Acoustics of enclosures

Professor Phil Joseph

ACOUSTIC MODES OF ENCLOSED SPACES

One-dimensional closed pipe, length *L*

Acoustic natural frequencies are given by

$$f_n = \frac{c_0}{\lambda} = \frac{nc_0}{2L}$$

for integer values of *n*

Pressure mode shapes given by

$$A_n \cos\left(\frac{n\pi x}{L}\right)$$

Rectangular space $L_1 \times L_2 \times L_3$

Acoustic natural frequencies are given by

$$f_{l,m,n} = \frac{c_0}{2}\sqrt{\left(\frac{l}{L_1}\right)^2 + \left(\frac{m}{L_2}\right)^2 + \left(\frac{n}{L_3}\right)^2}$$

for integer values of (*l, m, n*) or

$$f_{l,m,n} = \sqrt{f_{l00}{}^2 + f_{0m0}{}^2 + f_{00n}{}^2}$$

Pressure mode shapes given by

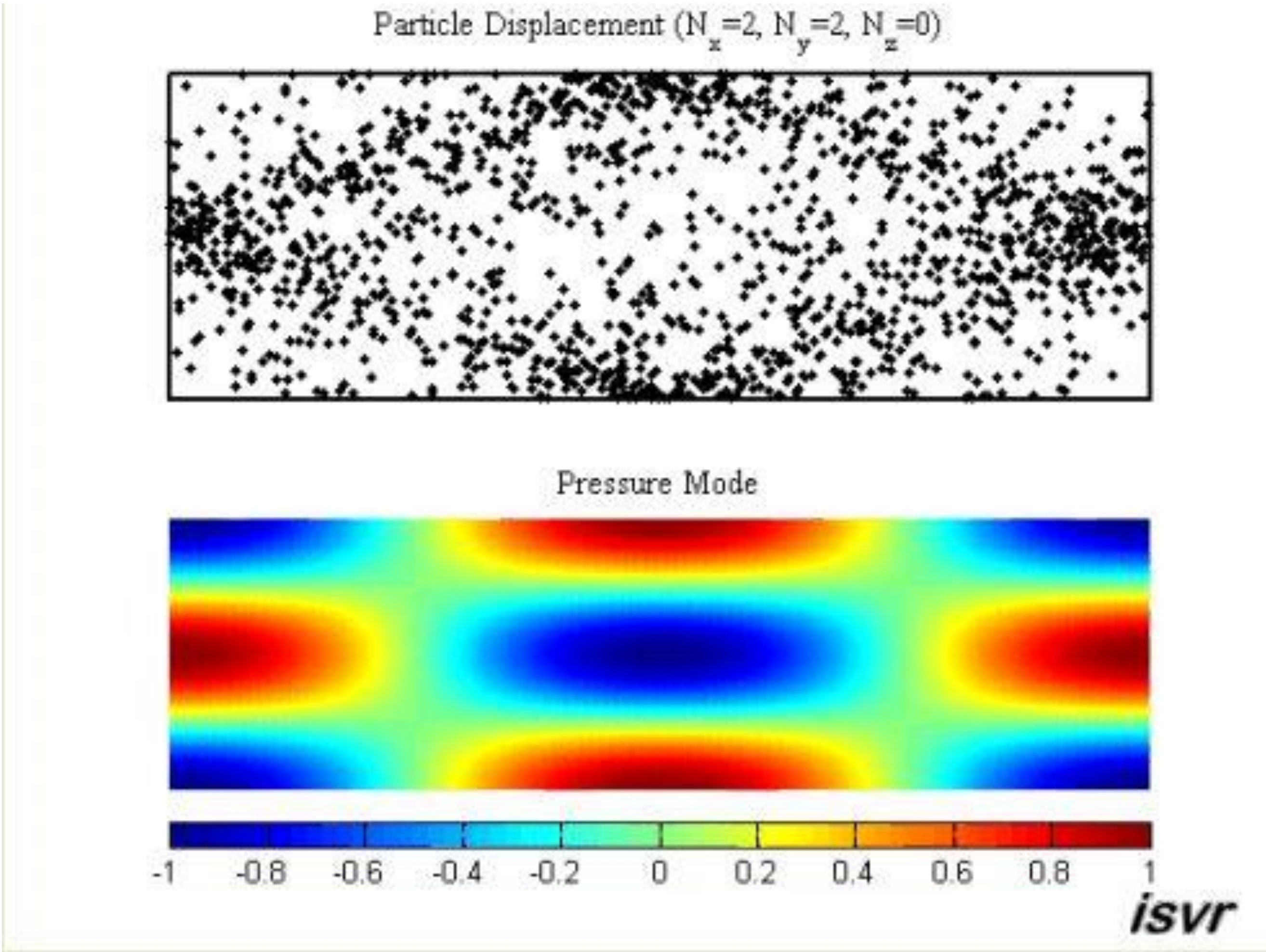
$$A_{l,m,n} \cos\left(\frac{l\pi x_1}{L_1}\right)\cos\left(\frac{m\pi x_2}{L_2}\right)\cos\left(\frac{n\pi x_3}{L_3}\right)$$

Limitations:

- irregular geometry (but the first few modes can be estimated quite well)
- wall/window flexibility (tends to *lower* cavity natural frequencies if panel modes are much lower than cavity modes).

ROOM MODES

The modes of a room can be defined by the number of nodal lines in each direction.



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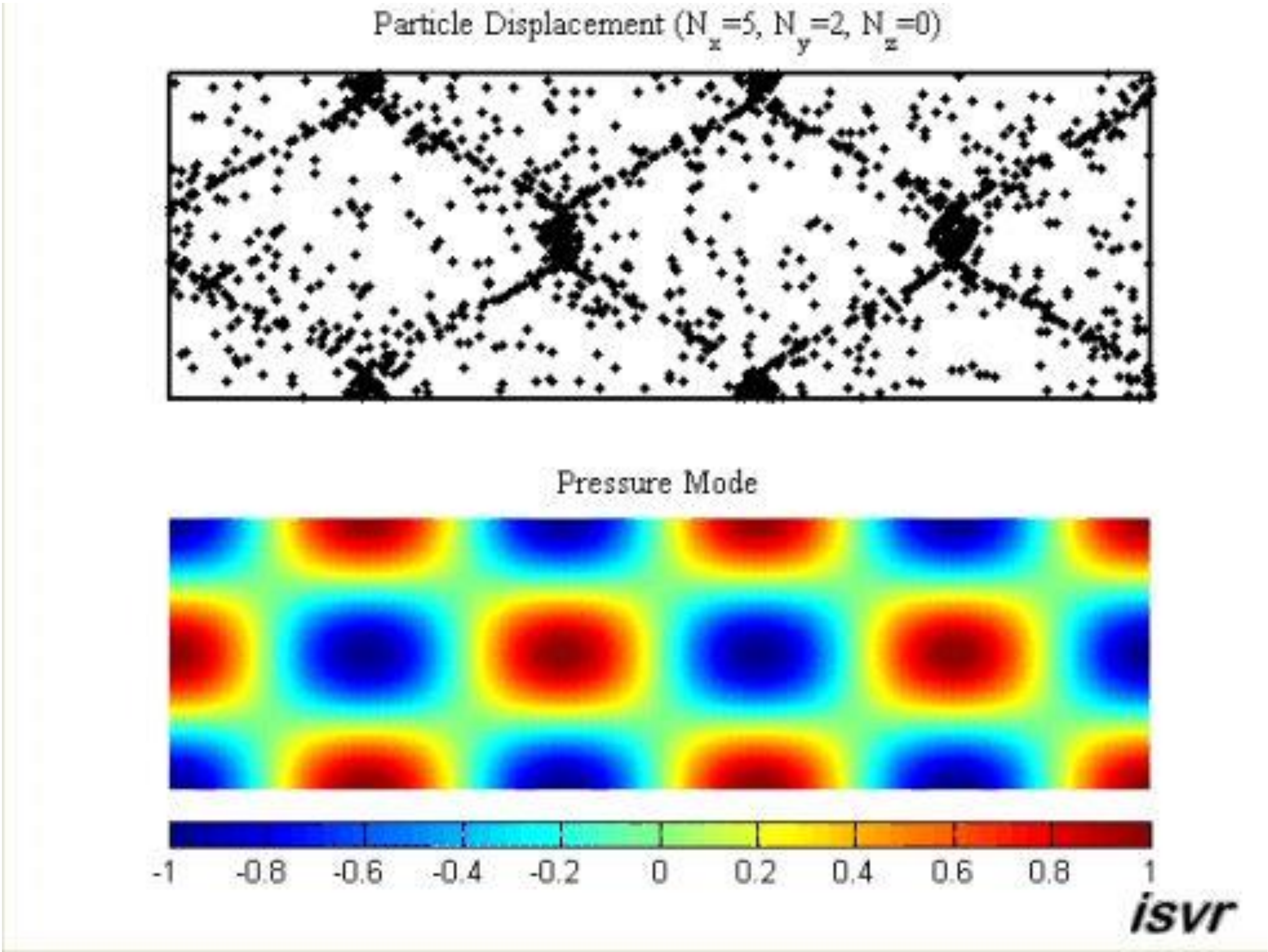
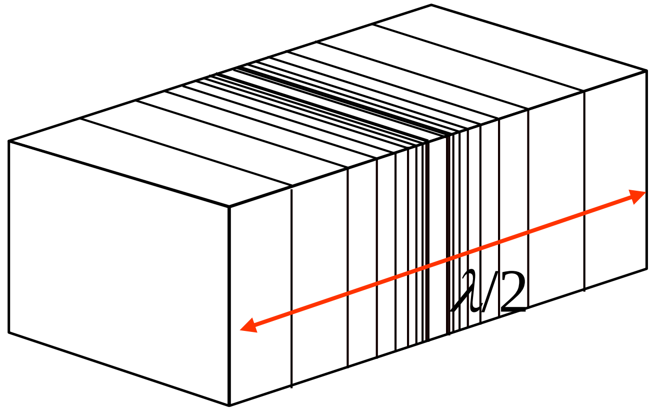


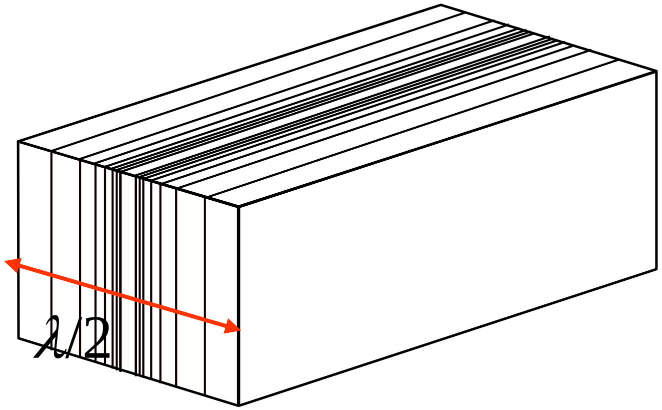
MODES OF A RECTANGULAR CAVITY

Example: rectangular space 2.3 m long x 1.3 m wide x 1.1 m high.
First 4 modes:

mode (1,0,0) : 73.91 Hz



mode (0,1,0) : 130.8 Hz



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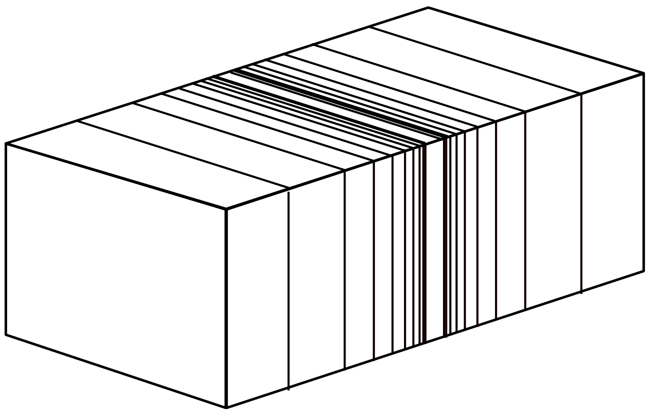

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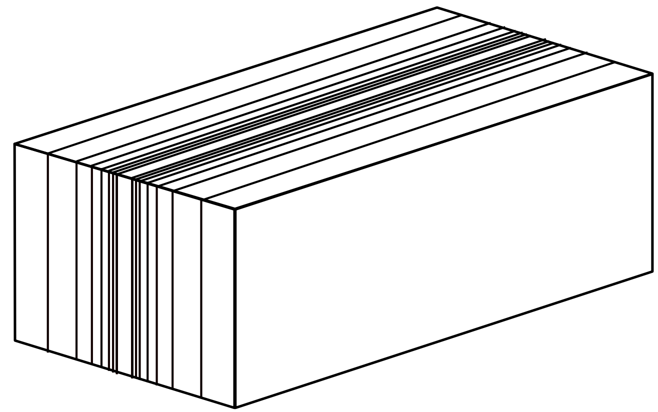
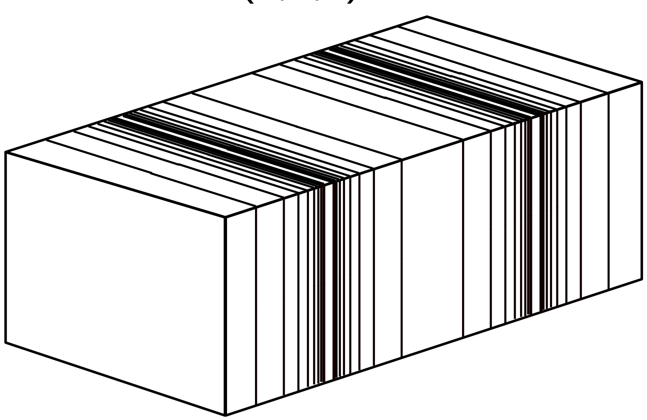
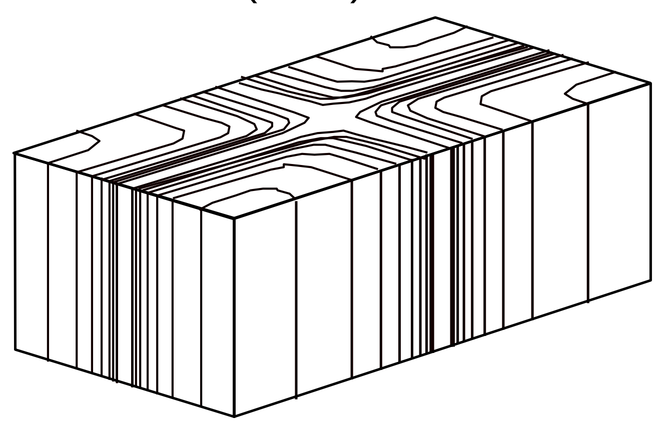


mode (2,0,0) : 147.8 Hz

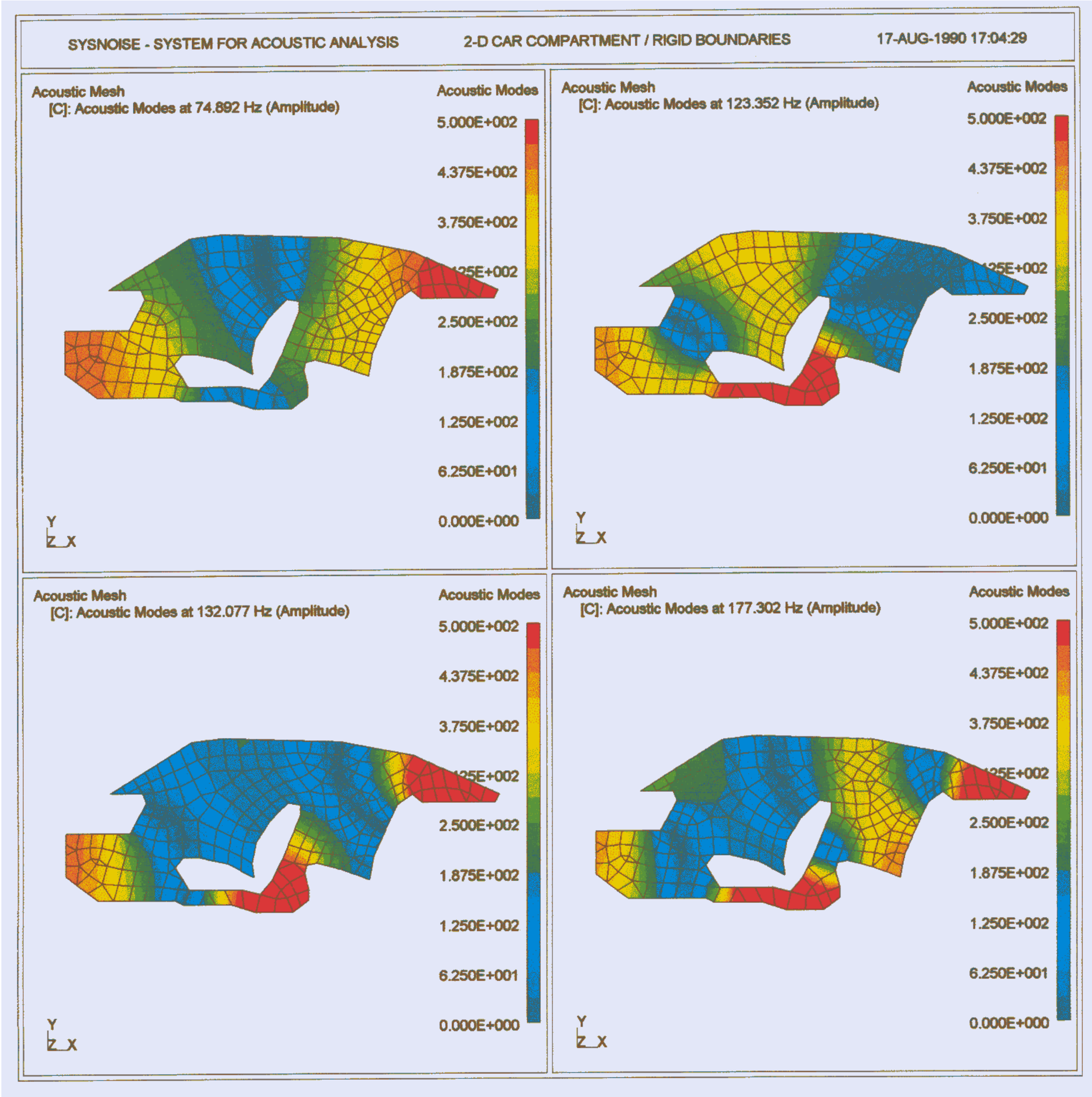


mode (1,1,0) : 150.2 Hz



MODES OF A CAR INTERIOR

- calculated using acoustic FEM
- also have modes in width direction (this is a 2D model)
- real modes more complicated than simple model
- but size of vehicle is main parameter



NOISE CONTOL APPROACHES IN ROOMS

Consider those techniques used to reduce noise levels other than noise control at source and direct protection of the listener's hearing.

1. The use of absorptive treatments

   (i) In "Sabine" spaces
   (ii) In "Non-Sabine" spaces

2. The use of barriers (screens)

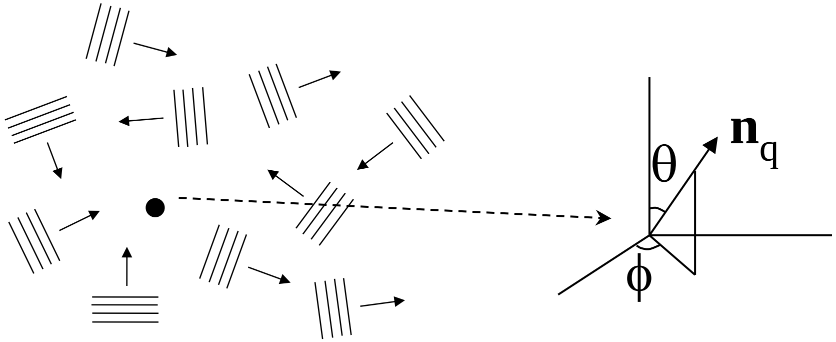
   (i) In free field conditions
   (ii) In the presence of ground reflections
   (iii) In enclosed spaces

3. The use of acoustic enclosures

DIFFUSE FIELD THEORY

Applicable to "large" rooms (dimensions L_1, $L_{2,}$, L_3 > l) of low aspect ratio (L_1~L_2 ~ L_3) and low average absorption coefficient. DIFFUSE FIELD MODEL (Morse & Ingard, 1968).

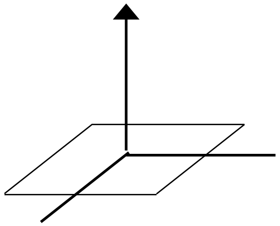
Represent sound field as an assemblage of plane waves, each travelling in a direction specified by the angles, ϕ, θ.



Local time averaged energy density

$$\overline{w} = \int_0^{2\pi} \int_0^{\pi} \frac{|\, p_q \,|^2}{\rho_0 c_0^2} \sin\theta \, d\theta \, d\phi$$

Local time averaged energy intensity



$$\bar{\mathbf{I}} = \int_0^{2\pi} \int_0^{\pi/2} \frac{|\, p_q \,|^2}{\rho_0 c_0} \sin\theta \cos\theta \, d\theta \, d\phi$$

DIFFUSE FIELD ASSUMPTIONS

1. Local time averaged energy density uniform throughout room (w independent of position).

2. Local time averaged acoustic intensity uniform throughout room and independent of direction. ($\bar{\mathbf{I}}$ independent of position and direction.)

   Under these conditions

$$\bar{\mathbf{I}} = \frac{c_0}{4}\bar{w} = \frac{\overline{p^2}}{4\rho_0 c_0}$$

3. Absorption properties of a wall surface can be represented by an absorption coefficient, α, such that the local rate at which energy is absorbed per unit area is given by

$$\alpha\bar{\mathbf{I}} = \frac{\alpha c_0 \bar{w}}{4} = \frac{\alpha\,\overline{p^2}}{4\rho_0 c_0}$$

ENERGY BALANCE EQUATION

The diffuse field assumptions lead to the energy balance given by

$$W_R = \frac{A\overline{p^2}}{4\rho_0 c_0} + \frac{d}{dt}\left[V \frac{\overline{p^2}}{\rho_0 c_0^2}\right] \qquad (A = S\overline{\alpha})$$

POWER IN = POWER LOST + RATE OF CHANGE OF ROOM ENERGY

Steady State Solution $$\overline{p^2} = \frac{4W_R \rho_0 c_0}{A}$$

Transient Solution $$\overline{p^2} = \overline{p_0^2} \exp\left(-\frac{Ac_0}{4V} t\right)$$

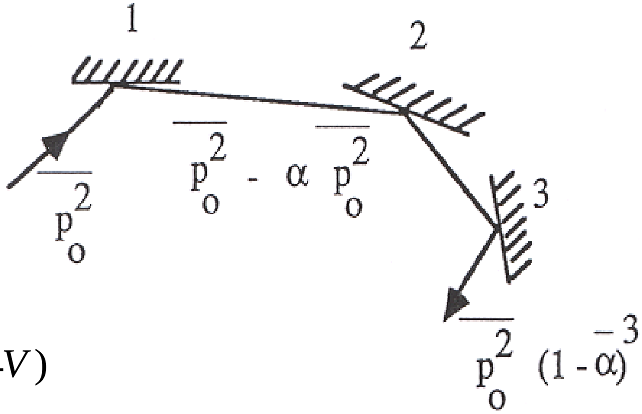
Time taken for $10\log_{10} \overline{p^2} / \overline{p_0^2} = -60\,\mathrm{dB}$ is the reverberation time

$$T = \frac{0.161V}{A}$$ - "Sabine's Formula"

LIMITATION OF SABINE'S FORMULA

Note that with perfectly absorbing surfaces $(A = S\overline{\alpha} = S)$ with a finite reverberation time is predicted. Eyring (1930) points out that Sabine's theory assumes a *continuous* decay of room energy, i.e. energy loss through walls assumed to occur at same instant as energy loss from room volume.

Eyring's formula assumes energy loss occurs at each discrete reflection of a sound ray.



No. of reflections/s = $\dfrac{c_0}{\text{"}Mean\ free\ path\text{"}}$

$$\overline{p^2} = \overline{p_0^2}(1-\alpha)^{(c_0 S/4V)}$$

$$\therefore T = \frac{0.161V}{-S\ln(1-\overline{\alpha})}$$ - "Eyring's formula"

DIRECT AND REVERBERANT SOUND

Sabine's theory does not account for the form of the sound field close to the source.

Assume the source power *W* is partitioned between the "direct" and "reverberant" fields

$$W = W_D + W_R$$

rate at which energy is lost on first reflection

rate at which energy is fed to "diffuse" reverberant field

Assume

$$W_D = \bar{\alpha} W$$

$$W_R = (1 - \bar{\alpha}) W$$

REDUCTION IN REVERBERANT FIELD SOUND PRESSURE DUE TO THE ADDITION OF ABSORPTION

From the steady state solution to the energy balance equation

$$\overline{p^2} = \frac{4W\rho_0 c_0 (1-\overline{\alpha})}{S\overline{\alpha}}$$

enables the calculation of the reduction in *reverberant* field SPL due to increased absorption

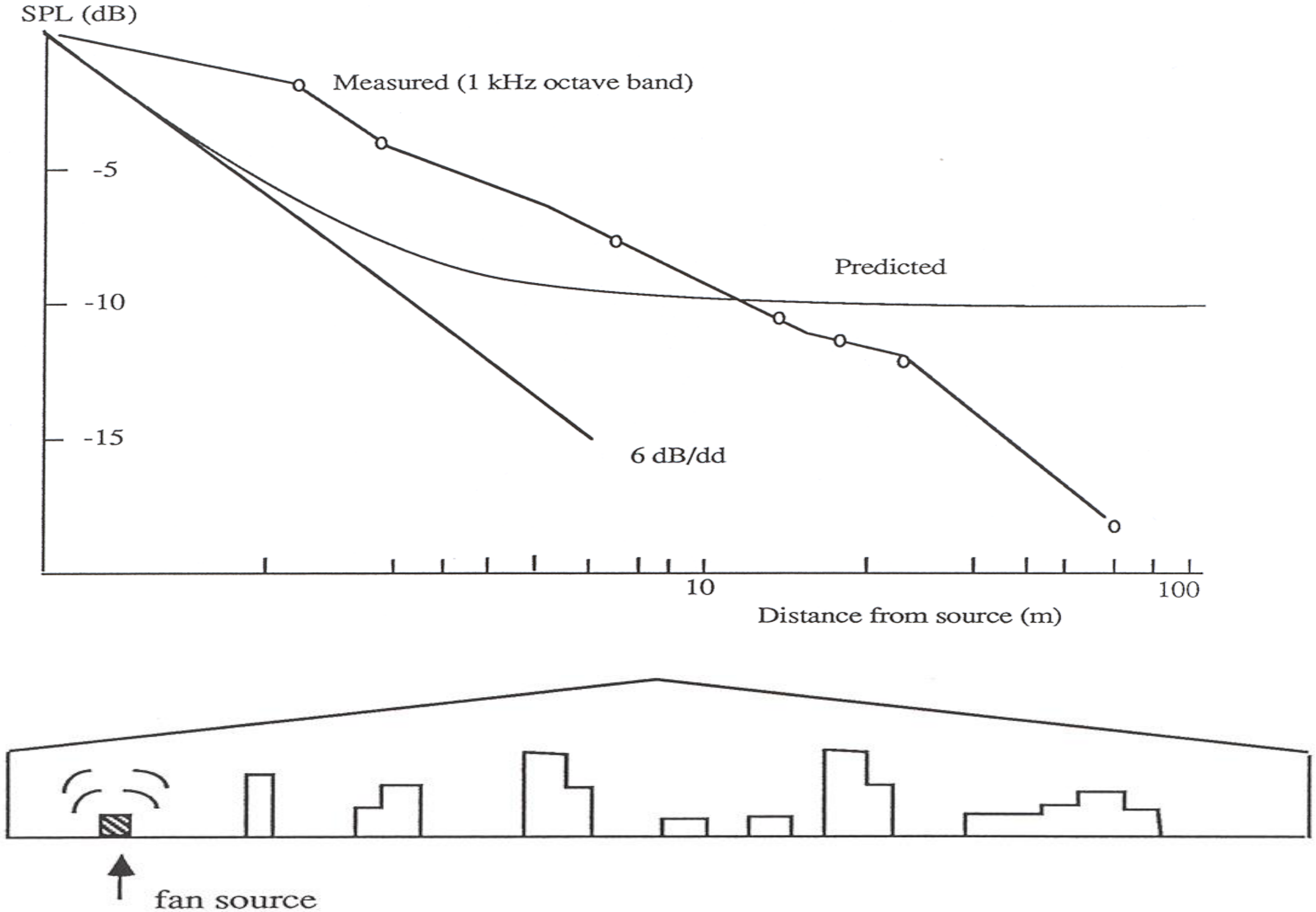
$$\Delta \mathrm{SPL} = 10\log_{10} \frac{(1-\overline{\alpha}_2)\overline{\alpha}_1}{(1-\overline{\alpha}_1)\overline{\alpha}_2}$$

e.g. $\overline{\alpha}_1 = 0.2$ $\quad \overline{\alpha}_2 = 0.7$ $\quad \Delta \mathrm{SPL} = 9.7\,\mathrm{dB}$

$\overline{\alpha}_1 = 0.5$ $\quad \overline{\alpha}_2 = 0.7$ $\quad \Delta \mathrm{SPL} = 3.7\,\mathrm{dB}$

THE USE OF DIFFUSE FIELD THEORY FOR PRACTICAL PREDICTIONS

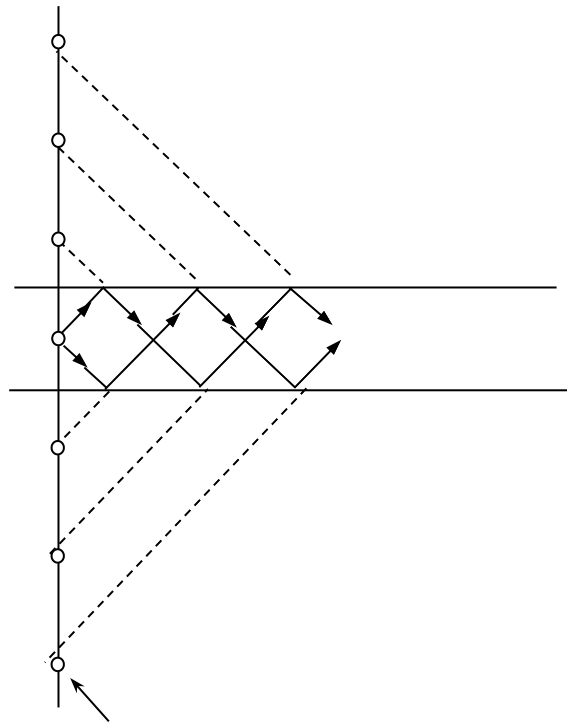
Typical comparison of measured sound field and that predicted from measured '*T*' in a large "flat" space.



THE IMAGE MODEL

Assumes "geometrical acoustics"; enclosures consist of plane boundaries with specular reflections.

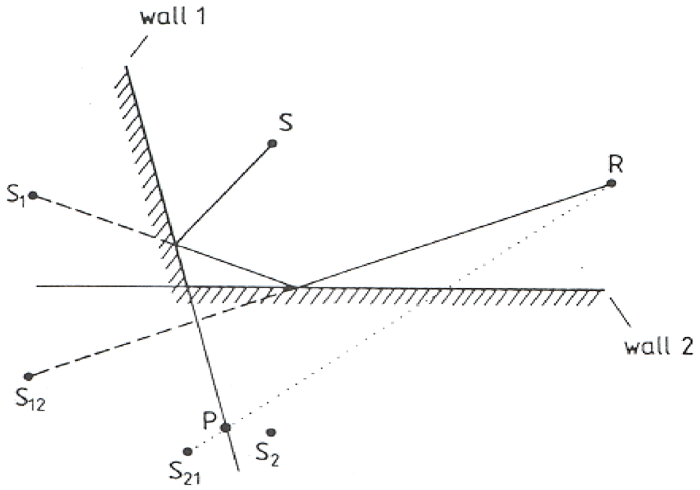
Example for two parallel plane surfaces:



“Image” sources used to represent specular reflections

VISIBILITY OF IMAGE SOURCES

When applied to complex geometries, it becomes important to establish the "visibility" of each image at a given receiver position.



Construct all image sources sequentially up to a prescribed order.

Determine visibility by geometrical back tracing of sound paths from the Receiver *R* to the original sound source.

Image source S_{12} is visible, but S_{21} is not, since the intersection of the straight line RS_{21} (dotted) with plane 1 is not within the boundaries of wall 1.

COMPUTATIONAL COMPLEXITY

The computation time mainly is dependent on the desired maximum order of image sources. The number of sources up to the order *i* of a room with n_w walls increases exponentially with *i* :

$$N_{IS} = \sum_{k=0}^{i-1} n_w (n_w - 1)^k$$

$$\therefore N_{IS} = \frac{n_w}{n_w - 2}\left[(n_w - 1)^i - 1\right]$$

e.g. for an impulse response duration of 400 ms for a room consisting of 30 walls enclosing a volume of 15000m^3 (average number of reflections per second = 25.5s^{-1})

$$N_{IS} \sim 4.5 \times 10^{14}$$

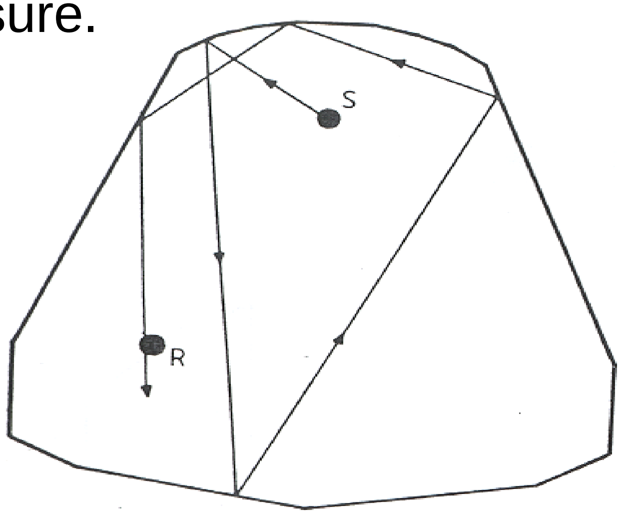
-but the ratio of N_{IS} to the number of *visible* sources is typically 10^{12} !

THE RAY TRACING MODEL

Geometrical acoustics is again assumed. The paths of a number of specularly reflected "particles" are traced.

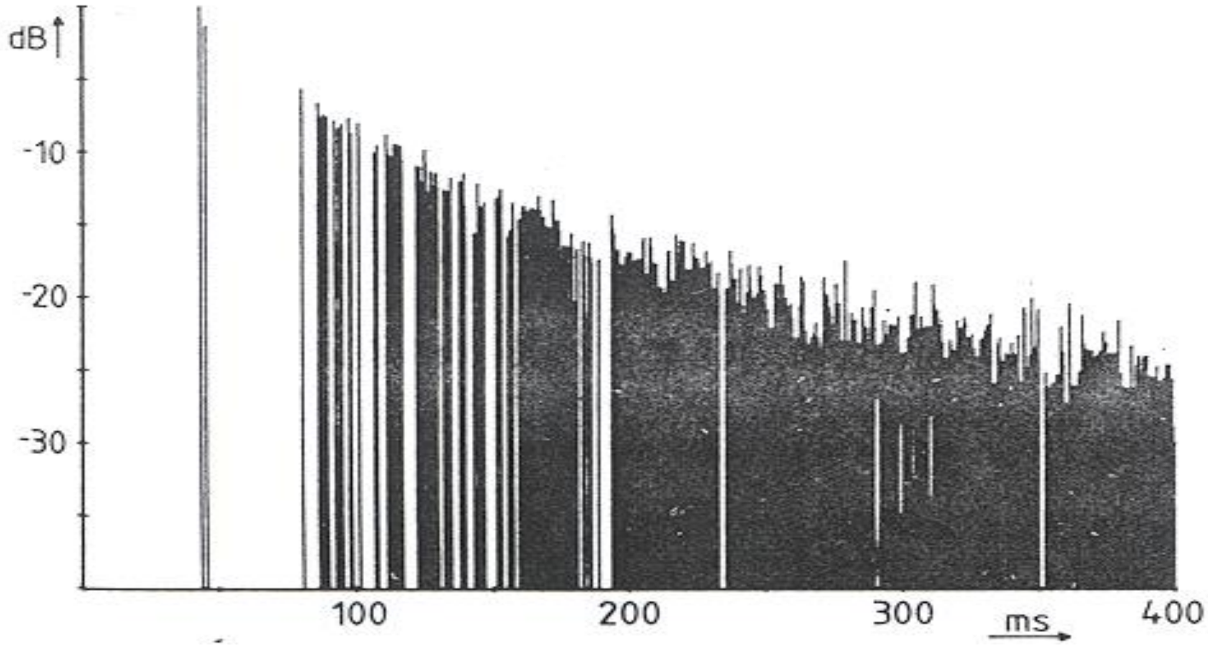
Particles travel at the speed of sound and carry a certain energy, which is reduced on each reflection.

The presence of a particle is detected within a finite (usually spherical) detector volume; this enables the construction of the "energy impulse response" of the enclosure.



HYBRID RAY TRACING / IMAGE MODELS

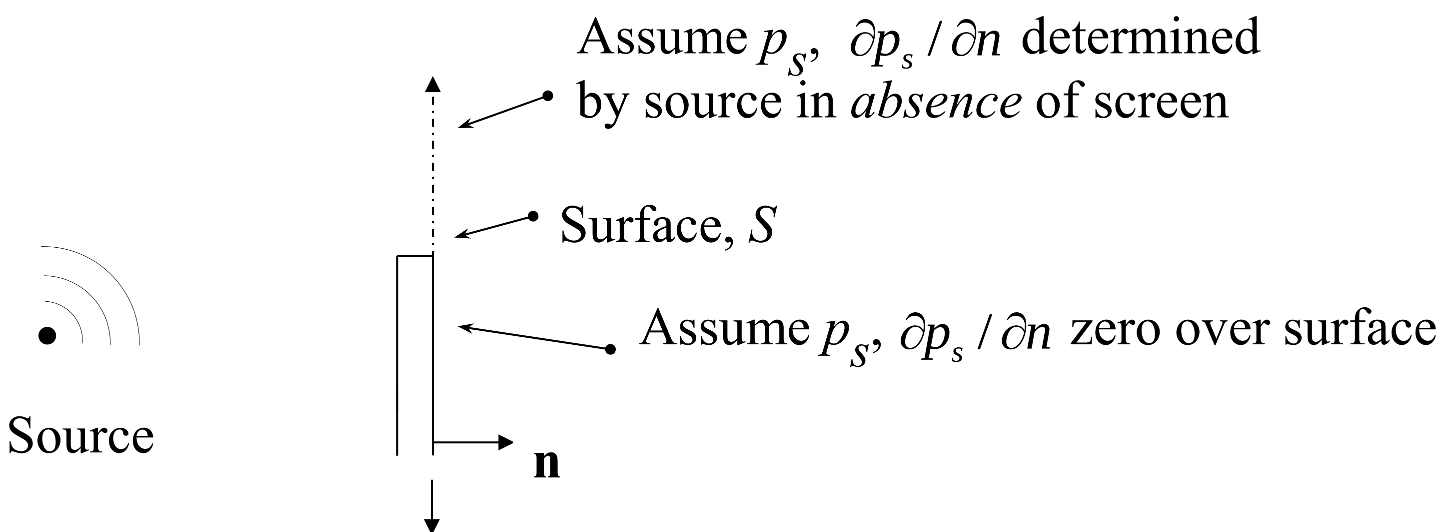
The ray tracing algorithm can be used to identify efficiently the visible sources in an image model giving considerable savings in computational cost.



Example of a highly resolved energy impulse response obtained with the new sound particle-image-source method

THE PERFORMANCE OF ACOUSTIC SCREENS

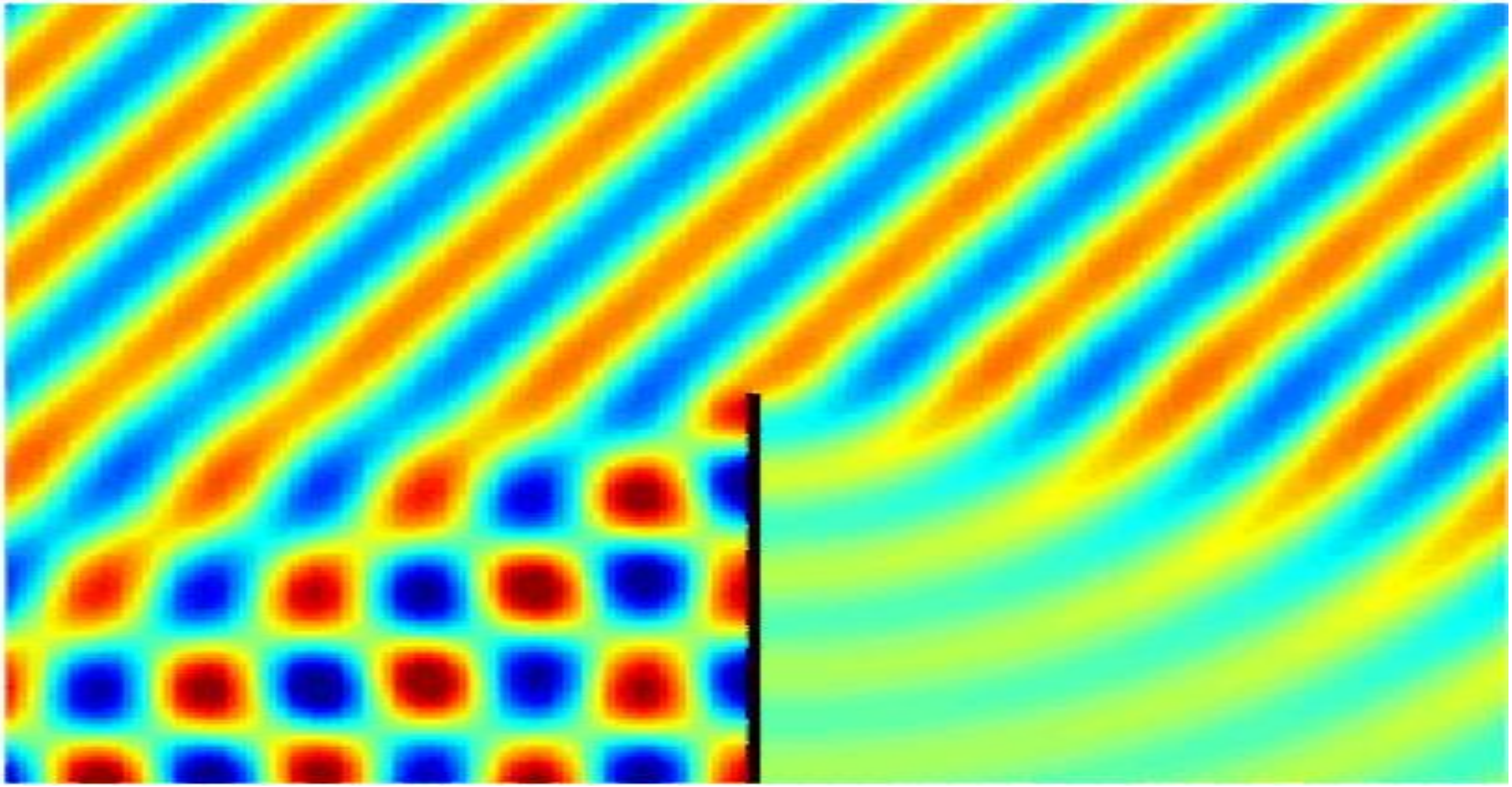
Diffraction by a semi-infinite rigid barrier; exact solution given by Sommerfeld (1896). Can also apply Kirchhoff diffraction theory



$$p = \int_S \left[g \frac{\partial p_s}{\partial n} - p_s \frac{\partial g}{\partial n} \right] dS$$

where *g* is the free space Green function $g = e^{-jkr_s / 4\pi r_s}$ and r_s is the distance to the observer from points on the surface *S*.

DIFFRACTION OF A PLANE WAVE BY A RIGID BARRIER



EXPERIMENTS DUE TO MAEKAWA (1968)

Compares results using pulse tone technique with Kirchhoff diffraction theory. Results plotted against Fresnel number $N = 2\delta/\lambda$ where δ is "path length difference".

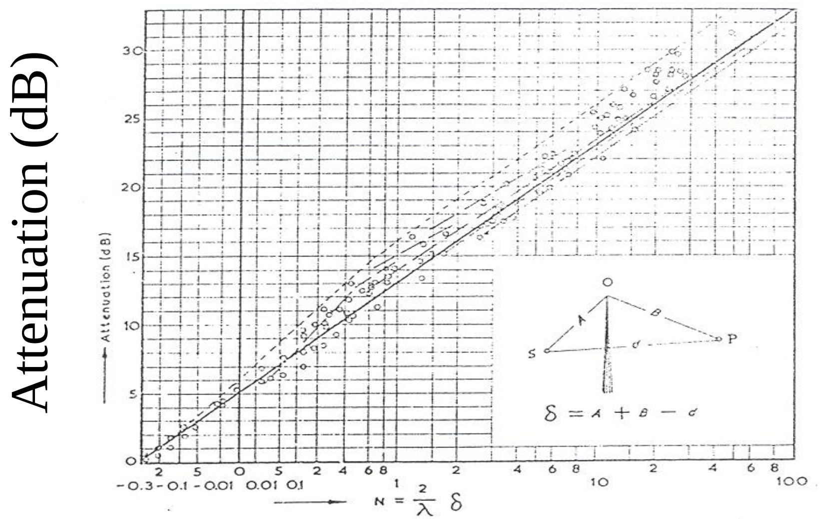
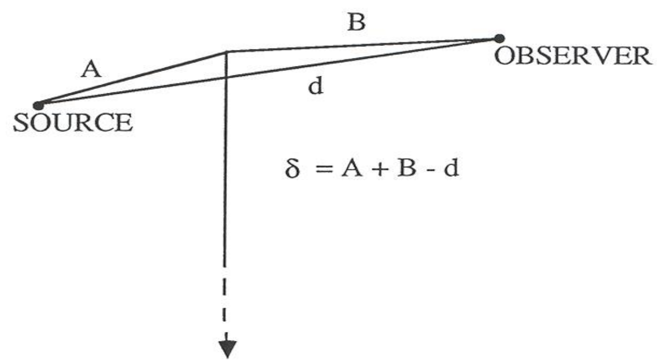
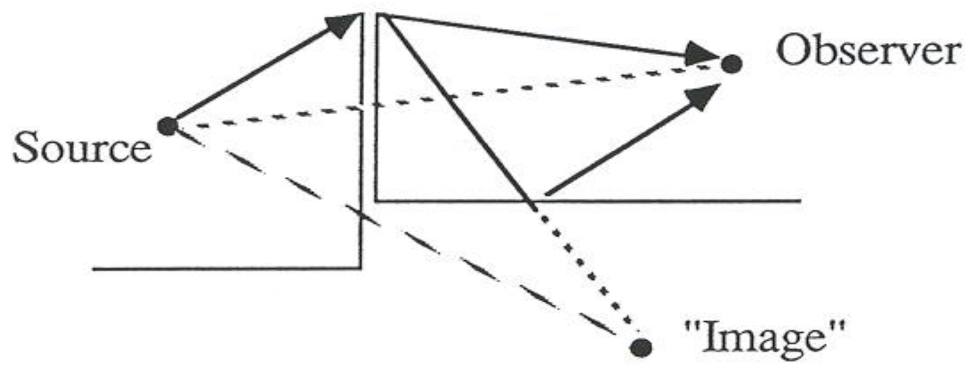


Fig. 4. Sound attenuation by a semi-infinite screen in free space. Horizontal scale, logarithmic scale in the region of $N > 1$, adjusted so that the experimental curve becomes a straight solid line in the region of $N < 1$. Depending on whether $N > 0$ or $N < 0$, the receiving point P lies in the illuminated region or in the geometrical shadow, respectively. Attenuation is relative to propagation in free space. — — —, Redfearn's theory; ——, Maekawa's work; - - - -, Kirchhoff's theory; o, experimental value measured by pulsed tone.



BARRIER PERFORMANCE ON REFLECTING GROUND SURFACES

Maekawa suggests the use of image technique for allowance of ground reflections.

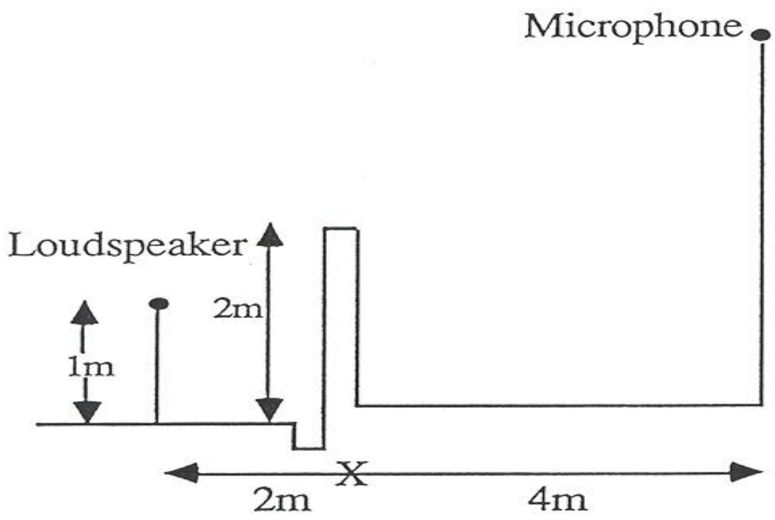


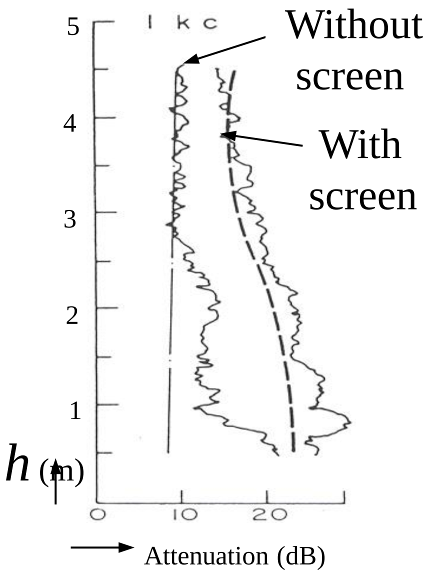
SPLs calculated at observer and "image observer" using appropriate "path length difference".

Total SPL calculated using energy addition at observer position.

BARRIER PERFORMANCE ON REFLECTING GROUND SURFACES

Typical experimental results; (1 kHz)

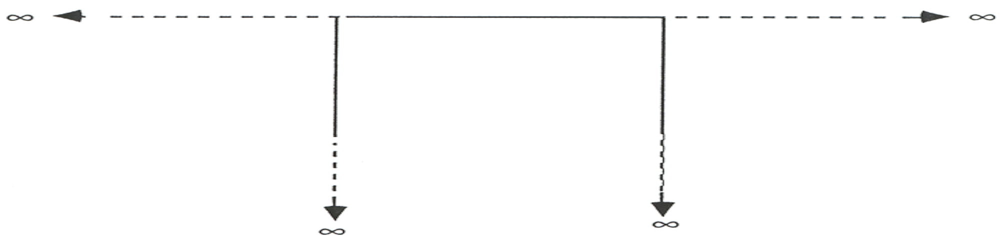




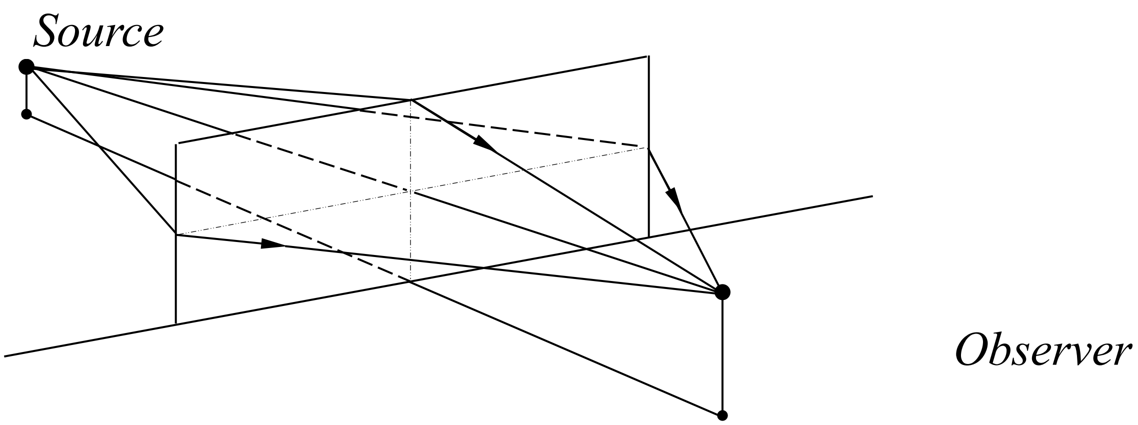
In general, large errors (~ 10 dB) at low frequencies (< 500 Hz)

ATTENUATION DUE TO BARRIERS OF FINITE WIDTH

Maekawa suggests energy addition of component waves diffracted by "half infinite" open surface and two "quarter infinite" open surfaces.



Ground reflections again accounted for using image observers.



Experimental results given reasonable agreement (± 5 dB) with predictions based on this method

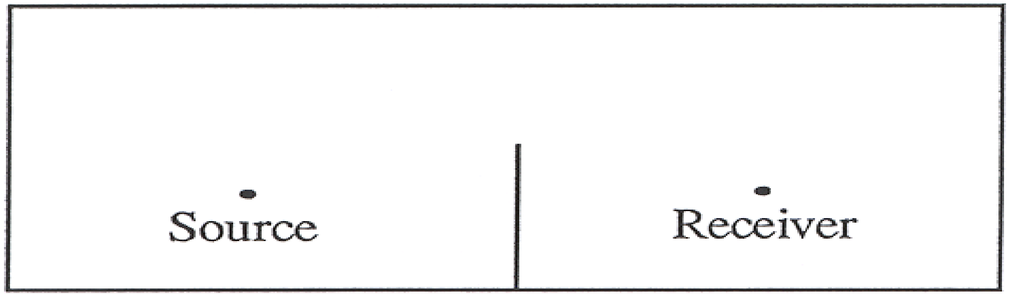
THE PEFORMANCE OF BARRIERS IN ENCLOSED SPACES

Comprehensive series of measurements undertaken by Kurze (1978) in open plan offices and industrial buildings.

Insertion loss of barriers in open plan offices (dB)

Barrier Height (m)	Source/ Receiver distance (m)		
	2-3	4-6	7-9
1.3 – 1.5	6.4 ± 2.8	5.4 ± 2.0	4.1 ± 2.4
1.5 – 2.2	8.3 ± 2.5	6.5 ± 1.6	6.0 ± 3.0

Mean values and standard deviations in 1 kHz octave band; based on 42 measurements.



THE PERFORMANCE OF BARRIERS IN ENCLOSED SPACES

After Kurze (1978)

Insertion loss of barriers in factory halls (dB)

$$\frac{\text{barrier height}}{\text{room height}} \qquad \frac{(\text{source - receiver distance})}{(\text{room height})}$$

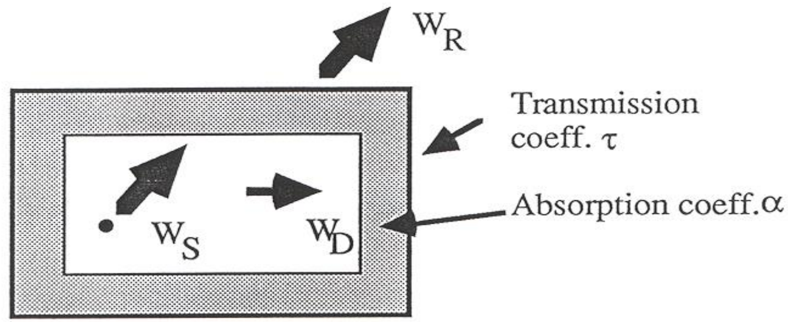
	0.3	0.3-1	1-3
0.3	7.4 ± 1.4	3.6 ± 2.1	
0.3-0.5	10	7.1 ± 1.8	4.5 ± 1.8
0.5		8.6 ± 1.7	6.3 ± 1.5

Mean values and standard deviations in 1 kHz octave band.

See also the work of Czarnecki (1978) who advocates the use of the "absorbing belt" in the region above a barrier.

ACOUSTIC ENCLOSURE PERFORMANCE: A DIFFUSE FIELD MODEL

Assume: Source power output unaffected by the presence of the enclosure



Total sound power radiated $W_R = \int_S \mathbf{I}\tau dS$

Total sound power absorbed $W_D = \int_S \mathbf{I}\alpha dS$

Total source power $W_s = W_R + W_D$

$$\therefore \frac{W_R}{W_S} = \frac{\bar{\tau}}{\bar{\tau} + \bar{\alpha}}$$

where the overbar indicates an average value.

ENCLOSURE INSERTION LOSS

Can be defined in terms of the reduction in sound power radiated

$$IL_W = 10\log_{10}\frac{W_S}{W_R} = SWL_S - SWL_R$$

If it is assumed that $\bar{\tau} << \bar{\alpha}$ (as is usually the case in practice, e.g. at low frequencies $\bar{\tau} \approx 0.01, \bar{\alpha} \approx 0.2$)

$$\frac{W_R}{W_S} \approx \frac{\bar{\tau}}{\bar{\alpha}}$$

Thus the expression for the insertion loss reduces to

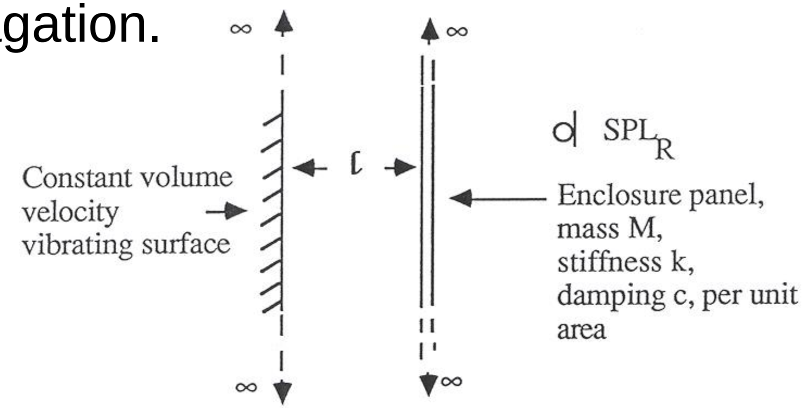
$$IL_W \approx 10\log_{10}\left(\frac{1}{\bar{\tau}}\right) + 10\log_{10}\bar{\alpha}$$

$$\boxed{IL_W \approx R_{\mathrm{field}} + 10\log_{10}\bar{\alpha}}$$

where R_{field} is the field S.R.I. Thus note that the maximum insertion loss is equal to the (field incidence) sound reduction index of the enclosure material and this is obtained when $\bar{\alpha} = 1$.

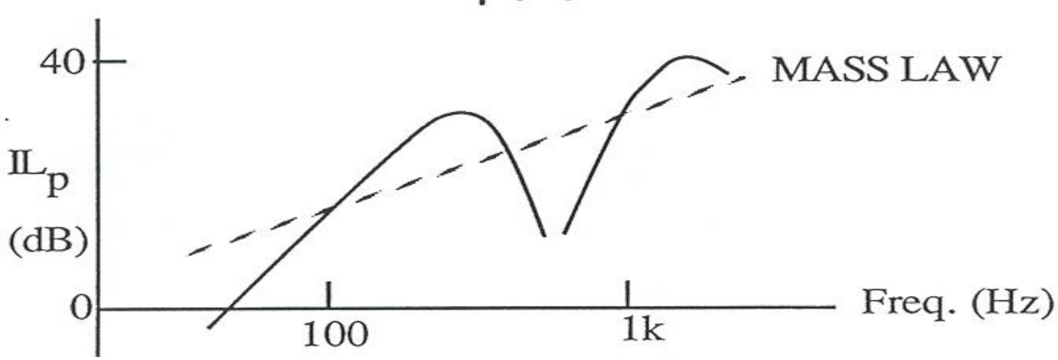
CLOSE FITTING ENCLOSURES: JACKSON'S THEORY

Jackson (1962, 1966) considers a model of an infinite plane vibrating surface parallel to an infinite plane enclosure surface – assumes only plane wave propagation.



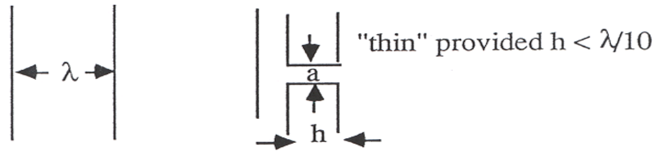
Expression given for enclosure insertion loss SPL_S - SPL_R

$$IL_p = 10\log_{10}\left\{\left(1+\frac{c}{\rho_0 c_0}\right)^2 \sin^2 kL + \left[\left(\frac{\omega M - k/\omega}{\rho_0 c_0}\right)\sin kL - \cos kL\right]^2\right\}$$



SOUND TRANSMISSION THROUGH APERTURES

The transmission coefficient has been deduced for plane wave incidence on "thick" or "thin" apertures.



For thin apertures

$$\lambda << a,\ \ \lambda \approx a,\ \ SRI \approx 0\,\mathrm{dB}$$

$\lambda >> a,$ Theories of Lamb (1931), Rayleigh (1877), Gomperts (1964), Wilson and Soroka (1965), Boukamp (1954), *all predict*

$$\mathrm{SRI} \approx 1.5\,\mathrm{dB}$$

Experiments of Mulholland and Parbrook (1967), and theory of Cook (1957), predict

$$\mathrm{SRI} \approx 1.5\,\mathrm{dB}$$

For thick apertures

SRI is highly frequency-dependent; approximate theory due to Wilson and Soroka (1965) Experimental results for a circular aperture between two reverberant chambers