

# Solving Systems of Equations

Algebraically

# Solving systems of equations

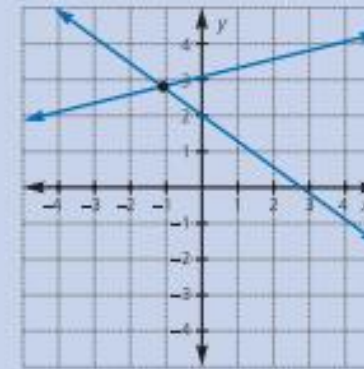
All these things are still true:

- A system of equations is two or more equations
- (We will work with 2 mostly)
- Types of systems of equations
  - Dependent (infinite solutions)
  - Consistent (at least one solution)
  - Inconsistent (no solutions)

## Consistent Systems of Equations

### Independent

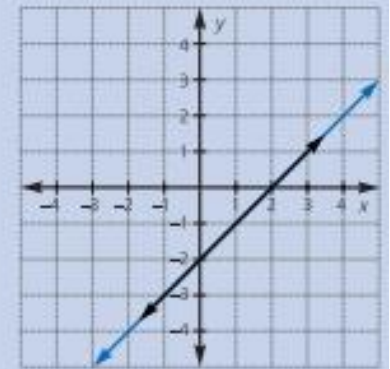
#### Intersecting Lines



*One solution*

### Dependent

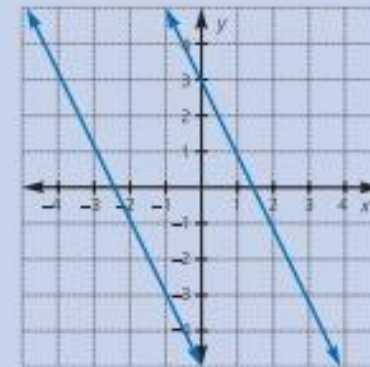
#### Coinciding Lines



*Infinitely many solutions*

## Inconsistent System of Equations

### Parallel Lines



*No solution*

# Solving systems of equations – the goal

- The goal of solving systems of equations is to see what point(s) they have in common.
- We can do this algebraically with:
  - Substitution method
  - Elimination (Subtraction) method

# Solving systems of equations – Substitution

- Steps in the substitution method:
  - Solve one equation for one of the variables (stick with  $y$ )
  - Substitute that into the other equation and solve for the remaining variable ( $x$ )
  - Put that value of  $x$  back into one of the equations to find  $y$ .
  - Verify that the pair  $(x, y)$  works in both original equations

# Solving systems of equations – Substitution

- Example

$$\begin{aligned}y - 3x &= 17 \\ x + 5y &= 9\end{aligned}$$

Solve for y in first equation:

$$\begin{array}{r}y - 3x = 17 \\ + 3x = + 3x \\ \hline y = 3x + 17\end{array}$$

Substitute for y in the second equation:

$$\begin{aligned}x + 5(3x + 17) &= 9 \\ x + 5 * 3x + 5 * 17 &= 9 \\ x + 15x + 85 &= 9 \\ -85 &= -85 \\ \hline (1 + 15)x &= 9 - 85 = -76 \\ 16x &= -76 \\ \hline 16 &= 16 \\ x &= \frac{-76}{16} = -4.75\end{aligned}$$

# Solving systems of equations – Substitution

- Example

$$\begin{aligned}y - 3x &= 17 \\ x + 5y &= 9\end{aligned}$$

Put value of  $x$  into first equation, solve for  $y$ :

$$y - 3(-4.75) = 17$$

$$y + 14.25 = 17$$

$$-14.25 = -14.25$$

---

$$y = 17 - 14.25 = 2.75$$

Solution:  $(-4.75, 2.75)$

# Solving systems of equations – Substitution

- Example

$$\begin{aligned}y - 3x &= 17 \\ x + 5y &= 9\end{aligned}$$

Check solution in both equations:

$$\begin{aligned}y - 3x &= 17 \\ 2.75 - 3(-4.75) &=? 17 \\ 2.75 + 14.25 &=? 17 \\ 17 &= 17 \checkmark\end{aligned}$$

$$\begin{aligned}x + 5y &= 9 \\ -4.75 + 5(2.75) &=? 9 \\ -4.75 + 13.75 &=? 9 \\ 9 &= 9 \checkmark\end{aligned}$$

Solution:  $(-4.75, 2.75)$

# Solving systems of equations – Elimination

- Steps in the elimination (subtraction) method:
  - Match the coefficient of one of the variables in both equations
  - Subtract one equation from the other
  - Solve for that single variable
  - Put that value into one of the equations and solve for the other variable
  - Verify that the pair  $(x, y)$  works in both equations



# Solving systems of equations – Elimination

- Example

$$10s + 30w = 300$$

$$14s + 27w = 315$$

Find a common coefficient so you can eliminate  $s$  or  $w$  when subtracting. (70)

Multiply whole equations by the number that will get you to have the proper coefficients:

$$7(10s + 30w) = 7(300)$$

$$5(14s + 27w) = 5(315)$$

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$$70s + 210w = 2100$$

$$70s + 135w = 1575$$

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$$0s + 75w = 525$$

*Divide by 75*

$$75w = 525$$

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$$75 \qquad 75$$

$$w = \frac{525}{75} = 7$$

# Solving systems of equations – Elimination

- Example

$$10s + 30w = 300$$

$$14s + 27w = 315$$

Once you find the answer for one variable, find the answer for the other by putting it into one equation.

$$10s + 30 * 7 = 300$$

$$10s + 210 = 300$$

$$10s = 300 - 210 = 90$$

$$\frac{10s}{10} = \frac{90}{10} = 9 = s$$

$$s = 9; w = 7$$

Check:

$$14s + 27w = 315$$

$$14 * 9 + 27 * 7 = ? 315$$

$$126 + 189 = ? 315$$

$$315 = 315$$

# Solving systems of equations – practice

8.2

- 5, 6, 7, 12, 14
  - 18, 19, 21, 22, 23
- CIRCLED  
DONE IN CLASS
- 

# Solving systems of equations – practice 5

$$\begin{aligned} y - 3x &= -7 \\ 5x - 2y &= 12 \end{aligned}$$

$$\textcircled{1} \quad \begin{array}{r} y - 3x = -7 \\ \phantom{y} + 3x = \phantom{-7} + 3x \end{array}$$

$$\begin{array}{r} y + 0 = -7 + 3x \\ y = -7 + 3x \end{array}$$

$$\textcircled{2} \quad \begin{array}{r} 5x - 2(3x - 7) = 12 \\ 5x - 2 \cdot 3x + (-2)(-7) = 12 \end{array}$$

$$5x - 6x + 14 = 12$$

$$(5 - 6)x + 14 = 12$$

$$\phantom{(5 - 6)x} - 14 \quad -14$$

$$\begin{array}{r} -x + 0 = -2 \end{array}$$

$$-x = -2$$

$$x = 2$$

$$\textcircled{3} \quad \begin{array}{r} 5(2) - 2y = 12 \\ 10 - 2y = 12 \\ \phantom{10 - 2y} - 10 \\ \hline 0 - 2y = 2 \\ \phantom{0 - 2y} -2 \\ \hline y = -1 \end{array}$$

$$\textcircled{4} \quad \begin{array}{r} y - 3x = -7 \\ \phantom{y} - 3(2) = -7 \\ \phantom{y} - 6 = -7 \\ \phantom{y} -7 = -7 \checkmark \end{array}$$

→ (2, -1)

# Solving systems of equations – practice 7

$$\begin{aligned} 6x - y &= -2 \\ -18x + 3y &= 4 \end{aligned}$$

$$\begin{array}{r} \textcircled{1} \quad 6x - y = -2 \\ \quad -6x \qquad \qquad -6x \\ \hline 0 - y = -2 - 6x \\ \quad -1 \qquad \quad -1 \\ \hline y = \frac{-2}{-1} + \frac{-6x}{-1} \\ \qquad \qquad \qquad = 2 + 6x \end{array}$$

$$\begin{array}{r} \textcircled{2} \quad -18x + 3(6x + 2) = 4 \\ -18x + (3 \cdot 6)x + (3 \cdot 2) = 4 \\ -18x + 18x + 6 = 4 \\ 0 + 6 = 4 \\ \boxed{6 = 4} \end{array}$$

No Solution  
(parallel lines)

always addition

# Solving systems of equations – practice 21

1D-99

The sum of the digits of a 2-digit number is 12. The second digit is 6 more than the first digit. What was the original number?

xy

$$x + y = 12$$
$$y = x + 6$$

$$\begin{array}{r} x + y = 12 \\ 3 + y = 12 \\ -3 \quad -3 \\ \hline 0 + y = 9 \end{array}$$

(3, 9)

39

$$\begin{array}{r} x + x + 6 = 12 \\ 2x + 6 = 12 \\ -6 \quad -6 \\ \hline 2x + 0 = 6 \\ \frac{2x}{2} = \frac{6}{2} \\ x = 3 \end{array}$$

# Solving systems of equations – practice 23

0.045

X, y

→ 0.06

Amber invested \$6000 in two accounts. Some of the money was invested at 4.5% and the remainder was invested at 6%. The total annual interest earned from the two accounts was \$279. How much was deposited at each rate?

$$0.045x + 0.06y = 279$$

$$0.045x + 0.06(6000 - x) = 279$$

$$0.045x + (0.06)(6000) - 0.06x = 279$$

$$(0.045x - 0.06x) + 360 = 279$$

$$-0.015x + 360 = 279$$

$$\begin{array}{r} -0.015x + 360 = 279 \\ -360 \quad -360 \\ \hline -0.015x = -81 \end{array}$$

$$x + y = 6000$$

$$\begin{array}{r} x + y = 6000 \\ -x \quad \quad -x \\ \hline \end{array}$$

$$y = 6000 - x$$

$$\begin{array}{r} -0.015x = -81 \\ \hline -0.015 \quad -0.015 \\ \hline \end{array}$$

$$x = 5400$$

# Solving systems of equations – practice 23

$$X = \overset{\$}{5400}$$

Amber invested \$6000 in two accounts. Some of the money was invested at 4.5% and the remainder was invested at 6%. The total annual interest earned from the two accounts was \$279. How much was deposited at each rate?

$$\begin{aligned} 0.045x + 0.06y &= 279 \\ (0.045)(5400) + (0.06)(600) &? 279 \\ 243 + 36 &? 279 \\ 279 &= 279 \checkmark \end{aligned}$$
$$\begin{aligned} x + y &= 6000 \\ y &= 6000 - x \\ &= 6000 - 5400 \\ &= \$600 \end{aligned}$$

(\$5400	\$600)
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# Solving systems of equations – practice 6

$$\begin{array}{r} x + 2y = 5 \\ -x \phantom{+ 2y} = 0 \\ \hline 0 + 2y = 5 - x \\ 2y = \frac{5 - x}{2} \\ y = \boxed{\frac{5}{2} - \frac{x}{2}} \end{array}$$

$$3x - y = 1$$

$$\begin{array}{r} 3x - \left(\frac{5}{2} - \frac{x}{2}\right) = 1 \\ 3x - \frac{5}{2} + \frac{x}{2} = 1 \\ \phantom{3x} + \frac{5}{2} \phantom{+ \frac{x}{2}} \phantom{=} \phantom{1} \\ \hline 3x + \frac{x}{2} = \frac{7}{2} \end{array}$$

$$3x + \frac{x}{2} = \frac{7}{2}$$

$$\frac{2}{7} \times \frac{7x}{2} = \frac{7}{2} \times \frac{2}{7} \Rightarrow x = 1$$

$$\frac{2 \cdot 1}{2} + \frac{5}{2} = \frac{2+5}{2} = \frac{7}{2}$$

$$\begin{array}{r} 2 \cdot 3x + \frac{x}{2} = \frac{6x+x}{2} \\ \phantom{2 \cdot 3x} = \frac{7x}{2} \end{array}$$

$$x=1$$

# Solving systems of equations – practice 6

$$x + 2y = 5$$

$$\begin{array}{r} 1 + 2 \cdot 2 \quad ? \quad 5 \\ 1 + 4 \quad ? \quad 5 \\ 5 = 5 \checkmark \end{array}$$

$$3x - y = 1$$

$$\begin{array}{r} 3 \cdot 1 - y = 1 \\ 3 - y = 1 \\ -3 \quad -3 \\ \hline -y = -2 \\ -y = -2 \\ y = 2 \end{array}$$

$$(1, 2)$$

# Solving systems of equations – answers

8.2

- 5 (2, -1)
- 6 (1, 2)
- 7 No solution
- 12 (-5, -6)
- 14 No solution

8.2

- 18 Infinitely many solutions
- 19 (-3, 4.5)
- 21 (39)
- 22 (15 MC, 5 essay)
- 23 (\$5400 at 4.5%, \$600 at 6%)