MATH2009 Assignment 8, due on May 8

(1) In test $E[\epsilon_i] = 0$ with repeated measures, we have used the equation

$$\sum_{i=1}^{k}\sum_{j=1}^{n_i}(Y_{ij} - \hat{Y}_i)^2 = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(Y_{ij} - \bar{Y}_i)^2 + \sum_{i=1}^{k} n_i(\bar{Y}_i - \hat{Y}_i)^2.$$

Prove the equality by showing that

$$\sum_{i=1}^{k}\sum_{j=1}^{n_i}\{(Y_{ij} - \bar{Y}_i)(\bar{Y}_i - \hat{Y}_i)\} = 0.$$

(2) In test $E[\epsilon_i] = 0$ without repeated measures, we have used the equation

$$\sum_{i=1}^{k}\sum_{j=1}^{n_i}(Y_{ij} - \hat{Y}_{ij})^2 = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(Y_{ij} - \hat{Y}_{ij}(i))^2 + \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\hat{Y}_{ij}(i) - \hat{Y}_{ij})^2,$$

Prove the equality by showing that

$$\sum_{i=1}^{k}\sum_{j=1}^{n_i}\{(Y_{ij} - \hat{Y}_{ij}(i))(\hat{Y}_{ij}(i) - \hat{Y}_{ij})\} = 0.$$

[Hint: Use $\hat{Y}_{ij}(i) = \hat{a}_i + \hat{b}_i X_{ij}$ and $\hat{Y}_{ij} = \hat{a} + \hat{b}X_{ij}$.]

(3) Use the $n = 11$ earthquakes in below:

Magnitude	5.9	6.2	6.7	6.8	7.0	7.0	7.0	7.3	7.6	8.0	8.0
Energy	23.4	24.01	24.6	24.57	24.53	25.6	25.48	25.42	26.09	26.73	26.59

Using the test method with repeated measures.

a). Find least squares estimates $\hat{a}$ and $\hat{b}$, compute the value of SS_{res};

b). Construct a plot of residual versus the fitted values and a normal probability plot of the standardized residulals;

c). Compute SS_{PE} directly from the data and determine its degrees of freedom. Find SS_{LOF} by subtraction. Perform an F test for $E[\epsilon_i] = 0$.