Michael Prigmore

Project Overview

For my project I decided to build a custom app. The app I chose to build is a calculator that performs elementary row operations on a matrix transforming it into Reduced Echelon Form. These concepts come from the study of Linear Algebra. Obtaining the Reduced Echelon Form of a matrix has many uses including solving systems of linear equations. For example, consider the system of linear equations below:

$$-4x_1 + 5x_2 + 9x_3 = -9$$

$$x_1 - 2x_2 + x_3 = 0$$

$$2x_2 - 8x_3 = 8$$

This system of linear equations can be rewritten as an augmented matrix. The coefficients of each equation along with the values on the right side of the equals sign make up the rows of the matrix. The augmented matrix is given below for this system of linear equations:

$$\begin{bmatrix} -4 & 5 & 9 & -9 \\ 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \end{bmatrix}$$

Matrices can be modified to form new matrices by using elementary row operations. When a matrix is transformed into another matrix using elementary row operations, the two matrices are said to be row equivalent. There are three elementary row operations and are given below:

- Rows can be interchanged
- Rows can be replaced by a row with the sum of itself and a multiple of another row
- Entire rows can be multiplied by a nonzero scalar

Elementary row operations can be used, along with an algorithm, to row reduce a matrix into a row equivalent Echelon Form, as well as a row equivalent Reduced Echelon Form. A matrix in Echelon Form has the value of 0 below all leading (left most) row entries. These left most entries are called pivot positions. For example, the matrix below is in Echelon Form:

$$\begin{bmatrix} 1 & -5/4 & -9/4 & 9/4 \\ 0 & 1 & -13/3 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

A matrix is in Reduced Echelon Form if all pivot position values are 1, and all values above and below the pivot positions are 0. The matrix below is in Reduced Echelon Form:

$$\begin{bmatrix} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

This is the Reduced Echelon Form of the original matrix given above. If the matrix were now to be rewritten as a system of linear equations, we would have:

$$x_1 + 0x_2 + 0x_3 = 29, \text{ which simplifies to } x_1 = 29$$

$$0x_1 + x_2 + 0x_3 = 16, \text{ which simplifies to } x_2 = 16$$

$$0x_1 + 0x_2 + x_3 = 3, \text{ which simplifies to } x_3 = 3$$

As you can see, row reducing a matrix into Row Echelon Form provides the solution to the system of linear equations, provided it has one. Matrices have many Echelon Forms, but only one unique Reduced Echelon Form.

My custom app will implement the algorithm used to row reduce a matrix into Reduced Echelon Form. The program requirements are given below:

1. The program must prompt the user to enter a matrix by entering the number of rows the matrix has, the number of columns the matrix has, and the values in each position in the matrix. Rows and columns must be entered as a positive integer greater than 1. Values can be any real number.
2. The program will then display the original matrix, a row equivalent matrix in Echelon Form, and the row equivalent matrix in Reduced Echelon Form.
3. The program must have scripts related to presentation logic and scripts related to domain logic saved in separate packages.
4. The program must be written such that 100% code coverage can be achieved from testing the domain layer.

Sources

1. Lay, D. (2011). *Linear Algebra and its Applications (4th ed.).* Pearson.