Suppose we have 2 sets of points $X, Y \in \mathbb{R}^{n\times d}$ with rows $X_i, Y_i \in S^{d-1}$, centered (in terms of spherical coordinates) at $m_x, m_y \in S^{d-1}$. Define $\tilde{X}_i = Exp^{-1}_{m_x}(X_i) - m_x$ and $\tilde{Y}_i = Exp^{-1}_{m_y}(Y_i) - m_y$. Let $b_x \in \mathbb{R}^{d\times d-1}$ (resp. b_y) be (the first d-1 basis vectors of) the PCA basis for $\tilde{X}$ (resp. $\tilde{Y}$).

Let $\alpha(t)$ denote the geodesic path along S^{d-1} from m_x to m_y. Let $b_x^{||} + m_y$ denote the resulting vectors after parallel transport of $b_x + m_x$ along α. Then we can calculate the sample covariance of the vectors in X, after parallel transport along α, in terms of b_y, as ;

$$\Sigma^{||}_{\tilde{X}} = (b_y^T b_x^{||})\tilde{X}^T\tilde{X}((b_x^{||})^T b_y) \tag{1}$$