Suppose we have 2 sets of points $X, Y \in \mathbb{R}^{n \times d}$ with rows $X_i, Y_i \in S^{d-1}$, centered (in terms of spherical coordinates) at $m_x, m_y \in S^{d-1}$. Let $b_x \in \mathbb{R}^{d \times d-1}$ (resp. b_y) be (the first d-1 basis vectors of) the PCA basis for $Exp_{m_x}^{-1}(X_i) - m_x$ (resp. $(Exp_{m_y}^{-1}(Y_i) - m_y)$. Define $\tilde{X}_i = (Exp_{m_x}^{-1}(X_i) - m_x) * b_x$ and $\tilde{Y}_i = (Exp_{m_y}^{-1}(Y_i) - m_y) * b_y$.

Let $\alpha(t)$ denote the geodesic path along S^{d-1} from m_x to m_y. Let $b_x^{||} + m_y$ denote the resulting vectors after parallel transport of $b_x + m_x$ along α. Then we can calculate the sample covariance of the vectors in X, after parallel transport along α, in terms of b_y, as ;

$$\Sigma_{\tilde{X}}^{||} = (b_y^T b_x^{||}) \tilde{X}^T \tilde{X} ((b_x^{||})^T b_y) \tag{1}$$