Subject: ACS 597 - Module 1 Assignement

Date: January 20, 2025 (Submitted)

Problem 1a

The lowest cut-on frequency for a rectangular duct with air flow is given by equation,

$$f_{\mathrm{cut-on}} = 0.5 \cdot \frac{c}{L} \tag{1}$$

where c is the speed of sound in air, 343 $\frac{m}{s}$, and L is the largest side of the rectangular cross-section.

With cross-sectional dimensions of $L_x = 12$ cm and $L_y = 20$ cm, the lowest cut-on frequency for this rectangular duct is,

$$f_{\mathrm{cut-on}} = 0.5 \cdot \frac{343\ \frac{m}{s}}{0.20\ m} = \mathbf{857.5\ Hz}$$

Problem 1b

The lowest cut-on frequency for a circular duct with air flow with the same cross-sectional area as the rectangular duct in part (a.) can be calculated using equation,

$$f_{\mathrm{cut-on}} = 0.568 \cdot \frac{c}{d} \tag{2}$$

where c is the speed of sound in air, 343 $\frac{m}{s}$, and d is diameter of the circular duct.

The cross-sectional area of the rectangular duct is,

$$\text{Area}_{\text{ rectangular duct}} = 0.12\ m\ \cdot 0.20\ m = 0.024\ m$$

The corresponding diameter for this area is,

$$\text{diameter} = \sqrt{\frac{0.24\ m^2}{\pi}} \cdot 2 = 0.17\ m$$

Using Eq. 2, the lowest cut-on frequency for this circular duct with air flow is,

$$f_{\mathrm{cut-on}} = 0.568 \cdot \frac{1,500\ \frac{m}{s}}{0.17\ m} = \mathbf{1,114.5\ Hz}$$

Problem 1c

The lowest cut-on frequency for this circular duct with water flow can be calculated using Eq. 2,

$$f_{\mathrm{cut-on}} = 0.568 \cdot \frac{1,500\ \frac{m}{s}}{0.17\ m} = \mathbf{4,873.9\ Hz}$$

The lowest cut-on frequency for water is considerable larger than it is for air flow.

Problem 1d

The speed of sound in air is calculated by,

$$c = \sqrt{\gamma \cdot R \cdot T_K} \tag{3}$$

where $\gamma = 1.4$ is the ratio of specific heats, $R = 287\ \frac{J}{kg \cdot K}$ is the gas constant, and T_K is the absolute temperature in Kelvin.

Figure 1 illustrates how the lowest cut-on frequency changes as the air heats from 0° to 500° Celsius.

The square-root relationship between temperature and the speed of sound in air is apparent and governs the behaviour of the cut-on frequency.

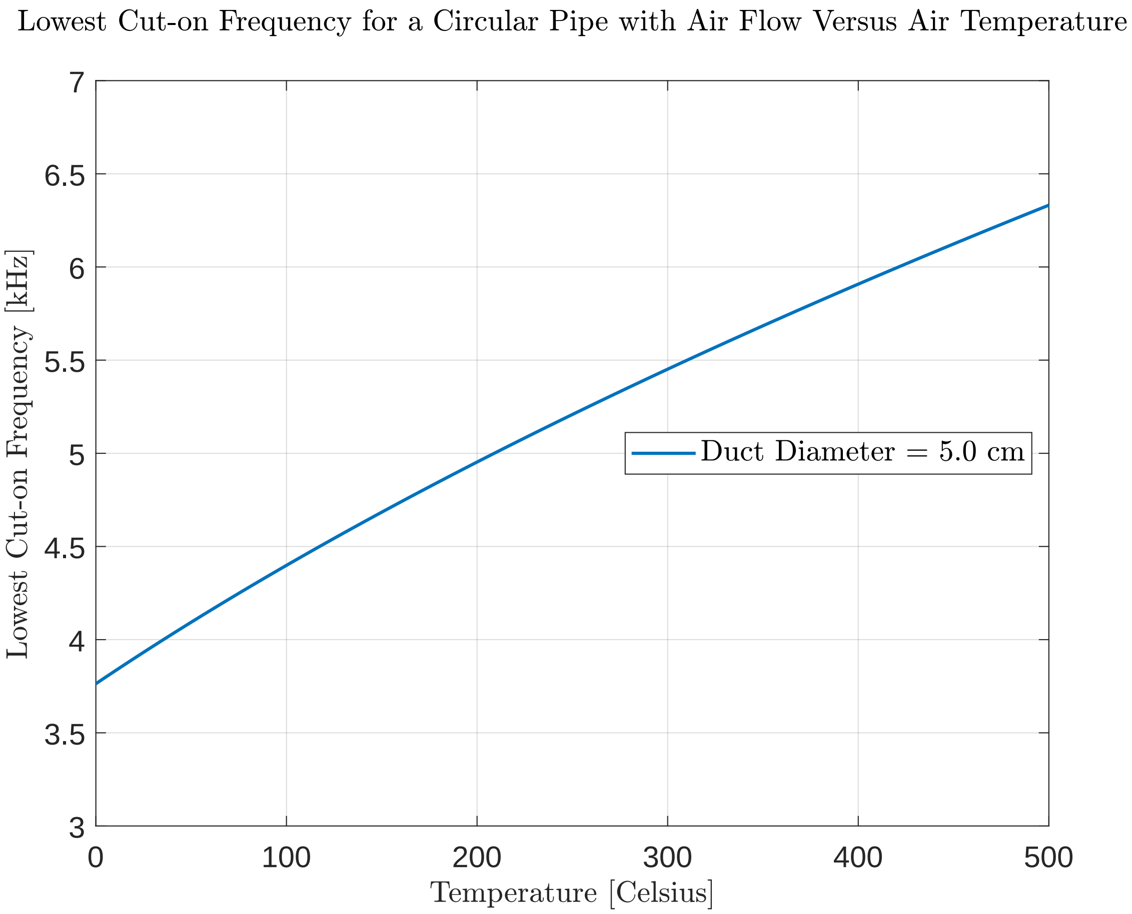

Figure 1: Lowest cut-on frequency for a circular 5 cm diameter duct versus air temperature.

Problem 1e

Question: Are cut-on frequencies higher for a circular or rectangular duct for a given cross-sectional area?

The lowest cut-on frequency is higher for a circular duct than for a rectangular duct for a given cross-sectional area.

For the dimensions given in class, the rectangular duct is not square. This produces a larger dimension and thus a smaller, lowest cut-on frequency. If the rectangular duct is square dimensions on the order of the circular duct diameter with the same cross-sectional area, the cut-on frequencies are approximately equal.

Question: What about in air versus water?

The lowest cut-on frequency is larger for water than for air. The cut-on frequency is proportional to the speed of sound and the speed of sound in water is greater than the speed of sound in air.

Question: What about cold versus hot air?

The lowest cut-on frequency is higher for warm air than it is for cold air.