

Задача:

Вопрос: $\{f_1, f_2, f_3\} \models f_4$, верно?

$$f_1: \forall x \neg \forall y (q(y, x) \Rightarrow \exists z (q(y, z) \& \neg r(z, x)))$$

$$f_2: \forall x (\forall y (q(x, y) \Rightarrow \exists z (q(y, z) \& q(x, z))) \Rightarrow \neg \exists z q(x, z))$$

$$f_3: \forall z \forall y (r(z, y) \Rightarrow \neg \exists x (q(y, x) \& \neg q(z, x)))$$

$$f_4: \forall x \exists y (q(x, x) \Rightarrow \exists z (r(z, y) \& \neg r(z, y)))$$

Решение:

Узнаем: $A \Rightarrow B \models \neg A \vee B$. Тогда

$$f_1 \models \forall x \neg \forall y (\neg q(y, x) \vee \exists z (q(y, z) \& \neg r(z, x)))$$

$$\models \forall x \exists y (q(y, x) \& \forall z (\neg q(y, z) \vee r(z, x)))$$

$$\models \forall x \exists y \forall z (q(y, x) \& (\neg q(y, z) \vee r(z, x)))$$

$$f_1 \stackrel{\text{соедин}}{\sim} \forall x \forall z (q(f(x), x) \& (\neg q(f(x), z) \vee r(z, x)))$$

$$y = f(x)$$

$$D_1 = \{q(f(x_1), x_2)\}$$

$$D_2 = \{\neg q(f(x_2), z_2), r(z_2, x_2)\}$$

$$f_2 \models \forall x (\forall y (\neg q(x, y) \vee \exists z (q(y, z) \& q(x, z))) \\ \Rightarrow \neg \exists z q(x, z))$$

$$\models \forall x (\neg \forall y (\neg q(x, y) \vee \exists z (q(y, z) \& q(x, z))) \\ \vee \neg \exists z q(x, z))$$

$$\models \forall x (\exists y (q(x, y) \& \forall z (\neg q(y, z) \vee \neg q(x, z))) \\ \vee \forall z \neg q(x, z))$$

$$\models \forall x \exists y \forall z \forall t ((q(x, y) \vee \neg q(x, t)) \& \\ (\neg q(y, z) \vee \neg q(x, z) \vee \neg q(x, t)))$$

$$f_2 \stackrel{\text{skolem}}{\sim} \forall x \forall z \forall t ((q(x, g(x)) \vee \neg q(x, t)) \& \\ (\neg q(g(x), z) \vee \neg q(x, z) \vee \neg q(x, t)))$$

$$D_3 = \{q(x_3, g(x_3)), \neg q(x_3, t_3)\}$$

$$D_4 = \{\neg q(g(x_4), z_4), \neg q(x_4, z_4), \neg q(x_4, t_4)\}$$

$$f_3 \models \forall z \forall y (\neg r(z, y) \vee \neg \exists x (q(y, x) \& \neg q(z, x)))$$

$$\models \forall z \forall y (\neg r(z, y) \vee \forall x (\neg q(y, x) \vee q(z, x)))$$

$$\models \forall z \forall y \forall x (\neg r(z, y) \vee \neg q(y, x) \vee q(z, x))$$

$$D_5 = \{\neg r(z_5, y_5), \neg q(y_5, x_5), q(z_5, x_5)\}$$

$$\neg f_4 \models \neg \forall x \exists y \neg q(x, x) \\ \models \exists x \forall y q(x, x)$$

$$\neg f_4 \overset{\text{scolem}}{\sim} \forall y q(a, a) \\ x=a \quad D_6 = \{q(a, a)\}$$

$$1. \text{ Hena } \sigma: \begin{cases} x_3 = f(x_1) \\ t_3 = x_1 \end{cases}, \text{ corolara}$$

$$\text{Res}(D_1, D_3) = \{q(f(x_1), q(f(x_1)))\}$$

$$D_7 = \{q(f(x_7), q(f(x_7)))\}$$

$$2. \text{ Hena } \sigma: \begin{cases} x_2 = x_7 \\ z_2 = q(f(x_7)) \end{cases}, \text{ corolara}$$

$$q(f(x_2), z_2) = q(f(x_7), q(f(x_7))) \sim$$

$$\text{Res}(D_7, D_2) = \{r(q(f(x_7)), x_7)\}$$

$$D_8 = \{r(q(f(x_8)), x_8)\}$$

$$3. \text{ Hena } \sigma: \begin{cases} z_5 = q(f(x_8)) \\ x_8 = y_5 \end{cases}, \text{ corolara } q(f(y_5))$$

$$\text{Res}(D_8, D_5) = \{\neg q(y_5, x_5), q(z_5, x_5)\}$$

$$D_9 = \{\neg q(y_9, x_9), q(q(f(y_9)), x_9)\}$$

4. Here $\sigma: \begin{cases} f(yg) = x_4, \text{ number} \\ x_3 = z_4 \end{cases}$

$$\begin{aligned} \text{Res}(D_g, D_4) &= \{ \neg q(yg, x_3), \neg q(x_4, z_4), \neg q(x_4, t_4) \} \\ &= \{ \neg q(yg, x_3), \neg q(f(yg), x_3), \\ &\quad \neg q(f(yg), z_4) \} \end{aligned}$$

$$D_{10} = \{ \neg q(y_{10}, x_{10}), \neg q(f(y_{10}), x_{10}), \neg q(f(y_{10}), t_{10}) \}$$

5. Here $\sigma: \begin{cases} yg = a \\ x_3 = a \end{cases}, \text{ number}$

$$\text{Res}(D_6, D_g) = \{ q(g(f(a)), a) \} = D_{11}$$

6. Here $\sigma: \begin{cases} x_4 = f(a) \\ z_4 = a \end{cases}, \text{ number}$

$$\text{Res}(D_{11}, D_4) = \{ \neg q(f(a), a), \neg q(f(a), t_4) \}$$

$$D_{12} = \{ \neg q(f(a), a), \neg q(f(a), t_{12}) \}$$

7. $D_{13} = \text{Col}(D_{12}) = \{ \neg q(f(a), a) \}$
 $t_{12} = a$

8. $\text{Res}(D_{13}, D_1) = \boxed{\text{X}}$