Linear Maps

2024 年 1 月 10 日

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1 The Vector Space of Linear Maps

1.1 Definition linear map

A **linear map** form V to W is a function T:$V \Rightarrow W$ with the following peiperties:

- **additivity**: $T(u+v) = Tu = Tv$ for all u,v $\in V$;
- **homogeneity**: $T(\lambda v) = \lambda(Tv)$ for all $\lambda \in F$ and all $v \in V$;

1.2 Example lihnear maps

from R^3 to R^2
define T $\in L(R^3, R^2)$ by

$$T(x, y, z) = (2x - y + 3z, 7x + 5y - 6z)$$

from F^n to F^m
generalizing the previous example, let m and n be positive intgers, let $A_{i,k} \in F$ for $j = 1, ..., m$ and $k = 1, ..., n$ and define T $\in l(F^n, F^m)$ by $T(x_1, ..., x_n) = (A_{1,1}x_1 + ... + A_{1,n}x_n, ..., A_{m,1}x_1 + ... + A_{m,n}x_n)$ actually every linear map from $F^n to F^m$ is of this form.

1.3 linear maps and basis of domain

Suppose $v_1, ..., v_m$ is a basis of the V and $w_1, ..., w_n \in W$. then there exists a unqiue linear map $T : V \rightarrow W$ sunch that

$$Tv_j = w_j$$

for each j = 1,...,n.

1.4 Algebraic Orerations on L(V, W)

1.4.1 Definition addition and scalar multiplication on L(V,W)

Suppose $S, T \in L(V, W)$ and $lambda \in F$ the sum S + T and the product λT are the linear maps from V to W defined by

$$(S+T)(v) = S(v) + T(v) \text{ and } (\lambda T)(v) = \lambda(Tv)$$

for all v ∈ V

1.4.2 L(V,W) is a vector space

With the operations of addition and scalar multiplication as defined above,L(v,w) is a vector space.

1.4.3 Definition product of linear maps

if T ∈ $L(U, V)$ ans $S \in L(V, W)$, then the product ST ∈ L(U,W) is defined by

$$(ST)(u) = S(Tu)$$

for all u ∈ U

1.4.4 linear maps take 0 to 0

Suppose T is a linear map from V to W. then T(0) = 0.