THE UNIVERSITY
OF ADELAIDE
AUSTRALIA

Semester 1 2017

Artificial Intelligence, Assignment 2
COMPSCI 3007, 7059

Instructions and submission guidelines:

- Answer all questions in a report.
- Make sure that your writing is legible and clear, and the mathematical symbols are consistent.
- You must sign an assessment declaration coversheet to submit with your assignment. The assessment declaration coversheet is included in the zip file.
- Submit via myuni.

Questions	Due day	Marks
Answer all 2 questions	See myuni	100 marks
		100 Total

Probabilities

Question 1

(a) John enjoys a repeated random game of taking a ball from one of three containers. He always randomly picks a container (uniformly) first. Then he randomly picks a ball from that container. Assuming the first draw has 1 blue ball and 1 red ball, the second drawer has 1 blue ball and 2 red balls, and the third drawer has 2 red balls. Let $D \in \{1, 2, 3\}$ be the random variable that represents which container that John would pick, and let $C \in \{b, r\}$ (b for blue, r for red) be the random variable representing the colour of the ball that John would pick.

Table 1: $P(D)$ (Left) and $P(C|D)$ (Right) tables

1	2	3

	b	r
$D = 1$		
$D = 2$		
$D = 3$		

i. Please write down $P(D)$ in the same table format in Table 1.

[3 marks]

ii. Please write down $P(C|D)$ in the same table format in Table 1.

[6 marks]

iii. What is $P(C = r, D = 2)$?

[2 marks]

(b) Independence $X \perp\!\!\!\perp Y$ means $P(X, Y) = P(X)P(Y)$, and Conditional Independence $X \perp\!\!\!\perp Y|Z$ means $P(X, Y|Z) = P(X|Z)P(Y|Z)$. Prove the following properties of independence:

i. Symmetry: $X \perp\!\!\!\perp Y|Z \Rightarrow Y \perp\!\!\!\perp X|Z$,

[10 marks]

ii. Decomposition: $X \perp\!\!\!\perp Y, W|Z \Rightarrow X \perp\!\!\!\perp Y|Z$ and $X \perp\!\!\!\perp W|Z$.

[10 marks]

iii. Weak union: $X \perp\!\!\!\perp Y, W|Z \Rightarrow X \perp\!\!\!\perp Y|Z, W$.

[10 marks]

[Total for Question 1: 41 marks]

Inference

Question 2

(a) A Bayesian network with 3 boolean variables A, B, C has a graph in Figure 1 with the local (conditional) distributions provided in the Table 2.

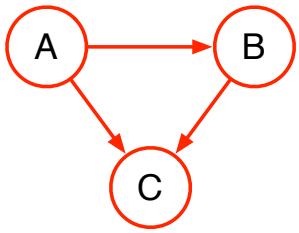

Figure 1: Bayesian Network

Table 2: Local (conditional) distributions $P(A), P(B|A), P(C|A,B)$

a	$\neg a$
0.4	0.6

	b	$\neg b$
a	0.6	0.4
$\neg a$	0.3	0.7

	c	$\neg c$
$a,\ b$	0.8	0.2
$a, \neg b$	0.4	0.6
$\neg a,\ b$	0.3	0.7
$\neg a, \neg b$	0.1	0.9

i. What is $P(A = a, B = b, C = \neg c)$? Please write down the derivation and intermediate result instead of the final number.

[6 marks]

ii. What is $P(B = b, C = \neg c)$? Please write down the derivation and intermediate result instead of the final number.

[6 marks]

iii. What is $P(A = a|B = b, C = \neg c)$? Please write down the derivation and intermediate result instead of the final number.

[6 marks]

iv. If the edge from A to B is deleted, will the local distribution tables be changed? If not, why? If yes, which one will be changed and becomes what?

[6 marks]

v. If the edge from A to B is deleted, is A independent to B? Prove it.

[6 marks]

vi. Which one of the following two inference methods can also be used to compute $P(C)$: max-product or sum-product?

[2 marks]

(b) Given a Bayesian Network in Figure 2, please answer the following questions.

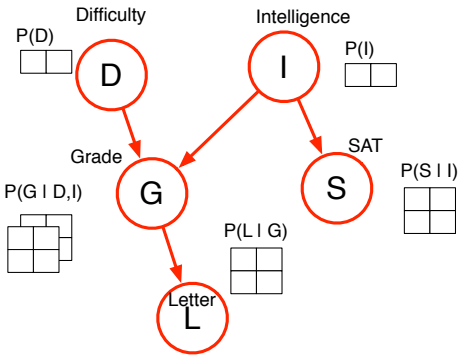

Figure 2: a Bayesian Network modelling student performance

i. Write down the factorisation of the joint distribution.

[3 marks]

ii. Write down variable elimination for marginal inference to compute $P(G)$.

[6 marks]

iii. Write down variable elimination for MAP inference.

[6 marks]

(c) You are given a Markov Random Field represented by Figure 3. Please answer the following questions.

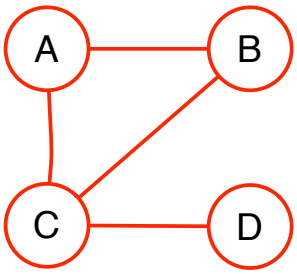

Figure 3: Markov Random Field

i. Is B independent to D?

[3 marks]

ii. Is B independent to D conditioned on C?

[3 marks]

iii. What messages are needed to compute message $m_{C \to B}(B)$ (*i.e.* the message from C to B)?

[3 marks]

iv. What messages are needed to compute message $m_{C \to D}(D)$ (*i.e.* the message from C to D)?

[3 marks]

[Total for Question 2: 59 marks]

End of Assignment