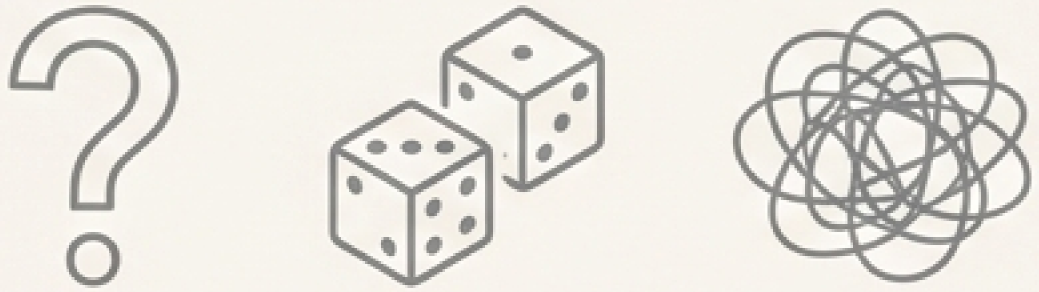
A Thinking Instrument for Attribution

A first-principles system for defensible, interpretable, and mathematically rigorous decision-making.

This platform is built on a single vision: to move beyond heuristic guesswork and black-box opacity. It is a thinking instrument designed for those who require attribution to be derivable from declared assumptions, with a complete audit trail from raw events to final conclusions. It combines probabilistic path modeling, axiomatic game theory, and comprehensive uncertainty quantification to make attribution not just accurate, but defensible.

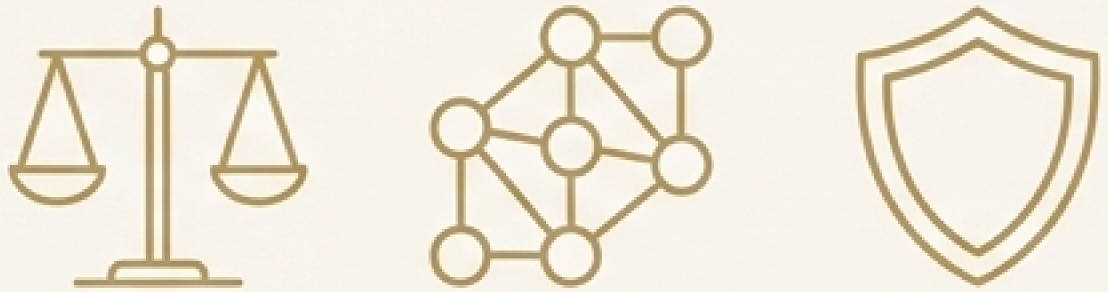
From Heuristic Guesswork to First-Principles Attribution

Heuristic Guesswork



- Relies on arbitrary rules (last-touch, linear, time-decay).
- Ignores sequence and interaction effects.
- Cannot answer counterfactuals (“what if?”).
- Violates fairness axioms.
- Provides no measure of confidence or uncertainty.

First-Principles Attribution



- Derives credit from probabilistic and game-theoretic models.
- Explicitly models path structure and sequence.
- Built on counterfactual reasoning (removal effects).
- Guarantees fairness through mathematical axioms.
- Quantifies uncertainty as a core tenet of intellectual honesty.

The Three Pillars of a Defensible Model

Absorbing Markov Chains

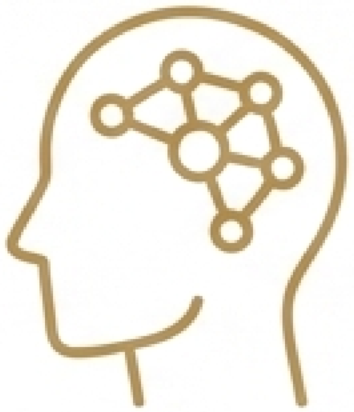
Probabilistic Causality

Models customer journeys as a stochastic process. It captures the critical role of sequence and path dependency, allowing for true counterfactual analysis by measuring “removal effects”.

Shapley Value Theory

Axiomatic Fairness

Treats attribution as a cooperative game. It provides the only unique, axiomatically fair method to distribute credit based on a channel’s marginal contribution to every possible coalition.



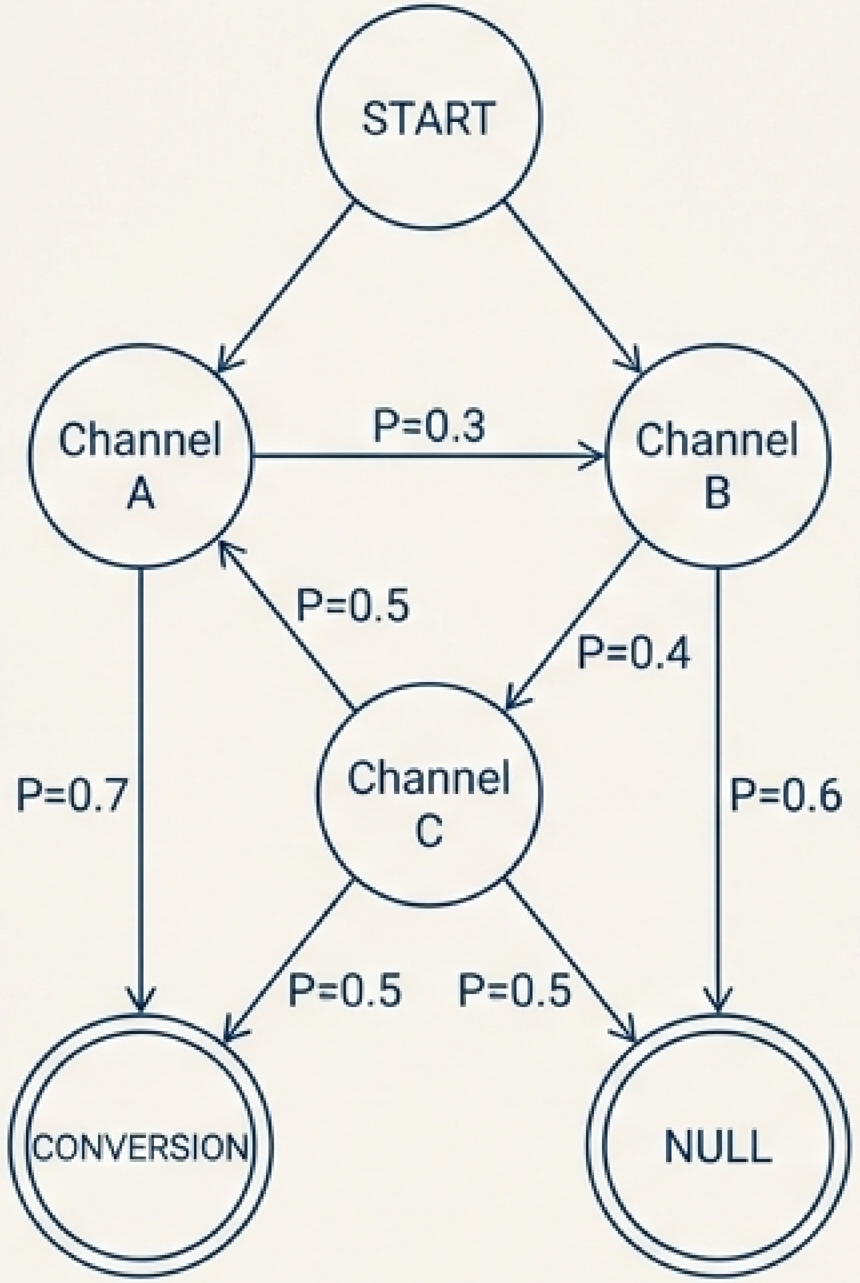
Psychographic Priors

Behavioral Context

Moves beyond raw frequencies by allowing domain knowledge to inform the model. It modulates transition probabilities based on behavioral context, such as user intent or device type.

Pillar 1: Modeling Journeys with Probabilistic Causality

"The order of touchpoints matters."



How it Works

We model user journeys as a series of transitions between states. The probability of the next step depends only on the a current state (a first-order Markov assumption).

What it Solves

This directly addresses the failure of heuristics by encoding path structure into a transition matrix T.

The Causal Question

By calculating absorption probabilities, we can precisely measure the causal impact of removing a channel, known as the **Markov Removal Effect:** $v(N) - v(N\backslash\{i\})$. This provides a direct answer to "How much did this channel contribute to the outcome?"

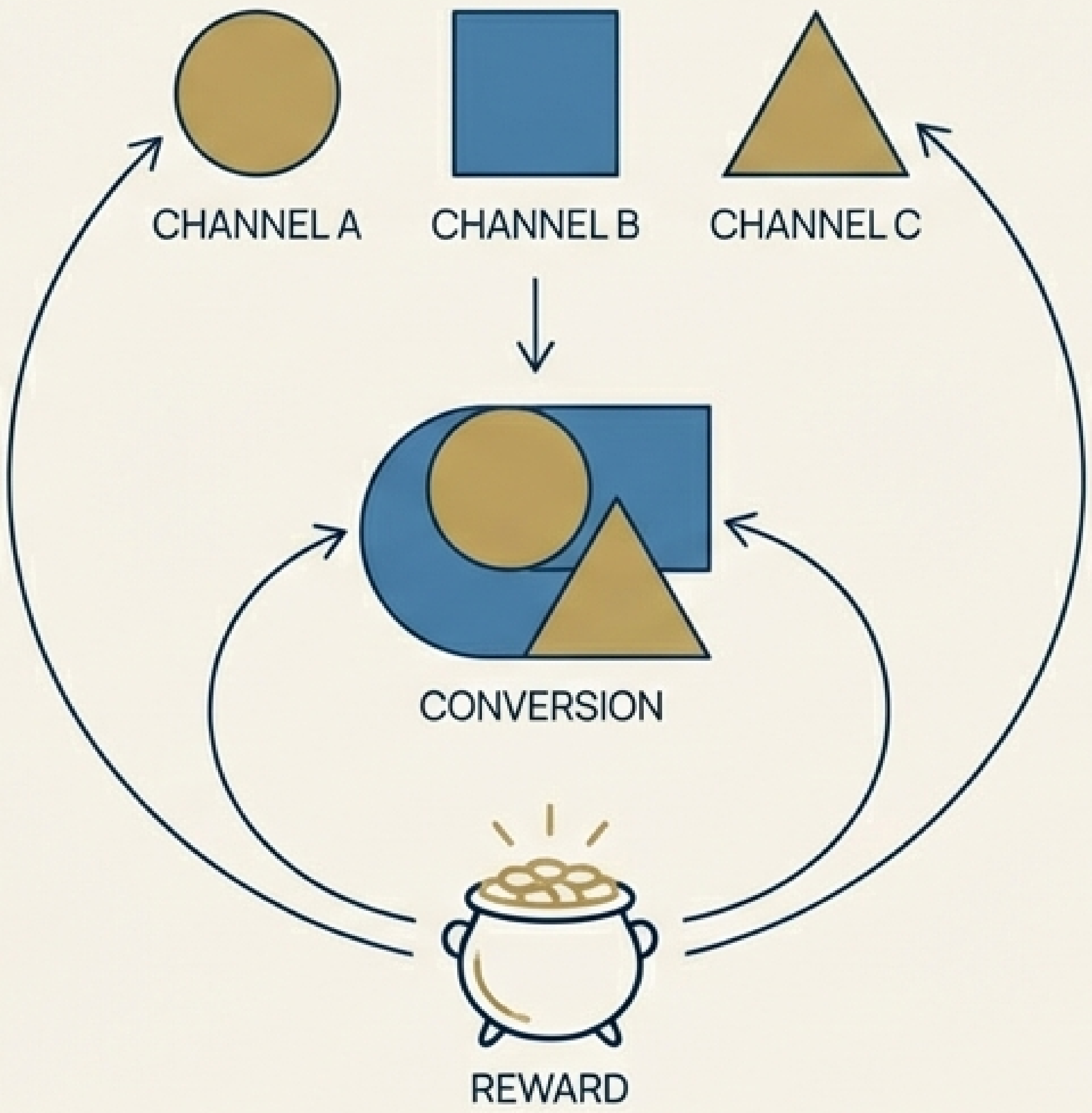
Pillar 2: Distributing Credit with Axiomatic Fairness

"Fairness is not a heuristic; it is a mathematical guarantee."

Shapley Value treats channels as players in a cooperative game where the payoff is a conversion. It calculates the expected marginal contribution of each channel, averaged over all possible sequences in which it could have appeared. It is the only allocation method that satisfies four critical axioms of fairness.

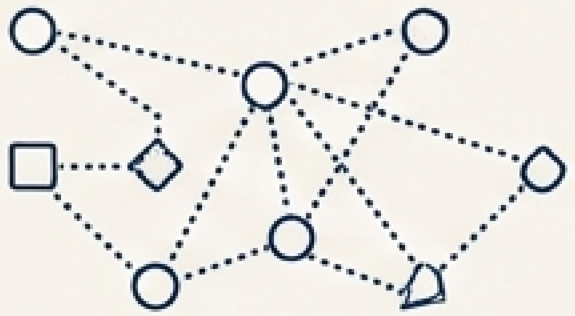
The Shapley Axioms

Efficiency The sum of all channel credits equals the total value of the conversion. ($\sum_i \phi_i = v(N)$)	**Symmetry** Channels that contribute equally receive equal credit.
Dummy Player A channel that adds no value to any coalition receives zero credit.	**Additivity** The model handles multiple games or value functions consistentiy.



The Synthesis: Stacking Causality and Fairness

This is not model averaging. It is model stacking.



Raw Path Data → **Markov Engine** Computes Causal Value $\xrightarrow{v(S)}$ **Shapley Engine** Distributes Value Fairly → **Hybrid Attribution Shares**

A pure Markov model understands causality but can be unfair. A pure Shapley model is fair but order-agnostic.
Our hybrid approach preserves the strengths of both.

Step 1: Markov defines the value.

The absorbing Markov chain is used to compute the characteristic function $v(S)$—the total conversion probability for any given coalition S of channels. This encodes the critical sequence and path information.

Step 2: Shapley distributes the value.

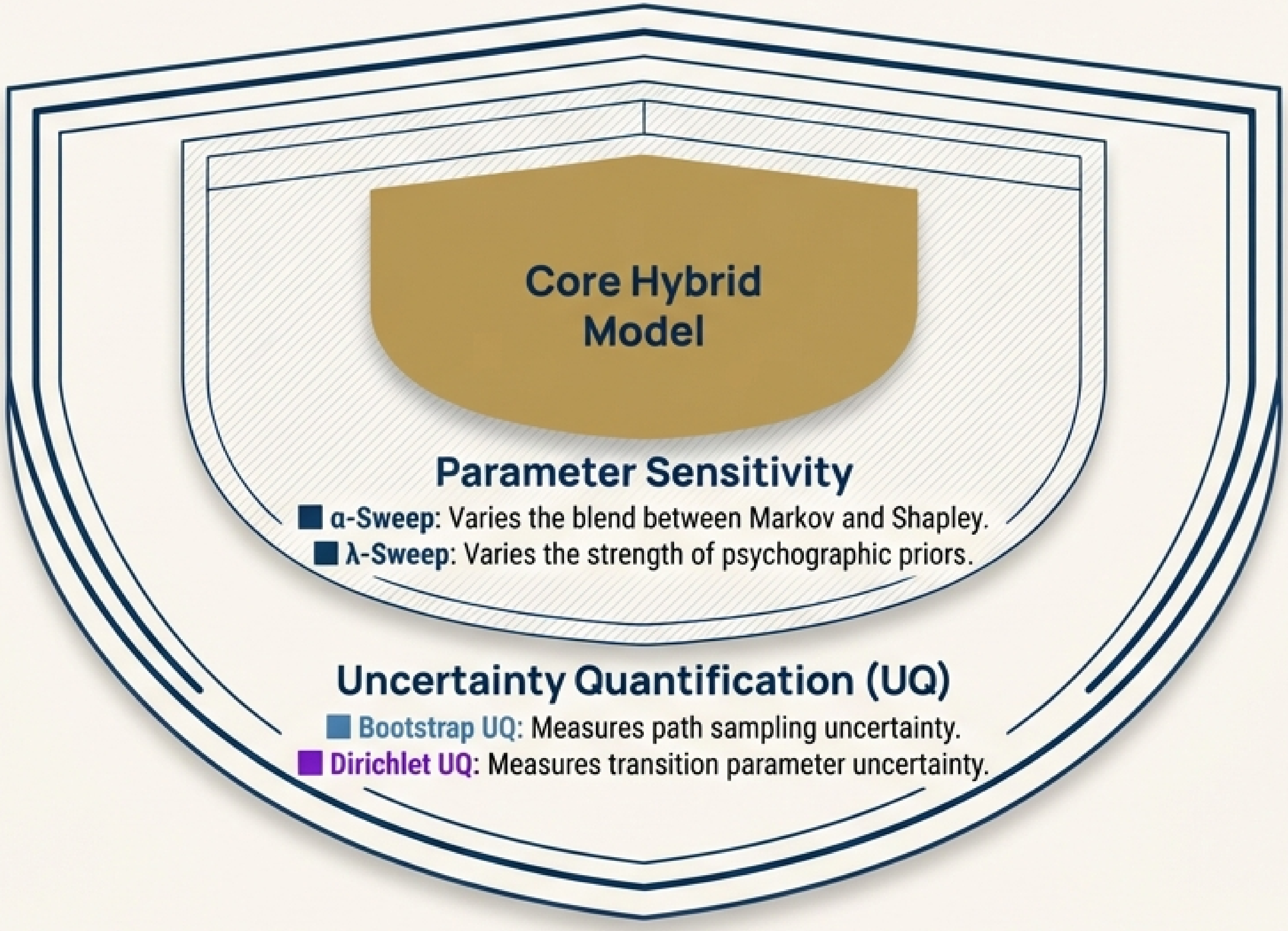
The Shapley formula then takes the Markov-generated $v(S)$ values as its input to distribute the total credit with axiomatic fairness.

```
Hybrid Share = α × Markov Share + (1-α) × Shapley Share
```

α is the blend parameter, controlling the emphasis on pure causality (α=1) versus pure fairness (α=0). The default is a balanced α=0.5.

Making Truth Defensible: The Robustness Stack

An answer **without a measure of confidence** is an **opinion**, not an analysis.
We make our models defensible by quantifying uncertainty from first principles.



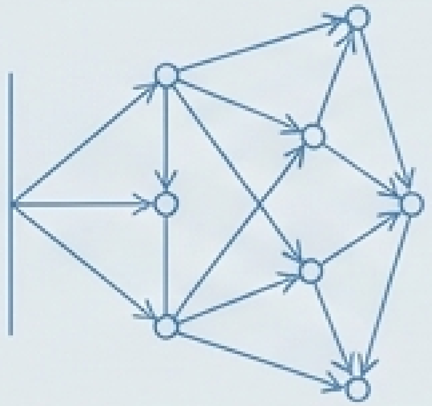
This stack provides a complete picture of a result's stability, protecting against both flawed assumptions and insufficient data.

The Two Lenses of Uncertainty

Bootstrap UQ: Path Sampling Uncertainty

The Question

"How much would the results change if we had a slightly different sample of user journeys?"



The Method

Paths are resampled with replacement B times (e.g., B=500), and the full attribution model is re-run for each sample. This creates a distribution of possible outcomes.

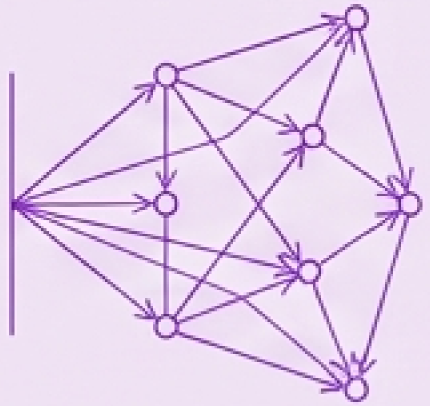
The Insight

Reveals uncertainty driven by the diversity and volume of the observed paths. If this is wide, you need more varied data.

Dirichlet UQ: Transition Parameter Uncertainty

The Question

"How confident are we in the transition probabilities (the T matrix) themselves?"



The Method

Treats the transition counts for each state as parameters of a Dirichlet distribution. We sample new T matrices B times and re-run the model.

The Insight

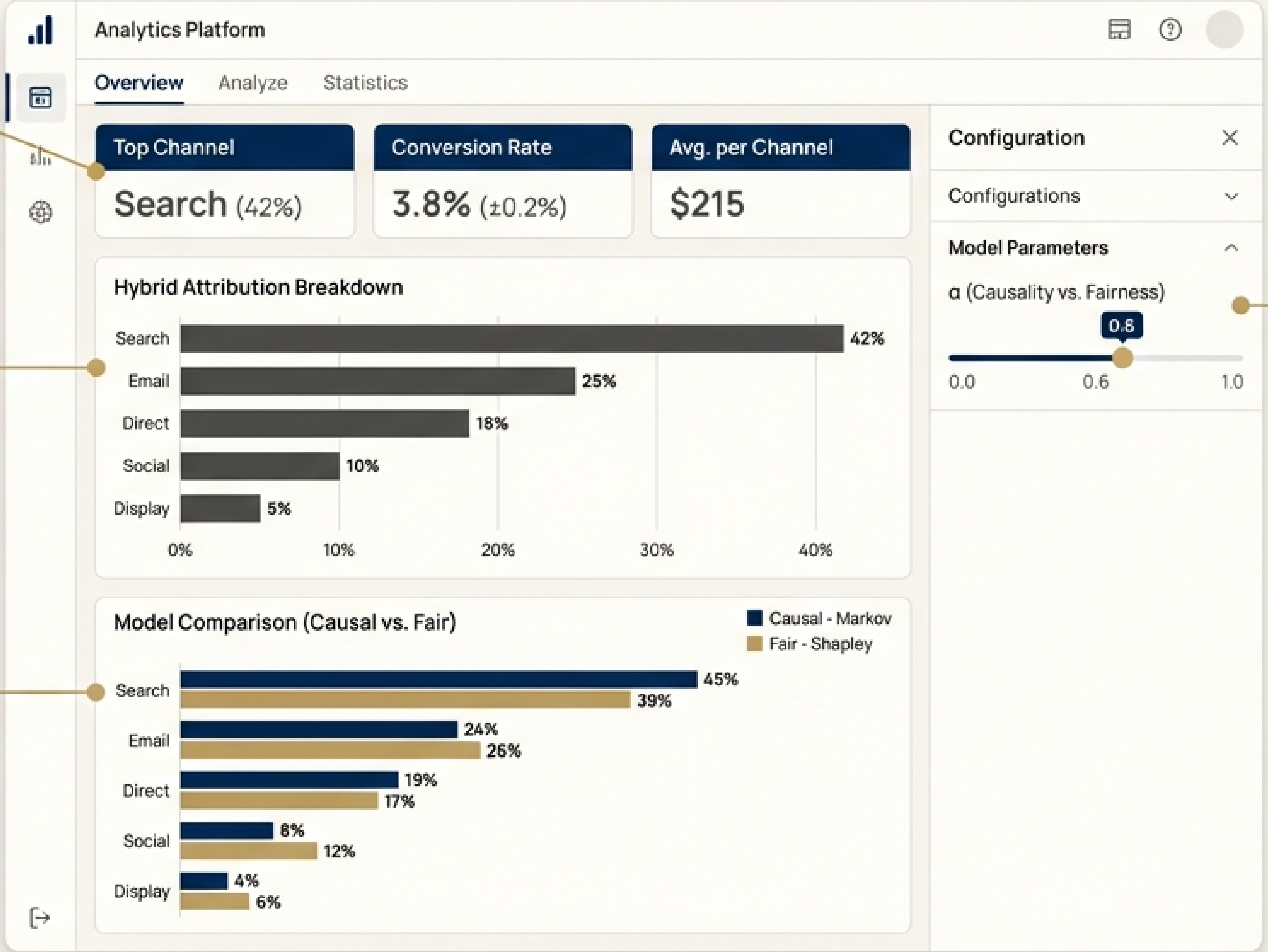
Reveals uncertainty driven by low-traffic transitions. If this is wide, you need more observations for specific state-to-state movements.

The Instrument’s Readouts: From Analysis to Action

Key Metrics Card: Call out “Top Channel,” "Conversion Rate," "Avg. per Channel."

Main Bar Chart: “Hybrid Attribution Breakdown: The final, blended attribution score for each channel."

Comparison Chart: 'Causal (Markov) vs. Fair (Shapley) Scores: Instantly see where the two models agree or diverge."

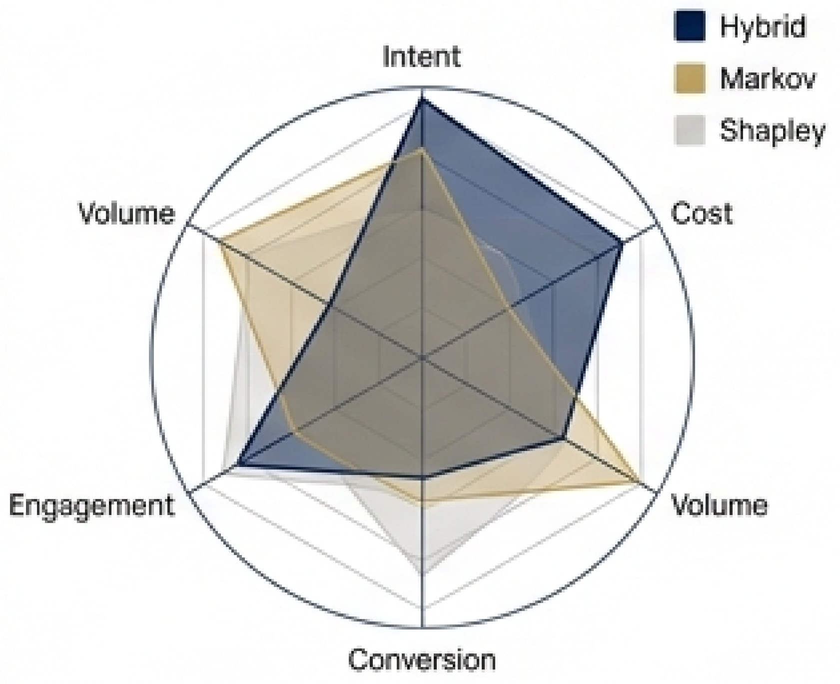


Model Configuration Slider: "Calibrate the Instrument: Adjust the α-parameter to explore the tradeoff between causality and fairness in real-time.”

Reading the Context and Flow

Answering ‘Why did it happen?’

Context Profiling Tab



Psychographic Weights

'high-intent search': 1.5x

'branded keyword': 1.2x

'returning visitor': 1.1x

Context Profiling Tab

Profile the behavioral context driving performance. The radar chart compares channels across Markov, Shapley, and Hybrid scores, while weight cards show the impact of psychographic priors like 'high-intent search'.

Answering ‘How did it happen?’

Flow Analysis Tab

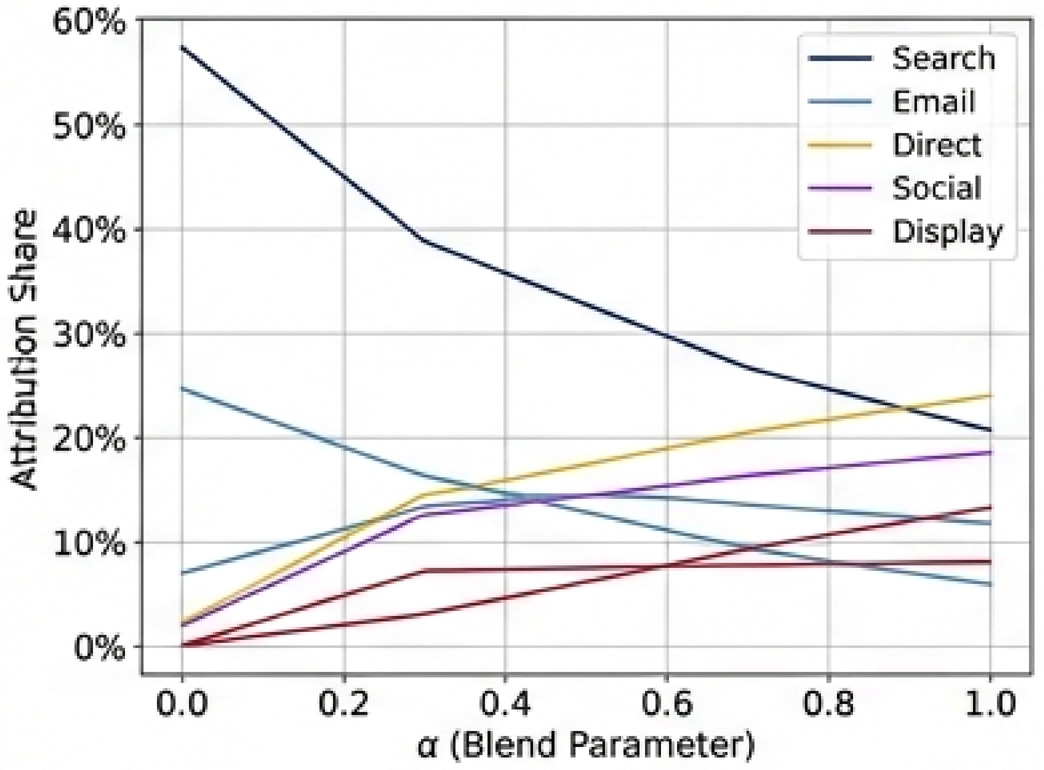
	Search	Email	Direct	Social	Display
Search	0.66	0.57	0.50	0.01	0.02
Email	0.55	0.80	0.29	0.02	0.02
Direct	0.01	0.03	0.98	0.33	0.01
Social	0.02	0.03	0.35	0.55	0.07
Display	0.02	0.02	0.05	0.07	1.00

Flow Analysis Tab

Visualize the structure of user journeys. The heatmap reveals the strongest and weakest transition probabilities between channels, identifying common paths and drop-off points.

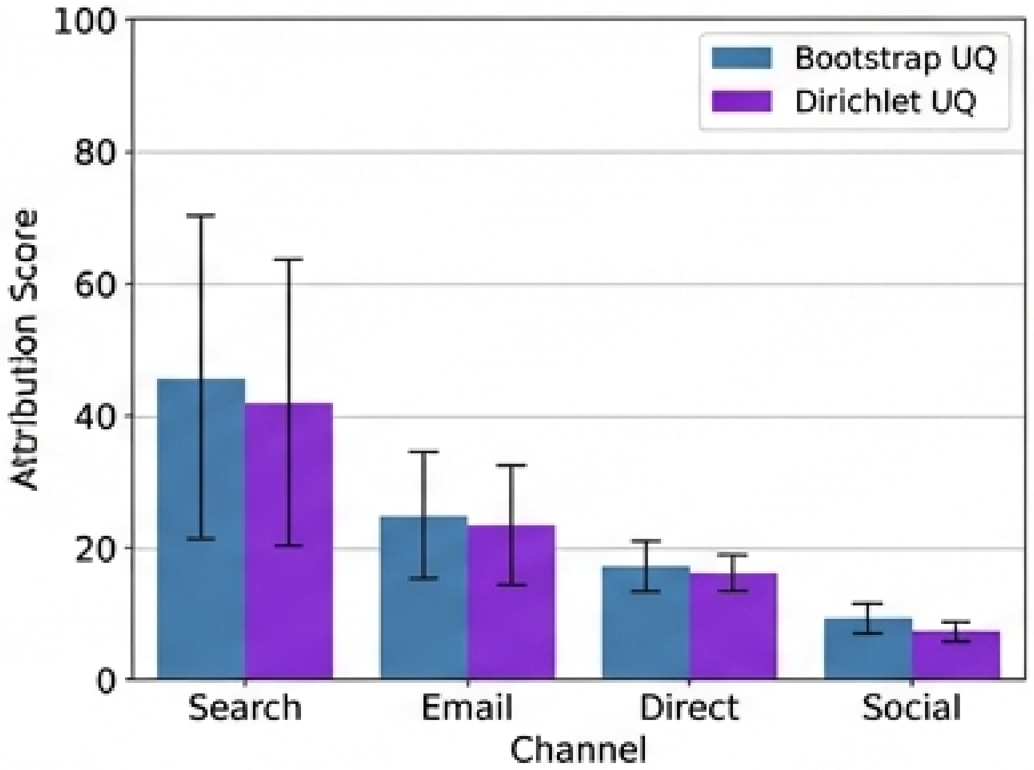
Reading the Robustness: How Sure Are We?

α-Sweep Sensitivity



Shows how channel attribution changes as the model shifts from pure Shapley (α=0) to pure Markov (α=1). Flat lines indicate robust attribution.

UQ Confidence Intervals



Compares 90% confidence intervals from Bootstrap (path uncertainty) and Dirichlet (parameter uncertainty). Wide bars signal high uncertainty and a need for more data.

Rank Stability

	Rank #1	Top 2	Top 3
Search	85%	98%	100%
Email	10%	80%	95%
Direct	3%	15%	60%
Social	1%	5%	30%
Display	1%	2%	15%

Quantifies decision-making confidence. Shows the percentage of UQ samples where a channel ranked #1, in the top 2, etc. A `top1` score > 80% indicates a dominant a dominant, reliable winner.

The Invariants of a Correct System

Mathematical and computational guarantees enforced at runtime.

Mathematical Invariants

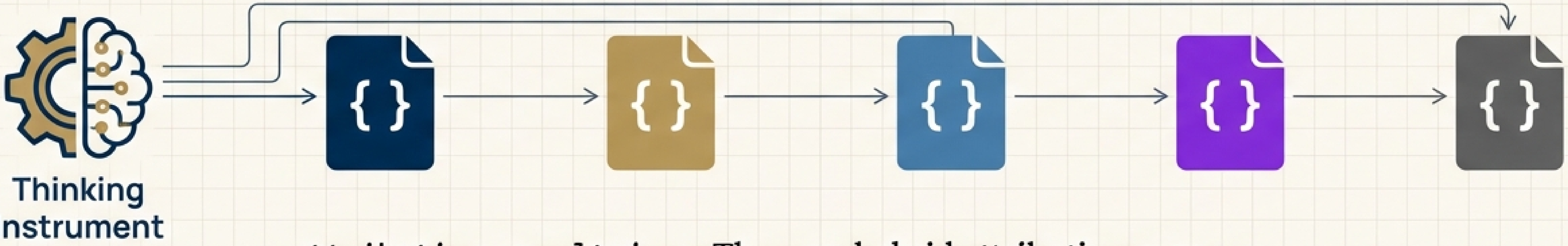
- **Shares Sum to 1:** All attribution shares must sum to 1.0 (Tolerance: ≤ 1e-6).
- **Values Sum to Total:** All monetary attributions must sum to the total conversion value (Tolerance: ≤ $1.00).
- **Row-Stochastic Matrix:** All rows in the transition matrix `T` must sum to 1.0.
- **Quantile Ordering:** Confidence interval quantiles must be correctly ordered (p05 ≤ p25 ≤ p50 ...).
- **Non-Negative Probabilities:** All transition probabilities must be ≥ 0.

Computational Guardrails

- **Shapley Complexity:** Exact enumeration is enforced for n ≤ 12 channels to guarantee tractable runtimes. An error is thrown for n > 12.
- **Reproducibility:** All random processes (Bootstrap, Dirichlet) are seeded, ensuring that the same input always produces the same output.
- **Privacy-First:** No PII is ever stored in the intermediate representation (IR). Path resolution is based on non-reversible fingerprints.
- **Schema Validation:** All exported artifacts are validated against a strict JSON schema.

The Permanent Record: Frozen & Auditable Artifacts

Analysis is reproducible only when its outputs are canonical. The platform exports five schema-validated Intermediate Representation (IR) artifacts, each with version stamping and generation timestamps.



`attribution_result.json`: The core hybrid attribution scores.
`sensitivity_alpha.json`: The full data from the α-sweep analysis.
`sensitivity_lambda.json`: The full data from the psychographic prior sweep.
`uq_bootstrap.json:`: The full distribution of results from path resampling UQ.
`uq_transition_dirichlet.json`: The full distribution of results from transition parameter UQ.

Timestamped filenames, **`version: '1.0.0'`** stamping in every file, and full compliance with the published **`ir-schema.json`**.

An Instrument for Explanation, Not Prediction

The platform's rigor comes from its deliberate focus.
By design, we are explicitly out of scope for the following:

- ❌ **Predictive Modeling:** Does not answer "what channel will convert next?"
- ❌ **Behavioral Clustering:** Does not group users into "lookalike" audiences.
- ❌ **Automated Recommendations:** Does not prescribe budget allocation.
- ❌ **Real-time Streaming Attribution:** Designed for batch analysis of historical data.
- ❌ **External Data Enrichment:** Does not use third-party APIs to augment user data.

Its purpose is to provide a defensible explanation of what has happened, empowering users to make better decisions.

The Guarantees of First-Principles Attribution

Axiomatically Fair: Credit allocation satisfies the Shapley axioms of Efficiency, Symmetry, and Dummy Player.

Sequence-Aware: Path order is mathematically encoded in the transition matrix.

Transparent: A full audit trail exists from raw events to the version-stamped, schema-validated final artifacts.

Counterfactual: The model is built to answer “what if?” via removal effects.

Defensible: All outputs are accompanied by a comprehensive robustness stack, including dual uncertainty quantification.

A complete, correct, and defensible attribution system.