1.1 $se(3) \rightarrow SE(3)$

$se(3) = \{ \xi = \begin{bmatrix} p \\ \phi \end{bmatrix} \in \mathbb{R}^6, \; p \in \mathbb{R}^3, \; \phi \in so(3) \}$

$SE(3) = \{ T = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} \in \mathbb{R}^{4\times4}, \; R \in SO(3), \; t \in \mathbb{R}^3 \}$

$\xi^\wedge = \begin{bmatrix} \phi^\wedge & p \\ 0^T & 0 \end{bmatrix}$ so $\exp(\xi^\wedge) = \begin{bmatrix} \exp(\phi^\wedge) & \exp(p) \\ 0^T & 1 \end{bmatrix} = \begin{bmatrix} R & Jp \\ 0^T & 1 \end{bmatrix}$

ϕ is a vector define norm, 单位向量 a

$\phi = \theta a$

1.2 $J = \frac{\sin\theta}{\theta} \cdot I + (1 - \frac{\sin\theta}{\theta}) a \cdot a^T + \frac{1-\cos\theta}{\theta} a^\wedge$

1.3 $SE(3) \rightarrow se(3)$

$J \cdot p = t \overset{so}{\Rightarrow} p = t \cdot J^{-1}$

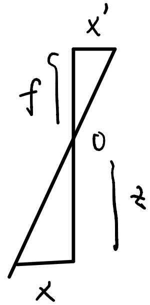
$\theta = 2 \arccos q_0$

$(n_x \; n_y \; n_z) \cdot \theta = \phi$ 旋转向量

1.4 $\frac{\partial(Tp)}{\partial \xi} = \lim\limits_{\delta\xi \to 0} \frac{\exp(\delta\xi^\wedge)\exp(\xi^\wedge) \cdot p - \exp(\xi^\wedge) \cdot p}{\delta\xi}$

$= \lim\limits_{\delta\xi \to 0} \frac{\delta\xi^\wedge \exp(\xi^\wedge) \cdot p}{\delta\xi}$

$= \lim\limits_{\delta\xi \to 0} \begin{bmatrix} \delta\phi^\wedge & \delta p \\ 0^T & 0 \end{bmatrix} \begin{bmatrix} Rp + t \\ 1 \end{bmatrix} = \begin{bmatrix} I & (Rp+t)^\wedge \\ 0^T & 0^T \end{bmatrix}$

1.5 

so $\frac{z}{f} = \frac{x}{x'}$ $\Rightarrow$ $\underset{u}{\overset{\shortparallel}{x'}} = f\frac{x}{z} + c_x$ 相机坐标系

1.6 投影函数对 f_x, f_y, c_x, c_y 的导数

$$\begin{aligned} u &= f_x \frac{x}{z} + c_x \\ v &= f_y \frac{y}{z} + c_y \end{aligned} \overset{d}{\Rightarrow} \begin{bmatrix} \frac{x}{z} & 0, & 1, & 0 \\ 0 & \frac{y}{z}, & 0, & 1 \end{bmatrix}$$

1.7 投影方程对点的导数关系

$u = f_x \frac{x}{z} + c_x$, $v = f_y \frac{y}{z} + c_y$

$$\frac{\partial e}{\partial p} = \begin{bmatrix} \frac{f_x}{z}, & 0, & \frac{-x f_x}{(z)^2} \\ 0 & \frac{f_y}{z}, & \frac{-y f_y}{(z)^2} \end{bmatrix}$$