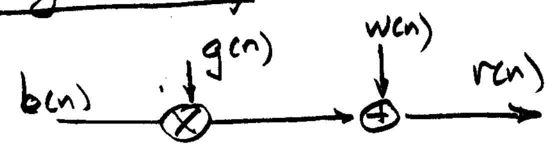
9.10 Sample Applications of Kalman Filters

9.10.1 Rayleigh Fading

- 

b data bit

g complex Gaussian gain

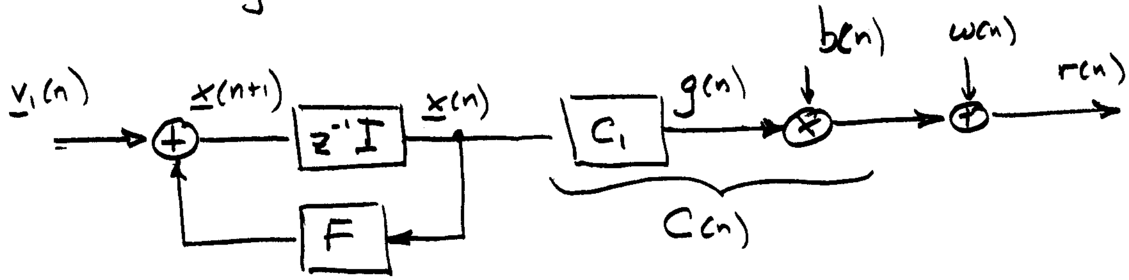
w white Gaussian noise

r observation.

Assume data is known.

Estimate $g(n)$ recursively from $r(1), \cdots, r(n)$

- Model: $g(n)$ process is ARMA



Identify:

$$\underline{v}_1(n) \longleftrightarrow \underline{v}_1(n)$$
$$\underline{x}(n) \longleftrightarrow \underline{x}(n)$$
$$C(n) \longleftrightarrow C_1\, b(n)$$
$$v_2(n) \longleftrightarrow w(n)$$
$$y(n) \longleftrightarrow r(n)$$

Then $\hat{g}(n|R_n) = C_1\,\hat{\underline{x}}(n|R_n)$

- If data is not known, then we can try every possible sequence, and $p_r(R_n|\underline{b}) \sim \sum_{i=1}^{n} |\alpha(i|R_{i-1}, \underline{b})|^2$

The sequence with smallest metric is the ML choice.

Do this recursively with Viterbi algorithm.

9.10.2 Omega Navigation System

This example is taken directly from A. Gelb, *Applied Optimal Estimation*, and transliterated into our notation. The technology is a little dated, but it's a good, simple example.

Omega is a world-wide navigation system, utilizing phase comparison of 10.2 kHz continuous-wave radio signals. The user employs a propagation correction, designed to account for globally predictable phenomena (diurnal effects, earth conductivity variations, etc.) to bring theoretical phase values into agreement with observed phase measurements. The residual Omega phase errors are known to exhibit correlation over large distances (e.g. > 1000 mi). Design a data processor that, operating in a limited geographic area, can process error data gathered at two measurement sites and infer best estimates of phase error at other locations nearby.

In the absence of other information, assume that the phase errors are well modelled by a zero mean Markov process in space, with variance σ_ϕ^2. Further assume that the phase error process is isotropic - i.e., that is posessed the same statistics in all directions - and that the measurement error is very small.

We proceed as follows. First, denote by ϕ_2 and ϕ_3 the two phase error measurements, and by ϕ_1 the phase error to be estimated. Then we can form $\boldsymbol{x}$ and $\boldsymbol{y}$ by

$$\mathbf{x}=\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\end{bmatrix} \qquad \mathbf{y}=\begin{pmatrix}\phi_2\\ \phi_3\end{pmatrix} \qquad \text{which makes} \quad \mathbf{C}=\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

Also, by definition

$$\mathbf{K}(1,0)=\sigma_\phi^{\,2}\cdot\begin{bmatrix}1 & \exp\left(-\frac{r_{12}}{d}\right) & \exp\left(-\frac{r_{13}}{d}\right)\\ \exp\left(-\frac{r_{12}}{d}\right) & 1 & \exp\left(-\frac{r_{23}}{d}\right)\\ \exp\left(-\frac{r_{13}}{d}\right) & \exp\left(-\frac{r_{23}}{d}\right) & 1\end{bmatrix}$$

where r_{ij} is the distance between sites i and j, and d is the correlation distance.

We'll take $\boldsymbol{x}(0)=\mathbf{0}$, and form the best estimate of its value after the measurement. This makes ***y_hat***(1|0)=**0**, and the innovation is then $\alpha(1)=\mathbf{y}(1)$. The updated estimate of the state is therefore

$$\mathbf{x\_hat}(1 \mid \mathbf{y}(1))=\mathbf{L}\cdot\alpha=\mathbf{L}\cdot\mathbf{y}(1)$$

where the Kalman gain matrix is

$$\mathbf{L}=\mathbf{K}(1,0)\cdot\overline{\left(\mathbf{C}^T\right)}\cdot\left(\mathbf{C}\cdot\mathbf{K}(1,0)\cdot\overline{\left(\mathbf{C}^T\right)}\right)^{-1}=\begin{pmatrix}a & b\\ 1 & 0\\ 0 & 1\end{pmatrix}$$

Note that the identity matrix in the lower part reflects the fact that the best estimates of ϕ_2 and ϕ_3 are themselves; that is, the measurements.

and we have defined

$$a = \frac{\left[\exp\left[\frac{-(-2\cdot r_{23}+r_{12})}{d}\right] - \exp\left[\frac{-(-r_{23}+r_{13})}{d}\right]\right]}{\exp\left(2\cdot\frac{r_{23}}{d}\right)-1}$$

These coefficients let us estimate ϕ_1 from ϕ_2 and ϕ_3 .

$$b = \frac{-\left[\exp\left[\frac{-(-r_{23}+r_{12})}{d}\right] - \exp\left[\frac{-(-2\cdot r_{23}+r_{13})}{d}\right]\right]}{\exp\left(2\cdot\frac{r_{23}}{d}\right)-1}$$

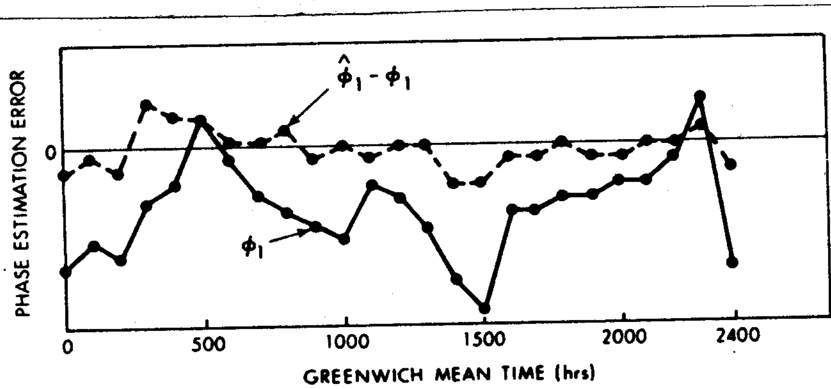


Figure 4.2-4 Phase Estimation Error – Kings Point, N.Y. (Ref. 14)

9.10.3 Inertial Navigation System (also taken directly from A. Gelb)

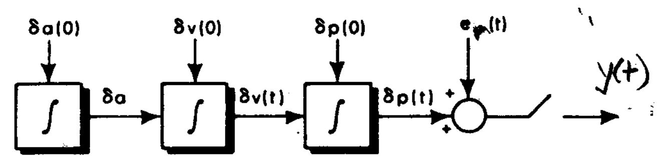
Study the design of an optimal data processing scheme by which an INS in a balloon-launched ionospheric probe can be initialized in early flight using radio position updates. In view of the intended application, it is appropriate to choose a model of INS error dynamics valid only over a several minute period. Taking single-axis errors into consideration, we may write

$$\delta p(t) = \delta p(0) + \delta v(0)\cdot t + \delta a(0)\cdot \frac{t^2}{2} \qquad \text{(approx)}$$

where $\delta p(0)$, $\delta v(0)$ and $\delta a(0)$ represent initial values of position, velocity and acceleration errors. The radio position error is denoted $e_r(t)$, and we make a scalar position measurement by subtracting the INS-indicated position from the radio-indicated position, giving

$$y(t) = \left(p_t + e_r(t)\right) - \left(p_t + \delta p(t)\right) = e_r(t) - \delta p(t) \qquad \text{(where } p_t \text{ is the true position)}$$

This results in the system and measurement model shown below



This model is described by the state vector at t, the state transition matrix from 0 to t and the measurement and noise covariance matrices

$$\mathbf{x}(t) = \begin{pmatrix} \delta p(t) \\ \delta v(t) \\ \delta a(t) \end{pmatrix} \qquad \mathbf{F}(t_2, t_1) = \begin{bmatrix} 1 & t_2 - t_1 & \frac{(t_2 - t_1)^2}{2} \\ 0 & 1 & t_2 - t_1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{C} = (-1 \;\; 0 \;\; 0)$$

$$\mathbf{Q}_1 = 0 \qquad \mathbf{v}_2(t) = e_r(t) \qquad \mathbf{Q}_2 = \sigma_r^2$$

If the first fix is at time T, then for any initial estimate of the errors ***x_hat***(0), we have

$$\mathbf{x\_hat}(T \mid 0) = \mathbf{F}(T,0)\cdot \mathbf{x\_hat}(0)$$

Assume that ***K***(0) is diagonal, for uncorrelated errors

$$\mathbf{K}(0) = \mathrm{diag}\left(\sigma_{p0}^2, \sigma_{v0}^2, \sigma_{a0}^2\right)$$

so that just before the radio fix, we have

$$\mathbf{K}(T,0) = \mathbf{F}(T,0)\cdot\mathbf{K}(0)\cdot\overline{\left(\mathbf{F}(T,0)^T\right)}$$

$$= \begin{bmatrix} \sigma_{p0}^2 + \sigma_{v0}^2\cdot T^2 + \sigma_{a0}^2\cdot\frac{T^4}{4} & \sigma_{v0}^2 + \sigma_{a0}^2\cdot\frac{T^3}{2} & \sigma_{a0}^2\cdot\frac{T^2}{2} \\ \sigma_{v0}^2 + \sigma_{a0}^2\cdot\frac{T^3}{2} & \sigma_{v0}^2 + \sigma_{a0}^2\cdot T^2 & \sigma_{a0}^2\cdot T \\ \sigma_{a0}^2\cdot\frac{T^2}{2} & \sigma_{a0}^2\cdot T & \sigma_{a0}^2 \end{bmatrix}$$ errors now correlated

To update INS errors from the radio fix, we use the Kalman gain

$$\mathbf{L}(T) = \mathbf{K}(T,0)\cdot\overline{\left(\mathbf{C}^T\right)}\cdot\left(\mathbf{C}\cdot\mathbf{K}(T,0)\cdot\overline{\left(\mathbf{C}^T\right)} + \mathbf{Q}_2\right)^{-1} = \begin{bmatrix} L_1(T) \\ L_2(T) \\ L_3(T) \end{bmatrix}$$

where the matrix to be inverted is just a scalar. Its components are

$$\mathbf{L}(T) = \frac{1}{\sigma_{p0}^2 + \sigma_{v0}^2\cdot T^2 + \frac{1}{4}\cdot\sigma_{a0}^2\cdot T^4 + \sigma_r^2}\cdot\begin{bmatrix} -\sigma_{p0}^2 - \sigma_{v0}^2\cdot T^2 - \frac{1}{4}\cdot\sigma_{a0}^2\cdot T^4 \\ -\sigma_{v0}^2 - \frac{1}{2}\cdot\sigma_{a0}^2\cdot T^3 \\ \frac{-1}{2}\cdot\sigma_{a0}^2\cdot T^2 \end{bmatrix}$$

Immediately after the fix, we update the covariance matrix to

$$\mathbf{K}(T) = (\mathbf{I} - \mathbf{L}(T)\cdot\mathbf{C})\cdot\mathbf{K}(T,0)$$

Substitution of the quantities on the right yields a bulky matrix. However, if T is small enough that the accumulated errors from velocity and acceleration are small, then the position error variance in $K(T)$ is approximately equal to the radio error variance σ_r^2.

Extrapolation to immediately before the next fix takes place in the usual way, and that fix nails the velocity error.

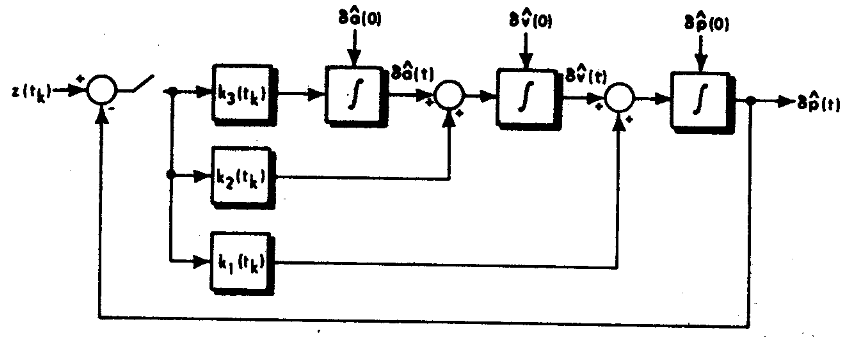
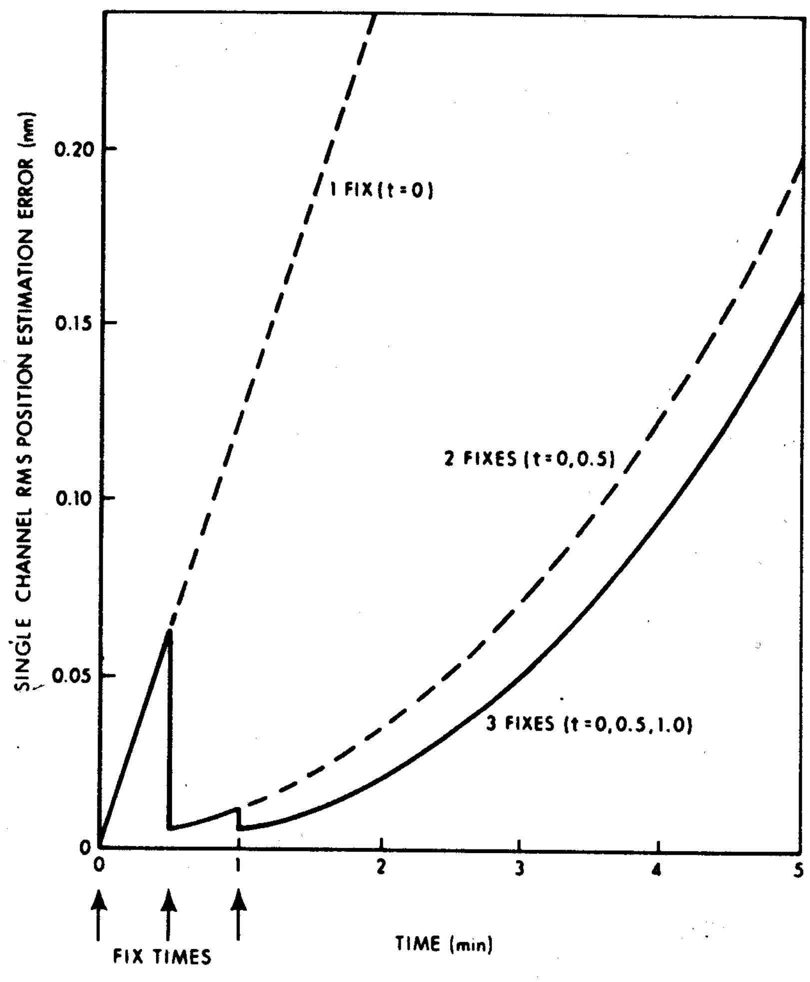


Figure 4.2-6 Optimal Filter Configuration



from J. V. Candy Signal Processing: The Modern Approach McGraw Hill 1998

We summarize the *Kalman filter estimator* as follows:

Criterion:	$J(t \mid t) = \text{trace } \tilde{P}(t \mid t)$
Models:	
Signal:	$x(t) = A(t-1)x(t-1) + B(t-1)u(t-1) + W(t-1)w(t-1)$
Measurement:	$y(t) = C(t)x(t) + v(t)$
Noise:	w and v are zero-mean and white with covariances R_{ww} and R_{vv}
Initial state:	$x(0)$ has mean $\hat{x}(0 \mid 0)$ and covariance $\tilde{P}(0 \mid 0)$
Algorithm (Kalman filter):	$\hat{x}(t \mid t) = \hat{x}(t \mid t-1) + K(t)e(t)$
Quality:	$\tilde{P}(t \mid t) = [I - K(t)C(t)]\tilde{P}(t \mid t-1)$

Kalman filters can be designed using the *tuning analysis procedure*, which is developed on sound theoretical grounds:

1. Innovations sequence is *zero-mean*.
2. Innovations sequence is *white*.
3. Innovations sequence is *uncorrelated* in time and with input u.
4. Innovations sequence *lies within* the confidence limits constructed from R_{ee} of the Kalman filter algorithm.
5. Innovations variance is *reasonably close* to the sample variance estimate $\hat{R}_{ee}$.
6. Tracking or estimation error *lies within* the confidence limits constructed from $\tilde{P}$ of the Kalman filter algorithm.
7. Error variance is *reasonably close* to the sample variance estimate $\hat{\tilde{P}}$.

Consider an application of model-based processing to an RLC circuit design problem.

Example 9.3–1. Consider the design of an estimator for a series RLC circuit (second-order system) excited by a pulse train [2]. The circuit diagram is shown in Fig. 9.3–3. Using Kirchhoff's voltage law, we can obtain the circuit equations with $i = C(de/dt)$:

$$\frac{d^2e}{dt^2} + \frac{R}{L}\frac{de}{dt} + \frac{1}{LC}e = \frac{1}{LC}e_{\text{in}}$$

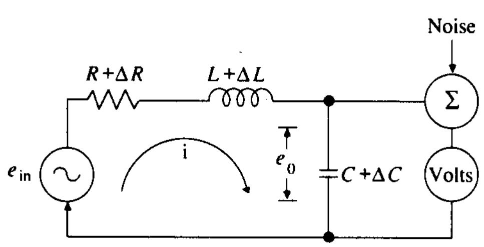


FIGURE 9.3–3
RLC circuit problem

where e_{in} is a unit pulse train. This equation is that of a second-order system which characterizes an electrical RLC circuit, or a mechanical vibration system, or a hydraulic flow system, etc. The dynamic equations can be placed in state-space form by choosing $x := [e \mid de/dt]'$ and $u = e_{in}$:

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix} x + \begin{bmatrix} 0 \\ -\frac{1}{LC} \end{bmatrix} u + \begin{bmatrix} 0 \\ -\frac{1}{LC} \end{bmatrix} w$$

where $w \sim N(0, R_{ww})$ is used to model component inaccuracies.

A high-impedance voltmeter is placed in the circuit to measure the capacitor voltage e. We assume that it is a digital (sampled) device contaminated with noise of variance R_{vv}; that is,

$$y(t) = e(t) + v(t)$$

where $v \sim N(0, R_{vv})$. For our problem we have the following parameters: $R = 5\ \text{k}\Omega$, $L = 2.5$ H, $C = 0.1\ \mu$F, and $T = 0.1$ ms (the problem will be scaled in milliseconds). We assume that the component inaccuracies can be modeled using $R_{ww} = 0.01$, characterizing a deviation of ± 0.1 V uncertainty in the circuit representation. Finally, we assume that the precision of the voltmeter measurements are ($e \pm 0.2$ V), the two standard deviation value, so that $R_{vv} = 0.01\ (\text{V})^2$. Summarizing the circuit model, we have the continuous-time representation

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ -4 \end{bmatrix} u + \begin{bmatrix} 0 \\ -4 \end{bmatrix} w$$

and the discrete-time measurements

$$y(t) = [1 \quad 0] x(t) + v(t)$$

where $$R_{ww} = \frac{(0.1)^2}{T} = 0.1(\text{V})^2 \quad \text{and} \quad R_{vv} = 0.01(\text{V})^2$$

Before we design the discrete Kalman estimator, we must convert the system or process model to a sampled-data (discrete) representation. Using SSPACK (see Appendix E, [10]), this is accomplished automatically with the Taylor-series approach to approximating the matrix exponential. For an error tolerance of 1×10^{-12}, a 15-term series expansion yields the following discrete-time Gauss-Markov model:

$$x(t) = \begin{bmatrix} 0.98 & 0.09 \\ -0.36 & 0.801 \end{bmatrix} x(t-1) + \begin{bmatrix} -0.019 \\ -0.36 \end{bmatrix} u(t-1) + \begin{bmatrix} -0.019 \\ -0.36 \end{bmatrix} w(t-1)$$

$$y(t) = [1 \quad 0] x(t) + v(t)$$

where $$R_{ww} = 0.1(\text{V})^2 \quad \text{and} \quad R_{vv} = 0.01(\text{V})^2$$

Using SSPACK with initial conditions $x(0) = 0$ and $P = \text{diag}\ (0.01, 0.04)$, the simulated system is depicted in Fig. 9.3–4. In Fig. 9.3–4(a) through (c) we see the simulated states and measurements with corresponding confidence limits about the mean (true) values. In each case, the simulation satisfies the statistical properties of the model. The corresponding true (mean) trajectories are also shown, along with the pulse-train excitation. Note that the measurements are merely a noisier version (process and measurement noise) of the voltage x_1.

9.10.9

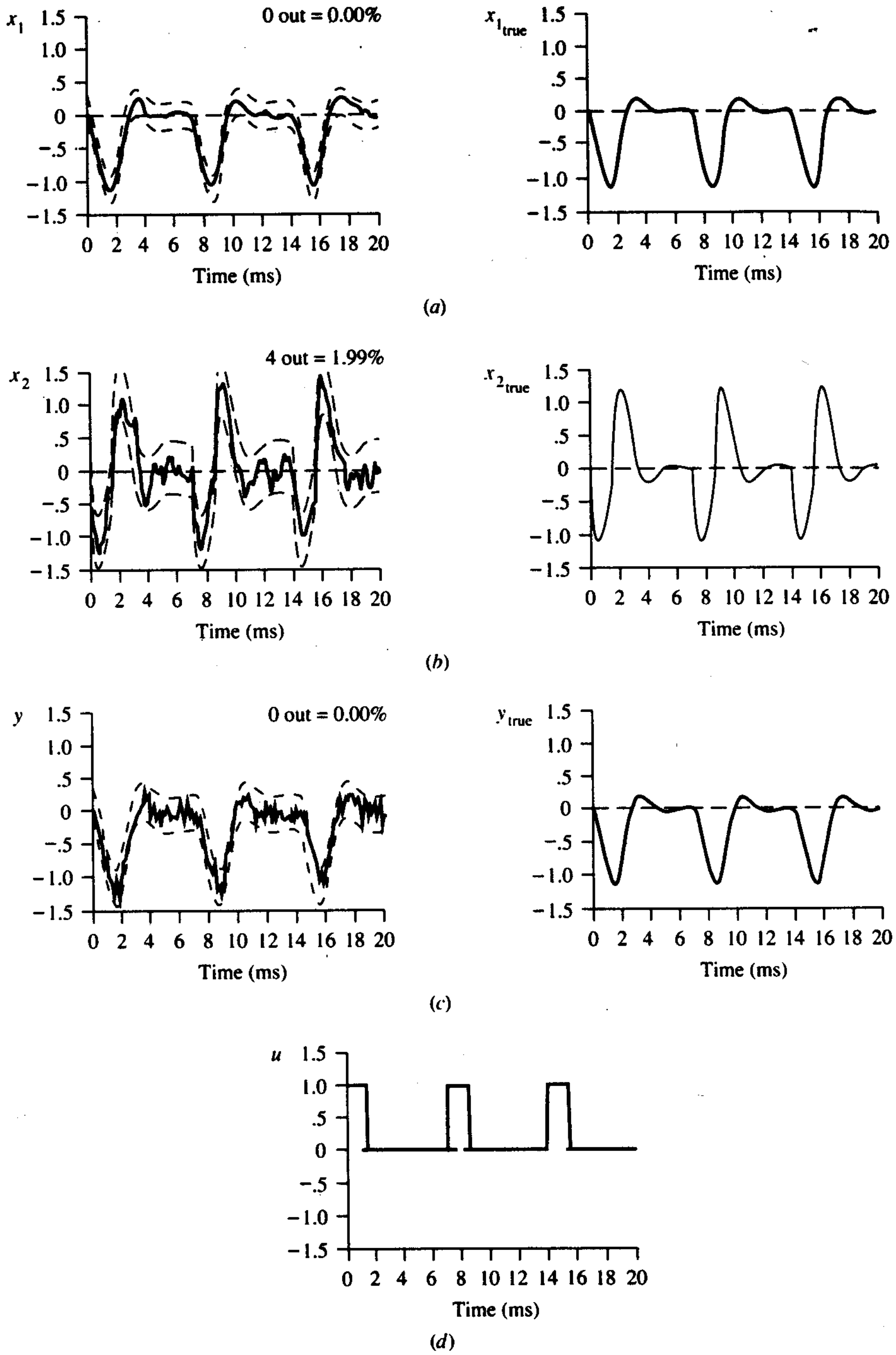


FIGURE 9.3-4
Gauss-Markov simulation of RLC circuit problem (*a*) Simulated and true state (voltage) (*b*) Simulated and true state (current) (*c*) Simulated and true measurement (*d*) Pulse-train excitation

9.10.10

A discrete Kalman estimator was designed using SSPACK (Appendix E) to improve the estimated voltage $\hat{x}_1$. The results are shown in Fig. 9.3–5. In Fig. 9.3–5(*a*) through (*c*), we see the filtered states and measurements, as well as the corresponding estimation errors. The true (mean) states are superimposed as well, to indicate the tracking capability of the estimator. The estimation errors lie within the bounds (3 percent out), for the second state, but the error covariance is slightly underestimated for the first state (14 percent out). The predicted and sample variances are close ($0.002 \approx 0.004$ and $0.028 \approx 0.015$) in both cases. The innovations lie within the bounds (3 percent out), with the predicted sample variances close ($0.011 \approx 0.013$). The innovations are statistically zero-mean ($0.0046 \ll 0.014$) and white (5 percent out, WSSR below threshold),† indicating a well-tuned estimator. We summarize the results of this problem as follows:

Criterion: $J(t \mid t) = \text{trace } \tilde{P}(t \mid t)$

Models:

Signal:

$$x(t) = \begin{bmatrix} 0.98 & 0.09 \\ -0.36 & 0.801 \end{bmatrix} x(t-1) + \begin{bmatrix} -0.019 \\ -0.36 \end{bmatrix} u(t-1) + \begin{bmatrix} -0.019 \\ -0.36 \end{bmatrix} w(t-1)$$

Measurement: $y(t) = [1 \quad 0]x(t) + v(t)$

Noise: w and v are zero-mean and white with covariances R_{ww} and R_{vv}

Algorithm: $\hat{x}(t \mid t) = \hat{x}(t \mid t-1) + K(t)e(t)$

Quality: $\tilde{P}(t \mid t) = [I - K(t)c(t)]\tilde{P}(t \mid t-1)$

9.4 KALMAN FILTER IDENTIFIER

The Kalman filter can easily be formulated to solve the well-known *system identification problem* [11]:

> Given a set of noisy measurements $\{y(t)\}$ and inputs $\{u(t)\}$, find the minimum (error) variance estimate $\hat{\Theta}(t)$ of $\Theta(t)$, a vector of unknown parameters of a linear system.

The identification problem is depicted in Fig. 9.4–1. The scalar linear system is given by the autoregressive model with exogenous inputs (ARX) of Sec. 4.5 as

$$A(q^{-1})y(t) = B(q^{-1})u(t) + e(t) \tag{9.4–1}$$

where A and B are polynomials in the backward-shift operator q^{-i} and $\{e(t)\}$ is white.

We must convert this model to the state-space/measurement-system framework required by the Kalman filter. The measurement model can be expressed as

†WSSR is the weighted-sum squared residual statistic which aggregates the innovation vector information over a window to perform a vector-type whiteness test (see [2] for details).

9.10.11

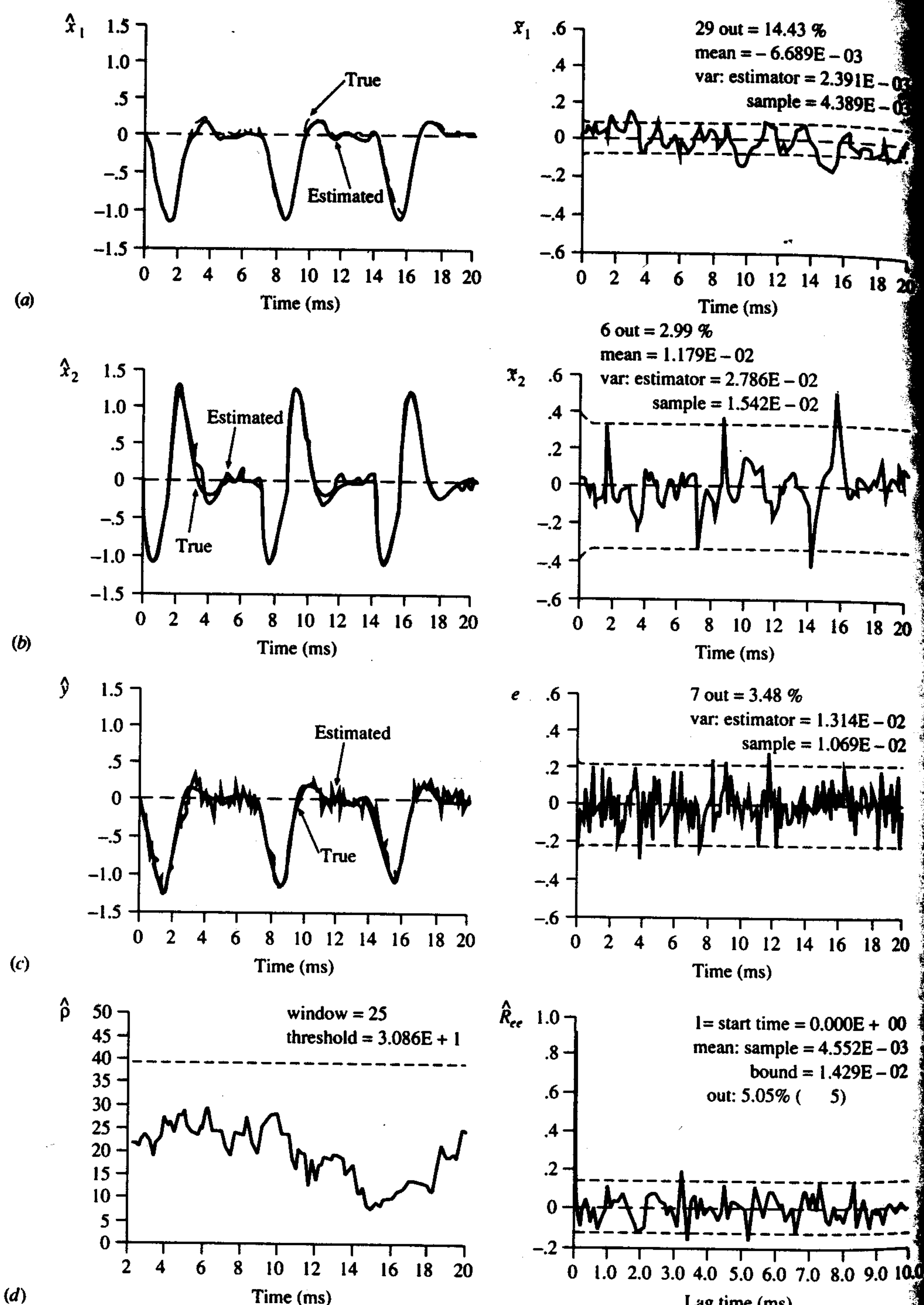


FIGURE 9.3-5
Kalman filter design for RLC circuit problem (*a*) Estimated state (voltage) and error (*b*) Estimated state (current) and error (*c*) Filtered and true measurement (voltage) and error (innovation) (*d*) WSSR and whiteness test

9.10.12

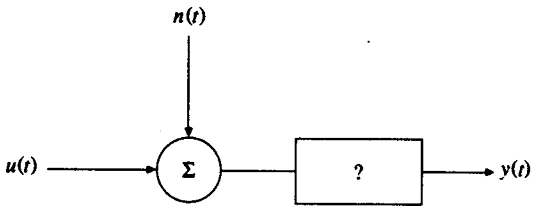


FIGURE 9.4–1
The identification problem

$$y(t) = c'(t)\Theta(t) + n(t) \tag{9.4–2}$$

where $\quad c'(t) = [-y(t-1) \cdots -y(t-N_a) \mid u(t) \cdots u(t-N_b)]$

for $n \sim N(0, R_{nn})$, and

$$\Theta(t) = [a_1 \cdots a_{N_a} \mid b_0 \cdots b_{N_b}]'$$

which represents the measurement equation in the Kalman filter formulation. The parameters can be modeled as constants with uncertainty $w(t)$; i.e., the state-space model used in this formulation takes the form

$$\Theta(t) = \Theta(t-1) + w(t) \tag{9.4–3}$$

where $w \sim N(0, R_{ww})$. We summarize the Kalman filter used as an identifier in Table 9.4–1. Thus, the Kalman filter provides the minimum-variance estimates of the parameters of the ARX model.

TABLE 9.4–1
Kalman filter identifier

	Prediction	
	$\hat{\Theta}(t \mid t-1) = \hat{\Theta}(t-1 \mid t-1)$	(Parameter prediction)
	$\tilde{P}(t \mid t-1) = \tilde{P}(t-1 \mid t-1) + R_{ww}(t-1)$	(Covariance prediction)
	Innovation	
	$e(t) = y(t) - \hat{y}(t \mid t-1) = y(t) - c'(t)\hat{\Theta}(t \mid t-1)$	(Innovation)
	$R_{ee}(t) = c'(t)\tilde{P}(t \mid t-1)c(t) + R_{vv}(t)$	(Innovation covariance)
	Gain	
	$K(t) = \tilde{P}(t \mid t-1)c(t)R_{ee}^{-1}(t)$	(Kalman gain or weight)
	Correction	
	$\hat{\Theta}(t \mid t) = \hat{\Theta}(t \mid t-1) + K(t)e(t)$	(Parameter correction)
	$\tilde{P}(t \mid t) = [I - K(t)c'(t)]\tilde{P}(t \mid t-1)$	(Covariance correction)
	Initial conditions: $\hat{\Theta}(0 \mid 0)$ and $\tilde{P}(0 \mid 0)$	
where	$c'(t) = [-y(t-1) \cdots -y(t-N_a) \mid u(t) \cdots u(t-N_b)]$	
and	$\Theta(t) = [a_1 \cdots a_{N_a} \mid b_0 \cdots b_{N_b}]'$	

9.10.13

Example 9.4–1. Suppose we have the RC circuit model of Chapter 1, that is,

$$x(t) = 0.97x(t-1) + 100u(t-1) + w(t-1)$$

and

$$y(t) = 2.0x(t) + v(t)$$

where $x(0) = 2.5$, $R_{ww} = 10^{-6}$, and $R_{vv} = 10^{-12}$. The "identification" model is given by

$$\Theta(t) = \Theta(t-1) + w^*(t)$$

$$y(t) = c'(t)\Theta(t) + v(t)$$

where $c'(t) = [-y(t-1) \mid u(t)u(t-1)]$, $\Theta'(t) = [a_1 \mid b_0 \quad b_1]$, $v \sim N(0, 10^{-12})$, and $w^* \sim N(0, 10^{-6})$.

Using the Gauss-Markov model and SSPACK (see Appendix E, [10]), we simulated the data with the specified variances. The performance of the Kalman identifier is shown in Fig. 9.4–2. The parameter estimates in Fig. 9.4–2(*a*) and (*c*) show that the estimator identifies the *a* parameter in about 10 samples, but b_0 does not converge at all in this time interval. The innovations are clearly zero-mean ($2.3 \times 10^{-4} < 2.8 \times 10^{-4}$) and white (0 percent lie outside), as depicted in Fig. 9.4–2(*c*). The measurement filtering property of the filter is shown in Fig. 9.4–2(*b*). We summarize the results as follows:

Criterion:	$J(t \mid t) = \text{trace}\, \tilde{P}(t \mid t)$
Models:	
Signal:	$\Theta(t) = \Theta(t-1) + w^*(t-1)$
Measurement:	$y(t) = c'(t)\Theta(t) + v(t)$
Noise:	$w^* \sim N(0, 10^{-6}),\ v \sim N(0, 10^{-12})$
Initial state:	$\hat{\Theta}(0 \mid 0) = [0.0 \quad 0.0 \quad 0.0],\ \tilde{P}(0 \mid 0)$
Algorithm:	$\hat{\Theta}(t \mid t) = \hat{\Theta}(t \mid t-1) + K(t)e(t)$
Quality:	$\tilde{P}(t \mid t) = [I - K(t)c'(t)]\tilde{P}(t \mid t-1)$

Note that the Kalman identifier is identical to the recursive least-squares algorithm of Chapter 7, with the exception that the parameters can include process noise $w^*(t)$. Although this difference appears minimal, avoiding estimator divergence actually becomes very significant and necessary (see [11] or [5] for details).

9.5 KALMAN FILTER DECONVOLVER

In this section, we consider extending the Kalman filter algorithm to solve the problem of estimating an unknown input from data that have been "filtered." This problem is called *deconvolution* in signal-processing literature and occurs commonly in seismic and speech processing [12,13] as well as transient problems [14].

In many measurement systems, it is necessary to deconvolve or estimate the input to an instrument, given that the data are noisy. The basic deconvolution problem is depicted in Fig. 9.5–1(*a*) for deterministic inputs $\{u(t)\}$ and outputs $\{y(t)\}$. The problem can be simply stated as follows:

9.10.14

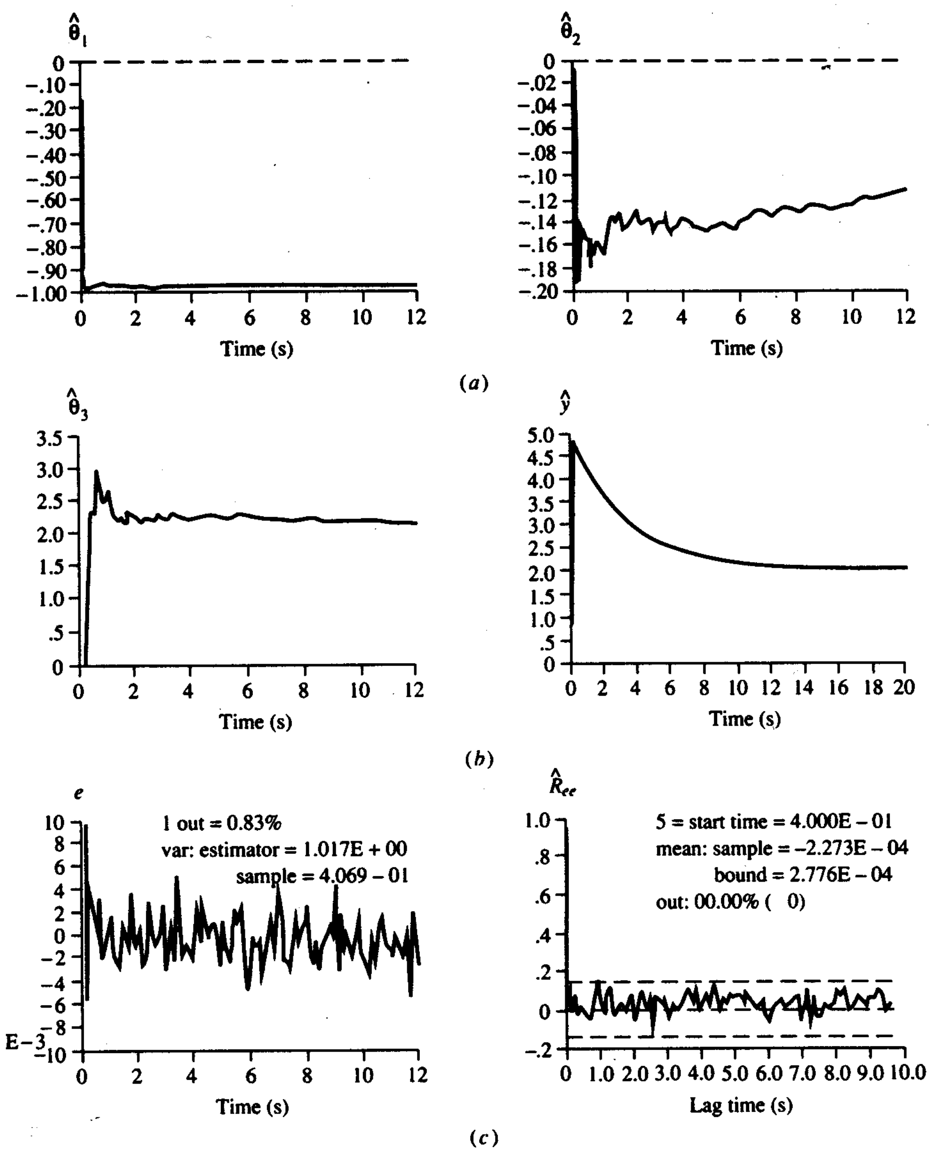


FIGURE 9.4-2
Kalman filter identifier output for tuning example (*a*) Parameter estimates for a_1 and b_0 (*b*) Parameter b_1 and filtered measurement (*c*) Prediction error (innovation) and whiteness test

Given the impulse response $H(t)$ of a linear system and outputs $\{y(t)\}$, find the unknown input $\{u(t)\}$ over some time interval.

In practice, this problem is complicated by the fact that the data are noisy, and impulse-response models are uncertain. Therefore, a more pragmatic view of the problem would account for these uncertainties. The uncertainties lead us