**Claim #1:**

$T_K(n) \leq 207\ n^{\log 3}$

**Given:**

$T_K(n) \leq \{\ 3n^2 + 2n$ for $n \leq 3$

$3T_K(\text{ceil}(n/2) + 1) + 6 \times 2 \times n$ for $n \geq 4$

**Note:**

$n = 2^k + 2 \quad (k \geq 1)$

$\text{ceil}(n/2) + 1 = \text{ceil}((2^k + 2)/2) + 1$ (substitute $n = 2^k + 2$)

$= 2^{k-1} + 1 + 1$

$= 2^{k-1} + 2$

**Claim #2:**

$T_K(2^k + 2) \leq 69 \times 3^k - 24 \times 2^k - 12$

**k = 0:**

$T_K(2^0 + 2) = T_K(3) < 3 \times 3^2 + 2 \times 3$

$= \underline{33}$

$= 69 \times 3^0 - 24 \times 2^0 - 12 = \underline{33}$

**k > 0:**

$T_K(2^k + 2) \leq 3 \times T_K(\text{ceil}(n/2) + 1) + 12\ (2^k + 2)$ (sub definition)

$\leq 3 \times T_K(2^{k-1} + 2) + 12\ (2^k + 2)$

$\leq 3 \times (69 \times 3^{k-1} - 24 \times 2^{k-1} - 12) + 12\ (2^k + 2)$

$\leq 3 \times 69 \times 3^{k-1} - 3 \times 24 \times 2^{k-1} + 12 \times 2^k - 3 \times 12 + 12 \times 2$

$\leq 69 \times 3^k - 72 \times 2^{k-1} + 12 \times 2^k - 36 + 24$

$\leq 69 \times 3^k - 36 \times 2^k + 12 \times 2^k - 12$

$\leq \underline{69 \times 3^k - 24 \times 2^k - 12}$ (see Claim #2)

**Note:**

Let k be a minimal integer such that $n \leq 2^k + 2$

$\Rightarrow\ k \leq \log(n) + 1$

**Finally:**

$T_K(n) \leq T_K(2^k + 2)$

$T_K(n) \leq 69 \times 3^k - 24 \times 2^k - 12$

$\leq 69 \times 3^{\log(n) + 1} \cancel{- 24 \times 2^k - 12}$ (removing negative things[1])

$\leq 69 \times 3 \times 3^{\log(n)}$

$\leq 207 \times 3^{\log(n)}$

$\leq 207 \times n^{\log(3)}$ (Prove $3^{\log(n)} = n^{\log(3)}$)

1) Clearly if we make the right side bigger, we don't violate the inequality