

University of Waterloo

CS240 - Fall 2016

Assignment 1 Part 2

Due Date: Wednesday September 28 at 5:00pm

Please read <http://www.student.cs.uwaterloo.ca/~cs240/f16/guidelines.pdf> for guidelines on submission. Problems 5 – 8(a) are written problems; submit your solutions electronically as a PDF with file name `a01p2wp.pdf` using MarkUs. We will also accept individual question files named `a01q5w.pdf`, `a01q6w.pdf`, ... , `a01q8w.pdf` if you wish to submit questions as you complete them. Submit your solution to 8(b) electronically as a file named `report.cpp`.

Note: Assignment 1 has been split into 2 parts both worth 2.5% each. Keep in mind that part 1 is due before part 2. There are 39 marks for part 1 and 34 marks for part 2.

Problem 5 [2+2+4+4=12 marks]

Consider the following procedure.

```
pre: n is a positive integer
pre: v[1..n] is a binary vector of length n,
      i.e., each entry is either 0 or 1
foo(v,n)
1.  i := 1;
2.  while i<=n and v[i]=0 do
3      i := i+1
4  od;
5.  for j from 1 to i do
6      print("Hello world!")
7.  od;
```

- a) How many inputs are there are of size n ?

Solution: There are totally 2^n inputs of size n .

- b) What is the worst case number of calls to print? Give an exact formula in terms of n and justify your answer by giving an example of a worst case input of size n .

Solution: The worst case happens when all entries in v are 0, which takes $n+1$ iterations.

- c) For $i \in \{1, 2, \dots, n\}$, let S_i denote the subset of inputs of size n for which the number of calls to print is i . Describe and enumerate S_i .

Solution: Before enumerating S_i , I will give an simple example. Eg: $v = [0, 0, 0, 0, 1, x, x, x, x]$, where v has size 9 and the first loop will terminate when i goes to

the fifth entry in the array, which is 1. Thus, $i = 5$ after the first loop. Therefore, the print is called 5 times totally. Because the first five entries is unchanged, the last four entries can be enumerated. Thus, the enumeration of S_i is $2^{9-5} = 2^4 = 16$. Conclusively, the enumeration of $S_i = 2^{n-i}$.

- d) What is the average case number of calls to print? Derive an exact closed form formula in terms of n .

Solution:

$$\begin{aligned}
 T_A^{avg}(n) &= \frac{1}{|I: size(I)=n|} \sum_{I: size(I)=n} T_A(I) \\
 &= \frac{1}{2^n} \sum_{i=1}^n i(2^{n-i}) \\
 &= \sum_{i=1}^n \frac{i}{2^i} \\
 &= 2 - (n+2)/2^n \text{ from A1P4}
 \end{aligned}$$

Problem 6 [5 marks]

Prove that the following code fragment will always terminate.

```

s := 3*n // n is an integer
while (s>0)
    if (s is even)
        s := floor(s/4)
    else
        s := 2*s

```

Solution:

If s is even, then it takes one operation to get one-fourth of its value. Because there is a $s > 0$ condition, if n is always a multiple of four, program will terminate in $\frac{1}{2} \log n + 4$ operations, where 4 means when n becomes 1, there are still four operations left, which is $s = 3 * 2 = 6 \rightarrow s = \frac{6}{4} = 1 \rightarrow s = 1 * 2 = 2 \rightarrow s = \frac{2}{4} = 0$, . Otherwise, if s is always odd, which is the worst case, there is one more operation in each iteration. Thus, it will take $\text{floor}(\log 3n)$ operations to complete the program. So, the program will terminate in both the best case and the worst case.

Problem 7 [5 marks]

Analyze the following piece of pseudo-code and give a tight bound (i.e. Θ notation) on the running time as a function of n . Show your work. A formal proof is not required, but you should justify your answer.

```

1.  mystery  $\leftarrow$  0
2.  for  $i \leftarrow 1$  to  $3n$  do
3.      mystery  $\leftarrow$  mystery  $\times$  4
4.      for  $j \leftarrow 1388$  to 2010 do
5.          for  $k \leftarrow 4i$  to  $6i$  do
6.              mystery  $\leftarrow$  mystery +  $k$ 

```

Solution:

$$\begin{aligned}
 & \Theta(\sum_{i=1}^{3n} \sum_{j=1388}^{2010} 2i) \\
 &= \Theta(\sum_{i=1}^{3n} (2010 - 1388 + 1) * 2i) \\
 &= \Theta(\sum_{i=1}^{3n} 1246i) \\
 &= \Theta(1246 * \frac{(3n+1)*3n}{2}) \\
 &= \Theta(n^2)
 \end{aligned}$$

Problem 8 [6+6=12 marks]

You are given an array $A[0 \dots n-1]$ of integers (not necessarily distinct) that forms a max-heap of size n .

- a) Describe an algorithm that takes as input an integer c , not necessarily in the heap, and reports all integers in the heap that are greater than or equal to c . The running time of your algorithm should be $O(k)$, where k is the number of integers reported. Provide a brief explanation for why the running time of your algorithm is $O(k)$.

Solution:

I will use the property of max-heap, and use recursion to find all nodes greater than integer c . If there is a function $findgreater(maxheap, c)$, the root needs to be checked first. If $root < c$, then return. Otherwise, prints the root, and do $findgreater(maxheap \rightarrow left, c)$ and $findgreater(maxheap \rightarrow right, c)$ at the same time. When either parts reach a node smaller than c , then recursion stops. Otherwise, prints all nodes greater than or equal to c .

- b) Implement your algorithm from part (a). Your program should read from `cin` the size n , then the n integers in the heap $A[0 \dots n-1]$, and finally the integer c , and then write to `cout` the integers in the heap that are greater than or equal to c . You may assume that every integer in the input is at least 0 and at most $2^{31} - 1$ (so every integer will fit into a variable of type `int`).

Every integer in the input and output should be on a separate line. So for instance if the input consists of the following lines:

```
5
17
15
13
10
3
12
```

then your program should print out the integers 17, 15, and 13 in any order (and on separate lines).

Submit the code for your `main` function, along with any helper functions, in a file called `report.cpp`.