Exercises

8.1 A logical knowledge base represents the world using a set of sentences with no explicit structure. An **analogical** representation, on the other hand, has physical structure that corresponds directly to the structure of the thing represented. Consider a road map of your country as an analogical representation of facts about the country—it represents facts with a map language. The two-dimensional structure of the map corresponds to the two-dimensional surface of the area.

a. Give five examples of *symbols* in the map language.

b. An *explicit* sentence is a sentence that the creator of the representation actually writes down. An *implicit* sentence is a sentence that results from explicit sentences because of properties of the analogical representation. Give three examples each of *implicit* and *explicit* sentences in the map language.

c. Give three examples of facts about the physical structure of your country that cannot be represented in the map language.

d. Give two examples of facts that are much easier to express in the map language than in first-order logic.

e. Give two other examples of useful analogical representations. What are the advantages and disadvantages of each of these languages?

8.2 Consider a knowledge base containing just two sentences: $P(a)$ and $P(b)$. Does this knowledge base entail $\forall\, x\; P(x)$? Explain your answer in terms of models.

8.3 Is the sentence $\exists\, x, y \;\; x = y$ valid? Explain.

8.4 Write down a logical sentence such that every world in which it is true contains exactly one object.

8.5 Consider a symbol vocabulary that contains c constant symbols, p_k predicate symbols of each arity k, and f_k function symbols of each arity k, where $1 \leq k \leq A$. Let the domain size be fixed at D. For any given model, each predicate or function symbol is mapped onto a relation or function, respectively, of the same arity. You may assume that the functions in the model allow some input tuples to have no value for the function (i.e., the value is the invisible object). Derive a formula for the number of possible models for a domain with D elements. Don't worry about eliminating redundant combinations.

8.6 Which of the following are valid (necessarily true) sentences?

a. $(\exists x\; x = x) \;\Rightarrow\; (\forall\, y \;\; \exists z\; y = z)$.

b. $\forall\, x \;\; P(x) \lor \neg P(x)$.

c. $\forall\, x \;\; Smart(x) \lor (x = x)$.

8.7 Consider a version of the semantics for first-order logic in which models with empty domains are allowed. Give at least two examples of sentences that are valid according to the standard semantics but not according to the new semantics. Discuss which outcome makes more intuitive sense for your examples.

8.8 Does the fact $\neg Spouse(George, Laura)$ follow from the facts $Jim \neq George$ and $Spouse(Jim, Laura)$? If so, give a proof; if not, supply additional axioms as needed. What happens if we use $Spouse$ as a unary function symbol instead of a binary predicate?

8.9 This exercise uses the function $MapColor$ and predicates $In(x, y)$, $Borders(x, y)$, and $Country(x)$, whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3) is syntactically valid but does not express the meaning of the English sentence.

a. Paris and Marseilles are both in France.

(i) $In(Paris \wedge Marseilles, France)$.

(ii) $In(Paris, France) \wedge In(Marseilles, France)$.

(iii) $In(Paris, France) \vee In(Marseilles, France)$.

b. There is a country that borders both Iraq and Pakistan.

(i) $\exists\, c \quad Country(c) \wedge Border(c, Iraq) \wedge Border(c, Pakistan)$.

(ii) $\exists\, c \quad Country(c) \Rightarrow [Border(c, Iraq) \wedge Border(c, Pakistan)]$.

(iii) $[\exists\, c \quad Country(c)] \Rightarrow [Border(c, Iraq) \wedge Border(c, Pakistan)]$.

(iv) $\exists\, c \quad Border(Country(c), Iraq \wedge Pakistan)$.

c. All countries that border Ecuador are in South America.

(i) $\forall\, c \;\; Country(c) \wedge Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)$.

(ii) $\forall\, c \;\; Country(c) \Rightarrow [Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)]$.

(iii) $\forall\, c \;\; [Country(c) \Rightarrow Border(c, Ecuador)] \Rightarrow In(c, SouthAmerica)$.

(iv) $\forall\, c \;\; Country(c) \wedge Border(c, Ecuador) \wedge In(c, SouthAmerica)$.

d. No region in South America borders any region in Europe.

(i) $\neg[\exists\, c, d \;\; In(c, SouthAmerica) \wedge In(d, Europe) \wedge Borders(c, d)]$.

(ii) $\forall\, c, d \;\; [In(c, SouthAmerica) \wedge In(d, Europe)] \Rightarrow \neg Borders(c, d)]$.

(iii) $\neg\forall\, c \;\; In(c, SouthAmerica) \Rightarrow \exists\, d \;\; In(d, Europe) \wedge \neg Borders(c, d)$.

(iv) $\forall\, c \;\; In(c, SouthAmerica) \Rightarrow \forall\, d \;\; In(d, Europe) \Rightarrow \neg Borders(c, d)$.

e. No two adjacent countries have the same map color.

(i) $\forall\, x, y \;\; \neg Country(x) \vee \neg Country(y) \vee \neg Borders(x, y) \vee$
$\neg(MapColor(x) = MapColor(y))$.

(ii) $\forall\, x, y \;\; (Country(x) \wedge Country(y) \wedge Borders(x, y) \wedge \neg(x = y)) \Rightarrow$
$\neg(MapColor(x) = MapColor(y))$.

(iii) $\forall\, x, y \;\; Country(x) \wedge Country(y) \wedge Borders(x, y) \wedge$
$\neg(MapColor(x) = MapColor(y))$.

(iv) $\forall\, x, y \;\; (Country(x) \wedge Country(y) \wedge Borders(x, y)) \Rightarrow MapColor(x \neq y)$.

8.10 Consider a vocabulary with the following symbols:

$Occupation(p, o)$: Predicate. Person p has occupation o.
$Customer(p1, p2)$: Predicate. Person $p1$ is a customer of person $p2$.
$Boss(p1, p2)$: Predicate. Person $p1$ is a boss of person $p2$.
$Doctor$, $Surgeon$, $Lawyer$, $Actor$: Constants denoting occupations.
$Emily$, Joe: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

a. Emily is either a surgeon or a lawyer.
b. Joe is an actor, but he also holds another job.
c. All surgeons are doctors.
d. Joe does not have a lawyer (i.e., is not a customer of any lawyer).
e. Emily has a boss who is a lawyer.
f. There exists a lawyer all of whose customers are doctors.
g. Every surgeon has a lawyer.

8.11 Complete the following exercises about logical senntences:

a. Translate into *good, natural* English (no xs or ys!):

$$\forall\, x, y, l \;\; SpeaksLanguage(x, l) \land SpeaksLanguage(y, l) \\ \Rightarrow \; Understands(x, y) \land Understands(y, x).$$

b. Explain why this sentence is entailed by the sentence

$$\forall\, x, y, l \;\; SpeaksLanguage(x, l) \land SpeaksLanguage(y, l) \\ \Rightarrow \; Understands(x, y).$$

c. Translate into first-order logic the following sentences:

(i) Understanding leads to friendship.
(ii) Friendship is transitive.

Remember to define all predicates, functions, and constants you use.

8.12 Rewrite the first two Peano axioms in Section 8.3.3 as a single axiom that defines $NatNum(x)$ so as to exclude the possibility of natural numbers except for those generated by the successor function.

8.13 Equation (8.4) on page 306 defines the conditions under which a square is breezy. Here we consider two other ways to describe this aspect of the wumpus world.

DIAGNOSTIC RULE

a. We can write **diagnostic rules** leading from observed effects to hidden causes. For finding pits, the obvious diagnostic rules say that if a square is breezy, some adjacent square must contain a pit; and if a square is not breezy, then no adjacent square contains a pit. Write these two rules in first-order logic and show that their conjunction is logically equivalent to Equation (8.4).

CAUSAL RULE

b. We can write **causal rules** leading from cause to effect. One obvious causal rule is that a pit causes all adjacent squares to be breezy. Write this rule in first-order logic, explain why it is incomplete compared to Equation (8.4), and supply the missing axiom.

9.3 Suppose a knowledge base contains just one sentence, $\exists\, x\ AsHighAs(x, Everest)$. Which of the following are legitimate results of applying Existential Instantiation?

a. $AsHighAs(Everest, Everest)$.

b. $AsHighAs(Kilimanjaro, Everest)$.

c. $AsHighAs(Kilimanjaro, Everest) \wedge AsHighAs(BenNevis, Everest)$ (after two applications).

9.4 For each pair of atomic sentences, give the most general unifier if it exists:

a. $P(A, B, B)$, $P(x, y, z)$.

b. $Q(y, G(A, B))$, $Q(G(x, x), y)$.

c. $Older(Father(y), y)$, $Older(Father(x), John)$.

d. $Knows(Father(y), y)$, $Knows(x, x)$.

9.5 Consider the subsumption lattices shown in Figure 9.2 (page 329).

a. Construct the lattice for the sentence $Employs(Mother(John), Father(Richard))$.

b. Construct the lattice for the sentence $Employs(IBM, y)$ ("Everyone works for IBM"). Remember to include every kind of query that unifies with the sentence.

c. Assume that STORE indexes each sentence under every node in its subsumption lattice. Explain how FETCH should work when some of these sentences contain variables; use as examples the sentences in (a) and (b) and the query $Employs(x, Father(x))$.

9.6 Write down logical representations for the following sentences, suitable for use with Generalized Modus Ponens:

a. Horses, cows, and pigs are mammals.

b. An offspring of a horse is a horse.

c. Bluebeard is a horse.

d. Bluebeard is Charlie's parent.

e. Offspring and parent are inverse relations.

f. Every mammal has a parent.

9.7 These questions concern concern issues with substitution and Skolemization.

a. Given the premise $\forall\, x\ \ \exists\, y\ \ P(x, y)$, it is not valid to conclude that $\exists\, q\ \ P(q, q)$. Give an example of a predicate P where the first is true but the second is false.

b. Suppose that an inference engine is incorrectly written with the occurs check omitted, so that it allows a literal like $P(x, F(x))$ to be unified with $P(q, q)$. (As mentioned, most standard implementations of Prolog actually do allow this.) Show that such an inference engine will allow the conclusion $\exists\, y\ \ P(q, q)$ to be inferred from the premise $\forall\, x\ \ \exists\, y\ \ P(x, y)$.

c. Suppose that a procedure that converts first-order logic to clausal form incorrectly Skolemizes $\forall x \;\; \exists y \;\; P(x,y)$ to $P(x, Sk0)$—that is, it replaces y by a Skolem constant rather than by a Skolem function of x. Show that an inference engine that uses such a procedure will likewise allow $\exists q \;\; P(q,q)$ to be inferred from the premise $\forall x \; \exists y \; P(x,y)$.

d. A common error among students is to suppose that, in unification, one is allowed to substitute a term for a Skolem constant instead of for a variable. For instance, they will say that the formulas $P(Sk1)$ and $P(A)$ can be unified under the substitution $\{Sk1/A\}$. Give an example where this leads to an invalid inference.

9.8 Explain how to write any given 3-SAT problem of arbitrary size using a single first-order definite clause and no more than 30 ground facts.

9.9 Suppose you are given the following axioms:

1. $0 \leq 3$.
2. $7 \leq 9$.
3. $\forall x \quad x \leq x$.
4. $\forall x \quad x \leq x + 0$.
5. $\forall x \quad x + 0 \leq x$.
6. $\forall x, y \quad x + y \leq y + x$.
7. $\forall w, x, y, z \quad w \leq y \land x \leq z \;\Rightarrow\; w + x \leq y + z$.
8. $\forall x, y, z \quad x \leq y \land y \leq z \;\Rightarrow\; x \leq z$

a. Give a backward-chaining proof of the sentence $7 \leq 3 + 9$. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that leads to success, not the irrelevant steps.

b. Give a forward-chaining proof of the sentence $7 \leq 3 + 9$. Again, show only the steps that lead to success.

9.10 A popular children's riddle is "Brothers and sisters have I none, but that man's father is my father's son." Use the rules of the family domain (Section 8.3.2 on page 301) to show who that man is. You may apply any of the inference methods described in this chapter. Why do you think that this riddle is difficult?

9.11 Suppose we put into a logical knowledge base a segment of the U.S. census data listing the age, city of residence, date of birth, and mother of every person, using social security numbers as identifying constants for each person. Thus, George's age is given by $Age(\text{443-65-1282}, 56)$. Which of the following indexing schemes S1–S5 enable an efficient solution for which of the queries Q1–Q4 (assuming normal backward chaining)?

- **S1**: an index for each atom in each position.
- **S2**: an index for each first argument.
- **S3**: an index for each predicate atom.
- **S4**: an index for each *combination* of predicate and first argument.