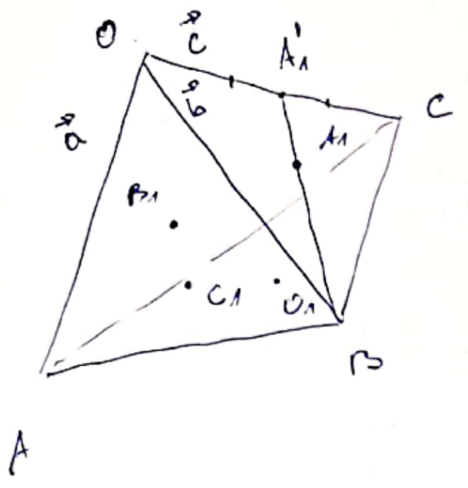
Вариант 1



$\vec{AA_1}, \vec{BB_1}, \vec{CC_1}$?

a) $\vec{AA_1} = \vec{AB} + \vec{BA_1}$

Нека BA_1' е медиана в $\triangle OBC$

A_1 медицентър в $\triangle OBC$
BA_1' медиана в $\triangle OBC$ $\Rightarrow \vec{BA_1} = \frac{2}{3} BA_1'$

$$BA_1' = \frac{1}{2}\left(\vec{BO} + \vec{BC}\right) = \frac{1}{2}\left(-\vec b + \vec{OC} - \vec{OB}\right) = \frac{1}{2}\left(-2\vec b + \vec c\right)$$

$$BA_1 = \frac{2}{3} \cdot BA_1' = \frac{1}{3}\left(\vec c - 2\vec b\right)$$

$$\vec{AA_1} = \vec{OB} - \vec{OA} + \vec{BA_1} = \vec b - \vec a + \frac{1}{3}\vec c - \frac{2}{3}\vec b = \underline{-\vec a + \frac{1}{3}\left(\vec c + \vec b\right)}$$

$$\vec{BB_1} = \vec{BO} + \vec{OB_1} = -\vec b + \frac{1}{3}\left(\vec a + \vec c\right)$$

$$\vec{CC_1} = \vec{CO} + \vec{OC_1} = -\vec c + \frac{1}{3}\left(\vec a + \vec b\right)$$

$$\vec{OO_1} = \vec{OB} + \vec{BO_1} = \vec b + \frac{2}{3} \cdot \frac{1}{2}\left(\vec{BA} + \vec{BC}\right) =$$

$$= \vec b + \frac{1}{3}\left(\vec{OA} - \vec{OB} + \vec{OC} - \vec{OB}\right) =$$

$$= \vec b + \frac{1}{3}\vec a + \frac{1}{3}\vec c - \frac{2}{3}\vec b = \underline{\frac{1}{3}\left(\vec a + \vec b + \vec c\right)}$$

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б) Док., че $\vec{AA_1} \cap \vec{BB_1} = M$, $AM : MA_1 = BM : MB_1 = 3:1$

$\vec{OM} = \vec{OA} + \vec{AM} = \vec{OB} + \vec{BM}$

т. $M \in \vec{AA_1} \iff \exists!\, \lambda \in \mathbb{R} : \vec{AM} = \lambda \vec{AA_1}$

т. $M \in \vec{BB_1} \iff \exists!\, \mu \in \mathbb{R} : \vec{BM} = \mu \vec{BB_1}$

$\vec{OM} = \vec{OA} + \lambda \vec{AA_1} = \vec{a} + \lambda \left(\frac{1}{3}(\vec{b} + \vec{c}) - \vec{a} \right)$

$\vec{OM} = \vec{OB} + \mu \vec{BB_1} = \vec{b} + \mu \left(\frac{1}{3}(\vec{a} + \vec{c}) - \vec{b} \right)$

$\vec{a}(1-\lambda) + \vec{b} \cdot \frac{\lambda}{3} + \vec{c} \cdot \frac{\lambda}{3} = \vec{a}\frac{\mu}{3} + \vec{b}(1-\mu) + \vec{c}\frac{\mu}{3}$

От $\vec{a}, \vec{b}, \vec{c}$ ЛНЗ $\Rightarrow$
$$\left| \begin{array}{l} 1-\lambda = \frac{\mu}{3} \\ \frac{\lambda}{3} = 1-\mu \\ \frac{\lambda}{3} = \frac{\mu}{3} \end{array} \right. \Rightarrow \boxed{\lambda = \mu = \frac{3}{4}}$$

$\Rightarrow \vec{AA_1} \cap \vec{BB_1} =$ т. M

$$\left. \begin{array}{l} \vec{AM} = \frac{3}{4}\vec{AA_1} \Rightarrow \vec{MA_1} = \frac{1}{4}\vec{AA_1} \\ \vec{BM} = \frac{3}{4}\vec{BB_1} \Rightarrow \vec{MB_1} = \frac{1}{4}\vec{BB_1} \end{array} \right| \Rightarrow \boxed{\begin{array}{l} \vec{AM} : \vec{MA_1} = 3:1 \\ \vec{BM} : \vec{MB_1} = 3:1 \end{array}}$$

в) O, M, O_1 док., че са колин.

от б) $\vec{OM} = \vec{OA} + \frac{3}{4}\vec{AA_1} = \frac{1}{4}(\vec{a} + \vec{b} + \vec{c})$

$$\left. \begin{array}{l} \vec{OM} = \frac{1}{4}(\vec{a} + \vec{b} + \vec{c}) \\ \vec{OO_1} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c}) \end{array} \right\} \Rightarrow \vec{OM} = \frac{3}{4}\vec{OO_1} \Rightarrow O, M, O_1 \text{ – колинеарни.}$$

Зад. 2 $\vec{a}, \vec{b}$

$|\vec{a}| = 3$, $|\vec{b}| = 2$, $\sphericalangle(\vec{a}, \vec{b}) = \frac{\pi}{3}$

$\vec{OA} = \vec{a} + \vec{b}$, $\vec{OB} = \vec{a} - \vec{b}$, $\vec{OC} = \vec{a} \times \vec{b}$

а) $\exists$ тетраедърът $OABC$ $\Leftrightarrow$ $\vec{OA}, \vec{OB}, \vec{OC}$ не са компланарни $\Leftrightarrow$ смесеното им произведение не е нула

$$(\vec{OA}\,\vec{OB}\,\vec{OC}) = \left((\vec{a}+\vec{b})(\vec{a}-\vec{b})(\vec{a}\times\vec{b})\right) =$$

$$= \left[(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})\right] \cdot (\vec{a}\times\vec{b}) =$$

$$= \left[\vec{a}\times(\vec{a}-\vec{b}) + \vec{b}\times(\vec{a}-\vec{b})\right] \cdot (\vec{a}\times\vec{b}) =$$

$$= \left[\vec{a}\times\vec{a} - \vec{a}\times\vec{b} + \vec{b}\times\vec{a} - \vec{b}\times\vec{b}\right] \cdot (\vec{a}\times\vec{b})$$

$$= \left[-2(\vec{a}\times\vec{b})\right] \cdot (\vec{a}\times\vec{b}) = \underline{-2(\vec{a}\times\vec{b})^2}$$

$|\vec{a}\times\vec{b}| \neq 0$, защото $\vec{a} \nparallel \vec{b}$, $\vec{a} \neq \vec{0}$, $\vec{b} \neq \vec{0}$ (по усл.)

$\rightarrow$ $\exists$ тетраедър $OABC$

б) $V_{OABC} = \frac{1}{6}\left|(\vec{OA}\,\vec{OB}\,\vec{OC})\right| = \frac{1}{6}\left|(-2)(\vec{a}\times\vec{b})^2\right|$

$|\vec{a}\times\vec{b}| = |\vec{a}|.|\vec{b}|.\sin\sphericalangle(\vec{a},\vec{b}) = 2.3.\frac{\sqrt{3}}{2} = \underline{3\sqrt{3}}$

$V_{OABC} = +\frac{1}{3}(3\sqrt{3})^2 = +\frac{9.3}{3} = \underline{9}$

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Зад. 3

$$a: \begin{cases} 2x+y+2z-10=0 \\ 4x-y+z-11=0 \end{cases} \qquad b: \begin{cases} x=-5+3q \\ y=5-2q \\ z=3-2q \end{cases}, \; q \in \mathbb{R}$$

$$a: \begin{cases} 2x+y+2z-10=0 \\ 4x-y+z-11=0 \end{cases} \Rightarrow \begin{array}{l} z=7-2x \\ y=2x-4 \end{array}$$

$$\Rightarrow \quad a: \begin{cases} x=\lambda \\ y=\cancel{2}-4+2\lambda \\ z=7-2\lambda \end{cases}, \; \lambda \in \mathbb{R}$$

? дължината на ~~трансверзалата~~ отсечката (AB), $AB \perp a$, $AB \perp b$

$A \in a$, $B \in b$.

Нека $\alpha \perp \vec{a} \Rightarrow \alpha: x+2y-2z+D=0$ $(K = Oxyz$ ОКС$)$.

$b \cap \alpha =$ т. B

$$\begin{cases} x+2y-2z+D=0 \\ x=-5+3q_B \\ y=5-2q_B \\ z=3-2q_B \end{cases} \Rightarrow 3q_B-1+D=0, \; \boxed{q_B = \frac{-D+1}{3}}$$

$$\Rightarrow \text{т. } B\left(-4-D, \; 5+\frac{2}{3}(D-1), \; 3+\frac{2}{3}(D-1)\right)$$

$a \cap \alpha = т.\, A$

$$\begin{cases} x + 2y - 2z + D = 0 \\ x = \lambda_1 \\ y = -4 + 2\lambda_1 \\ z = 7 - 2\lambda_1 \end{cases} \Rightarrow \quad 9\lambda_1 - 22 + D = 0$$

$$\lambda_1 = \frac{-D + 22}{9}$$

$$\Rightarrow A\left(\frac{-D+22}{9},\ -4 + \frac{2}{9}(-D+22),\ 7 - \frac{2}{9}(-D+22) \right)$$

$$\vec{AB} = \vec{OB} - \vec{OA} \Rightarrow$$

$$\Rightarrow \vec{AB}\left(\frac{-8D-58}{9},\ \frac{8D+31}{9},\ \frac{4D+2}{9} \right)$$

Искаме $\vec{AB} \perp \vec{b} \Rightarrow \vec{AB} \cdot \vec{b} = 0$, $\vec{b}(3, -2, -2)$

$$\frac{-8D-58}{9} \cdot 3 - \frac{8D+31}{9} \cdot 2 - \frac{4D+2}{9} \cdot 2 = 0$$

$\Rightarrow \underline{D_2 = -5}$ заместване $\Rightarrow \vec{AB}(-2, -1, -2)$

$$\boxed{|(AB)| = |\vec{AB}| = \sqrt{(-2)^2 + (-1)^2 + (-2)^2} = \sqrt{9} = 3}$$

б) $a \cap \gamma = P$

$$\Rightarrow \begin{cases} x + 2y - 2z - 23 = 0 \\ x = \lambda_p \\ y = -4 + 2\lambda_p \\ z = 7 - 2\lambda_p \end{cases} \Rightarrow \quad \lambda_p + 2(-4 + 2\lambda_p) - 2(7 - 2\lambda_p) - 23 = 0$$

$$9\lambda_p - 8 - 14 - 23 = 0$$

$$9\lambda_p = 45$$

5.1 $\boxed{\lambda_p = -5}$

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$P(-5, -14, 17)$

$b \cap \pi = Q$

$$\Rightarrow \begin{cases} 3x - 2y - 2z + 14 = 0 \\ x = -5 + 3q_a \\ y = 5 - 2q_a \\ z = 3 - 2q_a \end{cases} \Rightarrow \begin{array}{l} 17q_a - 17 = 0 \\ q_a = 1 \end{array}$$

$Q(-2, 3, 1)$

$S_{\Delta PQR} = \frac{1}{2} \left| \vec{PQ} \times \vec{PR} \right|$

$\vec{PQ} (3, 17, -16)$, $\vec{PR}(7, 18, \overset{-18}{\cancel{18}})$

$\vec{PR} (7, 18, -18)$.

$$\vec{PQ} \times \vec{PR} \left(\begin{vmatrix} 17 & -16 \\ 18 & -18 \end{vmatrix}, \begin{vmatrix} -16 & 3 \\ -18 & 7 \end{vmatrix}, \begin{vmatrix} 3 & 17 \\ 7 & 18 \end{vmatrix} \right)$$

$\vec{PQ} \times \vec{PR} (-18, -58, -65)$

$S_{\Delta PQR} = \frac{1}{2} \sqrt{(-18)^2 + (-58)^2 + (-65)^2}$

Index of comments

5.1 Защо 9.(-5)=45???????