Assignment 4 Solution

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Answer 1

You can find self-dual graphs in Wikipedia or Mathworld. Or you can just find one by trial and error. Below is an example. It is a K_4, which is also a wheel graph. All the wheel graphs are also self-dual.

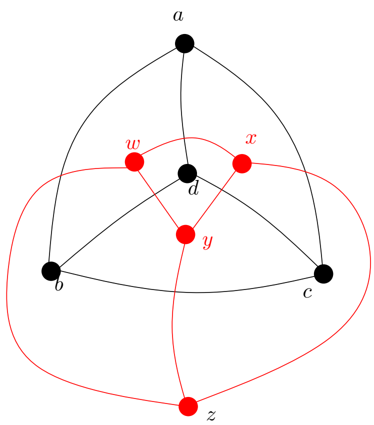

Figure 1: A self dual graph. The bijection is: $a \to z, b \to w, c \to x, d \to y$.

Answer 2

Assuming clockwise embedding, the given graph is shown in Figure 2(a). All the other embeddings are shown in Figures 2(b)–(d). You can assume that the embedding is given counterclockwise. But you have to stick to the rotation system you choose. If you have clockwise embedding around one vertex and counterclockwise embedding around another, then you don't have the right drawing for the graph.

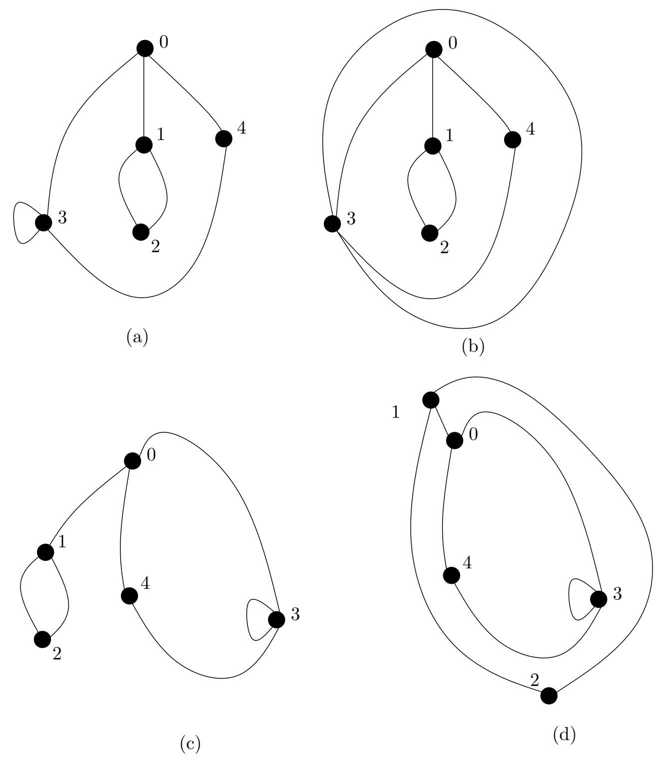

Figure 2: Four planar embeddings of the same graph

Answer 3

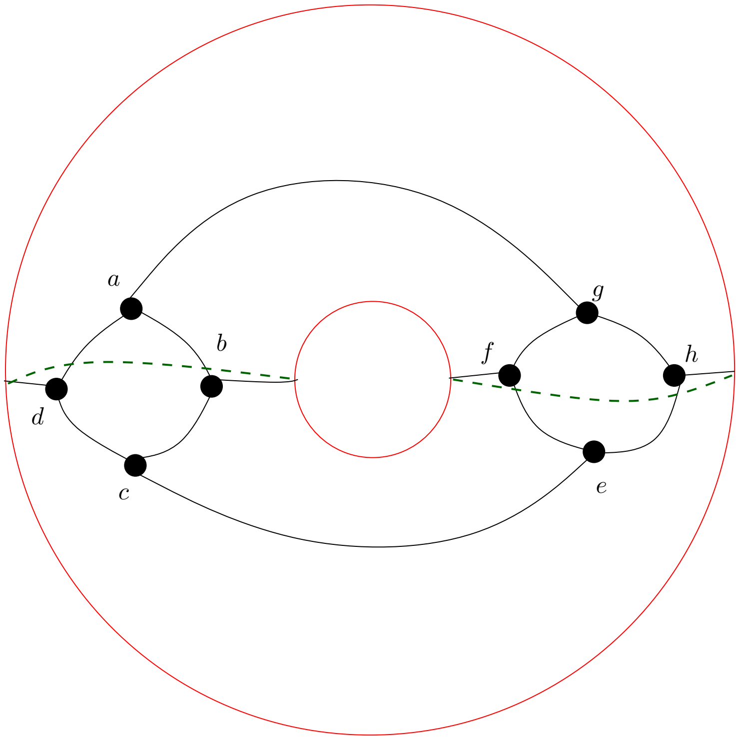

Figure 3: Embedding on a torus.

Here, the number of vertices n is 8.
The number of edges is 12.
The number of faces is 4.

So the genus is $(2 - n + m - f)/2 = 1$, which means it can't be embedded on the plane or on a sphere. It can be embedded on a torus. The embedding is shown in Figure 3.

Answer 4

An example is shown in Figure 4. It is biconnected, bipartite and balanced. Now we prove by contradiction that it doesn't have a Hamiltonian cycle.

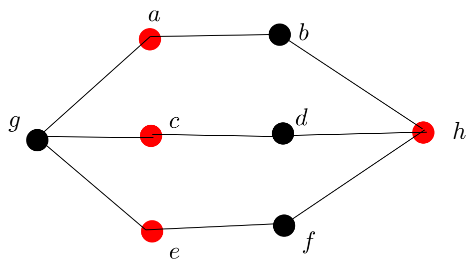

Figure 4: A balanced biconnected bipartite non-Hamiltonian graph.

If there is a Hamiltonian cycle, then it should have eight edges on the cycle, where all the vertices have degree exactly two. Now, the vertices $a - f$ have degree two, so both the edges incident to those vertices should be on the cycle. But that means we would have at least 9 edges on the cycle which is impossible. So, there can't be any Hamiltonian cycle.

Answer 5

If there is a negative cycle, then the Bellman-Ford algorithm goes into an infinite loop trying to minimize the cycle weight.