Assignment 4

Due date: 10:00am, Monday, 15th of September 2014

General Instructions

You have to do this assignment as a **team of two students** if you are an undergraduate student, or **individually** if you are a postgraduate student. Individual work of an undergraduate student is also allowed and will be treated same as team work. Team members should be from the same tutorial group.

Submissions have to include coversheet including names, student ids, and your tutorial group such that submissions can get marked.

Submit your solutions for Exercises 1, 2, and 3 to the box "ADSA" on level 4, Ingkarni Wardli (close to reception) **by the deadline**. **No late submissions will be accepted.**

Exercise 1 *AVL Trees (4+2+1+1 points)*

- Starting from an empty AVL tree, draw a sequence of diagrams showing the insertion of the values:

  `[11,4,1,9,13,7,15]`

  You must:

  - Show the resulting tree immediately after each insertion step (that is *before* any balancing has taken place).
  - Indicate the node(s) at which each rotation is performed.
  - Where there is a double rotation, show the tree after each single rotation.
  - Show the resulting tree after balancing operation(s).

- Draw a sequence of diagrams showing the deletion of the values:

  `[15,7,13]`

  from the tree formed in the question above. The deletions must occur in the order given. Again, show the tree after each deletion and rotation (if any).

- Give the best possible upper bound on the worst-case height of an AVL-tree consisting of n elements.

- Give a best possible lower bound on the height of a binary search tree consisting of n elements.

Exercise 2 *Skip Lists(3+2+2 points)*

- Given the following sequence of coin tosses, where H stands for head and T stands for tail:
  T H T T H T T H T T T H H T H T T T T H T H T T H T T T H T H T.

  Build a skip list containing the following values:

  `[20,6,101,1,35,19]`

  Show the full state of the skip list after each insertion.

- Give a pseudocode description of the element removal operation for a skip-list data structure.

- Assume that you currently have n elements to construct a skip-list data structure. What is the probability that the height h of the skip-list is larger than $4 \log n$? Provide an explanation.

Exercise 3 *Hash Tables(3+2 points)*

- Given is a dynamic set S represented by a direct-address table T of length m.
  - Provide the pseudocode of a procedure to find the minimum key used by the dynamic set S. What is the best and the worst case performance of your procedure?
  - Modify your procedure to obtain the difference between the maximum and the minimum keys used by the dynamic set S. What is the best and the worst case performance of your second procedure?
  - Describe a procedure to find the maximum element of the dynamic set S. What is the best and the worst case performance of your procedure?

- Draw the 11-item hash table that results from using the hash function

  $$h(i) = (2i + 5) \, mod 11,$$

  to hash the keys

  `[12,44,13,88,23,94,11,39,20,16,5],`

  assuming collisions are handled by chaining.

End of Questions