

Course 1 - Week 3/4 - Probability and Distribution

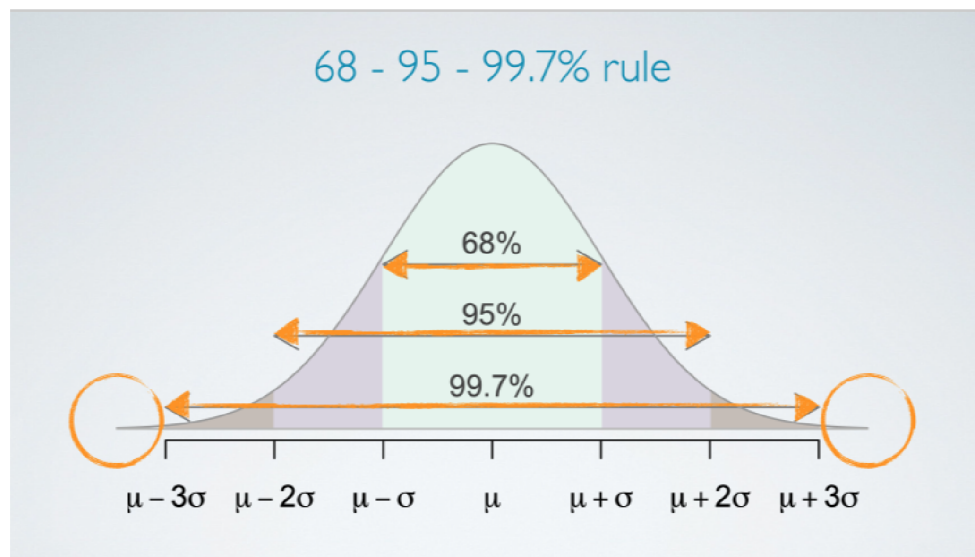
Probability

Some basics such as conditional/marginal/joint probability, product rule, add rule will not be included.

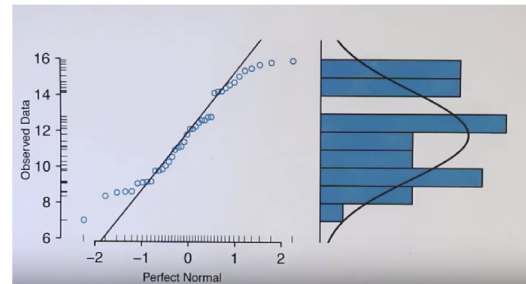
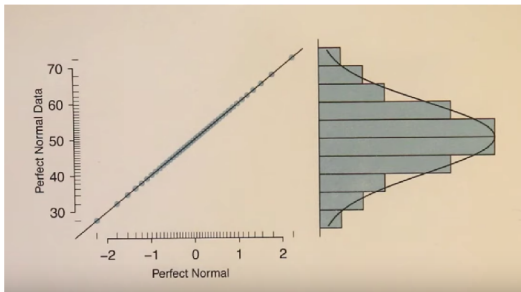
1. What's the difference between disjoint events and complementary events?
 - a. Disjoint events means any two of these events cannot happen at the same time
 - b. The sum of the probabilities of two complementary events must equal one
 - c. The sum of the probabilities of two disjoint events may be necessarily to be one
2. Steps of Bayesian Inference
 - a. Procedure
 - i. Set up prior probability
 - ii. Construct a probability tree listing all possible outcomes in each leaf
 - iii. Perform one experiment and see which leaf did the outcome fall into
 - iv. Compute posterior probability; $P(\text{outcome} \mid \text{experiment}) = P(\text{hypothesis} \mid \text{data})$
 - v. Update prior = posterior
 - vi. Repeat from b
 - b. There are two probabilities involved here: first is the one you ultimately want to calculate, another is a probability that happens after the experiment is performed
3. Re-do the Bayesian Inference practice problem!

Distributions

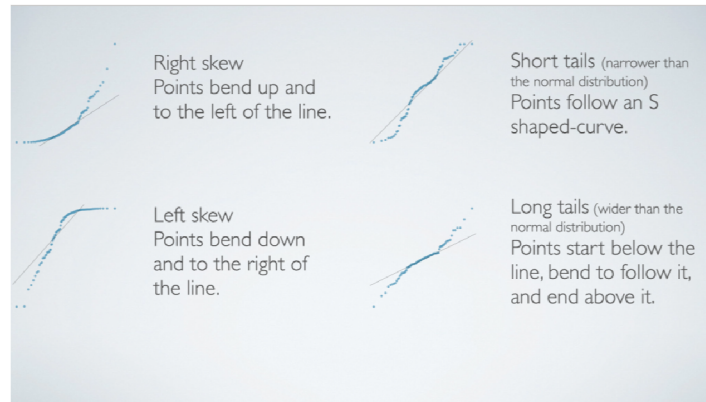
4. What is 68 - 95 - 99.7% rule?



5. What is standardized Z score and why do we use it?
 - a. Z score is defined as (data point - mean) / standard deviation; simply speaking, how many SD's are there between the observed data and the mean of the distribution.
 - b. It is defined for distribution of **any** shape (not just normal distribution)
 - c. Unusual observation: $|Z| > 2$
 - d. Motivation: To compare two data points from scales (e.g., compare student's exam performance on GRE and SAT)
6. What is percentile in normal distribution and how do we use Z score to calculate percentile or vice visa?
 - a. Define as the percentage of observations that fall below a data point (area of curve before a point)
 - b. Use [standard normal table](#) to do the mutual conversion
7. How to evaluate whether a distribution is normal distribution?
 - a. Normal probability plot: <https://www.youtube.com/watch?v=smJBsZ4YQZw>
 - b. Procedure



- i. Set up number of samples you want to work on (say 40 points)
- ii. Set up x-axis as the “theoretical” normal distribution quantiles, which is a perfect normal distribution. Generate 40 points on the x-axis that follow a perfect normal distribution.
- iii. Set up y-axis as the real data distribution. Plot the 40 data points (different from the prior 40 points) on the y-axis.
- iv. Map each point on the x-axis to each point on the y-axis **in order**



8. Conditions for binomial distribution

- Each trial must be independent from each other
- Each trial must have a binary outcome
- The probability of success must be the same for every trial
- Number of trials n must be fixed

9. What's the expectation and standard deviation of a binomial distribution?

- Expectation: $\mu = np$
- Standard deviation: $\sigma = \sqrt{np(1-p)}$

10. What's the relationship between binomial distribution and normal distribution?

- First plot success probability versus trials barplot (calculate the probability of success)
- Increase number of trials and then we find binomial distribution approaches normal distribution
- Success-failure Rule: $np \geq 10$ and $n(1-p) \geq 10$, which means both the expectation of success and expectation of failure must be greater than or equal to 10

