

Problem 1 - Cut-on Frequencies in Ducts and Pipes

Problem 1a

The lowest cut-on frequency for a rectangular duct with air flow is given by equation,

$$f_{\text{cut-on}} = 0.5 \cdot \frac{c}{L} \quad (1)$$

where c is the speed of sound in air, $343 \frac{\text{m}}{\text{s}}$, and L is the largest side of the rectangular cross-section.

With cross-sectional dimensions of $L_x = 12 \text{ cm}$ and $L_y = 20 \text{ cm}$, the lowest cut-on frequency for this rectangular duct is,

$$f_{\text{cut-on}} = 0.5 \cdot \frac{343 \frac{\text{m}}{\text{s}}}{0.20 \text{ m}} = \mathbf{857.5 \text{ Hz}}$$

Problem 1b

The lowest cut-on frequency for a circular duct with air flow with the same cross-sectional area as the rectangular duct in part (a.) can be calculated using equation,

$$f_{\text{cut-on}} = 0.568 \cdot \frac{c}{d} \quad (2)$$

where c is the speed of sound in air, $343 \frac{\text{m}}{\text{s}}$, and d is diameter of the circular duct.

The cross-sectional area of the rectangular duct is,

$$\text{Area}_{\text{rectangular duct}} = 0.12 \text{ m} \cdot 0.20 \text{ m} = 0.024 \text{ m}^2$$

The corresponding diameter for this area is,

$$\text{diameter} = \sqrt{\frac{0.024 \text{ m}^2}{\pi}} \cdot 2 = 0.17 \text{ m}$$

Using Eq. 2, the lowest cut-on frequency for this circular duct with air flow is,

$$f_{\text{cut-on}} = 0.568 \cdot \frac{1,500 \frac{\text{m}}{\text{s}}}{0.17 \text{ m}} = \mathbf{1,114.5 \text{ Hz}}$$

Problem 1c

The lowest cut-on frequency for this circular duct with water flow can be calculated using Eq. 2,

$$f_{\text{cut-on}} = 0.568 \cdot \frac{1,500 \frac{\text{m}}{\text{s}}}{0.17 \text{ m}} = \mathbf{4,873.9 \text{ Hz}}$$

The lowest cut-on frequency for water is considerable larger than it is for air flow.

Problem 1d

The speed of sound in air is calculated by,

$$c = \sqrt{\gamma \cdot R \cdot T_K} \quad (3)$$

where $\gamma = 1.4$ is the ratio of specific heats, $R = 287 \frac{\text{J}}{\text{kg} \cdot \text{K}}$ is the gas constant, and T_K is the absolute temperature in Kelvin.

Figure 1 illustrates how the lowest cut-on frequency changes as the air heats from 0° to 500° Celsius.

The square-root relationship between temperature and the speed of sound in air is apparent and governs the behaviour of the cut-on frequency.

Lowest Cut-on Frequency for a Circular Pipe with Air Flow Versus Air Temperature

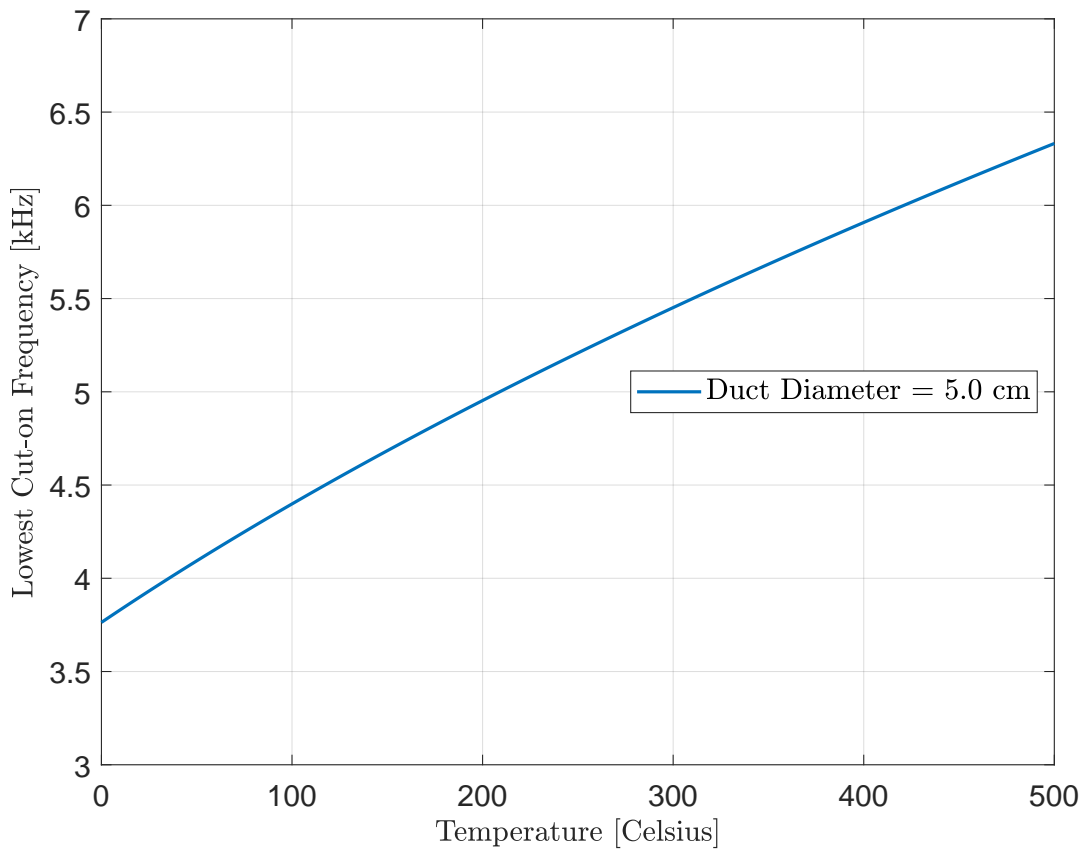


Figure 1: Lowest cut-on frequency for a circular 5 cm diameter duct versus air temperature.

Problem 1e

Question: Are cut-on frequencies higher for a circular or rectangular duct for a given cross-sectional area?

The lowest cut-on frequency is higher for a circular duct than for a rectangular duct for a given cross-sectional area.

For the dimensions given in class, the rectangular duct is not square. This produces a larger dimension and thus a smaller, lowest cut-on frequency. If the rectangular duct is square dimensions on the order of the circular duct diameter with the same cross-sectional area, the cut-on frequencies are approximately equal.

Question: What about in air versus water?

The lowest cut-on frequency is larger for water than for air. The cut-on frequency is proportional to the speed of sound and the speed of sound in water is greater than the speed of sound in air.

Question: What about cold versus hot air?

The lowest cut-on frequency is higher for warm air than it is for cold air.

Problem 2 - Muffler Design Comparison

For these muffler comparisons, it is assumed that there is no resistive terms (i.e., damping) and no flow. Since transmission losses are plotted, the end corrections (i.e., load impedance at outlet of the system) have no physical meaning and are not accounted for in the computations.

Problem 2a

Figure 2 shows the transmission loss profile for the simple expansion chamber (red, dashed line).

Peak occur at a quarter wavelength; extension tube at outlet.

Extension tube aids quarter wavelenth.

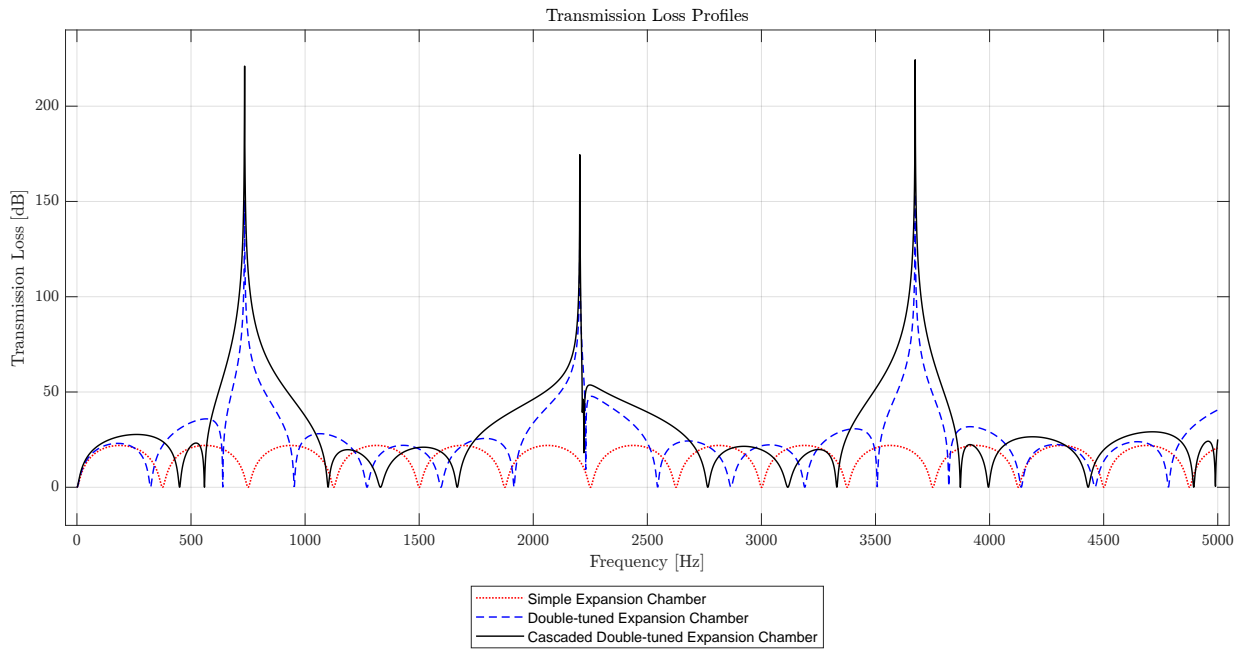


Figure 2: Transmission loss profiles for a simple expansion chamber, a double-tuned expansion chamber, and a cascaded double-tuned expansion chamber mufflers.

Problem 2b

There is no damping in the system; resonances will be artificially high.

Problem 2c

There is no damping in the system; resonances will be artificially high.

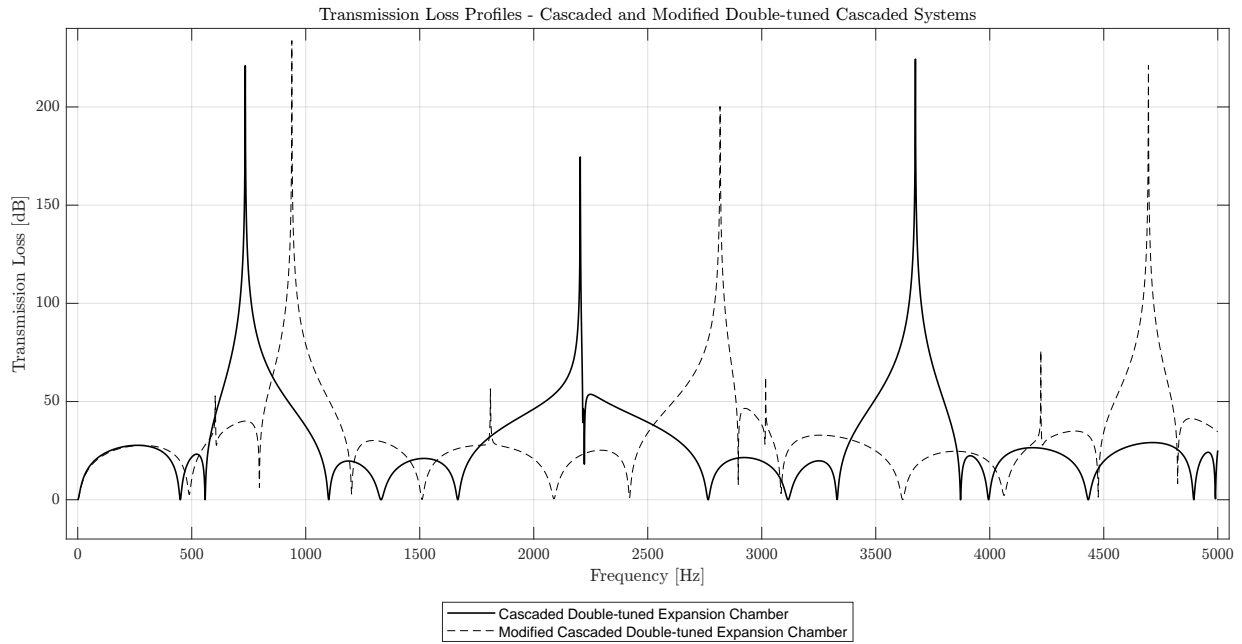


Figure 3: Transmission loss profiles for a cascaded double-tuned expansion chamber muffler and a modified, cascaded double-tuned expansion chamber mufflers.

Problem 3 - Bugle Recorder

Diameters of holes should be smaller than a wavelength.

R_A is neglected (energy loss).

Problem 3a

Problem 3b

Problem 4 - Intake Duct

Problem 4a

Problem 4b

Problem 4c

Problem 4d

Problem 5 - Intake Duct Silencer

Problem 5a

Problem 5b

Problem 5c

Problem 5d

Problem 5e

Appendix A: Matlab Code - Problem 1

%% Synopsis

% Homework Set 1 — Cut-on Frequencies in Ducts and Pipes

% Note: Send draft of report before submission for comments.

%

% Dimensions are annotated in the class notes.

%% To Do

% Focus on interpretation.

%% Environment

```
close all; clear; clc;
% restoredefaultpath;

% set( 0, 'DefaultFigurePosition', [ 400 400 900 400 ] ); % [ left bottom width
    height ]
set( 0, 'DefaultFigurePaperPositionMode', 'manual' );
set( 0, 'DefaultFigureWindowStyle', 'normal' );
set( 0, 'DefaultLineLineWidth', 1.5 );
set( 0, 'DefaultTextInterpreter', 'Latex' );

format ShortG;

pause( 1 );

PRINT_FIGURES = 0;
```

%% Define Values and Functions

```
c_air = 343; % The speed of sound in air in meters per second.
c_water = 1500; % The speed of sound in water in meters per second.

gamma = 1.4; % The ratio of specific heats [unitless].
R = 287; % The gas constant [Joules per ( kilogram * Kelvin)].

h_f_cut_on_rectangular_duct = @( c, L ) 0.5 .* c ./ L;
%
% c — The speed of sound.
% L — The largest cross-section dimension of the rectangular duct.

h_f_cut_on_circular_duct = @( c, d ) 0.568 .* c ./ d;
```



```
%
% c — The speec of sound.
% L — The diameter of the circular duct.

h_speed_of_sound_in_air = @( gamma, R, temperature_Kelvin) sqrt( gamma .* R .*
    temperature_Kelvin );
```

%% Problem 1a

```
% The cross-sectional dimensions for the rectangular duct are: Lx = 12 cm and Ly = 20
    cm.

% The largest dimension is Ly = 20 cm or 0.2 m.

% The cut-on frequency is,
h_f_cut_on_rectangular_duct( c_air, 0.2 ); % 857.5 Hz (shown in class 858 Hz)
    fprintf( 1, '\n Problem 1a: The lowest cut-on frequency for the rectangular pipe
        with air is %3.1f Hz.\n', h_f_cut_on_rectangular_duct( c_air, 0.2 ) );
```

%% Problem 1b

```
% The cross-sectional dimensions for the rectangular duct are: Lx = 12 cm and Ly = 20
    cm.

% The cross-sectional area of the rectangular duct is 12 cm * 20 cm = 240 cm^2 or 0.024
    m^2.
rectangular_duct_cross_sectional_area = 0.12 * 0.20; % 0.024 m^2

% The diameter of the circulat pipe is,
circular_duct_diameter = sqrt( 0.024 / pi ) * 2; % 0.17481 meters
%
% Check:
    % pi * ( circular_duct_diameter / 2 )^2 CHECKED

% The cut-on frequency for the circular duct is,
h_f_cut_on_circular_duct( c_air, circular_duct_diameter ); % 1,114.5 Hz
    fprintf( 1, '\n Problem 1b: The lowest cut-on frequency for the circular pipe (of
        equal area) with air is %3.1f Hz.\n', h_f_cut_on_circular_duct( c_air,
        circular_duct_diameter ) );
```

%% Problem 1c

```
% The cut-on frequency for the circular duct with water is,
h_f_cut_on_circular_duct( c_water, circular_duct_diameter ); % 4,873.9 Hz
    fprintf( 1, '\n Problem 1c: The lowest cut-on frequency for the circular pipe (of
        equal area) with water is %3.1f Hz.\n', h_f_cut_on_circular_duct( c_water,
```

```

circular_duct_diameter ) );

% The cut-on frequency should be higher because it is proportional to the
% speed of sound in a given medium.



---


%% Problem 1d

fprintf( 1, '\n Problem 1d: See the figure.\n' );

temperature_range_celsius = 0:0.1:500; % Celsius
temperature_range_kelvin = temperature_range_celsius + 273.15; % Kelvin

FONT_SIZE = 14;

figure( ); ...
    plot( temperature_range_celsius, h_f_cut_on_circular_duct( h_speed_of_sound_in_air(
gamma, R, temperature_range_kelvin ), 0.05 ) ./ 1e3 ); grid on;
    legend( 'Duct Diameter = 5.0 cm', 'Location', 'East', 'FontSize', FONT_SIZE, '
Interpreter', 'Latex' );
    set( gca, 'FontSize', FONT_SIZE );
%
xlabel( 'Temperature [Celsius]', 'FontSize', FONT_SIZE );
    % xl = get( gca, 'xlabel' );    pxl = get( xl, 'position' );    pxl( 2 ) = 1.1 *
pxl( 2 );
    %    set( xl, 'position', pxl );
%
ylabel( 'Lowest Cut-on Frequency [kHz]', 'FontSize', FONT_SIZE );
    % yl = get( gca, 'ylabel' );    pyl = get( yl, 'position' );    pyl( 1 ) = 1.2 * pyl
( 1 );
    %    set( yl, 'position', pyl );
%
caption = sprintf( 'Lowest Cut-on Frequency for a Circular Pipe with Air Flow Versus
Air Temperature\n' );
    title( caption, 'FontSize', FONT_SIZE );
%
ylim( [ 3 7 ] );

% if ( PRINT_FIGURES == 1 )
%     saves( gcf, 'Cut-on Frequency Versus Temperature – Sunday, January 19, 2025.pdf'
);
% end

```

%% Problem 1e

```

fprintf( 1, '\n Problem 1e: See Section Problem 1e of the Matlab script for the answers
.\n\n' );

```

%% Clean-up

```
if ( ~isempty( findobj( 'Type', 'figure' ) ) )
    monitors = get( 0, 'MonitorPositions' );
    if ( size( monitors, 1 ) == 1 )
        autoArrangeFigures( 2, 2, 1 );
    elseif ( 1 < size( monitors, 1 ) )
        autoArrangeFigures( 2, 2, 1 );
    end
end

if ( PRINT_FIGURES == 1 )
    saveas( gcf, 'Cut-on Frequency Versus Temperature — Sunday, January 19, 2025.pdf' );
end

fprintf( 1, '\n\n\n*** Processing Complete ***\n\n\n' );
```

%% Reference(s)