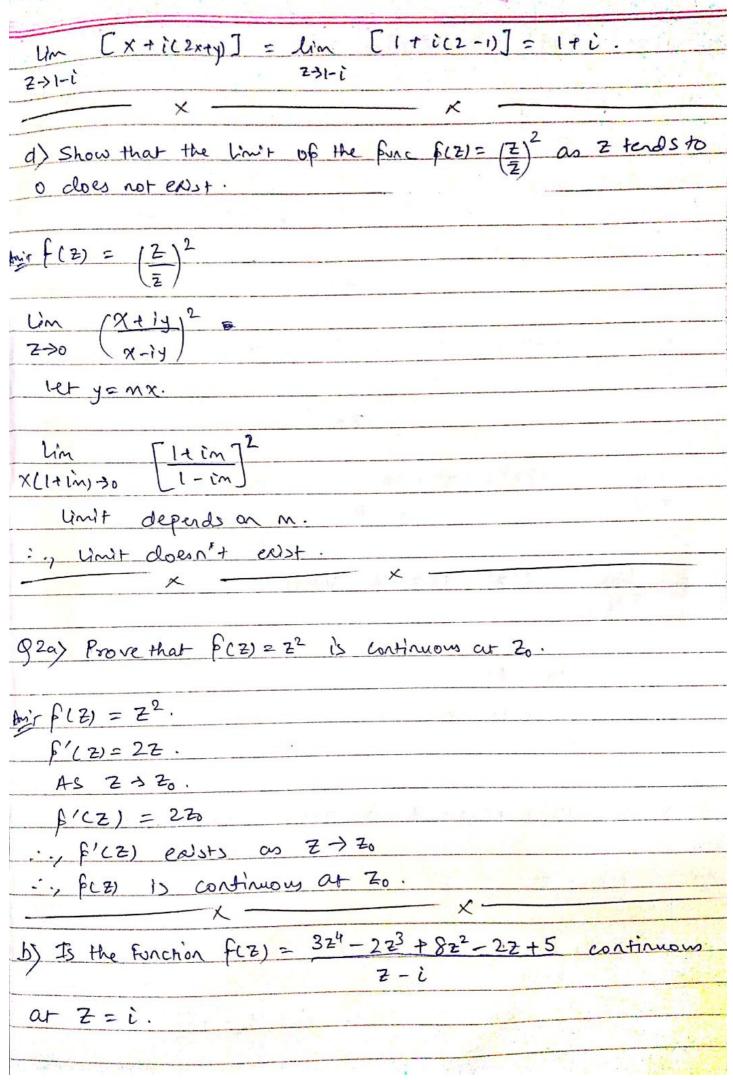
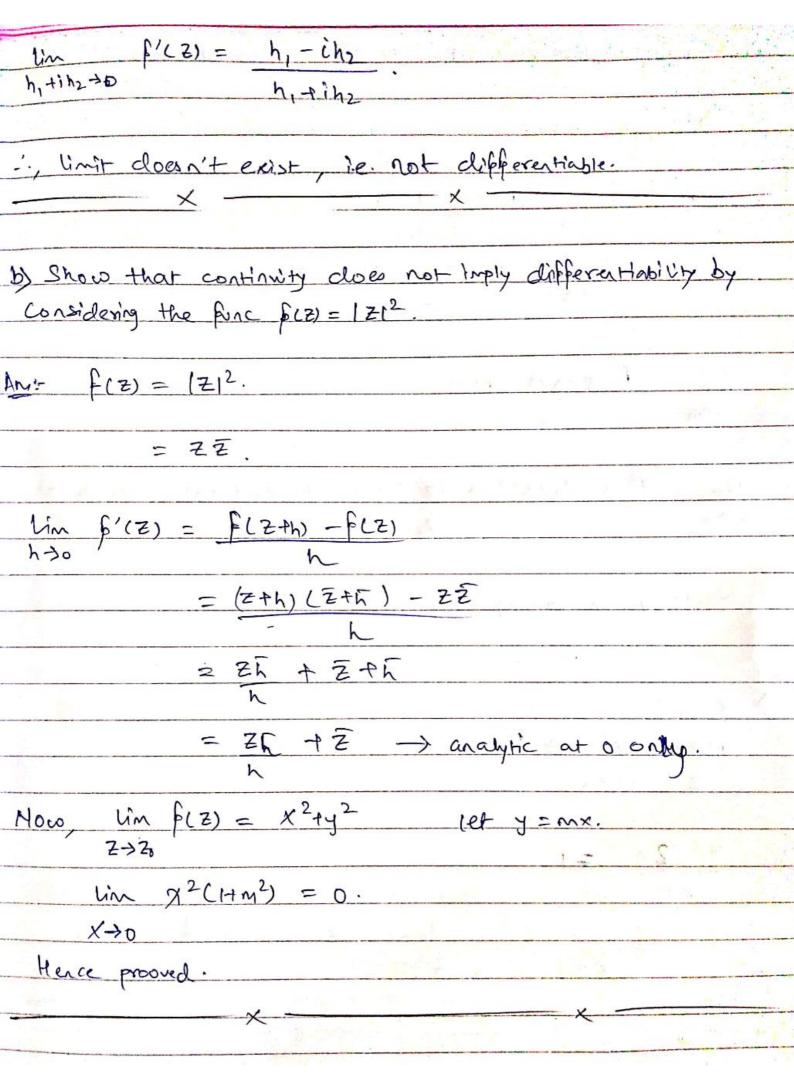
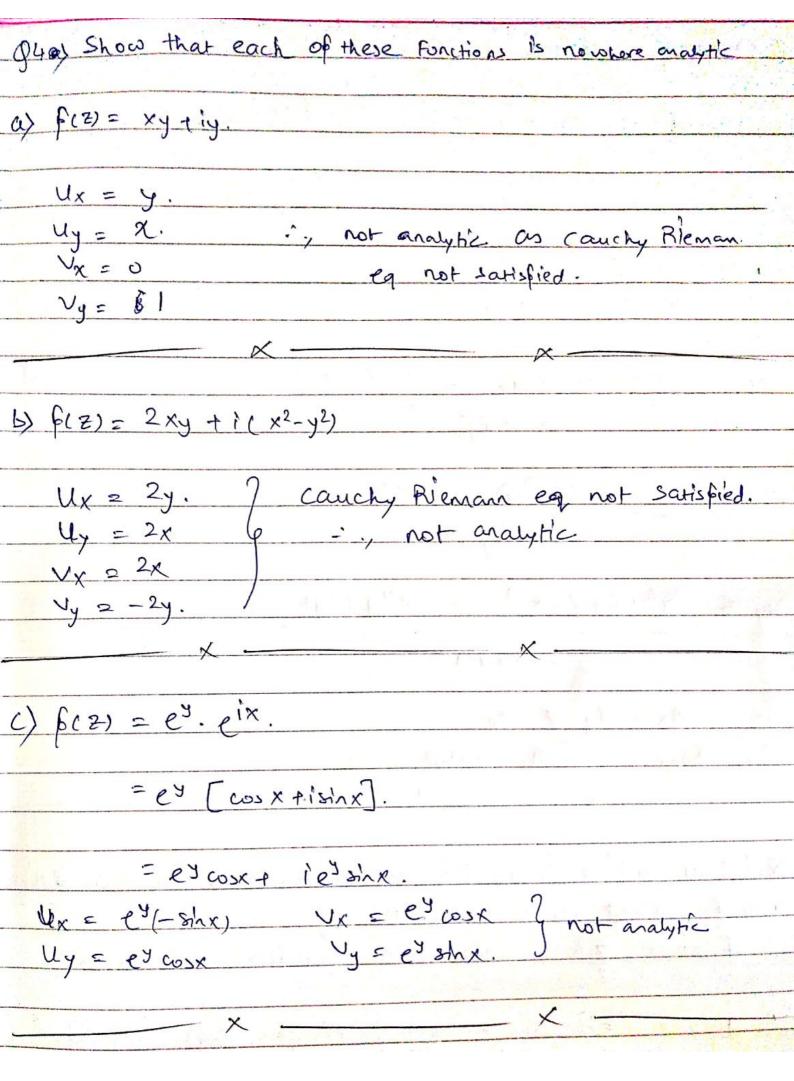
MODULE: 4	
$91/a$ ) Show that $\lim_{z\to 0} \frac{\overline{z}^2}{z} = 0$ .	
Z->0 Z	
$\frac{\text{Lin}}{2+0} \frac{(x-iy)^2}{(x+iy)} = \frac{\text{Lin}}{2+0} \frac{x^2-y^2-2xyi}{(x+iy)}$	
230 (Xtiy) 230 Xtly	1
let y=mx.	
$V_{in} = x^2 - m^2 x^2 - 2x^2 m i = 1 = x^2 + 1 = m^2 = 11$	,
Z->0 X+imx. Z->0 X(1+m)	
Now Z=x+iy.	
As 230 = x+iy >0 = X+ 1'mx >0.	
- XT (MX -) 0.	
$\chi(1-m^2-2mi)=0\times \text{ fivile}=0.$	17
x(1+im)→0 (1+mi)	
b) $\lim \overline{z} = \overline{z}_0$	
7-2	
And the second s	
ty: let Z= X+'y.	
Zo = xo + lyo.	
= x - 1y.	
$\lim_{z \to \infty} \overline{z} = x_0 - iy_0 = \overline{z}_0$ .	
こうる	
~ ~ ~ ~ ~	
9 let a, b and c denote complex constants. Show that lim[x-	+ 11244)
2-31-2	= iti.
by's Z=x+iy.	
と う 1 - に	
Comparing above 2 equations.	
9->1: 4-3-1.	
Scanned by Cam	Scanner



$\frac{Am!}{f(z)} = 3z^4 - 2z^3 + 8z^2 - 2z + 5$
(E) = SE = ZE
Z-i
g lin f(2) - 6(2)
t->i Z-i
1 2 - 3 5 - 2
\$ Um \[ \begin{array}{cccccccccccccccccccccccccccccccccccc
= 2-11 2-1
2-i
= lin (324-223+822-22+5) - (4+4i)(2-i)
2->i ==i
$(z-i)^2.$
$= \lim_{n \to \infty} 12z^3 - 6z^2 + 16z - 2 - (4+4i)$
1 1 1
2(2-1)2
= lim 3621 - 127 +16 - (4-12)
= lim 362 - 122 +16 -(4+16)
2
= -10 - 6i
Hence limit exists at Z=i.
X
(3a) Show that f(z) = Z is non-analytic anywhere.
101- analytic anywhere
Am: f(z) = = = ; Z= X+iy.
lim 6'(Z) = 6(Z+h) - 6(Z)
h-30
$= \frac{2\lambda - \lambda}{\lambda} \qquad h = h_1 + ih_2$
h 102



() Using the definition find the derivative of w= f(2) at the point where is z= Zo and is Z=-1. f'(Z) = Um f(Z) - f(Z0) Z-Z6  $= \lim_{z \to z_0} \frac{z^2 - 2z - z_0^2 + 2z_0}{z - z_0}$ FLX+DX) - F(X) lim 23 + (Zo+h)3 + 322 (Zo+h) + 32 (Zo+h)2-220+h-23 4-10 Similarly on solving we get  $3z_0^2-2$ . û) Z=-1 P(Z) = Z3-2Z Z = -lth. 23-22 =1  $\frac{1}{2^3 - 2z - 1} = \frac{1}{2-p_1}$  $\lim_{z \to 0} 3z^2 - 2 = 1$ . 2-3-1 2-1-1



(95) Verify whether the func

$$f(z) = \begin{cases} x^3y(y-ix) & , 270 \text{ is non-analyth} \\ x^0 + y^2 & \text{at } 2 \ge 0. \end{cases}$$

In:

$$f(z) = x^3y^2 - \frac{ix^4y}{x^4+y^2}$$

$$(x^4+y^2) (3x^2y^2) - x^3y^2 \cdot 6x^5$$

$$(x^6+y^2)^2 \cdot (x^6+y^2)^2 \cdot (x^4+y^2)^2 \cdot (x^4+y^2)^2 \cdot (x^6+y^2)^2 \cdot (x^6+y^$$

$$f(z) = x^3 - xy^2 - 2x^2y^2 - x^2y^2 + iy^3 - 2xy^2$$

$$x^2 - y^2.$$

$$f(2) = \frac{x^3 - 3xy^2}{x^2 - y^2} + i \left( \frac{y^3 - 3x^2y}{x^2 - y^2} \right)$$

$$(4)$$

$$(4)$$

$$(V)$$

$$u_{x} = (x^{2} - y^{2})(3x^{2} - 3y^{2}) - (x^{3} - 3xy^{2}) \cdot 2x$$

$$(x^{2} - y^{2})^{2}$$

$$U_{y} := (\chi^{2} - y^{2}) (-6xy) - (\chi^{3} - 3xy^{2}) (-2y)$$

$$(\chi^{2} - y^{2})^{2}.$$

$$V_{X} = (x^{2} - y^{2}) (-6xy) - (y^{3} - 3x^{2}y)(2x)$$

$$(x^{2} - y^{2})^{2}.$$

$$= 0.$$

$$v_{y} = (\chi^{2} - y^{2}) (3y^{2} - 3x^{2}) - (y^{3} - 3x^{2}y)(-2y).$$

$$(\chi^{2} - y^{2})^{2}$$

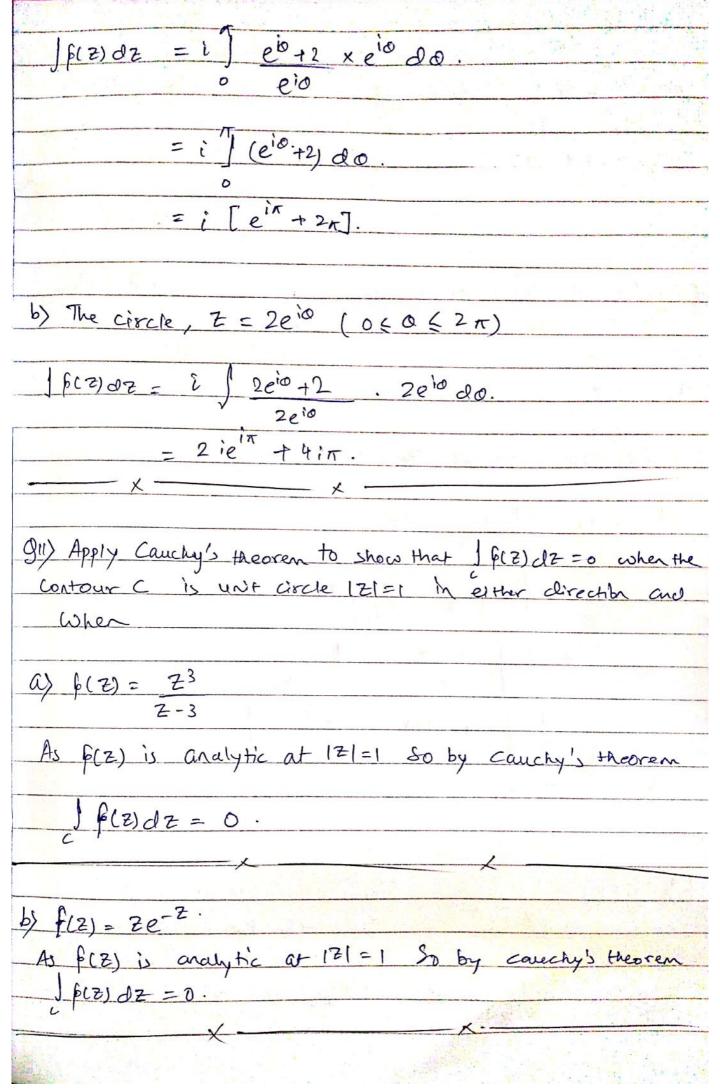
$$= -3.$$

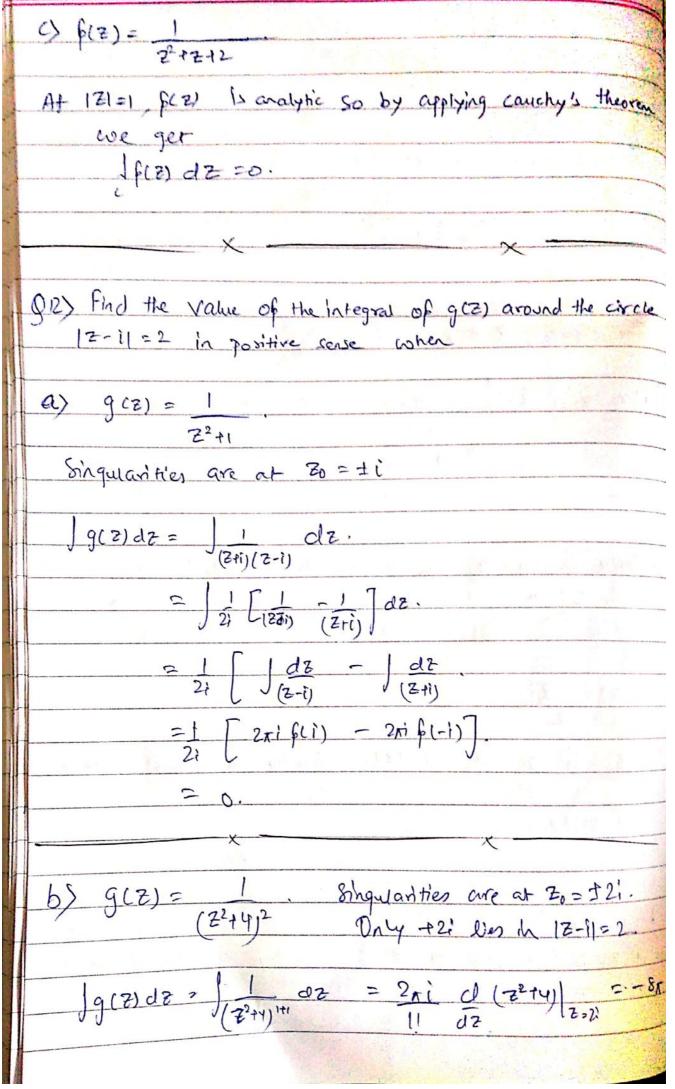
97) Show that 
$$u(x,y)$$
 is harmonic in some domain and find a harmonic Conjugate  $v(x,y)$  when i)  $u(x,y) = 2x(1-y)$ 

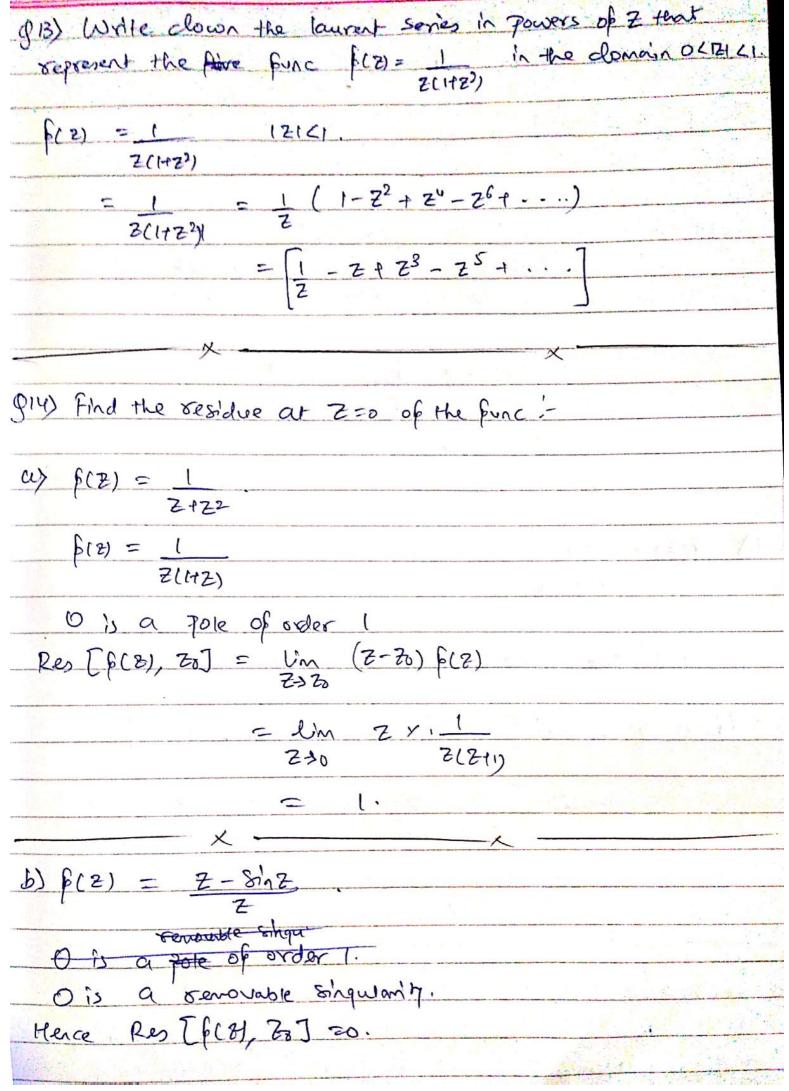
-		
-	$\frac{8y}{8x} = 2(1-y)$ $\frac{8y}{5y} = -2x$ .	- 0
	Sx Sy	
	$S^{2}4 = 0.$ $S^{2}4 = 0.$	
-	5x2 5y2	
	Co . C) Hormonic.	
	$82y + 52u = 0. \rightarrow Harmonic.$ $8x^2 = 5y^2$	
-	The state of the s	
-	Now according to couchy Riemann eq:	
1	Ux = Vy.	
	Uy = -Vx	
	$\frac{dx}{dy} = 2 - 2y$ , $\frac{dy}{dx} = 2x$ .	
	dy ax	
	$V = 2y - y^2 + \phi(x)$	
	$\frac{dv}{dx} = \phi'(x) = 2x$	e de la seco
	$dx \qquad \phi(x) = \chi^2 + C.$	Con Vigge
		197
	Now, V(x,y) = 2y - y2 +x2+c.	
		1 Y
-	Ogy Proof that Sy + 1 Su + 1 Sh = 0.	
	285 LEOUR HOT ON 4 1 PM = 0.	
	$\frac{2}{5r^2} + \frac{1}{8} + \frac{1}{8r} + \frac{1}{8r^2} + \frac{2}{80^2}$	
1	$u(x,y) = 2x - x^3 + x^3 + x^2$	
$\parallel$		to be
+	$\frac{4m^2}{8x} = \frac{5u}{2} = \frac{2-3x^2+3y^2}{8x}$	
-		
	$\frac{8^2 y}{6x^2} = -6x$	
	8x2 S24 + 524 = 0.	
	Sy 2 6 xy. 5x2 Sy2	
	84	
	5°4 = 6x	
-	842	4

Now according to cauchy Riemann ec	<b>1</b>
UX = Vy	
Uy = - Vx.	
$clv = 2-3x^2 + 3y^2 / v = 2y - 3y$	x2y + y3 + &(x)
dy	J
$\frac{dv = -6xy}{0x} \qquad \frac{dv = -6x}{0x}$	4 + o'(x)
dx	
$\phi(x) = C$	
The state of the s	
V2 2y - 3x2y + y3+c	- A -
98) Proof that 524 + 1 Su + 1	C <sup>2</sup> 11 = - 1
98) Proof that 524 + 1 84 + 1 5x2 8 8x 82	80 <sup>2</sup>
SY2 8 8x 82 502	
n 1 7 - NA 1 - 10	
Ami- Z= Xxiy = reio.	
$u + iv = \beta(z) = \beta(re^{i0})$	
0, 0, 11	
$\frac{3}{5} \frac{Su}{sr} + \frac{i}{s} \frac{8v}{sr} = \frac{f'(reio) \cdot e^{io}}{sr}$	•
8u + 18u = 6'(reio) · ireio	= 18 80 + 1,80
	L 84 2. J
Equating imaginary and real part	
<u>Su = 1 Sv</u>	the state of the state of the
E8 0 90	
$\frac{g_{\lambda}}{g_{\Lambda}} = \frac{g_{\lambda}}{-1} \frac{g_{\lambda}}{g_{\Lambda}}$	
St 2 -1 SV + 1 82V St 82 80 8 8088	S24 = -8 82N
	802 8r80

524 + 1 84 + 1 824 = 0.
Sr2 8 88 80
$\frac{y_0w_0}{s_0} = \frac{1}{s_0} \frac{s_0}{s_0} - \frac{1}{s_0} \frac{s_0^2}{s_0}$
8r2 72 80 8 Sr80
Similarly: 820 + 1 SV + 1 820 =0.
Similarly: 820 + 1 SV + 1 82V = 0.
X
091 Tr 1 constitution is constituted that
99) To func f(Z) = u(x,y) + iv(x,y) is analytic habe
D then it's component functions is and vote harmonical
λ. β.σ.
Ani- f(z) = U(x,y) + iv(x,y)
Walte
$\frac{1}{8x} = \frac{5u}{8x} = \frac{5^2v}{5x^2} = \frac{5^2v}{5x^3y}$
$\frac{\delta u}{sy} = -\frac{\delta u}{sx} \qquad \frac{\delta^2 u}{sy^2} = -\frac{\delta^2 v}{sx sy}$
) Dray
Um 2 11 a
$\frac{U_{xx} + U_{yy} = 0}{S^2 V} = S^2 V$
Syz SxSy ( =) 82.1 c2
$S^{2}v = -8^{2}v \qquad S^{2}v = 0.$
$\delta x^2 \qquad \delta x^2$
gio) For the Functions & and Cout a
gio) for the Functions ford Contours C, use parametric sepresentations for C to commune If(z) dz where
(12) = (2+2)/2 and c is (2) dz where
a) Semicircle Z=eio (O(O(T)
Scanned by CamScanne







915) We residue theorem to evaluate the hotografie of each of these func around 121=3. a) 6(2) = e-2 O is a gole of order 2. J f(2) 02 2 2 xi Per [f(2), Zo]. z - 2×1. b) P(2) = (Z+1) O and 2 are poles of order 1. in 171=3. [ P(2) dz = 2xi [ Res ( F(2), 0) + Res ( F(2), 2)]. = 24i [lim Z(Z+1) + lim (Z-Z(Z+1)] Z+30 Z(Z-2) Z+2 Z(Z-24)  $=2\pi i \left( -\frac{1}{2} + \frac{3}{2} \right) = 2\pi i$ c) 6(Z) = Z2 e1/2. As the func is analytic in 121 = 3 so by applying Cauchy's theorem | f(z)dz =0.