THE UNIVERSITY OF ADELAIDE
AUSTRALIA

Primary Examination, Semester 1, 2011

Data Structures and Algorithms
COMPSCI 2004, 7082

Official Reading Time:	10 mins
Writing Time:	120 mins
Total Duration:	130 mins

Questions	Time	Marks
Answer all 6 questions	120 mins	80 marks
		80 Total

Instructions

- Begin each answer on a new page
- Examination material must not be removed from the examination room
- No Calculators Allowed

Materials

- 1 Blue book
- 1 double-sided A4-page of handwritten notes
- 1 Dictionary

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO

Right or Wrong?

Question 1

(a) Indicate whether each of the following statements is true or false.

	Statement
1	$100 \cdot \sqrt{n} = o(n)$
2	$n^3 + 120n^2 = O(n^2)$
3	Karatsuba Integer Multiplication takes time $o(n^2)$.
4	The PostOrder traversal for trees can be implemented in linear time.
5	$P \subseteq NP$
6	The Vertex Cover Problem is known to be NP-hard.
7	The Eulerian cycle problem is known to be NP-hard.
8	There is a known polynomial time algorithm which solves the Traveling Salesman Problem.
9	Heapsort runs in time $O(n \log n)$.
10	The worst-case height of a Binary Search Tree is $O(\log n)$.

[10 marks]

[Total for Question 1: 10 marks]

Integer Multiplication

Question 2

(a) Give the Karatsuba algorithm for integer multiplication.

[5 marks]

(b) Derive the recursive formula for analysing its runtime. Give the best possible upper bound on the runtime.

[4 marks]

(c) Explain why the Karatsuba Multiplication is asymptotically faster than the School Multiplication.

[3 marks]

(d) Assume that you want to multiply two 10-digit numbers. In terms of runtime, would you prefer the Karatsuba Multiplication or the School Multiplication? Explain your decision.

[3 marks]

[Total for Question 2: 15 marks]

Heaps

Question 3

(a) Give an algorithm that establishes the heap property for a given array consisting of n integers in time $O(n)$. If you are using sub-methods give an algorithm for them as well.

[5 marks]

(b) Prove that your algorithm for establishing the heap property has runtime $O(n)$.

[5 marks]

(c) Give the heapsort algorithm and analyze its runtime.

[5 marks]

[Total for Question 3: 15 marks]

Binary trees

Question 4

The balance of a binary search tree depends on the insertion order of its values. Given the set of values

$$\{1, 2, 3, 4, 5, 6, 7\}$$

perform the following tasks.

(a) i. Write *two* different orders of insertion of the elements from the set above that would result in a perfectly balanced binary search tree.

[3 marks]

ii. Write *two* different orders of insertion of the elements from the set above that would result in a maximally *unbalanced* binary search tree.

[3 marks]

(b) Construct the binary tree by inserting the 7 elements in the order $5, 3, 7, 6, 2, 4, 1$. You only have to show the resulting tree, not the intermediate steps.

[2 marks]

i. Compute the cost of this binary search tree.

[3 marks]

ii. What is the average cost of a find operation for this binary search tree?

[2 marks]

[Total for Question 4: 13 marks]

AVL Trees

Question 5

(a) Draw a sequence of diagrams showing the insertion of the values:

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[8,5,4,7,9,6,10]
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into an empty AVL tree.

You must:

- Show the resulting tree immediately after each insertion step (that is *before* any balancing has taken place).
- Indicate the node(s) at which each rotation is performed.
- Where there is a double rotation, show the tree after each single rotation.
- Show the resulting tree after balancing operation(s).

[8 marks]

(b) Give the best possible upper bound on the worst-case height of an AVL-tree consisting of n elements.

[2 marks]

(c) Give a best possible lower bound on the height of a binary search tree consisting of n elements.

[2 marks]

[Total for Question 5: 12 marks]

Graphs

Question 6

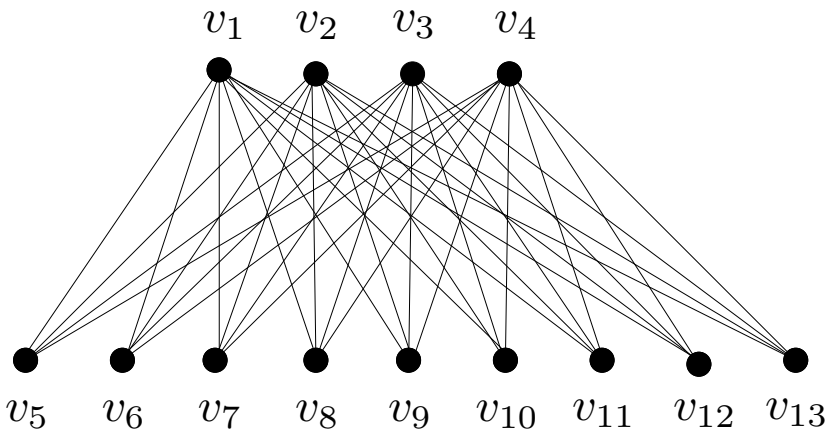
(a) Give 3 applications that can be modelled as graphs. State precisely what are the nodes and the edges of the graph.

[6 marks]

(b) A graph is connected if there exists a path between any pair of nodes. Give an algorithm that checks whether a given undirected graph is connected. Analyze the runtime of your algorithm. Your algorithm should be as efficient as possible in terms of O-notation.

[5 marks]

(c) Give a minimum vertex cover for the following graph.



Explain why your solution is a minimum vertex cover.

[4 marks]

[Total for Question 6: 15 marks]

End of exam