# Exam Preparation 1

**Exercise 1** *Big O-Notation*

Right or wrong?

- $5n^3 + 100n^2 \in O(n^3)$
- $n^{1/2} \in O(\log n)$
- $2n^2 + 3n^4 \in O(n^2)$
- $2n^2 + 3n^4 \in \Omega(n^2)$
- $2n^2 + 3n^4 \in o(n^5)$

**Exercise 2** *Integer Multiplication*

1. Describe the recursive school multiplication algorithm for multiplying two n-digit numbers. Develop the recursive formula for the number of primitive operations.
2. Describe the Karatsuba multiplication algorithm for multiplying two n-digit numbers. Develop the recursive formula for the number of primitive operations. Why is the Karatsuba multiplication asymptotically faster than the school multiplication?

**Exercise 3** *Heaps and Heapsort*

- What are the basic properties of a heap?
- Describe the operations insert and delete for heaps.
- Give an algorithm that makes an array heap-ordered in linear time. Analyze the runtime of your algorithm.
- Describe the Heapsort algorithm. Analyze its runtime.

**Exercise 4** *Binary Search Trees*

- What are the basic properties of binary search trees?
- Describe the operations find, insert, and remove for binary search trees.
- What is the height of a perfectly balanced binary search tree?
- What is the worst case height of a binary search tree?

- What is the average time for find in a binary search tree that is generated from a sequence of $n$ randomly chosen integers?

**Exercise 5** *AVL-Trees*

- State the basic property of an AVL-tree.
- What is the maximum height of an AVL-tree?
- Consider an example sequence where the insertion into the AVL-tree involves all possible rotation operations.

**Exercise 6** *Skip Lists*

- Describe the basic properties of a Skip List.
- What is the probability that the Skip Lists exceeds a certain height h after the insertion of $n$ elements?
- Show by example how find, insertion, and deleting works for Skip Lists.