THE UNIVERSITY OF ADELAIDE
AUSTRALIA

**Primary Examination, Semester 1, 2012**

# Algorithm and Data Structure Analysis
# COMPSCI 2201, 7201

| | |
|---|---|
| Official Reading Time: | 10 mins |
| Writing Time: | 120 mins |
| Total Duration: | 130 mins |

| Questions | Time | Marks |
|---|---|---|
| Answer all 6 questions | 120 mins | 80 marks |
| | | 80 Total |

Instructions

- Begin each answer on a new page
- Examination material must not be removed from the examination room
- No Calculators Allowed

Materials

- 1 Blue book
- 1 double-sided A4-page of handwritten notes
- 1 Dictionary

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO

**Right or Wrong?**

**Question 1**

(a) Indicate whether each of the following statements is true or false. There is one mark for each correct answer and zero marks for each incorrect answer.

| | Statement |
|---|---|
| 1 | $n^{7/8} = o(n)$ |
| 2 | $2.1 \cdot n^{2.1} + 120n^2 = O(n^2)$ |
| 3 | Karatsuba Integer Multiplication takes time $o(n^2)$. |
| 4 | The single-source-shortest path problem can be solved in time $O(n^2)$. |
| 5 | $NP \subset P$ |
| 6 | The Clique problem is known to be NP-hard. |
| 7 | The Minimum Spanning Tree problem is known to be NP-hard. |
| 8 | There is a known polynomial time algorithm which solves the Maximum Clique Problem. |
| 9 | Heapsort runs in time $O(n \log n)$. |
| 10 | The worst-case height of a Binary Search Tree is $O(n)$. |

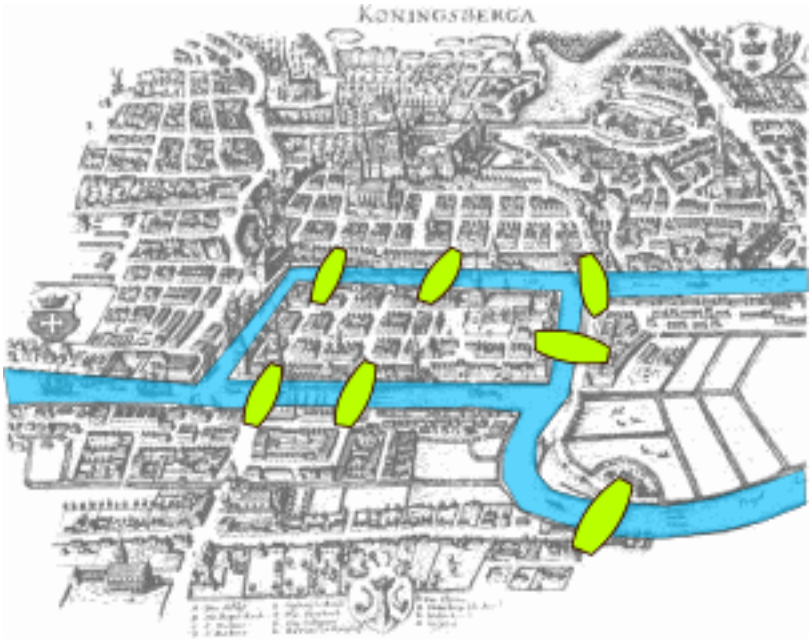
[10 marks]

**[Total for Question 1: 10 marks]**

## Graphs

### Question 2

(a) The following map shows the map of Königsberg. There are seven bridges over the river Preger which connect the different parts of the city.




The Königsberg bridge problem asks if the seven bridges of the city of Königsberg over the river Preger can all be traversed in a single trip without doubling back, with the additional requirement that the trip ends in the same place it began.

i. Model the Königsberg bridge problem as a graph problem and state the decision problem for the graph.

[5 marks]

ii. Determine whether there is a solution to the Königsberg bridge problem and justify your solution by either presenting a solution or arguing why there is no solution to the problem.

[5 marks]

**[Total for Question 2: 10 marks]**

**Trees**

**Question 3**

(a) Draw a sequence of diagrams showing the insertion of the values:

```
[23, 21, 20, 19, 25, 26, 22]
```

into a binary search tree. You must show the resulting tree immediately after each insertion step.

[5 marks]

(b) Draw a sequence of diagrams showing the insertion of the values:

```
[18,12,10,27,19,21,27]
```

into an empty AVL tree.

You must:

- Show the resulting tree immediately after each insertion step (that is *before* any balancing has taken place).
- Indicate the node(s) at which each rotation is performed.
- Where there is a double rotation, show the tree after each single rotation.
- Show the resulting tree after balancing operation(s).

[6 marks]

**[Total for Question 3: 11 marks]**

**Directed Graphs**

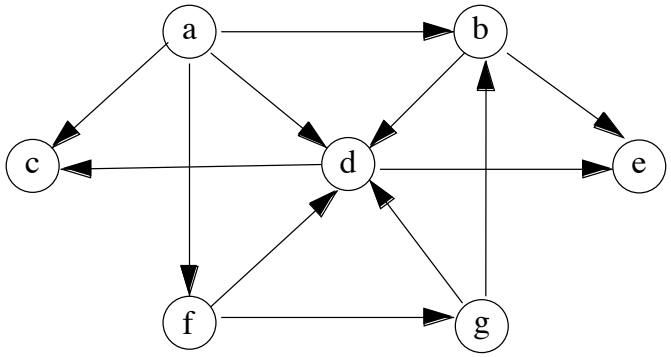
**Question 4**

(a) Prove that a directed graph is acyclic iff DFS scans of the graph do not have back edges.

[5 marks]

(b) A Reversed Topological Sort of a directed acyclic graph $G = (V, E)$ is a linear ordering of the vertices of $V$ such that if $G$ contains an edge $(u, v)$, then $u$ appears after $v$ in the ordering.

i. Find a reversed topological sort of the following graph




[2 marks]

ii. Show that the termination order of a DFS scan of a directed acyclic graph is a reveresed topological sort.

[8 marks]

**[Total for Question 4: 15 marks]**

**Minimum Spanning Trees**

**Question 5**

We consider the minimum spanning tree problem for a given undirected connected weighted graph $G = (V, E)$ with cost function $c\colon E \to \mathbb{R}^+$ on the edges.

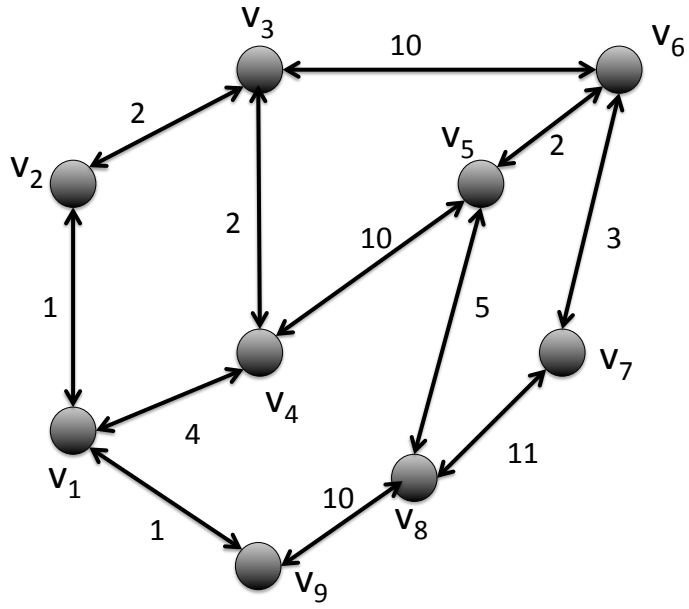
(a) State the cut property for the computation of minimum spanning trees.

[2 marks]

(b) Give Jarník-Prim's algorithm for computing a minimum spanning tree and analyze its runtime.

[5 marks]

(c) Compute a minimum spanning of the following graph by using Kruskal's algorithm. Show the status of your partial minimum spanning tree after each edge insertion.




[8 marks]

(d) Is there a unique minimum spanning tree for this graph? If so, explain why. If not, give two different minimum spanning trees.

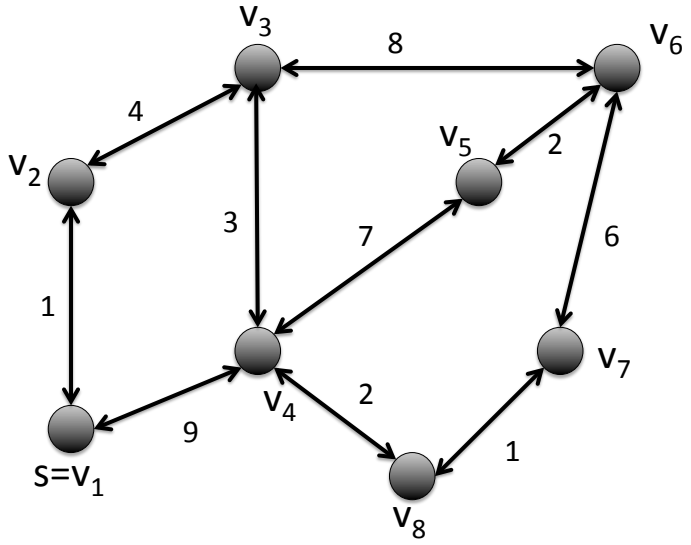
[3 marks]

**[Total for Question 5: 18 marks]**

**Shortest Paths**

**Question 6**

Consider the following graph.




(a) Solve the single-source-shortest path problem for the start node s using Dijkstra's algorithm. Show for each iteration which nodes becomes scanned and which edges are relaxed.

[8 marks]

(b) Solve the all-pairs-shortest-path problem for the given graph by using the Floyd-Warshall Algorithm. Show for each iteration $k$, the distance matrix giving the current cost of a shortest path for any pair of nodes.

[8 marks]

**[Total for Question 6: 16 marks]**

**End of exam**