Tutorial 2: Discussion Questions

Exercise 1 *Induction Proofs*

Let $N = \{0, 1, 2, \ldots\}$ be the set of natural numbers.

1. (from Melhorn 2.10). Access to data structures is often governed by the recurrence $T(1) = a, T(n) = c + T(n/2)$. Prove by induction that $T(n) \in O(logn)$. Do not attempt to use the Master Theorem for this proof.

2. Can the statement above be proven by the Master Theorem? If so, show your proof. If not then explain why not.

3. Let $F(n)$ the n-th Fibonacci number. We have that $F(1) = F(2) = 1$ and $F(n) = F(n-1) + F(n-2)$. Find an a value so that $F(n) \in O(a^n)$. We want a to be as small as possible.

Exercise 2 *Approximation Algorithms*

1. Suppose your task is to implement a boolean function called `badSign` that takes an integer input and returns `true` if the input is positive and `false` otherwise. **However**, you are required to write `badSign` so that it, randomly, 25% of the time returns the incorrect boolean value. How would you implement such a function?

2. Suppose your second task is to implement a boolean function called `betterSign` that takes an integer input and returns `true` if the input is positive and `false` otherwise. **However**, this time, you are not allowed to directly inspect the input integer x. You are only allowed to inspect $badSign(x)$. How would you write the `betterSign` function, in order to obtain a function that is more accurate than `badSign` itself?

3. What happens when `badSign` is incorrect 49% of the time? What happens when `badSign` is incorrect 51% of the time?