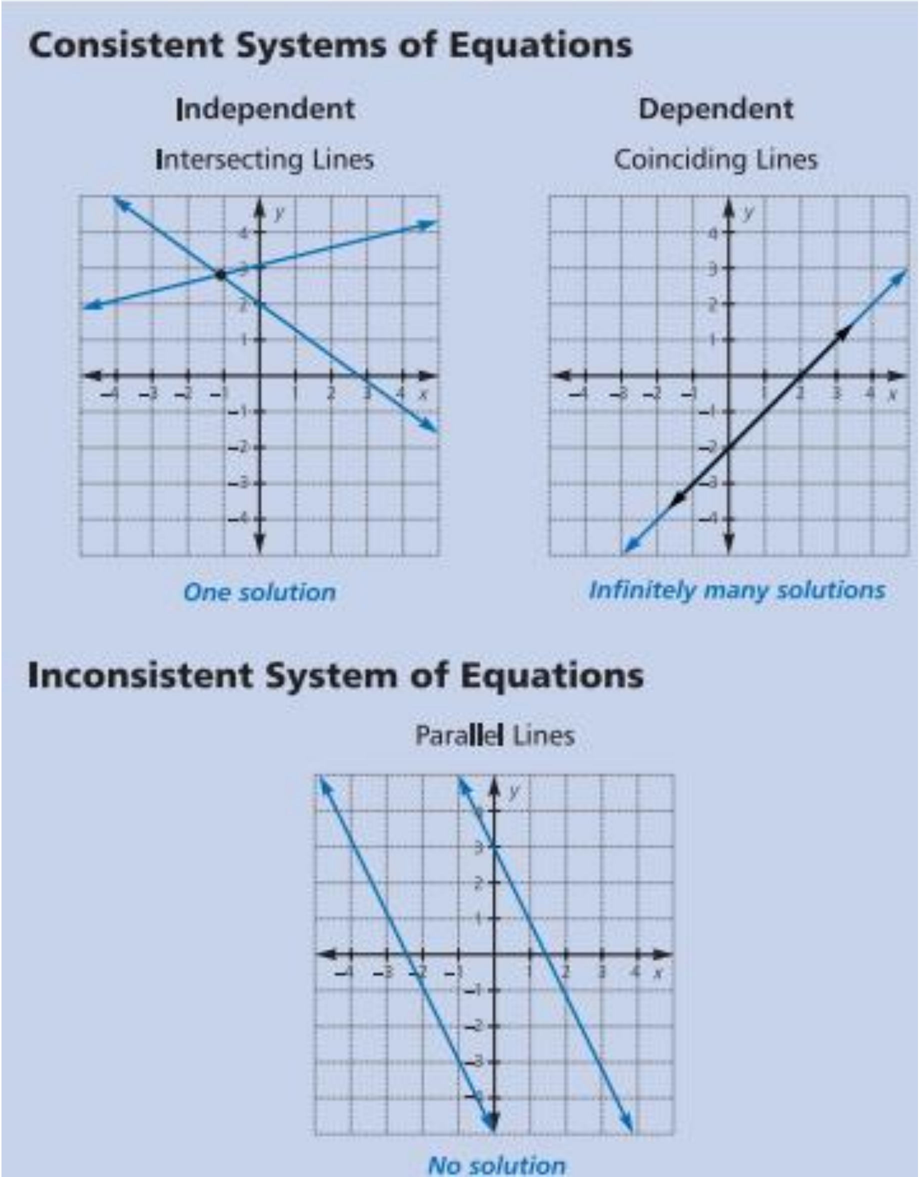
Solving Systems of Equations

Algebaically

Solving systems of equations

All these things are still true:

- A system of equations is two or more equations
- (We will work with 2 mostly)
- Types of systems of equations
    - Dependent (infinite solutions)
    - Consistent (at least one solution)
    - Inconsistent (no solutions)



Solving systems of equations – the goal

- The goal of solving systems of equations is to see what point(s) they have in common.
- We can do this algebraically with:
  - Substitution method
  - Elimination (Subtraction) method

Solving systems of equations – Substitution

- Steps in the substitution method:
  - Solve one equation for one of the variables (stick with y)
  - Substitute that into the other equation and solve for the remaining variable (x)
  - Put that value of x back into one of the equations to find y.
  - Verify that the pair (x, y) works in both original equations

Solving systems of equations – Substitution

- Example

$y \ - 3x = 17$
$x + 5y = 9$

Solve for y in first equation:

$$
\begin{array}{r}
y \ - 3x = 17 \\
+ 3x = \ + 3x \\
\hline
y \ = 3x + \ 17
\end{array}
$$

Substitute for y in the second equation:

$x + 5(3x + 17) = 9$
$x + 5 * 3x + 5 * 17 = 9$
$x + 15x + 85 = 9$

$$
\begin{array}{c}
-85 = -85 \\
\hline
(1 + 15)x = 9 - 85 = -76 \\
16x = -76 \\
\hline
16 = 16 \\
x = \frac{-76}{16} = -4.75
\end{array}
$$

Solving systems of equations – Substitution

- Example

$y - 3x = 17$
$x + 5y = 9$

Put value of x into first equation, solve for y:

$$y - 3(-4.75) = 17$$
$$y + 14.25 = 17$$
$$-14.25 = -14.25$$
$$y = 17 - 14.25 = 2.75$$

Solution: $(-4.75, 2.75)$

Solving systems of equations – Substitution

- Example

$$y - 3x = 17$$
$$x + 5y = 9$$

Check solution in both equations:

$$y - 3x = 17$$
$$2.75 - 3(-4.75) \;?\!=\!?\; 17$$
$$2.75 + 14.25 \;?\!=\!?\; 17$$
$$17 = 17 \checkmark$$

$$x + 5y = 9$$
$$-4.75 + 5(2.75) \;?\!=\!?\; 9$$
$$-4.75 + 13.75 \;?\!=\!?\; 9$$
$$9 = 9 \checkmark$$

Solution: $(-4.75, 2.75)$

Solving systems of equations – Elimination

- Steps in the elimination (subtraction) method:
  - Match the coefficient of one of the variables in both equations
  - Subtract one equation from the other
  - Solve for that single variable
  - Put that value into one of the equations and solve for the other variable
  - Verify that the pair (x, y) works in both equations

Solving systems of equations – Elimination

- Example

$$10s + 30w = 300$$
$$14s + 27w = 315$$

Find a common coefficient so you can eliminate s or w when subtracting. (70)

Multiply whole equations by the number that will get you to have the proper coefficients:

$$7(10s + 30w) = 7(300)$$
$$5(14s + 27w) = 5(315)$$

$$70s + 210w = 2100$$
$$70s + 135w = 1575$$
$$0s + 75w = 525$$

Divide by 75

$$\frac{75w}{75} = \frac{525}{75}$$

$$w = \frac{525}{75} = 7$$

Solving systems of equations – Elimination

- Example

$$10s + 30w = 300$$
$$14s + 27w = 315$$

Once you find the answer for one variable, find the answer for the other by putting it into one equation.

$$10s + 30 * 7 = 300$$
$$10s + 210 = 300$$
$$10s = 300 - 210 = 90$$
$$\frac{10s}{10} = \frac{90}{10} = 9 = s$$
$$s = 9; w = 7$$

Check:

$$14s + 27w = 315$$
$$14 * 9 + 27 * 7 ?=? 315$$
$$126 + 189 ?=? 315$$
$$315 = 315$$

Solving systems of equations – practice

8.2

- 5, 6, 7, 12, 14
- 18, 19, 21, 22, 23

CIRCLED DONE IN CLASS

Solving systems of equations – practice 5

$y - 3x = -7$
$5x - 2y = 12$

① $y - 3x = -7$
$+3x = \quad +3x$
$y + 0 = -7 + 3x$
$y = \underbrace{-7 + 3x}$

② $5x - 2(3x - 7) = 12$
$5x - (2 \cdot 3)x + (-2)(-7) = 12$
$5x - 6x + 14 = 12$
$(5-6)x + 14 = 12$
$-14 \quad -14$
$-x + 0 = -2$
$-x = -2$
$x = 2$

③ $5(2) - 2y = 12$
$10 - 2y = 12$
$-10 \quad -10$
$0 - 2y = 2$
$\frac{-2y}{-2} = \frac{2}{-2}$
$y = -1$

④ $y - 3x = -7$
$-1 - 3(2) \stackrel{?}{=} -7$
$-1 - 6 = -7$
$-7 = -7$ ✓

$(2, -1)$

Solving systems of equations – practice 7

$6x - y = -2$
$-18x + 3y = 4$

① $6x - y = -2$

$-6x \qquad -6x$

$0 - y = -2 - 6x$

$\frac{-y}{-1} = \frac{-2-6x}{-1}$

$y = \frac{-2}{-1} + \frac{-6x}{-1}$

$= 2 + 6x$

② $-18x + 3(6x + 2) = 4$

$-18x + (3 \cdot 6)x + (3 \cdot 2) = 4$

$-18x + 18x + 6 = 4$

$0 + 6 = 4$

$\boxed{\cancel{6 = 4}}$

No Solution

(parallel lines)

always addition

Solving systems of equations – practice 21

10 – 99

y

The sum of the digits of a 2-digit number is 12. The second digit is 6 more than the first digit. What was the original number?

x

xy

$x + y = 12$

$y = x + 6$

$x + x + 6 = 12$

$2x + 6 = 12$

$-6 \quad -6$

$2x + 0 = 6$

$\frac{2x}{2} = \frac{6}{2}$

$x = \frac{6}{2} = 3$

$x + y = 12$

$3 + y = 12$

$-3 \quad -3$

$0 + y = 9$

$(3, 9)$

$\boxed{39}$

Solving systems of equations – practice 23

0.045 x, y 0.06

Amber invested $\boxed{\$6000}$ in $\boxed{\text{two accounts}}$. Some of the money was invested at $\boxed{4.5\%}$ and the remainder was invested at $\boxed{6\%}$. The total annual interest earned from the two accounts was $\boxed{\$279}$. How much was deposited at each rate?

$$0.045x + 0.06y = 279$$
$$0.045x + 0.06(6000 - x) = 279$$
$$0.045x + (0.06)(6000) - 0.06x = 279$$
$$(0.045x - 0.06x) + 360 = 279$$
$$-0.015x + 360 = 279$$
$$-360 \quad -360$$
$$-0.015x = -81$$

$$x + y = 6000$$
$$-x \qquad -x$$
$$y = 6000 - x$$

$$\frac{-0.015x}{-0.015} = \frac{-81}{-0.015} = 5400$$
$$x = 5400$$

Solving systems of equations – practice 23

$x = \$5400$

Amber invested \$6000 in two accounts. Some of the money was invested at 4.5% and the remainder was invested at 6%. The total annual interest earned from the two accounts was \$279. How much was deposited at each rate?

$0.045x + 0.06y = 279$

$(0.045)(5400) + (0.06)(600) \stackrel{?}{=} 279$

$243 + 36 \stackrel{?}{=} 279$

$279 = 279$ ✓

$x + y = 6000$

$y = 6000 - x$

$= 6000 - 5400$

$= \$600$

$\boxed{(\$5400, \$600)}$

Solving systems of equations – practice 6

$x + 2y = 5$

$-x \qquad -x$

$0 + 2y = 5 - x$

$\frac{2y}{2} = \frac{5-x}{2}$

$y = \boxed{\frac{5}{2} - \frac{x}{2}}$

$3x - y = 1$

$3x - \left(\frac{5}{2} - \frac{x}{2}\right) = 1$

$3x - \frac{5}{2} + \frac{x}{2} = 1$

$+\frac{5}{2} \qquad +\frac{5}{2}$

$3x + \frac{x}{2} = \frac{7}{2}$

$\frac{2}{7} \times \frac{7x}{2} = \frac{7}{2} \times \frac{2}{7} \Rightarrow x = 1$

$\frac{2 \cdot 1}{2} + \frac{5}{2} = \frac{2+5}{2} = \frac{7}{2}$

$\frac{2 \cdot 3x}{2} + \frac{x}{2} = \frac{6x + x}{2}$

$= \frac{7x}{2}$

$x = 1$

Solving systems of equations – practice 6

$x + 2y = 5$

$1 + 2 \cdot 2 \overset{?}{=} 5$

$1 + 4 \overset{?}{=} 5$

$5 = 5$ ✓

$3x - y = 1$

$3 \cdot 1 - y = 1$

$3 - y = 1$

$-3 \qquad -3$

$-y = -2$

$\frac{-y}{-1} = \frac{-2}{-1}$

$y = 2$

$\boxed{(1,2)}$