grant or contribution numbers may be acknowledged.

Please refer to Journal-level guidance for any specific requirements.

Declarations

Some journals require declarations to be submitted in a standardised format. Please check the Instructions for Authors of the journal to which you are submitting to see if you need to complete this section. If yes, your manuscript must contain the following sections under the heading 'Declarations':

- Funding
- Conflict of interest/Competing interests (check journal-specific guidelines for which heading to use)
- Ethics approval
- Consent to participate
- Consent for publication
- Availability of data and materials
- Code availability
- Authors' contributions

If any of the sections are not relevant to your manuscript, please include the heading and write 'Not applicable' for that section.

Editorial Policies for:

Springer journals and proceedings:
https://www.springer.com/gp/editorial-policies

Nature Portfolio journals:
https://www.nature.com/nature-research/editorial-policies

Scientific Reports:
https://www.nature.com/srep/journal-policies/editorial-policies

BMC journals:
https://www.biomedcentral.com/getpublished/editorial-policies

Appendix A Section title of first appendix

An appendix contains supplementary information that is not an essential part of the text itself but which may be helpful in providing a more comprehensive understanding of the research problem or it is information that is too cumbersome to be included in the body of the paper.

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