

## Problem 1 - Cut-on Frequencies in Ducts and Pipes

### Problem 1a

The lowest cut-on frequency for a rectangular duct with air flow is given by equation,

$$f_{\text{cut-on}} = 0.5 \cdot \frac{c}{L} \quad (1)$$

where  $c$  is the speed of sound in air,  $343 \frac{\text{m}}{\text{s}}$ , and  $L$  is the largest side of the rectangular cross-section.

With cross-sectional dimensions of  $L_x = 12 \text{ cm}$  and  $L_y = 20 \text{ cm}$ , the lowest cut-on frequency for this rectangular duct is,

$$f_{\text{cut-on}} = 0.5 \cdot \frac{343 \frac{\text{m}}{\text{s}}}{0.20 \text{ m}} = \mathbf{857.5 \text{ Hz}}$$

### Problem 1b

The lowest cut-on frequency for a circular duct with air flow with the same cross-sectional area as the rectangular duct in part (a.) can be calculated using equation,

$$f_{\text{cut-on}} = 0.568 \cdot \frac{c}{d} \quad (2)$$

where  $c$  is the speed of sound in air,  $343 \frac{\text{m}}{\text{s}}$ , and  $d$  is diameter of the circular duct.

The cross-sectional area of the rectangular duct is,

$$\text{Area}_{\text{rectangular duct}} = 0.12 \text{ m} \cdot 0.20 \text{ m} = 0.024 \text{ m}^2$$

The corresponding diameter for this area is,

$$\text{diameter} = \sqrt{\frac{0.024 \text{ m}^2}{\pi}} \cdot 2 = 0.17 \text{ m}$$

Using Eq. 2, the lowest cut-on frequency for this circular duct with air flow is,

$$f_{\text{cut-on}} = 0.568 \cdot \frac{1,500 \frac{\text{m}}{\text{s}}}{0.17 \text{ m}} = \mathbf{1,114.5 \text{ Hz}}$$

### Problem 1c

The lowest cut-on frequency for this circular duct with water flow can be calculated using Eq. 2,

$$f_{\text{cut-on}} = 0.568 \cdot \frac{1,500 \frac{\text{m}}{\text{s}}}{0.17 \text{ m}} = \mathbf{4,873.9 \text{ Hz}}$$

The lowest cut-on frequency for water is considerable larger than it is for air flow.

### Problem 1d

The speed of sound in air is calculated by,

$$c = \sqrt{\gamma \cdot R \cdot T_K} \quad (3)$$

where  $\gamma = 1.4$  is the ratio of specific heats,  $R = 287 \frac{\text{J}}{\text{kg} \cdot \text{K}}$  is the gas constant, and  $T_K$  is the absolute temperature in Kelvin.

Figure 1 illustrates how the lowest cut-on frequency changes as the air heats from  $0^\circ$  to  $500^\circ$  Celsius.

The square-root relationship between temperature and the speed of sound in air is apparent and governs the behaviour of the cut-on frequency.

Lowest Cut-on Frequency for a Circular Pipe with Air Flow Versus Air Temperature

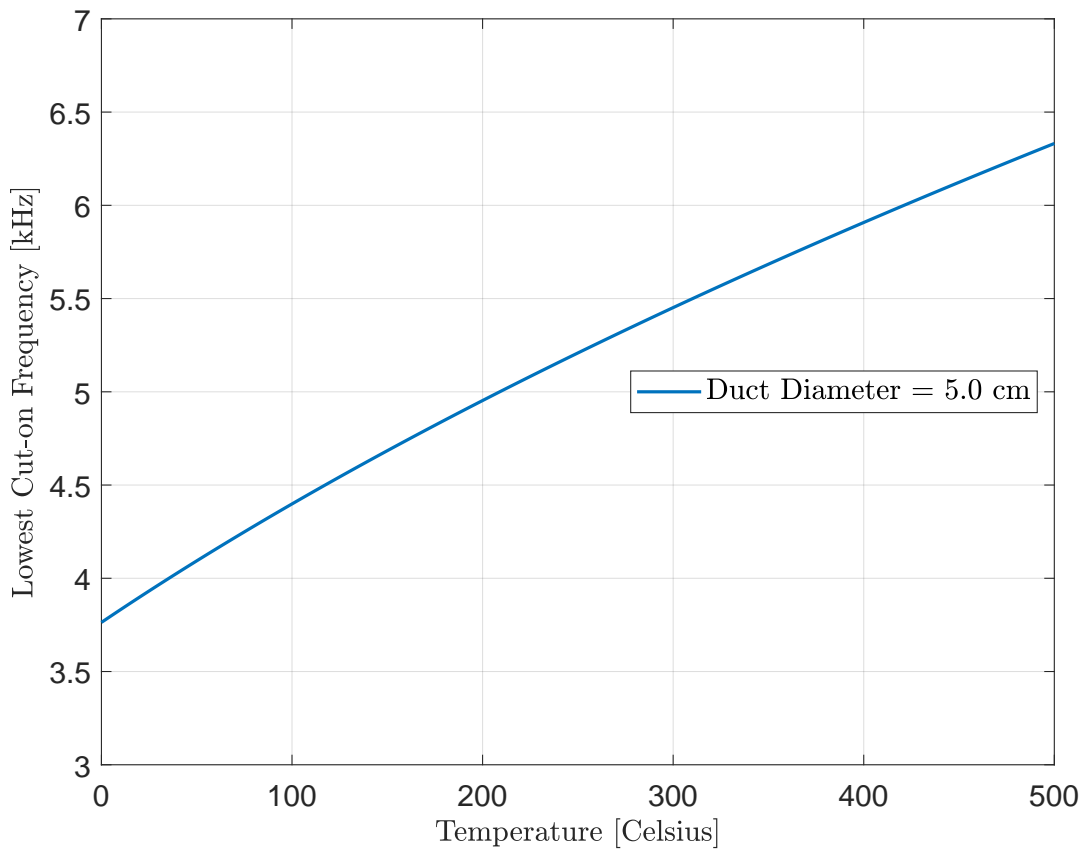


Figure 1: Lowest cut-on frequency for a circular 5 cm diameter duct versus air temperature.

### Problem 1e

**Question:** Are cut-on frequencies higher for a circular or rectangular duct for a given cross-sectional area?

The lowest cut-on frequency is higher for a circular duct than for a rectangular duct for a given cross-sectional area.

For the dimensions given in class, the rectangular duct is not square. This produces a larger dimension and thus a smaller, lowest cut-on frequency. If the rectangular duct is square dimensions on the order of the circular duct diameter with the same cross-sectional area, the cut-on frequencies are approximately equal.

**Question: What about in air versus water?**

The lowest cut-on frequency is larger for water than for air. The cut-on frequency is proportional to the speed of sound and the speed of sound in water is greater than the speed of sound in air.

**Question: What about cold versus hot air?**

The lowest cut-on frequency is higher for warm air than it is for cold air.

## **Problem 2 - Muffler Design Comparison**

Assumptions:

No resistive terms (i.e.,  $R_A$  is zero). No flow.

There is no damping in the system; resonances will be artificially high.

End corrections; no physical meaning.

Peak occur at a quarter wavelength; extension tube at outlet.

1 foot or 0.3048 meters.

Extension tube aids quarter wavelenths.

**Problem 2a**

**Problem 2b**

**Problem 2c**

### Problem 3 - Bugle Recorder

Diameters of holes should be smaller than a wavelength.

$R_A$  is neglected (energy loss).

#### Problem 3a

#### Problem 3b

## Problem 4 - Intake Duct

Problem 4a

Problem 4b

Problem 4c

Problem 4d

## Problem 5 - Intake Duct Silencer

Problem 5a

Problem 5b

Problem 5c

Problem 5d

Problem 5e

## Appendix A: Matlab Code - Problem 1

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### %% Synopsis

% Homework Set 1 — Cut-on Frequencies in Ducts and Pipes

% Note: Send draft of report before submission for comments.

%

% Dimensions are annotated in the class notes.

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### %% To Do

% Focus on interpretation.

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### %% Environment

```
close all; clear; clc;
% restoredefaultpath;

% set( 0, 'DefaultFigurePosition', [ 400 400 900 400 ] ); % [ left bottom width
    height ]
set( 0, 'DefaultFigurePaperPositionMode', 'manual' );
set( 0, 'DefaultFigureWindowStyle', 'normal' );
set( 0, 'DefaultLineLineWidth', 1.5 );
set( 0, 'DefaultTextInterpreter', 'Latex' );

format ShortG;

pause( 1 );

PRINT_FIGURES = 0;
```

---

### %% Define Values and Functions

```
c_air = 343; % The speed of sound in air in meters per second.
c_water = 1500; % The speed of sound in water in meters per second.

gamma = 1.4; % The ratio of specific heats [unitless].
R = 287; % The gas constant [Joules per ( kilogram * Kelvin)].

h_f_cut_on_rectangular_duct = @( c, L ) 0.5 .* c ./ L;
%
% c — The speed of sound.
% L — The largest cross-section dimension of the rectangular duct.

h_f_cut_on_circular_duct = @( c, d ) 0.568 .* c ./ d;
```



```
%
% c — The speec of sound.
% L — The diameter of the circular duct.

h_speed_of_sound_in_air = @( gamma, R, temperature_Kelvin) sqrt( gamma .* R .*
    temperature_Kelvin );
```

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### %% Problem 1a

```
% The cross-sectional dimensions for the rectangular duct are: Lx = 12 cm and Ly = 20
    cm.

% The largest dimension is Ly = 20 cm or 0.2 m.

% The cut-on frequency is,
h_f_cut_on_rectangular_duct( c_air, 0.2 ); % 857.5 Hz (shown in class 858 Hz)
    fprintf( 1, '\n Problem 1a: The lowest cut-on frequency for the rectangular pipe
        with air is %3.1f Hz.\n', h_f_cut_on_rectangular_duct( c_air, 0.2 ) );
```

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### %% Problem 1b

```
% The cross-sectional dimensions for the rectangular duct are: Lx = 12 cm and Ly = 20
    cm.

% The cross-sectional area of the rectangular duct is 12 cm * 20 cm = 240 cm^2 or 0.024
    m^2.
rectangular_duct_cross_sectional_area = 0.12 * 0.20; % 0.024 m^2

% The diameter of the circulat pipe is,
circular_duct_diameter = sqrt( 0.024 / pi ) * 2; % 0.17481 meters
%
% Check:
    % pi * ( circular_duct_diameter / 2 )^2 CHECKED

% The cut-on frequency for the circular duct is,
h_f_cut_on_circular_duct( c_air, circular_duct_diameter ); % 1,114.5 Hz
    fprintf( 1, '\n Problem 1b: The lowest cut-on frequency for the circular pipe (of
        equal area) with air is %3.1f Hz.\n', h_f_cut_on_circular_duct( c_air,
        circular_duct_diameter ) );
```

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### %% Problem 1c

```
% The cut-on frequency for the circular duct with water is,
h_f_cut_on_circular_duct( c_water, circular_duct_diameter ); % 4,873.9 Hz
    fprintf( 1, '\n Problem 1c: The lowest cut-on frequency for the circular pipe (of
        equal area) with water is %3.1f Hz.\n', h_f_cut_on_circular_duct( c_water,
```

```

circular_duct_diameter ) );

% The cut-on frequency should be higher because it is proportional to the
% speed of sound in a given medium.



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%% Problem 1d

fprintf( 1, '\n Problem 1d: See the figure.\n' );

temperature_range_celsius = 0:0.1:500; % Celsius
temperature_range_kelvin = temperature_range_celsius + 273.15; % Kelvin

FONT_SIZE = 14;

figure( ); ...
    plot( temperature_range_celsius, h_f_cut_on_circular_duct( h_speed_of_sound_in_air(
gamma, R, temperature_range_kelvin ), 0.05 ) ./ 1e3 ); grid on;
    legend( 'Duct Diameter = 5.0 cm', 'Location', 'East', 'FontSize', FONT_SIZE, '
Interpreter', 'Latex' );
    set( gca, 'FontSize', FONT_SIZE );
%
xlabel( 'Temperature [Celsius]', 'FontSize', FONT_SIZE );
    % xl = get( gca, 'xlabel' );    pxl = get( xl, 'position' );    pxl( 2 ) = 1.1 *
pxl( 2 );
    %    set( xl, 'position', pxl );
%
ylabel( 'Lowest Cut-on Frequency [kHz]', 'FontSize', FONT_SIZE );
    % yl = get( gca, 'ylabel' );    pyl = get( yl, 'position' );    pyl( 1 ) = 1.2 * pyl
( 1 );
    %    set( yl, 'position', pyl );
%
caption = sprintf( 'Lowest Cut-on Frequency for a Circular Pipe with Air Flow Versus
Air Temperature\n' );
    title( caption, 'FontSize', FONT_SIZE );
%
ylim( [ 3 7 ] );

% if ( PRINT_FIGURES == 1 )
%     saves( gcf, 'Cut-on Frequency Versus Temperature – Sunday, January 19, 2025.pdf'
);
% end

```

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#### %% Problem 1e

```

fprintf( 1, '\n Problem 1e: See Section Problem 1e of the Matlab script for the answers
.\n\n' );

```

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## %% Clean-up

```
if ( ~isempty( findobj( 'Type', 'figure' ) ) )
    monitors = get( 0, 'MonitorPositions' );
    if ( size( monitors, 1 ) == 1 )
        autoArrangeFigures( 2, 2, 1 );
    elseif ( 1 < size( monitors, 1 ) )
        autoArrangeFigures( 2, 2, 1 );
    end
end

if ( PRINT_FIGURES == 1 )
    saveas( gcf, 'Cut-on Frequency Versus Temperature – Sunday, January 19, 2025.pdf' );
end

fprintf( 1, '\n\n\n*** Processing Complete ***\n\n\n' );
```

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## %% Reference(s)