# Assignment 1

Due date: 09:00am, Wednesday, 3rd of August 2016

## General Instructions

Assignment 1 will be discussed in the tutorials taking place in the second week of the semester. You should prepare solutions, but you have not to hand them in and they will not get marked.

**Exercise 1** *Induction Proofs*

Recall the principle of doing proofs by mathematical induction.

1. Prove by mathematical induction that $n! \geq \mathrm{fib}(n)$ for all $n \geq 0$. Note that $\mathrm{fib}(n)$ denotes the $n^{th}$ Fibonacci number.
2. Let $a$ and $r \neq 1$ be real numbers. Prove by mathematical induction the geometric series, i.e. that
$$\sum_{i=0}^{n} a \cdot r^i = \frac{a\left(1 - r^{(n+1)}\right)}{1 - r}$$
holds for all natural numbers $n$.

**Exercise 2** *Asymptotic notations*

1. Solve Exercise 2.1 in the book of Mehlhorn&Sanders (page 22).
2. Solve Exercise 2.2 in the book of Mehlhorn&Sanders (page 23).
3. Is it true that if $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n))$, then $h(n) = \Theta(f(n))$? Provide your explanation.
4. Is it true that if $f(n) = \mathcal{O}(g(n))$ and $g(n) = \mathcal{O}(h(n))$, then $h(n) = \Omega(f(n))$? Provide your explanation.
5. Is it true that $\Theta(n^2)$ algorithm always takes longer to run than a $\Theta(\log n)$ algorithm? Provide your explanation.
6. For each pair of functions given below, point out the asymptotic relationships that apply: $f = \mathcal{O}(g)$, $f = \Theta(g)$, and $f = \Omega(g)$.
   - $f(n) = \sqrt{n}$ and $g(n) = \log n$;

- $f(n) = 1$ and $g(n) = 2$;
- $f(n) = 1000 \cdot 2^n$ and $g(n) = 3^n$;
- $f(n) = 4^{n+4}$ and $g(n) = n^{2n+2}$;
- $f(n) = 5n \log n$ and $g(n) = n \log 5n$;
- $f(n) = n!$ and $g(n) = (n+1)!$.

7. Prove that $n^k = o(c^n)$ for any integer k and any $c > 1$.

# End of Questions