Subject: Noise Control Applications - Module 1 Assignement

Date: February 6, 2025 (Submitted)

Problem 1 - Cut-on Frequencies in Ducts and Pipes

The Matlab code for this problem is listed in Appendix 1.

Problem 1a

The lowest cut-on frequency for a rectangular duct with air flow is given by equation,

$$f_{cut-on} = 0.5 \cdot \frac{c}{L} \tag{1}$$

where c is the speed of sound in air, 343 $\frac{m}{s}$, and L is the largest side of the rectangular cross-section.

With cross-sectional dimensions of L_x = 12 cm and L_y = 20 cm, the lowest cut-on frequency for this rectangular duct is,

$$f_{cut-on} = 0.5 \cdot \frac{343 \ \frac{m}{s}}{0.20 \ m} = \mathbf{857.5 \ Hz}$$

Problem 1b

The lowest cut-on frequency for a circular duct with air flow with the same cross-sectional area as the rectangular duct in part (a.) can be calculated using equation,

$$f_{cut-on} = 0.568 \cdot \frac{c}{d} \tag{2}$$

where c is the speed of sound in air, 343 $\frac{m}{s}$, and d is diameter of the circular duct.

The cross-sectional area of the rectangular duct is,

$$\text{Area}_{\text{ rectangular duct}} = 0.12 \ m \ \cdot 0.20 \ m = 0.024 \ m$$

The corresponding diameter for this area is,

$$\text{diameter} = \sqrt{\frac{0.24 \ m^2}{\pi}} \cdot 2 = 0.17 \ m$$

Using Eq. 2, the lowest cut-on frequency for this circular duct with air flow is,

$$f_{cut-on} = 0.568 \cdot \frac{1,500 \ \frac{m}{s}}{0.17 \ m} = \mathbf{1,114.5 \ Hz}$$

Problem 1c

The lowest cut-on frequency for this circular duct with water flow can be calculated using Eq. 2,

$$\mathrm{f}_{\mathrm{cut-on}} = 0.568 \cdot \frac{1,500\ \frac{\mathrm{m}}{\mathrm{s}}}{0.17\ \mathrm{m}} = \mathbf{4,873.9\ Hz}$$

The lowest cut-on frequency for water is considerable larger than it is for air flow.

Problem 1d

The speed of sound in air is calculated by,

$$c = \sqrt{\gamma \cdot R \cdot T_K} \tag{3}$$

where $\gamma = 1.4$ is the ratio of specific heats, $R = 287 \frac{J}{kg \cdot K}$ is the gas constant, and T_K is the absolute temperature in Kelvin.

Figure 1 illustrates how the lowest cut-on frequency changes as the air heats from 0° to 500° Celsius.

The square-root relationship between temperature and the speed of sound in air is apparent and governs the behaviour of the cut-on frequency.

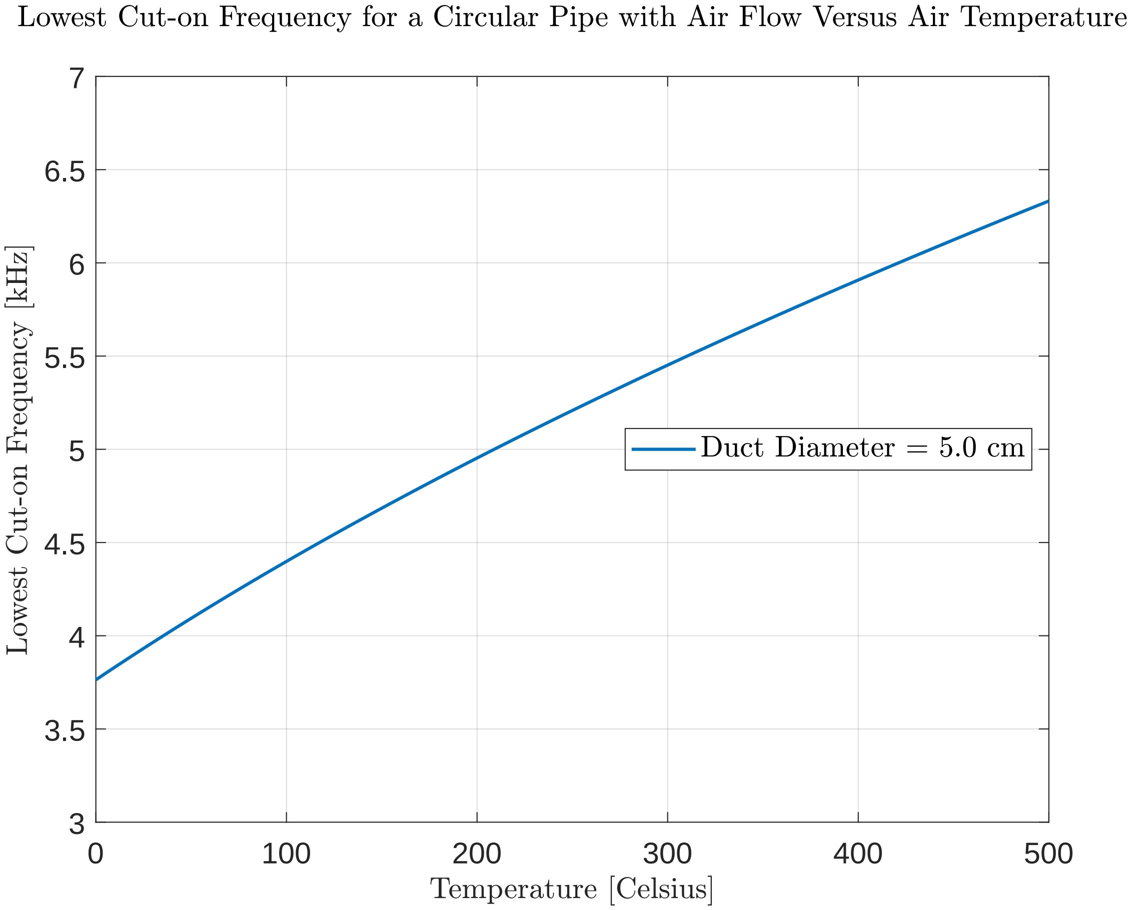

Figure 1: Lowest cut-on frequency for a circular 5 cm diameter duct versus air temperature.

Problem 1e

Question: Are cut-on frequencies higher for a circular or rectangular duct for a given cross-sectional area?

The lowest cut-on frequency is higher for a circular duct than for a rectangular duct for a given cross-sectional area.

For the dimensions given in class, the rectangular duct is not square. This produces a larger dimension and thus a smaller, lowest cut-on frequency. If the rectangular duct is square dimensions on the order of the circular duct diameter with the same cross-sectional area, the cut-on frequencies are approximately equal.

Question: What about in air versus water?

The lowest cut-on frequency is larger for water than for air. The cut-on frequency is proportional to the speed of sound and the speed of sound in water is greater than the speed of sound in air.

Question: What about cold versus hot air?

The lowest cut-on frequency is higher for warm air than it is for cold air.

Problem 2 - Muffler Design Comparison

The Matlab code for this problem is listed in Appendix 2.

For these muffler comparisons, it is assumed that there is no resistive terms and no flow. Since transmission losses are considered, the end corrections (i.e., load impedance at outlet of the system) have no physical meaning and are not accounted for in the computations.

Problems 2a, 2b, and 2c

Figure 2 shows the transmission loss profiles for a simple expansion chamber, a double-tuned expansion chamber, and a cascaded double-tuned expansion chamber muffler.

The peaks for the simple expansion chamber (red, dashed line) are approximately 22 dB and occur at frequencies with a wavelength that is a quarter of the length of the expansion chamber. Minimal loss occurs at half wavelength multiples.

The addition of the extension tube inside the muffler produces a quarter wavelength resonator. The side branch of Ji (2005; Slide 11, Lecture 3 notes) was used to calculate L_o. For the cascaded double-tuned expansion chamber, the extension tubes produce a secondary quarter wavelength resonator.

As noted in the office hours session, there is no damping which produces artificially high resonances.

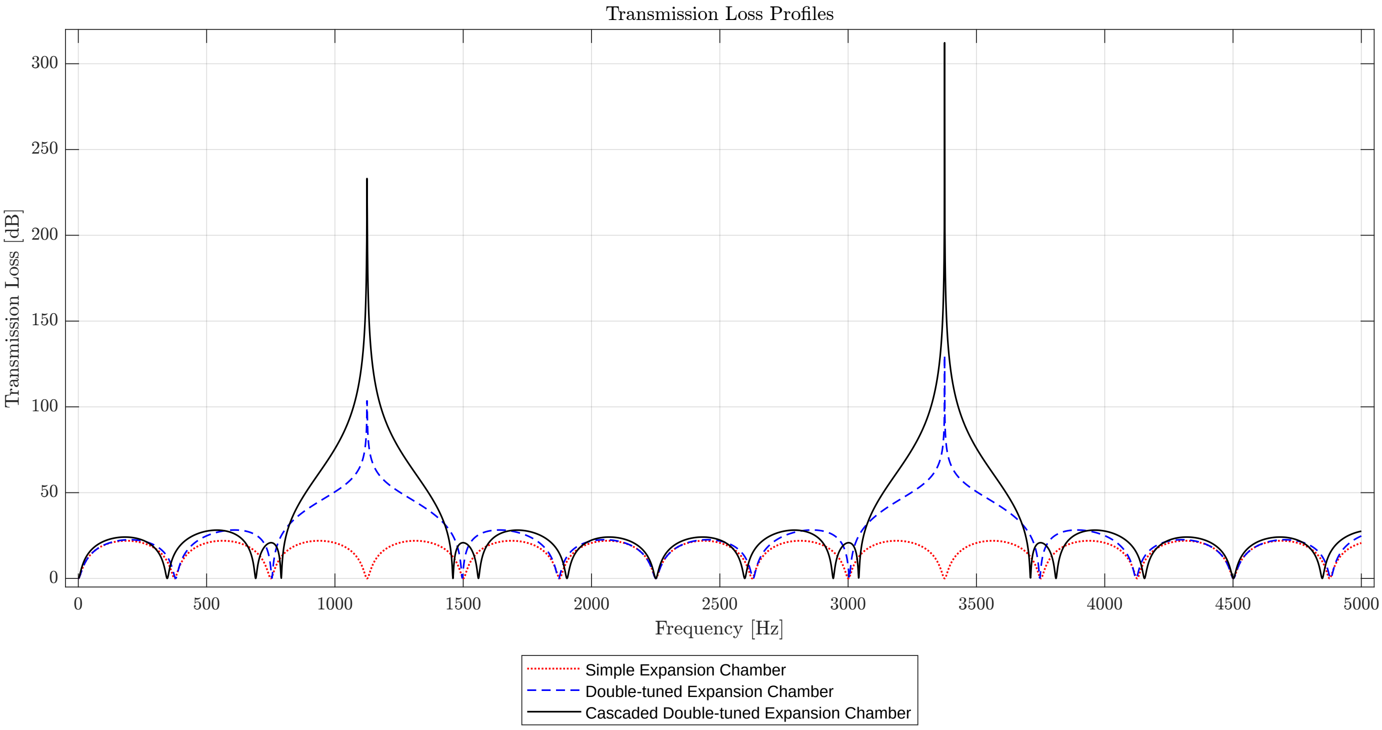

Figure 2: Transmission loss profiles for a simple expansion chamber, a double-tuned expansion chamber, and a cascaded double-tuned expansion chamber mufflers.

Problem 2d

Figure 3 shows the transmission loss profiles for a cascaded double-tuned expansion chamber, and a modified cascaded double-tuned expansion chamber muffler.

Two modifications were made to the original system:

1. The left 3" extension tube in the left chamber was shortened to 2" inches, making the respective muffler section 1" longer.

2. The left 3" extension tube in the right chamber was lengthened to 4", making the respective muffler section 1" shorter.

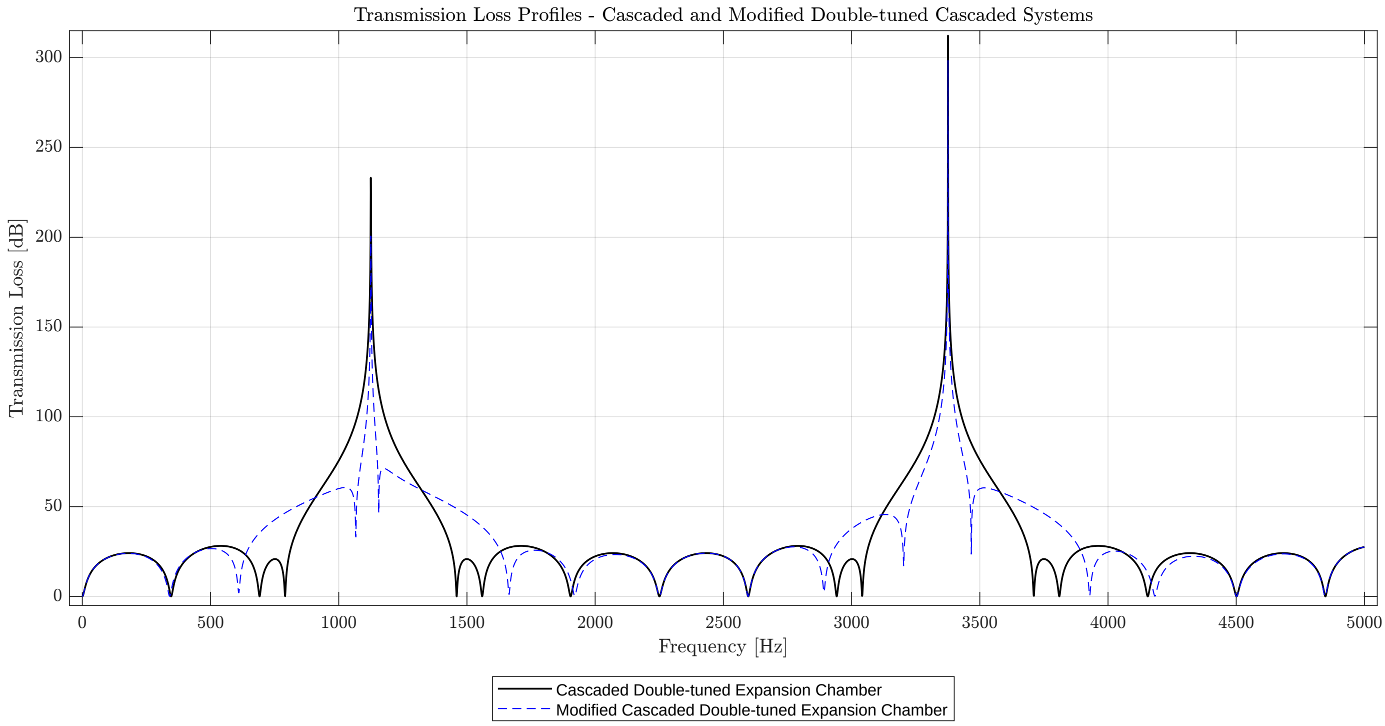

Figure 3: Transmission loss profiles for a cascaded double-tuned expansion chamber muffler and a modified version of this muffler.

These modifications change the symmetry of the cascaded system, and allow the resonate frequencies to be independently changed.

Problem 3 - Bugle Recorder

Diameters of holes should be smaller than a wavelength.

R_A is neglected (energy loss).

Problem 3a

Problem 3b

Problem 4 - Intake Duct

Problem 4a

Problem 4b

Problem 4c

Problem 4d

Problem 5 - Intake Duct Silencer

Problem 5a

Problem 5b

Problem 5c

Problem 5d

Problem 5e

1 Appendix - Matlab Code for Problem 1

```
%% Synopsis

% Question 1 − Cut−on Frequencies in Ducts and Pipes


%% Environment

close all; clear; clc;
% restoredefaultpath;

% addpath( genpath( '' ), '−begin' );
addpath( genpath( '../40 Assignments/00 Support' ), '−begin' );

% set( 0, 'DefaultFigurePosition', [  400  400  900  400  ] ); % [ left bottom width height ]
set( 0, 'DefaultFigurePaperPositionMode', 'manual' );
set( 0, 'DefaultFigureWindowStyle', 'normal' );
set( 0, 'DefaultLineLineWidth', 1.5 );
set( 0, 'DefaultTextInterpreter', 'Latex' );

format ShortG;

pause( 1 );

PRINT_FIGURES = 0;


%% Define Constants and Anonymous Functions

c_air = 343;  % The speed of sound in air (meters per second).
c_water = 1500;  % The speed of sound in water (meters per second).

gamma = 1.4;  % The ratio of specific heats [unitless].
R = 287;  % The gas constant [Joules per ( kilogram * Kelvin)].


h_f_cut_on_rectangular_duct = @( c, L )  0.5 .* c ./ L;
%
% c − The speed of sound.
% L − The largest cross−section dimension of the rectangular duct.


h_f_cut_on_circular_duct = @( c, d )  0.568 .* c ./ d;
%
% c − The speec of sound.
% L − The diameter of the circular duct.


h_speed_of_sound_in_air = @( gamma, R, temperature_Kelvin)  sqrt( gamma .* R .*
    temperature_Kelvin );


%% Problem 1a

% The cross−sectional dimensions for the rectangular duct are:  Lx = 12 cm and Ly = 20 cm.

% The largest dimension is Ly = 20 cm or 0.2 m.

% The cut−on frequency is ,
h_f_cut_on_rectangular_duct( c_air, 0.2 );  % 857.5 Hz (shown in class 858 Hz)
    fprintf( 1, '\n Problem 1a:  The lowest cut−on frequency for the rectangular pipe with air is
     %3.1f Hz.\n', h_f_cut_on_rectangular_duct( c_air, 0.2 ) );


%% Problem 1b

% The cross−sectional dimensions for the rectangular duct are:  Lx = 12 cm and Ly = 20 cm.

% The cross−sectional area of the rectangular duct is 12 cm * 20 cm = 240 cm^2 or 0.024 m^2.
rectangular_duct_cross_sectional_area = 0.12 * 0.20;  % 0.024 m^2
```

```
% The diameter of the circulat pipe is,
circular_duct_diameter = sqrt( 0.024 / pi ) * 2;  % 0.17481 meters
%
% Check:
    % pi * ( circular_duct_diameter / 2 )^2  CHECKED


% The cut-on frequency for the circular duct is,
h_f_cut_on_circular_duct( c_air, circular_duct_diameter );  % 1,114.5 Hz
    fprintf( 1, '\n Problem 1b:  The lowest cut-on frequency for the circular pipe (of equal area
    ) with air is %3.1f Hz.\n', h_f_cut_on_circular_duct( c_air, circular_duct_diameter ) );
```

%% Problem 1c

```
% The cut-on frequency for the circular duct with water is,
h_f_cut_on_circular_duct( c_water, circular_duct_diameter );  % 4,873.9 Hz
    fprintf( 1, '\n Problem 1c:  The lowest cut-on frequency for the circular pipe (of equal area
    ) with water is %3.1f Hz.\n', h_f_cut_on_circular_duct( c_water, circular_duct_diameter ) );

% The cut-on frequency should be higher because it is proportional to the
% speed of sound in a given medium.
```

%% Problem 1d

```
fprintf( 1, '\n Problem 1d:  See the figure.\n' );

temperature_range_celsius = 0:0.1:500;  % Celsius
    temperature_range_kelvin = temperature_range_celsius + 273.15;  % Kelvin


FONT_SIZE = 14;

figure( ); ...
    plot( temperature_range_celsius, h_f_cut_on_circular_duct( h_speed_of_sound_in_air( gamma, R,
     temperature_range_kelvin ), 0.05 ) ./ 1e3 );  grid on;
        legend( 'Duct Diameter = 5.0 cm', 'Location', 'East', 'FontSize', FONT_SIZE, 'Interpreter
    ', 'Latex' );
        set( gca, 'FontSize', FONT_SIZE );
    %
    xlabel( 'Temperature [Celsius]', 'FontSize', FONT_SIZE );
        % xl = get( gca, 'xlabel' );    pxl = get( xl, 'position' );  pxl( 2 ) = 1.1 * pxl( 2 );
        %     set( xl, 'position', pxl );
    %
    ylabel( 'Lowest Cut-on Frequency [kHz]', 'FontSize', FONT_SIZE );
        % yl = get( gca, 'ylabel' );  pyl = get( yl, 'position' );  pyl( 1 ) = 1.2 * pyl( 1 );
        %     set( yl, 'position', pyl );
    %
    caption = sprintf( 'Lowest Cut-on Frequency for a Circular Pipe with Air Flow Versus Air
    Temperature\n' );
        title( caption, 'FontSize', FONT_SIZE );
    %
    ylim( [ 3  7 ] );
```

%% Problem 1e

```
fprintf( 1, '\n Problem 1e:  See Section Problem 1e of the Matlab script for the answers.\n\n' );


% Question:  Are cut-on frequencies higher for a circular or rectangular duct for a given cross-
    sectional area?

% The lowest cut-on frequency is higher for a circular duct than for a
% rectangular duct for a given cross-sectional area.

% For the dimensions given in class, the rectangular duct is not square.
% This produces a larger dimension and thus a smaller, lowest cut-on
% frequency.

% If the rectangular duct is square dimensions on the order of the circular
% duct diameter with the same cross-sectional area, the the cut-on
% frequencies are approximately equal.
```

```
% Question:  What about in air versus water?

% The lowest cut-on frequency is larger with water than air.  This due to
% the fact that the cut-on frequency is proportional to the speed of sound
% and the speed of sound in water is greater than it is in air.


% Question:  What about cold versus hot air?

% For a circular pipe, the cut-on frequency is higher in warm air than cold
% air.
```

```
%% Clean-up

if ( ~isempty( findobj( 'Type', 'figure' ) ) )
    monitors = get( 0, 'MonitorPositions' );
        if ( size( monitors, 1 ) == 1 )
            autoArrangeFigures( 2, 2, 1 );
        elseif ( 1 < size( monitors, 1 ) )
            autoArrangeFigures( 2, 2, 1 );
        end
end


if ( PRINT_FIGURES == 1 )
    saveas( gcf, 'Cut-on Frequency Versus Temperature - Sunday, January 19, 2025.pdf' );
end


fprintf( 1, '\n\n\n*** Processing Complete ***\n\n\n' );
```

```
%% Reference(s)
```

2 Appendix - Matlab Code for Problem 2

```
%% Synopsis

% Question 2 — Muffler Design Comparison


%% Environment

close all; clear; clc;
% restoredefaultpath;

% addpath( genpath( '' ), '-begin' );
addpath( genpath( '../00 Support' ), '-begin' );

% set( 0, 'DefaultFigurePosition', [ 400 400 900 400 ] ); % [ left bottom width height ]
set( 0, 'DefaultFigurePaperPositionMode', 'manual' );
set( 0, 'DefaultFigureWindowStyle', 'normal' );
set( 0, 'DefaultLineLineWidth', 1.5 );
set( 0, 'DefaultTextInterpreter', 'Latex' );

format ShortG;

pause( 1 );

PRINT_FIGURES = 0;


%% Constants

rho0 = 1.21; % Air density (kg per m^3).
c = 343; % Speed of sound in air (meters per second).

frequency_set = 0:1:5e3; % Hertz


%% Dimensions

convert.inches_to_meters = 0.0254;
convert.foot_to_meters = 0.3048;

dimensions.inlet_diameter_meters = 2 * convert.inches_to_meters; % 0.0508 meters
dimensions.inlet_length_meters = 6 * convert.foot_to_meters; % 1.82 meters

dimensions.muffler_diameter_meters = 10 * convert.inches_to_meters; % 0.254 meters
dimensions.muffler_length_meters = 18 * convert.inches_to_meters; % 0.4572 meters

dimensions.outlet_diameter_meters = 2 * convert.inches_to_meters; % 0.0508 meters
dimensions.outlet_length_meters = 1 * convert.foot_to_meters; % 0.3048 meters

outlet_flanged = false;

dimensions.overhang = 3 *convert.inches_to_meters; % 0.0762 meters

segment_diameters = [ ...
    dimensions.outlet_diameter_meters, ...
    dimensions.muffler_diameter_meters, ...
    dimensions.inlet_diameter_meters, ...
    ].';
%
h_area_from_diameter = @( d ) pi .* d.^2 ./ 4;
%
segment_areas = h_area_from_diameter( segment_diameters );

segment_lengths = [ ...
    dimensions.outlet_length_meters, ...
    dimensions.muffler_length_meters, ...
    dimensions.inlet_length_meters, ...
    dimensions.overhang, ...
    ].';
```

```
%% Part a - Simple Expansion Chamber

nFreq = length( frequency_set );
    TL = zeros( nFreq, 1 );

for frequency_index = 1:1:nFreq

    f = frequency_set( frequency_index );

    T_total = [ 1 0; 0 1 ];

    T1 = duct_segment_transfer_matrix( f, rho0, c, 0.3048, 0.0020268 );  % Duct - Outlet
    T2 = duct_segment_transfer_matrix( f, rho0, c, 0.4572, 0.050671 );  % Duct
    T3 = duct_segment_transfer_matrix( f, rho0, c, 1.8288, 0.0020268 );  % Duct - Inlet

    T_net = T3 * T2 * T1 * T_total;
    % T_net = T_inlet *  T_total;  % Zero transmission loss for a straight duct.

    T11 = T_net(1, 1);  T12 = T_net(1, 2);  T21 = T_net(2, 1);  T22 = T_net(2, 2);
        TL( frequency_index ) = 10 * log10( abs( ( T11 +  0.0020268*T12/(rho0*c) + (rho0*c)*
    T21/0.0020268 + T22 ) / 2 )^2 );
        %
        % The transmission loss calculation does not require a load impedance.

end

TL_parta = TL;  % The maximum peak value should be about 22 (21.952) dB.
%
% Expected behaviour:
%
%   1.)  0 dB at 0 Hz.
%   2.)  The transmission loss of a straight duct section is zero;  energy out equals energy in.

max( TL_parta );  % 22 dB


%% Part b - Double-tuned Expansion Chamber

annulus_area_squared_meters = pi/4 * ( 0.254^2 - 0.0508^2 );

L_o = 0;  % Assume that the Lo extension is neglible.


nFreq = length( frequency_set );
    TL = zeros( nFreq, 1 );

for frequency_index = 1:1:nFreq

    f = frequency_set( frequency_index );

    T_total = [ 1 0; 0 1 ];

    T1 = duct_segment_transfer_matrix( f, rho0, c, (0.3048 + 0.0762), 0.0020268 );
    T3 = duct_segment_transfer_matrix( f, rho0, c, (0.4572 - 2*0.0762), 0.050671 );
    T5 = duct_segment_transfer_matrix( f, rho0, c, (1.8288 + 0.0762), 0.0020268 );

    k = 2*pi*f/c;
        Z_A = -1j*rho0*c/annulus_area_squared_meters*cot(k * ( 0.0762 + L_o ) );
            T2 = [ 1  0;  1/Z_A  1 ];
                T4 = T2;

    T_net = T5 * T4 * T3 * T2 * T1 * T_total;


    T11 = T_net(1, 1);  T12 = T_net(1, 2);  T21 = T_net(2, 1);  T22 = T_net(2, 2);
        TL( frequency_index ) = 10 * log10( abs( ( T11 + 0.0020268*T12/(rho0*c) + (rho0*c)*
    T21/0.0020268 + T22 ) / 2 )^2 );
        %
        % The transmission loss calculation does not require a load impedance.

end

TL_partb = TL;
%
% Expected behaviour:
```

```
%
%   1.)  0 dB at 0 Hz.
%   2.)  0 dB at same locations as a simple expansion chamber.
%   3.)  Peaks at 1,125 Hz and 3,376 Hz;

% Frequency at which the quarter-wavelength is 0.0762 meters.
% 343 / ( 4 * 0.0762 );  % 1,125 Hz.

% Also work at three-quarter-wavelength.
% 3 * 1125;  % 3,375 Hz
```

```
%% Part c - Cascaded, Double-tuned Expansion Chamber

annulus_area_squared_meters = pi/4 * ( 0.254^2 - 0.0508^2 );

L_o = 0;  % Assume that the Lo extension is neglible.


nFreq = length( frequency_set );
    TL = zeros( nFreq, 1 );

for frequency_index = 1:1:nFreq

    f = frequency_set( frequency_index );
        k = 2*pi*f/c;
            Z_A = -1j*rho0*c/annulus_area_squared_meters*cot(k * ( 0.0762 + L_o ) );

    T_total = [ 1 0; 0 1 ];

    T1 = duct_segment_transfer_matrix( f, rho0, c, (0.3048 + 0.0762), 0.0020268 );  % Duct -
    Outlet
    T2 = [ 1  0;  1/Z_A  1 ];  % Straight Side Branch
    T3 = duct_segment_transfer_matrix( f, rho0, c, (0.2286 - 2*0.0762), 0.050671 );  % Duct
    T4 = T2;  % Straight Side Branch
    T5 = duct_segment_transfer_matrix( f, rho0, c, 2*0.0762, 0.050671 );  % Duct
    T6 = T2;  % Straight Side Branch
    T7 = T3;  % Duct
    T8 = T2;  % Straight Side Branch
    T9 = duct_segment_transfer_matrix( f, rho0, c, (1.8288 + 0.0762), 0.0020268 );  % Duct -
    Inlet

    T_net = T9 * T8 * T7 * T6 * T5 * T4 * T3 * T2 * T1 * T_total;

    T11 = T_net(1, 1);  T12 = T_net(1, 2);  T21 = T_net(2, 1);  T22 = T_net(2, 2);
        TL( frequency_index ) = 10 * log10( abs( ( T11  +  0.0020268*T12/(rho0*c)  +  (rho0*c)*
    T21/0.0020268  +  T22 ) / 2 )^2 );

end

TL_partc = TL;
```

```
%% Part d - Cascaded, Double-tuned Expansion Chamber

nFreq = length( frequency_set );
    TL = zeros( nFreq, 1 );

for frequency_index = 1:1:nFreq

    f = frequency_set( frequency_index );
        k = 2*pi*f/c;
            Z_A = -1j*rho0*c/annulus_area_squared_meters*cot(k * ( 0.0762 + L_o ) );

    T_total = [ 1 0; 0 1 ];

    T1 = duct_segment_transfer_matrix( f, rho0, c, (0.3048 + 0.0762), 0.0020268 );  % Duct -
    Outlet
    T2 = [ 1  0;  1/Z_A  1 ];  % Straight Side Branch
    T3 = duct_segment_transfer_matrix( f, rho0, c, 0.0076, 0.050671 );  % Duct
    T4 = T2;  % Straight Side Branch
    T5 = duct_segment_transfer_matrix( f, rho0, c, 2*0.0762, 0.050671 );  % Duct
    T6 = T2;  % Straight Side Branch
    T7 = duct_segment_transfer_matrix( f, rho0, c, (0.29718 - 2*0.0762), 0.050671 );  % Duct
    T8 = T2;  % Straight Side Branch
    T9 = duct_segment_transfer_matrix( f, rho0, c, (1.8288 + 0.0762), 0.0020268 );  % Duct -
```

```
    Inlet

    T_net = T9 * T8 * T7 * T6 * T5 * T4 * T3 * T2 * T1 * T_total;

    T11 = T_net(1, 1);  T12 = T_net(1, 2);  T21 = T_net(2, 1);  T22 = T_net(2, 2);
        TL( frequency_index ) = 10 * log10( abs( ( T11  +  0.0020268*T12/(rho0*c)  +  (rho0*c)*
    T21/0.0020268  +  T22 ) / 2 )^2 );

end

TL_partd = TL;


%% Plot Transmission Loss Profiles

Y_LIMITS = [ -5  320 ];

h_figure_1 = figure( ); ...
    plot( frequency_set, TL_parta, 'LineWidth', 1.0, 'LineStyle', ':', 'Color', 'r' );  hold on;
    plot( frequency_set, TL_partb, 'LineWidth', 0.9, 'LineStyle', '--', 'Color', 'b' );
    plot( frequency_set, TL_partc, 'LineWidth', 0.9, 'LineStyle', '-', 'Color', 'k' );  grid on;
    grid on;
        legend( ...
            'Simple Expansion Chamber', ...
            'Double-tuned Expansion Chamber', ...
            'Cascaded Double-tuned Expansion Chamber', ...
            'Location', 'SouthOutside' );
    xlabel( 'Frequency [Hz]' );  ylabel( 'Transmission Loss [dB]' );
    title( 'Transmission Loss Profiles' );
    %
    Ax = gca;
        Ax.XAxis.TickLabelInterpreter = 'latex';
        Ax.YAxis.TickLabelInterpreter = 'latex';
    %
    axis( [ -50  5e3+50  Y_LIMITS ] );


Y_LIMITS = [ -5  315 ];

h_figure_2 = figure( ); ...
    plot( frequency_set, TL_partc, 'LineWidth', 1.0, 'LineStyle', '-', 'Color', 'k' );  hold on;
    plot( frequency_set, TL_partd, 'LineWidth', 0.6, 'LineStyle', '--', 'Color', 'b' );  grid on;
        legend( ...
            'Cascaded Double-tuned Expansion Chamber', ...
            'Modified Cascaded Double-tuned Expansion Chamber', ...
            'Location', 'SouthOutside' );
    xlabel( 'Frequency [Hz]' );  ylabel( 'Transmission Loss [dB]' );
    title( 'Transmission Loss Profiles - Cascaded and Modified Double-tuned Cascaded Systems' );
    %
    Ax = gca;
        Ax.XAxis.TickLabelInterpreter = 'latex';
        Ax.YAxis.TickLabelInterpreter = 'latex';
    %
    axis( [ -50  5e3+50  Y_LIMITS ] );


%% Clean-up

if ( ~isempty( findobj( 'Type', 'figure' ) ) )
    monitors = get( 0, 'MonitorPositions' );
        if ( size( monitors, 1 ) == 1 )
            autoArrangeFigures( 2, 2, 1 );
        elseif ( 1 < size( monitors, 1 ) )
            autoArrangeFigures( 2, 2, 1 );
        end
end


if ( PRINT_FIGURES == 1 )
        exportgraphics( h_figure_1, 'Assignment 1 - Question 2 Figure All TL Profiles.pdf', '
    Append', true );
        exportgraphics( h_figure_2, 'Assignment 1 - Question 2 Figure Comparison TL Plot For
    Cascaded Systems.pdf', 'Append', true );
end
```

```
fprintf( 1, '\n\n\n*** Processing Complete ***\n\n\n' );
```

```
%% Reference(s)
```