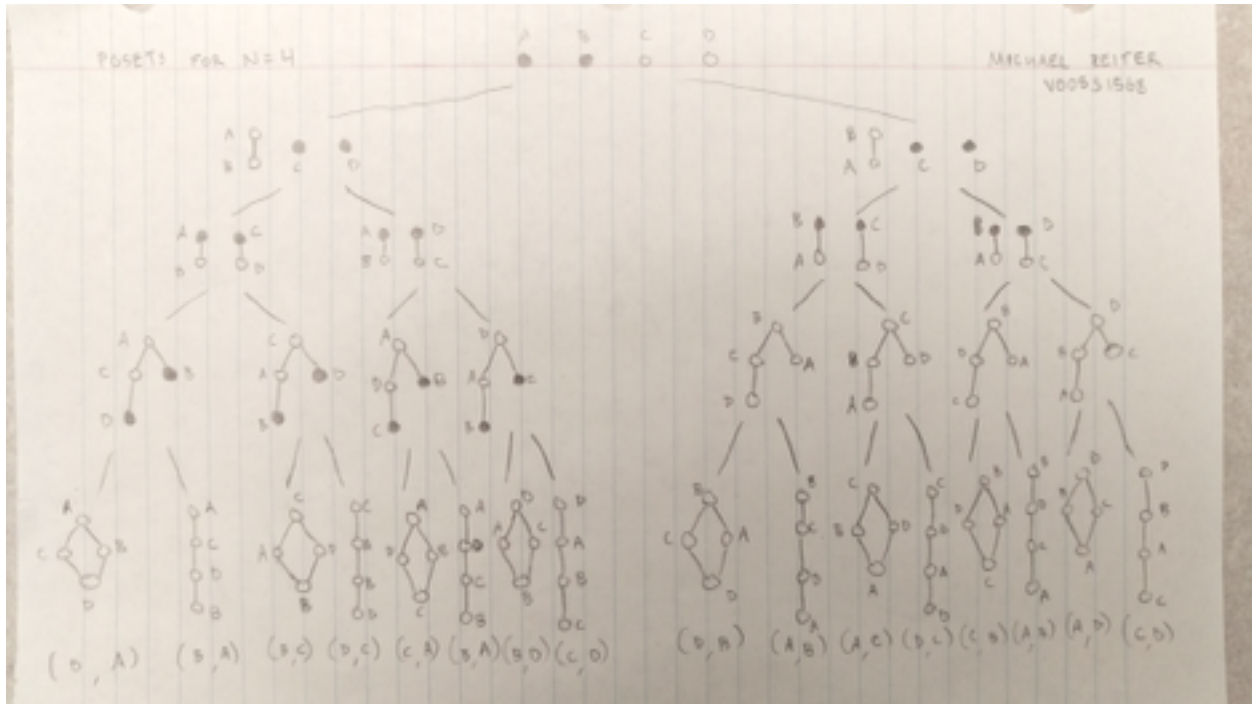


**CSC 226 Assignment 2 Written**



The smallest value of  $n$  for which  $mmA$  and  $mmB$  use a different number of comparisons is 6.

$$C(n) = \begin{cases} 0 & \text{for } n=1 \\ 1 & \text{for } n=2 \\ 2 + C\left(2^{\lfloor \log_2 \left(\frac{n}{2}\right) \rfloor}\right) + C\left(n - 2^{\lfloor \log_2 \left(\frac{n}{2}\right) \rfloor}\right) & \text{for } n \geq 3 \end{cases}$$

The minimum number of inversions of a permutation of  $1, 2, \dots, n$  is 0 (provided its already in order).

The maximum number of inversions of a permutation of  $1, 2, \dots, n$  is  $n \text{ choose } 2 = n(n-1) / 2$ .

To compute inversions in a permutation, you can create a Red Black tree and keep track of a sum of inversions. Each item in the permutation is added to the tree. Whenever a node is attached to the left subtree, after doing any necessary rotations, we add the amount of nodes to the right of it to the sum (i.e. size of right subtree+1). After the tree is complete, the sum will equal the number of inversions. Since we are adding  $n$  items to the tree, we are calling `put()`  $n$  times. `put()` runs in  $O(\log n)$  time since the Red Black tree is a balanced binary tree. The modification to `put` to keep track of the inversions incurs no extra run time asymptotically, so the total run time is  $O(n \log n)$ . The code for the modification to `put` would look something like this:

```
if (cmp < 0) { h.left = put(h.left, key, val); inversions +=
size(h.right)+1; }
```