Д.р. 3

Домашна работа 3. ДИС 1

Задача 1.

а) $\int (10x+15)^6\,dx = \frac{1}{10}\int (10x+15)^6\,d(10x+6) =$

$= \frac{1}{10}\cdot\frac{(10x+6)^7}{7} + c = \underline{\frac{(10x+6)^7}{70} + c}$

б) $\int \frac{\ln^4(2x)}{x}\,dx = \int \frac{\ln^4(2x).2}{2x}\,dx = \int \ln^4(2x)\,d\ln(2x) =$

$= \underline{\frac{\ln^5(2x)}{5} + c}$

в) $\int \frac{dx}{4x^2+15} = \frac{1}{4}\int \frac{1}{\frac{4x^2}{4}+\frac{15}{4}}\,dx = \frac{1}{4}\int \frac{1}{\frac{15}{4}\left(1+\left(\frac{2x}{\sqrt{15}}\right)^2\right)}\,dx =$

$= \frac{\sqrt{15}}{15.2}\int \frac{1}{1+\left(\frac{2x}{\sqrt{15}}\right)^2}\,d\frac{2x}{\sqrt{15}} = \underline{\frac{\sqrt{15}}{30}.\operatorname{arctg}\left(\frac{2x}{\sqrt{15}}\right) + c}$

г) $\int (x^2+1)\sin x\,dx = \int x^2+1\,d(-\cos x) =$

$= -\cos x.(x^2+1) + \int \cos x\,d(x^2+1) =$

$= -\cos x.(x^2+1) + \int 2x\,d\sin x =$

$= -\cos x(x^2+1) + 2x\sin x - \int \sin x\,d2x =$

$= \underline{-\cos x(x^2+1) + 2x\sin x + \cancel{2\sin} + 2\cos x}$

$\sqrt{1}$

i) $\int x^3 e^{2x} dx = \frac{1}{2}\int x^3 \, de^{2x} = \frac{e^{2x} \cdot x^3}{2} - \frac{1}{2}\int e^{2x} dx^3 =$

$= \frac{e^{2x}}{2} \cdot x^3 - \frac{1}{4}\int 3x^2 \, de^{2x} =$

$= \frac{e^{2x}}{2} \cdot x^3 - \frac{3x^2 e^{2x}}{4} + \frac{1}{4}\int e^{2x} \, d3x^2 =$

$= \frac{e^{2x}}{2}\left(x^3 - \frac{3}{2}x^2\right) + \frac{1}{4}\int 3x \, de^{2x} =$

$= \frac{e^{2x}}{2}\left(x^3 - \frac{3}{2}x^2\right) + \frac{3x e^{2x}}{4} - \frac{1}{4}\int e^{2x} \, d3x =$

$= \frac{e^{2x}}{2}\left(x^3 - \frac{3}{2}x^2 + \frac{3x}{2}\right) - \frac{3}{8}e^{2x} + c =$

$= \frac{e^{2x}}{2}\left(x^3 - \frac{3}{2}x^2 + \frac{3}{2}x - \frac{3}{4}\right) + c$

iii) $\int \cos^4 x \, dx = \int \frac{1}{4}(1 + \cos 2x)^2 dx = \frac{1}{4}\int 1 + 2\cos 2x + \cos^2 2x \, dx =$

$= \frac{x}{4} + \frac{1}{2}\int \cos 2x \, dx + \frac{1}{4}\int \frac{1 + \cos 4x}{2} dx =$

$= \frac{x}{4} + \frac{\sin 2x}{4} + \frac{x}{8} + \frac{1}{8}\int \cos 4x \, dx =$

$= \frac{3x}{8} + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + c$

3) $\underset{I}{\int e^x \sin(2x)\,dx} = \int \sin(2x)\,de^x = e^x\sin(2x) - \int e^x\,d\sin(2x) =$ (D.P. B)

$= e^x\sin(2x) - \int 2\cos(2x)\,de^x = e^x\sin(2x) - 2e^x\cos(2x) +$

$+ 2\int e^x\,d\cos(2x) =$

$= e^x\sin(2x) - 2e^x\cos(2x) - 4\underbrace{\int \sin(2x)\,e^x\,dx}_{I}$

$5I = e^x\sin(2x) - 2e^x\cos(2x) \quad |:5$

$I = \frac{e^x}{5}\left(\sin(2x) - 2\cos(2x)\right)$

Zag. 2 $\int \frac{4x^2-x}{(x+1)^2(x^2+1)}\,dx$

$\frac{4x^2-x}{(x+1)^2(x^2+1)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+1}$

$4x^2-x = A(x+1)(x^2+1) + B(x^2+1) + (Cx+D)(x^2+2x+1)$

$\begin{array}{l|l} x^0 & 0 = A+B+D \\ x^1 & -1 = A+C+2D \\ x^2 & 4 = A+B+2C+D \\ x^3 & 0 = A+C \end{array}$

$A=-C$

$4=2C$, $\boxed{C=2,\ A=-2}$

$\boxed{D=-\frac{1}{2}}$ $\boxed{B=\frac{5}{2}}$

$\int \frac{-2}{x+1} + \frac{5}{2(x+1)^2} + \frac{2x-\frac{1}{2}}{x^2+1}\,dx =$

$= -2\ln|x+1| - \frac{5}{2(x+1)} + 2\int\frac{x}{x^2+1}\,dx - \frac{1}{2}\int\frac{1}{x^2+1}\,dx =$

$$= -2\ln|x+1| - \frac{5}{2(x+1)} + \ln(x^2+1) - \frac{1}{2}\operatorname{arctg} x + C$$