Вектори - обобщение

1 зад. $\vec{a}, \vec{b}$: $|\vec{a}|=2,\ |\vec{b}|=1,\ \sphericalangle(\vec{a},\vec{b})=\frac{2\pi}{3}$ $\Rightarrow \cos\frac{2\pi}{3}=-\frac{1}{2}$

$\vec{OA}=(\underset{1}{\vec{a}}\times\underset{2}{\vec{b}})\times\underset{3}{\vec{a}}$, $\vec{OB}=\vec{b}\times(\vec{a}\times\vec{b})$ $\qquad \vec{a}^2=4,\ \vec{b}^2=1$

$\vec{OA}=(\underset{1}{\vec{a}}.\underset{3}{\vec{a}}).\underset{2}{\vec{b}}-(\underset{2}{\vec{b}}.\underset{3}{\vec{a}}).\underset{1}{\vec{a}}=\boxed{4.\vec{b}+\vec{a}}$ ✓ $\qquad (\vec{a}.\vec{b})=2.1.(-\frac{1}{2})=-1$

$\vec{OB}=\underset{1}{\vec{b}}\times(\underset{2}{\vec{a}}\times\underset{3}{\vec{b}})=(\underset{1}{\vec{b}}.\underset{3}{\vec{b}}).\underset{2}{\vec{a}}-(\underset{1}{\vec{b}}.\underset{2}{\vec{a}}).\underset{3}{\vec{b}}=1.\vec{a}+1.\vec{b}$

$\boxed{\vec{OB}=\vec{a}+\vec{b}}$ ✓ $\qquad \vec{AB}=-3\vec{b}$ ✓

а) $P_{\triangle OAB}=?$ $\quad S_{\triangle OAB}=?$

$P=|\vec{OA}|+|\vec{OB}|+|\vec{AB}|$ $\qquad |\vec{OA}|^2=(4\vec{b}+\vec{a})^2=16.\vec{b}^2+8(\vec{a}.\vec{b})+\vec{a}^2=$
$=16+8(-1)+4=12$

$|\vec{OA}|=2\sqrt{3}$

$|\vec{OB}|^2=(\vec{a}+\vec{b})^2=\vec{a}^2+2.(\vec{a}\vec{b})+\vec{b}^2=4+2.(-1)+1=3$

$|\vec{OB}|=\sqrt{3}$

$\vec{AB}=\vec{OB}-\vec{OA}=-3.\vec{b}\Rightarrow |\vec{AB}|=3.|\vec{b}|=3$

$P_{\triangle ABO}=3.\sqrt{3}+3$

! $S_{\triangle OAB}=\frac{|\vec{OB}\times\vec{AB}|}{2}$ $\qquad \vec{OB}\times\vec{AB}=(\vec{a}+\vec{b})\times(-3\vec{b})=-3.[(\vec{a}\times\vec{b})+\underbrace{(\vec{b}\times\vec{b})}_{=\vec{0}}]$

$S_{\triangle OAB}=\frac{3\sqrt{3}}{2}$ кв.ед $\qquad \vec{OB}\times\vec{AB}=-3.(\vec{a}\times\vec{b})$

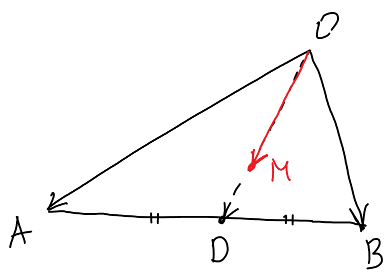
$|\vec{OB}\times\vec{AB}|=3.|\vec{a}\times\vec{b}|=3.\ 2.1.\frac{\sqrt{3}}{2}=3\sqrt{3}$

$\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$

$\boxed{a\times b=2.1.\frac{\sqrt{3}}{2}}$ НЕ

б) Нека т. M е медицентърът на $\triangle OAB$

$\vec{OM}=?$, $|\vec{OM}|=?$



$\vec{OM}=\frac{1}{3}.(\vec{OA}+\vec{OB}+\vec{OO})=\frac{1}{3}.(\vec{OA}+\vec{OB})$

$\vec{OM}=\frac{2}{3}.\vec{OD}=\frac{2}{3}.\frac{1}{2}.(\vec{OA}+\vec{OB})=\frac{1}{3}.(\vec{OA}+\vec{OB})$

$\vec{OM}=\frac{1}{3}.(4\vec{a}+\vec{b}+\vec{a}+\vec{b})=\frac{1}{3}.(5\vec{a}+2\vec{b})$

Упр. $|\vec{OM}| = ?$

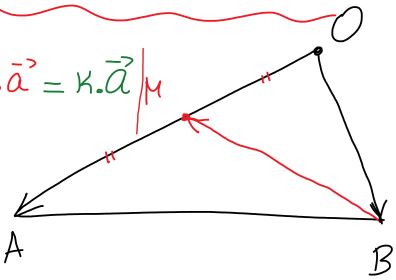
2 зад. $\vec{a}, \vec{b}$: $|\vec{a}| = |\vec{b}| = 1$, $\measuredangle(\vec{a}, \vec{b}) = \varphi$, $\varphi \in (0; \pi)$

$\vec{OA} = \vec{b}$ $\quad \vec{OB} = (\vec{a} \times \vec{b}) \times (\vec{a} + \vec{b})$ $\quad \vec{a}^2 = \vec{b}^2 = 1$, $(\vec{a}.\vec{b}) = \cos\varphi$

а) $\varphi = ?$: медианата $\vec{BM}$ в $\triangle OAB$ да е колинеарна на $\vec{a}$

$$\vec{OB} = (\vec{a} \times \vec{b}) \times \vec{a} + (\vec{a} \times \vec{b}) \times \vec{b} = (\vec{a}^2).\vec{b} - (\vec{b}.\vec{a}).\vec{a} + (\vec{a}.\vec{b}).\vec{b} - \vec{b}^2.\vec{a} =$$

$$= 1.\vec{b} - \cos\varphi.\vec{a} + \cos\varphi.\vec{b} - 1.\vec{a} = (1 + \cos\varphi).\vec{b} - (1 + \cos\varphi).\vec{a}$$



$$\vec{BM} = \vec{OM} - \vec{OB} = \frac{\vec{b}}{2} - (1 + \cos\varphi).\vec{b} + (1 + \cos\varphi).\vec{a} = k.\vec{a}$$

$\vec{a}, \vec{b}$ – ЛНЗ

$$\vec{b}.\left(\frac{1}{2} - 1 - \cos\varphi\right) + (1 + \cos\varphi).\vec{a} = k.\vec{a} + 0.\vec{b}$$

$$\frac{1}{2} - 1 - \cos\varphi = 0 \Rightarrow \cos\varphi = -\frac{1}{2} \Rightarrow \varphi = \frac{2\pi}{3}$$

б) Нека $\varphi = \frac{\pi}{3}$ $\quad \vec{OC} = [(\vec{a} \times \vec{b}) \times \vec{a}] \times [(\vec{a} \times \vec{b}) \times \vec{b}]$

$V_{OABC} = ?$ $\quad \vec{OA} = \vec{b}$

$\vec{OB} = (1 + \cos\varphi).(\vec{b} - \vec{a}) = \frac{3}{2}.(\vec{b} - \vec{a})$

$$\vec{OC} = [\underbrace{\vec{a}^2}_{1}.\vec{b} - \underbrace{(\vec{b}.\vec{a})}_{1/2}.\vec{a}] \times [\underbrace{(\vec{a}\vec{b})}_{\frac{1}{2}}.\vec{b} - \underbrace{\vec{b}^2}_{1}.\vec{a}] =$$

$$= \left(\vec{b} - \frac{\vec{a}}{2}\right) \times \left(\frac{\vec{b}}{2} - \vec{a}\right) = -\vec{b} \times \vec{a} - \frac{\vec{a} \times \vec{b}}{4} = \vec{a} \times \vec{b} - \frac{\vec{a} \times \vec{b}}{4} = \boxed{\frac{3}{4}.(\vec{a} \times \vec{b})}$$

$$V_{OABC} = \frac{1}{6}.|(\vec{OA}\ \vec{OB}\ \vec{OC})|$$

$$\vec{OA} \times \vec{OB} = \vec{b} \times \left[\frac{3}{2}.(\vec{b} - \vec{a})\right] = \frac{3}{2}.(-\vec{b} \times \vec{a}) = \frac{3}{2}(\vec{a} \times \vec{b}) \,/.\, \vec{OC} = \frac{3}{4}(\vec{a} \times \vec{b})$$

$$V = \frac{1}{6}.\frac{9}{8}.(\vec{a} \times \vec{b})^2 = \frac{3}{16}\left[\vec{a}^2.\vec{b}^2 - (\vec{a}\vec{b})^2\right] = \frac{3}{16}.\left(1.1 - \frac{1}{4}\right) =$$

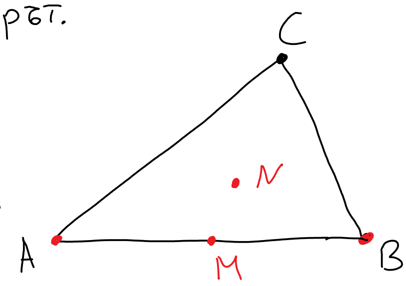
$$= \frac{3}{16}.\frac{3}{4} = \frac{9}{64}$$

3 зад. ОКС $K = Oxyz$

$A(3,4,-2)$, $M(0,2,1)$, $N(4,2,3)$

а) ?, коорд. на върховете B и C на $\triangle ABC$, за който M е средата на AB, а N е медицентърът.

Решение:



1) M е средата на AB

$$x_M = \frac{x_A + x_B}{2} \qquad y_M = \frac{y_A + y_B}{2} \qquad z_M = \frac{z_A + z_B}{2}$$

$$0 = \frac{3 + x_B}{2} \qquad 2 = \frac{4 + y_B}{2} \qquad 1 = \frac{-2 + z_B}{2}$$

$B(-3, 0, 4)$

2) ? т. $C(x_C, y_C, z_C)$

$$\vec{ON} = \frac{1}{3}\cdot(\vec{OA} + \vec{OB} + \vec{OC})$$

$$x_N = \frac{x_A + x_B + x_C}{3}, \quad y_N = \frac{y_A + y_B + y_C}{3}, \quad z_N = \frac{z_A + z_B + z_C}{3}$$

$$4 = \frac{3 + (-3) + x_C}{3} \qquad 2 = \frac{4 + 0 + y_C}{3} \qquad 3 = \frac{-2 + 4 + z_C}{3}$$

$C(12, 2, 7)$

б) $P_{\triangle ABC} = ?$ Да се определи вида на $\triangle ABC$ спрямо ъглите му.

в) $S_{\triangle ABC} = ?$ $\quad S_{\triangle ABC} = \dfrac{|\vec{AB} \times \vec{AC}|}{2}$

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4 зад. ОКС $K = Oxyz$

$A(6,0,1)$ $B(-1,3,2)$, $C(5,1,3)$ $D(6,1,3)$

а) Определете взаимното положение на правите AB и CD;

б) Ако същ. тетраедър $ABCD$, $V_{ABCD} = ?$

Ако $ABCD$ е четириъгълник, $S_{ABCD} = ?$

Решение:

а) Взаимни положения на 2 прави в пространството:

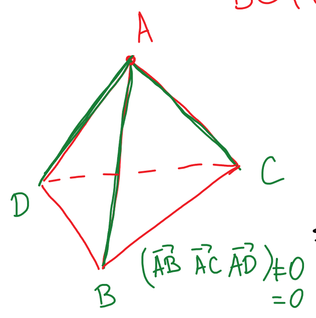
1) $AB \equiv CD$; 2) $AB \parallel CD$; 3) $AB \cap CD = $ т. S ; 4) AB и CD да са кръстосани

Не / Не

I. Разгл. $\vec{AB}(-7,3,1)$, $\vec{CD}(1,0,0)$ $\Rightarrow$ $\vec{AB}$ и $\vec{CD}$ са ЛНЗ $\{\vec{AB} \times \vec{CD} \neq \vec{0}\}$ $\Rightarrow$ $AB \not\equiv CD$, $AB \nparallel CD$

$\vec{BC}(6,-2,1)$

$\vec{BC}(6,-2,1)$



$$(\vec{AB}\ \vec{CD}\ \vec{BC}) = \begin{vmatrix} -7 & 3 & 1 \\ 1 & 0 & 0 \\ 6 & -2 & 1 \end{vmatrix} \begin{matrix} -7 & 3 \\ 1 & 0 \\ 6 & -2 \end{matrix} = 0+0+(-2) - 3 = -5 \neq 0 \Rightarrow$$

смесено произв.

$\Rightarrow \vec{AB}\ \vec{CD}\ \vec{BC}$ са ЛНЗ $\Rightarrow A, B, C$ и D са върхове на тетраедър

AB и CD са кръстосани

$$V_{ABCD} = \frac{|(\vec{AB}\ \vec{CD}\ \vec{BC})|}{6} = \frac{5}{6} \qquad |(\vec{AB}\ \vec{AC}\ \vec{AD})| = 5\ !$$

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4 зад. В успоредника $ABCD$ точките M и N са средите съответно на страните BC и CD. Точката P е такава, че $AMPN$ е успоредник. Докажете, че точката P принадлежи на правата AC.

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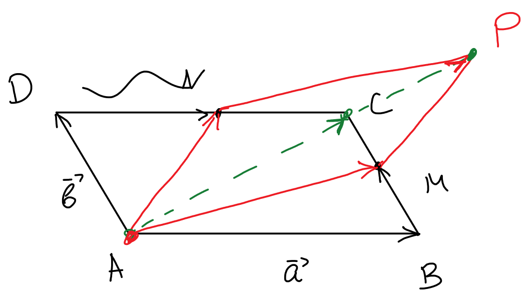
1. P : $AMPN$ – успоредник

$\vec{AC}$ и $\vec{AP}$

Нека $\vec{AB} = \vec{a}$, $\vec{AD} = \vec{b}$

$\vec{AC} = \vec{a} + \vec{b}$

$\vec{AP} = \vec{AM} + \vec{AN}$



$\vec{AM} = \vec{a} + \frac{\vec{b}}{2}$, $\vec{AN} = \vec{b} + \frac{\vec{a}}{2} \Rightarrow \vec{AP} = \vec{a} + \frac{\vec{b}}{2} + \vec{b} + \frac{\vec{a}}{2} = \frac{3}{2}\cdot(\vec{a}+\vec{b}) = \frac{3}{2}\cdot\vec{AC}$

$\vec{AP} \parallel \vec{AC} \Rightarrow A, C, P$ са колин.

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$$2.(\vec{b}\times\vec{a}).(\vec{a}\times\vec{b}) = -2.(\vec{a}\times\vec{b}).(\vec{a}\times\vec{b}) = -2.(\vec{a}\times\vec{b})^2 = -2.\left(\vec{a}^2.\vec{b}^2 - \underbrace{(\vec{a}\vec{b})^2}_{\vec{a}^2.\vec{b}^2.\cos^2\varphi}\right)\ !$$

1 зад. В четириъгълника $ABCD$ точките M и N са средите съответно на страните AD и CB.

Да се докаже, че $\overline{MN} = \frac{1}{2}(\overline{AB} + \overline{CD})$

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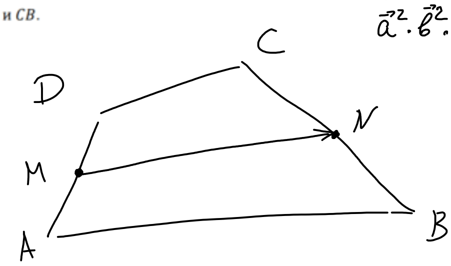


$\vec{OM} = \frac{1}{2}\cdot(\vec{OA}+\vec{OD})$

$\vec{ON} = \frac{1}{2}\cdot(\vec{OC}+\vec{OB})$

$\vec{MN} = \vec{ON} - \vec{OM} = \frac{1}{2}\cdot(\vec{OC}+\vec{OB}-\vec{OA}-\vec{OD}) =$

$= \frac{1}{2}\cdot(\vec{AB}+\vec{DC})$

3ад. $\vec{a}, \vec{b}$: $|\vec{a}|=3$, $|\vec{b}|=2$, $\varphi \in (0;\pi)$

$\vec{OA} = \vec{a}+\vec{b}$ $\quad \vec{OB} = \vec{a}-\vec{b}$ $\quad \vec{OC} = \vec{a}\times\vec{b}$

а) $\varphi = ?$: $V_{OABC} = 9$

б) $S_{\triangle OAB} = ?$

а) $(\vec{OA}\ \vec{OB}\ \vec{OC}) = \left[(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right].(\vec{a}\times\vec{b}) = \cdot = -2.(\vec{a}\times\vec{b})^2$

$$V = \frac{|(\vec{OA}\ \vec{OB}\ \vec{OC})|}{6} = 9 \Rightarrow \boxed{\frac{|-2.(\vec{a}\times\vec{b})^2|}{6} = 9}$$

$(\vec{a}\times\vec{b})^2 = 27$

$\vec{a}^2.\vec{b}^2 - \vec{a}^2.\vec{b}^2.\cos^2\varphi = 27$

$9.4 - 9.4.\cos^2\varphi = 27$

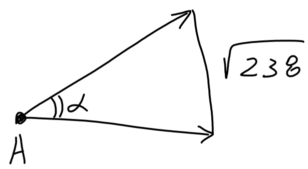
$36-27 = 36.\cos^2\varphi$

$9 = 36.\cos^2\varphi$

$\cos^2\varphi = \frac{1}{4} \Rightarrow \cos\varphi = \pm\frac{1}{2} \Rightarrow \varphi_1 = \frac{\pi}{3} \quad \varphi_2 = \frac{2\pi}{3}$

$|\vec{AB}| = \sqrt{88}$, $|\vec{AC}| = \sqrt{166}$, $|\vec{BC}| = \sqrt{238}$

$$\cos\angle(\vec{AB},\vec{AC}) = \frac{(\vec{AB}.\vec{AC})}{|\vec{AB}|.|\vec{AC}|}$$



$\vec{AB}(-6,-4,6)$
$\vec{AC}(9,-2,9)$ $\Rightarrow (\vec{AB}.\vec{AC}) = -54+8+54 = 8 \Rightarrow \cos\alpha = \frac{8}{\sqrt{88}.\sqrt{166}} > 0$

$\Downarrow$

α е остър

$\Downarrow$

$\triangle ABC$ е остроъгълен

$\boxed{\text{ОКС} \quad K = Oxyz}$ A, B, C, D – дали са компланарни или
координати
същ. тетраедър ABCD?

$(\vec{DA}\ \vec{DB}\ \vec{DC}) = \begin{vmatrix} - - - - & — \\ . - - - - & \\ . - - - & - \end{vmatrix}$ $= 0 \Rightarrow$ A, B, C, D са компланарни

$\neq 0 \Rightarrow \exists$ тетр. ABCD

$\vec{e}_1\ \vec{e}_2\ \vec{e}_3 \longrightarrow \vec{DA}\ \vec{DB}\ \vec{DC}$

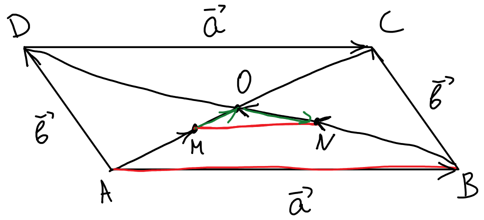
Детерминанта на Грам

$\vec{a}, \vec{b}, \vec{c}$ - не знам координати
знам дължини и ъгли $\Rightarrow \vec{a}^2, \vec{b}^2, \vec{c}^2$
$(ab)(ac)(bc)$

$$(\vec{a}\vec{b}\vec{c})^2 = \Gamma(a,b,c) = \begin{vmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{vmatrix}$$

$$(\overrightarrow{DA}\ \overrightarrow{DB}\ \overrightarrow{DC}) = \underbrace{\begin{vmatrix} \cdots\ \overrightarrow{DA}\ \cdots \\ \cdots\ \overrightarrow{DB}\ \cdots \\ \cdots\ \overrightarrow{DC}\ \cdots \end{vmatrix}}_{\text{число}} = \underbrace{\begin{vmatrix} \vdots & \vdots & \vdots \\ \overrightarrow{DA} & \overrightarrow{DB} & \overrightarrow{DC} \\ \vdots & \vdots & \vdots \end{vmatrix}}_{\text{число}}$$

$$\det T = \det T^t$$



ABCD – успоредник

M – медиц. на △ABD

N – медиц. на △ABC

?, че $MN \parallel AB$

Решение:

Нека $\overrightarrow{AB} = \vec a$, $\overrightarrow{AD} = \vec b$

$$\overrightarrow{AM} = \frac{2}{3}\cdot\overrightarrow{AO} = \frac{2}{3}\cdot\frac{1}{2}\cdot\overrightarrow{AC} = \frac{1}{3}\cdot\overrightarrow{AC} = \frac{1}{3}\cdot(\vec a + \vec b)$$

$$\boxed{\vec a - \vec b = \overrightarrow{AB} - \overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{DA} = \overrightarrow{DA} + \overrightarrow{AB} = \overrightarrow{DB}}$$

$$\overrightarrow{BN} = \frac{2}{3}\cdot\overrightarrow{BO} = \frac{2}{3}\cdot\frac{1}{2}\cdot\overrightarrow{BD} = \frac{1}{3}\cdot\overrightarrow{BD} = \frac{1}{3}\cdot(\overrightarrow{AD} - \overrightarrow{AB}) = \frac{1}{3}\cdot(\vec b - \vec a)$$

$$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AB} + \overrightarrow{BN} = -\frac{1}{3}\cdot(\vec a + \vec b) + \vec a + \frac{1}{3}\cdot(\vec b - \vec a) = \vec a\left(1 - \frac{2}{3}\right) = \frac{\vec a}{3}$$

$$\boxed{\overrightarrow{MN} = \frac{\vec a}{3},\quad \overrightarrow{AB} = \vec a} \Rightarrow \boxed{\overrightarrow{MN} = \frac{1}{3}\cdot\overrightarrow{AB}} \Rightarrow \underline{MN \parallel AB}$$

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5 зад. Даден е тетраедър $OABC$, за който $\overrightarrow{OA} = \vec a$, $\overrightarrow{OB} = \vec b$ и $\overrightarrow{OC} = \vec c$. Точките A_1, C_1 и O_1 са медицентровете съответно на триъгълниците: BOC, AOB и ABC.

a) Да се изразят медианите на тетраедъра $\overrightarrow{AA_1}, \overrightarrow{CC_1}, \overrightarrow{OO_1}$ чрез $\vec a$, $\vec b$ и $\vec c$;

b) Да се докаже, че векторите $\overrightarrow{AA_1}$ и $\overrightarrow{CC_1}$ са линейно независими;

c) Да се докаже, че векторите $\overrightarrow{AA_1}, \overrightarrow{CC_1}$ и $\overrightarrow{AC}$ са линейно зависими, т.е. четирите точки A, C, A_1 и C_1 лежат в една равнина. От двете подусловия b) и c) следва, че двете прави AA_1 и CC_1 се пресичат в единствена точка M;

d) Да се докаже, че намерената по-горе точка M лежи и на третата медиана OO_1 и да се намерят отношенията, в които т. M дели всяка от медианите.

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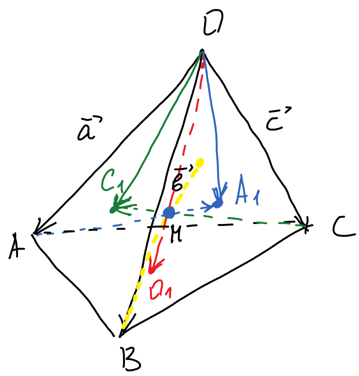

a) $\overrightarrow{OO_1} = ?$ т. O_1 – медиц. на △ABC

$$\overrightarrow{OO_1} = \frac{1}{3}\cdot(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}) = \boxed{\frac{1}{3}\cdot(\vec a + \vec b + \vec c)}$$

$\overrightarrow{CC_1} = ?$ т. C_1 – медиц. на △OAB

$$\overrightarrow{CC_1} = \frac{1}{3}\cdot(\overrightarrow{CO} + \overrightarrow{CA} + \overrightarrow{CB})$$

$$\left.\begin{aligned} \overrightarrow{CO} &= -\vec c \\ \overrightarrow{CA} &= \overrightarrow{OA} - \overrightarrow{OC} = \vec a - \vec c \\ \overrightarrow{CB} &= \overrightarrow{OB} - \overrightarrow{OC} = b - c \end{aligned}\right\} \to \overrightarrow{CC_1}$$

$$\boxed{\overrightarrow{CC_1} = \frac{1}{3}\cdot(\vec a + \vec b - 3.\vec c)}$$

Упр. $\overrightarrow{AA_1} = ?$ и $\overrightarrow{BB_1} = ?$

б) ?, че $\vec{AA_1} = \frac{1}{3}\cdot(\vec{b}+\vec{c}-3\vec{a})$

$\vec{CC_1} = \frac{1}{3}\cdot(\vec{a}+\vec{b}-3\vec{c})$ са ЛНЗ

$\alpha\cdot\vec{AA_1} + \beta\cdot\vec{CC_1} = \vec{0}$ $\quad \alpha = ?\ \beta = ?$

$\frac{\alpha}{3}(\vec{b}+\vec{c}-3\vec{a}) + \frac{\beta}{3}(\vec{a}+\vec{b}-3\vec{c}) = \vec{0}$

$\vec{a}\cdot\left(-\alpha+\frac{\beta}{3}\right) + \vec{b}\cdot\left(\frac{\alpha}{3}+\frac{\beta}{3}\right) + \vec{c}\cdot\left(\frac{\alpha}{3}-\beta\right) = \vec{0}$

$\vec{a}, \vec{b}, \vec{c}$ – ЛНЗ

$$\Rightarrow \begin{cases} -\alpha+\frac{\beta}{3} = 0 \\ \frac{\alpha+\beta}{3} = 0 \\ \frac{\alpha}{3} - \beta = 0 \end{cases}$$

$\ldots\ \boxed{\alpha = \beta = 0} \Rightarrow \vec{AA_1}$ и $\vec{CC_1}$ са $(*)$ ЛНЗ

$\Downarrow$

правите AA_1 и CC_1 могат да са кръстосани или пресичащи се

c) Ще док., че точките A, A_1, C и C_1 лежат в 1 равнина

I н. ?, че $AC \parallel A_1C_1$

$\vec{AC} = \vec{OC} - \vec{OA} = \vec{c} - \vec{a}$

$\vec{A_1C_1} = \vec{OC_1} - \vec{OA_1} = \underset{\text{Защо?}}{\frac{1}{3}\cdot(\vec{a}+\vec{b})} - \underset{\text{Защо?}}{\frac{1}{3}\cdot(\vec{b}+\vec{c})} = \frac{1}{3}(\vec{a}-\vec{c})$

$\vec{A_1C_1} = -\frac{1}{3}\cdot\vec{AC} \Rightarrow \vec{A_1C_1} \parallel \vec{AC} \Rightarrow A_1C_1 \parallel AC \Rightarrow A_1, C_1, A$ и C са компланарни $\left(\begin{smallmatrix}*\\*\end{smallmatrix}\right)$

От $(*)$ и $\left(\begin{smallmatrix}*\\*\end{smallmatrix}\right) \Rightarrow AA_1 \cap CC_1 =$ т. M

II н. $\alpha\cdot\vec{AA_1} + \beta\cdot\vec{CC_1} + \gamma\cdot\vec{CA} = \vec{0}$ $\quad \alpha = ?,\ \beta = ?,\ \gamma = ?$ $\quad (\alpha, \beta, \gamma) \neq (0,0,0)$

$\vec{AA_1}, \vec{CC_1}, \vec{CA}$ – л.з. $\Rightarrow$

$\Rightarrow A, C, A_1$ и C_1 компланарни

d) ?, че M, O, O_1 лежат на 1 права

$\vec{OO_1} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$ $\quad \vec{OM} = ?$

$\vec{OM} = \underbrace{\vec{OA}}_{a} + \underbrace{\vec{AM}}_{x\cdot\vec{AA_1}} = \underbrace{\vec{OC}}_{c} + \underbrace{\vec{CM}}_{y\cdot\vec{CC_1}}$

$\vec{a} + x\cdot\vec{AA_1} = \vec{c} + y\cdot\vec{CC_1}$

$\vec{a} + \frac{x}{3}\cdot(\vec{b}+\vec{c}-3\vec{a}) = \vec{c} + \frac{y}{3}\cdot(\vec{a}+\vec{b}-3\vec{c})$

$\vec{a}\underbrace{\left(1 - x - \frac{y}{3}\right)}_{0} + \vec{b}\underbrace{\left(\frac{x}{3} - \frac{y}{3}\right)}_{0} + \vec{c}\cdot\underbrace{\left(\frac{x}{3} - 1 + y\right)}_{0} = \vec{0}$

$$\begin{cases} x + \frac{y}{3} = 1 \\ x = y \\ \frac{x}{3} + y = 1 \end{cases} \Rightarrow y + \frac{y}{3} = 1 \qquad y = \frac{3}{4} = x$$

$$\begin{cases} x = y \\ \frac{x}{3} + y = 1 \end{cases} \qquad y = \frac{3}{4} = x$$

$$\vec{AM} = \frac{3}{4} \cdot \vec{AA_1} \qquad \vec{CM} = \frac{3}{4} \cdot \vec{CC_1}$$

$$\vec{OM} = \frac{1}{4} \cdot (\vec{a} + \vec{b} + \vec{c}) = \frac{3}{4} \cdot \vec{OO_1}$$