Q2

a) radius 1cm:

$A = \pi(0.01)^2$

$= 3.14 \times 10^{-4}$ m^2

$L = 2.5$ m

$E = 200 \times 10^9$ Pa

$F = 50 \times 10^3$ N

$\Delta L = ?$ m

$E = \frac{FL}{A \Delta L}$

$\Delta L = \frac{FL}{AE}$

$\Delta L = \frac{50 \times 10^3 \times 2.5}{3.14 \times 10^{-4} \times 200 \times 10^9}$

$= 1.99 \times 10^{-3}$ m

radius 3.5cm

$A = \pi(0.035)^2 = 3.848 \times 10^{-3}$

$L = 2.5$ m

$E = 200 \times 10^9$ Pa

$F = 50 \times 10^3$ N

$\Delta L = ?$ m

$E = \frac{FL}{A \Delta L}$

$\Delta L = \frac{FL}{EA}$

$\Delta L = \frac{50 \times 10^3 \times 2.5}{3.14 \times 10^{-4} \times 200 \times 10^9}$

$\Delta L = 1.628 \times 10^{-4}$ m

b) Find average ~~area~~ cross sectional area of the shaft:

$$\int_0^{2.5} e^{0.5x}\, dx = \left[2e^{0.5x}\right]_0^{2.5}$$

~~$= 3.87 \times 10^{54}$ cm~~

~~$\frac{3.87 \times 10^{54}}{250}$~~

$= 4.980685\ldots$

$\frac{4.98}{2.5} = 1.992274\ldots$

average radius $= 1.992$ cm

Extension calculation:

$A = \pi (0.01992)^2 = 1.24\cancel{7}\ \cancel{\ }\times 10^{-3}\ m^2$

$L = 2.5m \quad m$

$E = 200 \times 10^9 \quad Pa$

$F = 50 \times 10^3 \quad N$

$\Delta L = ? \quad m$

$$\Delta L = \frac{FL}{AE}$$

$$\Delta L = \frac{50 \times 10^3 \times 2.5}{1.247 \times 10^{-3} \times 200 \times 10^9}$$

$$\Delta L = 5.012 \times 10^{-4} \quad m$$

$5.012 \times 10^{-4} > 1.628 \times 10^{-4}$

$5.012 \times 10^{-4} < 1.99 \times 10^{-3}$

$\therefore$ Answer is in correct range

Extension of shaft is 5.012×10^{-4} m

c) Calculating Youngs modulus for each material:

Steel: 200 GPa (given)

PVC: $L = 1.5$ m

$A = \pi(0.02)^2 = 1.2566 \times 10^{-3}$ m^2

$F = 90$ kN

$\Delta L = 0.030$ m

$$E = \frac{FL}{A\Delta L}$$

$E = 3.58$ ~~M~~ GPa

Titanium:

$L = 1.5$ m

$A = 1.2566 \times 10^{-3}$ m^2

$F =$ ~~90~~ 640 kN

$\Delta L = 0.007$ m

$E = ?$

$$E = \frac{FL}{A\Delta L} = \frac{640 \times 10^3 \times 1.5}{1.2566 \times 10^{-3} \times 0.007}$$

$= 109$ ~~M~~ GPa

Alumina:

$L = 1.5$ m

$A = 1.2566 \times 10^{-3}$ m^2

$F = 830$ kN

$\Delta L =$ ~~1.8~~ 0.0018 m

$E = ?$

$$E = \frac{FL}{A\Delta L} = \frac{830 \times 10^3 \times 1.5}{1.2566 \times 10^{-3} \times 0.0018} = 550 \text{ ~~M~~ GPa}$$

Finding minimum Youngs modulus
In exponentially varying system:

$\Delta L = 0.001$ m
$L = 2.5$ m
$A = 1.247 \times 10^{-3}$ m^2
$F = 50 \times 10^3$ N
$E = ?$

$$E = \frac{50 \times 10^3 \times 2.5}{0.001 \times 1.247 \times 10^{-3}}$$

$$= 1.002405774 \times 10^{11}$$

$$= 100 \text{ GPa}$$

For a maximum extension of less than 1mm, the Youngs modulus of the material should be at least 100 GPa.
This means only steel, titanium, and alumina could be used.
Titanium is both heavier and more expensive than alumina so is not a sensible choice.
Steel is cheaper than alumina but much heavier, Therefore the choice is between steel and alumina depending on budget, ~~w~~ with a low budget the obvious choice is steel, however if ~~more performance~~ a low ~~weight~~ weight is more important alumina should be chosen.