8.6 Which of the following are valid (necessarily true) sentences?

a. $(\exists x\ x = x) \Rightarrow (\forall y\ \exists z\ y = z)$.

Valid

Trivially, $\exists x(x = x) = T$

In any model with an object, $\forall y \exists z(y = z) = T$ (the same object is referred to by y and z).

Thus, $(\exists x(x = x) \Rightarrow \forall y \exists z(y = z)) = T$

b. $\forall x\ P(x) \vee \neg P(x)$.

Valid

When $P(x) = T$, $\neg P(x) = F$, $P(x) \vee \neg P(x) = T$

When $P(x) = F$, $\neg P(x) = T$, $P(x) \vee \neg P(x) = T$

Thus, $\forall x\ P(x) \vee \neg P(x) = T$

c. $\forall x\ Smart(x) \vee (x = x)$.

Valid

$\forall x(x = x) = T$

Thus, $(\forall x\ Smart(x) \vee (x = x)) = T$

8.9 This exercise uses the function $MapColor$ and predicates $In(x, y)$, $Borders(x, y)$, and $Country(x)$, whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3) is syntactically valid but does not express the meaning of the English sentence.

a. Paris and Marseilles are both in France.

(i) $In(Paris \wedge Marseilles, France)$.

(ii) $In(Paris, France) \wedge In(Marseilles, France)$.

(iii) $In(Paris, France) \vee In(Marseilles, France)$.

i. Invalid. Paris and Marseilles is not a geographical region **2**

ii. Correct **1**

iii. Valid, but not correct expression. Paris and Marseilles are both in France, not one or the other. **3**

b. There is a country that borders both Iraq and Pakistan.

(i) $\exists c \quad Country(c) \wedge Border(c, Iraq) \wedge Border(c, Pakistan)$.

(ii) $\exists c \quad Country(c) \Rightarrow [Border(c, Iraq) \wedge Border(c, Pakistan)]$.

(iii) $[\exists c \quad Country(c)] \Rightarrow [Border(c, Iraq) \wedge Border(c, Pakistan)]$.

(iv) $\exists c \quad Border(Country(c), Iraq \wedge Pakistan)$.

i. Correct **1**

ii. Valid, but is still true when c is not a country **3**

iii. Invalid. C is not defined on right **2**

iv. Invalid. Iran and Pakistan is not a geographical region. **2**

8.10 Consider a vocabulary with the following symbols:

$Occupation(p, o)$: Predicate. Person p has occupation o.
$Customer(p1, p2)$: Predicate. Person $p1$ is a customer of person $p2$.
$Boss(p1, p2)$: Predicate. Person $p1$ is a boss of person $p2$.
$Doctor$, $Surgeon$, $Lawyer$, $Actor$: Constants denoting occupations.
$Emily$, Joe: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

a. Emily is either a surgeon or a lawyer.

$$Occupation(Emily, Surgeon) \oplus Occupation(Emily, Lawyer)$$

$$\equiv \big(\big(Occupation(Emily, Surgeon) \wedge \neg Occupation(Emily, Lawyer)\big) \\ \vee \big(\neg Occupation(Emily, Surgeon) \wedge Occupation(Emily, Lawyer)\big)\big)$$

b. Joe is an actor, but he also holds another job.

$$(\exists o \ (o \neq Actor) \ \wedge Occupation(Joe, o)) \wedge Occupation(Joe, Actor)$$

c. All surgeons are doctors.

$$\forall p \ Occupation(p, Surgeon) \Rightarrow Occupation(p, Doctor)$$

d. Joe does not have a lawyer (i.e., is not a customer of any lawyer).

$$\neg \exists p Occupation(p, Lawyer) \wedge Customer(Joe, p)$$

$$\equiv \forall p \ \neg(Occupation(p, Lawyer) \wedge Customer(Joe, p))$$

$$\equiv \forall p \ \neg Occupation(p, Lawyer) \vee \neg Customer(Joe, p)$$

$$\equiv \forall p \ Occupation(p, Lawyer) \Rightarrow \neg Customer(Joe, p)$$

e. Emily has a boss who is a lawyer.

$$\exists p \ Boss(p, Emily) \wedge Occupation(p, Lawyer)$$

f. There exists a lawyer all of whose customers are doctors.

$$\exists p1 \ Occupation(p1, Lawyer) \wedge (\forall p2 \ Customer(p2, p1) \Rightarrow Occupation(p2, Doctor))$$

g. Every surgeon has a lawyer.

$$\forall p1 \ Occupation(p1, Surgeon) \Rightarrow (\exists p2 \ Customer(p1, p2) \wedge Occupation(p2, Lawyer))$$

9.3 Suppose a knowledge base contains just one sentence, $\exists x\ AsHighAs(x, Everest)$. Which of the following are legitimate results of applying Existential Instantiation?

a. $AsHighAs(Everest, Everest)$.

b. $AsHighAs(Kilimanjaro, Everest)$.

c. $AsHighAs(Kilimanjaro, Everest) \land AsHighAs(BenNevis, Everest)$ (after two applications).

Existential Instantiation: Replace each variable with a *single* new constant

a. Illegitimate. Not a new constant.
b. Legitimate. A single new constant.
c. Illegitimate. Not a single constant.

9.7 These questions concern concern issues with substitution and Skolemization.

a. Given the premise $\forall x\ \exists y\ \ P(x, y)$, it is not valid to conclude that $\exists q\ \ P(q, q)$. Give an example of a predicate P where the first is true but the second is false.

Not equal, XOR, less than, greater than

b. Suppose that an inference engine is incorrectly written with the occurs check omitted, so that it allows a literal like $P(x, F(x))$ to be unified with $P(q, q)$. (As mentioned, most standard implementations of Prolog actually do allow this.) Show that such an inference engine will allow the conclusion $\exists y\ \ P(q, q)$ to be inferred from the premise $\forall x\ \exists y\ \ P(x, y)$.

$\forall x \exists y P(x, y)$ $\equiv \forall x \exists y P(x, F(x))$, Skolemization

$\equiv \exists y P(x, F(x))$, As everything depends on x (would normally drop y too)

$\equiv \exists y P(q, q)$, Incorrect inference engine

■

De Morgan's Rule	Generalized De Morgan's Rule
$P \land Q \equiv \neg(\neg P \lor \neg Q)$	$\forall x\, P \equiv \neg \exists x (\neg P)$
$P \lor Q \equiv \neg(\neg P \land \neg Q)$	$\exists x\, P \equiv \neg \forall x (\neg P)$
$\neg(P \land Q) \equiv \neg P \lor \neg Q$	$\neg \forall x\, P \equiv \exists x (\neg P)$
$\neg(P \lor Q) \equiv \neg P \land \neg Q$	$\neg \exists x\, P \equiv \forall x (\neg P)$