EE 342 Control Systems Laboratory

Experiments and Supporting Information

Preliminaries

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California Polytechnic State University
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Dedication

This laboratory manual is dedicated to the memory of Prof. William Horton, a founder and leader of the Cal Poly Electrical Electrical Engineering Department as it exists today. Prof. Horton taught in the Power and Controls area for over 30 years. He brought to this laboratory and EE department a wealth of theoretical as well as practical experience. Prof. Horton passed away in 2008. His contributions will be greatly missed.

Acknowledgments

This manual builds upon the cumulative insight, expertise and experience of many Cal Poly EE Dept. faculty over the previous 25 years. In particular, late Profs. William Horton, Richard Thorpe, and Ben Patrick, as well as emeritus faculty Jerome Breightenbach, Saul Goldberg and Shyama Tandon. Recent editorial input and beta-testing by Profs. Xiao-Hua Yu, Gary Perks, Taufik, and Wayne Pilkington have also been invaluable. All of their contributions are gratefully acknowledged.

Electrical Engineering Department General Laboratory Policy

Required per University and EE Department Policies: http://www.ee.calpoly.edu/policies/

Introduction

At the discretion of the laboratory instructor, all experiments in this compendium/manual may be tailored to best meet individual teaching styles and the needs of the students by variation of experimental specifications.

Although extensive debugging has gone into the experimental procedures and the text of this document, errors will most surely be found. Please report any corrections or suggestions for improvement to the author. It is faculty and student feedback that makes the laboratories meaningful. I can be contacted via email at Art's Cal Poly email address.

Version 1 of this manual was first approved for use in course EE 342 by the EE Power and Control Area Committee in Spring 1989. It has been updated through five major several minor revisions.

This manual, including all Experiments and supporting materials, can be ***downloaded at no cost*** *from "**Prof. MacCarley's Control Systems Web Site"** http://telab.ee.calpoly.edu/~amaccarl/ under the EE342 tab. The site is password-protected for enrolled students only, however, I am happy to share with anyone if requested. Usernames and passwords will be distributed at the first lab meeting, or by email request (see above).*

This document is distributed in PDF format which can be read and printed using the public domain Acobat Reader available from http://www.adobe.com/prodindex/acrobat/readstep.html . If your browser software has the Acrobat reader already registered, double-clicking on any experiment document will immediately display the document and allow you to save or print. If this doesn't work, try using the "Save Target as" dropdown menu option of most browsers.

Course EE 302 *Control Systems* is the co-requisite for the EE 342 laboratory. The student is expected to be familiar with basic linear systems concepts, frequency response and Bode analysis. In addition, students are expected to know basic circuit analysis (specifically nodal analysis of op-amp circuits), and kinematic physics (f = ma, force/torque balance) from prerequisite EE courses or service courses taken by all engineering majors.

The experiments in this lab have been designed to synchronize with the progression of the course outline for the co-requisite EE 302 lecture. There may be significant variations in the order of material presentation in individual sections of EE 302. Therefore, students may be required to read ahead in the EE302 course text or notes in the event that a topic has not yet been covered. Some misalignment of topics is unavoidable, since many of the basic control system analysis tools used in EE 342 may not be presented in the EE 302 lecture until late in the course.
All notes from Prof. MacCarley's offerings of EE302 and all Controls Courses are online at *http://telab.ee.calpoly.edu/~amaccarl/ under the EE302 tab.*

Each experiment includes a recommended *Preparation* section at the end. At the discretion of the instructor, this may be assigned as a graded Prelab requirement, or students may simply be entrusted to complete this work prior to the lab meeting, which will greatly assist the students in completing all experiments within the allotted three-hour lab period. Students *are encouraged* to work together on the Preparation assignment to help assure full understanding prior to the lab meeting. The Preparation assignment will be reviewed at the start of each lab meeting.

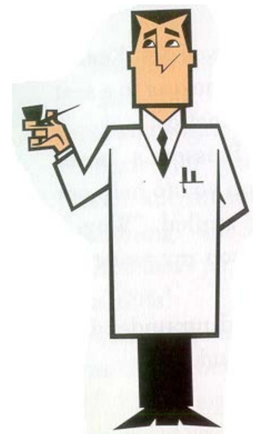
The experiments are based upon the venerable Motomatic Control System Laboratory (MCSL) manufactured in 1980 by Electro-Craft Corporation of Hopkins, Minnesota. Despite its age, this apparatus remains an excellent example of a **real** electromechanical control system, including all of the nonlinearities and unmodeled modes that are typical of the actual systems that you will very likely encounter in practice.

We use the Simulink and SISOTool features of MATLAB for simulation of the dynamic response of the MCSL and for design support, respectively. Each lab bench is equipped with a Windows PC capable of running MATLAB, MS Office, importing data from the oscilloscope, and printing to a network laser printer. All are Internet connected and can download the lab materials described above. If you are unfamiliar with MATLAB, extensive tutorials can be found online, for example: http://www.mathworks.com/academia/student_center/tutorials/ .

Assessment and grading, **lab report format and expectations**, **laboratory deliverables**, and **course policies** will be stated in the **EE342 Course Syllabus of the individual course instructor.** ***These may vary significantly according to the professional judgment and experience of each instructor.*** The ultimate content and evaluation criteria for this lab or ANY course are at the sole discretion of the instructor.

Course Learning Objectives

Narrative



EE course EE342 is intended to complement basic linear control system theory developed in course EE302, Linear Control Systems. The complete list of these topics need not be redundantly restated here. Experiments are conducted to provide practical experience with an actual servo-control system, and general familiarization with the technology and nomenclature of linear control systems.

Prerequisite material for this laboratory (beyond that provided in EE 302) includes knowledge of operational amplifiers and their use as analog computer elements, basic kinematic physics (Newton's Laws), use of an oscilloscope, digital multimeter, function generator, and standard laboratory methods including lab report format, data collection and presentation methods.

Enumerated

1. Master the vernacular and nomenclature of control systems.
2. Create accurate linear Laplace models for electrical and mechanical systems.
3. Express these models in block diagram form, and be able to manipulate diagrams to enhance understanding of system attributes.
4. Analyze and design basic single-loop servocontrol systems.
5. Identify system Type number, and predict static errors and error coefficients.
6. Apply and draw conclusions from Root-Locus Analysis for a second order system regarding expected transient (step) response.
7. Recognize situations in which the dominant pole approximation is valid, and utilize it to predict transient response characteristics for a second order system.
8. Design a lead compensator using root locus methods, and predict the transient response.
9. Recognize, predict and avoid asymptotic instability in linear control systems.
10. Design, implement in analog form, and tune a PID control using a modified Zeigler-Nichols approach.
11. Recognize and mitigate the effects of integrator wind-up in a PID-controlled system.
12. Use Bode methods to predict relative stability characteristics in the form of phase and gain margin, for a third-order linear system.
13. Design feedback coefficients of a full-state-feedback control system to achieve a desired transient response.
14. Recognize, name and ameliorate if possible/appropriate nonlinear effects in actual systems.
15. Recognize the limitations of linear system models, tools and methods, and those situations in which they can be safely applied to real control systems.

Laboratory Experiments

The EE342 Laboratory Course includes 7 experiments and a written final exam administered during the last week of instruction. Note that in the recommended course activity table below, eleven weeks would be required for all activities, if the first meeting is just introductory and a lab final is administered. For this reason, the instructor may choose to eliminate one or more activities at their discretion. Here is the complete list and recommended time allocations:

Weeks Allowed	Exp #	Experiment Title or Activity	Points
1	Prelims	Introduction	0
1	1	System Modeling	10
1	2	Proportional Control	10
1	3	Root Locus and Stability	10
2	4	Lead Compensation	20
2	5	PID Control of Unstable System	20
1	6	Frequency Response	10
1	7	State Variable Feedback	10
1		Final Exam	10
Total: 11			100

Experiment 4 “Lead Compensation” has been allocated two weeks for completion, but actually only requires approximately 1.5 lab periods. It is suggested that the second week of Exp 4 be split between Exp 4 and the start of Exp 5 (below) which is very time-consuming.
Experiment 5 focuses on the design of a Proportional-Integral-Derivative (PID) controller for an unstable system. The PID control is ubiquitous among all linear controls, and knowledge of the design and tuning of a PID control is considered essential undergraduate knowledge in almost all fields of engineering. In the author’s opinion, this is a critical experiment, but a potentially time-consuming one since it requires the modification and re-modeling of the MCSL with an inverted pendulum attachment, uses the largest number of resistor and capacitor decade boxes, and introduces several concepts that may not appear until later in the EE302 Textbook. There is also a potential problem in the design of the MCSL that may affect results: the center position of the Step Input switch is NOT zero volts (short to ground) as depicted for simplicity in Figure 1 below; it is an open circuit. This design error has minimal impact on the other experiments which do not use “feed-forward” networks between the op amp input and the switch, but its effect may be more noticeable in this experiment.

Experiment 6 “Frequency Response of Feedback Control Systems” is included because of the importance of the concepts of control system bandwidth and phase or gain margin. However, the very low frequency response and Type 1 classification of the MCSL, compounded by the unusual complexity of the HP 35665A Dynamic Signal Analyzer, make these measurements difficult and often inaccurate. Instructors, at their discretion, may choose to forgo this experiment.

Experiment 7 “State Variable Feedback” is the only experiment that uses tachometer feedback. It is a relatively short experiment, and it will become obvious to the student that almost all of the concepts involved have been previously demonstrated in prior experiments. However, since State Variable Feedback is not always covered by instructors in EE302 (typically delayed to EE513), Experiment 7 may be considered optional at the discretion of the lab instructor.

The Laboratory Apparatus

All experiments will be based upon the *Motomatic Control System Laboratory* (MCSL), which is a small-scale rotational servomechanism incorporating a linear analog controller, a DC permanent magnet motor, and position and velocity feedback sensors. The MCSL consists of two assemblies. The *control console* includes an operational amplifier configured as an analog summer, a DC-coupled power amplifier, and a dual ± 15 volt power supply. The *electromechanical assembly* consists of a double-rail platform upon which is mounted a permanent magnet DC motor with built-in tachometer generator, an adjustable speed reducer, and a shaft rotational position sensor.

In addition to the MCSL, basic electronic instrumentation will be used throughout the experiments, including the HP 54603B Digital Oscilloscope, the HP 35665A Dynamic Signal Analyzer (an enhanced spectrum analyzer), and the Fluke Digital Multimeter.
Figure 1 is schematic diagram of the key components of the MCSL, showing the electronic components explicitly and the mechanical components as blocks in the style of a simulation diagram. The operation of the MCSL will be described with reference to this figure.

Figure 2 shows the MATLAB Simulink model of the MCSL, which will be used for simulation of the response of various configurations of the system. In each case, you will replace the nominal coefficients in each block with the values for your apparatus which you will determine in Experiment 1. This model, filename "MotomaticAnalogWithTach.mdl" may be downloaded from "Prof. MacCarley's Control Systems Web Site" under the EE342 tab. To edit and run the model, just double click on the file name in the Windows folder listing, and it will open in MATLAB. Due to the size of the MATLAB program, the loading time may be over 60 seconds.

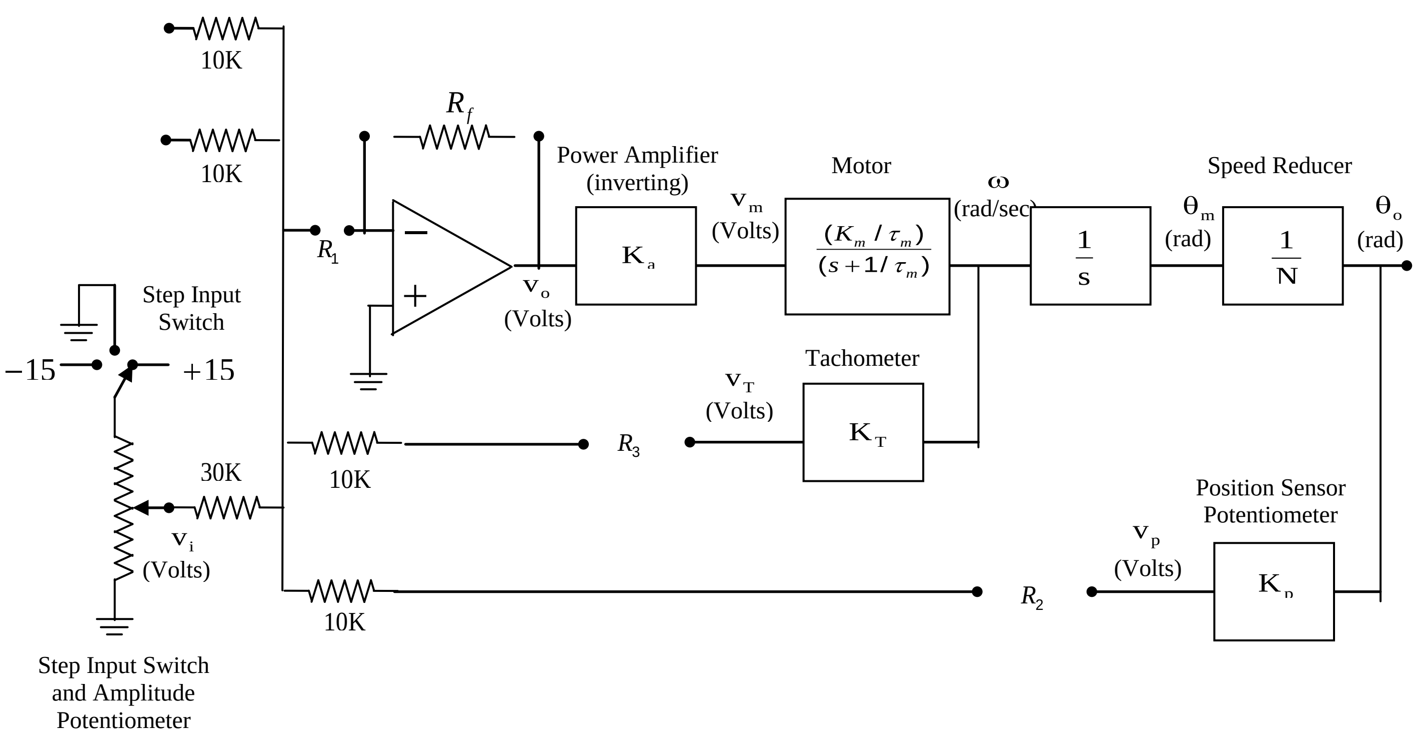


Figure 1. Schematic and Block Diagram of MCSL Electrical and Mechanical Components.

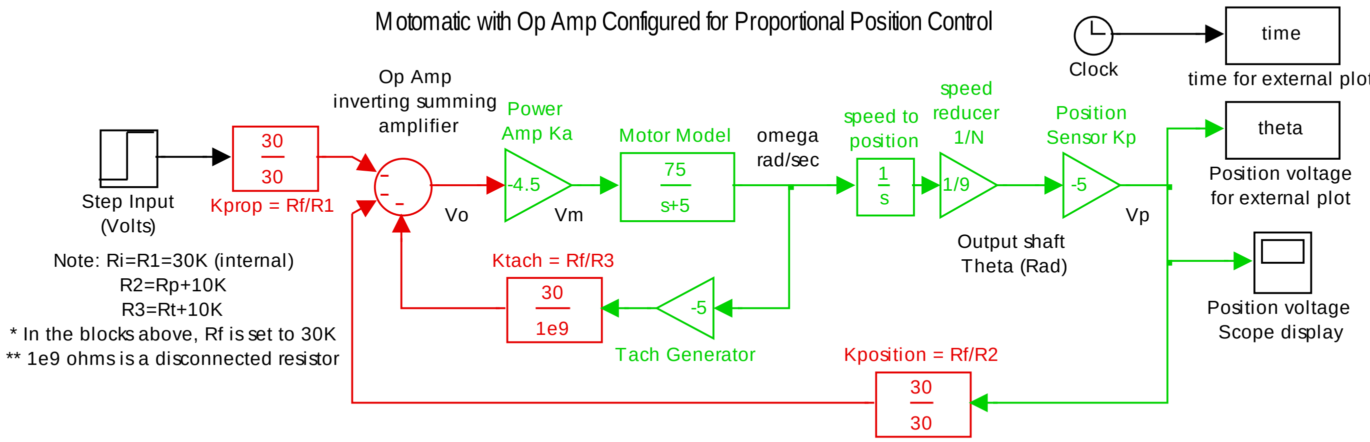


Figure 2. MATLAB Simulink model of MCSL, including nominal values for components (screen capture from MATLAB Simulink).

Many feedback control and compensator configurations are possible with the MCSL, all implemented using a single Op Amp. The operational amplifier performs the dual functions of analog summation and various compensation (active filter) functions depending upon the arrangement of resistors and capacitors. It is configured as an inverting amplifier, and its output is v_o. The operational amplifier is treated as ideal, which assumes three electronic attributes:

1) Open loop gain $A = \infty$
2) Input impedance $Z_{in} = \infty$
3) Output impedance $Z_{out} = 0$

Using these assumptions, we apply Kirchoff's Current Law (KCL) at the inverting input (v_-) of the op amp:

$$v_+ = v_- = 0 \text{ (virtual ground via assumption (1))}$$

$$\frac{v_o}{R_f} + \frac{v_i}{R_1} + \frac{v_p}{R_2} + \frac{v_T}{R_3} = 0 \text{ (KCL and assumption (2))}$$

Therefore,

$$v_o = -\left[\frac{R_f}{R_1} v_i + \frac{R_f}{R_2} v_p + \frac{R_f}{R_3} v_T\right]$$

The output of the op amp, v_o, is the input to the power amplifier which has gain K_a. The inverting DC power amplifier is capable of providing an output voltage within the nominal range of ± 15 volts at a high enough current level to drive the DC motor. It is internally current- limited to approximately +/- 4.5 amps to protect the electronics, especially in the case of an accidental short circuit of its output, v_m. If the output current exceeds these limits, the power amplifier LED changes from green to red, indicating an overload condition. The power amplifier gain K_a is negative due to the signal inversion. The output of the power amplifier $\underline{v_m}$ is the input to the DC permanent magnet motor. The motor is armature controlled such that the shaft torque is roughly proportional to the armature current. The transfer function from the motor armature voltage v_m to the motor shaft rotational speed ω is derived below from both the rotational kinematics and the electrical model of the motor.

Our rotational convention shall be that *counterclockwise* shaft rotation, viewed from the *motor* end of the MCSL, will be considered the positive rotation direction of the motor shaft θ_m,the motor speed ω_m and the system output position after the speed reducer, θ_o which is measured by the position sensor. Positive rotation in this sense will correspond to an *increasing* position sensor voltage v_p when the position feedback switch is in the positive (+) position (making the position sensor coefficient K_p negative). A positive voltage v_m applied to the motor will induce a torque which tends to rotate the shaft in a positive direction. The "zero position" of the output shaft is the center of the range of the position sensor potentiometer, which yields v_p=0.

Motor Kinematics

The motor torque T_m is proportional to the motor armature current, and must be equal to the opposing torque contributions of the rotational friction and inertia of the system. The torque balance equation is

$$J_m\dot{\omega} + K_{fm}\omega \quad = \quad T_m \quad = \quad K_c i_m \tag{1}$$

where

$\omega = \dfrac{d\theta_m}{dt}$ = rotational velocity [radians / second]

$T_m =$ motor torque [N-m]

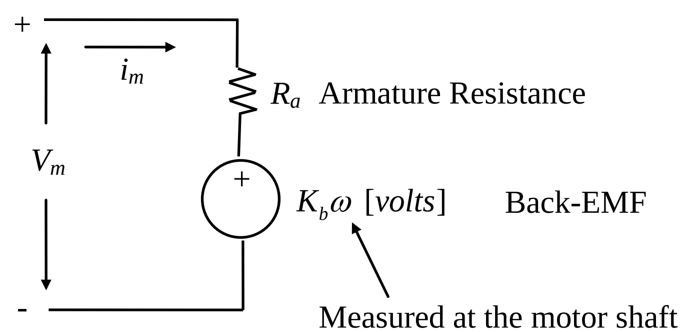
$K_c =$ motor torque constant [N-m / amp]

$K_{fm} =$ frictional coefficient $\left[\dfrac{\mathrm{N-m-s}}{\mathrm{rad}}\right]$

J_m = polar moment of inertia $\left[\dfrac{\mathrm{N-m-s^2}}{\mathrm{rad}}\right]$

Motor Electrical Circuit

The relationship of the motor current i_m to the motor voltage v_m can be derived from inspection of the equivalent electrical model of the DC motor armature:



Analysis of this simple circuit shows the relationship between the applied motor voltage and the motor current:

$$i_m = \frac{v_m - K_b\omega}{R_a} \tag{2}$$

Overall Motor System Model

Putting together the motor electrical model (2) and the motor/system torque balance (1), we get the overall differential equation relating the motor shaft speed ω to the applied motor voltage v_m:

$$J_m\dot{\omega} + K_{fm}\omega \quad = K_c\left(\frac{v_m - K_b\omega}{R_a}\right) \tag{3}$$

Re-arranged, this is written:

$$\left(\frac{J}{K_f+\dfrac{K_cK_b}{R_a}}\right)\dot{\omega}+\omega = \left(\frac{1}{K_b+\dfrac{K_fR_a}{K_c}}\right)v_m \tag{4}$$

Laplace transformed, and written as a transfer function:

$$\left(\tau_m s+1\right)\omega = K_m v_m$$

$$\frac{\omega}{v_m}=\frac{K_m}{\left(\tau_m s+1\right)} \tag{5}$$

where

$$\tau_m=\frac{J_m}{K_{fm}+\dfrac{K_cK_b}{R_a}} \quad \text{[seconds]} \quad = \text{ motor time constant}$$

$$K_m=\frac{1}{K_b+\dfrac{K_fR_a}{K_c}} \quad \left[\frac{\text{rad}}{\text{volt-sec}}\right] \quad = \text{ motor gain constant}$$

The composite parameters K_m and τ_m can vary widely between individual MCSL units.

The tachometer is a small DC generator built into the same housing as the motor. It provides a voltage $\mathrm{v_T}$ proportional to the shaft speed ω. The constant of proportionality is K_T. The tachometer or "tach generator" is internally wired such that, according to our rotational conventions, $\mathrm{K_T}$ is negative, yielding a negative velocity feedback voltage for positive (counterclockwise viewed from motor end) shaft rotation. This is the reason that the tachometer (motor speed meter) on the MCSL control panel reads negatively when the motor is rotating positively.

The motor shaft speed ω, in radians/second, is the rate of change of the motor shaft position θ_m in radians. The motor shaft position θ_m is the integral over any given time period of ω. Expressed in terms of the Laplace operator s,

$$\theta_m=\frac{1}{s}\omega \quad .$$

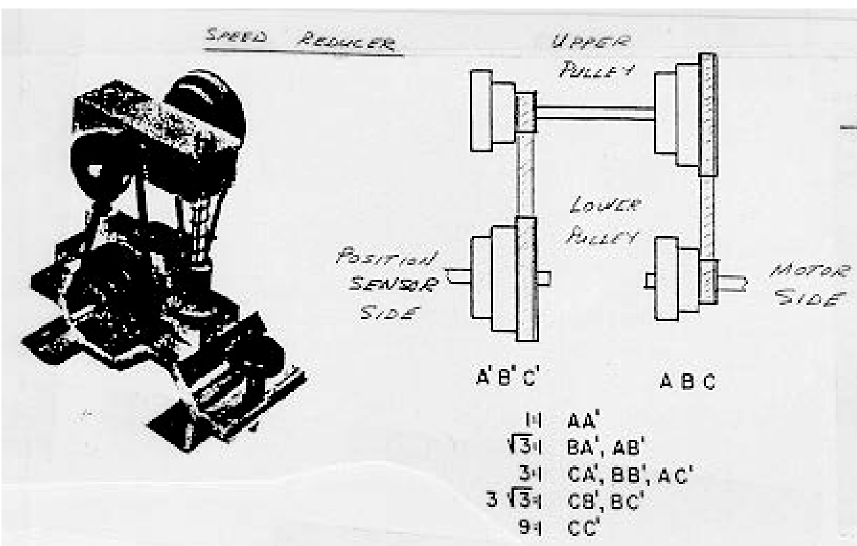
A multi-step pulley speed reducer provides several possible reduction ratios from the motor shaft position θ_m to the output shaft position θ_o. Referring to Figure 3a, each of the two drive belts may be placed in three different positions, yielding nine possible drive ratios, some of which are redundant. A 9:1 drive ratio corresponding to belt locations C and C′ will be used exclusively in this laboratory. The reduction ratio is specified by *N*, which in this case is nominally equal to 9. The speed reduction pulleys do not change the direction of rotation, only the relative input and output shaft speeds or incremental angles.

$$\theta_o = \frac{1}{N}\theta_m \text{ [radians]}$$

If we replace ω with $N\dot{\theta}_o$ in equation (5), and rearrange in normal form, the motor transfer function including the speed reduction, from the motor input voltage v_m to the shaft output angle θ_o is:

$$\frac{\theta_o}{v_m} = \frac{\dfrac{K_m}{N\tau_m}}{\left(s + \dfrac{1}{\tau_m}\right)} \tag{6}$$

A 10K ohm potentiometer (variable resistor) is used as a shaft position sensor, shown in Figure 3b. The potentiometer serves as a variable voltage divider, providing an output v_p proportional to θ_o over a range of slightly less than one complete rotation. The amount less than a complete rotation is called the "dead-zone" of the potentiometer.



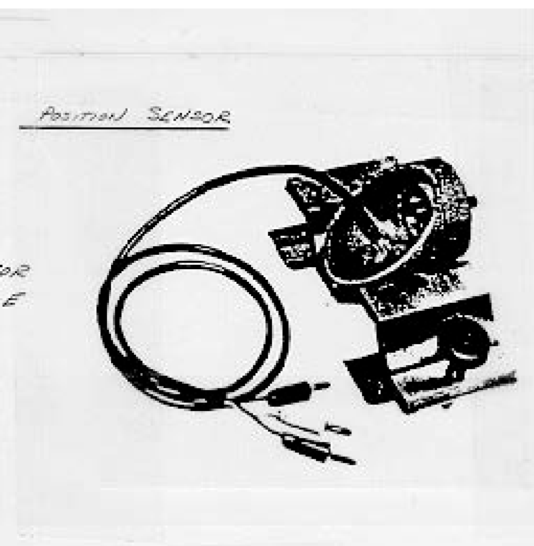


Figure 3a. Speed Reducer.[1]

Figure 3b. Position Sensor.

Figure 4 illustrates the electrical action of the potentiometer relative to the output shaft position θ_o. The voltage vs. angular position proportionality constant is K_p. K_p may be positive or negative depending on the polarity of the voltage applied across the potentiometer, which is set by the positive/negative position feedback switch on the controller console.

[1] Figures 3a and 3b were photocopied from the (now non-existent) MCSL product manual.

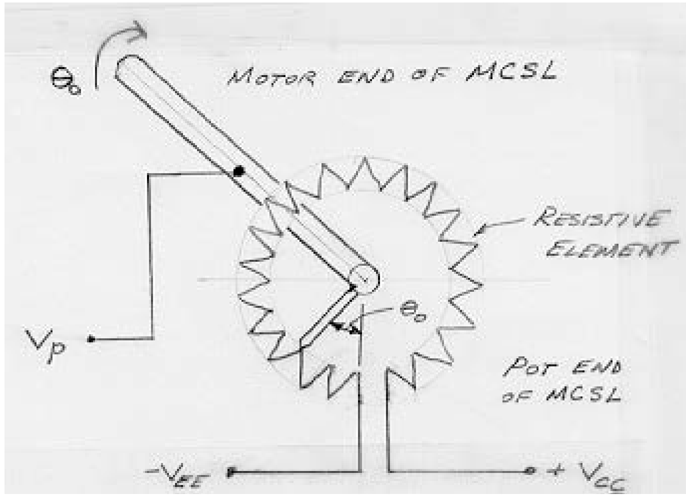


Figure 4. Position Sensor Potentiometer Model.

Consistent with our rotational convention, *counterclockwise* rotation of the potentiometer viewed from the motor end is considered positive θ_o. v_p will increase with positive rotation when the position feedback switch is in the positive (+) setting.
For completeness, let us consider the possibility of loading effects on the position sensor potentiometer. The Thevenin equivalent resistance of this voltage divider varies from 2.5K when $v_p = 0$, to 0 when $v_p = \pm 15$ volts. If R_2 is selected to be less than approximately 10K ohms, v_p can no longer be considered independent of the load due to current flowing in R_2 to the virtual ground at the inverting input of the op amp. In such cases, a complete electrical model of the potentiometer resistance divider and the R_2 load to ground must be used, and the simple relationship $v_p = K_p \theta_o$ is not really valid. An internal 10K resistor is built into the MCSL in the position feedback path to prevent excessive loading of the position sensor potentiometer $\left(R_2 = R_p + 10K\right)$. We will assume that R_2 will be sufficiently large that the loading effects can be ignored. This allows us to assume that v_p is proportional to the shaft position.

In all experiments, negative position feedback (feedback switch set to "-") is required to yield a stable system. Therefore, K_p will always be a negative number for experiments in this laboratory. Keep this in mind when using the voltage v_p as an indicator of θ_o. They have opposite signs when the position feedback switch is set negatively.

For convenience, in all later sections of this manual θ will be used in place of θ_o.

A reference control input voltage $\mathrm{v_i}$ is provided for servocontrol experiments. A voltage of either polarity may be selected using the *step input switch* on the control console. In the middle (neutral) position, this switch selects ground or $v_i = 0$. The step input potentiometer may be adjusted to set the magnitude of $\mathrm{v_i}$, over a nominal range of from 0 to ± 15 volts. An internal 30K ohm resistor R_1

connects the output of the reference input potentiometer to the inverting input of the op Amp. R_1 is sufficiently large to avoid loading effects on the reference input potentiometer.

All supply voltages for the MCSL are provided by an internal ± 15 volt power supply. It should be noted that the $+v_{cc}$ and $-v_{ee}$ voltages are only *nominally* +15 and -15 respectively. Significant variations have been observed on some units. These voltages directly affect the position sensor calibration and the reference input voltage.

Built-in 10K resistances are provided in the position and velocity feedback paths, and the external input paths. These must be added to the user-installed resistors R_p and R_T when specifying R_2 and R_3 respectively, that is:

$$R_2 = R_p + 10K$$
$$R_3 = R_T + 10K$$

As mentioned previously, the internal 10K resistances reduce the electrical loading of the feedback sensors. But they also serve as safety features to prevent the possibility of driving the inverting input of the op amp directly from a voltage source, which could damage the op amp. Recall that if the non-inverting input of an op amp (with reasonable feedback resistance) is grounded, as in this case, the inverting input is a *virtual ground*, i.e., zero volts.

In Experiments 1 we will determine the actual model parameters for your particular MCSL apparatus. In Experiments 2 through 7, we will mechanize several compensators and feedback configurations. In Experiment 5 we will fundamentally modify the mechanical system by adding an inverted pendulum to the output shaft, which will require that we re-derive the kinematic model. In Experiment 7 we will preview advanced linear control methods by investigating the use of full-state feedback. ***We will follow a consistent sequence in experiments 2 through 7:***

1. **Design.** Start by designing a particular configuration of the system to achieve a particular response using analysis and design methods taught in the EE302 lecture.

2. **Simulate** the system using MATLAB Simulink, starting with the downloadable simulation file MotomaticAnalogWith Tach.mdl . Compare the predicted response with the calculated expectations – they should match exactly since your simulation and (paper) analysis/design models are the same.

3. **Test and Tune**. Configure the MCSL hardware per our design. Test with controller as designed in Step 1. Compare the actual response with that predicted by calculations and simulation. The results will almost never match those of steps (1) and (2), but should be reasonably close. This is typical of any control system design since a system model can never be exactly correct. Tune the controller (vary resistor values) until the desired system response is obtained.

Experiment 1: System Modeling

Methodology

In this experiment we seek to experimentally derive a fully specified control system model for the complete MCSL apparatus, including both the control console and the electromechanical assembly.

A description of the electrical model and rotational kinematics of the MCSL has already been presented. Unfortunately, there is not an exact one-to-one correspondence between the actual system components illustrated in the schematic of Figure 2 of the Preliminaries section, and the block diagram of the system as shown in Figure 1 of this experiment. Block diagrams such as Figure 1 are the standard means for describing linear control systems.

The constants K_a, K_m, τ_m, N, K_p and K_T were defined in the ***Preliminaries***. The input path gain K_1 and the feedback path gains K_2 and K_3 are selected by the ratios of R_f to each resistor R_1, R_2 and R_3 respectively.

$$K_1 = -\frac{R_f}{R_1}$$

$$K_2 = -\frac{R_f}{R_2}$$

$$K_3 = -\frac{R_f}{R_3}$$

Note that K_1, K_2 and K_3 are always negative, due to the inverting configuration of the op amp. These will become parameters that we can vary in order to achieve some desired behavior of the control system. As discussed in the Preliminaries, we will always set $R_2=R_1$ so that $K_2=K_1$ in all experiments.

We have no control over the system model constants K_m, τ_m, K_p and K_T. These are characteristics of the apparatus, including the motor, position sensor and velocity sensor (tach generator). In this experiment, we will experimentally measure these constants and the reduction ratio N for each individual MCSL.

As noted in the Preliminaries, substantial differences exist between different MCSLs. The model measurements made in this experiment will be used throughout all the remaining experiments. It is extremely important that all experiments be conducted using the same MCSL control console and electromechanical assembly. Equipment is often moved between benches since other courses and senior project students also use the MCSL, or parts thereof. *Be sure to record the equipment numbers from both these assemblies in this and ALL experiment reports, to verify that the same equipment has been located and used for each lab*.

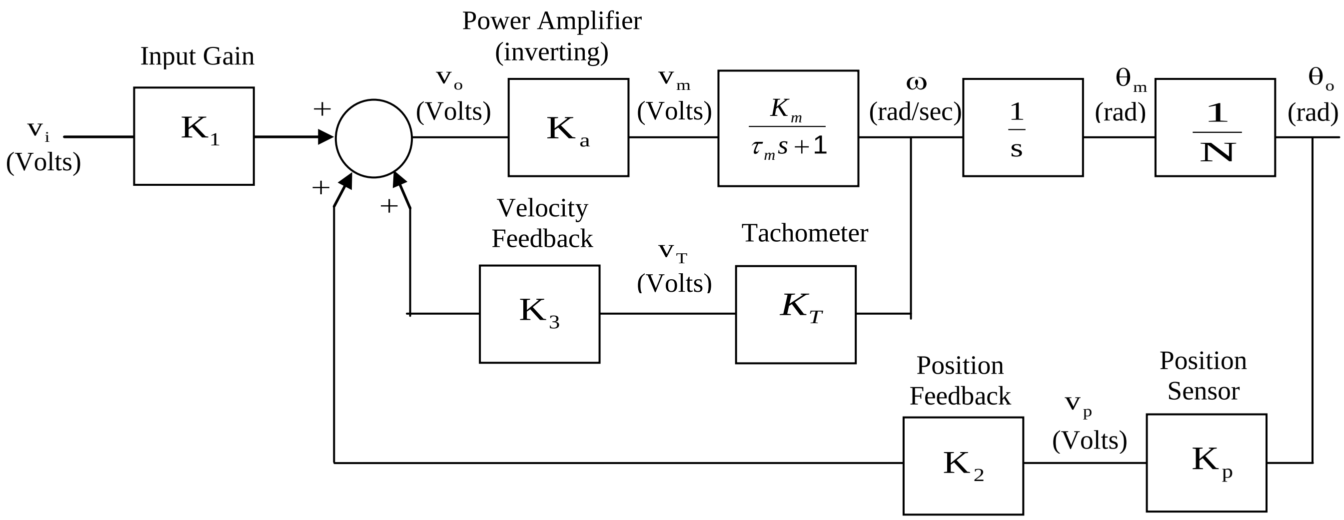


Figure 1. Physical Block Diagram for MCSL, comparable to Figure 1 of *Preliminaries*.

Linear Modeling of a Nonlinear System

We will be fitting a *linear* model to a physical system which is actually nonlinear. For small perturbations about an operating point, the linear model can usually be relied upon to predict the system behavior with reasonable accuracy. However, when larger excursions between system states are required (say, due to a large step function input), or when phenomena such as static friction or dead zone in the rotating parts must be considered, the physical system exhibits *nonlinear behavior* which is not accurately predicted by the linear model. It is important to always be aware of the limitations of a linear model, and when it may be legitimately used. This will become evident in the course of this and subsequent experiments, when the behavior of the system predicted by our linear model may diverge from actual observations.

In general, a model parameters will be dependent upon the operating point, that is, the position and velocity conditions at which the linear model was determined. The following procedures for measuring the linear model coefficients of the MCSL must be conducted under conditions spanning a range of operating points. Therefore, the composite linear model will not be truly correct at *any* particular operating point, but must be assume valid over *all* operating points out of necessity. Note the test conditions applicable during each measurement. When system behavior in later experiments fails to agree with model predictions based upon the measurements made in this experiment, a difference in operating conditions is often a contributing factor for the discrepancy.

Transfer Function for Position Servocontrol

If the output variable (the thing we want to control) is the output shaft position θ_o, and the input v_i is an indicator of the desired output shaft position, we have a *position servocontrol system*. The objective of the position servocontrol system is to make the shaft position track as closely as possible the reference input voltage v_i. We construct an overall closed-loop Laplace transfer function relating θ_o to v_i. The block diagram of Figure 1 may be reduced to the more familiar form of Figure 2, in which all blocks have net positive gains. Note that K_1, K_2, K_3, K_a, K_p, and K_T are *negative* constants.

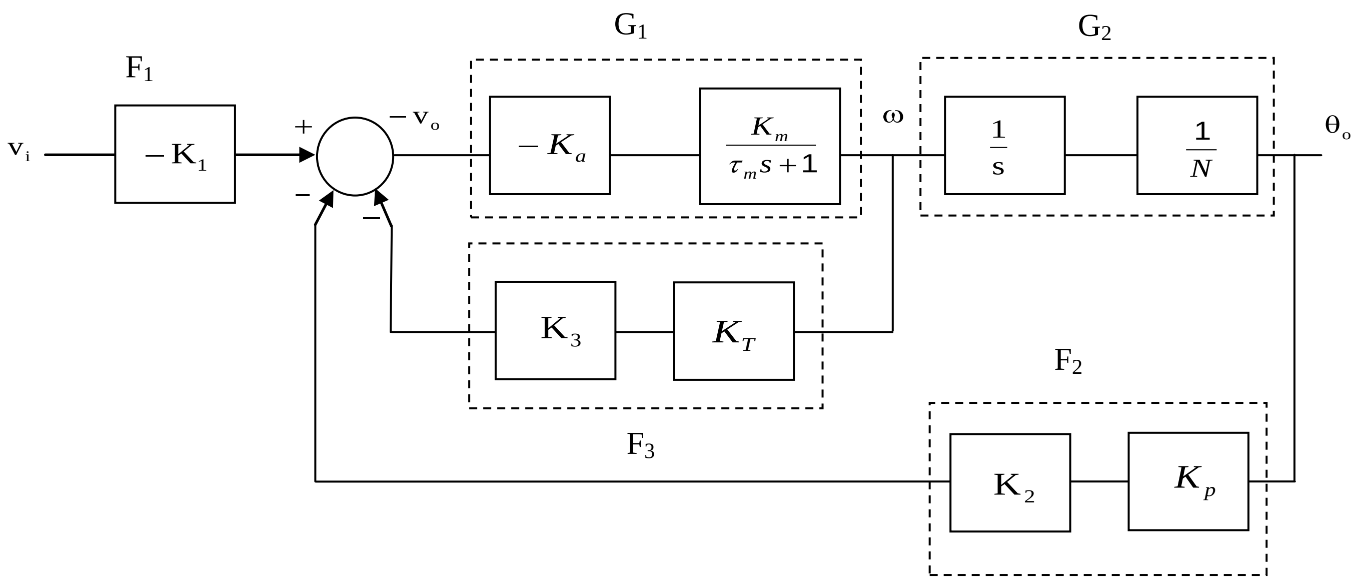


Figure 2: Block Diagram for Closed-Loop System with Proportional Position and Velocity Feedback, in Standard Servocontrol Form.

The closed-loop transfer function H(s) may be written from direct inspection of the block diagram of Figure 2. Note that the time domain variables θ, ω, and v_i are now Laplace Transform quantities with magnitude and phase.

$$\theta_o = G_1 G_2 \left[F_1 v_i - F_3 \omega - F_2 \theta_o\right] = G_1 G_2 \left[F_1 V_i - \frac{F_3}{G_2}\theta_o - F_2 \theta_o\right]$$

$$\theta_o\left[1 + G_1 F_3 + G_1 G_2 F_2\right] = G_1 G_2 F_1 v_i$$

$$H(s) = \frac{\theta_o(s)}{v_i(s)} = \frac{G_1 G_2 F_1}{1 + G_1 F_3 + G_1 G_2 F_2}$$

Substituting the actual transfer functions for each block G_1, G_2, F_1, F_2, and F_3 from Figure 2 and simplifying, we get the closed-loop position servocontrol transfer function in terms of the actual system parameters:

$$\frac{\theta_o(s)}{v_i(s)} = H(s) = \frac{\frac{K_m K_a K_1}{\tau_m N}}{s^2 + \frac{1}{\tau_m}\left(1 - K_m K_a K_3 K_T\right)s - \frac{K_m K_a K_2 K_p}{\tau_m N}}$$

If the system is underdamped or critically damped, it is possible to identify the damping factor ζ and the natural frequency ω_n by comparison with the standard underdamped second order transfer function

$$H(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{1}$$

Comparing terms, we see that:

$$-\frac{K_m K_a K_2 K_p}{\tau_m N} = \omega_n^2 \left[\sec^{-2}\right]$$

$$\frac{1}{\tau_m}\left[1 - K_m K_a K_3 K_T\right] = 2\zeta\omega_n \left[\sec^{-1}\right]$$

$$\frac{K_m K_a K_1}{\tau_m N} = K\omega_n^2 \left[\frac{rad}{V - \sec^2}\right]$$

Therefore

$$\omega_n = \sqrt{\frac{-K_m K_a K_2 K_p}{\tau_m N}} \tag{2}$$

$$\zeta = \frac{1}{2\omega_n \tau_m}\left[1 - K_m K_a K_3 K_T\right] \tag{3}$$

ω_n cannot be measured directly from observation of the transient response. However, the *damped natural frequency* ω_d can be measured graphically as the frequency of oscillation during the decaying underdamped response:

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \tag{4}$$

Note that $\omega_d = \omega_n$ only when the system is completely *undamped*, meaning that the oscillatory response neither decays nor increases.

The steady state gain constant in (1) is

$$K = \frac{K_m K_a K_1}{\tau_m N \omega_n^2} = -\frac{K_1}{K_2 K_p} \tag{5}$$

Since we measure and display the position voltage v_p rather than θ_o on the oscilloscope, we are interested in the modified transfer function

$$\frac{v_p(s)}{v_i(s)} = K_p \frac{\theta_o(s)}{v_p(s)} = \frac{K_p K \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Because $K = -\frac{K_1}{K_2 K_p}$ and $K_1 = K_2$ in all experiments, as discussed in the ***Preliminaries***.

Determining K_m and τ_m Experimentally

K_m and τ_m are the motor gain constant and time constant respectively. K_m relates the steady state motor speed ω to the applied motor voltage v_m. τ_m is the time constant of the complete rotating system including the motor and speed reducer - an indicator of how long it takes to accelerate or decelerate between different speeds. This metric includes the effects of the overall polar moment of inertia and the back-EMF of the motor. Both of these constants can be measured by a single experiment: the velocity step response of the motor.

A voltage step, from zero to some constant v_{step} is applied to the motor input v_m, with no load on the motor. No feedback is used. From the system equation:

$$\tau_m \dot{\omega} + \omega = K_m v_m \quad t \geq 0$$

Solution for $v_m = v_{step} u(t)$, where u(t) is the unit step function:

$$\omega = K_m \left(1 - e^{-\frac{t}{\tau_m}}\right) v_{step}$$

$$\lim_{t\to\infty} \omega = \omega_{ss} = K_m v_m = \text{steady state motor speed}$$

To measure:

K_m: At several different motor speeds, measure the steady state motor speed ω vs v_m after a long time (several seconds) has elapsed.

$$\lim_{t\to\infty} \frac{\omega}{v_{step}} = \lim_{s\to 0} \frac{K_m}{\tau_m s + 1} = K_m \left(\frac{rad}{volt - \sec}\right)$$

τ_m : τ_m can be observed as the time it takes for $\frac{\omega}{\omega_{ss}} = 63\%$, since when $t = \tau_m$,

$$\frac{\omega}{\omega_{ss}} = 1 - e^{-\frac{t}{\tau_m}} = 1 - e^{-1} = 0.6321$$

Determining K_T, K_p and N Experimentally

The tachometer provides a voltage output proportional to the motor shaft speed ω:

$$v_T = K_T\,\omega$$

where K_T is the tachometer constant, with units of volt-seconds/radian. According to our rotational convention (positive rotation is counterclockwise viewed from the motor end), K_T is negative. K_T is found by observing v_T (using a voltmeter) and ω (using the MCSL tachometer readout) at several data points, and calculating the ratio. At each datum, the motor is driven by a constant positive voltage v_m.

The potentiometer (position sensor) rotates freely and provides a unique position voltage v_p over a range of slightly less than one revolution (2π radians). If the output voltage $v_p = K_p\theta_o$ is plotted as a function of the shaft position while the shaft rotates at a constant speed, a saw-tooth-like pattern is displayed on the oscilloscope. Figure 3 illustrates this periodic voltage-angle relationship, as well as the dead zone of the sensor. The slope (voltage vs. angle) of the continuous parts of the plot is equal to the constant K_p.

N is the exact ratio of the motor shaft position θ_m measured at the input of the speed reducer, to the output shaft position $\theta_o = \dfrac{\theta_m}{N}$ measured at the output of the speed reducer. It is most easily measured visually by rotating the input shaft an integer number of complete rotations, and observing the exact angular change of the output shaft. It will be observed that the ratio may not be exactly the same as that prescribed by Figure 3a in the *Preliminaries*. Inexact dimensions of the pulleys and belts may yield slight differences from the ideal reduction ratios.

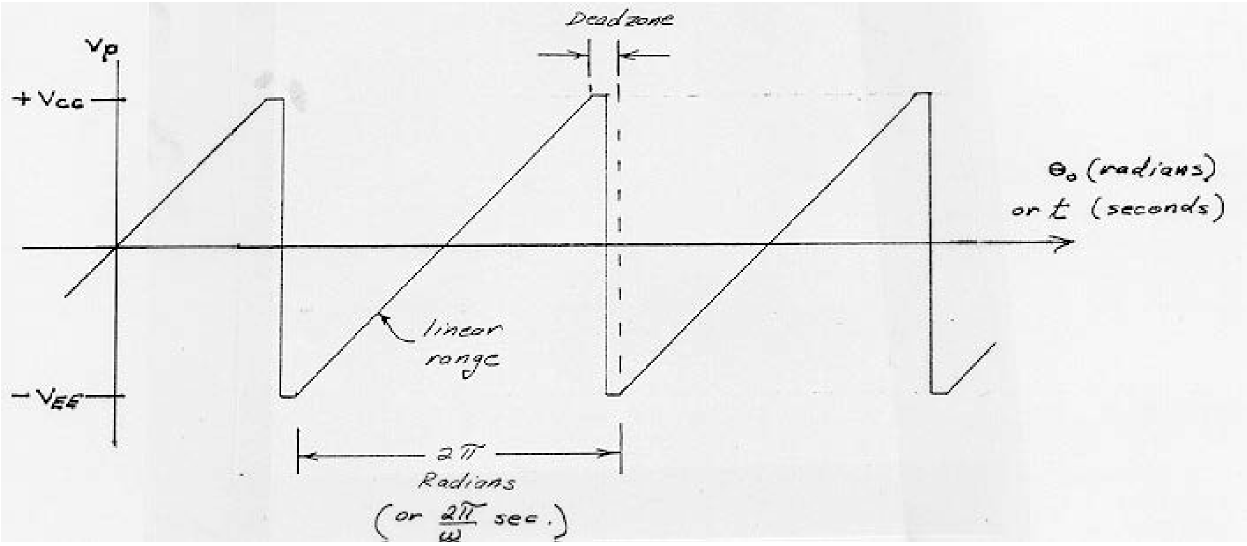


Figure 3. Output v_p of Position Sensor for Constant Rate of Rotation ω

MCSL Operating Precautions

1) Perform all equipment hookup and wiring with the MCSL control console power OFF. The motor, tachometer and potentiometer plugs are color coded to match the jacks on the control console. Verify that these are connected correctly before turning on the power.

2) When connecting the speed reducer and position sensors on the MCSL, do not press the shaft couplings tightly together. This restricts the action of the couplers which must be slightly loose in order for them to correctly handle small shaft misalignments. Carefully align the shafts by hand by pushing them together while holding the plastic coupler in place, and tightening the knurled clamp knobs. The quality of the assembly of each coupling has been found to have a significant effect on the experimentally measured system parameters.

3) DO NOT allow the motor to run continuously with the position sensor potentiometer coupled. This causes excessive wear of the pot element. Never direct-couple the pot to the motor. Always couple through the speed reducer.

4) Avoid operating the control console without some resistance (less than one Megohm) in the op amp feedback path. Without a finite feedback resistor R_f, the op amp gain is (ideally) infinite, such that the output saturates at either v_{cc} or v_{ee}, overdriving the power amplifier and possibly causing damage.

5) When the power amplifier is operating in current-overload mode, the LED changes from green to red. Do not allow the power amplifier to operate continuously in current-overload mode for periods greater than about one second. Possible damage to components could occur. A brief red flash of this LED during step response tests is normal.

6) Use common sense at all times. Fully understand the experimental procedure and underlying theory prior to operating the MCSL.

Procedure

1. If not already done, connect all leads from the MCSL electromechanical apparatus to the control console, observing the color coded jacks and plugs. Decouple the potentiometer from the speed reducer by loosening the clamp screw and sliding the pot bracket away from the speed reducer. The plastic coupler may fall off; save it in a safe place. Leave the speed reducer coupled to the motor shaft. Check that the speed reducer is set for a nominal 9:1 reduction ratio. All electrical connections are to be made using double-banana plug test leads. Connect the summing node output to the op amp input. Connect the op amp output to the power amp input. Connect a 30K resistance, using a resistance decade box, from the op amp output to its input. Since internally $\mathrm{R}_1 = 30\mathrm{K}$, this 30K feedback resistance fixes the op amp gain at -1 for the reference voltage input v_i from the step input switch $(\mathrm{K}_1 = -1)$.

2. The motor will be operated open-loop for these tests, that is, without any feedback from the position or velocity sensors. Set the step input switch to the (+) position. Vary the input voltage v_i using the input voltage adjustment pot on the console. Each setting will correspond to a different motor speed, which may be monitored using the tachometer meter on the control console. Adjust v_i to establish each of the following motor speeds, and record v_o, v_m and v_T using the bench digital voltmeter. All voltage measurements are made with respect to the floating ground (black jacks) of the MCSL console, NOT the chassis ground (green jack). These two ground points are not the same (although some defective units may have them internally shorted together). We include the 1 ohm shunt resistor in the ground leg of the motor as part of the motor model, purely in the interest of keeping the model as simple as possible. This internal resistor is used for measurement of the motor current.

   Note that the motor speed ω is measured in radians per second. The tachometer displays the motor speed in revolutions per minute (RPM). Convert RPM to radians per second by multiplying RPM by $\pi / 30$. Recall from previous discussion that for positive rotation the tachometer will display negatively. Report your speed measurements as positive, regardless of the negative meter reading. Compile your data in a table containing the following information:

RPM	$\omega[rad/\mathrm{sec}]$	$v_o[volts]$	$v_m[volts]$	$v_T[volts]$
0				
500				
1000				
1500				
2000				
2500				

   Draw, either by hand or using a computer plotting program, the following plots using the data just taken:

a) v_T vs ω

b) ω vs v_m

c) v_m vs v_o

Interpolate the data by sketching straight lines on each plot, lying through (or nearly through) data points in a region where a fairly linear relationship exists (typically in the mid-speed range). Make sure that the data points used for the straight line in each plot are the same for all three plots: this assures that the same operating point is being used for determination of all three linear model parameters. Identify this "linear" measurement region on each plot.

The slope of the line in plot (a) is the coefficient K_T. Similarly, the slope of the line in plot (b) is K_m, and for plot (c) the slope is K_a. Record K_T, K_m and K_a for your apparatus. Note the proper units and sign in each case, consistent with those defined in the ***Preliminaries***.

3. The motor time constant τ_m is found by applying step voltage inputs v_i and observing the speed response ω via the tachometer voltage v_T using the storage oscilloscope. The op amp configuration remains unchanged from previous steps.

   Set up the digital oscilloscope in SINGLE SWEEP mode with an EXTERNAL TRIGGER with DC coupling. Set for rising edge triggering. Use the input voltage v_i as the trigger signal. There is no terminal at the v_i position on the MCSL, but it can be accessed at the terminals just below the test meter on the MCSL console, with the "Selector Switch" set to position 3. With DC triggering, it is necessary to adjust the trigger level to just above the ground level, such that triggering will occur the moment that the step input is applied.

   Display v_T on either channel of the scope. Since K_T is negative, a positive voltage step will yield a negative v_T trace. It is therefore advisable to use the "invert" feature of the digital oscilloscope to display a positive v_T vs. time trace. Set the vertical axis scaling to 5 Volts/division and start with a time axis scaling of 0.2 seconds/division (you may have to adjust this later so that the trace reaches steady state within the display interval). Set the 0 volt (ground) level of the trace at the bottom graticule line on the scope display, and the trigger point at the left side of the display.

   In SINGLE SWEEP mode, the trigger must be rest before each new trace. To overlay multiple traces, use the AUTOSTORE button to reset without erasing the prior trace.

   Generate and overlay on the scope three transient response plots of the motor speed voltage v_T from 0 RPM to each of the following speeds: 500, 1000 and 2000 RPM.

   The reference input voltage v_i corresponding to each must be found in advance by running the motor and adjusting the step input voltage knob until the desired speed is achieved. Transcribe this data into your lab report.

   As previously discussed, τ_m is the time required for the motor to achieve $1 - e^{-1} = 63\%$ of its steady-state speed in each case. The 63% point should be determined graphically for each of the three traces on the printed plot. Indicate the time at which 63% of steady state is reached for each trace. The average of these three time measurements should be reported as the value of the motor time constant τ_m.

4. The position sensor potentiometer coefficient K_p is determined by comparing the sensor voltage to the shaft position in the linear range of the pot.

   Couple the pot to the speed reducer. Set the position feedback switch to the negative (-) position. Connect v_p (the output of the position sensor potentiometer) to the scope. Set the scope for AUTO TRIGGER mode, the time base at 0.02 seconds per division and vertical axis scaling to 5 Volts/division.

   Apply a small constant input voltage $\mathrm{v_i}$ such that the motor rotates at a constant speed of approximately 500 RPM. Allow the motor to come fully up to steady state speed. Press the STOP button on the scope to freeze the display. At least one complete rotation of the pot must be displayed. Note that one complete cycle of the observed waveform is equivalent to 2π radians of shaft rotation. It may be necessary to adjust the time base and volts/division such that the trace looks like Figure 3.

   Again, import and copy into your lab report of this oscilloscope trace. Graphically measure and indicate the slope of the sawtooth linear regions in units of volts per radian. This slope is the constant K_p:

$$K_p = \frac{-\left(v_{cc} + v_{ee}\right)}{2\pi - \text{deadzone}} \quad \left[\frac{\text{volts}}{\text{radian}}\right].$$

   Both v_{cc} and v_{ee} are positive numbers, since the negative supply is designated $-v_{ee}$. The overall minus sign is included since for all experiments in this laboratory, the position feedback switch will be set negative (-). Also report the deadzone in radians of output shaft rotation.

5. Experimentally measure the exact reduction ratio N of the speed reducer pulley system, as described previously. The *nominal* ratio should be 9:1. Rotate by hand the input shaft an exact integer multiple of nine revolutions. Eighteen revolutions are suggested. Observe the integer and fractional number of output shaft revolutions to the closest 5 degrees. A protractor may be helpful in making this measurement. Although the protractor may read in degrees, report your measurement in radians.

$$\text{N} = \frac{\text{number of input shaft revolutions } \times 2\pi \text{ radians (or } \times 360 \text{ degrees)}}{\text{actual output shaft angular displacement in radians (or degrees)}}$$

6. Using all the experimentally determined model parameters, specify the closed loop system transfer function H(s) including both position and velocity feedback, as previously derived. Leave K_1, K_2 and K_3 as undetermined parameters, since these are set by the values of resistors that can be selected for each experiment.

7. Reproduce the table below in your report, filling in the values of each of your system parameters with their respective error bounds.

Parameter	Value	Units
K_m		radians/volt-sec
K_a		Volts/Volt
τ_m		seconds
N		radians/radian
K_p		Volts/radian
K_T		Volt-sec/radian

Notes

- Each member of the lab group should keep a copy of this table for use in later experiments, which will all utilize the parameters determined in this experiment.
- Don't forget to record the numbers of both the console and the rotating apparatus of the MCSL and use the same components for all subsequent experiments. (Stamped numbers are found on each subassembly.) This is important, since in all later experiments, you will be comparing your measured results with calculated and computer simulated results based upon the parameters you measured in this first experiment.
- If any components of the MCSL are replaced during the quarter (e.g., the position sensor potentiometer or power supply) you may have to redo the parameter measurement procedure affected by that/those component(s), as described in this experiment.

Preparation

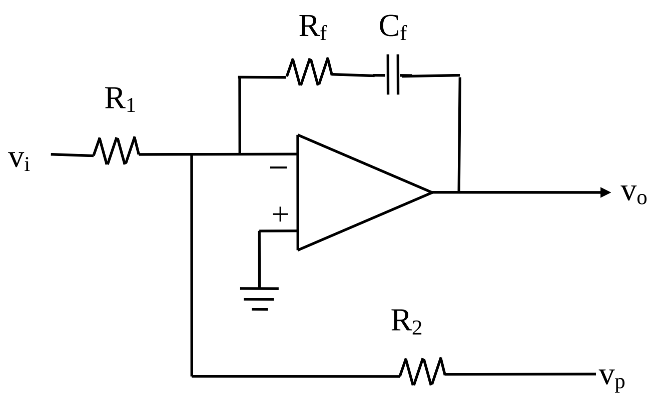
Read the Preliminaries section and this Experiment completely. You should be able to solve the following problems:

1. Review modeling of rotational kinematic systems from earlier physics courses. Here's the differential equation for the MCSL plant, from the motor input voltage v_m to the position output θ.

$$\left(\frac{J}{K_f + \frac{K_c K_b}{R_a}} \right) \ddot{\theta} + \dot{\theta} = \left(\frac{1}{K_b + \frac{K_f R_a}{K_c}} \right) v_m$$

   If we were to add a linear clock (torsion) spring to the output shaft, with spring force equal to $K_s\theta$ and zero force for $\theta = 0$, how would this equation change?

2. Review the analysis and design of operational amplifier circuits. For the following circuit, derive the transfer function, from the two inputs v_i and v_p, to the op amp output v_o, in terms of R_1 , R_2, R_f and C_f :



Experiment 2: Proportional Control

Introduction

In this experiment, the MCSL will be used as a closed-loop feedback position servocontrol system. A simple proportional feedback configuration will be used, by selection of appropriate resistors in the op amp circuit of the MCSL console.

This experiment focuses on the transient response of the closed loop system, in particular, the ***step response***. A step change in the reference input voltage v_i is applied, and the output shaft position is observed via the voltage output of the position sensor, v_p. These are compared, and the difference, known as the error signal, is amplified by a proportional gain to generate the input voltage to the electric motor, v_m. The objective is to achieve an output shaft position that follows as closely as possible the reference input change. This illustrates the basic purpose of servocontrol systems: the ability of the system to achieve a desired output that tracks the reference input. The "closeness" of the output response to the applied reference input step can be assessed in terms of several performance metrics such as the rise time τ_r, peak time τ_p, settling time τ_s and maximum overshoot M_p.

From the Preliminaries 1 that the closed loop transfer function of the MCSL may be expressed in the form below, for which you determined all the model parameters for your apparatus in Experiment 1.

$$H(s) = \frac{\theta(s)}{v_i(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{1a}$$

Where

$$K = -\frac{K_1}{K_2 K_p} \tag{1b}$$

$$\omega_n = \sqrt{\frac{-K_m K_a K_2 K_p}{\tau_m N}} \tag{2a}$$

$$\zeta = \frac{1}{2\omega_n \tau_m}\left(1 - K_m K_a K_3 K_T\right) \tag{2b}$$

In this experiment, we will not be using the speed feedback signal from the tachometer, so K_3=0. This simplifies the damping factor to:

$$\zeta = \frac{1}{2\omega_n \tau_m}$$

The oscilloscope can display only voltages, not actual radial positions $\theta(s)$. v_p is an indicator of the actual shaft position while v_i is an indicator of the *target* position, the position that you wish the shaft to go to. However, H(s) as derived above relates the output shaft position $\theta(s)$ [radians] to the input voltage v_i. To express the output position voltage v_p in terms of the input v_i we must include the position transducer factor K_p :

$$\frac{v_p(s)}{v_i(s)} = K_p H(s)$$

In most of our step response experiments (except Experiment 7), $R_1=R_2$, and therefore $K_1 = K_2$. This assures that the output voltage scaling v_p matches that of the input v_i. In other words, a 6 volt step input v_i will result in a 6 volt steady state output voltage v_p. With this equivalence, $K_p K = -1$ in equation (1b), and equation (1a) reduces to the simpler closed-loop relationship:

$$\frac{v_p(s)}{v_i(s)} = \frac{-\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{3}$$

The negative sign is due to the fact that K_p is negative when the feedback switch is set for negative feedback. A *positive* step in v_i will yield a proportional *negative* step in v_p . We can correct for this on the scope display by using the trace inversion feature available on the digital oscilloscope.

If $0 < \zeta < 1$, the system is considered *underdamped* and the following *transient response metrics* may be calculated directly from knowledge of ζ and ω_n :

$$\tau_r = \frac{\pi - \cos^{-1}\zeta}{\omega_n\sqrt{1-\zeta^2}} \quad \text{[sec]} \quad \text{0-100\% Rise Time} \tag{4a}$$

$$\tau_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \quad \text{[sec] Peak Time} \tag{4b}$$

$$\tau_s = \frac{3}{\zeta\omega_n} \quad \text{[sec]} \quad \text{5\% Settling Time} \tag{4c}$$

$$M_p = \frac{y_p - y_{ss}}{y_{ss}} = e^{\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)} \quad \text{[unitless]} \quad \text{Maximum Overshoot Fraction} \tag{4d}$$

Not that if $\zeta > 1.0$, the system is *overdamped*, and overshoot will not be observed, so that the above metrics cannot be defined.

For $\zeta < 1.0$, the system is underdamped. M_p is a function of ζ alone, while τ_r, τ_p, and τ_s depend on both ζ and ω_n. Figure 1 illustrates the influence of ζ on the step response.

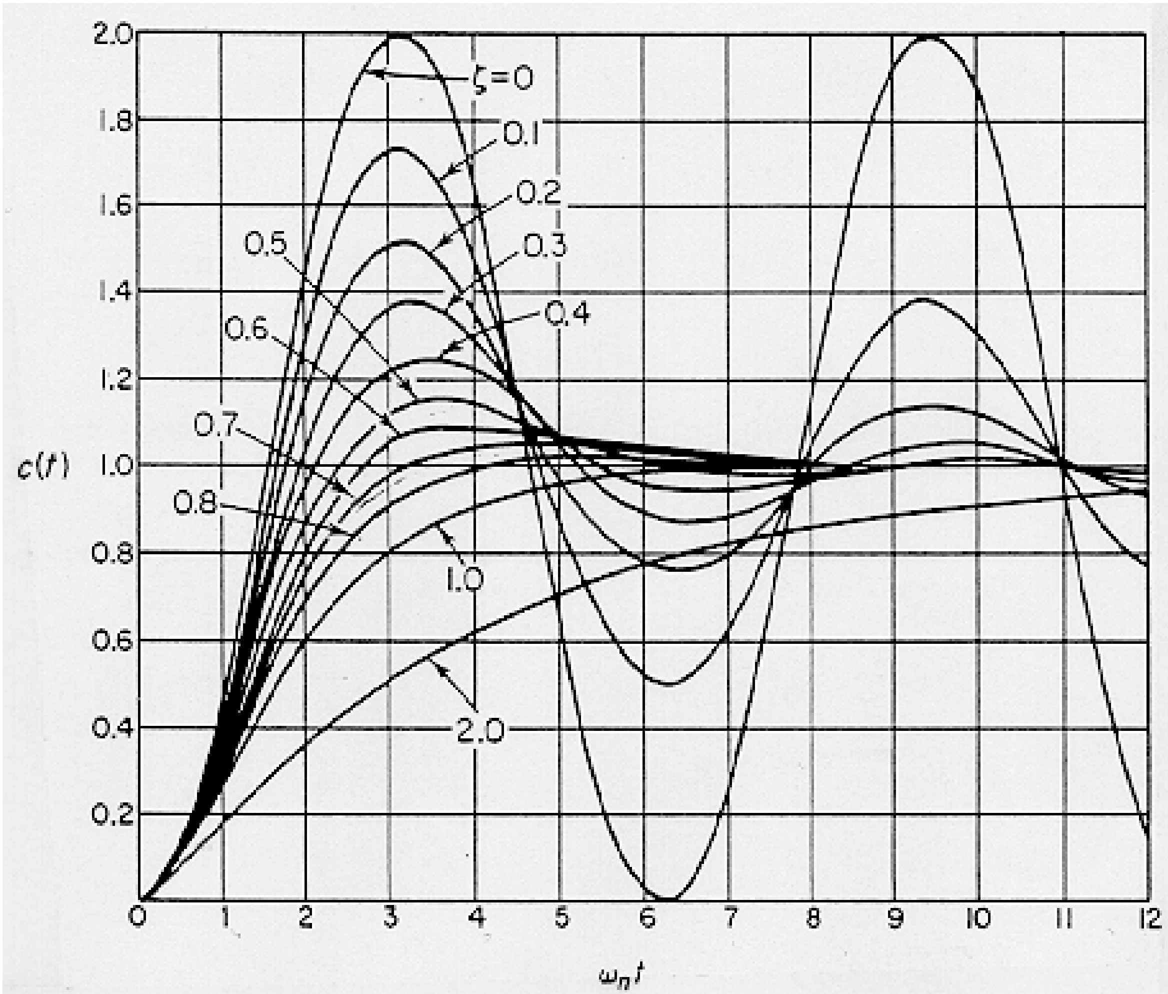


Figure 1. Normalized Step Responses, Parametric with Damping Factor ζ

The observed *steady state error* of the closed-loop system may be expressed as

$$e_{ss} = v_i - (-v_p) \text{ [volts]}$$

or normalized to a percentage,

$$e_{ss}(\%) = \frac{v_i - (-v_p)}{v_i} \times 100\% \tag{5}$$

Applying the Laplace Final Value Theorem to equation (3), the steady state gain of the closed-loop system with respect to the position voltage v_p is:

$$\lim_{t\to\infty} \frac{v_p}{v_i} = \lim_{s\to 0} K_p H(s) = \lim_{s\to 0} \frac{-\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = -1 \text{ [volt/volt]}$$

The steady state error is theoretically zero for the MCSL since it does not have any type of "spring return" that generates an opposing force as the position deviates from the center position. This is classified as a Type 1 plant[1] which will be discussed in the EE302 lecture. However, nonlinear effects such as static friction in the apparatus may cause steady values of v_p that are not exactly the same as v_i .

Figure 2 illustrates how each step response metric (4) may be graphically determined from an oscilloscope display of the position sensor voltage $v_p = y(t)$. Note that in general, $y(t)$ in Figure 2 could represent either the shaft position θ_o (in radians) or v_p (in volts), related by the position sensor gain K_p .

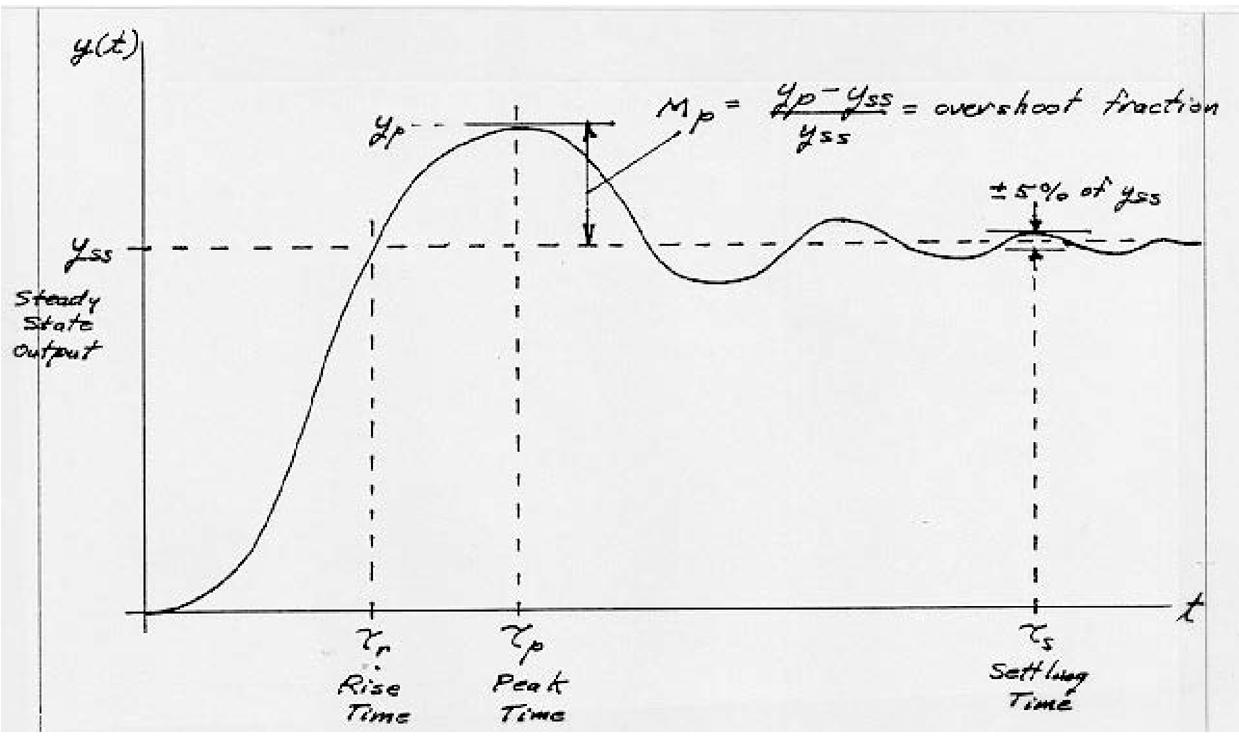


Figure 2. Measurement of Transient Response Metrics

Methodology

Our design objective is to achieve a step response that has a 50% overshoot, i.e., $M_p = 0.50$. In this experiment we will first calculate the value of R_f the we expect will achieve the desired overshoot for *our* system, with model parameters determined in Experiment 1. We will also attempt to predict τ_p, τ_s *and* τ_r . We will then simulate the system with these values using MATLAB Simulink. Finally, we will configure the MCSL apparatus and observe the actual

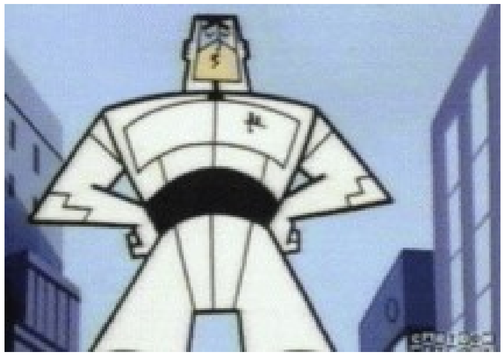
[1] System Type numbers will be discussed later in Experiment 6.

system response. It will not exactly match predictions, so we will "tune" the response of the system by iteratively varying the value of R_f until we achieve the desired overshoot.

For our proportional control feedback configuration, the input v_i will be a positive step function, going from 0 to 6 volts, connected through the internal 30K input resistance R_1 to the summing junction of the op amp. Set $R_2 = R_1$ = 30K by insertion of a 20K resistance between the yellow position feedback terminals. This assures that the output position voltage v_p is scaled the same as the reference input voltage v_i. Since we are using position feedback only, the tachometer feedback resistor R_3 is not connected, such that $K_3 = 0$.

Be certain that you are using the same MCSL control console and electromechanical assembly as used in Experiment 1, or the comparisons will be meaningless.

Procedure



1. For $M_p = 0.50$, calculate ζ. Then using the model parameters that you found in Experiment 1 with this value of ζ, calculate ω_n. From these, calculate the predicted τ_r, τ_s, and τ_p. Finally, calculate the required value of R_f to achieve this overshoot specification and write the closed loop transfer function $\frac{v_p(s)}{v_i(s)}$ with all of these values inserted.

2. Download the MATLAB Simulink model MotomaticAnalogWithTach.mdl from ***Prof. MacCarley's Control Systems Web Site*** under the EE342 tab, using the link

http://telab.ee.calpoly.edu/~amaccarl/students/EE342/MotomaticAnalogWithTach.mdl

This can be done (for the Internet Explorer web browser) by using the "Save Target As" menu selection item after right clicking the file name on the web site. Store it on the PC desktop. Launch the program by double-clicking the downloaded file. This should start MATLAB and bring up the model in Simulink. Edit the model blocks to match your parameters measured in Experiment 1. Simulate a 0 to 6 volt input step in v_i by clicking the triangular "run" icon on the Simulink Toolbar. Observe the plot of the output voltage response v_p (not θ_o) on the Simulink "Oscilloscope" . Note that MATLAB puts the scope display behind the model window each time the simulation is run, so be sure to move the model window aside and bring the plot to the front if it is obscured by the model window. If the plot auto-scales, change the vertical plot axis to 2 a range of 10 volts, and the horizontal (time) axis to display enough of the response to allow measurement of all the transient response metrics. Copy and paste the simulated response display directly into your lab report.

3. Set up the MCSL with your calculated value of R_f using a resistor substitution box. Set the input for a 0-6 volt input step voltage v_i using the MCSL's voltage adjustment knob and the step input switch. v_i can be accessed by setting the rotary selector switch on the MCSL to "3", and measuring the voltage at the terminals below the "Test Meter" on the console.

Be sure that the position feedback switch is in the negative position. This configures the system for negative position feedback (negative K_p). Therefore, a positive-going step response will appear as a negative-going position voltage response. Display v_p on either channel of the oscilloscope. For display purposes, invert the trace on the oscilloscope so the response appears positive. Use SINGLE sweep TRIG mode for the scope triggering. Externally trigger the scope from the step input voltage v_i . Use DC coupling for both the scope input and external trigger. Adjust the DC trigger level to react to the step motor voltage change while avoiding premature triggering. Use a voltage axis scaling of 2 volts/division, and set the ground level to the bottom of the display. The time base scaling should be selected such that the step response trace reaches steady state in each case. Assure that the step response starts from zero by rotating by hand the MCSL until $v_p = 0$ as measured by the digital multimeter.

Copy the oscilloscope display using the HP screen copy icon on the PC desktop, and paste it into your lab report. Graphically measure M_p, τ_r, τ_p *and* τ_s and on the plot using a pencil and ruler. τ_s may be difficult to measure since the point at which the decaying oscillation drops within 5% of the 6 volt steady state value is within the range of the static friction of the apparatus. Do your best to estimate it.

The actual system response will differ significantly from that predicted by your calculations and the simulation, due to simplified second-order approximation of a system we are using for the system model, which actually contains several nonlinearities and unmodeled modes.

4. Tune the proportional gain by iteratively varying the feedback resistor R_f to achieve a 50% overshoot (peaking at 9.0 volts). Copy the final oscilloscope display into your report and again, graphically measure M_p, τ_r, τ_p *and* τ_s and on the plot using a pencil and ruler.

5. Fill in the table below, in which the *calculated*, *simulated*, and *actual* response (before and after tuning) compared. Transcribe this table in your report.

	τ_r [sec]	τ_s [sec]	τ_p [sec]	M_p [unitless]
Calculated				
Simulation				
Initial System				
Tuned System				

6. List the numbers of the MCSL equipment that you used, including both the control console and the mechanical apparatus.

Preparation

Read the Experiment completely. You should be able to solve the following problem:

2. Using R_f=R_1=R_2=30K and the model parameters you measured in Experiment 1, calculate the closed-form time response $v_p(t)$ $t \geq 0$ from the closed-loop transfer function to a 6 volt step input $v_i(t)$, assuming the system is initially at rest. You may use either differential equation methods or Laplace Transforms for the second order system. By hand or using a plotting program such as Excel, plot the response from zero to two seconds, with enough individual points to be able to interpolate the plot clearly.

Experiment 3: Root Locus and Stability

Introduction

In this experiment, root locus methods will be used to analyze a second order servocontrol system. The stability limit of a third order system will also be studied.

The step response of the actual closed loop system will be analyzed to determine the damping factor ζ for the second order system. This will be compared with ζ calculated from the pole locations on the root locus based upon previously determined linear model parameters. The root locus will be used to specify the forward path gain of the servo control system such that a particular damping factor ζ is achieved.

The order of the system will then be artificially increased to three by the addition of a capacitor in the feedback loop of the operational amplifier. This deliberately degrades the system performance, as will be observed experimentally. The root locus and Routh-Hurwitz criteria will be used to predict the stability limit of this third order system. That is, we will find the value of the forward path gain *K* such that the system commences continuous oscillation. Comparison will be made with experimental observation of the actual gain at the stability limit.

The open-loop transfer function $G_c G_p F$ of the system can be found by opening the loop somewhere in the feedback path and expressing the variable at that point as a function of an input applied at that point. In this case, we open the path at the position voltage v_p, and derive a transfer function around the loop from the position feedback summing junction input, which we designate v_p', to the position sensor output, v_p. This illustrated in Figure 1. The negative sign of F_2 in Figure 1 undoes the inverting effect of the subtractive input to the summing junction. Note that in Experiment 3, the position feedback transfer function F_2 was just $K_{p,}$ since K_2 was treated separately as the forward loop gain.

$$G_1 G_2 F_2 = \frac{K_0}{s\left(s + \frac{1}{\tau_m}\right)} \tag{1}$$

where

$$K_0 = \frac{-K_2 K_p K_a K_m}{\tau_m N} \text{ (positive)} \quad , \quad K_2 = -\frac{R_f}{R_2}$$

Since K_p, K_a, and K_2 are all negative, the numerator is positive.

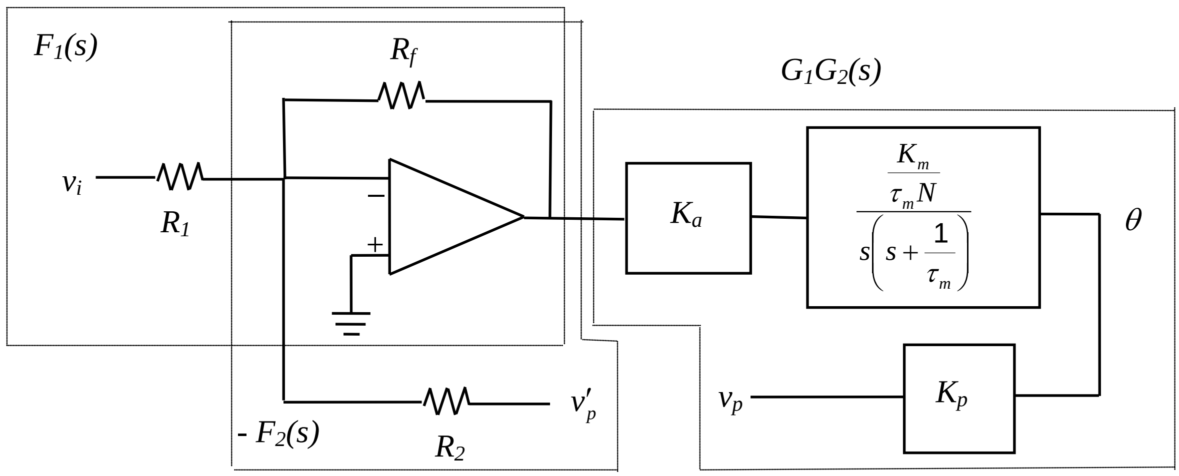


Figure 1: Opening the Loop to Identify $G_cG_pF_2$

$F_1 = -K_1 = \frac{R_f}{R_1}$ is the reference input gain defined in Experiment 1. It is not part of the open loop transfer function. When assessing the stability of a system, F_1 is not considered because the reference input v_i is always set to zero.

$F_2 = -K_2 = \frac{R_f}{R_2}$ is the feedback gain, which is part of the open loop transfer function

In almost all cases we will set $R_1 = R_2$, so that $F_1 = F_2 = F$

The *closed loop* transfer function from v_i to v_p may be written:

$$H(s) = \frac{G_cG_p}{1+G_cG_pF} \tag{2}$$

The root locus equation comes from the denominator of equation (2):

$$1+G_cG_pF = 0$$

With G_c , G_p , and F replaced by their transfer functions, this reduces to the *characteristic equation* of the system, with adjustable gain parameter K_0:

$$s^2 + \frac{1}{\tau_m}s + K_0 = 0 \tag{3}$$

Where

$$K_0 = \frac{R_f}{R_2}\frac{K_aK_mK_p}{\tau_m N} \quad \text{which is positive.} \tag{5}$$

The *root locus* is a trace of the roots of equation (4) in the complex s-plane as the gain K_0 varies, usually from zero to some sufficiently large value. K_0 can be adjusted by variation of either R_f or R_2, the only two parameters that we are free to change in equation (5).

Figure 2 illustrates the relationship between the closed loop pole locations, the damping factor ζ and the natural frequency ω_n.

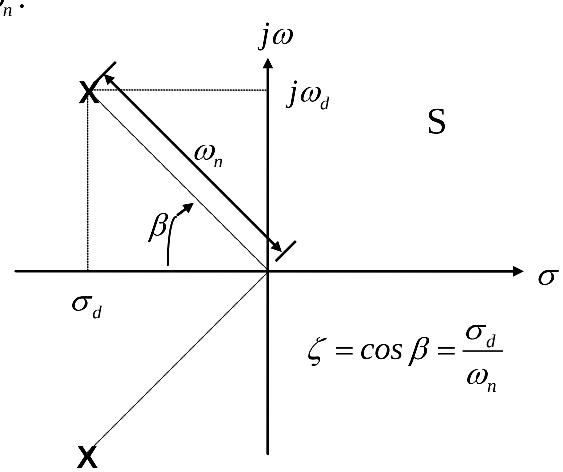
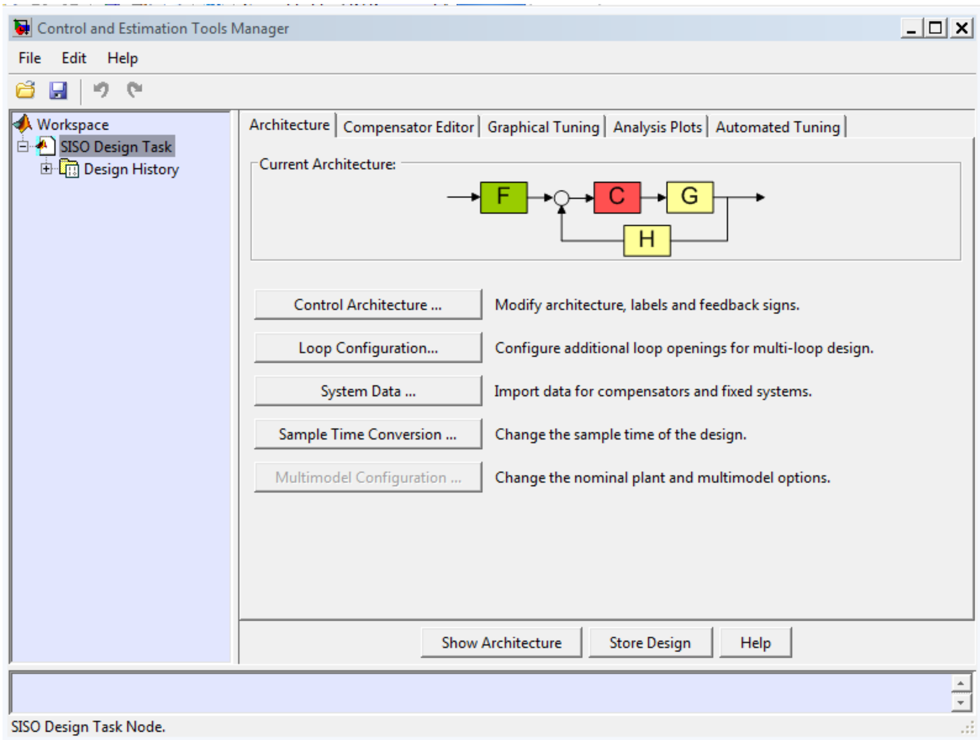


Figure 2. ζ and ω_n from complex pole locations in S plane

Procedure

1. *Write the open loop transfer function for your system from* v_p' *to* v_p with the values for the system model parameters that you determined in Experiment 1. Plot by hand the root locus for your system. Let the gain K_0 vary from zero to a maximum value $K_{\max}$ that results in complex conjugate poles with a pole angle equivalent to $\zeta = 0.1$. Let $R_2 = 30K$. What value of R_f corresponds to $K_0 = K_{max}$?

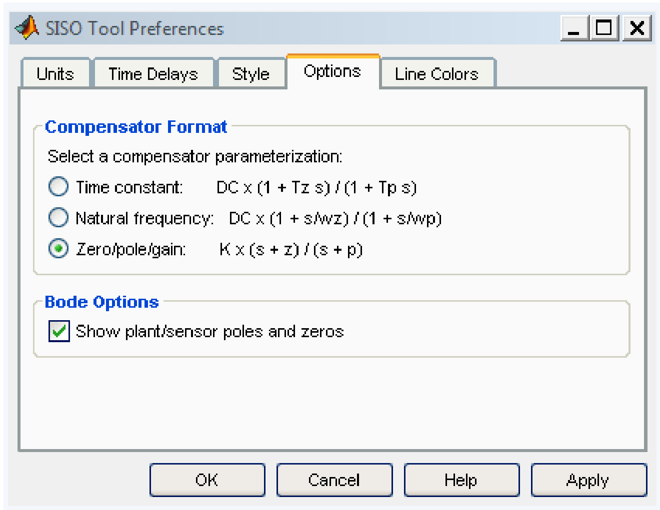
2. SISOTOOL is a component of the MATLAB Control Systems Toolbox, that allows direct manipulation of the root locus while concurrently observing the effect of the step response and frequency response of the system. We will use SISOTOOL to iteratively plot the root locus until the closed-loop poles are positioned to achieve $\zeta = 0.1$. Note that the *open loop* transfer function is used for root locus plots, even though we are interested in the *closed loop* system response. Start by entering at the MATLAB command prompt "**sisotool**". Some experimentation with this MATLAB feature is the best way to become familiar with it. The opening screen of SISOTOOL is shown below.



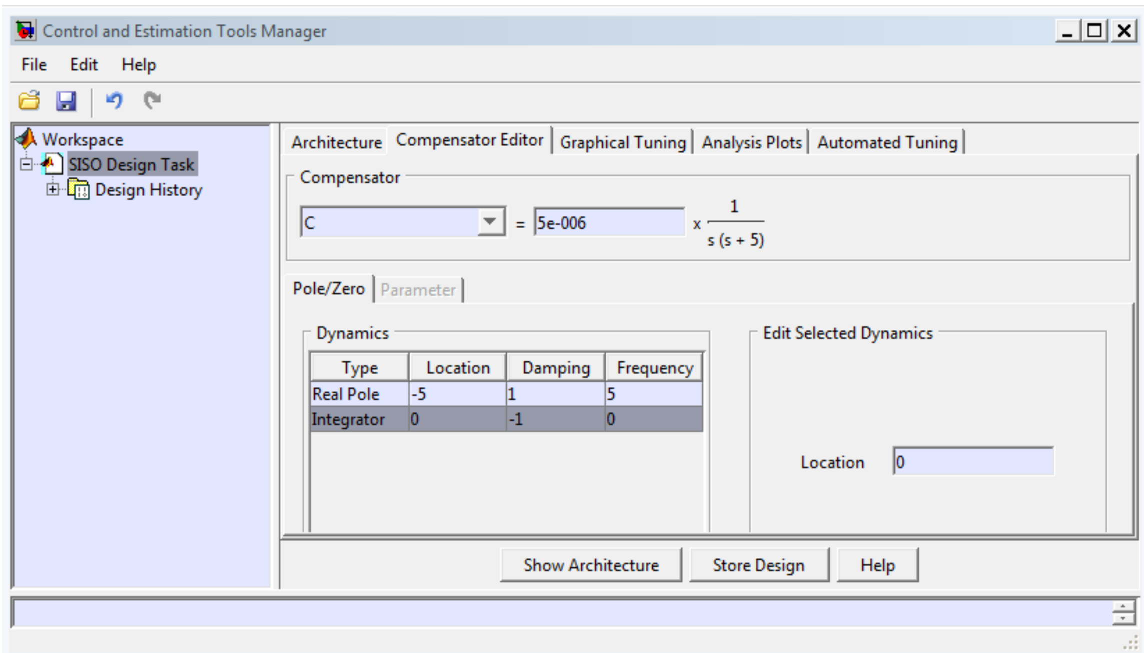
Click the Compensator Editor tab and enter the *open loop* positive transfer function (positive) for your system into the **Compensator Editor**. We will enter the entire open-loop transfer function as the **C** block as depicted in the **Current Architecture** diagram. SISOTOOL will assume that the G and H blocks are unity. Your open loop transfer function should have the form:

$$G_1 G_2 F_2 = \frac{K_0}{s\left(s + \dfrac{1}{\tau_m}\right)}$$

and it should be overall positive. By default, SISTOOL displays transfer functions in “Time Constant” format, which is non-standard for control system analysis and design. To get SISTOOL to display transfer functions in the more common zero/pole/zero/gain format, select **Edit,** then **SISO Tool Preferences**, and click the **Zero/pole/gain** button on the menu, as shown below.

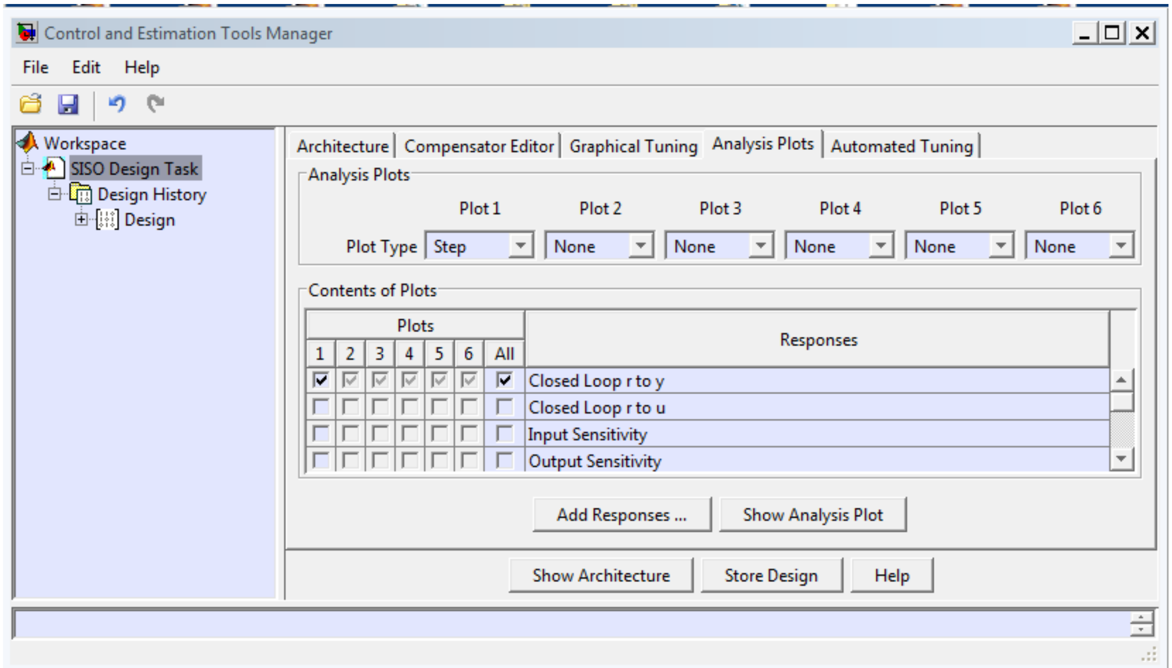


For a typical MCSL transfer function, the **Compensator Editor** window should look like:

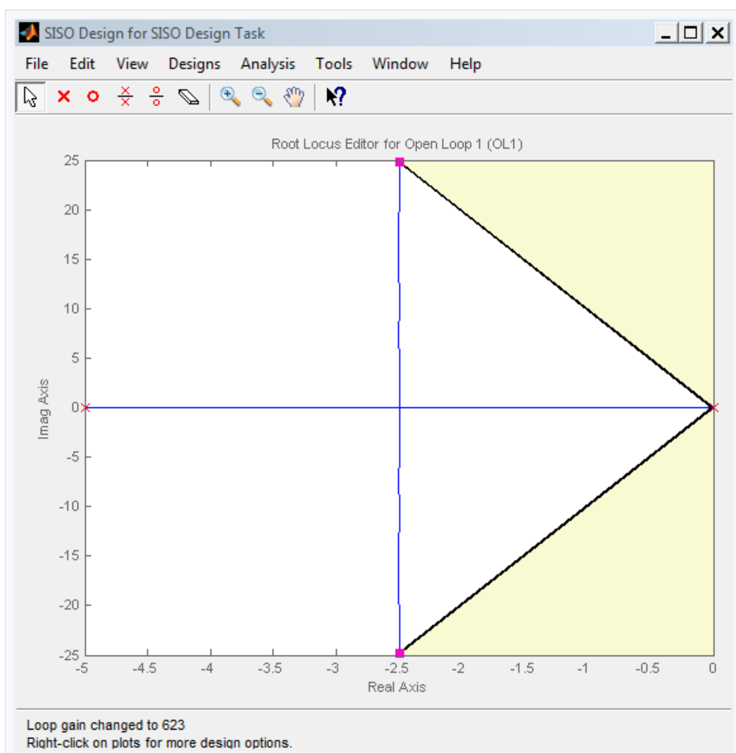


Right click in the **Dynamics** area of this window to add the open-loop system poles.

Click the **Analysis Plots** tab and select **Closed Loop r to y** and then click **Show Analysis Plot** to plot the locus of closed loop poles as the loop gain K_0 is swept from 0 to some maximum value. Also select **Step** for Plot 1 to produce a separate window with the corresponding closed-loop system step response.

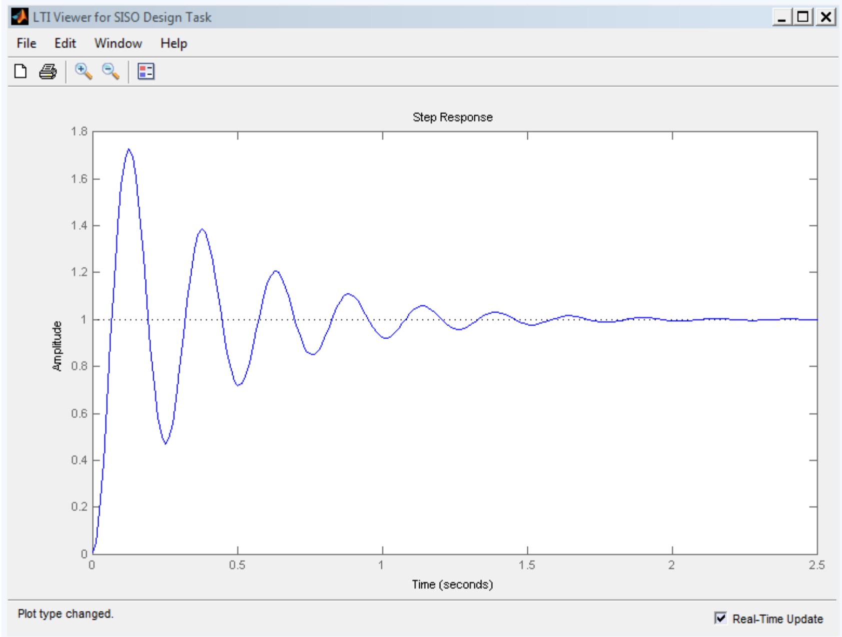


The root locus window should look like this:



Right click inside this plot to display the pole angle line for a specified overshoot. The loop gain at a current pole positions is displayed at the bottom of the plot when the cursor is in the white (left) area of the plot.

The step response window for this pole placement should display a step response that looks something like:



Using the computer mouse you can drag the closed loop poles around on the root locus plot, and *immediately* see the effect on the step response.

Record in your report the value of K_0 *when the closed loop pole locations are in the desired locations (*$\zeta = 0.1$*). What value of* R_f *would yield this gain if* $R_2 = 30K$ *?*

3. Using Alt-PrintScreen, capture both the root locus plot and the corresponding step response plot for your system with the poles located such that $\zeta = 0.1$. ζ and ω_n can be graphically measured for a left-half-plane complex conjugate pole pair, $s = -\sigma_d \pm j\omega_d$. ζ can be calculated as:

$$\zeta = \frac{\sigma_d}{\sqrt{\sigma^2{}_d + \omega^2{}_d}}$$

However, SISOTOOL directly reports ζ and ω_n for the complex conjugate poles. *Record these numbers and use them to calculate the expected values of* M_P*,* τ_r *and* τ_p for the step response of the closed-loop system. Then measure and report the values of each by graphical measurements on the step response plot.

4. Configure the MCSL apparatus for position-only negative feedback. Let R_1 and R_2 be 30K, and select R_f as determined from the loop gain value found using SISOTOOL for $\zeta = 0.1$. Apply a 6 volt step input and observe v_p on the oscilloscope, with the scope set up for

recording a single sweep as in Experiments 1 and 2. *Using the scope trace import utility on the PC, copy and paste the scope display data into your report.* Note that M_P, τ_r and τ_p will probably be more damped that predicted from the root locus or step response simulation response.

5. We now deliberately destabilize the system by adding a third pole to the system and increase the loop gain until the system starts to spontaneously oscillate, indicating that it has become unstable. The reference input v_i should be zero. We add a pole by placing a capacitor C_f *in parallel* with R_f in the feedback circuit of the operational amplifier. This increases the system order from two to three. The reference input v_i will be zero in this test – we are only examining the stability of the system, which does not depend on any particular input.

   Derive the new open loop transfer function $G_1G_2F_2$ for this configuration, using standard analysis methods for operational amplifiers. Recall, as discussed in Experiment 1, that the gain of the inverting op amp configuration is

$$\frac{v_o(s)}{v_p'(s)} = -\frac{Z_f(s)}{R_2}$$

   where Z_f is the impedance of the parallel resistor R_f and capacitor C_f:

$$Z_f(s) = \frac{\dfrac{1}{C_f}}{s + \dfrac{1}{R_f C_f}}$$

   $G_1G_2F_2$ will have the form:

$$G_1G_2F_2 = \frac{\dfrac{K_0}{\tau_f}}{s\left(s + \dfrac{1}{\tau_f}\right)\left(s + \dfrac{1}{\tau_m}\right)}$$

   where $\tau_f = R_f C_f$ is the time constant of the op amp feedback network. Fix $G_1G_2F_2$ using your model parameters, and $R_f = 30K$ and $C_f = 5.0\,\mu F$. Write the new *closed-loop* transfer function *H(s)* for this *open-loop* transfer function.

6. In SISOTOOL add the new third open-loop pole using the **Compensator Editor** and display the root locus and step response plots. Drag one of the complex conjugate poles on the root locus plot to the location where the root locus intersects the $j\omega$ axis. This is the stability limit. SISOTOOL will display the value of the loop gain (in this case $\dfrac{K_0}{\tau_f}$) at the bottom of

the root locus plot. If R_f remains fixed at 30K (so that τ_f remains constant), what value of R_2 does this loop gain correspond to? Record $\frac{K_0}{\tau_f}$ and R_2 in your report.

7. We now test the physical MCSL to check how accurately our predicted stability-limit loop gain matches that of the actual apparatus. Adjust the resistor substitution box for R_2 starting with $R_2 = 500K$ and incrementally decreasing R_2 (thus increasing K_0) until the system becomes unstable. To accurately observe the threshold of instability, it is necessary to overcome the static friction of the apparatus. We do this by initially displacing the motor shaft by hand some initial angle, releasing it, and observing whether the resultant oscillation decays or grows in time. You can initially displace the motor shaft up to two turns, but don't hold it for more than a second since the power amplifier will go into current overload while you do this. Instability is indicated when the system continues to oscillate indefinitely after manually displacing and releasing the motor shaft.

   As you decrease R_2, note the first value for which instability is first observed. Report this value, and the value of the loop gain $\frac{K_0}{\tau_f}$ that this corresponds to. Compare these with the values predicted in the simulation of Step 6.

Preparation

Read completely the experiment. Check out root locus plotting techniques in the EE 302 text and notes. If this topic has not yet been covered in lecture, it may be necessary read ahead in the text to supplement the material presented in this experiment. Try the following problem to check your knowledge of manual root locus plotting techniques:

1. Plot the root locus for the following open loop MCSL plant as K_0 varies from 0 to some large value. Use your own values for each of the model parameters. (This is part of Step 1 of the experimental procedure.)

$$G_1 G_2 F_2 = \frac{K_0}{s\left(s + \frac{1}{\tau_m}\right)}$$

$$\text{where } K_0 = \frac{R_f}{R_2} \frac{K_a K_m K_p}{\tau_m N}$$

Experiment 4: Lead Compensation

Introduction

This experiment introduces one of the primary tasks of the control system engineer: the design of a *compensator* to improve the step response of a position feedback control system. A forward loop *compensator* is a controller which acts upon the control error *e* (difference between what you want and what you get) and produces a control input to the plant. The compensator is located just ahead of the plant. This is sometimes referred to as a cascade compensator, and is illustrated in Figure 1.

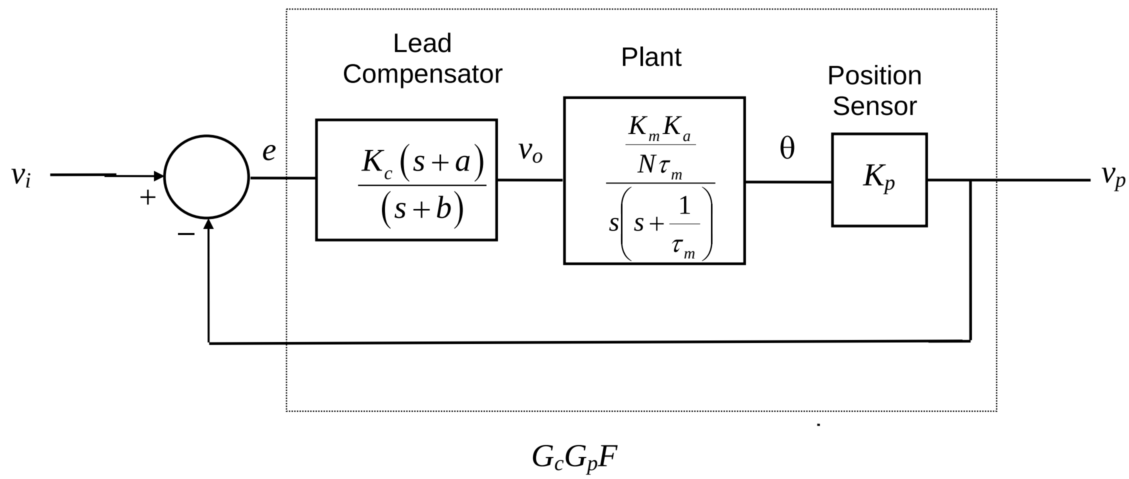


Figure 1. Block Diagram of Closed-Loop System with Lead Compensator.

In Figure 1, $R_1 = R_2$ so that $K_1 = K_2$, and the effect of both gains are represented by the gain K_c of the compensator. Since the output is v_p which is compared directly with the input v_i, the system takes the form of a unity-gain feedback system (i.e., $F = 1$).

In Experiment 2, the compensator was nothing more than a proportional gain which we adjusted by varying the op amp feedback resistor R_f. This allowed us to achieve only one design target M_p, the overshoot ratio of the step response. We simply accepted whatever system response times τ_r, τ_p and τ_s that resulted from the proportional gain. In this experiment we will be constructing a *phase-lead* compensator by adding a simple RC network at the input to the op amp, as shown in Figure 2. The lead compensator introduces an additional zero and pole to the open loop transfer function, with the zero at a lower frequency (closer to $j\omega$ axis) than the pole). With the ability to adjust both the capacitor value as well as the op amp gain, we can now achieve not only the desired overshoot ratio, but also the desired response times τ_r, τ_p and τ_s.

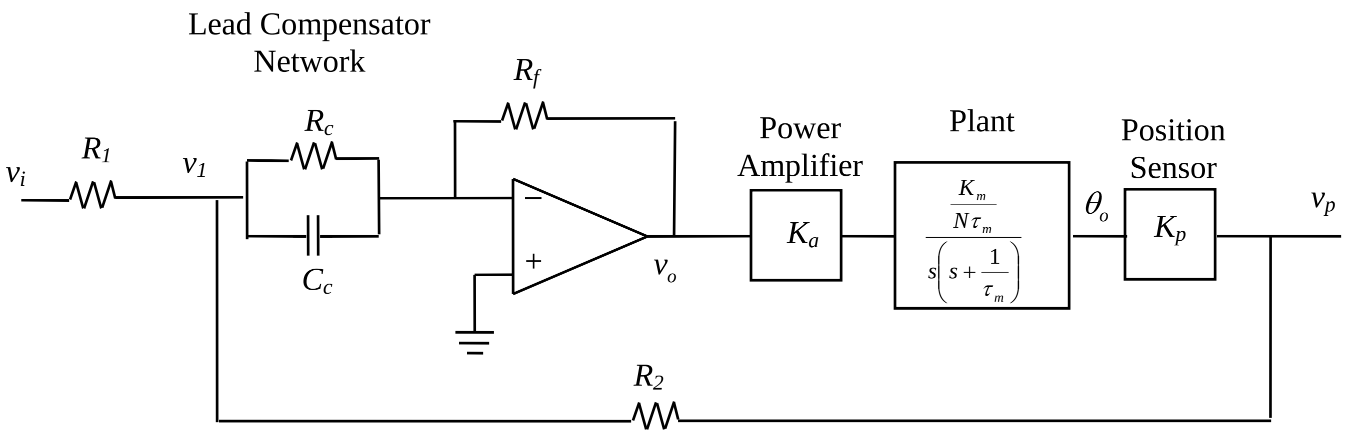


Figure 2. Schematic for Implementation of Lead Compensator

A KCL nodal analysis is performed at v_1 and at the inverting input of the op amp. With $R_1 = R_2 = R$, the relationship between the op amp output v_o and the reference input v_i and feedback v_p is:

$$v_o = \frac{-\frac{R_f}{R}(s+a)}{(s+b)}(v_i + v_p) \tag{1}$$

where $$a = \frac{1}{R_c C_c} \quad \text{and} \quad b = \frac{1}{R_c C_c} + \frac{2}{R C_c} \tag{2}$$

We complete the open-loop transfer function by multiplying this relationship by the plant and position sensor transfer functions as depicted in Figure 1. The overall result is:

$$v_o = \frac{-K_0(s+a)}{s\left(s+\frac{1}{\tau_m}\right)(s+b)}(v_i + v_p) \tag{3}$$

so that the open-loop transfer function is

$$G_c G_p F = \frac{K_0(s+a)}{s\left(s+\frac{1}{\tau_m}\right)(s+b)} \tag{4}$$

where K_o was previously defined in Experiment 3 as $K_0 = \frac{R_f}{R}\frac{K_a K_m K_p}{\tau_m N}$ which is positive.

With the open-loop poles and zeros viewed in the s-plane, the resulting root locus plot shows the locus for the dominant pole pair deflected further into the left half plane compared with the uncompensated locus of Experiment 3. This results in a larger value of ω_n for a given value of ζ (see Figure 3). Since ω_n appears in the denominator of all three transient response time constants, a larger ω_n decreases all of these time constants while maintaining the desired overshoot.

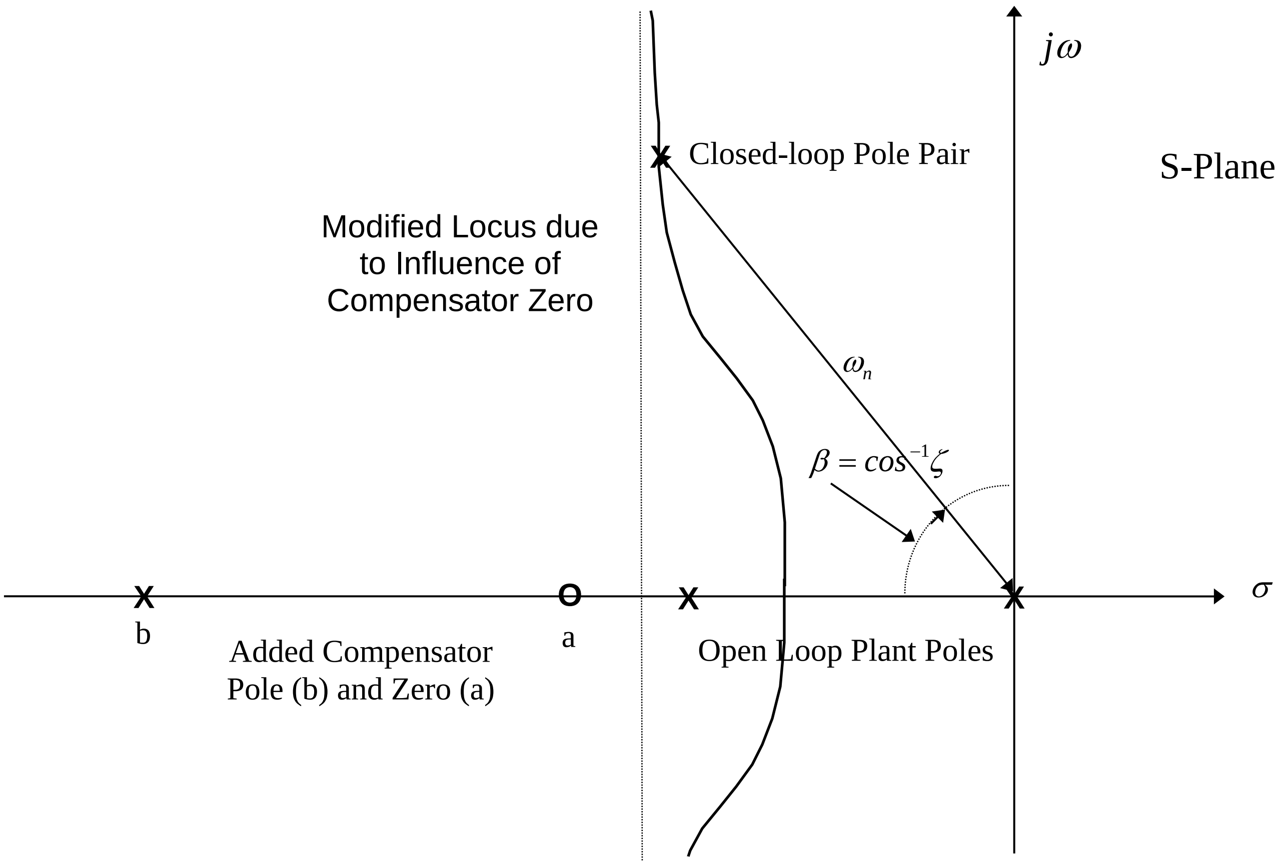


Figure 3. Sample Root Locus for System with Lead Compensator.

Procedure

1. Confirm equation (1) by analysis of the op amp circuit, and then confirm the complete *open loop* transfer function G_cG_pF of equation (4). Leave the model parameters, resistances and capacitance unspecified for now. When you analyze the circuit with the parallel resistor R_c and capacitor C_c connected to the inverting input of the op amp, don't neglect R_1 since current will flow through it to the reference input $v_i = 0$.

2. Now derive the *closed loop* transfer function *H(s)* for the compensated system. Leave the model parameters, resistances and capacitance unspecified. Express *H(s)* in standard form as a normalized rational polynomial in s.

3. Let $R_1 = R_2 = 30K$. Use your model parameter measurements from Experiment 1. The design objective is determine values for R_c , C_c and R_f such that the following specifications are met by the closed loop system:

   i) The magnitude of the compensator pole s = -b must be at least four times the magnitude of the plant pole at $1/\tau_m$.

   ii) The magnitude of the compensator zero s = -a must be at least 1.25 times the magnitude of the plant pole at $1/\tau_m$. (You cannot cancel the plant pole with the compensator zero.)

   iii) The overshoot is M_p= 33%.

   iv) Natural frequency ω_n at least twice that of the uncompensated value (refer to your root locus plots from Experiment 3).

   Use MATLAB SISOTOOL and enter the plant poles at 0 and $1/\tau_m$ with the *compensator editor*. Then add the zero and pole of the lead compensator to meet the specifications above. On the SISOTOOL root locus plot, using the computer mouse, move the pole and zero locations, to achieve the desired root locus. Try to get the locus to deflect to left, which increases ω_n for a given ζ . When you have shaped a locus that meets the specifications above, drag the closed loop poles along the locus until the desired overshoot specification is obtained. Report the forward loop gain K_0 for these closed loop pole locations.

   As long as the dominant pole approximation is reasonably valid, the intersection of a line at the required pole angle β from the negative real axis to the locus will dictate the required value of K_0 to achieve a particular ζ .

   When the specifications have been met, print your final computer generated root locus plot, and graphically show your measurement of ζ and ω_n on the hardcopy plot. A sample root locus plot is shown in Figure 4.

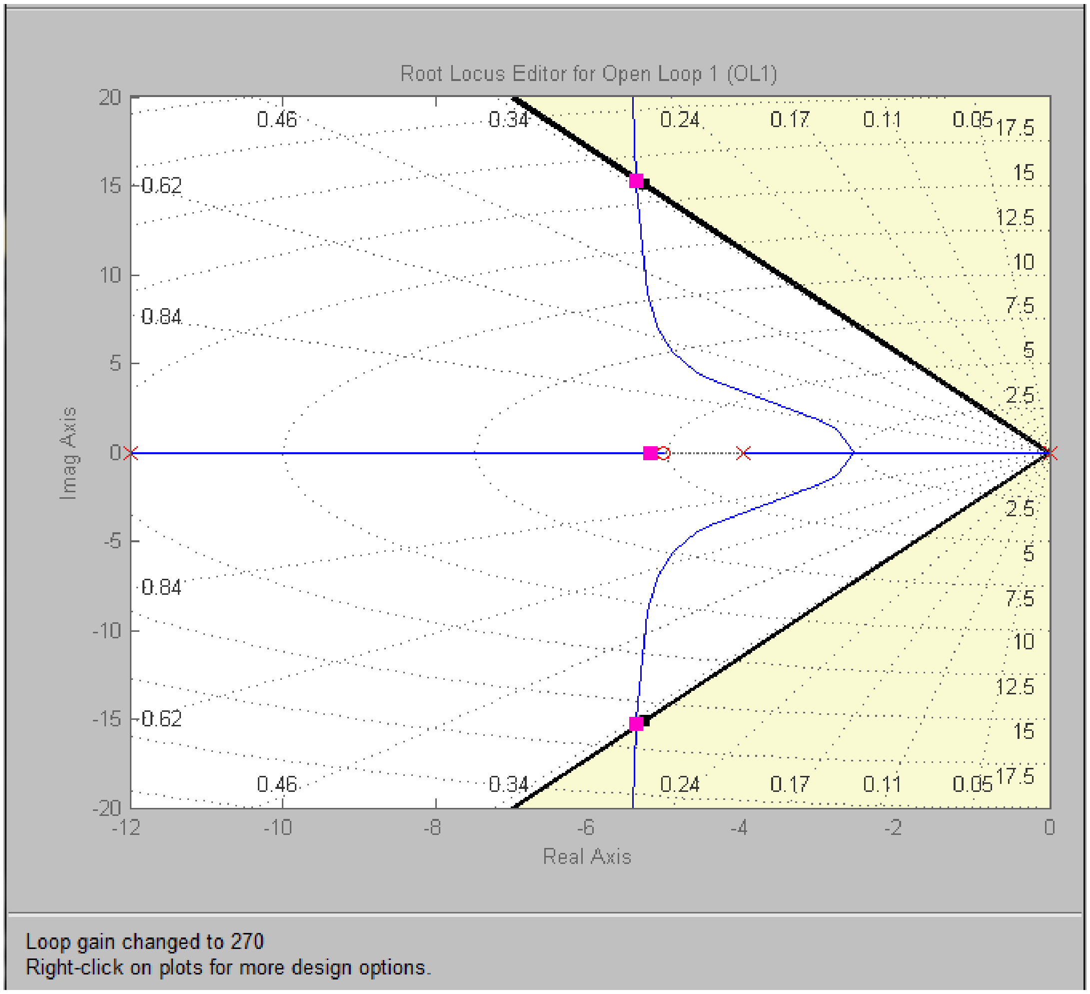


Figure 4. Sample Root Locus Plot.

4. With the addition of the compensator, the second order all-pole plant has been increased to third order, and now has a zero. If the compensator pole is sufficiently far into the left half-plane, it's effect on the system transient response can be nearly ignored (i.e., the dominant pole approximation applies). The compensator zero appears in the numerator of both the open and closed-loop transfer functions. Because of its proximity to the dominant system poles (and the influence it exerts on locus), its effects on the step response cannot be ignored.

In Experiment 3, formulas relating the rise time τ_r, peak time τ_p, overshoot ratio M_p and settling time τ_s to the damping factor ζ and natural frequency ω_n were presented for a second order all-pole system. With the addition of the compensator zero and non-dominant pole, these formulas are no longer correct, although they may still provide a reasonable approximation for step response characteristics. Use these formulas with your values for ω_n and ζ found from SISOTOOL to estimate the overshoot ratio, rise time, peak time, and settling time for your closed loop system. Show your calculations in your report along with your estimated results.

5. Observe the step response plot of v_p produced by SISOTOOL for your final root locus. SISTOOL will assume a unit step input at v_i, but you may rescale the plot for a 6 volt step input, Use a sufficient time duration such that the response settles to approximately steady state. This is typically about 5 seconds. Copy the plot into your report and graphically measure τ_r, τ_p, M_p and τ_s on the plot.

6. For the open loop gain and the compensator zero and pole that you determined to meet the design specifications, calculate R_c, C_c and R_f. Leave $R_1 = R_2 = 30K$. Now implement the compensator on the MCSL with these component values, using decade boxes for the resistances and capacitor. Three decade resistance boxes and one decade capacitance box are needed. Apply a 0 to 6 volt step input, and display the v_p response on the digital oscilloscope. Import the data into your report. Graphically measure the overshoot and the three time constants on this plot.

7. Finally, adjust R_c, C_c and R_f by trial and error until the system meets at least the overshoot and rise time specification. Try to get the step response as close as possible to that predicted by the simulation. *Record the final values of* R_c, C_c, R_f, *K_0, a, and b.* Copy the final scope trace into your report. Graphically measure τ_r, τ_p, τ_s and M_p on this plot.

8. Summarize your results from steps 4 through 7 in the table below, showing τ_r, τ_p, τ_s and M_p found for each scenario.

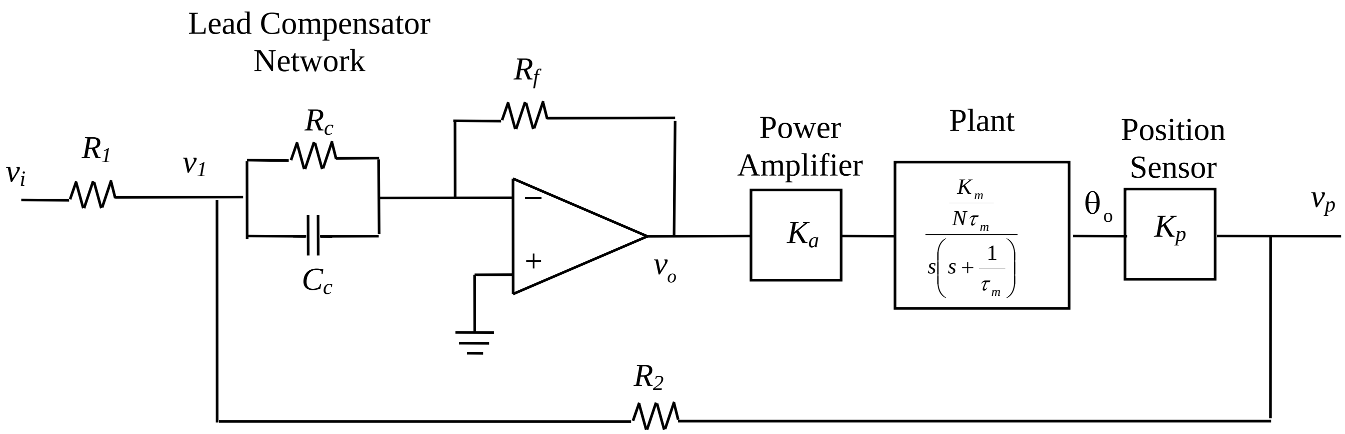
Data Source	τ_r [sec]	τ_p [sec]	τ_s [sec]	M_p [unitless]
Approximations based upon ζ and ω_n for the dominant pole pair.				
Measurements of step response simulation plot.				
Actual System with initial component values				
Actual system after optimally tuning the component values				

Preparation

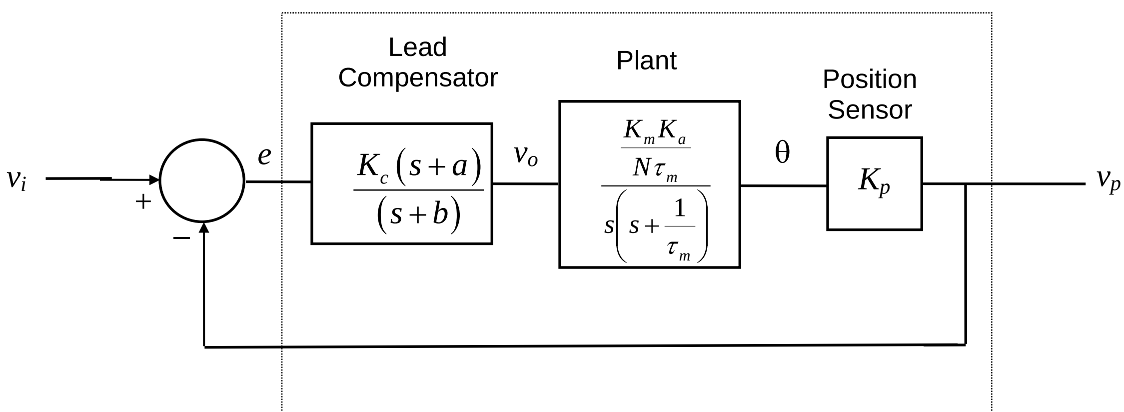
1. Review two-node circuit analysis and ideal op-amp circuits.

2. Check your knowledge of circuits and block diagrams by deriving Figure 2 from Figure 1, as repeated below:

Be sure you can convert this:



into this:



G_cG_pF

3. Perform Procedure Step 1 (open-loop transfer function). Do not yet numerically specify the parameters in the transfer function.

4. Perform Procedure Step 2 (closed-loop transfer function). Do not yet numerically specify the parameters in the transfer function.

Experiment 5: PID Control of an Unstable System

Introduction[1]

The Proportional - Integral - Derivative (PID) control is one of the most common programmable control laws in actual use. PID controls are employed in a wide range of applications, from elevator controls to diesel engine governors. The popularity is due that the fact that the control has an intuitive "feel" to it that makes it tunable by experience rather than formal analysis and design, often without complete knowledge of the plant model. The integrator in the PID control generates a growing control action with even the smallest control error, eventually nulling the position error to zero. Control applications such as robotics, factory automation such as welding robots, elevators, generator governors, and aircraft autopilots require this type of control action.

A general PID control has a transfer function

$$G_{PID}(s) = K_{Prop} + \frac{K_{Int}}{s} + K_{Der}s = \frac{K_{Der}s^2 + K_{Prop}s + K_{Int}}{s}$$

Its implementation as three parallel signal paths is depicted in the block diagram of Figure 1.

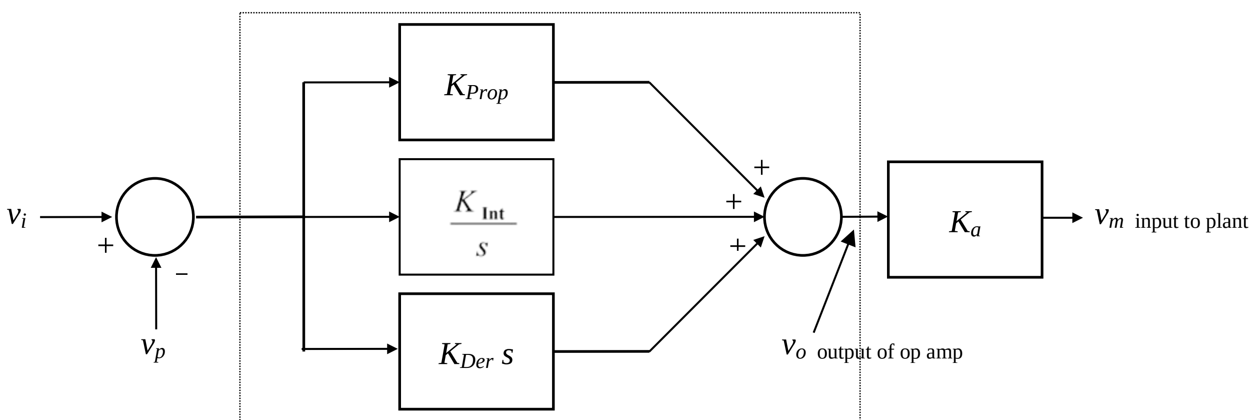


Figure 1. PID Control

[1] Special thanks to Gary Perks for proof-reading and testing this experiment.

To justify the use of a PID control we need a Type 0 system, which is subject to steady state position error. We will modify the Motomatic plant by the addition of an inverted pendulum attachment which not only makes it a Type 0 plant, but an unstable one at that. The PID control will be implemented on the Motomatic Control Console, and used to achieve three design objectives:

- A stable system. The pendulum should be able to transition from the vertical position to a position of approximately 45 degrees off-center, and stabilize at this position.
- Zero steady-state position error (a Type 1 system response).
- A “reasonably good” transient response to a step input. Not an exact spec, but generally overshoot less than about 25%, and as fast a step response as possible.

With three degrees of freedom in the control design, we should be able to achieve all three requirements.

The plant for this experiment is a modification of the basic MCSL used in previous experiments. Recall that the basic MCSL servo control is a Type 1 system with respect to position because there is no restoring (spring) force in the system. Thus, even a non-integrating feedback control can achieve a zero steady state position error (except for the effects of the static friction of the MCSL). The addition of the inverted pendulum attachment degrades the plant to Type 0, for which a finite steady-state position error could be expected if simple proportional control was used, for positions other than the zero (vertical) position.

The inverted pendulum is a wood dowel with a weight at the end, which pivots with the rotation of the motor shaft. It is attached to the output shaft of the speed reducer as pictured at the right. The center position of the pendulum is vertical. The inverted pendulum is unstable, since the natural position of the pendulum is away from the center position, at either extreme rotation angle. While this is actually a nonlinear system, for small angular displacements about the vertical position, the model can be treated as linear, with the pendulum torque approximately proportional to the shaft angle. The integrator of the PID control gives it the ability to reduce the steady state position error to zero. Although there is no position error in the zero position (vertical), the integral term is needed to overcome the static friction that might otherwise prevent the pendulum from reaching a fully vertical position when the input voltage v_i is zero.

With addition of the inverted pendulum, we will have to re-determine the parameters of the modified plant as we did in the first experiment. Fortunately, we need to perform only one additional test, and can otherwise use all the previously determined plant parameter values. The test is a little tricky, and will require the use of a practical method called *dither* to assure an accurate measurement in the presence of static friction.

After characterization of the modified plant, a MATLAB Simulink™ model will be used to design the values of the PID control coefficients using an intuitive approach – a modification of the Zeigler-Nichols method described in the EE302 textbook. The control coefficients will be tuned to achieve a brisk step response with zero steady state error and minimal overshoot.

The actual PID compensator is mechanized using the single op amp on the MCSL control console. Unfortunately, a clean parallel implementation like that shown in Figure 1 is not possible with a single op amp. However, we can implement the PID control with a network of resistors and capacitors in the feedback and input circuits of the op amp, as shown in the schematic of Figure 2. The three coefficients, K_{Prop}, K_{Int} and K_{Der} will depend upon the selected values of R_i, C_i, R_f, *and* C_f .

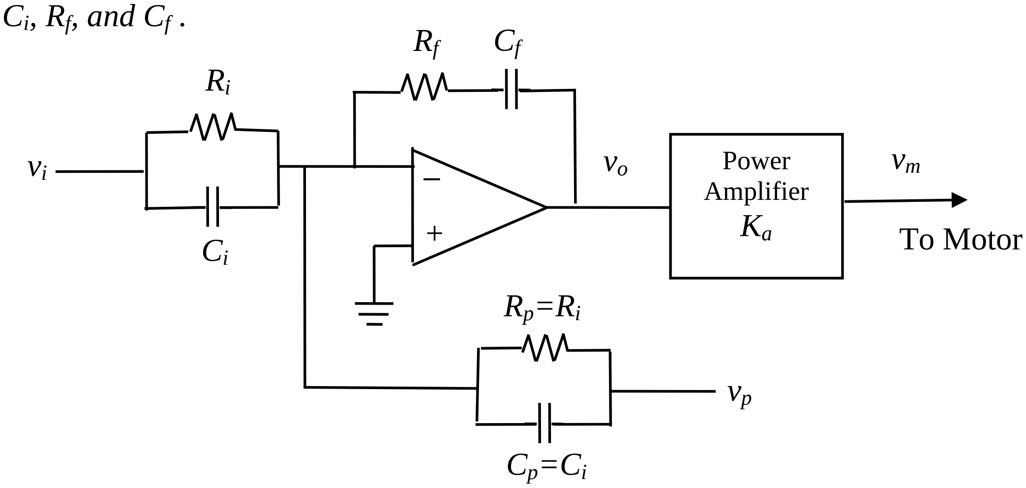


Figure 2. PID Compensator Schematic

Note: It important that C_p=C_i and R_p=R_i to assure that the output v_p follows the input v_i.

Position Transfer Function with Inverted Pendulum

Let's re-derive the MCSL plant model must with the inverted pendulum added as depicted in Figure 3 below.

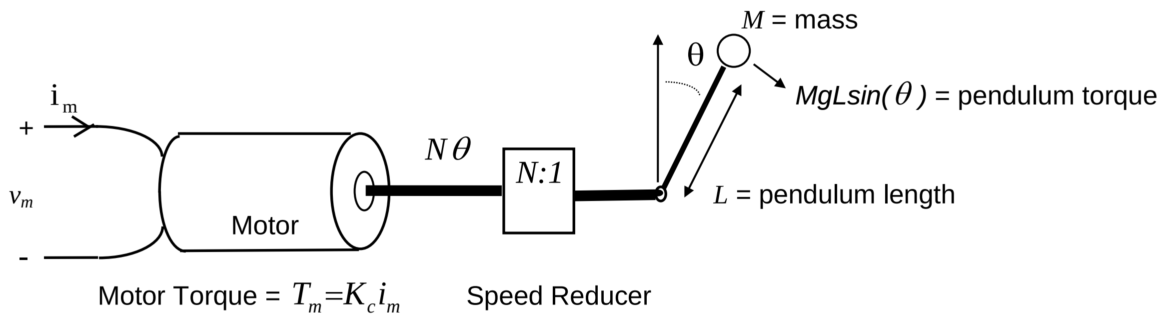


Figure 3. MCSL Model with Inverted Pendulum

The electrical model for the DC motor and kinematic model for the MCSL were described in the *Preliminaries* and *Experiment 1*. The differential equation for the complete plant was a simple sum of torques. It is re-written below (without the added inverted pendulum), slightly re-arranged, such that each additive term has units of torque in Newton-meters (Nm). The left side of the equation is the driving torque provided by the motor current. The first term on the right is the inertial opposing torque due to the rotational acceleration of the motor shaft. The second term on the right is the opposing torque due to frictional and the back-EMF of the motor, both proportional to rotational velocity.

$$\frac{K_c}{R_a} v_m = J_m \dot{\omega} + \left(K_{fm} + \frac{K_b K_c}{R_a} \right) \omega \tag{1}$$

The polar moment of inertia J and linear coefficient of friction K_f from the *Preliminaries* have been subscripted J_m and K_{fm} to make clear that these refer to the motor (with speed reducer), but without the pendulum attachment. Note that $\omega = \dot{\theta}_m$, where θ_m is the *motor shaft* position. Its relationship to the *output shaft* position θ is $\theta_m = N\theta$. Making this substitution we have the torque balance at the *output shaft* of the speed reducer:

$$\frac{K_c}{R_a} v_m = J_m N \ddot{\theta} + \left(K_{fm} + \frac{K_b K_c}{R_a} \right) N \dot{\theta} \tag{2}$$

The addition of the inverted pendulum to the output shaft adds additional inertia and friction to the equation. But most significantly, it adds another term $MgL\sin(\theta)$ to the torque balance that is a function of position. The modified differential equation for the kinematic system and motor with the inverted pendulum attached is:

$$\frac{K_c}{R_a} v_m = \left(NJ_m + J_P \right) \ddot{\theta} + \left(NK_{fm} + K_{fP} + N\frac{K_b K_c}{R_a} \right) \dot{\theta} - MgL\sin(\theta) \tag{3}$$

In this model,

v_m = motor armature voltage (input to plant)
θ = pendulum deflection angle, radians/sec (output of plant)
K_c = motor torque constant (N-m/amp, NOT the same as K_m the motor constant in radian-sec/volt)
R_a = motor armature resistance
K_b = motor back-emf coefficient
J_m = polar moment of inertia of motor and speed reducer
J_p = polar moment of inertia of inverted pendulum = L^2M
L = pendulum shaft length
M = mass at end of pendulum
g = gravitational acceleration = 9.8 m/sec^2
K_{fm} = friction coefficient for motor and speed reducer
K_{fp} = additional friction coefficient due to pendulum
N = Speed reduction ratio

Note the augmentation of the previous polar moment of inertia: NJ_m for the motor is supplemented by the added inertia of the pendulum J_p , which is not multiplied by N since it is measured with respect to the output shaft position θ (after the speed reducer) rather than the motor shaft position θ_m . The previous frictional term, now called NK_{fm} (motor friction), is supplemented by an additive term K_{fp} (pendulum friction) due to the pendulum. It is also measured with respect to the output shaft position θ rather than the motor shaft position θ_m . Since the pendulum does not add bearing surfaces, and air friction can be assumed small for short excursions of the pendulum, we will assume that the additional friction is so small that K_{fp} can be ignored in our model.

The added term in Equation 3, $-MgL\sin(\theta)$, is a torque that is a function of the shaft position. Because of the *sin(*θ*)* term, the system model is actually *nonlinear*. However, we will use the *small angle approximation* to **replace** *sin(*θ*)* **by just** θ , measured in radians. With this approximation and setting $K_{fp}=0$ as noted above, we get a linearized model for the MCSL with the inverted pendulum:

$$\frac{K_c}{R_a} v_m = \left(NJ_m + J_P \right) \ddot{\theta} + N\left(K_{fm} + \frac{K_b K_c}{R_a} \right) \dot{\theta} - MgL\,\theta \tag{4}$$

This model is unstable because the last term (the torque due to the pendulum) is negative. The additional polar moment of inertia J_p added by the pendulum is significant. J_p can be calculated by assuming that the pendulum is a massless shaft of length L with a point mass M at the end. With this assumption, we may calculate the additional polar moment of inertia due to the pendulum, $J_p = L^2M$.

In deriving the modified MCSL model, we can continue to use almost all of the parameter measurements made in Experiment 1, but must conduct one additional test to fully specify equation (4) for the MCSL with the pendulum.

First, let's rewrite the system equation in terms of the originally-determined system parameters. Starting with the full nonlinear equation (3), we divide through by the total system inertia to normalize to standard form:

$$\frac{K_c}{R_a(NJ_m + J_p)} v_m = \ddot{\theta} + \frac{NK_{fm} + K_{fP} + N\dfrac{K_b K_c}{R_a}}{NJ_m + J_P} \dot{\theta} - \frac{MgL}{NJ_m + J_P} \sin(\theta) \tag{5}$$

In prior experiments (without the pendulum), $M = 0$ and $J_p = 0$ so that equation (5) previously reduced to

$$\frac{K_c}{R_a J_m} v_m = N\,\ddot{\theta} + \frac{K_{fm} + \dfrac{K_b K_c}{R_a}}{J_m} N\dot{\theta} = \dot{\omega} + \frac{K_{fm} + \dfrac{K_b K_c}{R_a}}{J_m} \omega \tag{6}$$

Recall in the *Preliminaries* that we modeled the motor as:

$$K_m v_m = \tau_m \dot{\omega} + \omega$$

or

$$\frac{K_m}{\tau_m} v_m = \dot{\omega} + \frac{1}{\tau_m}\omega \qquad (7)$$

where

$$K_m = \frac{\dfrac{K_c}{R_a}}{K_{fm} + \dfrac{K_b K_c}{R_a}} \quad \text{and} \quad \tau_m = \frac{J_m}{K_{fm} + \dfrac{K_b K_c}{R_a}} \quad .$$

In *Experiment 1* we measured $K_m \left(\frac{\text{volt-sec}}{\text{radian}}\right)$ and the motor time constant τ_m (sec) directly by physical tests performed on the system. But now we need to measure values for each because of the added inverted pendulum. How to do this?

Suppose the system (5) is at rest with the pendulum at an angle of $-\frac{\pi}{2}$ or -90 degrees (horizontal). At this angle $sin(\theta) = -1$. Rest implies that $\ddot{\theta}$ and $\dot{\theta}$ are zero. In this state, equation (5) reduces to

$$\frac{\dfrac{K_c}{R_a}}{NJ_m + J_P} v_{m,0} = \frac{MgL}{NJ_m + J_P} \quad .$$

or

$$\frac{K_c}{R_a} = \frac{MgL}{v_{m,0}} \quad . \qquad (8)$$

Since we know *M, g* and *L*, we can solve for the ratio $\frac{K_c}{R_a}$ by determining the motor voltage $v_m = v_{m,0}$ that is just sufficient to keep the pendulum at rest in this position, balancing the motor torque with gravitational force on the pendulum[2]. The ratio $\frac{K_c}{R_a}$ has units of N-m/volt and has intuitive significance as the *static torque-voltage constant*. With $\frac{K_c}{R_a}$ found using this test, and K_m known from *Experiment 1*, we can calculate the polar moment of inertia of the motor alone:

[2] We actually measure $v_{m,0}$ as the minimum voltage that will just barely move the pendulum up from the horizontal position with a small *dither* signal added to it. The dither signal is just a zero-mean sinusoidal signal of small amplitude that keeps the shaft in constant motion during the measurement, so the shaft never has a chance to get stuck in one place.

$$J_m = \frac{K_c}{R_a}\frac{\tau_m}{K_m} \tag{9}$$

Combining this with the relationship for τ_m from the *Preliminaries*, repeated in equation (7), we can replace the middle (friction) term in equation (4) using:

$$K_{fm} + \frac{K_b K_c}{R_a} = \frac{J_m}{\tau_m} = \frac{K_c}{R_a K_m} \quad . \tag{10}$$

If we know the pendulum length L and mass M , we can determine the added polar moment of inertia due to the pendulum is $J_P = L^2 M$, or from Equation 8, calculate the ratio $\frac{K_c}{R_a}$.

Substituting both of these relationships into Equation (4), we get a linear (near the vertical position) model for the system including the inverted pendulum in terms of just the previously-measured *Experiment 1* parameters and the ratio $\frac{K_c}{R_a}$:

$$\frac{1}{\left(\frac{N\tau_m}{K_m} + \frac{L^2 M}{\frac{K_c}{R_a}}\right)} v_m = \ddot{\theta} + \frac{1}{\left(\tau_m + \frac{L^2 M K_m}{N\frac{K_c}{R_a}}\right)}\dot{\theta} - \frac{MgL}{\left(\frac{N}{K_m}\frac{K_c}{R_a}\tau_m + L^2 M\right)}\theta \tag{11}$$

For convenience, we define composite model constants (the mp subscript designates *motor with pendulum*):

$$K_{mp} = \frac{1}{\left(\frac{N\tau_m}{K_m} + \frac{L^2 M}{\left(\frac{K_c}{R_a}\right)}\right)} \quad , \quad \tau_{mp} = \tau_m + \frac{L^2 M K_m}{N\left(\frac{K_c}{R_a}\right)} \quad , \quad \text{and } K_S = -\frac{MgL}{\frac{N}{K_m}\left(\frac{K_c}{R_a}\right)\tau_m + L^2 M}$$

Then we can write Equation (11) in compact form as a simple second order linear differential equation:

$$K_{mp} v_m = \ddot{\theta} + \frac{1}{\tau_{mp}}\dot{\theta} + K_S \theta \tag{12}$$

The **plant** transfer function, from the motor input voltage v_m to the shaft position voltage v_p, can therefore be written:

$$\frac{v_p}{v_m} = K_p G_p(s) = \frac{K_p K_{mp}}{s^2 + \frac{1}{\tau_{mp}} s + K_S} \qquad (13)$$

where K_p is the position sensor coefficient which converts shaft position θ to the position voltage v_p. K_S may be thought of as similar to a linear spring constant, normalized by division by the total polar moment of inertia. It is negative, since the inverted pendulum naturally tends to fall away from the vertical equilibrium point, making the plant naturally unstable.

The op amp PID controller produces an output voltage v_o that is passed through the power amplifier with gain K_a to produce the motor voltage v_m that drives the plant. Note that the signal inversion through the inverting op amp circuit is corrected by the inverting power amplifier.

Apparatus

MCSL Electromechanical Apparatus and MCSL Control Console

HP Digital Oscilloscope

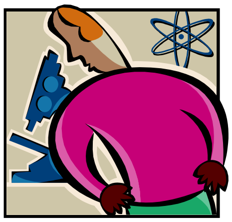
HP3314 Signal Generator

PC with IEEE 488 communications card and MATLAB/Simulink

Check out from Student Project Lab:

- Inverted Pendulum Assembly with hex keys for set screws on MCSL shaft couplings
- Misc. banana-end and BNC cables

Procedure



Our design objective will be to obtain a desirable step response starting at a vertical initial position corresponding to v_p = 0 Volts and going to a position approximately 45^o from vertical, corresponding to 4 Volts. In its final tuned configuration, the pendulum should be able to transition to the target position as quickly as possible, with a maximum allowable overshoot of 25%, and zero steady state position error. The design process begins using the instructor-provided MATLAB Simulink model for the inverted pendulum system into which you insert your plant parameter values. The tuning task is facilitated by adjusting the three digital PID control parameters, following a modified version of the ***Zeigler-Nichols*** method[3]. The initial control parameters to be used in the actual apparatus are those determined via the Simulink simulation.

[3] Please refer to EE 302 course textbook Modern Control Engineering, Twelfth ed., by Richard Dorf, pg. 483.

1. From the schematic of Figure 2, perform a KCL circuit analysis at the inverting input of the op amp to derive the transfer function of the PID circuit, from $\underline{v_i}$ and v_p to the op amp output v_o in terms of the resistors and capacitors of Figure 2. Leave the resistor and capacitor values unspecified at this time.

2. Match the terms in the controller transfer function derived in Step 1 with the generic PID compensator transfer function,

$$v_m = K_a \left(\frac{K_{Der} s^2 + K_{\mathrm{Prop}} s + K_{Int}}{s} \right) \left(v_i + v_p \right) \quad (14)$$

to determine formulas for the proportional, integral and derivative gains K_{Prop}, K_{Int}, *and* K_{Der} in terms of the resistor and capacitor values of Figure 2. *Record these relationships in your report*.

These formulas will be used later to set each of the PID gains by adjustment of resistor and capacitor decade boxes. Note from Figure 2 that for this experiment, the power amplifier gain K_a is incorporated into the controller. Since K_a is negative, the signal inversion of the op amp is re-inverted, yielding a net positive transfer function for the controller. Also, recall that v_p is negative with respect to θ and v_i because K_p is negative. This is why v_p is added rather than subtracted from v_i to generate the position error voltage in Equation 14.

3. Add the inverted pendulum to the MCSL as illustrated in Figure 4.

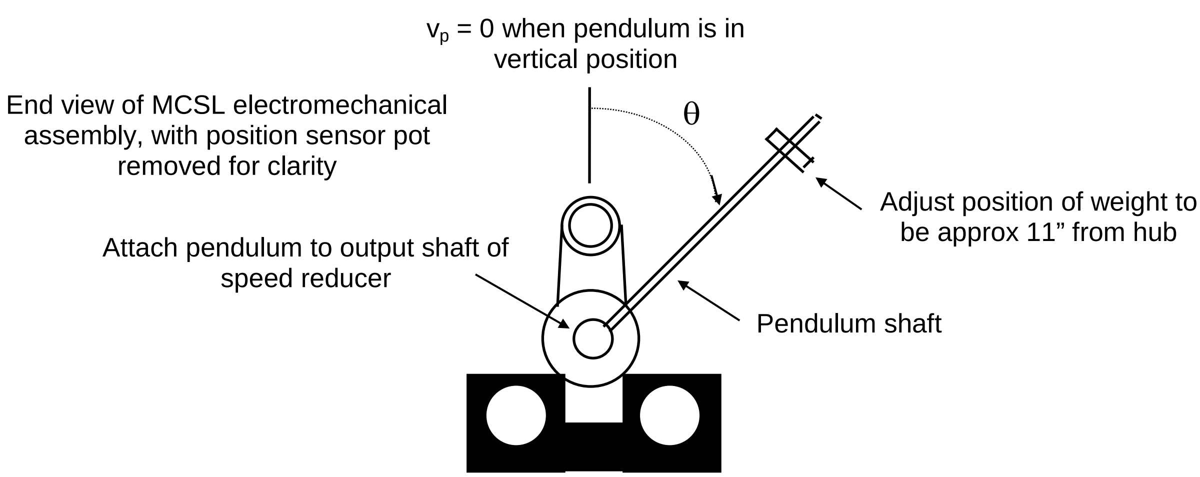


Figure 4. Inverted Pendulum Added to the MCSL

The pendulum and two small Allen wrenches (hex keys) are available packaged together from the senior project room checkout window. The position sensor pot is temporarily decoupled by sliding it back from its universal joint (retain the plastic slotted spacer). The half of the universal joint coupling on the speed reducer output shaft is removed by loosening two small set screws with the smaller hex key. The pendulum hub is inserted in place of the coupling on the output shaft of the speed reducer, and secured by tightening its set screw with the larger hex key. The half of the universal joint coupling that you just removed is then reinstalled on the end of the pendulum hub, and secured by tightening its two set screws. The position sensor pot is then re-coupled to the universal joint by sliding it forward, engaging the plastic slotted spacer lightly - do not engage firmly as this could cause the universal joint to bind or not turn freely.

With the position sensor pot connected to the MCSL control console, but the motor disconnected from the console, turn the MCSL on. Measure the voltage at the wiper (center terminal) of the pot using the digital voltmeter. Loosen any of the set screws on the couplings that you just installed, and rotate the shaft of the pot until the voltmeter reading is approximately zero. Then hold the shaft of the pot in this position while you position the pendulum shaft vertically (straight up). Tighten the set screw(s) in this position. This adjustment assures that the position sensor output v_p equals zero volts when the pendulum is vertical. Make sure that all coupling set screws are sufficiently tight to prevent slippage on the shaft during a step response of the pendulum.

4. With the pendulum now added to the system, we will perform the test described on Page 6 to determine the values of the unknown coefficients for the plant with pendulum. From equation (8), we will determine the ratio

$$\frac{K_c}{R_a} = \frac{1}{v_{m,0}} MgL \qquad (15)$$

by measuring the minimum motor voltage $v_m = v_{m,0}$ necessary to counteract the gravitational force on the pendulum when it is at 90^0 to the vertical (zero) position.

To perform this test, we set up the MCSL without any feedback (disconnect v_p from the op amp summing junction). Let R_f = 10K. Set the step input adjustment pot to zero volts. Set the step input toggle switch to the positive position. Make the usual remaining connections between the op amp and the power amplifier, and between the power amplifier and the motor.

We seek to measure the minimum voltage motor v_m that will start to move the pendulum up from an initial horizontal position. Unfortunately, static friction will grossly interfere with this delicate torque balance unless we figure a way to overcome it. We will use a “trick” often used in making measurements in such situations: We overcome the effects of the static friction in the apparatus by adding a small AC “dither” signal to the DC input. Connect the output of the HP function generator to auxiliary input “1” at the upper left corner of the MCSL console. Set the function generator for a sinusoidal output, initially at 5 Hz and zero volts. Make sure that the DC offset on the HP function generator is zero. (Instructions for the function generator can be found on the pull-out card below the unit.)

Power up the MCSL and move the pendulum by hand to the negative-side horizontal (90 degree) position. Initially support the pendulum with your hand, then increase the amplitude of the function generator until the pendulum oscillates up and down approximately one to two inches peak-to-peak. Typically, a sinusoidal signal amplitude of 1-3 volts is sufficient, but this varies for each apparatus. The exact amplitude or frequency of the dither oscillation is not important. The sole objective is to keep the pendulum moving a small amount at all times. Finally, increase the step input amplitude pot on the MCSL slowly until the vibrating pendulum just starts to move definitively up from the horizontal position toward the vertical position. After the pendulum just starts to move upward, disconnect the function generator, but leave everything else as-is (especially the step amplitude voltage setting). Measure and report the *motor* voltage v_m using the digital voltmeter. Remember that v_m is measured between the red motor terminal and an actual ground point, not the black terminal of the motor. This voltage measurement is the calibrated value $v_{m,0}$.

The pendulum mass M is approximately 0.028 kG (1.0 ounce). We assume that the wood stick on which it is mounted has no mass when we calculate the polar moment of inertia of this assembly. Measure the pendulum length L in meters from the hub to the weight with a ruler. If necessary, adjust it to be about 0.280 m (11”). The speed reducer ratio N was determined in *Experiment 1*. The gravitational constant g is 9.8 m/sec^2. Calculate $\frac{K_c}{R_a}$ from equation (8), using consistent metric units (M in kg, L in meters, g in m/sec^2 , $v_{m,0}$ in volts). You may check the result by noting that the ratio $\frac{K_c}{R_a}$ has units of Newton-meters/volt, and that one Newton is equivalent to one kg-m/sec^2.

With M, L and the ratio $\frac{K_c}{R_a}$ known, we have sufficient information to fully specify the motor/pendulum transfer function $G_p(s)$ of Equation (13). Calculate and record the composite parameters K_{mp}, K_s, and τ_{mp} , and write the complete open-loop motor/pendulum plant transfer function in the form of equation (13).

5. Now that you know $\frac{K_c}{R_a}$ as well as the plant parameters from Experiment 1, combine the result of the previous steps with the linearized plant transfer function from equation (13) for the plant and (14) for the controller, assuming $v_i = 0$ to find the *open-loop* transfer function $G_cG_p(s)$. Note that as shown in Figure 2, the PID controller in this experiment incorporates the gain of the power amplifier K_a *, and that the system output is* v_p *so the* K_p *is included as part of the plant.* Leave the coefficients K_{Prop}, K_{Int}, *and* K_{Der} in the PID controller undetermined at this time, but use your actual plant parameters in the transfer function.

$$\frac{v_p}{v_p^{'}} = G_cG_p(s) = K_a\left(\frac{K_{Der}s^2 + K_{\mathrm{Prop}}s + K_{Int}}{s}\right)\frac{K_pK_{mp}}{s^2 + \frac{1}{\tau_{mp}}s + K_S} \qquad (16)$$

Rewrite $G_cG_p(s)$ from Equation 16 as a single rational polynomial in normal form (descending powers of s, and unity coefficient of highest power of s in the denominator).

6. Download the MATLAB Simulink model EE302_MCSLInvPendExp5.mdl from http://telab.ee.calpoly.edu under the EE342 tab, to the PC desktop. Double click to run this model with Simulink. Replace the default plant model coefficients with those determined for your own apparatus including the inverted pendulum. Simulate the system step response going from an initial position of $v_i = 0$ volts to a final position of $v_i = 4$ volts.
We will implement the PID control progressively, by adding each of the PID control terms until all terms are active. I have provided suggested conservative values below as starting points; iteratively adjusting these to achieve the best possible response in that configuration. You do not need to meet the 25% overshoot specification until the final configuration, for which K_{Prop}, K_{Int}, *and* K_{Der} are all non-zero.

The three configurations to be examined are given below. For each, screen capture and paste into your report the corresponding step response. Make graphical (pencil) measurements on the printed plots in your report for the rise time τ_r, peak time τ_p, 5% settling time τ_s, and the overshoot ratio M_p. If no overshoot is observed for a configuration, just note this in your report, confirming with the step response plot.

i) *Proportional control only*. Start with $K_{Prop} = 0.5$. $K_{Int} = 0$, $K_{Der} = 0$.

With proportional control only, attempt only to stabilize the system. You will not be able to obtain a very good tradeoff between overshoot and response time. Also, a positive steady state position error will be observed. It is positive because the system is unstable, and the steady state value of v_p will be greater than its commanded value v_i, although it may not be observed due to the static friction of the system.

ii) *Proportional plus integral control*. Start with $K_{Prop} = 0.5$ and $K_{Int} = 0.5$. $K_{Der} = 0$.

The only improvement added by the integral term is the elimination of the steady state error. For small integral gain values, it may take several seconds for the steady state error to null out in the step response. Larger values of integral gain can cause the system to exhibit excessive overshoot. Even higher values can cause the system to become unstable again, as can be expected by the addition of a pole at the origin (an integrator).

iii) *Proportional, integral and derivative control*. Start with $K_{Prop} = 0.5$, $K_{Int} = 0.5$ and $K_{Der} = 0.1$.

The addition of a small amount of derivative gain tends to reduce the slope of the step response transient, and counteracts the destabilizing effects of the integral control. But it does not reduce overshoot due to integrator wind-up. In this configuration, iteratively adjust K_{Prop}, K_{Int} and K_{Der} to achieve the best possible rise time within the overshoot limit specification. The response must achieve close to zero steady state error within no more than ten seconds. (This means you cannot use a trivial value for the integrator gain.) You should be able to obtain a fairly good step response in this configuration, easily achieving the overshoot specification.

A “good” step response in this case is that with the shortest values of rise time, peak time, and settling time that you can obtain without exceeding an overshoot of 25% (1 volt overshoot for a 4 volt step transient), with the position error reduced to zero within no more than ten seconds. The step responses for the first two configurations need not (and probably won’t) meet these specifications, but for the final configuration with all PID coefficients non-zero, it should be possible for all MCSL apparatuses in the Controls Lab. Remember that the system is intrinsically unstable, so that some minimum control action is necessary to even stabilize the system, much less to achieve an acceptable step response. Copy and paste the best simulation plot for each of the three configurations into your lab report. *Only the full PID control of the last configuration needs to conform to the required specifications.* Record your data from each simulation configuration in the table below:

PID Control *Simulation* Data Table

Config	K_{Prop}	K_{Int}	K_{Der}	R_i (KΩ)	R_f (KΩ)	C_i [uF]	C_f [uF]	τ_r [sec]	τ_p [sec]	τ_s [sec]	M_p [%]
(i) P											
(ii) PI											
(iii) PID											

7. Build the actual controller. Construct the PID compensator network using three decade resistor boxes and three decade capacitor boxes. (You will need a lot of room on the lab bench for everything.) The input is the step input switch on the MCSL control console.

   The feedback parallel resistor/capacitor network is connected between the op amp input and the position sensor output v_p, bypassing the internal 10K position feedback resistor. We also need to bypass the internal 30K resistor between the step input switch and the op amp input junction terminal. To do this, connect the input-side parallel resistor/capacitor network between the Test Meter terminal with the selector switch set to position "3" and the input junction terminal of the op amp.

   The complete setup is shown in Figure 5.

   As usual, the series-connected resistor/capacitor R_f and C_f is connected between the output of the op amp (v_o) and the input junction terminal of the op amp. Set the position feedback switch for negative feedback as usual.

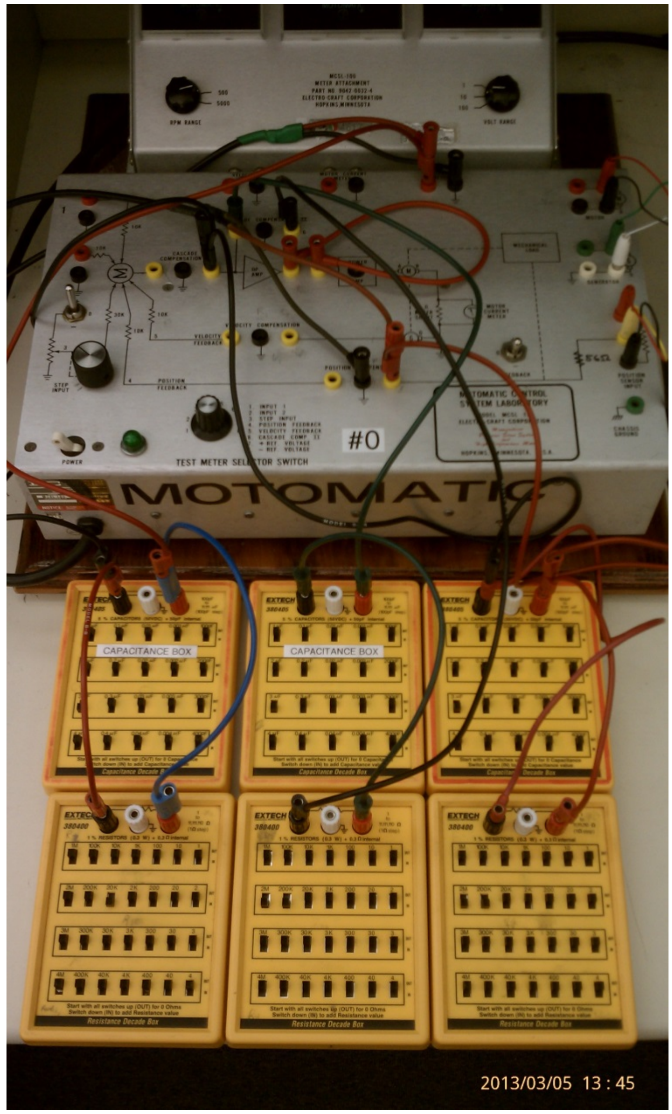


Figure 5. Wiring for PID Control.

   Monitor the output v_p on the digital oscilloscope, using the usual single-sweep setup for recording the step response. Use 2 volt/div axis scaling, and 0.5 sec/div time base scaling. You may rescale the time/division setting of the scope as necessary to display the entire step response. Externally trigger the scope from the input voltage v_i using DC, rising edge trigger.

8. Configure the actual inverted pendulum plant with the PID control. Use the values of K_{Prop}, K_{Int} and K_{Der} that you found to work best from your final simulation configuration as the initial values for the actual system. Calculate R_i, C_i, R_f, and C_f, using the results of Step 2. With four adjustable component values, you have four degrees of freedom to determine three constraints. Try to keep C_i and C_f in the range of 0.5 to 10.0 μF if possible, and note that a zero capacitance value is an open circuit. Note that changing the value of one component will affect more than one control coefficient. Record K_{Prop}, K_{Int} and K_{Der} and R_i, C_i, R_f, and C_f.

Test the system in this initial configuration with a step input v_i going from 0 volts to 4 volts (45^o). As in prior experiments, the actual system response may differ significantly from the simulated response. Import the step response trace from the scope and copy into your report.

9. Tune the control. Adjust K_{Prop}, K_{Int} and K_{Der} iteratively by variation of R_i, C_i, R_f, and C_f until a reasonably good step response is obtained, as described above. Again note that the system is unstable when uncontrolled, so intuition from previous experiments that less proportional gain implies a more damped response not necessarily true. Record your initial and final values of K_{Prop}, K_{Int} and K_{Der} and R_i, C_i, R_f, and C_f in the table below:

PID Control *Initial and Tuned System* Data Table

Config	K_{Prop}	K_{Int}	K_{Der}	R_i (KΩ)	R_f (KΩ)	C_i [uF]	C_f [uF]	τ_r [sec]	τ_p [sec]	τ_s [sec]	M_p [%]
Initial											
Tuned											

Notes from experience…

- The belts of the speed reducer sometimes slip during a step response transient, changing the zero position of the pendulum from straight up to some angle. If this occurs, push the pendulum back to the vertical position corresponding to $v_p = 0$ volts. If slippage is excessive, it can be improved somewhat by use of belt dressing or rubber cement on the belts, available in the lab.

- When experimenting with R_i, C_i, R_f, and C_f , be careful to not select values that could cause oscillatory behavior of the pendulum.

- In practical PID controls, the integrator contribution to the control output is usually clamped to reduce “integrator wind-up” and the overshoot that results from this. In this experiment, you are not asked to include integrator clamping because it is very difficult to implement with this single-op-amp implementation.

Preparation

1. Read the experiment completely.

2. Perform Procedure Step 1 (analysis of op amp PID circuit).

3. Perform Procedure Step 2 (PID coefficients in terms of circuit component values).

4. Download from Control System Web Site “EE302_MCSLInvPendExp5.mdl” under the EE342 tab, and run it with MATLAB Simulink to become familiar with this model. Try various PID control coefficients to get a sense for their relative effects.

Experiment 6
Frequency Response and Stability Margins

Introduction

The *frequency response* of a linear system is an important design consideration. Frequency-based metrics are used to characterize the performance and relative stability of open and closed-loop systems. An "ideal" servocontrol system would exhibit constant magnitude and zero phase shift for all frequencies. In other words, the output would follow the reference input in a proportional sense with no lag. Real systems, however, usually exhibit a low-pass type of response, with constant magnitude and zero phase shift at low frequencies, degrading as the frequency increases. The so-called *bandwidth* of a control system may be generically described as the frequency at which the magnitude response becomes excessively attenuated, and the phase shift becomes excessive. In this experiment we will investigate two frequency-related stability metrics for closed-loop systems: *gain margin* and *phase margin*.

Consider again the closed loop transfer function derived in Experiment 1:

$$H(s) = \frac{\theta(s)}{v_i(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{1}$$

The frequency response is measured by substituting $s = j\omega$ in H(s), and sweeping the radian frequency $\omega = 2\pi f$ from some low value to a high value. The magnitude response as a function of frequency is found by taking the magnitude $|H(j\omega)|$, usually in decibels. The phase response is found by taking the phase $\angle H(j\omega)$ in either degrees or radians. These two function of frequency together characterize the frequency response of the linear system. The DC or zero-frequency response of the system may be found by evaluating $H(j\omega)$ at $\omega = 0$:

$$H(0) = \frac{K\omega_n^2}{0 + 2\zeta\omega_n 0 + \omega_n^2} = K = -\frac{K_1}{K_2 K_p} \tag{2}$$

By inspection, K is the DC or low frequency magnitude of H(s). Since K is real and positive, zero phase shift occurs at $\omega = 0$.

At very high frequencies, we note that $\lim_{\omega \to \infty} H(j\omega) = 0$. Therefore, the MCSL position feedback control system has zero magnitude frequency response at very high frequencies.

The Bode Approximation

A Bode plot provides straight-line approximations to logarithmic magnitude and linear phase plots as a function of logarithmic frequency. This topic may be reviewed in the EE 301 or EE 302 textbook. In particular, you may wish to review the methods for handling factors of s in the denominator, and resonant peaks for quadratic underdamped terms. You may also need to read the section in the EE 302 text on gain and phase margins, in advance of the formal coverage of these topics in class, although the presentation herein is designed to be self-contained.

While the corner frequencies corresponding to the poles and zeros of H(s) appear explicitly in the Bode plot, the magnitude of the resonant peak at ω_m for an underdamped second order system is not specifically indicated. The ratio of the magnitude M_m of this peak relative to the low frequency gain H(0) may be calculated from

$$M_m = \frac{|G(\omega_m)|}{|G(0)|} = \frac{1}{2\zeta\left(1-\zeta^2\right)^{1/2}} \quad , \quad \zeta < 0.707 \tag{3}$$

and

$$\omega_m = \omega_n\left(1-2\zeta^2\right)^{1/2} \ , \ \ \zeta < 0.707 \, . \tag{4}$$

For $\zeta \geq 0.707$, no resonant peak is observed so that M_m and ω_m are undefined.

Figure 1 illustrates typical Bode magnitude and phase plots for a system with $\zeta < 0.707$ and H(0) = 1 (0 dB).

Phase and Gain Margin

The phase and gain margins of a closed-loop system are indicators of the relative stability. Generally speaking, the larger the gain margin or phase margin, the more stable the system is in closed loop operation. While stability is a necessity for any servocontrol system, an excessive gain or phase margin is not necessarily desirable, as the system may exhibit sluggish tracking behavior. The usual goal is to assure an adequate phase margin, typically in the range of 10 to 60 degrees, and an adequate gain margin, usually at least 6 dB.

We consider again the position feedback control system. The phase and gain margin of the closed loop system are found by evaluating the frequency response of the open loop path $G_1G_2K_p$ as defined in Figure 2 of Experiment 1. ($F_2 = K_p$ in this case, since the feedback path transfer function F_2 is just the sensor gain K_p .) The magnitude and phase response plots are usually plotted above each other on the same page, sharing the same frequency axis.

Figure 2 below illustrates this method. The phase margin is defined as

$$180° + \angle G_1 G_2\left(\omega_g\right) K_p \, . \tag{5}$$

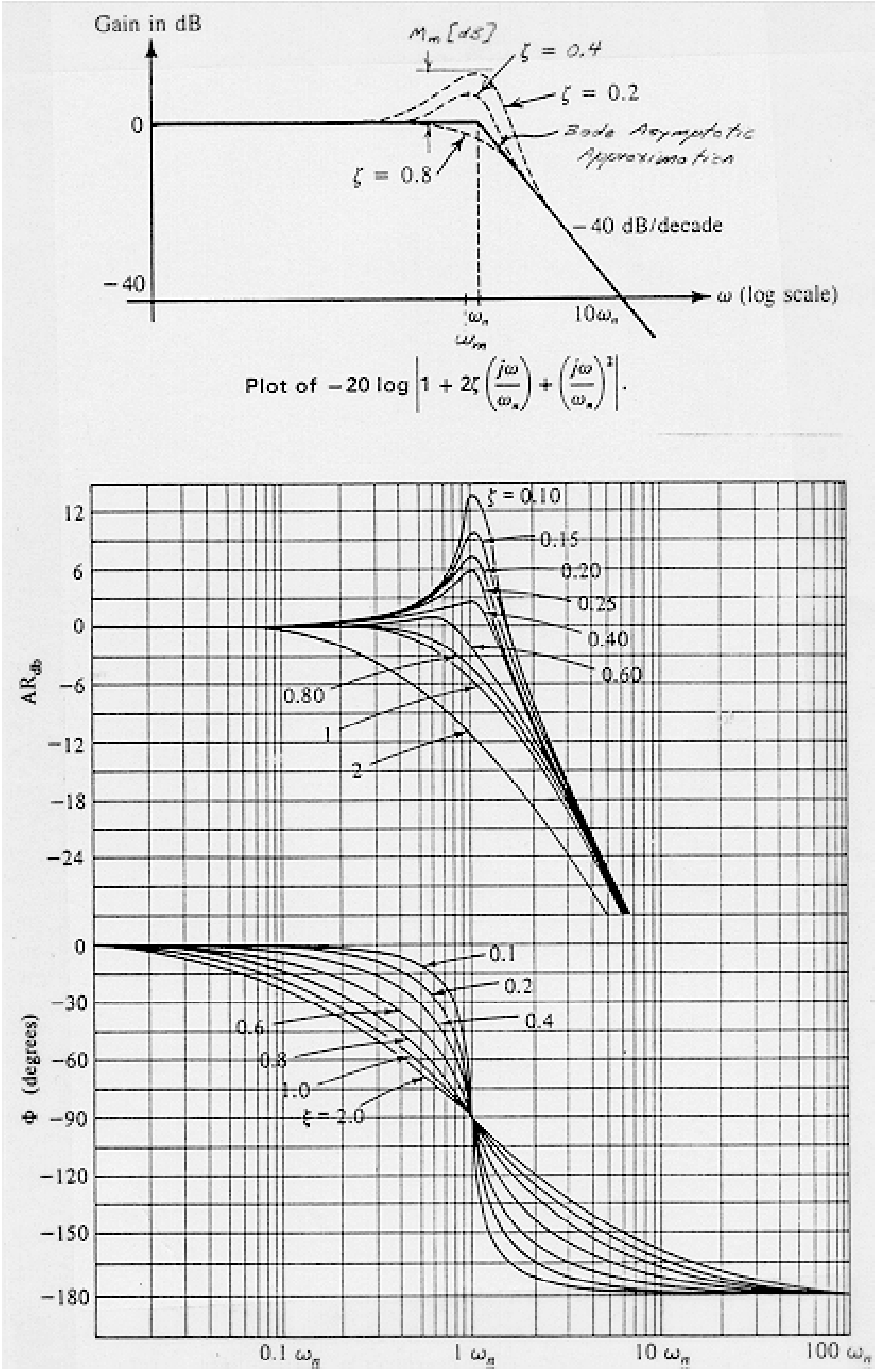


Figure 1. Example of Bode Magnitude and Phase Plots for Underdamped System

where ω_g is the frequency at which the gain $|G_1G_2(\omega_g)K_p| = 1$. Note that $\angle G_1G_2K_p$ will be a negative number since it is phase *lag*.

Similarly, gain margin is defined by $-20\ \log|G_1G_2(\omega_p)K_p|$ where ω_p is the frequency at which the phase shift $\angle G_1G_2(\omega_p)K_p = -180°$ (or $-\pi$ radians).

In this experiment, we will examine first the frequency response of $G_1G_2K_p$ to determine the phase and gain margins for a position-only feedback configuration of the MCSL. Then the loop will be closed and the closed-loop frequency response will be evaluated. Three methods of generating the frequency response plots will be used in each case:

- Computer analysis.
- Analysis of the physical system with the aid of the HP 3582A spectrum analyzer.
- Bode plot by hand.

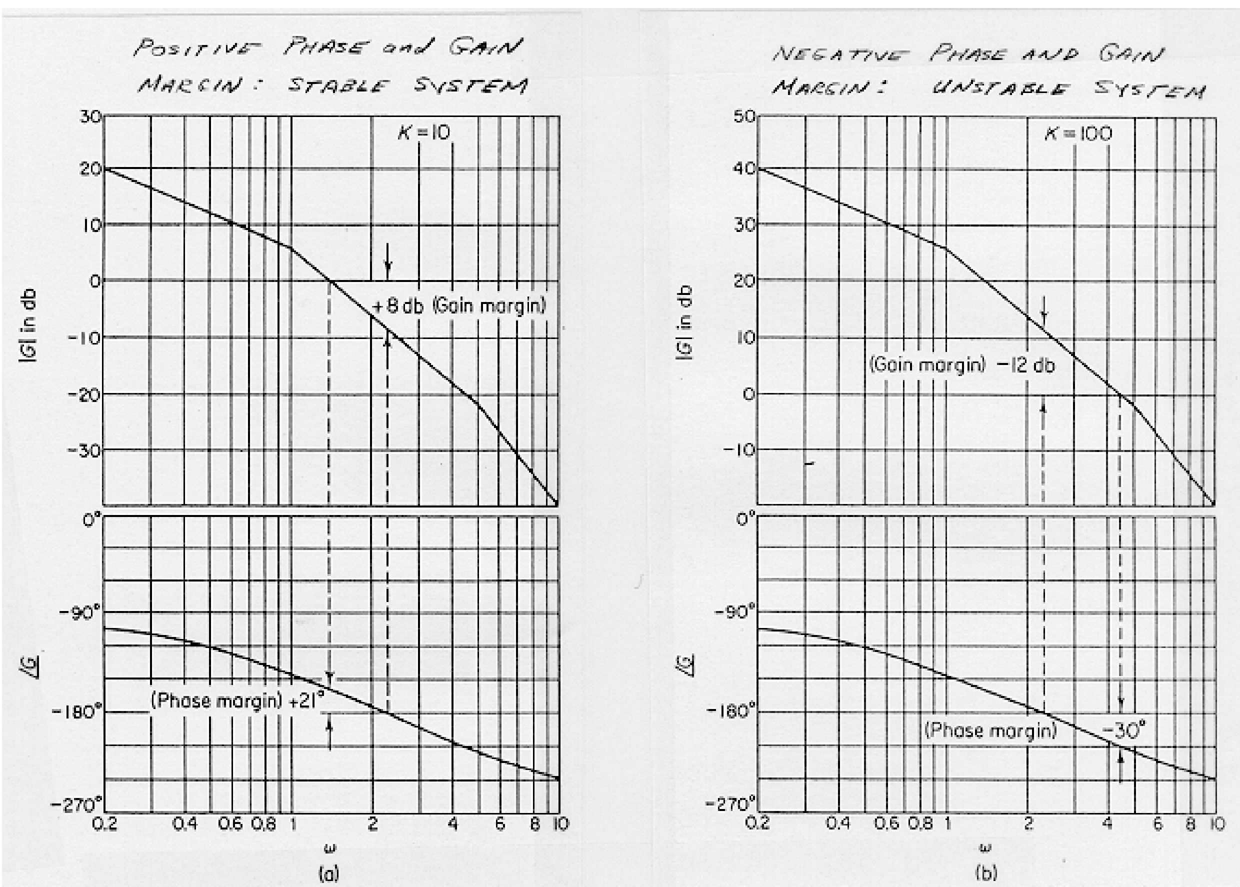


Figure 2: Measurement of Gain and Phase Margin from Magnitude and Phase Plots

Frequency Response Measurement Using the Spectrum Analyzer

A common method for testing the frequency response of a linear system is to apply a pure sinusoidal signal of known amplitude to the input, and observe the relative amplitude and phase of the output. Comparison of the input and output sinusoids yields the magnitude and phase response of the system at a single frequency. Many frequencies are checked to establish data points on frequency response magnitude and phase plots.

A faster approach is made possible by use of a spectrum analyzer, which is capable of displaying the magnitude and phase characteristics as a function of frequency, for an arbitrary periodic signal over a continuum of frequencies. In other words, it can produce an entire frequency response plot at one time. In this experiment we will be using the HP 3582A spectrum analyzer to display the magnitude and phase response vs. frequency of the MCSL system in both open-loop and closed-loop configurations.

Rather than apply a single sinusoid at a time to the input of the system, we apply a special signal which consists of all frequency components from 0 to 25 kHz combined, equally weighted. Such a signal is called white noise. It is generated by the 3582A at the *random noise* output. This signal is applied to the input of system under test and to the Channel A input of the spectrum analyzer. The output of the system is connected to the Channel B input of the spectrum analyzer. With the instrument in "Xfer Function" mode, a magnitude and/or phase plot of the Channel B signal relative to the Channel A signal is displayed over any selected range of frequencies.

The Spectrum Analyzer performs a Fast Fourier Transform, using time samples of an input signal to produce frequency samples of the output transform. 256 time domain samples are taken to produce 256 Fourier frequency domain samples, each a magnitude and phase sample of the frequency response of the system under test. In order to produce an acceptably smooth plot, several analysis runs must be made and averaged together. This multiple-run averaging can be done automatically by the spectrum analyzer using its averaging features.

In this experiment, the magnitude and phase response vs. frequency of the open loop and closed loop servocontrol systems will be tested using the spectrum analyzer. The experimentally measured response plots will be compared with computer-generated frequency response plots and Bode plots based upon the linear system model determined in Experiment 1. The results of these three analysis methods will be compared.

Be certain that you use the same MCSL (both control module and electromechanical assembly) that you used in Experiment 1, or these comparisons will be meaningless.

The lab instructor may wish to demonstrate the use of the HP 3582A spectrum analyzer. Additional information is available on the pull-out quick reference cards underneath each unit, or in the instruction manual in the side drawer of the lab bench. The instrument is delicate and expensive (>$10K) so please treat it with care.

DC Offset Precalibration of the Spectrum Analyzer

The differential input amplifiers of the spectrum analyzer tend to drift and accumulate DC offset voltages over time and with equipment wear. This is of little concern for spectral measurements at high frequencies, since AC-coupling can be used to reject the DC (zero frequency) component. But when measurements must be made at low frequencies approaching DC, the inputs must be DC-coupled, so that the following pre-calibration procedure is necessary to null the input amplifiers, assuring accurate measurements at low frequency.

Set the coupling switches for both Channel A and B to DC (the straight line symbol rather than the wavy one). Short both inputs to ground with jumpers between each red and black input terminal pair. Set up the spectrum analyzer as follows:

Channel A:	Amplitude
Channel B:	Amplitude
Amplitude Scale:	10 dB V/div
Amplitude Reference	Level: adjust for best display
Freq Span:	0 - 50 Hz (Zero Start Mode)
Ch. A Sensitivity:	30 Volts
Ch. B Sensitivity:	30 Volts
Input Mode:	A and B
Coupling:	DC, Channel A and B
Passband Window:	Flat Top
Trigger Settings:	Level = Free Run, Slope = N/A, Repetitive = On
Averaging:	Off

First perform this procedure for Channel A, with Channel B Amplitude off, then repeat for Channel B with Channel A off. Adjust the small DC offset adjustment screw next to the input terminals (using a small screwdriver or a fingernail) to minimize the spectral bar at 0 Hz (DC). You should be able to get it to less than 20 dB above the noise floor (the fuzzy level at the bottom of the display).

Procedure

1. Derive the loop transfer function $G_1G_2K_p$ for your system, by breaking the loop at the position sensor voltage v_p. Note that, as depicted in Figure 2 of Experiment 1, $G_1G_2K_p$ is the *negative* of the transfer function from v_p around the loop, though the OP amp, power amplifier, and position sensor, back to itself. $G_1G_2K_p$ should have a net positive low frequency gain. Figure 1 of Experiment 4 may be helpful in seeing how the loop is broken for the open loop measurement. Use your model parameter measurements from Experiment 1, to assign numeric values to the coefficients of $G_1G_2K_p$.

2. Use the frequency response analysis capability of the SIM342 program to generate the exact magnitude and phase plots for $G_1G_2K_p$ based upon the linear system model and your model parameters. Perform the analysis over two frequency decades from 0.1 Hz to 10 Hz. A tutorial is provided in the Appendix of this experiment to demonstrate how to use SIM342 to perform a frequency analysis. Leave the magnitude and phase plots on the computer screen for now.

3. Perform the DC offset precalibration of the spectrum analyzer described above.

4. Set up the spectrum analyzer as follows:

Channel A:	Xfer Function Amplitude or Phase (depends on the measurement you're making).
Amplitude Scale:	10 dB V/div
Amplitude Reference Level:	Adjust for best display
Freq Span:	0 - 5 Hz (Zero Start Mode)
Ch. A and Ch. B Sensitivity:	30 Volts
Input Mode:	Both
Coupling:	DC, both Channels
Passband Window:	Flat Top
Trigger Settings:	Level = Free Run, Repetitive Mode = On
Averaging:	RMS Average, 16 Runs

5. Configure the MCSL with *no* feedback, except for the special network described below. Let R_f =10K. Connect the random noise source output of the spectrum analyzer to auxiliary input #1 on the MCSL. Note that the internal resistance in this path is 10K. Also connect the noise input to the Channel A input of the spectrum analyzer. Set the noise source amplitude initially to about 60% of maximum. Connect the shaft position sensor pot output v_p to the Channel B input of the spectrum analyzer. The spectrum analyzer is now set up to display the magnitude and phase of the output voltage v_p relative to the to the summing junction input where v_p would normally be connected to close the loop. This setup is used to evaluate the *open loop* characteristics $G_1G_2K_p$ of the servocontrol system, with the objective of determining the phase and gain margins of the closed loop system.

   Since the open loop transfer function contains a free integrator, the output v_p will tend to drift whenever the mean of the input signal is non-zero. Even though we are applying a zero-mean white noise signal, the asymmetric nonlinearities of the system will eventually cause the output shaft angle to rotate more than π radians from its center position where $v_p = 0$. If this occurs, the pot wiper crosses the dead zone, and v_p jumps from +15 to -15 volts, or vice versa, instantaneously. If this ever happens during a data run, the test is invalid.

The spectrum analyzer will be performing a 16 run frequency response RMS average, which will take several minutes to complete due to the low frequency range. During the entirety of this test averaging period, we cannot permit the output shaft to drift to the point where the pot contacts the dead zone in either direction, since the resulting voltage wraparound would destroy the validity of the frequency response data.

To reduce the possibility of this occurrence, we provide a very small amount of negative position feedback, through a DC-pass filter, which essentially only corrects for long-term drift of the shaft angle θ_o from the middle position of the sensor. This gently keeps the output shaft within the $\pm\pi$ range, preventing wraparound. As illustrated in Figure 3, a low-pass network with a -3 dB point of 0.3 Hz is inserted in the position feedback loop between the position sensor output v_p and the summing junction. While this network does have an effect on the open loop transfer function $G_1 G_2 K_p$, we will ignore it in our analysis of the system, since its characteristics show up at about an order of magnitude lower frequency than the dynamics of the system. The feedback network is fabricated using two resistance decade boxes and one capacitance box.

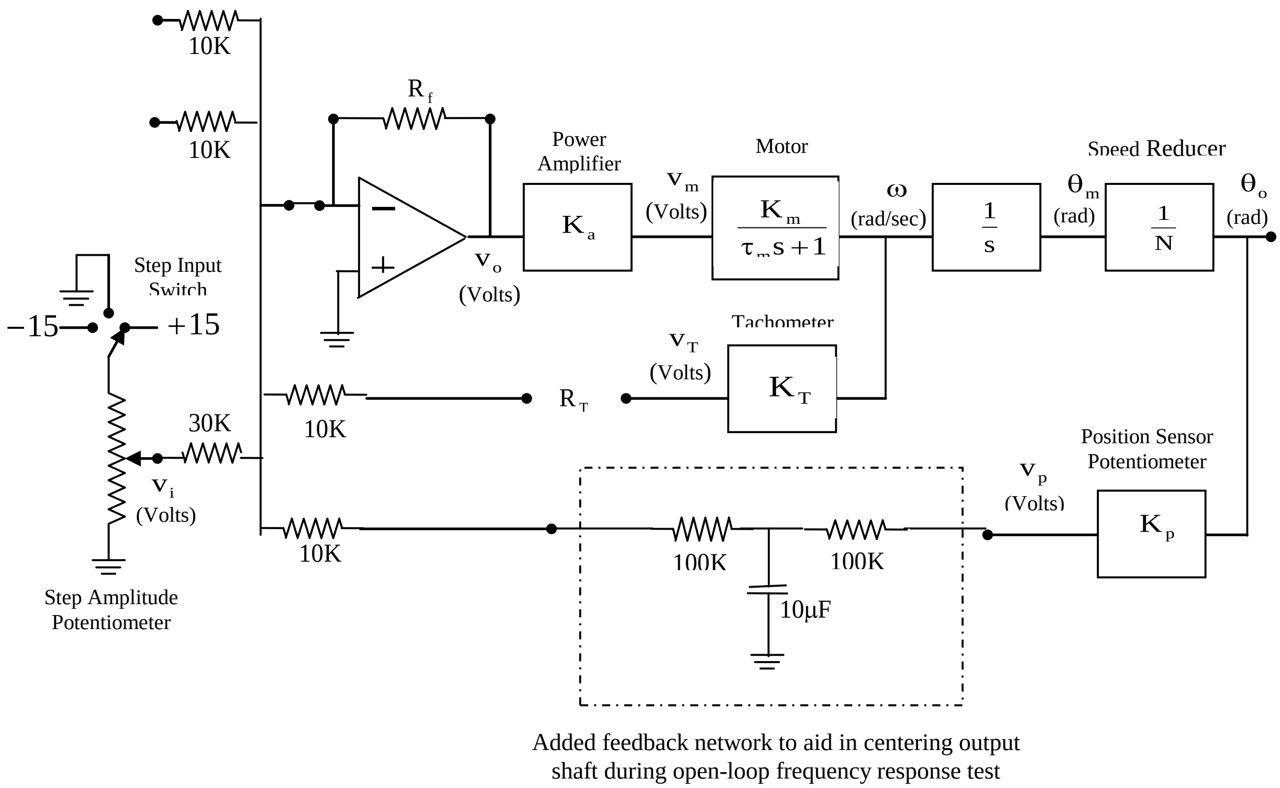


Figure 3: Low-pass Filter Feedback Network to Keep Position Sensor Centered in Range.

6. In this step we will perform a frequency response test with the open-loop system driven by the noise source. The motor shaft position will appear to move randomly during this test, but should remain generally centered about the starting position. Adjust the noise amplitude level such that θ_o varies as much as possible without wrapping around through the dead zone of the pot. It is helpful to monitor v_p on the oscilloscope, auto-triggered and set to a very slow sweep speed (e.g., 1 sec/div) to check how close v_p may be getting to ± 15 volts. The greater the amplitude of variation of the noise, the better (and smoother) the frequency response plots generated by the spectrum analyzer will be, since the nonlinear effects of the MCSL will be minimized. But if the shaft crosses the pot dead zone even once, the analysis will be ruined and you must start it again. We therefore must find, possibly by trail and error, the largest noise amplitude that does not result in hitting the dead zone of the pot. Avoid adjusting the noise amplitude after the test run has been started, although small corrections should not have an effect on the data (larger adjustments could change the measured transfer function slightly due to the nonlinear large-signal behavior of the system).

   Sixteen successive analysis runs will be automatically performed by the spectrum analyzer, which will accumulate the root mean square (RMS) average of all runs. Watch the spectrum analyzer display as each new analysis improves the smoothness of the frequency response plot. The cumulative effects of 16 runs will be displayed in succession, then the spectrum analyzer will stop, leaving the final 16-run average plot on the display and in memory available to download to the PC via SIM342.

   You may select either the magnitude or phase (or both) on the spectrum analyzer display, but both will always be acquired and stored internally by the spectrum analyzer. Note that since we are monitoring $-v_p$ (negative feedback switch setting), the phase displayed by the spectrum analyzer is offset by $180°$ or π radians. Keep this in mind during the next step, when you overlay the spectrum analyzer data on the simulated frequency response plot.

7. When the 16-run average is complete, use SIM342 to import the data directly from the spectrum analyzer, and display it overlaid on the same plot as the previously generated simulated magnitude and phase plots. This procedure is described in the SIM342 tutorial in the Appendix to this experiment. The SIM342 data import routine requires that both the magnitude and phase simulation results must be displayed on the spectrum analyzer at the time the spectrum analyzer data is imported. Using the zoom feature of SIM342, re-scale the frequency axes of the overlaid magnitude and phase plots to cover two decades, from 0.1 to 10 Hz. Note that the spectrum analyzer uses linear frequency axis scaling rather than the logarithmic one normally used in frequency response plots. This is due to the way the discrete frequency samples are measured via the Fast Fourier Transform algorithm. The SIM342 program will correctly warp the linear frequency axis of the spectrum analyzer data to match the logarithmic frequency axis of the simulated response plots. Thus, the spectrum analyzer magnitude and phase plots will look somewhat different after they have been imported to the PC and displayed by SIM342.

As previously mentioned, the spectrum analyzer phase plot is offset by $180°$, so it will be necessary later, when graphically measuring the phase margin from the magnitude and phase plots, to manually add (or subtract) the $180°$ phase offset to/from the spectrum analyzer phase data. With the spectrum analyzer and simulation plots overlaid, print the magnitude and phase plots on the network laser printer using the **print** feature of SIM342.

8. Draw (by hand) on the hard copy plot of the actual and simulated frequency response data, the Bode approximate magnitude and phase plots for $G_1G_2K_p$, over two decades from 0.1 Hz to 10 Hz. Scale your Bode plots to match exactly the scaling of the previous plots. Note that it is often more convenient to use radian frequency when creating Bode plots, but be sure to use Hertz when drawing the plot, so that it matches the frequency axes of the actual and simulated data plots.

9. Show graphically on each of the three overlaid (spectrum analyzer, simulated, and Bode) magnitude/phase plot pairs the phase margin for the open-loop transfer function $G_1G_2K_p$. Report these in tabular form. Use degrees for reporting phase (π Radians = 180 degrees):

	Phase Margin [degrees]
Actual System	
Simulation	
Bode Approximation	

Note that since a second order system cannot generate more than a 180 degree phase shift, the gain margin is undefined on the Bode and computer generated plots, although the experimental data may reflect higher order unmodeled modes which contribute greater phase lag.

10. The frequency response of the *closed loop* servocontrol system will now be evaluated. Connect the position feedback v_p through a total resistance $R_2 = 10K \left(R_p = 0\right)$. As before, provide the input to the system at the number 1 auxiliary input to the summing junction on the MCSL, through the internal 10K resistance. All other connections and settings of the spectrum analyzer remain unchanged. Unlike the open-loop situation, there is no danger of position sensor wrap-around during this test, since the closed loop feedback always centers the pot in its midrange. Therefore, the auxiliary low-pass feedback network can be removed.

11. Use SIM342 to perform a simulated frequency analysis of the closed-loop system model as presently configured. Again, plot the analysis over two decades, from 0.1 to 10 Hz. Leave the magnitude and phase plots on the screen for now.

12. Use the spectrum analyzer to again perform a 16-run average frequency response test, similar to the procedure of step 5. A resonant peak in the magnitude response

characteristic of an underdamped system should be observed, although the actual response varies substantially between different MCSL units. Use SIM342 to import the plotted data from the spectrum analyzer and overlay it on the computer generated data. Print the combined plots.

13. Draw (by hand) on the hard copy plot of the actual and computer simulated frequency response data, the Bode magnitude and phase plots for the closed loop system, over two decades from 0.1 Hz to 10 Hz. Scale your Bode plots to match exactly the scaling of the previous plots.

14. Note the substantially different frequency response plots for the closed loop vs. open loop systems. Identify on each of the three (spectrum analyzer, simulated, Bode) closed loop magnitude and phase plots the following: KKp (the low frequency closed loop gain), f_m (resonant frequency), and M_m (peak amplitude fraction). Note that $f_m = \frac{\omega_m}{2\pi}$. Report these measurements in tabular form:

	KK_p [dB]	f_m [Hertz]	M_m [unitless]
Actual System			
Simulation			
Bode Approximation			

Appendix

Tutorial on frequency response simulation using SIM342.

1. Log onto the computer. Select **EE342**, then **MATLAB** from the startup menu. The SIM342 program will start automatically. If at any time you accidentally exit SIM342, type "sim342" on the command line of the MATLAB command window that will remain visible. This will restart the SIM342 program.

2. Once the startup process has completed, you will see the **New Plot** menu window. Under the **Analysis** heading click **Freq Resp**. You will see blank Magnitude and Phase response plots. On the right, click **Simulate**. You will see the **Simulation** window in which you enter information about the system to analyze and the test parameters.

   As an example, we will analyze the frequency response of the system (which could be open or closed loop) described by the transfer function:

$$\frac{100}{s^2 + 5s + 100}$$

In the **Numerator** box (select box by clicking on it with the mouse). enter:

100

Select the **Denominator** box and enter three numbers separated by spaces:

1 5 100

These are the coefficients of the numerator and denominator polynomials of the transfer function. The coefficients are entered in decreasing powers of s, with the highest power term first. A zero must be entered for a power of s that does not appear in the polynomial.

Leave the **Parameter** box empty for now (or clear it by clicking on the box and using backspace to clear it). This will cause the resulting plot to be autoscaled.

3. Click **Done** to run the frequency response analysis for the specified transfer function, producing magnitude (in dB) and phase (in degrees) response curves as functions of frequency on a logarithmically scaled axis**. Frequency is displayed in Hertz, not radians per second**.

4. To check the exact value of a particular point on either the magnitude of phase plots, click the **GetPoint** button, and wait until the mouse arrow icon changes into crosshairs. Position the crosshairs at the point you wish to read on the plot, and the exact x,y coordinates will be displayed on the right side of the window.

5. For the same transfer function, let's now manually set the frequency axis limits and number of points to be calculated and plotted by the analysis. Click the **Simulate** button to recall the **Simulation** window. In the **Parameter** box type:

   0.1 10 100

   This will turn off autoscaling and force the analysis to run between f = 0.1 and f = 10 Hz, with 100 points plotted. The resulting plot will include the specified frequency range, within the nearest decade divisions. Click the **Done** button to re-run the simulation with these new specifications.

6. To run your own frequency response analysis, click **Simulate**. And enter the system and simulation information. For autoscaling leave the **Parameter** box empty. This is a good choice the first time you analyze a new transfer function, since autoscaling checks the response for significant features and makes sure that they are all displayed within the viewable plot limits. After an initial autoscaled run, you can specify an exact frequency range and plot resolution (number of points) in the **Parameter** box, and re-run to produce a customized plot.

To import frequency response data from the Spectrum Analyzer:

SIM342 has the capability to import magnitude and phase response data directly from the HP3582A spectrum analyzer, and overlay this upon the magnitude and phase plots produced by the simulation. The download is accomplished over an IEEE488 bus connection.

1. The spectrum analyzer *must* be set up as follows:
   i) Set Channel A to **AMPLITUDE XFR FCTN**
   ii) Set Channel B to **PHASE XFR FCTN**
   (Both traces must be displayed on the analyzer at the same time to download to S im342.)
   iii) The **STATUS** indicators will flash **TALK, LISTEN** and **REMOTE** during the download, but will be blank or display only **LISTEN** at other times.

2. Perform a frequency response analysis on the actual system using the spectrum analyzer, as described in the experimental procedure. You *must* have *both the Magnitude and Phase* display on the spectrum analyzer screen prior to importing. Otherwise, the computer could hang up requiring a cold reboot (turn power off, then on again).

3. When you have the magnitude and phase response that you want displayed on the analyzer screen, click the **Import** button of SIM342. This will import the magnitude and phase data from the analyzer and display it on the computer screen, overlaid upon any previously generated simulated or imported data plots. The spectrum analyzer data will appear as a solid blue trace. The simulation data will appear as a dashed black trace.

4. The **Zoom**, **Get Point** and **Add Text** buttons can be used to modify and enhance the plots as described in the previous tutorials.

Prelab Assignment

1. Read completely the Experiment, and review Bode plotting techniques from course EE 301.

2. On log-axis paper, generate a *Bode magnitude and phase plot* for the following transfer function. Use a sufficient frequency range (in Hertz) to show the point where the magnitude of $G(j\omega)$ drops below zero dB.

$$G(s) = \frac{20}{s(s+10)}$$

3. Assuming this G(s) is a complete open-loop transfer function, *determine the phase margin* of the corresponding closed loop transfer function.

Experiment 7
State Variable Feedback

Introduction

This experiment demonstrates the design of control systems using the methods *of modern control theory*. If all state variables of a system are accessible or estimable, these may be gain weighted and summed to provide *full state feedback* compensation of the system. Theoretically, this feedback permits the arbitrary placement of the closed loop system poles, programmable via the feedback gain constants.

In this experiment we will investigate the compensation of the MCSL (without the inverted pendulum that we attached in Experiment 6) via full state feedback. As in the previous experiment, the objective is to achieve the best possible step response from the closed loop system. The feedback of all states of the plant through gain factors will permit the location of the closed loop system poles at any desired values (within the linear limitations of the system).

Both states of the second order MCSL system (position and velocity) are accessible for feedback via appropriate transducers. The position state variable is sensed in the form of the position feedback voltage v_p. The velocity state variable is sensed in the form of the tachometer feedback voltage v_T. These voltages are gain weighted and summed with the input voltage v_i via the resistances R_2 and R_3 connected to the summing junction of the op amp, and the op amp feedback resistor R_f. The state feedback gains K_2 and K_3 are programmed by selection of R_2 and R_3 relative to R_f. This is illustrated in Figure 1.

The state variable feedback design problem is formulated beginning with the matrix state equation of the plant. The plant (including the motor, power amplifier and speed reducer) has a transfer function of the form:

$$\frac{\theta_o}{V_o} = \frac{K'}{s\left(s+\frac{1}{\tau_m}\right)} \qquad (1)$$

where

$$K' = \frac{K_a K_m}{N\tau_m}$$

The corresponding differential equation is:

$$\ddot{\theta}_o + \frac{1}{\tau_m}\dot{\theta}_o = K' v_o \qquad (2)$$

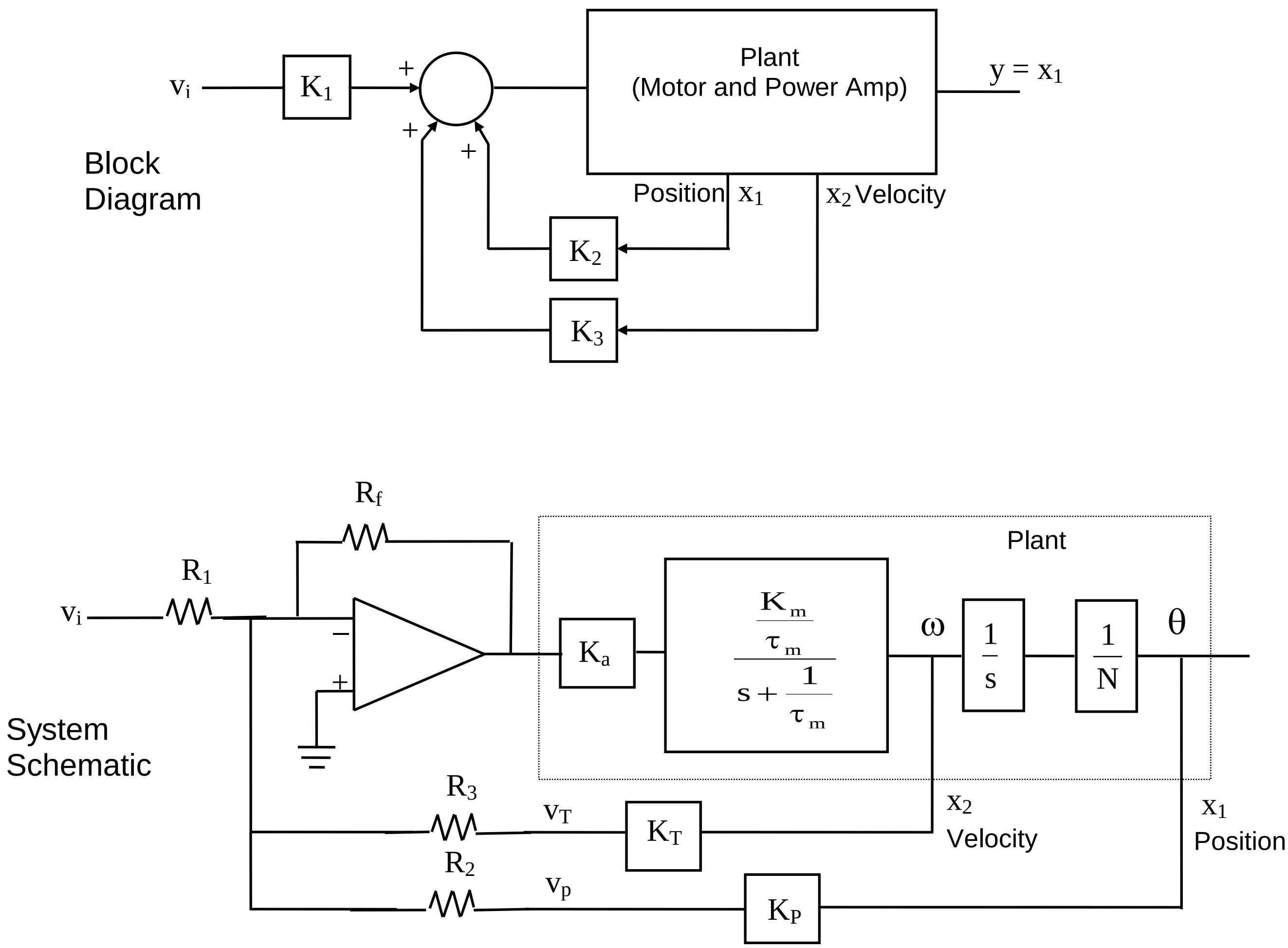


Figure 1. Full State Feedback: System Block Diagram and Corresponding Schematic

Define the state variables,

$$x_1 = \theta_o \qquad x_2 = \dot{\theta}_o \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{3}$$

the input variable

$$u = v_o \qquad \text{(the output of the op amp)}$$

and the output

$$y = \theta_o$$

The state equations are:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{1}{\tau_m} x_2 + K' u \end{aligned} \tag{4}$$

In matrix form,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\mathrm{u}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{1}{\tau_m} \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ K' \end{bmatrix}$$

The full state feedback control law is:

$$\begin{aligned} u &= K_1 v_i + K_2 v_p + K_3 v_T \\ &= K_1 v_i + K_2 K_p \theta_o + K_3 K_T \omega \end{aligned} \tag{5}$$

Note that the tachometer measurement is taken at the motor shaft rather than after the pulley speed reducer. Therefore,

$$\theta_m = N\theta_o \quad , \quad \omega = \dot{\theta}_m = N\dot{\theta}_o = Nx_2$$

Thus,

$$u = K_1 v_i + K_2 K_p x_1 + K_3 K_T N x_2$$

or in matrix form,

$$u = \begin{bmatrix} K_2 K_p & K_3 K_T N \end{bmatrix} \mathbf{x} + K_1 v_i \tag{6}$$

Applying this input to the plant, we get the closed loop system with the modified matrices:

$$\widehat{\mathbf{A}} = \begin{bmatrix} 0 & 1 \\ K'K_2K_p & K'K_3K_TN - \dfrac{1}{\tau_m} \end{bmatrix}$$

$$\widehat{\mathbf{b}} = \begin{bmatrix} 0 \\ K'K_1 \end{bmatrix}$$

Since the system matrix is in Controller Canonical Form, the characteristic equation may be written by inspection of the bottom row of $\widehat{\mathbf{A}}$:

$$s^2 - \left(K'K_3K_TN - \frac{1}{\tau_m} \right) s - K'K_2K_p = (s - p_1)(s - p_2) \tag{7}$$

The values of roots (poles) p_1 and p_2 may therefore be programmed by appropriate selection of K_2 and K_3. The objective of this experiment is to use this compensation method to locate the closed loop system poles p_1 and p_2 such that a desirable step response is obtained.

Recall that for underdamped systems the characteristic equation may be written in terms of the damping factor ζ and the natural frequency ω_n:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = (s - p_1)(s - p_2) \tag{8}$$

We select K_2 and K_3 to obtain the desired values of ζ and ω_n, which will characterize the step response of the system.

Procedure

1. Set up the MCSL apparatus the same as in Experiment 2, Configuration 3, including tachometer feedback via $R_3 = R_T + 10K$. Let $R_f = R_1 = 30K$. (This sets $K_1 = -1$.)

2. Using your model parameters from Experiment 1, select appropriate values for K_2 and K_3 (and therefore R_2 and R_3) such that $\zeta = 0.6$ and ω_n is at least as large as that obtained using cascade compensation in Experiment 5. Report the resistor values and show all calculations. Limit your resistor selection to reasonable values within the ranges provided by the decade boxes. Since K_2 effects the steady state gain of the closed-loop system, try to keep K_2 in the range of 0.5 to 2.0. What are the calculated closed loop pole locations?

3. Implement your design using the MCSL apparatus and the selected values of the feedback resistors. Apply a voltage step input of appropriate height and observe the time response using the digital storage oscilloscope. Note that, depending on the steady state gain you ended up with, a 6 volt step may be too large, causing the output shaft angle to exceed the $\pm\pi$ limits of the position sensor pot. Import the data into Sim342 and print the step response plot. Compare your observations with the expected rise time, settling time, peak time, and peak overshoot for your design values of ζ and ω_n. Report calculated vs. experimental values of each. Identify each transient response metric on the printed oscilloscope data plot. Note that since K_p is negative, v_p changes in the opposite sense of v_i. The steady state output voltage v_p is equal to $-\frac{K_1}{K_2}v_i$. Derive this relationship.

   If zero steady state error is desired (at least theoretically), how might you change K_1 as a function of K_2? Would this affect the system response in any way other than scaling the closed loop gain? If so, how?

4. Adjust R_2 and R_3 iteratively until the desired step response characteristics are obtained. To obtain reasonable values of these resistances, first determine the effect of varying each individually on the steady state v_p, and τ_r and M_p. Report your final values of R_2 and R_3 and the corresponding values of K_2 and K_3. Using Sim342, import and print the actual step response showing the values of the rise time, peak time, settling time and peak overshoot. Also report the final experimentally determined closed loop pole locations.

   Compare your experimental with your calculated values of K_2 and K_3. How does the performance of the state feedback compensation compare with the lead compensation of Experiment 5?

Prelab Assignment

1. Read the experiment. Review state variable descriptions of linear systems in your EE 302 textbook (Dorf & Bishop, *Modern Control Systems, 7th ed.*, Chapter 3). Although the material in this experiment is intended to be self-contained, it would also be helpful to review/preview Chapter 11 in the text on "The Design of State Variable Systems".

2. Using the design technique described in this experiment, select R_2 and R_3 such that both poles of your closed-loop system are at s = -2 . Use the model parameter values for your system. (Find K_2 and K_3 first, then use these to find R_2 and R_3.)