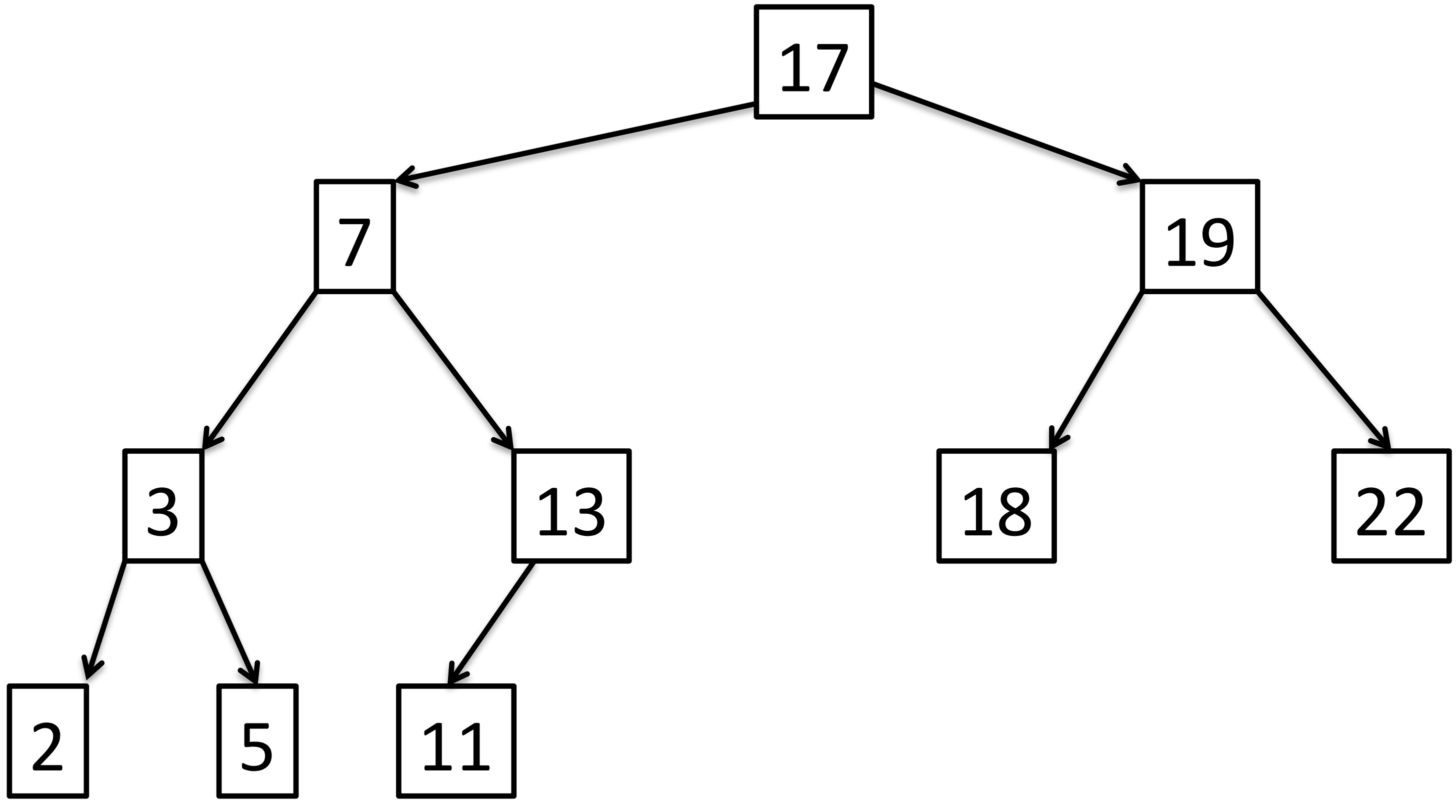
Algorithm and Data Structure Analysis (ADSA)

Lecture 7: Binary Search Trees

Overview

- Trees
- Sorted Sequences
- Binary Search Trees

Tree



Representing Trees

- Heaps are special trees (can be stored efficiently in an array)
- Want to have a pointer based representation of trees
- Each node stores
  - an element
  - has a pointer to the left subtree (might be empty)
  - has a point to the right subtree (might be empty)

TreeItem

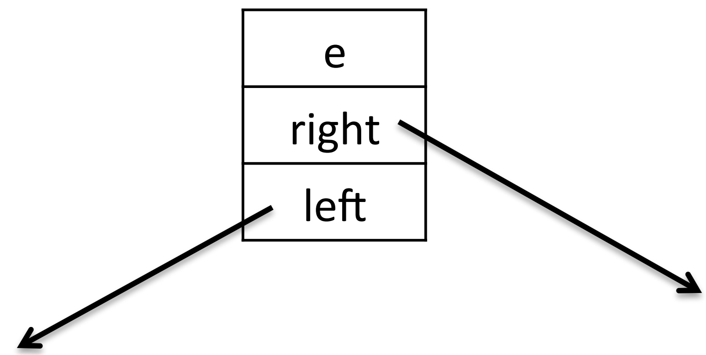
Class Handle = **Pointer to** TreeItem

Class TreeItem **of** Element

e: Element

right: Handle

left: Handle



Tree Traversal

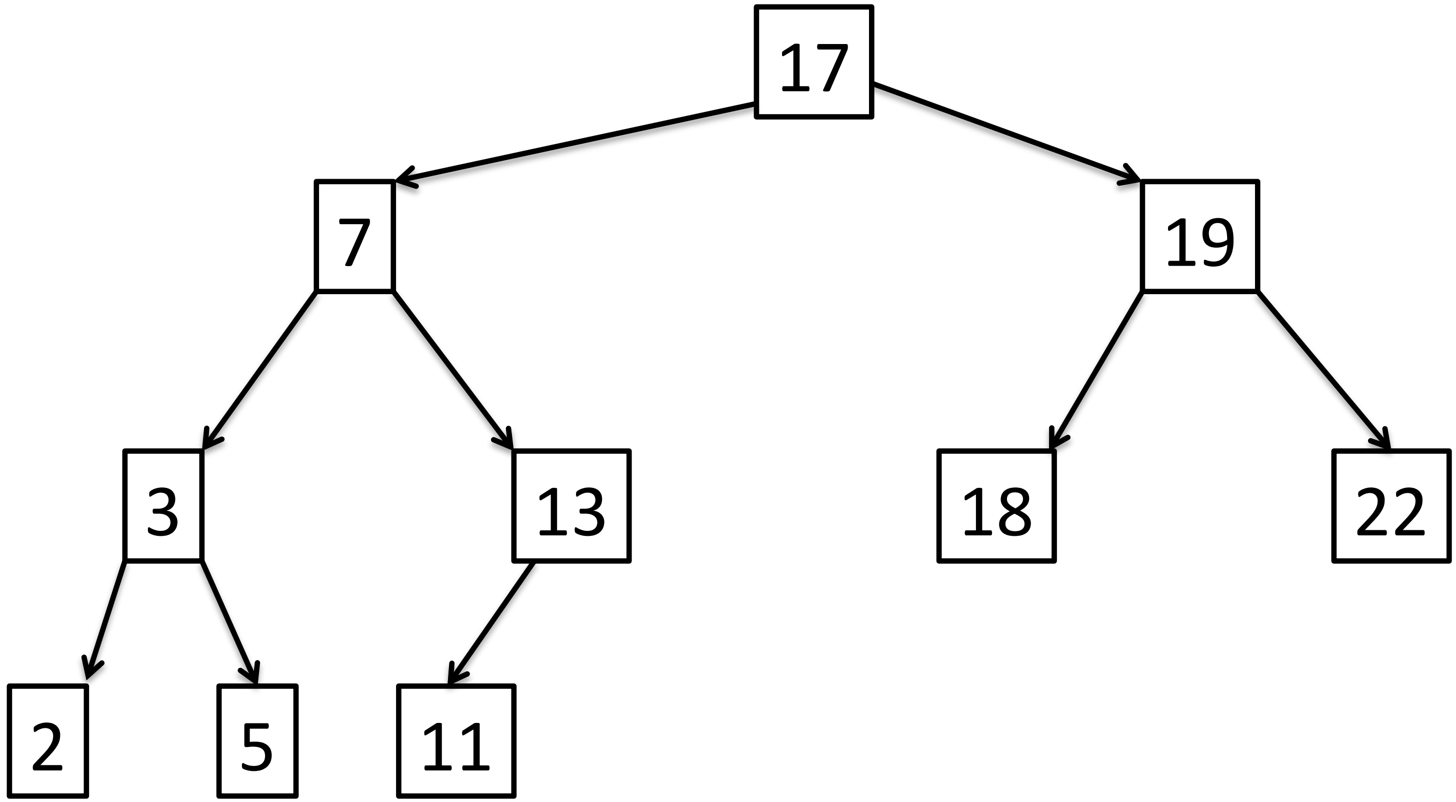
- Want to visit every node in the tree (and print out the elements).
- Recursive formulation for tree traversal

Preorder Traversal

Preorder(Tree T)

1. Visit the root (and print out the element)
2. If (T->left !=null) Preorder(T->left)
3. If (T->right !=null) Preorder(T->right)

Preorder Traversal



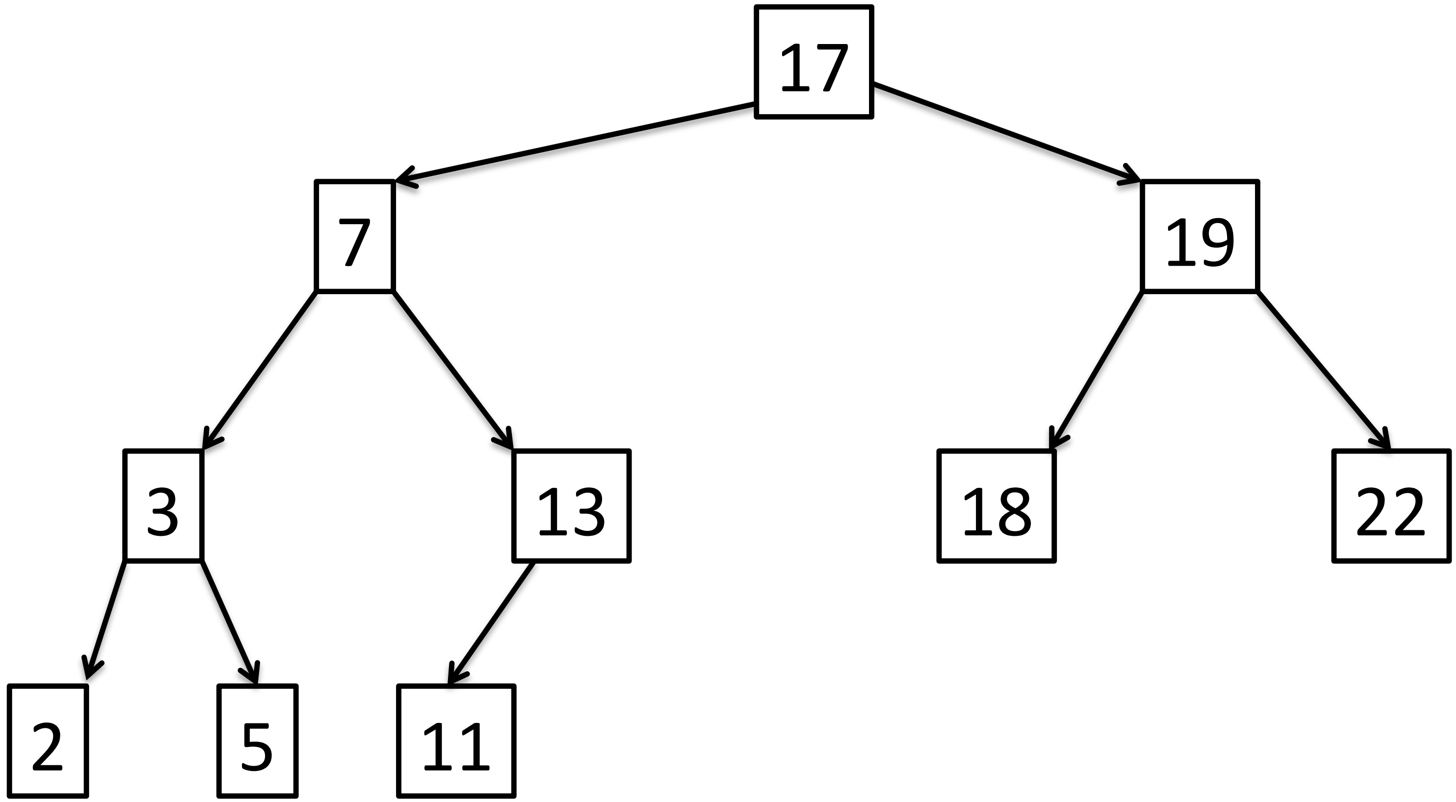
Order nodes are visited: 17, 7, 3, 2, 5, 13, 11, 19, 18, 22

Postorder Traversal

Postorder(Tree T)

1. If (T->left !=null) Postorder(T->left)
2. If (T->right !=null) Postorder(T->right)
3. Visit the root (and print out the element)

Postorder Traversal



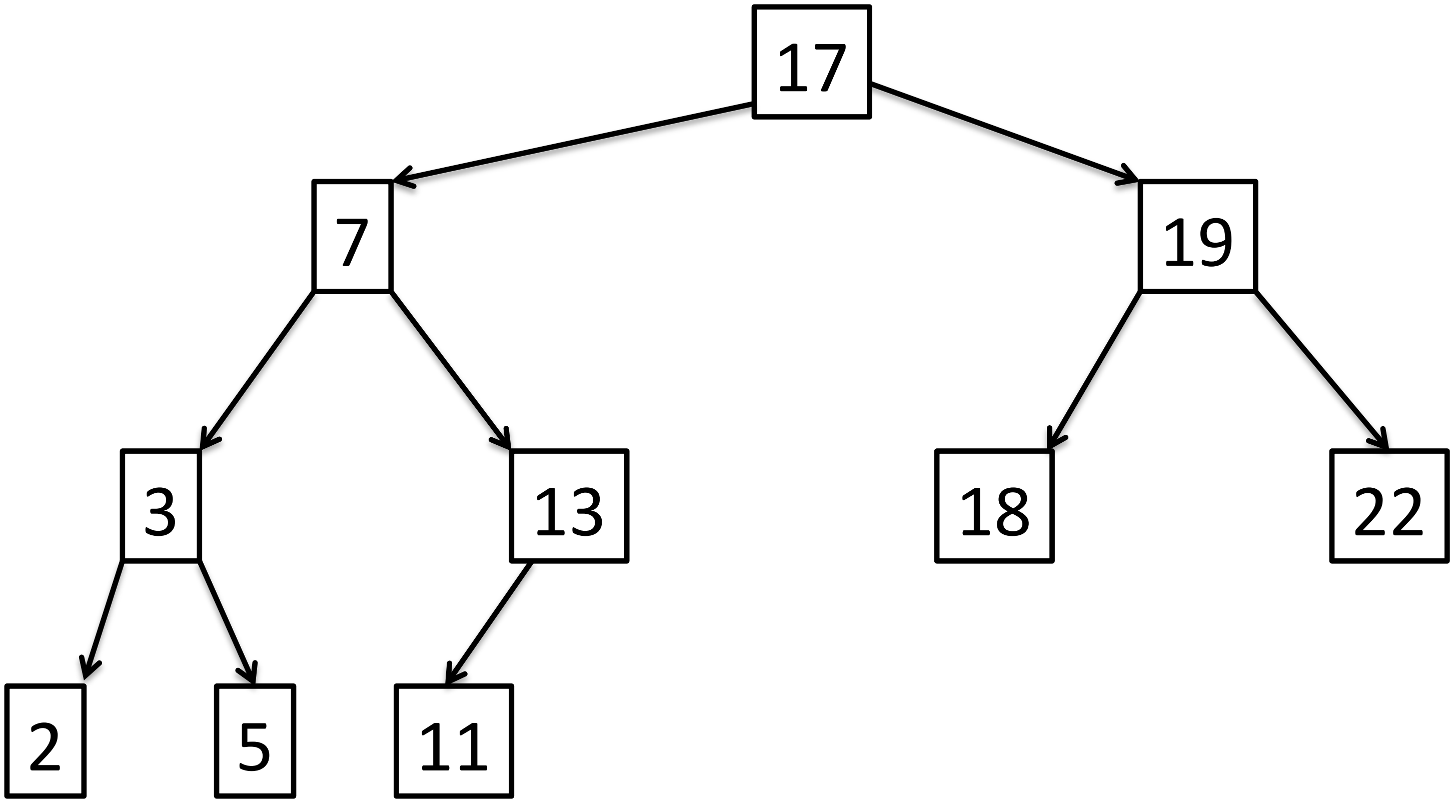
Order nodes are visited: 2, 5, 3, 11, 13, 7, 18, 22, 19, 17

Inorder Traversal

Inorder(Tree T)

1. If (T->left !=null) Inorder(T->left)
2. Visit the root (and print out the element)
3. If (T->right !=null) Inorder(T->right)

Inorder Traversal



Order nodes are visited: 2, 3, 5, 7, 11, 13, 17, 18, 19, 22

Observation: This sequence is sorted

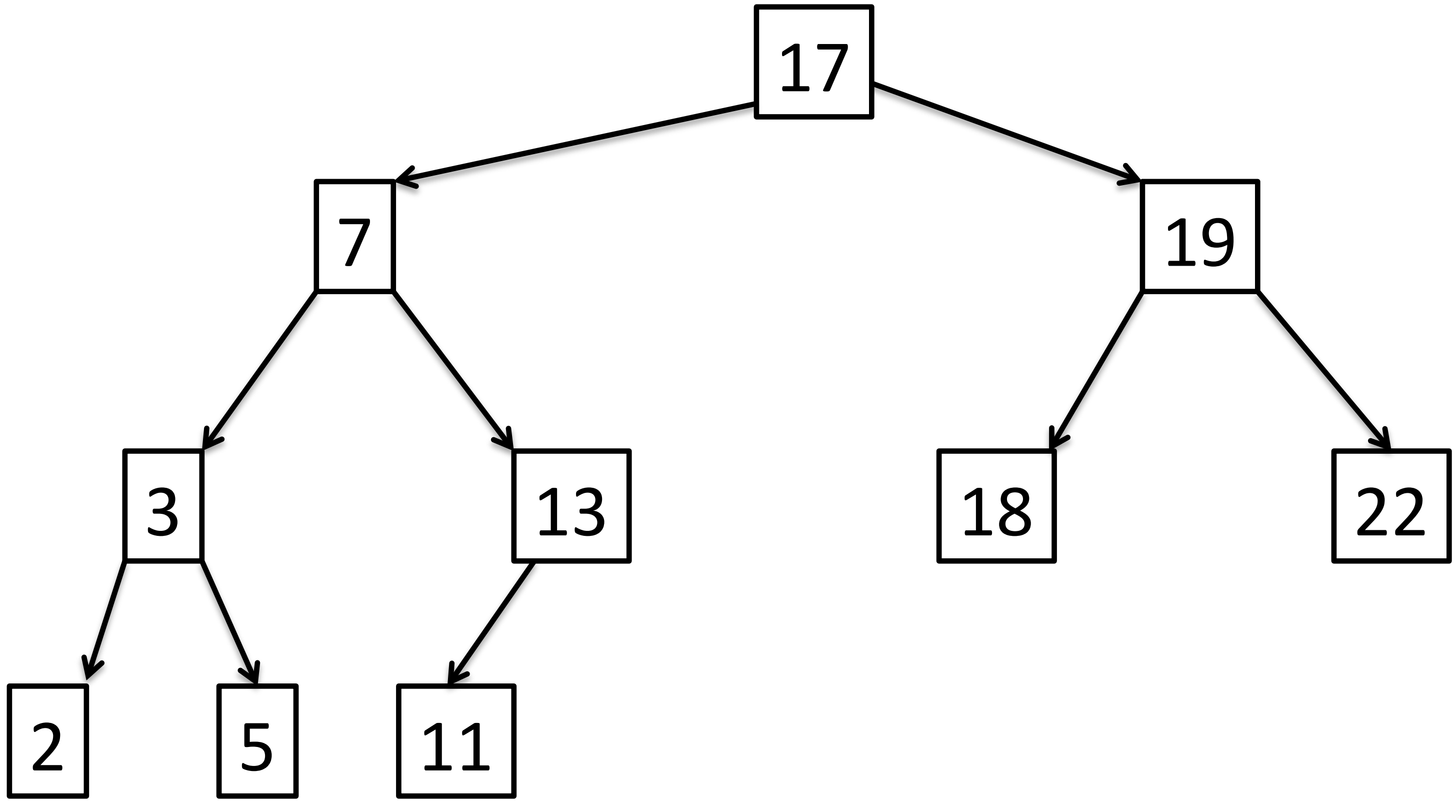
Sorted Sequences

Operations for Sorted Sequences

- Find an element e in the sorted sequence
- Insert an element e into the sorted sequence
- Delete an element e from the sorted sequence.

Want to have all these operations implemented in time O(log n).

Binary Search Tree



Sorted sequence by Inorder Traversal: 2, 3, 5, 7, 11, 13, 17, 18, 19, 22

Properties of Binary Search Trees

- All elements in the left subtree of a node k have value smaller than k.
- All elements in the right subtree of a node k have value larger than k.

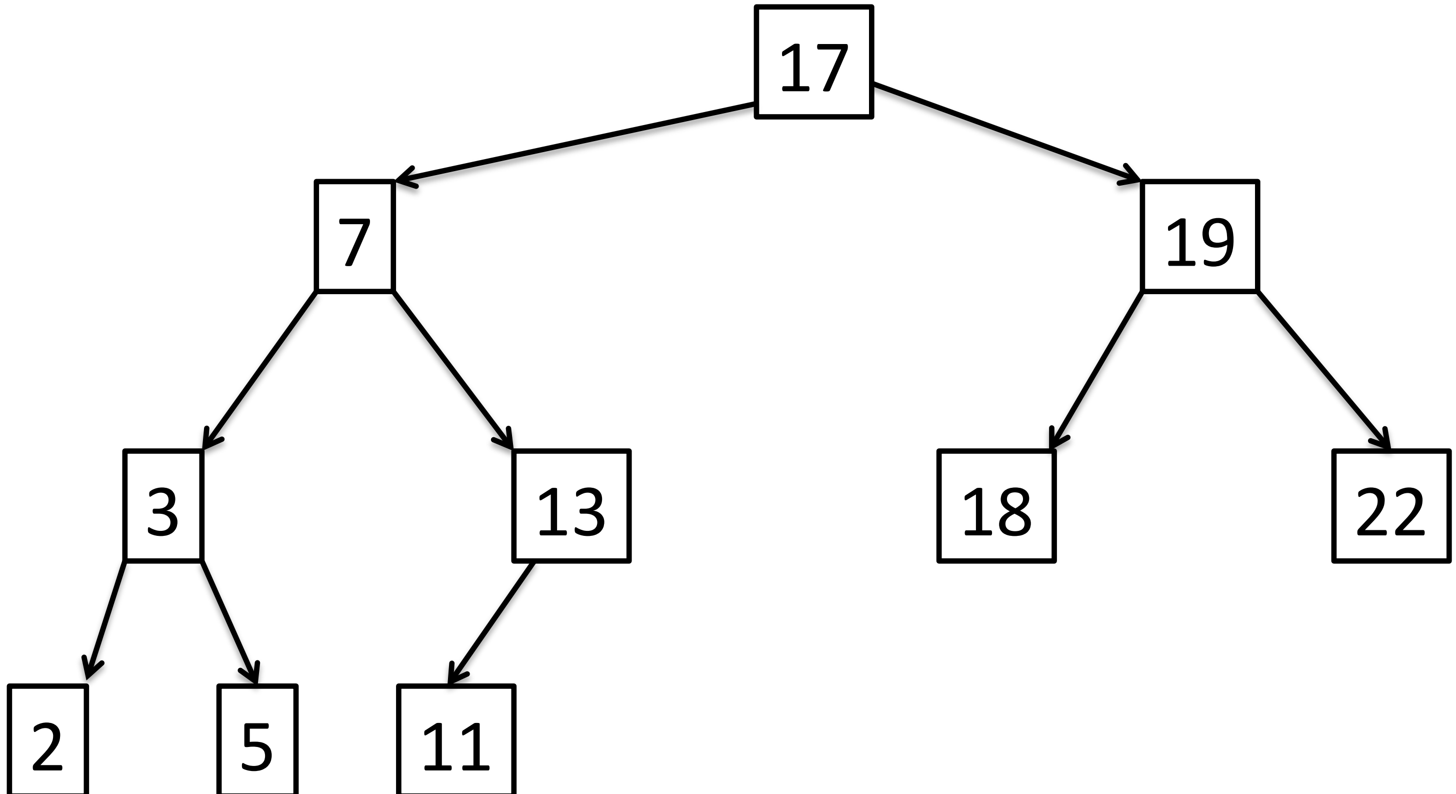
Method find

find(k)

- Start at the root.
- At a node x, compare x and k.

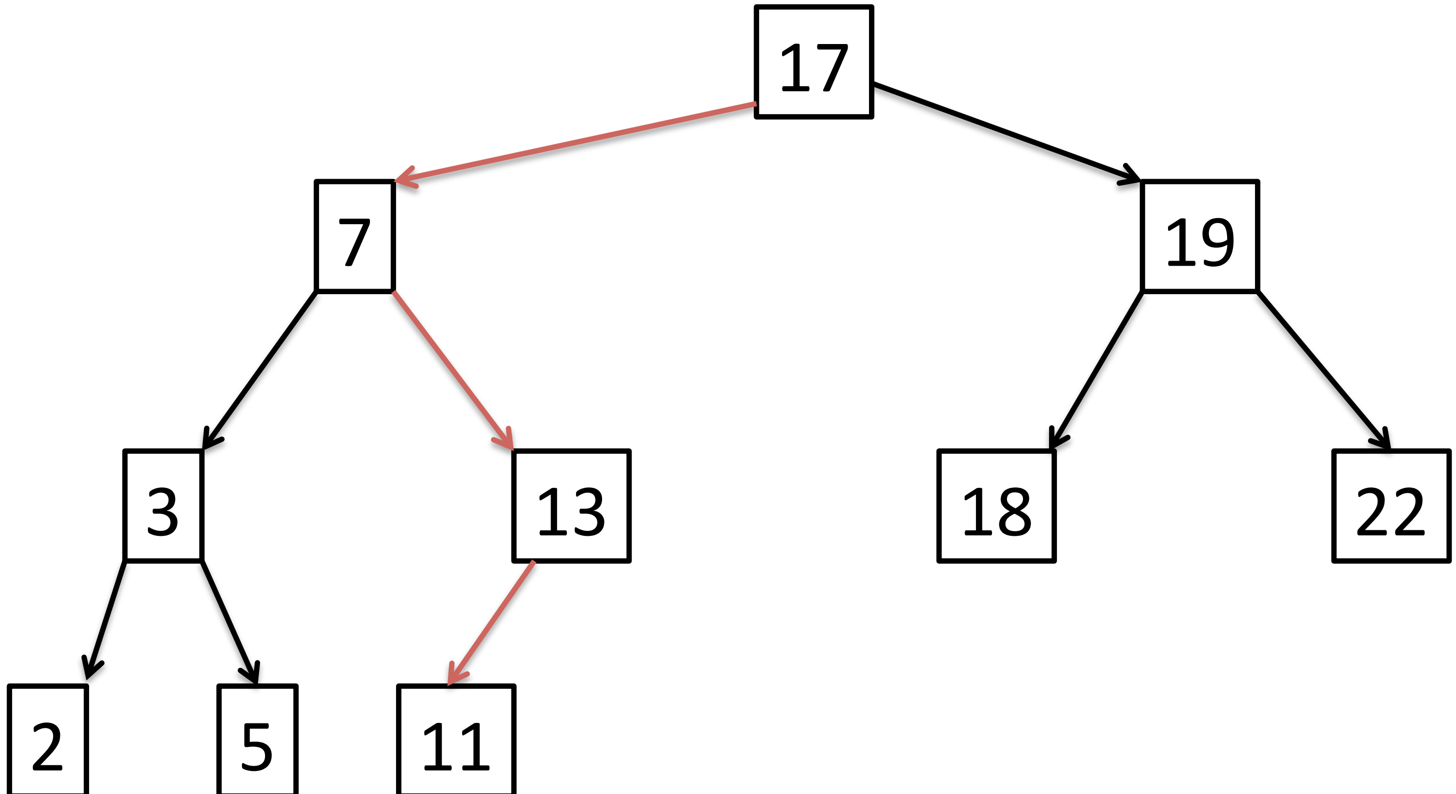
1. If k=x, then found
2. If k < x, search in the left subtree of x. If subtree does not exist return not found.
3. If k > x, search in the right subtree of x. If subtree does not exist, return not found

Find



Find 11

Find



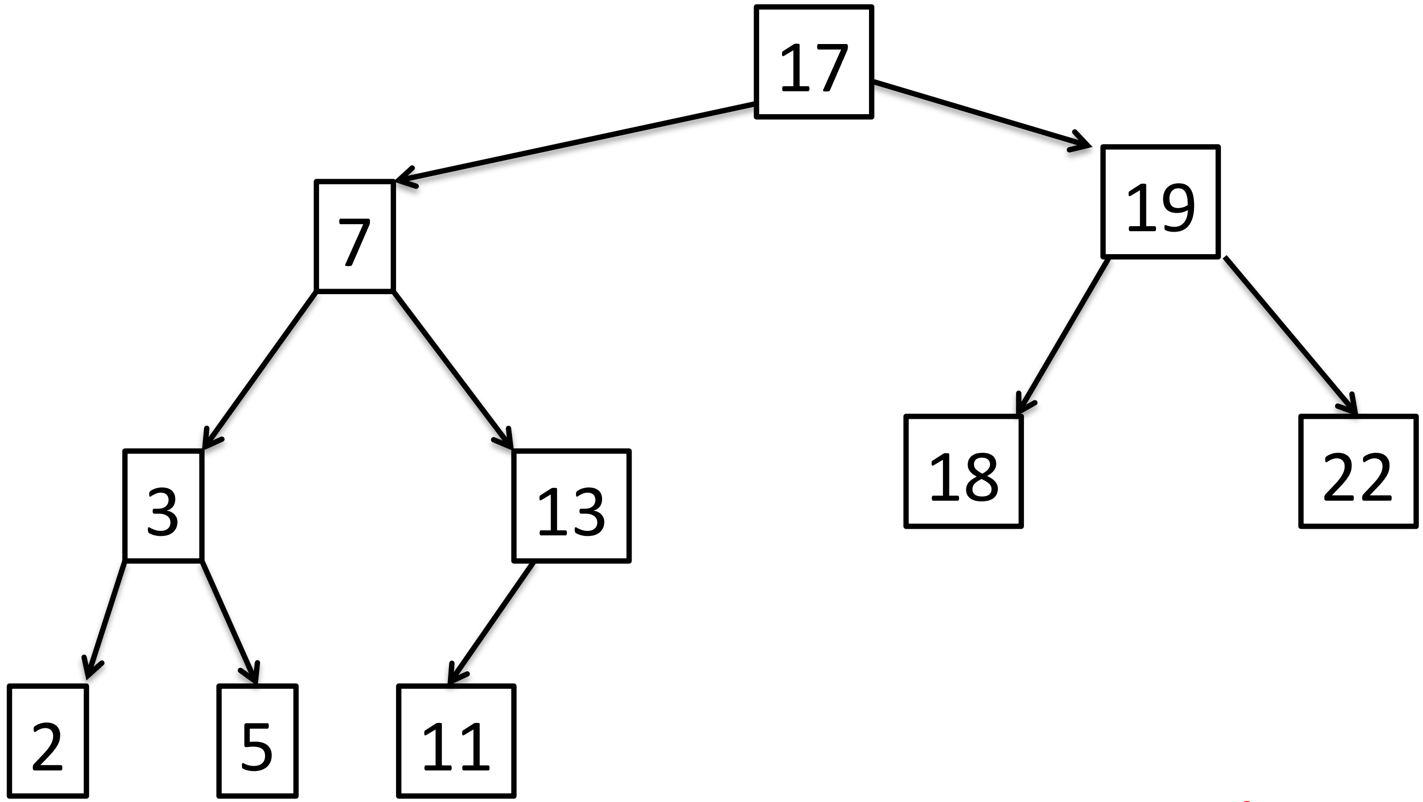
Find 11

Insertion

insert(k)

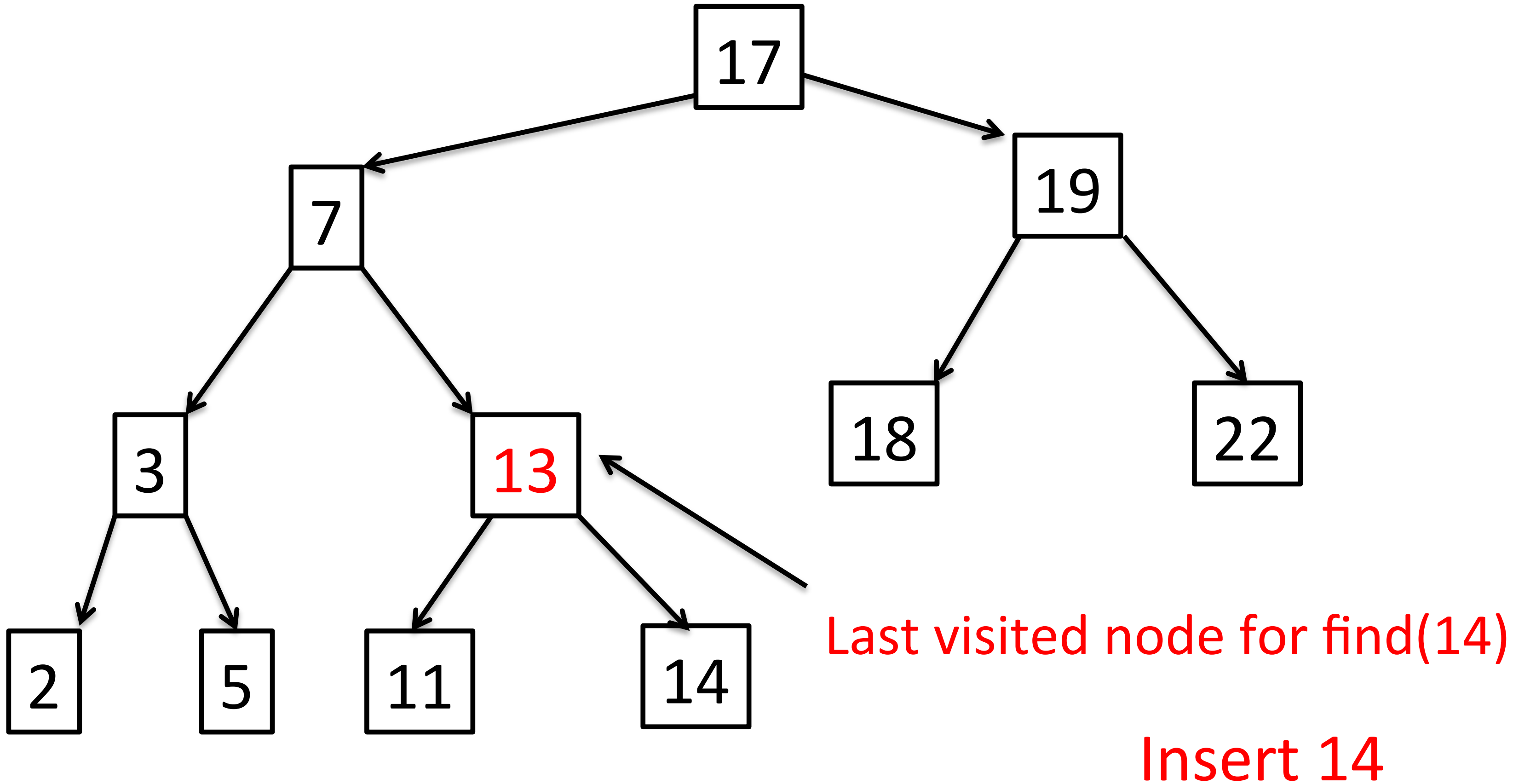
1. find(k)
2. If not found, element with key k becomes child of the last visited node of find(k).

Insert



Insert 14

Insert


17
7
19
3
13
18
22
2
5
11
14
Last visited node for find(14)
Insert 14

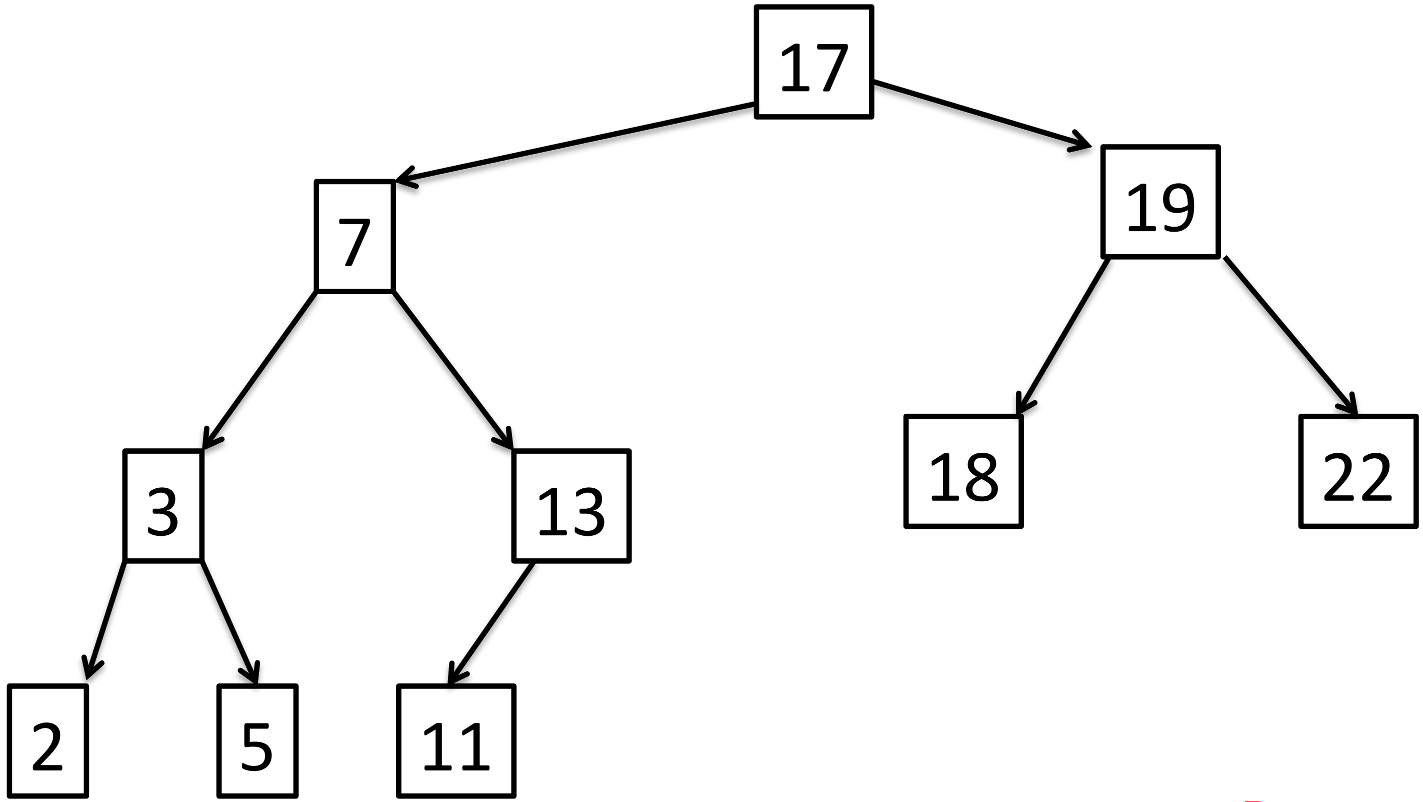
Remove

remove(k)

1. find(k)

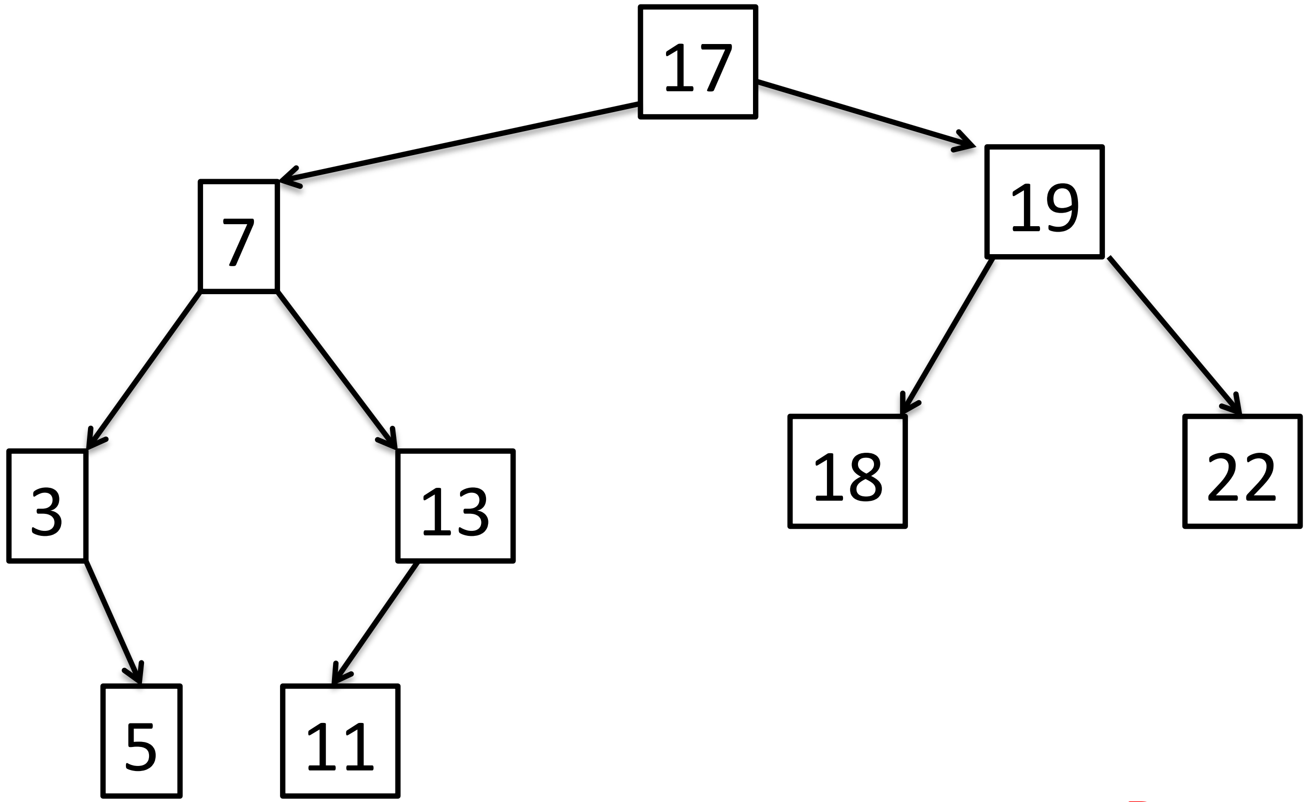
- If k is stored at a leaf, delete this leaf and the incoming edge.
- If k has one child x, redirect pointer pointing to k to x and delete k.
- If k has two children
  - search in the tree for the largest element x smaller than k.
  - swap x and k and delete(k)

Remove leaf



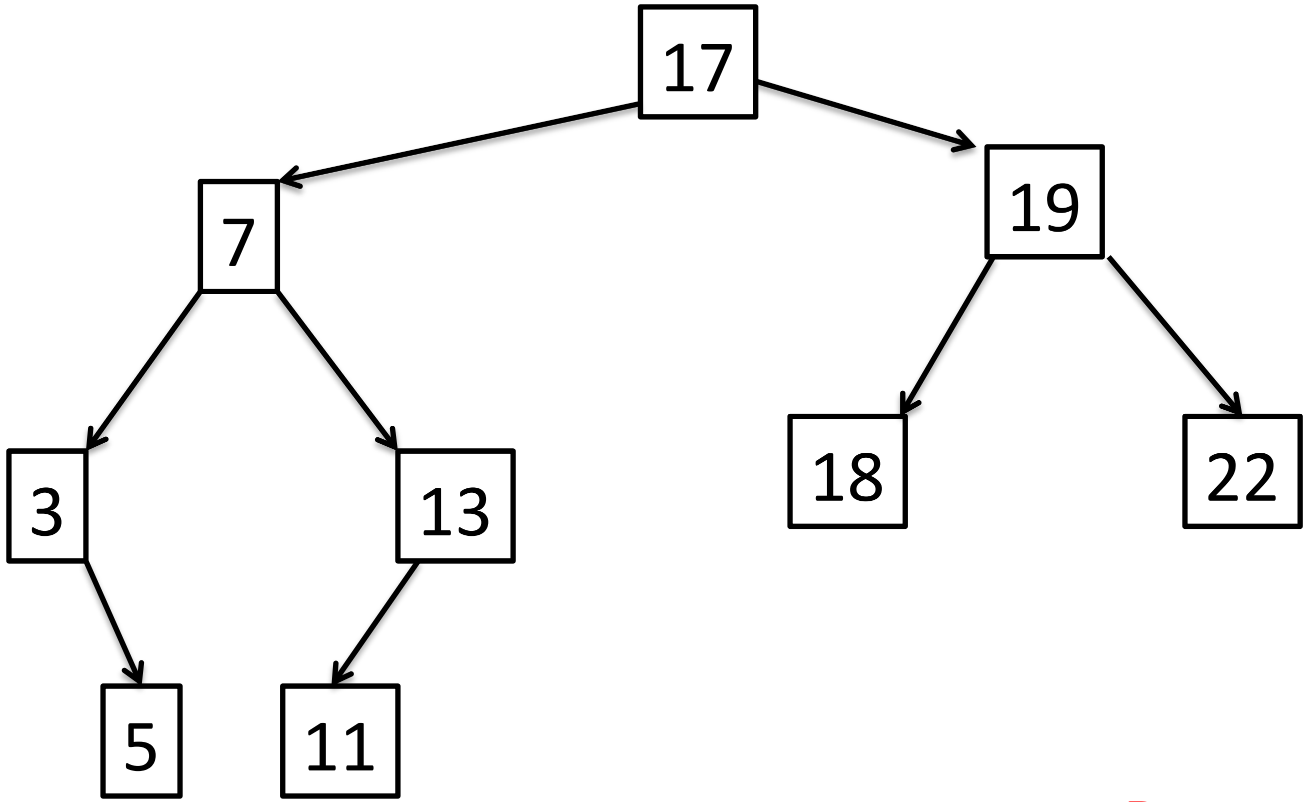
Remove 2

Remove leaf



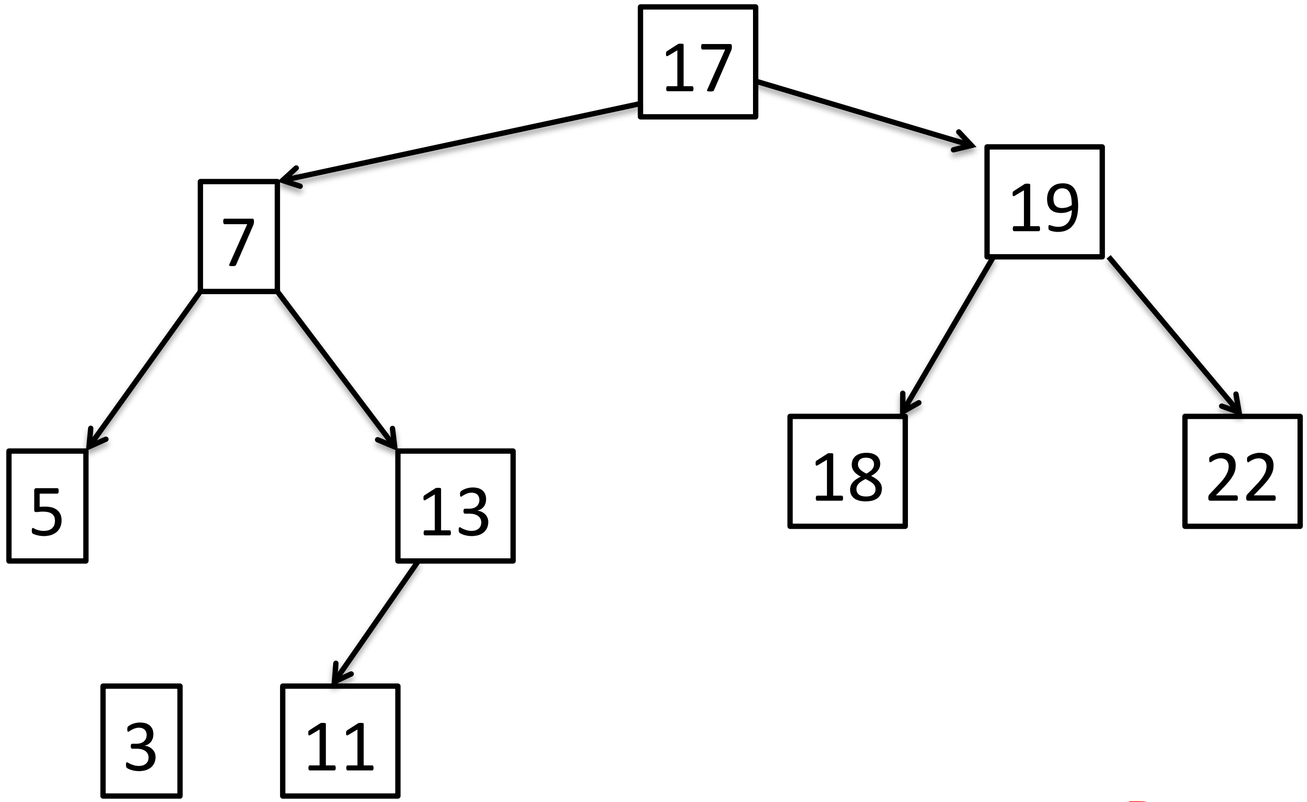
Remove 2

Remove node with 1 child



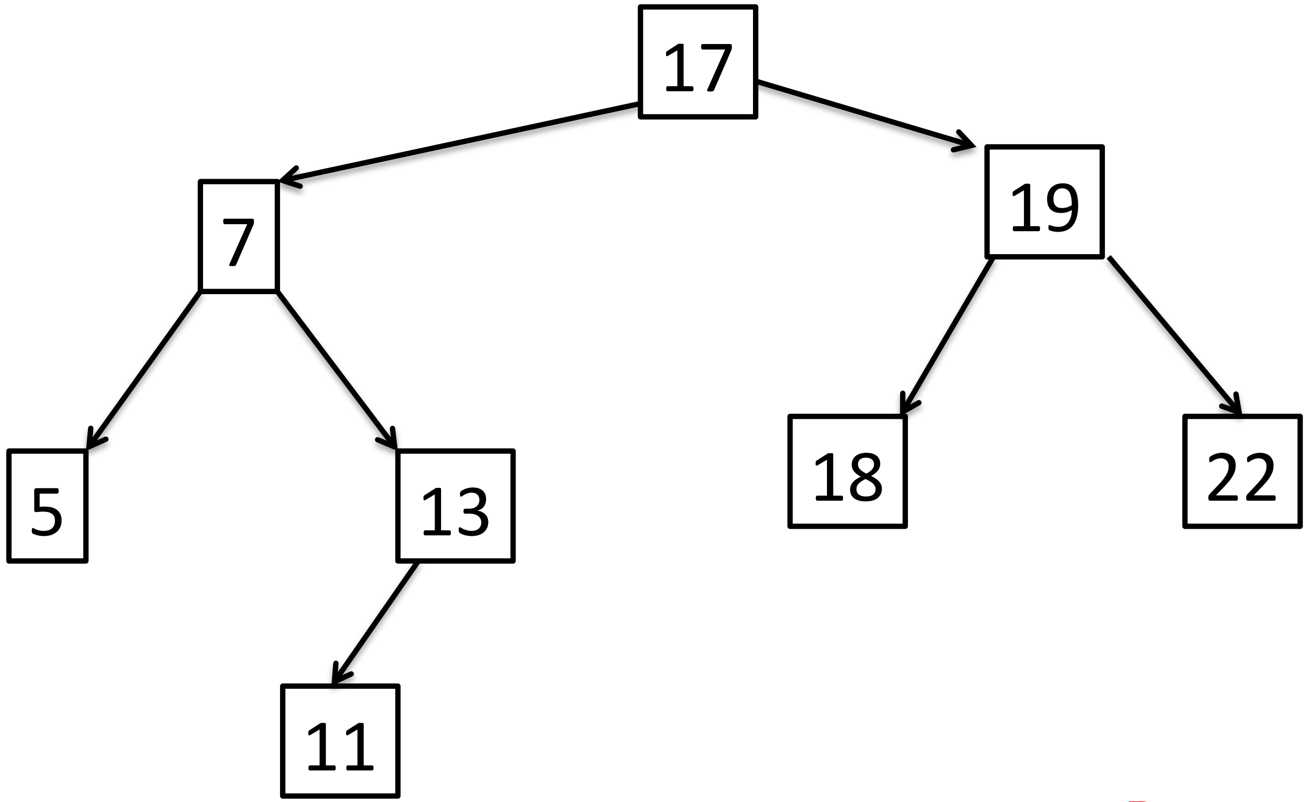
Remove 3

Remove node with 1 child



Remove 3

Remove node with 1 child



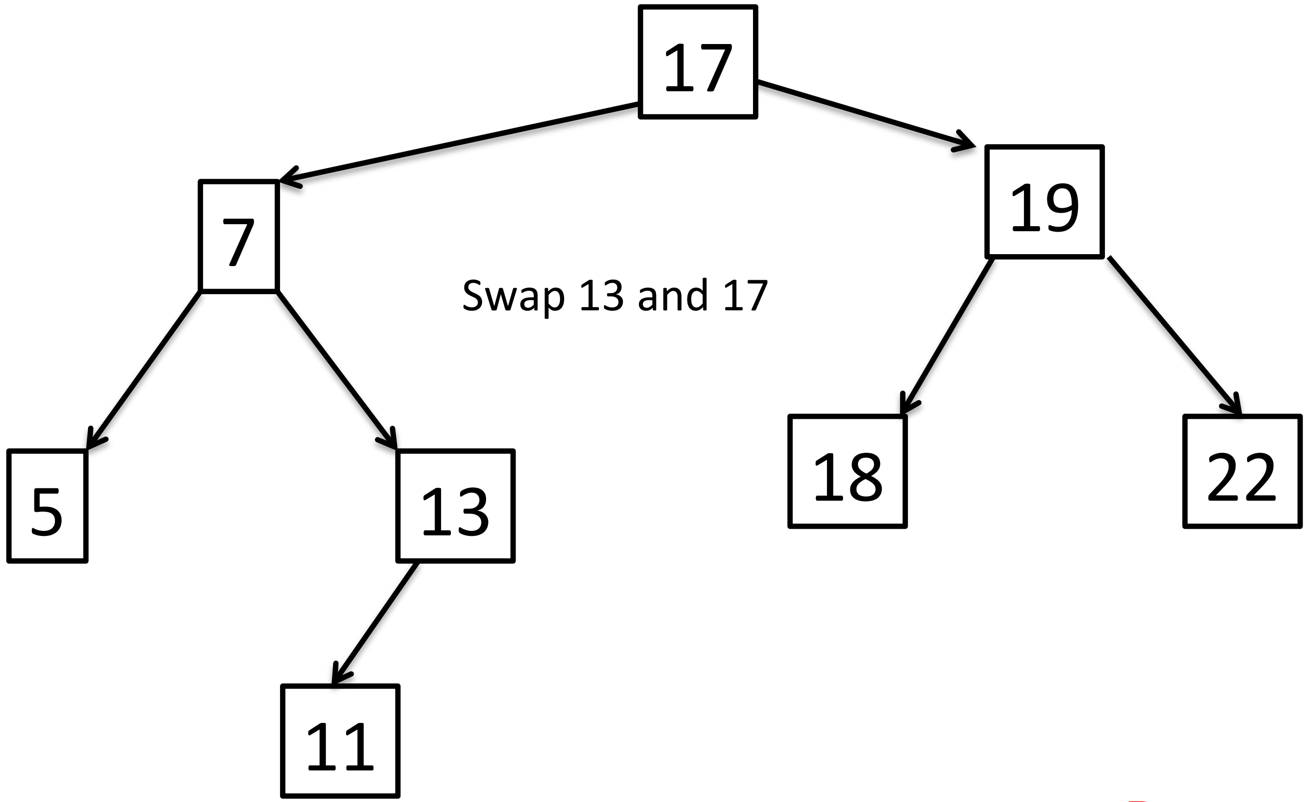
Remove 3

Remove node with 2 children

How to find the largest element smaller than k?

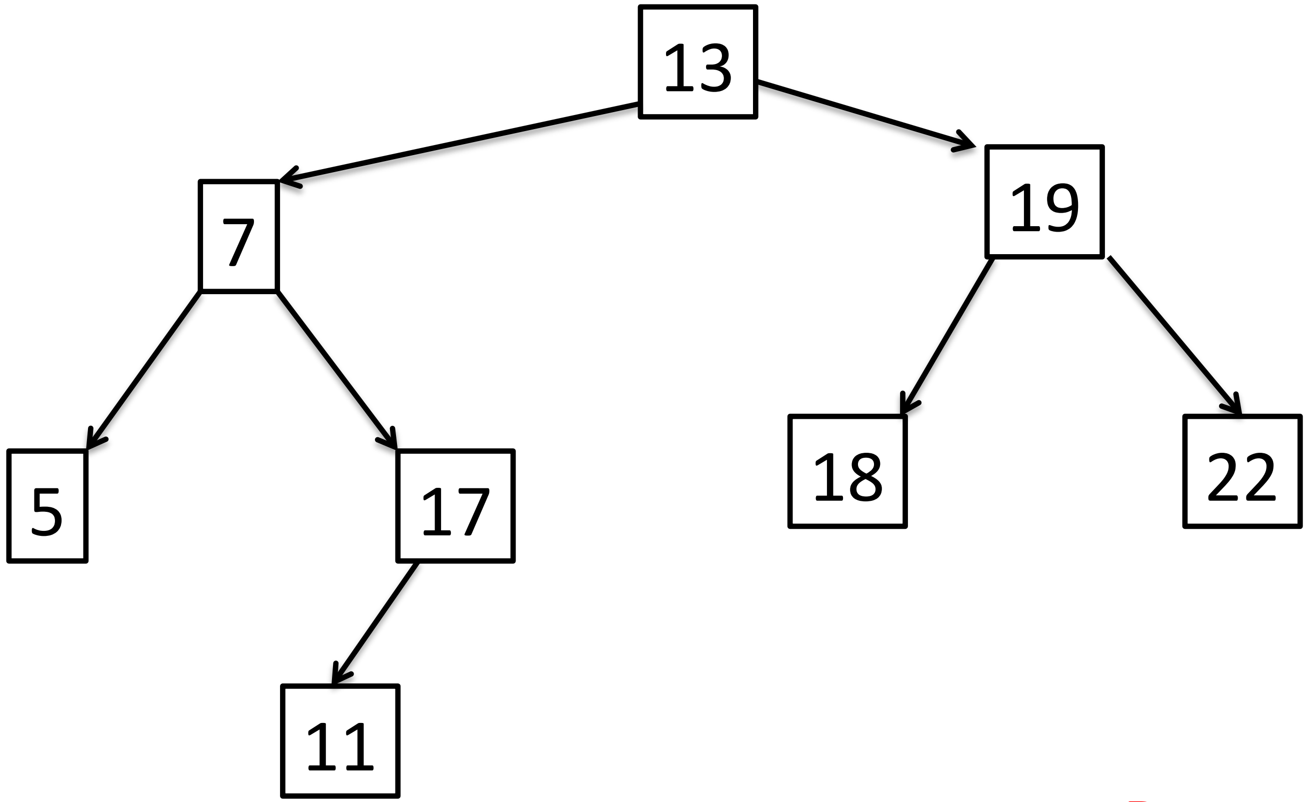
- Go to the left child of k (has to exist as k has two children.
- Follow the pointer to the right as long as possible.

Remove node with 2 children



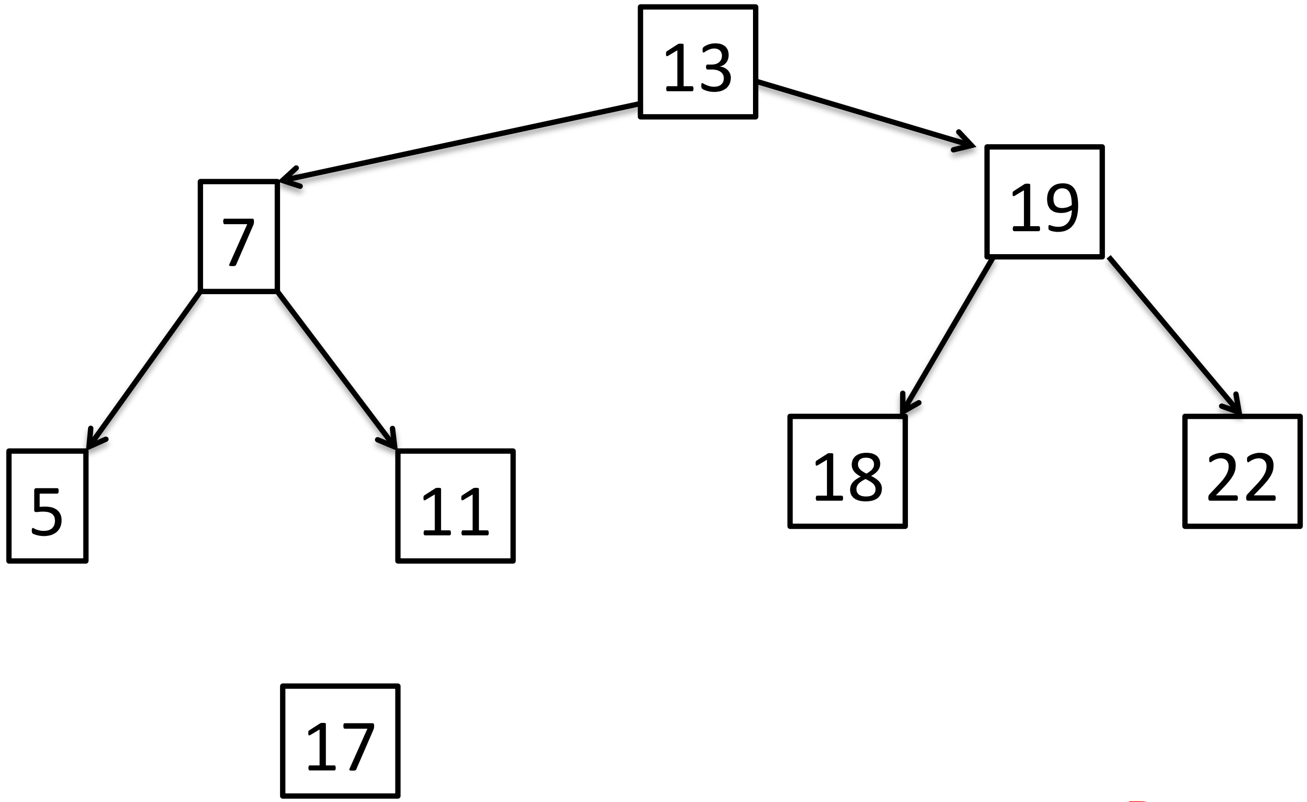
Remove 17

Remove node with 2 children



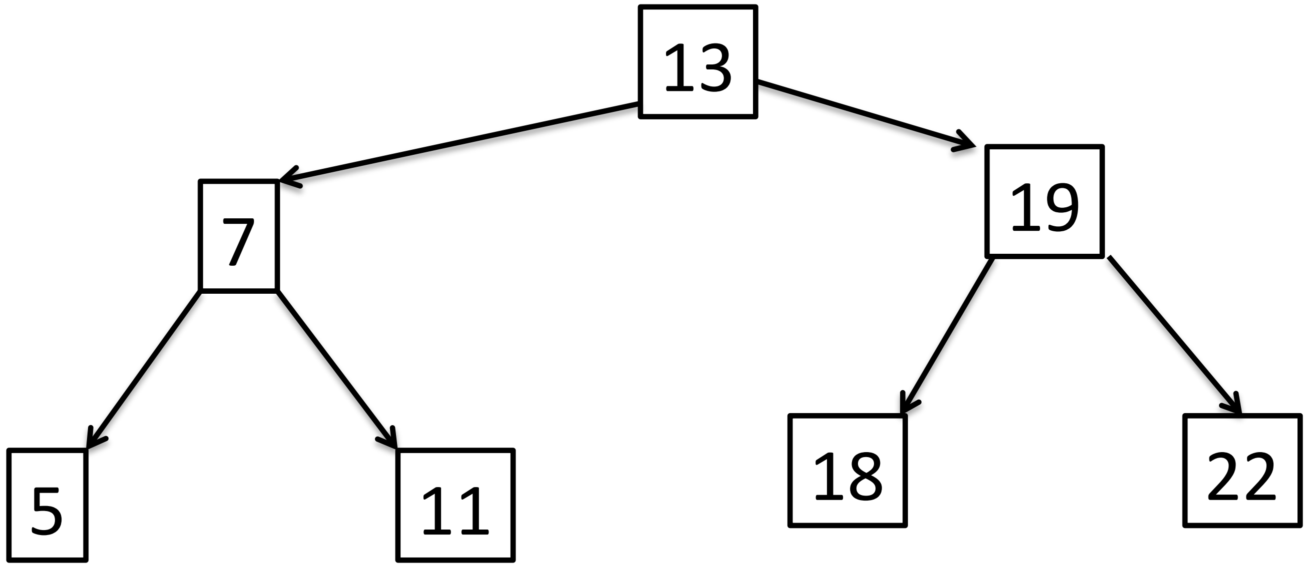
Remove 17

Remove node with 2 children



Remove 17

Remove node with 2 children



Remove 17

Perfectly Balanced Binary Search Trees

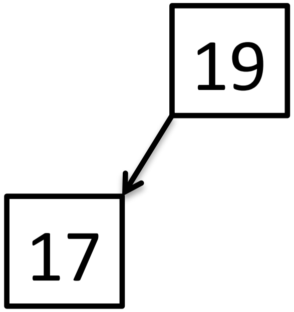
- A binary search tree is perfectly balanced if it has height $\lfloor \log n \rfloor$ (height is the length of the longest path from the root to a leaf)

Insertion

19

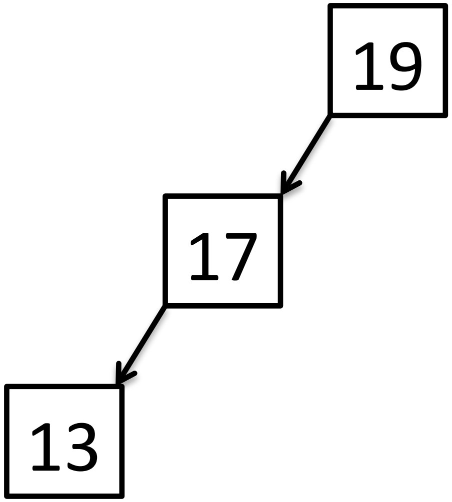
Insert 17

Insertion



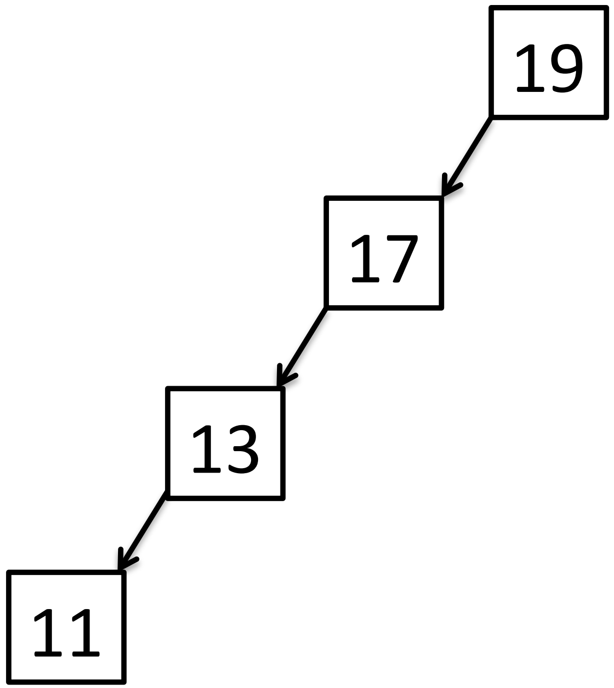
Insert 13

Insertion



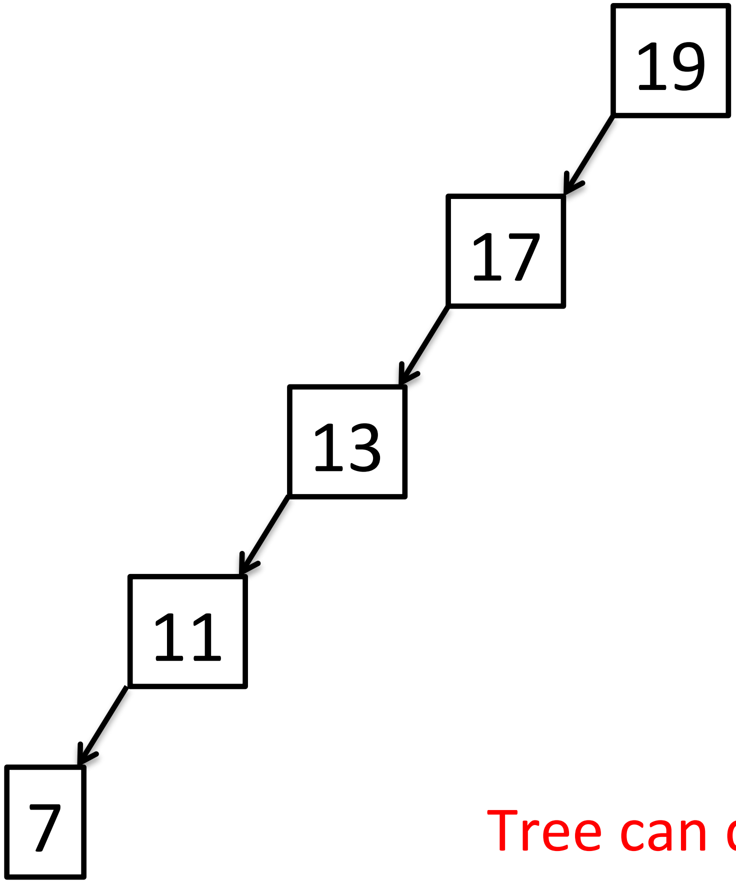
Insert 11

Insertion



Insert 7

Insertion



Problem?

Tree can degenerate to a list.

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