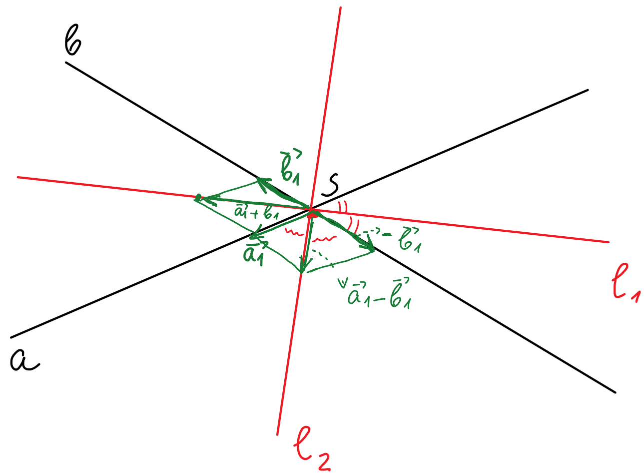
22.11. Ъглополовящи на ъгли между две прави



$a \cap b = $ т. S

Търсим уравнения на l_1 и l_2

$a \parallel \vec{a} \Rightarrow |\vec{a}| \Rightarrow \vec{a}_1 = \dfrac{\vec{a}}{|\vec{a}|} \Rightarrow |\vec{a}_1| = 1$

$b \parallel \vec{b} \Rightarrow |\vec{b}| \Rightarrow \vec{b}_1 = \dfrac{\vec{b}}{|\vec{b}|} \Rightarrow |\vec{b}_1| = 1$

$l_1 \begin{cases} \ni S \\ \parallel (\vec{a}_1 + \vec{b}_1) \end{cases}$ $\qquad l_2 : \begin{cases} \ni S \\ \parallel (\vec{a}_1 - \vec{b}_1) \end{cases}$

Ъглополовяща на остър и тъп ъгъл

$l_1 \parallel \vec{a}_1 + \vec{b}_1$ $\qquad l_2 \parallel \vec{a}_1 - \vec{b}_1$

$\measuredangle(\vec{a}_1, \vec{b}_1) = \varphi$ $\qquad \measuredangle(\vec{a}_1, -\vec{b}_1) = 180^\circ - \varphi$

$\cos\varphi = (\underbrace{\vec{a}_1 \cdot \vec{b}_1}_{\text{координати}})$

1сл. Ако $(\vec{a}_1 \cdot \vec{b}_1) > 0 \Rightarrow$ l_1 е ъглоп. на острия $\measuredangle(a, b)$
l_2 е −||− ||− тъпия $\measuredangle(a, b)$

2сл. Ако $(\vec{a}_1 \cdot \vec{b}_1) < 0 \Rightarrow$ l_1 е ъглоп. на тъпия $\measuredangle(a, b)$
l_2 − на острия

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1 зад. ОКС $K = Oxy$

$a: 3x - 4y + 5 = 0$

$b: 4x - 3y - 5 = 0$

Да се намерят уравнения на ъглополовящите l_1 и l_2 на ъглите м/у a и b.
Да се определи коя е ъглоп. на острия и коя − на тъпия ъгъл м/у a и b.

1) т. $S = a \cap b$ $\quad \left| \begin{array}{l} 3x - 4y + 5 = 0 \\ 4x - 3y - 5 = 0 \end{array} \right. \Rightarrow S(5, 5)$

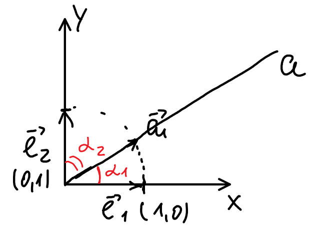
2) $a \parallel \vec{a}(4, 3) \Rightarrow |\vec{a}| = \sqrt{4^2 + 3^2} = 5$ $\qquad \vec{a}_1 = \dfrac{\vec{a}}{5} \Rightarrow \vec{a}_1\left(\dfrac{4}{5}, \dfrac{3}{5}\right)$

2) $a \parallel \vec{a}(4,3) \Rightarrow |\vec{a}| = \sqrt{4^2+3^2} = 5$ $\quad \vec{a}_1 = \dfrac{\vec{a}}{5} \Rightarrow \vec{a}_1\left(\dfrac{4}{5}, \dfrac{3}{5}\right)$

$b \parallel \vec{b}(3,4) \Rightarrow |\vec{b}| = 5$ $\quad \vec{b}_1 = \dfrac{\vec{b}}{5} \Rightarrow \vec{b}_1\left(\dfrac{3}{5}, \dfrac{4}{5}\right)$

$\vec{a}_1\left(\dfrac{4}{5}, \dfrac{3}{5}\right)$!

$\vec{b}_1\left(\dfrac{3}{5}, \dfrac{4}{5}\right)$



$\alpha_1 = \sphericalangle(\vec{e}_1, \vec{a}_1)$

$\alpha_2 = \sphericalangle(\vec{e}_2, \vec{a}_1)$

$\left(\vec{a}_1 \; \underset{\cos\alpha_1}{\dfrac{4}{5}}, \; \underset{\cos\alpha_2}{\dfrac{3}{5}}\right)$ директорни косинуси

$$l_1 \begin{cases} \ni S(5,5) \\ \parallel \vec{a}_1 + \vec{b}_1\left(\underset{1}{\dfrac{7}{5}}, \underset{1}{\dfrac{7}{5}}\right) \parallel (1,1) \end{cases}$$

$$l_1: \begin{cases} x = 5 + \lambda \cdot \dfrac{7}{5} \\ y = 5 + \lambda \cdot \dfrac{7}{5} \end{cases} (-) \quad \lambda \in \mathbb{R}$$

$\boxed{l_1: x - y = 0}$ Oxy

$$l_2: \begin{cases} \ni S(5,5) \\ \parallel \vec{a}_1 - \vec{b}_1\left(\underset{1}{\dfrac{1}{5}}, \underset{-1}{-\dfrac{1}{5}}\right) \end{cases}$$

$$l_2: \begin{cases} x = 5 + \mu \cdot \dfrac{1}{5} \\ y = 5 + \mu \cdot \left(-\dfrac{1}{5}\right) \end{cases} (+) \quad \mu \in \mathbb{R} \Rightarrow \boxed{l_2: x + y - 10 = 0} \text{ Oxy}$$

$\vec{a}_1\left(\dfrac{4}{5}, \dfrac{3}{5}\right)$

$\vec{b}_1\left(\dfrac{3}{5}, \dfrac{4}{5}\right)$

$$(\vec{a}_1 \cdot \vec{b}_1) = \frac{4}{5} \cdot \frac{3}{5} + \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25} = \cos\varphi > 0 \Rightarrow$$

$\Rightarrow l_1 \to$ остър ъгъл

$l_2 \to$ тъп ъгъл

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2 зад. (Упр.) ОКС $K = Oxy$

$a: x - 3y = 0$ $\qquad l_1 = ?, \; l_2 = ?$

$b: 3x - y + 8 = 0$ $\qquad$ Коя е на остър и коя - на тъп ъгъл?

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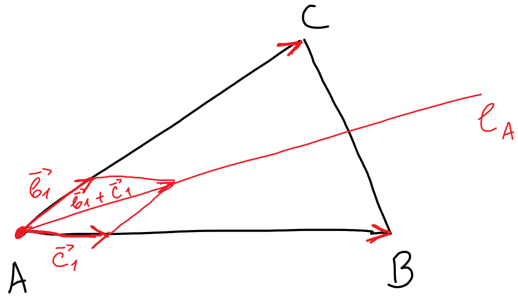
3 зад ОКС $K = Oxy$

$A(1,2)$ $\quad B(-1,3)$ $\quad C(5,4)$

?, уравнение на ℓ_A - ъглопол. на вътрешния ∡ при върха A на $\triangle ABC$.

Решение:



$\vec{AB}(-2, 1) \Rightarrow |\vec{AB}| = \sqrt{5}$

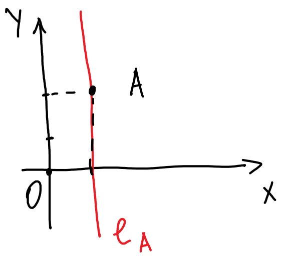
$\vec{AC}(4, 2) \Rightarrow |\vec{AC}| = 2.\sqrt{5}$

$\vec{c}_1 = \dfrac{\vec{AB}}{|\vec{AB}|} \Rightarrow \vec{c}_1\left(-\dfrac{2}{\sqrt{5}}, \dfrac{1}{\sqrt{5}}\right)$

$\vec{b}_1 = \dfrac{\vec{AC}}{|\vec{AC}|} \Rightarrow \vec{b}_1\left(\dfrac{2}{\sqrt{5}}, \dfrac{1}{\sqrt{5}}\right)$

$$\ell_A \begin{cases} \ni A(1, 2) \\ \parallel \vec{b}_1+\vec{c}_1\left(0, \frac{2}{\sqrt{5}}\right) \parallel \vec{e}_2(0,1) \end{cases}$$

$$\ell_A: \begin{cases} x = 1 \\ y = 2 + \frac{2}{\sqrt{5}}\cdot s \end{cases}, s \in \mathbb{R}$$



$\ell_A: x = 1$

Нормално уравнение на права. Разстояние от точка до права

ОКС $K = Oxy$

$\boxed{g: A.x+B.y+C=0}$ - общо

$(A,B) \neq (0,0)$

$g \perp \vec{n}_g(A,B) \Rightarrow |\vec{n}_g| = \sqrt{A^2+B^2}$

Търсим вектор $\vec{n}_1 \begin{cases} \perp g \\ |\vec{n}_1| = |\vec{e}_1| = |\vec{e}_2| = 1 \end{cases} \Rightarrow \vec{n}_1 = \dfrac{\vec{n}_g}{|\vec{n}_g|}$

$$\vec{n}_1\left(\frac{A}{\sqrt{A^2+B^2}}, \frac{B}{\sqrt{A^2+B^2}}\right)$$

$$\boxed{g: \frac{A.x+B.y+C}{\sqrt{A^2+B^2}} = 0}$$ - нормално уравнение на g

Пример:

$g: \ 2x - y + 4 = 0$

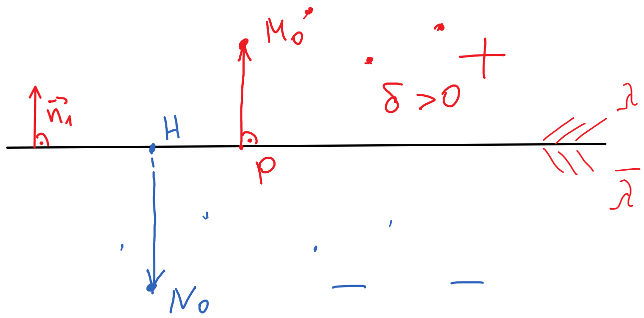
$g: (2k)x - k.y + 4.k = 0 \quad k \neq 0 \Rightarrow$ безброй много общи уравнения

$g: \pm \dfrac{2x - y + 4}{\sqrt{5}} = 0$ → две нормални уравнения

Ориентирано разстояние от т. $M_0(x_0, y_0)$ до права g

$M_0(x_0, y_0)$

$g: \dfrac{A.x + B.y + C}{\sqrt{A^2 + B^2}} = 0$



$$\delta(M_0, g) = \frac{A.x_0 + B.y_0 + C}{\sqrt{A^2 + B^2}}$$

$\delta(M_0, g)$ → > 0 $\quad \overrightarrow{PM_0} \uparrow\uparrow \vec{n}_1$

→ $= 0$ $\quad$ ако $M_0 \in g$

→ < 0 $\quad \overrightarrow{HN_0} \uparrow\downarrow \vec{n}_1$

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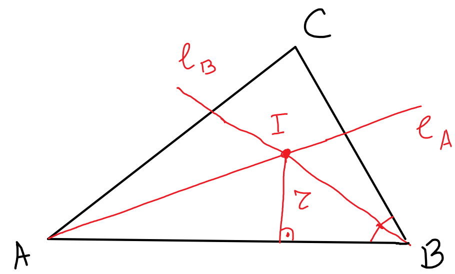
4 зад. (Упр.) ОКС $K = Oxy$

$A(1, -2) \quad B(2, 0) \quad C\left(-\frac{2}{3}, \frac{4}{3}\right)$

$k(I, r)$ - вписана в $\triangle ABC$

Търсим коорд. на т. I и $r = ?$



Упътване:

1) ℓ_A и ℓ_B

2) т. $I = \ell_A \cap \ell_B$

3) $r = |\delta(I, AB)|$

Отг. $I\left(1, -\frac{1}{3}\right)$

$r = \dfrac{\sqrt{5}}{3}$

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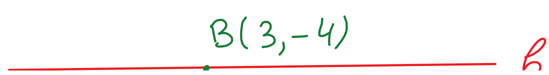
5 зад. ОКС $K = Oxy$

$g: \ 2x - 3y - 5 = 0 \qquad b \xrightarrow{G_g} b'$

Да се намери уравнение на b', симетрична на b относно g.

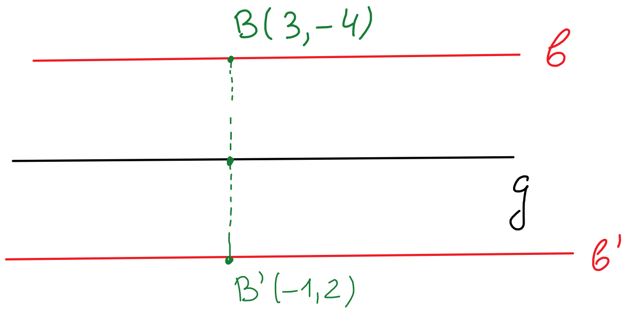
a) $b: 2\overset{3}{x} - 3\overset{-4}{y} - 18 = 0$



a) $b: 2\overset{3}{x} - 3\overset{-4}{y} - 18 = 0$

$b \parallel g \Rightarrow b' \parallel g \Rightarrow$

$\Rightarrow b': 2x - 3y + D = 0$



Изб. т. $B(3,-4) \in b$

$B(3,-4) \xrightarrow{\sigma_g} B'(-1,2)$ (Да се пресметне)

$b' \begin{cases} \ni B'(-1,2) \\ \parallel g \parallel b \end{cases} \Rightarrow b': 2\overset{-1}{x} - 3\overset{2}{y} + D = 0$

$-2 - 6 + D = 0 \Rightarrow D = 8$

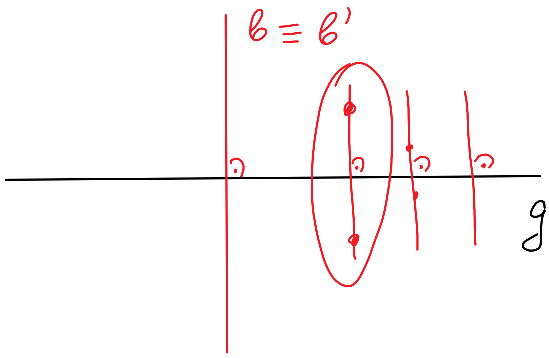
$b \xrightarrow{\sigma_g} b': 2x - 3y + 8 = 0$

б) $g: 2x - 3y - 5 = 0$

$b: 3x + 2y - 1 = 0$

$b \perp g$

$b \xrightarrow{\sigma_g} b$ $\qquad \sigma_g(b) = b$



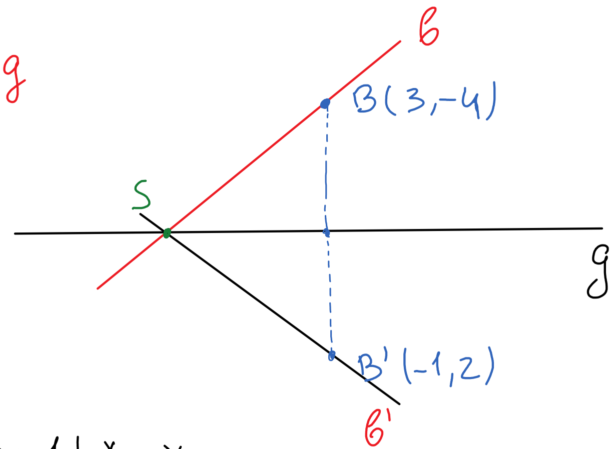
в) $g: 2\overset{4}{x} - 3\overset{1}{y} - 5 = 0$

$b: 5\underset{4}{\overset{3}{x}} - \underset{1}{\overset{-4}{y}} - 19 = 0$ $\qquad b \not\parallel g, \; b \not\perp g$

т. $S = g \cap b \Rightarrow S(4,1)$

Изб. т. B от b

Нека $B(3,-4) \xrightarrow{\sigma_g} B'(-1,2)$
смятат се



Тогава $b' \begin{cases} \ni S(4,1) \\ \ni B'(-1,2) \end{cases}$

$b': \begin{vmatrix} x & y & 1 \\ 4 & 1 & 1 \\ -1 & 2 & 1 \end{vmatrix} \begin{matrix} x & y \\ 4 & 1 \\ -1 & 2 \end{matrix} = x - y + 8 + 1 - 2x - 4y$

$b': -x - 5y + 9 = 0 \; | \cdot (-1)$

$b': \underset{\underset{-1}{4}}{x} + 5\underset{\underset{2}{1}}{y} - 9 = 0$