ELE2024 COURSEWORK - GROUP 3

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0.1. **Problem A1.**

$$m\ddot{x} = mg\sin(\phi) + F_{magnet} - k(x-d) - b\dot{x} + F_{friction} \tag{1}$$

$$F_{magnet} = c\frac{I^2}{y^2} \tag{2}$$

We need to find equations for I and y.

$$y = \delta - x \tag{3}$$

Equation for y found.

$$\begin{aligned} V &= IR + \dot{I}L \\ V &= IR + \frac{dI}{dt}L \\ V - IR &= \frac{dI}{dt}L \\ \frac{V-IR}{dI} &= \frac{L}{dt} \\ \frac{1}{V-IR}dI &= \frac{1}{L}dt \\ \int \frac{1}{V-IR}dI &= \int \frac{1}{L}dt \end{aligned}$$

Let $u = V - IR$

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Some note goes here. Version 0.0.1. Last updated: November 23, 2020.

$$
\begin{aligned}
\implies \frac{du}{dI} &= -R \\
dI &= \frac{1}{-R} du \\
\int_{u_0}^{u} \frac{1}{u} \times \frac{1}{-R} du &= \int_0^t \frac{1}{L} dt \\
\frac{1}{-R} \int_{u_0}^{u} \frac{1}{u} du &= \int_0^t \frac{1}{L} dt \\
\frac{1}{-R} \left[\ln(u)\right]_{u_0}^{u} &= \frac{1}{L} t \\
\frac{1}{-R} \left[\ln(V - IR)\right]_{I_0}^{I} &= \frac{1}{L} t \\
\left[\ln(V - IR)\right]_{I_0}^{I} &= \frac{-R}{L} t \\
\ln(V - IR) - \ln(V - I_0 R) &= \frac{-R}{L} t \\
\ln\left(\frac{V - IR}{V - I_0 R}\right) &= \frac{-R}{L} t \\
\frac{V - IR}{V - I_0 R} &= e^{\frac{-R}{L} t} \\
V - IR &= (V - I_0 R)\, e^{\frac{-R}{L} t} \\
V - (V - I_0 R)\, e^{\frac{-R}{L} t} &= IR \\
I &= \frac{V}{R} - \frac{V - I_0 R}{R} e^{\frac{-R}{L} t}
\end{aligned}
$$

Equation for I found.
Substituting I and y into F_{magnet}.

$$
F_{magnet} = c \frac{\left(\frac{V}{R} - \frac{V - I_0 R}{R} e^{\frac{-R}{L} t}\right)^2}{(\delta - x)^2} \tag{4}
$$