finite dimensional space

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1 span and linear independence

1.1 Notation list of vectors

we will usually wirite lists of vectors without surrounding parentheses.

1.2 definition linear combination

a **linear combination** of list $v_1, ..., v_m$ of vectors in V is a vector of the form

$$a_1v_1 + ... + a_mv_m,$$

where $a_1, ..., a_m \in F$

1.3 definition span

the set of all linear combination of a list of vectors $v_1, ..., v_m$ in V is called the span of $v_1, ..., v_m$, denoted span$(v_1, ..., v_m)$. in other words,

$$span(v_1, ..., v_m) = \{a_1v_1 + a_2v_2 + ... + a_mv_m : a_1, ..., a_m \in F\}.$$

the span of the empty list () is defined to be { 0}

1.4 span is the smallest containing subspace

the span of a list of vectors in V is the smallest subspace in of V containing all the vectors in the list.

1.5 definition spans

if span $(v_1, ..., v_m)$ equals V, we say that $v_1, ..., v_m$ spans V.

1.6 definition polynoomial, $\rho(F)$

- A function ρ : F -> F is called a polynoomial with coefficients in F if there exists $a_0, ..., a_m \in F$ sunch that

$$\rho(z) = a_o + a_1 z + a_2 z^2 + ... + a_m z^m$$

for all z ∈ F

- ρ is set of all polynoomial with coefficients ∈ F

1.7 degree of a polynoomial, deg(p)

- A polynoomial $p \in \rho(F)$ is said to hve degree m if there exists scalars $a_0, ..., a_m \in F$ with $a_m \neq 0$ sunch that

$$\rho(z) = a_0 + a_1 z + ... + a_m z^m$$

for all $z \in F$ if p has degree m, we write deg(p) = m

- the polynoomial that is identically 0 is said to have degree $-\infty$

1.8 Linear independence

- A list $v_z, ..., v_m$ of vectors in V is called **linearly independent** if the only choice of $a_1, ..., a_m \in F$ that makes $a_1 v_1 + ... + a_m v_m$ equal 0 is $a_1 = ... = a_m = 0$

- the empty list() is also declared to be linearly independent

1.9 linearly dependent

- A list of vectors in V is called linearly dependent if it is not linearly independent.

- in other words, a list $v_1, ..., v_m$ of vectors in V is linearly dependent if there exists $a_1, ..., a_m \in F$, not all 0, sunch that $a_1 v_1 + ... + a_m v_m = 0$

1.10 length of linearly independent list $\leq$ length of spanning list

In a finite-dimensional vector space, the length of every linearly independent list of vectors is less than or equal to length of every spanning list of vectors.

Bases

1.11 basis

a **basis** of V is list of vectors in V that is linearly independent and spans V.

1.12 criterion for basis

A list of $v_1, ..., v_n$ of vectors in V is a basis of V if and only if every $v \in V$ can be written uniquely in the form

$$v = a_1v_1 + ... + a_nv_n$$

where $a_1, ..., a_n \in F$

1.13 spanning list contains a basis

Every spanning list in a vector space can be reduced to a basis of the vector space.

1.14 linearly independent list extends to a basis

Every linearly independent list of vectors in a finnite-dimensional vector space can be extended to a basis of vector space.

1.15 every subspace of V is part of a direct sum equal to V

suppose V is finite-dimensional and U is subspace of V. then there is a subspace W of V sunch that V = U ⊕ w

1.16 Basis length does not depend on basis

any two bases of a finite-dimensional vector space have the same length.

1.17 Definition dimension, dim$V$

- The dimension of a finite-dimensional vector space is the length of any basis of the vector space
- the dimension of V (if V is finite-dimensional) is denoted by dim$V$

1.18 Linear independent list of the right length is a basis

Suppose V is finite-dimensional. then every linearly independent list of vectors in V with length dim$V$ is a basis of V.

1.19 Spanning list of the right length is a basis

Suppose V is finnite-dimensional. then every spanning list of vectors in V with length dim$V$ is basis of V.

1.20 Demension of a sum

if U_1 and U_2 are subspace of a finite-dimensional vector space, then

$$dim(U_1 + U_2) = dimU_1 + dimU_2 - dim(U_1 \cap U_2).$$