Continuous 3D Scene Representations with Implicit Functions

Michaël Ramamonjisoa, Van Nguyen Nguyen

ENPC's Imagine Seminars 16/12/2020

Outline

I. Implicit functions: an illustration with 3D surface representation

2. Neural Radiance Field (NeRF)

Explicit vs Implicit Representation (2D)

Explicit:

$$\mathbf{f}(\alpha) = (r\cos(\alpha), r\sin(\alpha))^T$$

Domain: $[0,2\pi]$

Implicit:

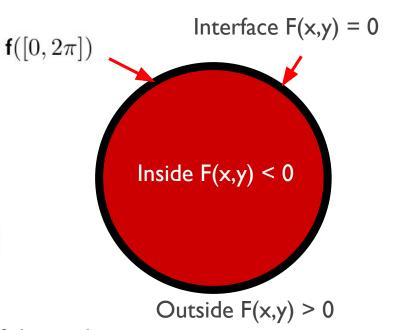
$$F(x,y) = \sqrt{x^2 + y^2} - r$$

Domain: $(x,y) \in \mathbb{R}^2$

 \implies Circle is implicitly defined by $\{(x,y)|F(x,y)=0\}$

 $\mathbf{f}(\alpha)$ defines the interface

F(x,y) defines the **Signed Distance Function** of the circle



Explicit vs Implicit Representation (3D)

Explicit:

$$\mathbf{f}(\alpha,\beta) = (r\sin(\alpha)\cos(\beta), -r\cos(\beta), r\sin(\alpha)\sin(\beta))$$
 Domain: $\alpha \in [0; 2\pi], \beta \in [0; \pi]$

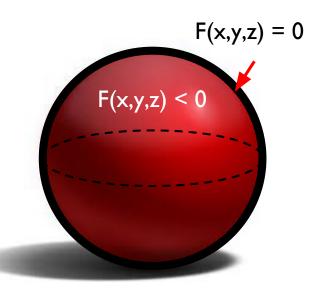
Implicit:

$$F(x, y, z) = \sqrt{x^2 + y^2 + z^2} - r$$

Domain: $(x,y,z) \in \mathbb{R}^3$

 \implies Sphere is implicitly defined by $\{(x,y,z)|F(x,y,z)=0\}$

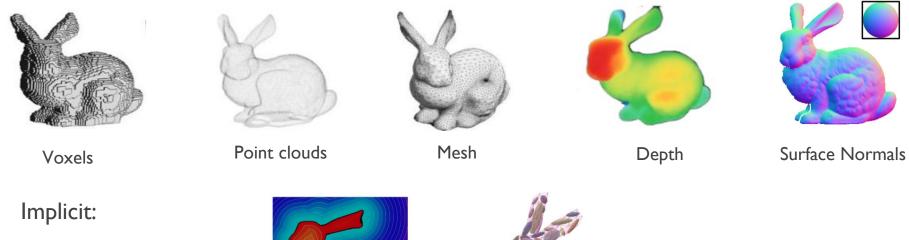
 $\mathbf{f}(\alpha,\beta)$ defines the 3D surface F(x,y,z) defines the **Signed Distance Function** of the sphere

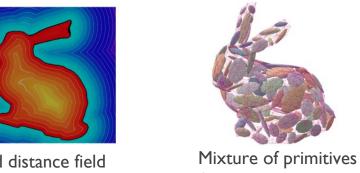


F(x,y,z) > 0

Representing 3D surfaces

Explicit:





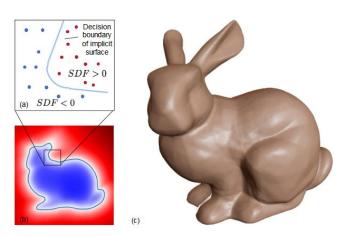
Signed distance field Mixture of primitives (e.g gaussian mixtures)

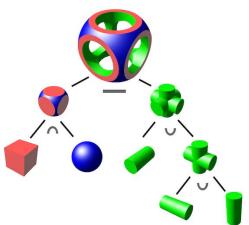
Signed Distance Field (SDF)

• Maps each 3D points p to it's signed distance from to the object surface S

$$SDF(\mathbf{p}) = \min_{q \in S} \|\mathbf{p} - q\|$$

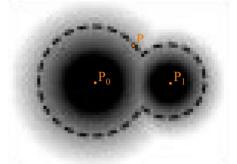
- Sign indicates whether the point p is inside (-) or outside (+) of the shape
- Shape's boundary as the zero-level-set of SDF
- Allows for Constructive Solid Geometry (CSG) through boolean operations





Mixture of Gaussians

Represents a shape as a mixture of local implicit functions (3D gaussians)



$$F(\mathbf{x}, \mathbf{\Theta}) = \sum_{i \in [N]} f_i(\mathbf{x}, \theta_i)$$

$$f_i(\mathbf{x}, \theta_i) = c_i \exp\left(\sum_{d \in \{x, y, z\}} \frac{-(\mathbf{p}_{i, d} - \mathbf{x}_d)^2}{2\mathbf{r}_{i, d}^2}\right)$$

• Shape's boundary is defined as an iso-level of the global implicit function



[I] Genoval9

[2] Genova20

Representing 3D surfaces with Implicit Functions

Pros:

- Compared to point clouds: clearly defines the (iso-)surface
- Compared to meshes: can continuously adapt to arbitrary topology
- Compared to **voxels**: can be represented with **few parameters** (e.g. mixture of simple implicit functions)
- They are continuous in 3D
- Can give analytic normals, can be applied with boolean operations, etc

Representing 3D surfaces with Implicit Functions

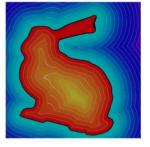
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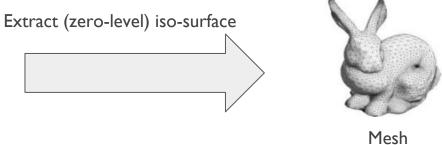
Cons:

- SDF is well-defined for only watertight meshes (there is an interior and an exterior)
- Need extra steps to visualize

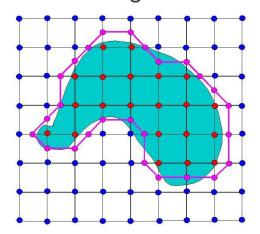
Converting Implicit Surfaces to meshes



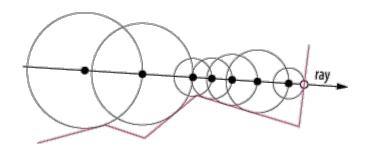




Marching Cubes



Ray marching



Representing 3D surfaces with Implicit Functions

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- Compared to meshes: can continuously adapt to arbitrary topology
- Compared to voxels: can be represented with few parameters (e.g. mixture of simple implicit functions
- They are **continuous** in 3D
- Can give analytic normals, can be applied with boolean operations, etc.

Cons:

- Implicit functions is well-defined for only watertight meshes (there is an interior and an exterior)
- Need extra steps to visualize
- Not all complex shapes can be efficiently / accurately represented with simple primitives

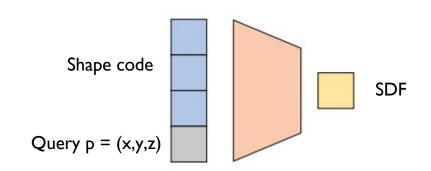
Representing 3D surfaces

DeepSDF: Efficiently representing complex shapes by learning their SDF

Idea: Learn a continuous representation of 3D implicit surfaces

Query p = (x,y,z), Shape latent code
$$\boldsymbol{Z}$$

$$F(p;\boldsymbol{Z}) = SDF(p,\mathcal{M})$$

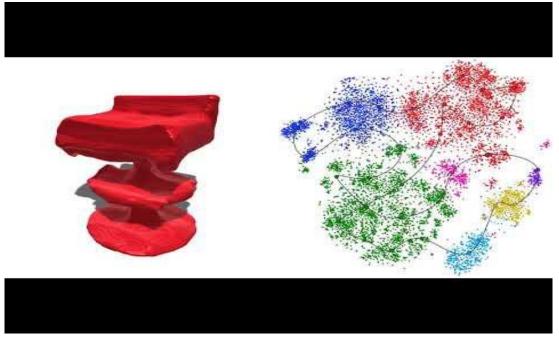


=> Continuity in 3D space AND shapes space

[3] Park 19

Representing 3D surfaces

DeepSDF: Representing complex shapes by learning their SDF

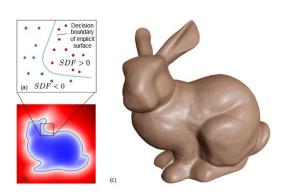


Take home message on Implicit Functions

Representation of a continuous field

Learned implicit functions:

- Can represent complex shapes
- Are continuous mappings because they use MLPs
- Are applicable to N-D data: 2D images, 3D shapes, radiance fields

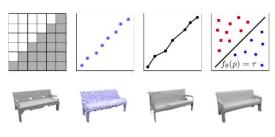


Visualization of implicit functions is done by extracting iso-surfaces:

- I. Running inference for multiple queries in input space
- 2. Rendering the result by combining the queries

More works on Implicit Functions for 3D shape

Occupancy Networks



[4] Mescheder 19

PiFu and PiFuHD



References

- [1] Genova et al., Learning Shape Templates with Structured Implicit Functions, ICCV 2019
- [2] Genova et al., Local Deep Implicit Functions for 3D Shape, CVPR 2020
- [3] Park et al., DeepSDF: Learning Continuous Signed Distance Functions for Shape Representation, CVPR 2019
- [4] Mescheder et al., Occupancy Networks: Learning 3D Reconstruction in Function Space, CVPR 2019
- [5] Saito, Huang, Natsume et al., PIFu: <u>Pixel-Aligned Implicit Function for High-Resolution Clothed Human Digitization</u>, ICCV 2019
- [6] Saito et al., <u>PIFuHD: Multi-Level Pixel-Aligned Implicit Function for High-Resolution 3D Human Digitization</u>, CVPR 2020

Courses and Seminars

Lecture on **Implicit geometry**

Lecture on **Implicit surface**

Lecture on Explicit & Implicit Surfaces

Thomas Funkhouser's talk at 3DGV seminar

Princeton COS 426, Spring 2014 on <u>Implicit Surfaces & Solid Representations</u>

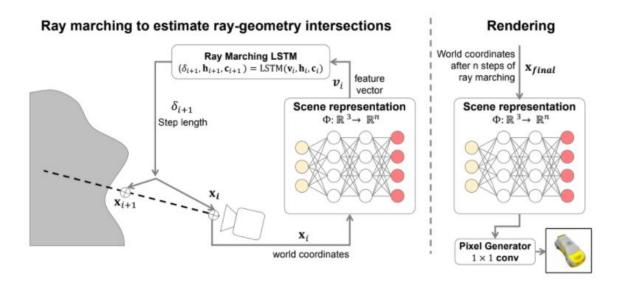
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Scene Representation Networks: Continuous 3D-Structure-Aware Neural Scene Representations

 One of the first relevant works on scene representation, also benchmark for most of NeRF's paper

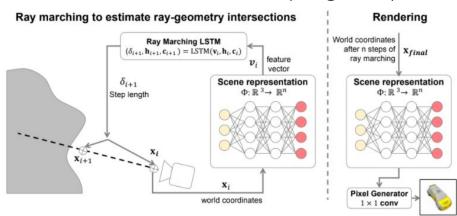


Scene Representation Networks: Continuous 3D-Structure-Aware Neural Scene Representations

Represent a scene as a function Φ which maps a spatial location x to a feature representation v

$$\Phi: \mathbb{R}^3 \to \mathbb{R}^n, \quad \mathbf{x} \mapsto \Phi(\mathbf{x}) = \mathbf{v}$$

- v may encode:
 - visual information: surface color or reflectance
 - geometry: signed distance of x
- Then learn a differentiable renderer to render v (using LSTM)

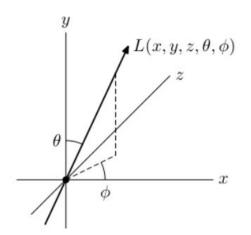


Definition of radiance field

• Radiance field is a 5-dimensional function which maps a 3D location $\underline{\mathbf{x}}$ and a direction in 3D sphere $\underline{\mathbf{d}}$ to a color $(\mathbf{r},\mathbf{g},\mathbf{b})$:

$$L: R^3 x S^2 \to R^3$$

$$L(\underline{\mathbf{x}},\underline{\mathbf{d}}) = (r,g,b)$$

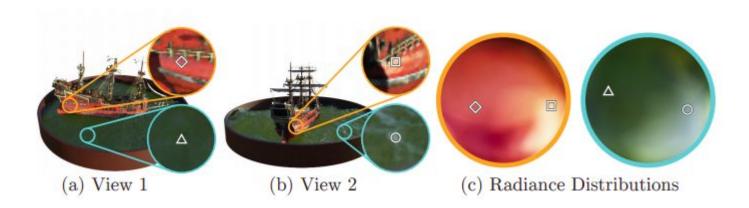


- Intuitively, "radiance" is the amount of light energy passing through a given point in space, heading in a given direction
- ullet In NeRF, there is an additional output is volume density $oldsymbol{\sigma} \in \mathsf{R}$

$$L(\underline{\mathbf{x}},\underline{\mathbf{d}}) = (\mathsf{r},\mathsf{g},\mathsf{b},\boldsymbol{\sigma})$$

Definition of radiance field

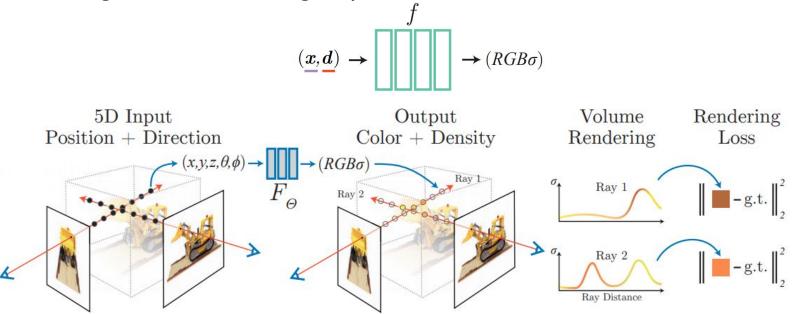
• Radiance field is a 5-dimensional function which maps a 3D location $\underline{x,y,z}$ and a direction in 3D sphere \underline{d} to a color (r,g,b):



[7] Mildenhall20

Idea:

- Continuous neural networks as a view-dependent volumetric scene representation (xyz + view direction d)
- Using volumetric rendering to synthesize new views



Volumetric rendering with ray tracing:

Opacity Predicted colors
$$C(\mathbf{r}) = \int_{t_n}^{t_f} T(t) \sigma(\mathbf{r}(t)) \mathbf{c}(\mathbf{r}(t), \mathbf{d}) dt \,, \text{ where } T(t) = \exp\left(-\int_{t_n}^t \sigma(\mathbf{r}(s)) ds\right)$$
 Volume density

Rendering model for ray r(t) = o + td:

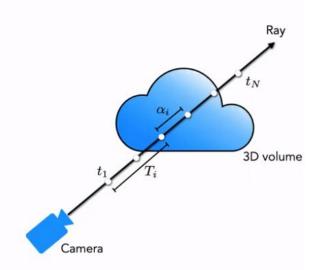
$$Cpprox \sum_{i=1}^{N} T_i lpha_i c_i$$
 colors

How much light is blocked earlier along ray:

$$T_i = \prod_{j=1}^{i-1} (1 - \alpha_j)$$

How much light is contributed by ray segment i:

$$\alpha_i = 1 - e^{-\sigma_i \delta t_i}$$



[7] Mildenhall20

Volumetric rendering with ray tracing:

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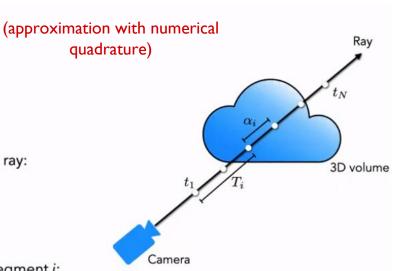
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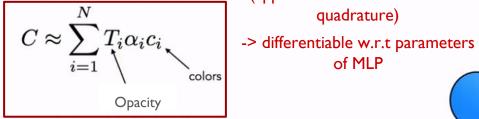


[7] Mildenhall20

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 Volume density





(approximation with numerical quadrature)

of MLP

Camera

How much light is blocked earlier along ray:

$$T_i = \prod_{j=1}^{i-1} (1-lpha_j)$$

How much light is contributed by ray segment i:

$$\alpha_i = 1 - e^{-\sigma_i \delta t_i}$$



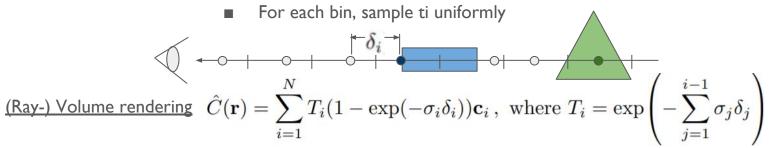
Ray

3D volume

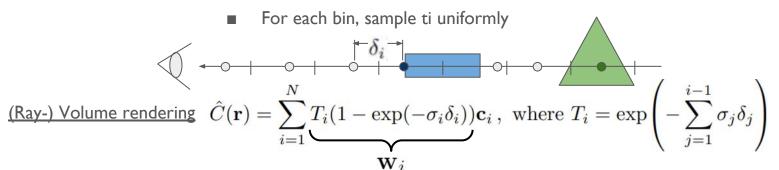
- <u>Hierarchical Sampling:</u> coarse to fine importance sampling
 - First sample coarsely along the ray with stratified sampling
 - Create Nc bins between tn and tf
 - For each bin, sample ti uniformly

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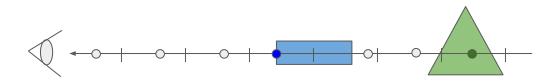
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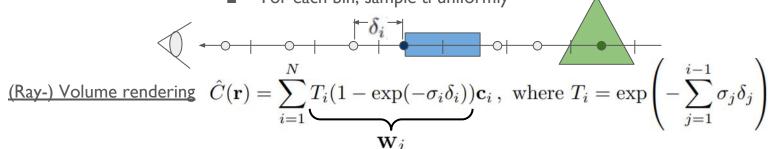


- Hierarchical Sampling: coarse to fine importance sampling
 - First sample coarsely along the ray with stratified sampling
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- For each bin, sample ti uniformly $\hat{C}(\mathbf{r}) = \sum_{i=1}^{N} T_i (1 \exp(-\sigma_i \delta_i)) \mathbf{c}_i \,, \text{ where } T_i = \exp\left(-\sum_{j=1}^{i-1} \sigma_j \delta_j\right)$
 - \circ Then do importance sampling based on color weight \mathbf{W}_i

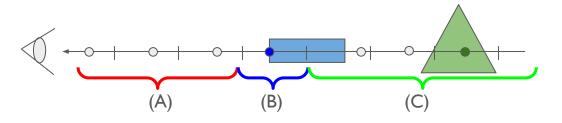


Tricks:

- <u>Hierarchical Sampling:</u> coarse to fine importance sampling
 - First sample coarsely along the ray with stratified sampling
 - Create Nc bins between tn and tf
 - For each bin, sample ti uniformly



 \circ Then do importance sampling based on color weight \mathbf{W}_i



(A) $T_i \approx 1, \ \sigma_i \approx 0$

(B) $T_i > 0, \ \sigma_i > 0$

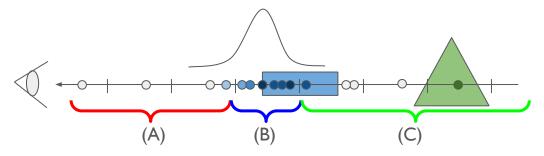
(C) $T_i \approx 0 \quad \mathbf{w}_i \approx 0$

Tricks:

- Hierarchical Sampling: coarse to fine importance sampling
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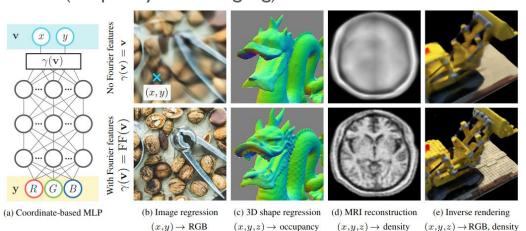
 \mathbf{W}_{i}



- (A) $T_i \approx 1, \ \sigma_i \approx 0$
 - $T_i > 0, \ \sigma_i > 0$
- (C) $T_i \approx 0 \quad \mathbf{w}_i \approx 0$

Tricks:

- Positional encoding to map each input 5D coordinate into a higher dimensional space
 - Learning in high-frequency mappings is difficult to learn $\gamma(p) = \left(\sin\left(2^0\pi p\right), \cos\left(2^0\pi p\right), \cdots, \sin\left(2^{L-1}\pi p\right), \cos\left(2^{L-1}\pi p\right)\right)$
 - Fourier Basis feature mapping allocates neurons to different spatial frequency bands (frequency disentangling)



[8] Tancik20

	Input	#Im.	L	(N_c, N_f)	PSNR [↑]	SSIM†	LPIPS.
1) No PE, VD, H	xyz	100	-	(256, -)	26.67	0.906	0.136
2) No Pos. Encoding	$xyz\theta\phi$	100	=	(64, 128)	28.77	0.924	0.108
3) No View Dependence	xyz	100	10	(64, 128)	27.66	0.925	0.117
4) No Hierarchical	$xyz\theta\phi$	100	10	(256, -)	30.06	0.938	0.109
5) Far Fewer Images	$xyz\theta\phi$	25	10	(64, 128)	27.78	0.925	0.107
6) Fewer Images	$xyz\theta\phi$	50	10	(64, 128)	29.79	0.940	0.096
7) Fewer Frequencies	$xyz\theta\phi$	100	5	(64, 128)	30.59	0.944	0.088
8) More Frequencies	$xyz\theta\phi$	100	15	(64, 128)	30.81	0.946	0.096
9) Complete Model	$xyz\theta\phi$	100	10	(64, 128)	31.01	0.947	0.081







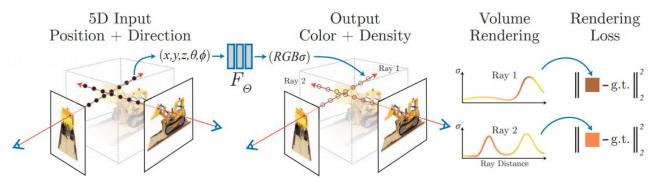


Ground Truth		Complete Model		No View Dependence No Positional End					coding	
	Diffuse S	Synthetic	360° [41]	Realistic Synthetic 360°			Real Forward-Facing [28]			
Method	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS.	
SRN [42]	33.20	0.963	0.073	22.26	0.846	0.170	22.84	0.668	0.378	5
NV [24]	29.62	0.929	0.099	26.05	0.893	0.160	-	-	(946)	[7] M
LLFF [28]	34.38	0.985	0.048	24.88	0.911	0.114	24.13	0.798	0.212	F. J
Ours	40.15	0.991	0.023	31.01	0.947	0.081	26.50	0.811	0.250	

[7] Mildenhall20



NeRF in a nutshell:



- Learn the radiance field of a scene based on a collection of calibrated images
 - Use an MLP to learn continuous geometry and view-dependent appearance
- Use fully differentiable volume rendering with reconstruction loss
- Combines <u>importance sampling</u> and <u>Fourier-basis encoding</u> of 5D query to produce high-fidelity novel view synthesis results
- Allows efficient storage of scenes (x3000 gain over voxelized representations)

Neural Radiance field (NeRF)

Remaining challenges

- Handling dynamic scenes when acquiring calibrated views
- One network trained per scene no generalization

Neural Radiance field (NeRF)

Remaining challenges

- Handling dynamic scenes when acquiring calibrated views
 - D-NeRF: Neural Radiance Fields for Dynamic Scenes
 - Deformable Neural Radiance Fields
- One network trained per scene no generalization

NeRF

- Only applicable to rigid scenes
- **-** 5D continuous function
- Requiring multiple views of a rigid scene

D-NeRF

- Applicable for rigid and non-rigid scenes
- + 6D continuous function by considering time-component as an additional input
- + Requiring a single view per time instant for non-rigid scenes.

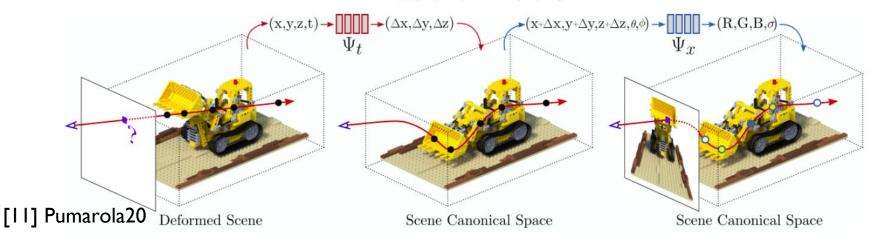


• **Deformation network** Ψ_t : to predict deformation field between the scene at time instant t and the scene in canonical space (t=0)

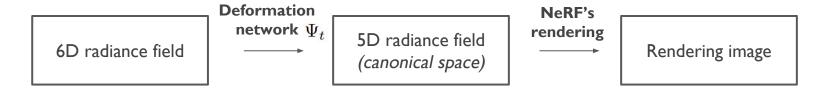
$$\Psi_t(\mathbf{x}, t) = \begin{cases} \Delta \mathbf{x}, & \text{if } t \neq 0 \\ 0, & \text{if } t = 0 \end{cases}$$

ullet Canonical network Ψ_x : to predict color and density in canonical configuration

$$\Psi_x(\mathbf{x}, \mathbf{d}) \mapsto (\mathbf{c}, \sigma)$$

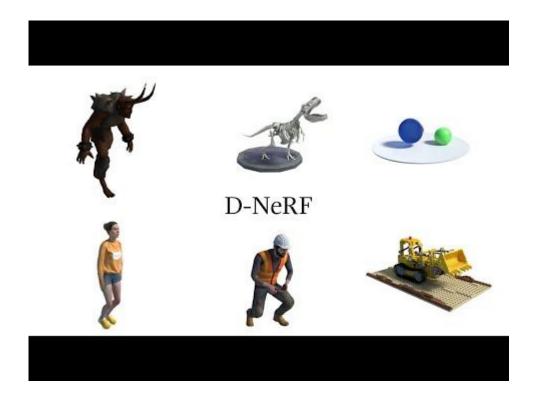


Volumetric rendering is the same as NeRF in canonical space:

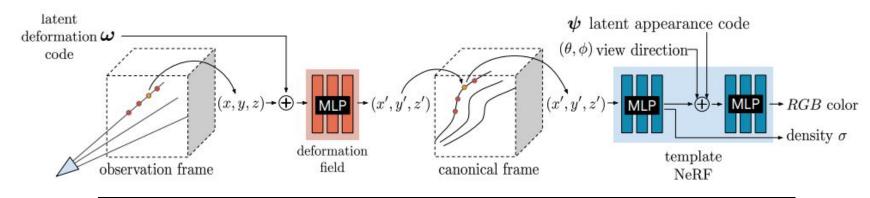


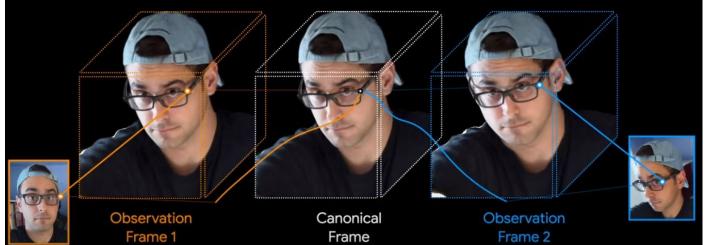
$$C(p,t) = \int_{h_n}^{h_f} \Im(h,t) \sigma(\mathbf{p}(h,t)) \mathbf{c}(\mathbf{p}(h,t),\mathbf{d}) dh$$
 Volume density

where
$$\mathbf{p}(h,t) = \mathbf{x}(h) + \Psi_t(\mathbf{x}(h),t)$$
, $[\mathbf{c}(\mathbf{p}(h,t),\mathbf{d}), \sigma(\mathbf{p}(h,t))] = \Psi_x(\mathbf{p}(h,t),\mathbf{d})$, and $\Im(h,t) = \exp\left(-\int_{h_n}^h \sigma(\mathbf{p}(s,t))ds\right)$.



Deformable Neural Radiance Fields





Deformable Neural Radiance Fields



[12] Park20

Deformable Neural Radiance Fields vs D-NeRF

Deformable Neural Radiance Fields

Submission history

From: Keunhong Park [view email] [v1] Wed, 25 Nov 2020 18:55:04 UTC (47,887 KB) [v2] Thu, 26 Nov 2020 01:52:45 UTC (47,887 KB)

We present the first method capable of photorealistically reconstructing a non-rigidly deforming scene using photos/videos captured casually from mobile phones. Our approach—D-NERF—augments neural radiance fields (NeRF)



- Works on real data
- Relies on pretrained foreground dynamic object segmentation
- Formulation of elastic deformation regularization
- Does not explore time dependency

D-NeRF

Submission history

From: Albert Pumarola [view email]
[v1] Fri, 27 Nov 2020 19:06:50 UTC (16,352 KB)

ages. In this paper we introduce D-NeRF, a method that extends neural radiance fields to a dynamic domain, allowing to reconstruct and render novel images of objects under rigid and non-rigid motions from a single camera moving around the scene. For this purpose we consider time as an

- Works on synthetic data
- Works on scenes with isolated object

Time as input

Neural Radiance field (NeRF)

Remaining challenges

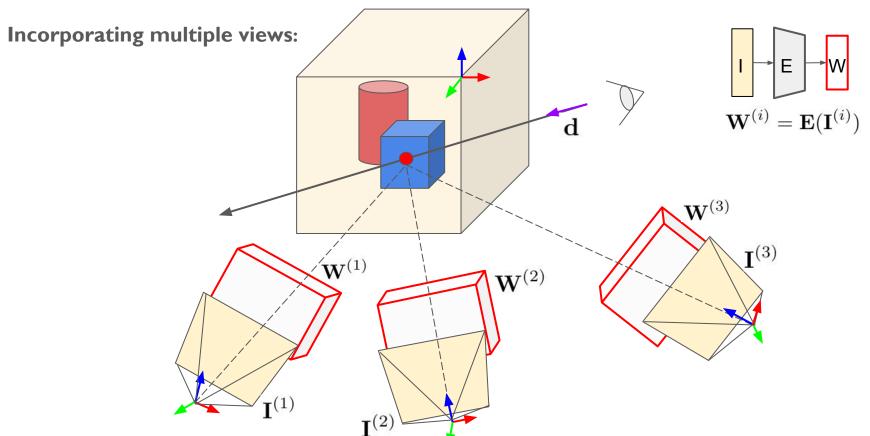
- Handling dynamic scenes when acquiring calibrated views
- One network trained per scene no generalization
 - PixelNeRF (CVPR'21 submission)
 - General radiance field (ICLR'21 submission)

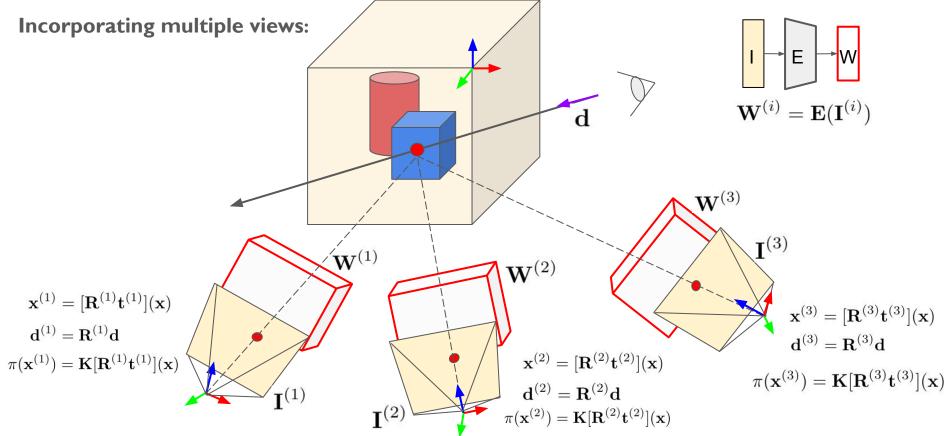
NeRF [7] Mildenhall20

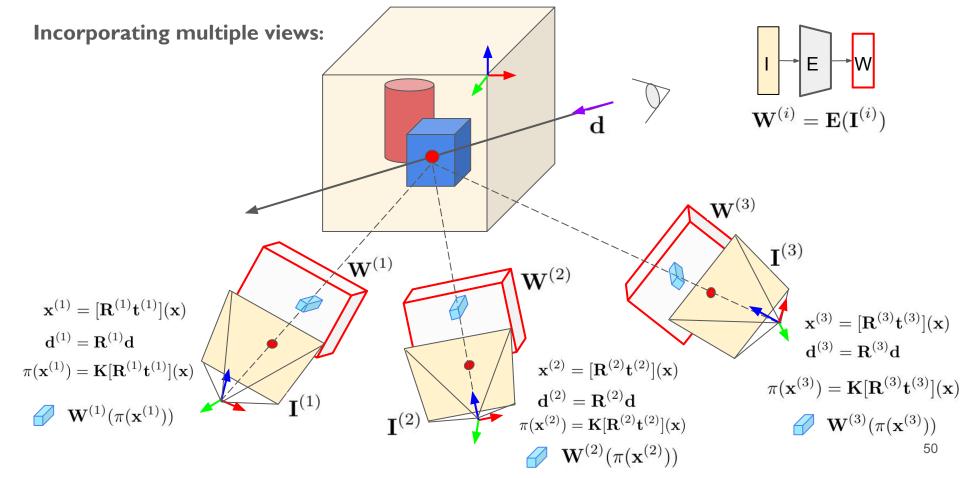
- Optimizing NeRF of each scene independently
- Requiring many calibrated views (TODO put number estimate)
- Using canonical coordinate frame

PixelNeRF [9] Yu20

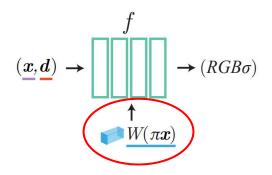
- Training across multiple scenes to learn a (image(s) conditioned?) scene prior
- Address few-shot view synthesis task with sparse set of views
- Predicting a NeRF representation in the camera coordinate system
- + Incorporate a variable number of posed input views







Incorporating multiple views:

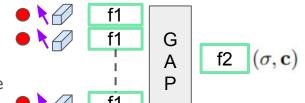


- First, transform 5D input into coordinate system of each view given camera transform
- Then, calculate intermediate feature vector for each view:

$$\mathbf{V}^{(i)} = f_1\left(\mathbf{y}(\mathbf{x}^{(i)}), \mathbf{d}^{(i)}; \mathbf{W}^{(i)}(\pi(\mathbf{x}^{(i)}))\right)$$
 f1

Finally, aggregate with the average pooling operator ψ and passed into a the final layer

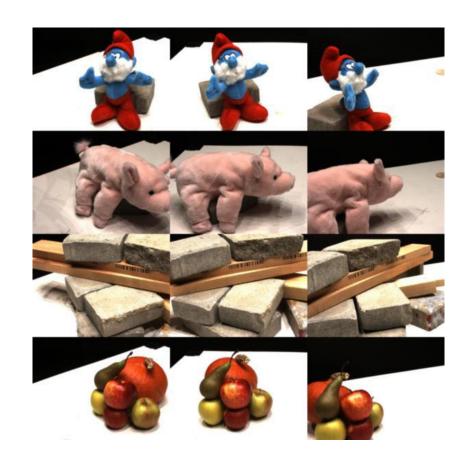
$$(\sigma, \mathbf{c}) = f_2\left(\psi\left(\mathbf{V}^{(1)}, \dots, \mathbf{V}^{(n)}\right)\right)$$

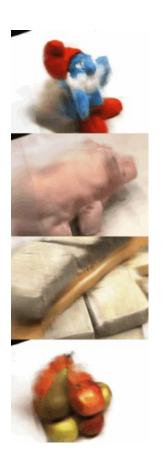


2.	1-view			2-view		
	↑ PSNR	↑ SSIM	↓ LPIPS	↑ PSNR	↑ SSIM	↓ LPIPS
- Local	20.39	0.848	0.196	21.17	0.865	0.175
- Dirs	21.93	0.885	0.139	23.50	0.909	0.121
Full	23.43	0.911	0.104	25.95	0.939	0.071

Table 3: **Ablation studies for ShapeNet chair reconstruction.** We show the benefit of using local features over a global code to condition the NeRF network (—Local vs Full), and of providing view directions to the network (—Dirs vs Full).

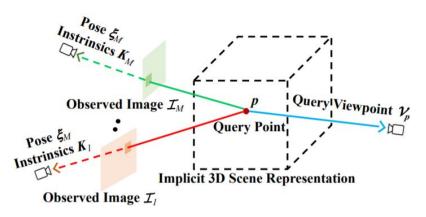
[9] Yu20





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GRF: Learning a General Radiance Field for 3D Scene Representation and Rendering



PixelNeRF

GRF

[v1] Fri, 9 Oct 2020 14:21:43 UTC (7,696 KB) [v2] Sun, 29 Nov 2020 06:33:25 UTC (25,183 KB)

ICLR21 submission

OpenReview grades: 7, 6, 5, 4

[10] Trevithick20

[v1] Thu, 3 Dec 2020 18:59:54 UTC (9,768 KB)

IEEE International Conference on Neural Radiance Fields (ICNeRF)

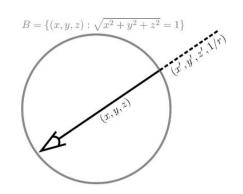
Related Work

Lastly, note that concurrent work [42] adds image features to NeRF. A key difference is that we operate in view rather than canonical space, which makes our approach applicable in more general settings.

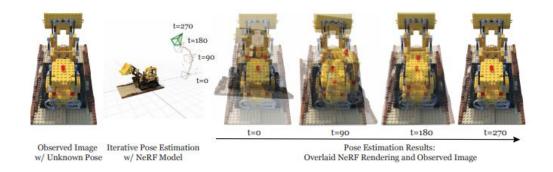
Moreover, we extensively demonstrate our method's performance in few-shot view synthesis, while GRF shows very limited quantitative results for this task.

More works on NeRF

NeRF++: Analyzing and Improving Neural Radiance Fields [Zhang20]



iNeRF: Inverting Neural Radiance Fields for Pose Estimation [Yen-Chen20]



NeRF in the Wild [Ricardo20]...

References

- [7] Mildenhall, Srinivasan, Tancik et al., NeRF: Representing Scenes as Neural Radiance Fields for View Synthesis, ECCV 2020
- [8] Tancik, Srinivasan, Mildenhall et al., <u>Fourier Features Let Networks Learn High Frequency Functions in Low Dimensional Domains</u>, NeurlPS 2020
- [9] Yu et al., PixelNeRF: Neural Radiance Fields from One or Few Images, Arxiv preprint 2020
- [10] Trevithick and Yang, GRF: Learning a General Radiance Field for 3D Scene Representation and Rendering, Arxiv preprint 2020
- [11] Pumarola et al., D-NeRF: Neural Radiance Fields for Dynamic Scenes, Arxiv preprint 2020
- [12] Park et al., Deformable Neural Radiance Fields, Arxiv preprint 2020
- [13] Ricardo et al., NeRF in the Wild: Neural Radiance Fields for Unconstrained Photo Collections, Arxiv preprint [14] Zhang et al., NeRF++: Analyzing and improving neural radiance fields, Arxiv preprint
- [15] Yen-Chen et al., iNeRF: Inverting Neural Radiance Fields for Pose Estimation, Arxiv preprint

Matthew Tancik's 1h talk at Tübingen seminar of the Autonomous Vision Group

Awesome Neural Radiance Fields: https://github.com/yenchenlin/awesome-NeRF

NeRF papers with code: https://paperswithcode.com/method/nerf