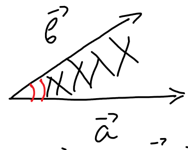
Скаларно произведение на вектори

$\vec a \ne \vec 0$, $\vec b \ne \vec 0$

$\langle \vec a, \vec b \rangle = |\vec a| . |\vec b| . \cos \angle(\vec a, \vec b)_e = (\vec a . \vec b)$ → число

$|\vec a|$ – дължина

$\angle(\vec a, \vec b)_e$ – елементарно геом. ъгъл : $\angle(\vec a, \vec b)_e = \varphi$: 1)



$\angle(\vec a, \vec b)_e = \angle(\vec b, \vec a)_e$

2) $\varphi \in [0; \pi]$

$-1 \le \cos\varphi \le 1$

3) $\cos\varphi = a$

$\exists!\, \varphi_0 \in [0; \pi]$:

$\cos\varphi_0 = a$

$\boxed{\langle \vec a, \vec b \rangle \text{ е метрика}}$

Свойства:

1) $\langle \vec a, \vec b \rangle = \langle \vec b, \vec a \rangle$, $(\vec a . \vec b) = (\vec b . \vec a)$

2) $(\vec a + \vec b) . \vec c = \underbrace{(\vec a . \vec c)}_{\text{число}} + \underbrace{(\vec b . \vec c)}_{\text{число}}$

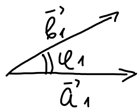
3) $(k . \vec a) . \vec b = k . \underbrace{(\vec a . \vec b)}_{\text{число}}$

4) $(\vec a . \vec b) = 0 \iff \vec a \perp \vec b$ $\{ \angle(\vec a, \vec b) = \frac{\pi}{2} \}$

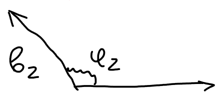
5) $(\vec a . \vec a) = \vec a^{\,2}$ – скаларен квадрат на $\vec a$

$(\vec a . \vec a) = \vec a^{\,2} = |\vec a| . |\vec a| . \underbrace{\cos 0^\circ}_{1} = |\vec a|^2$ $\qquad |\vec a| = \sqrt{\vec a^{\,2}}$

6) $\cos \angle(\vec a, \vec b) = \dfrac{(\vec a . \vec b)}{|\vec a| . |\vec b|}$



$\cos\varphi_1 > 0$



$\cos\varphi_2 < 0$

Задачи

1 зад. Дадени са вект. $\vec a, \vec b, \vec c$ – ЛНЗ

$|\vec a| = 1$, $|\vec b| = 2$, $|\vec c| = \sqrt 2$

$\angle(\vec a, \vec b) = \frac{\pi}{2}$, $\angle(\vec b, \vec c)_e = \frac{\pi}{2}$, $\angle(\vec a, \vec c)_e = \frac{\pi}{4}$

$\vec p = \vec a + \vec b - \vec c$

$\vec q = 2\vec a - 3\vec b + \vec c$

$\vec r = \vec a + \lambda . \vec b - \vec c$

а) $|\vec p| = ?$ Упр. , $|\vec q| = ?$

$|\vec q|^2 = (2\vec a - 3 . \vec b + \vec c)^2 = (2\vec a)^2 + (3\vec b)^2 + (\vec c)^2 - 2.(2\vec a).(3\vec b) + 2.(2\vec a).\vec c - 2.(3\vec b).\vec c =$

$= 4.\vec a^{\,2} + 9\,\vec b^{\,2} + \vec c^{\,2} - 12\,(\vec a . \vec b) + 4.(\vec a . \vec c) - 6.(\vec b . \vec c)$

$|\vec{q}|^2 = (2\vec{a}-3.\vec{b}+\vec{c})^2 = (2\vec{a})^2+(3\vec{b})^2+(\vec{c})^2 - 2.(2\vec{a}).(3\vec{b}) + 2.(2\vec{a}).\vec{c} - 2.(3\vec{b}).\vec{c}$

$$= 4.\vec{a}^2 + 9.\vec{b}^2 + \vec{c}^2 - 12.(\vec{a}.\vec{b}) + 4.(\vec{a}.\vec{c}) - 6.(\vec{b}.\vec{c})$$

$|\vec{a}|=1 \Rightarrow \vec{a}^2=1$
$|\vec{b}|=2 \Rightarrow \vec{b}^2=4$
$|\vec{c}|=\sqrt{2} \Rightarrow \vec{c}^2=2$

$|\vec{a}|=1,\ |\vec{b}|=2,\ |\vec{c}|=\sqrt{2}$
$\angle(\vec{a},\vec{b})=\frac{\pi}{2},\ \angle(\vec{b},\vec{c})_e=\frac{\pi}{2},\ \angle(\vec{a},\vec{c})_e=\frac{\pi}{4}$ $\Rightarrow$

$(\vec{a}.\vec{b})=0$!
$(\vec{b}.\vec{c})=0$
$(\vec{a}.\vec{c})=1.\sqrt{2}.\frac{\sqrt{2}}{2}=1$

$|\vec{q}|^2 = 4.1+9.4+2-12.0+4.1-6.0 = 46$ $\quad |\vec{q}|=\sqrt{46}$

б) $(\vec{p}.\vec{q}) = (\vec{a}+\vec{b}-\vec{c}).(2\vec{a}-3\vec{b}+\vec{c}) =$

$$= 2.(\underbrace{\vec{a}^2}_{1}) - 3.(\underbrace{\vec{a}.\vec{b}}_{0}) + (\underbrace{\vec{a}.\vec{c}}_{1}) + 2.(\underbrace{\vec{b}.\vec{a}}_{0}) - 3.\underbrace{\vec{b}^2}_{4} + (\underbrace{\vec{b}.\vec{c}}_{0}) - 2.(\underbrace{\vec{c}.\vec{a}}_{1}) + 3.(\underbrace{\vec{c}.\vec{b}}_{0}) - \underbrace{\vec{c}^2}_{2}$$

$(\vec{p}.\vec{q}) = 2+1-12-2-2 = -13 \Rightarrow \angle(\vec{p},\vec{q}) > 90°$

в) Упр. $\cos\angle(\vec{p},\vec{q}) = ?$

г) Да се определи за $\lambda=?$ $\vec{p}\perp\vec{\tau}$; $\to$ Упр. $(\vec{p}.\vec{\tau})=0$

д) За $\lambda=?$ $|\vec{\tau}|=\sqrt{5}$.

$\vec{\tau}^2=5 \qquad \vec{\tau}^2=(\vec{a}+\lambda.\vec{b}-\vec{c})^2=5$

$$\underbrace{\vec{a}^2}_{1} + \lambda^2.\underbrace{\vec{b}^2}_{4} + \underbrace{\vec{c}^2}_{2} + 2.\lambda.(\underbrace{\vec{a}.\vec{b}}_{0}) - 2(\underbrace{\vec{a}.\vec{c}}_{1}) - 2.\lambda.(\underbrace{\vec{b}.\vec{c}}_{0}) = 5$$

$1+4\lambda^2+2-2=5$
$4\lambda^2=4$
$\lambda^2=1$
$\lambda=\pm 1$ $\quad \vec{\tau}_1=\vec{p}=\vec{a}+\vec{b}-\vec{c}$, $\quad \vec{\tau}_2=\vec{a}-\vec{b}-\vec{c}$

$\tau=\vec{a}+\lambda.b-\vec{c}$ $\quad 1 \quad -1$

$\langle\vec{a},\vec{b}\rangle$ $\quad (\vec{a}.\vec{b})\quad (\vec{a}\vec{b})$
$5.(6.\vec{a}).\vec{b}$
$5.\langle 6\vec{a},\vec{b}\rangle$

2 зад.
$\vec{a},\vec{b},\vec{c}$ – ЛНЗ
За вектор $\vec{p}$ е дадено, че $\vec{p}\perp\vec{a},\ \vec{p}\perp\vec{b},\ \vec{p}\perp\vec{c}$!!!

?, че $\vec{p}=\vec{o}$

Решение

..., че $p = 0$

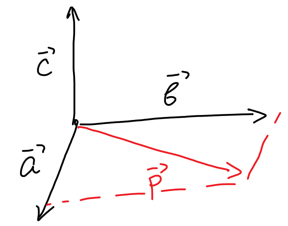
Решение

$\vec a, \vec b, \vec c$ – ЛНЗ

$\vec a, \vec b, \vec c, \vec p$ – ЛЗ $\Rightarrow$ $\vec p = \alpha.\vec a + \beta.\vec b + \gamma.\vec c \quad | \cdot \vec p$

$$\vec p^{\,2} = \alpha.\underbrace{(\vec a.\vec p)}_{0} + \beta.\underbrace{(\vec b.\vec p)}_{0} + \gamma.\underbrace{(\vec c.\vec p)}_{0} = 0 \Rightarrow \vec p = \vec o$$

$$\overbrace{\underbrace{\vec a, \vec b, \vec c}_{ЛНЗ}, \vec p}^{\forall \vec p} \text{ – ЛЗ} \overset{?}{\Rightarrow} \vec p = \vec o \quad \text{Не}$$



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3 зад.

$\vec a, \vec b, \vec c$: $|\vec a| = 2$, $|\vec b| = 1$, $|\vec c| = 3$, $\measuredangle(\vec a, \vec b) = \measuredangle(\vec b, \vec c) = \measuredangle(\vec c, \vec a) = \frac{\pi}{3}$

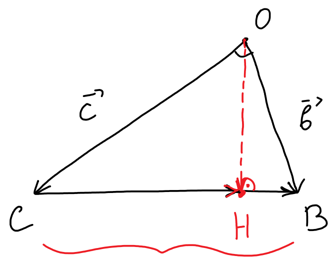
$\vec a^{\,2} = 4 \qquad \vec b^{\,2} = 1 \qquad \vec c^{\,2} = 9$

$(\vec a.\vec b) = 2.1.\frac{1}{2} = 1 \qquad (\vec b.\vec c) = 1.3.\frac{1}{2} = \frac{3}{2} \qquad (\vec c.\vec a) = 2.3.\frac{1}{2} = 3$!

Нека $\vec{OA} = \vec a$, $\vec{OB} = \vec b$, $\vec{OC} = \vec c$

а) Ако т. H $\begin{cases} \in BC \\ OH \perp BC \end{cases}$, „$\in$" $\to$ $H \in g$ – лежи на; $g \ni H$ – минава през

$\vec{OH} = ?$ чрез $\cancel{\vec a}$, $\vec b$ и $\vec c$ и $|\vec{OH}| = ?$



$\vec{OH} = ?$ чрез $\vec b$ и $\vec c$

$\vec{OH} = \underbrace{\vec{OC}}_{\vec c} + \underbrace{\vec{CH}}_{?}$

$\vec{CH} = x.\vec{CB}$

$\vec{CB} = \vec{OB} - \vec{OC} = \vec b - \vec c$

$\vec{CH} = x.(\vec b - \vec c) \qquad x = ?$

$\vec{OH} \perp \vec{CB} \Rightarrow (\vec{OH}.\vec{CB}) = 0$

$\vec{OH} = \vec c + x.(\vec b - \vec c)$

$\vec{CB} = \vec b - \vec c$

$[\vec c + x.(\vec b - \vec c)] \cdot (\vec b - \vec c) = 0$

$\vec c.(\vec b - \vec c) + x.(\vec b - \vec c)^2 = 0$

$(\vec c.\vec b) - \vec c^{\,2} + x.(\vec b^{\,2} - 2.(\vec b.\vec c) + \vec c^{\,2}) = 0$

$\frac{3}{2} - 9 + x.(1 - 2.\frac{3}{2} + 9) = 0$

$-\frac{15}{2} + x.(7) = 0 \Rightarrow x = \frac{15}{14} \to \vec{OH}$

$\vec{OH} = \vec c + \frac{15}{14}.(\vec b - \vec c)$

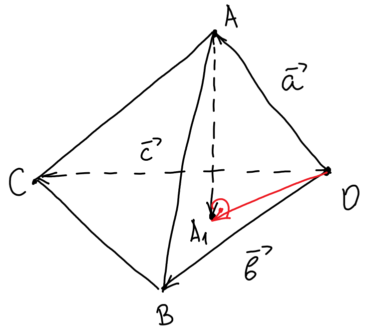
$\left| \begin{array}{l} \vec{OH} = \underbrace{\frac{15}{14}} \cdot \vec{b} - \underbrace{\frac{1}{14}} \cdot \vec{c} + \underbrace{0} . \vec{a} \end{array} \right.$

$|\vec{OH}|^2 = \left(\frac{15.\vec{b} - \vec{c}}{14}\right)^2 = \frac{1}{14^2} \cdot \left(15^2 . \vec{b}^2 - 30.(\vec{b}.\vec{c}) + \vec{c}^{\,2}\right) =$

$= \frac{1}{14^2} \cdot \left(225.1 - 30.\frac{3}{2} + 9\right) = \frac{1}{14^2} \cdot (225 - 45 + 9) = \frac{189}{14^2}$

$|\vec{OH}| = \frac{3.\sqrt{21}}{14}$

б) Нека т. $A_1 \in (BOC)$
$AA_1 \perp (BOC)$



$\vec{OA_1} = ?$ чрез $\vec{b}$ и $\vec{c}$

$\vec{OA_1}$ е компл. с $\vec{b}$ и $\vec{c}$ $\Rightarrow$

$\Rightarrow \vec{OA_1} = \beta . \vec{b} + \gamma . \vec{c}$ $\beta = ?$ $\gamma = ?$

$AA_1 \perp (BOC) \Rightarrow \left| \begin{array}{l} \vec{AA_1} \perp \vec{b} \\ \vec{AA_1} \perp \vec{c} \end{array} \right.$

$\vec{AA_1} = \vec{OA_1} - \vec{OA} = \beta . \vec{b} + \gamma . \vec{c} - \vec{a}$ $\left| \begin{array}{l} (\vec{AA_1} . \vec{b}) = 0 \\ (\vec{AA_1} . \vec{c}) = 0 \end{array} \right.$

$\left| \begin{array}{l} (\beta . \vec{b} + \gamma . \vec{c} - \vec{a}) . \vec{b} = 0 \\ (\beta . \vec{b} + \gamma . \vec{c} - \vec{a}) . \vec{c} = 0 \end{array} \right.$

$\left| \begin{array}{l} \beta . \vec{b}^2 + \gamma . (\vec{c}.\vec{b}) - (\vec{a}.\vec{b}) = 0 \\ \beta . (\vec{b}.\vec{c}) + \gamma . \vec{c}^{\,2} - (\vec{a}.\vec{c}) = 0 \end{array} \right.$ $\vec{a}^2 = 4$ ✓ $\vec{b}^2 = 1$ ✓ $\vec{c}^{\,2} = 9$ ✓

$(\vec{a}.\vec{b}) = 2.1.\frac{1}{2} = 1$ $(\vec{b}.\vec{c}) = 1.3.\frac{1}{2} = \frac{3}{2}$ $(\vec{c}.\vec{a}) = 2.3.\frac{1}{2} = 3$

$\left| \begin{array}{l} \beta . 1 + \gamma . \frac{3}{2} - 1 = 0 \quad | .2 \\ \beta . \frac{3}{2} + \gamma . 9 - 3 = 0 \quad | : 3 \end{array} \right.$

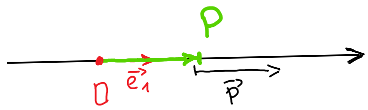
$\left| \begin{array}{l} 2\beta + 3\gamma = 2 \\ \frac{\beta}{2} + 3\gamma = 1 \end{array} \right. (-)$ $\frac{3}{2} . \beta = 1 \Rightarrow \beta = \frac{2}{3}$

$3\gamma = 2 - 2 . \frac{2}{3} = 2 - \frac{4}{3} = \frac{2}{3}$

$\gamma = \frac{2}{9}$

$\vec{OA_1} = \frac{2}{3} . \vec{b} + \frac{2}{9} . \vec{c}$, упр. $|\vec{OA_1}| = ?$

28.10.21г. Координатни системи

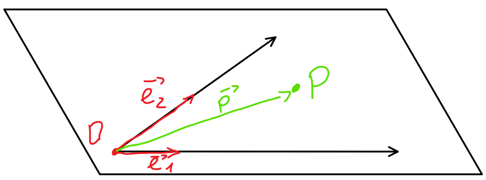
I 

$K = O\vec{e}_1$ $\vec{p} \parallel \vec{e}_1 \Rightarrow \exists!\, x_1 : \vec{p} = x_1.\vec{e}_1 \Leftrightarrow \vec{p}(x_1)$ спр. $K = O\vec{e}_1$

$\vec{OP} = \vec{p}$ – радиус-вектор на т. P

$\vec{OP} = x_1.\vec{e}_1 \Leftrightarrow$ т. $P(x_1)$ спр. $K = O\vec{e}_1$

II Равнина



$K = O\underbrace{\vec{e}_1\vec{e}_2}_{ЛНЗ}$

$\vec{p}, \vec{e}_1, \vec{e}_2$ – л.з. $\Rightarrow \exists!\, (x_1, x_2)$:

$\vec{p} = x_1.\vec{e}_1 + x_2.\vec{e}_2 \Leftrightarrow \vec{p}(x_1, x_2)$ спр. K

т. $P(x_1, x_2)$ спр. $K \Leftrightarrow \vec{OP}(x_1, x_2)$ спр. K

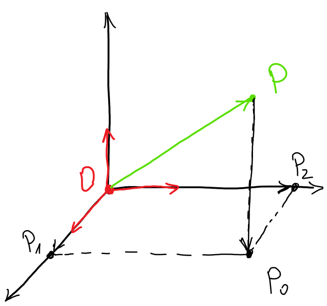
Кога $K = O\vec{e}_1\vec{e}_2$ е ортонормирана ?

$|\vec{e}_1| = |\vec{e}_2| = 1 \Rightarrow (\vec{e}_1)^2 = 1 \quad (\vec{e}_2)^2 = 1 \quad (\vec{e}_1.\vec{e}_2) = 0$

$\vec{e}_1 \perp \vec{e}_2$

III $K = O\underbrace{\vec{e}_1\vec{e}_2\vec{e}_3}_{ЛНЗ}$, $\vec{p}$ – произволен $\Rightarrow \exists!\, (x_1, x_2, x_3)$: $\vec{p} = x_1.\vec{e}_1 + x_2.\vec{e}_2 + x_3.\vec{e}_3 \Leftrightarrow \vec{p}(x_1, x_2, x_3)$ спр. K

$\vec{p} = \vec{OP} \Rightarrow \vec{OP}(x_1, x_2, x_3) \Rightarrow$ т. $P(x_1, x_2, x_3)$ спр. K



Кога $K = O\vec{e}_1\vec{e}_2\vec{e}_3$ е ОКС ?

$\vec{e}_1 \perp \vec{e}_2 \perp \vec{e}_3 \perp \vec{e}_1$

$|\vec{e}_1| = |\vec{e}_2| = |\vec{e}_3| = 1$

Скаларно произведение спрямо ОКС

ОКС $K = O\vec{e}_1\vec{e}_2\vec{e}_3$

$\vec{a}(a_1, a_2, a_3) \Leftrightarrow \vec{a} = (a_1.\vec{e}_1 + a_2.\vec{e}_2 + a_3.\vec{e}_3)$

$\vec{b}(b_1, b_2, b_3) \Leftrightarrow \vec{b} = (b_1.\vec{e}_1 + b_2.\vec{e}_2 + b_3.\vec{e}_3)$

$$\boxed{(\vec{a}.\vec{b}) = a_1.b_1 + a_2.b_2 + a_3.b_3} \quad \text{ОКС}$$

$(\vec{a}.\vec{a}) = \vec{a}^2 = a_1^2 + a_2^2 + a_3^2$

$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$ ОКС

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4 зад. ОКС $K = O\vec{e}_1\vec{e}_2\vec{e}_3 = Oxyz$

$A(-1,-1,1)$ ✓
$B(2,-7,4)$ ✓
$C(4,-2,6)$

а) $P_{\triangle ABC} = ?$;

б) Да се определи вида на $\triangle ABC$ спр. ъглите му;

в) т. $H \begin{cases} \in AB \\ CH \perp AB \end{cases}$ Да се намерят коорд. на т. H.

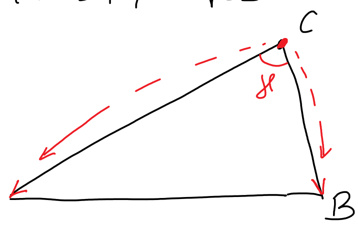
Решение:

а) $\vec{AB} = \vec{OB} - \vec{OA} \Rightarrow \vec{AB}(3,-6,3) \Rightarrow |\vec{AB}| = \sqrt{3^2+(-6)^2+3^2} = \sqrt{54}$

$\vec{AC}(5,-1,5) \Rightarrow |\vec{AC}| = \sqrt{25+1+25} = \sqrt{51}$

$\vec{BC}(2,5,2) \Rightarrow |\vec{BC}| = \sqrt{4+25+4} = \sqrt{33}$

б) $\cos\gamma = \dfrac{(\vec{CA}\cdot\vec{CB})}{|\vec{CA}|\cdot|\vec{CB}|}$



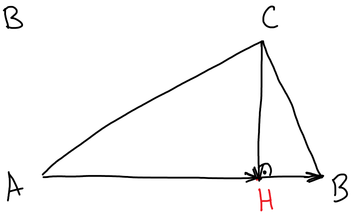
$\vec{CA}(-5,\ 1,\ -5)$
$\vec{CB}(-2,\ -5,\ -2)$ $\Rightarrow (\vec{CA}\cdot\vec{CB}) = 10+(-5)+10$

$(\vec{CA}\cdot\vec{CB}) = 15 > 0 \Rightarrow$

$\cos\gamma = \dfrac{15}{\sqrt{51}\cdot\sqrt{33}} > 0 \Rightarrow \gamma$ е остър $\Rightarrow \triangle ABC$ е остроъгълен

в) $\vec{CH} = \vec{CA} + \underset{?}{\vec{AH}}$, $\vec{AH} \parallel \vec{AB} \Rightarrow \exists!\, x$ $\vec{AH} = x.\vec{AB}$

$x = ?$



$\vec{CH} = \vec{CA} + x.\vec{AB}$, $\vec{CH} \perp \vec{AB}$

$(\vec{CH}.\vec{AB}) = 0$

$(\vec{CA} + x.\vec{AB}).\vec{AB} = 0$

$(\vec{CA}.\vec{AB}) + x.\vec{AB}^2 = 0$

$\vec{CA}(-5,1,-5)$
$\vec{AB}(3,-6,3)$

$(\vec{CA}.\vec{AB}) = -36$
$\vec{AB}^2 = 54$

$-36 + x.54 = 0$

$x = 2/3 \rightarrow \vec{AH} = \frac{2}{3}.\vec{AB}$

$\vec{OH} - \vec{OA} = \frac{2}{3}.\vec{AB}$

$\vec{OH} = \vec{OA} + \frac{2}{3}.\vec{AB}$

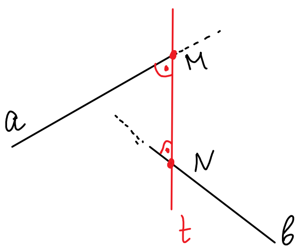
т.$H(x_H, y_H, z_H)$
т.$A(-1,-1,1)$
$\vec{AB}(3,-6,3)$

$x_H = -1 + \frac{2}{3}.3$

$y_H = -1 + \frac{2}{3}.(-6)$ $\Rightarrow$ т.$H(1,-5,3)$

$z_H = 1 + \frac{2}{3}.3$

Ос на кръстосани прави



$$\begin{cases} t \cap a = M \\ t \cap b = N \\ t \perp a \\ t \perp b \end{cases} \Rightarrow t \text{ е ос на } a \text{ и } b$$

MN е ос-отсечка

$|MN| = d(a, b)$

5 зад. ОКС $K = Oxyz$

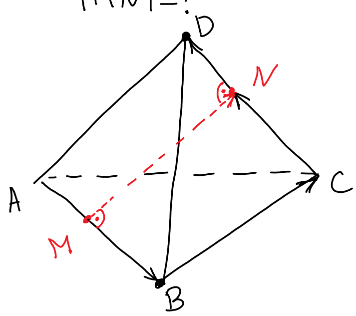
$A(0,0,-2)$ $ABCD$ – тетраедър

$B(4,0,-4)$ а) ?, коорд. на т. M и т. N :

$$\begin{array}{|l} M \in AB \\ N \in CD \\ \vec{MN} \perp \vec{AB} \ ! \\ \vec{MN} \perp \vec{CD} \ ! \end{array}$$

$C(2,0,0)$

$D(5,3,-3)$ $|\vec{MN}| = ?$



$\vec{MN} = \vec{MB} + \vec{BC} + \vec{CN}$

$\vec{MB} = x \cdot \vec{AB}$ (1)

$\vec{CN} = y \cdot \vec{CD}$ (2)

$\vec{MN} = x \cdot \vec{AB} + \vec{BC} + y \cdot \vec{CD}$

$$\begin{cases} (\vec{MN} \cdot \vec{AB}) = 0 \\ (\vec{MN} \cdot \vec{CD}) = 0 \end{cases} \Rightarrow \begin{cases} (x \cdot \vec{AB} + y \cdot \vec{CD} + \vec{BC}) \cdot \vec{AB} = 0 \\ (x \cdot \vec{AB} + y \cdot \vec{CD} + \vec{BC}) \cdot \vec{CD} = 0 \end{cases}$$

$\vec{AB}(4,0,-2)$ $\vec{AB}^2 = 20$

$\vec{CD}(3,3,-3)$ $(\vec{CD} \cdot \vec{AB}) = 18$

$\vec{BC}(-2,0,4)$ $(\vec{BC} \cdot \vec{AB}) = -16$

$\vec{CD}^2 = 27$

$(\vec{BC} \cdot \vec{CD}) = -18$

$$\begin{cases} x \cdot (\vec{AB}^2) + y \cdot (\vec{CD} \cdot \vec{AB}) + (\vec{BC} \cdot \vec{AB}) = 0 \\ x \cdot (\vec{AB} \cdot \vec{CD}) + y \cdot \vec{CD}^2 + (\vec{BC} \cdot \vec{CD}) = 0 \end{cases}$$

$$\begin{cases} 20 \cdot x + 18y = 16 \quad |:2 \\ 18x + 27 \cdot y = 18 \quad |:3 \end{cases}$$

$$\begin{cases} 10x + 9y = 8 \\ 6 \cdot x + 9 \cdot y = 6 \end{cases} (-) \Rightarrow x = \frac{1}{2} \quad y = \frac{1}{3}$$

$\downarrow$ (1) $\downarrow$ (2)

$\vec{MB} = \frac{1}{2} \cdot \vec{AB} \Rightarrow$ т. M е средата на AB $\Rightarrow \vec{OM} = \frac{1}{2} \cdot (\vec{OA} + \vec{OB})$

$M(2, 0, -3)$

$x_M = \frac{1}{2} \cdot (0 + 4) = 2$

$y_M = \frac{1}{2} \cdot (0 + 0) = 0$

$z_M = \frac{1}{2} \cdot (-2 + (-4)) = -3$

$\vec{CN} = \frac{1}{3} \cdot \vec{CD} \Rightarrow \vec{ON} = \vec{OC} + \frac{1}{3} \cdot \vec{CD}$

$x_N = 2 + \frac{1}{3} \cdot 3 = 3$

$\vec{OC}(2, 0, 0)$

$y_N = 0 + \frac{1}{3} \cdot 3 = 1$

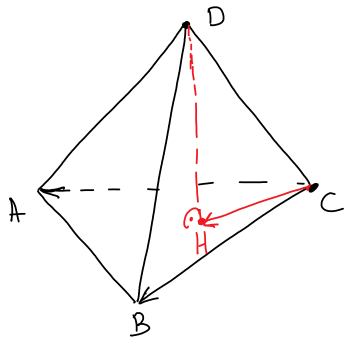
$\vec{CD}(3, 3, -3)$

$z_N = 0 + \frac{1}{3} \cdot (-3) = -1$

$\vec{MN}(1,1,2) \Rightarrow |\vec{MN}| = \sqrt{6}$

$N(3, 1, -1)$
$M(2, 0, -3)$

б) ?, коорд. на т. H $\begin{cases} \in (ABC) \\ DH \perp (ABC) \end{cases}$



$\vec{CH}$ може да се изрази чрез $\vec{CA}$ и $\vec{CB}$

$\vec{CH} = x.\vec{CA} + y.\vec{CB}$ (3) $x = ?$, $y = ?$

$\vec{DH} = \vec{DC} + \vec{CH} = \vec{DC} + x.\vec{CA} + y.\vec{CB}$

$\vec{DH} \perp \vec{CA}$ $\quad \left| (\vec{DH}.\vec{CA}) = 0 \right.$

$\vec{DH} \perp \vec{CB}$ $\quad \left| (\vec{DH}.\vec{CB}) = 0 \right.$

Отг.: т. $H(5, 0, -3)$