

Suppose we have 2 sets of points $X, Y \in \mathbb{R}^{n \times d}$ with rows $X_i, Y_i \in S^{d-1}$, centered (in terms of spherical coordinates) at $m_x, m_y \in S^{d-1}$. Let $b_x \in \mathbb{R}^{d \times d-1}$ (resp. b_y) be (the first d-1 basis vectors of) the PCA basis for \tilde{X} (resp. \tilde{Y}). Define $\tilde{X}_i = (\text{Exp}_{m_x}^{-1}(X_i) - m_x) * b_x$ and $\tilde{Y}_i = (\text{Exp}_{m_y}^{-1}(Y_i) - m_y) * b_y$.

Let $\alpha(t)$ denote the geodesic path along S^{d-1} from m_x to m_y . Let $b_x^{\parallel} + m_y$ denote the resulting vectors after parallel transport of $b_x + m_x$ along α . Then we can calculate the sample covariance of the vectors in X , after parallel transport along α , in terms of b_y , as ;

$$\Sigma_{\tilde{X}}^{\parallel} = (b_y^T b_x^{\parallel}) \tilde{X}^T \tilde{X} ((b_x^{\parallel})^T b_y) \quad (1)$$