Subject: ACS 597 - Module 1 Assignement

Date: January 22, 2025 (Submitted)

Problem 1a

The lowest cut-on frequency for a rectangular duct with air flow is given by equation,

$$\mathrm{f_{cut-on}} = 0.5 \cdot \frac{\mathrm{c}}{\mathrm{L}} \tag{1}$$

where c is the speed of sound in air, 343 $\frac{\mathrm{m}}{\mathrm{s}}$, and L is the largest side of the rectangular cross-section.

With cross-sectional dimensions of $\mathrm{L_x} = 12$ cm and $\mathrm{L_y} = 20$ cm, the lowest cut-on frequency for this rectangular duct is,

$$\mathrm{f_{cut-on}} = 0.5 \cdot \frac{343\ \frac{\mathrm{m}}{\mathrm{s}}}{0.20\ \mathrm{m}} = \mathbf{857.5\ Hz}$$

Problem 1b

The lowest cut-on frequency for a circular duct with air flow with the same cross-sectional area as the rectangular duct in part (a.) can be calculated using equation,

$$\mathrm{f_{cut-on}} = 0.568 \cdot \frac{\mathrm{c}}{\mathrm{d}} \tag{2}$$

where c is the speed of sound in air, 343 $\frac{\mathrm{m}}{\mathrm{s}}$, and d is diameter of the circular duct.

The cross-sectional area of the rectangular duct is,

$$\mathrm{Area}_{\ \mathrm{rectangular\ duct}} = 0.12\ \mathrm{m}\ \ \cdot 0.20\ \mathrm{m} = 0.024\ \mathrm{m}$$

The corresponding diameter for this area is,

$$\mathrm{diameter} = \sqrt{\frac{0.24\ \mathrm{m}^2}{\pi}} \cdot 2 = 0.17\ \mathrm{m}$$

Using Eq. 2, the lowest cut-on frequency for this circular duct with air flow is,

$$\mathrm{f_{cut-on}} = 0.568 \cdot \frac{1,500\ \frac{\mathrm{m}}{\mathrm{s}}}{0.17\ \mathrm{m}} = \mathbf{1,114.5\ Hz}$$

Problem 1c

The lowest cut-on frequency for this circular duct with water flow can be calculated using Eq. 2,

$$\mathrm{f_{cut-on}} = 0.568 \cdot \frac{1,500\ \frac{\mathrm{m}}{\mathrm{s}}}{0.17\ \mathrm{m}} = \mathbf{4,873.9\ Hz}$$

The lowest cut-on frequency for water is considerable larger than it is for air flow.

Problem 1d

The speed of sound in air is calculated by,

$$c = \sqrt{\gamma \cdot R \cdot T_K} \tag{3}$$

where $\gamma = 1.4$ is the ratio of specific heats, $R = 287\ \frac{J}{kg \cdot K}$ is the gas constant, and T_K is the absolute temperature in Kelvin.

Figure 1 illustrates how the lowest cut-on frequency changes as the air heats from 0° to 500° Celsius.

The square-root relationship between temperature and the speed of sound in air is apparent and governs the behaviour of the cut-on frequency.

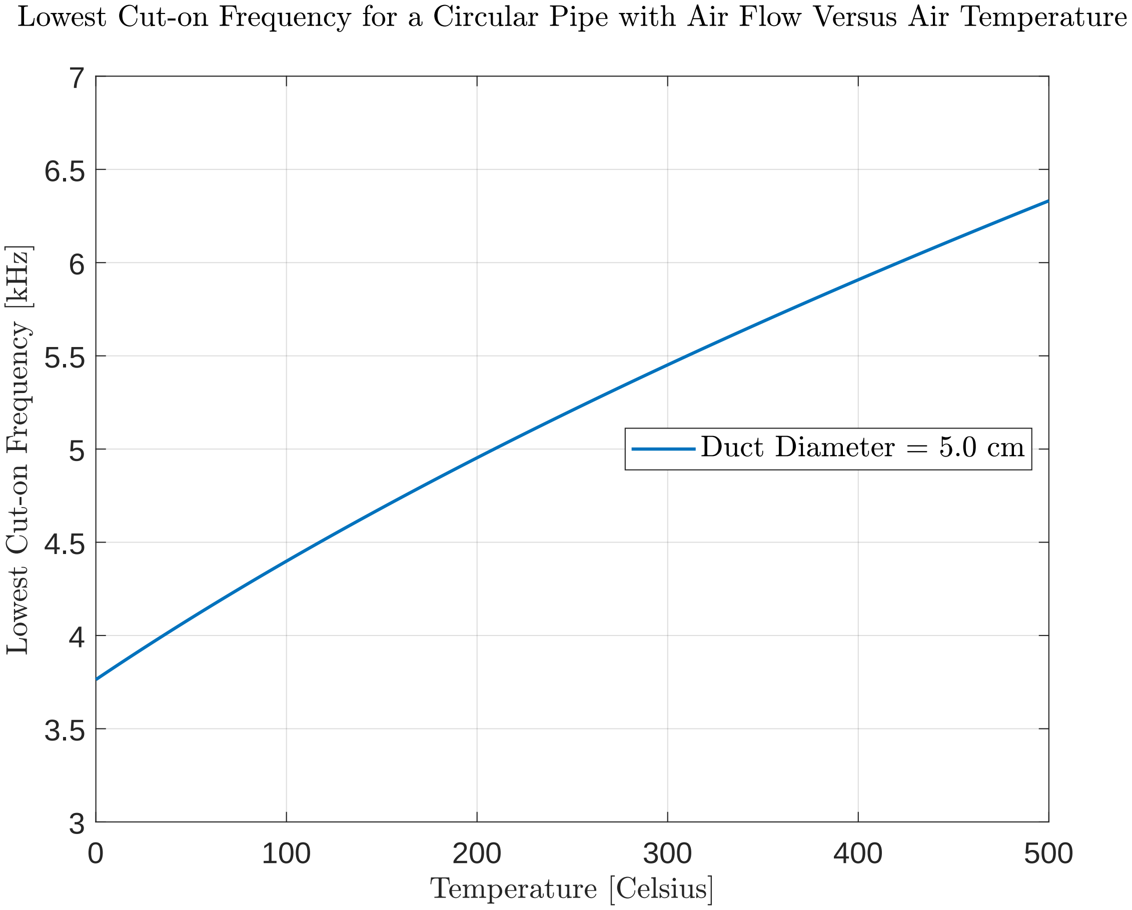

Figure 1: Lowest cut-on frequency for a circular 5 cm diameter duct versus air temperature.

Problem 1e

Question: Are cut-on frequencies higher for a circular or rectangular duct for a given cross-sectional area?

The lowest cut-on frequency is higher for a circular duct than for a rectangular duct for a given cross-sectional area.

For the dimensions given in class, the rectangular duct is not square. This produces a larger dimension and thus a smaller, lowest cut-on frequency. If the rectangular duct is square dimensions on the order of the circular duct diameter with the same cross-sectional area, the cut-on frequencies are approximately equal.

Question: What about in air versus water?

The lowest cut-on frequency is larger for water than for air. The cut-on frequency is proportional to the speed of sound and the speed of sound in water is greater than the speed of sound in air.

Question: What about cold versus hot air?

The lowest cut-on frequency is higher for warm air than it is for cold air.

Appendix A: Matlab Code

```
%% Synopsis

% Homework Set 1 — Cut—on Frequencies in Ducts and Pipes

% Note:  Send draft of report before submission for comments.
%
% Dimensions are annontated in the class notes.



%% To Do

% Focus on interpretation.



%% Environment

close all; clear; clc;
% restoredefaultpath;

% set( 0, 'DefaultFigurePosition', [  400  400  900  400  ] );  % [ left bottom width
    height ]
set( 0, 'DefaultFigurePaperPositionMode', 'manual' );
set( 0, 'DefaultFigureWindowStyle', 'normal' );
set( 0, 'DefaultLineLineWidth', 1.5 );
set( 0, 'DefaultTextInterpreter', 'Latex' );

format ShortG;

pause( 1 );

PRINT_FIGURES = 0;



%% Define Values and Functions

c_air = 343;  % The speed of sound in air in meters per second.
c_water = 1500;  % The speed of sound in water in meters per second.

gamma = 1.4;  % The ratio of specific heats [unitless].
R = 287;  % The gas constant [Joules per ( kilogram * Kelvin)].


h_f_cut_on_rectangular_duct = @( c, L )  0.5 .* c ./ L;
%
% c — The speed of sound.
% L — The largest cross—section dimension of the rectangular duct.


h_f_cut_on_circular_duct = @( c, d )  0.568 .* c ./ d;
```

```
%
% c — The speec of sound.
% L — The diameter of the circular duct.


h_speed_of_sound_in_air = @( gamma, R, temperature_Kelvin)  sqrt( gamma .* R .*
    temperature_Kelvin );
```

```
%% Problem 1a

% The cross-sectional dimensions for the rectangular duct are:  Lx = 12 cm and Ly = 20
    cm.

% The largest dimension is Ly = 20 cm or 0.2 m.

% The cut-on frequency is,
h_f_cut_on_rectangular_duct( c_air, 0.2 );  % 857.5 Hz (shown in class 858 Hz)
    fprintf( 1, '\n Problem 1a:  The lowest cut-on frequency for the rectangular pipe
    with air is %3.1f Hz.\n', h_f_cut_on_rectangular_duct( c_air, 0.2 ) );
```

```
%% Problem 1b

% The cross-sectional dimensions for the rectangular duct are:  Lx = 12 cm and Ly = 20
    cm.

% The cross-sectional area of the rectangular duct is 12 cm * 20 cm = 240 cm^2 or 0.024
    m^2.
rectangular_duct_cross_sectional_area = 0.12 * 0.20;  % 0.024 m^2

% The diameter of the circulat pipe is,
circular_duct_diameter = sqrt( 0.024 / pi ) * 2;  % 0.17481 meters
%
% Check:
    % pi * ( circular_duct_diameter / 2 )^2  CHECKED


% The cut-on frequency for the circular duct is,
h_f_cut_on_circular_duct( c_air, circular_duct_diameter );  % 1,114.5 Hz
    fprintf( 1, '\n Problem 1b:  The lowest cut-on frequency for the circular pipe (of
    equal area) with air is %3.1f Hz.\n', h_f_cut_on_circular_duct( c_air,
    circular_duct_diameter ) );
```

```
%% Problem 1c

% The cut-on frequency for the circular duct with water is,
h_f_cut_on_circular_duct( c_water, circular_duct_diameter );  % 4,873.9 Hz
    fprintf( 1, '\n Problem 1c:  The lowest cut-on frequency for the circular pipe (of
    equal area) with water is %3.1f Hz.\n', h_f_cut_on_circular_duct( c_water,
```

```
    circular_duct_diameter ) );

% The cut-on frequency should be higher because it is proportional to the
% speed of sound in a given medium.
```

```
%% Problem 1d

fprintf( 1, '\n Problem 1d:  See the figure.\n' );

temperature_range_celsius = 0:0.1:500;  % Celsius
    temperature_range_kelvin = temperature_range_celsius + 273.15;  % Kelvin


FONT_SIZE = 14;

figure( ); ...
    plot( temperature_range_celsius, h_f_cut_on_circular_duct( h_speed_of_sound_in_air(
    gamma, R, temperature_range_kelvin ), 0.05 ) ./ 1e3 );  grid on;
        legend( 'Duct Diameter = 5.0 cm', 'Location', 'East', 'FontSize', FONT_SIZE, '
    Interpreter', 'Latex' );
        set( gca, 'FontSize', FONT_SIZE );
    %
    xlabel( 'Temperature [Celsius]', 'FontSize', FONT_SIZE );
        % xl = get( gca, 'xlabel' );    pxl = get( xl, 'position' );  pxl( 2 ) = 1.1 *
    pxl( 2 );
        %     set( xl, 'position', pxl );
    %
    ylabel( 'Lowest Cut-on Frequency [kHz]', 'FontSize', FONT_SIZE );
        % yl = get( gca, 'ylabel' );  pyl = get( yl, 'position' );  pyl( 1 ) = 1.2 * pyl
    ( 1 );
        %     set( yl, 'position', pyl );
    %
    caption = sprintf( 'Lowest Cut-on Frequency for a Circular Pipe with Air Flow Versus
     Air Temperature\n' );
        title( caption, 'FontSize', FONT_SIZE );
    %
    ylim( [ 3  7 ] );


% if ( PRINT_FIGURES == 1 )
%     saveas( gcf, 'Cut-on Frequency Versus Temperature - Sunday, January 19, 2025.pdf'
    );
% end
```

```
%% Problem 1e

fprintf( 1, '\n Problem 1e:  See Section Problem 1e of the Matlab script for the answers
    .\n\n' );
```

```matlab
%% Clean-up

if ( ~isempty( findobj( 'Type', 'figure' ) ) )
    monitors = get( 0, 'MonitorPositions' );
        if ( size( monitors, 1 ) == 1 )
            autoArrangeFigures( 2, 2, 1 );
        elseif ( 1 < size( monitors, 1 ) )
            autoArrangeFigures( 2, 2, 1 );
        end
end


if ( PRINT_FIGURES == 1 )
    saveas( gcf, 'Cut-on Frequency Versus Temperature - Sunday, January 19, 2025.pdf' );
end


fprintf( 1, '\n\n\n*** Processing Complete ***\n\n\n' );
```

```matlab
%% Reference(s)
```