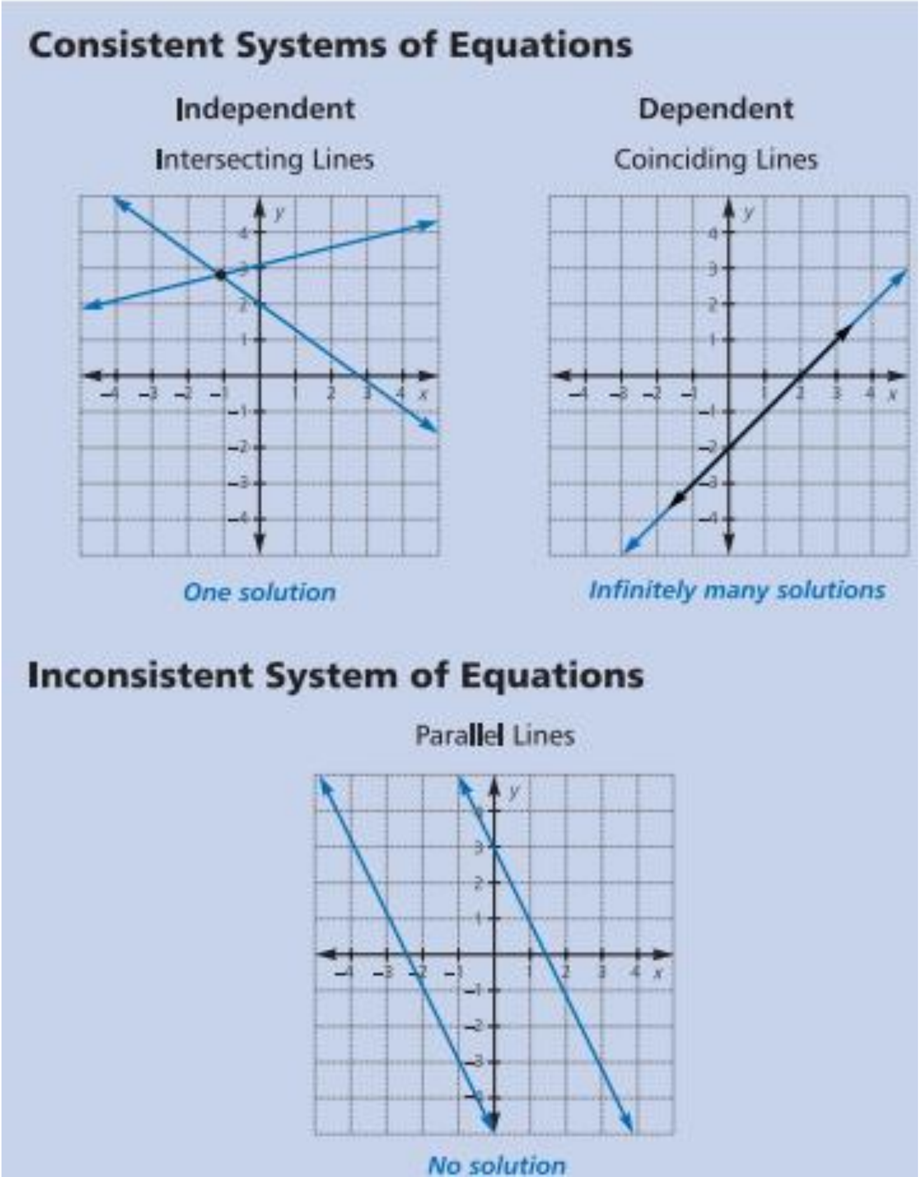
# Solving Systems of Equations

Algebaically

# Solving systems of equations

All these things are still true:

- A system of equations is two or more equations
- (We will work with 2 mostly)
- Types of systems of equations
  - Dependent (infinite solutions)
  - Consistent (at least one solution)
  - Inconsistent (no solutions)



# Solving systems of equations – the goal

- The goal of solving systems of equations is to see what point(s) they have in common.
- We can do this algebraically with:
  - Substitution method
  - Elimination (Subtraction) method

# Solving systems of equations – Substitution

- Steps in the substitution method:
  - Solve one equation for one of the variables (stick with y)
  - Substitute that into the other equation and solve for the remaining variable (x)
  - Put that value of x back into one of the equations to find y.
  - Verify that the pair (x, y) works in both original equations

# Solving systems of equations – Substitution

- Example

$$y - 3x = 17$$
$$x + 5y = 9$$

Solve for y in first equation:

$$y - 3x = 17$$
$$+3x = +3x$$
$$y = 3x + 17$$

Substitute for y in the second equation:

$$x + 5(3x + 17) = 9$$
$$x + 5 * 3x + 5 * 17 = 9$$
$$x + 15x + 85 = 9$$
$$-85 = -85$$
$$(1 + 15)x = 9 - 85 = -76$$
$$16x = -76$$
$$16 = 16$$
$$x = \frac{-76}{16} = -4.75$$

# Solving systems of equations – Substitution

- Example

$y \ - 3x = 17$
$x + 5y = 9$

Put value of x into first equation, solve for y:

$$y \ - 3(-4.75) = 17$$
$$y + 14.25 = 17$$
$$-14.25 = -14.25$$
$$y = 17 \ - 14.25 = 2.75$$

Solution: $(-4.75, 2.75)$

# Solving systems of equations – Substitution

- Example

$$y - 3x = 17$$
$$x + 5y = 9$$

Check solution in both equations:

$$y - 3x = 17$$
$$2.75 - 3(-4.75) \mathrel{?=?} 17$$
$$2.75 + 14.25 \mathrel{?=?} 17$$
$$17 = 17 \checkmark$$

$$x + 5y = 9$$
$$-4.75 + 5(2.75) \mathrel{?=?} 9$$
$$-4.75 + 13.75 \mathrel{?=?} 9$$
$$9 = 9 \checkmark$$

Solution: $(-4.75, 2.75)$

# Solving systems of equations – Elimination

- Steps in the elimination (subtraction) method:
  - Match the coefficient of one of the variables in both equations
  - Subtract one equation from the other
  - Solve for that single variable
  - Put that value into one of the equations and solve for the other variable
  - Verify that the pair (x, y) works in both equations

# Solving systems of equations – Elimination

- Example

$$10s + 30w = 300$$
$$14s + 27w = 315$$

Find a common coefficient so you can eliminate s or w when subtracting. (70)

Multiply whole equations by the number that will get you to have the proper coefficients:

$$7(10s + 30w) = 7(300)$$
$$5(14s + 27w) = 5(315)$$

$$70s + 210w = 2100$$
$$70s + 135w = 1575$$
$$0s + 75w = 525$$

*Divide by 75*

$$\frac{75w}{75} = \frac{525}{75}$$

$$w = \frac{525}{75} = 7$$

# Solving systems of equations – Elimination

- Example

$$10s + 30w = 300$$
$$14s + 27w = 315$$

Once you find the answer for one variable, find the answer for the other by putting it into one equation.

$$10s + 30 * 7 = 300$$
$$10s + 210 = 300$$
$$10s = 300 \ - 210 = 90$$
$$\frac{10s}{10} = \frac{90}{10} = 9 = s$$
$$s = 9; w = 7$$

Check:

$$14s + 27w = 315$$
$$14 * 9 + 27 * 7 ?=? 315$$
$$126 + 189 ?=? 315$$
$$315 = 315$$

# Solving systems of equations – practice

8.2

- 5, 6, 7, 12, 14
- 18, 19, 21, 22, 23

# Solving systems of equations – practice 5

$$y - 3x = -7$$
$$5x - 2y = 12$$

① 
$$y - 3x = -7$$
$$+3x = \quad +3x$$
$$y + 0 = -7 + 3x$$
$$y = \underbrace{-7 + 3x}$$

② 
$$5x - 2(3x - 7) = 12$$
$$5x - (2 \cdot 3)x + (-2)(-7) = 12$$
$$5x - 6x + 14 = 12$$
$$(5-6)x + 14 = 12$$
$$-14 \quad -14$$
$$-x + 0 = -2$$
$$-x = -2$$
$$x = 2$$
$$(2, -1)$$

③ 
$$5(2) - 2y = 12$$
$$10 - 2y = 12$$
$$-10 \quad -10$$
$$0 - 2y = 2$$
$$\frac{-2y}{-2} = \frac{2}{-2}$$
$$y = -1$$

④ 
$$y - 3x \stackrel{?}{=} -7$$
$$-1 - 3(2) \stackrel{?}{=} -7$$
$$-1 - 6 = -7$$
$$-7 = -7 \checkmark$$

# Solving systems of equations – practice 7

$$6x - y = -2$$
$$-18x + 3y = 4$$

① 
$$6x - y = -2$$
$$-6x \qquad -6x$$
$$0 - y = -2 - 6x$$
$$\frac{-y}{-1} = \frac{-2-6x}{-1}$$
$$y = \frac{-2}{-1} + \frac{-6x}{-1}$$
$$= 2 + 6x$$

②
$$-18x + 3(6x+2) = 4$$
$$-18x + (3\cdot 6)x + (3\cdot 2) = 4$$
$$-18x + 18x + 6 = 4$$
$$0 + 6 = 4$$
$$\boxed{\cancel{6 = 4}}$$

No Solution
(parallel lines)

always addition

# Solving systems of equations – practice 21

10 – 99

y

The sum of the digits of a 2-digit number is 12. The second digit is 6 more than the first digit. What was the original number?

x

$xy$

$x + y = 12$

$y = x + 6$

$x + x + 6 = 12$

$2x + 6 = 12$

$-6 \quad -6$

$2x + 0 = 6$

$\frac{2x}{2} = \frac{6}{2}$

$x = \frac{6}{2} = 3$

$x + y = 12$

$3 + y = 12$

$-3 \quad -3$

$0 + y = 9$

$(3, 9)$

$\boxed{39}$

# Solving systems of equations – practice 23

Amber invested \$6000 in two accounts. Some of the money was invested at 4.5% and the remainder was invested at 6%. The total annual interest earned from the two accounts was \$279. How much was deposited at each rate?