# Аксонометрия

## I Проекционен апарат

$\bar{K} = \bar{O}\bar{x}\bar{y}\bar{z}$ – ОКС

$\pi$ – проекционна равнина

$\ell$ – проектиращо направление

$\ell \nparallel \pi$, $\ell \nparallel \bar{O}\bar{x}\bar{y}, \bar{O}\bar{y}\bar{z}, \bar{O}\bar{x}\bar{z}$

$\bar{E}_1, \bar{E}_2, \bar{E}_3$ – единични точки

$\bar{O} \xrightarrow{\psi_\pi^{\text{ме}}} O$, $\bar{E}_i \xrightarrow{\psi_\pi^{\text{ме}}} \bar{E}_i$, $i = \overline{1,3}$

$\vec{OE_i} = \vec{e_i} \Rightarrow K = O\vec{e_1}\vec{e_2}\vec{e_3}$ – аксоном. к.с.

$O\vec{e_1}, O\vec{e_2}, O\vec{e_3}$ – аксоном. оси

$|\vec{e_1}| = p$, $|\vec{e_2}| = q$, $|\vec{e_3}| = r$

$p, q, r$ – коефициенти на изменение

При $p = q = r$ $\psi$ се нарича изометрия

При $p = r \neq q$ $\psi$ е диметрия

При $p \neq q \neq r \neq p$ $\psi$ е триметрия

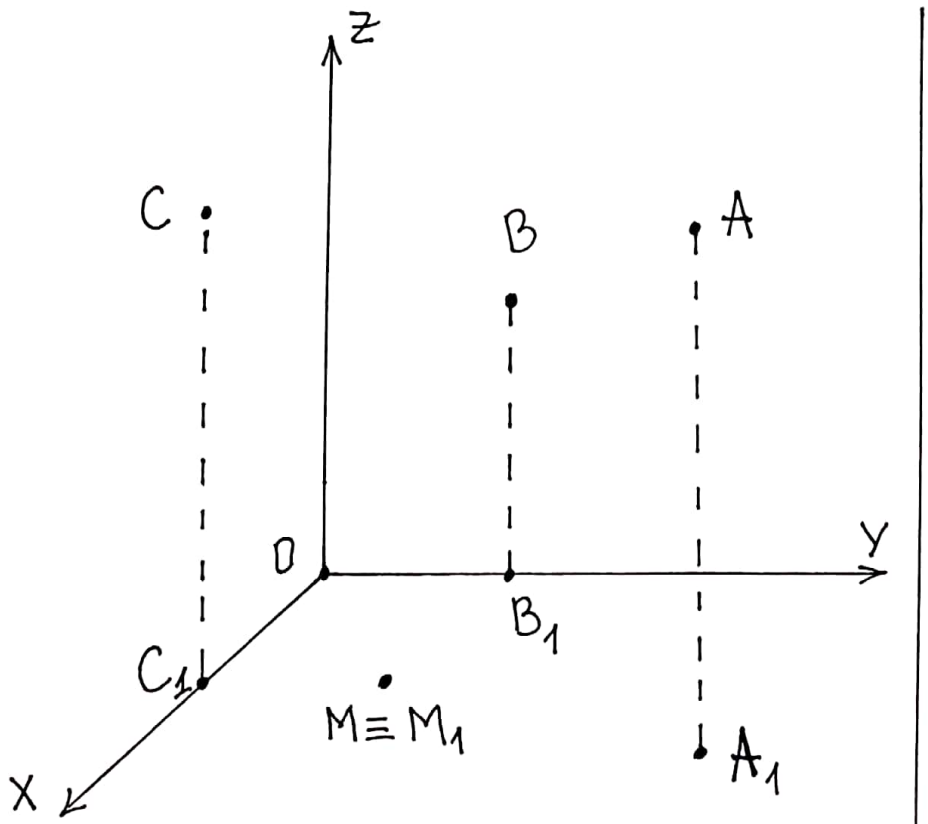
## II Изобразяване на точки

$\bar{A}$ – точка от пространството
$\bar{A}_1$ = орт. пр. $\bar{A}$ в $\bar{O}\bar{x}\bar{y}$
$\Rightarrow \bar{A}\bar{A}_1 \parallel \bar{O}\bar{z}$

$A: \begin{cases} \supset \pi \\ A\bar{A} \parallel \ell \end{cases}$, A – аксонометрична проекция на $\bar{A}$

$A_1: \begin{cases} \supset \pi \\ \bar{A}_1 A_1 \parallel \ell \end{cases}$, $A_1$ – първа вторична проекция на $\bar{A}$

При аксонометрия $\bar{A}(A; A_1)$ : $AA_1 \parallel O_z$




$\bar{M} \supset \bar{O}\bar{x}\bar{y} \Leftrightarrow$
$M \equiv M_1$

$\bar{B} \supset \bar{O}\bar{y}\bar{z} \Leftrightarrow B_1 \equiv O$

$\bar{C} \supset \bar{O}\bar{x}\bar{z} \Leftrightarrow C_1 \supset O_x$

# III Изобразяване на прави

$\bar{a}$ – права в пространството

$\bar{a}_1$ – орт. проекция на $\bar{a}$ в $\bar{O}\bar{X}\bar{Y}$

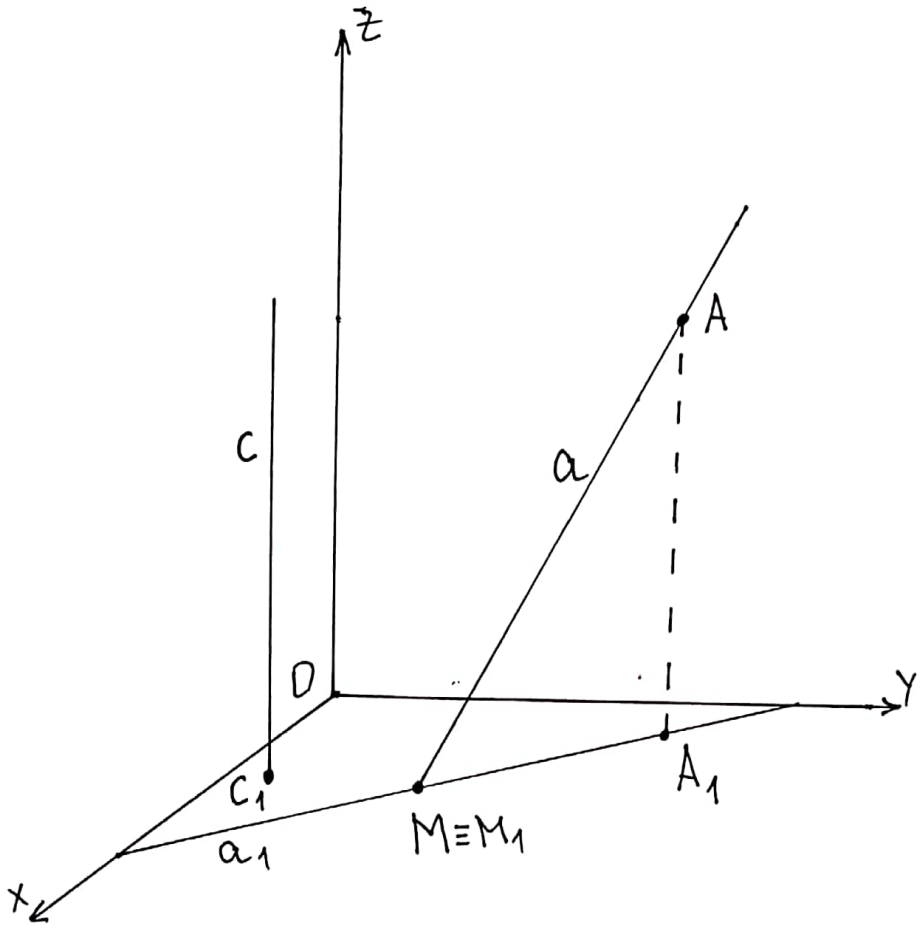
$a$ – образ на $\bar{a}$ върху $\pi$ при $\psi_\pi^{ue}$

$a$ – аксонометрична проекция на $\bar{a}$

$a_1$ – образ на $\bar{a}_1$ върху $\pi$ при $\psi$

$a_1$ – първа вторична проекция на $\bar{a}$

$\bar{a}(a, a_1)$



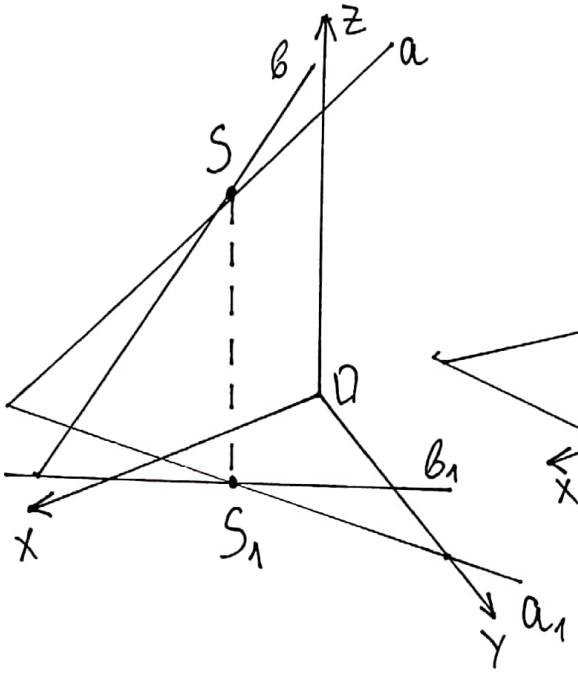

$\bar{A}(A, A_1) \in \bar{a} \Leftrightarrow$
$A \in a$ и $A_1 \in a_1$

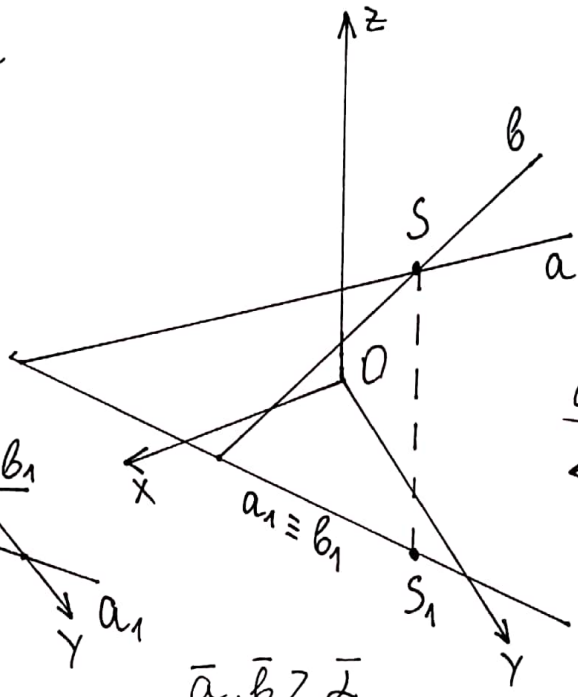
$\bar{C}(C, C_1)$
↳ точка $\Leftrightarrow$
$\bar{C} \parallel \bar{O}\bar{z}$

$\bar{B}(B, B_1)$
↳ точка $\Leftrightarrow \bar{B} \parallel \ell$

## IV Взаимни положения на две прави $\bar{a}(a, a_1)$ и $\bar{b}(b, b_1)$

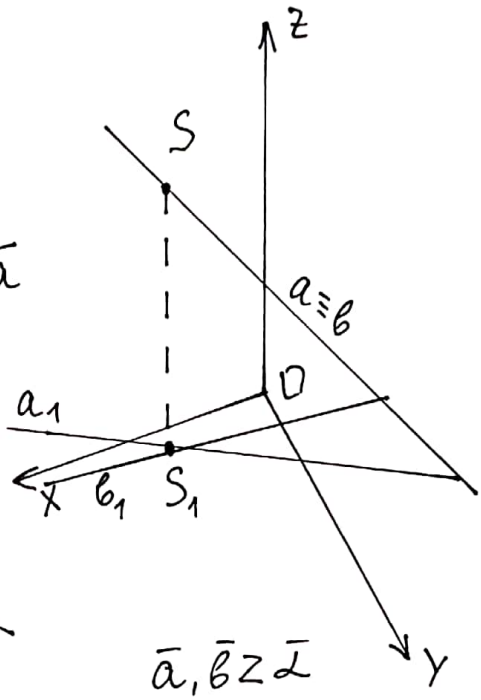
1) $\bar{a} \cap \bar{b} = \bar{S} \Rightarrow \bar{S}(S, S_1)$: $SS_1 \parallel Oz$




$\bar{a}, \bar{b} \subset \bar{\alpha}$

$\bar{\alpha} \parallel \bar{Oz}$




$\bar{a}, \bar{b} \subset \bar{\alpha}$

$\bar{\alpha} \parallel \ell$

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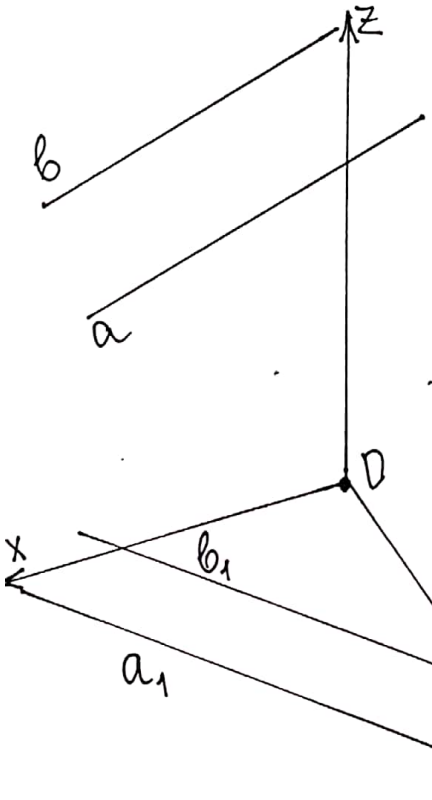
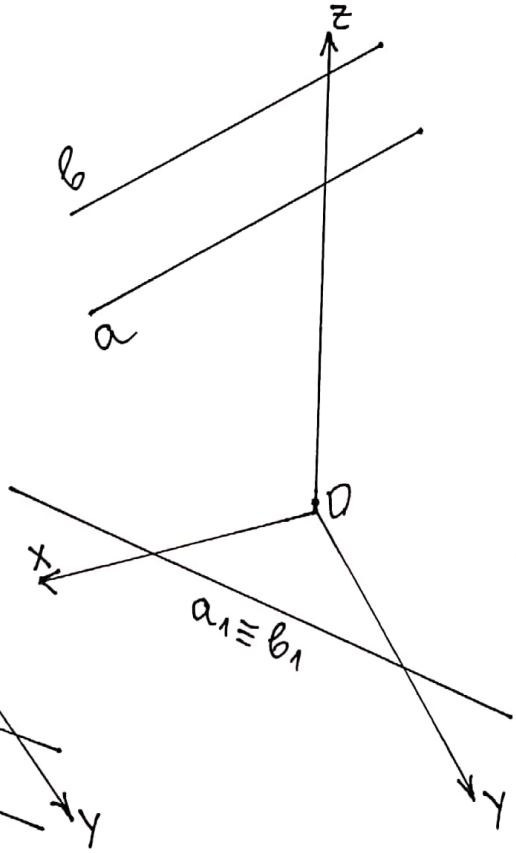
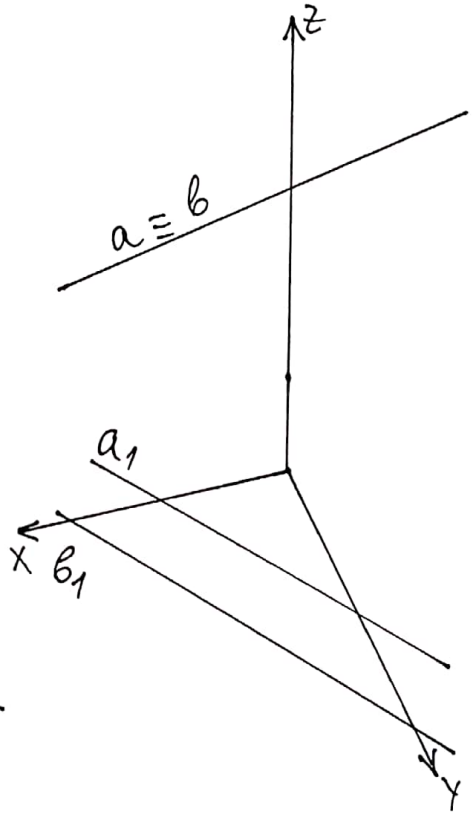
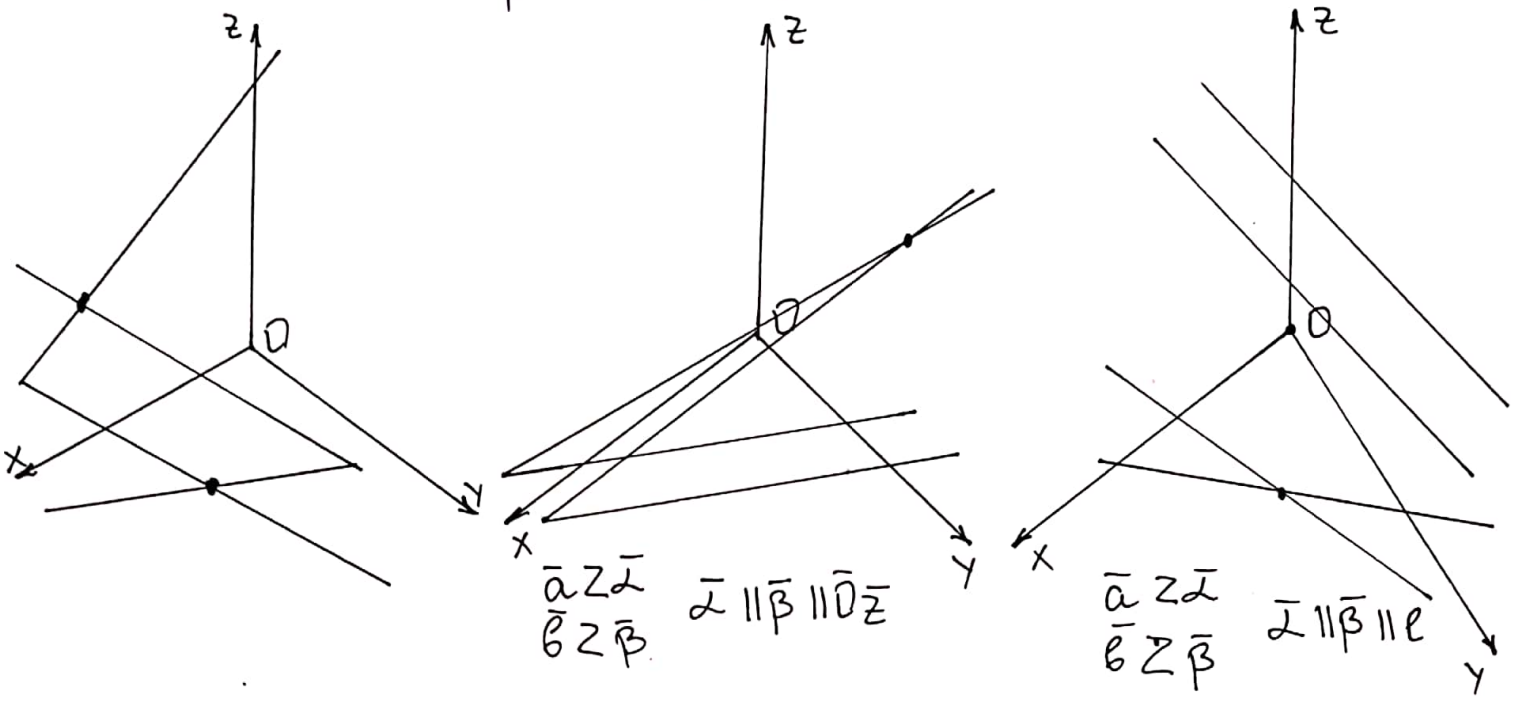
2) $\bar{a} \parallel \bar{b}$



3) $\bar{a}$ и $\bar{b}$ – кръстосани

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V Стъпки на права: $\bar{a}\,(a, a_1)$

$\bar{a} \cap \bar{\Omega}_{\bar{x}\bar{y}} \Rightarrow \bar{M}^{\bar{a}}(M^{\bar{a}}, M_1^{\bar{a}})$, $M^{\bar{a}} \equiv M_1^{\bar{a}} = a \cap a_1$
($M^{\bar{a}} \in z_a$, $M_1^{\bar{a}} \in z_{a_1}$)

$\bar{a} \cap \bar{\Omega}_{\bar{x}\bar{z}} = \bar{N}^{\bar{a}}(N^{\bar{a}}, N_1^{\bar{a}})$, $N_1^{\bar{a}} \subset \Omega_x \Rightarrow N_1^{\bar{a}} = a_1 \cap \Omega_x$
($N^{\bar{a}} \in z_a$, $N_1^{\bar{a}} \in z_{a_1}$)

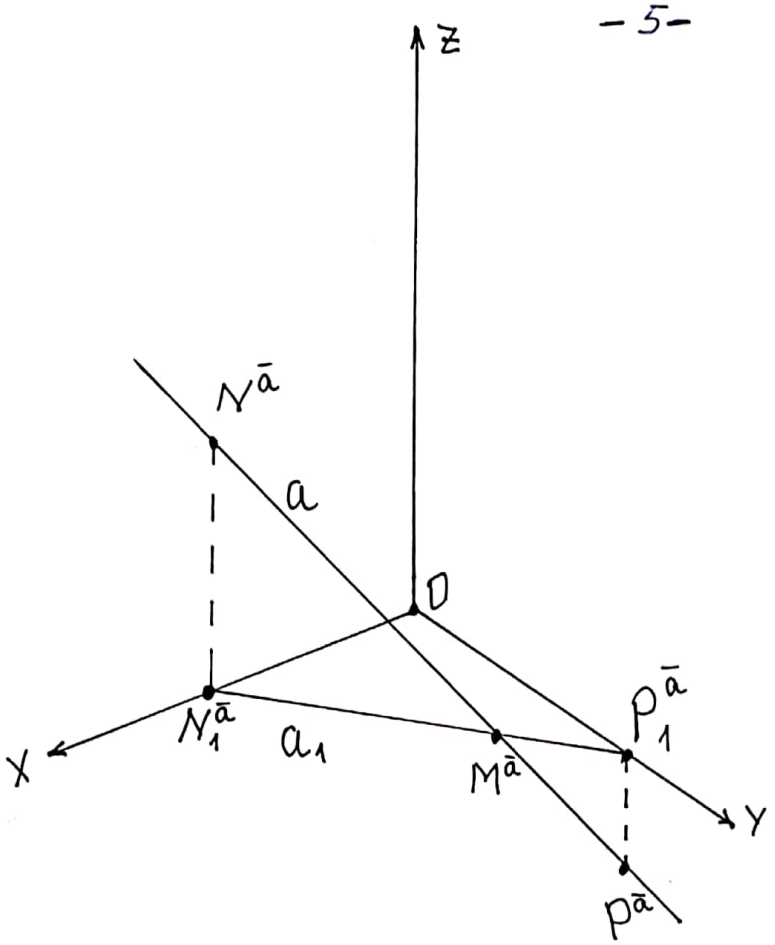
$\bar{a} \cap \bar{\Omega}_{\bar{y}\bar{z}} = \bar{P}^{\bar{a}}(P^{\bar{a}}, P_1^{\bar{a}})$, $P_1^{\bar{a}} \subset \Omega_y \Rightarrow P_1^{\bar{a}} = a_1 \cap \Omega_y$
($P^{\bar{a}} \in z_a$, $P_1^{\bar{a}} \in z_{a_1}$)

/Задача:

В аксонометрия е дадена права $\bar{a}\,(a, a_1)$.

Да се изобразят стъпките на $\bar{a}$

Построение:




1) $M^{\bar{a}} \equiv M_1^{\bar{a}} = a \cap a_1$

2) $N_1^{\bar{a}} = a_1 \cap O_X$

3) $N^{\bar{a}}$: $\begin{cases} Z\, a \\ N^{\bar{a}} N_1^{\bar{a}} \parallel O_Z \end{cases}$

4) $P_1^{\bar{a}} = a_1 \cap O_Y$

5) $P^{\bar{a}}$: $\begin{cases} Z\, a \\ P^{\bar{a}} P_1^{\bar{a}} \parallel O_Z \end{cases}$

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## VI Изобразяване на равнина

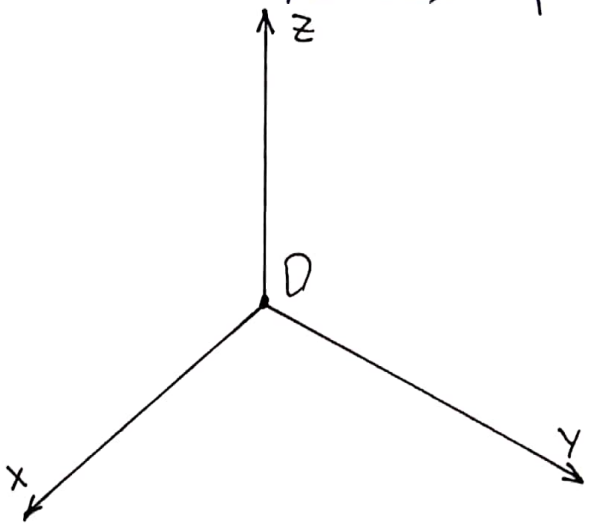
$\bar{\alpha}$ – равнина от пространството

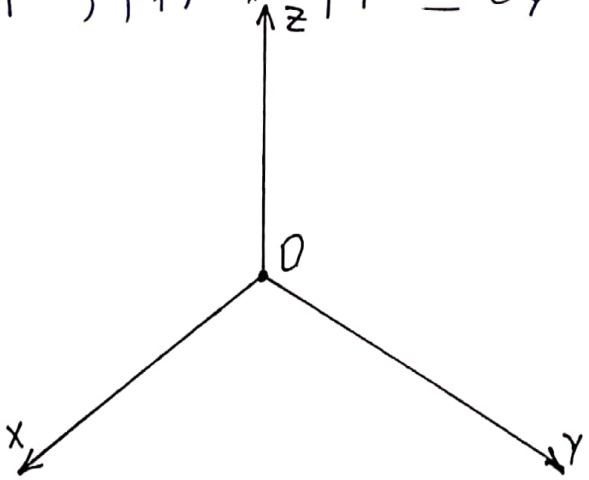
Следи (дири) на $\bar{\alpha}$:

$\bar{m}^{\bar{\alpha}} = \bar{\alpha} \cap \bar{O}_{\bar{X}\bar{Y}} \Rightarrow \bar{m}^{\bar{\alpha}} (m^{\bar{\alpha}}, m_1^{\bar{\alpha}}) : m^{\bar{\alpha}} \equiv m_1^{\bar{\alpha}}$

$\bar{n}^{\bar{\alpha}} = \bar{\alpha} \cap \bar{O}_{\bar{X}\bar{Z}} \Rightarrow \bar{n}^{\bar{\alpha}} (n^{\bar{\alpha}}, n_1^{\bar{\alpha}}) : n_1^{\bar{\alpha}} \equiv O_X$

$\bar{p}^{\bar{\alpha}} = \bar{\alpha} \cap O_{\bar{Y}\bar{Z}} \Rightarrow \bar{p}^{\bar{\alpha}} (p^{\bar{\alpha}}, p_1^{\bar{\alpha}}) : p_1^{\bar{\alpha}} \equiv O_Y$



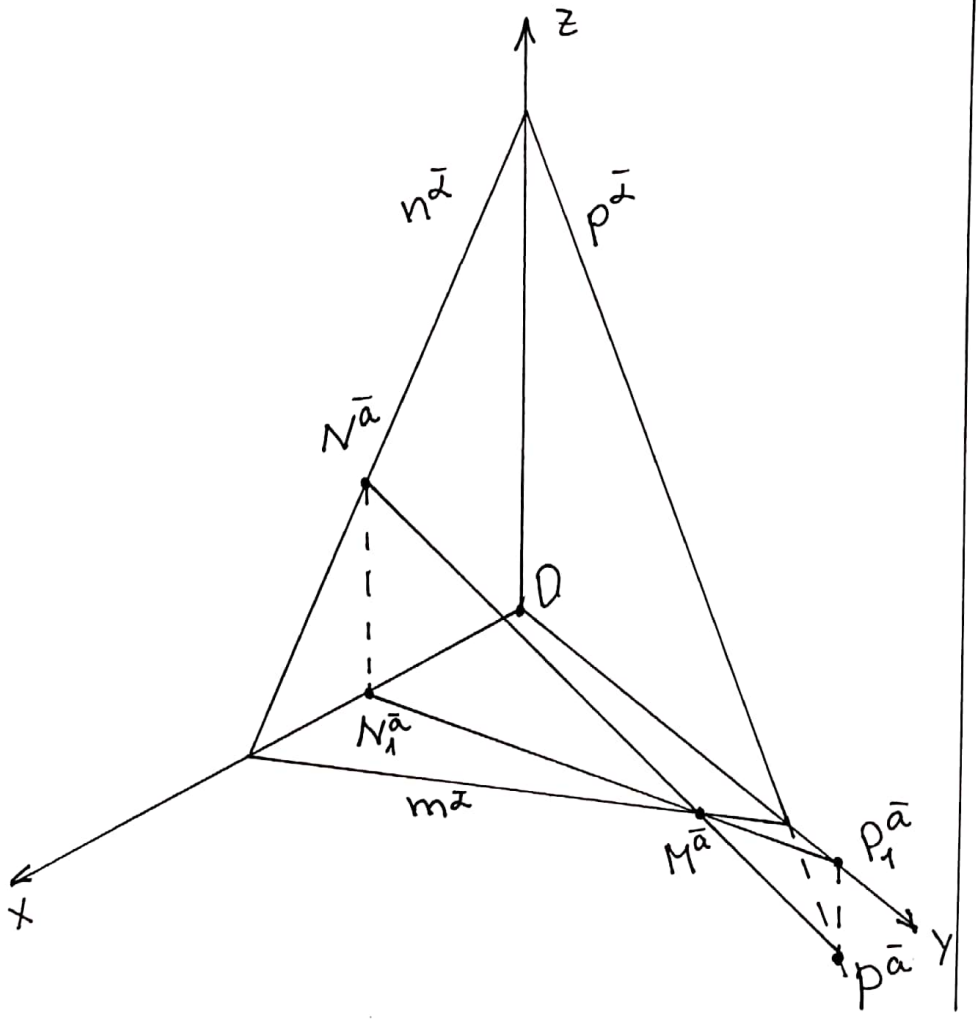



## VII Инцидентност на права и равнина

$$\bar{a} \subset \bar{\alpha} \Leftrightarrow \begin{cases} \bar{M}^{\bar{a}} \subset \bar{m}^{\bar{\alpha}} \Rightarrow M^{\bar{a}} \cap M_1^{\bar{a}} = a \cap a_1 \cap m^{\bar{\alpha}} \\ \bar{N}^{\bar{a}} \subset \bar{n}^{\bar{\alpha}} \Rightarrow N_1^{\bar{a}} = a_1 \cap O_x,\ N^{\bar{a}} \subset n^{\bar{\alpha}} \\ \bar{P}^{\bar{a}} \subset \bar{p}^{\bar{\alpha}} \Rightarrow P_1^{\bar{a}} = a_1 \cap O_y,\ P^{\bar{a}} \subset p^{\bar{\alpha}} \end{cases}$$

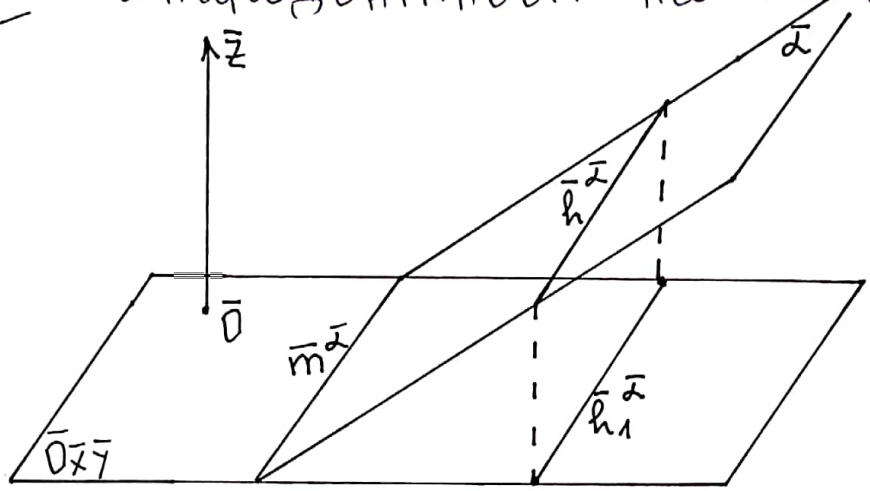
**Задача:** В аксонометрия са дадени равнина $\alpha(m^{\alpha}, n^{\alpha})$ и права $a_1$.

Да се построи права $a : (a, a_1) \to \bar{a} \subset \bar{\alpha}$.




1) $M^{\bar{a}} \equiv M_1^{\bar{a}} = a_1 \cap m^{\bar{\alpha}}$
2) $N_1^{\bar{a}} = a_1 \cap O_x$
3) $N^{\bar{a}} : \begin{cases} \subset n^{\bar{\alpha}} \\ N^{\bar{a}} N_1^{\bar{a}} \parallel O_z \end{cases}$
4) $a \begin{cases} \subset M^{\bar{a}} \\ \subset N^{\bar{a}} \end{cases}$
5) $P_1^{\bar{a}}$
6) $P^{\bar{a}}$

## VIII Инцидентност на точка и равнина



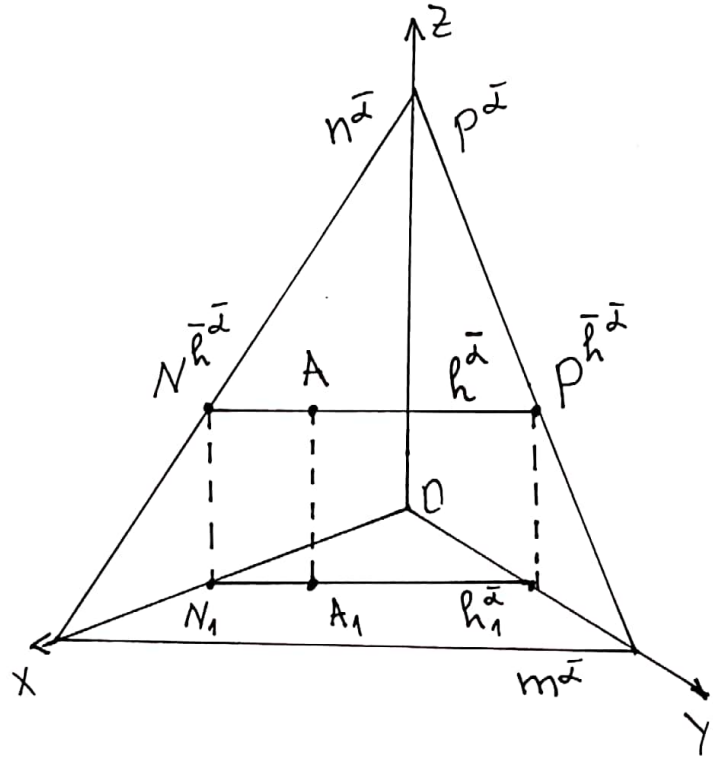

Главна права от I система за $\bar{\alpha}$

$h^{\bar{\alpha}} \begin{cases} \mathrm{Z}\, \bar{\alpha} \\ \parallel \bar{0}\bar{x}\bar{y} \end{cases} \Rightarrow$

$h^{\bar{\alpha}} \parallel m^{\bar{\alpha}} \Rightarrow$

$\Rightarrow h^{\bar{\alpha}}(h^{\bar{\alpha}}, h_1^{\bar{\alpha}})$: $h^{\bar{\alpha}} \parallel h_1^{\bar{\alpha}} \parallel m^{\bar{\alpha}}$

**Задача:** В аксонометрия са дадени равнина $\bar{\alpha}(m^{\bar{\alpha}}, n^{\bar{\alpha}})$ и точка $A_1$. Да се построи точка $A : (A, A_1) \to \bar{A} \in \bar{\alpha}$.




1) $h_1^{\bar{\alpha}} \begin{cases} \mathrm{Z}\, A_1 \\ \parallel m^{\bar{\alpha}} \end{cases}$

2) $N_1^{h^{\bar{\alpha}}} = h_1^{\bar{\alpha}} \cap 0x$

3) $N^{h^{\bar{\alpha}}} \begin{cases} \mathrm{Z}\, n^{\bar{\alpha}} \\ N^{h^{\bar{\alpha}}} N_1^{h^{\bar{\alpha}}} \parallel 0z \end{cases}$

4) $h^{\bar{\alpha}} \begin{cases} \mathrm{Z}\, N^{h^{\bar{\alpha}}} \\ \parallel m^{\bar{\alpha}} \end{cases}$

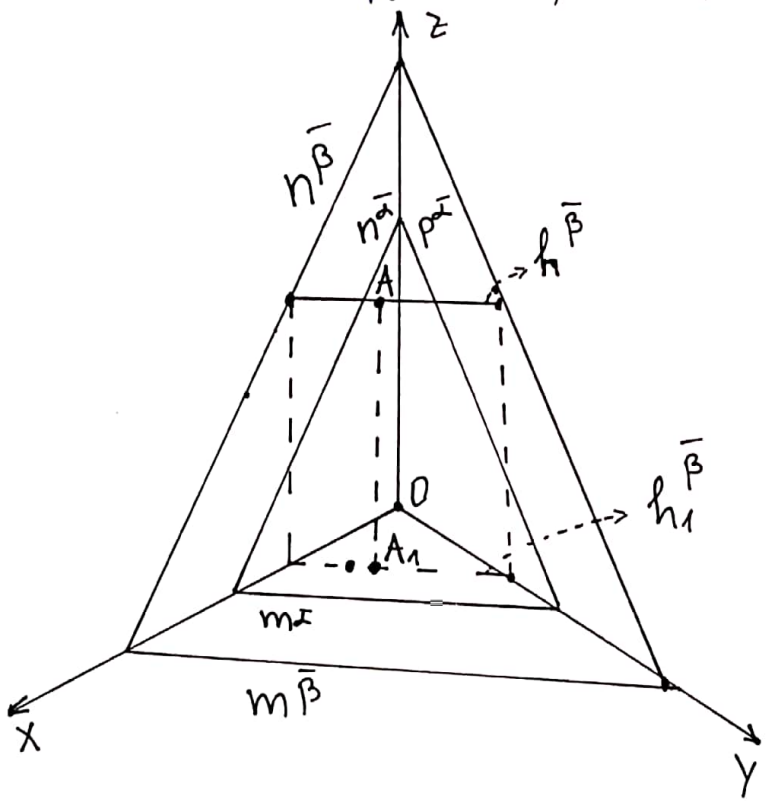
5) $A \begin{cases} \mathrm{Z}\, h^{\bar{\alpha}} \\ AA_1 \parallel 0z \end{cases}$

# IX Взаимни положения на две равнини

**Задача:** $\bar{\alpha} \parallel \bar{\beta} \Leftrightarrow m^{\bar{\alpha}} \parallel m^{\bar{\beta}}, n^{\bar{\alpha}} \parallel n^{\bar{\beta}}, p^{\bar{\alpha}} \parallel p^{\bar{\beta}}$

В аксонометрия са дадени $\bar{\alpha}(m^{\bar{\alpha}}, n^{\bar{\alpha}})$ и точка $\bar{A}(A, A_1) : \bar{A} \not\geq \bar{\alpha}$. Да се изобрази р-на $\bar{\beta} \begin{cases} \supset \bar{A} \\ \parallel \bar{\alpha} \end{cases}$. Ще използваме $h^{\bar{\beta}} \geq \bar{A}$:

$h^{\bar{\beta}}(h^{\bar{\beta}}, h_1^{\bar{\beta}}), h^{\bar{\beta}} \parallel h_1^{\bar{\beta}} \parallel m^{\bar{\beta}} \parallel m^{\bar{\alpha}}$




1) $h_1^{\bar{\beta}} \begin{cases} \supset A_1 \\ \parallel m^{\bar{\alpha}} \end{cases}$

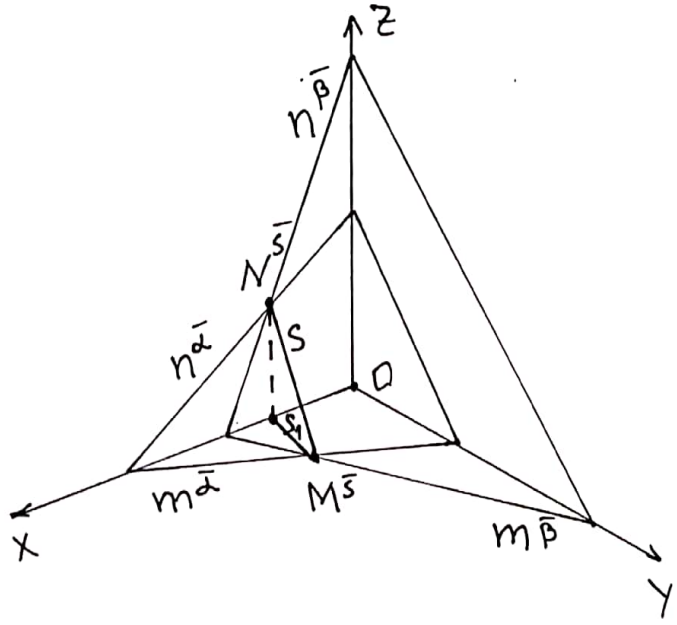
$h^{\bar{\beta}} \begin{cases} \supset A \\ \parallel m^{\bar{\alpha}} \end{cases}$

2) $N_1^{\bar{h}^{\bar{\beta}}} = h_1^{\bar{\beta}} \cap O_X$

3) $N^{\bar{h}^{\bar{\beta}}} \begin{cases} \supset h^{\bar{\beta}} \\ N N_1 \parallel O_Z \end{cases}$

4) $n^{\bar{\beta}} \begin{cases} \parallel n^{\bar{\alpha}} \\ \supset N^{\bar{h}^{\bar{\beta}}} \end{cases}$

5) $m^{\bar{\beta}}, p^{\bar{\beta}}$

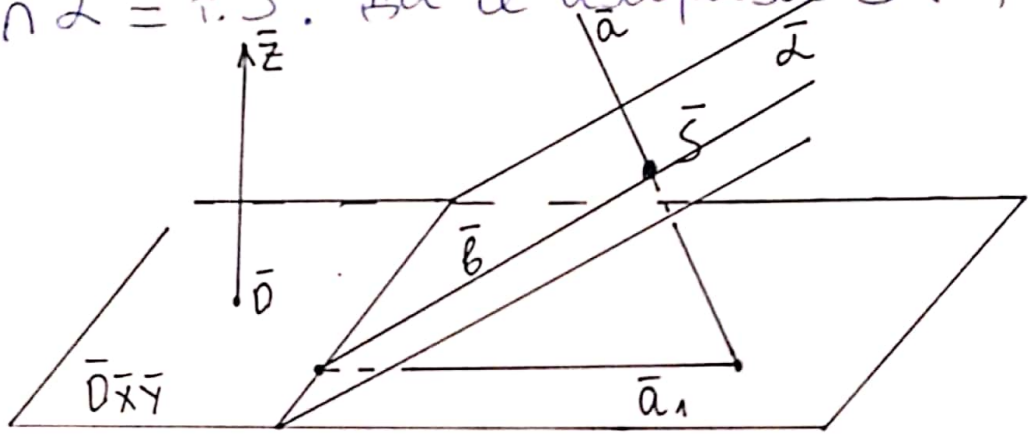



$\bar{\alpha} \cap \bar{\beta} = \bar{s} \Rightarrow \bar{s}(s, s_1)$

Построение:

## X Пробод на права и равнина

**Задача:** В аксонометрия са дадени равнина $\bar{\alpha}(m^{\bar{\alpha}}, n^{\bar{\alpha}})$ и права $\bar{a}(a, a_1)$: $\bar{a} \cap \bar{\alpha} = т.\bar{S}$. Да се изобрази $\bar{S}(S; S_1)$.




1) $\bar{\beta} \begin{cases} \supset \bar{a} \\ \supset \bar{a}_1 \end{cases} \Rightarrow$
$\bar{\beta} \parallel \bar{D}\bar{z}$
2) $\bar{\beta} = \bar{\alpha} \cap \bar{\beta}$
3) $\bar{S} = \bar{a} \cap \bar{\beta}$

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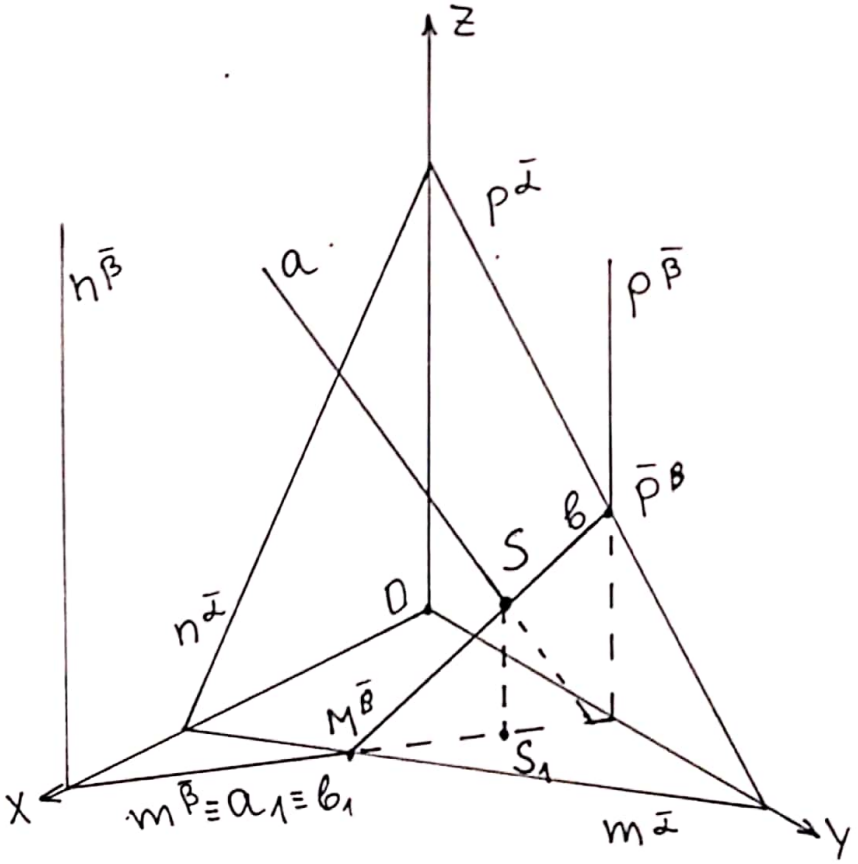


1) $m^{\bar{\beta}} \equiv a_1$
2) $n^{\bar{\beta}} \parallel p^{\bar{\beta}} \parallel Oz$
3) $M^{\bar{\beta}} = m^{\bar{\alpha}} \cap m^{\bar{\beta}}$
4) $p^{\bar{\beta}} = p^{\bar{\alpha}} \cap p^{\bar{\beta}}$
5) $\beta \begin{cases} M^{\bar{\beta}} \\ p^{\bar{\beta}} \end{cases}, \beta_1 \equiv a_1$
6) $т.S = a \cap \beta$