A General Framework Based on a Hybrid Analytical Model for the Analysis and Design of Permanent Magnet Machines

S. Ouagued, A. Aden Diriye, Y. Amara, and G. Barakat

Université du Havre, Le Havre 76600, France

The aim of this paper is to highlight the capabilities of a new hybrid analytical modeling (HAM) approach for the analysis and design of various electromagnetic structures. After an introduction, where the aim of the development of this modeling approach is presented, the modeling approach is described. The new HAM is based on a strong coupling of the magnetic equivalent circuits method and analytical models (AMs). The AM, based on the formal solution of Maxwell's equations, is established thanks to the separation of variables method. The HAM is applied to different permanent magnet electric machine structures, and the results are validated extensively by comparison with the finite-element analyses.

Index Terms—**Electric machines, electromagnetic fields, electromagnetic modeling, magnetic equivalent circuits (MECs).**

I. Introduction

IN HIGHLY concurrential markets, the time to market is reduced, which implies a need of design tools having relatively high accuracy to computation time ratio. It is to address this problem that the presented modeling approach has been developed [1]–[3]. In this contribution, a general framework for the analysis and design of electromagnetic devices is presented. This framework is based on the hybrid analytical model (HAM) [1]–[3]. The HAM can be applied to different electric machine types (asynchronous [1] and synchronous [2]) and structures (rotating with radial field or axial field, linear flat or tubular structures, or multidegree-of-freedom structures). It is based on a direct coupling of the analytical solution of Maxwell's equations [analytical model (AM)] [4] with mesh-based generated reluctance networks (MBGRN) [5]. Ghoizad *et al.* [1] developed a hybrid model, where the analytical solution is based on magnetic vector potential, and the approach adopted in this contribution is based on the analytical solution of magnetic scalar potential. Pluk *et al.* [3] adopted the same approach, but magnetic saturation has been neglected. In [2], the presented work was limited to the study of open-circuit performance of a linear permanent magnet (PM) machine, and magnetic saturation was neglected. This new technique allows to combine the advantages of both methods and should help reduce computation time as compared with the finite-element method (FEM) [2], [3]. This modeling approach is relatively simple and allows the accurate prediction of the performances of electromagnetic devices.

In this contribution, this technique is used to develop a general framework for the analysis and design of PM machines. It will be shown throughout the study of some examples that it can be advantageously used in the predesign of electric machines.

Manuscript received March 20, 2015; revised May 8, 2015; accepted June 3, 2015. Date of publication June 15, 2015; date of current version October 22, 2015. Corresponding author: Y. Amara (e-mail: yacine.amara@univ-lehavre.fr).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TMAG.2015.2442214

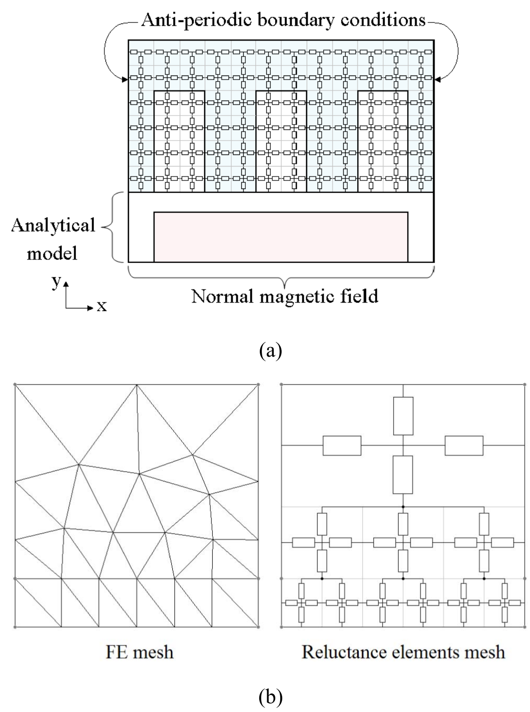

Fig. 1. HAM approach illustration. (a) Illustration of modeling approach. (b) Comparison of finite elements and reluctance elements meshes.

II. Hybrid Analytical Model

The HAM is based on a strong coupling between the magnetic equivalent circuit (MEC) method [5] and the AMs based on the formal solution of Maxwell's equations [4]. This coupling can help solve the problem of air-gap modeling in the MEC method, and the consideration of the local magnetic saturation in modeling approaches involving an analytical technique.

Fig. 1(a) shows how the two approaches are combined for the case of a flat linear structure. In this example, the analytical solution is used for modeling the magnetic air-gap region (PM and mechanical air-gap regions), and the RN method is used to model the stator part (slots and stator iron core).

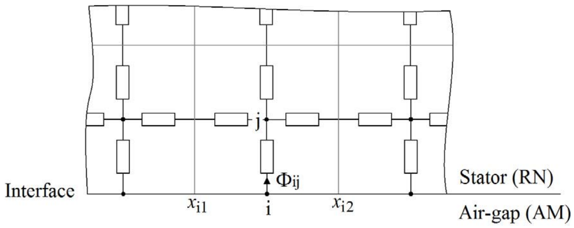

Fig. 2. Illustration of the direct coupling at models interface.

The AM solution is obtained using the separation of variables method along with the Fourier series expression of the magnetic field sources [2]. For this structure [Fig. 1(a)], the general solution of Maxwell's equations based on magnetic scalar potential formulation, in PM and mechanical air-gap regions, can be expressed for a region i by

$$\begin{aligned}U^{(i)}(x, y) &= a_0^{(i)} + a_1^{(i)}x + a_2^{(i)}y + a_3^{(i)}xy \\ &+ \sum_{n=1}^{+\infty}\left[\begin{array}{l}\left(\begin{array}{l}a_{4n}^{(i)}\sinh(n\pi y/\tau_p) \\ \quad + a_{5n}^{(i)}\cosh(n\pi y/\tau_p)\end{array}\right)\sin(n\pi x/\tau_p) \\ + \left(\begin{array}{l}a_{6n}^{(i)}\sinh(n\pi y/\tau_p) \\ \quad + a_{7n}^{(i)}\cosh(n\pi y/\tau_p)\end{array}\right)\cos(n\pi x/\tau_p)\end{array}\right]. \quad (1)\end{aligned}$$

Considering the geometric and electromagnetic periodicities of studied structure (end effects are not considered), it is easy to demonstrate that $a_1^{(i)} = a_2^{(i)} = 0$ and $a_3^{(i)} = 0$, for both the regions. $a_0^{(i)}$ is an arbitrary constant, which can be set to null.

The magnetic field $\mathbf{H}$ can be written in terms of the magnetic scalar potential U as

$$\mathbf{H} = -\nabla U. \quad (2)$$

In order to have a more generic approach, the stator is modeled using an MBGRN technique [5]. This technique, as a classical RN method, can be used with a minimum number of reluctances for regions, where flux tubes are not highly affected by topology changes whether it is due to motion or to geometric dimension variations. As for the FEM, the studied domain in MBGRN should be finely meshed in some regions (e.g., air gap) and coarsely meshed in other regions. However, in contrast to the FEM, the mesh relaxation in MBGRN can be done more easily conducting to reduced system matrix dimensions, and consequently reduced computation time. Indeed, while in the FEM two adjacent elements should share an edge, it is no more necessary for the RN method, as shown in Fig. 1(b).

Along with Fig. 1(a), Fig. 2 shows more precisely the coupling between both the approaches (AM and MEC) for an example formulated in the Cartesian coordinates (x and y). The coupling between both the models is obtained by equalizing magnetic scalar potential at $y = y_{\text{int}}$ (3), and by computing the incoming fluxes for the nodes at the interface using the y-component of air-gap flux density vector obtained from the analytical solution (4). By doing so, the interface conditions, which should be respected by electromagnetic field components between two domains are fulfilled (5).

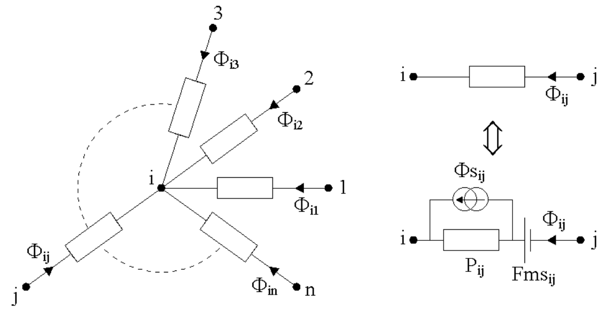

Fig. 3. Equation setting for the ith node.

Indeed, (3) and (4) are, respectively, equivalent to equalizing the tangential component of magnetic field $\mathbf{H}$, and the normal component of magnetic flux density $\mathbf{B}$ (5)

$$U^{\text{AM}}(x, y = y_{\text{int}}) = U^{\text{RN}}(x) \quad (3)$$

$$P_{ij}(U_i - U_j) = -\mu_0 l_a \int_{x_{i1}}^{x_{i2}} \left.\frac{\partial U^{\text{AM}}}{\partial y}\right|_{y=y_{\text{int}}} dx \quad (4)$$

where U^{AM} and U^{RN} refer to the scalar magnetic potentials obtained from the AM model (in the air-gap) and the Fourier series expression obtained from the discrete values of magnetic scalar potential of nodes located at the interface, respectively. y_{int} is the y-coordinate of the interface between both the models. l_a is the machine's active length. P_{ij}, U_i, and U_j are the permeance between the nodes i and j, and the magnetic scalar potential values at nodes i and j, respectively (Fig. 2)

$$\begin{cases} H_t(\text{AM}) = H_t(\text{RN}) \\ B_n(\text{AM}) = B_n(\text{RN}). \end{cases} \quad (5)$$

The nodal method is used for formulating the RN equations system. The unknowns for the generated circuit equations system are the magnetic scalar potentials at each node. Fig. 3 shows how the equations corresponding to nodes, which are not located at the interface between the air gap and the stator (Fig. 2), are determined from Kirchhoffs's laws.

According to Kirchoff's laws, it can be established that

$$\begin{cases} \displaystyle\sum_{\substack{j=1 \\ j\neq i}}^{n} \Phi_{ij} = 0Wb \\ U_i - U_j = Fms_{ij} - \dfrac{(\Phi_{ij} - \Phi s_{ij})}{P_{ij}} \end{cases} \quad (6)$$

and then

$$\left(\sum_{\substack{j=1 \\ j\neq i}}^{n} P_{ij}\right) U_i + \sum_{\substack{j=1 \\ j\neq i}}^{n}(-P_{ij})U_j = \sum_{\substack{j=1 \\ j\neq i}}^{n}(\Phi s_{ij} + P_{ij} Fms_{ij}). \quad (7)$$

Combining (3), (4), and the equations issued from the RN model (7), along with the equations issued from the boundary conditions at external limits of the modeled domain in the y-direction, yields to the algebraic equations system allowing to obtain the magnetic field solution. This model will

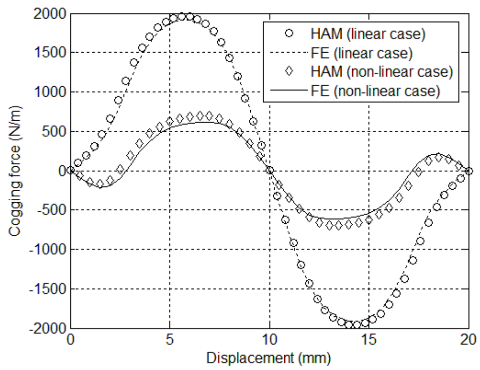

Fig. 4. Comparison of cogging force waveforms.

result on a set of equations, which can be expressed in matrix form

$$[A][X] = [E] \tag{8}$$

where $[A]$ is the topological matrix, where the elements are depending on the geometrical shape of the limits between the different regions of the machine and the magnetic properties of the different materials; $[E]$ is the source vector, elements of which are related to geometry distribution and physical properties of magnetic field sources (magnetic remanence and current density distributions); and $[X]$ is the unknowns vector, elements of which are the series coefficients of the analytical solution of scalar potential (AM) and unknowns issued from the MEC model. In case magnetic saturation is considered, the obtained set of nonlinear equations can be solved in an iterative way using classical methods: 1) fixed point iteration method and 2) Newton–Raphson method [6]. The consideration of magnetic saturation will only concern the equations issued from the reluctance network model in ferromagnetic regions.

Once the magnetic field solution is established, it is possible to have access to local quantities (magnetic field in all machines regions), and subsequently to global quantities (flux linkages, electromotive force, self-inductance and mutual inductance, forces and torques, and losses) [7]. Calculation of global quantities allows evaluating machines performance. Furthermore, global quantities can be used for the coupling of developed models with electric circuit equations. Details about the way the armature current MMF sources are considered in the MBGRN can be found in [8]. It can then be used to study the behavior of electric machines when connected to power converters and so for sizing and optimization purposes [9].

In Section III, different structures are studied using the HAM. Results issued from the HAM are validated extensively by comparison with the finite-element analyses (FEAs).

III. HAM Validation

In the following sections, the HAM is used to analyze the operation of different PM synchronous machine structures: 1) a planar liner PM machine; 2) a rotating radial flux PM synchronous machine; and 3) a rotating radial flux hybrid excited synchronous machine [10].

A. Planar Linear Machine

Details about the structure studied in this section can be found in [2]. Fig. 4 compares cogging force waveforms in linear and saturated cases. While the results presented in [2] were obtained in linear case ($\mu_r = 7500$), the results shown in Fig. 4 also concern nonlinear case. For the nonlinear case, Fig. 4 compares the results obtained from the FEA and those of the HAM. As can be seen good agreement is obtained.

For the HAM, the stator is meshed using a uniform mesh with 96 elements in the x-direction and 15 elements in the y-direction, and the number of considered harmonics in analytical solutions of the scalar magnetic potential is chosen equal to 70. The total number of unknowns is then equal to 1676 [$= (16 \times 96) + (2 \times 70)$].

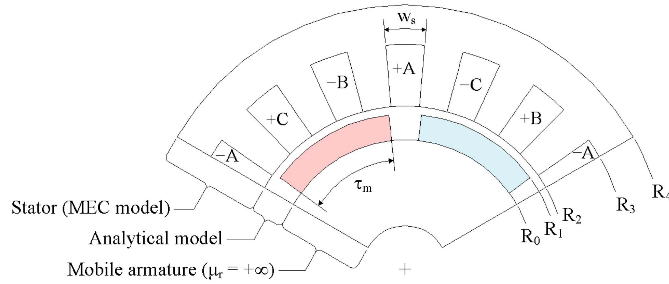

Fig. 5. Cross-sectional view of the studied machine.

TABLE I
MACHINE CHARACTERISTICS

PM magnetic remanence (T)	1.2
Magnetisation type	Radial magnetisation
Number of pole pairs	3
Pole pitch τ_p, τ_m, and w_s (rad)	$\pi/3$, $0.9\cdot\tau_p$, $\pi/18$
R_0, R_1, R_2, R_3 and R_4 (mm)	50, 60, 61, 81, 91

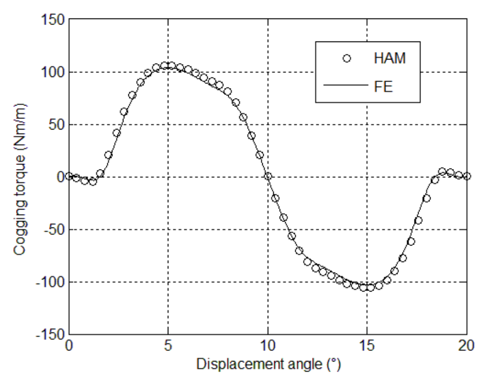

Fig. 6. Comparison of cogging torque waveforms.

B. Radial Flux PM Machine

Fig. 5 shows the rotating radial flux PM machine studied in this section. Table I lists the main machine's characteristics. As for the linear machine, the stator has been uniformly meshed with 120 elements in the circumferential direction and 60 in the radial direction, and the number of considered harmonics in analytical solutions of the scalar magnetic potential is chosen equal to 70. The total number of unknowns is then equal to 7460 [$= (61 \times 120) + (2 \times 70)$].

Fig. 6 compares cogging torque waveforms obtained from the FEA and the HAM in nonlinear case. In both the cases, the Maxwell stress tensor method is used. As can be seen, good agreement is again achieved.

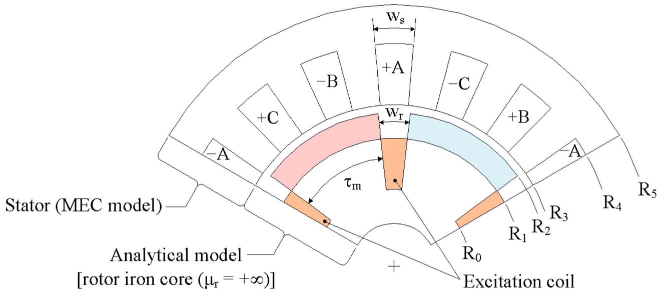

Fig. 7. Cross-sectional view of the studied hybrid excited machine.

TABLE II

MACHINE CHARACTERISTICS

PM magnetic remanence (T)	1
Magnetisation type	Radial magnetisation
Number of pole pairs	3
Stator core relative permeability μ_r	7500
Pole pitch τ_p, τ_m, w_s and w_r (rad)	$\pi/3$, $0.9\cdot\tau_p$, $\pi/18$, $\pi/15$
R_0, R_1, R_2, R_3, R_4 and R_5 (mm)	26.7, 50, 56, 57.5, 77.5, 87.5

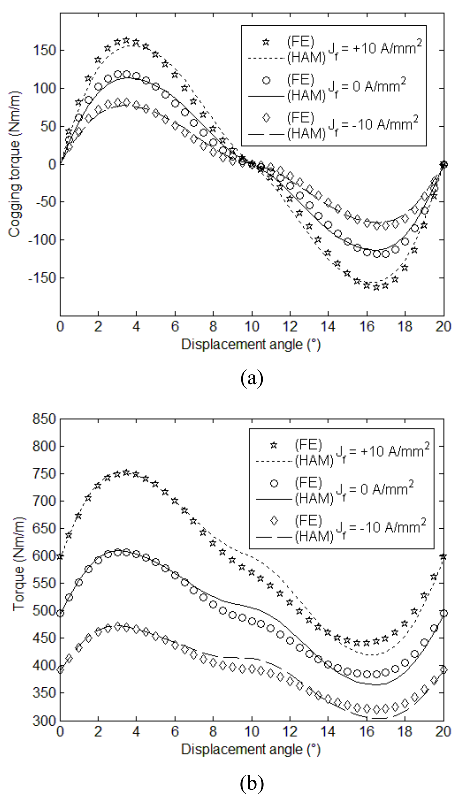

Fig. 8. Performance analysis of the hybrid excited machine. (a) Cogging torque waveforms. (b) On-load torque waveforms (maximum torque control, $J_{a\text{Max}} = 5$ A/mm^2).

Cogging force/torque (Figs. 4 and 6) has been chosen as a reference quantity for the comparisons because it is very sensitive to errors in the local quantities and constitutes, therefore, a good measure of the accuracy of the new HAM approach.

C. Radial Flux Hybrid Excited Machine

Fig. 7 shows the studied hybrid excited machine [10]. Table II lists its main characteristics. Fig. 8 compares different global quantities obtained from the FEA and the HAM. J_f and $J_{a\text{Max}}$ are the excitation current density and the maximum armature current density, respectively. Again, good agreement is obtained. The stator has been uniformly meshed with 120 elements in the circumferential direction and 60 elements in the radial direction, and the number of considered harmonics in analytical solutions of the scalar magnetic potential is chosen equal to 70. The total number of unknowns is then equal to 7400 [$= (61 \times 120) + (4 \times 70)$].

IV. CONCLUSION

A general framework for the analysis and design of electric machines has been exposed. The HAM has been used for analyzing different electric machine structures. Its use for the analysis of tubular linear machines and rotating axial flux machines will be soon reported. The results obtained from this new technique have been extensively validated by finite-element calculations. This technique should, therefore, be useful for comparative studies, design optimization, and dynamic modeling of a variety of electric machines.

ACKNOWLEDGMENT

This work was supported by Groupement de Recherche Systèmes d'Energie Electrique dans leur Dimension Sociétale through The French National Centre for Scientific Research, France.

REFERENCES

[1] H. Ghoizad, M. Mirsalim, M. Mirzaei, and W. Cheng, "Coupled magnetic equivalent circuits and the analytical solution in the air-gap of squirrel cage induction machines," *Int. J. Appl. Electromagn. Mech.*, vol. 25, nos. 1–4, pp. 749–754, 2007.

[2] Y. Laoubi, M. Dhifli, G. Verez, Y. Amara, and G. Barakat, "Open circuit performance analysis of a permanent magnet linear machine using a new hybrid analytical model," *IEEE Trans. Magn.*, vol. 51, no. 3, Mar. 2015, Art. ID 8102304.

[3] K. J. W. Pluk, J. W. Jansen, and E. A. Lomonova, "Hybrid analytical modeling: Fourier modeling combined with mesh-based magnetic equivalent circuits," *IEEE Trans. Magn.*, to be published.

[4] J. W. Jansen, J. P. C. Smeets, T. T. Overboom, J. M. M. Rovers, and E. A. Lomonova, "Overview of analytical models for the design of linear and planar motors," *IEEE Trans. Magn.*, vol. 50, no. 11, Nov. 2014, Art. ID 8206207.

[5] C. B. Rasmussen and E. Ritchie, "A magnetic equivalent circuit approach for predicting PM motor performance," in *Proc. 32nd IAS Annu. Meeting Conf. Rec. IEEE Ind. Appl.*, vol. 1. New Orleans, LA, USA, Oct. 1997, pp. 10–17.

[6] S. Ouagued, A. Diriyé, Y. Amara, and G. Barakat, "Consideration of magnetic saturation in a new hybrid semi-numerical model," in *Proc. COMPUMAG*, Montréal, QC, Canada, Jun./Jul. 2015, pp. 1–2.

[7] J. L. Coulomb, "A methodology for the determination of global electromechanical quantities from a finite element analysis and its application to the evaluation of magnetic forces, torques and stiffness," *IEEE Trans. Magn.*, vol. 19, no. 6, pp. 2514–2519, Nov. 1983.

[8] J. Perho, "Reluctance network for analysing induction machines," *Acta Polytech. Scand., Electr. Eng. Ser.*, vol. 110, pp. 1–147, Dec. 2002.

[9] M.-F. Hsieh, I.-H. Lin, and D. G. Dorrell, "An analytical method combining equivalent circuit and magnetic circuit for BDFRG," *IEEE Trans. Magn.*, vol. 50, no. 11, Nov. 2014, Art. ID 8206605.

[10] Y. Amara, L. Vido, M. Gabsi, E. Hoang, A. Hamid Ben Ahmed, and M. Lécrivain, "Hybrid excitation synchronous machines: Energy-efficient solution for vehicles propulsion," *IEEE Trans. Veh. Technol.*, vol. 58, no. 5, pp. 2137–2149, Jun. 2009.