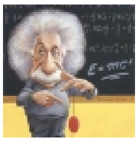

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The Kalman Filter - An Introduction

47 votes

published**14 years ago**, **16,785** views, comments: **11** ,

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Reason for the article

During my university studies, later in various specialization courses and often, trying to solve some control problems in companies, I came across the **Kalman** filter .
There are thousands of books and articles on the subject but few immediately provide a clear, conceptual and simple vision of this **very powerful tool** used in many applications.
This surprised me a lot and I asked myself if it was just a personal perception related to the literature I had consulted or if the embarrassment in dealing with the subject was generalized.
Many friends and colleagues asked me for information on the subject because they had not grasped the essence of the tool and in a few words I tried to explain its potential.
I would like to point out that:

- It is a complex topic from a mathematical point of view.
- Sooner or later you have to read the sacred texts and understand the theory behind the scenes.
- It is not always the best tool to use and even less the simplest as many would have you believe.
- Understanding the topic requires rudiments of *Automatic Control* , *Signal Theory* and *Systems Theory*

This premise should not frighten the reader as this document only wants to introduce the topic from a conceptual point of view so the complex mathematical details will be deliberately omitted.
There are however a certain number of concepts that must be clear before starting reading:

- What are state variables in a dynamical system?
- The definition of random variable and random process

The mathematical model of reality

At the engineering level, the physical/mathematical model is a very powerful tool that has as its main purpose that of describing the object that one wants to create and control.

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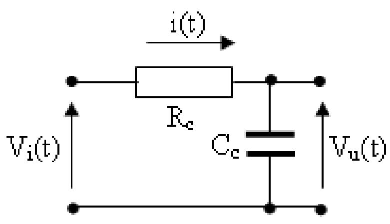
Kalman filter mathematical model

The model (in all its forms) is a representation of physical reality.
In fact, one must never forget that:

- There are many ways to describe an object depending on the aspect you want to emphasize.
- None of them is perfect in the sense that no matter how hard you try, you always have to "settle" for describing a physical entity by formulating simplifying hypotheses since reality is still too complex to be portrayed faithfully.

Let's take a very simple example.



circuit_rc.png

When we draw an electrical circuit like the one shown in the figure we are implicitly making many approximations; I will mention some of them as examples.

1. The contact resistances of the components are implicitly neglected.
2. There is no reference to the physical location of the components and to the phenomena of magnetic induction in the surrounding space due to the field generated by the currents flowing in the conductors.
3. Propagating electromagnetic fields are neglected since it is assumed that at operating frequencies the model is a good approximation of reality.
4. Resistor and capacitor are not time variants so it is assumed that their characteristics and therefore the nominal value are always preserved
5. Component tolerances are not indicated
6. The equivalent circuits of the various components are not indicated, therefore internal losses and high frequency behaviour are not considered.
7. Disturbances that may come from outside such as electromagnetic interference from other electrical circuits close to the one indicated are not modelled.

We could go on for a long time but we will stop here as we already have the elements to assert the following:

> *A physical/mathematical model, however scrupulous, is always imprecise and differs from reality by a quantity that we will indicate as a model error.*

So if we indicate the real object with the letter R and the representative model of the latter with the letter M we can say that:

$$R - M = \delta M$$

It is precisely δM that we will focus on in the next paragraph as it plays a fundamental role in the structure of the *Kalman* filter and beyond.

The model error

The model error **is not a deterministic quantity** since it is not possible to establish its value a priori using well-defined quantities.
Conceptually, the meaning of this statement is very simple to understand in light of the next example.
Let's consider the *RC* circuit previously illustrated. The value of the resistance R_c is not known a priori, at most we can assume that we know its nominal value. Let's suppose that the resistance has a nominal value equal to $R_{c\ nom}$ =470Ω.

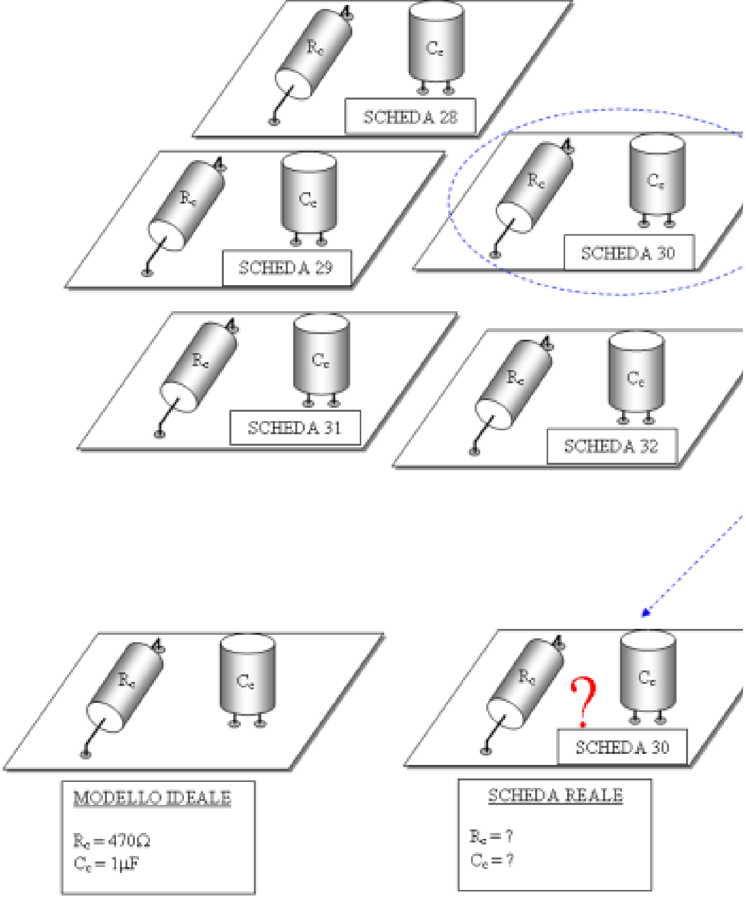
The same applies to the capacitor for which only the desired nominal value $C_{c\ nom}$ *is known .* =1μF. Let's also suppose that the resistor manufacturer guarantees a tolerance of ±5% on the nominal value while the capacitor manufacturer guarantees a tolerance of ±10% on the nominal value of the component.
At this point, every time you build an *RC* circuit and you require the assembly of a resistor R_c = 470Ω and a capacitor $C_c = 1\mu F$, you are actually creating something different with resistive components that go from a minimum value $R_{c\ min}$ to a maximum value $R_{c\ max}$ around the nominal value and similarly capacitive components that go from a minimum value $C_{c\ min}$ to a maximum value $C_{c\ max}$.

Now pay close attention to the following step .

If we have produced a lot of *RC* boards forcing the manufacturer to use the nominal values for the components, we can choose **one at random** and ask ourselves what the error is in the assembled components.

<u>on that particular board</u> than what we designed on paper.

Let us clearly assume as an ideal reference model the *RC* circuit in which **the components are mounted exactly equal to the required nominal value** .



card_choice.png

It is therefore evident that the sample card could be one of many with any pair of components in a possible range of choices. We will call this pair (R_{c_i} , C_{c_i}).

At this point, the following question arises spontaneously:

- Is it possible to somehow describe all the possible cards produced with a single model?
- Since the possible cards are practically infinite, they cannot be described one by one with a deterministic model, so what is the mathematical means that allows us to do this?
- A general method should be used to describe the uncertainties of components that are not perfectly known, but which can be adapted to all classical engineering cases so that the tool does not have to be varied according to the problem.

This tool exists and it is **noise** . In fact, noise is by definition something that **makes the determination of a quantity uncertain** and this is exactly what we need. In fact, by using two *independent* noise sources we can pollute the value of the nominal resistance and capacity, obtaining random values that are clearly close to the nominal value; this is exactly what we wanted.
Pay attention to the word " *independent* " which was not introduced randomly into the discussion. In the example in question, the suppliers of resistors and capacitors are different, so there is no relationship between the production error of the resistor and that of the capacitor. If the production lines of the components were somehow linked, two independent noise sources could not be used, but in some way one would be related to the other and this should be taken into account.

Let us write the equations of the system in the time domain.

$$
\begin{cases}
i(t) = C_c \cdot \dfrac{\partial v_c(t)}{\partial t} = C_c \cdot \dfrac{\partial V_u(t)}{\partial t} \\
V_u(t) = V_i(t) - R_c \cdot i(t) \\
R_c = R_{C_{nom}} + \eta_R \\
C_c = C_{C_{nom}} + \eta_C
\end{cases}
\qquad
\begin{cases}
i(t) = \\
i(t) = \\
R_c = \\
C_c =
\end{cases}
$$

$$
\begin{cases}
\dfrac{\partial V_u(t)}{\partial t} = -\dfrac{1}{R_c \cdot C_c} \cdot V_u(t) + \dfrac{1}{R_c \cdot C_c} \cdot V_i(t) \\
R_c = R_{C_{nom}} + \eta_R \\
C_c = C_{C_{nom}} + \eta_C
\end{cases}
$$

$$
\begin{cases}
\dfrac{\partial V_u(t)}{\partial t} = -\dfrac{1}{\tau} \cdot V_u(t) + \dfrac{1}{\tau} \cdot V_i(t) \\
\tau = R_c \cdot C_c = (R_{C_{nom}} + \eta_R) \cdot (C_{C_{nom}} + \eta_C)
\end{cases}
$$

equation_rc_1.png

Now let's rewrite them as equations of state in the typical *Systems Theory* sense :

$$
\begin{aligned}
& x(t) = V_u(t) \\
& u(t) = V_i(t) \\
& A_\tau = -\frac{1}{\tau} \quad B_\tau = \frac{1}{\tau} \\
& C_\tau = 1 \quad D_\tau = 0 \\
& \begin{cases}
\dfrac{\partial x(t)}{\partial t} = A_\tau \cdot x(t) + B_\tau \cdot u(t) \\
\quad y \quad = C_\tau \cdot x(t) + D_\tau \cdot u(t)
\end{cases}
\end{aligned}
$$

equation_rc_2.png

It is evident from the writing as:

- x(t) : State variable
- u(t) : Input
- (A_T , B_T , C_T , D_T) : Model matrices

At this point we are faced with a **very important conceptual problem** .

In fact, the matrices $\mathbf{A}_T$ and $\mathbf{B}_T$ are not constant but depend on two *random* variables and in an evidently **non-linear** way .
In fact, it is enough to review the complete writing of $\mathbf{A}_T$ as a function of the noises:.

$$A_\tau = -\frac{1}{\tau} = -\frac{1}{(\boxed{R_{Cnom}} + \boxed{\eta_R}) \cdot (\boxed{C_{Cnom}} + \boxed{\eta_C})}$$

Parte deterministica

Parte statistica

equation_rc_3.png

The equations just written simulate all the circuits that can be created with the various resistors and capacitors but unfortunately they are not expressed in a suitable way.
The aim is to physically distinguish the deterministic part of the system from the statistical part . In the formulation just made the two parts are **strictly connected** .
In practice we would like the matrices **A** and **B** (also **C** and **D** if they were not constant as in the particular example) of the system to be *dependent only and exclusively on the nominal values of the components* (**deterministic component**) and the statistical part that corrupts the ideal model *to be modeled separately* (**random component**).
Let's rewrite the equations as we would like them to be structured:

$$\begin{cases} \dfrac{\partial x(t)}{\partial t} = \boxed{A \cdot x(t) + B \cdot u(t)} + \boxed{\eta} & (1) \\ y = \boxed{C \cdot x(t) + D \cdot u(t)} + \boxed{\xi} & (2) \end{cases}$$

Parte deterministica

Parte statistica

equation_rc_4.png

At this point we see that the model noise η corrupts the value of the derivative of the state and consequently the state and therefore the output. The noise ξ will be treated soon since it has a very precise meaning; **for now note that the latter does not act on the state but directly on the output.**
In practice we **simulate the effects of modeling errors (imprecisions in the drafting of the equations) and parametric uncertainty (not exact knowledge of the geometric physical quantities and so on) by corrupting the state generated by the ideal system with noise.**
Let's make some fundamental observations:

- The model noise η is not directly related to the noises simulating the uncertainties of the components i.e. η_R and η_C . It is a simple mathematical tool that simulates the ignorance of the errors introduced in the model. Think of it as a **potentiometer** that you can adjust to declare how good your model is compared to the physical reality.

- The model noise η acts directly on the derivative of the state variable and therefore on the state variable itself while in the previous model the noises η_R and η_C corrupted the matrix A which, when multiplied by the current state, corrupted the derivative. **The mechanism of action is completely different** .

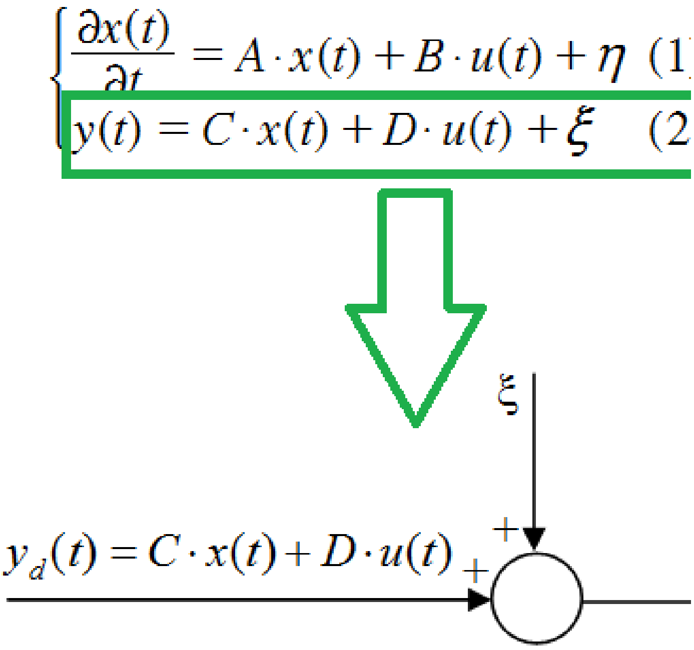
In conclusion the basic concept is that the model noise η does not necessarily have a simple and direct connection with the physical uncertainties but is an effective tool to simulate the global effect of these uncertainties on the state variables of the system and therefore also on the system output.

The considerations just expressed lead to two reflections linked to experience in the industrial field:

- The devices to be modeled are often specified by the manufacturer via nominal values; the deviation from these values is generally specified in a much less precise and punctual manner (when it comes to sensors this does not apply, it is more frequent in actuators). This means that it is simpler and more physiological to write the deterministic model of the *nominal* machine than the statistical one that describes its uncertainty.
- Providing a measure of how much the model differs from reality is not only a function of the tolerances of the components but also and **above all of the skill of the person writing the equations** . In fact, even the approximations that are imposed in writing the model must be taken into account. Since it is very difficult to evaluate the repercussions due to the approximation errors of the equations on the final result, a very simple technique is used; **they are written as best as possible respecting a balance between functionality and simplicity** and then some noise is added to the state to explicitly declare the existence of a certain modeling error.

The measurement noise

Measurement noise is something more physiological to describe and intuit. In fact, anyone who has observed a signal coming from an industrial device with a simple oscilloscope will certainly have noticed the typology of the disturbances.
It is therefore clear that measurement noise must also be modeled as a random variable (*aleatory* precisely) that overlaps with the useful signal (ideal measurement).



composition_of_y.png

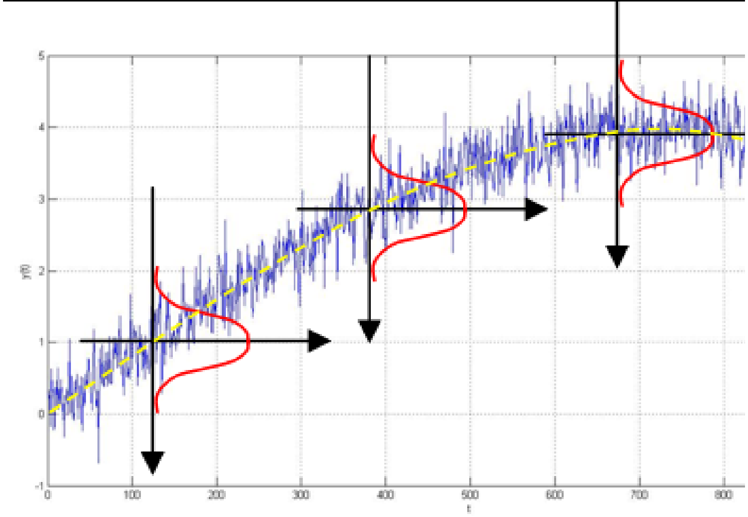
In the figure it can be seen how equation (2) is composed of two terms; one of deterministic type **$y_d(t)$** and one of statistical type **ξ** . While the signal can be described with a classical mathematical function, the noise is necessarily a random variable and its description is done through statistics.

At this point, doubts may arise about the nature of the noise and we can ask ourselves the following questions:

- What kind of noise do we use?
- *Do we represent both impulsive and burst* noise and background noise?

To the first question we could answer that the statistical description of the noise of a sensor should be hypothetically obtained by sampling the output signal of the same and verifying its statistical characteristics. In reality we proceed as a general criterion assuming that the noise has a *Gaussian* distribution with zero mean. In fact this hypothesis is not too risky as we could discuss the subject at length but in general the background noise superimposed on a measured signal is often assimilable to a distribution of this type by virtue of the following characteristics:

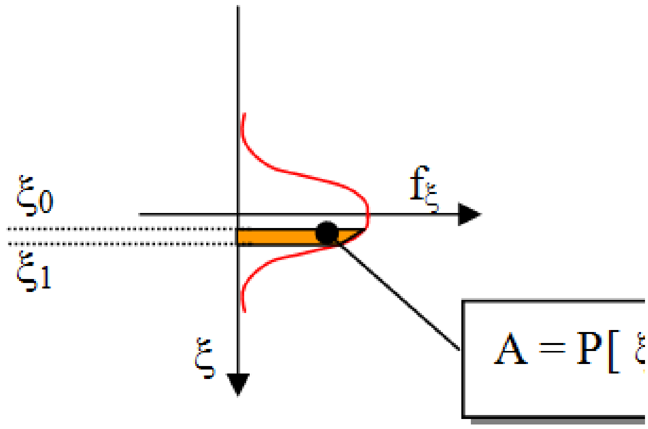
- It has zero average
- Noise is more likely to have an absolute value close to zero than large values.
- It is symmetrical with respect to zero



signal_with_noise.png

In the figure, a typical y(t) signal is represented in which a zero-mean **Gaussian noise is present. The ideal trajectory can be identified by the dotted yellow line and represents the y_d (t)** part of the model.

The blue part is the **y(t)** signal given by **y_d (t) to which the random variable ξ** has been added to simulate the noise. If we choose some instants at random, we can look at the statistical distribution of the error at that specific point of the curve; all we have to do is **place** the Gaussian curve on the ideal trajectory, in fact the signal is formed by the ideal trajectory to which a random variable **ξ** is added , identified by its statistical distribution. Let's suppose that the distribution does not change over time, that is, that the **bell always has the same shape and intensity. In this case, the y(t)** signal turns out to be a stationary process (in reality, the mathematical definition would be more complex, but we are only trying to give a basic idea of the concepts behind the modeling).



noise_statistics.png

Remember that the orange area indicated in the figure represents the probability that the disturbance (noise) has values between ξ_0 and ξ_1 .

As for pulses and **bursts** or other types of non-continuous noise, they will be simulated later as a disturbance signal in the system by injecting the signal into the model and verifying its effects.

In reality, if strictly necessary, these disturbances can also be modeled, but in this context we will not deal with them since they require rather complex mathematical tools for a simple treatment of the problem in

question.
The fundamental concept to understand is that the measurement noise **ξ** represents another virtual **potentiometer** that serves to indicate the quality of the measurement.

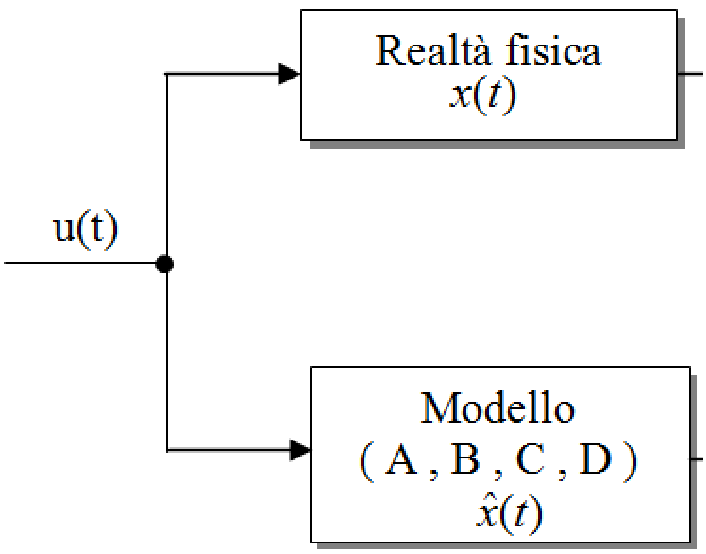
Structure of the Kalman estimator

The estimator is nothing more than **a tool capable of reconstructing the state of the real system** .
In practice we have two cards to play to reconstruct the state:

- The mathematical model we wrote and declared to be good through model noise.
- The measurements coming from the sensors that give us the value of the real output of the system (corrupted by noise obviously)

Let's try to use them consistently to arrive at the estimate of the state.
First we use the **model** .
In the figure we can observe the following:

- The real system and the model are fed with the same input **u(t)**
- The symbol "^" used in the model indicates the state and output estimation



prediction_correction_1.png

If the equations are written 'well enough' it is expected that by stimulating the input of the real system and the mathematical model in the same way, the outputs will be very similar.
This is a **very serious mistake** that is often made when one begins to handle real systems.
In fact, there is the problem of the indeterminacy of the initial conditions:

- We know nothing about the internal state of the real system at the beginning of the observation. The state variables have values different from those of the real system. In the **RC** circuit example, the capacitor could be partially charged and we do not know it.
- Not knowing the previous history of the system leads to an indeterminacy of the initial state. In fact, we begin to observe the real system at any moment and we know nothing about the

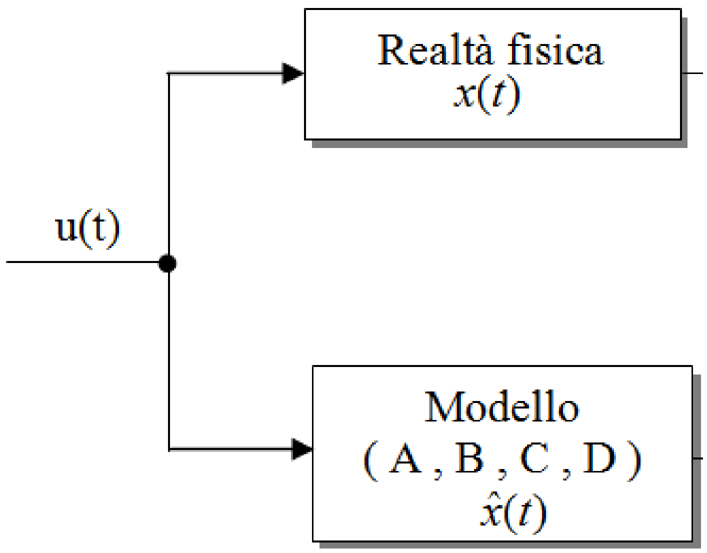
behavior of the state variables up to that moment. We now set ourselves the goal of estimating them but we do not know what their value is at the moment in which we begin to observe them, therefore the mathematical model starts from a completely different state (this obviously means that the output is also different since it depends linearly on the state due to how the system of equations is defined).

The correct statement would therefore be the following:
If the capacitor is initially discharged and I apply the same input to the real system and to the mathematical model, I expect the outputs to be similar, assuming that the equations are written correctly.

If the two outputs are not similar, the causes could be the following:

- The model is not written correctly and therefore does not provide the same output as the real system for the same input
- The initial conditions of the two objects were different and therefore, even if stimulated by the same input, they followed different output trajectories.

Let's now take a step forward.
Consider the difference between the two outputs as a parameter indicating the "goodness" of the estimate.



prediction_correction_2.png

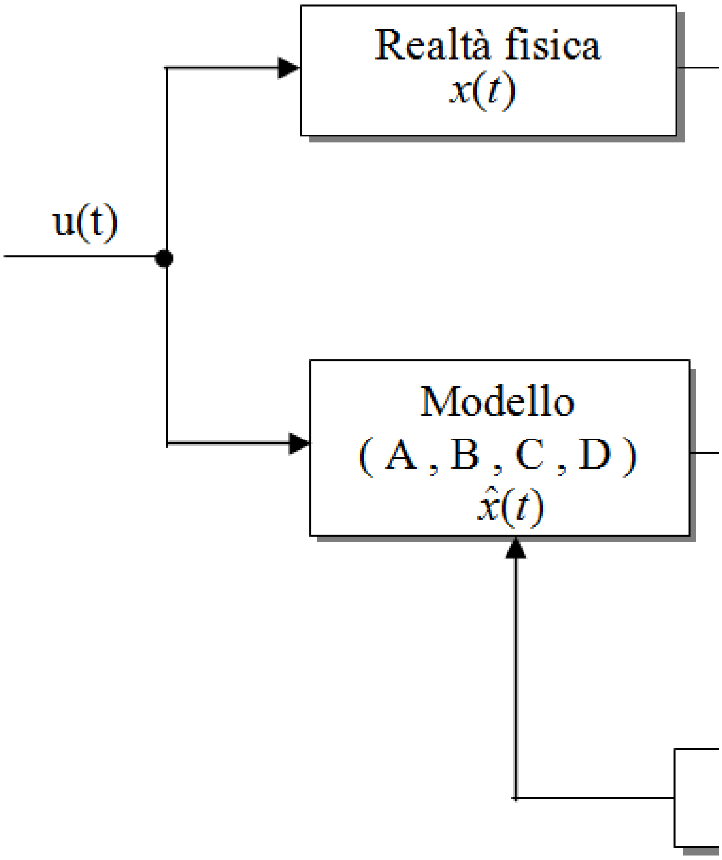
The error **e(t)** is commonly called **innovation** .
Let's think about its characteristics for a moment. For the estimate of **x(t)** and therefore of **y(t)** to be good, the prediction error **e(t)** should be:

- ' **Small'** : in fact this would mean that the estimate and the real value are very similar
- ' **Zero-mean'** : in fact, if *y(t)* and the estimate of *y(t)* differed by a constant, it would mean that the model was poorly written since the constant could be predicted and inserted into the model equations.
- ' **White'** : Basically this is a statistical feature that essentially implies this; between the error *e(t)* at time **t** and the error *e(t+dt)* at time **t+dt** there must be no relationship, that is, they must be independent of each other. If this were not the case, it would mean

that there is a rule that allows us to estimate part of *e(t+dt)* starting from *e(t)* using the rule itself and this, once again, **implies that there are still relationships between the signals that are not written in the model** .

In practice, everything that can be said a priori in terms of relationships between signals must be written in the model, otherwise we lose pieces of dynamics that end up in error and do not contribute to giving a good prediction of the state and therefore of the output .

Let's take it one step further, that is, let's use the error *e(t)* to improve the state estimate.



prediction_correction_3.png

The K gain allows us to weight the error *e(t)* and bring it back into the model to correct the state estimate.
The figure does not explicitly explain how *e(t)* will be used , that is, at what point in the model it will actually be injected, but for now we do not care.
The fundamental thing to understand is that we have created a feedback on our model to improve the state estimate using the error . This scheme is completely general. For now, we have done nothing more than create a state predictor by first exploiting the model and then refining the estimate using the error *e(t)* .

The main question that arises at this point is the following:

- How is **K** calculated ?
- Is it unique?

K can be calculated in various ways and is obviously not unique.

In case:

- The system of equations (A , B , C , D) is linear and time invariant
- Let the model noise μ and the measurement noise ξ be Gaussian.

There is an optimal algorithm that allows us to calculate **K** which becomes the **Kalman gain** .

At this point we can conclude with the following summary procedure:

- Let's write the model of the real system
- We estimate the model error or indicate the variance of the same
- We estimate the measurement error or indicate its variance
- We calculate the Kalman gain using an algorithm that takes as input the model matrices (A, B, C, D) and the variances of the model and measurement errors.
- We realize the structure just illustrated using the Kalman gain calculated in the previous step.

If everything is done carefully, after a certain transient period the estimated state converges to the state of the real system and thus we have realized a state estimator.

Some important notes

Throughout the discussion, a state variable model in the *continuous* time domain has been proposed . In reality, the *Kalman* filter was initially developed in the *discrete* time domain , i.e. for sampled signals, and subsequently extended to the continuous domain.
In the following article, the discussion has been conducted in the continuous time domain because physical systems are generally more easily modeled. At this point, however, some clarifications must be made.

- *The Kalman* filter is implemented through a sequence of matrix operations performed by a numerical algorithm implemented on any processor. This implies that the transformation from continuous time to discrete time must be done sooner or later (it is essentially necessary to pass from the domain of the *s-Transform* to that of the *z-Transform*).
- When studying the *Kalman filter* **it is always better to start from the formulation in the discrete domain** .
- *Kalman* filter synthesis is best performed in the discrete domain for reasons of numerical stability related to the algorithms that calculate the gain.

In short, **I strongly recommend the reader to study the discrete Kalman filter before the continuous one** .

But what is the Kalman estimator ultimately for? What do I do with the estimated state?

Two things basically:

- Estimate what happens inside the system without installing sensors. In fact, estimating the state is the equivalent of having a virtual instrument that reads a certain variable. If the state variable is a physical variable such as a water level or an electrical voltage, we have the possibility of estimating its value without installing additional instruments. In the case of the water level we could verify that the fluid does not overflow while in the case of the electrical voltage we could avoid the saturation of some component at a certain point of the circuit.

- The feedback theory of dynamic systems teaches that *feedback* through the state is more powerful than *feedback* from the output because it allows to precisely regulate the dynamics. Furthermore, it is indispensable when the systems are no longer *SISO* (Single Input Single Output) but become *MIMO (Multi Input Multi Output). If the state is not directly measurable, it must be estimated in some way and the Kalman* filter is a very powerful tool that satisfies this requirement by implicitly adding stability and robustness.

Conclusions

In this paper, a basic description of the *Kalman* estimator has been given , focusing on some general aspects not related to the mathematical formalism but equally important for application purposes to understand the potential of the tool.
The underlying reason why the model is formulated by separating the deterministic and statistical parts, thus avoiding generating matrices *(A , B , C , D)* related to the noise statistics, has been explained.
The model *(A , B , C , D)* is therefore deterministic and is corrupted by the model noise and the measurement noise in order to weigh the uncertainties related to the writing of the equations and those due to a noisy measurement of the output of the real system.
At this point, the interested reader could start reading the "official" documentation trying to solve some very simple academic exercises. It is important to familiarize yourself with the following concepts:

- How to write the template
- How to parameterize model and measurement noises
- How to calculate Kalman gain
- **Check the convergence conditions**

The third point is generally the most difficult and is often neglected causing unwanted behaviors of the estimator. The convergence conditions **must always be verified** otherwise there is no certainty regarding the truthfulness of the estimate.

Bibliography

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[6] **Optimal state estimation: Kalman, Hinf and nonlinear approaches** Dan Simon
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[8] An Introduction to the Kalman Filter, SIGGRAPH 2001 Course, Greg Welch and Gary Bishop

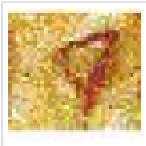
11 Comments and notes

Insert a comment

by JTRobbins , 12 years ago

Thanks. You managed to give an idea even to a pilot who is particularly bad at math. Application: INS (Inertial Navigation System) and GPS integration on military aircraft. The Kalman filter determines for me the data set that provides the most accurate solution: in practice it gives me the best of GPS and INS, combining it into a single set of information that I ultimately read on the displays. Hats off

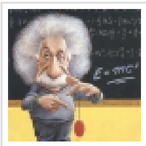
Answers



of sebago , 14 years ago

Damn, what a rambling! In addition to the much-deserved compliments to the author, a round of applause also to Admin, who is increasingly known as a very skilled magnet for prodigious talents.

Answers



of dimaios , 14 years ago

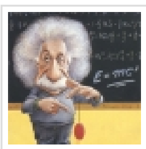
Thank you very much for the positive comment. It is an honor and a pleasure to be part of this community. I will try as much as possible to find the time to publish the continuation of the article going into detail to promote the use of the illustrated tool. Thanks again for the appreciation.

Answers

by admin , 14 years ago

When articles of this kind arrive on ElectroYou, I feel emotion and pride. Emotion in thinking about the author's knowledge; pride in seeing that this virtual space has managed to convince him to participate. I experience it as a reward for the patience and perseverance with which we have tried to create a community that is a reliable reference of scientific knowledge, in the tumultuous growth of the network. So, welcome, Stefano, and thank you for this important article, which has already received qualified approval ;-)

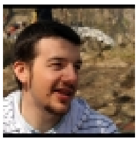
Answers



of dimaios , 14 years ago

As a first approach I definitely recommend the following book: [3] Theory of prediction and filtering Bittanti Sergio. Prof. Bittanti is an excellent teacher and you will find the reading certainly smooth and the subject treated with simplicity. Immediately after, for practice you could look at the following book: [4] Kalman filtering: theory and practice using MATLAB Mohinder S. Grewal, Angus P. Andrews In fact there are exercises developed with MATLAB which often turns out to be a very useful tool for dealing with modeling and control problems. Best regards

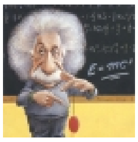
Answers



of crestus , 14 years ago

I had just done a search in the library of the polytechnic a few days ago on books that deal with the Kalman filter, promising myself to take some after the exams!! Now I will read with pleasure this article that comes at the right time....:)

Answers



of dimaios , 14 years ago

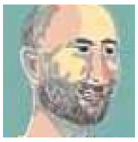
I am glad you appreciated the work. I had some doubts about the interest it could arouse so I introduced the subject in the simplest way without trivializing. Given the positive feedback I intend to elaborate on what I started with the introduction by moving on to algorithms and some practical examples. Thanks to everyone.

Answers

by DirtyDeeds , 14 years ago

What can I say... WOW!, magnificent article!

Answers



by carloc , 14 years ago

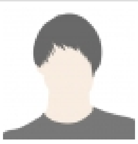
Congratulations!!! Really a nice article.

Answers

by etec83 , 14 years ago

I remember that this topic was part of the second part of the course of Fundamentals of Automatic Control, that is, basically all that part of the course that I never really understood a thing about. I practically did the exercises of the second part of the course for the exam by heart without ever understanding a thing. Partly because I didn't have the mathematical basis, partly because time was limited and I had a thousand other exams, partly because the professor didn't want to explain or didn't know how to explain these topics at all... and he's not incompetent, in fact I won't mention his name, but he's famous and he's one of the few who wrote a book just about optimal control in Italian. In any case, I'm happy that someone had the courage to write an article on this topic, there's already a chance that he'll understand something. Really congratulations!!

Answers



Of, 14 years ago

Chapeau! Waiting for the part about algorithms with examples :)

Answers

Insert a comment

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