Introduction

Measurements are routinely made in a multitude of daily activities ranging from trade to manufacturing or operation of engineering systems. Instrumentation systems, used for such measurements, usually consist of a sensor and the associated electronic circuitry for signal conditioning. Strain gauge sensors are widely used in mechanical engineering applications. In this laboratory, measurement circuits associated with strain gauges will be used to illustrate the principle of a digital weight scale and the effect of electronic circuits on the measurement accuracy. The objectives of this experiment are:

- To use a beam instrumented with strain gauges for weight measurement.
- To investigate the effects of electronic circuits on the measurement uncertainty.
- To relate the measurements using strain gauge to the material property.

Theory

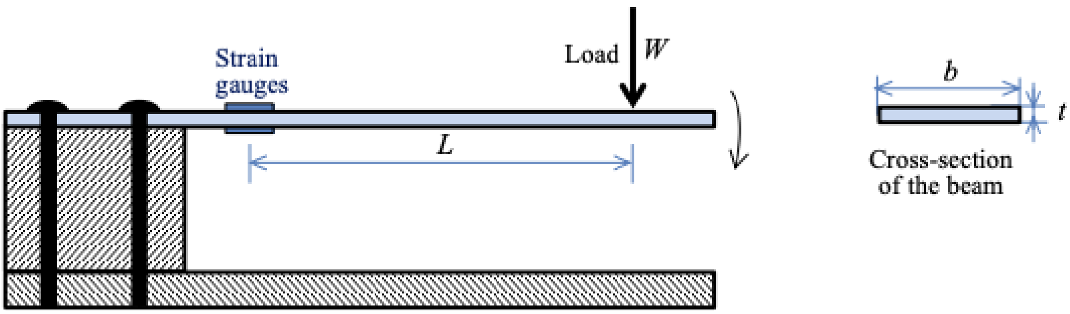


Figure 1 – Cantilever beam instrumented with strain gauges.

Considering the cantilever beam in Figure 1. Stress at the surface is given by the following formula:

$$\sigma = \frac{6WL}{bt^2}$$

where σ is the stress at the surface of the beam, W is the applied force, L is the distance to the applied force, b is the width and t is the thickness of the beam.

Hooke's law relates the stress and strain in the beam:

$$\varepsilon = \frac{\sigma}{E}$$

Where E is the material Young's Modulus

Strain Gauges can be mounted on a beam to measure the strain at the surface. The change in the resistance of the strain gauge is given by the following formula:

$$\frac{\Delta R}{R} = G_F \varepsilon$$

where R is the resistance of the gauge, ΔR is the change in the gauge resistance, G_F is the gauge factor ($G_F = 2$) and ε is the strain.

Measurement circuits such as a Potential Divider, a Wheatstone Bridge, and a Wheatstone Bridge plus amplifier are used to allow accurate measurement of the change in resistance of the strain gauges. This difference causes a measurable voltage change.

Several combinations of strain gauges are possible, providing variable levels of accuracy, these being a 'quarter', 'half' and 'full' bridge. The output voltage of the bridge is related to the excitation voltage and given by the following equation.

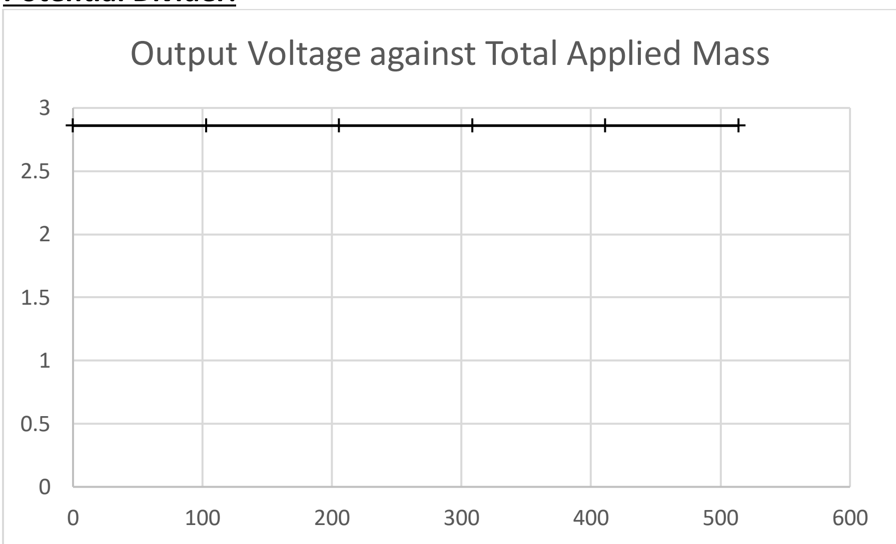
$$\Delta V = K\frac{\Delta R}{R}V$$

Where $K = \frac{1}{4}$ for a quarter bridge, $K = \frac{1}{2}$ for a half bridge configuration and K = 1 for a full bridge configuration, R is the resistance of the gauge, ΔR is the change in the gauge resistance due to strain.

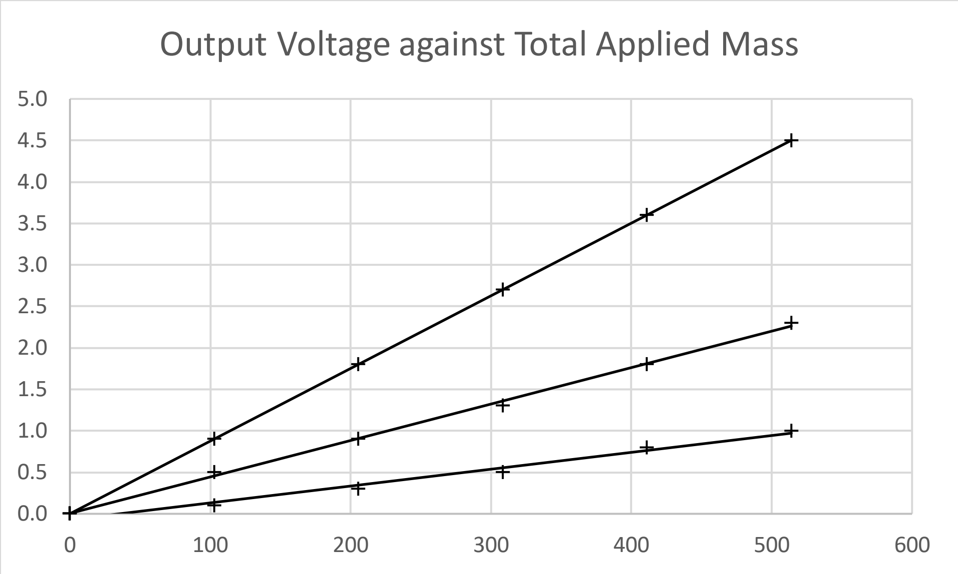
Table of results:

Total Applied Mass/g	Potential Divider/V	Quarter Bridge/mV	Half Bridge/mV	Full Bridge/mV	Full Bridge + Amplifier/V
0	2.86	0.0	0.0	0.0	0.000
102.7	2.86	0.1	0.5	0.9	0.451
205.4	2.86	0.3	0.9	1.8	0.902
308.2	2.86	0.5	1.3	2.7	1.351
411.1	2.86	0.8	1.8	3.6	1.815
514.0	2.86	1.0	2.3	4.5	2.260
M_x	2.86	0.4	1.1	2.1	1.041

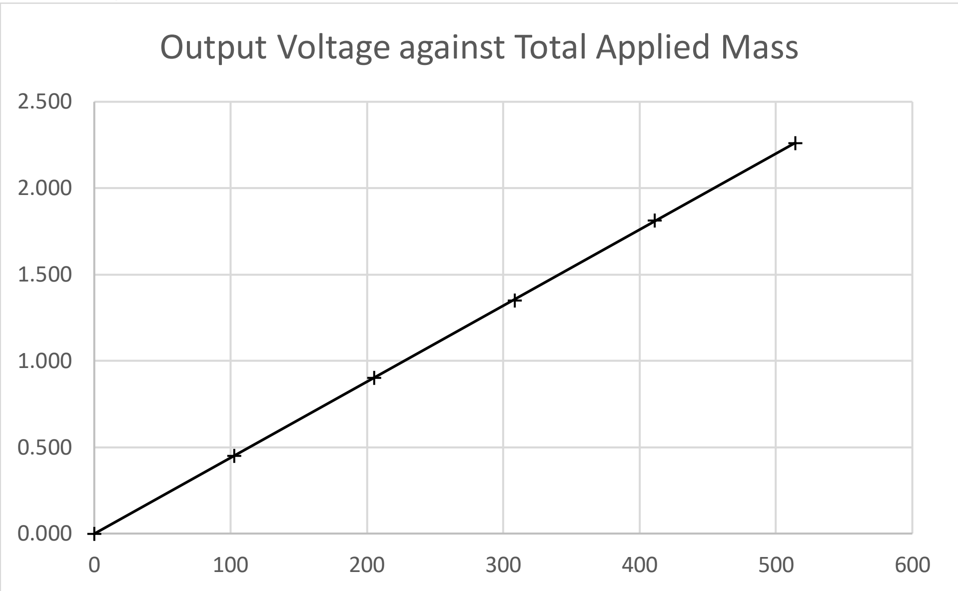
Potential Divider:



Quarter, Half, and Full Bridge:



Full Bridge with Amplifier:



Resolutions:
Calculating linear equations for each line of best fit, they are as follows:

Potential Divider:	y = 2.86 (constant result)
Quarter Bridge:	y = 0.002x – 0.0713
Half Bridge:	y = 0.0044x + 0.0052
Full Bridge:	y = 0.0088x + 0.0009
FB + Amplifier:	y = 0.0044x – 0.0012

Using the smallest possible change in voltage measurement, the mass resolution of each setup can be found:

Quarter Bridge:	+- 50 g
Half Bridge:	+- 22.71 g
Full Bridge:	+- 11.37 g
FB + Amplifier:	+- 0.22 g

Unknown Mass Calculations:

Quarter Bridge:

$y = 0.002x - 0.0713$

$0.4 = 0.002x - 0.0713$

$x = 235.65 \pm 50.00$ g

Half Bridge:

$y = 0.0044x - 0.0052$

$1.1 = 0.0044x - 0.0052$

$x = 22.73 \pm 22.71$ g

Full Bridge:

$y = 0.0088x + 0.0009$

$2.1 = 0.0088x + 0.0009$

$x = 238.53 \pm 11.37$ g

Full Bridge + Amplifier:

$y = 0.0044x - 0.0012$

$1.041 = 0.0044x - 0.0012$

$x = 236.64 \pm 0.22$ g

Beam Dimensions:

Length	0.2 m
Width	0.0192 m
Thickness	0.00334 m

Using the equation:

$$\sigma = \frac{6WL}{bt^2}$$

The stress in the bar can be calculated as follows:

Mass/g	Stress/MPa
0	0
102.7	5.64
205.4	11.29
308.2	16.94
411.1	22.59
514.0	28.25

Amplifier Gain:

Full Bridge	Full Bridge + Amplifier	Gain
0.0	0.000	501.11
0.9	0.451	501.11
1.8	0.902	500.37
2.7	1.351	504.17
3.6	1.815	502.22
4.5	2.260	495.714

Average gain = 500.78

The Following formulas can be used to calculate the strain in the beam:

$$\Delta V = K\frac{\Delta R}{R}V$$

$$\frac{\Delta R}{R} = G_F\varepsilon$$

Using the following values of K, V, G_F :

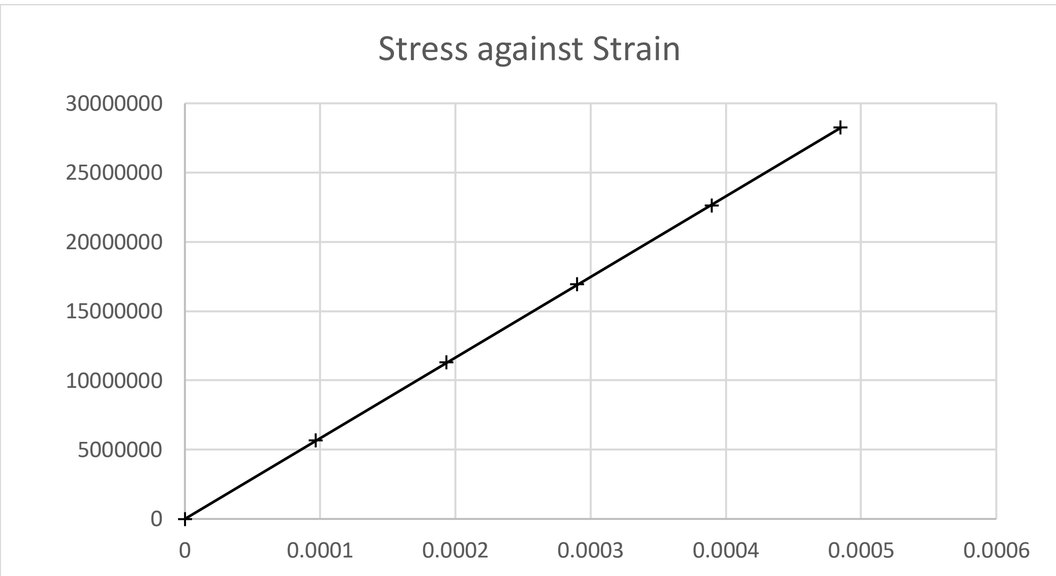
K	1
V	4.66 V
G_F	2

The following values can be found:

Mass/g	Change in V*/mV	$\frac{\Delta R}{R}$	Strain, ε
0	0	0	0
102.7	0.902	0.0001936	0.00009678
205.4	1.804	0.0003871	0.0001936
308.2	2.702	0.0005798	0.0002899
411.1	3.630	0.0007790	0.0003895
514.0	4.520	0.0009700	0.0004850

*Voltage values adjusted based on an amplifier gain of 500.

Using the Stress and Strain Values calculated a Graph may be plotted to find the Young's Modulus of the Material.



To find the Young's Modulus, the gradient of the graph must be calculated.
In this case it is found to be 60GPa.

Discussion

5.1. Explain the role of the 1.5kΩ and 507Ω resistors in the circuits (Figures 2, 3, & 4)

Potential Dividers are simple circuits which work based on the way voltage drops across resistors in series. The total Potential Difference across a circuit is V_{in}, however the total Potential Difference is split between resistors depending on their relative resistance values. The purpose of the 1.5kΩ resistor is to scale the Potential Difference drop in the strain gauge.

In a Wheatstone bridge circuit, the purpose of the resistor is the balance the bridge. The Wheatstone bridge is a circuit that is used to measure an unknown resistance by comparing it with a known resistance. It consists of four resistors arranged in a diamond shape with the unknown resistance connected between two of the corners, and a source of voltage connected across the other two corners.

The 507Ω resistor is used to adjust the voltage across the bridge to zero when the resistance of the unknown resistor is equal to the known resistor. This is done by adjusting the value of the resistor until the voltage across the bridge is zero, which indicates that the resistance of the unknown resistor is equal to the known resistor.

Without the resistor, the bridge would be unbalanced, and the voltage across the bridge would not be zero, making it difficult to determine the value of the unknown resistance accurately. The resistor is therefore essential or accurate measurements in a Wheatstone bridge circuit.

5.2. Compare the measurement uncertainties in sections 3.1 to 3.5. Are the results as predicted by theory? What would be possible reasons for discrepancy?

Based on theory, it can be assumed that the sensitivity of the Wheatstone bridge setup will increase as the number of active resistors does. The measurement uncertainty of each setup can be seen above, and these uncertainty values confirm the predicted trend. The precision of the experiment is limited by the resolution of the Voltmeter, as a small change in the resistance of the strain gauge(s) and therefore the subsequent change in potential difference

5.3. From the experimental data, determine the best estimate of the unknown weight and discuss what factors affect the uncertainty of that estimated value.

The best estimate of the unknown mass comes from the Amplified Full Bridge setup, according to the data from that combination, the unknown mass weighs 236.64 ± 0.22 g. Factors affecting the uncertainty of this value are the resolution of the multi-meter, as well as the accuracy of the uncertainty in weight increments.

5.4 Assuming the beam is made of Aluminium. Is the young's modulus in the expected textbook range? What would be possible reasons for discrepancy?

Based on research, I have found the accepted Young's modulus of Aluminium to be 69 GPa (Engineering ToolBox, 2003) (Kenneth G. Budinski, 2010). My experimental value of the Young's modulus for Aluminium can be calculated by finding the gradient of the stress/strain graph plotted above. The gradient is found to be $6\text{x}10^{10}$, this gives a Young's Modulus of 60 GPa. This is a 13% percent discrepancy, likely due to calibration, measurement, and rounding error.

Conclusion

The purpose of this experiment was to demonstrate an understanding of material properties, as well as the measurement of such properties using electrical apparatus, specifically variations of a Wheatstone Bridge.

In the calculation of the experimental young's modulus, the result was a value of 60 GPa, this is a 13% discrepancy to the commonly accepted value of 69 GPa. As stated previously, this error is likely due to calibration, measurement, and rounding errors. However, this result has the same order of magnitude and is close enough for this experiment to be deemed successful in its goal of demonstrating an understanding of material properties, as well as measuring the properties using electrical apparatus.

The Lab, Using strain gauges, made evident that proper lab techniques provide insight into useful material properties. This allows engineers to design and test designs to ensure their suitability for a given task as well as their safety.

Bibliography

Kenneth G. Budinski, M. K. (2010). Engineering Materials: Properties and Selection. Pearson.

ToolBox, E. (2003). *Young's Modulus, Tensile Strenght and Yield Strength Values for some Materials*. Retrieved from The Engineering ToolBox: engineeringtoolbox.com