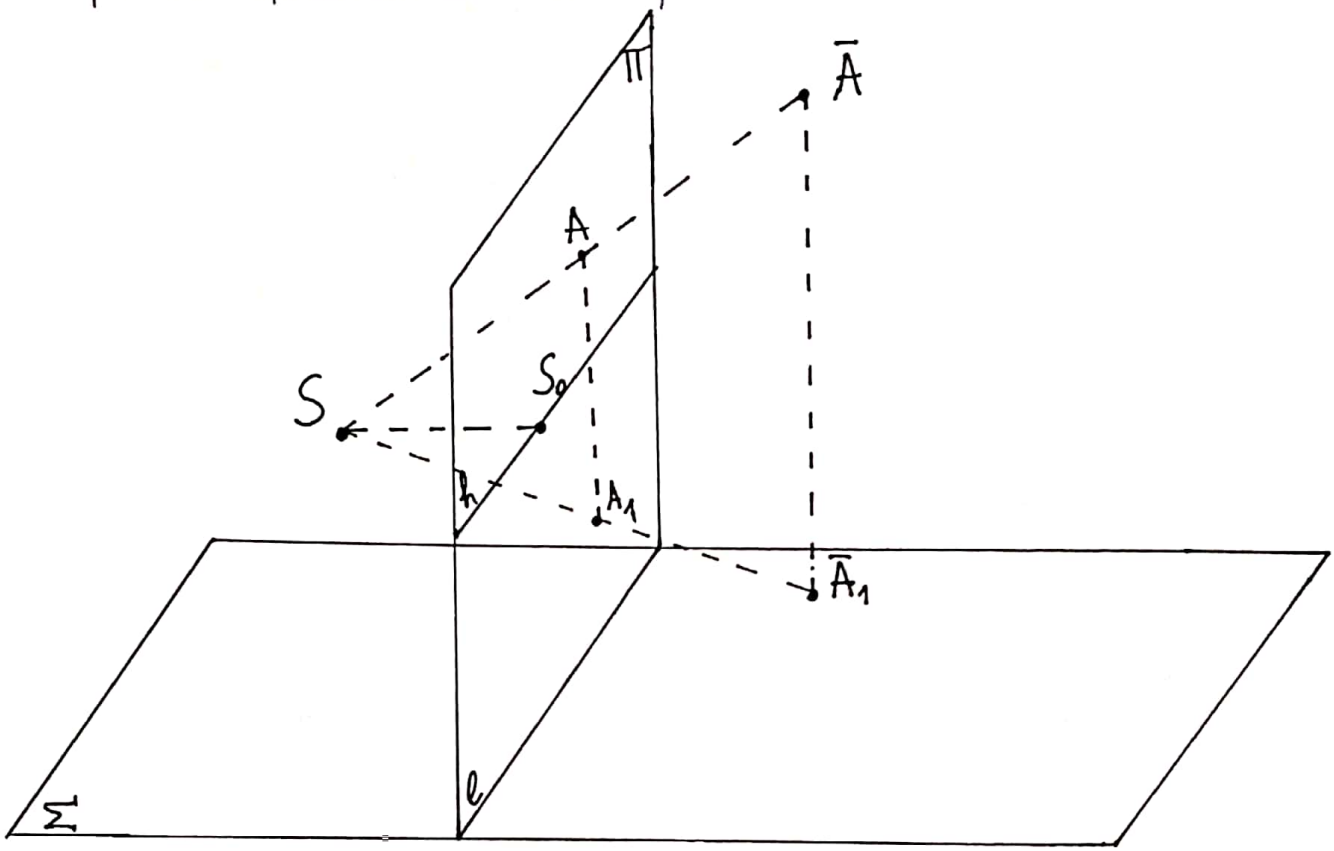
Перспектива

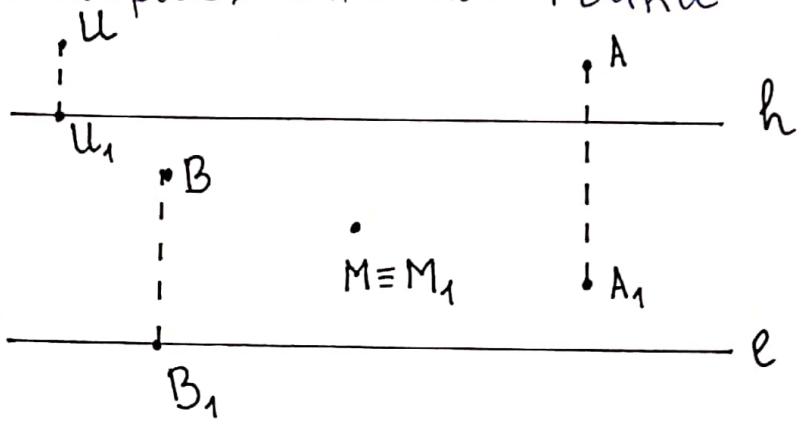
I Проекционен апарат



* Σ' - предметна равнина
* Π - картинна равнина (проекционна) $\Sigma' \perp \Pi$
* S - гледна точка, проекционен център
  $S \notin \Sigma'$, $S \notin \Pi$, S - крайна точка
* $S_0 = \text{орт. пр.}_{\Pi} S$
  S_0 - главна точка на картината
* $\ell = \Sigma' \cap \Pi$ - основа на картината
* Нека $\Sigma'_0 \begin{cases} \ni S \\ \parallel \Sigma' \end{cases} \Rightarrow h = \Sigma'_0 \cap \Pi$

  h - хоризонт

II Изобразяване на точки



* $\bar{A}$ - точка от пространството, крайна
  $A = S\bar{A} \cap \Pi$, A - перспектива на т. $\bar{A}$
  $\bar{A}_1 =$ орт. пр.$_{\Sigma'}$ $\bar{A}$
  $A_1 = S\bar{A}_1 \cap \Pi$, A_1 - вторична проекция на $\bar{A}$
  $\bar{A}(A, A_1) \Leftrightarrow AA_1 \perp e$
* $\bar{M}(M, M_1)$, $\bar{M} \supset \Sigma \Leftrightarrow M \equiv M_1$
* $\bar{B}(B, B_1)$, $\bar{B} \supset \Pi \Leftrightarrow B_1 \supset e$
* $\bar{U}(U, U_1)$, $\bar{U} \supset \Omega \Leftrightarrow U_1 \supset h$

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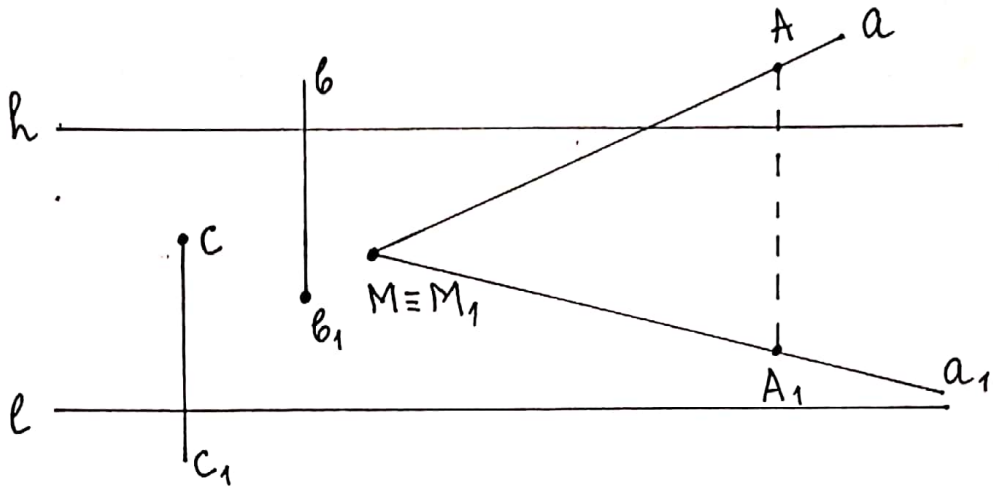
III Изобразяване на прави

$\bar{a}$ - крайна права
$\bar{a} \cap \Sigma' = \bar{M}$
$\bar{a}_1 =$ орт. пр.$_{\Sigma'}$ $\bar{a} \Rightarrow \bar{a}_1 \supset \bar{M}$
Нека $\bar{\alpha} = (S, \bar{a})$, $a = \bar{\alpha} \cap \Pi$
a - перспектива на $\bar{a}$
$\bar{\alpha}_1 = (S, \bar{a}_1)$, $a_1 = \bar{\alpha}_1 \cap \Pi$
a_1 - вторична проекция на $\bar{a}$

$\bar{a}(a, a_1)$
$\bar{a} \supset \bar{A}(A, A_1) \Leftrightarrow$
$A \supset a$
$A_1 \supset a_1$
$AA_1 \perp e$



* Ако $\bar{b} \perp \bar{\Sigma}_1$, $\bar{b}(b, b_1)$ => b_1 е точка
b – права $\perp \ell$
$b \geq b_1$

* Ако $\bar{c} \geq S$, $\bar{c}(c, c_1)$ => c е точка
c_1 – права $\perp \ell$
$c_1 \geq c$

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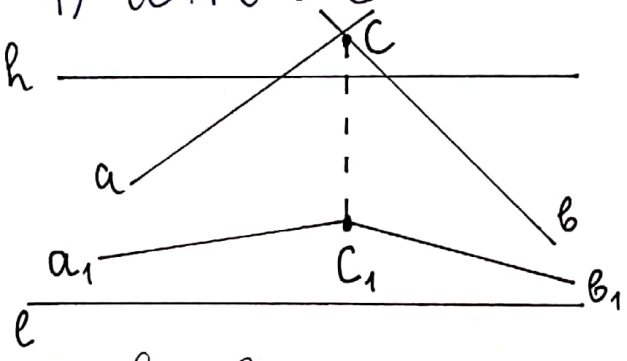
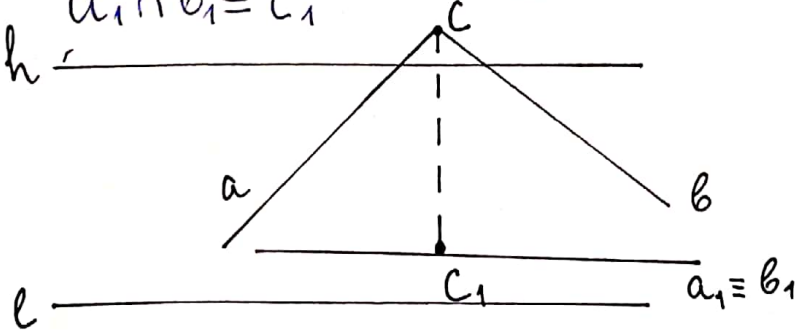
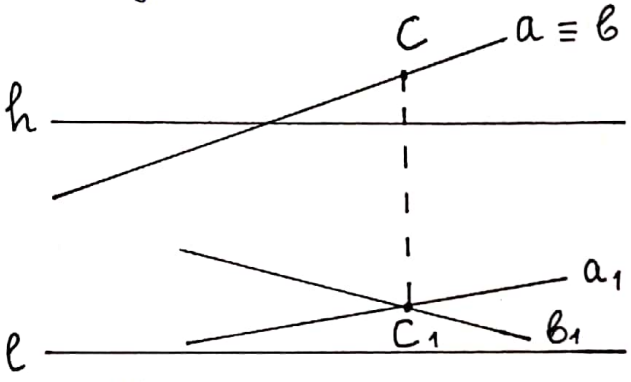
III Взаимни положения на две прави

1) $\bar{a} \cap \bar{b} = \bar{C}$



$a \cap b = C$
$a_1 \cap b_1 = C_1$
$CC_1 \perp \ell$





$\bar{a}, \bar{b} \geq \bar{\lambda}$, $\bar{\lambda} \geq S$ => $a \equiv b$
$CC_1 \perp \ell$

$\bar{a}, \bar{b} \geq \bar{\beta}$, $\bar{\beta} \perp \bar{\Sigma}_1$ => $a_1 \equiv b_1$
$CC_1 \perp \ell$

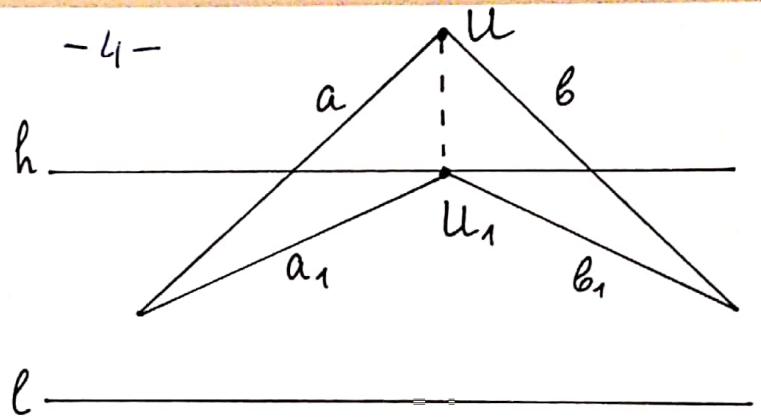
2) $\bar{a} \parallel \bar{b}$, $\bar{a} \cap \bar{b} = \bar{u}$

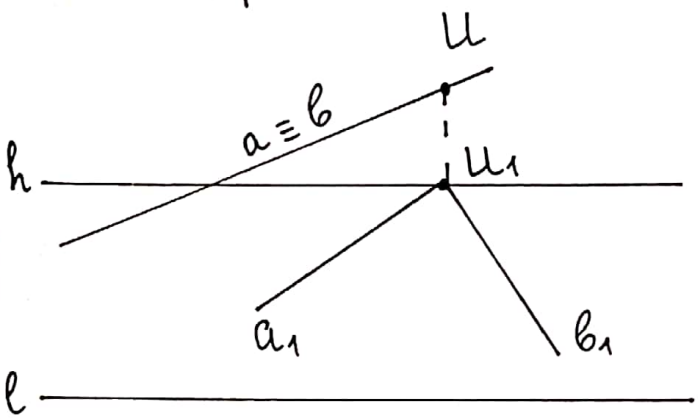
$a \cap b = u$

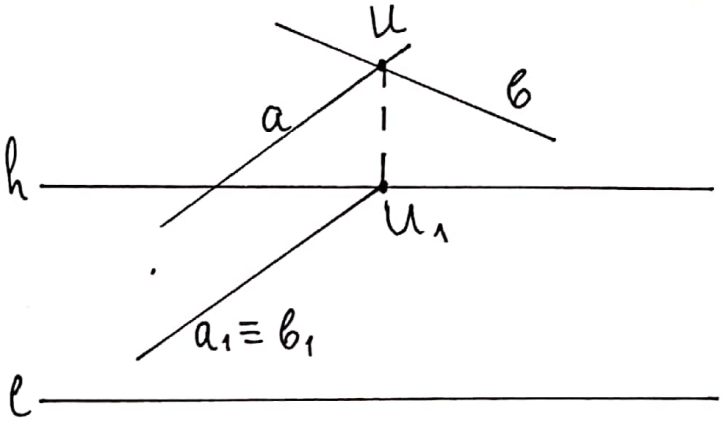
$a_1 \cap b_1 = u_1$

$uu_1 \perp h$

$u_1 \subset h$







$\bar{a}, \bar{b} \subset \bar{\alpha}, \bar{\alpha} \subset S$

$uu_1 \perp h$, $u_1 \subset h$

$\bar{a}, \bar{b} \subset \bar{\beta}, \bar{\beta} \perp \bar{\Sigma}$

$uu_1 \perp h$, $u_1 \subset h$

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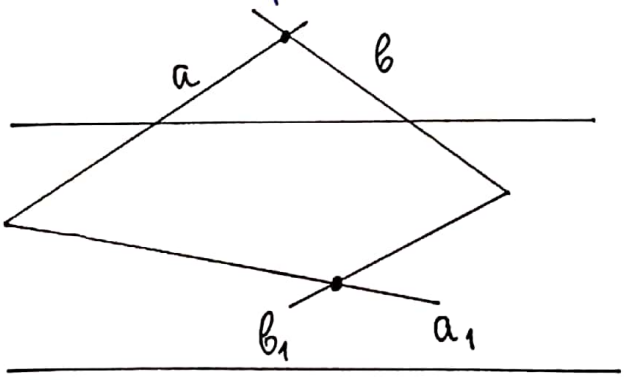
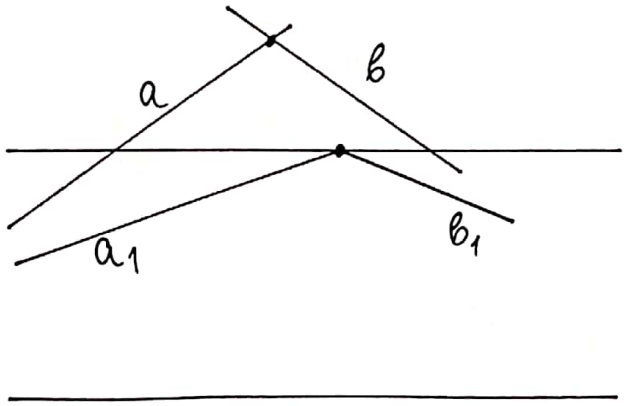
3) $\bar{a}, \bar{b}$ – кръстосани





$\bar{a}_1 \parallel \bar{b}_1 \subset \bar{\Sigma}$

IV Стопки на права: $\bar{a}\,(a, a_1)$

* $\bar{M}^{\bar{a}} = \bar{a} \cap \Sigma$, $\bar{M}^{\bar{a}}(M^{\bar{a}}, M_1^{\bar{a}})$, то $M^{\bar{a}} \equiv M_1^{\bar{a}} = a \cap a_1$
  $M^{\bar{a}} \in a$, $M_1^{\bar{a}} \in a_1$

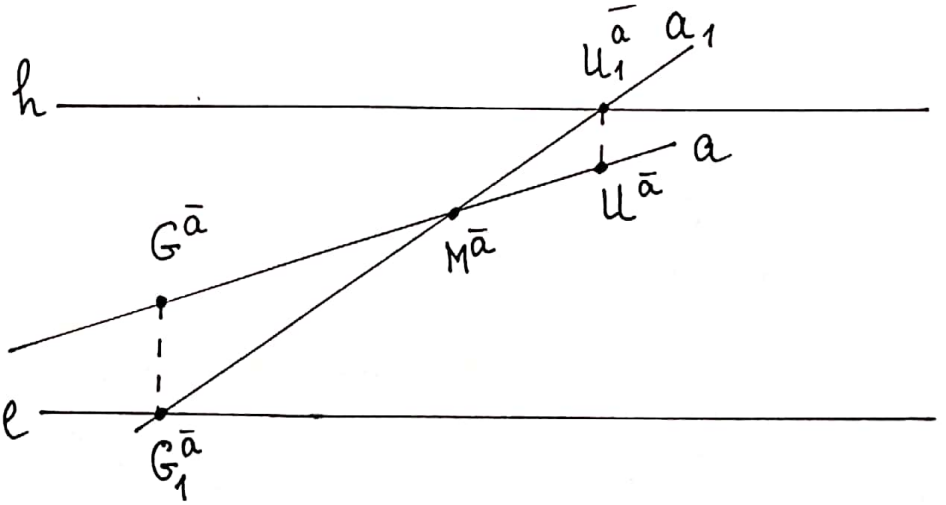
* $\bar{G}^{\bar{a}} = \bar{a} \cap \Pi$, $\bar{G}^{\bar{a}}(G^{\bar{a}}, G_1^{\bar{a}})$, то $G_1^{\bar{a}} \in \ell \Rightarrow G_1^{\bar{a}} = \ell \cap a_1$
  ↳ прабод
  $G^{\bar{a}} \in a$, $G_1^{\bar{a}} \in a_1$

* $\bar{U}^{\bar{a}} = \bar{a} \cap \Omega$, $\bar{U}^{\bar{a}}(U^{\bar{a}}, U_1^{\bar{a}})$, то $U_1^{\bar{a}} \in h \Rightarrow U_1^{\bar{a}} = h \cap a_1$
  ↳ безкрайна точка на $\bar{a}$
  ↳ убежна точка на $\bar{a}$

Задача: В перспектива е дадена $\bar{a}\,(a, a_1)$
Да се изобразят стопките на правата.

Построение:

1) $M^{\bar{a}} \equiv M_1^{\bar{a}} = a \cap a_1$
2) $G_1^{\bar{a}} = a_1 \cap \ell$
3) $G^{\bar{a}} \begin{cases} \in a \\ G_1^{\bar{a}} G^{\bar{a}} \perp \ell \end{cases}$
4) $U_1^{\bar{a}} = a_1 \cap h$
5) $U^{\bar{a}} \begin{cases} \in a \\ U_1^{\bar{a}} U^{\bar{a}} \perp h \end{cases}$



Задача: В перспектива са дадени $\bar{a}(a,a)$ и $\bar{A}(A,A_1)$: $\bar{A} \not\subset \bar{a}$. Да се изобрази $\bar{B}\begin{cases} \in \bar{A} \\ \parallel \bar{a} \end{cases}$.

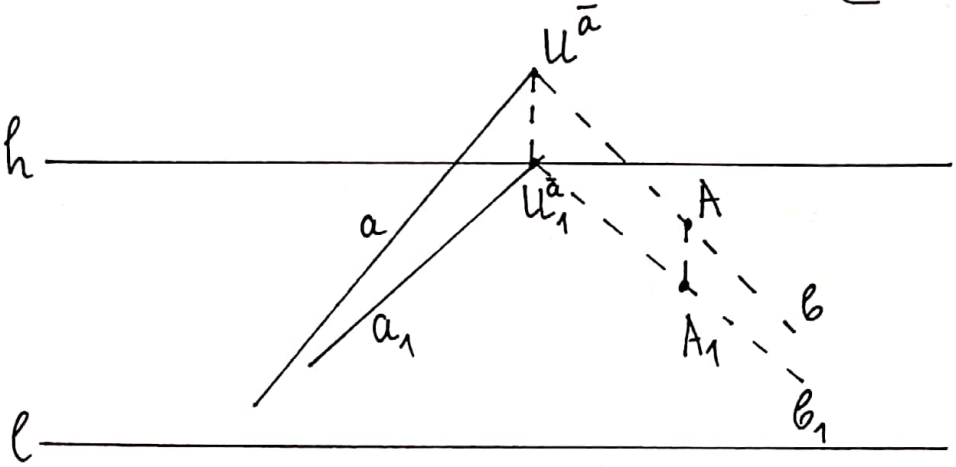
Построение:

1) $U_1^{\bar{a}} = a_1 \cap h$

2) $U^{\bar{a}}\begin{cases} \in a \\ U^{\bar{a}}U_1^{\bar{a}} \perp h \end{cases}$

3) $B_1\begin{cases} \in A_1 \\ \in U_1^{\bar{a}} \end{cases}$

4) $B\begin{cases} \in A \\ \in U^{\bar{a}} \end{cases}$



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V Изобразяване на равнина

$\bar{\alpha}$ - равнина

$\bar{\alpha} \cap \pi = \bar{g}^{\bar{\alpha}}$, ако $\bar{g}^{\bar{\alpha}}(g^{\bar{\alpha}}, g_1^{\bar{\alpha}})$, то $\bar{g}^{\bar{\alpha}} \equiv g^{\bar{\alpha}}$ и $g_1^{\bar{\alpha}} \equiv \ell$

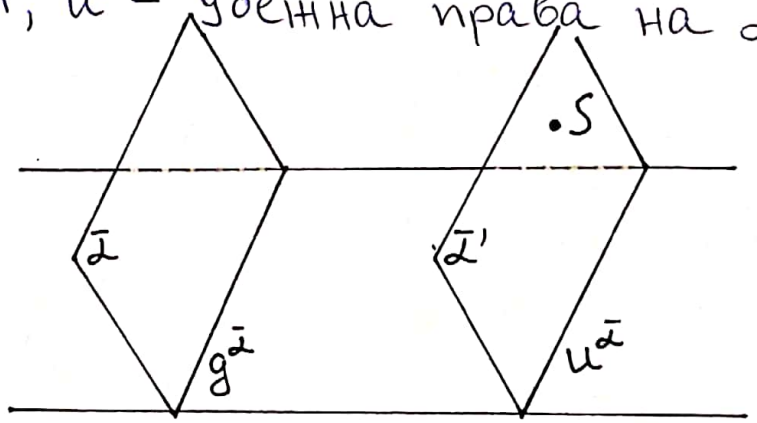
$\bar{\alpha} \cap \Omega = \bar{u}^{\bar{\alpha}}$ → безкрайна права на $\bar{\alpha}$

$\bar{\alpha}'\begin{cases} \parallel \bar{\alpha} \\ \in S \end{cases} \Rightarrow u^{\bar{\alpha}} = \bar{\alpha}' \cap \pi$, $u^{\bar{\alpha}}$ - убежна права на $\bar{\alpha}$

$\bar{u}^{\bar{\alpha}}(u^{\bar{\alpha}}, u_1^{\bar{\alpha}})$, то $u_1^{\bar{\alpha}} \equiv h$

$\bar{\alpha}[g^{\bar{\alpha}}, u^{\bar{\alpha}}]$

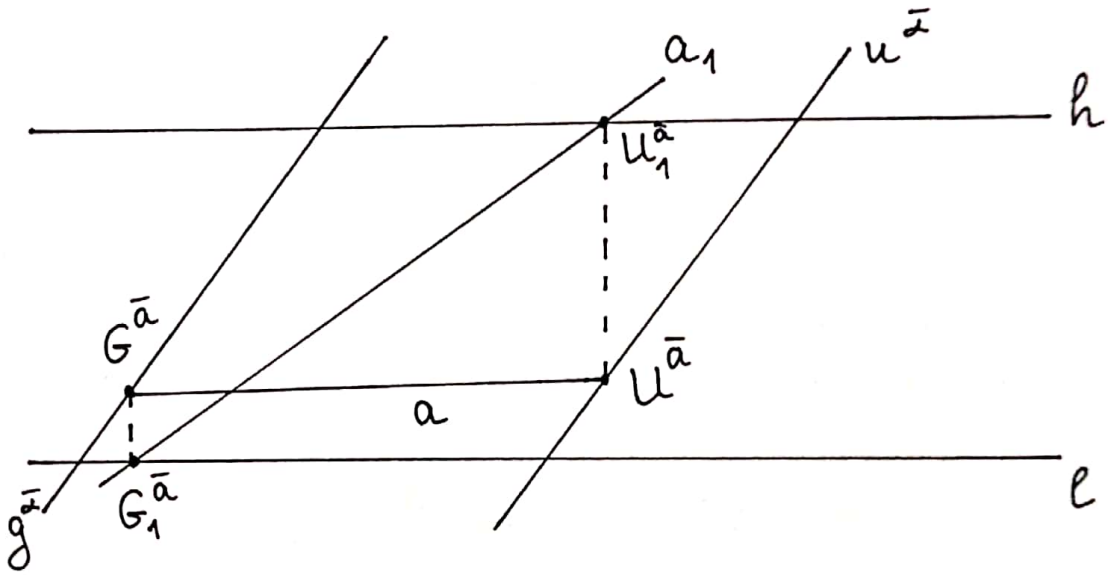
$g^{\bar{\alpha}} \parallel u^{\bar{\alpha}}$



VI Инцидентност на права и равнина

$\bar{a} \mathrel{Z} \bar{\alpha} \Leftrightarrow \begin{vmatrix} \bar{G}^{\bar{a}} \mathrel{Z} \bar{g}^{\bar{\alpha}} \\ \bar{u}^{\bar{a}} \mathrel{Z} \bar{u}^{\bar{\alpha}} \end{vmatrix} \Leftrightarrow \begin{vmatrix} G^{\bar{a}} \mathrel{Z} g^{\bar{\alpha}}, & G_1^{\bar{a}} = a_1 \cap \ell \\ u^{\bar{a}} \mathrel{Z} u^{\bar{\alpha}}, & u_1^{\bar{a}} = a_1 \cap h \end{vmatrix}$

Задача: В перспектива са дадени $\alpha\,[g^{\alpha}, u^{\alpha}]$, $g^{\alpha} \parallel u^{\alpha}$ и a_1 – вторична проекция на $\bar{a}$. Да се построи а-перспектива на $\bar{a}$: $\bar{a} \mathrel{Z} \bar{\alpha}$.

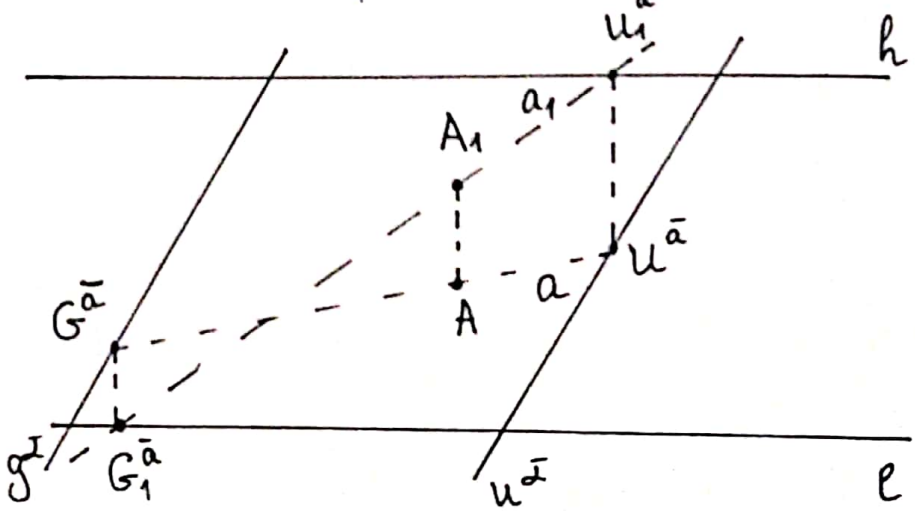


Построение:

1) $G_1^{\bar{a}} = a_1 \cap \ell$
2) $G^{\bar{a}} \begin{cases} Z\, g^{\alpha} \\ G^{\bar{a}} G_1^{\bar{a}} \perp \ell \end{cases}$
3) $u_1^{\bar{a}} = a_1 \cap h$
4) $u^{\bar{a}} \begin{cases} Z\, u^{\alpha} \\ u^{\bar{a}} u_1^{\bar{a}} \perp h \end{cases}$
5) $a \begin{cases} Z\, G^{\bar{a}} \\ Z\, u^{\bar{a}} \end{cases}$

VII Инцидентност на точка и равнина

Задача: $\begin{cases} \alpha\,[g^{\alpha}, u^{\alpha}] \\ A_1 \end{cases}$. Да се изобрази A : $\bar{A}(A, A_1) \mathrel{Z} \bar{\alpha}$



Построение:

1) a_1 – произволна права през т. A_1
2) a – от предх. задача
3) $A \begin{cases} Z\, a \\ AA_1 \perp \ell \end{cases}$

VIII Успоредни равнини

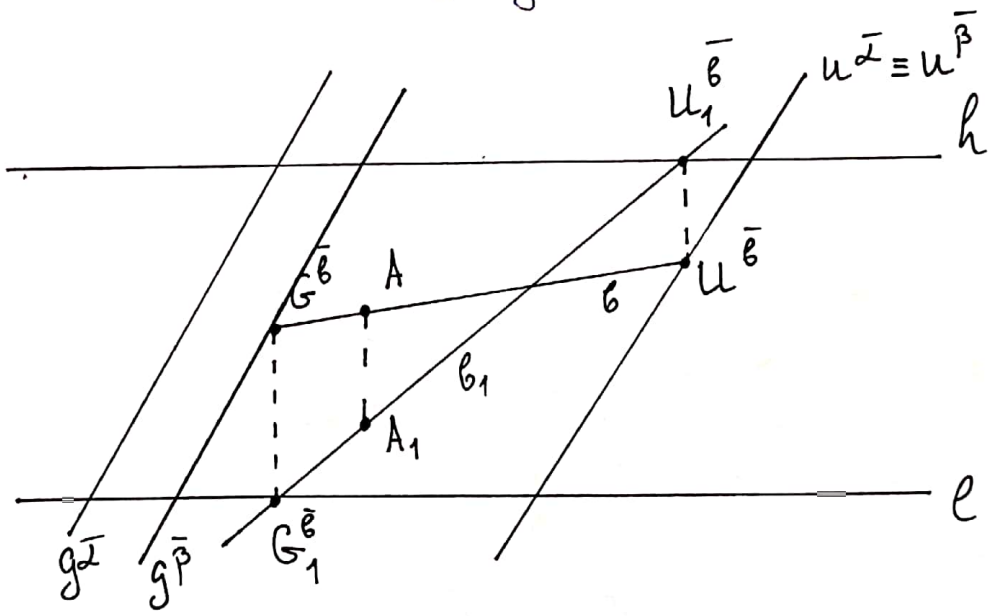
$\bar{\alpha} \parallel \bar{\beta} \Leftrightarrow \bar{u}^{\bar{\alpha}} \equiv \bar{u}^{\bar{\beta}} \Rightarrow \begin{matrix} \bar{\alpha}\,[g^{\bar{\alpha}}, u^{\bar{\alpha}}] \\ \bar{\beta}\,[g^{\bar{\beta}}, u^{\bar{\beta}}] \end{matrix} \Rightarrow \begin{matrix} g^{\bar{\alpha}} \parallel g^{\bar{\beta}} \\ u^{\bar{\alpha}} \equiv u^{\bar{\beta}} \end{matrix}$

Задача: Дадени са:
$\bar{\alpha}\,[g^{\bar{\alpha}}, u^{\bar{\alpha}}]$ и т. $\bar{A}(A, A_1) : \bar{A} \not\in \bar{\alpha}$.
Да се изобрази равнината $\bar{\beta} \begin{cases} \supset \bar{A} \\ \parallel \bar{\alpha} \end{cases}$

! Необходима е помощна права $\bar{b} \begin{cases} \subset \bar{\beta} \\ \supset \bar{A} \end{cases}$.
Ще построим $G^{\bar{b}}$. Тогава $g^{\bar{\beta}} \begin{cases} \supset G^{\bar{b}} \\ \parallel g^{\bar{\alpha}} \end{cases}$.

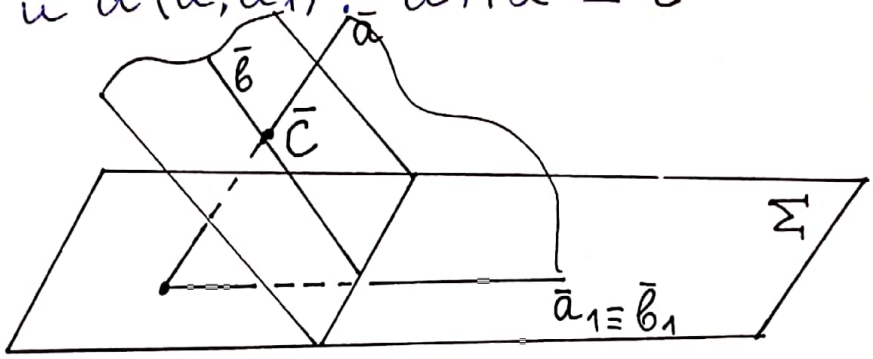
Построение:

1) Изб. $u^{\bar{b}} \supset u^{\bar{\beta}}$
2) $u_1^{\bar{b}} \begin{cases} \supset h \\ u^{\bar{b}} u_1^{\bar{b}} \perp h \end{cases}$
3) $b_1 \begin{cases} \supset u_1^{\bar{b}} \\ \supset A_1 \end{cases}$
4) $b \begin{cases} \supset u^{\bar{b}} \\ \supset A \end{cases}$
5) $G_1^{\bar{b}} = b_1 \cap \ell$
6) $G^{\bar{b}} \begin{cases} \supset b \\ G^{\bar{b}} G_1^{\bar{b}} \perp \ell \end{cases}$
7) $g^{\bar{\beta}} \begin{cases} \supset G^{\bar{b}} \\ \parallel g^{\bar{\alpha}} \end{cases}$



IX Пробод на права и равнина

Задача: В перспектива са дадени $\bar{\alpha}[g^{\bar{\alpha}}, u^{\bar{\alpha}}]$ и $\bar{a}(a, a_1)$; $\bar{a} \cap \bar{\alpha} = \bar{C}$



Да се изобрази $\bar{C}(C, C_1)$

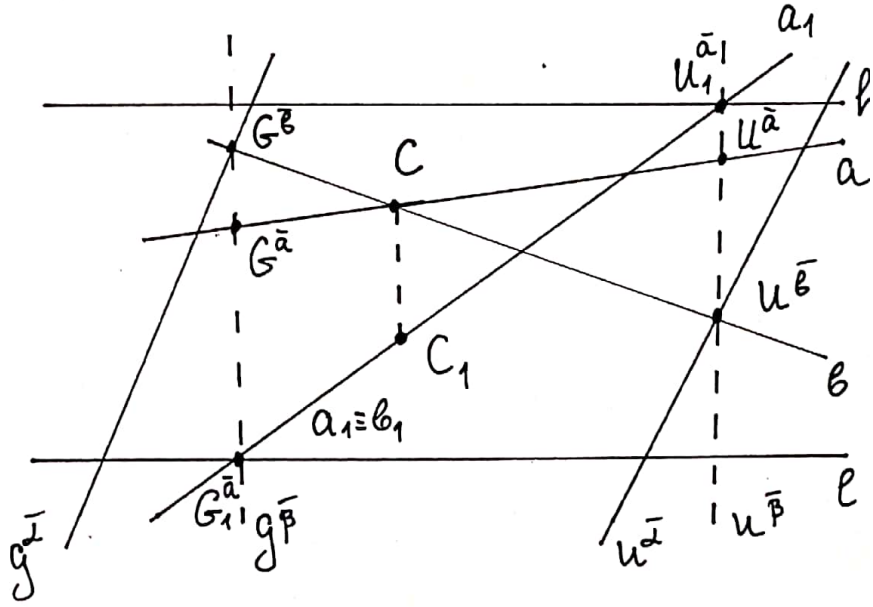
План:

1) Равнина $\bar{\beta}\begin{cases}\ni \bar{a} \\ \ni \bar{a}_1\end{cases} \Rightarrow \bar{\beta} \perp \Sigma_1 \Rightarrow g^{\bar{\beta}} \parallel u^{\bar{\beta}} \perp \ell$

2) $\bar{b} = \bar{\alpha} \cap \bar{\beta}$

3) $\bar{C} = \bar{a} \cap \bar{b}$

Построение:



1) $G_1^{\bar{a}} = a_1 \cap \ell$

$G^{\bar{a}}\begin{cases}\ni a \\ G^{\bar{a}}G_1^{\bar{a}} \perp \ell\end{cases}$

2) $u_1^{\bar{a}} = a_1 \cap h$

$u^{\bar{a}}\begin{cases}\ni a \\ u^{\bar{a}}u_1^{\bar{a}} \perp h\end{cases}$

3) $g^{\bar{\beta}}\begin{cases}\ni G^{\bar{a}} \\ \perp \ell\end{cases}$

4) $u^{\bar{\beta}}\begin{cases}\ni u^{\bar{a}} \\ \perp h\end{cases}$

5) $G^{\bar{b}} = g^{\bar{\alpha}} \cap g^{\bar{\beta}}$

6) $u^{\bar{b}} = u^{\bar{\alpha}} \cap u^{\bar{\beta}}$

7) $b\begin{cases}\ni G^{\bar{b}} \\ \ni u^{\bar{b}}\end{cases}$

8) $C = a \cap b$

$C_1\begin{cases}\ni a_1 \\ CC_1 \perp \ell\end{cases}$