

Calculations Report

Group 13¹

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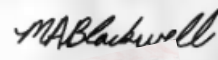
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This calculations report contains all calculations used for design of the individual sections contributing to final design. The table below details how the work was distributed, and which members did what.

Calculations Completed:	Description:	Member(s) Responsible:	Checked by:
Gears	Calculations and SMATH compilation. Iteration of these calculations for maximum performance	*Matthew *Michael *Will-Mari *Emeale	*Michael *Matthew
Shafts	Calculations and SMATH compilation. Iteration of these calculations for optimum performance	*Michael *Matthew	*Cross-Checked
Bearings	Calculations and SMATH compilation. Selection of bearing from catalogue	*Michael *Matthew	*Matthew *Michael
Keyways	Calculations and SMATH compilation. Dimensioning of keyways	*Will-Mari	*Matthew
Retaining Rings	Calculations and SMATH compilation. Selection of retaining rings from catalogue	*Will-Mari *Emeale	*Matthew
Seals	Calculations and SMATH compilation. Selection of seals from catalogue	*Will-Mari	Michael

Signatures

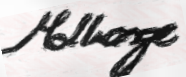
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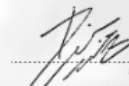
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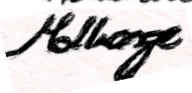
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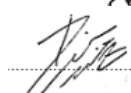
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Appendix A

GEARS

Overview

1. Geartrain Properties
2. Gear Force Analysis
3. Checks for Interference
4. Contact and Allowable Contact
5. Final Contact
6. Final Bending Stress



A1. Geartrain Properties

Geartrain Properties:

Input Characteristics

$$P := 168000 \quad \omega_{input_rpm} := 2600$$

Power and speed as specified on page 5 of the project brief

$$\omega_{input} := \left(\omega_{input_rpm} \right) \cdot \frac{\pi}{30} = 272.2714$$

Quality factor

Quality factor as specified on page 5 of the project brief

$$Q_v := 7$$

Gear Angles

Helix angle of 20 degrees was chosen to minimise axial forces

$$\psi := 20$$

$$\phi_n := 20$$

Transverse Pressure Angle:

Shigley 13-19 (Page 864)

$$\phi_t := \frac{\operatorname{atan} \left(\frac{\tan \left(\phi_n \cdot \frac{\pi}{180} \right)}{\cos \left(\psi \cdot \frac{\pi}{180} \right)} \right)}{\left(\frac{\pi}{180} \right)} = 21.1728$$

Number of teeth

The tooth selection method can be found in the 'Gear Tooth Selection' Appendix

$$\begin{aligned} N1 &:= 40 & N3 &:= 40 \\ N2 &:= 63 & N4 &:= 63 \end{aligned}$$

Gear Ratios

Total gear ratio = 2.480625 which is 99.225% of 2.5

$$GR_{12} := 1.575$$

$$GR_{34} := 1.575$$

Normal Module

$$m_n := 4$$

Transverse Module

Shigley 13-18 (Page 684)

$$m_t := \frac{m_n}{\cos \left(\psi \cdot \frac{\pi}{180} \right)} = 4.2567$$

A2. Gear Force Analysis

Gear Pitch Diameters

$$d1 := 0.001 \cdot m_n \cdot \frac{N1}{\cos\left(\psi \cdot \frac{\pi}{180}\right)} = 0.1703$$

$$d3 := 0.001 \cdot m_n \cdot \frac{N3}{\cos\left(\psi \cdot \frac{\pi}{180}\right)} = 0.1703$$

$$d2 := 0.001 \cdot m_n \cdot \frac{N2}{\cos\left(\psi \cdot \frac{\pi}{180}\right)} = 0.2682$$

$$d4 := 0.001 \cdot m_n \cdot \frac{N4}{\cos\left(\psi \cdot \frac{\pi}{180}\right)} = 0.2682$$

$$Hb := 300$$

Hardness as specified in project description

Gear Force Analysis:

Gear 1:

$$\omega_1 := \omega_{input} = 272.2714$$

Shigley 13-33 (Page 698)

$$T_1 := \frac{P}{\omega_1} = 617.0315$$

$$W_{t21} := \frac{(2 \cdot T_1)}{d1} = 7247.749$$

$$W_{21} := \frac{W_{t21}}{\cos\left(\phi_n \cdot \frac{\pi}{180}\right) \cdot \cos\left(\psi \cdot \frac{\pi}{180}\right)} = 8207.8897$$

Shigley 13-39 (Page 704)

$$W_{r21} := W_{21} \cdot \sin\left(\phi_n \cdot \frac{\pi}{180}\right) = 2807.2636$$

$$W_{a21} := W_{21} \cdot \cos\left(\phi_n \cdot \frac{\pi}{180}\right) \cdot \sin\left(\psi \cdot \frac{\pi}{180}\right) = 2637.9649$$

Gear 2:

$$\omega_2 := \frac{\omega_1}{GR_{12}} = 172.8707$$

$$T_2 := T_1 \cdot GR_{12} = 971.8246$$

$$W_{r12} := W_{r21} = 2807.2636$$

$$W_{t12} := W_{t21} = 7247.749$$

$$W_{12} := W_{21} = 8207.8897$$

$$W_{a12} := W_{a21} = 2637.9649$$

A3. Checks for Interference

Gear 3:

$$W_{t43} := \frac{(2 \cdot T_2)}{d3} = 11415.2047$$

$$W_{43} := \frac{W_{t43}}{\cos\left(\phi_n \cdot \frac{\pi}{180}\right) \cdot \cos\left(\psi \cdot \frac{\pi}{180}\right)} = 12927.4263$$

$$W_{r43} := W_{43} \cdot \sin\left(\phi_n \cdot \frac{\pi}{180}\right) = 4421.4402$$

$$W_{a43} := W_{43} \cdot \cos\left(\phi_n \cdot \frac{\pi}{180}\right) \cdot \sin\left(\psi \cdot \frac{\pi}{180}\right) = 4154.7947$$

Gear 4:

$$\omega_3 := \frac{\omega_2}{GR_{34}} = 109.7592$$

$$T_3 := T_2 \cdot GR_{34} = 1530.6237$$

$$W_{r34} := W_{r43} = 4421.4402$$

$$W_{t34} := W_{t43} = 11415.2047$$

$$W_{34} := W_{43} = 12927.4263$$

$$W_{a34} := W_{a43} = 4154.7947$$

Checks for Interference:

Smallest number of pinion teeth (N_{p_test}) for a given gear ratio mg :

Shigley 13-22 (Page 686)

$$m_{g1} := GR_{12} = 1.575$$

$$k := 1$$

$$m_{g2} := GR_{34} = 1.575$$

$$N_{p_test_GR.1} := \frac{\left(2 \cdot k \cdot \cos\left(\psi \cdot \frac{\pi}{180}\right)\right)}{\left(1 + 2 \cdot m_{g1}\right) \cdot \left(\sin\left(\phi_t \cdot \frac{\pi}{180}\right)\right)^2} \cdot \left(m_{g1} + \sqrt{m_{g1}^2 + \left(1 + 2 \cdot m_{g1}\right) \cdot \left(\sin\left(\phi_t \cdot \frac{\pi}{180}\right)\right)^2}\right) = 11.5024$$

$$N_{p_test_GR2} := \frac{\left(2 \cdot k \cdot \cos\left(\psi \cdot \frac{\pi}{180}\right)\right)}{\left(1 + 2 \cdot m_{g2}\right) \cdot \left(\sin\left(\phi_t \cdot \frac{\pi}{180}\right)\right)^2} \cdot \left(m_{g2} + \sqrt{m_{g2}^2 + \left(1 + 2 \cdot m_{g2}\right) \cdot \left(\sin\left(\phi_t \cdot \frac{\pi}{180}\right)\right)^2}\right) = 11.5024$$

A4. Contact and Allowable Contact Stresses

Largest gear (Ng_test) for a specified Np:

Shigley 13-23 (Page 686)

$$N_{p1} := N1 = 40$$

$$N_{p2} := N3 = 40$$

$$N_{g_test_GR1} := \frac{\left(\left(N_{p1}^2 \cdot \left(\sin \left(\phi_t \cdot \frac{\pi}{180} \right) \right)^2 \right) - \left(4 \cdot k^2 \cdot \left(\cos \left(\psi \cdot \frac{\pi}{180} \right) \right)^2 \right) \right)}{\left(4 \cdot k \cdot \cos \left(\psi \cdot \frac{\pi}{180} \right) \right) - \left(2 \cdot N_{p1} \cdot \left(\sin \left(\phi_t \cdot \frac{\pi}{180} \right) \right)^2 \right)} = -30.7291$$

$$N_{g_test_GR2} := \frac{\left(\left(N_{p2}^2 \cdot \left(\sin \left(\phi_t \cdot \frac{\pi}{180} \right) \right)^2 \right) - \left(4 \cdot k^2 \cdot \left(\cos \left(\psi \cdot \frac{\pi}{180} \right) \right)^2 \right) \right)}{\left(4 \cdot k \cdot \cos \left(\psi \cdot \frac{\pi}{180} \right) \right) - \left(2 \cdot N_{p2} \cdot \left(\sin \left(\phi_t \cdot \frac{\pi}{180} \right) \right)^2 \right)} = -30.7291$$

Contact and Allowable Contact Stresses:

More Geartrain and Material Properties:

Poissons ratio, gearset 1&2

$$\nu_{p1} := 0.29 \quad \nu_{p2} := 0.29$$

$$\nu_{g1} := 0.29 \quad \nu_{g2} := 0.29$$

Youngs Modulus, gearset 1&2

$$E_{p1} := 200 \cdot 10^9 \quad E_{p2} := E_{p1}$$

$$E_{g1} := E_{p1} \quad E_{g2} := E_{g1}$$

Axial Pitch

$$P_x := \frac{\left(\pi \cdot m_n \right)}{\sin \left(\psi \cdot \frac{\pi}{180} \right)} = 36.7416$$

Shigley 13-17 (Page 684)

Sc, Contact strength, table 14-6

$$SC := 2.22 \cdot Hb + 200 = 866 \text{ MPa}$$

Shigley Figure 14-5 (Page 742)

Elastic Coefficient (ZE), gearset 1&2

Shigley 14-13 (Page 736)

$$Z_{E1} := \frac{1}{\sqrt{\pi \cdot \left(\frac{1 - \nu_{p1}}{E_{p1}} + \frac{1 - \nu_{g1}}{E_{g1}} \right)}} = 1.8642 \cdot 10^5$$

$$Z_{E2} := \frac{1}{\sqrt{\pi \cdot \left(\frac{1 - \nu_{p2}}{E_{p2}} + \frac{1 - \nu_{g2}}{E_{g2}} \right)}} = 1.8642 \cdot 10^5$$

Overload Factor (Ko)

As specified on page 5 of project description

$$K_o := 1.05$$

Velocity of gears

$$V1 := \omega_1 \cdot \left(\frac{d1}{2} \right) = 23.1796 \quad V3 := \omega_2 \cdot \left(\frac{d3}{2} \right) = 14.7172$$

$$V2 := V1 = 23.1796 \quad V4 := V3 = 14.7172$$

Dynamic factor (Kv), gearset 1&2

Shigley 14-27 and 14-28 (Page 748)

$$B := 0.25 \cdot \left(12 - Q_v \right)^{\frac{2}{3}} = 0.731 \quad A := 50 + 56 \cdot (1 - B) = 65.0638$$

$$K_{v1} := \left(\frac{A + \sqrt{200 \cdot V1}}{A} \right)^B = 1.6879 \quad K_{v2} := \left(\frac{A + \sqrt{200 \cdot V3}}{A} \right)^B = 1.5578$$

$$V_{max} := \frac{\left(A + (Q_v - 3) \right)^2}{200} = 23.849$$

Face width (b)

$$b1 := 3.3 \cdot p_x = 121.2473$$

$$b2 := 5 \cdot p_x = 183.708$$

Shigley 14-19 (Page 743)

Form factor (Lewis-Y)

$$Y_{p1} := 0.3892 \quad Y_{p2} := 0.3892$$

$$Y_{g1} := 0.425 \quad Y_{g2} := 0.425$$

Shigley Table 14-2 (Page)

Size Factor (Ks), gearset 1&2[Shigley a \(Page 751\)](#)

$$K_{s_{p1}} := 0.8433 \cdot \left(b1 \cdot m_t \cdot \sqrt{Y_{p1}} \right)^{0.0535} = 1.1486$$

$$K_{s_{g1}} := 0.8433 \cdot \left(b1 \cdot m_t \cdot \sqrt{Y_{g1}} \right)^{0.0535} = 1.1513$$

$$K_{s_{p2}} := 0.8433 \cdot \left(b2 \cdot m_t \cdot \sqrt{Y_{p2}} \right)^{0.0535} = 1.1744$$

$$K_{s_{g2}} := 0.8433 \cdot \left(b2 \cdot m_t \cdot \sqrt{Y_{g2}} \right)^{0.0535} = 1.1771$$

Load distribution factor (Kh), gearset 1&2

$$C_{mc} := 1$$

[Shigley 14-31 \(Page 752\)](#)

$$C_{pf1} := \frac{b1}{10 \cdot (d1 \cdot 1000)} - 0.0375 + 0.000492 \cdot b1 = 0.0934$$

[Shigley 14-32 \(Page 752\)](#)

$$C_{pf2} := \frac{b2}{10 \cdot (d3 \cdot 1000)} - 0.0375 + 0.000492 \cdot b2 = 0.1608$$

$$C_{pm} := 1$$

[Shigley 14-13 \(Page 752\)](#)

$$C_e := 1$$

[Shigley 14-35 \(Page 752\)](#)

$$a := 0.127$$

The abc values are based on the assumption that our gears are "commercial enclosed units

It is also based on a centre line distance greater than 17cm. Gears are assumed to be

$$b := 0.000622$$

crowned and the face width between 125mm and 425mm

$$c := -0.000000169$$

[Shigley Table 14-9 \(Page 752\)](#)

$$C_{ma1} := a + b \cdot b1 + c \cdot b1^2 = 0.1999$$

$$C_{ma2} := a + b \cdot b2 + c \cdot b2^2 = 0.2356$$

[Shigley 14-34 \(Page 752\)](#)

$$K_{H1} := 1 + C_{mc} \cdot (C_{pf1} \cdot C_{pm} + C_{ma1} \cdot C_e) = 1.2933$$

[Shigley 14-30 \(Page 751\)](#)

$$K_{H2} := 1 + C_{mc} \cdot (C_{pf2} \cdot C_{pm} + C_{ma2} \cdot C_e) = 1.3963$$

Surface condition factor[Shigley \(Page 750\)](#)

$$Z_R := 1$$

Length L-ab, gearset 1&2

$$T1 := \frac{((d1 + d2) \cdot 1000)}{2} \cdot \sin\left(\phi_t \cdot \frac{\pi}{180}\right) = 79.1786$$

$$T2 := \frac{((d3 + d4) \cdot 1000)}{2} \cdot \sin\left(\phi_t \cdot \frac{\pi}{180}\right) = 79.1786$$

$$CP1 := \sqrt{\left(\frac{d1 \cdot 1000}{2} + aden\right)^2 - (R1^2)} = 40.5287$$

$$PD1 := \sqrt{\left(\frac{d2 \cdot 1000}{2} + aden\right)^2 - (R2^2)} = 58.6014$$

$$CD1 := CP1 + PD1 - T1 = 19.9514$$

$$CP2 := \sqrt{\left(\frac{d3 \cdot 1000}{2} + aden\right)^2 - (R3^2)} = 40.5287$$

$$PD2 := \sqrt{\left(\frac{d4 \cdot 1000}{2} + aden\right)^2 - (R4^2)} = 58.6014$$

$$CD2 := CP2 + PD2 - T2 = 19.9514$$

Theory Slides: Week 1 - Gear Theory (Page 30)

Shigley 14-25 (Page 747)

Load sharing ratio

$$m_{N1} := \frac{P_{N1}}{0.95 \cdot CD1} = 0.623$$

$$m_{N2} := \frac{P_{N2}}{0.95 \cdot CD2} = 0.623$$

Shigley 14-21 (Page 745)

Geometry factor, gearset 1&2

$$Z_{I1} := \frac{\left(\cos\left(\phi_t \cdot \frac{\pi}{180}\right) \cdot \sin\left(\phi_t \cdot \frac{\pi}{180}\right)\right)}{2 \cdot m_{N1}} \cdot \left(\frac{m_{G1}}{(m_{G1} + 1)}\right) = 0.1653$$

$$Z_{I2} := \frac{\left(\cos\left(\phi_t \cdot \frac{\pi}{180}\right) \cdot \sin\left(\phi_t \cdot \frac{\pi}{180}\right)\right)}{2 \cdot m_{N2}} \cdot \left(\frac{m_{G2}}{(m_{G2} + 1)}\right) = 0.1653$$

Shigley 14-23 (Page 747)

Number of Cycles:

$$N_{c_{p1}} := 10 \cdot 365 \cdot 2 \cdot 60 \cdot 2600 = 1.1388 \cdot 10^9$$

$$N_{c_{g1}} := 10 \cdot 365 \cdot 2 \cdot 60 \cdot \frac{2600}{1.575} = 7.2305 \cdot 10^8$$

$$N_{c_{p2}} := N_{c_{g1}} = 7.2305 \cdot 10^8$$

$$N_{c_{g2}} := 10 \cdot 365 \cdot 2 \cdot 60 \cdot \frac{2600}{2.480625} = 4.5908 \cdot 10^8$$

Calculated for operation at 2600 rpm for 2 hours per day for 10 years (excluding leap years)

Load cycle factors (ZN)

$$Z_{N_{p1}} := 2.466 \cdot (N_{c_{p1}})^{-0.056} = 0.7671$$

Shigley Figure 14-15 (Page 755)

$$Z_{N_{g1}} := 2.466 \cdot (N_{c_{g1}})^{-0.056} = 0.7868$$

$$Z_{N_{p2}} := 2.466 \cdot (N_{c_{p2}})^{-0.056} = 0.7868$$

$$Z_{N_{g2}} := 2.466 \cdot (N_{c_{g2}})^{-0.056} = 0.8071$$

Hardness ratio ZW

$$Z_W := 1 \quad \begin{array}{l} \text{Both pinion and gear will be made from the} \\ \text{same material and so hardness factors will be} \\ \text{the same making this ratio one} \end{array}$$

Shigley 14-36 (Page 753)

Reliability factor

$$Y_Z := 1 \quad \text{This is based on a 99\% reliability}$$

Shigley Table 14-10 (Page 756)

Temperature factor

$$Y_T := 1 \quad \text{this only becomes a factor above 120 degrees celcius}$$

Shigley (Page 756)

Final Contact Stress Answers:

Contact Stresses:

Shigley 14-16 (Page 738)

$$\sigma_{c_p1} := \left(Z_{E1} \cdot 10^{-3} \right) \cdot \sqrt{W_{t21} \cdot K_O \cdot K_V1 \cdot K_{s_p1} \cdot \left(\frac{K_{H1}}{(d1 \cdot 1000) \cdot b1} \right) \cdot \left(\frac{Z_R}{Z_{I1}} \right)} = 440.7758$$

$$\sigma_{c_g1} := \left(Z_{E2} \cdot 10^{-3} \right) \cdot \sqrt{W_{t12} \cdot K_O \cdot K_V1 \cdot K_{s_g1} \cdot \left(\frac{K_{H1}}{(d2 \cdot 1000) \cdot b1} \right) \cdot \left(\frac{Z_R}{Z_{I1}} \right)} = 351.6322$$

$$\sigma_{c_p2} := \left(Z_{E1} \cdot 10^{-3} \right) \cdot \sqrt{W_{t43} \cdot K_O \cdot K_V2 \cdot K_{s_p2} \cdot \left(\frac{K_{H2}}{(d3 \cdot 1000) \cdot b2} \right) \cdot \left(\frac{Z_R}{Z_{I2}} \right)} = 453.6185$$

$$\sigma_{c_g2} := \left(Z_{E2} \cdot 10^{-3} \right) \cdot \sqrt{W_{t34} \cdot K_O \cdot K_V2 \cdot K_{s_g2} \cdot \left(\frac{K_{H2}}{(d4 \cdot 1000) \cdot b2} \right) \cdot \left(\frac{Z_R}{Z_{I2}} \right)} = 361.8775$$

Allowable contact stresses:

Shigley 14-18 (Page 742)

$$\sigma_{all_p1} := \left(\frac{SC}{1.5} \right) \cdot \left(\frac{Z_{N_p1} \cdot Z_W}{Y_t \cdot Y_Z} \right) = 442.852$$

$$\sigma_{all_g1} := \left(\frac{SC}{1.5} \right) \cdot \left(\frac{Z_{N_g1} \cdot Z_W}{Y_t \cdot Y_Z} \right) = 454.2619$$

$$\sigma_{all_p2} := \left(\frac{SC}{1.5} \right) \cdot \left(\frac{Z_{N_p2} \cdot Z_W}{Y_t \cdot Y_Z} \right) = 454.2619$$

$$\sigma_{all_g2} := \left(\frac{SC}{1.5} \right) \cdot \left(\frac{Z_{N_g2} \cdot Z_W}{Y_t \cdot Y_Z} \right) = 465.9658$$

Safety Factor:

Shigley 14-42 (Page 757)

$$S_{H_p1} := \frac{\left(\sigma_{all_p1} \right)}{\sigma_{c_p1}} = 1.0047$$

$$S_{H_g1} := \frac{\left(\sigma_{all_g1} \right)}{\sigma_{c_g1}} = 1.2919$$

$$S_{H_p2} := \frac{\left(\sigma_{all_p2} \right)}{\sigma_{c_p2}} = 1.0014$$

$$S_{H_g2} := \frac{\left(\sigma_{all_g2} \right)}{\sigma_{c_g2}} = 1.2876$$

A6. Additional Formula for Bending Stress

Gear Weights:

$$Weight_{p1} := \left(\pi \cdot \left(\frac{d1}{2} \right)^2 \right) \cdot \frac{b1}{1000} \cdot 8000 = 22.0862$$

$$Weight_{g1} := \left(\pi \cdot \left(\frac{d2}{2} \right)^2 \right) \cdot \frac{b1}{1000} \cdot 8000 = 54.7875$$

$$Weight_{p2} := \left(\pi \cdot \left(\frac{d3}{2} \right)^2 \right) \cdot \frac{b2}{1000} \cdot 8000 = 33.4639$$

$$Weight_{g2} := \left(\pi \cdot \left(\frac{d4}{2} \right)^2 \right) \cdot \frac{b2}{1000} \cdot 8000 = 83.0114$$

Additional Formulas Required for Bending Stress Formulas:

Bending geometry factor (YJ)

$$Y_{J_{p1}} := 0.59 \cdot 1 = 0.59$$

Shigley Figures 14-7 and 14-8 (Pages 746,747)

$$Y_{J_{g1}} := 0.61 \cdot 0.993 = 0.6057$$

$$Y_{J_{p2}} := 0.59 \cdot 1 = 0.59$$

$$Y_{J_{g2}} := 0.61 \cdot 0.993 = 0.6057$$

Allowable bending stress number

$$St := 0.533 \cdot Hb + 88.3 = 248.2 \text{ MPa}$$

Shigley Figure 14-2 (Page 739)

Rim Thickness Factor:

$$Kb := 1$$

Gears do not have rims

Load cycle factors (YN)

$$Y_{N_{p1}} := 1.6831 \cdot \left(N_{c_{p1}} \right)^{-0.0323} = 0.8582$$

Shigley Figure 14-14 (Page 755)

$$Y_{N_{g1}} := 1.6831 \cdot \left(N_{c_{g1}} \right)^{-0.0323} = 0.8709$$

$$Y_{N_{p2}} := 1.6831 \cdot \left(N_{c_{p2}} \right)^{-0.0323} = 0.8709$$

$$Y_{N_{g2}} := 1.6831 \cdot \left(N_{c_{g2}} \right)^{-0.0323} = 0.8838$$

Final Bending Stress Answers:

Bending Stress

Shigley 14-15

$$\sigma_{b_p1} := W_{t21} \cdot K_o \cdot K_v1 \cdot K_{s_p1} \cdot \frac{1}{b1 \cdot m_t} \cdot \frac{K_{H1} \cdot Kb}{Y_{J_p1}} = 62.6599$$

$$\sigma_{b_g1} := W_{t12} \cdot K_o \cdot K_v1 \cdot K_{s_g1} \cdot \frac{1}{b1 \cdot m_t} \cdot \frac{K_{H1} \cdot Kb}{Y_{J_g1}} = 61.1766$$

$$\sigma_{b_p2} := W_{t43} \cdot K_o \cdot K_v2 \cdot K_{s_p2} \cdot \frac{1}{b2 \cdot m_t} \cdot \frac{K_{H2} \cdot Kb}{Y_{J_p2}} = 66.3645$$

$$\sigma_{b_g2} := W_{t34} \cdot K_o \cdot K_v2 \cdot K_{s_g2} \cdot \frac{1}{b2 \cdot m_t} \cdot \frac{K_{H2} \cdot Kb}{Y_{J_g2}} = 64.7934$$

Allowable Bending Stress

Shigley 14-17 (Page 741)

$$\sigma_{b_all_p1} := \frac{St}{1.5} \cdot \frac{Y_{N_p1}}{Y_t \cdot Y_Z} = 142.0029$$

$$\sigma_{b_all_g1} := \frac{St}{1.5} \cdot \frac{Y_{N_g1}}{Y_t \cdot Y_Z} = 144.1018$$

$$\sigma_{b_all_p2} := \frac{St}{1.5} \cdot \frac{Y_{N_p2}}{Y_t \cdot Y_Z} = 144.1018$$

$$\sigma_{b_all_g2} := \frac{St}{1.5} \cdot \frac{Y_{N_g2}}{Y_t \cdot Y_Z} = 146.2317$$

Safety Factor:

Shigley 14-41 (Page 757)

$$S_{F_p1} := \frac{(\sigma_{b_all_p1})}{\sigma_{b_p1}} = 2.2662 \quad S_{F_g1} := \frac{(\sigma_{b_all_g1})}{\sigma_{b_g1}} = 2.3555$$

$$S_{F_p2} := \frac{(\sigma_{b_all_p2})}{\sigma_{b_p2}} = 2.1714 \quad S_{F_g2} := \frac{(\sigma_{b_all_g2})}{\sigma_{b_g2}} = 2.2569$$

Appendix B

SHAFTS

Overview

1. Endurance Limit
2. Stress Concentration
3. Notch Sensitivity
4. DE-Soberberg
5. Input Shaft Deflection
6. Intermediate Shaft Deflection
7. Output Shaft Deflection

The gear seat was selected as the most critical location since it is the location of maximum bending moment and torque is also present.

Although there are sections of the shaft that have diameters smaller than the minimum allowable diameter calculated in this document, care was taken to ensure that these diameters were in locations where the bending moment had drastically decreased or there was no torque in that portion of the shaft. This can be seen by comparing the BMD of the shafts to the shaft diagrams. There were also lower/no stress concentrations in those sections. Following a brief comparison, (not included) and into account the above reasons. It became clear that the gear seat was the most critical location by a large margin.

B1. Endurance Limit

Inputs

Shaft Material: AISI 4130 Q&T @ 425 C

Ultimate tensile strength (Sut):

$$S_{ut} := 1280000000 \text{ Pa}$$

Shigley Table A-21 (Page 1050)

Yield stress (Sy):

$$S_y := 1190000000 \text{ Pa}$$

Diameters:

$$d_1 := 65 \text{ mm}$$

Chosen as second iteration values

$$d_2 := 65 \text{ mm}$$

$$d_3 := 65 \text{ mm}$$

Endurance Limit:

$$S'_e := 0,5 \cdot S_{ut} = 6,4 \cdot 10^8 \text{ Pa}$$

Shigley 6-8 (Page 290)

Endurance Limit Modifying Factors:

Surface factor, Ka

Get from shaft design document

$$A := 4,51$$

$$B := -0,265$$

Assuming Machined or Cold drawn

Shigley 6-19 (Page 295)

$$K_a := A \cdot \left(\frac{S_{ut}}{1000000} \right)^B = 0,6773$$

This is assuming Sut is less than 1400Mpa

Shigley Table 6-2

Size Factor:

Shigley 6-20 (Page 295)

$$K_{b1} := 1,24 \cdot d_1^{-0,107} = 0,7933$$

$$K_{b2} := 1,24 \cdot d_2^{-0,107} = 0,7933$$

$$K_{b3} := 1,24 \cdot d_3^{-0,107} = 0,7933$$

This is assuming the shafts
are between 51 and 254 mm

Loading Factor:

Shigley 6-26 (Page 298)

$$K_c := 1$$

Temperature Factor:

Shigley 6-27 (Page 299)

$$T_c := 60 \text{ Celsius}$$

where T.c is the temperature in celcius

$$T_f := \left(T_c \cdot \frac{9}{5} \right) + 32 = 140 \text{ Fahrenheit}$$

$$K_d := 0,975 + 0,000432 \cdot T_f - 0,00000115 \cdot T_f^2 + 0,00000000104 \cdot T_f^3 + 0,000000000000595 \cdot T_f^4 = 1,016$$

B2. Stress Concentration and Notch Sensitivity

Reliability Factor:

Shigley Table 6-5

$$K_e := 0,702$$

Assuming 99.99% reliability

Miscellaneous Effects Factor:

No other negative effects known
(Not applicable)

$$K_f := 1$$

Marin Equation: (Final Endurance Limit)

$$S_{e1} := K_a \cdot K_{b1} \cdot K_c \cdot K_d \cdot K_e \cdot K_f \cdot S'_e = 2,4526 \cdot 10^8 \text{ Pa}$$

Shigley 6-18 (Page 295)

$$S_{e2} := K_a \cdot K_{b2} \cdot K_c \cdot K_d \cdot K_e \cdot K_f \cdot S'_e = 2,4526 \cdot 10^8 \text{ Pa}$$

$$S_{e3} := K_a \cdot K_{b3} \cdot K_c \cdot K_d \cdot K_e \cdot K_f \cdot S'_e = 2,4526 \cdot 10^8 \text{ Pa}$$

Stress Concentration and Notch Sensitivity:

Notch Sensitivities:

Assuming Notch Radius of 2mm

Q values

$$q := 0,9$$

Shigley Figure 6-20 (Page 303)

$$q_{shear} := 0,91$$

Shigley Figure 6-21 (Page 304)

Check document

Stress Concentration Factors:

Kt values (Diameter Change)

Kts values (Diameter Change)

Assuming smaller diameter = 60mm
and larger diameter = 75mm radius of 2mm

Shigley Figures A-15-8 and A-15-9 and (Page 1036)

$$K_{t11} := 2,1$$

$$K_{ts11} := 1,8$$

$$K_{t21} := 2,1$$

$$K_{ts21} := 1,8$$

$$K_{t31} := 2,1$$

$$K_{ts31} := 1,8$$

Kt values (Keyway)

Kts values (Keyway)

Assuming smaller diameter = 53mm
and larger diameter = 60mm

Shigley Table 7-1 (Page 365)

$$K_{t12} := 2,14$$

$$K_{ts12} := 3$$

$$K_{t22} := 2,14$$

$$K_{ts22} := 3$$

$$K_{t32} := 2,14$$

$$K_{ts32} := 3$$

Kt values (Final)

$$K_{t1} := \max \left(\left[K_{t11} \ K_{t12} \right] \right) = 2,14$$

$$K_{t2} := \max \left(\left[K_{t21} \ K_{t22} \right] \right) = 2,14$$

$$K_{t3} := \max \left(\left[K_{t31} \ K_{t32} \right] \right) = 2,14$$

Kts values (Final)

$$K_{ts1} := \max \left(\left[K_{ts11} \ K_{ts12} \right] \right) = 3$$

$$K_{ts2} := \max \left(\left[K_{ts21} \ K_{ts22} \right] \right) = 3$$

$$K_{ts3} := \max \left(\left[K_{ts31} \ K_{ts32} \right] \right) = 3$$

Adjusted Stress Concentration Factors:

Shigley 6-32 (Page 303)

$$K_{ff1} := 1 + q \cdot (K_{t1} - 1) = 2,026$$

$$K_{ff2} := 1 + q \cdot (K_{t2} - 1) = 2,026$$

$$K_{ff3} := 1 + q \cdot (K_{t3} - 1) = 2,026$$

$$K_{fs1} := 1 + q_{shear} \cdot (K_{ts1} - 1) = 2,82$$

$$K_{fs2} := 1 + q_{shear} \cdot (K_{ts2} - 1) = 2,82$$

$$K_{fs3} := 1 + q_{shear} \cdot (K_{ts3} - 1) = 2,82$$

Midrange and alternating bending moments and torques

See Matlab section

$$M_{a1} := \sqrt{(317,9)^2 + (432,1)^2} = 536,4427 \text{ Nm}$$

$$M_{m1} := 0 \text{ Nm}$$

$$M_{a2} := \sqrt{(586,5)^2 + (1008)^2} = 1166,2102 \text{ Nm}$$

$$M_{m2} := 0 \text{ Nm}$$

$$M_{a3} := \sqrt{(697,5)^2 + (876,5)^2} = 1120,16 \text{ Nm}$$

$$M_{m3} := 0 \text{ Nm}$$

$$T_{m1} := 617,0315 \text{ Nm} \quad T_{a1} := 30,851575 \text{ Nm}$$

$$T_{m2} := 974,9097 \text{ Nm} \quad T_{a2} := 48,745485 \text{ Nm}$$

$$T_{m3} := 1540,3574 \text{ Nm} \quad T_{a3} := 77,01787 \text{ Nm}$$

Safety Factors:

Specified by project brief

$$n_{sf1} := 1,5$$

$$n_{sf2} := n_{sf1}$$

$$n_{sf3} := n_{sf1}$$

$$n_{sf4} := n_{sf1}$$

DE - Soberberg

Shigley 7-13 and 7-14 (Page 361)

Minimum Diameter:

$$d_{sf1} := \left(\frac{16 \cdot n_{sf1}}{\pi} \cdot \left(\frac{1}{S_{e1}} \cdot \left(4 \cdot (K_{ff1} \cdot M_{a1})^2 + 3 \cdot (K_{fs1} \cdot T_{a1})^2 \right)^{\frac{1}{2}} + \frac{1}{S_y} \cdot \left(4 \cdot (K_{ff1} \cdot M_{m1})^2 + 3 \cdot (K_{fs1} \cdot T_{m1})^2 \right)^{\frac{1}{2}} \right) \right)^{\frac{1}{3}} = 0,0443$$

Chosen Diameter:

$$d_{sf1} := 58 \text{ mm}$$

$$n_{sf1} := \left(\frac{16}{\pi \cdot \left(\frac{d_{sf1}}{1000} \right)^3} \cdot \left(\frac{1}{S_{e1}} \cdot \left(4 \cdot (K_{ff1} \cdot M_{a1})^2 + 3 \cdot (K_{fs1} \cdot T_{a1})^2 \right)^{\frac{1}{2}} + \frac{1}{S_y} \cdot \left(4 \cdot (K_{ff1} \cdot M_{m1})^2 + 3 \cdot (K_{fs1} \cdot T_{m1})^2 \right)^{\frac{1}{2}} \right) \right)^{-1} = 3,3557$$

Minimum Diameter:

$$d_{sf2} := \left(\frac{16 \cdot n_{sf2}}{\pi} \cdot \left(\frac{1}{S_{e2}} \cdot \left(4 \cdot (K_{ff2} \cdot M_{a2})^2 + 3 \cdot (K_{fs2} \cdot T_{a2})^2 \right)^{\frac{1}{2}} + \frac{1}{S_y} \cdot \left(4 \cdot (K_{ff2} \cdot M_{m2})^2 + 3 \cdot (K_{fs2} \cdot T_{m2})^2 \right)^{\frac{1}{2}} \right) \right)^{\frac{1}{3}} = 0,0562$$

Chosen Diameter:

$$d_{sf2} := 58 \text{ mm}$$

$$n_{sf2} := \left(\frac{16}{\pi \cdot \left(\frac{d_{sf2}}{1000} \right)^3} \cdot \left(\frac{1}{S_{e2}} \cdot \left(4 \cdot (K_{ff2} \cdot M_{a2})^2 + 3 \cdot (K_{fs2} \cdot T_{a2})^2 \right)^{\frac{1}{2}} + \frac{1}{S_y} \cdot \left(4 \cdot (K_{ff2} \cdot M_{m2})^2 + 3 \cdot (K_{fs2} \cdot T_{m2})^2 \right)^{\frac{1}{2}} \right) \right)^{-1} = 1,6447$$

Minimum Diameter:

$$d_{sf3} := \left(\frac{16 \cdot n_{sf3}}{\pi} \cdot \left(\frac{1}{S_{e3}} \cdot \left(4 \cdot (K_{ff3} \cdot M_{a3})^2 + 3 \cdot (K_{fs3} \cdot T_{a3})^2 \right)^{\frac{1}{2}} + \frac{1}{S_y} \cdot \left(4 \cdot (K_{ff3} \cdot M_{m3})^2 + 3 \cdot (K_{fs3} \cdot T_{m3})^2 \right)^{\frac{1}{2}} \right) \right)^{\frac{1}{3}} = 0,0575$$

Chosen Diameter:

$$d_{sf3} := 58 \text{ mm}$$

$$n_{sf3} := \left(\frac{16}{\pi \cdot \left(\frac{d_{sf3}}{1000} \right)^3} \cdot \left(\frac{1}{S_{e3}} \cdot \left(4 \cdot (K_{ff3} \cdot M_{a3})^2 + 3 \cdot (K_{fs3} \cdot T_{a3})^2 \right)^{\frac{1}{2}} + \frac{1}{S_y} \cdot \left(4 \cdot (K_{ff3} \cdot M_{m3})^2 + 3 \cdot (K_{fs3} \cdot T_{m3})^2 \right)^{\frac{1}{2}} \right) \right)^{-1} = 1,539$$

The below code is used to determine the shear forces, bending moments, as well as the deflections and angles of the shafts

Bearing Forces

```
Way = 1738.7538;  
Waz = 2359.3328;  
Wby = 1068.5098;  
Wbz = 4888.4162;  
Wcy = -3429.1078;  
Wcz = -2341.851;  
Wdy = -3799.596;  
Wdz = 6509.3067;  
Wey = 1173.7476;  
Wez = -7327.588;  
Wfy = 3247.6926;  
Wfz = -4087.6167;
```

Gear Forces

```
Wr21 = -2807.2636;  
Wr12 = 2807.2636;  
Wr43 = 4421.4402;  
Wr34 = -4421.4402;
```

```
Wa21 = -2637.9649;  
Wa12 = 2637.9649;  
Wa43 = -4154.7947;  
Wa34 = 4154.7947;
```

```
Wt21 = -7247.749;  
Wt12 = 7247.749;  
Wt43 = -11415.2047;  
Wt34 = 11415.2047;
```

Bearing and shaft Dimensions

```
Wa = 0.026;  
Wb = 0.026;  
Wc = 0.026;  
Wd = 0.026;  
We = 0.026;  
Wf = 0.026;
```

```
l = 0.112;  
la = 0.110;  
lb = 0.015;
```

B4. Input shaft (Shear, Moment, Slope, and deflection)

```
lc = 0.05;  
ld = 0.05;  
le = 0.015;  
lf = 0.110;  
bp1 = 0.1212473;  
bp2 = 0.183708;  
bg1 = 0.1212473;  
bg2 = 0.183708;
```

```
d1 = 0.1703;  
d2 = 0.2682;  
d3 = 0.1703;  
d4 = 0.2682;
```

Calculations:

```
d = 0.045; % Using the minimum shaft diameter provides a conservative estimate  
E = 200*10^9; % Young's Modulus  
I = (1/4)*pi*(d/2)^4; % Area Moment of Inertia
```

Input Shaft:

```
distance1 = 0.5*Wa + la + bp1 + lb + 0.5*Wb;  
X = linspace(0,distance1,1000);
```

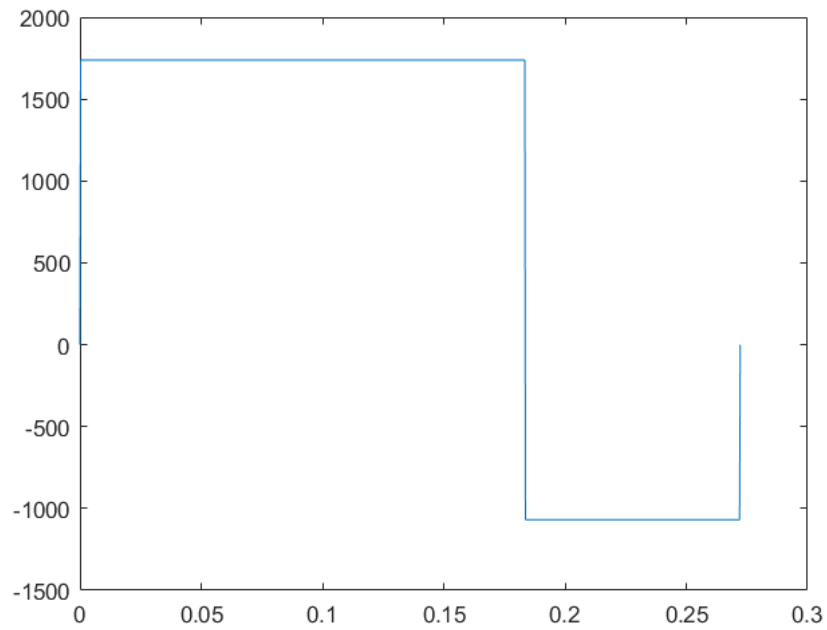
```
M1 = @(X) X;  
M2 = @(X) X - (0.5*Wa+la+(0.5*bp1));  
M3 = @(X) X - ((0.5*Wa)+la+bp1+lb+(0.5*Wb));
```

```
C = -((Way/6).*(M1(distance1).^3) + (Wr21/6).*(M2(distance1).^3) + ...  
      (Wa21/2).*(d1./2).*(M2(distance1).^2) +  
(Wby/6).*(M3(distance1).^3))/distance1;  
D = 0;  
Et = -((Waz/6).*(M1(distance1).^3) + (Wt21/6).*(M2(distance1).^3) + ...  
      (Wbz/6).*(M3(distance1).^3))/distance1;  
F = 0;
```

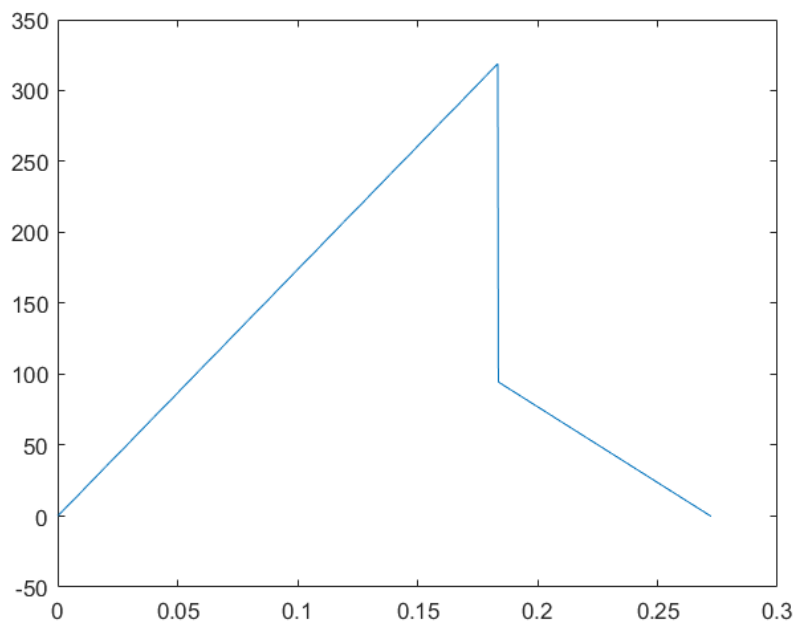
```
Shear_1y = Way.*(M1(X)>0) + Wr21.*(M2(X)>0) + Wby.*(M3(X)>0);  
Shear_1y(1000) = 0;  
Moment_1y = Way.*(M1(X)).*(M1(X)>0) + Wr21.*(M2(X)).*(M2(X)>0) + ...  
            Wa21.*(d1./2).*(M2(X)>0) + Wby.*(M3(X)).*(M3(X)>0);  
Slope_1y = ((Way/2).*(M1(X).^2).*(M1(X)>0) + (Wr21/2).*(M2(X).^2).*(M2(X)>0) +  
...  
            Wa21.*(d1./2).*(M2(X)).*(M2(X)>0) + (Wby/2).*(M3(X).^2).*(M3(X)>0) +  
C)/(E*I);  
Deflection_1y = ((Way/6).*(M1(X).^3).*(M1(X)>0) +  
(Wr21/6).*(M2(X).^3).*(M2(X)>0) + ...
```

```
(Wa21/2).*(d1./2).*(M2(X).^2).*(M2(X)>0) + (Wby/6).*(M3(X).^3).*(M3(X)>0) +  
C*X + D)/(E*I);
```

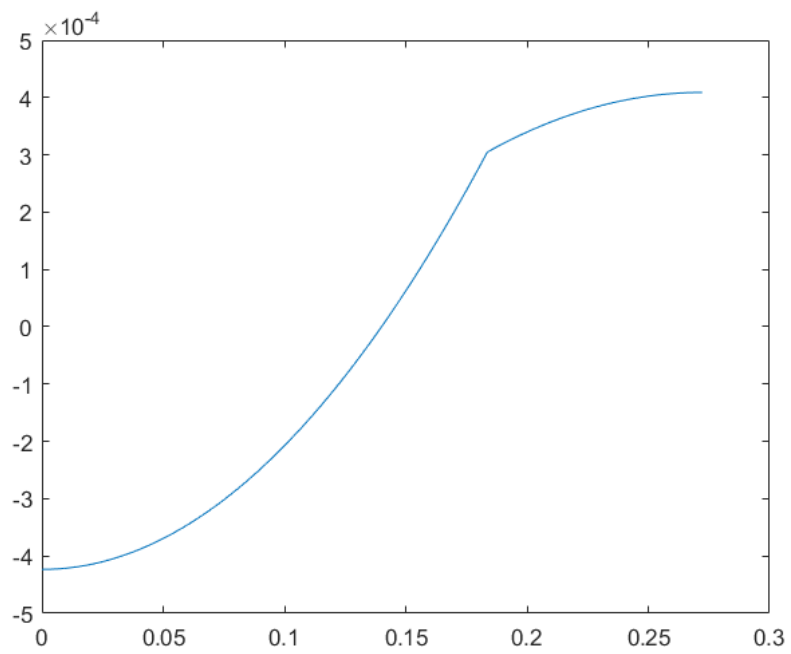
```
plot(X,Shear_1y)
```



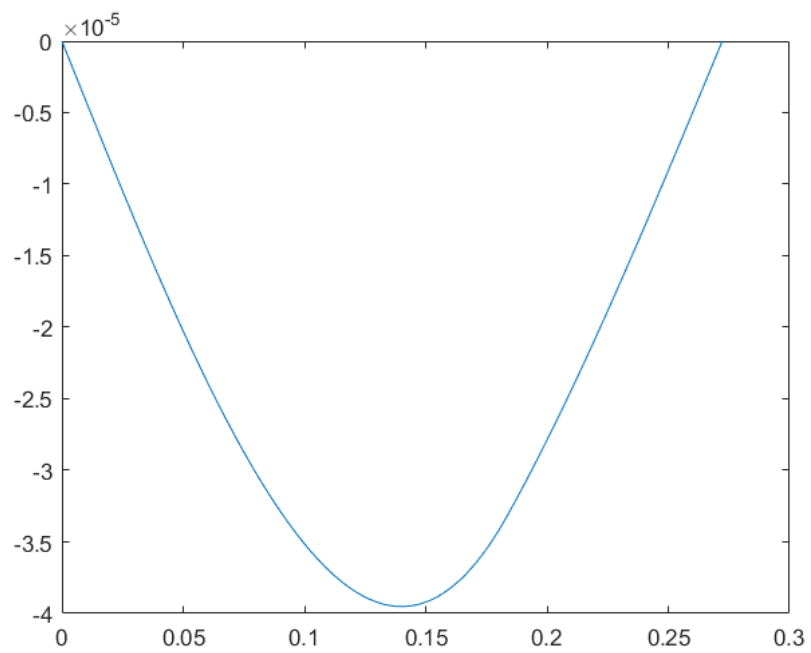
```
plot(X,Moment_1y)
```




```
plot(X,Slope_1y)
```



```
plot(X,Deflection_1y)
```

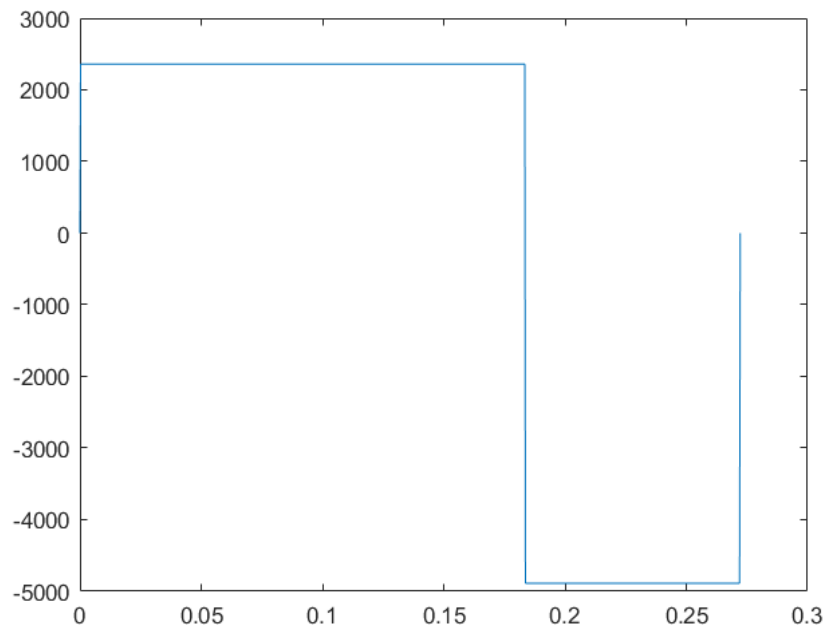


```

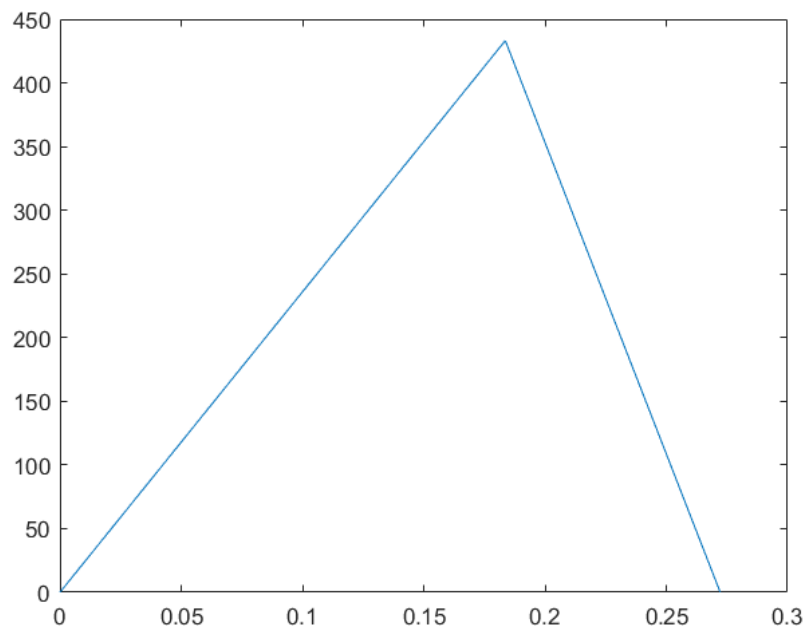
Shear_1z = Waz.*(M1(X)>0) + Wt21.*(M2(X)>0) + Wbz.*(M3(X)>0);
Shear_1z(1000) = 0;
Moment_1z = Waz.*(M1(X)).*(M1(X)>0) + Wt21.*(M2(X)).*(M2(X)>0) +
Wbz.*(M3(X)).*(M3(X)>0);
Slope_1z = ((Waz/2).*(M1(X).^2).*(M1(X)>0) + (Wt21/2).*(M2(X).^2).*(M2(X)>0) +
...
(Wbz/2).*(M3(X).^2).*(M3(X)>0) + Et)/(E*I);
Deflection_1z = ((Waz/6).*(M1(X).^3).*(M1(X)>0) +
(Wt21/6).*(M2(X).^3).*(M2(X)>0) ...
+ (Wbz/6).*(M3(X).^3).*(M3(X)>0) + Et*X + F)./(E*I);
Deflection_1z(1000) = 0;

plot(X,Shear_1z)

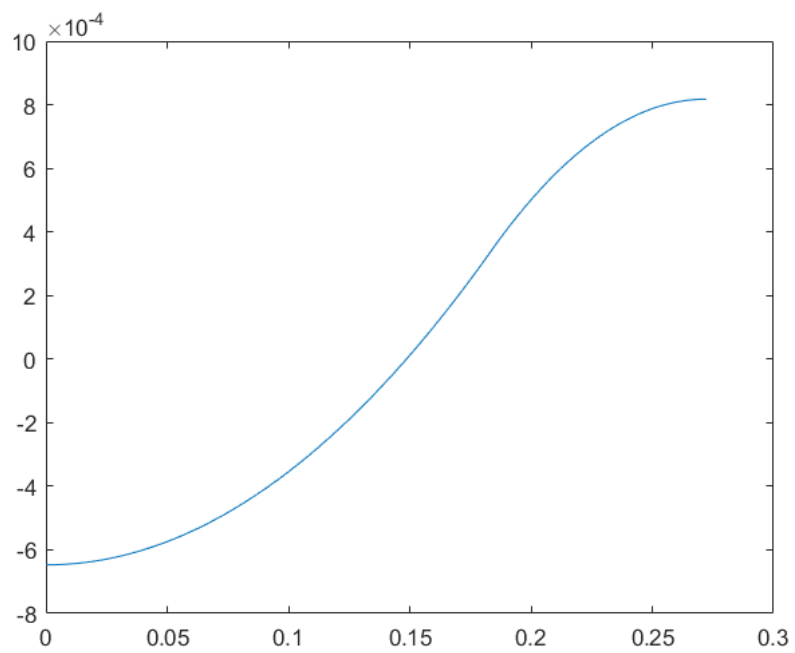
```



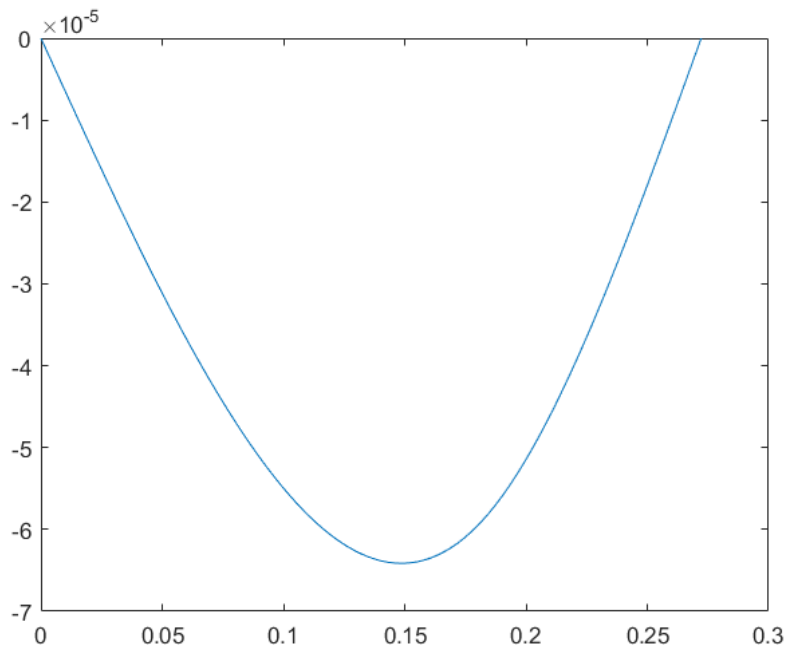
```
plot(X,Moment_1z)
```



```
plot(X,Slope_1z)
```



```
plot(X,Deflection_1z)
```



B5. Intermediate shaft (Shear, Moment, Slope, and deflection)

Intermediate Shaft:

```
distance2 = 0.5*Wc + lc + bg1 + l + bp2 + ld + 0.5*Wd;
X2 = linspace(0,distance2,1000);
```

```
M1 = @(X2) X2;
M2 = @(X2) X2 - (0.5*Wc + lc + (0.5*bg1));
M3 = @(X2) X2 - ((0.5*Wc) + lc + bg1 + l + (0.5*bp2));
M4 = @(X2) X2 - ((0.5*Wc) + lc + bg1 + l + bp2 + ld + (0.5*Wd));
```

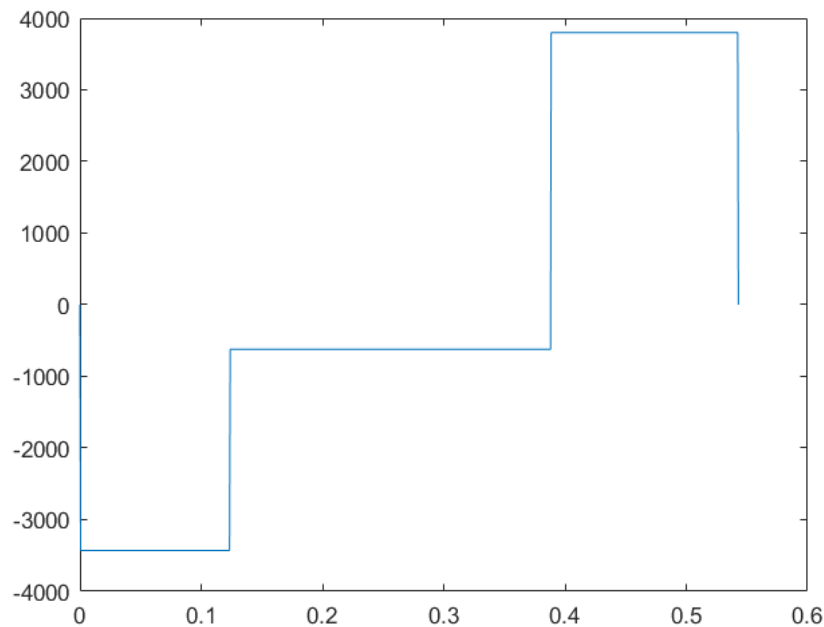
```
G = -((Wcy/6).*(M1(distance2).^3) + (Wr12/6).*(M2(distance2).^3) + ...
      (Wa12/2).*(d2/2).*(M2(distance2).^2) + (Wr43/6).*(M3(distance2).^3) + ...
      (Wa43/2).*(d3/2).*(M3(distance2).^2) +
      (Wdy/6).*(M4(distance2).^3))/distance2;
H = 0;
It = -((Wcz/6).*(M1(distance2).^3) + (Wt12/6).*(M2(distance2).^3) + ...
      (Wt43/6).*(M3(distance2).^3) + (Wdz/6).*(M4(distance2).^3))/distance2;
J = 0;
```

```

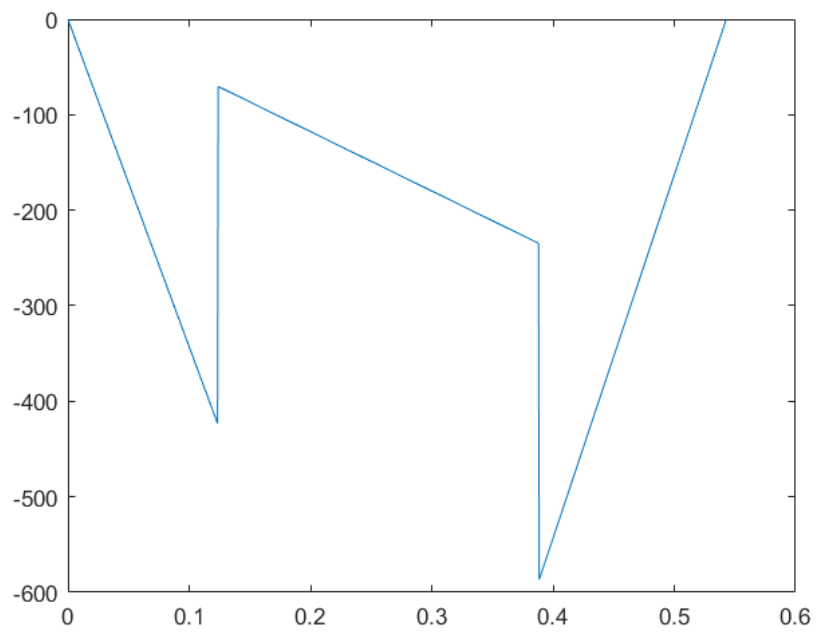
Shear_2y = Wcy.*(M1(X2)>0) + Wr12.*(M2(X2)>0) + Wr43.*(M3(X2)>0) +
Wdy.*(M4(X2)>0);
Shear_2y(1000) = 0;
Moment_2y = Wcy.*(M1(X2)).*(M1(X2)>0) + Wr12.*(M2(X2)).*(M2(X2)>0) + ...
    Wa12.*(d2/2).*(M2(X2)>0) + Wr43.*(M3(X2)).*(M3(X2)>0) +
Wa43.*(d3/2).*(M3(X2)>0) ...
    + Wdy.*(M4(X2)).*(M4(X2)>0);
Slope_2y = ((Wcy/2).*(M1(X2).^2).*(M1(X2)>0) +
(Wr12/2).*(M2(X2).^2).*(M2(X2)>0)...
    + Wa12.*(d2/2).*(M2(X2)).*(M2(X2)>0) + (Wr43/2).*(M3(X2).^2).*(M3(X2)>0)
...
    + Wa43.*(d3/2).*(M3(X2)).*(M3(X2)>0) + (Wdy/2).*(M4(X2).^2).*(M4(X2)>0) +
G)./(E*I);
Deflection_2y = ((Wcy/6).*(M1(X2).^3).*(M1(X2)>0) +
(Wr12/6).*(M2(X2).^3).*(M2(X2)>0) ...
    + (Wa12/2).*(d2/2).*(M2(X2).^2).*(M2(X2)>0) +
(Wr43/6).*(M3(X2).^3).*(M3(X2)>0) +...
    (Wa43/2).*(d3/2).*(M3(X2).^2).*(M3(X2)>0) +
(Wdy/6).*(M4(X2).^3).*(M4(X2)>0) + G*X2 + H)./(E*I);

plot(X2,Shear_2y)

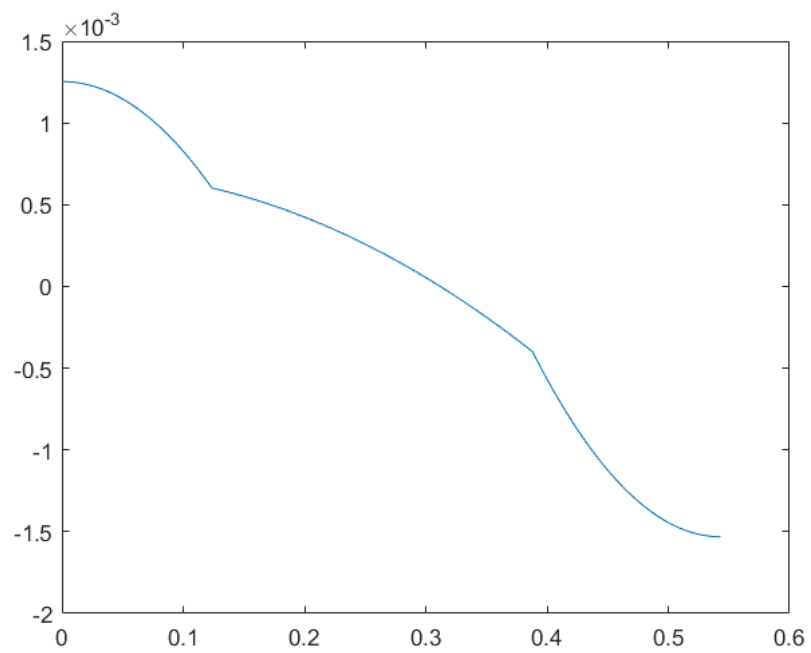
```



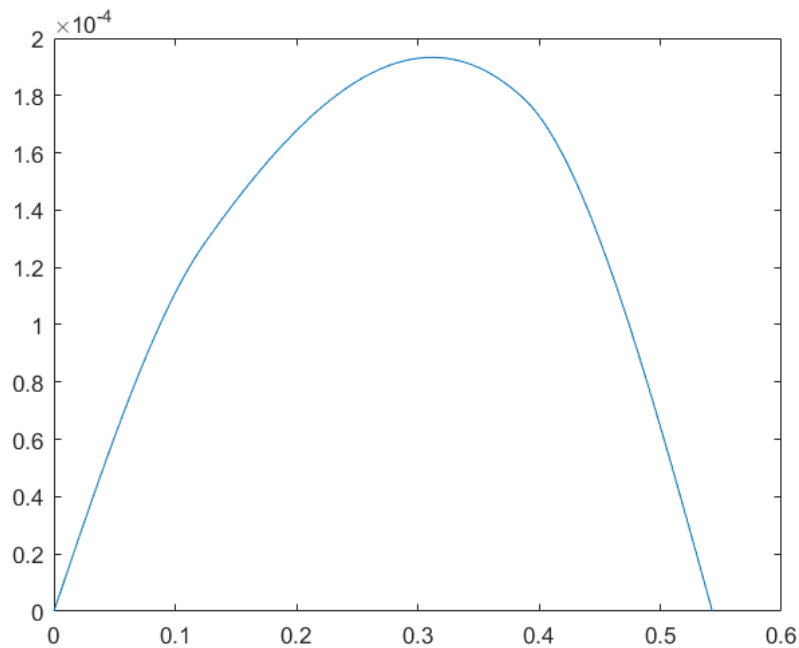
```
plot(X2,Moment_2y)
```



```
plot(X2,Slope_2y)
```

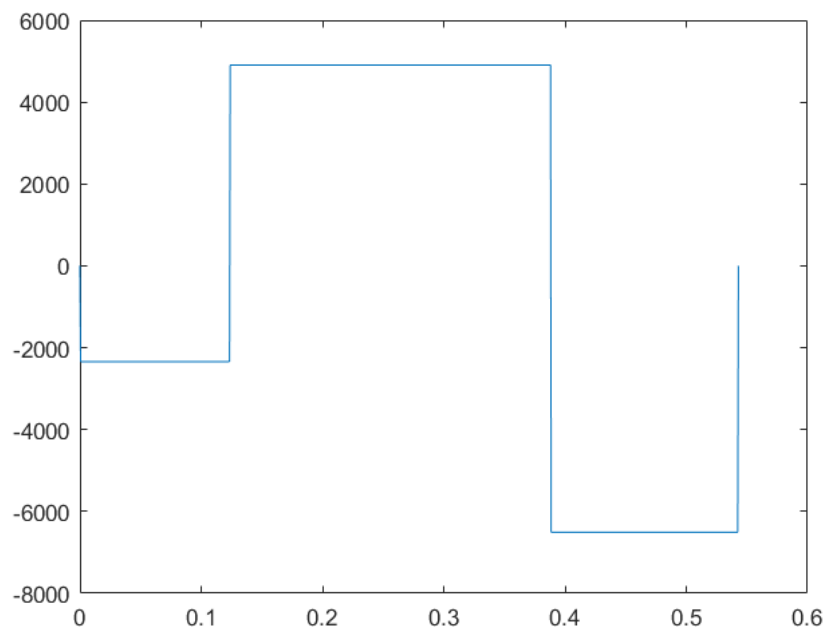


```
plot(X2,Deflection_2y)
```

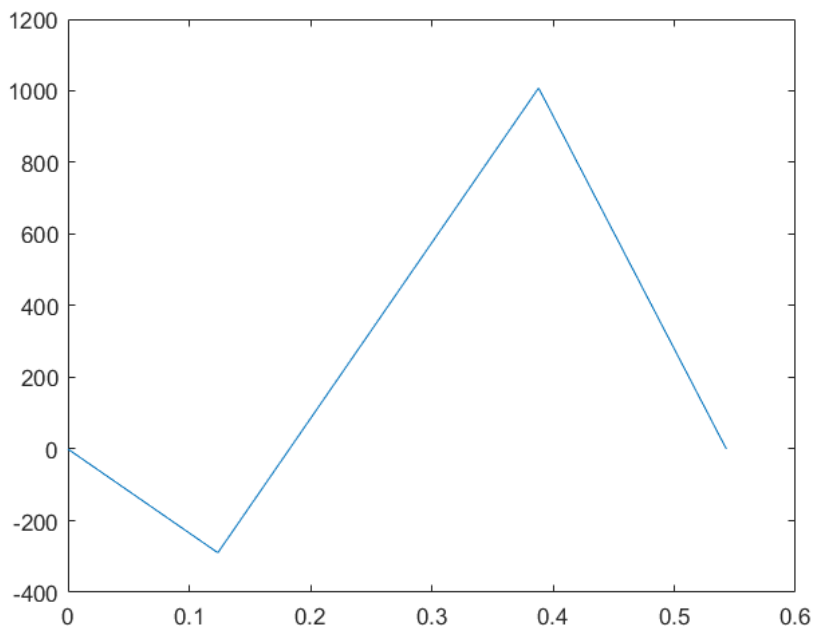


```
Shear_2z = Wcz.*(M1(X2)>0) + Wt12.*(M2(X2)>0) + Wt43.*(M3(X2)>0) +
Wdz.*(M4(X2)>0);
Shear_2z(1000) = 0;
Moment_2z = Wcz.*(M1(X2)).*(M1(X2)>0) + Wt12.*(M2(X2)).*(M2(X2)>0) + ...
Wt43.*(M3(X2)).*(M3(X2)>0) + Wdz.*(M4(X2)).*(M4(X2)>0);
Slope_2z = ((Wcz/2).*(M1(X2).^2).*(M1(X2)>0) +
(Wt12/2).*(M2(X2).^2).*(M2(X2)>0)...
+ (Wt43/2).*(M3(X2).^2).*(M3(X2)>0) + (Wdz/2).*(M4(X2).^2).*(M4(X2)>0) +
It)./(E*I);
Deflection_2z = ((Wcz/6).*(M1(X2).^3).*(M1(X2)>0) +
(Wt12/6).*(M2(X2).^3).*(M2(X2)>0)...
+ (Wt43/6).*(M3(X2).^3).*(M3(X2)>0) + (Wdz/6).*(M4(X2).^3).*(M4(X2)>0) +
It*X2 + J)./(E*I);

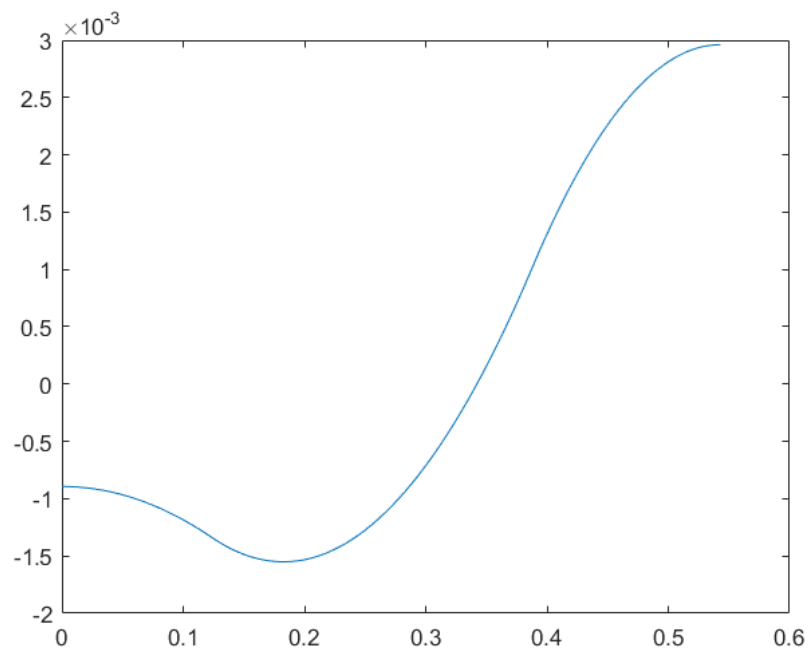
plot(X2,Shear_2z)
```



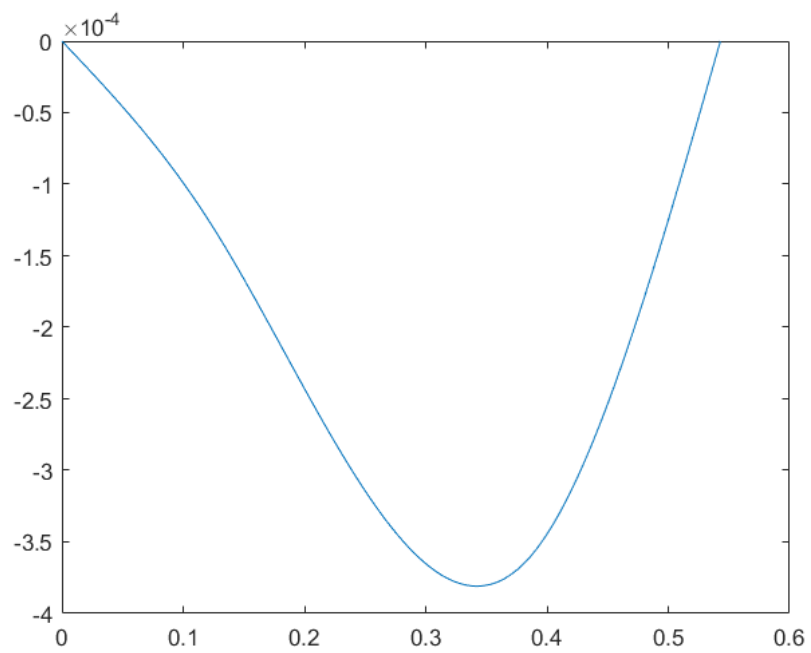
```
plot(X2,Moment_2z)
```




```
plot(X2,Slope_2z)
```



```
plot(X2,Deflection_2z)
```



Output Shaft:

```

distance3 = 0.5*We + le + bg2 + lf + 0.5*Wf;
X3 = linspace(0,distance3,1000);

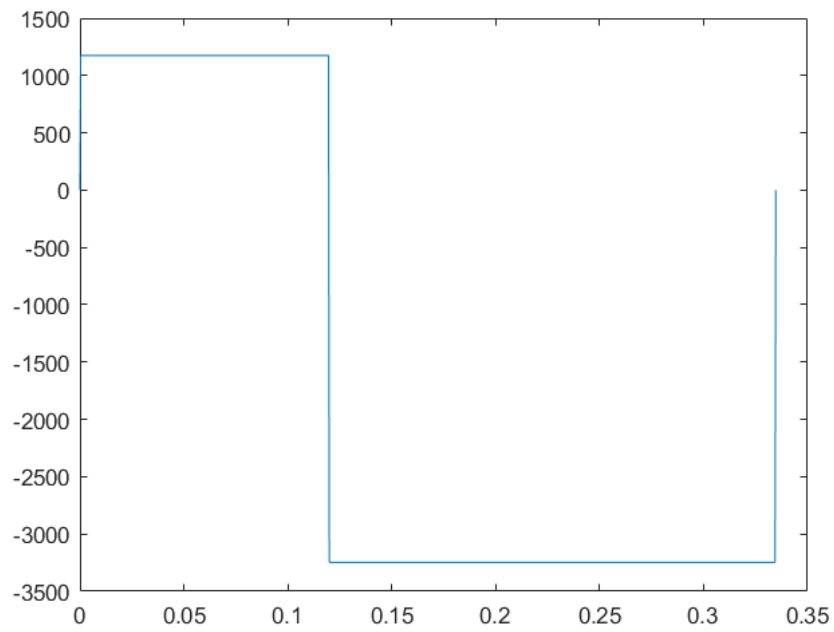
M1 = @(X3) X3;
M2 = @(X3) X3 - (0.5*We+le+(0.5*bg2));
M3 = @(X3) X3 - ((0.5*We)+le+bg2+lf+(0.5*Wf));

K = -((Wey/6).*(M1(distance3).^3) + (Wr34/6).*(M2(distance3).^3) + ...
      (Wa34/2).*(d4/2).*(M2(distance3).^2) +
(Wfy/6).*(M3(distance3).^3))/distance3;
L = 0;
Mt = -((Wez/6).*(M1(distance3).^3) + (Wt34/6).*(M2(distance3).^3) +
(Wfz/6).*(M3(distance3).^3))/distance3;
N = 0;

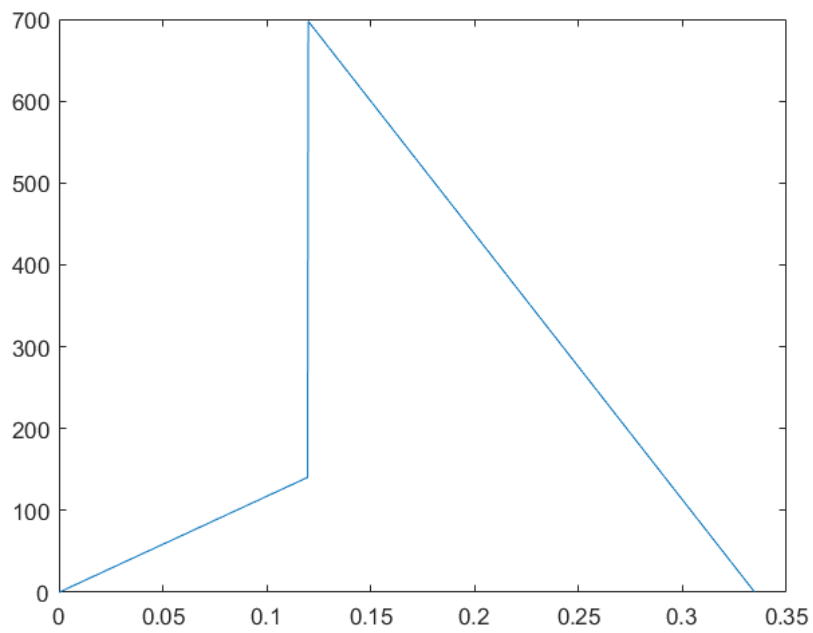
Shear_3y = Wey.*(M1(X3)>0) + Wr34.*(M2(X3)>0) + Wfy.*(M3(X3)>0);
Shear_3y(1000) = 0;
Moment_3y = Wey.*(M1(X3)).*(M1(X3)>0) + Wr34.*(M2(X3)).*(M2(X3)>0) + ...
      Wa34.*(d4/2).*(M2(X3)>0) + Wfy.*(M3(X3)).*(M3(X3)>0);
Slope_3y = ((Wey/2).*(M1(X3).^2).*(M1(X3)>0) +
(Wr34/2).*(M2(X3).^2).*(M2(X3)>0)...
      + Wa34.*(d4/2).*(M2(X3)).*(M2(X3)>0) + (Wfy/2).*(M3(X3).^2).*(M3(X3)>0) +
K)./(E*I);
Deflection_3y = ((Wey/6).*(M1(X3).^3).*(M1(X3)>0) +
(Wr34/6).*(M2(X3).^3).*(M2(X3)>0)...
      + (Wa34/2).*(d4/2).*(M2(X3).^2).*(M2(X3)>0) +
(Wfy/6).*(M3(X3).^3).*(M3(X3)>0) + K*X3 + L)./(E*I);

plot(X3,Shear_3y)

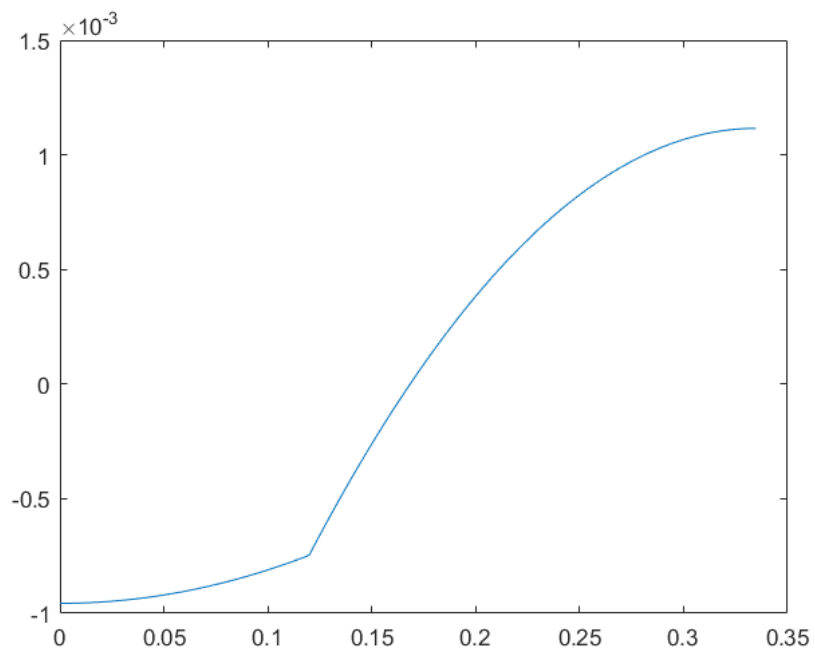
```



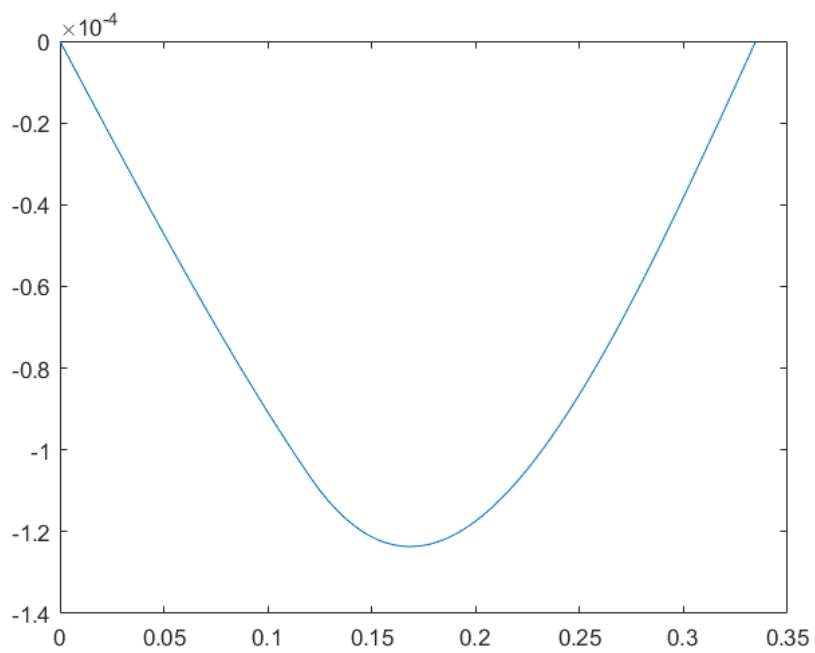
```
plot(X3,Moment_3y)
```



```
plot(X3,Slope_3y)
```



```
plot(X3,Deflection_3y)
```

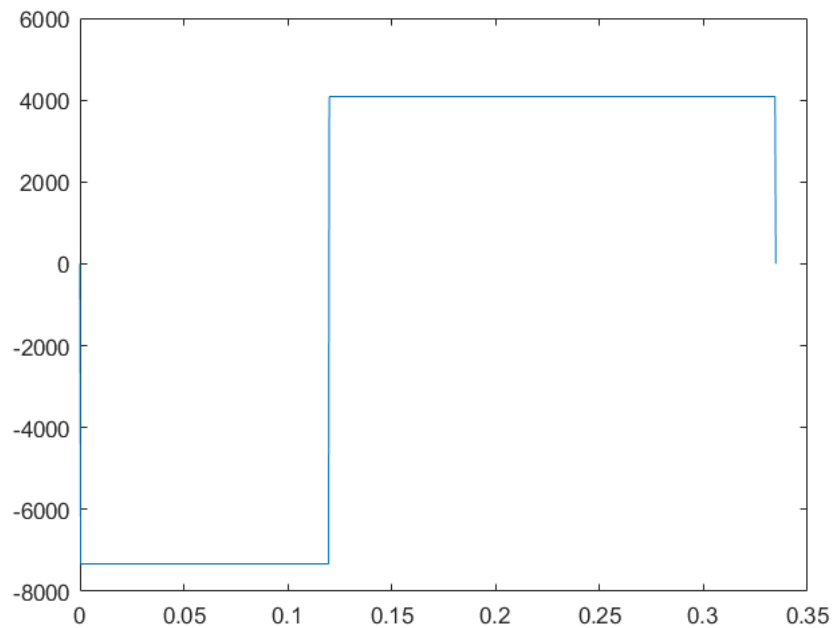


```

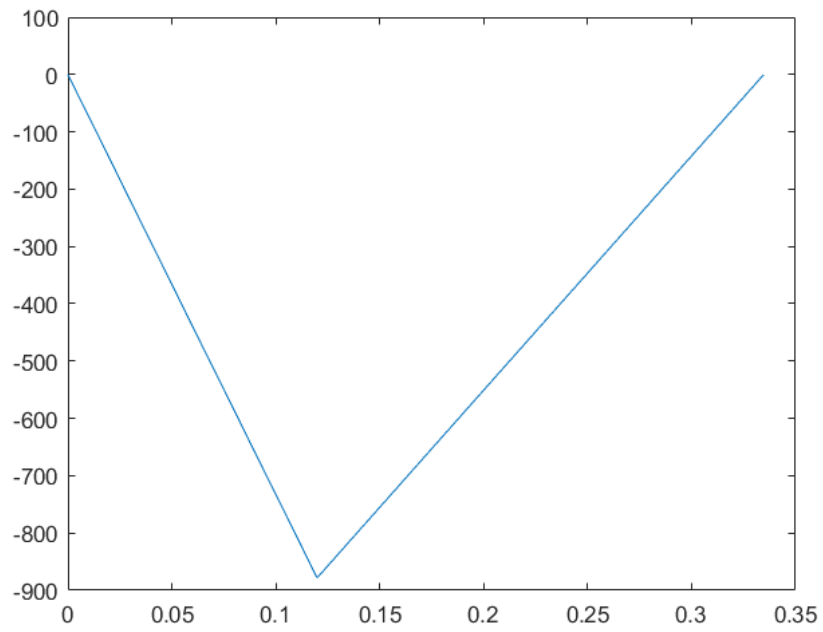
Shear_3z = Wez.*(M1(X3)>0) + Wt34.*(M2(X3)>0) + Wfz.*(M3(X3)>0);
Shear_3z(1000) = 0;
Moment_3z = Wez.*(M1(X3)).*(M1(X3)>0) + Wt34.*(M2(X3)).*(M2(X3)>0)...
    + Wfz.*(M3(X3)).*(M3(X3)>0);
Slope_3z = ((Wez/2).*(M1(X3).^2).*(M1(X3)>0) +
(Wt34/2).*(M2(X3).^2).*(M2(X3)>0)...
    + (Wfz/2).*(M3(X3).^2).*(M3(X3)>0) + Mt)./(E*I);
Deflection_3z = ((Wez/6).*(M1(X3).^3).*(M1(X3)>0) +
(Wt34/6).*(M2(X3).^3).*(M2(X3)>0)...
    + (Wfz/6).*(M3(X3).^3).*(M3(X3)>0) + Mt*X3 + N)./(E*I);

plot(X3,Shear_3z)

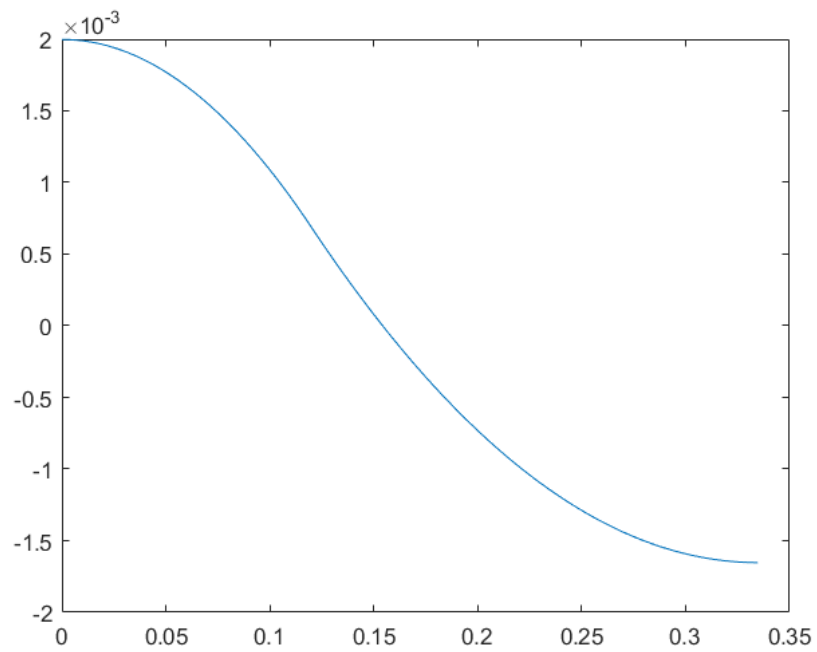
```



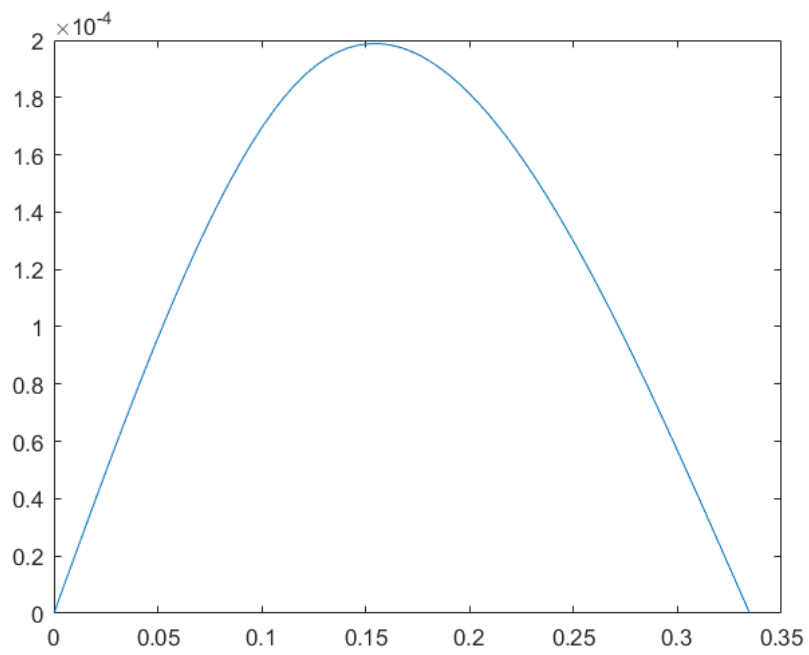
```
plot(X3,Moment_3z)
```



```
plot(X3,Slope_3z)
```



```
plot(X3,Deflection_3z)
```



B7. Shaft Deflection Summary

Shaft Deflection Calculations:

In order to determine whether the deflection is within an acceptable range, two methods must be used.

1. Assume Input and output shaft are rigid and determine whether Intermediate shaft deflection will cause damage.
2. Assume Intermediate shaft is rigid and determine whether Input and Output shaft deflections will cause incorrect meshing,

(All cases will fall between these extremes)

Method 1: Using the length averaged diameter of 66mm

Intermediate Shaft:

Deflection at the gear (due to bending) = 2.82×10^{-5} m

[Matlab Deflection Graphs](#)

Allowable gear misalignment = (centre - centre missalignment)/2 = 3.8×10^{-5} m

[Specified by AGMA quality number](#)

Safety Factor = 1.35

Deflection at the pinion (due to bending) = 3.76×10^{-5} m

[Matlab Deflection Graphs](#)

Allowable gear misalignment = (centre - centre missalignment)/2 = 3.8×10^{-5} m

[Specified by AGMA quality number](#)

Safety Factor = 1.01

Method 2: Using the length averaged diameter of 67mm

Input Shaft:

Deflection at the pinion (due to bending) = 7.06×10^{-6} m

[Matlab Deflection Graphs](#)

Allowable gear misalignment = (centre - centre missalignment)/2 = 3.8×10^{-5} m

[Specified by AGMA quality number](#)

Safety Factor = 5.38

Output Shaft: Using the length averaged diameter of 66mm

Deflection at the gear (due to bending) = 2.27×10^{-5} m

[Matlab Deflection Graphs](#)

Allowable gear misalignment = (centre - centre missalignment)/2 = 3.8×10^{-5} m

[Specified by AGMA quality number](#)

Safety Factor = 1.67

Appendix C

BEARINGS

Overview

1. Radial Forces
2. Axial Forces
3. Equivalent Dynamic Load
4. Basic Dynamic Load
5. Safety factors

C1. Radial and Axial Forces

Bearing Analysis:

Propellor Thrust:

$$Thrust := 13700 \text{ N}$$

Number of cycles:

$$L1 := 1138800000 \text{ Cycles}$$

$$L2 := 723047619 \text{ Cycles}$$

$$L3 := 459077853.4 \text{ Cycles}$$

[See Gear Appendix](#)

Total bearing radial forces:

$$Ra := \sqrt{1738.7538^2 + 2359.3328^2} = 2930.8217 \text{ N}$$

$$Rb := \sqrt{1068.5098^2 + 4888.4162^2} = 5003.8311 \text{ N}$$

$$Rc := \sqrt{3429.1078^2 + 2341.851^2} = 4152.4747 \text{ N}$$

$$Rd := \sqrt{3799.596^2 + 6509.3067^2} = 7537.1084 \text{ N}$$

$$Re := \sqrt{1173.7476^2 + 7327.588^2} = 7420.9992 \text{ N}$$

$$Rf := \sqrt{3247.6926^2 + 4087.6167^2} = 5220.7392 \text{ N}$$

[See Matlab section](#)

Total bearing axial forces:

$$Ta := 2637.9649 \text{ N}$$

$$Tb := Ta$$

$$Tc := 2341.851 \text{ N}$$

$$Td := Tc$$

$$Te := 4154.7947 \text{ N}$$

$$Tf := Te$$

$$Te_{actual} := Thrust - Te = 9545.2053 \text{ N}$$

[See Matlab section](#)

Bearing life adjustment factors:

$$A_{SKF} := 2.5$$

$$A_1 := 0.25$$

[SKF catalogue](#)

[Diagram 8 \(Page 95\)](#)

Bearing specific force multiplication factors

$$k := \frac{10}{3}$$

$$V := 1$$

$$X := 0.37$$

$$Y := 1.6$$

[SKF catalogue \(Page 697\)](#)

C2. Equivalent Dynamic Load

C3. Basic Dynamic Load

Equivalent dynamic bearing load:

$$P1 := (V \cdot X \cdot Ra) + (Y \cdot Ta) = 5305.1479 \text{ N}$$

$$P2 := (V \cdot X \cdot Rb) + (Y \cdot Tb) = 6072.1614 \text{ N}$$

$$P3 := (V \cdot X \cdot Rc) + (Y \cdot Tc) = 5283.3773 \text{ N}$$

$$P4 := (V \cdot X \cdot Rd) + (Y \cdot Td) = 6535.6917 \text{ N}$$

$$P5 := (V \cdot X \cdot Re) + (Y \cdot Te_{actual}) = 18018.0982 \text{ N}$$

$$P6 := (V \cdot X \cdot Rf) + (Y \cdot Tf) = 8579.345 \text{ N}$$

SKF catalogue (Page 92)

Basic dynamic load rating:

$$C1 := P1 \cdot \left(\frac{L1}{10^6 \cdot A_{SKF} \cdot A_I} \right)^{\left(\frac{1}{k} \right)} = 50450.7616 \text{ N}$$

$$C2 := P2 \cdot \left(\frac{L1}{10^6 \cdot A_{SKF} \cdot A_I} \right)^{\left(\frac{1}{k} \right)} = 57744.8871 \text{ N}$$

$$C3 := P3 \cdot \left(\frac{L2}{10^6 \cdot A_{SKF} \cdot A_I} \right)^{\left(\frac{1}{k} \right)} = 43842.7399 \text{ N}$$

$$C4 := P4 \cdot \left(\frac{L2}{10^6 \cdot A_{SKF} \cdot A_I} \right)^{\left(\frac{1}{k} \right)} = 54234.7476 \text{ N}$$

$$C5 := P5 \cdot \left(\frac{L3}{10^6 \cdot A_{SKF} \cdot A_I} \right)^{\left(\frac{1}{k} \right)} = 1.3047 \cdot 10^5 \text{ N}$$

$$C6 := P6 \cdot \left(\frac{L3}{10^6 \cdot A_{SKF} \cdot A_I} \right)^{\left(\frac{1}{k} \right)} = 62123.5164 \text{ N}$$

SKF catalogue (Page 89)

C5 is much larger due to the thrust force and is therefore used only for Bearing 5

C6 is used in the selection of Bearings 1,2,3,4,6

C4. Required Hours and life

Safety Factors

Required life in hours

10 years at 2 hours a day

Bredell (2020), Project Description

$Hours := 2$

$Days := 7$

$Weeks := 52$

$Years := 10$

$TotalHours := Hours \cdot Days \cdot Weeks \cdot Years = 7280 \text{ Hours}$

Recommended Life in hours

Recommended 60 000 to 100 000 for marine propulsion

SKF bearing catalogue

Life in hours

Calculated for bearing with maximum force
all other safety factors will then be below this value
The max value occurs at bearing six as seen above
(Excluding Bearing five)

$P6 := 8579,345 \text{ N}$

SKF catalogue

Diagram 8 (Page 95)

Life modification factor will be set to 2 and 3
since this is used as 2.5 for the force
calculations and will give a good average for the life of the bearings.

$a_1 := 0,25$

$A_{skf1} := 2$

$A_{skf2} := 3$

The bearing chosen has a dynamic load of
 $C := 104000 \text{ N}$ (for all bearings except P5)
For P5 this value is $C1=196000 \text{ N}$

SKF catalogue

$$Lnm1 := a_1 \cdot A_{skf1} \cdot \left(\frac{C}{P6} \right)^{\frac{10}{3}} = 2045,9767$$

$$Lnm2 := a_1 \cdot A_{skf2} \cdot \left(\frac{C}{P6} \right)^{\frac{10}{3}} = 3068,9651$$

Now converting these values to hours

we need the output shaft speed calculated as

$$n := 2600 \cdot \frac{1}{2,5} = 1040$$

$$Lnmh1 := \left(\frac{10^6}{60 \cdot n} \right) \cdot Lnm1 = 32788,0889 \quad \text{Hours}$$

$$Lnmh2 := \left(\frac{10^6}{60 \cdot n} \right) \cdot Lnm2 = 49182,1334 \quad \text{Hours}$$

Life range of between 33 000 Hours and 50 000 hours

C5. Safety Factors in both Force and Life

For Bearing P5 the basic dynamic load is given as $C1 := 196000 \text{ N}$ and P5 as

SKF catalogue

$$P5 := 18018,0982$$

$$Lnm16 := a_1 \cdot A_{skf1} \cdot \left(\frac{C1}{P5} \right)^{\frac{10}{3}} = 1426,0248$$

$$Lnm26 := a_1 \cdot A_{skf2} \cdot \left(\frac{C1}{P5} \right)^{\frac{10}{3}} = 2139,0372$$

Now converting these values to hours

we need the output shaft speed calculated as

$$n := 2600 \cdot \frac{1}{2,5} = 1040$$

$$Lnmh16 := \left(\frac{10^6}{60 \cdot n} \right) \cdot Lnm16 = 22852,9615 \text{ Hours}$$

$$Lnmh26 := \left(\frac{10^6}{60 \cdot n} \right) \cdot Lnm26 = 34279,4422 \text{ Hours}$$

Life range of between 23 000 Hours and 35 000 hours for bearing P6

Safety factor in hours

This leaves a minimum safety factor in hours of

$$Sf_{min1} := \frac{Lnmh1}{TotalHours} = 4,5039 \quad (\text{All except P5})$$

$$Sf_{min2} := \frac{Lnmh16}{TotalHours} = 3,1391 \quad (P5)$$

Safety factor in force

Since the bearing can handle 104kN and the max bearing supports only 68kN

$$C6 := 62123,5164$$

$$Sf_{force} := \frac{C}{C6} = 1,6741$$

P5 can support 196kN when the force applied is only $C = 130.47\text{kN}$

$$C5 := 130470 \text{ N}$$

$$Sf_{force} := \frac{C1}{C5} = 1,5023$$

Both are above the required 1.5 which in itself has leeway

C5. Bearing Angle

Bearing Angle Calculations:

Allowable Internal Bearing Slope: 0.0012 radians

[Shigley Table 7-2 \(Page 371\) - Tapered Roller Bearings](#)

Bearing 1 Slope: 2.92×10^{-6} radians

[Shaft Matlab Section](#)

Safety Factor: 411.31

Bearing 2 Slope: 3.44×10^{-6}

Safety Factor: 348.84

Bearing 3 Slope: 5.8×10^{-6}

Safety Factor: 206.90

Bearing 4 Slope: 1.26×10^{-5}

Safety Factor: 95.24

Bearing 5 Slope: 8.36×10^{-6}

Safety Factor: 143.54

Bearing 6 Slope: 7.52×10^{-6}

Safety Factor: 159.57

Appendix D

KEYWAYS

Overview

1. Shear Stress Failure
2. Checks of lengths
3. Normal Stress Failure



D1. Input Shaft

Keyway analysis

Variables

d = diameter of shaft

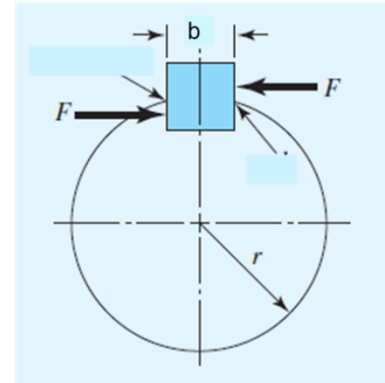
b = thickness of key

Maximum length of key = l_{\max}

h = height of key

t_1 = distance from bottom of key to top of shaft

Material for all keys = AISI 1020 CD



Keys and keyways										
Shaft Diameter		Key width "b"	Key thickness "h"	Key length "L"		Depth of Keyway			Height of Gib Head h ₂	
						Shaft t ₁	Hub t ₂			
From	To and including			From	To and including			Feather key	Taper key	
6	8	2	2	6	20	1,2	1,0	0,5		
8	10	3	3	6	36	1,8	1,4	0,9		
10	12	4	4	8	45	2,5	1,8	1,2		7
12	17	5	5	10	56	3,0	2,3	1,7		8
17	22	6	6	14	70	3,5	2,8	2,0		10
22	30	8	7	18	90	4,0	3,3	2,5		11
30	38	10	8	22	110	5,0	3,3	2,5		12
38	44	12	8	28	140	5,0	3,3	2,5		12
44	50	14	9	36	160	5,5	3,8	3,0		14
50	58	16	10	45	180	6,0	4,3	3,5		16
58	65	18	11	50	200	7,0	4,4	3,3		18
65	75	20	12	56	220	7,5	4,9	3,8		20
75	85	22	14	63	250	9,0	5,4	4,3		22
85	95	25	14	70	280	9,0	5,4	4,3		22
95	110	28	16	80	320	10,0	6,4	5,3		25

Trinchero, P. 2019. Introduction to Machine Design 244. Class lecture (Keyways). Stellenbosch: Stellenbosch University.

Torques calculated in AGMA analysis

$$T_1 := 617,0315 \text{ Nm}$$

$$T_3 := T_2 \text{ Nm}$$

$$T_2 := 971,8246 \text{ Nm}$$

$$T_4 := 1530,6237 \text{ Nm}$$

Calculation of length of key for input shaft at gear 1

From the table above at $d = 65$

$$b := 18 \text{ key width}$$

$$SF := 1,5$$

$$h := 11 \text{ key height}$$

$$d_1 := 60$$

$$t_1 := 7 \text{ depth of groove into shaft}$$

$$l_{\max} := 1,5 \cdot d_1$$

Failure due to shear stress

$$F_{\text{shear}} := \frac{T_1}{0,001 \cdot \frac{d_1}{2}} = 20567,7167 \text{ N}$$

Reference for all equations used:

Shigley (2015, p116, p239)

From distortion-energy theory,

$$S_y := 390 \cdot 10^6 \text{ pa}$$

$$S_{sy} := 0,577 \cdot S_y = 2,2503 \cdot 10^8 \text{ pa}$$

D2. Intermediate Shaft

$$l_{1s} := \frac{(F_{shear} \cdot SF)}{0,001 \cdot b \cdot S_{sy}} = 0,0076 \text{ mm}$$

Check

$$l_{max} \cdot 0,001 = 0,09 \text{ mm}$$

$$l_{1s} \leq l_{max}$$

Failure due to crushing (normal) stress

$$l_{1c} := \frac{(F_{shear} \cdot SF)}{0,001 \cdot \frac{b}{2} \cdot S_y} = 0,0088 \text{ mm}$$

$$l_{max} \cdot 0,001 = 0,09 \text{ mm}$$

$$l_{1c} \leq l_{max}$$

$$l_1 := l_{1c} = 0,0088 \text{ mm}$$

Calculation of length of key for intermediate shaft at gear 2 and 3

From the table above at d = 65

$$b := 18$$

$$h := 11$$

$$t_1 := 7$$

$$SF := 1,5$$

$$d_2 := 60$$

$$l_{max} := 1,5 \cdot d_2$$

Failure due to shear stress

$$F_{shear} := \frac{T_2}{0,001 \cdot \frac{d_2}{2}} = 32394,1533 \text{ N}$$

$$l_{2s} := \frac{(F_{shear} \cdot SF)}{0,001 \cdot b \cdot S_{sy}} = 0,012 \text{ mm}$$

Check

$$l_{max} \cdot 0,001 = 0,09 \text{ mm}$$

$$l_{2s} \leq l_{max}$$

Failure due to crushing (normal) stress

$$l_{2c} := \frac{(F_{shear} \cdot SF)}{0,001 \cdot \frac{b}{2} \cdot S_y} = 0,0138 \text{ mm}$$

$$l_{max} \cdot 0,001 = 0,09 \text{ mm}$$

$$l_{2c} \leq l_{max}$$

$$l_2 := l_{2c} = 0,0138 \text{ mm}$$

$$l_3 := l_2$$

D3. Output Shaft

Calculation of length of key for output shaft at gear 4

From the table above at $d = 65$

$$b := 18$$

$$h := 11$$

$$t_1 := 7$$

$$SF := 1,5$$

$$d_4 := 60$$

$$l_{max} := 1,5 \cdot d_4$$

Failure due to shear stress

$$F_{shear} := \frac{T_4}{0,001 \cdot \frac{d_4}{2}} = 51020,79 \text{ N}$$

$$l_{4s} := \frac{(F_{shear} \cdot SF)}{0,001 \cdot b \cdot S_{sy}} = 0,0189 \text{ mm}$$

Check

$$l_{max} \cdot 0,001 = 0,09 \text{ mm}$$

$$l_{4s} \leq l_{max}$$

Failure due to crushing (normal) stress

$$l_{4c} := \frac{(F_{shear} \cdot SF)}{0,001 \cdot \frac{b}{2} \cdot S_y} = 0,0218 \text{ mm}$$

$$l_{max} \cdot 0,001 = 0,09 \text{ mm}$$

$$l_{4c} \leq l_{max}$$

$$l_4 := l_{4c} = 0,0218 \text{ mm}$$

To simplify the manufacturing process a nominal length is selected for all the keyways. This length will be the longest length to accommodate the highest crushing stress.

Note: all keyways will be at a distance $d/10$ from shoulders
all keyways will be end-milled

$$l := l_{4c} = 0,0218 \text{ mm}$$

Keyway dimensions

Width = 18 mm

Height = 11 mm

Groove depth = 7 mm

Length = 21.8 mm

Appendix E

RETAINING

Overview

1. Bearing Stress
Concentration Factors
2. Mod-Goodman Bearings
3. Gear Stress
Concentration Factors
4. Mod-Goodman Gears



Retaining Ring

Reference for all equations used: Shigley (2015, p340, p360)

Moments at gear & bearing shoulder

$$\begin{aligned} M_{a1} &:= 536.4427 & M_{as1} &:= 75.628 & M_{m1} &:= 0 \\ M_{a2} &:= 1166.2102 & M_{as2} &:= 150.473 & M_{m2} &:= 0 \\ M_{a3} &:= 1120.16 & M_{as3} &:= 200.733 & M_{m3} &:= 0 \end{aligned}$$

See Matlab Section

Torques

$$\begin{aligned} T_{a1} &:= 30.851575 & T_{m1} &:= 617.0315 \\ T_{a2} &:= 48.745485 & T_{m2} &:= 974.9097 \\ T_{a3} &:= 77.01787 & T_{m3} &:= 1540.3574 \end{aligned}$$

See Matlab Section

Diameters

$$d_1 := 65 \quad d_A := 45$$

See Shaft Calculations

For diameters d_A, d_B, d_C, d_D, d_E and d_F (which are all 45 mm)

Ring properties


Preview Image	Part Number	Application Diameter (mm)	Ring Shear (N)	Groove Yield (N)	Ring Diameter (mm)	Radial Wall (mm)	Ring Thickness (mm)	Groove Diameter (mm)	Min Groove Width (mm)	Turns
	DNS-45 External, Metric DIN, Spirolux	45.00	69186	27414	42.06	4.01	1.69	42.50	1.85	2

Table 1: Catalogue ring description (Source: Smalley catalogue, 2020)

This ring will need a custom width however, due to the fact that this is a spiral ring the number of turns may be increased at an economical cost rate. Four (4) turns will be used instead of 2 to accomodate the design requirements.

Groove dimensions from catalogue

$$\begin{aligned} \text{width} = w &= 3.7 \\ \text{diameter} = D_g &= 42.5 \end{aligned}$$

Ring dimensions from catalogue

$$\begin{aligned} \text{thickness} = t &= 3.38 \\ \text{ring inner diameter} = D_i &= 42.06 \end{aligned}$$

Stress concentration factors

$$K_{t1} := 3.7$$

$$K_{ts1} := 2.4$$

$$q_1 := 0.82$$

$$q_{s1} := 0.82$$

$$K_{f1} := 1 + (K_{t1} - 1) \cdot q_1 = 3.214$$

$$K_{fs1} := 1 + q_{s1} \cdot (K_{ts1} - 1) = 2.148$$

Stress Concentration Figures and References at the end of this Section.

Figure 4: Catalogue ring preview image (Source: Smalley catalogue, 2020)



E1. Bearings

At bearing A and B

$$\sigma_{aa} := \sqrt{\left(\frac{32 \cdot K_{f1} \cdot M_{as1}}{\pi \cdot (0.001 \cdot d_A)^3} \right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs1} \cdot T_{a1}}{\pi \cdot (0.001 \cdot d_A)^3} \right)^2} = 2.7917 \cdot 10^7$$

Shigley equation 7-5 (Page 360)

$$\sigma_{m1} := \sqrt{\left(\frac{32 \cdot K_{f1} \cdot M_{m1}}{\pi \cdot (0.001 \cdot d_A)^3} \right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs1} \cdot T_{m1}}{\pi \cdot (0.001 \cdot d_A)^3} \right)^2} = 1.283 \cdot 10^8$$

Shigley equation 7-6 (Page 360)

$$S_e := 2.1745 \cdot 10^8$$

$$S_{ut} := 1030 \cdot 10^6$$

Mod-Goodman

$$n_a := \frac{1}{\frac{\sigma_{aa}}{S_e} + \frac{\sigma_{m1}}{S_{ut}}} = 3.9534$$

$n \geq 1.5$ Acceptable

Shigley equation 6-46 (Page 314)

At bearing C and D

$$\sigma_{ac} := \sqrt{\left(\frac{32 \cdot K_{f1} \cdot M_{as2}}{\pi \cdot (0.001 \cdot d_A)^3} \right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs1} \cdot T_{a2}}{\pi \cdot (0.001 \cdot d_A)^3} \right)^2} = 5.5001 \cdot 10^7$$

Shigley equation 7-5 (Page 360)

$$\sigma_{mc} := \sqrt{\left(\frac{32 \cdot K_{f1} \cdot M_{m2}}{\pi \cdot (0.001 \cdot d_A)^3} \right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs1} \cdot T_{m2}}{\pi \cdot (0.001 \cdot d_A)^3} \right)^2} = 2.0272 \cdot 10^8$$

Shigley equation 7-6 (Page 360)

$$n_c := \frac{1}{\frac{\sigma_{ac}}{S_e} + \frac{\sigma_{mc}}{S_{ut}}} = 2.2235$$

Shigley equation 6-46 (Page 314)

At bearing E and F

$$\sigma_{ae} := \sqrt{\left(\frac{32 \cdot K_{f1} \cdot M_{as3}}{\pi \cdot (0.001 \cdot d_A)^3} \right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs1} \cdot T_{a3}}{\pi \cdot (0.001 \cdot d_A)^3} \right)^2} = 7.3872 \cdot 10^7$$

Shigley equation 7-5 (Page 360)

$$\sigma_{me} := \sqrt{\left(\frac{32 \cdot K_{f1} \cdot M_{m3}}{\pi \cdot (0.001 \cdot d_A)^3} \right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs1} \cdot T_{m3}}{\pi \cdot (0.001 \cdot d_A)^3} \right)^2} = 3.2029 \cdot 10^8$$

Shigley equation 7-6 (Page 360)

$$n_e := \frac{1}{\frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_{ut}}} = 1.5368$$

Shigley equation 6-46 (Page 314)

E2. Gears

At Gears


Preview Image	Part Number	Application Diameter (mm)	Ring Shear (N)	Groove Yield (N)	Ring Diameter (mm)	Radial Wall (mm)	Ring Thickness (mm)	Groove Diameter (mm)	Min Groove Width (mm)	Turns	Crimp
	DNS-65 External, Metric DIN, Spirolox	65.00	135725	47518	61.39	5.08	2.41	62.00	2.65	2	Y

Table 2: Catalogue ring description (Source: Smalley catalogue, 2020)

This ring will need a custom width however, due to the fact that this is a spiral ring the number of turns may be increased at an economical cost rate. Six (6) turns will be used instead of 2 to accomodate the design requirements

Groove dimensions from catalogue

width = $w = 7.23$

diameter = $D_g = 62$

Ring dimensions from catalogue

thickness = $t = 6.625$

ring inner diameter = $D_i = 61.39$

Stress concentration factors

$$K_{t2} := 3.2$$

$$K_{ts2} := 2.5$$

$$q_2 := 0.82$$

$$q_{s2} := 0.82$$

$$K_{f2} := 1 + (K_{t2} - 1) \cdot q_2 = 2.804$$

$$K_{fs2} := 1 + q_{s1} \cdot (K_{ts2} - 1) = 2.23$$

At Gear 1

$$\sigma_{a1} := \sqrt{\left(\frac{32 \cdot K_{f2} \cdot M_{a1}}{\pi \cdot (0.001 \cdot d_1)^3} \right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs2} \cdot T_{a1}}{\pi \cdot (0.001 \cdot d_1)^3} \right)^2} = 5.5834 \cdot 10^7$$

Shigley equation 7-5 (Page 360)

$$\sigma_{m1} := \sqrt{\left(\frac{32 \cdot K_{f2} \cdot M_{m1}}{\pi \cdot (0.001 \cdot d_1)^3} \right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs2} \cdot T_{m1}}{\pi \cdot (0.001 \cdot d_1)^3} \right)^2} = 4.4198 \cdot 10^7$$

Shigley equation 7-6 (Page 360)

$$n_1 := \frac{1}{\frac{\sigma_{a1}}{S_e} + \frac{\sigma_{m1}}{S_{ut}}} = 3.3369$$

$n \geq 1.5$ Acceptable

Shigley equation 6-46 (Page 314)

At Gear 2 and 3

$$\sigma_{a2} := \sqrt{\left(\frac{32 \cdot K_{f2} \cdot M_{a2}}{\pi \cdot (0.001 \cdot d_1)^3} \right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs2} \cdot T_{a2}}{\pi \cdot (0.001 \cdot d_1)^3} \right)^2} = 1.2134 \cdot 10^8$$

Shigley equation 7-5 (Page 360)

$$\sigma_{m2} := \sqrt{\left(\frac{32 \cdot K_{f2} \cdot M_{m2}}{\pi \cdot (0.001 \cdot d_1)^3} \right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs2} \cdot T_{m2}}{\pi \cdot (0.001 \cdot d_1)^3} \right)^2} = 6.9833 \cdot 10^7$$

Shigley equation 7-6 (Page 360)

$$n_{g2} := \left(\frac{1}{\frac{\sigma_{a2}}{S_e} + \frac{\sigma_{m2}}{S_{ut}}} \right) = 1.598$$

$n \geq 1.5$ Acceptable

Shigley equation 6-46 (Page 314)

At Gear 4

$$\sigma_{a3} := \sqrt{\left(\frac{32 \cdot K_{f2} \cdot M_{a3}}{\pi \cdot (0.001 \cdot d_1)^3} \right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs2} \cdot T_{a3}}{\pi \cdot (0.001 \cdot d_1)^3} \right)^2} = 1.1663 \cdot 10^8$$

Shigley equation 7-5 (Page 360)

$$\sigma_{m3} := \sqrt{\left(\frac{32 \cdot K_{f2} \cdot M_{m3}}{\pi \cdot (0.001 \cdot d_1)^3} \right)^2 + 3 \cdot \left(\frac{16 \cdot K_{fs2} \cdot T_{m3}}{\pi \cdot (0.001 \cdot d_1)^3} \right)^2} = 1.1034 \cdot 10^8$$

Shigley equation 7-6 (Page 360)

$$n_g := \frac{1}{\frac{\sigma_{a3}}{S_e} + \frac{\sigma_{m3}}{S_{ut}}} = 1.5541$$

$n \geq 1.5$ Acceptable

Shigley equation 6-46 (Page 314)

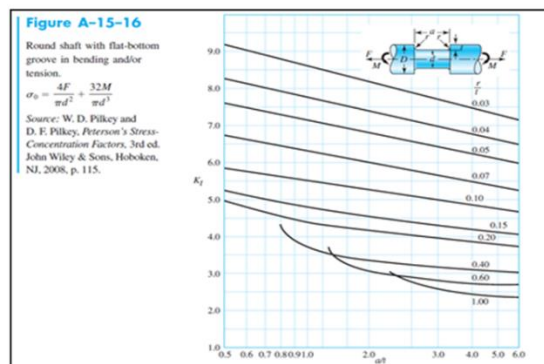


Figure 1: Figure A-15-17 Groove shear stress concentration (Source: Shiegly, 2015)

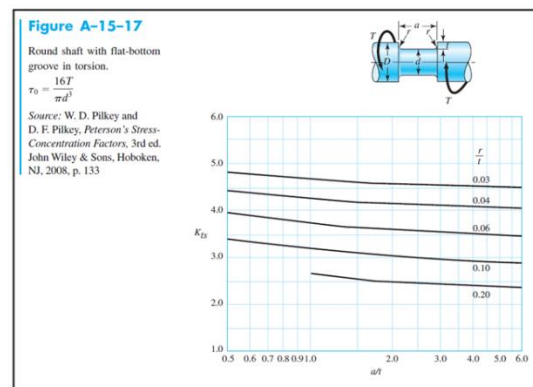


Figure 2: Figure A-15-17 Groove shear stress concentration (Source: Shiegly, 2015)

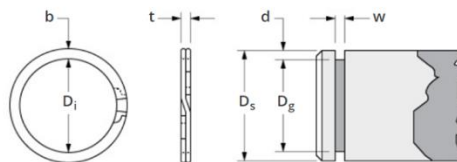


Figure 3: Catalogue ring layout (Source: Smalley catalogue, 2020)

Appendix F

SEALS

Overview

1. Seal Selection
2. Dimensions
3. Maximum speed
4. Tolerances



Seal selection (HMS5)

Selection criterion

1. Minimum interface speed of 2700 rpm
2. Must be compatible with the selected lubrication
3. Must have the ability to retain liquid
4. Must have the ability to resist water penetration
5. Bore diameter = 80 mm
6. Must have the ability to withstand shaft misalignment

The HMS5 seal meets all the requirements specified above. This seal is compatible with most oil and grease lubrications, is able to withstand interfacial speeds of 4500 rpm and has good lubrication retention and water penetration resistance properties. According to the SKF catalogue (2019, p106) additional benefits include, good resistance to ageing and wear. The outer diameter is lined with beads to improve the sealing ability. The seal design allows for high dynamic run out as well as shaft to bore misalignment. Below, figure 1 and table 1 present the layout of seal along with the required dimensions.

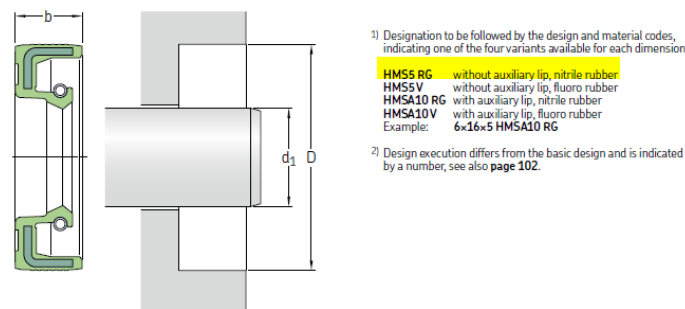


Figure 1: Seal shaft and bore dimensions
(SKF Group, 2019)

Dimensions	Shaft	Bore	Nominal seal width	Designation ¹⁾	ISO / DIN
	d ₁	D	b		
	mm			–	–
45	62	7	45x62x7		
cont.	62	8	45x62x8		•
	62	10	45x62x10		
	65	8	45x65x8		•
	65	10	45x65x10		
	68	7	45x68x7		
	68	10	45x68x10		
	68	12	45x68x12		
	72	8	45x72x8		
	72	10	45x72x10		
	75	8	45x75x8		
	75	10	45x75x10		
	80	10	45x80x10		

Table 1: Seal shaft and bore dimensions (SKF Group, 2019)

F2. Tolerances

Table 2 shows the circumferential speeds the seal is able to withstand

Pumping ability			
Speed Rotating	Circumferential	Pumping time Standard NBR	SKF compound RG
r/min	m/s	s	
1 000	3,1	–	117
1 500	4,7	280	69
2 000	6,3	186	50
2 500	7,9	130	40
3 000	9,4	102	31
3 500	11,0	82	25
4 000	12,6	68	21
4 500	14,1	57	18

Table 2: Speed requirements

(SKF Group, 2019)

Table 3 depicts the tolerances required on the shaft to ensure compatibility with the seal.

table - Counterface tolerances for metric shafts			
Nominal		Deviation	
d ₁			
over	incl.	high	low
mm		μm	
6	10	0	-90
10	18	0	-110
18	30	0	-130
30	50	0	-160

Table 3: Tolerance required

(SKF Group, 2019)

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