Assignment2

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2025-10-27

```
## Warning: package 'magick' was built under R version 4.5.1

## Linking to ImageMagick 6.9.13.29
## Enabled features: cairo, freetype, fftw, ghostscript, heic, lcms, pango, raw, rsvg, webp
## Disabled features: fontconfig, x11

## Warning: package 'GA' was built under R version 4.5.1

## Loading required package: foreach

## Loading required package: iterators

## Package 'GA' version 3.2.4

## Type 'citation("GA")' for citing this R package in publications.

## Attaching package: 'GA'

## The following object is masked from 'package:utils':
## ## de
```

#Q1

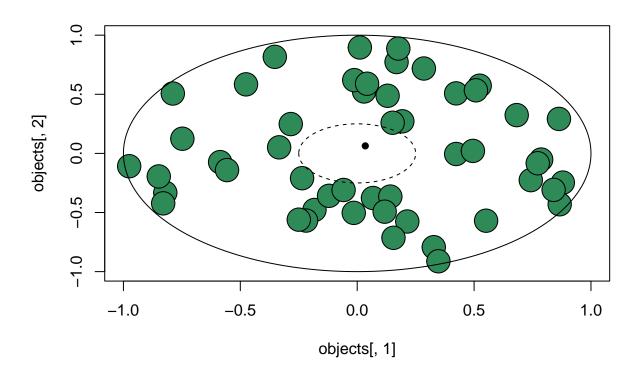
Q1a

```
game_status=function(xt,objects, rad)
     = rep(0,dim(xt)[1])
  min_dists = rep(0,dim(xt)[1])
  J = dim(objects)[1]
           = matrix(1,J,1)
  ones
  for(i in 1:dim(xt)[1])
    min_dists[i] = min(sqrt(rowSums((objects-ones%*%xt[i,])^2)))
    outRad = ((xt[i, 1])^2) + ((xt[i, 2])^2)
    if (min_dists[i] < rad)</pre>
    {
     sk[i] = -1
    else if ((outRad >= 1))
     sk[i] = 1
    }
    else
    {
      sk[i] = 0
    }
  }
  \#sk = 1*(xt[,2]>1)-1*((xt[,1]<-1)|(xt[,1]>+1)|(xt[,2]<-1)|(min\_dists< rad))
  ret = list(status = sk, minDist = min_dists)
  return(ret)
}
```

Q₁b

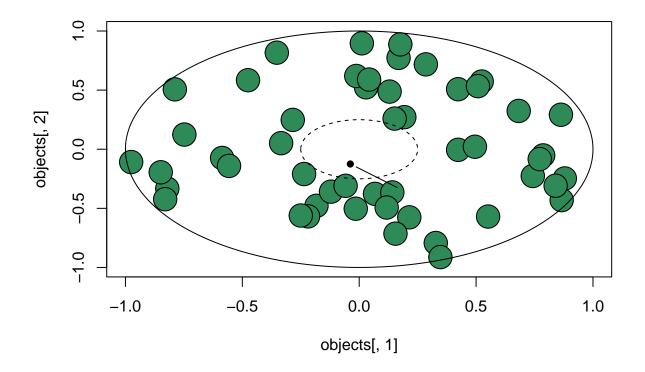
i

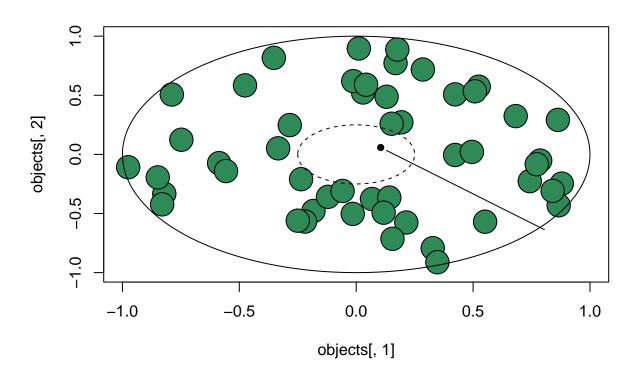
```
set.seed(2024)
J = 50
objects = draw_objects(J)
xt = draw_starts()
delt = 0.025  # How big are the steps you can take?
rad = 0.05  # How close to an object before you crash?
plot_game(xt,objects,rad,'black')
```

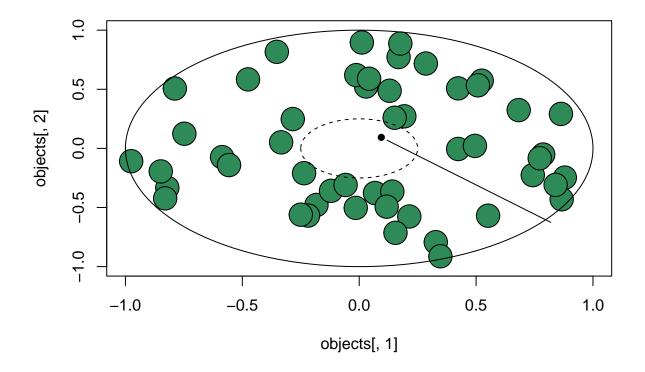


```
play_a_game = function(theta)
  хt
        = draw_starts()
        = play(xt,delt,objects,rad,theta, trace = TRUE)
  score = mean(res$status==1)
  return(score)
}
# play_a_game(theta_rand)
obj = play_a_game
GA = ga(type = 'real-valued', fitness = play_a_game, lower = rep(-10, npars), upper = rep(10, npars), popSi
#plot(GA)
theta_hat = GA@solution[1,]
\#theta\_hat
win = c()
for(i in 1:100)
{
  #objects = draw_objects(J)
          = draw_starts()
  xt_try
  res_final = play(xt_try,delt,objects,rad,theta_hat,plt = FALSE,trace = TRUE)
  #print(typeof(res_final$trajectories))
 trajs = na.omit(res_final$trajectories)
```

```
if (i == 1 || i == 50 || i == 100)
{
    plot_game(xt_try,objects,rad,'black')
    lines(trajs[, 2,]~trajs[, 1, ])
}
win[i] = (res_final$status==1)
}
```







```
propWins = data.frame("Proportion of successful navigations" = mean(win))
kable(propWins)
```

 $\underline{ Proportion. of. successful. navigations } \\$

0.46

ii

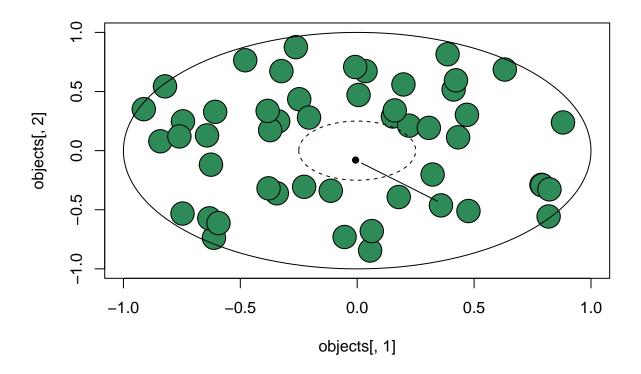
```
winNewObjs = c()
objectsNew = draw_objects(J)
for(i in 1:100)
{

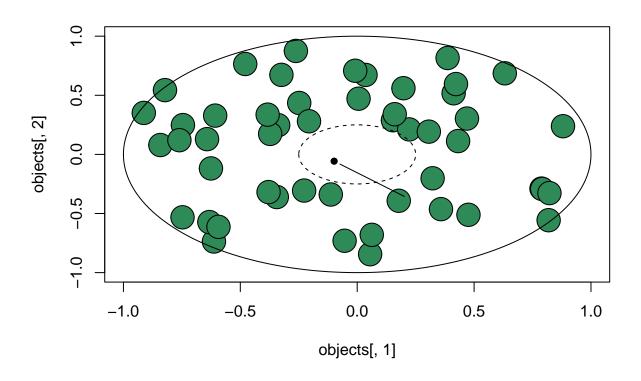
    #objects = draw_objects(J)
    xt_try = draw_starts()
    res_final = play(xt_try,delt,objectsNew,rad,theta_hat,plt = FALSE,trace = TRUE)
    #print(typeof(res_final$trajectories))

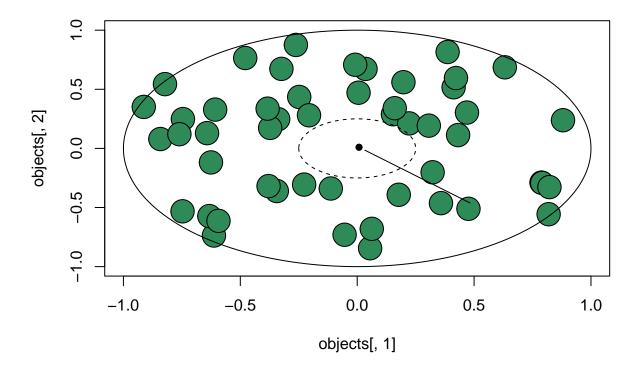
trajs = na.omit(res_final$trajectories)

if (i == 1 || i == 50 || i == 100)
{
```

```
plot_game(xt_try,objectsNew,rad,'black')
    lines(trajs[, 2,]~trajs[, 1, ])
}
winNewObjs[i] = (res_final$status==1)
}
```







```
propNewWins = data.frame("Proportion of successful navigations" = mean(winNewObjs))
kable(propNewWins)
```

 $\frac{ \text{Proportion.of.successful.navigations}}{0.07}$

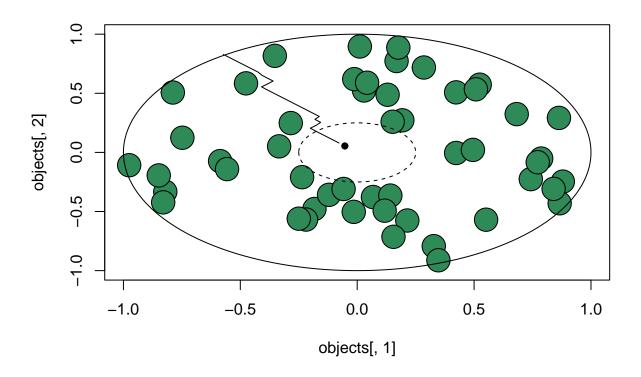
Can see that when measuring performance against the same trees, the model performs relatively well with a proportion of 0.73 passing. However, when using new trees, the model performs much worse with only a proportion of 0.21 passing. This means that the model has not learned how to avoid objects, but simply how to avoid the given set of objects.

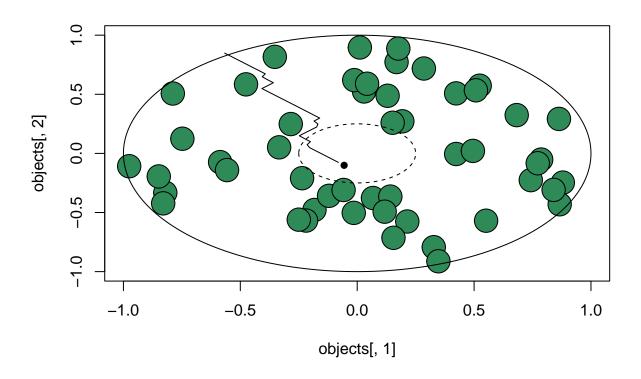
$\mathbf{Q1c}$

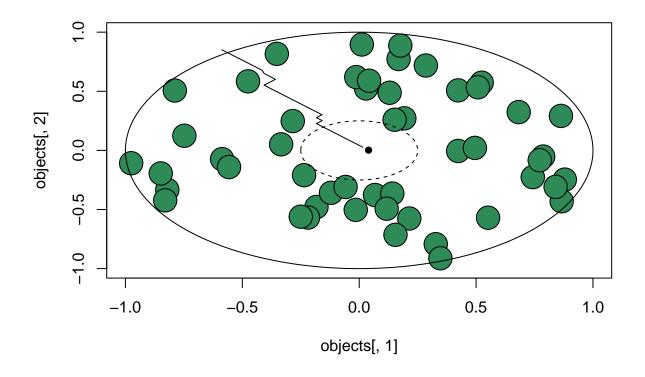
i

```
#print(rad)
  res_status = game_status(xt, objects, rad)
             = res_status$status
  minDists = res_status$minDist
  #print(minDists)
  inverseDist = 1/pmax((minDists - rad), 1e-6)
  # Check which games are still active:
              = (status != 0)
  terminal
  if(trace)
    trajectories = array(dim = c(dim(xt)[1],dim(xt)[2],101))
   trajectories[,,1] = xt
  if(plt){plot_game(xt,objects,rad,'black')}
  while((any(status==0))&(k<100))</pre>
    k = k + 1
    # Now, let the model update the position of the pieces:
    #print("1")
    ct = controlAdjusted(xt, theta, inverseDist)
    #print(ct)
    xt = xt+ct*delt*cbind(1-terminal,1-terminal)
    # Checkk the game status after the positions are updates:
    res_status = game_status(xt, objects, rad)
    status
               = res_status$status
    minDists = res_status$minDist
    inverseDist = 1/(minDists-rad)
             = (status != 0)
    terminal
    if(trace){trajectories[,,k] = xt}
    if(plt){plot_game(xt,objects,rad,c('red','black','green')[status+2])}
  }
  return(list(k = k, status = status, xt= xt, trajectories = trajectories))
controlAdjusted = function(xt,pars, inverseDist)
  xt_aug = cbind(xt, inverseDist)
  res_model = model(xt_aug,pars,rep(nodes,2))
  #print(res model)
 return(res_model$a3)
play_a_game_adj = function(theta)
        = draw_starts()
 xt
        = playAdj(xt,delt,objects,rad,theta, trace = TRUE)
  score = mean(res$status==1)
  return(score)
```

```
= 3
p
q = 2
nodes = 3
npars = p*nodes+nodes*nodes+nodes+q
# npars
theta_rand = runif(npars,-1,1)
objAdj = play_a_game_adj
GA_adj = ga(type = 'real-valued', fitness = play_a_game_adj,lower = rep(-10,npars),upper = rep(10,npars
# plot(GA_adj)
theta_hat_adj = GA_adj@solution[1,]
# theta_hat_adj
win_adj = c()
for(i in 1:100)
{
  #objects = draw_objects(J)
  xt_try = draw_starts()
 res_final = playAdj(xt_try,delt,objects,rad,theta_hat_adj,plt = FALSE,trace = TRUE)
  #print(typeof(res_final$trajectories))
 trajs = na.omit(res_final$trajectories)
  if (i == 1 || i == 50 || i == 100)
      plot_game(xt_try,objects,rad,'black')
      lines(trajs[, 2,]~trajs[, 1, ])
  }
  win_adj[i] = (res_final$status==1)
```







```
propWinsAdj = data.frame("Proportion of successful navigations" = mean(win_adj))
kable(propWinsAdj)
```

 $\frac{\text{Proportion.of.successful.navigations}}{0.93}$

```
winNewObjs_adj = c()

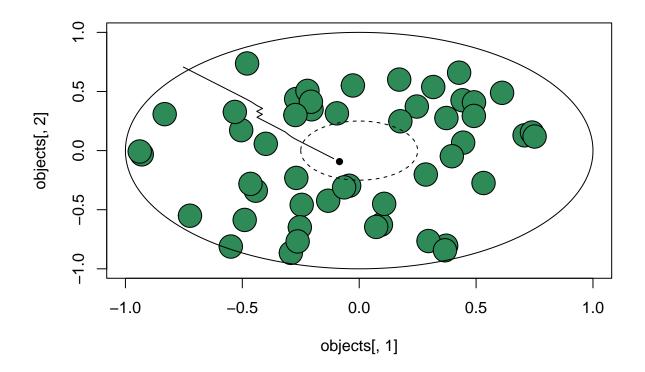
objectsNew = draw_objects(J)
for(i in 1:100)
{
    xt_try = draw_starts()

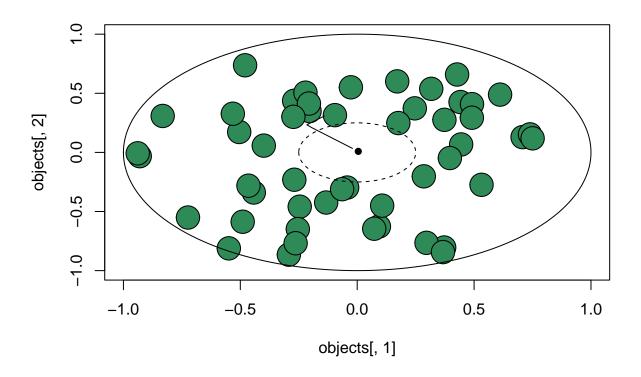
    res_final = playAdj(xt_try,delt,objectsNew,rad,theta_hat_adj,plt = FALSE,trace = TRUE)
    #print(typeof(res_final$trajectories))

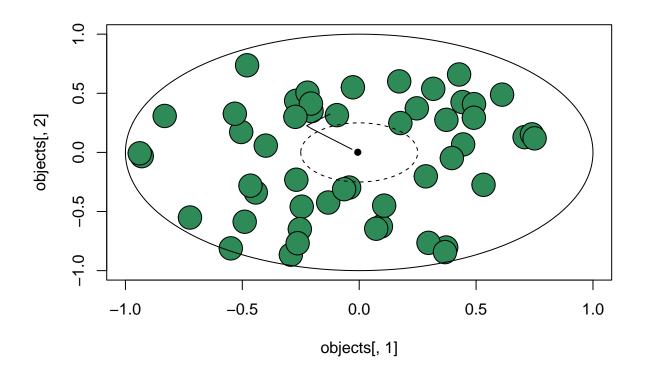
trajs = na.omit(res_final$trajectories)

if (i == 1 || i == 50 || i == 100)
{
    plot_game(xt_try,objectsNew,rad,'black')
    lines(trajs[, 2,]~trajs[, 1, ])
}
```

```
winNewObjs_adj[i] = (res_final$status==1)
}
```







```
#mean(winNewObjs_adj)
propNewWinsAdj = data.frame("Proportion of successful navigations" = mean(winNewObjs_adj))
kable(propNewWinsAdj)
```

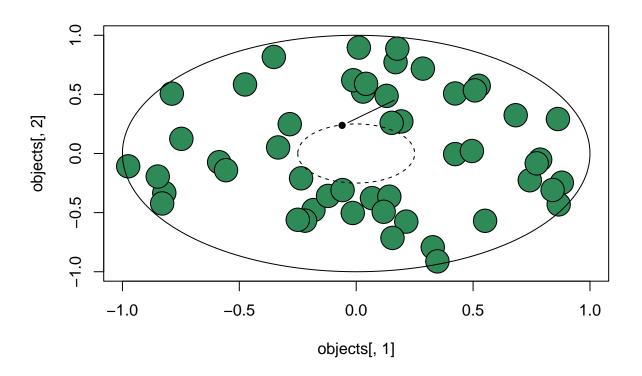
 $\frac{\text{Proportion.of.successful.navigations}}{0.18}$

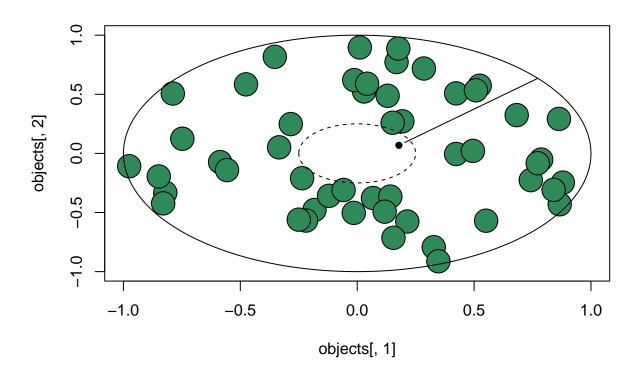
Q1d

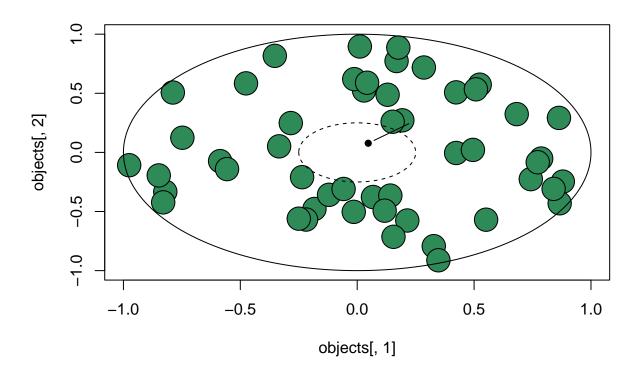
```
### d
play_a_game_adj_newObs = function(theta)
{
    xt = draw_starts()
    objects = draw_objects(J)
    res = playAdj(xt,delt,objects,rad,theta, trace = TRUE)
    score = mean(res$status==1)
    return(score)
}

objAdj = play_a_game_adj_newObs
GA_adj_newObs = ga(type = 'real-valued',fitness = play_a_game_adj_newObs,lower = rep(-10,npars),upper = rep(-10,npars)
```

```
# plot(GA_adj_newObs)
theta_hat_adj_newObs = GA_adj_newObs@solution[1,]
# theta_hat_adj_newObs
win_adj_newObs = c()
for(i in 1:100)
  #objects = draw_objects(J)
          = draw_starts()
  xt_try
  res_final = playAdj(xt_try,delt,objects,rad,theta_hat_adj_newObs,plt = FALSE,trace = TRUE)
  #print(typeof(res_final$trajectories))
  trajs = na.omit(res_final$trajectories)
  if (i == 1 || i == 50 || i == 100)
      plot_game(xt_try,objects,rad,'black')
      lines(trajs[, 2,]~trajs[, 1, ])
  }
  win_adj_newObs[i] = (res_final$status==1)
```







prop_win_adj_newObs = data.frame("Proportion of successful navigations" = mean(win_adj_newObs))
kable(prop_win_adj_newObs)

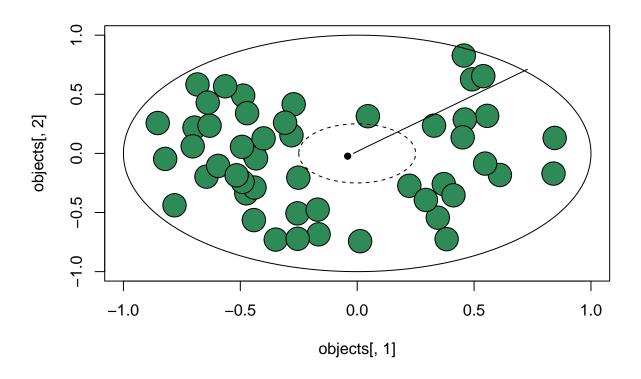
 $\frac{\text{Proportion.of.successful.navigations}}{0.5}$

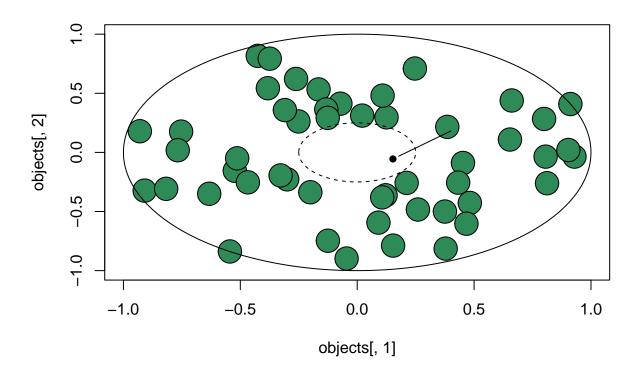
```
winNewObjs_adj_genNewObs = c()

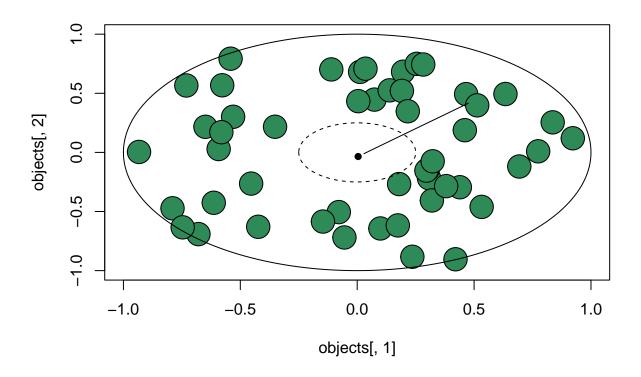
for(i in 1:100)
{

   objects = draw_objects(J)
   xt_try = draw_starts()
   res_final = playAdj(xt_try,delt,objects,rad,theta_hat_adj_newObs,plt = FALSE,trace = TRUE)
   #print(typeof(res_final$trajectories))

   trajs = na.omit(res_final$trajectories)
   if (i == 1 || i == 50 || i == 100)
   {
      plot_game(xt_try,objects,rad,'black')
      lines(trajs[, 2,]~trajs[, 1, ])
   }
   winNewObjs_adj_genNewObs[i] = (res_final$status==1)
}
```







prop_winNewObjs_adj_genNewObs = data.frame("Proportion of successful navigations" = mean(winNewObjs_adj kable(prop_winNewObjs_adj_genNewObs)

 $\frac{\text{Proportion.of.successful.navigations}}{0.23}$

 $\mathbf{Q2}$

 $\mathbf{Q2a}$

```
game_tree= function(m,k, currMoves = 0, maxMoves = 9)
{
    g = c()
    game_state = rho(m,S)
    if(game_state$terminal)
    {
        g = c(g,game_state$winner)
        return(g)
    }
    else if(currMoves == maxMoves)
    {
}
```

```
g = c(g, 2)
    return(g)
  else{
    Index = which(m == 0)
    for(i in 1:length(Index))
    {
      x = m
      x[Index[i]]=k
      g = c(g,game\_tree(x,-1*k, currMoves + 1, maxMoves))
    return(g)
}
m = as.matrix(c(0,0,0,0,0,0,0,0,0))
res5 = game_tree(m, 1, 0, 5)
n_g5 =
          length(res5)
Xwins5 = sum(res5 == -1)
Draws5 = sum(res5 == 0)
0 \text{wins5} = \text{sum}(\text{res5}==+1)
Unfinished5 = sum(res5==+2)
\#Games5 = c(n_g5, Xwins5, Draws5, Owins5, Unfinished5)
games5 = data.frame("Number of moves" = 5, "Number of games" = n_g5, "X wins" = Xwins5, "O wins" = Owin
res8 = game_tree(m, 1, 0, 8)
n_g8 = length(res8)
Xwins8 = sum(res8 == -1)
Draws8 = sum(res8 == 0)
Owins8 = sum(res8 == +1)
Unfinished8 = sum(res8==+2)
\#Games8 = c(n_g8, Xwins8, Draws8, Owins8, Unfinished8)
games8 = data.frame("Number of moves" = 8, "Number of games" = n_g8, "X wins" = Xwins8, "O wins" = Owin
res9 = game_tree(m, 1, 0, 9)
n_g9 = length(res9)
Xwins9 = sum(res9 == -1)
Draws9 = sum(res9 == 0)
Owins9 = sum(res9 == +1)
Unfinished9 = sum(res9==+2)
CombinedFull = data.frame("Number of Games" = n_g9,"X wins" = Xwins9,"Draws" = Draws9, "O wins" = Owins
CombinedFullWinProb = data.frame("Prop X wins" = Xwins9/n_g9, "Prop Draws" = Draws9/n_g9, "Prop O wins" =
datQ2A = rbind(games5, games8)
kable(datQ2A)
```

Number.of.moves	Number.of.games	X.wins	O.wins	Draws
5	15120	0	1440	0
8	255168	77904	49392	23040

```
mcts = function(m, k, alpha)
 g = c()
 game_state = rho(m,S)
 randAlpha = runif(1, 0, 1)
 if(game_state$terminal)
   g = c(g,game_state$winner)
   return(g)
 }
 else
 {
   if (randAlpha <= alpha)</pre>
     Index = which(m == 0)
     for(i in 1:length(Index))
       x = m
      x[Index[i]]=k
       g = c(g,mcts(x,-1*k, alpha))
     return(g)
   }
   else
     g = c(g,2)
     return(g)
   }
 }
m = as.matrix(c(1,1,0,0,0,0,0,0,-1))
resMCTS = mcts(m,-1, 0.95)
n_gMCTS = length(resMCTS)
XwinsMCTS = sum(resMCTS==-1)
DrawsMCTS = sum(resMCTS== 0)
OwinsMCTS = sum(resMCTS==+1)
UnfinishedMCTS = sum(resMCTS==+2)
MCTS = data.frame("Games" = n_gMCTS,"X Wins" = XwinsMCTS, "Draws" = DrawsMCTS, "O wins" = OwinsMCTS, "U
kable(MCTS, caption = "Table of results using Monte Carlo Tree Search, alpha = 0.95")
```

Table 8: Table of results using Monte Carlo Tree Search, alpha = 0.95

Games	X.Wins	Draws	O.wins	Unfinished.Games
307	119	47	126	15

```
kable(CombinedFull, caption = "Table of results using Tree Search")
```

Table 9: Table of results using Tree Search

Number.of.Games	X.wins	Draws	O.wins	Unfinished.games
255168	77904	46080	131184	0

kable(MCTSWinProb, caption = "Table of proportions using Monte Carlo Tree Search, alpha = 0.95")

Table 10: Table of proportions using Monte Carlo Tree Search, alpha=0.95

Prop.X.wins	Prop.Draws	Prop.O.wins
0.3876221	0.1530945	0.4104235

```
kable(CombinedFullWinProb, caption = "Table of proportions using Tree Search")
```

Table 11: Table of proportions using Tree Search

Prop.X.wins	Prop.Draws	Prop.O.wins
0.3053047	0.1805869	0.5141084

Q2c

```
m = as.matrix(c(1,1,-1,0,0,0,0,0,-1))

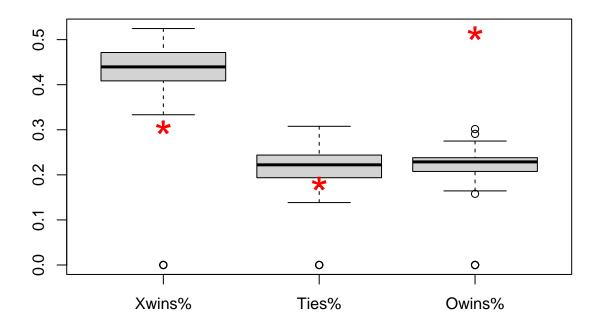
df90 = data.frame()

df70 = data.frame()

for(i in 1:100)
{
    resMCTS90 = mcts(m,1, 0.9)
    resMCTS70 = mcts(m,1, 0.7)

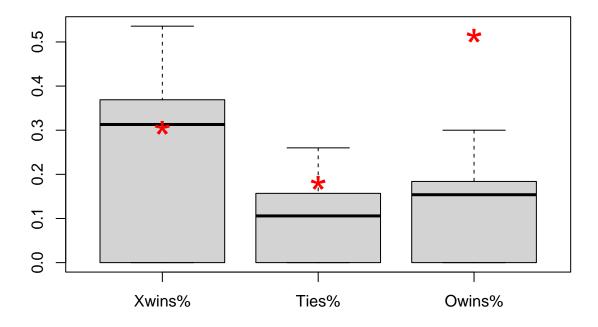
    n_gMCTS90 = length(resMCTS90)
    XwinsMCTS90 = sum(resMCTS90=-1)
    DrawsMCTS90 = sum(resMCTS90=-1)
    UnfinishedMCTS90 = sum(resMCTS90=+1)
    UnfinishedMCTS90 = sum(resMCTS=+2)
```

Box plot of MCTS, alpha = 0.9



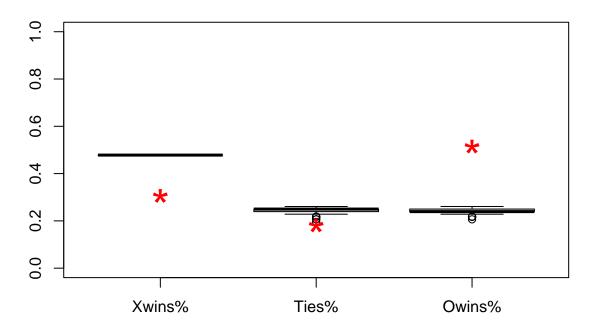
```
boxplot(df70[, 2:4], main = "Box plot of MCTS, alpha = 0.7")
points(c(CombinedFullWinProb$Prop.X.wins, CombinedFullWinProb$Prop.Draws, CombinedFullWinProb$Prop.O.win
```

Box plot of MCTS, alpha = 0.7



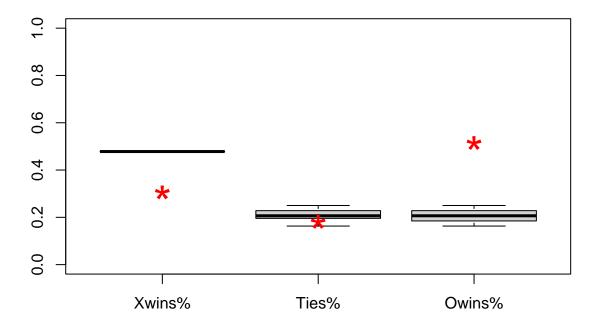
```
else
      if (randAlpha <= alpha)</pre>
       Index = which(m == 0)
        for(i in 1:length(Index))
          x = m
         x[Index[i]]=k
          g = c(g,mcts_minBranch(x,-1*k, alpha, currMoves + 1, minBranch))
       return(g)
      }
      else
       g = c(g,2)
       return(g)
   }
 }
\# m = as.matrix(c(1,1,0,0,0,0,0,0,-1))
# resMCTS_Branch = mcts_minBranch(m,-1, 0.95, 3, 9)
\# n_qMCTS\_Branch = length(resMCTS\_Branch)
# XwinsMCTS_Branch = sum(resMCTS_Branch==-1)
# DrawsMCTS_Branch = sum(resMCTS_Branch== 0)
# OwinsMCTS_Branch = sum(resMCTS_Branch==+1)
# UnfinishedMCTS_Branch = sum(resMCTS_Branch==+2)
# c(n_qMCTS_Branch, XwinsMCTS_Branch, DrawsMCTS_Branch, OwinsMCTS_Branch, UnfinishedMCTS_Branch)
m = as.matrix(c(1,1,-1,0,0,0,0,0,-1))
df90_Branch_new = data.frame()
df70_Branch_new = data.frame()
for(i in 1:100)
  resMCTS90_new = mcts_minBranch(m,1, 0.9, 4, 7)
  resMCTS70_new = mcts_minBranch(m,1, 0.7, 4, 7)
  n_gMCTS90_new = length(resMCTS90_new)
  XwinsMCTS90_new = sum(resMCTS90_new==-1)
  DrawsMCTS90_new = sum(resMCTS90_new== 0)
  OwinsMCTS90_new = sum(resMCTS90_new==+1)
  UnfinishedMCTS90_new = sum(resMCTS90_new==+2)
  Combined90_new = c(n_gMCTS90_new, XwinsMCTS90_new/n_gMCTS90_new,
                     DrawsMCTS90_new/n_gMCTS90_new,OwinsMCTS90_new/n_gMCTS90_new, UnfinishedMCTS90_new)
  n_gMCTS70_new = length(resMCTS70_new)
  XwinsMCTS70_new = sum(resMCTS70_new==-1)
```

Box plot of MCTS, alpha = 0.9 with minimum of 3 steps



boxplot(df70_Branch_new[, 2:4], ylim = c(0, 1), main = "Box plot of MCTS, alpha = 0.7 with minimum of 3 points(c(CombinedFullWinProb\$Prop.X.wins, CombinedFullWinProb\$Prop.Draws, CombinedFullWinProb\$Prop.O.win

Box plot of MCTS, alpha = 0.7 with minimum of 3 steps



$\mathbf{Q3}$

Q3a

```
library(quadprog)

polKern = function(x1, x2, gm, cf, dg)
{
    return((cf+gm*t(x1)%*%x2)^dg)
}

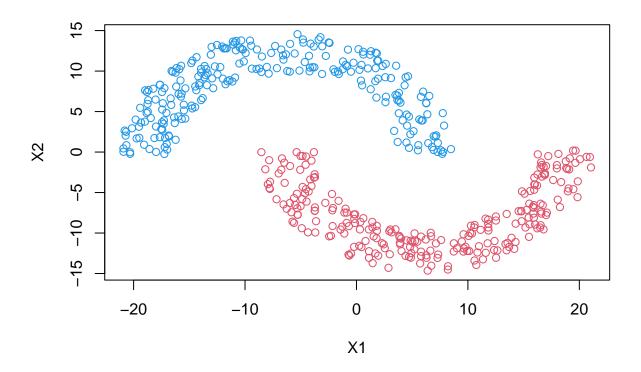
radialKern = function(x1, x2, gm)
{
    return(exp(-gm*(sum((x1 - x2)^2))))
}

my_svm = function(y, x, kern, cost = 0, softMarg = FALSE, gm = 0, cf = 0, dg = 0, plt = FALSE)
{
    N = dim(x)[1]
    DD = matrix(0,N,N)

    if(kern == "none")
{
```

```
for(i in 1:N)
   for(j in 1:N)
     DD[i,j] = y[i]*y[j]*(t(x[i,])%*%x[j,])
  }
}
else if (kern == "poly")
  for(i in 1:N)
    for(j in 1:N)
     KK = polKern(x[i,], x[j,], gm, cf, dg)
     DD[i,j] = y[i]*y[j]*KK
  }
}
else if (kern == "radial")
  for(i in 1:N)
   for(j in 1:N)
     KK = radialKern(x[i,], x[j,], gm)
     DD[i,j] = y[i]*y[j]*KK
  }
}
eps = 5e-6
DD = DD+eps*diag(N)
Amat = cbind(y,diag(N)) # y will be on first row of t(Amat)
bvec = matrix(0, N+1, 1)
d = matrix(1,N,1)
if (softMarg == TRUE)
 negativeC = (-1)*cost
 Amat = cbind(Amat, -diag(N))
  Cvec = rep(negativeC, N)
 vec0 = rep(0, N+1)
 bvec = matrix(c(vec0, Cvec), ncol = 1, nrow = (2*N + 1))
#print(dim(Amat))
#print(dim(bvec))
res = solve.QP(Dmat = DD, dvec = d, Amat = Amat, bvec = bvec, meq = 1, factorized = FALSE)
a = res$solution
if (plt == TRUE)
```

```
{
    plot(a,type= 'h', main = expression(alpha[i]), xlab = expression(i), lwd = 2)
           = round(a,3)
  pad.a
  wh
            = which.max(a)
  if (kern == "none")
   ww = t(a*y)%*%X
   intercept = 1/y[wh] - X[wh, ]%*%t(ww)
    yhat = sign(X%*%t(ww) + intercept[1])
  else if (kern == "poly")
    T1 = rep(0,N)
    for(i in 1:N)
     KK = polKern(x[i, ], t(X), gm, cf, dg)
     T1[i] = sum(a*y*(KK))
    }
    #print(KK)
    KKNew = polKern(x[wh,], t(x), gm, cf, dg)
    intercept = 1/y[wh]-sum(a*y*KKNew)
              = sign(T1+intercept[1])
    yhat
  else if (kern == "radial")
   T1 = rep(0,N)
   for(i in 1:N)
      T1[i] = sum(a*y*(x[i,]%*%t(x)))
    KKNew = radialKern(x[wh,], t(x), gm)
    intercept = 1/y[wh]-sum(a*y*KKNew)
              = sign(T1+intercept[1])
    yhat
  }
  res = list("yhat" = yhat, "padA" = pad.a, "a" = a, "intercept" = intercept)
  return(res)
}
PLADat = read.table("PLA Dynamics.txt")
plot(X2 ~ X1, col = (Y + 3), data = PLADat)
```

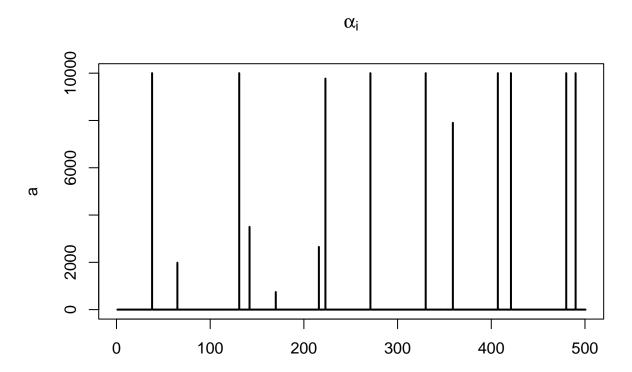


```
# Create data matrix:
X = cbind(PLADat$X1,PLADat$X2)
Y = PLADat$Y

#Y[Y == -1] = 0

gm = 2
cf = 1
dg = 2

mySVM = my_svm(Y, X, "poly", 10000, TRUE, gm, cf, dg, plt = TRUE)
```

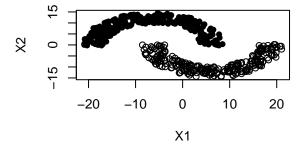


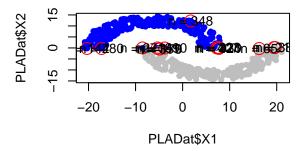
```
yhat = mySVM$yhat
padA = mySVM$padA

par(mfrow = c(2,2))
plot(X2~X1,pch = c(1,16)[(Y+1)/2+1], data = PLADat)
plot(PLADat$X2~PLADat$X1, pch = 16, col = c('grey','blue')[(yhat+1)/2+1])

wh.text = which(padA!=0)
points(PLADat$X2~PLADat$X1, pch = 1, col = c(NA,'red')[(padA>0)+1],cex=2)
text(PLADat$X2[wh.text]~PLADat$X1[wh.text],labels = paste0('n = ',wh.text))
```

i





$\mathbf{Q3b}$

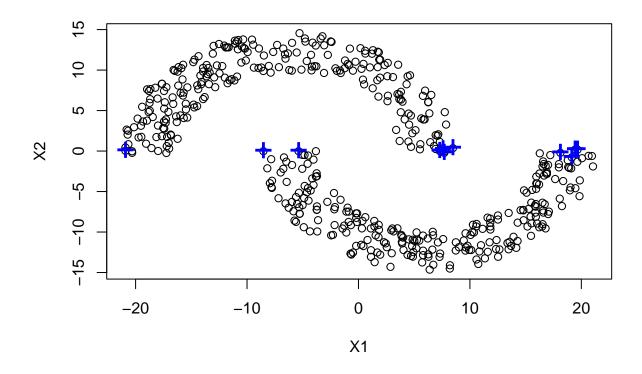
```
library('e1071')

## Warning: package 'e1071' was built under R version 4.5.1

model = svm(Y~(X1 + X2), data = PLADat, scale = FALSE,kernel = 'polynomial',degree =dg,gamma = gm,coef

# Our solution
plot(X2~X1, data = PLADat)

points(model$SV[,2]~model$SV[,1],pch = '+', col = 'blue',cex = 2)
```



```
N = dim(X)[1]
plot(model$coefs~model$index,type = 'h',xlim = c(0,N))
```

