each of the following sentences is true:

> α_2 = "There is no pit in [2,2]."
> α_3 = "There is a wumpus in [1,3]."

Hence show that $KB \models \alpha_2$ and $KB \models \alpha_3$.

7.2 (Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

7.3 Consider the problem of deciding whether a propositional logic sentence is true in a given model.

  a. Write a recursive algorithm PL-TRUE?(s, m) that returns *true* if and only if the sentence s is true in the model m (where m assigns a truth value for every symbol in s). The algorithm should run in time linear in the size of the sentence. (Alternatively, use a version of this function from the online code repository.)

  b. Give three examples of sentences that can be determined to be true or false in a *partial* model that does not specify a truth value for some of the symbols.

  c. Show that the truth value (if any) of a sentence in a partial model cannot be determined efficiently in general.

  d. Modify your PL-TRUE? algorithm so that it can sometimes judge truth from partial models, while retaining its recursive structure and linear run time. Give three examples of sentences whose truth in a partial model is *not* detected by your algorithm.

  e. Investigate whether the modified algorithm makes TT-ENTAILS? more efficient.

7.4 Which of the following are correct?

  a. $False \models True$.
  b. $True \models False$.
  c. $(A \land B) \models (A \Leftrightarrow B)$.
  d. $A \Leftrightarrow B \models A \lor B$.
  e. $A \Leftrightarrow B \models \neg A \lor B$.
  f. $(A \land B) \Rightarrow C \models (A \Rightarrow C) \lor (B \Rightarrow C)$.
  g. $(C \lor (\neg A \land \neg B)) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))$.
  h. $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B)$.
  i. $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B) \land (\neg D \lor E)$.
  j. $(A \lor B) \land \neg(A \Rightarrow B)$ is satisfiable.
  k. $(A \Leftrightarrow B) \land (\neg A \lor B)$ is satisfiable.
  l. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

7.5   Prove each of the following assertions:

a. α is valid if and only if $True \models \alpha$.

b. For any α, $False \models \alpha$.

c. $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.

d. $\alpha \equiv \beta$ if and only if the sentence $(\alpha \Leftrightarrow \beta)$ is valid.

e. $\alpha \models \beta$ if and only if the sentence $(\alpha \wedge \neg\beta)$ is unsatisfiable.

7.6   Prove, or find a counterexample to, each of the following assertions:

a. If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \wedge \beta) \models \gamma$

b. If $\alpha \models (\beta \wedge \gamma)$ then $\alpha \models \beta$ and $\alpha \models \gamma$.

c. If $\alpha \models (\beta \vee \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).

7.7   Consider a vocabulary with only four propositions, $A, B, C,$ and D. How many models are there for the following sentences?

a. $B \vee C$.

b. $\neg A \vee \neg B \vee \neg C \vee \neg D$.

c. $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$.

7.8   We have defined four binary logical connectives.

a. Are there any others that might be useful?

b. How many binary connectives can there be?

c. Why are some of them not very useful?

7.9   Using a method of your choice, verify each of the equivalences in Figure 7.11 (page 249).

7.10   Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11 (page 249).

a. $Smoke \Rightarrow Smoke$

b. $Smoke \Rightarrow Fire$

c. $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

d. $Smoke \vee Fire \vee \neg Fire$

e. $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$

f. $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \wedge Heat) \Rightarrow Fire)$

g. $Big \vee Dumb \vee (Big \Rightarrow Dumb)$

7.11   Any propositional logic sentence is logically equivalent to the assertion that each possible world in which it would be false is not the case. From this observation, prove that any sentence can be written in CNF.

7.12   Use resolution to prove the sentence $\neg A \wedge \neg B$ from the clauses in Exercise 7.20.

7.13   This exercise looks into the relationship between clauses and implication sentences.

a. Show that the clause $(\neg P_1 \vee \cdots \vee \neg P_m \vee Q)$ is logically equivalent to the implication sentence $(P_1 \wedge \cdots \wedge P_m) \Rightarrow Q$.

b. Show that every clause (regardless of the number of positive literals) can be written in the form $(P_1 \wedge \cdots \wedge P_m) \Rightarrow (Q_1 \vee \cdots \vee Q_n)$, where the Ps and Qs are proposition symbols. A knowledge base consisting of such sentences is in **implicative normal form** or **Kowalski form** (Kowalski, 1979).

IMPLICATIVE
NORMAL FORM

c. Write down the full resolution rule for sentences in implicative normal form.

7.14 According to some political pundits, a person who is radical (R) is electable (E) if he/she is conservative (C), but otherwise is not electable.

a. Which of the following are correct representations of this assertion?

(i) $(R \wedge E) \iff C$

(ii) $R \Rightarrow (E \iff C)$

(iii) $R \Rightarrow ((C \Rightarrow E) \vee \neg E)$

b. Which of the sentences in (a) can be expressed in Horn form?

7.15 This question considers representing satisfiability (SAT) problems as CSPs.

a. Draw the constraint graph corresponding to the SAT problem

$$(\neg X_1 \vee X_2) \wedge (\neg X_2 \vee X_3) \wedge \ldots \wedge (\neg X_{n-1} \vee X_n)$$

for the particular case $n = 5$.

b. How many solutions are there for this general SAT problem as a function of n?

c. Suppose we apply BACKTRACKING-SEARCH (page 215) to find *all* solutions to a SAT CSP of the type given in (a). (To find *all* solutions to a CSP, we simply modify the basic algorithm so it continues searching after each solution is found.) Assume that variables are ordered $X_1, \ldots, X_n$ and *false* is ordered before *true*. How much time will the algorithm take to terminate? (Write an $O(\cdot)$ expression as a function of n.)

d. We know that SAT problems in Horn form can be solved in linear time by forward chaining (unit propagation). We also know that every tree-structured binary CSP with discrete, finite domains can be solved in time linear in the number of variables (Section 6.5). Are these two facts connected? Discuss.

7.16 Explain why every nonempty propositional clause, by itself, is satisfiable. Prove rigorously that every set of five 3-SAT clauses is satisfiable, provided that each clause mentions exactly three distinct variables. What is the smallest set of such clauses that is unsatisfiable? Construct such a set.

7.17 A propositional *2-CNF* expression is a conjunction of clauses, each containing *exactly* 2 literals, e.g.,

$$(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G) \ .$$

a. Prove using resolution that the above sentence entails G.