$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

(3 зад), 2 тип   Решение:

$$B_{n+1}(f;x) = \sum_{k=0}^{n+1} f\left(\frac{k}{n+1}\right)\binom{n+1}{k} x^k (1-x)^{n+1-k}$$

$$B'_{n+1}(f;x) = \sum_{k=0}^{n+1} f\left(\frac{k}{n+1}\right)\binom{n+1}{k}\left[k x^{k-1}(1-x)^{n+1-k} - (n+1-k)x^k(1-x)^{n-k}\right]$$

$$= \sum_{k=1}^{n+1} f\left(\frac{k}{n+1}\right)\binom{n+1}{k} k x^{k-1}(1-x)^{n+1-k} - \sum_{k=0}^{n} f\left(\frac{k}{n+1}\right)\binom{n+1}{k}(n+1-k)x^k(1-x)^{n-k}$$

$$= \sum_{k=0}^{n} f\left(\frac{k+1}{n+1}\right)\boxed{\binom{n+1}{k+1}(k+1)} x^k(1-x)^{n-k} - \sum_{k=0}^{n} f\left(\frac{k}{n+1}\right)\boxed{\binom{n+1}{k}(n+1-k)} x^k(1-x)^{n-k}$$

$$\binom{n+1}{k+1}\cdot(k+1) = \frac{(n+1)!\,(k+1)}{(k+1)!\,(n-k)!} = \frac{(n+1)!}{k!\,(n-k)!} = (n+1)\cdot\binom{n}{k}$$

$$\binom{n+1}{k}(n+1-k) = \frac{(n+1)!\,(n+1-k)}{k!\,(n+1-k)!} = \frac{(n+1)!}{k!\,(n-k)!} = (n+1)\binom{n}{k}$$

Следователно

$$B'_{n+1}(x) = \sum_{k=0}^{n}\left[\frac{f\left(\frac{k+1}{n+1}\right) - f\left(\frac{k}{n+1}\right)}{\frac{1}{n+1}}\right]\binom{n}{k} x^k(1-x)^{n-k}$$

По теоремата за крайните нараствания, съществува $\xi_k \in \left[\frac{k}{n+1}, \frac{k+1}{n+1}\right]$, за която

$$\frac{f\left(\frac{k+1}{n+1}\right) - f\left(\frac{k}{n+1}\right)}{\frac{1}{n+1}} = f'(\xi_k)$$

Окончателно

$$B'_{n+1}(x) = \sum_{k=0}^{n} f'(\xi_k)\binom{n}{k} x^k(1-x)^{n-k}$$

Условие: Докажете, че ако $f \in C^1[0,1]$, то за производната на полинома на Бернщайн е изпълнено:

$$B'_{n+1}(f;x) = \sum_{k=0}^{n} f'(\xi_k)\binom{n}{k} x^k(1-x)^{n-k},$$ където

$$\xi_k \in \left[\frac{k}{n+1}, \frac{k+1}{n+1}\right], \quad k = 0, \dots, n$$