Зад. 1

$$a_n = \left(\frac{n^2+9n+16}{n^2+4n+7}\right)^n$$

$$\lim_{n\to\infty}\left(\frac{n^2+9n+16}{n^2+4n+7}\right)^n = \lim_{n\to\infty}\left(1+\frac{\frac{(5n+9)n}{n^2+4n+7}}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{\frac{5n^2+9n}{n^2+4n+7}}{n}\right)^n =$$

$$= e^{5/1} = e^5$$

$$b_n = \frac{n^3+2^n n+n^6}{n^2+3^n n^2-n^7}$$

$$\lim_{n\to\infty}\frac{n^3+2^n n+n^6}{n^2+3^n n^2-n^7} = \lim_{n\to\infty}\frac{2^n\cdot \cancel{n}\left(1+\frac{n^2}{2^n}+\frac{n^5}{2^n}\right)}{3^n n^{\cancel{2}}\left(1+\frac{1}{3^n}-\frac{n^5}{3^n}\right)} = 0 \quad \boxed{2^n < 3^n}$$

Зад. 2 $\lim\limits_{x \to 0} \frac{1 - \cos(4x)}{x^2} = \lim\limits_{x \to 0} \frac{1 - \cos^2 2x + \sin^2 2x}{x^2} =$

$$= \lim\limits_{x \to 0} \frac{2\sin^2 2x}{x^2} = 2 . \lim\limits_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = 2 . \lim\limits_{x \to 0} \left(\frac{2\sin x . \cos x}{x}\right)^2 =$$

$$= 8 . \lim\limits_{x \to 0} \left(\frac{\sin x}{x}\right)^2 = 8$$

$$\lim\limits_{x \to 0} \frac{e^{2x} - 1}{3} = \lim\limits_{x \to 0} \left(\frac{e^{2x}}{3} - \frac{1}{3}\right) = \lim\limits_{x \to 0} \frac{e^{2x}}{3} + \lim\limits_{x \to 0} \left(-\frac{1}{3}\right) = \frac{1}{3} - \frac{1}{3} = 0$$

Зад. 3

$$f'(x) = \left(e^{-x} . tg^2 x\right)' = \left(e^{-x}\right)' . tg^2 x + e^{-x} . \left(tg^2 x\right)' =$$

$$= -e^{-x} . tg^2 x + e^{-x}\left(2 . tg x . \frac{1}{\cos^2 x}\right) = -e^{-x} tg^2 x + \underline{\frac{2e^{-x} . tg x}{\cos^2 x}}$$

$$g'(x) = \left((\sin x)^{\cos x}\right)' = \left(e^{\cos x . \ln \sin x}\right)' = (\sin x)^{\cos x}\left(\cos x . \ln \sin x\right)' =$$

$$= \underline{(\sin x)^{\cos x}\left(-\sin x . \ln \sin x + \frac{\cos^2 x}{\sin x}\right)}$$

Зад. 4

$$\lim_{x\to 0} \frac{e^{2x}+e^{-2x}-3-6x^2+2x^3}{\sin x - x\cos x} = \underbrace{\lim_{x\to 0} e^{2x}+e^{-2x}-3-6x^2+2x^3}_{-1} \cdot \lim_{x\to} \frac{1}{\sin x - x\cos x} =$$

$$= -1 \cdot \lim_{x\to 0} \frac{1}{\underbrace{\sin x - x\cos x}_{0}} = -\infty$$

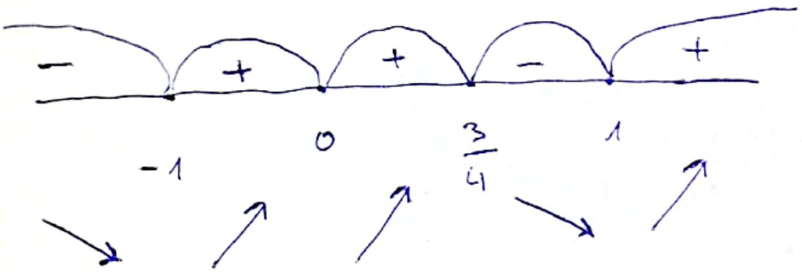
Зад. 5 $h(x) = 10x^6 - 9x^5 - 15x^4 + 15x^3$

$$h'(x) = 60x^5 - 45x^4 - 60x^3 + 45x^2 = 60x^3(x^2-1) - 45x^2(x^2-1) =$$

$$= (x^2-1)(60x^3 - 45x^2) = (x-1)(x+1)(4x-3)\cdot 15x^2$$

$h'(x) = 0$ при $x = 1$, $x = -1$, $x = 0$, $x = \frac{3}{4}$

$\exists f'(x)\ \forall x \in \mathbb{R}$



лок. min: $x = -1$, $x = 1$

лок. max: $x = \frac{3}{4}$