CSE 6140 / CX 4140

Computational Science & Engineering Algorithms

Homework 4

1.
Diverse Subset is in NP:
Given the subset array, examine each column to see if there is only one non-zero value present. O(MN) where M is the number of customers and N is the number of products.
Independent Set $\leq_p$ Diverse Subset:
An arbitrary graph G can be represented as an adjacency matrix. The adjacency matrix can be regarded as the 2-D array A in the Diverse Subset. To find an independent set is to identify rows of the array such that no two rows have a non-zero value in the same column, which can also be viewed as a Diverse Subset problem. Since Independent Set is NP-Complete, Diverse Subset is NP-Complete.

2.
Efficient Recruiting is in NP:
Given the subset, examine each sport to see if there is at least one counselor qualified. O(MN) where M is the number of counselors and N is the number of sports.
Vertex Cover $\leq_p$ Efficient Recruiting:
An arbitrary graph G can be represented as an adjacency matrix. The adjacency matrix can correspond to Efficient Recruiting by viewing rows as counselor applicants and non-zero columns as different sports they skill at. If we can find a vertex cover S of size at most k such that every edge has at least one end in S for the graph, we can find a set of applicants of size at most k such that every sport has at least one counselor in S qualified for the corresponding recruiting problem.
Since Vertex Cover is NP-Complete, Efficient Recruiting is NP-Complete.

3.
Truck Loading is in NP:
Given the containers and trucks, for each truck, simply go through all containers on it to see if there are two containers are placed in the same truck when they are not supposed to be. O(N) where N is the total number of containers.
3-Coloring $\leq_p$ Truck Loading:
Given an arbitrary undirected graph G. For a coloring problem, if there is an edge between two vertices, they can't be assigned the same color. Regarding the vertices as containers and the colors as trucks, if we can find a way to color the vertices with 3 colors, we can load the containers into k(k$\geq$3) trucks. Since 3-Coloring is NP-Complete, Truck Loading is NP-Complete.

4.
Perfect Assembly is NP:
Given the ordered S and set T, for each string s_i and its latter s_{i+1}, simply go through the set T and find if there is a string t_k corroborates the pair (s_i, s_{i+1}).

O(NK) where N is the number of strings in S and K is the number of strings in T.
Hamiltonian Path $\leq_p$ Perfect Assembly:
Given an arbitrary directed graph G. Each vertex can be viewed as a string in S, and each edge can be viewed as a string in T. If there is an edge between two vertices, there is a t_k in T corroborate the string pair. To find a Hamiltonian path for this graph is the same as to find a perfect assembly for the corresponding string set.
Since Hamiltonian Path is NP-Complete, Perfect Assembly is NP-Complete.