Zad. 1

$$\int \frac{10+2x+4x^2-x^3}{\sqrt{x}}\,dx = \int \frac{10}{x^{1/2}} + \frac{2x}{x^{1/2}} + \frac{4x^{\frac{4}{2}}}{x^{1/2}} - \frac{x^{6/2}}{x^{1/2}}\,dx =$$

$$= 10\,\frac{x^{1/2}}{1/2} + 2\,\frac{x^{3/2}}{3/2} + 4\,\frac{x^{5/2}}{5/2} - \frac{x^{7/2}}{7/2} =$$

$$= 20x^{1/2} + \frac{4}{3}x^{3/2} + \frac{8}{5}x^{5/2} - \frac{2}{7}x^{7/2} + C$$

$$\int x^2 \ln(8x)\,dx = \int \ln(8x)\,d\frac{x^3}{3} = \frac{x^3}{3}\ln(8x) - \int \frac{x^3}{3}\,d\ln(8x) =$$

$$= \frac{x^3}{3}\ln(8x) - \frac{1}{3}\int \frac{x^3}{x}\,dx = \underline{\frac{x^3}{3}\ln(8x) - \frac{x^3}{9} + C}$$

$$\int \cos^3(5x)\,dx = \frac{1}{5}\int \cos(5x)\cos^2(5x)\,d(5x) =$$

$$= \frac{1}{5}\int 1-\sin^2(5x)\,d\sin(5x) = \frac{1}{5}\int 1\,d\sin(5x) - \frac{1}{5}\int \sin^2(5x)\,d\sin(5x) =$$

$$= \underline{\frac{\sin(5x)}{5} - \frac{\sin^3(5x)}{15} + C}$$

$$I = \int e^{-2x}\cos(2x)\,dx = \frac{1}{2}\int e^{-2x}\,d\sin(2x) =$$

$$= \frac{e^{-2x}}{2}\cdot\sin(2x) - \frac{1}{2}\int \sin(2x)\,de^{-2x} = \frac{e^{-2x}}{2}\cdot\sin(2x) - \frac{1}{2}\int \sin(2x)(-2)e^{-2x}\,dx =$$

$$= \frac{e^{-2x}}{2}\cdot\sin(2x) + \frac{1}{2}\int e^{-2x}\,d(-\cos(2x)) =$$

$$= \frac{e^{-2x}}{2}\cdot\sin(2x) + \frac{e^{-2x}}{2}(-\cos(2x)) - \frac{1}{2}\int -\cos(2x)\,de^{-2x} = \longrightarrow$$

$$\rightarrow = \frac{e^{-2x}}{2}\left(\sin(2x) - \cos(2x)\right) - \underbrace{\int \cos(2x)\, e^{-2x}\, dx}_{= -I}$$

$$\Rightarrow I = \frac{e^{-2x}}{4}\left(\sin(2x) - \cos(2x)\right) + C$$

$$\int \frac{dx}{x^2-2x+5} \qquad D = 4-20 < 0 \Rightarrow \text{полагаме } x = t - \frac{b}{2a}$$

$$x = t + \frac{2}{2} = t+1$$

$$t = x-1$$

$$\int \frac{dt}{(t+1)^2 - 2(t+1)+5} = \int \frac{dt}{t^2+2t+1-2t-2+5} = \int \frac{dt}{4+t^2} =$$

$$= \int \frac{1}{4\left(1+\left(\frac{t}{2}\right)^2\right)}\, dt = \frac{2}{4}\int \frac{1}{1+\left(\frac{t}{2}\right)^2}\, d\frac{t}{2} = \frac{1}{2}\operatorname{arctg}\left(\frac{t}{2}\right) =$$

$$= \frac{1}{2}\operatorname{arctg}\left(\frac{x-1}{2}\right) + C$$

Зад. 2 $\displaystyle\int \frac{dx}{4\sin x + 3\cos x + 4}$ $\quad t = \operatorname{tg}\frac{x}{2} \quad x = 2\operatorname{arctg} t$

$$dx = \frac{2}{1+t^2}\, dt \qquad \sin x = \frac{2t}{1+t^2}$$

Заместваме: $\quad \cos x = \dfrac{1-t^2}{1+t}$

$$\int \frac{1}{4\cdot\frac{2t}{1+t^2} + \frac{3(1-t^2)}{1+t^2} + \frac{4(1+t^2)}{1+t^2}} \cdot \frac{2}{1+t^2}\, dt =$$

$$= \int \frac{2}{t^2+8t+7}\, dt = 2\int \frac{1}{(t+7)(t+1)}\, dt$$

$$\frac{1}{(t+7)(t+1)} = \frac{A}{t+7} + \frac{B}{t+1}$$

$$1 = A(t+1) + B(t+7)$$

$$\begin{cases} 1 = A + 7B \\ 0 = A + B \end{cases} \Rightarrow A = -\frac{1}{6} \quad B = \frac{1}{6}$$

$$2\int \frac{1}{-6(t+7)}dt + 2\int \frac{1}{6(t+1)}dt = -\frac{1}{3}\int \frac{1}{t+7}\,d(t+7) + \frac{1}{3}\int \frac{1}{t+1}\,d(t+1) =$$

$$= -\frac{1}{3}\ln|t+1| + \frac{1}{3}\ln|t+1| = -\frac{1}{3}\ln\left|\operatorname{tg}\frac{x}{2}+7\right| + \frac{1}{3}\ln\left|\operatorname{tg}\frac{x}{2}+1\right| + C$$