Зад. 1

$$f(x) = |x+1| \sqrt{\frac{x-3}{x-4}}$$

$x \neq 4$

~~1. ДМ: $\frac{x-3}{x-4} \geq 0 \iff (x-3)(x-4) \geq 0$~~

1. ДМ: $x \neq 4$, $\frac{x-3}{x-4} \geq 0 \Rightarrow \boxed{x \in (-\infty, 3] \cup (4, +\infty)}$

2. Пресечни точки с осите:

$x = 0$, $y = \sqrt{\frac{-3}{-4}} = \frac{\sqrt{3}}{2}$

$y = 0$, $x_1 = -1$, $x_2 = 3$

$$\boxed{\left(0, \frac{\sqrt{3}}{2}\right), (-1, 0), (3, 0)}$$

3. Четност и периодичност - няма

$$f(-x) = |-x+1| \sqrt{\frac{-x-3}{-x-4}} = |1-x| \sqrt{\frac{x+3}{x+4}} \Rightarrow \begin{array}{l} f(-x) \neq f(x) \\ f(-x) \neq -f(x) \end{array}$$

4. Асимптоти:

- вертикална: $\boxed{x = 4}$

$$\lim_{x \to 4^+} |x+1| \sqrt{\frac{x-3}{x-4}} = 5 \cdot \lim_{x \to 4^+} \sqrt{\frac{4-3}{4-4+0^+}} = +\infty$$

- хоризонтална - няма

$$\lim_{x \to \pm\infty} |x+1| \sqrt{\frac{x-3}{x-4}} = +\infty$$

- наклонена $\boxed{g_1: y = x + \frac{3}{2}}$ $\boxed{g_2: y = -x - \frac{3}{2}}$

$$k_1 = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{(x+1)\sqrt{\frac{x-3}{x-4}}}{x} = \lim_{x \to +\infty} \left(\sqrt{\frac{x-3}{x-4}} + \sqrt{\frac{x-3}{x^2(x-4)}} \right) =$$

$= 1 + 0$

$$b_1 = \lim_{x\to+\infty} \left(f(x) - kx \right) = \lim_{x\to+\infty} \left((x+1)\sqrt{\frac{x-3}{x-4}} - x \right) =$$

$$= \lim_{x\to+\infty} \left(\underbrace{x\cdot\left(\sqrt{\frac{x-3}{x-4}} - 1\right)} + \sqrt{\frac{x-3}{x-4}} \right) = \boxed{\frac{1}{2} + 1 = \frac{3}{2}}$$

$\sqrt{x-3} + \sqrt{x-4}$

$$\lim_{x\to+\infty} x \left(\sqrt{\frac{x-3}{x-4}} - \sqrt{\frac{x-4}{x-4}} \right) = \lim_{x\to+\infty} x \cdot \frac{\sqrt{x-3} - \sqrt{x-4}}{\sqrt{x-4}} =$$

$$= \lim_{x\to+\infty} \frac{x}{\sqrt{(x-3)(x-4)} + \sqrt{(x-4)^2}} = \lim_{x\to+\infty} \frac{x}{x\left(1-\frac{4}{x}\right) + x\sqrt{1-\frac{7}{x}+\frac{12}{x^2}}} =$$

$$= \frac{1}{2}$$

$$\Rightarrow \boxed{g_1: y = x + \frac{3}{2} \text{ наклонена асимптота при } x \to +\infty}$$

$$k_2 = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{(-x-1)\sqrt{\frac{x-3}{x-4}}}{x} = \lim_{x\to-\infty} \left(\frac{-x\sqrt{\frac{x-3}{x-4}}}{x} - \frac{\sqrt{\frac{x-3}{x-4}}}{x} \right) =$$

$$= -1$$

$$b_2 = \lim_{x\to-\infty} \left(f(x) - kx \right) = \lim_{x\to-\infty} \left((-x-1)\sqrt{\frac{x-3}{x-4}} + x \right) =$$

$$= \lim_{x\to-\infty} \left(-x\cdot\left(\sqrt{\frac{x-3}{x-4}} - 1\right) - \sqrt{\frac{x-3}{x-4}} \right) = -\frac{1}{2} - 1 = -\frac{3}{2}$$

$$\Rightarrow \boxed{g_2: y = -x - \frac{3}{2} \text{ наклонена асимп. при } x \to -\infty}$$

5. Екстремуми.

За $x \geq -1$: $f'(x) = \left((x+1)\sqrt{\frac{x-3}{x-4}} \right)' =$

$$= \sqrt{\frac{x-3}{x-4}} + (x+1) \cdot \frac{1}{2} \cdot \left(\frac{x-3}{x-4} \right)^{-\frac{1}{2}} \cdot \left(\frac{x-3}{x-4} \right)' =$$

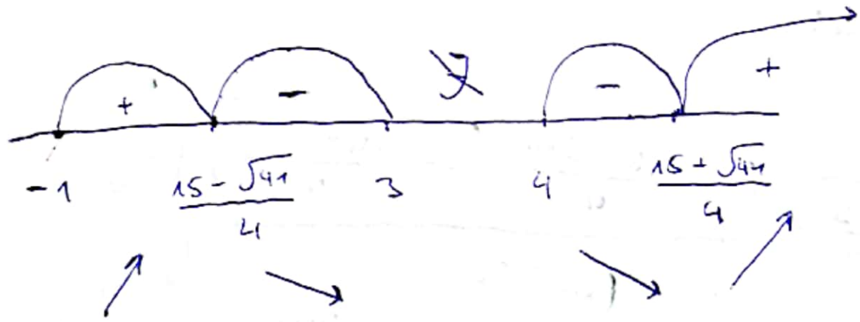
$$= \sqrt{\frac{x-3}{x-4}} + \frac{x+1}{2} \sqrt{\frac{x-4}{x-3}} \cdot \frac{x-4-(x-3)}{(x-4)^2} =$$

$$= \sqrt{\frac{x-3}{x-4}} - \frac{x+1}{2(x-4)^2} \sqrt{\frac{x-4}{x-3}} = \sqrt{\frac{x-3}{x-4}} \left(1 - \frac{(x+1)(x-4)}{2(x-4)^2(x-3)} \right) =$$

$$= \sqrt{\frac{x-3}{x-4}} \cdot \frac{2x^2 - 15x + 23}{2(x-4)(x-3)} = \boxed{\sqrt{\frac{x-3}{x-4}} \cdot \frac{\left(x - \frac{15-\sqrt{41}}{4}\right)\left(x - \frac{15+\sqrt{41}}{4}\right)}{(x-4)(x-3)}}$$

$f'(x) = 0$ при $x = \frac{15-\sqrt{41}}{4}$, $x = \frac{15+\sqrt{41}}{4}$

$f'(x) \nexists$ при $x \in [3, 4]$



лок. max: $x = \frac{15-\sqrt{41}}{4}$

лок. min: $x = 3$, $x = \frac{15+\sqrt{41}}{4}$.

За $x \leq -1$

$$f'(x) = \left((-x-1)\sqrt{\frac{x-3}{x-4}} \right)' = \left(-(x+1)\sqrt{\frac{x-3}{x-4}} \right)' =$$

$$= -\sqrt{\frac{x-3}{x-4}} \cdot \frac{\left(x - \frac{15-\sqrt{41}}{4}\right)\cdot\left(x - \frac{15+\sqrt{41}}{4}\right)}{(x-4)(x-3)}$$

$f'(x)\ \exists\ \forall x \leq -1$, $f'(x) < 0$, $\forall x \leq -1$

В $x = -1$ лявата и дясната производна не съвпадат

$f'(-1^+) = -f'(-1^-) \Rightarrow \nexists f'(-1) \Rightarrow$ $x = -1$ е лок. min

6. Инфлексни точки:

За $x \geq -1$:

$$f''(x) = \left(\sqrt{\frac{x-3}{x-4}} \cdot \frac{\left(x - \frac{15-\sqrt{41}}{4}\right)\left(x - \frac{15+\sqrt{41}}{4}\right)}{(x-4)(x-3)} \right)' =$$

$$= \frac{1}{2}\sqrt{\frac{x-4}{x-3}} \cdot \frac{-1}{(x-4)^2} \cdot \frac{\left(x - \frac{15-\sqrt{41}}{4}\right)\left(x - \frac{15+\sqrt{41}}{4}\right)}{(x-4)(x-3)} +$$

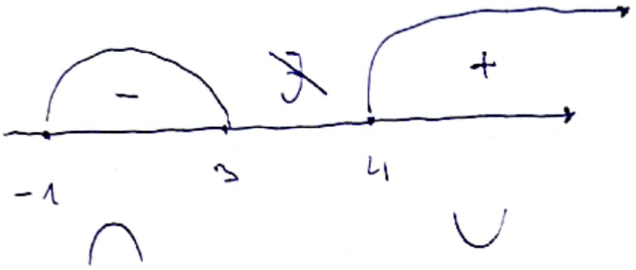
$$+ \sqrt{\frac{x-3}{x-4}} \cdot \frac{\left(2x - \frac{15}{2}\right)(x-3)(x-4) - \left(x - \frac{15-\sqrt{41}}{4}\right)\left(x - \frac{15+\sqrt{41}}{4}\right)(2x-7)}{(x-4)^2(x-3)^2} =$$

$$= \frac{1}{4(x-4)^2(x-3)^2} \cdot \sqrt{\frac{x-3}{x-4}} \cdot \left[-2\left(x - \frac{15-\sqrt{41}}{4}\right)\left(x - \frac{15+\sqrt{41}}{4}\right) + 2(4x-15)(x-3)(x-4) - 2(4x-14)\left(x - \frac{15+\sqrt{41}}{4}\right)\left(x - \frac{15-\sqrt{41}}{4}\right) \right]$$

$$= \frac{1}{4(x-4)^2(x-3)^2} \cdot \sqrt{\frac{x-3}{x-4}} \cdot \left[-2\cdot(4x-13)\cdot\left(x^2-\frac{15}{2}x+\frac{23}{2}\right)+2(4x-15)\left(x^2-7x+12\right)\right] =$$

$$\boxed{= \frac{19}{4(x-4)^2(x-3)^2} \cdot \sqrt{\frac{x-3}{x-4}} \cdot \left(x-\frac{61}{19}\right) = f''(x)}$$

$f''(x) \nexists \quad x \in [3,4]$, $\quad f''(x) = 0$ при $x = \frac{61}{19}$, но $\frac{61}{19} \in [3,4]$



от $f'' \neq 0 \Rightarrow$ няма инфлексни точки.

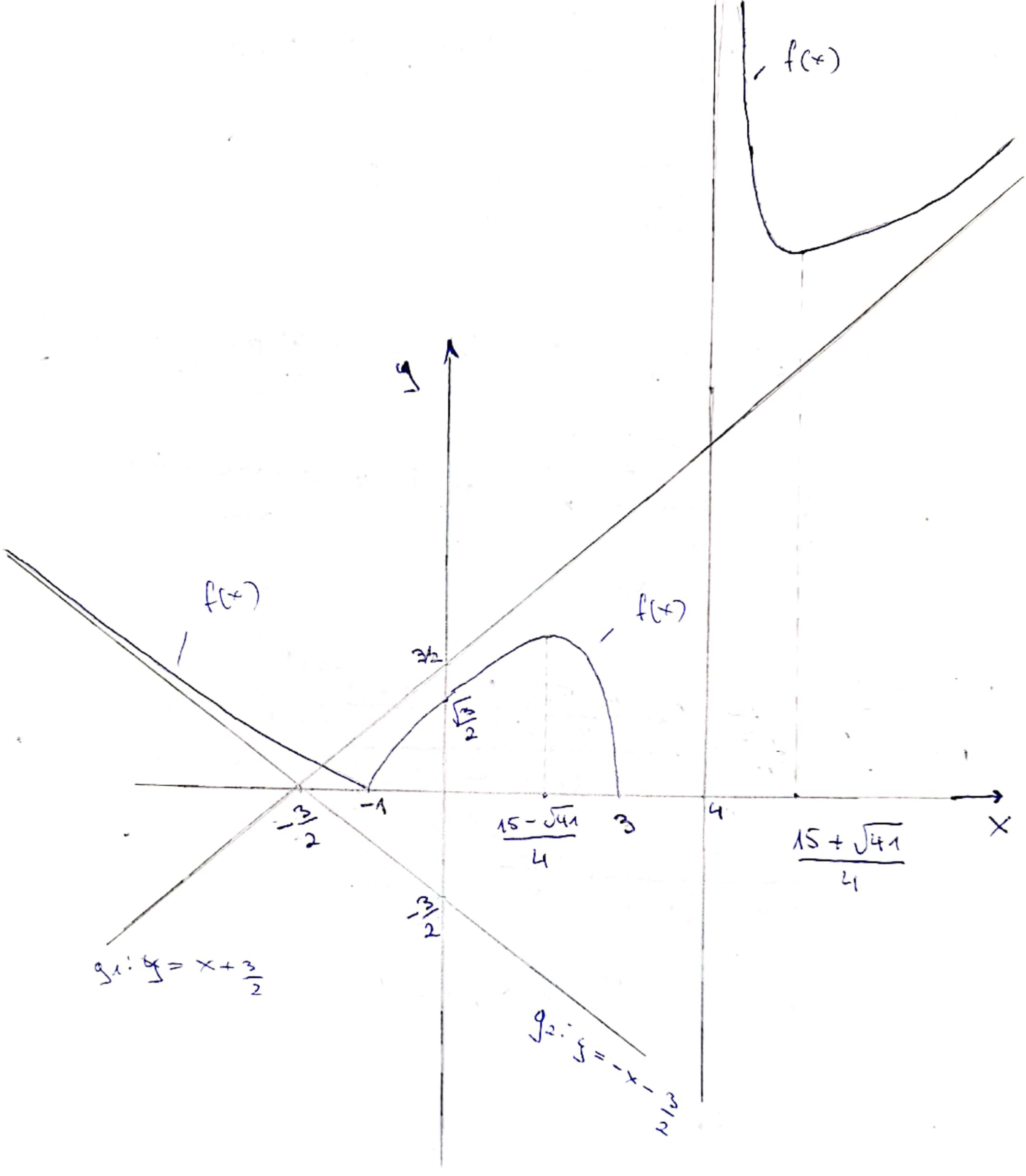
За $x \leq -1$

$$f''(x) = \left(-\sqrt{\frac{x-3}{x-4}} \cdot \frac{\left(x-\frac{15-\sqrt{41}}{4}\right)\cdot\left(x-\frac{15+\sqrt{41}}{4}\right)}{(x-4)(x-3)}\right)' =$$

$$= -\frac{19}{4(x-4)^2(x-3)^2} \cdot \sqrt{\frac{x-3}{x-4}} \left(x-\frac{61}{19}\right)$$

$\exists f''(x) \ \forall x \leq -1$, $\quad f''(x) \neq 0$, $x \leq -1$

$f''(-1^+) = -f''(-1^-) \Rightarrow f''(-1) \nexists$ – инфлексна точка



f(x)
y
f(x)
f(x)
3/2
$\frac{\sqrt{3}}{2}$
-1
$-\frac{3}{2}$
$\frac{15-\sqrt{41}}{4}$
3
4
$\frac{15+\sqrt{41}}{4}$
X
$-\frac{3}{2}$
$g_1: y = x + \frac{3}{2}$
$g_2: y = -x - \frac{3}{2}$

Зад. 2

$$f(x) = \frac{2x^2+4x+5}{x+3}$$

$$k = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{2x^2+4x+5}{x(x+3)} = \lim_{x\to\pm\infty} \frac{x^2(2+4/x+5/x^2)}{x^2(1+\frac{3}{x})} = \frac{2}{1} = 2$$

$$b = \lim_{x\to\pm\infty} (f(x) - kx) = \lim_{x\to\pm\infty} \left(\frac{2x^2+4x+5}{x+3} - \overset{x+3}{2x}\right) =$$

$$= \lim_{x\to\pm\infty} \frac{2x^2+4x+5-2x^2-6x}{x+3} = \lim_{x\to\pm\infty} \frac{-2x+5}{x+3} = -2$$

$\Rightarrow$ $\boxed{y = 2x - 2 \text{ - наклонена асимптота при } x \to \pm\infty}$

Зад. 3

$f(x) = \cos(2x)$, $x_0 = 0$, $f(1/100) = ?$

$$f^{(1)}(x) = (\cos(2x))' = -2\sin(2x)$$

$$f^{(2)}(x) = (-2\sin(2x))' = -4\cos(2x)$$

$$f^{(3)}(x) = (-4\cos(2x))' = 8\sin(2x)$$

$$f^{(4)}(x) = (8\sin(2x))' = 16\cos(2x)$$

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0)^1 + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots$$

$$\cos(2x) \approx \cos 0 + 0 + \frac{-4\cos 0}{2!}x^2 + 0 + \frac{16\cos 0 \cdot x^4}{4!} =$$

$$= 1 - 2x^2 + \frac{2}{3}x^4$$

$$f(1/100) = 1 - \frac{2}{100^2} + \frac{2}{3\cdot 100^4} = \frac{299\,940\,002}{300\,000\,000} \approx 0,9998$$

Зад. 4

$$\lim_{x\to+\infty} (2x+3)^{\frac{8}{3+\ln(2x+3)}} = \lim_{x\to+\infty} e^{\ln\left((2x+3)^{\frac{8}{3+\ln(2x+3)}}\right)} =$$

$$= e^{\lim\limits_{x\to+\infty}\frac{8\cdot\ln(2x+3)}{3+\ln(2x+3)}} = e^{\lim\limits_{x\to+\infty}\frac{8\cdot\ln(2x+3)}{\ln(2x+3)\left(1+\frac{3}{\ln(2x+3)}\right)}} = e^{8}$$