Chapter 05

Association Rule Mining

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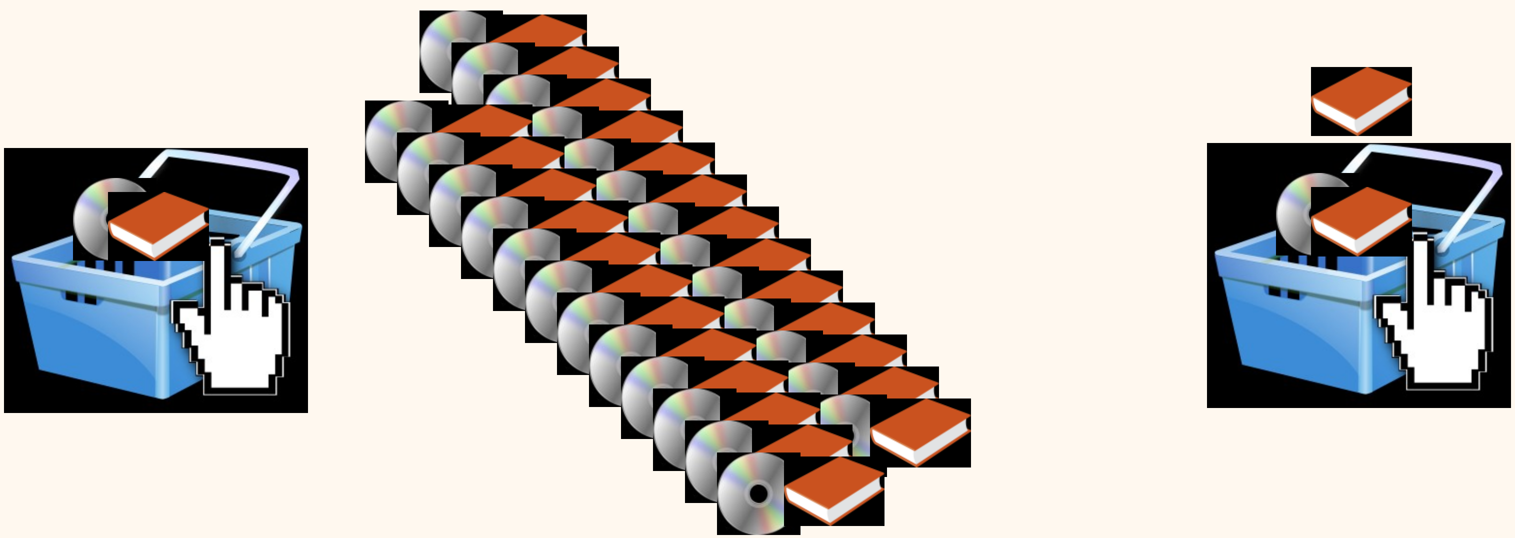
Outline

- Overview
- The Apriori Algorithm
- Summary

Example of Association Rules

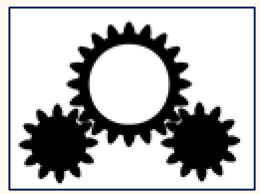
Items already in basket + Available items → Item likely to be added



The General Problem

Set of Transactions

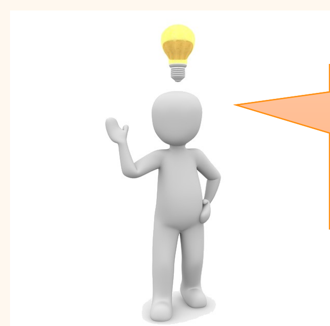
- item1,item2,item3
- item2,item4
- item1,item5
- item6,item7
- item2,item3,item4,item7
- item2,item3,item4,item8
- item2,item4,item5
- item2,item3,item4
- item4,item5
- …



Association Rule Mining

Association Rules

- item2 → item3
- item2 → item4
- item2,item3 → item4
- …



Rules describe „interesting relationships“

The Formal Problem

- Items
  - $I = \{i_1, i_2, \dots, i_m\}$
- Transactions
  - $T = \{t_1, \dots, t_n\}$ with $t_i \subseteq I$

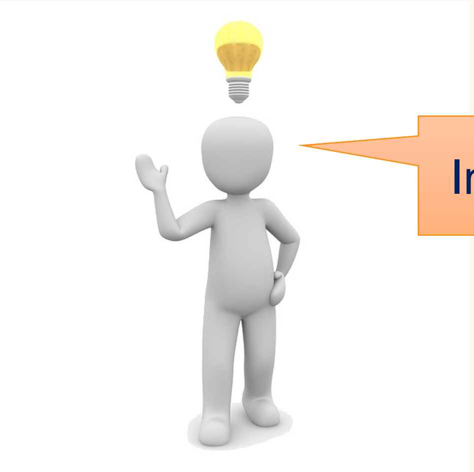
- Rules
  - $X \Rightarrow Y$ such that $X, Y \subseteq I$ and $X \cap Y = \emptyset$
  - X is also called antecedent or left-hand-side
  - Y is also called consequent or right-hand-side

Defining „Interesting Relationships“

Set of Transactions

- item1,item2,item3
- item2,item4
- item1,item5
- item6,item7
- item2,item3,item4,item7
- item2,item3,item4,item8
- item2,item4,item5
- item2,item3,item4
- item4,item5
- …

item2, item3, item4 occur often together



Interesting == often together

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- **The Apriori Algorithm**
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Support and Frequent Item Sets

- Support
  - Percentage of occurances of an itemset
  - $support(i) = \frac{|\{t \in T : i \subseteq t\}|}{|T|}$
- Frequent item set
  - Itemsets that appear together „often enough“
  - Defined using a threshold
  - $i \in I$ is frequent if $support(i) > minsupp$
- Rules can be generated by splitting itemsets
  - $i = X \cup Y$

Example for Generating Rules

$minsupp = 0.3$

- $\text{support}(\{item2, item3, item4\}) = \frac{3}{10} \geq 0.3$

Set of Transactions

- item1,item2,item3
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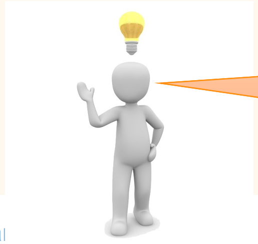
- item2,item3,item4,item8
- item2,item4,item5
- item2,item3,item4
- item4,item5
- item6,item7

- Eight possible rules:
  - $\emptyset \Rightarrow \{item2, item3, item4\}$
  - $\{item2\} \Rightarrow \{item3, item4\}$
  - $\{item3\} \Rightarrow \{item2, item4\}$
  - $\{item4\} \Rightarrow \{item2, item3\}$
  - $\{item2, item3\} \Rightarrow \{item4\}$
  - $\{item2, item4\} \Rightarrow \{item3\}$
  - $\{item3, item4\} \Rightarrow \{item2\}$
  - $\{item2, item3, item4\} \Rightarrow \emptyset$

Are all rules interesting?

Confidence, Lift, and Leverage

- Confidence
  - Percentage of transactions that contain the antecedent, which also contain consequent
  - $confidence(X \Rightarrow Y) = \frac{support(X \cup Y)}{support(X)} = \frac{|\{t \in T : X \cup Y \subseteq T\}|}{|\{t \in T : X \subseteq T\}|}$
- Lift
  - Ratio of the probability of X and Y together and independently
  - $lift(X \Rightarrow \mathrm{Y}) = \frac{support(X \cup Y)}{support(X) \cdot support(Y)}$
- Leverage
  - Difference in the probability of X and Y together and independently
  - $leverage(X \Rightarrow Y) = support(X \cup Y) - support(x) \cdot support(Y)$



Usually, lift favors itemsets with lower support, leverage with higher support

Confidence for the Example Rules

Set of Transactions

- item1,item2,item3
- item2,item4
- item1,item5
- item6,item7
- item2,item3,item4,item7
- item2,item3,item4,item8
- item2,item4,item5
- item2,item3,item4
- item4,item5
- item6,item7

- $confidence(\emptyset \Rightarrow \{item2, item3, item4\}) = \frac{0.3}{1} = 0.3$
- $confidence(\ \{item2\} \Rightarrow \{item3, item4\}) = \frac{0.3}{0.6} = 0.5$
- $confidence(\ \{item3\} \Rightarrow \{item2, item4\}) = \frac{0.3}{0.4} = 0.75$
- $confidence(\{item4\} \Rightarrow \{item2, item3\}) = \frac{0.3}{0.6} = 0.5$
- $confidence(\{item2, item3\} \Rightarrow \{item4\}) = \frac{0.3}{0.4} = 0.75$
- $confidence(\{item2, item4\} \Rightarrow \{item3\}) = \frac{0.3}{0.5} = 0.6$
- $confidence(\{item3, item4\} \Rightarrow \{item2\}) = \frac{0.3}{0.3} = 1$
- $confidence(\{item2, item3, item4\} \Rightarrow \emptyset) = \frac{0.3}{0.3} = 1$

Lift for the Example Rules

Set of Transactions

- item1,item2,item3
- item2,item4
- item1,item5
- item6,item7
- item2,item3,item4,item7
- item2,item3,item4,item8
- item2,item4,item5
- item2,item3,item4
- item4,item5
- item6,item7

- $lift(\emptyset \Rightarrow \{item2, item3, item4\}) = \frac{0.3}{1 \cdot 0.3} = 1$
- $lift(\{item2\} \Rightarrow \{item3, item4\}) = \frac{0.3}{0.6 \cdot 0.3} = 1.66$
- $lift(\{item3\} \Rightarrow \{item2, item4\}) = \frac{0.3}{0.4 \cdot 0.5} = 1.5$
- $lift(\{item4\} \Rightarrow \{item2, item3\}) = \frac{0.3}{0.6 \cdot 0.4} = 1.25$
- $lift(\{item2, item3\} \Rightarrow \{item4\}) = \frac{0.3}{0.4 \cdot 0.6} = 1.25$
- $lift(\{item2, item4\} \Rightarrow \{item3\}) = \frac{0.3}{0.5 \cdot 0.4} = 1.5$
- $lift(\{item3, item4\} \Rightarrow \{item2\}) = \frac{0.3}{0.3 \cdot 0.6} = 1.66$
- $lift(\{item2, item3, item4\} \Rightarrow \emptyset) = \frac{0.3}{0.3 \cdot 1} = 1$

Overview of Scores for Example

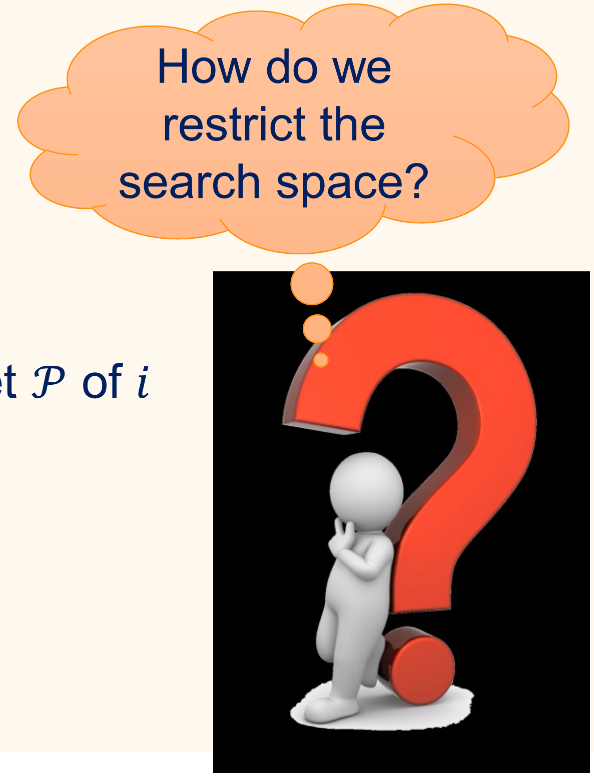
Rule	Confidence	Lift	Leverage
$\emptyset \Rightarrow \{item2, item3, item4\}$	0.30	1.00	0.00
$\{item2\} \Rightarrow \{item3, item4\}$	0.50	1.66	0.12
$\{item3\} \Rightarrow \{item2, item4\}$	0.75	1.50	0.10
$\{item4\} \Rightarrow \{item2, item3\}$	0.50	1.25	0.06
$\{item2, item3\} \Rightarrow \{item4\}$	0.75	1.25	0.06
$\{item2, item4\} \Rightarrow \{item3\}$	0.60	1.50	0.10
$\{item3, item4\} \Rightarrow \{item2\}$	1.00	1.66	0.12
$\{item2, item3, item4\} \Rightarrow \emptyset$	1.00	1.00	0.00

Perfect confidence, but no gain over randomness

Perfect confidence, and 1.66 times more likely than randomness

Itemsets and Rules = Exponential

- Number of itemset is exponential
  - All possible itemsets are the powerset $\mathcal{P}$ of I
    - $|\mathcal{P}(I)| = 2^{|I|}$
  - Still exponential if we restrict the size
    - $|I|$ itemsets with $k = 1$ items
    - $\frac{|I| \cdot (|I|-1)}{2}$ itemsets with $k = 2$ items
    - $\binom{|I|}{k} = \frac{|I|!}{(|I|-k)!k!}$ itemsets with k items
- Number of rules per itemset is exponential
  - Possible antecedents of itemset i are the powerset $\mathcal{P}$ of i
    - $|\mathcal{P}(i)| = 2^{|i|}$
- Example: $|I| = 100, k = 3$
  - 161,700 possible itemsets
  - 1,293,600 possible rules



Pruning the Search Space

- Apriori Property
  - **All subsets of a frequent itemset are also frequent**
  - $support(i') \geq support(i)$ for all $i' \subseteq i, i \in I$

- „Grow“ itemsets and prune search space by applying Apriori property
  - Start with itemsets of size $\mathrm{k} = 1$
  - Drop all itemsets that do not have minimal support
  - Build all combinations of size $\mathrm{k} + 1$
  - Repeat until
    - No itemsets with minimal support are found
    - A threshold for k is reached
- Still has exponential complexity, but better bounded

Example for Growing Itemsets (k=1)

$minsupp = 0.3$

Set of Transactions

- item1,item2,item3
- item2,item4
- item1,item5
- item6,item7
- item2,item3,item4,item7
- item2,item3,item4,item8
- item2,item4,item5
- item2,item3,item4
- item4,item5
- item6,item7

Rule	Support	
$\{item1\}$	0.2	Drop
$\{item2\}$	0.6	
$\{item3\}$	0.4	
$\{item4\}$	0.5	
$\{item5\}$	0.3	
$\{item6\}$	0.2	Drop
$\{item7\}$	0.3	
$\{item8\}$	0.1	Drop

Example for Growing Itemsets (k=2)

$minsupp = 0.3$

Set of Transactions

- item1,item2,item3
- item2,item4
- item1,item5
- item6,item7
- item2,item3,item4,item7
- item2,item3,item4,item8
- item2,item4,item5
- item2,item3,item4
- item4,item5
- item6,item7

Rule	Support	
$\{item2, \text{item3}\}$	0.4	
$\{item2, \text{item4}\}$	0.5	
$\{item2, \text{item5}\}$	0.1	**Drop**
$\{item2, \text{item7}\}$	0.1	**Drop**
$\{item3, \text{item4}\}$	0.3	
$\{item3, \text{item5}\}$	0.0	**Drop**
$\{item3, \text{item7}\}$	0.1	**Drop**
…	…	**Drop**

Example for Growing Itemsets (k=3)

$minsupp = 0.3$

Set of Transactions

- item1,item2,item3
- item2,item4
- item1,item5
- item6,item7
- item2,item3,item4,item7
- item2,item3,item4,item8
- item2,item4,item5
- item2,item3,item4
- item4,item5
- item6,item7

Rule	Support
$\{item2, \text{item3}, \text{item4}\}$	0.3

Only itemset remaining, growing terminates.

- Found the following frequent itemsets with at least two items:
  - $\{item2, item3\}, \{item2, item4\}, \{item3, item4\}$
  - $\{item2, item3, item4\}$

Candidates for Rules

- Usually, not all possible rules are considered
- Two common restrictions:
  - No empty antecedents and consequents
    - $X \neq \emptyset$ and $Y \neq \emptyset$
  - Only one item as consequent
    - $|Y| = 1$
- Example:

$$\cancel{\emptyset \Rightarrow \{item2, item3, item4\}}$$
$$\cancel{\{item2\} \Rightarrow \{item3, item4\}}$$
$$\cancel{\{item3\} \Rightarrow \{item2, item4\}}$$
$$\cancel{\{item4\} \Rightarrow \{item2, item3\}}$$
$$\{item2, item3\} \Rightarrow \{item4\}$$
$$\{item2, item4\} \Rightarrow \{item3\}$$
$$\{item3, item4\} \Rightarrow \{item2\}$$
$$\cancel{\{item2, item3, item4\} \Rightarrow \emptyset}$$

Evaluating Association Rules

- Use different criteria, not just support and confidence
  - Lift and leverage can tell you if rules are coincidental
- Validate if rules hold on test data
  - Check if the rules would also be found on the test data
- For basket prediction, use incomplete itemsets on test data
  - Example: remove item4 from all itemsets and see if the rules would correctly predict where it is associated
- Manually inspect rules
  - Do they make sense?
  - Ask domain experts

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Summary

- Associations are interesting relationships between items
- Interesting usually means appear together and not coincidental
- Number of possible itemsets/rules exponential
  - Apriori property for bounding itemsets
  - Restrictions on rule structures
- Test data and manual inspections for validation