vector space

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1 R^n and C^n

1.1 definition complex number

* ***A complex number*** is an ordered pair (a,b), where $a, b \in R$, but we will write this as $a + bi$

* the set of all complex numbers is denoted by C:

$$C = \{a + bi : a, b \in R\}.$$

* ***addition and multiplication*** on C are defined by

$$(a + bi) + (c + d)i = (a + c) + (b + d)i,$$
$$(a + bi) * (c + d)i = (ac - bd) + (ab + bc)i,$$
$$here a, b, c, d \in R.$$

1.2 Properties of complex arithmetic

commutativity

$$\alpha + \beta = \beta + \alpha \text{ and } \alpha\beta = \beta\alpha \text{ for all } \alpha, \beta \in R$$

associativity

$$(\alpha + \beta) + \lambda = \alpha + (\beta + \lambda) \text{ and } (\alpha\beta)\lambda = \alpha(\beta\lambda) \text{ for all } \alpha, \beta, \lambda \in R$$

indentities

$$\lambda + 0 \text{ and } \lambda 1 = \lambda \text{ for all } \lambda \in C;$$

indentities

$$\text{for every } \alpha \in C, \text{ there exists a unique } \beta \in C \text{ such that } \alpha + \beta = 0;$$

multiplicative inverse

$$\text{for every} \alpha \in C, \text{ with } \alpha \neq 0, \text{ there exists a unique } \beta \in C \text{ such that } \alpha\beta = 1;$$

1.3 Notation F

Throughout this book,F stand for either R or C

1.4 Example

- The set R^3, which you can think of as ordinary space, is the set of all ordered triples of real numbers:

$$R^3 = \{(x, y, z) : x, y, z \in R\}$$

1.5 definition list,length

Suppose n is a nonnegative interger. A **list** of **length** n is an ordered collection of n elements(witch might be numbers, other lists, or more abstract entities) separated by commas and surrounded by parentheses. A list of length n looks like this:

$$(x_1, ..., x_n)$$

Two lists are equal if and only if they have the same length and the some elements in the same order.

1.6 definition F^n

F^n is the set of all lists of length n of elements of F:

$$F^n = \{(x_1, ..., x_n) : x_j \in R for j = 1, ...n\}$$

For $(x_1, ..., x_n) \in F^n$ and $j \in \{1, ..., n\}$, we say that x_j is the j^{th} coordinate of $(x_1, ..., x_n)$

1.6.1 definition in F^n

$$(x_1, ..., x_n) + (y_1, ...y_n) = (x_1 + y_1, ..., x_n + y_n)$$

1.6.2 scalar multiplication in F^n

The product of a number λ and a vector in F^n is computed by multiplying each coordinate of the vector by λ:

$$\lambda(x_1, ..., x_n) = (\lambda x_1, ..., \lambda x_n)$$

here $\lambda \in F$ and $(x_1, ..., x_n) \in F^n$

2 definition of vector space

2.1 definition textbfaddition, scalar multiplication

- An **addition** on a set V is a function that assigns an element $u+v \in V$ to each pair of elements $u, v \in R$
- A **scalar multiplication** on a set V is a function that assigns an element $\lambda v \in F$ to each $v \in R$

2.1.1 definition vector space

A **vector space** is a set V along with addition on V and a scalar multiplication on V sunch taht following Properties hold:
commutativity

$$u + v = v + u \text{ for all } u, v \in V;$$

associativity

$$(u + v) + w = v + (u + w) \text{ and } (ab)v = a(bv) \text{ for all } u, v, w \in V\text{,and all } a, v \in F;$$

additive identity
there exists an element $0 \in R$ sunch that $v + 0 = v$ for all $v \in V$.
additive inverse
for every $v \in V$, there exists $w \in R$ sunch that $v + w = 0$
multiplicative indentity

$1v = v$ for all $v \in V$;

distribution Properties

$a(u + v) = au + av$ and $(a + b)u = au + bu$ for all $a, b \in F$ and all $u, v \in V$

3 Subspaces

3.1 definition subspaces

A subset U of V is called a **subspaces** of V if U is also a vector space (using the same addition and scalar multiplication as on V)

3.1.1 Example

$$\{(x_1, x_2, 0) : x_1, x_2 \in F\} \text{is a subspace of } F^3$$

3.2 Condiitions for a subspaces

A subset of U of V is subspace of V if and noly if U statisfies the following three Condiitions:

additive indentity

$$0 \in U$$

closed under addition

$$u, w \in U \text{ implies } u + w \in U$$

closed under scalar multiplication

$$a \in F \text{ and } u \in U \text{ implies } au \in U$$

3.3 Sum of subspace

3.3.1 definition sum of subsets

Suppose $U_1, ..., U_m$ are subsets of V. the sum of $U_1, ..., U_m$, denoted $U_1, ..., U_m$, is the set of all possible sum of elements of $U_1, ..., U_m$ More precisely,

$$U_1 + ... + U_m = \{u_1 + ... + u_m : u_1 \in U_1, ..., u_m \in U_m\}.$$

3.3.2 sum of subsoace is the smallest containing subspace

Suppose $U_1, ...U_m$ are the subspaces of V. Then $U_1 + ... + U_m$ is the smallest subspace of V containing $U_1, ..., U_m$

3.3.3 diretct sums

Suppose $U_1, ..., U_m$ are subspace of V. Every element of $U_1, ..., U_m$ can be written in the form

$$u_1 + ... + u_m,$$

Where each u_j is in U_j. We will be especially interested in cases where each vector in $U_1, ..., U_m$ can represented in the form above in only one way. this situation is so important that we give it a special name:direct sum.

3.3.4 condititons for a direct sum

Suppose $U_1, ..., U_m$ are subspaces of V. then $U_1 + ... + U_m$ is a direct sum if and only way to wirite 0 as sum $u_1 + ... + u_m$, where each u_j is in U_j, is by taking each u_j equal to 0.

3.3.5 direct sum of two subspace

Suppose U and W are subspace of V. then U + W is a direct sum if and only $U \cap V = \{0\}$