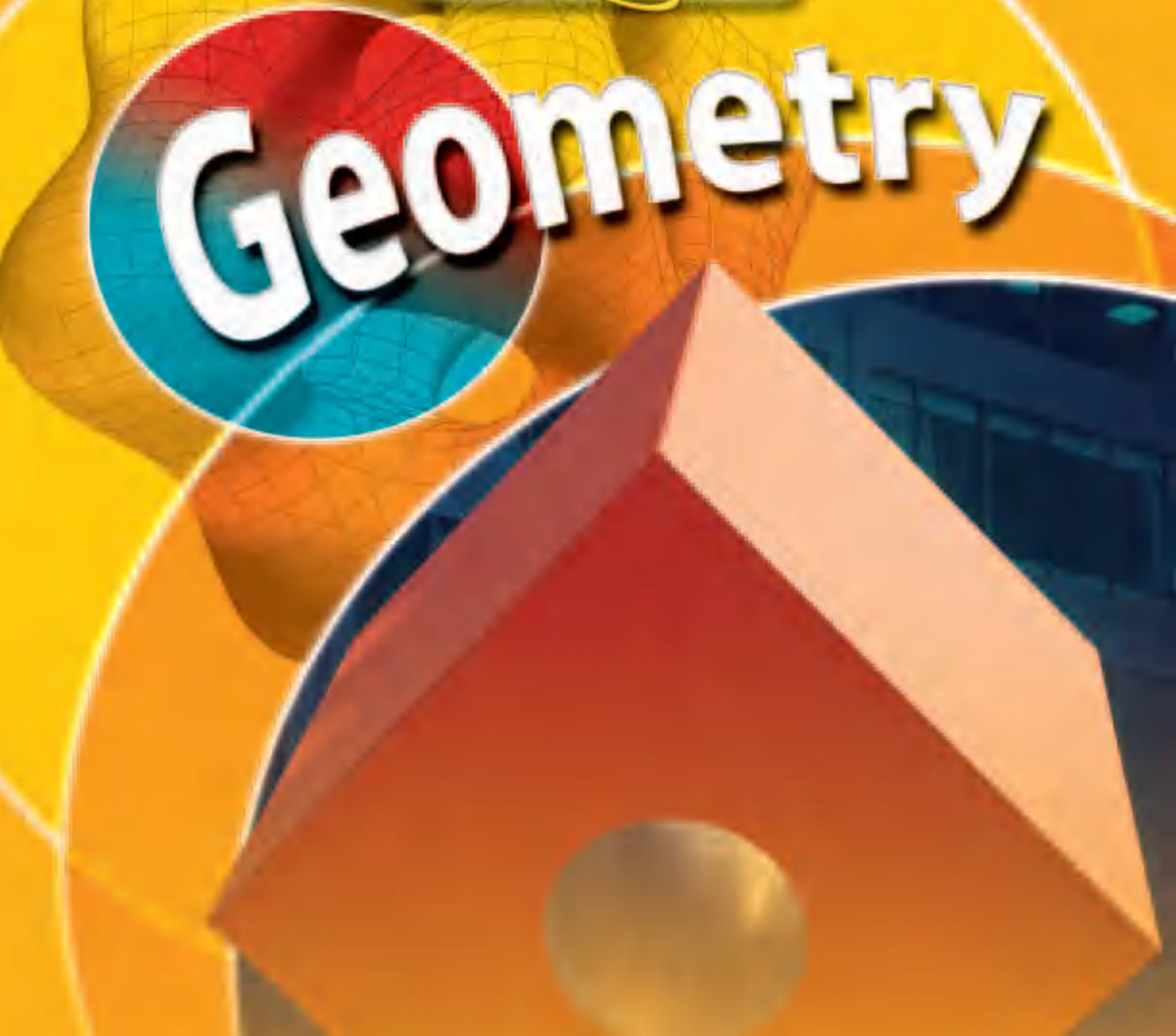
Glencoe McGraw-Hill

Geometry

Interactive Student Edition

Glencoe McGraw-Hill

Geometry



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McGraw Hill Glencoe

New York, New York   Columbus, Ohio   Chicago, Illinois   Woodland Hills, California

About the Cover

In New York City, the bright red painted steel of Isamu Noguchi's *Red Cube* stands out in contrast to the blacks, browns, and whites of the buildings and the sidewalks of Broadway. The sculpture is not actually a cube, but instead seems as though it has been stretched along its vertical axis. Through the center of the sculpture is a cylindrical hole, revealing an inner surface of gray with evenly-spaced parallel lines. You'll learn more about three-dimensional objects such as cubes and cylinders in Chapters 12 and 13.

About the Graphics

Created with *Mathematica*.
A Cmutov surface is generated using the equation
$(8x^4 - 8x^2 + 1) + (8y^4 - 8y^2 + 1) + (8z^4 - 8z^2 + 1) = 0$.
For more information, and for programs to construct such graphics, see: www.wolfram.com.

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UNIT 1
Geometric Structure

Focus

Undrestand basic geometric terms, such as lines, planes, and angles and how they can be used to prove theorems.

CHAPTER 1
Tools of Geometry

BIG Idea Identify and give examples of undefined terms.

BIG Idea Solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures.

CHAPTER 2
Reasoning and Proof

BIG Idea Identify and give examples of axioms, theorems, and inductive and deductive reasoning.

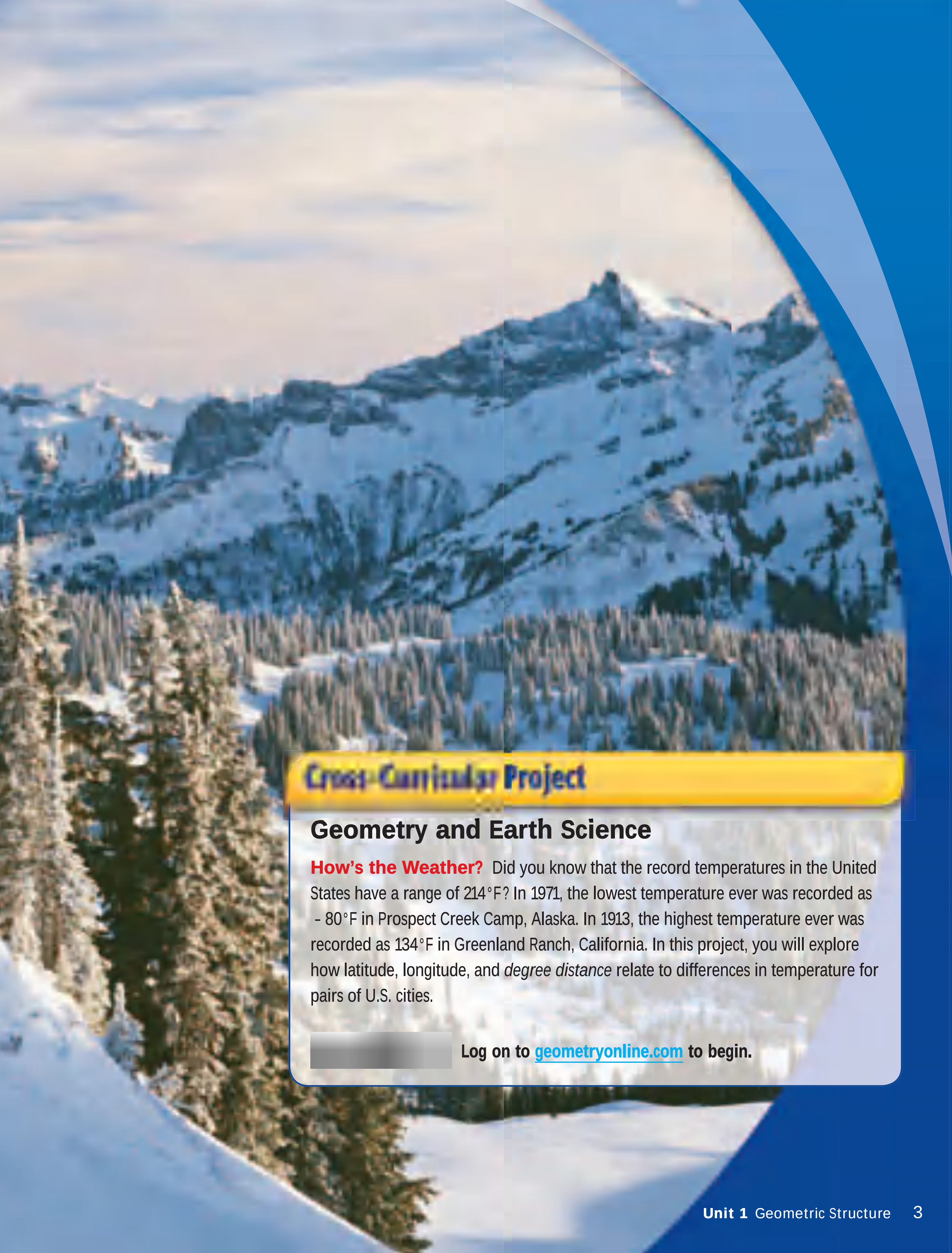
BIG Idea Write geometric proofs, including proofs by contradiction, give counterexamples to disprove a statement, and prove basic theorems involving congruence.

CHAPTER 3
Parallel and Perpendicular Lines

BIG Idea Prove and use theorems involving the properties of parallel lines cut by a transversal.

BIG Idea Perform basic constructions with a straightedge and compass.

Alan Majchrowicz/Getty Images



Cross-Curricular Project

Geometry and Earth Science

How's the Weather? Did you know that the record temperatures in the United States have a range of 214°F? In 1971, the lowest temperature ever was recorded as −80°F in Prospect Creek Camp, Alaska. In 1913, the highest temperature ever was recorded as 134°F in Greenland Ranch, California. In this project, you will explore how latitude, longitude, and *degree distance* relate to differences in temperature for pairs of U.S. cities.

Log on to geometryonline.com to begin.

CHAPTER 1

Tools of Geometry

BIG Ideas

- Measure segments and determine accuracy of measurements.
- Find the distances between points and the midpoints of segments.
- Measure and classify angles and identify angle relationships.
- Identify polygons and find their perimeters.
- Identify three-dimensional figures and find their surface areas and volumes.

Key Vocabulary

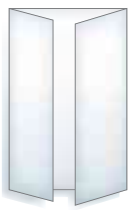
line segment (p. 13)
congruent (p. 15)
bisector (pp. 25, 35)
perpendicular (p. 43)

Real-World Link

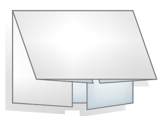
Kites A kite can model lines, angles, and planes.

Lines and Angles Make this Foldable to help you organize your notes. Begin with a sheet of 11" × 17" paper.

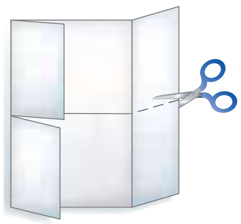
1 **Fold** the short sides to meet in the middle.



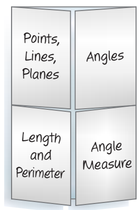
2 **Fold** the top to the bottom.



3 **Open.** Cut flaps along the second fold to make four tabs.



4 **Label** the tabs as shown.



Roy Morsch/CORBIS

GET READY for Chapter 1

Diagnose Readiness **You have two options for checking Prerequisite Skills.**

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Graph and label each point in the coordinate plane. (Prerequisite Skill)

1. $A(3, -2)$
2. $B(4, 0)$
3. $C(-4, -4)$
4. $D(-1, 2)$
5. **GEOGRAPHY** Joaquin is making a map on a coordinate grid with his school at the center. His house is 4 blocks north and 2 blocks west of the school. The library is 6 blocks east of his house and 2 blocks south. Graph and label the house and library. (Prerequisite Skill)

Find each sum or difference. (Prerequisite Skill)

6. $\frac{3}{4} + \frac{3}{8}$
7. $2\frac{5}{16} + 5\frac{1}{8}$
8. $\frac{7}{8} - \frac{9}{16}$
9. $11\frac{1}{2} - 9\frac{7}{16}$
10. **BAKING** A recipe calls for $\frac{3}{4}$ cup of flour plus $1\frac{1}{2}$ cups of flour. How much flour is needed? (Prerequisite Skill)

Evaluate each expression. (Prerequisite Skill)

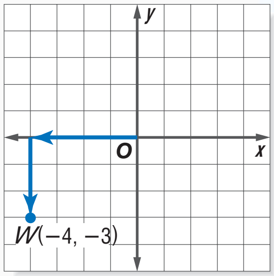
11. $-4 - (-9)$
12. $23 - (-14)$
13. $(18 + 20)^2$
14. $[-7 - (-2)]^2$
15. the sum of 13 and -8 squared

QUICKReview

EXAMPLE 1

Graph and label the point $W(-4, -3)$ in the coordinate plane.

Start at the origin. Move 4 units left, since the x-coordinate is -4. Then move 3 units down, since the y-coordinate is -3. Draw a dot, and label it W.



EXAMPLE 2

Find the sum of $\frac{3}{7} + \frac{5}{6}$.

$$\frac{3}{7} + \frac{5}{6} = \frac{3}{7}\left(\frac{6}{6}\right) + \frac{5}{6}\left(\frac{7}{7}\right) \quad \text{Rename.}$$

$$= \frac{18}{42} + \frac{35}{42} \quad \text{Multiply.}$$

$$= \frac{53}{42} \text{ or } 1\frac{11}{42} \quad \text{Simplify.}$$

EXAMPLE 3

Evaluate the expression $[8 - (-2)^4]^2$.

Follow the order of operations.

$$[8 - (-2)^4]^2 = [8 - 16]^2 \quad (-2)^4 = 16$$

$$= [-8]^2 \quad \text{Subtract.}$$

$$= 64 \quad \text{Square } -8.$$

1-1 Points, Lines, and Planes

Have you ever noticed that a four-legged chair sometimes wobbles, but a three-legged stool never wobbles? This is an example of points and how they lie in a plane. All geometric shapes are made of points. In this book, you will learn about those shapes and their characteristics.

Main Ideas

- Identify and model points, lines, and planes.
- Identify collinear and coplanar points and intersecting lines and planes in space.

New Vocabulary

undefined term
point
line
collinear
plane
coplanar
space
locus

Name Points, Lines, and Planes You are familiar with the terms *plane*, *line*, and *point* from algebra. You graph on a coordinate *plane*, and ordered pairs represent *points* on *lines*. In geometry, these terms have similar meanings.

Unlike objects in the real world that model these shapes, points, lines, and planes do not have any actual size. In geometry, *point*, *line*, and *plane* are considered **undefined terms** because they are only explained using examples and descriptions.

- A **point** is simply a location.
- A **line** is made up of points and has no thickness or width. Points on the same line are said to be **collinear**.
- A **plane** is a flat surface made up of points. Points that lie on the same plane are said to be **coplanar**. A plane has no depth and extends infinitely in all directions.

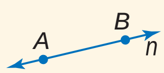
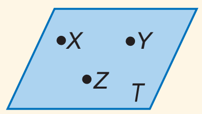
Points are often used to name lines and planes.

Reading Math

Noncollinear and Noncoplanar

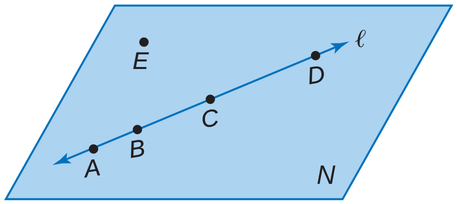
The word *noncollinear* means not collinear or not lying on the same line. Likewise, *noncoplanar* means not lying in the same plane.

KEY CONCEPT *Points, Lines, and Planes*

	Point	Line	Plane
Model			
Drawn	as a dot	with an arrowhead at each end	as a shaded, slanted 4-sided figure
Named by	a capital letter	the letters representing two points on the line or a lowercase script letter	a capital script letter or by the letters naming three noncollinear points
Facts	A point has neither shape nor size.	There is exactly one line through any two points.	There is exactly one plane through any three noncollinear points.
Words/ Symbols	point *P*	line *n*, line *AB* or $\overleftrightarrow{AB}$, line *BA* or $\overleftrightarrow{BA}$	plane $\mathcal{T}$, plane *XYZ*, plane *XZY*, plane *YXZ*, plane *YZX*, plane *ZXY*, plane *ZYX*

EXAMPLE 1 Name Lines and Planes

Use the figure to name each of the following.



a. a line containing point *A*

The line can be named as line ℓ. There are four points on the line. Any two of the points can be used to name the line.

$\overleftrightarrow{AB}$ $\overleftrightarrow{BA}$ $\overleftrightarrow{AC}$ $\overleftrightarrow{CA}$ $\overleftrightarrow{AD}$ $\overleftrightarrow{DA}$ $\overleftrightarrow{BC}$ $\overleftrightarrow{CB}$ $\overleftrightarrow{BD}$ $\overleftrightarrow{DB}$ $\overleftrightarrow{CD}$ $\overleftrightarrow{DC}$

b. a plane containing point *C*

The plane can be named as plane $\mathcal{N}$. You can also use the letters of any three *noncollinear* points to name the plane.

plane *ABE* plane *ACE* plane *ADE* plane *BCE* plane *BDE* plane *CDE*

The letters of each name can be reordered to create other names for this plane. For example, *ABE* can be written as *AEB*, *BEA*, *BAE*, *EBA*, and *EAB*.

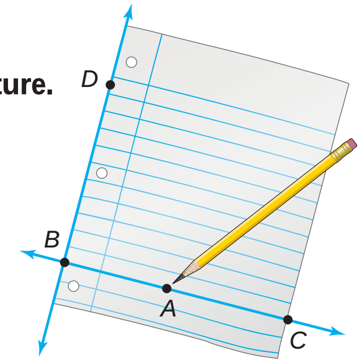
Study Tip

Dimension
A point has no dimension. A line exists in one dimension. However, a square is two-dimensional, and a cube is three-dimensional.

1. Use the figure to name a plane containing points *A* and *D*.

EXAMPLE 2 Model Points, Lines, and Planes

Name the geometric shapes modeled by the picture.



The pencil point models point *A*.

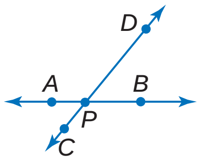
The blue rule on the paper models line *BC*.

The edge of the paper models line *BD*.

The sheet of paper models plane *ADC*.

2. Name the geometric shape modeled by stripes on a sweater.

Two lines intersect in a point. In the figure at the right, point *P* represents the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. Lines can intersect planes, and planes can intersect each other.



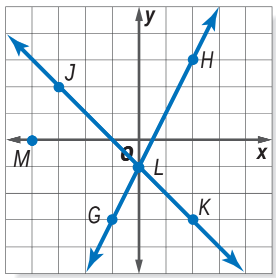
Study Tip

Naming Points
Recall that points on the coordinate plane are named using *rectangular coordinates* or *ordered pairs*. Point *G* can be named as $G(-1, -3)$.

EXAMPLE 3 Draw Geometric Figures

Draw and label a figure for each relationship.

a. ALGEBRA Lines *GH* and *JK* intersect at *L* for $G(-1, -3)$, $H(2, 3)$, $J(-3, 2)$, and $K(2, -3)$ on a coordinate plane. Point *M* is coplanar with these points, but not collinear with $\overleftrightarrow{GH}$ or $\overleftrightarrow{JK}$.



Graph each point and draw $\overleftrightarrow{GH}$ and $\overleftrightarrow{JK}$.

Label the intersection point as *L*.

An infinite number of points are coplanar with *G*, *H*, *J*, *K*, and *L*, but not collinear with $\overleftrightarrow{GH}$ or $\overleftrightarrow{JK}$. In the graph, one such point is $M(-4, 0)$.

Study Tip

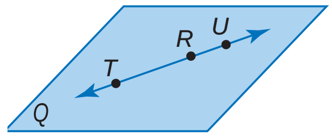
Three-Dimensional Drawings

Because it is impossible to show space or an entire plane in a figure, edged shapes with different shades of color are used to represent planes. If the lines are hidden from view, the lines or segments are shown as dashed lines or segments.

b. $\overleftrightarrow{TU}$ lies in plane *Q* and contains point *R*.

Draw a surface to represent plane *Q* and label it.
Draw a line anywhere on the plane.
Draw dots on the line for points *T* and *U*.
Since $\overleftrightarrow{TU}$ contains *R*, point *R* lies on $\overleftrightarrow{TU}$.
Draw a dot on $\overleftrightarrow{TU}$ and label it *R*.

The locations of points *T*, *R*, and *U* are totally arbitrary.

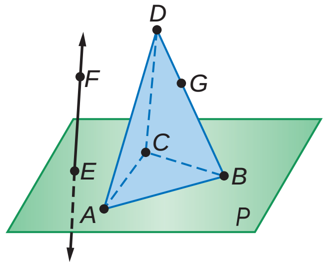


3. Draw and label a figure in which points *A*, *B*, and *C* are coplanar and *B* and *C* are collinear.

Personal Tutor at geometryonline.com

Points, Lines, and Planes in Space **Space** is a boundless, three-dimensional set of all points. Space contains lines and planes.

EXAMPLE 4 Interpret Drawings



a. How many planes appear in this figure?

There are four planes: plane *P*, plane *ADB*, plane *BCD*, plane *ACD*.

b. Name three points that are collinear.

Points *D*, *B*, and *G* are collinear.

c. Are points *G*, *A*, *B*, and *E* coplanar? Explain.

Points *A*, *B*, and *E* lie in plane *P*, but point *G* does not lie in plane *P*. Thus, they are not coplanar. Points *A*, *G*, and *B* lie in a plane, but point *E* does not lie in plane *AGB*.

d. At what point do $\overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ intersect?

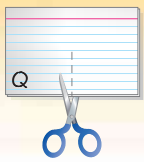
$\overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ do not intersect. $\overleftrightarrow{AB}$ lies in plane *P*, but only point *E* lies in *P*.

4. Name the intersection of plane *BCD* and plane *P*.

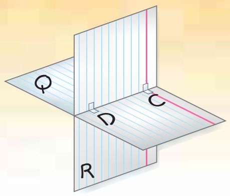
GEOMETRY LAB

Modeling Intersecting Planes

- Label one index card as *Q* and another as *R*.



- Hold the two index cards together and cut a slit halfway through both cards.
- Hold the cards so that the slits meet and insert one card into the slit of the other. Use tape to hold the cards together.
- Where the two cards meet models a line. Draw the line and label two points, *C* and *D*, on the line.



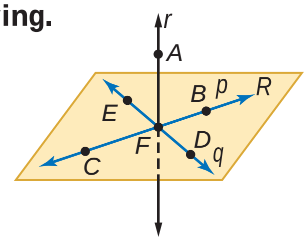
ANALYZE

1. Draw a point *F* on your model so that it lies in *Q* but not in *R*. Can *F* lie on $\overleftrightarrow{DC}$? Explain.
2. If point *H* lies in both *Q* and *R*, where would it lie? Draw point *H* on your model.
3. Draw a sketch of your model on paper. Label all points, lines, and planes.

Check Your Understanding

Example 1 (p. 7)

Use the figure at the right to name each of the following.



1. a line containing point *B*
2. a plane containing points *D* and *C*

Example 2 (p. 7)

Name the geometric term modeled by each object.

3. the beam from a laser
4. a ceiling

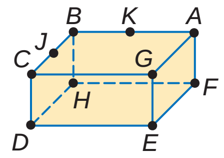
Example 3 (pp. 7–8)

Draw and label a figure for each relationship.

5. A line in a coordinate plane contains *X*(3, - 1), *Y*(- 3, - 4), and *Z*(- 1, - 3) and a point *W* that does not lie on $\overleftrightarrow{XY}$.
6. Plane *Q* contains lines *r* and *s* that intersect in *P*.

Example 4 (p. 8)

For Exercises 7–9, refer to the figure.

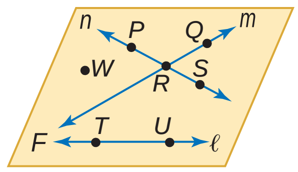


7. How many planes are shown in the figure?
8. Name three points that are collinear.
9. Are points *A*, *C*, *D*, and *J* coplanar? Explain.

Exercises

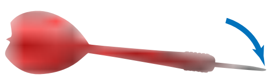
HOMEWORK HELP	
For Exercises	See Examples
10–15	1
16–22	2
23–30	3
31–34	4

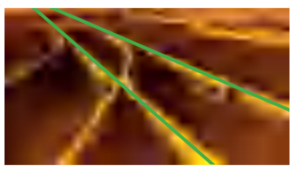
Refer to the figure.

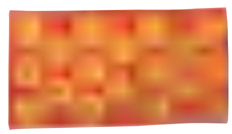


10. Name a line that contains point *P*.
11. Name the plane containing lines *n* and *m*.
12. Name the intersection of lines *n* and *m*.
13. Name a point not contained in lines ℓ, *m*, or *n*.
14. What is another name for line *n*?
15. Does line ℓ intersect line *m* or line *n*? Explain.

Name the geometric term(s) modeled by each object.

16. 

17. 

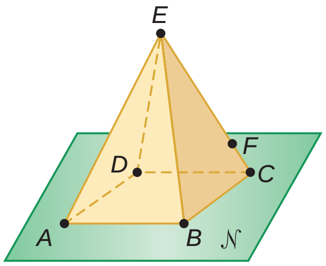
18. 

19. a tablecloth
20. a partially-opened newspaper
21. woven threads in a piece of cloth
22. a knot in a string

Draw and label a figure for each relationship.

23. Line *AB* intersects plane *Q* at *W*.
24. Point *T* lies on $\overleftrightarrow{WR}$.
25. Points *Z*(4, 2), *R*(- 4, 2), and S are collinear, but points *Q*, *Z*, *R*, and *S* are not.
26. The coordinates for points *C* and *R* are (- 1, 4) and (6, 4), respectively. $\overleftrightarrow{RS}$ and $\overleftrightarrow{CD}$ intersect at *P*(3, 2).

Refer to the figure.



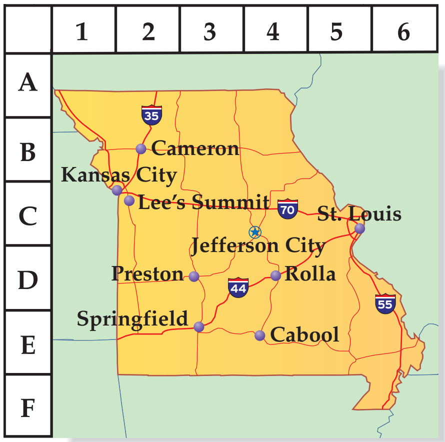
27. How many planes are shown in the figure?

28. How many planes contain points B, C, and E?

29. Name three collinear points.

30. Where could you add point G on plane N so that A, B, and G would be collinear?

MAPS For Exercises 31–34, refer to the map, and use the following information. A map represents a plane. Locations on this plane are named using a letter / number combination.



31. Name the letter / number combination where St. Louis is located.

32. Name the letter / number combination where Springfield is located.

33. What city is located at (B, 2)?

34. What city is located at (D, 4)?

Real-World Career

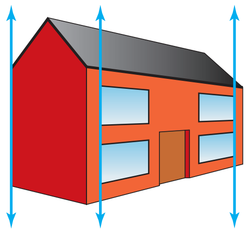
Engineering Technician

Engineering technicians or drafters use perspective to create drawings used in construction, and manufacturing. Technicians must have knowledge of math, science, and engineering.

Math Online

For more information, go to geometryonline.com.

ONE-POINT PERSPECTIVE One-point perspective drawings use lines to convey depth in a picture. Lines representing horizontal lines in the real object can be extended to meet at a single point called the *vanishing point*.



35. Trace the figure at the right. Draw all of the vertical lines. Three are already drawn for you.

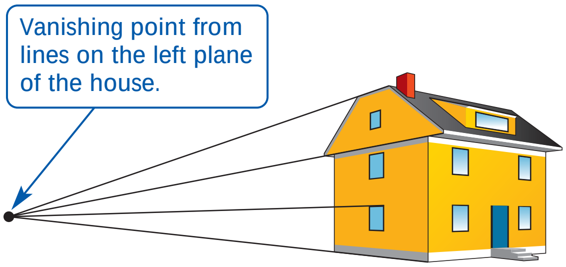
36. Draw and extend the horizontal lines to locate the vanishing point and label it.

37. Draw a one-point perspective of your classroom or a room in your house.

38. **RESEARCH** Use the Internet or other research resources to investigate one-point perspective drawings in which the vanishing point is in the center of the picture. How do they differ from the drawing for Exercises 35–37?

TWO-POINT PERSPECTIVE Two-point perspective drawings also use lines to convey depth, but two sets of lines can be drawn to meet at two vanishing points.

39. Trace the outline of the house. Draw all of the vertical lines.



40. Draw and extend the lines on your sketch representing horizontal lines in the real house to identify the vanishing point on the right plane in this figure.

41. Which type of lines seems to be unaffected by any type of perspective drawing?

Michael Newman/PhotoEdit

EXTRA PRACTICE
See pages 800, 828.

Math Online
Self-Check Quiz at
geometryonline.com

Another way to describe a group of points is called a locus. A locus is a set of points that satisfy a particular condition.

42. Find five points that satisfy the equation $4 - x = y$. Graph them on a coordinate plane and describe the geometric figure they suggest.

43. Find ten points that satisfy the inequality $y > -2x + 1$. Graph them on a coordinate plane and describe the geometric figure they suggest.

H.O.T. Problems

44. **OPEN ENDED** Fold a sheet of paper. Open the paper and fold it again in a different way. Open the paper and label the geometric figures you observe. Describe the figures.

45. **FIND THE ERROR** Raymond and Micha were looking for patterns to determine how many ways there are to name a plane given a certain number of points. Who is correct? Explain your reasoning.

Raymond	Micha
If there are 4 points, then there are $4 \cdot 3 \cdot 2$ ways to name the plane.	If there are 5 noncollinear points, then there are $5 \cdot 4 \cdot 3$ ways to name the plane.

46. **CHALLENGE** Describe a real-life example of three lines in space that do not intersect each other and no two of which lie in the same plane.

47. **Writing in Math** Refer to the information about chairs on page 6. Explain how the chair legs relate to points in a plane. Include how many legs would create a chair that does not wobble.

48. Four lines are coplanar. What is the *greatest* number of intersection points that can exist?

A 4

B 5

C 6

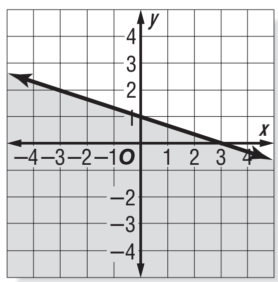
D 7

49. **REVIEW** What is the value of x if $-5x + 4 = -6$?

F -5 **H** 2

G -2 **J** 5

50. **REVIEW** Which inequality is shown on the graph below?



A $y > -\frac{1}{3}x + 1$ **C** $y \leq -\frac{1}{3}x + 1$

B $y < -\frac{1}{3}x + 1$ **D** $y \geq -\frac{1}{3}x + 1$

PREREQUISITE SKILL **Replace each ● with >, <, or = to make a true statement.**

51. $\frac{1}{2}$ in. ● $\frac{3}{8}$ in.

52. $\frac{4}{16}$ in. ● $\frac{1}{4}$ in.

53. $\frac{4}{5}$ in. ● $\frac{6}{10}$ in.

54. 10 mm ● 1 cm

55. 2.5 cm ● 28 mm

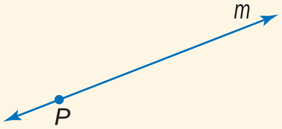
56. 0.025 cm ● 25 mm

READING MATH

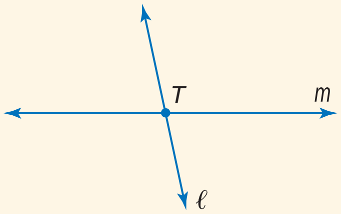
Describing What You See

Figures play an important role in understanding geometric concepts. It is helpful to know what words and phrases can be used to describe figures. Likewise, it is important to know how to read a geometric description and be able to draw the figure it describes.

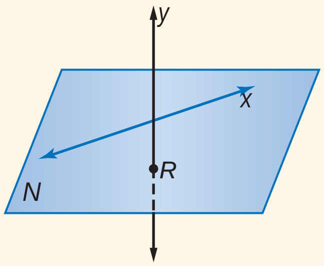
The figures and descriptions below help you visualize and write about points, lines, and planes.



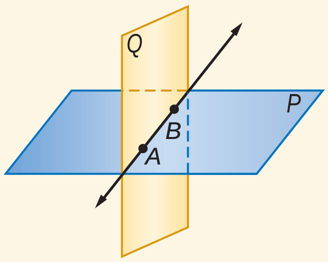
Point P is on line m.
Line m contains P.
Line m passes through P.



Lines ℓ and m intersect in T.
Point T is the interesection of lines ℓ and m.
Point T is on line m. Point T is on line ℓ.



Line x and point R are in N.
Point R lies in N.
Plane N contains R and line x.
Line y intersects N at R.
Point R is the intersection of line y with N.
Lines y and x do not intersect.

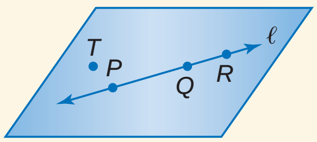


$\overleftrightarrow{AB}$ is in P and Q.
Points A and B lie in both P and Q.
Planes P and Q both contain $\overleftrightarrow{AB}$.
Planes P and Q intersect in $\overleftrightarrow{AB}$.
$\overleftrightarrow{AB}$ is the intersection of P and Q.

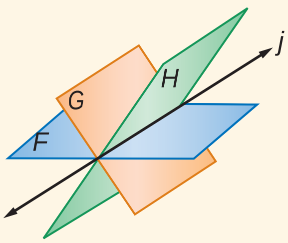
Reading to Learn

Write a description for each figure.

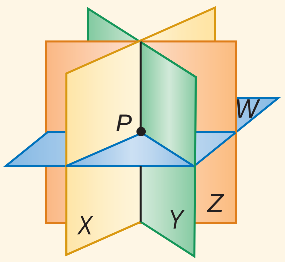
1.



2.



3.



4. Draw and label a figure for the statement *Planes A, B, and C do not intersect.*

1-2 Linear Measure and Precision

Main Ideas

- Measure segments and determine accuracy of measurement.
- Compute with measures.

New Vocabulary

line segment
precision
betweenness of points
between
congruent
construction
absolute error
relative error

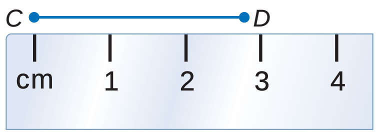
With the abundance of rulers, yardsticks, laser measuring devices, and other instruments for measurement, it is easy to take our system of measurement for granted. When the ancient Egyptians found a need for a measurement system, they used the human body as a guide. The cubit was the length of an arm from the elbow to the fingertips. Eventually the Egyptians standardized the length of a cubit.

Measure Line Segments Unlike a line, a **line segment**, or *segment*, can be measured because it has two endpoints. A segment with endpoints A and B can be named as $\overline{AB}$ or $\overline{BA}$. The *measure* of $\overline{AB}$ is written as AB. The length or measure of a segment always includes a unit of measure, such as meter or inch.

EXAMPLE 1 Length in Metric Units

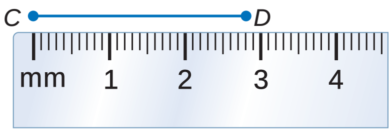
Find the length of $\overline{CD}$ using each ruler.

a.



The ruler is marked in centimeters. Point D is closer to the 3-centimeter mark than to 2 centimeters.
Thus, $\overline{CD}$ is about 3 centimeters long.

b.

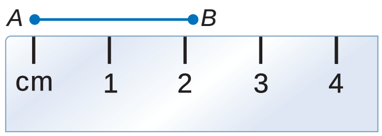


The long marks are centimeters, and the shorter marks are millimeters. There are 10 millimeters for each centimeter. Thus, $\overline{CD}$ is about 28 millimeters long.

1A. Measure the length of a dollar bill in centimeters.

1B. Measure the length of a pencil in millimeters.

1C. Find the length of $\overline{AB}$.



Nebamum Hunting in the Marshes with His Wife and Daughter. c. B.C.E. 1350, Egyptian. Wall painting from the tomb of Nebamum, Thebes/British Museum, London/Bridgeman Art Library

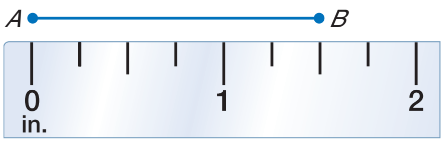
Study Tip

Using a Ruler The zero point on a ruler may not be clearly marked. For some rulers, zero is the left edge of the ruler. On others, it may be a line farther in on the scale. If it is not clear where zero is, align one endpoint on 1 and subtract 1 from the measurement at the other endpoint.

EXAMPLE 2 Length in Customary Units

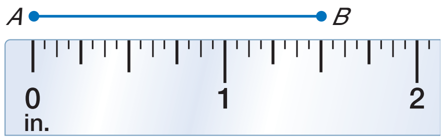
Find the length of $\overline{AB}$ using each ruler.

a.



Each inch is divided into fourths. Point B is closer to the $1\frac{2}{4}$-inch mark. Thus, $\overline{AB}$ is about $1\frac{2}{4}$ or $1\frac{1}{2}$ inches long.

b.



Each inch is divided into sixteenths. Point B is closer to the $1\frac{8}{16}$-inch mark. Thus, $\overline{AB}$ is about $1\frac{8}{16}$ or $1\frac{1}{2}$ inches long.

2A. Measure the length of a dollar bill in inches.

2B. Measure the length of a pencil in inches.

The **precision** of any measurement depends on the smallest unit available on the measuring tool. The measurement should be precise to within 0.5 unit of measure. For example, in part **a** of Example 1, 3 centimeters means that the actual length is no less than 2.5 centimeters, but no more than 3.5 centimeters.

Measurements of 28 centimeters and 28.0 centimeters indicate different precision in measurement. A measurement of 28 centimeters means that the ruler is divided into centimeters. However, a measurement of 28.0 centimeters indicates that the ruler is divided into millimeters.

The precision of a measurement in customary units is determined before reducing the fraction. For example, if you measure the length of an object to be $2\frac{2}{4}$ inches, then the measurement is precise to within $\frac{1}{8}$ inch. In this book, the given measurements are indicative of the precision of the measuring instrument.

Study Tip

Units of Measure A measurement of 38.0 centimeters on a ruler with millimeter marks means a measurement of 380 millimeters. So the actual measurement is between 379.5 millimeters and 380.5 millimeters, not 37.5 centimeters and 38.5 centimeters. The range of error in the measurement is called the **tolerance** and can be expressed as ± 0.5.

EXAMPLE 3 Precision

Find the precision for each measurement. Explain its meaning.

a. 5 millimeters

The measurement is precise to within 0.5 millimeter. So, a measurement of 5 millimeters could be 4.5 to 5.5 millimeters.

b. $8\frac{1}{2}$ inches

The measurement is precise to within $\frac{1}{2}\left(\frac{1}{2}\right)$ or $\frac{1}{4}$ inch. Therefore, the measurement could be between $8\frac{1}{4}$ inches and $8\frac{3}{4}$ inches.

3A. Find the precision for the measure $10\frac{1}{4}$ inches. Explain its meaning.

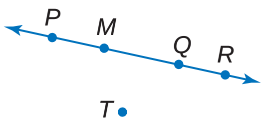
3B. Find the precision for the measure 12.2 centimeters. Explain its meaning.

Study Tip

Comparing Measures

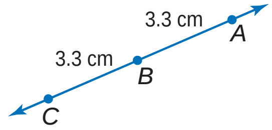
Because measures are real numbers, you can compare them. If X, Y, and Z are collinear in that order, then one of these statements is true. $XY = YZ$, $XY > YZ$, or $XY < YZ$.

Calculate Measures Measures are real numbers, so all arithmetic operations can be used with them. Recall that for any two real numbers a and b, there is a real number n between a and b such that $a < n < b$. This relationship also applies to points on a line and is called **betweenness of points**. Point M is **between** points P and Q if and only if P, Q, and M are collinear and $PM + MQ = PQ$. Notice that points R and T are not between points P and Q.



EXAMPLE 4 Find Measurements

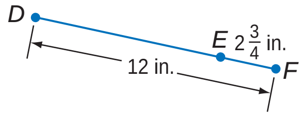
a. Find AC.



Point B is between A and C. AC can be found by adding AB and BC.

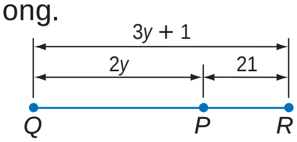
$AB + BC = AC$	Betweenness of points
$3.3 + 3.3 = AC$	Substitution
$6.6 = AC$	So, AC is 6.6 centimeters long.

b. Find DE.



$DE + EF = DF$	Betweenness of points
$DE + 2\frac{3}{4} = 12$	Substitution
$DE + 2\frac{3}{4} - 2\frac{3}{4} = 12 - 2\frac{3}{4}$	Subtract $2\frac{3}{4}$ from each side.
$DE = 9\frac{1}{4}$	So, DE is $9\frac{1}{4}$ inches long.

c. Find y and QP if P is between Q and R, $QP = 2y$, $QR = 3y + 1$, and $PR = 21$.



Draw a figure to represent this information.

$QR = QP + PR$	Betweenness of points
$3y + 1 = 2y + 21$	Substitute known values.
$3y + 1 - 1 = 2y + 21 - 1$	Subtract 1 from each side.
$3y = 2y + 20$	Simplify.
$3y - 2y = 2y + 20 - 2y$	Subtract $2y$ from each side.
$y = 20$	Simplify.

Now find QP.

$QP = 2y$	Given
$= 2(20)$ or 40	$y = 20$

4. Find x and BC if B is between A and C, $AC = 4x - 12$, $AB = x$, and $BC = 2x + 3$.

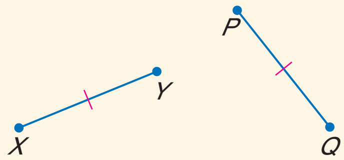
Look at the figure in part **a** of Example 4. Notice that $\overline{AB}$ and $\overline{BC}$ have the same measure. When segments have the same measure, they are said to be **congruent**.

KEY CONCEPT — Congruent Segments

Words Two segments having the same measure are congruent.

Symbol $\cong$ is read *is congruent to*.
Red slashes on the figure also indicate that segments are congruent.

Model $\overline{XY} \cong \overline{PQ}$

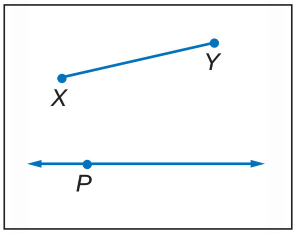


Constructions are methods of creating geometric figures without the benefit of measuring tools. Generally, only a pencil, straightedge, and compass are used. You can construct a segment that is congruent to a given segment.

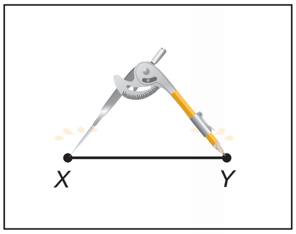
CONSTRUCTION

Copy a Segment

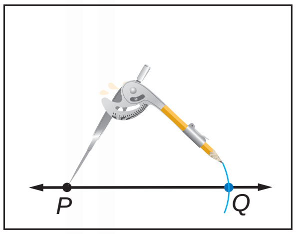
Step 1 Draw a segment $\overline{XY}$. Elsewhere on your paper, draw a line and a point on the line. Label the point P.



Step 2 Place the compass at point X and adjust the compass setting so that the pencil is at point Y.



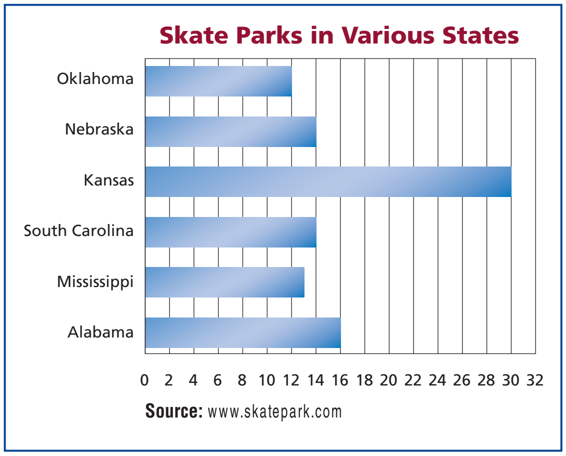
Step 3 Using that setting, place the compass point at P and draw an arc that intersects the line. Label the point of intersection Q. Because of identical compass settings, $\overline{PQ} \cong \overline{XY}$.



Congruent Segments

5 SKATE PARKS **In the graph at the right, suppose a segment was drawn along the top of each bar. Which states would have segments that are congruent? Explain.**

The segments on the bars for Nebraska and South Carolina would be congruent because they both represent the same number of skate parks.



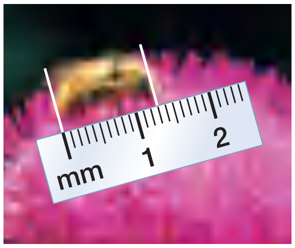
5. Suppose Oklahoma added another skate park. The segment drawn along the bar representing Oklahoma would be congruent to which other segment?

Check Your Understanding

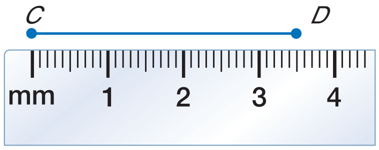
Example 1 (p. 13)

Find the length of each line segment or object.

1.

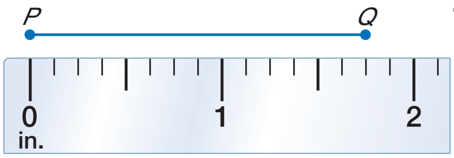


2.

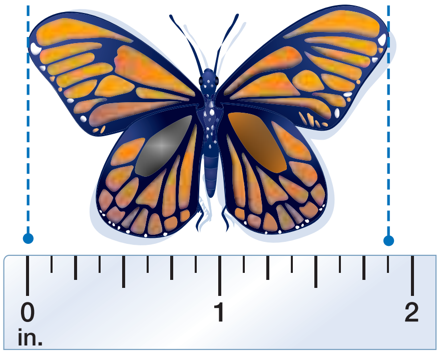


Example 2 (p. 14)

3.



4.



Example 3 (p. 14)

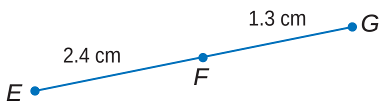
5. Find the precision for a measurement of 14 meters. Explain its meaning.

6. Find the precision for a measurement of $3\frac{1}{4}$ inches. Explain its meaning.

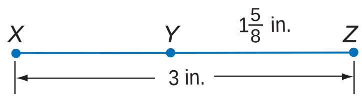
Example 4 (p. 15)

Find the measurement of each segment. Assume that each figure is not drawn to scale.

7. $\overline{EG}$



8. $\overline{XY}$



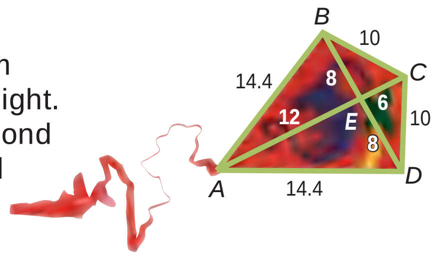
Example 5 (p. 16)

Find the value of *x* and *LM* if *L* is between *N* and *M*.

9. $NL = 5x$, $LM = 3x$, and $NL = 15$

10. $NL = 6x - 5$, $LM = 2x + 3$, and $NM = 30$

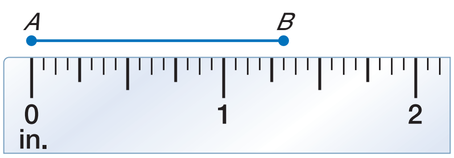
11. **KITES** Kite making has become an art form using numerous shapes and designs for flight. The figure at the right is known as a diamond kite. The measures are in inches. Name all of the congruent segments in the figure.



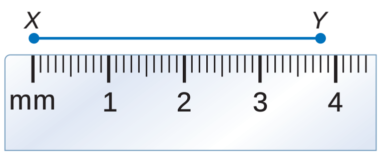
Exercises

Find the length of each line segment or object.

12.



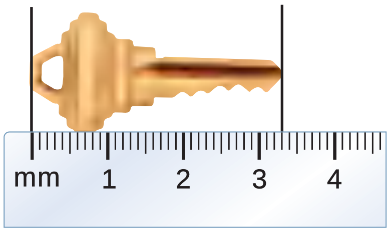
13.



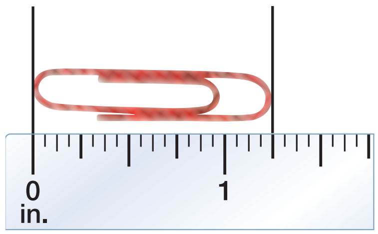
HOMEWORK HELP	
For Exercises	See Examples
13, 14	1
12, 15	2
16–21	3
22–27	4
28–33	5

Find the length of each line segment or object.

14.



15.



Find the precision for each measurement. Explain its meaning.

16. 80 in. **17.** 22 mm **18.** $16\frac{1}{2}$ in.

19. 308 cm **20.** 3.75 meters **21.** $3\frac{1}{4}$ ft

Study Tip

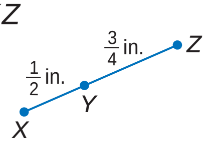
Information from Figures

Segments that have the same measure are congruent. Congruence marks are used to indicate this.

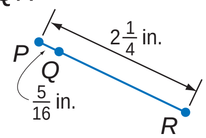
Find the measurement of each segment.

22. AC

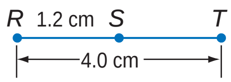
23. XZ



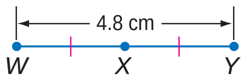
24. QR



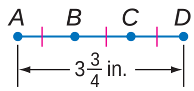
25. ST



26. WX



27. BC



Find the value of the variable and *ST* if *S* is between *R* and *T*.

28. $RS = 7a$, $ST = 12a$, $RT = 76$

29. $RS = 12$, $ST = 2x$, $RT = 34$

30. $RS = 2x$, $ST = 3x$, $RT = 25$

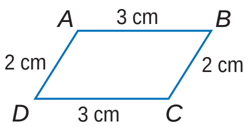
31. $RS = 16$, $ST = 2x$, $RT = 5x + 10$

32. $RS = 3y + 1$, $ST = 2y$, $RT = 21$

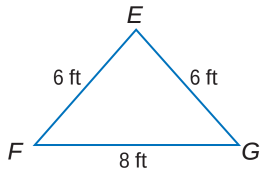
33. $RS = 4y - 1$, $ST = 2y - 1$, $RT = 5y$

Use the figures to determine whether each pair of segments is congruent.

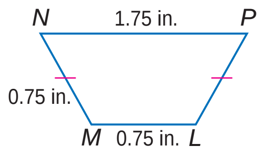
34. $\overline{AB}, \overline{CD}$



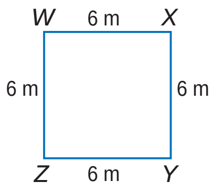
35. $\overline{EF}, \overline{FG}$



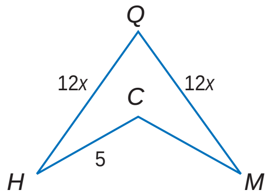
36. $\overline{NP}, \overline{LM}$



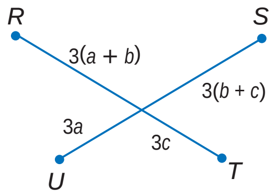
37. $\overline{WX}, \overline{XY}$



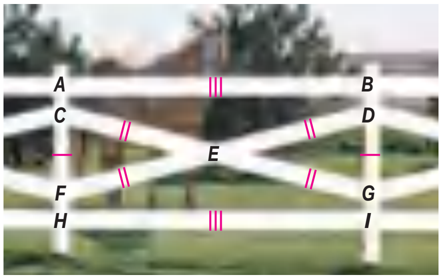
38. $\overline{CH}, \overline{CM}$



39. $\overline{TR}, \overline{SU}$



40. FENCES Name all segments in the crossbuck pattern in the picture that appear to be congruent.



41. MUSIC A CD has a single spiral track of data, circling from the inside of the disc to the outside. Use a metric ruler to determine the full width of a music CD.

42. **JEWELRY** Roxanna sells the jewelry that she makes. One necklace is made from a $14\frac{3}{4}$-inch length of cord. What are the greatest and least possible lengths for the cords to make these necklaces? Assume the same measuring device is used to measure each cord. Explain.

Real-World Link

There are more than 3300 state parks, historic sites, and natural areas in the United States. Most of the parks are open to visitors year round.

Source: *Parks Directory of the United States*

PERIMETER For Exercises 43 and 44, use the following information.
The *perimeter* of a geometric figure is the sum of the lengths of its sides. Pablo used a ruler divided into centimeters and measured the sides of a triangle as 3 centimeters, 5 centimeters, and 6 centimeters. Use what you know about the precision of any measurement to answer each question.

43. What is the least possible perimeter of the triangle? Explain.

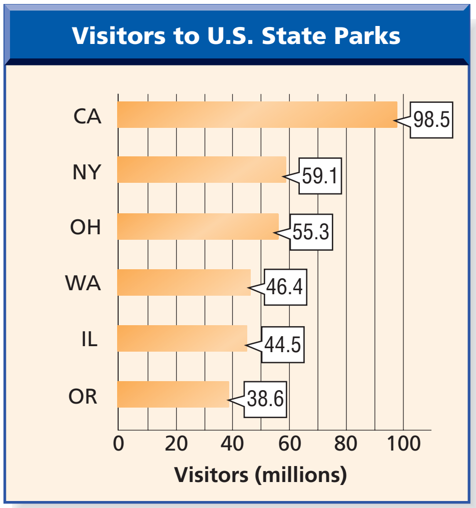
44. What is the greatest possible perimeter of the triangle? Explain.

RECREATION For Exercises 45–47, refer to the graph that shows the states with the greatest number of visitors to state parks in a recent year.

45. To what number can the precision of the data be measured?

46. Find the precision for the California data.

47. Can you be sure that 1.9 million more people visited Washington state parks than Illinois state parks? Explain.



Source: National Association of Park Directors

ERROR Accuracy is an indication of error. The absolute value of the difference between the actual measure of an object and the allowable measure is the **absolute error**. The **relative error** is the ratio of the absolute error to the actual measure. The relative error is expressed as a percent. For a length of 11 inches and an allowable error of 0.5 inch, the absolute error and relative error can be found as follows.

$$\frac{\text{absolute error}}{\text{measure}} = \frac{|11 \text{ in.} - 11.5 \text{ in.}|}{11 \text{ in.}} = \frac{0.5 \text{ in.}}{11 \text{ in.}} \approx 0.045 \text{ or } 4.5\%$$

Determine the relative error for each measurement.

48. 27 ft

49. $14\frac{1}{2}$ in.

50. 42.3 cm

51. 63.7 km

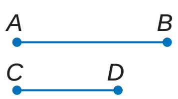
EXTRA PRACTICE

See pages 800, 828.

Math Online

Self-Check Quiz at geometryonline.com

CONSTRUCTION For Exercises 52 and 53, refer to the figure.



52. Construct a segment with a measure of 4(CD).

53. Construct a segment that has length 3(AB) - 2(CD).

H.O.T. Problems

54. **REASONING** Explain how to measure a segment with a ruler divided into eighths of an inch.

55. **OPEN ENDED** Give two examples of some geometric figures that have congruent segments.

CHALLENGE **Significant digits** represent the accuracy of a measurement.

- Nonzero digits are always significant.
- In whole numbers, zeros are significant if they fall between nonzero digits.
- In decimal numbers greater than or equal to 1, every digit is significant.
- In decimal numbers less than 1, the first nonzero digit and every digit to its right are significant.

For example, 600.070 has six significant digits, but 0.0210 has only three. How many significant digits are there in each measurement below?

56. 83,000 miles **57.** 33,002 miles **58.** 450.0200 liters

59. **Writing in Math** Why is it important to have a standard of measure? Refer to the information on ancient Egyptian measurement on page 13. Include an advantage and disadvantage to the ancient Egyptian measurement system.

60. A 36-foot-long ribbon is cut into three pieces. The first piece of ribbon is half as long as the second piece of ribbon. The third piece of ribbon is 1 foot longer than twice the length of the second piece of ribbon. What is the length of the longest piece of ribbon?

A 10 feet **C** 21 feet

B 12 feet **D** 25 feet

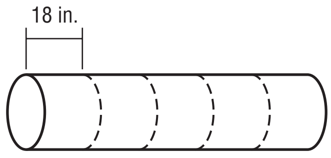
61. **REVIEW** The pipe shown is divided into five equal sections. How long is the pipe in feet (ft) and inches (in.)?

F 6 ft 0 in.

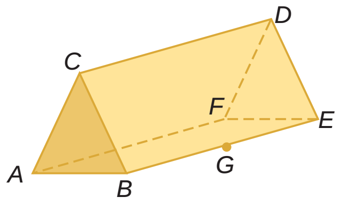
G 6 ft 6 in.

H 7 ft 5 in.

J 7 ft 6 in.



Refer to the figure at the right. (Lesson 1-1)



62. Name three collinear points.

63. Name two planes that contain points B and C.

64. Name another point in plane DFA.

65. How many planes are shown?

TRAVEL **The Hernandez family is driving from Portland, Oregon, to Seattle, Washington. They are using maps to navigate a route. Name the geometric term modeled by each object.** (Lesson 1-1)

66. map **67.** highway **68.** city

PREREQUISITE SKILL **Evaluate each expression if $a = 3$, $b = 8$, and $c = 2$.** (Page 780)

69. $2a + 2b$

70. $ac + bc$

71. $\frac{a - c}{2}$

72. $\sqrt{(c - a)^2}$

1-3 Distance and Midpoints

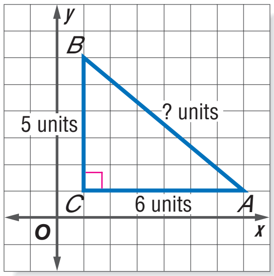
Main Ideas

- Find the distance between two points.
- Find the midpoint of a segment.

New Vocabulary

midpoint
segment bisector

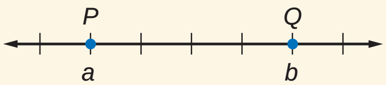
When you connect two points on a number line or on a plane, you have graphed a line segment. Distance on a number line is determined by counting the units between the two points. On a coordinate plane, you can use the Pythagorean Theorem to find the distance between two points. In the figure, to find the distance from A to B, use $(AC)^2 + (CB)^2 = (AB)^2$.



Distance Between Two Points The coordinates of the endpoints of a segment can be used to find the length of the segment. Because the distance from A to B is the same as the distance from B to A, the order in which you name the endpoints makes no difference.

KEY CONCEPT — *Distance Formulas*

Number Line



$PQ = |b - a|$ or $|a - b|$

Coordinate Plane

The distance d between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

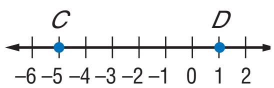
Study Tip

Alternative Method

$CD = |1 - (-5)|$
$= |6|$ or 6

EXAMPLE Find Distance on a Number Line

1 Use the number line to find *CD*.

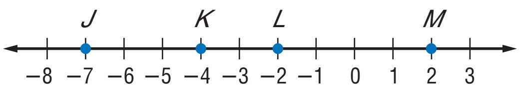


The coordinates of C and D are –5 and 1.

$CD = |-5 - 1|$ Distance Formula

$= |-6|$ or 6 Simplify.

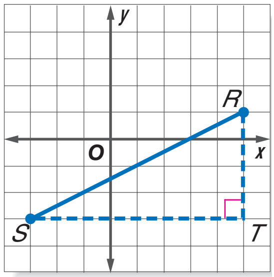
Use the number line to find each measure.



1A. KM **1B.** JM **1C.** KL **1D.** JL

EXAMPLE Find Distance on a Coordinate Plane

1 **Find the distance between $R(5, 1)$ and $S(-3, -3)$.**



Method 1 Pythagorean Theorem

Use the gridlines to form a triangle.

$(RS)^2 = (RT)^2 + (ST)^2$ Pythagorean Theorem

$(RS)^2 = 4^2 + 8^2$ $RT = 4$ units, $ST = 8$ units

$(RS)^2 = 80$ Simplify.

$RS = \sqrt{80}$ Take the square root of each side.

Method 2 Distance Formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$RS = \sqrt{(-3 - 5)^2 + (-3 - 1)^2}$ $(x_1, y_1) = (5, 1)$ and $(x_2, y_2) = (-3, -3)$

$RS = \sqrt{(-8)^2 + (-4)^2}$ Subtract.

$RS = \sqrt{64 + 16}$ or $\sqrt{80}$ Simplify.

The distance from R to S is $\sqrt{80}$ units. Use a calculator to find that $\sqrt{80}$ is approximately 8.94.

2. Find the distance between $D(-5, 6)$ and $E(8, -4)$.

Study Tip

Pythagorean Theorem

Recall that the Pythagorean Theorem is often expressed as $a^2 + b^2 = c^2$, where a and b are the measures of the shorter sides (legs) of a right triangle and c is the measure of the longest side (hypotenuse) of a right triangle.

Midpoint of a Segment The **midpoint** of a segment is the point on the segment that divides the segment into two congruent segments. If X is the midpoint of $\overline{AB}$, then $AX = XB$. In the lab you will derive the Midpoint Formula.

GEOMETRY LAB

Midpoint of a Segment

MODEL

- Graph points $A(5, 5)$ and $B(-1, 5)$ on grid paper. Draw $\overline{AB}$.
- Hold the paper up to the light and fold the paper so that points A and B match exactly. Crease the paper slightly. Then open the paper.
- Put a point where the crease intersects $\overline{AB}$. Label this midpoint as C.
- Repeat using endpoints $X(-4, 3)$ and $Y(2, 7)$. Label the midpoint Z.

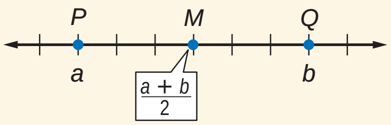
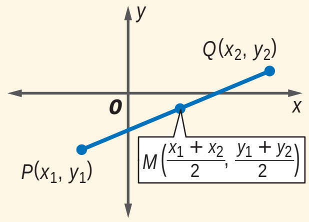
MAKE A CONJECTURE

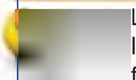
1. What are the coordinates of point C? What are the measures of $\overline{AC}$ and $\overline{CB}$?

2. What are the coordinates of point Z? What are the measures of $\overline{XZ}$ and $\overline{ZY}$?

3. Study the coordinates of points A, B, and C. Write a rule that relates these coordinates. Then use points X, Y, and Z to verify your conjecture.

KEY CONCEPT — Midpoint

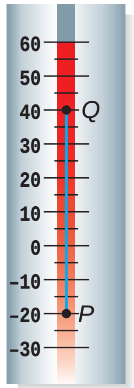
Words	The midpoint M of $\overline{PQ}$ is the point between P and Q such that $PM = MQ$.	
Symbols	**Number Line**	**Coordinate Plane**
	The coordinate of the midpoint of a segment with endpoints that have coordinates a and b is $\frac{a+b}{2}$.	The coordinates of the midpoint of a segment with endpoints that have coordinates (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.
Models		



Cross-Curricular Project Latitude and longitude form another coordinate system. The latitude, longitude, degree distance, and monthly high temperature can be used to create several different scatter plots. Visit geometryonline.com to continue work on your project.

EXAMPLE 3 Find Coordinates of Midpoint

TEMPERATURE Find the coordinate of the midpoint of $\overline{PQ}$.



The coordinates of P and Q are -20 and 40.

Let M be the midpoint of $\overline{PQ}$.

$M = \frac{-20 + 40}{2}$ $a = -20, b = 40$

$= \frac{20}{2}$ or 10 Simplify.

The midpoint is 10.

3. TEMPERATURE The temperature dropped from 25° to -8°. Find the midpoint of these temperatures.

EXAMPLE 4 Find Coordinates of Midpoint

Find the coordinates of M, the midpoint of $\overline{JK}$, for $J(-1, 2)$ and $K(6, 1)$.

Let J be (x_1, y_1) and K be (x_2, y_2).

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = M\left(\frac{-1+6}{2}, \frac{2+1}{2}\right) \quad (x_1, y_1) = (-1, 2), (x_2, y_2) = (6, 1)$$

$$= M\left(\frac{5}{2}, \frac{3}{2}\right) \text{ or } M\left(2\tfrac{1}{2}, 1\tfrac{1}{2}\right) \quad \text{Simplify.}$$

4. Find the coordinates of the midpoint of $\overline{AB}$ for $A(5, 12)$ and $B(-4, 8)$.

You can also find the coordinates of an endpoint of a segment if you know the coordinates of its other endpoint and its midpoint.

EXAMPLE Find Coordinates of Endpoint

5 **Find the coordinates of X if $Y(-1, 6)$ is the midpoint of $\overline{XZ}$ and Z has coordinates $(2, 8)$.**

Let Z be (x_2, y_2) in the Midpoint Formula.

$$Y(-1, 6) = Y\left(\frac{x_1 + 2}{2}, \frac{y_1 + 8}{2}\right) \quad (x_2, y_2) = (2, 8)$$

Write two equations to find the coordinates of X.

$-1 = \dfrac{x_1 + 2}{2}$

$-2 = x_1 + 2$ Multiply each side by 2.

$-4 = x_1$ Subtract 2 from each side.

$6 = \dfrac{y_1 + 8}{2}$

$12 = y_1 + 8$ Multiply each side by 2.

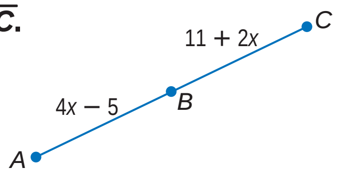
$4 = y_1$ Subtract 8 from each side.

The coordinates of X are $(-4, 4)$.

5. Find the coordinates of G if $P(-5, 10)$ is the midpoint of $\overline{EG}$ and E has coordinates $(-8, 6)$.

EXAMPLE Use Algebra to Find Measures

6 **Find the measure of $\overline{BC}$ if B is the midpoint of $\overline{AC}$.**



You know that B is the midpoint of $\overline{AC}$, and the figure gives algebraic measures for $\overline{AB}$ and $\overline{BC}$. You are asked to find the measure of $\overline{BC}$.

Because B is the midpoint, you know that $AB = BC$. Use this equation and the algebraic measures to find a value for x.

$AB = BC$ Definition of midpoint

$4x - 5 = 11 + 2x$ $AB = 4x - 5$, $BC = 11 + 2x$

$4x = 16 + 2x$ Add 5 to each side.

$2x = 16$ Subtract $2x$ from each side.

$x = 8$ Divide each side by 2.

Now substitute 8 for x in the expression for BC.

$BC = 11 + 2x$ Original measure

$= 11 + 2(8)$ $x = 8$

$= 11 + 16$ or 27 The measure of $\overline{BC}$ is 27.

6. Find the measure of $\overline{XY}$ if Y is the midpoint of $\overline{XZ}$, and $XY = 2x + 3$, and $YZ = 6 - 4x$.

Interactive Lab
geometryonline.com

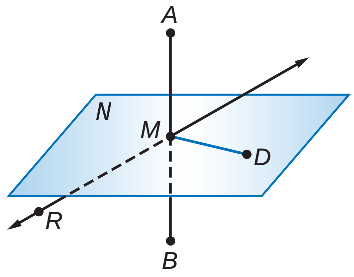
Online Personal Tutor at geometryonline.com

Study Tip

Segment Bisectors

There can be an infinite number of bisectors and each must contain the midpoint of the segment.

Any segment, line, or plane that intersects a segment at its midpoint is called a **segment bisector**. In the figure at the right, M is the midpoint of $\overline{AB}$. Plane N, $\overline{MD}$, $\overleftrightarrow{RM}$, and point M are all bisectors of $\overline{AB}$. We say that they *bisect* $\overline{AB}$.

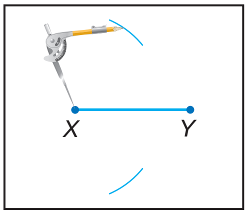


You can construct a line that bisects a segment without measuring to find the midpoint of the given segment.

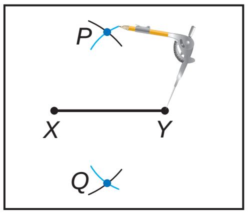
CONSTRUCTION

Bisect a Segment

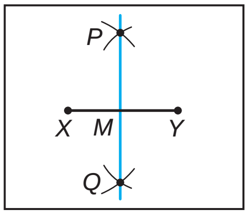
Step 1 Draw a segment and name it $\overline{XY}$. Place the compass at point X. Adjust the compass so that its width is greater than $\frac{1}{2}XY$. Draw arcs above and below $\overline{XY}$.



Step 2 Using the same compass setting, place the compass at point Y and draw arcs above and below $\overline{XY}$ that intersect the two arcs previously drawn. Label the points of intersection as P and Q.



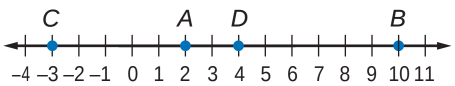
Step 3 Use a straightedge to draw $\overleftrightarrow{PQ}$. Label the point where it intersects $\overline{XY}$ as M. Point M is the midpoint of $\overline{XY}$, and $\overleftrightarrow{PQ}$ is a bisector of $\overline{XY}$. Also $XM = MY = \frac{1}{2}XY$.



✓ CHECK Your Understanding

Example 1 (p. 21)

Use the number line to find each measure.



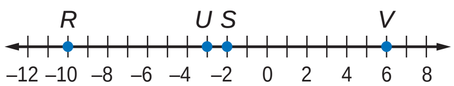
1. AB
2. CD

Example 2 (p. 22)

3. Use the Pythagorean Theorem to find the distance between $X(7, 11)$ and $Y(-1, 5)$.
4. Use the Distance Formula to find the distance between $D(2, 0)$ and $E(8, 6)$.

Example 3 (p. 23)

Use the number line to find the coordinate of the midpoint of each segment.



5. $\overline{RS}$
6. $\overline{UV}$

Example 4 (p. 23)

Find the coordinates of the midpoint of a segment having the given endpoints.

7. $X(-4, 3), Y(-1, 5)$

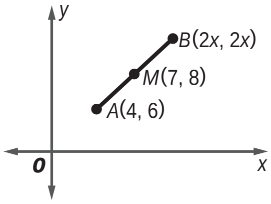
8. $A(2, 8), B(-2, 2)$

Example 5 (p. 24)

9. Find the coordinates of A if $B(0, 5.5)$ is the midpoint of $\overline{AC}$ and C has coordinates $(-3, 6)$.

Example 6 (p. 24)

10. Point M is the midpoint of $\overline{AB}$. What is the value of x in the figure?



Exercises

HOMEWORK HELP	
For Exercises	See Examples
11–16	1
17–28	2
29–34	3
35–42	4
43–45	5
46–49	6

Use the number line to find each measure.

11. DE

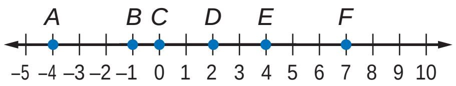
12. CF

13. AB

14. AC

15. AF

16. BE



Use the Pythagorean Theorem to find the distance between each pair of points.

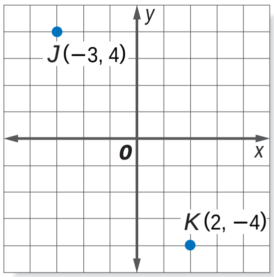
17. $A(0, 0), B(8, 6)$

18. $C(-10, 2), D(-7, 6)$

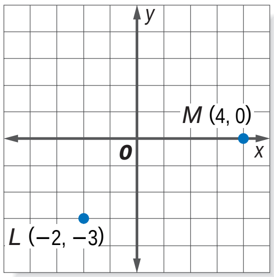
19. $E(-2, -1), F(3, 11)$

20. $G(-2, -6), H(6, 9)$

21.



22.

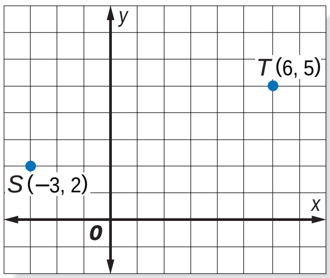


Use the Distance Formula to find the distance between each pair of points.

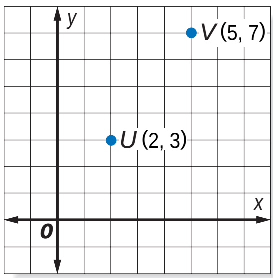
23. $J(0, 0), K(12, 9)$

24. $L(3, 5), M(7, 9)$

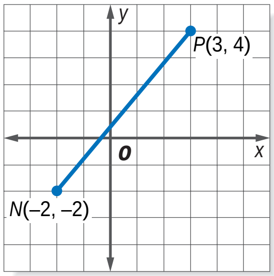
25.



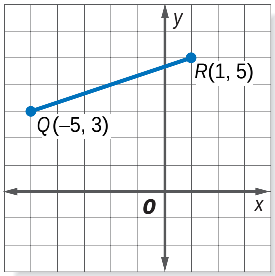
26.



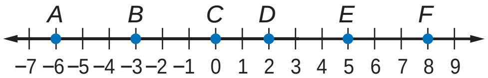
27.



28.



Use the number line to find the coordinate of the midpoint of each segment.

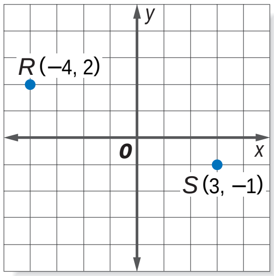


29. $\overline{AC}$ **30.** $\overline{DF}$ **31.** $\overline{CE}$

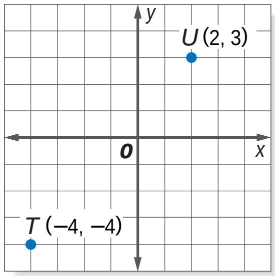
32. $\overline{BD}$ **33.** $\overline{AF}$ **34.** $\overline{BE}$

Find the coordinates of the midpoint of a segment having the given endpoints.

35.



36.



37. $A(8, 4), B(12, 2)$ **38.** $C(9, 5), D(17, 4)$

39. $E(-11, -4), F(-9, -2)$ **40.** $G(4, 2), H(8, -6)$

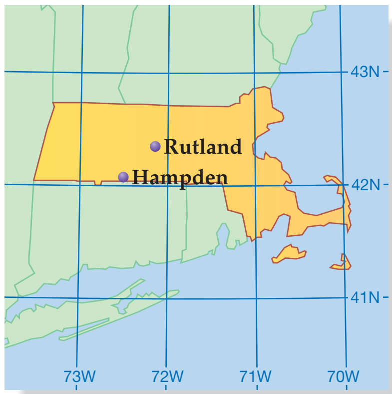
41. $J(3.4, 2.1), K(7.8, 3.6)$ **42.** $L(-1.4, 3.2), M(2.6, -5.4)$

Find the coordinates of the missing endpoint given that *S* is the midpoint of $\overline{RT}$.

43. $T(-4, 3), S(-1, 5)$ **44.** $T(2, 8), S(-2, 2)$ **45.** $R\left(\frac{2}{3}, -5\right), S\left(\frac{5}{3}, 3\right)$

GEOGRAPHY For Exercises 46–49, use the following information.

The geographic center of Massachusetts is in Rutland at $(42.4°, 71.9°)$, which represents north latitude and west longitude. Hampden is near the southern border of Massachusetts at $(42.1°, 72.4°)$.



46. If Hampden is one endpoint of a segment and Rutland is its midpoint, find the latitude and longitude of the other endpoint.

47. Use an atlas or the Internet to find a city near the location of the other endpoint.

48. If Hampden is the midpoint of a segment with one endpoint at Rutland, find the latitude and longitude of the other endpoint.

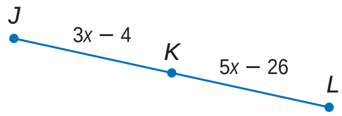
49. Use an atlas or the Internet to find a city near the location of the other endpoint.

Study Tip

Distance on Earth

Actual distances on Earth are calculated along the curve of the Earth's surface. This uses *spherical geometry*. When points are close together, you can use plane geometry to approximate the distance.

50. ALGEBRA Find the measure of $\overline{JK}$ if K is the midpoint of $\overline{JL}$.



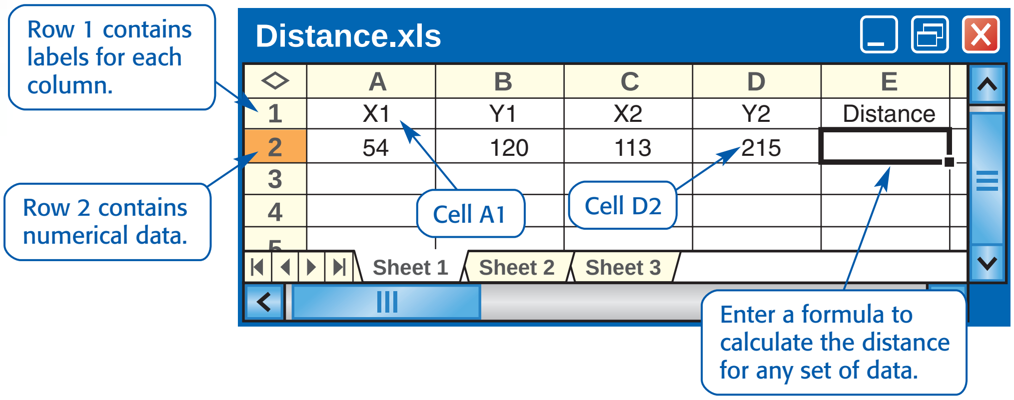
Study Tip

Spreadsheets

Spreadsheets often use special commands to perform operations. For example, $\sqrt{x_1 - x_2}$ would be written as = SQRT(A2 - C2). Use the ^ symbol to raise a number to a power. For example, x^2 is written as x^2.

SPREADSHEETS For Exercises 51 - 55, refer to the information at the left and below.

Spreadsheets can be used to perform calculations quickly. The spreadsheet below can be used to calculate the distance between two points. Values are used in formulas by using a specific cell name. The value of x_1 is used in a formula using its cell name, A2.



51. Write a formula for cell E2 that could be used to calculate the distance between (x_1, y_1) and (x_2, y_2).

EXTRA PRACTICE
See pages 800, 828.
Math Online
Self-Check Quiz at geometryonline.com

Find the distance between each pair of points to the nearest tenth.

52. (54, 120), (113, 215)

53. (68, 153), (175, 336)

54. (421, 454), (502, 798)

55. (837, 980), (612, 625)

H.O.T. Problems

56. REASONING Explain three ways to find the midpoint of a segment.

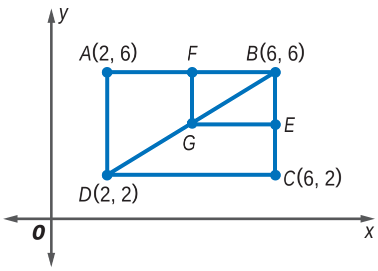
57. OPEN ENDED Draw a segment. Construct the bisector of the segment and use a millimeter ruler to check the accuracy of your construction.

58. CHALLENGE In the figure, $\overline{GE}$ bisects $\overline{BC}$, and $\overline{GF}$ bisects $\overline{AB}$. $\overline{GE}$ is a horizontal segment, and $\overline{GF}$ is a vertical segment.

a. Find the coordinates of points F and E.

b. Name the coordinates of G and explain how you calculated them.

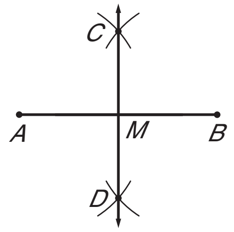
c. Describe what relationship, if any, exists between $\overline{DG}$ and $\overline{GB}$. Explain.



59. CHALLENGE $\overline{WZ}$ has endpoints $W(-3, -8)$ and $Z(5, 12)$. Point X lies between W and Z, such that $WX = \frac{1}{4}WZ$. Find the coordinates of X.

60. Writing in Math Explain how you can find the distance between two points without a ruler. Include how to use the Pythagorean Theorem and the Distance Formula to find the distance between two points, and the length of $\overline{AB}$ from the figure on page 21.

61. Which of the following *best* describes the first step in bisecting segment AB?



A From point A, draw equal arcs on $\overline{CD}$ using the same compass width.

B From point A, draw equal arcs above and below $\overline{AB}$ using a compass width of $\frac{1}{3}AB$.

C From point A, draw equal arcs above and below $\overline{AB}$ using a compass width greater than $\frac{1}{2}AB$.

D From point A, draw equal arcs above and below $\overline{AB}$ using a compass width less than $\frac{1}{2}AB$.

62. Madison paid \$138.16 for 4 pairs of jeans. All 4 pairs of jeans were the same price. How much did each pair of jeans cost?

F \$34.54

G \$42.04

H \$135.16

J \$142.16

63. **REVIEW** Simplify

$(3x^2 - 2)(2x + 4) - 2x^2 + 6x + 7.$

A $4x^2 + 14x - 1$

B $4x^2 - 14x + 15$

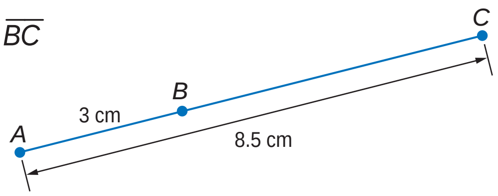
C $6x^3 + 12x^2 + 2x - 1$

D $6x^3 + 10x^2 + 2x - 1$

Find the measurement of each segment. (Lesson 1-2)

64. $\overline{WY}$

65. $\overline{BC}$



Draw and label a figure for each relationship. (Lesson 1-1)

66. four noncollinear points A, B, C, and D that are coplanar

67. line m that intersects plane A and line n in plane A

68. Lines a, b, and c are coplanar, but do not intersect.

69. Lines a, b, and c are coplanar and meet at point F.

70. Point C and line r lie in M. Line r intersects line s at D. Point C, line r, and line s are not coplanar.

71. Planes A and B intersect in line s. Plane C intersects A and B, but does not contain line s.

PREREQUISITE SKILL **Solve each equation.** (Pages 781–782)

72. $2k = 5k - 30$

73. $14x - 31 = 12x + 8$

74. $180 - 8t = 90 + 2t$

75. $12m + 7 = 3m + 52$

76. $8x + 7 = 5x + 20$

77. $13n - 18 = 5n + 32$

EXTEND 1-3

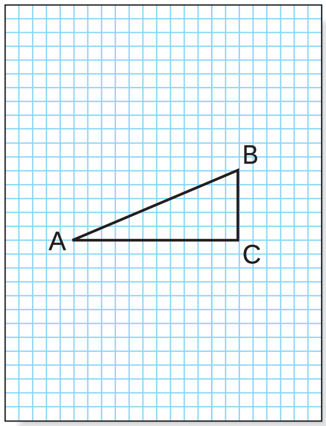
Geometry Lab

Modeling the Pythagorean Theorem

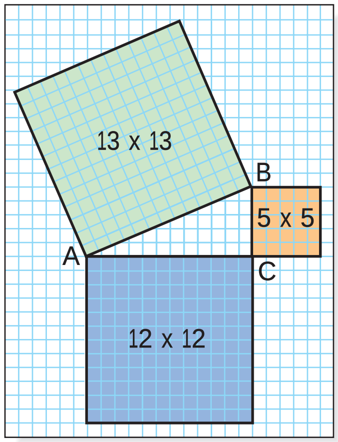
In Chapter 8, you will prove the Pythagorean Theorem, but this activity will suggest that the Pythagorean Theorem holds for any right triangle.

ACTIVITY

- Draw right triangle *ABC* in the center of a piece of grid paper.



- On another piece of grid paper, draw a square that is 5 units on each side, a square that is 12 units on each side, and a square that is 13 units on each side. Use colored pencils to shade each of these squares. Cut out the squares. Label them as 5×5, 12×12, and 13×13, respectively.
- Place the squares so that a side of the square matches up with a side of the right triangle.



ANALYZE THE RESULTS

1. Determine the number of grid squares in each square you drew.

2. How do the numbers of grid squares relate?

3. If $AB = c$, $BC = a$, and $AC = b$, write an expression to describe each of the squares.

4. Compare this expression with what you know about the Pythagorean Theorem.

5. **MAKE A CONJECTURE** Repeat the activity for triangles with each of the side measures listed below. What do you find is true of the relationship of the squares on the sides of the triangle?

a. 3, 4, 5 **b.** 8, 15, 17 **c.** 6, 8, 10

6. Repeat the activity with a right triangle with shorter sides that are both 5 units long. How could you determine the number of grid squares in the larger square?

1-4 Angle Measure

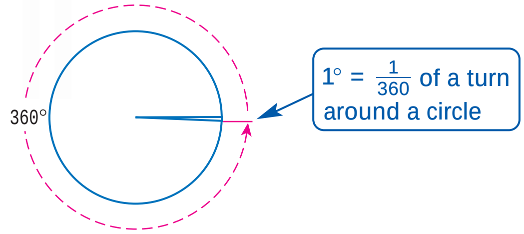
Main Ideas

- Measure and classify angles.
- Identify and use congruent angles and the bisector of an angle.

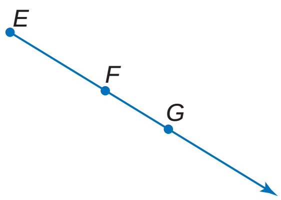
New Vocabulary

degree
ray
opposite rays
angle
sides
vertex
interior
exterior
right angle
acute angle
obtuse angle
angle bisector

Astronomer Claudius Ptolemy based his observations of the solar system on a unit that resulted from dividing the circumference, or the distance around, a circle into 360 parts. This later became known as a **degree**. In this lesson, you will learn to measure angles in degrees.



Measure Angles A **ray** is part of a line. It has one endpoint and extends indefinitely in one direction. Rays are named stating the endpoint first and then any other point on the ray. The figure at the right shows ray *EF*, which can be symbolized as $\overrightarrow{EF}$. This ray could also be named as $\overrightarrow{EG}$, but not as $\overrightarrow{FE}$ because *F* is not the endpoint of the ray.



If you choose a point on a line, that point determines exactly two rays called **opposite rays**. Line *m*, shown below, is separated into two opposite rays, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. Point *P* is the common endpoint of those rays. $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are collinear rays.

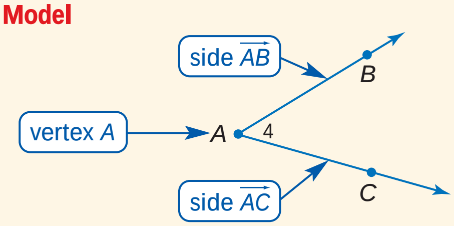
An **angle** is formed by two *noncollinear* rays that have a common endpoint. The rays are called **sides** of the angle. The common endpoint is the **vertex**.

KEY CONCEPT — Angle

Words An angle is formed by two noncollinear rays that have a common endpoint.

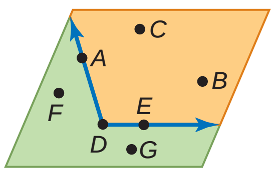
Symbols $\angle A$
$\angle BAC$
$\angle CAB$
$\angle 4$

Model



An angle divides a plane into three distinct parts.

- Points *A*, *D*, and *E* lie on the angle.
- Points *C* and *B* lie in the **interior** of the angle.
- Points *F* and *G* lie in the **exterior** of the angle.

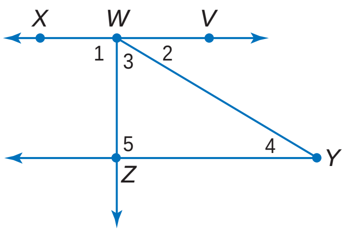


Study Tip

Naming Angles

You can name an angle by a single letter *only* when there is one angle shown at that vertex.

EXAMPLE 1 Angles and Their Parts



a. Name all angles that have *W* as a vertex.

$\angle 1$, $\angle 2$, $\angle 3$, $\angle XWY$, $\angle ZWV$, $\angle YWV$

b. Name the sides of $\angle 1$.

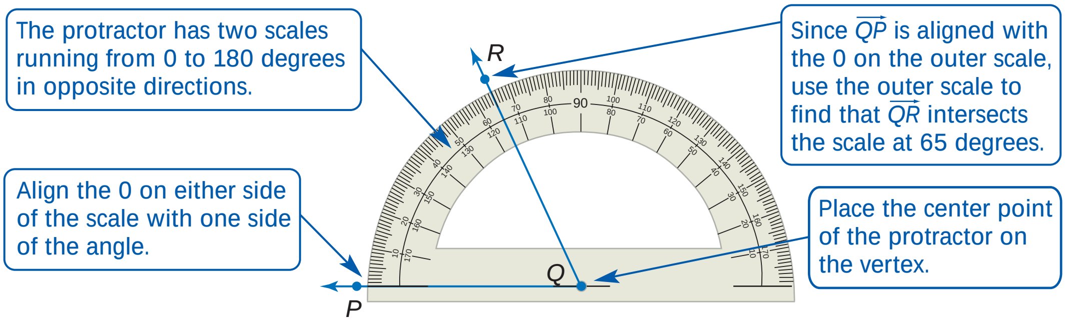
$\overrightarrow{WZ}$ and $\overrightarrow{WX}$ are the sides of $\angle 1$.

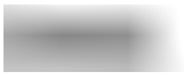
c. Write another name for $\angle WYZ$.

$\angle 4$, $\angle Y$, and $\angle ZYW$ are other names for $\angle WYZ$.

1. Name a pair of opposite rays.

To measure an angle, you can use a *protractor*. Angle PQR is a 65 degree (65°) angle. We say that the *degree measure* of $\angle PQR$ is 65, or simply $m\angle PQR = 65$.





Animation
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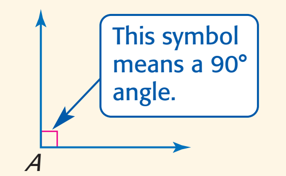
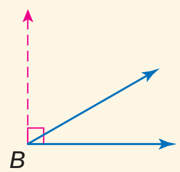
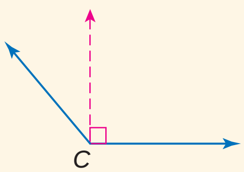
Angles can be classified by their measures.

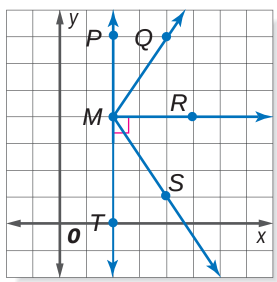
Reading Math

Angles Opposite rays are also known as a *straight angle*. Its measure is 180°. Unless otherwise specified in this book, the term *angle* means a nonstraight angle.

KEY CONCEPT *Classify Angles*

Name	right angle	acute angle	obtuse angle
Measure	$m\angle A = 90$	$m\angle B < 90$	$180 > m\angle C > 90$
Model			

EXAMPLE 2 Measure and Classify Angles



Measure each angle and classify as *right, acute,* or *obtuse.*

a. $\angle PMQ$

Use a protractor to find that $m\angle PMQ = 30$.
$30 < 90$, so $\angle PMQ$ is an acute angle.

b. $\angle TMR$

$\angle TMR$ is marked with a right angle symbol, so measuring is not necessary; $m\angle TMR = 90$.

2. Measure $\angle QMT$ and classify it as *right*, *acute*, or *obtuse*.

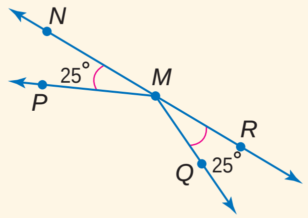
Congruent Angles Just as segments that have the same measure are congruent, angles that have the same measure are congruent.

KEY CONCEPT — Congruent Angles

Words Angles that have the same measure are congruent angles. Arcs on the figure indicate which angles are congruent.

Symbols $\angle NMP \cong \angle QMR$

Model



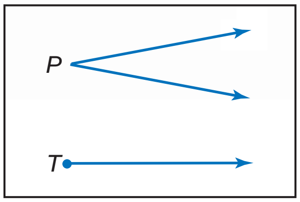
You can construct an angle congruent to a given angle without knowing the measure of the angle.

CONSTRUCTION

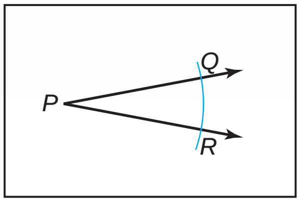
Copy an Angle

Animation geometryonline.com

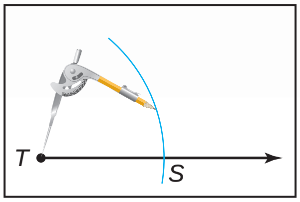
Step 1
Draw an angle like $\angle P$ on your paper. Use a straightedge to draw a ray on your paper. Label its endpoint T.



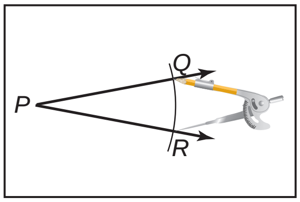
Step 2
Place the tip of the compass at point P and draw a large arc that intersects both sides of $\angle P$. Label the points of intersection Q and R.



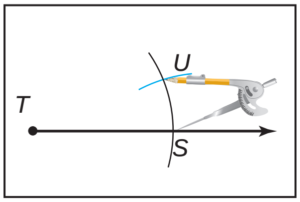
Step 3
Using the same compass setting, put the compass at T and draw a large arc that intersects the ray. Label the point of intersection S.



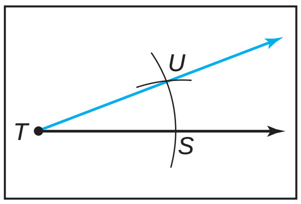
Step 4
Place the point of your compass on R and adjust so that the pencil tip is on Q.



Step 5
Without changing the setting, place the compass at S and draw an arc to intersect the larger arc you drew in Step 3. Label the point of intersection U.

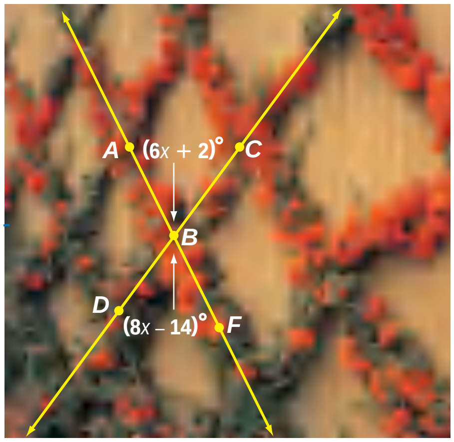


Step 6
Use a straightedge to draw $\overrightarrow{TU}$.



EXAMPLE 3 Use Algebra to Find Angle Measures

GARDENING **A trellis is often used to provide a frame for vining plants. Some of the angles formed by the slats of the trellis are congruent angles. In the figure, $\angle ABC \cong \angle DBF$. If $m\angle ABC = 6x + 2$ and $m\angle DBF = 8x - 14$, find the actual measurements of $\angle ABC$ and $\angle DBF$.**



$\angle ABC \cong \angle DBF$	Given
$m\angle ABC = m\angle DBF$	Definition of congruent angles
$6x + 2 = 8x - 14$	Substitution
$6x + 16 = 8x$	Add 14 to each side.
$16 = 2x$	Subtract $6x$ from each side.
$8 = x$	Divide each side by 2.

Use the value of x to find the measure of one angle.

$m\angle ABC = 6x + 2$	Given
$= 6(8) + 2$	$x = 8$
$= 48 + 2$ or 50	Simplify.

Since $m\angle ABC = m\angle DBF$, $m\angle DBF = 50$. Both $\angle ABC$ and $\angle DBF$ measure 50.

Study Tip

Checking Solutions

Check that you have computed the value of x correctly by substituting the value into the expression for $\angle DBF$. If you don't get the same measure as $\angle ABC$, you have made an error.

3. Suppose $\angle JKL \cong \angle MKN$. If $m\angle JKL = 5x + 4$ and $m\angle MKN = 3x + 12$, find the actual measurements of the two angles.

Personal Tutor at geometryonline.com

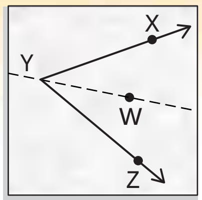
GEOMETRY LAB

Bisect an Angle

Animation geometryonline.com

MAKE A MODEL

- Draw any $\angle XYZ$ on patty paper or tracing paper.
- Fold the paper through point Y so that $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are aligned together.
- Open the paper and label a point on the crease in the interior of $\angle XYZ$ as point W.



ANALYZE THE MODEL

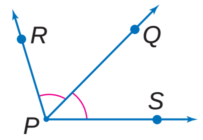
1. What seems to be true about $\angle XYW$ and $\angle WYZ$?

2. Measure $\angle XYZ$, $\angle XYW$, and $\angle WYZ$.

3. You learned about segment bisectors in Lesson 1-3. **Make a conjecture** about the term *angle bisector*.

Red Habegger/Grant Heilman Photography

A ray that divides an angle into two congruent angles is called an **angle bisector**. If $\overrightarrow{PQ}$ is the angle bisector of $\angle RPS$, then point Q lies in the interior of $\angle RPS$, and $\angle RPQ \cong \angle QPS$. A line segment can also bisect an angle.



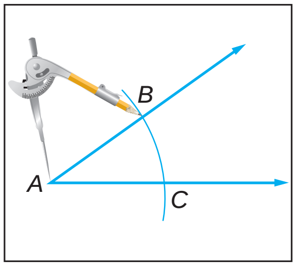
Just as with segments, when a line divides an angle into smaller angles, the sum of the measures of the smaller angles equals the measure of the largest angle. So in the figure, $m\angle RPS = m\angle RPQ + m\angle QPS$.

You can construct the angle bisector of any angle without knowing the measure of the angle.

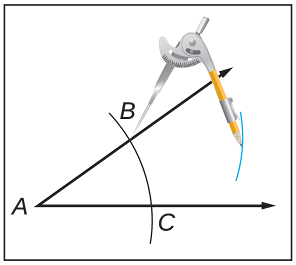
CONSTRUCTION

Bisect an Angle

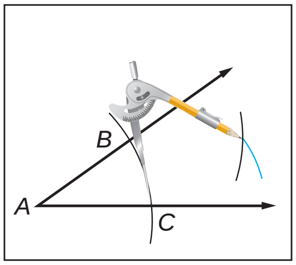
Step 1
Draw an angle and label the vertex as A. Put your compass at point A and draw a large arc that intersects both sides of $\angle A$. Label the points of intersection B and C.



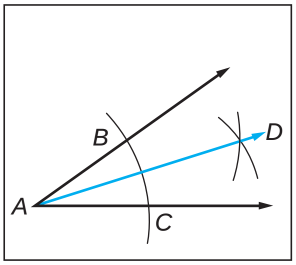
Step 2
With the compass at point B, draw an arc in the interior of the angle.



Step 3
Keeping the same compass setting, place the compass at point C and draw an arc that intersects the arc drawn in Step 2.



Step 4
Label the point of intersection D. Draw $\overrightarrow{AD}$. $\overrightarrow{AD}$ is the bisector of $\angle A$. Thus, $m\angle BAD = m\angle DAC$ and $\angle BAD \cong \angle DAC$.

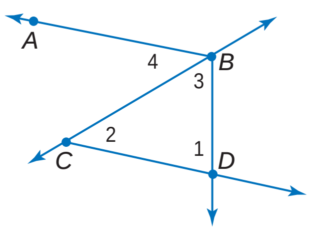


Check Your Understanding

Example 1
(p. 32)

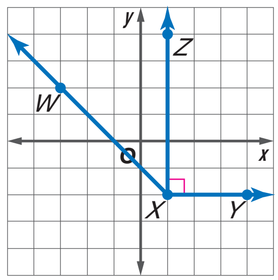
For Exercises 1–3, use the figure at the right.

1. Name the vertex of $\angle 2$.
2. Name the sides of $\angle 4$.
3. Write another name for $\angle BDC$.



Example 2 (p. 32)

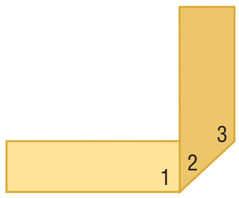
Measure each angle and classify as *right*, *acute*, or *obtuse*.



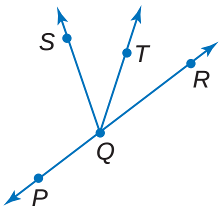
4. $\angle WXY$

5. $\angle WXZ$

Example 3 (p. 34)



6. **ORIGAMI** The art of origami involves folding paper at different angles to create designs and three-dimensional figures. One of the folds in origami involves folding a strip of paper so that the lower edge of the strip forms a right angle with itself. Identify each numbered angle as *right*, *acute*, or *obtuse*.



ALGEBRA **In the figure, $\overrightarrow{QP}$ and $\overrightarrow{QR}$ are opposite rays, and $\overrightarrow{QT}$ bisects $\angle RQS$.**

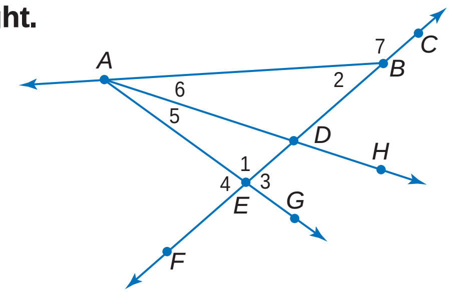
7. If $m\angle RQT = 6x + 5$ and $m\angle SQT = 7x - 2$, find $m\angle RQT$.

8. Find $m\angle TQS$ if $m\angle RQS = 22a - 11$ and $m\angle RQT = 12a - 8$.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–24	1
25–30	2
31–36	3

For Exercises 9–24, use the figure on the right. Name the vertex of each angle.



9. $\angle 1$ **10.** $\angle 2$

11. $\angle 6$ **12.** $\angle 5$

Name the sides of each angle.

13. $\angle ADB$ **14.** $\angle 6$

15. $\angle 3$ **16.** $\angle 5$

Write another name for each angle.

17. $\angle 7$ **18.** $\angle AEF$ **19.** $\angle ABD$ **20.** $\angle 1$

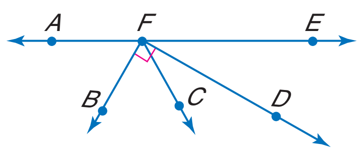
21. Name a point in the interior of $\angle GAB$.

22. Name an angle with vertex B that appears to be acute.

23. Name a pair of angles that share exactly one point.

24. Name a point in the interior of $\angle CEG$.

Measure each angle and classify as *right*, *acute*, or *obtuse*.

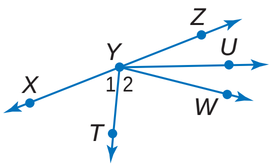


25. $\angle BFD$ **26.** $\angle AFB$

27. $\angle DFE$ **28.** $\angle EFC$

29. $\angle AFD$ **30.** $\angle EFB$

ALGEBRA **In the figure, $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are opposite rays. $\overrightarrow{YU}$ bisects $\angle ZYW$, and $\overrightarrow{YT}$ bisects $\angle XYW$.**



31. If $m\angle ZYU = 8p - 10$ and $m\angle UYW = 10p - 20$, find $m\angle ZYU$.

32. If $m\angle 1 = 5x + 10$ and $m\angle 2 = 8x - 23$, find $m\angle 2$.

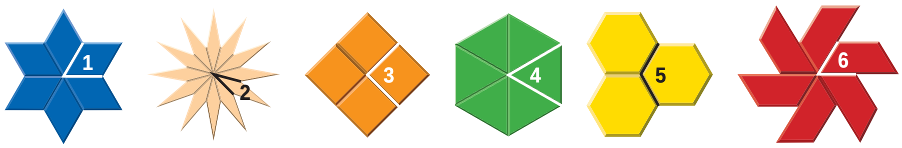
33. If $m\angle 1 = y$ and $m\angle XYW = 6y - 24$, find y.

34. If $m\angle WYZ = 82$ and $m\angle ZYU = 4r + 25$, find r.

35. If $m\angle WYX = 2(12b + 7)$ and $m\angle ZYU = 9b - 1$, find $m\angle UYW$.

36. If $\angle ZYW$ is a right angle and $m\angle ZYU = 13a - 7$, find a.

37. **PATTERN BLOCKS** Pattern blocks can be arranged to fit in a circular pattern without leaving spaces. Remember that the measurement around a full circle is 360°. Determine the degree measure of the numbered angles shown below.



EXTRA PRACTICE
See pages 801, 828.
Math Online
Self-Check Quiz at
geometryonline.com

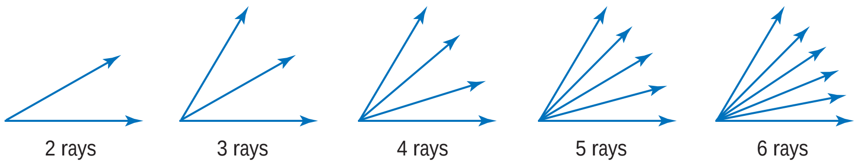
38. **RESEARCH** The words *obtuse* and *acute* have other meanings in the English language. Look these words up in a dictionary and write how the everyday meaning relates to the mathematical meaning.

H.O.T. Problems

39. **OPEN ENDED** Draw and label a figure to show $\overrightarrow{PR}$ that bisects $\angle SPQ$ and $\overrightarrow{PT}$ that bisects $\angle SPR$. Use a protractor to measure each angle.

40. **REASONING** Are all right angles congruent? What information would you use to support your answer?

CHALLENGE **For Exercises 41–44, use the following information.**
Each figure below shows noncollinear rays with a common endpoint.



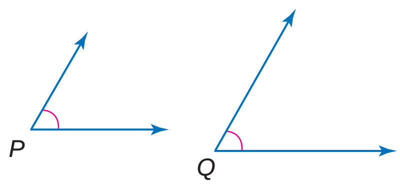
41. Count the number of angles in each figure.

42. Describe the pattern between the number of rays and the number of angles.

43. **Make a conjecture** about the number of angles that are formed by 7 noncollinear rays and by 10 noncollinear rays.

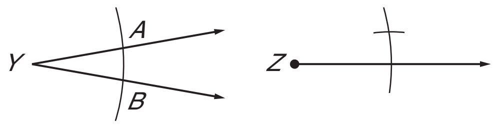
44. Write a formula for the number of angles formed by n noncollinear rays with a common endpoint.

45. **REASONING** How would you compare the sizes of $\angle P$ and $\angle Q$? Explain.



46. **Writing in Math** Refer to page 31. Describe the size of a degree. Include how to find degree measure with a protractor.

47. Dominic is using a straightedge and compass to do the construction shown below.



Which *best* describes the construction Dominic is doing?

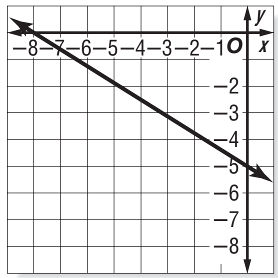
A a line through Z that bisects $\angle AYB$

B a line through Z parallel to $\overrightarrow{YA}$

C a ray through Z congruent to $\overrightarrow{YA}$

D an angle Z congruent to $\angle AYB$

48. **REVIEW** Which coordinate points represent the x- and y-intercepts of the graph below?



F (–5, –8), (0, 0)

G (0, –8), (–5, 0)

H (–8, 0), (0, –5)

J (0, –5), (0, –8)

Find the distance between each pair of points. Then find the coordinates of the midpoint of the line segment between the points. (Lesson 1-3)

49. $A(2, 3)$, $B(5, 7)$

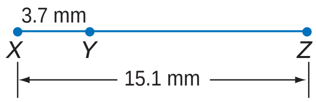
50. $C(-2, 0)$, $D(6, 4)$

51. $E(-3, -2)$, $F(5, 8)$

Find the measurement of each segment. (Lesson 1-2)

52. $\overline{WX}$

53. $\overline{YZ}$



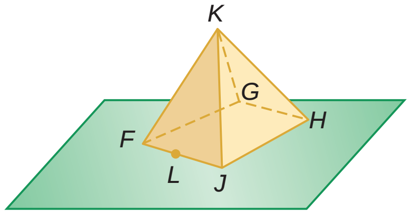
54. Find PQ if Q lies between P and R, $PQ = 6x - 5$, $QR = 2x + 7$, and $PQ = QR$. (Lesson 1-2)

Refer to the figure at the right. (Lesson 1-1)

55. How many planes are shown?

56. Name three collinear points.

57. Name a point coplanar with J, H, and F.



PREREQUISITE SKILL **Solve each equation.** (Pages 781–782)

58. $14x + (6x - 10) = 90$

59. $2k + 30 = 180$

60. $180 - 5y = 90 - 7y$

61. $90 - 4t = \frac{1}{4}(180 - t)$

62. $(6m + 8) + (3m + 10) = 90$

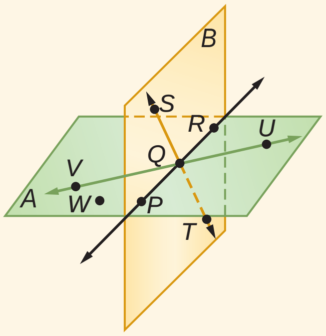
63. $(7n - 9) + (5n + 45) = 180$

CHAPTER 1

Mid-Chapter Quiz

Lessons 1-1 through 1-4

For Exercises 1–2, refer to the figure. (Lesson 1-1)



1. Name another point that is collinear with points S and Q.

2. Name a line that is coplanar with $\overleftrightarrow{VU}$ and point W.

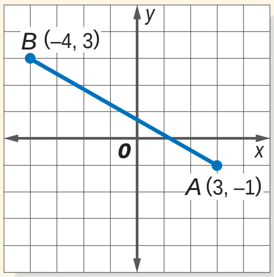
Find the value of x and SR if R is between S and T. (Lesson 1-2)

3. $SR = 3x$, $RT = 2x + 1$, $ST = 6x - 1$

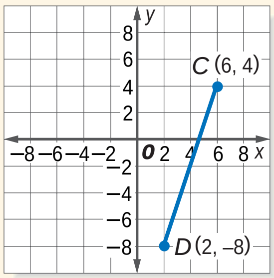
4. $SR = 5x - 3$, $ST = 7x + 1$, $RT = 3x - 1$

Find the coordinates of the midpoint of each segment. Then find the distance between the endpoints. (Lesson 1-3)

5.



6.



Find the coordinates of the midpoint of a segment having the given endpoints. Then find the distance between the endpoints. (Lesson 1-3)

7. $E(10, 20)$, $F(-10, -20)$
8. $A(-1, 3)$, $B(5, -5)$
9. $C(4, 1)$, $D(-3, 7)$
10. $F(4, -9)$, $G(-2, -15)$
11. $H(-5, -2)$, $J(7, 4)$

12. **MULTIPLE CHOICE** $\overline{AB}$ has endpoints $A(n, 4n)$ and $B(3n, 6n)$. Which of the following is true?

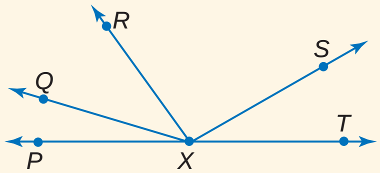
    A $AB = 4n$

    B The midpoint of $\overline{AB}$ is $(2n, 2n)$.

    C $AB = n\sqrt{8}$

    D The midpoint of $\overline{AB}$ is $(4n, 10n)$.

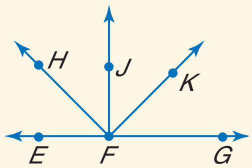
In the figure, $\overrightarrow{XP}$ and $\overrightarrow{XT}$ are opposite rays. (Lesson 1-4)



13. If $m\angle SXT = 3a - 4$, $m\angle RXS = 2a + 5$, and $m\angle RXT = 111$, find $m\angle RXS$.

14. If $m\angle QXR = a + 10$, $m\angle QXS = 4a - 1$, and $m\angle RXS = 91$, find $m\angle QXS$.

Measure each angle and classify as *right*, *acute*, or *obtuse*. (Lesson 1-4)



15. $\angle KFG$
16. $\angle HFG$
17. $\angle HFK$
18. $\angle JFE$
19. $\angle HFJ$
20. $\angle EFK$

1-5 Angle Relationships

Main Ideas

- Identify and use special pairs of angles.
- Identify perpendicular lines.

New Vocabulary

adjacent angles
vertical angles
linear pair
complementary angles
supplementary angles
perpendicular

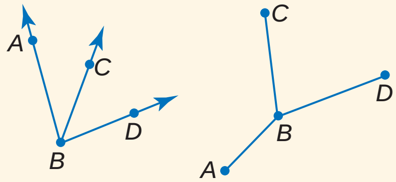
When two lines intersect, four angles are formed. In some cities, more than two streets might intersect to form even more angles. All of these angles are related in special ways.

Pairs of Angles Certain pairs of angles have special names.

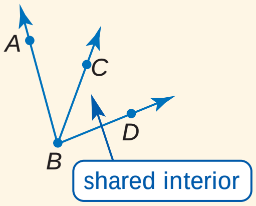
KEY CONCEPT — Angle Pairs

Words **Adjacent angles** are two angles that lie in the same plane, have a common vertex and a common side, but no common interior points.

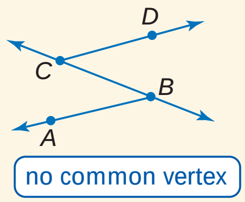
Examples
$\angle ABC$ and $\angle CBD$



Nonexamples
$\angle ABC$ and $\angle ABD$

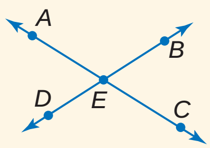


$\angle ABC$ and $\angle BCD$

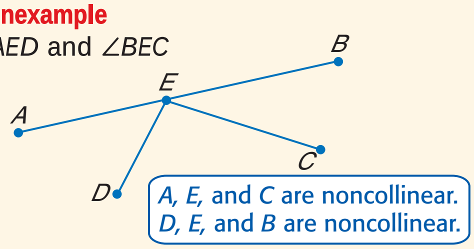


Words **Vertical angles** are two nonadjacent angles formed by two intersecting lines.

Examples
$\angle AEB$ and $\angle CED$
$\angle AED$ and $\angle BEC$

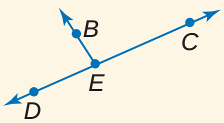


Nonexample
$\angle AED$ and $\angle BEC$

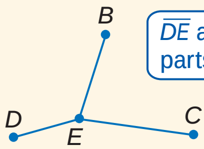


Words A **linear pair** is a pair of adjacent angles with noncommon sides that are opposite rays.

Example
$\angle DEB$ and $\angle BEC$



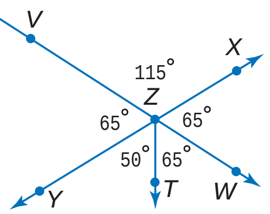
Nonexample
$\angle DEB$ and $\angle BEC$



$\overline{DE}$ and $\overline{EC}$ are not parts of opposite rays.

EXAMPLE Identify Angle Pairs

1 **Name an angle pair that satisfies each condition.**



a. two obtuse vertical angles

$\angle VZX$ and $\angle YZW$ are vertical angles.

They each have measures greater than 90, so they are obtuse.

b. two acute adjacent angles

There are four acute angles shown.

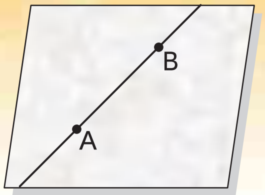
Adjacent acute angles are $\angle VZY$ and $\angle YZT$, $\angle YZT$ and $\angle TZW$, and $\angle TZW$ and $\angle WZX$.

1. Name an angle pair that is a linear pair.

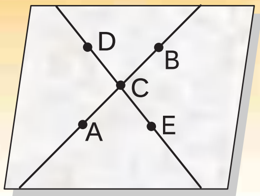
The measures of angles formed by intersecting lines have a special relationship.

GEOMETRY LAB

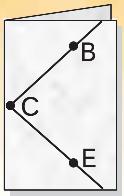
Angle Relationships



Step 1
Fold a piece of patty paper so that it makes a crease across the paper. Open the paper, trace the crease with a pencil, and name two points on the crease *A* and *B*.



Step 2
Fold the paper again so that the new crease intersects $\overleftrightarrow{AB}$ between the two labeled points. Open the paper, trace this crease, and label the intersection *C*. Label two other points, *D* and *E*, on the second crease so that *C* is between *D* and *E*.



Step 3
Fold the paper again through point *C* so that $\overrightarrow{CB}$ aligns with $\overrightarrow{CD}$.

ANALYZE THE MODEL

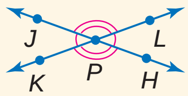
1. What did you notice about $\angle BCE$ and $\angle DCA$ when you made the last fold?
2. Fold again through *C* so that $\overrightarrow{CB}$ aligns with $\overrightarrow{CE}$. What do you notice?
3. Use a protractor to measure each angle. Label the measures on your model.
4. Name pairs of vertical angles and their measures.
5. Name linear pairs of angles and their measures.
6. Compare your results with those of your classmates. Write a “rule” about the measures of vertical angles and another about the measures of linear pairs.

The Geometry Lab suggests that all vertical angles are congruent. It also supports the concept that the sum of the measures of a linear pair is 180.

KEY CONCEPT ***Vertical Angles***

Words Vertical angles are congruent.

Examples $\angle JPK \cong \angle HPL$
$\angle JPL \cong \angle HPK$

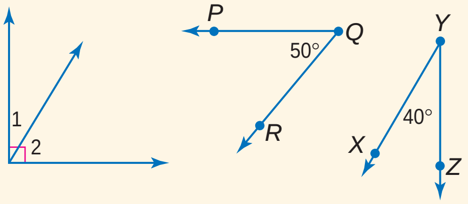


There are other angle relationships that you may remember from previous math courses. These are complementary angles and supplementary angles.

KEY CONCEPT ***Angle Relationships***

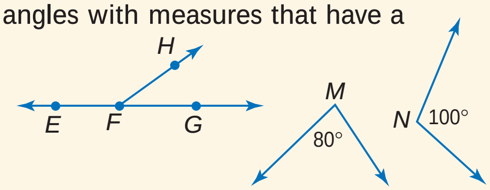
Words **Complementary angles** are two angles with measures that have a sum of 90.

Examples
$\angle 1$ and $\angle 2$ are complementary.
$\angle PQR$ and $\angle XYZ$ are complementary.



Words **Supplementary angles** are two angles with measures that have a sum of 180.

Examples
$\angle EFH$ and $\angle HFG$ are supplementary.
$\angle M$ and $\angle N$ are supplementary.



Study Tip

Complementary and Supplementary Angles

While the other angle pairs in this lesson share at least one point, complementary and supplementary angles need not share any points.

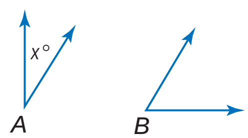
Remember that angle measures are real numbers. So, the operations for real numbers and algebra can be used with angle measures.

EXAMPLE Angle Measure

2 **ALGEBRA** **Find the measures of two complementary angles if the difference in the measures of the two angles is 12.**

Explore The problem relates the measures of two complementary angles. You know that the sum of the measures of complementary angles is 90.

Plan Draw two figures to represent the angles.
Let the measure of one angle be x.
If $m\angle A = x$, then, because $\angle A$ and $\angle B$ are complementary, $m\angle B + x = 90$ or $m\angle B = 90 - x$.



The problem states that the difference of the two angle measures is 12, or $m\angle B - m\angle A = 12$.

Online Problem Solving Handbook at geometryonline.com

Solve $m\angle B - m\angle A = 12$ Given

$(90 - x) - x = 12$ $m\angle A = x$, $m\angle B = 90 - x$

$90 - 2x = 12$ Simplify.

$-2x = -78$ Subtract 90 from each side.

$x = 39$ Divide each side by -2.

Use the value of x to find each angle measure.

$m\angle A = x$	$m\angle B = 90 - x$
$m\angle A = 39$	$m\angle B = 90 - 39$ or 51

Check Add the angle measures to verify that the angles are complementary.

$m\angle A + m\angle B = 90$

$39 + 51 = 90$

$90 = 90$

2. Find the measures of two supplementary angles if the difference in the measures of the two angles is 32.

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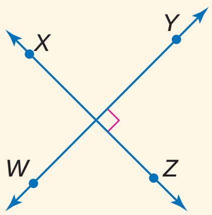
Perpendicular Lines Lines, segments, or rays that form right angles are **perpendicular**.

KEY CONCEPT — *Perpendicular Lines*

Words

- Perpendicular lines intersect to form four right angles.
- Perpendicular lines intersect to form congruent adjacent angles.
- Segments and rays can be perpendicular to lines or to other line segments and rays.
- The right angle symbol in the figure indicates that the lines are perpendicular.

Example



$\overleftrightarrow{XZ} \perp \overleftrightarrow{WY}$

Symbol $\perp$ is read *is perpendicular to*.

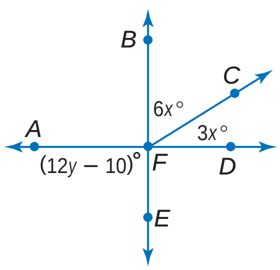
Study Tip

Interpreting Figures

Never assume that two lines are perpendicular because they appear to be so in the figure. The only sure way to know if they are perpendicular is if the right angle symbol is present or if the problem states angle measures that allow you to make that conclusion.

EXAMPLE Perpendicular Lines

3 **ALGEBRA Find x and y so that $\overrightarrow{BE}$ and $\overrightarrow{AD}$ are perpendicular.**



If $\overrightarrow{BE} \perp \overrightarrow{AD}$, then $m\angle BFD = 90$ and $m\angle AFE = 90$. To find x, use $\angle BFC$ and $\angle CFD$.

$m\angle BFD = m\angle BFC + m\angle CFD$ Sum of parts = whole

$90 = 6x + 3x$ Substitution

$90 = 9x$ Add.

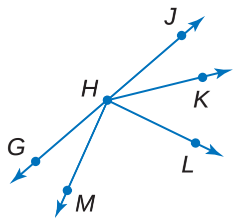
$10 = x$ Divide each side by 9.

To find y, use $\angle AFE$.

$m\angle AFE = 12y - 10$	Given
$90 = 12y - 10$	Substitution
$100 = 12y$	Add 10 to each side.
$\frac{25}{3} = y$	Divide each side by 12, and simplify.

3. Suppose $m\angle D = 3x - 12$. Find x so that $\angle D$ is a right angle.

While two lines may appear to be perpendicular in a figure, you cannot assume this is true unless other information is given. In geometry, figures are used to depict a situation. They are not drawn to reflect total accuracy of the situation. There are certain relationships you can assume to be true, but others that you cannot. Study the figure at the right and then compare the lists below.



Study Tip

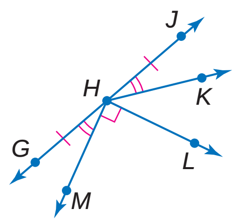
Naming Figures

The list of statements that can be assumed is not a complete list. There are more special pairs of angles than those listed. Also remember that all figures except points usually have more than one way to name them.

Can Be Assumed	Cannot Be Assumed
All points shown are coplanar.	Perpendicular segments: $\overline{HL} \perp \overline{GJ}$
G, H, and J are collinear.	Congruent angles: $\angle JHK \cong \angle GHM$
$\overrightarrow{HM}$, $\overrightarrow{HL}$, $\overrightarrow{HK}$, and $\overleftrightarrow{GJ}$ intersect at H.	$\angle JHK \cong \angle KHL$
H is between G and J.	$\angle KHL \cong \angle GHM$
L is in the interior of $\angle MHK$.	Congruent segments: $\overline{GH} \cong \overline{HJ}$
$\angle GHM$ and $\angle MHL$ are adjacent angles.	$\overline{HJ} \cong \overline{HK}$
$\angle GHL$ and $\angle LHJ$ are a linear pair.	$\overline{HK} \cong \overline{HL}$
$\angle JHK$ and $\angle KHG$ are supplementary.	$\overline{HL} \cong \overline{HG}$

EXAMPLE 4 Interpret Figures

Determine whether each statement can be assumed from the figure at the right.



a. $\angle GHM$ and $\angle MHK$ are adjacent angles.

Yes; they share a common side and vertex and have no interior points in common.

b. $\angle KHJ$ and $\angle GHM$ are complementary.

No; they are congruent, but we do not know anything about their exact measures.

c. $\angle GHK$ and $\angle JHK$ are a linear pair.

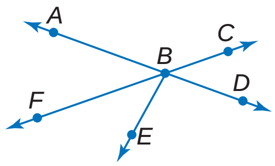
Yes; they are adjacent angles whose noncommon sides are opposite rays.

4. Determine whether the statement *$\angle GHL$ and $\angle LHJ$ are supplementary* can be assumed from the figure.

CHECK Your Understanding

Example 1 (p. 41)

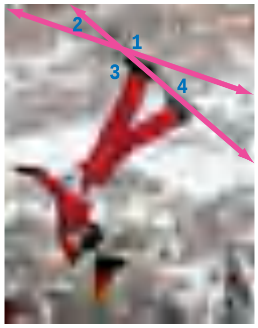
For Exercises 1 and 2, use the figure at the right and a protractor.



1. Name two acute vertical angles.
2. Name two obtuse adjacent angles.

Example 2 (pp. 42–43)

3. **SKIING** Alisa Camplin won a gold medal in the 2002 Winter Olympics with a triple-twisting, double backflip jump in the women's freestyle skiing event. While she is in the air, her skis give the appearance of intersecting lines. If $\angle 4$ measures 60°, find the measures of the other angles.

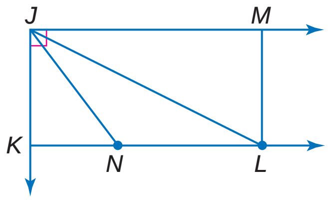


Example 3 (pp. 43–44)

4. The measures of two complementary angles are $16z - 9$ and $4z + 3$. Find the measures of the angles.
5. Find $m\angle T$ if $m\angle T$ is 20 more than four times the measure of its supplement.

Example 4 (p. 44)

Determine whether each statement can be assumed from the figure. Explain.



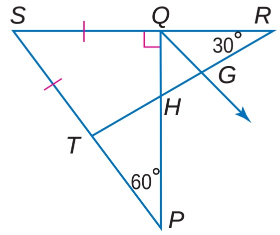
6. $\angle MLJ$ and $\angle JLN$ are complementary.
7. $\angle KJN$ and $\angle NJL$ are adjacent, but neither complementary nor supplementary.

Exercises

HOMEWORK HELP

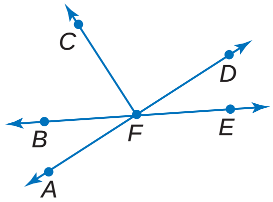
For Exercises	See Examples
8–13	1
14–19	2
20–22	3
23–27	4

For Exercises 8–13, use the figure at the right and a protractor.



8. Name two acute vertical angles.
9. Name two obtuse vertical angles.
10. Name a pair of complementary adjacent angles.
11. Name a pair of complementary nonadjacent angles.
12. Name a linear pair whose vertex is G.
13. Name an angle supplementary to $\angle HTS$.
14. Rays PQ and QR are perpendicular. Point S lies in the interior of $\angle PQR$. If $m\angle PQS = 4 + 7a$ and $m\angle SQR = 9 + 4a$, find $m\angle PQS$ and $m\angle SQR$.
15. The measure of the supplement of an angle is 60 less than three times the measure of the complement of the angle. Find the measure of the angle.
16. Lines p and q intersect to form adjacent angles 1 and 2. If $m\angle 1 = 3x + 18$ and $m\angle 2 = -8y - 70$, find the values of x and y so that p is perpendicular to q.
17. The measure of an angle's supplement is 44 less than the measure of the angle. Find the measure of the angle and its supplement.
18. Two angles are supplementary. One angle measures 12° more than the other. Find the measures of the angles.
19. The measure of $\angle 1$ is five less than four times the measure of $\angle 2$. If $\angle 1$ and $\angle 2$ form a linear pair, what are their measures?

ALGEBRA **For Exercises 20–22, use the figure at the right.**

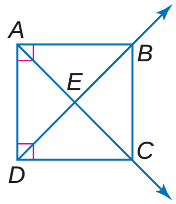


20. If $m\angle CFD = 12a + 45$, find a so that $\overrightarrow{FC} \perp \overrightarrow{FD}$.

21. If $m\angle AFB = 8x - 6$ and $m\angle BFC = 14x + 8$, find the value of x so that $\angle AFC$ is a right angle.

22. If $m\angle BFA = 3r + 12$ and $m\angle DFE = -8r + 210$, find $m\angle AFE$.

Determine whether each statement can be assumed from the figure. Explain.



23. $\angle DAB$ is a right angle.

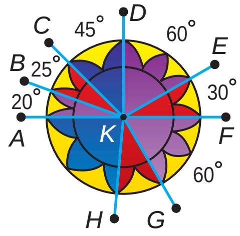
24. $\overline{AB} \perp \overline{BC}$

25. $\angle AEB \cong \angle DEC$

26. $\angle DAE \cong \angle ADE$

27. $\angle ADB$ and $\angle BDC$ are complementary.

28. **STAINED GLASS** In the stained glass pattern at the right, determine which segments are perpendicular.



Determine whether each statement is *sometimes*, *always*, or *never* true.

29. If two angles are supplementary and one is acute, the other is obtuse.

30. If two angles are complementary, they are both acute angles.

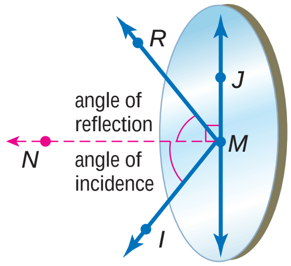
31. If $\angle A$ is supplementary to $\angle B$, and $\angle B$ is supplementary to $\angle C$, then $\angle A$ is supplementary to $\angle C$.

32. If $\overline{PN} \perp \overline{PQ}$, then $\angle NPQ$ is acute.

Real-World Link

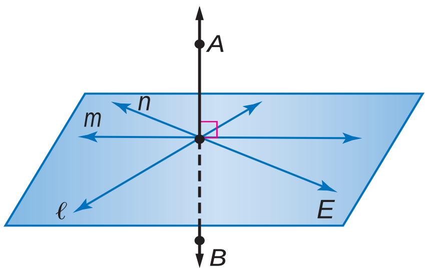
As light from the Sun travels to Earth, it is reflected or refracted by many different surfaces. Light that strikes a smooth, flat surface is very bright because the light is being reflected at the same angle.

33. **PHYSICS** As a ray of light meets a mirror, the light is reflected. The angle that the light strikes the mirror is the *angle of incidence*. The angle that the light is reflected is the *angle of reflection*. The angle of incidence and the angle of reflection are congruent. In the diagram at the right, if $m\angle RMI = 106$, find the angle of reflection and $m\angle RMJ$.



34. **RESEARCH** Look up the words *complementary* and *complimentary* in a dictionary. Discuss the differences in the terms and determine which word has a mathematical meaning.

35. The concept of perpendicularity can be extended to include planes. If a line, line segment, or ray is perpendicular to a plane, it is perpendicular to every line, line segment, or ray in that plane at the point of intersection. In the figure at the right, $\overleftrightarrow{AB} \perp E$. Name all pairs of perpendicular lines.



EXTRA PRACTICE

See pages 801, 828.

Math Online

Self-Check Quiz at geometryonline.com

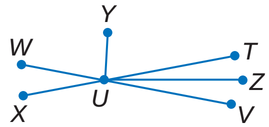
H.O.T. Problems

36. **OPEN ENDED** Draw two angles that are supplementary, but not adjacent.

37. **REASONING** Explain the statement *If two adjacent angles form a linear pair, they must be supplementary*.

38. CHALLENGE A *counterexample* is used to show that a statement is not necessarily true. Draw a counterexample for the statement *Supplementary angles form linear pairs*.

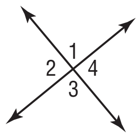
39. CHALLENGE In the figure, $\angle WUT$ and $\angle XUV$ are vertical angles, $\overline{YU}$ is the bisector of $\angle WUT$, and $\overline{UZ}$ is the bisector of $\angle TUV$. Write a convincing argument that $\overline{YU} \perp \overline{UZ}$.



40. Writing in Math Refer to page 40. What kinds of angles are formed when streets intersect? Include the types of angles that might be formed by two intersecting lines, and a sketch of intersecting streets with angle measures and angle pairs identified.

STANDARDIZED TEST PRACTICE

41. In the diagram below, $\angle 1$ is an acute angle.



Which conclusion is *not* true?

A $m\angle 2 > m\angle 3$

B $m\angle 2 = m\angle 4$

C $m\angle 1 < m\angle 4$

D $m\angle 3 > m\angle 4$

42. REVIEW Solve: $5(x - 4) = 3x + 18$

Step 1: $5x - 4 = 3x + 18$

Step 2: $2x - 4 = 18$

Step 3: $2x = 22$

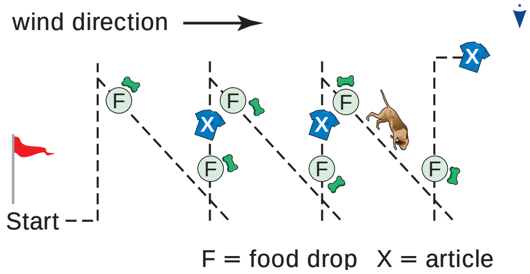
Step 4: $x = 11$

Which is the first *incorrect* step in the solution shown above?

F Step 1 **H** Step 3

G Step 2 **J** Step 4

43. DOG TRACKING A dog is tracking when it is following the scent trail left by a human being or other animal that has passed along a certain route. One of the training exercises for these dogs is a tracking trail. The one shown is called an acute tracking trail. Explain why it might be called this. (Lesson 1-4)



Find the distance between each pair of points. (Lesson 1-3)

44. $A(3, 5), B(0, 1)$ **45.** $C(5, 1), D(5, 9)$ **46.** $E(-2, -10), F(-4, 10)$

47. $G(7, 2), H(-6, 0)$ **48.** $J(-8, 9), K(4, 7)$ **49.** $L(1, 3), M(3, -1)$

Find the value of the variable and *QR*, if *Q* is between *P* and *R*. (Lesson 1-2)

50. $PQ = 1 - x, QR = 4x + 17, PR = -3x$ **51.** $PR = 7n + 8, PQ = 4n - 3, QR = 6n + 2$

PREREQUISITE SKILL Evaluate each expression if $\ell = 3$, $w = 8$, and $s = 2$. (Page 780)

52. $2\ell + 2w$ **53.** ℓw **54.** $4s$ **55.** $\ell w + ws$ **56.** $s(\ell + w)$

EXTEND 1-5

Geometry Lab

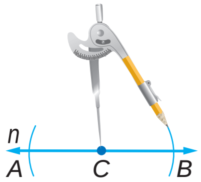
Constructing Perpendiculars

You can use a compass and a straightedge to construct a line perpendicular to a given line through a point on the line, or through a point not on the line.

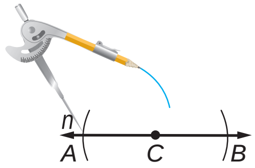
ACTIVITY 1 Perpendicular Through a Point on the Line

Construct a line perpendicular to line *n* and passing through point *C* on *n*.

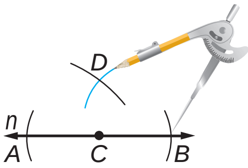
1. Place the compass at point *C*. Using the same compass setting, draw arcs to the right and left of *C*, intersecting line *n*. Label the points of intersection *A* and *B*.



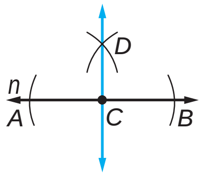
2. Open the compass to a setting greater than *AC*. Put the compass at point *A* and draw an arc above line *n*.



3. Using the same compass setting as in Step 2, place the compass at point *B* and draw an arc intersecting the arc drawn in Step 2. Label the point of intersection *D*.



4. Use a straightedge to draw $\overleftrightarrow{CD}$.

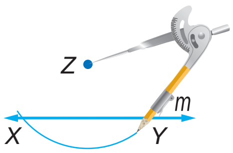


ACTIVITY 2 Perpendicular Through a Point not on the Line

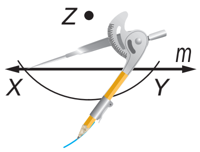
Animation geometryonline.com

Construct a line perpendicular to line *m* and passing through point *Z* *not* on *m*.

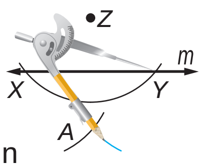
1. Place the compass at point *Z*. Draw an arc that intersects line *m* in two different places. Label the points of intersection *X* and *Y*.



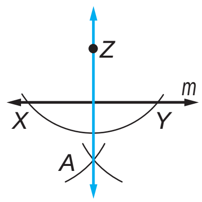
2. Open the compass to a setting greater than $\frac{1}{2}XY$. Put the compass at point *X* and draw an arc below line *m*.



3. Using the same compass setting, place the compass at point *Y* and draw an arc intersecting the arc drawn in Step 2. Label the point of intersection *A*.



4. Use a straightedge to draw $\overleftrightarrow{ZA}$.



Model and Analyze the Results

1. Draw a line and construct a line perpendicular to it through a point on the line. Repeat with a point not on the line.

2. How is the second construction similar to the first one?

1-6 Two-Dimensional Figures

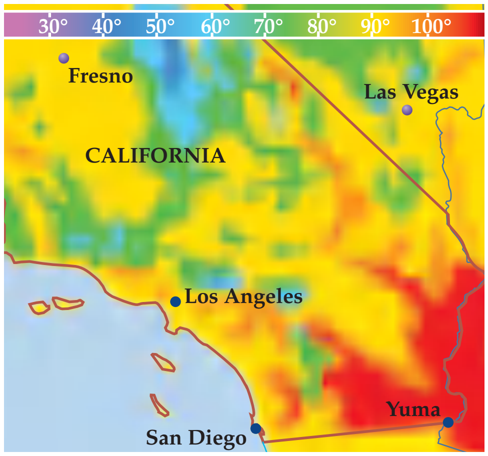
Main Ideas

- Identify and name polygons.
- Find perimeter or circumference and area of two-dimensional figures.

New Vocabulary

polygon
concave
convex
n-gon
regular polygon
perimeter
circumference
area

To predict weather conditions, meteorologists divide the surface into small geographical cells. Meteorologists use computer programs and mathematical models to track climate and other weather conditions for each cell. The weather in each cell is affected by the weather in surrounding cells. Maps like the one on the right show the temperature for each cell.



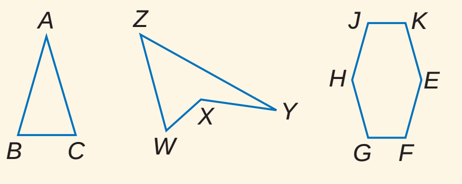
Polygons Each closed figure shown in the map is a **polygon**. A polygon is a closed figure whose sides are all segments. The sides of each angle in a polygon are called *sides* of the polygon, and the vertex of each angle is a *vertex* of the polygon.

KEY CONCEPT — Polygon

Words A polygon is a closed figure formed by a finite number of coplanar segments such that
(1) the sides that have a common endpoint are noncollinear, and
(2) each side intersects exactly two other sides, but only at their endpoints.

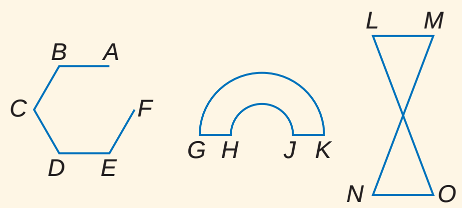
Symbol A polygon is named by the letters of its vertices, written in order of consecutive vertices.

Examples

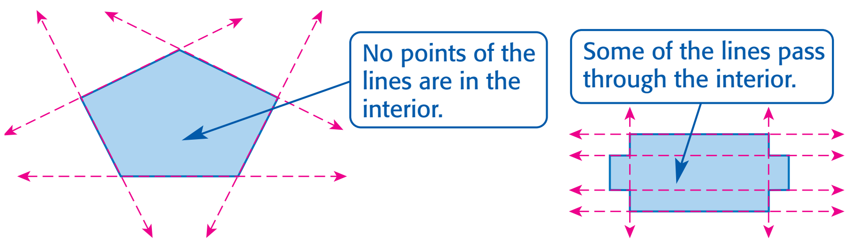


polygons *ABC*, *WXYZ*, *EFGHJK*

Nonexamples



Polygons can be **concave** or **convex**. Suppose the line containing each side is drawn. If any of the lines contain any point in the interior of the polygon, then it is concave. Otherwise it is convex.



convex polygon concave polygon

Reading Math

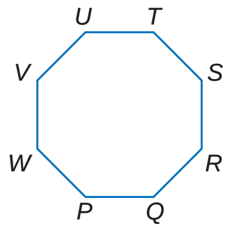
Root Word The term *polygon* is derived from a Greek word meaning *many angles*. Since *hexa-* means 6, *hexagon* means 6 angles. Every polygon has the same number of angles as it does sides.

You are already familiar with many polygon names, such as triangle, square, and rectangle. In general, polygons can be classified by the number of sides they have. A polygon with n sides is an ***n*-gon**. The table lists some common names for various categories of polygon.

Number of Sides	Polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
12	dodecagon
n	n-gon

Polygons and circles are examples of *simple closed curves*.

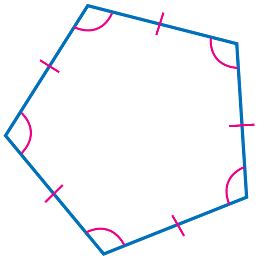
A convex polygon in which all the sides are congruent and all the angles are congruent is called a **regular polygon**. Octagon *PQRSTUVW* below is a regular octagon.



EXAMPLE 1 Identify Polygons

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

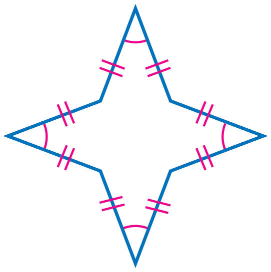
a.



There are 5 sides, so this is a pentagon. No line containing any of the sides will pass through the interior of the pentagon, so it is convex.

The sides are congruent, and the angles are congruent. It is regular.

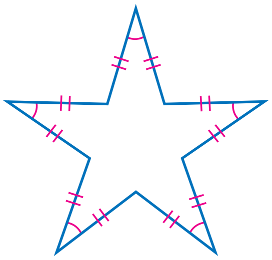
b.



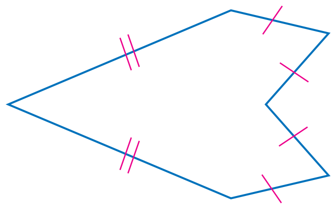
There are 8 sides, so this is an octagon. A line containing any of the sides will pass through the interior of the octagon, so it is concave.

The sides are congruent. However, since it is concave, it cannot be regular.

1A.



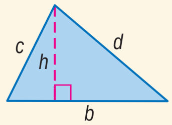
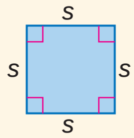
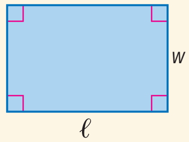
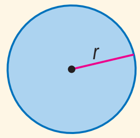
1B.



Perimeter, Circumference, and Area Review the formulas for circles and three common polygons given below. Some shapes have special formulas for perimeter, but all are derived from the basic definition of perimeter. You will derive the area formulas in Chapter 11.

KEY CONCEPT — *Perimeter, Circumference, and Area*

Words The **perimeter** P of a polygon is the sum of the length of the sides of the polygon. The **circumference** C of a circle is the distance around the circle. The **area** A is the number of square units needed to cover a surface.

Examples	**triangle**	**square**	**rectangle**	**circle**
Perimeter/ Circumference	$P = b + c + d$	$P = s + s + s + s$ $= 4s$	$P = \ell + w + \ell + w$ $= 2\ell + 2w$	$C = 2\pi r$
Area	$A = \frac{1}{2}bh$	$A = s^2$	$A = \ell w$	$A = \pi r^2$
				

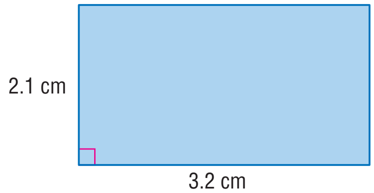
Reading Math

Math Symbols The symbol π is read *pi*. It is an irrational number. In this book, we will use a calculator to evaluate expressions involving π. If no calculator is available, 3.14 is a good estimate for π.

EXAMPLE 2 Find Perimeter and Area

Find the perimeter or circumference and area of each figure to the nearest tenth.

a.



$P = 2\ell + 2w$ Perimeter of a rectangle

$= 2(3.2) + 2(2.1)$ $\ell = 3.2$, $w = 2.1$

$= 10.6$ Simplify.

The perimeter is 10.6 centimeters.

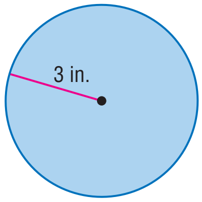
$A = \ell w$ Area of a rectangle

$= (3.2)(2.1)$ $\ell = 3.2$, $w = 2.1$

$= 6.72$ Simplify.

The area is about 6.7 square centimeters.

b.



$C = 2\pi r$ Circumference

$= 2\pi(3)$ $r = 3$

≈ 18.9 Use a calculator.

The circumference is about 18.85 inches.

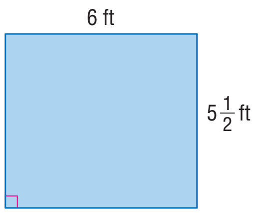
$A = \pi r^2$ Area of a circle

$= \pi(3)^2$ $r = 3$

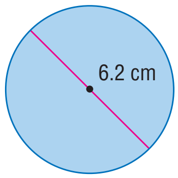
≈ 28.3 Use a calculator.

The area is about 28.3 square inches.

2A.



2B.



STANDARDIZED TEST EXAMPLE **Largest Area**

1 Winona has 26 centimeters of cording to frame a photograph in her scrapbook. Which of these shapes would use *most* or all of the cording and enclose the *largest* area?

A right triangle with each leg about 7 centimeters long

B circle with a radius of about 4 centimeters

C rectangle with a length of 8 centimeters and a width of 4.5 centimeters

D square with a side length of 6 centimeters

Test-Taking Tip

Mental Math When you are asked to compare measures for varying figures, it can be helpful to use mental math. By estimating the perimeter or area of each figure, you can check your calculations.

Read the Test Item

You are asked to compare the area and perimeter of four different shapes.

Solve the Test Item

Find the perimeter and area of each shape.

Right Triangle

Use the Pythagorean Theorem to find the length of the hypotenuse.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$c^2 = 7^2 + 7^2$ or 98	$a = 7, b = 7$
$c^2 = 98$	Simplify.
$c = \sqrt{98}$	Take the square root of each side.
$c \approx 9.9$	Use a calculator.
$P = a + b + c$	Perimeter of a triangle
$\approx 7 + 7 + 9.9$ or 23.9 cm	Substitution
$A = \frac{1}{2}bh$	Area of a triangle
$= \frac{1}{2}(7)(7)$ or 24.5 cm^2	Substitution

Circle	Rectangle	Square
$C = 2\pi r$	$P = 2\ell + 2w$	$P = 4s$
$\approx 2\pi(4)$	$= 2(8) + 2(4.5)$	$= 4(6)$
≈ 25.1 cm	$= 25$ cm	$= 24$ cm
$A = \pi r^2$	$A = \ell w$	$A = s^2$
$\approx \pi(4)^2$	$= (8)(4.5)$	$= 6^2$
≈ 50.3 cm^2	$= 36$ cm^2	$= 36$ cm^2

The shape that uses all of the cording and encloses the largest area is the circle. The answer is B.

3. Danny wants to fence in a play area for his dog. He has 32 feet of fencing. Which shape uses *most* or all of the fencing and encloses the *largest* area?

F circle with radius of about 5 feet

G rectangle with length 6 feet and width 10 feet

H right triangle with legs of length 10 feet

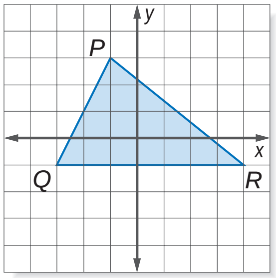
J square with a side length of 8 feet

Personal Tutor at geometryonline.com

You can use the Distance Formula to find the perimeter of a polygon graphed on a coordinate plane.

EXAMPLE Perimeter and Area on the Coordinate Plane

4 **COORDINATE GEOMETRY Refer to $\triangle PQR$ with vertices $P(-1, 3)$, $Q(-3, -1)$, and $R(4, -1)$.**



a. Find the perimeter of $\triangle PQR$.

Since $\overline{QR}$ is a horizontal line, we can count the squares on the grid. The length of $\overline{QR}$ is 7 units. Use the Distance Formula to find PQ and PR.

$$PQ = \sqrt{(-1 - (-3))^2 + (3 - (-1))^2} \quad \text{Substitute.}$$
$$= \sqrt{2^2 + 4^2} \quad \text{Subtract.}$$
$$= \sqrt{20} \approx 4.5 \quad \text{Simplify.}$$

$$PR = \sqrt{(-1 - 4)^2 + (3 - (-1))^2} \quad \text{Substitute.}$$
$$= \sqrt{(-5)^2 + 4^2} \quad \text{Subtract.}$$
$$= \sqrt{41} \approx 6.4 \quad \text{Simplify.}$$

The perimeter of $\triangle PQR$ is $7 + \sqrt{20} + \sqrt{41}$ or about 17.9 units.

b. Find the area of $\triangle PQR$.

The height is the perpendicular distance from P to $\overline{QR}$. Counting squares on the graph, the height is 4 units. The length of $\overline{QR}$ is 7 units.

$$A = \frac{1}{2}bh \quad \text{Area of a triangle}$$
$$= \frac{1}{2}(7)(4) \quad \text{Substitution}$$
$$= 14 \quad \text{Simplify.}$$

The area of $\triangle PQR$ is 14 square units.

Study Tip

Look Back

To review the **Distance Formula**, see Lesson 1-3.

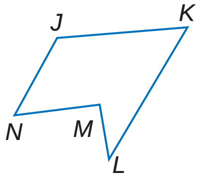
4. Find the perimeter and area of $\triangle ABC$ with vertices $A(-1, 4)$, $B(-1, -1)$, and $C(6, -1)$.

Check Your Understanding

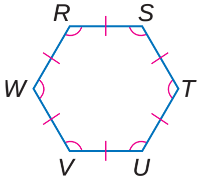
Example 1 (p. 50)

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

1.



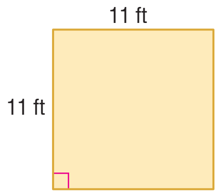
2.



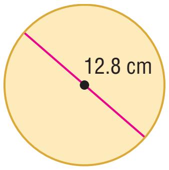
Example 2 (p. 51)

Find the perimeter or circumference and area of each figure. Round to the nearest tenth.

3.



4.



Example 3 (p. 52)

5. STANDARDIZED TEST PRACTICE Tara is building a playpen for the children next door. She has 15 square feet of fabric. What shape will use *most* or all of the fabric?

A a square with a side length of 4 feet
B a rectangle with a length of 4 feet and a width of 3.5 feet
C a circle with a radius of about 2.5 feet
D a right triangle with legs of about 5 feet

Example 4 (p. 53)

6. COORDINATE GEOMETRY Find the perimeter and area of $\triangle ABC$ with vertices $A(-1, 2)$, $B(3, 6)$, and $C(3, -2)$.

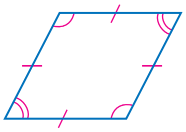
Exercises

HOMEWORK HELP

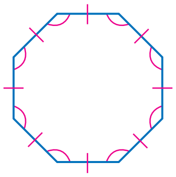
For Exercises	See Examples
7–11	1
12–17	2
18, 19	3
20–23	4

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

7.



8.



TRAFFIC SIGNS **Identify the shape of each traffic sign.**

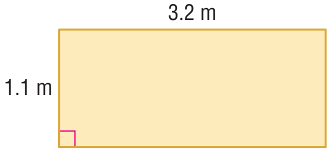
9. school zone

10. caution or warning

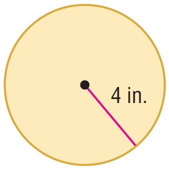
11. railroad

Find the perimeter or circumference and area of each figure. Round to the nearest tenth.

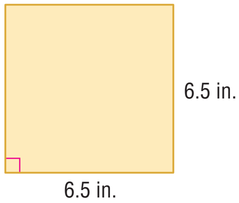
12.



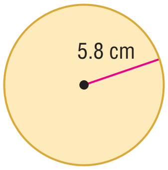
13.



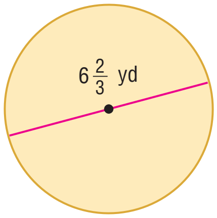
14.



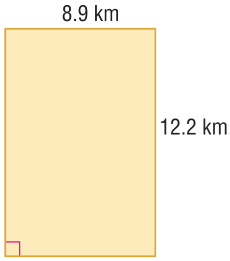
15.



16.



17.



Real-World Link

Some gardens are landscaped with shrubbery that has been sculpted or *topiary*. Columbus, Ohio, is home to a topiary garden that recreates Georges Seurat's painting *A Sunday Afternoon on the Ile De La Grande Jatte*. The garden includes 54 people sculpted in topiary.

Source: topiarygarden.org

18. **CRAFTS** Candace has a square picture that is 4 inches on each side. The picture is framed with a length of ribbon. She wants to use the same piece of ribbon to frame a circular picture. What is the maximum radius of the circular frame?

19. **LANDSCAPING** Mr. Hernandez has a circular garden with a diameter of 10 feet surrounded by edging. Using the same length of edging, he is going to create a square garden. What is the maximum side length of the square?

COORDINATE GEOMETRY **Find the perimeter and area of each figure with the given vertices.**

20. $D(-2, -2)$, $E(-2, 3)$, and $F(2, -1)$

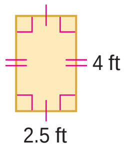
21. $J(-3, -3)$, $K(3, 2)$, and $L(3, -3)$

22. $P(-1, 1)$, $Q(3, 4)$, $R(6, 0)$, and $S(2, -3)$

23. $T(-2, 3)$, $U(1, 6)$, $V(5, 2)$, and $W(2, -1)$

24. **HISTORIC LANDMARKS** The Pentagon building in Arlington, Virginia, is so named because of its five congruent sides. Find the perimeter of the outside of the Pentagon if one side is 921 feet long.

CHANGING DIMENSIONS **For Exercises 25–27, use the rectangle at the right.**

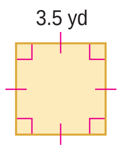


25. Find the perimeter of the rectangle.

26. Find the area of the rectangle.

27. Suppose the length and width of the rectangle are doubled. What effect does this have on the perimeter? Describe the effect on the area.

CHANGING DIMENSIONS **For Exercises 28–30, use the square at the right.**



28. Find the perimeter of the square.

29. Find the area of the square.

30. Suppose the length of a side of the square is divided by 2. What effect does this have on the perimeter? Describe the effect on the area.

Jim Baron

ALGEBRA Find the perimeter or circumference for each figure with the given information.

31. The area of a square is 36 square inches.

32. The length of a rectangle is half the width. The area is 25 square meters.

33. The area of a circle is 25π square units.

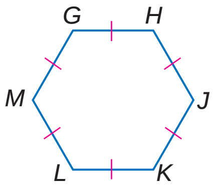
34. The area of a circle is 32π square units.

35. The length of a rectangle is three times the width. The area is 27 square inches.

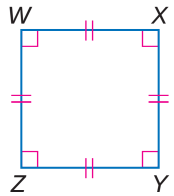
36. The length of a rectangle is twice the width. The area is 48 square inches.

ALGEBRA Find the length of each side of the polygon for the given perimeter.

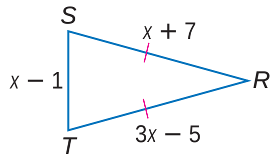
37. $P = 90$ centimeters



38. $P = 14$ miles

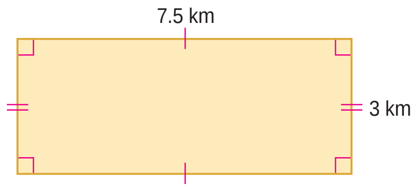


39. $P = 31$ units



40. CHANGING DIMENSIONS The perimeter of an n-gon is 12.5 meters. Find the perimeter of the n-gon if the length of each of its n sides is multiplied by 10.

CHANGING DIMENSIONS For Exercises 41–44, use the rectangle at the right.



41. Find the perimeter of the rectangle.

42. Find the area of the rectangle.

43. Suppose the length and width of the rectangle are doubled. What effect does this have on the perimeter?

44. Describe the effect of doubling the length and width on the area.

ENLARGEMENT For Exercises 45–48, use the following information.
The coordinates of the vertices of a triangle are $A(1, 3)$, $B(9, 10)$, and $C(11, 18)$.

45. Find the perimeter of triangle ABC.

46. Suppose each coordinate is multiplied by 2. What is the perimeter of this triangle?

47. Find the perimeter of the triangle when the coordinates are multiplied by 3.

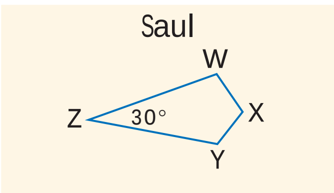
48. Make a conjecture about the perimeter of a triangle when the coordinates of its vertices are multiplied by the same positive factor.

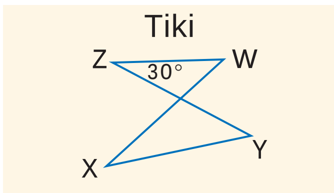
EXTRA PRACTICE
See pages 801, 828.
Math Online
Self-Check Quiz at geometryonline.com

H.O.T. Problems

49. OPEN ENDED Explain how you would find the length of a side of a regular decagon if the perimeter is 120 centimeters.

50. FIND THE ERROR Saul and Tiki were asked to draw quadrilateral $WXYZ$ with $m\angle Z = 30$. Who is correct? Explain your reasoning.





CHALLENGE **Use grid paper to draw all possible rectangles with length and width that are whole numbers and with a perimeter of 12. Record the number of grid squares contained in each rectangle.**

51. What do you notice about the rectangle with the greatest number of squares?

52. The perimeter of another rectangle is 36. What would be the dimensions of the rectangle with the greatest number of squares?

53. **Which One Doesn't Belong?** Identify the term that does not belong with the other three. Explain your reasoning.

square	circle	triangle	pentagon

54. **Writing in Math** Refer to page 49. Explain why dividing a state into geographical cells allows meteorologists to more accurately predict weather.

55. The circumferences of two circles are in the ratio of 9 to 16. What is the ratio between the areas of the two circles?

A 3 to 4

B 9 to 16

C 81 to 64

D 81 to 256

56. **REVIEW** A rectangle has an area of 1000 square meters and a perimeter of 140 meters. What are the dimensions of the rectangle?

F 100 m by 100 m

G 50 m by 20 m

H 40 m by 25 m

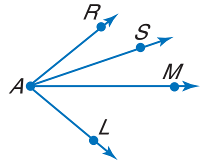
J 10 m by 100 m

Determine whether each statement is *always*, *sometimes*, or *never* true. (Lesson 1-5)

57. Two angles that form a linear pair are supplementary.

58. If two angles are supplementary, then one of the angles is obtuse.

In the figure, $\overrightarrow{AM}$ bisects $\angle LAR$, and $\overrightarrow{AS}$ bisects $\angle MAR$. (Lesson 1-4)



59. If $m\angle MAR = 2x + 13$ and $m\angle MAL = 4x - 3$, find $m\angle RAL$.

60. If $m\angle RAL = x + 32$ and $m\angle MAR = x - 31$, find $m\angle LAM$.

61. Find $m\angle LAR$ if $m\angle RAS = 25 - 2x$ and $m\angle SAM = 3x + 5$.

62. **CRAFTS** Martin makes pewter figurines. When a solid object with a volume of 1 cubic centimeter is submerged in water, the water level rises 1 milliliter. Martin pours 200 mL of water into a cup, submerges a figurine in it, and watches it rise to 343 mL. What is the maximum amount of molten pewter needed to make a figurine? Explain. (Lesson 1-2)

PREREQUISITE SKILL **Evaluate each expression if $a = 12$, $b = 16$, $c = 21$, and $d = 18$.** (Page 780)

63. $\frac{1}{2}(b^2) + 2d$

64. $\frac{1}{3}(cd)$

65. $\frac{1}{2}(2a + 3c^2)$

66. $\frac{1}{3}(ac) + d^2$

EXTEND 1-6

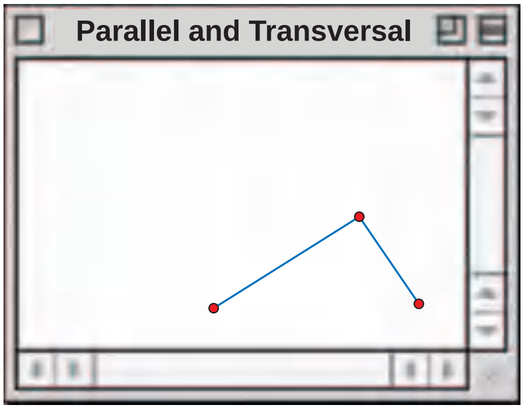
Geometry Software Lab

Measuring Two-Dimensional Figures

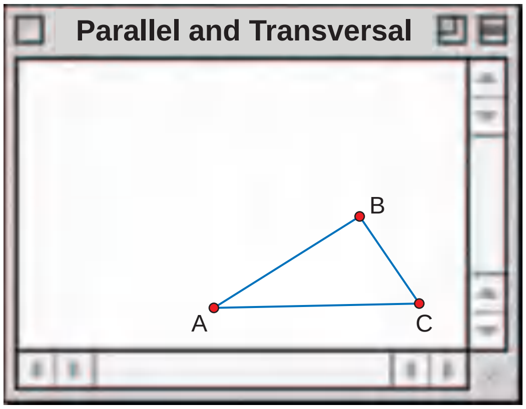
You can use The Geometer's Sketchpad® to draw and investigate polygons. It can be used to find the measures of the sides and the perimeter of a polygon. You can also find the measures of the angles in a polygon.

ACTIVITY

Step 1 Draw $\triangle ABC$.



- Select the segment tool from the toolbar, and click to set the first endpoint A of side $\overline{AB}$. Then drag the cursor and click again to set the other endpoint B.
- Click on point B to set the endpoint of $\overline{BC}$. Drag the cursor and click to set point C.

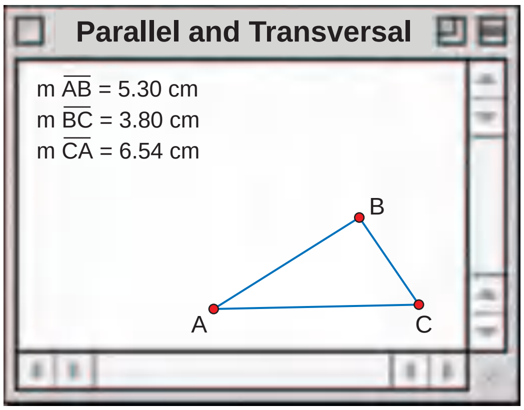


- Click on point C to set the endpoint of $\overline{CA}$. Then move the cursor to highlight point A. Click on A to draw $\overline{CA}$.
- Use the pointer tool to click on points A, B, and C. Under the Display menu, select Show Labels to label the vertices of your triangle.

Step 2 Find AB, BC, and CA.

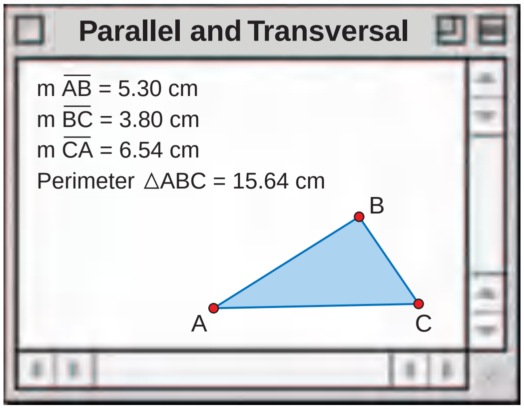
- Use the pointer tool to select $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
- Select the Length command under the Measure menu to display the lengths of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

$AB = 5.30$
$BC = 3.80$
$CA = 6.54$



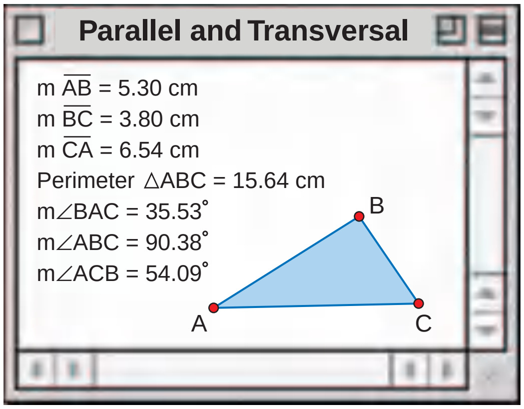
Step 3 Find the perimeter of $\triangle ABC$.

- Use the pointer tool to select points A, B, and C.
- Under the **Construct** menu, select **Triangle Interior**. The triangle will now be shaded.
- Choose the **Perimeter** command under the **Measure** menu to find the perimeter of $\triangle ABC$. The perimeter of $\triangle ABC$ is 15.64 centimeters.



Step 4 Find $m\angle A$, $m\angle B$, and $m\angle C$.

- Recall that $\angle A$ can also be named $\angle BAC$ or $\angle CAB$. Use the pointer to select points B, A, and C in order.
- Select the **Angle** command from the **Measure** menu to find $m\angle A$.
- Select points A, B, and C. Find $m\angle B$.
- Select points A, C, and B. Find $m\angle C$.



ANALYZE THE RESULTS

1. Add the side measures you found in Step 2. Compare this sum to the result of Step 3. How do these compare?
2. What is the sum of the angle measures of $\triangle ABC$?
3. Repeat the activities for each convex polygon.
   a. irregular quadrilateral **b.** square **c.** pentagon **d.** hexagon
4. Draw another quadrilateral and find its perimeter. Then enlarge your figure using the **Dilate** command. How does the change affect the perimeter?
5. Compare your results with those of your classmates.
6. **Make a conjecture** about the sum of the measures of the angles in any triangle.
7. What is the sum of the measures of the angles of a quadrilateral? pentagon? hexagon?
8. Make a conjecture about how the sums of the measures of the angles of polygons are related to the number of sides.
9. Test your conjecture on other polygons. Does your conjecture hold? Explain.
10. When the sides of a polygon are changed by a common factor, does the perimeter of the polygon change by the same factor as the sides? Explain.

1-7 Three-Dimensional Figures

Main Ideas

- Identify three-dimensional figures.
- Find surface area and volume.

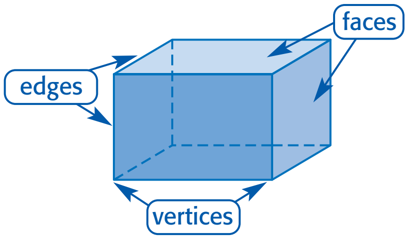
New Vocabulary

polyhedron
face
edges
prism
bases
vertex
regular prism
pyramid
regular polyhedron
Platonic solids
cylinder
cone
sphere
surface area
volume

Archaeologists and Egyptologists continue to study the Great Pyramids of Egypt. Even though some of the exterior materials used to build the pyramids have worn away, scientists can still speculate on the appearance of the pyramids when they were first built.

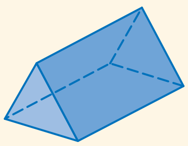
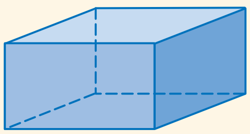
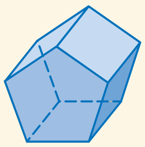
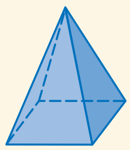
Identify Three-Dimensional Figures

A solid with all flat surfaces that enclose a single region of space is called a **polyhedron**. Each flat surface, or **face**, is a polygon. The line segments where the faces intersect are called **edges**. Edges intersect at a point called a *vertex*.



- A **prism** is a polyhedron with two parallel congruent faces called **bases**. The intersection of three edges is a **vertex**. Prisms are named by the shape of their bases. A **regular prism** is a prism with bases that are regular polygons. A cube is an example of a regular prism.
- A polyhedron with all faces (except for one) intersecting at one vertex is a **pyramid**. Pyramids are named for their bases, which can be any polygon. A polyhedron is a **regular polyhedron** if all of its faces are regular congruent polygons and all of the edges are congruent.

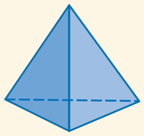
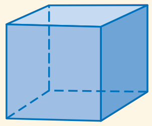
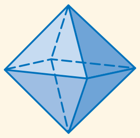
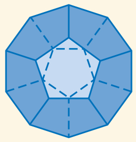
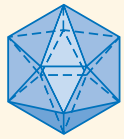
Some common polyhedrons are shown below.

Name	Triangular Prism	Rectangular Prism	Pentagonal Prism	Square Pyramid
Model				
Shape of Base(s)	triangle	rectangle	pentagon	square

There are exactly five types of regular polyhedra. These are called the **Platonic solids** because Plato described them extensively in his writings.

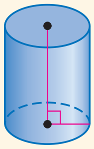
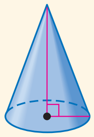
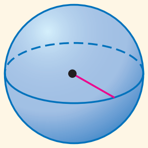
Study Tip

Common Misconception
Prisms can be oriented so the bases are not the top and bottom of the solid.

Platonic Solids					
Name	**Tetrahedron**	**Hexahedron**	**Octahedron**	**Dodecahedron**	**Icosahedron**
Model					
Faces	4	6	8	12	20
Shape of Face	equilateral triangle	square	equilateral triangle	regular pentagon	equilateral triangle

There are solids that are *not* polyhedrons. Some or all of the faces in each of these types of solids are not polygons.

- A **cylinder** is a solid with congruent circular bases in a pair of parallel planes.
- A **cone** has a circular base and a vertex.
- A **sphere** is a set of points in space that are a given distance from a given point.

Name	**Cylinder**	**Cone**	**Sphere**
Model			
Base(s)	2 circles	1 circle	none

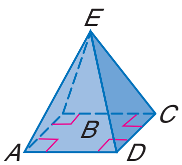
Reading Math

Symbols This symbol ▭ means rectangle. The symbol △ means triangle. The symbol ⊙ means circle.

EXAMPLE Identify Solids

1 **Identify each solid. Name the bases, faces, edges, and vertices.**

a.



The base is a rectangle, and the four faces meet in a point. So this solid is a rectangular pyramid.

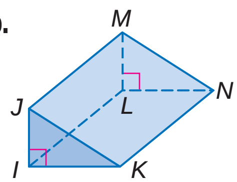
Base: ▭$ABCD$

Faces: ▭$ABCD$, $\triangle AED$, $\triangle DEC$, $\triangle CEB$, $\triangle AEB$

Edges: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$, $\overline{AE}$, $\overline{DE}$, $\overline{CE}$, $\overline{BE}$

Vertices: A, B, C, D, E

b.



The bases are right triangles. It is a triangular prism.

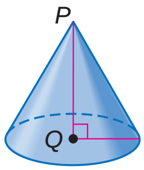
Bases: $\triangle IJK$, $\triangle LMN$

Faces: $\triangle IJK$, $\triangle LMN$, ▭$ILNK$, ▭$KJMN$, ▭$IJML$

Edges: $\overline{IL}$, $\overline{LN}$, $\overline{NK}$, $\overline{IK}$, $\overline{IJ}$, $\overline{LM}$, $\overline{JM}$, $\overline{MN}$, $\overline{JK}$

Vertices: I, J, K, L, M, N

c.



The base is a circle and there is one vertex. So it is a cone.

Base: $\odot Q$

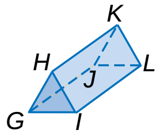
Vertex: P

(continued on the next page)

Identify each solid. Name the bases, faces, edges, and vertices.

1A.

1B.



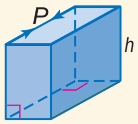
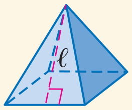
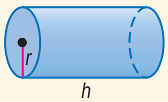
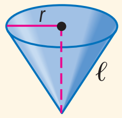
Surface Area and Volume The **surface area** is the sum of the areas of each face of a solid. **Volume** is the measure of the amount of space the solid encloses. You have studied surface area and volume in earlier math classes. The formulas for surface area and volume of four common solids are given below. You will derive these formulas in Chapters 12 and 13.

Study Tip

Formulas

To find the surface area for each solid, find the area of the base(s), and add the area of the face(s).

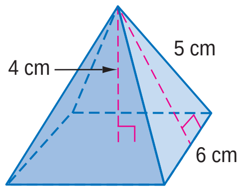
KEY CONCEPT *Surface Area and Volume*

Solid	Prism	Pyramid	Cylinder	Cone
				
Surface Area	$T = Ph + 2B$	$T = \frac{1}{2}P\ell + B$	$T = 2\pi rh + 2\pi r^2$	$T = \pi r\ell + \pi r^2$
Volume	$V = Bh$	$V = \frac{1}{3}Bh$	$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$

P = perimeter of the base
B = area of the base
T = total surface area
h = height of solid
ℓ = slant height
r = radius

EXAMPLE 2 Surface Area and Volume

a. Find the surface area of the square pyramid.



$T = \frac{1}{2}P\ell + B$ Surface area of pyramid

$= \frac{1}{2}(24)(5) + 36$ $P = 24$ cm, $\ell = 5$ cm, $B = 36$ cm^2

$= 96$ cm^2 Simplify.

The surface area of the square pyramid is 96 square centimeters.

b. Find the volume of the square pyramid.

$V = \frac{1}{3}Bh$ Volume of pyramid

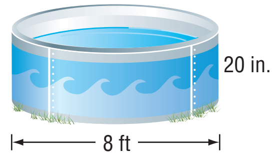
$= \frac{1}{3}(36)(4)$ Substitution

$= 48$ cm^3 Simplify.

The volume is 48 cubic centimeters.

2. Find the surface area and volume of a square pyramid that has a base with dimensions 10 centimeters, a height of 12 centimeters, and a slant height of 13 centimeters.

EXAMPLE 3 Surface Area and Volume



POOLS The diameter of the pool Mr. Diaz purchased is 8 feet. The height of the pool is 20 inches.

a. What is the surface area of the pool?

The pool's surface consists of the sides and one base. The height of the pool is 20 inches or $1\frac{2}{3}$ feet.

$$A = 2\pi rh + \pi r^2 \quad \text{Area of sides + Area of base}$$
$$= 2\pi(4)\left(1\frac{2}{3}\right) + \pi(4)^2 \quad \text{Substitution}$$
$$\approx 92.2 \quad \text{Use a calculator to simplify.}$$

The surface area of the pool is about 92.2 square feet.

b. If he fills the pool with water to a depth of 16 inches, what is the volume of the water in the pool, in cubic feet? Round to the nearest tenth.

The pool is a cylinder. The height of the water is 16 inches. To convert this measure to feet, divide by 12 to get $1\frac{1}{3}$ feet.

$$V = \pi r^2 h \quad \text{Volume of cylinder}$$
$$= \pi(4)^2\left(1\frac{1}{3}\right) \quad \text{Substitution}$$
$$\approx 67.0 \quad \text{Use a calculator to simplify.}$$

The volume of water in the pool is approximately 67.0 cubic feet.

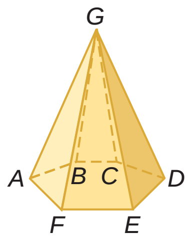
3. **CRAFTS** Jessica is making candles with a mold of a square pyramid. The measure of each side of the base is 2 inches and the height is 4.5 inches. What volume of wax will fill the mold?

Online Personal Tutor at geometryonline.com

CHECK Your Understanding

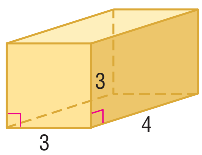
Example 1 (p. 61) **Identify each solid. Name the bases, faces, edges, and vertices.**

1.



2.

Example 2 (p. 62) **For Exercises 3 and 4, refer to the figure.**



3. Find the surface area of the square prism.
4. Find the volume of the square prism.

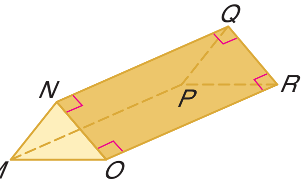
Example 3 (p. 63)

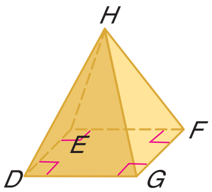
5. **PARTY FAVORS** Latasha is filling cone-shaped hats with candy for party favors. The base of each hat is 4 inches in diameter, and it is 6.5 inches deep. What volume of candy will fill the cone?

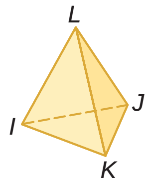
Exercises

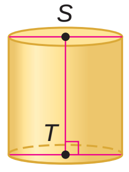
HOMEWORK H	
For Exercises	S Exa
6–11	1
12–19	2
20, 21	3

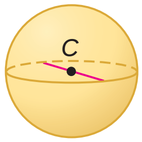
Identify each solid. Name the bases, faces, edges, and vertices.

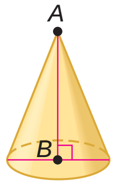
6.


7.


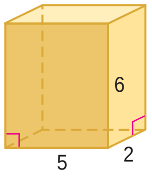
8.


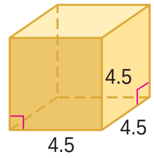
9.


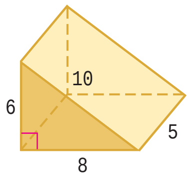
10.


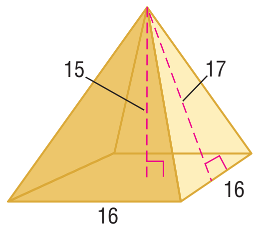
11.


Find the surface area and volume of each solid.

12.


13.


14.


15.


16. SANDBOX A rectangular sandbox is 3 feet by 4 feet. The depth of the box is 8 inches, but the depth of the sand is $\frac{3}{4}$ of the depth of the box. What is the volume of sand in the sandbox? Round to the nearest tenth.

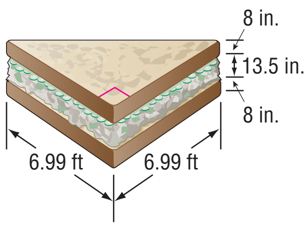
Real-World Link

The Campana brothers started a design workshop in which participants create art using objects that can be inflated, such as tires or balloons.

Source: arango-design.com

17. ART Fernando and Humberto Campana designed the Inflating Table shown at the left. The diameter of the table is $15\frac{1}{2}$ inches. Suppose the height of the cylinder is $11\frac{3}{4}$ inches. What volume of air will fully inflate the table? Round to the nearest tenth. Assume that the side of the table is perpendicular to the bases of the table.

FOOD In 1999, Marks & Spencer, a British department store, created the biggest sandwich ever made. The tuna and cucumber sandwich was in the form of a triangular prism. Suppose each slice of bread was 8 inches thick. Refer to the isometric view of the sandwich.



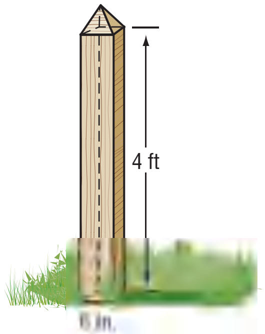
18. Find the surface area in square feet to the nearest tenth.

19. Find the volume, in cubic feet, of filling to the nearest tenth.

20. The surface area of a cube is 54 square inches. Find the length of each edge.

21. The volume of a cube is 729 cubic centimeters. Find the length of each edge.

22. **PAINTING** Desiree is painting her family's fence. Each post is composed of a square prism and a square pyramid. The height of the pyramid is 4 inches. Determine the surface area to be painted and the volume of each post.



23. **CHANGING DIMENSIONS** A rectangular prism has a length of 12 centimeters, width of 18 centimeters, and height of 22 centimeters. Describe the effect on the volume of a rectangular prism when each dimension is doubled.

For Exercises 24–26, use the following table.

24. Name the type of prism or pyramid that has the given number of faces.

Number of Faces	Prism	Pyramid
4	none	tetrahedron
5	a. ___?___	square or rectangular
6	b. ___?___	c. ___?___
7	pentagonal	d. ___?___
8	e. ___?___	heptagonal

25. Analyze the information in the table. Is there a pattern between the number of faces and the bases of the corresponding prisms and pyramids?

26. Is it possible to classify a polyhedron given only the number of faces? Explain.

27. **COLLECT DATA** Use a ruler or tape measure and what you have learned in this lesson to find the surface area and volume of a soup can.

EXTRA PRACTICE
See pages 802, 828.
Math Online
Self-Check Quiz at geometryonline.com

28. **EULER'S FORMULA** The number of faces F, vertices V, and edges E of a polyhedron are related by Euler's (OY luhrz) Formula: $F + V = E + 2$. Determine whether Euler's Formula is true for each of the figures in Exercises 6–11.

H.O.T. Problems

29. **OPEN ENDED** Draw a rectangular prism.

30. **REASONING** Compare a square pyramid and a square prism.

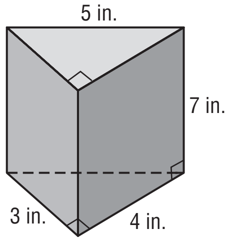
CHALLENGE **Describe the solid that results if the number of sides of each base increases infinitely. The bases of each solid are regular polygons inscribed in a circle.**

31. pyramid

32. prism

33. Writing in Math Refer to the information on archaeologists on page 60. Explain how classifying the shape of an ancient structure is helpful to their study.

34. What is the volume of the triangular prism shown?



A 42 in^3

B 60 in^3

C 84 in^3

D 210 in

35. **REVIEW** The data in the table show the cost of renting a moving truck for an in-town move. The cost is based on the miles driven and also includes a one-time fee.

Renting a Moving Truck	
Miles (m)	**Cost in Dollars (c)**
10	45
20	60
30	75

If miles m were graphed on the horizontal axis and cost c were graphed on the vertical axis, what would be the equation of a line that fits the data?

F $c = 1.5m + 30$

G $c = \frac{1}{30}m + 1.5$

H $c = 3m - 15$

J $c = 30m + 15$

Find the perimeter and area of each figure. (Lesson 1-6)

36. a square with length 12 feet

37. a rectangle with length 4.2 inches and width 15.7 inches

38. a square with length 18 centimeters

39. a rectangle with length 5.3 feet and width 7 feet

Find the measure of the angles. (Lesson 1-5)

40. The measures of two complementary angles are $(3x + 14)^\circ$ and $(5x - 8)^\circ$.

41. The measures of two supplementary angles are $(10x - 25)^\circ$ and $(15x + 50)^\circ$.

Find the value of the variable and *MP*, if *P* is between *M* and *N*. (Lesson 1-2)

42. $MP = 7x$, $PN = 3x$, $PN = 24$

43. $MP = 2c$, $PN = 9c$, $PN = 63$

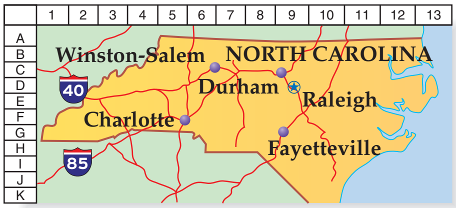
44. $MP = 4x$, $PN = 5x$, $MN = 36$

45. $MP = 6q$, $PN = 6q$, $MN = 60$

46. $MP = 4y + 3$, $PN = 2y$, $MN = 63$

47. $MP = 2b - 7$, $PN = 8b$, $MN = 43$

MAPS **For Exercises 48 and 49, refer to the map, and use the following information.** (Lesson 1-1)
A map represents a plane. Points on this plane are named using a letter / number combination.



48. Name the point where Raleigh is located.

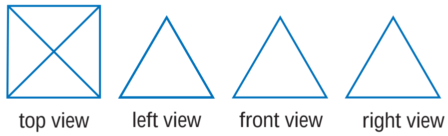
49. What city is located at (F, 5)?

EXTEND 1-7

Geometry Lab
Orthographic Drawings and Nets

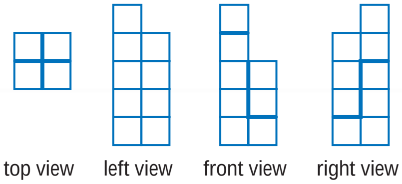
Animation geometryonline.com

If you see a three-dimensional object from only one viewpoint, you may not know its true shape. Here are four views of a square pyramid. The two-dimensional views of the top, left, front, and right sides of an object are called an **orthographic drawing**.

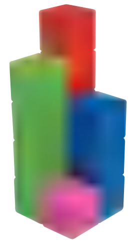


ACTIVITY 1

Make a model of a figure given the orthographic drawing.



- The top view indicates two rows and two columns of different heights.
- The front view indicates that the left side is 5 blocks high and the right side is 3 blocks high. The dark segments indicate breaks in the surface.
- The right view indicates that the right front column is only one block high. The left front column is 4 blocks high. The right back column is 3 blocks high.
- Check the left side of your model. All of the blocks should be flush.

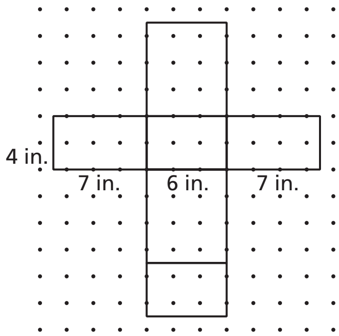


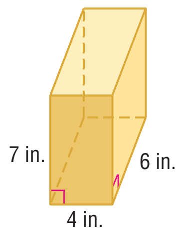
If you cut a cardboard box at the edges and lay it flat, you will have a pattern, or **net**, for the three-dimensional solid.

ACTIVITY 2

Make a model of a figure given the net.

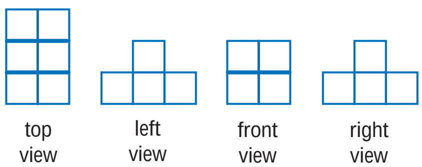
The figure is a rectangular prism. Use a large sheet of paper, a ruler, scissors, and tape. Measure the dimensions on the paper. Cut around the edges. Fold the pattern on the solid lines and secure the edges with tape.



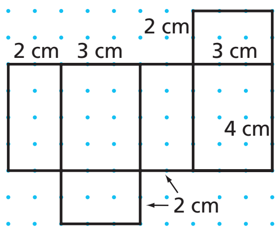


Model and Analyze

1. Make a model of a figure given the orthographic drawing. Then find the volume of the model.



2. Make a model of a figure given the net. Then find the surface area of the model.



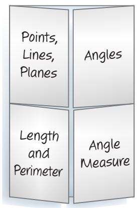
CHAPTER 1

Study Guide and Review

Download Vocabulary Review from geometryonline.com

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Points, Lines, and Planes (Lesson 1-1)

- There is exactly one line through any two points.
- There is exactly one plane through any three noncollinear points.

Distance and Midpoints (Lesson 1-3)

- On a number line, the measure of a segment with endpoint coordinates a and b is given by $|a - b|$.
- In the coordinate plane, the distance between two points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- On a number line, the coordinate of the midpoint of a segment with endpoints that have coordinates a and b is $\frac{a + b}{2}$.
- In the coordinate plane, the coordinates of the midpoint of a segment with endpoints that are (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Angles (Lessons 1-4 and 1-5)

- An angle is formed by two noncollinear rays that have a common endpoint. Angles can be classified by their measures.
- Adjacent angles are two angles that lie in the same plane and have a common vertex and a side but no common interior parts.
- Vertical angles are two nonadjacent angles formed by two intersecting lines.
- A linear pair is a pair of adjacent angles with noncommon sides that are opposite rays.
- Complementary angles are two angles with measures that have a sum of 90.
- Supplementary angles are two angles with measures that have a sum of 180.

Key Vocabulary

acute angle (p. 32)
adjacent angles (p. 40)
angle (p. 31)
angle bisector (p. 35)
area (p. 51)
bases (p. 60)
between (p. 15)
circumference (p. 51)
collinear (p. 6)
complementary angles (p. 42)
concave (p. 49)
cone (p. 61)
congruent (p. 15)
construction (p. 16)
convex (p. 49)
coplanar (p. 6)
cylinder (p. 61)
degree (p. 31)
edges (p. 60)
face (p. 60)
line (p. 6)
linear pair (p. 40)
line segment (p. 13)
midpoint (p. 22)
obtuse angle (p. 32)
opposite rays (p. 31)
perimeter (p. 51)
perpendicular (p. 43)
plane (p. 6)
point (p. 6)
polygon (p. 49)
polyhedron (p. 60)
prism (p. 60)
pyramid (p. 60)
ray (p. 31)
right angle (p. 32)
segment bisector (p. 25)
sides (p. 31)
space (p. 8)
sphere (p. 61)
supplementary angles (p. 42)
undefined term (p. 6)
vertex (p. 31)
vertical angles (p. 40)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or phrase to make a true sentence.

1. A <u>line</u> is determined by points and has no thickness or width. **True**
2. Points that lie on the same plane are said to be <u>collinear</u>. **True**
3. The symbol ≅ is read <u>*is equal to*</u>. **congruent**
4. Two angles whose measures have a sum of 180° are <u>complementary angles</u>. **supplementary angles**
5. A <u>ray</u> can be measured because it has two endpoints. **line segment**

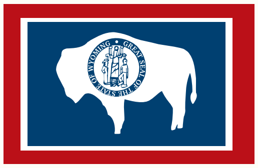
Vocabulary Review at geometryonline.com

Lesson-by-Lesson Review

1–1 Points, Lines, and Planes (pp. 6–11)

Draw and label a figure for each relationship.

6. Lines ℓ and m are coplanar and meet at point C.
7. Points S, T, and U are collinear, but points S, T, U, and V are not.
8. **FLAGS** The Wyoming state flag is shown below. Identify the geometric figures that could be represented by this flag.



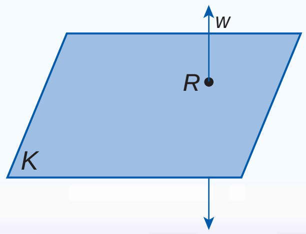
Example 1 **Draw and label a figure for the relationship below.**

Line *w* intersects plane K at *R*.

Draw a surface to represent plane K and label it.

Draw a line intersecting the plane and label it.

Draw a dot where the line and the plane meet and label it.



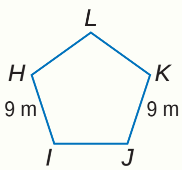
1–2 Linear Measure and Precision (pp. 13–20)

Find the value of the variable and *PB*, if *P* is between *A* and *B*.

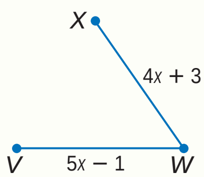
9. $AP = 7, PB = 3x, AB = 25$
10. $AP = s + 2, PB = 4s, AB = 8s - 7$
11. $AP = -2k, PB = k + 6, AB = 11$

Determine whether each pair of segments is congruent.

12. $\overline{HI}, \overline{KJ}$

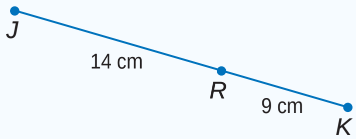


13. $\overline{VW}, \overline{WX}$



14. **HEIGHT** Gregory measured his height to be 71.5 inches. Find and explain the precision for this measurement.

Example 2 **Use the figure to find the measurement $\overline{JK}$.**



$JK = JR + RK$		Betweenness of points
$= 14 + 9$		Substitution
$= 23$		Simplify.

So, $\overline{JK}$ is 23 centimeters long.

Example 3 **Find the precision for 62 miles.**

The measurement is precise to within 0.5 miles. So, a measurement of 62 miles could be 61.5 to 62.5 miles.

1–3 Distance and Midpoints (pp. 21–29)

Use the Pythagorean Theorem to find the distance between each pair of points.

15. $A(1, 0), B(-3, 2)$

16. $G(-7, 4), L(3, 3)$

Use the Distance Formula to find the distance between each pair of points.

17. $J(0, 0), K(4, -1)$

18. $M(-4, 16), P(-6, 19)$

Find the coordinates of the midpoint of each segment.

19. $U(-6, -3), V(12, -7)$

20. $R(3.4, -7.3), S(-2.2, -5.4)$

21. WALKING Paul and Susan are standing outside City Hall. Paul walks three blocks north and two blocks west while Susan walks five blocks south and four blocks east. If City Hall represents the origin, find the coordinates of the midpoint of Paul and Susan's locations.

Example 4 Find the distance between $A(3, -4)$ and $B(-2, 10)$.

Let $(x_1, y_1) = (3, -4)$ and $(x_2, y_2) = (-2, -10)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2 - 3)^2 + (-10 - (-4))^2} \\ &= \sqrt{(-5)^2 + (-6)^2} \\ &= \sqrt{61} \end{aligned}$$

The distance from A to B is $\sqrt{61}$ units or about 7.8 units.

Example 5 Find the coordinates of the midpoint between $G(5, -2)$ and $N(-1, 6)$.

Let $(x_1, y_1) = (5, -2)$ and $(x_2, y_2) = (-1, 6)$.

$$\begin{aligned} &M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= M\left(\frac{5 + (-1)}{2}, \frac{-2 + 6}{2}\right) \\ &= M(2, 2) \end{aligned}$$

The coordinates of the midpoint are (2, 2).

1–4 Angle Measure (pp. 31–38)

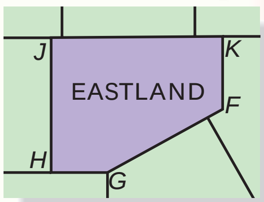
Refer to the figure in Example 6.

22. Name the vertex of $\angle 4$.

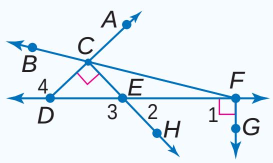
23. Name the sides of $\angle 2$.

24. Write another name for $\angle 2$.

25. COUNTIES Refer to the map of Eastland County. Measure each of the five angles and classify them as *right*, *acute*, or *obtuse*.



Example 6 Refer to the figure. Name all angles that have E as a vertex.

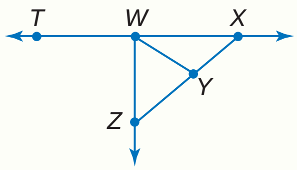


$\angle 3, \angle 2, \angle HEF, \angle DEH, \angle CED, \angle CEF, \angle CEH, \angle DEF$

Mixed Problem Solving
For mixed problem–solving practice, see page 828.

1–5 Angle Relationships (pp. 40–47)

Refer to the figure.



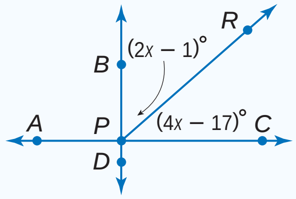
26. Name a linear pair whose vertex is Y.

27. Name an angle supplementary to $\angle XWY$.

28. If $m\angle TWZ = 2c + 36$, find c so that $\overline{TW} \perp \overline{WZ}$.

29. **DRIVING** At the intersection of 3rd and Main Streets, Sareeta makes a 110° turn from Main onto 3rd. Tyrone, behind her, makes a left turn onto 3rd. If 3rd and Main are straight lines, what is the angle measure of his turn?

Example 7 **Refer to the figure. Name a linear pair whose vertex is *P*.**

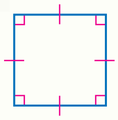


Sample answers: $\angle APR$ and $\angle RPC$, $\angle APD$ and $\angle DPC$, $\angle DPA$ and $\angle APB$, $\angle APB$ and $\angle BPC$, $\angle BPR$ and $\angle RPD$.

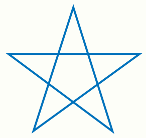
1–6 Two-Dimensional Figures (pp. 49–57)

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

30.

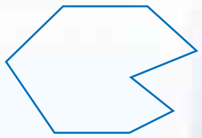


31.



32. **BEES** A honeycomb is formed by repeating regular hexagonal cells, as shown. The length of one side of a cell can range from 5.21 millimeters to 5.375 millimeters. Find the range of perimeters of one cell.

Example 8 **Name the polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.**

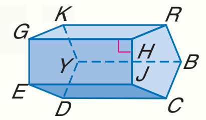


There are 8 sides, so this is an octagon. A line containing two of the sides will pass through the interior of the octagon, so it is concave. Since it is concave, it cannot be regular.

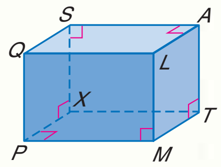
1–7 Three-Dimensional Figures (pp. 60–66)

Identify each solid. Name the bases, faces, edges, and vertices.

33.

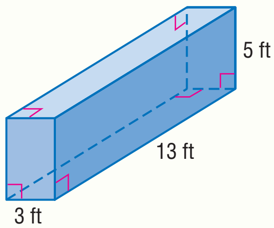


34.

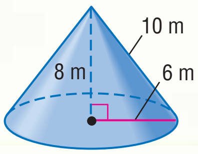


Find the surface area of each solid.

35.

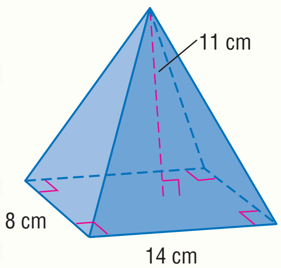


36.

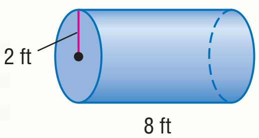


Find the volume of each solid.

37.

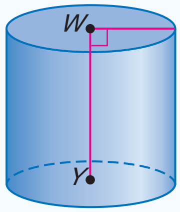


38.



39. CEREAL Company A created a cereal box with dimensions 8.5 by 1.75 by 11.5 inches. Their competitor, company B, created a cereal box with dimensions 8.5 by 2 by 11.25 inches. Which company created the box that would hold more cereal?

Example 9 **Identify the solid below. Name the bases, faces, edges, and vertices.**

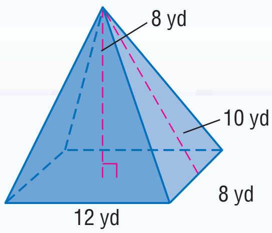


This solid has congruent circular bases in a pair of parallel planes. So, it is a cylinder.

Bases: $\odot W$ and $\odot Y$

A cylinder has no faces, edges, or vertices.

Example 10 **Find the surface area and volume of the rectangular pyramid below.**



$T = \frac{1}{3}P\ell + B$ Surface area of pyramid

$= \frac{1}{3}(40)(10) + 96$ Substitution

$\approx 229.3 \text{ yd}^2$

The surface area is 229.3 square yards.

$V = \frac{1}{3}Bh$ Volume of pyramid

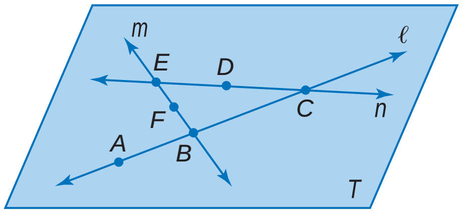
$= \frac{1}{3}(96)(8)$ Substitution

$= 256 \text{ yd}^3$

The volume is 256 cubic yards.

CHAPTER 1 Practice Test

For Exercises 1–3, refer to the figure below.



1. Name the line that contains points B and F.
2. Name a point that is not contained in lines ℓ or m.
3. Name the intersection of lines ℓ and n.

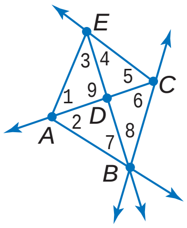
Find the value of the variable and *VW* if *V* is between *U* and *W*.

4. $UV = 2$, $VW = 3x$, $UW = 29$
5. $UV = r$, $VW = 6r$, $UW = 42$
6. $UV = 4p - 3$, $VW = 5p$, $UW = 15$
7. $UV = 3c + 29$, $VW = -2c - 4$, $UW = -4c$

Find the coordinates of the midpoint of a segment having the given endpoints. Then find the distance between the endpoints.

8. $G(0, 0)$, $H(-3, 4)$
9. $A(-4, -4)$, $W(-2, 2)$
10. $N(5, 2)$, $K(-2, 8)$

For Exercises 11–14, refer to the figure below.



11. Name the vertex of $\angle 6$.
12. Name the sides of $\angle 4$.
13. Write another name for $\angle 7$.
14. Write another name for $\angle ADE$.

15. **ALGEBRA** The measures of two supplementary angles are $(4r + 7)°$ and $(r - 2)°$. Find the measures of the angles.

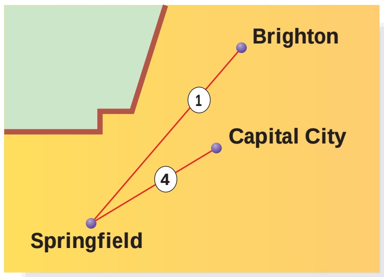
16. **ALGEBRA** Two angles are complementary. One angle measures 26 degrees more than the other. Find the measures of the angles.

Find the perimeter of each polygon.

17. triangle PQR with vertices $P(-6, -3)$, $Q(1, -1)$, and $R(1, -5)$
18. pentagon $ABCDE$ with vertices $A(-6, 2)$, $B(-4, 7)$, $C(0, 4)$, $D(0, 0)$, and $E(-4, -3)$

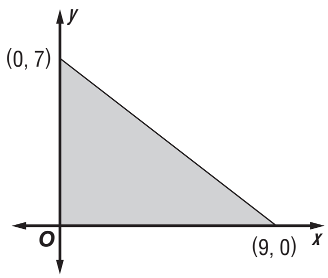
DRIVING For Exercises 19 and 20, use the following information and the diagram.

The city of Springfield is 5 miles west and 3 miles south of Capital City, while Brighton is 1 mile east and 4 miles north of Capital City. Highway 1 runs straight between Brighton and Springfield; Highway 4 connects Springfield and Capital City.



19. Find the length of Highway 1.
20. How long is Highway 4?

21. **MULTIPLE CHOICE** What is the area, in square units, of the triangle shown below?



A 63 **C** 27.4
B 31.5 **D** 8

Standardized Test Practice

CHAPTER 1

Chapter 1

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which of the following describes a plane?

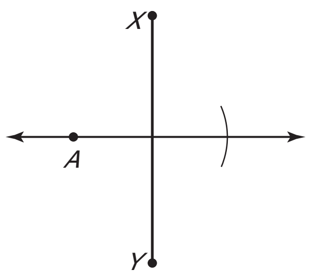
A a location having neither size nor shape

B a flat surface made up of points having no depth

C made up of points and has no thickness or width

D boundless, three-dimensional set of all points

2. Allie is using a straightedge and compass to do the construction shown below.



Which *best* describes the construction Allie is doing?

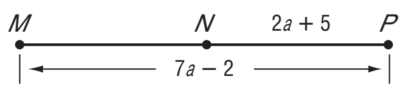
G a line through A parallel to $\overline{XY}$

F a segment starting at A congruent to $\overline{XY}$

H a line through A perpendicular to $\overline{XY}$

J a line through A bisecting $\overline{XY}$

3. What value of a makes N the midpoint of $\overline{MP}$?



4. ALGEBRA $3(x - 2)(x + 4) - 2(x^2 + 3x - 5) =$

A $x^2 - 14$

B $x^2 + 3x + 14$

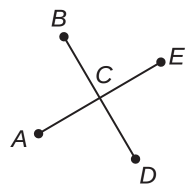
C $x^2 + 12x + 34$

D $5x^2 + 12x - 14$

TEST-TAKING TIP

Question 4 Most standardized tests allow you to write in the test booklet or on scrap paper. To avoid careless errors, work out your answers on paper rather than in your head.

5. In the diagram, $\overline{BD}$ intersects $\overline{AE}$ at C. Which of the following conclusions does *not* have to be true?



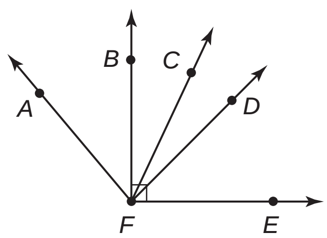
F $\angle ACB \cong \angle ECD$

G $\angle ACB$ and $\angle ACD$ form a linear pair.

H $\angle BCE$ and $\angle ACD$ are vertical angles.

J $\angle BCE$ and $\angle ECD$ are complementary angles.

6. In the figure below, $\overrightarrow{FC}$ bisects $\angle AFE$, and $\overrightarrow{FD}$ bisects $\angle BFE$.

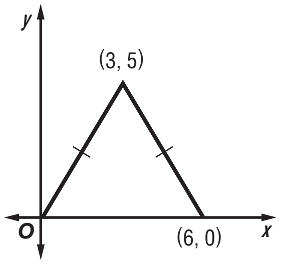


If $m\angle AFE = 130°$, what is $m\angle AFD$ in degrees?

Preparing for Standardized Tests

For test-taking strategies and more practice, see pages 841–856.

7. What is the perimeter of the triangle shown below?



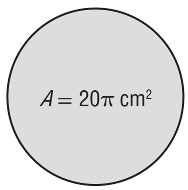
A 11

B 14

C $6 + 2\sqrt{34}$

D $6 + 4\sqrt{2}$

8. The area of a circle is 20π square centimeters. What is its circumference in centimeters?



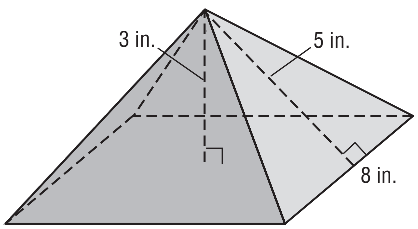
F $\sqrt{5\pi}$

G $2\sqrt{5\pi}$

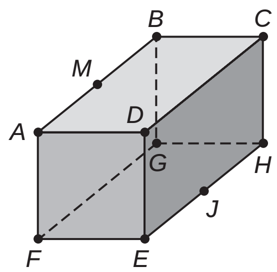
H $4\sqrt{5\pi}$

J 20π

9. GRIDDABLE What is the volume in cubic inches of the square pyramid shown? (Volume of pyramid $= \frac{1}{3}Bh$, where B = area of base)



10. In the figure below, which of the following points are noncoplanar?



A A, D, G, and H

B E, F, J, and H

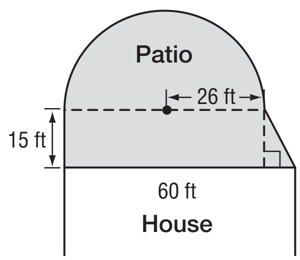
C A, B, J, and M

D C, D, B, and F

Pre AP

Record your answer on a sheet of paper. Show your work.

11. Suppose you work for a landscaping company and need to give a home owner a cost estimate for laying a patio and putting a stone border around the perimeter of the patio that does not share a side with the house. A scale drawing of the proposed patio is shown below.



The cost (labor and materials) for laying the patio is $48 per square yard. The cost (labor and materials) for the stone border is $22 per linear foot. What is your estimate?

NEED EXTRA HELP?											
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11
Go to Lesson or Page...	1-1	1-2	1-3	792	1-5	1-4	1-6	1-6	1-7	1-1	1-6

CHAPTER 2

Reasoning and Proof

- Make conjectures, determine whether a statement is true or false, and find counterexamples for statements.
- Use deductive reasoning to reach valid conclusions.
- Verify algebraic and geometric conjectures using informal and formal proof.
- Write proofs involving segment and angle theorems.

Key Vocabulary

inductive reasoning (p. 78)

deductive reasoning (p. 99)

postulate (p. 105)

theorem (p. 106)

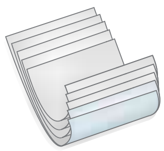
proof (p. 106)

Real-World Link

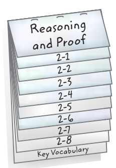
Health Professionals Doctors talk with patients and run tests. They analyze the results and use reasoning to diagnose and treat patients.

Reasoning and Proof Make this Foldable to help you organize your notes. Begin with five sheets of $8\frac{1}{2}$" × 11" plain paper.

1. **Stack** the sheets of paper with edges $\frac{3}{4}$ inch apart. Fold the bottom edges up to create equal tabs.



2. **Staple** along the fold. Label the top tab with the chapter title. Label the next 8 tabs with lesson numbers. The last tab is for Key Vocabulary.



Stewart Cohen/Getty Images

GET READY for Chapter 2

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Evaluate each expression for the given value of n. (Prerequisite Skill)

1. $3n - 2; n = 4$
2. $(n + 1) + n; n = 6$
3. $n^2 - 3n; n = 3$
4. $180(n - 2); n = 5$
5. $n\left(\frac{n}{2}\right); n = 10$
6. $\frac{n(n - 3)}{2}; n = 8$
7. Write the expression *three more than the square of a number*.
8. Write the expression *three less than the square of a number and two*.

Solve each equation. (Prerequisite Skill)

9. $6x - 42 = 4x$
10. $8 - 3n = -2 + 2n$
11. $3(y + 2) = -12 + y$
12. $12 + 7x = x - 18$
13. $3x + 4 = \frac{1}{2}x - 5$
14. $2 - 2x = \frac{2}{3}x - 2$
15. **MUSIC** Mark bought 3 CDs and spent $24. Write and solve an equation for the average cost of each CD. (Prerequisite Skill)

For Exercises 16–19, refer to the figure from Example 3. (Prerequisite Skill)

16. Identify a pair of vertical angles that appear to be acute.
17. Identify a pair of adjacent angles that appear to be obtuse.
18. If $m\angle AGB = 4x + 7$ and $m\angle EGD = 71$, find x.
19. If $m\angle BGC = 45$, $m\angle CGD = 8x + 4$, and $m\angle DGE = 15x - 7$, find x.

QUICKReview

EXAMPLE 1

Evaluate $n^3 - 3n^2 + 3n - 1$ for $n = 1$.

$n^3 - 3n^2 + 3n - 1$	Write the expression.
$= (1)^3 - 3(1)^2 + 3(1) - 1$	Substitute 1 for n.
$= 1 - 3(1) + 3(1) - 1$	Evaluate the exponents.
$= 1 - 3 + 3 - 1$	Multiply.
$= 0$	Simplify.

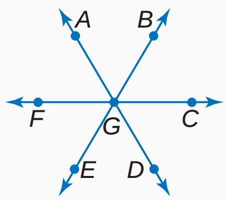
EXAMPLE 2

Solve $70x + 140 = 35x$.

$70x + 140 = 35x$	Write the equation.
$35x + 140 = 0$	Subtract $35x$ from each side.
$35x = -140$	Subtract 140 from each side.
$x = -4$	Divide each side by 35.

EXAMPLE 3

Refer to the figure. If $m\angle AGE = 6x + 2$ and $m\angle BGD = 110$, find x.



$\angle AGE$ and $\angle BGD$ are vertical angles.

$m\angle AGE = m\angle BGD$	Vert. ⦞ are ≅.
$6x + 2 = 110$	Substitution
$6x = 108$	Subtract 2 from each side.
$x = 18$	Divide each side by 6.

2-1 Inductive Reasoning and Conjecture

Main Ideas

- Make conjectures based on inductive reasoning.
- Find counterexamples.

New Vocabulary

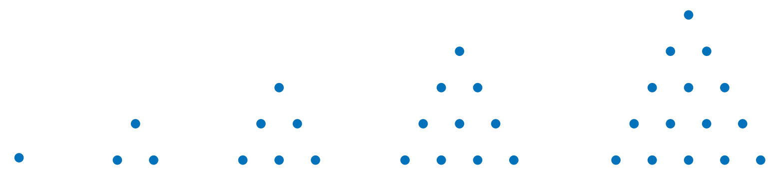
conjecture
inductive reasoning
counterexample

People in the ancient Orient developed mathematics to assist in farming, business, and engineering. Documents from that time show that they taught mathematics by showing several examples and looking for a pattern in the solutions. This process is called inductive reasoning.

Make Conjectures A **conjecture** is an educated guess based on known information. Examining several specific situations to arrive at a conjecture is called inductive reasoning. **Inductive reasoning** is reasoning that uses a number of specific examples to arrive at a plausible generalization or prediction.

EXAMPLE 1 Patterns and Conjecture

The numbers represented below are called *triangular numbers*. Make a conjecture about the next triangular number.



1 3 6 10 15

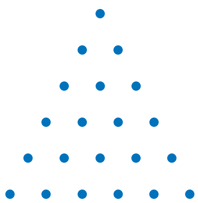
Observe: Each triangle is formed by adding a row of dots.

Find a Pattern: $1 \xrightarrow{+2} 3 \xrightarrow{+3} 6 \xrightarrow{+4} 10 \xrightarrow{+5} 15$

The numbers increase by 2, 3, 4, and 5.

Conjecture: The next number will increase by 6. So, it will be 15 + 6 or 21.

Check: Drawing the next triangle verifies the conjecture.



1. Make a conjecture about the next term in the sequence 20, 16, 11, 5, -2, -10.

Study Tip

Conjectures
List your observations and identify patterns before you make a conjecture.

Agricultural Scenes. B.C.E. 715-322, Egyptian. Detail from The Book of the Dead of Tsekohns./Egyptian Museum, Turin, Italy/Alinari/Bridgeman Art Library

In Chapter 1, you learned some basic geometric concepts. These concepts can be used to make conjectures in geometry.

EXAMPLE **Geometric Conjecture**

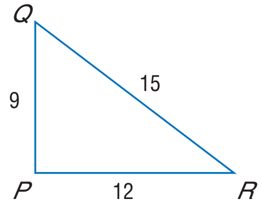
2 **For points *P*, *Q*, and *R*, *PQ* = 9, *QR* = 15, and *PR* = 12. Make a conjecture and draw a figure to illustrate your conjecture.**

Given: points P, Q, and R; $PQ = 9$, $QR = 15$, and $PR = 12$

Examine the measures of the segments.
Since $PQ + PR \neq QR$, the points cannot be collinear.

Conjecture: P, Q, and R are noncollinear.

Check: Draw $\triangle PQR$. This illustrates the conjecture.



2. K is the midpoint of $\overline{JL}$. Make a conjecture and draw a figure to illustrate your conjecture.

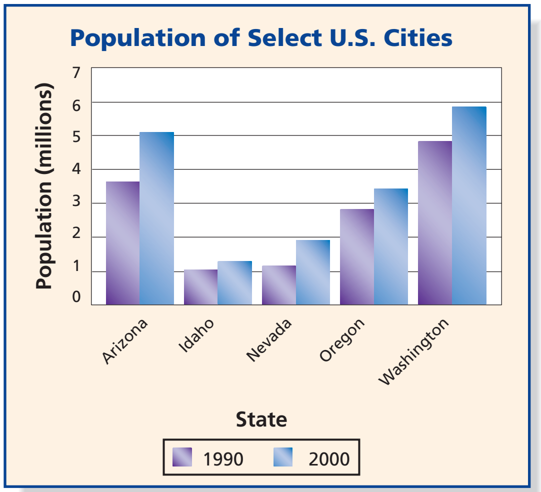
Find Counterexamples A conjecture based on several observations may be true in most circumstances, but false in others. It takes only one false example to show that a conjecture is not true. The false example is called a **counterexample**.

Vocabulary Link
Counterexample
Everyday use: the prefix *counter-* means *the opposite of*
Math use: a counterexample is the opposite of an example

3 **POPULATION** **Find a counterexample for the following statement based on the graph.**

The populations of these U.S. states increased by less than 1 million from 1990 to 2000.

Examine the graph. The statement is true for Idaho, Nevada, and Oregon. However, the populations of Arizona and Washington increased by more than 1 million from 1990 to 2000. Thus, either of these increases is a counterexample to the given statement.



Source: census.gov

3. Find a counterexample to the statement *The states with a population increase of less than 1 million people increased their population by more than 25% from 1990 to 2000.*

Personal Tutor at geometryonline.com

CHECK Your Understanding

Example 1 (p. 78)

Make a conjecture about the next item in each sequence.

1. 

2. $-8, -5, -2, 1, 4$

Example 2 (p. 79)

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

3. $PQ = RS$ and $RS = TU$
4. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at P.

Example 3 (p. 79)

FISHING For Exercises 5 and 6, refer to the graphic and find a counterexample for each statement.

5. The number of youth anglers in a state is less than one-fourth of the total anglers in that state.
6. Each state listed has at least 3,000,000 anglers.

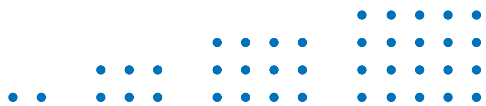
Fishing		
State	Number of Youth Anglers	Percent of Total Anglers per State
California	1,099,000	31
Florida	543,000	15
Michigan	452,000	25
North Carolina	353,000	21.5

Source: American Sportfishing Association

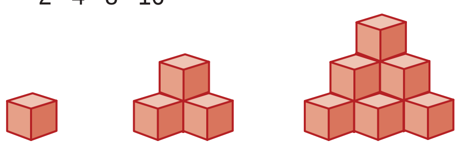
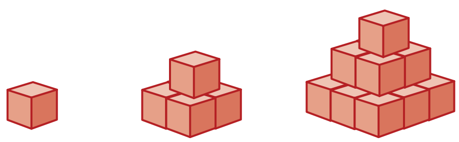
Exercises

HOMEWORK HELP	
For Exercises	See Examples
7–16	1
17–24	2
25–32	3

Make a conjecture about the next item in each sequence.

7. 

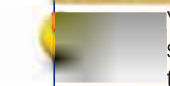
8. 

9. 1, 2, 4, 8, 16
10. 4, 6, 9, 13, 18
11. $\frac{1}{3}, 1, \frac{5}{3}, \frac{7}{3}, 3$
12. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$
13. $2, -6, 18, -54$
14. $-5, 25, -125, 625$
15. 
16. 

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

17. Lines ℓ and m are perpendicular.
18. $A(-2, -11)$, $B(2, 1)$, $C(5, 10)$
19. $\angle 3$ and $\angle 4$ are a linear pair.
20. $\overrightarrow{BD}$ is an angle bisector of $\angle ABC$.
21. $P(-1, 7)$, $Q(6, -2)$, $R(6, 5)$
22. $HIJK$ is a square.
23. $PQRS$ is a rectangle.
24. $\angle B$ is a right angle in $\triangle ABC$.

Cross-Curricular Project



You can use scatter plots to make conjectures about the relationships among latitude, longitude, degree distance, and the monthly high temperature. Visit geometryonline.com to continue work on your project.

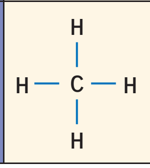
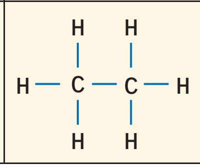
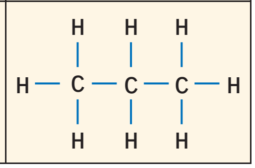
Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

25. **Given:** $\angle 1$ and $\angle 2$ are complementary angles.
    Conjecture: $\angle 1$ and $\angle 2$ form a right angle.
26. **Given:** $m + y \geq 10$, $y \geq 4$
    Conjecture: $m \leq 6$
27. **Given:** points W, X, Y, and Z
    Conjecture: W, X, Y, and Z are noncollinear.
28. **Given:** $A(-4, 8)$, $B(3, 8)$, $C(3, 5)$
    Conjecture: $\triangle ABC$ is a right triangle.
29. **Given:** n is a real number.
    Conjecture: n^2 is a nonnegative number.
30. **Given:** $DE = EF$
    Conjecture: E is the midpoint of $\overline{DF}$.

31. **HOUSES** Most homes in the northern United States have roofs made with steep angles. In the warmer southern states, homes often have flat roofs. Make a conjecture about why the roofs are different.

32. **MUSIC** Many people learn to play the piano by ear. This means that they first learned how to play without reading music. What process did they use?

CHEMISTRY For Exercises 33–35, use the following information.
Hydrocarbons are molecules composed of only carbon (C) and hydrogen (H) atoms. The simplest hydrocarbons are called alkanes. The first three alkanes are shown below.

Alkanes			
Compound Name	Methane	Ethane	Propane
Chemical Formula	CH_4	C_2H_6	C_3H_8
Structural Formula			

EXTRA PRACTICE
See pages 802, 829.
Math Online
Self-Check Quiz at geometryonline.com

33. **MAKE A CONJECTURE** about butane, which is the next compound in the group. Write its structural formula.
34. Write the chemical formula for the 7th compound in the group.
35. Develop a rule you could use to find the chemical formula of the nth substance in the alkane group.

H.O.T. Problems

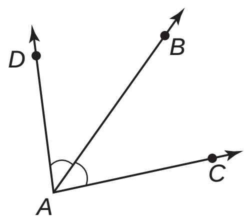
36. **REASONING** Determine whether the following conjecture is *always*, *sometimes*, or *never* true based on the given information. Justify your reasoning.
    Given: collinear points D, E, and F
    Conjecture: $DE + EF = DF$

37. **OPEN ENDED** Write a statement. Then find a counterexample for the statement. Justify your reasoning.

38. **CHALLENGE** The expression $n^2 - n + 41$ has a prime value for $n = 1$, $n = 2$, and $n = 3$. Based on this pattern, you might conjecture that this expression always generates a prime number for any positive integral value of n. Try different values of n to test the conjecture. Answer *true* if you think the conjecture is always true. Answer *false* and give a counterexample if you think the conjecture is false. Justify your reasoning.

39. **Writing in Math** Refer to the information on page 78. Compare the method used to teach mathematics in the ancient Orient to how you have been taught mathematics. Describe any similarities or differences.

40. In the diagram below, $\overrightarrow{AB}$ is an angle bisector of $\angle DAC$.



Which of the following conclusions does *not* have to be true?

A $\angle DAB \cong \angle BAC$

B $\angle DAC$ is a right angle.

C A and D are collinear.

D $2(m\angle BAC) = m\angle DAC$

41. **REVIEW** A chemistry student mixed some 30%-copper sulfate solution with some 40%-copper sulfate solution to obtain 100 mL of a 32%-copper sulfate solution. How much of the 30%-copper sulfate solution did the student use in the mixture?

F 90 mL

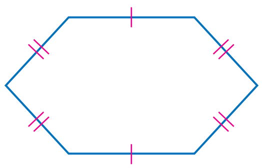
G 80 mL

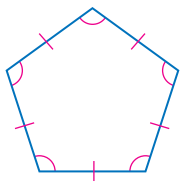
H 60 mL

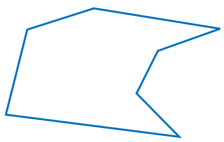
J 20 mL

42. **FISH TANKS** Brittany purchased a cylindrical fish tank. The diameter of the base is 8 inches, and it is 12 inches tall. What volume of water will fill the tank? (Lesson 1-7)

Name each polygon by its number of sides and then classify it as *convex* or *concave* and *regular* or *not regular*. (Lesson 1-6)

43. 

44. 

45. 

PREREQUISITE SKILL **Determine which values in the replacement set make the inequality true.**

46. $x + 2 > 5$
$\{2, 3, 4, 5\}$

47. $12 - x < 0$
$\{11, 12, 13, 14\}$

48. $5x + 1 > 25$
$\{4, 5, 6, 7\}$

2-2 Logic

Main Ideas

- Determine truth values of conjunctions and disjunctions.
- Construct truth tables.

New Vocabulary

statement
truth value
negation
compound statement
conjunction
disjunction
truth table

When you answer true-false questions on a test, you are using a basic principle of logic. For example, refer to the map, and answer *true* or *false*.

Frankfort is a city in Kentucky.

You know that there is only one correct answer, either true or false.

Determine Truth Values A **statement**, like the true-false example above, is any sentence that is either true or false, but not both. Unlike a conjecture, we know that a statement is either true or false. The truth or falsity of a statement is called its **truth value**.

Statements are often represented using a letter such as *p* or *q*. The statement above can be represented by *p*.

p: Frankfort is a city in Kentucky. This statement is true.

The **negation** of a statement has the opposite meaning as well as an opposite truth value. For example, the negation of the statement above is *not p*.

not p: Frankfort is not a city in Kentucky. In this case, the statement is false.

KEY CONCEPT **Negation**

Word	If a statement is represented by *p*, then *not p* is the negation of the statement.
Symbols	$\sim p$, read *not p*

Two or more statements can be joined to form a **compound statement**. Consider the following two statements.

p: Frankfort is a city in Kentucky.
q: Frankfort is the capital of Kentucky.

The two statements can be joined by the word *and*.

p and *q*: Frankfort is a city in Kentucky, *and* Frankfort is the capital of Kentucky.

The statement formed by joining *p* and *q* is an example of a conjunction.

KEY CONCEPT — ***Conjunction***

Words A **conjunction** is a compound statement formed by joining two or more statements with the word *and*.

Symbols $p \wedge q$, read *p and q*

A conjunction is true only when both statements in it are true. Since it is true that Frankfort is in Kentucky and it is the capital, the conjunction is also true.

EXAMPLE Truth Values of Conjunctions

1 **Use the following statements to write a compound statement for each conjunction. Then find its truth value.**

p: **January 1 is the first day of the year.**
q: $-5 + 11 = -6$
r: **A triangle has three sides.**

a. ***p* and *q***

January is the first day of the year, and $-5 + 11 = -6$.
p and *q* is false, because *p* is true and *q* is false.

b. $\sim q \wedge r$

$-5 + 11 \neq -6$, and a triangle has three sides.
$\sim q \wedge r$ is true because $\sim q$ is true and *r* is true.

1A. $r \wedge p$ **1B.** *p* and not *r*

Reading Math

Negations The negation of a statement is not necessarily false. It has the opposite truth value of the original statement.

Statements can also be joined by the word *or*. This type of statement is a disjunction. Consider the following statements.

p: Ahmed studies chemistry.

q: Ahmed studies literature.

p or *q*: Ahmed studies chemistry, *or* Ahmed studies literature.

KEY CONCEPT — ***Disjunction***

Words A **disjunction** is a compound statement formed by joining two or more statements with the word *or*.

Symbols $p \vee q$, read *p or q*

A disjunction is true if at least one of the statements is true. In the case of *p* or *q* above, the disjunction is true if Ahmed either studies chemistry or literature or both. The disjunction is false only if Ahmed studies neither chemistry nor literature.

EXAMPLE 2 Truth Values of Disjunctions

Use the following statements to write a compound statement for each disjunction. Then find its truth value.

***p*: 100 ÷ 5 = 20**

***q*: The length of a radius of a circle is twice the length of its diameter.**

***r*: The sum of the measures of the legs of a right triangle equals the measure of the hypotenuse.**

a. *p* or *q*

100 ÷ 5 = 20, or the length of a radius of a circle is twice the length of its diameter.

p or *q* is true because *p* is true. It does not matter that *q* is false.

b. $q \vee r$

The length of a radius of a circle is twice the length of its diameter, or the sum of the measures of the legs of a right triangle equals the measure of the hypotenuse.

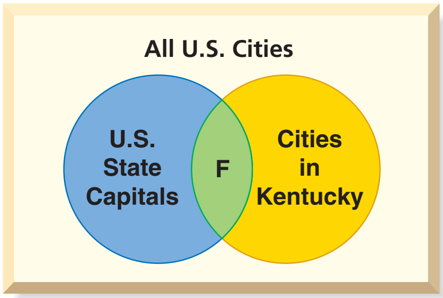
$q \vee r$ is false since neither statement is true.

2. $\sim q \vee r$

Study Tip

Venn Diagrams
The size of the overlapping region in a Venn diagram does not indicate how many items fall into that category.

Conjunctions can be illustrated with Venn diagrams. Refer to the statement at the beginning of the lesson. The Venn diagram at the right shows that Frankfort (F) is represented by the *intersection* of the set of cities in Kentucky and the set of state capitals. In other words, Frankfort is in both the set of cities in Kentucky and in the set of state capitals.



A disjunction can also be illustrated with a Venn diagram. Consider the following statements.

p: Jerrica lives in a U.S. state capital.

q: Jerrica lives in a Kentucky city.

$p \vee q$: Jerrica lives in a U.S. state capital, or Jerrica lives in a Kentucky city.

Reading Math

Intersection and Union The word *intersection* means the point at which more than one object overlap. The word *union* means to group together.

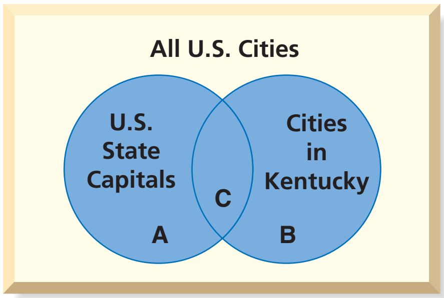
In the Venn diagrams, the disjunction is represented by the *union* of the two sets. The union includes all U.S. capitals and all cities in Kentucky.

The three regions represent

A U.S. state capitals excluding the capital of Kentucky,

B cities in Kentucky excluding the state capital, and

C the capital of Kentucky, which is Frankfort.



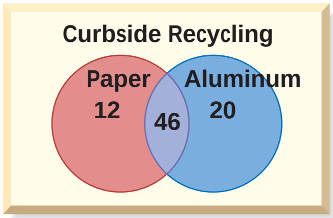
Real-World Link

Earth could be circled 20 times by the amount of paper produced by American businesses in one day.

Source: Resourcefulschools.org

EXAMPLE Use Venn Diagrams

3 **RECYCLING** **The Venn diagram shows the number of neighborhoods that have a curbside recycling program for paper or aluminum.**



a. **How many neighborhoods recycle both paper and aluminum?**

The neighborhoods that have paper and aluminum recycling are represented by the intersection of the sets. There are 46 neighborhoods that have paper and aluminum recycling.

b. **How many neighborhoods recycle paper or aluminum?**

The neighborhoods that have paper or aluminum recycling are represented by the union of the sets. There are 12 + 46 + 20 or 78 neighborhoods that have paper or aluminum recycling.

c. **How many neighborhoods recycle paper and not aluminum?**

The neighborhoods that have paper and not aluminum recycling are represented by the nonintersecting portion of the paper region. There are 12 neighborhoods that have paper and not aluminum recycling.

3. How many neighborhoods recycle aluminum and not paper?

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Truth Tables A convenient method for organizing the truth values of statements is to use a **truth table**.

Negation	
p	$\sim p$
T	F
F	T

If p is a true statement, then $\sim p$ is a false statement.

If p is a false statement, then $\sim p$ is a true statement.

Truth tables can also be used to determine truth values of compound statements.

Conjunction		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

A conjunction is true only when both statements are true.

Disjunction		
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

A disjunction is false only when both statements are false.

You can use the truth values for negation, conjunction, and disjunction to construct truth tables for more complex compound statements.

EXAMPLE Construct Truth Tables

4 **Construct a truth table for each compound statement.**

a. $p \wedge \sim q$

Step 1 Make columns with the headings p, q, $\sim q$, and $p \wedge \sim q$.

Step 2 List the possible combinations of truth values for p and q.

Step 3 Use the truth values of q to determine the truth values of $\sim q$.

Step 4 Use the truth values for p and $\sim q$ to write the truth values for $p \wedge \sim q$.

Step 1

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

Step 2 Step 3 Step 4

Study Tip

Truth Tables

Use the Fundamental Counting Principle to determine the number of rows necessary. In Example 4b, there are 2 possible values for each of the three statements, p, q, and r. So there should be $2 \cdot 2 \cdot 2$ or 8 rows in the table.

b. $(p \wedge q) \vee r$

Make columns for p, q, $p \wedge q$, r, and $(p \wedge q) \vee r$.

p	q	$p \wedge q$	r	$(p \wedge q) \vee r$
T	T	T	T	T
T	F	F	T	T
T	T	T	F	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T
F	T	F	F	F
F	F	F	F	F

4. $\sim p \vee \sim q$

CHECK Your Understanding

Examples 1–2 (pp. 84–85)

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

p: **9 + 5 = 14**

q: **February has 30 days.**

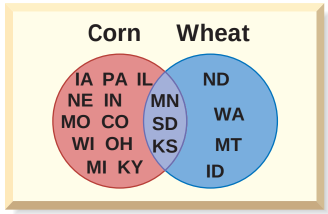
r: **A square has four sides.**

1. p and q
2. $p \wedge r$
3. $q \wedge r$
4. p or $\sim q$
5. $q \vee r$
6. $\sim p \vee \sim r$

Example 3
(p. 86)

AGRICULTURE For Exercises 7–9, refer to the Venn diagram that represents the states producing more than 100 million bushels of corn or wheat per year.

Grain Production



Source: U.S. Department of Agriculture

7. How many states produce more than 100 million bushels of corn?
8. How many states produce more than 100 million bushels of wheat?
9. How many states produce more than 100 million bushels of corn and wheat?

Example 4
(p. 87)

10. Copy and complete the truth table.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	
T	F		
F	T		
F	F		

Construct a truth table for each compound statement.

11. $p \wedge q$
12. $\sim p \wedge r$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
13–24	1, 2
25–31	3
32–41	4

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

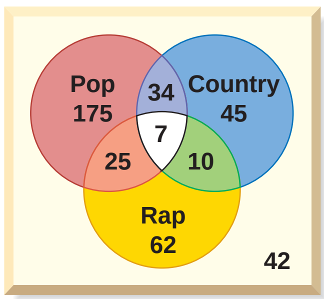
p: $\sqrt{-64} = 8$
q: A triangle has three sides.
r: $0 < 0$
s: An obtuse angle measures greater than 90 and less than 180.

13. p and q
14. p or q
15. p and $\sim r$
16. r and s
17. q or r
18. q and s
19. $p \wedge s$
20. $\sim q \wedge r$
21. $r \vee p$
22. $s \vee q$
23. $(\sim p \wedge q) \vee s$
24. $s \vee (q \wedge \sim r)$

MUSIC For Exercises 25–28, use the following information.

A group of 400 teens were asked what type of music they listened to. They could choose among pop, rap, and country. The results are shown in the Venn diagram.

Music Preference



25. How many said that they listened to none of these types of music?
26. How many said that they listened to all three types of music?
27. How many said that they listened to only pop and rap music?
28. How many said that they listened to pop, rap, or country music?

Real-World Link

Nationwide, approximately 80% of high school seniors participate in extracurricular activities. Athletics, performing arts, and clubs are the most popular.

Source: National Center for Education Statistics

SCHOOL **For Exercises 29–31, use the following information.**
In a school of 310 students, 80 participate in academic clubs, 115 participate in sports, and 20 students participate in both.

29. Make a Venn diagram of the data.

30. How many students participate in either academic clubs or sports?

31. How many students do not participate in either academic clubs or sports?

Copy and complete each truth table.

32.

p	q	$\sim p$	$\sim p \vee q$
T	T		
T	F		
F	T		
F	F		

33.

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T		F	F	
T		F	T	
F		T	F	
F		T	T	

Construct a truth table for each compound statement.

34. q and r **35.** p or q **36.** p or r **37.** p and q

38. $q \wedge \sim r$ **39.** $\sim p \wedge \sim q$ **40.** $\sim p \vee (q \wedge \sim r)$ **41.** $p \wedge (\sim q \vee \sim r)$

GEOGRAPHY **For Exercises 42–44, use the following information.**
A travel agency surveyed their clients about places they had visited. Of the participants, 60 had visited Europe, 45 visited England, and 50 visited France.

42. Make a Venn diagram of the data.

43. Write a conjunction from the data.

44. Write a disjunction from the data.

EXTRA PRACTICE
See pages 802, 829.

Math Online
Self-Check Quiz at geometryonline.com

RESEARCH **For Exercises 45–47, use the Internet or another resource to determine whether each statement is *true* or *false*.**

45. Dallas is not located on the Gulf of Mexico.

46. Either Cleveland or Columbus is located near Lake Erie.

47. It is false that Santa Barbara is located on the Pacific Ocean.

H.O.T. Problems

OPEN ENDED **Write a compound statement for each condition.**

48. a true disjunction

49. a false conjunction

50. a true statement that includes a negation

CHALLENGE **For Exercises 51 and 52, use the following information.**
All members of Team A also belong to Team B, but only some members of Team B also belong to Team C. Teams A and C have no members in common.

51. Draw a Venn diagram to illustrate the situation.

52. Which statement(s) are true? Justify your reasoning.

p: If a person is a member of Team C, then the person is not a member of Team A.

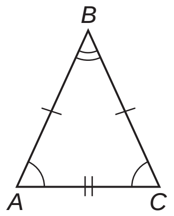
q: If a person is not a member of Team B, then the person is not a member of Team A.

r: No person that is a member of Team A can be a member of Team C.

53. **Writing in Math** Refer to page 83. Describe how you can apply logic to taking tests. Include the difference between a conjunction and a disjunction.

Bill Bachmann/PhotoEdit

54. Which statement about $\triangle ABC$ has the same truth value as $AB = BC$?



A $m\angle A = m\angle C$

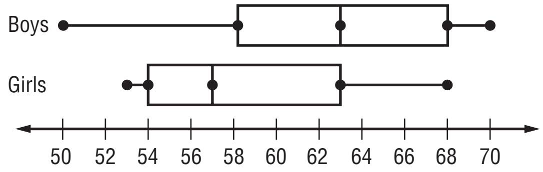
B $m\angle A = m\angle B$

C $AC = BC$

D $AB = AC$

55. **REVIEW** The box-and-whisker plot below represents the height of 9th graders at a certain high school.

Heights of 9th Graders (inches)



How much greater was the median height of the boys than the median height of the girls?

F 4 inches

G 5 inches

H 6 inches

J 7 inches

Make a conjecture about the next item in each sequence. (Lesson 2-1)

56. 3, 5, 7, 9

57. 1, 3, 9, 27

58. $6, 3, \frac{3}{2}, \frac{3}{4}$

59. 17, 13, 9, 5

60. 64, 16, 4, 1

61. 5, 15, 45, 135

62. Rayann has a glass paperweight that is a square pyramid. If the length of each side of the base is 2 inches and the slant height is 2.5 inches, find the surface area. (Lesson 1-7)

COORDINATE GEOMETRY **Find the perimeter of each polygon. Round to the nearest tenth.** (Lesson 1-6)

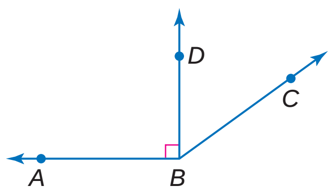
63. triangle ABC with vertices $A(-6, 7)$, $B(1, 3)$, and $C(-2, -7)$

64. square $DEFG$ with vertices $D(-10, -9)$, $E(-5, -2)$, $F(2, -7)$, and $G(-3, -14)$

65. quadrilateral $HIJK$ with vertices $H(5, -10)$, $I(-8, -9)$, $J(-5, -5)$, and $K(-2, -4)$

66. hexagon $LMNPQR$ with vertices $L(2, 1)$, $M(4, 5)$, $N(6, 4)$, $P(7, -4)$, $Q(5, -8)$, and $R(3, -7)$

Measure each angle and classify it as right, acute, or obtuse. (Lesson 1-4)



67. $\angle ABC$

68. $\angle DBC$

69. $\angle ABD$

70. **FENCING** Michelle wanted to put a fence around her rectangular garden. The front and back measured 35 feet each, and the sides measured 75 feet each. She plans to buy 5 extra feet of fencing to make sure that she has enough. How much should she buy? (Lesson 1-2)

PREREQUISITE SKILL **Evaluate each expression for the given values.** (Page 780)

71. $5a - 2b$ if $a = 4$ and $b = 3$

72. $4cd + 2d$ if $c = 5$ and $d = 2$

73. $4e + 3f$ if $e = -1$ and $f = -2$

74. $3g^2 + h$ if $g = 8$ and $h = -8$

2-3 Conditional Statements

Main Ideas

- Analyze statements in if-then form.
- Write the converse, inverse, and contrapositive of if-then statements.

New Vocabulary

conditional statement
if-then statement
hypothesis
conclusion
related conditionals
converse
inverse
contrapositive
logically equivalent

How are conditional statements used in advertisements? Advertisers often lure consumers into purchasing expensive items by convincing them that they are getting something for free in addition to their purchase.

If-Then Statements The statements above are examples of conditional statements. A **conditional statement** is a statement that can be written in *if-then form*. The second example above can be rewritten to illustrate this.

If you buy a car, *then* you get $1500 cash back.

KEY CONCEPT — *If-Then Statement*

Words An **if-then statement** is written in the form *if p, then q*. The phrase immediately following the word *if* is called the **hypothesis**, and the phrase immediately following the word *then* is called the **conclusion**.

Symbols $p \rightarrow q$, read *if p then q*, or *p implies q*.

Reading Math

If and Then The word *if* is not part of the hypothesis. The word *then* is not part of the conclusion.

EXAMPLE 1 Identify Hypothesis and Conclusion

Identify the hypothesis and conclusion of each statement.

a. If points *A*, *B*, and *C* lie on line ℓ, then they are collinear.

If $\underbrace{\text{points } A, B, \text{ and } C \text{ lie on line } \ell}_{\text{hypothesis}}$, then $\underbrace{\text{they are collinear}}_{\text{conclusion}}$.

Hypothesis: points *A*, *B*, and *C* lie on line ℓ

Conclusion: they are collinear

b. The Tigers will play in the tournament if they win their next game.

Hypothesis: the Tigers win their next game

Conclusion: they will play in the tournament

1A. If a polygon has six sides, then it is a hexagon.

1B. Another performance will be scheduled if the first one is sold out.

Some conditional statements are written without the "if" and "then." You can write these statements in if-then form by first identifying the hypothesis and the conclusion.

Real-World Link

Inline skating is the fastest-growing recreational sport. Participation has increased 630% over the past 15 years.

Source: International Inline Skating Association

EXAMPLE Write a Conditional in If-Then Form

2 **Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.**

a. An angle with a measure greater than 90 is an obtuse angle.

Hypothesis: an angle has a measure greater than 90

Conclusion: it is an obtuse angle

If an angle has a measure greater than 90, then it is an obtuse angle.

b. The length of the course for an inline skating marathon is 26.2 miles.

Hypothesis: a course is for an inline skating marathon

Conclusion: it is 26.2 miles

If a course is for an inline skating marathon, then it is 26.2 miles.

2A. An angle formed by perpendicular lines is a right angle.

2B. A cheetah has nonretractile claws.

Recall that the truth value of a statement is either true or false. The hypothesis and conclusion of a conditional statement, as well as the conditional statement itself, can also be true or false.

Study Tip

Common Misconception

A true hypothesis does not necessarily mean that a conditional is true. Likewise, a false conclusion does not guarantee that a conditional is false.

Truth Values of Conditionals

3 SCHOOL **Determine the truth value of the following statement for each set of conditions.**

If you get 100% on your test, then your teacher will give you an A.

a. You get 100%; your teacher gives you an A.

The hypothesis is true since you got 100%, and the conclusion is true because the teacher gave you an A. Since what the teacher promised is true, the conditional statement is true.

b. You get 100%; your teacher gives you a B.

The hypothesis is true, but the conclusion is false. Because the result is not what was promised, the conditional statement is false.

c. You get 98%; your teacher gives you an A.

The hypothesis is false, and the conclusion is true. The statement does not say what happens if you do not get 100% on the test. You could still get an A. It is also possible that you get a B. In this case, we cannot say that the statement is false. Thus, the statement is true.

3. You get 85%; your teacher gives you a B.

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The resulting truth values in Example 3 can be used to create a truth table for conditional statements. Notice that a conditional statement is true in all cases except where the hypothesis is true and the conclusion is false.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Converse, Inverse, and Contrapositive Other statements based on a given conditional statement are known as **related conditionals**. Consider the conditional *If you live in San Francisco, then you live in California*. The hypothesis is *you live in San Francisco*, and the conclusion is *you live in California*. If you reverse the hypothesis and conclusion, you form the conditional *If you live in California, then you live in San Francisco*. This is the converse of the conditional. The inverse and the contrapositive are formed using the negations of the hypothesis and the conclusion.

Animation
geometryonline.com

KEY CONCEPT *Related Conditionals*

Statement	Formed by	Symbols	Examples
Conditional	given hypothesis and conclusion	$p \rightarrow q$	If two angles have the same measure, then they are congruent.
Converse	exchanging the hypothesis and conclusion of the conditional	$q \rightarrow p$	If two angles are congruent, then they have the same measure.
Inverse	negating both the hypothesis and conclusion of the conditional	$\sim p \rightarrow \sim q$	If two angles do not have the same measure, then they are not congruent.
Contrapositive	negating both the hypothesis and conclusion of the converse statement	$\sim q \rightarrow \sim p$	If two angles are not congruent, then they do not have the same measure.

Study Tip

Contrapositive

The relationship of the truth values of a conditional and its contrapositive is known as the Law of Contrapositive.

If a given conditional is true, the converse and inverse are not necessarily true. However, the contrapositive of a true conditional is always true, and the contrapositive of a false conditional is always false. Likewise, the converse and inverse of a conditional are either both true or both false.

Statements with the same truth values are said to be **logically equivalent**. So, a conditional and its contrapositive are logically equivalent as are the converse and inverse of a conditional. These relationships are summarized in the table below.

p	q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\sim p \rightarrow \sim q$	Contrapositive $\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

EXAMPLE Related Conditionals

4 **Write the converse, inverse, and contrapositive of the following statement. Determine whether each statement is *true* or *false*. If a statement is false, give a counterexample.**
***Linear pairs of angles are supplementary*.**

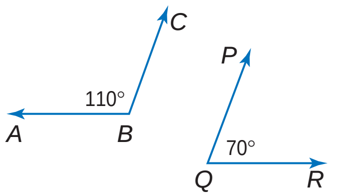
First, write the conditional in if-then form.

Conditional: If two angles form a linear pair, then they are supplementary. The conditional statement is true.

Write the converse by switching the hypothesis and conclusion.

Converse: If two angles are supplementary, then they form a linear pair. The converse is false. $\angle ABC$ and $\angle PQR$ are supplementary, but are not a linear pair.

Inverse: If two angles do not form a linear pair, then they are not supplementary. The inverse is false. $\angle ABC$ and $\angle PQR$ do not form a linear pair, but they are supplementary. The inverse is formed by negating the hypothesis and conclusion of the conditional.



The contrapositive is formed by negating the hypothesis and conclusion of the converse.

Contrapositive: If two angles are not supplementary, then they do not form a linear pair. The contrapositive is true.

4. *Vertical angles are congruent*.

CHECK Your Understanding

Example 1 (p. 91)

Identify the hypothesis and conclusion of each statement.

1. If it rains on Monday, then I will stay home.

2. If $x - 3 = 7$, then $x = 10$.

Example 2 (p. 92)

Write each statement in if-then form.

3. A 32-ounce pitcher holds a quart of liquid.

4. The sum of the measures of supplementary angles is 180.

5. **FORESTRY** In different regions of the country, different variations of trees dominate the landscape. Write the three conditionals in if-then form.

- In Colorado, aspen trees cover high areas of the mountains.
- In Florida, cypress trees rise from swamps.
- In Vermont, maple trees are prevalent.

Example 3 (p. 92)

Determine the truth value of the following statement for each set of conditions.
If you drive faster than 65 miles per hour, then you will receive a speeding ticket.

6. You drive 70 miles per hour, and you receive a speeding ticket.

7. You drive 62 miles per hour, and you do not receive a speeding ticket.

8. You drive 68 miles per hour, and you do not receive a speeding ticket.

Example 4 (p. 94)

Write the converse, inverse, and contrapositive of each conditional statement. Determine whether each related conditional is true or false. If a statement is false, find a counterexample.

9. If plants have water, then they will grow.

10. Flying in an airplane is safer than riding in a car.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
11–18	1
19–26	2
27–36	3
37–42	4

Identify the hypothesis and conclusion of each statement.

11. If you are a teenager, then you are at least 13 years old.

12. If you have a driver's license, then you are at least 16 years old.

13. If $2x + 6 = 10$, then $x = 2$.

14. If three points lie on a line, then they are collinear.

15. "If there is no struggle, there is no progress." (Frederick Douglass)

16. If the measure of an angle is between 0 and 90, then the angle is acute.

17. If a quadrilateral has four congruent sides, then it is a square.

18. If a convex polygon has five sides, then it is a pentagon.

Write each statement in if-then form.

19. Get a free water bottle with a one-year gym membership.

20. Math teachers love to solve problems.

21. "I think, therefore I am." (Descartes)

22. Adjacent angles have a common side.

23. Vertical angles are congruent.

24. Equiangular triangles are equilateral.

25. **MUSIC** Different instruments are emphasized in different types of music.
- Jazz music often incorporates trumpet or saxophone.
- Rock music emphasizes guitar and drums.
- In hip-hop music the bass is featured.

26. **ART** Several artists have their own museums dedicated to exhibiting their work. At the Andy Warhol Museum in Pittsburgh, Pennsylvania, most of the collection is Andy Warhol's artwork.

Real-World Link

The Andy Warhol Museum has over 4000 works of art including prints, paintings, films, photographs, and sculpture. About 500 of these pieces are on display at a time. The museum also has works by colleagues of Andy Warhol.

Source: warhol.org

Determine the truth value of the following statement for each set of conditions.

If you are over 18 years old, then you vote in all elections.

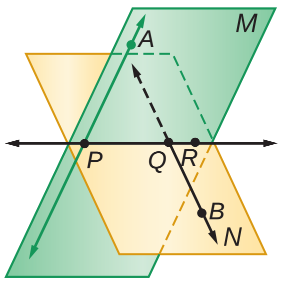
27. You are 19 years old and you vote.

28. You are 21 years old and do not vote.

29. You are 17 years old and do not vote.

30. Your dad is 45 years old and does not vote.

In the figure, *P*, *Q*, and *R* are collinear, *P* and *A* lie in plane *M*, and *Q* and *B* lie in plane *N*. Determine the truth value of each statement.



31. P, Q, and R lie in plane M.

32. $\overleftrightarrow{QB}$ lies in plane N.

33. Q lies in plane M.

34. P, Q, A, and B are coplanar.

35. $\overleftrightarrow{AP}$ contains Q.

36. Planes M and N intersect at $\overleftrightarrow{RQ}$.

"*Self Portrait 9*", 1986. Andy Warhol. Synthentic polymer paint and screenprint on canvas. National Gallery of Victoria, Melbourne, Australia.

Write the converse, inverse, and contrapositive of each conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

37. If you live in Dallas, then you live in Texas.

38. If you exercise regularly, then you are in good shape.

39. The sum of two complementary angles is 90.

40. All rectangles are quadrilaterals.

41. All right angles measure 90.

42. Acute angles have measures less than 90.

SEASONS **For Exercises 43 and 44, use the following information.**
Due to the movement of Earth around the Sun, summer days in Alaska have more hours of daylight than darkness, and winter days have more hours of darkness than daylight.

43. Write two true conditional statements in if-then form for summer days and winter days in Alaska.

44. Write the converse of the two true conditional statements. State whether each is *true* or *false*. If a statement is false, find a counterexample.

EXTRA PRACTICE
See pages 803, 829.

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H.O.T. Problems

45. **OPEN ENDED** Write an example of a conditional statement.

46. **REASONING** Compare and contrast the inverse and contrapositive of a conditional.

47. **CHALLENGE** Write a false conditional. Is it possible to insert the word *not* into your conditional to make it true? If so, write the true conditional.

48. **Writing in Math** Refer to page 91. Describe how conditional statements are used in advertisements. Include an example of a conditional statement in if-then form that could be used in an advertisement.

49. *"If the sum of the measures of two angles is 90, then the angles are complementary angles."*

Which of the following is the converse of the conditional above?

A If the angles are complementary angles, then the sum of the measures of two angles is 90.

B If the angles are not complementary angles, then the sum of the measures of two angles is 90.

C If the angles are complementary angles, then the sum of the measures of two angles is not 90.

D If the angles are not complementary angles, then the sum of the measures of two angles is not 90.

50. **REVIEW** What is $\frac{10a^2 - 15ab}{4a^2 - 9b^2}$ reduced to lowest terms?

F $\frac{5a}{2a - 3b}$

G $\frac{5a}{2a + 3b}$

H $\frac{a}{2a + 3b}$

J $\frac{a}{2a - 3b}$

Graphing Calculator

For Exercises 51–53, refer to the following information.
The program at the right assigns random single digit integers to A and B. Then the program evaluates A and B and assigns a value to C.

```
PROGRAM: BOOLEAN
:randInt (0, 9)→A
:randInt (0, 9)→B
:if A≥2 and B=3
:Then:4→C
:Else:5→C
:End
```

51. Copy the program into your graphing calculator. Execute the program five times.

52. Write the conditional statement used in the program that assigns the value 4 to C.

53. Write the conditional statement that assigns the value 5 to C.

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value. (Lesson 2-2)

***p*: George Washington was the first president of the United States.**
***q*: A hexagon has five sides.**
***r*: $60 \times 3 = 18$**

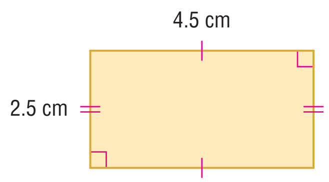
54. $p \wedge q$ **55.** $q \vee r$ **56.** $p \vee q$

57. $\sim q \vee r$ **58.** $p \wedge \sim q$ **59.** $\sim p \wedge \sim r$

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture. (Lesson 2-1)

60. *ABCD* is a rectangle. **61.** $J(-3, 2)$, $K(1, 8)$, $L(5, 2)$ **62.** In $\triangle PQR$, $m\angle PQR = 90$.

For Exercises 63–66, use the rectangle at the right. (Lesson 1-6)



63. Find the perimeter of the rectangle.

64. Find the area of the rectangle.

65. Suppose the length and width of the rectangle are each doubled. What effect does this have on the perimeter?

66. Describe the effect on the area.

Use the Distance Formula to find the distance between each pair of points. (Lesson 1-3)

67. $C(-2, -1)$, $D(0, 3)$ **68.** $J(-3, 5)$, $K(1, 0)$ **69.** $P(-3, -1)$, $Q(2, -3)$

For Exercises 70–72, draw and label a figure for each relationship. (Lesson 1-1)

70. $\overleftrightarrow{FG}$ lies in plane *M* and contains point *H*.

71. Lines *r* and *s* intersect at point *W*.

72. Line ℓ contains *P* and *Q*, but does not contain *R*.

PREREQUISITE SKILL Identify the operation used to change Equation (1) to Equation (2). (Pages 781–782)

73. (1) $3x + 4 = 5x - 8$
(2) $3x = 5x - 12$

74. (1) $\frac{1}{2}(a - 5) = 12$
(2) $a - 5 = 24$

75. (1) $8p = 24$
(2) $p = 3$

READING MATH

Biconditional Statements

Ashley began a new summer job, earning $10 an hour. If she works over 40 hours a week, she earns time and a half, or $15 an hour. If she earns $15 an hour, she has worked over 40 hours a week.

p: Ashley earns $15 an hour
q: Ashley works over 40 hours a week

$p \rightarrow q$: If Ashley earns $15 an hour, she has worked over 40 hours a week.
$q \rightarrow p$: If Ashley works over 40 hours a week, she earns $15 an hour.

In this case, both the conditional and its converse are true. The conjunction of the two statements is called a **biconditional**.

KEY CONCEPT — *Biconditional Statement*

Words A biconditional statement is the conjunction of a conditional and its converse.

Symbols $(p \rightarrow q) \wedge (q \rightarrow p)$ is written $(p \leftrightarrow q)$ and read *p if and only if q*.

If and only if can be abbreviated *iff*.

So, the biconditional statement is as follows.

$p \leftrightarrow q$: Ashley earns $15 an hour *if and only if* she works over 40 hours a week.

Examples

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

a. Two angle measures are complements if and only if their sum is 90.
Conditional: If two angle measures are complements, then their sum is 90.
Converse: If the sum of two angle measures is 90, then they are complements.
Both the conditional and the converse are true, so the biconditional is true.

b. $x > 9$ iff $x > 0$
Conditional: If $x > 9$, then $x > 0$. **Converse:** If $x > 0$, then $x > 9$.
The conditional is true, but the converse is not. Let $x = 2$. Then $2 > 0$ but $2 \not> 9$. So, the biconditional is false.

Reading to Learn

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

1. A calculator will run if and only if it has batteries.
2. Two lines intersect if and only if they are not vertical.
3. Two angles are congruent if and only if they have the same measure.
4. $3x - 4 = 20$ iff $x = 7$.

2-4 Deductive Reasoning

Main Ideas

- Use the Law of Detachment.
- Use the Law of Syllogism.

New Vocabulary

deductive reasoning
Law of Detachment
Law of Syllogism

When you are ill, your doctor may prescribe an antibiotic to help you get better. Doctors may use a dose chart like the one shown to determine the correct amount of medicine you should take.

Weight (kg)	Dose (mg)
10–20	150
20–30	200
30–40	250
40–50	300
50–60	350
60–70	400

Law Of Detachment The process that doctors use to determine the amount of medicine a patient should take is called **deductive reasoning**. Unlike inductive reasoning, which uses examples to make a conjecture, deductive reasoning uses facts, rules, definitions, or properties to reach logical conclusions. Inductive reasoning by itself does not prove anything, but deductive reasoning can be used to prove statements.

One form of deductive reasoning that is used to draw conclusions from true conditional statements is called the **Law of Detachment**.

Study Tip

Validity

When you apply the Law of Detachment, make sure that the conditional is true before you test the validity of the conclusion.

KEY CONCEPT — *Law of Detachment*

Words If $p \rightarrow q$ is true and p is true, then q is also true.

Symbols $[(p \rightarrow q) \wedge p] \rightarrow q$

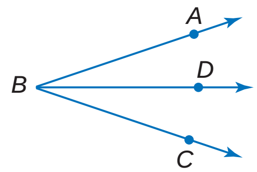
EXAMPLE — Determine Valid Conclusions

1 **The statement below is a true conditional. Determine whether each conclusion is valid based on the given information. Explain your reasoning.**

If a ray is an angle bisector, then it divides the angle into two congruent angles.

Given: $\overrightarrow{BD}$ bisects $\angle ABC$.

Conclusion: $\angle ABD \cong \angle CBD$



The hypothesis states that $\overrightarrow{BD}$ is the bisector of $\angle ABC$. Since the conditional is true and the hypothesis is true, the conclusion is valid.

1. *If segments are parallel, then they do not intersect.*
Given: $\overline{AB}$ and $\overline{CD}$ do not intersect.
Conclusion: $\overline{AB} \parallel \overline{CD}$

Law of Syllogism Another law of logic is the **Law of Syllogism**. It is similar to the Transitive Property of Equality.

KEY CONCEPT *Law of Syllogism*

Words If $p \rightarrow q$ and $q \rightarrow r$ are true, then $p \rightarrow r$ is also true.

Symbols $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Example If $2x = 14$, then $x = 7$ and if $x = 7$, then $\frac{1}{x} = \frac{1}{7}$. Therefore, if $2x = 14$ then $\frac{1}{x} = \frac{1}{7}$.

Study Tip

Conditional Statements
Label the hypotheses and conclusions of a series of statements before applying the Law of Syllogism.

EXAMPLE Determine Valid Conclusions From Two Conditionals

2 **CHEMISTRY** **Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements.**

a. (1) If the symbol of a substance is Pb, then it is lead.
(2) If a substance is lead, then its atomic number is 82.

Let p, q, and r represent the parts of the statements.

p: the symbol of a substance is Pb

q: it is lead

r: the atomic number is 82

Statement (1): $p \rightarrow q$

Statement (2): $q \rightarrow r$

Since the given statements are true, use the Law of Syllogism to conclude $p \rightarrow r$. That is, *If the symbol of a substance is Pb, then its atomic number is 82.*

b. (1) Water can be represented by H_2O.
(2) Hydrogen (H) and oxygen (O) are in the atmosphere.

There is no valid conclusion. While both statements are true, the conclusion of each statement is not used as the hypothesis of the other.

2A. (1) If you stand in line, then you will get to ride the new roller coaster.
(2) If you are at least 48 inches tall, you will get to ride the new roller coaster.

2B. (1) If a polygon has six congruent sides, then it is a regular hexagon.
(2) If a regular hexagon has a side length of 3 units, then the perimeter is 3(6) or 18 units.

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EXAMPLE 3 Analyze Conclusions

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

(1) Vertical angles are congruent.

(2) If two angles are congruent, then their measures are equal.

(3) If two angles are vertical, then their measures are equal.

p: two angles are vertical

q: they are congruent

r: their measures are equal

Statement (3) is a valid conclusion by the Law of Syllogism.

Interactive Lab
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3. (1) The length of a side of square A is the same as the length of a side of square B.
(2) If the lengths of the sides of two squares are the same, then the squares have the same perimeter.
(3) Square A and square B have the same perimeter.

CHECK Your Understanding

Example 1 (p. 99)

Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

If two angles are vertical angles, then they are congruent.

1. Given: $\angle A$ and $\angle B$ are vertical angles.
Conclusion: $\angle A \cong \angle B$

2. Given: $\angle C \cong \angle D$
Conclusion: $\angle C$ and $\angle D$ are vertical angles.

Example 2 (p. 100)

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it. If not, write *no conclusion*.

3. If you are 18 years old, you can vote.
You can vote.

4. The midpoint divides a segment into two congruent segments.
If two segments are congruent, then their measures are equal.

Example 3 (p. 101)

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

5. (1) If Molly arrives at school at 7:30 A.M., she will get help in math.
(2) If Molly gets help in math, then she will pass her math test.
(3) If Molly arrives at school at 7:30 A.M., then she will pass her math test.

6. (1) Right angles are congruent.
(2) $\angle X \cong \angle Y$
(3) $\angle X$ and $\angle Y$ are right angles.

INSURANCE For Exercises 7 and 8, use the following information.
An insurance company advertised the following monthly rates for life insurance.

If you are a:	Premium for $30,000 Coverage	Premium for $50,000 Coverage
Female, age 35	$14.35	$19.00
Male, age 35	$16.50	$21.63
Female, age 45	$21.63	$25.85
Male, age 45	$23.75	$28.90

7. If Marisol is 35 years old and she wants to purchase $30,000 of insurance from this company, then what is her premium?

8. Terry paid $21.63 for life insurance. Can you conclude that Terry is 35? Explain.

Exercises

HOMEWORK HELP

For Exercises	See Examples
9–16	1
17–20	2
21–26	3

For Exercises 9–16, determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.

If two numbers are odd, then their sum is even.

9. **Given:** The sum of two numbers is 22.
Conclusion: The two numbers are odd.

10. **Given:** The numbers are 5 and 7.
Conclusion: The sum is even.

11. **Given:** 11 and 23 are added together.
Conclusion: The sum of 11 and 23 is even.

12. **Given:** The numbers are 2 and 6.
Conclusion: The sum is odd.

If three points are noncollinear, then they determine a plane.

13. **Given:** A, B, and C are noncollinear.
Conclusion: A, B, and C determine a plane.

14. **Given:** E, F, and G lie in plane M.
Conclusion: E, F, and G are noncollinear.

15. **Given:** P and Q lie on a line.
Conclusion: P and Q determine a plane.

16. **Given:** $\triangle XYZ$
Conclusion: X, Y, and Z determine a plane.

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it. If not, write *no conclusion*.

17. If you interview for a job, then you wear a suit.
If you interview for a job, then you will be offered that job.

18. If the measure of an angle is less than 90, then it is acute.
If an angle is acute, then it is not obtuse.

19. If X is the midpoint of segment YZ, then $YX = XZ$.
If the measures of two segments are equal, then they are congruent.

20. If two lines intersect to form a right angle, then they are perpendicular.
Lines ℓ and m are perpendicular.

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

21. (1) Ballet dancers like classical music.
(2) If you like classical music, then you enjoy the opera.
(3) If you are a ballet dancer, then you enjoy the opera.

22. (1) If the measure of an angle is greater than 90, then it is obtuse.
(2) $m\angle ABC > 90$
(3) $\angle ABC$ is obtuse.

23. (1) Vertical angles are congruent.
(2) $\angle 3 \cong \angle 4$
(3) $\angle 3$ and $\angle 4$ are vertical angles.

24. (1) If an angle is obtuse, then it cannot be acute.
(2) $\angle A$ is obtuse.
(3) $\angle A$ cannot be acute.

25. (1) If you drive safely, then you can avoid accidents.
(2) Tika drives safely.
(3) Tika can avoid accidents.

26. (1) If you are athletic, then you enjoy sports.
(2) If you are competitive, then you enjoy sports.
(3) If you are competitive, then you are athletic.

Real-World Link

Carly Patterson was the first American gymnast to win the all-around competition since Mary Lou Retton 20 years before.

Source: www.dallas.about.com

27. LITERATURE John Steinbeck, a Pulitzer Prize-winning author, lived in Monterey, California, for part of his life. In 1945, he published the book, *Cannery Row*, about many of his working-class heroes from Monterey. If you visited Cannery Row in Monterey during the 1940s, then you could hear the grating noise of the fish canneries. Write a valid conclusion to the hypothesis *If John Steinbeck lived in Monterey in 1941, ...*

28. SPORTS In the 2004 Summer Olympics, gymnast Carly Patterson won the gold medal in the women's individual all-around competition. Use the two true conditional statements to reach a valid conclusion about the 2004 competition.

(1) If the sum of Carly Patterson's individual scores is greater than the rest of the competitors, then she wins the competition.

(2) If a gymnast wins the competition, then she earns a gold medal.

EXTRA PRACTICE
See pages 804, 829.
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H.O.T. Problems

29. OPEN ENDED Write an example to illustrate the correct use of the Law of Detachment.

30. REASONING Explain how the Transitive Property of Equality is similar to the Law of Syllogism.

31. FIND THE ERROR An article in a magazine states that if you get seasick, then you will get dizzy. It also says that if you get seasick, you will get an upset stomach. Suzanne says that this means that if you get dizzy, then you will get an upset stomach. Lakeisha says that she is wrong. Who is correct? Explain.

32. CHALLENGE Suppose all triangles that satisfy Property B satisfy the Pythagorean Theorem. Is the following statement *true* or *false*? Justify your answer using what you have learned in Lessons 2-3 and 2-4.
A triangle that is not a right triangle does not satisfy Property B.

33. Writing in Math Refer to page 99. Explain how a doctor uses deductive reasoning to diagnose an illness, such as strep throat or chickenpox.

34. Determine which statement follows logically from the given statements.

If you order two burritos, you also get nachos.

Michael ordered two burritos.

A Michael ordered one burrito.

B Michael will order two burritos.

C Michael ordered nachos and burritos.

D Michael got nachos.

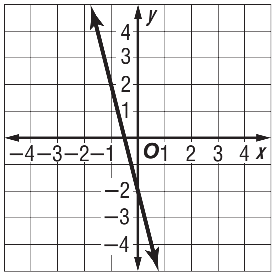
35. **REVIEW** What is the slope of this line?

F $\frac{1}{4}$

G $-\frac{1}{4}$

H 4

J -4



MARKETING **For Exercises 36–38, use the following information.** (Lesson 2-3)

Marketing executives use if-then statements in advertisements to promote products or services. An ad for a car repair shop reads, *If you're looking for fast and reliable car repair, visit AutoCare.*

36. Write the converse of the conditional.

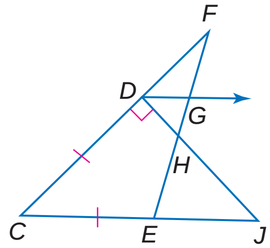
37. What do you think the advertiser wants people to conclude about AutoCare?

38. Does the advertisement say that AutoCare is fast and reliable?

Construct a truth table for each compound statement. (Lesson 2-2)

39. $q \wedge r$ **40.** $\sim p \vee r$ **41.** $p \wedge (q \vee r)$ **42.** $p \vee (\sim q \wedge r)$

For Exercises 43–47, refer to the figure at the right. (Lesson 1-5)



43. Which angle is complementary to $\angle FDG$?

44. Name a pair of vertical angles.

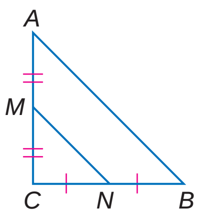
45. Name a pair of angles that are noncongruent and supplementary.

46. Identify $\angle FDH$ and $\angle CDH$ as congruent, adjacent, vertical, complementary, supplementary, and/or a linear pair.

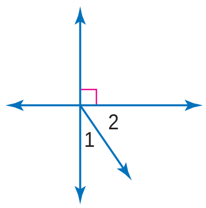
47. Can you assume that $\overline{DC} \cong \overline{CE}$? Explain.

PREREQUISITE SKILL **Write what you can assume about the segments or angles listed for each figure.** (Lesson 1-5)

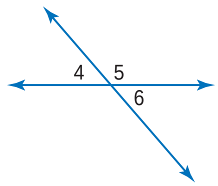
48. $\overline{AM}, \overline{CM}, \overline{CN}, \overline{BN}$



49. $\angle 1, \angle 2$



50. $\angle 4, \angle 5, \angle 6$



2-5 Postulates and Paragraph Proofs

Main Ideas

- Identify and use basic postulates about points, lines, and planes.
- Write paragraph proofs.

New Vocabulary

postulate
axiom
theorem
proof
paragraph proof
informal proof

U.S. Supreme Court Justice William Douglas stated, "The First Amendment makes confidence in the common sense of our people and in the maturity of their judgment the great postulate of our democracy." The writers of the constitution assumed that citizens would act and speak with common sense and maturity. Some statements in geometry also must be assumed or accepted as true.

Points, Lines, and Planes A **postulate** or **axiom** is a statement that is accepted as true. In Chapter 1, you studied basic ideas about points, lines, and planes. These ideas can be stated as postulates.

POSTULATES

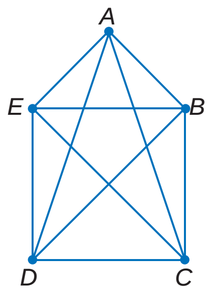
2.1	Through any two points, there is exactly one line.
2.2	Through any three points not on the same line, there is exactly one plane.

Points and Lines

1 COMPUTERS **Each of five computers needs to be connected to every other computer. How many connections need to be made?**

Explore There are five computers, and each is connected to four others.

Plan Draw a diagram to illustrate the solution.



Solve Let noncollinear points A, B, C, D, and E represent the five computers. Connect each point with every other point.

Between every two points there is exactly one segment. So, the connection between computer A and computer B is the same as between computer B and computer A. For the five points, ten segments can be drawn.

Check $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{BC}$, $\overline{BD}$, $\overline{BE}$, $\overline{CD}$, $\overline{CE}$, and $\overline{DE}$ each represent a connection. So, there will be ten connections in all.

1. Determine the number of segments that can connect 4 points.

Study Tip

Drawing Diagrams

When listing segments, start with one vertex and draw all of the segments from that vertex. Then move on to the other vertices until all possible segments have been drawn.

Extra Examples at geometryonline.com

Jeff Hunter/Getty Images

Other postulates are based on relationships among points, lines, and planes.

POSTULATES

2.3 A line contains at least two points.

2.4 A plane contains at least three points not on the same line.

2.5 If two points lie in a plane, then the entire line containing those points lies in that plane.

2.6 If two lines intersect, then their intersection is exactly one point.

2.7 If two planes intersect, then their intersection is a line.

EXAMPLE 2 Use Postulates

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

a. If points *A*, *B*, and *C* lie in plane *M*, then they are collinear.

Sometimes; *A*, *B*, and *C* do not have to be collinear to lie in plane *M*.

b. There is exactly one plane that contains noncollinear points *P*, *Q*, and *R*.

Always; Postulate 2.2 states that through any three noncollinear points, there is exactly one plane.

c. There are at least two lines through points *M* and *N*.

Never; Postulate 2.1 states that through any two points, there is exactly one line.

2. If two coplanar lines intersect, then the point of intersection lies in the same plane as the two lines.

Paragraph Proofs Undefined terms, definitions, postulates, and algebraic properties of equality are used to prove that other statements or conjectures are true. Once a statement or conjecture has been shown to be true, it is called a **theorem**, and it can be used to justify that other statements are true.

You will study and use various methods to verify or prove statements and conjectures in geometry. A **proof** is a logical argument in which each statement you make is supported by a statement that is accepted as true. One type of proof is called a **paragraph proof** or **informal proof**. In this type of proof, you write a paragraph to explain why a conjecture for a given situation is true.

Study Tip

Axiomatic System

An **axiomatic system** is a set of axioms, from which some or all axioms can be used together to logically derive theorems.

KEY CONCEPT

Five essential parts of a good proof:

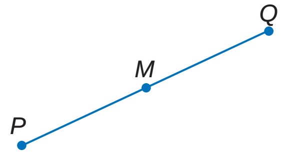
- State the theorem or conjecture to be proven.
- List the given information.
- If possible, draw a diagram to illustrate the given information.
- State what is to be proved.
- Develop a system of deductive reasoning.

Study Tip

Proofs
Before writing a proof, you should have a plan. One strategy is to *work backward*. Start with what you want to prove, and work backward step by step until you reach the given information.

EXAMPLE Write a Paragraph Proof

3 **Given that M is the midpoint of $\overline{PQ}$, write a paragraph proof to show that $\overline{PM} \cong \overline{MQ}$.**

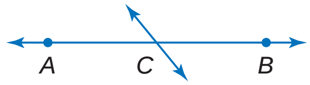


Given: M is the midpoint of $\overline{PQ}$.

Prove: $\overline{PM} \cong \overline{MQ}$

From the definition of midpoint of a segment, $PM = MQ$. This means that $\overline{PM}$ and $\overline{MQ}$ have the same measure. By the definition of congruence, if two segments have the same measure, then they are congruent. Thus, $\overline{PM} \cong \overline{MQ}$.

3. Given that $\overline{AC} \cong \overline{CB}$, and C is between A and B, write a paragraph proof to show that C is the midpoint of $\overline{AB}$.



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Once a conjecture has been proven true, it can be stated as a theorem and used in other proofs. The conjecture in Example 3 is the Midpoint Theorem.

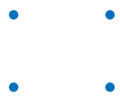
THEOREM 2.1 ***Midpoint Theorem***

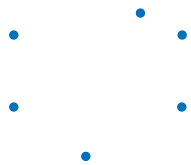
If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$.

CHECK Your Understanding

Example 1 (p. 105)

Determine the number of segments that can be drawn connecting each set of points.

1. 

2. 

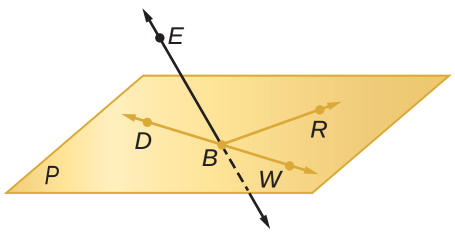
3. **DANCING** Six students will dance at the opening of a new community center. The students, each connected to each of the other students with wide colored ribbons, will move in a circular motion. How many ribbons are needed?

Example 2 (p. 106)

4. Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain. *The intersection of three planes is two lines.*

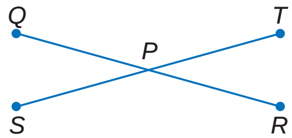
Example 3 (p. 107)

In the figure at the right, $\overleftrightarrow{BD}$ and $\overleftrightarrow{BR}$ are in plane P, and W is on $\overleftrightarrow{BD}$. State the postulate that can be used to show each statement is true.



5. B, D, and W are collinear.
6. E, B, and R are coplanar.

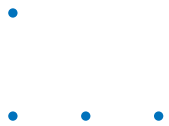
7. **PROOF** In the figure at the right, P is the midpoint of $\overline{QR}$ and $\overline{ST}$, and $\overline{QR} \cong \overline{ST}$. Write a paragraph proof to show that $PQ = PT$.

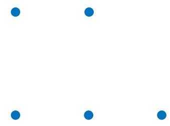


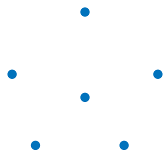
Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–10	1
11–14	2
15, 16	3

Determine the number of segments that can be drawn connecting each set of points.

8. 

9. 

10. 

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

11. Three points determine a plane.

12. Points G and H are in plane X. Any point collinear with G and H is in plane X.

13. The intersection of two planes can be a point.

14. Points S, T, and U determine three lines.

15. **PROOF** Point C is the midpoint of $\overline{AB}$ and B is the midpoint of $\overline{CD}$. Prove that $\overline{AC} \cong \overline{BD}$.

16. **PROOF** Point L is the midpoint of $\overline{JK}$. $\overline{JK}$ intersects $\overline{MK}$ at K. If $\overline{MK} \cong \overline{JL}$, prove that $\overline{LK} \cong \overline{MK}$.

Real-Life Career

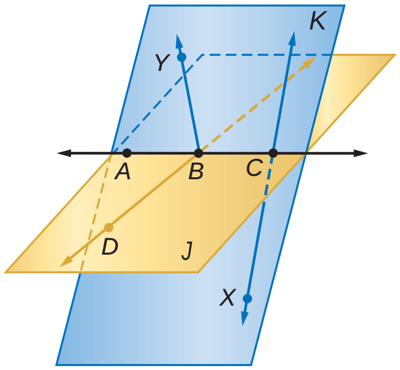
Detective

A police detective gathers facts and collects evidence for use in criminal cases. The facts and evidence are used together to prove a suspect's guilt in court.

Math Online

For more information, go to geometryonline.com.

In the figure at the right, $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ lie in plane J, and $\overleftrightarrow{BY}$ and $\overleftrightarrow{CX}$ lie in plane K. State the postulate that can be used to show each statement is true.



17. C and D are collinear.

18. $\overleftrightarrow{XB}$ lies in plane K.

19. Points A, C, and X are coplanar.

20. $\overleftrightarrow{AD}$ lies in plane J.

21. **CAREERS** Many professions use deductive reasoning and paragraph proofs. For example, a police officer uses deductive reasoning investigating a traffic accident and then writes the findings in a report. List a profession, and describe how it can use paragraph proofs.

22. **MODELING** Isabel's teacher asked her to make a model showing the number of lines and planes formed from four points that are noncollinear and noncoplanar. Isabel decided to make a mobile of straws, pipe cleaners, and colored sheets of tissue paper. She plans to glue the paper to the straws and connect the straws together to form a group of connected planes. How many planes and lines will she have?

H.O.T. Problems

23. **REASONING** Explain how deductive reasoning is used in a proof. List the types of reasons that can be used for justification in a proof.

24. **OPEN ENDED** Draw figures to illustrate Postulates 2.6 and 2.7.

EXTRA PRACTICE
See pages 803, 829.

Math Online
Self-Check Quiz at
geometryonline.com

25. **Which One Doesn't Belong?** Identify the term that does not belong with the other three. Explain your reasoning.

postulate	conjecture	theorem	axiom

26. **CHALLENGE** Three noncollinear points lie in a single plane. In Exercise 22, you found the number of planes defined by four noncollinear points. What are the least and greatest number of planes defined by five noncollinear points?

27. **Writing in Math** Refer to page 105. Describe how postulates are used in literature. Include an example of a postulate in historic United States' documents.

28. Which statement *cannot* be true?

A Three noncollinear points determine a plane.

B Two lines intersect at exactly one point.

C At least two lines can contain the same two points.

D A midpoint divides a segment into two congruent segments.

29. **REVIEW** Which is one of the solutions to the equation $3x^2 - 5x + 1 = 0$?

F $\frac{5 + \sqrt{13}}{6}$

G $\frac{-5 - \sqrt{13}}{6}$

H $\frac{5}{6} - \sqrt{13}$

J $-\frac{5}{6} + \sqrt{13}$

30. Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*. (Lesson 2-4)
(1) Part-time jobs require 20 hours of work per week.
(2) Diego has a part-time job.
(3) Diego works 20 hours per week.

31. Write the converse, inverse, and contrapositive of the conditional statement. Determine whether the related conditional is *true* or *false*. If a statement is false, find a counterexample. *If you have access to the Internet at your house, then you have a computer.* (Lesson 2-3)

PREREQUISITE SKILL **Solve each equation.** (Pages 781–782)

32. $m - 17 = 8$ 33. $3y = 57$ 34. $\frac{y}{6} + 12 = 14$ 35. $-t + 3 = 27$

CHAPTER 2 Mid-Chapter Quiz

Lessons 2-1 through 2-5

Determine whether each conjecture is *true* or *false*. Give a counterexample for any *false* conjecture. (Lesson 2-1)

1. **Given:** $WX = XY$
   Conjecture: W, X, Y are collinear.
2. **Given:** $\angle 1$ and $\angle 2$ are not complementary.
   $\angle 2$ and $\angle 3$ are complementary.
   Conjecture: $m\angle 1 = m\angle 3$

3. **PETS** Michaela took a survey of six friends and created the table shown below.

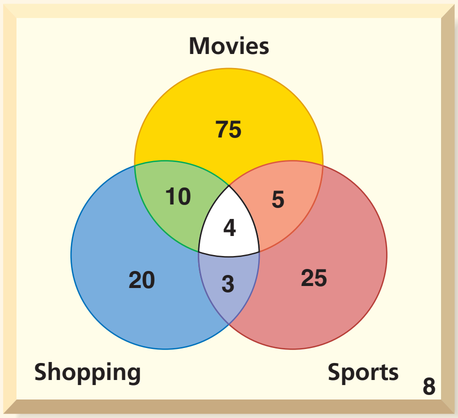
Name	Number of Cats	Number of Dogs	Number of Fish
Kristen	2	0	0
Jorge	0	0	5
Mark	0	2	2
Carissa	1	1	0
Alex	2	1	10
Akilah	1	1	1

Michaela reached the following conclusion. *If a person has 3 or more pets, then they have a dog*. Is this conclusion valid? If not, find a counterexample. (Lesson 2-1)

Construct a truth table for each compound statement. (Lesson 2-2)

4. $\sim p \wedge q$
5. $p \vee (q \wedge r)$

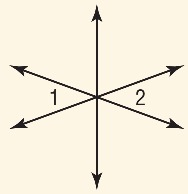
6. **RECREATION** A group of 150 students were asked what they like to do during their free time. How many students like going to the movies or shopping? (Lesson 2-2)



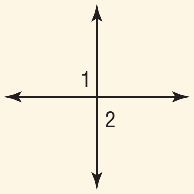
7. **MULTIPLE CHOICE** Which figure can serve as a *counterexample* to the conjecture below?

   If $\angle 1$ and $\angle 2$ share exactly one point, then they are vertical angles.

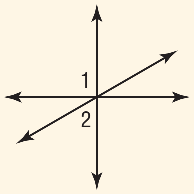
A



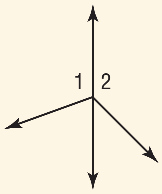
C



B



D



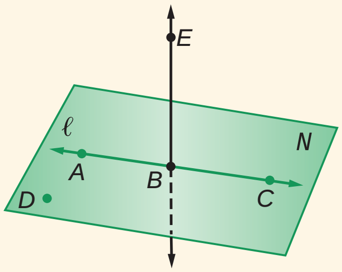
8. Write the converse, inverse, and contrapositive of the following conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample. (Lesson 2-3)

   If two angles are adjacent, then the angles have a common vertex.

9. Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*. (Lesson 2-4)

   (1) If n is an integer, then n is a real number.
   (2) n is a real number.
   (3) n is an integer.

In the figure below, *A*, *B*, and *C* are collinear. Points *A*, *B*, *C*, and *D* lie in plane *N*. State the postulate or theorem that can be used to show each statement is true. (Lesson 2-5)



10. A, B, and D determine plane N.
11. $\overleftrightarrow{BE}$ intersects $\overleftrightarrow{AC}$ at B.
12. ℓ lies in plane N.

2-6 Algebraic Proof

Main Ideas

- Use algebra to write two-column proofs.
- Use properties of equality in geometry proofs.

New Vocabulary

deductive argument
two-column proof
formal proof

GET READY for the Lesson

Lawyers develop their cases using logical arguments based on evidence to lead a jury to a conclusion favorable to their case. At the end of a trial, a lawyer makes closing remarks summarizing the evidence and testimony that they feel proves their case. These closing arguments are similar to a proof in mathematics.

Algebraic Proof Algebra is a system with sets of numbers, operations, and properties that allow you to perform algebraic operations. The following table summarizes several properties of real numbers that you studied in algebra.

Study Tip

Commutative and Associative Properties

Throughout this text, we shall assume the Commutative and Associative Properties for addition and multiplication.

CONCEPT SUMMARY *Properties of Real Numbers*

The following properties are true for any numbers a, b, and c.

Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.
Addition and Subtraction Properties	If $a = b$, then $a + c = b + c$ and $a - c = b - c$.
Multiplication and Division Properties	If $a = b$, then $a \cdot c = b \cdot c$ and if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then a may be replaced by b in any equation or expression.
Distributive Property	$a(b + c) = ab + ac$

The properties of equality can be used to justify each step when solving an equation. A group of algebraic steps used to solve problems form a **deductive argument**.

EXAMPLE 1 Verify Algebraic Relationships

Solve $3(x - 2) = 42$. Justify each step.

Algebraic Steps	Properties
$3(x - 2) = 42$	Original equation
$3x - 6 = 42$	Distributive Property
$3x - 6 + 6 = 42 + 6$	Addition Property
$3x = 48$	Substitution Property
$\frac{3x}{3} = \frac{48}{3}$	Division Property
$x = 16$	Substitution Property

CHECK Your Progress

1. Solve $2x + 3 = 5$. Justify each step.

Study Tip

Mental Math

If your teacher permits you to do so, some steps may be eliminated by performing mental calculations. For example, in Example 2, statements 4 and 6 could be omitted. Then the reason for statements 5 and 7 would be Addition Property and Division Property, respectively.

Example 1 is a proof of the conditional statement *If* $3(x - 2) = 42$, *then* $x = 16$. Notice that the column on the left is a step-by-step process that leads to a solution. The column on the right contains the reason for each statement. In geometry, a similar format is used to prove conjectures and theorems. A **two-column proof**, or **formal proof**, contains *statements* and *reasons* organized in two columns.

EXAMPLE 2 Write a Two-Column Proof

Write a two-column proof to show that if $3\left(x - \frac{5}{3}\right) = 1$, then $x = 2$.

Statements	Reasons
1. $3\left(x - \frac{5}{3}\right) = 1$	**1.** Given
2. $3x - 3\left(\frac{5}{3}\right) = 1$	**2.** Distributive Property
3. $3x - 5 = 1$	**3.** Substitution
4. $3x - 5 + 5 = 1 + 5$	**4.** Addition Property
5. $3x = 6$	**5.** Substitution
6. $\frac{3x}{3} = \frac{6}{3}$	**6.** Division Property
7. $x = 2$	**7.** Substitution

CHECK Your Progress

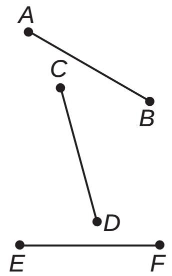
2. The Pythagorean Theorem states that in a right triangle ABC, $c^2 = a^2 + b^2$. Write a two-column proof to show that $a = \sqrt{c^2 - b^2}$.

Geometric Proof Segment measures and angle measures are real numbers, so properties of real numbers can be used to discuss their relationships.

Property	Segments	Angles
Reflexive	$AB = AB$	$m\angle 1 = m\angle 1$
Symmetric	If $AB = CD$, then $CD = AB$.	If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.
Transitive	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

STANDARDIZED TEST EXAMPLE **Justify Geometric Relationships**

3 If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then which of the following is a valid conclusion?



I $AB = CD$ and $CD = EF$

II $\overline{AB} \cong \overline{EF}$

III $AB = EF$

A I only

B I and II

C I and III

D I, II, and III

Test-Taking Tip

Analyzing Statements
More than one statement may be correct. Work through each problem completely before indicating your answer.

Read the Test Item

Determine whether the statements are true based on the given information.

Solve the Test Item

Statement I: Examine the given information, $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$. From the definition of congruent segments, $AB = CD$ and $CD = EF$. Thus, Statement I is true.

Statement II: By the definition of congruent segments, if $AB = EF$, then $\overline{AB} \cong \overline{EF}$. Statement II is true also.

Statement III: If $AB = CD$ and $CD = EF$, then $AB = EF$ by the Transitive Property. Thus, Statement III is true.

Because Statements I, II, and III are true, choice D is correct.

CHECK Your Progress

3. If $m\angle 1 = m\angle 2$ and $m\angle 2 = 90$, then which of the following is a valid conclusion?

F $m\angle 1 = 45$

G $m\angle 1 = 90$

H $m\angle 1 + m\angle 2 = 180$

J $m\angle 1 + m\angle 2 = 90$

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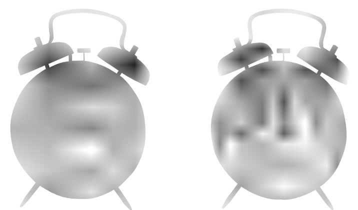
In Example 3, each conclusion was justified using a definition or property. This process is used in geometry to verify and prove statements.

EXAMPLE **Geometric Proof**

4 **TIME** **On a clock, the angle formed by the hands at 2:00 is a 60° angle. If the angle formed at 2:00 is congruent to the angle formed at 10:00, prove that the angle at 10:00 is a 60° angle.**



Given: $m\angle 2 = 60$
$\angle 2 \cong \angle 10$

Prove: $m\angle 10 = 60$

(continued on the next page)

Extra Examples at geometryonline.com

Aaron Haupt

Proof:

Statements	Reasons
1. $m\angle 2 = 60$; $\angle 2 \cong \angle 10$	**1.** Given
2. $m\angle 2 = m\angle 10$	**2.** Definition of congruent angles
3. $60 = m\angle 10$	**3.** Substitution
4. $m\angle 10 = 60$	**4.** Symmetric Property

CHECK Your Progress

4. It is given that $\angle A$ and $\angle B$ are congruent. The measure of $\angle A$ is 110. Write a two-column proof to show that the measure of $\angle B$ is 110.

CHECK Your Understanding

Example 1 (p. 112)

State the property that justifies each statement.

1. If $\frac{x}{2} = 7$, then $x = 14$.

2. If $x = 5$ and $b = 5$, then $x = b$.

3. If $XY - AB = WZ - AB$, then $XY = WZ$.

Example 2 (p. 112)

4. Complete the following proof.

Given: $5 - \frac{2}{3}x = 1$

Prove: $x = 6$

Proof:

Statements	Reasons
a. ___?___	**a.** Given
b. $3\left(5 - \frac{2}{3}x\right) = 3(1)$	**b.** ___?___
c. $15 - 2x = 3$	**c.** ___?___
d. ___?___	**d.** Subtraction Property
e. $x = 6$	**e.** ___?___

Example 3 (p. 113)

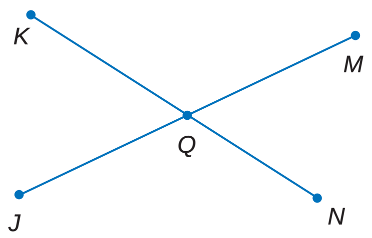
5. MULTIPLE CHOICE If $\overline{JM}$ and $\overline{KN}$ intersect at Q to form $\angle JQK$ and $\angle MQN$, which of the following is *not* a valid conclusion?

A $\angle JQK$ and $\angle MQN$ are vertical angles.

B $\angle JQK$ and $\angle MQN$ are supplementary.

C $\angle JQK \cong \angle MQN$

D $m\angle JQK = m\angle MQN$



Examples 2 and 4 (pp. 112–114)

PROOF Write a two-column proof for each conditional.

6. If $25 = -7(y - 3) + 5y$, then $-2 = y$.

7. If rectangle $ABCD$ has side lengths $AD = 3$ and $AB = 10$, then $AC = BD$.

Exercises

HOMEWORK HELP

For Exercises	See Examples
8–13	1
14–17	3
18–23	2
24, 25	4

State the property that justifies each statement.

8. If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, $m\angle A = m\angle C$.

9. If $HJ + 5 = 20$, then $HJ = 15$.

10. If $XY + 20 = YW$ and $XY + 20 = DT$, then $YW = DT$.

11. If $m\angle 1 + m\angle 2 = 90$ and $m\angle 2 = m\angle 3$, then $m\angle 1 + m\angle 3 = 90$.

12. If $\frac{1}{2}AB = \frac{1}{2}EF$, then $AB = EF$.

13. $AB = AB$

14. If $2\left(x - \frac{3}{2}\right) = 5$, then $2x - 3 = 5$.

15. If $m\angle 4 = 35$ and $m\angle 5 = 35$, then $m\angle 4 = m\angle 5$.

16. If $\frac{1}{2}AB = \frac{1}{2}CD$, then $AB = CD$.

17. If $EF = GH$ and $GH = JK$, then $EF = JK$.

Complete each proof.

18. Given: $\frac{3x + 5}{2} = 7$

Prove: $x = 3$

Proof:

Statements	Reasons
a. $\frac{3x + 5}{2} = 7$	**a.** ___?___
b. ___?___	**b.** Multiplication Property
c. $3x + 5 = 14$	**c.** ___?___
d. $3x = 9$	**d.** ___?___
e. ___?___	**e.** Division Property

19. Given: $2x - 7 = \frac{1}{3}x - 2$

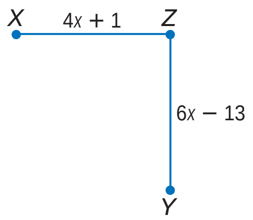
Prove: $x = 3$

Proof:

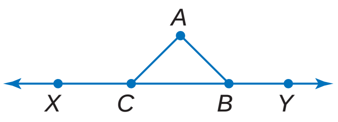
Statements	Reasons
a. ___?___	**a.** Given
b. ___?___	**b.** Multiplication Property
c. $6x - 21 = x - 6$	**c.** ___?___
d. ___?___	**d.** Subtraction Property
e. $5x = 15$	**e.** ___?___
f. ___?___	**f.** Division Property

PROOF **Write a two-column proof.**

20. If $XZ = ZY$, $XZ = 4x + 1$, and $ZY = 6x - 13$, then $x = 7$.



21. If $m\angle ACB = m\angle ABC$, then $\angle XCA \cong \angle YBA$.



EXTRA PRACTICE

See pages 804, 829.

Math Online

Self-Check Quiz at geometryonline.com

Real-World Link

The Formula Society of Automotive Engineers at University of California, Berkeley, holds a competition each year for the design and construction of a race car. The cars are judged on many factors, including acceleration.

Source: www.dailycal.org

PROOF **Write a two-column proof.**

22. If $-\frac{1}{2}m = 9$, then $m = -18$.

23. If $5 - \frac{2}{3}z = 1$, then $z = 6$.

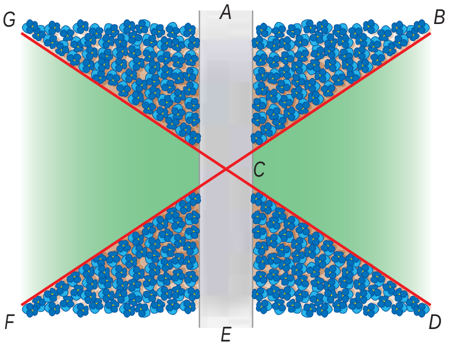
24. If $4 - \frac{1}{2}a = \frac{7}{2} - a$, then $a = -1$.

25. If $-2y + \frac{3}{2} = 8$, then $y = -\frac{13}{4}$.

26. PHYSICS Acceleration, distance traveled, velocity, and time are related in the formula $d = vt + \frac{1}{2}at^2$. Solve for a and justify each step.

27. CHEMISTRY The Ideal Gas law is given by the formula $PV = nRT$, where P = pressure, V = volume, n = the amount of a substance, R is a constant value, and T is the temperature. Solve the formula for T and justify each step.

28. GARDENING In the arrangement of pansies shown, the walkway divides the two sections of pansies into four beds of the same size. If $m\angle ACB = m\angle DCE$, what could you conclude about the relationship among $\angle ACB$, $\angle DCE$, $\angle ECF$, and $\angle ACG$?

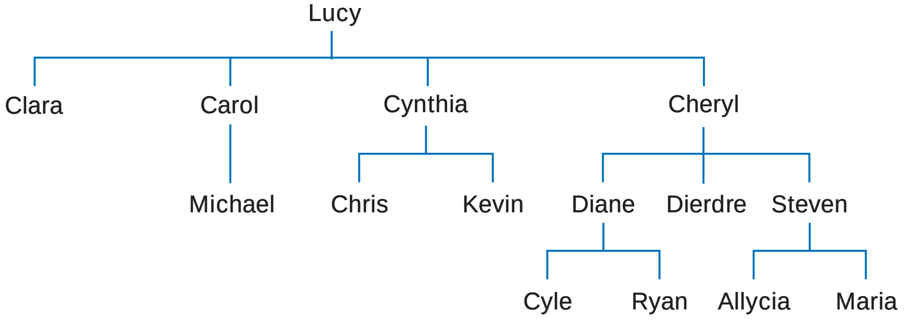


H.O.T. Problems

29. OPEN ENDED Write a statement that illustrates the Substitution Property of Equality.

30. REASONING Compare one part of a conditional to the *Given* statement of a proof. What part is related to the *Prove* statement?

31. CHALLENGE Below is a family tree of the Gibbs family. Clara, Carol, Cynthia, and Cheryl are all daughters of Lucy. Because they are sisters, they have a transitive and symmetric relationship. That is, Clara is a sister of Carol, Carol is a sister of Cynthia, so Clara is a sister of Cynthia.



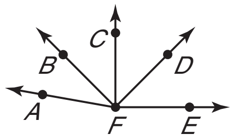
What other relationships in a family have reflexive, symmetric, or transitive relationships? Explain why. Remember that the child or children of each person are listed beneath that person's name. Consider relationships such as first cousin, ancestor or descendent, aunt or uncle, sibling, or any other relationship.

32. Writing in Math Compare proving a theorem of mathematics to proving a case in a court of law. Include a description of how evidence is used to influence jurors' conclusions in court and a description of the evidence used to make conclusions in mathematics.

Berkeley Formula SAE

STANDARDIZED TEST PRACTICE

33. In the diagram below, $m\angle CFE = 90°$ and $\angle AFB \cong \angle CFD$.



Which of the following conclusions does not have to be true?

A $m\angle BFD = m\angle BFD$

B $\overline{BF}$ bisects $\angle BFD$.

C $m\angle CFD = m\angle AFB$

D $\angle CFE$ is a right angle.

34. REVIEW Which expression can be used to find the values of $s(n)$ in the table?

n	-8	-4	-1	0	1
$s(n)$	1.00	2.00	2.75	3.00	3.25

F $-n + 7$

G $-2n + 3$

H $\frac{1}{2}n + 5$

J $\frac{1}{4}n + 3$

Spiral Review

35. CONSTRUCTION There are four buildings on the Medfield High School Campus, no three of which stand in a straight line. How many sidewalks need to be built so that each building is directly connected to every other building? (Lesson 2-5)

Determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.
A number is divisible by 3 if it is divisible by 6. (Lesson 2-4)

36. Given: 24 is divisible by 6. Conclusion: 24 is divisible by 3.

37. Given: 27 is divisible by 3. Conclusion: 27 is divisible by 6.

38. Given: 85 is not divisible by 3. Conclusion: 85 is not divisible by 6.

Write each statement in if-then form. (Lesson 2-3)

39. "He that can have patience can have what he will." (Benjamin Franklin)

40. "To be without some of the things you want is an indispensable part of happiness." (Bertrand Russell)

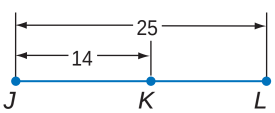
41. "Respect yourself and others will respect you." (Confucius)

42. "A fanatic is one who can't change his mind and won't change the subject." (Sir Winston Churchill)

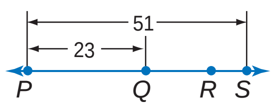
GET READY for the Next Lesson

PREREQUISITE SKILL Find the measure of each segment. (Lesson 1-2)

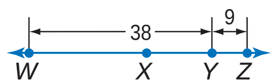
43. $\overline{KL}$



44. $\overline{QS}$



45. $\overline{WZ}$

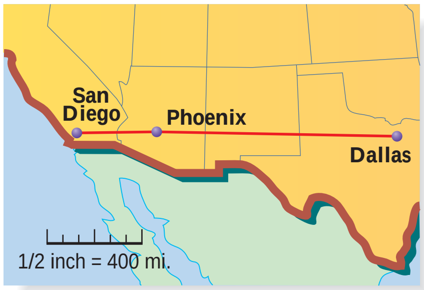


2-7 Proving Segment Relationships

Main Ideas

- Write proofs involving segment addition.
- Write proofs involving segment congruence.

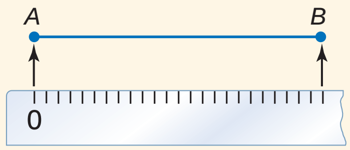
When leaving San Diego, the pilot said that the flight would be about 360 miles to Phoenix. When the plane left Phoenix, the pilot said that the flight would be about 1070 miles to Dallas. Distances in a straight line on a map are sometimes measured with a ruler.



Segment Addition In Lesson 1-2, you measured segments with a ruler by placing the mark for zero on one endpoint, then finding the distance to the other endpoint. This illustrates the **Ruler Postulate**.

POSTULATE 2.8 ***Ruler Postulate***

The points on any line or line segment can be paired with real numbers so that, given any two points *A* and *B* on a line, *A* corresponds to zero, and *B* corresponds to a positive real number.



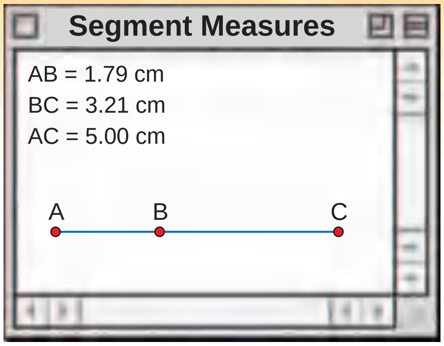
The Ruler Postulate can be used to further investigate line segments.

GEOMETRY LAB

Adding Segment Measures

CONSTRUCT A FIGURE

- Use The Geometer's Sketchpad to construct $\overline{AC}$.
- Place point *B* on $\overline{AC}$.
- Find *AB*, *BC*, and *AC*.



ANALYZE THE MODEL

1. What is the sum $AB + BC$?

2. Move *B*. Find *AB*, *BC*, and *AC*. What is the sum of $AB + BC$?

3. Repeat step 2 three times. Record your results.

4. What is true about the relationship of *AB*, *BC*, and *AC*?

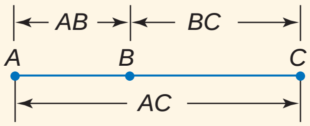
5. Is it possible to place *B* on $\overline{AC}$ so that this relationship is not true?

The Geometry Lab suggests the following postulate.

POSTULATE 2.9 ***Segment Addition Postulate***

If A, B, and C are collinear and B is between A and C, then $AB + BC = AC$.

If $AB + BC = AC$, then B is between A and C.



EXAMPLE 1 Proof With Segment Addition

Prove the following.

Given: $PQ = RS$

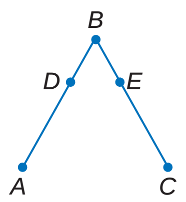
Prove: $PR = QS$

Proof:

Statements	**Reasons**
1. $PQ = RS$	**1.** Given
2. $PQ + QR = QR + RS$	**2.** Addition Property
3. $PQ + QR = PR$ $QR + RS = QS$	**3.** Segment Addition Postulate
4. $PR = QS$	**4.** Substitution

1. Given: $\overline{AD} \cong \overline{CE}$, $\overline{DB} \cong \overline{EB}$

Prove: $\overline{AB} \cong \overline{CB}$



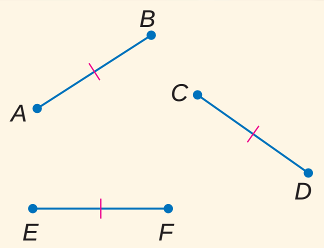
Segment Congruence In algebra, you learned about the properties of equality. The Reflexive Property of Equality states that a quantity is equal to itself. The Symmetric Property of Equality states that if $a = b$, then $b = a$. And the Transitive Property of Equality states that for any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$. These properties of equality are similar to the following properties of congruence.

Vocabulary Link
Symmetric
Everyday Use balanced or proportional
Math Use if $a = b$, then $b = a$

THEOREM 2.2 ***Segment Congruence***

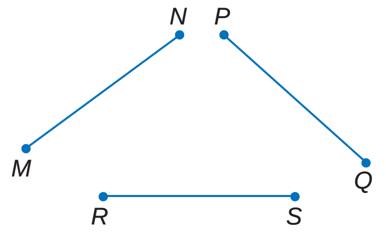
Congruence of segments is reflexive, symmetric, and transitive.

Reflexive Property	$\overline{AB} \cong \overline{AB}$
Symmetric Property	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive Property	If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.



You will prove the first two properties in Exercises 4 and 5.

PROOF Transitive Property of Congruence



Given: $\overline{MN} \cong \overline{PQ}$
$\overline{PQ} \cong \overline{RS}$

Prove: $\overline{MN} \cong \overline{RS}$

Proof:

Method 1 Paragraph Proof

Since $\overline{MN} \cong \overline{PQ}$ and $\overline{PQ} \cong \overline{RS}$, $MN = PQ$ and $PQ = RS$ by the definition of congruent segments. By the Transitive Property of Equality, $MN = RS$. Thus, $\overline{MN} \cong \overline{RS}$ by the definition of congruent segments.

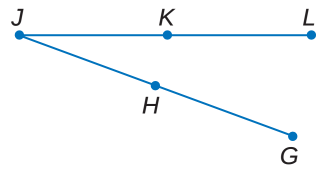
Method 2 Two-Column Proof

Statements	Reasons
1. $\overline{MN} \cong \overline{PQ}$, $\overline{PQ} \cong \overline{RS}$	**1.** Given
2. $MN = PQ$, $PQ = RS$	**2.** Definition of congruent segments
3. $MN = RS$	**3.** Transitive Property
4. $\overline{MN} \cong \overline{RS}$	**4.** Definition of congruent segments

Theorems about congruence can be used to prove segment relationships.

EXAMPLE Proof With Segment Congruence

2 **Prove the following.**



Given: $\overline{JK} \cong \overline{KL}$, $\overline{HJ} \cong \overline{GH}$, $\overline{KL} \cong \overline{HJ}$

Prove: $\overline{GH} \cong \overline{JK}$

Proof:

Method 1 Paragraph Proof

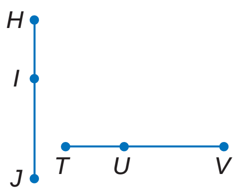
It is given that $\overline{JK} \cong \overline{KL}$ and $\overline{KL} \cong \overline{HJ}$. Thus, $\overline{JK} \cong \overline{HJ}$ by the Transitive Property. It is also given that $\overline{HJ} \cong \overline{GH}$. By the Transitive Property, $\overline{JK} \cong \overline{GH}$. Therefore, $\overline{GH} \cong \overline{JK}$ by the Symmetric Property.

Method 2 Two-Column Proof

Statements	Reasons
1. $\overline{JK} \cong \overline{KL}$, $\overline{KL} \cong \overline{HJ}$	**1.** Given
2. $\overline{JK} \cong \overline{HJ}$	**2.** Transitive Property
3. $\overline{HJ} \cong \overline{GH}$	**3.** Given
4. $\overline{JK} \cong \overline{GH}$	**4.** Transitive Property
5. $\overline{GH} \cong \overline{JK}$	**5.** Symmetric Property

2. Given: $\overline{HI} \cong \overline{TU}$
$\overline{HJ} \cong \overline{TV}$

Prove: $\overline{IJ} \cong \overline{UV}$



Personal Tutor at geometryonline.com

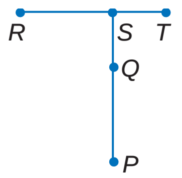
CHECK Your Understanding

Example 1
(p. 119)

1. Copy and complete the proof.

Given: $\overline{PQ} \cong \overline{RS}$, $\overline{QS} \cong \overline{ST}$

Prove: $\overline{PS} \cong \overline{RT}$



Proof:

Statements	**Reasons**
a. ___?___, ___?___	**a.** Given
b. $PQ = RS$, $QS = ST$	**b.** ___?___
c. $PS = PQ + QS$, $RT = RS + ST$	**c.** ___?___
d. ___?___	**d.** Substitution Property
e. ___?___	**e.** Substitution Property
f. $\overline{PS} \cong \overline{RT}$	**f.** ___?___

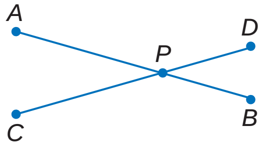
Example 2
(p. 120)

2. PROOF Prove the following.

Given: $\overline{AP} \cong \overline{CP}$

$\overline{BP} \cong \overline{DP}$

Prove: $\overline{AB} \cong \overline{CD}$



Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
3, 6	1
4, 5, 7, 8	2

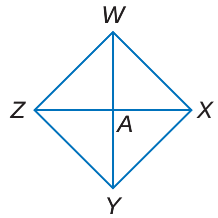
3. Copy and complete the proof.

Given: $\overline{WY} \cong \overline{ZX}$

A is the midpoint of $\overline{WY}$.

A is the midpoint of $\overline{ZX}$.

Prove: $\overline{WA} \cong \overline{ZA}$



Proof:

Statements	**Reasons**
a. $\overline{WY} \cong \overline{ZX}$ A is the midpoint of $\overline{WY}$. A is the midpoint of $\overline{ZX}$.	**a.** ___?___
b. $WY = ZX$	**b.** ___?___
c. ___?___	**c.** Def. of midpoint
d. $WY = WA + AY$, $ZX = ZA + AX$	**d.** ___?___
e. $WA + AY = ZA + AX$	**e.** ___?___
f. $WA + WA = ZA + ZA$	**f.** ___?___
g. $2WA = 2ZA$	**g.** ___?___
h. ___?___	**h.** Division Property
i. $\overline{WA} \cong \overline{ZA}$	**i.** ___?___

Real-World Link Windows come in many different shapes and sizes. Some commonly used shapes for windows are circles, rectangles, squares, triangles, pentagons, and octagons.

Source: www.pella.com

EXTRA PRACTICE
See pages 804, 829.

Math Online
Self-Check Quiz at
geometryonline.com

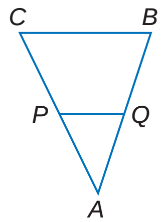
Prove the following.

4. Reflexive Property of Congruence (Theorem 2.2)
5. Symmetric Property of Congruence (Theorem 2.2)

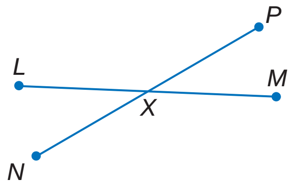
PROOF **Prove the following.**

6. If $AB = BC$, then $AC = 2BC$.

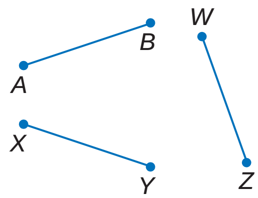
7. If $\overline{AB} \cong \overline{BC}$ and $\overline{PC} \cong \overline{QB}$, then $\overline{AB} \cong \overline{AC}$.



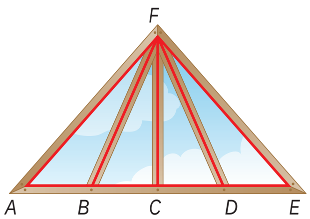
8. If $\overline{LM} \cong \overline{PN}$ and $\overline{XM} \cong \overline{XN}$, then $\overline{LX} \cong \overline{PX}$.



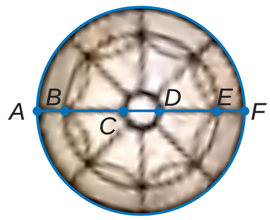
9. If $\overline{XY} \cong \overline{WZ}$ and $\overline{WZ} \cong \overline{AB}$, then $\overline{XY} \cong \overline{AB}$.



10. **DESIGN** The front of a building has a triangular window. If $\overline{AB} \cong \overline{DE}$ and C is the midpoint of $\overline{BD}$, prove that $\overline{AC} \cong \overline{CE}$.



11. **LIGHTING** In the light fixture, $\overline{AB} \cong \overline{EF}$ and $\overline{BC} \cong \overline{DE}$. Prove that $\overline{AC} \cong \overline{DF}$.

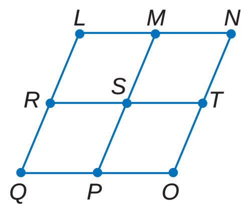


H.O.T. Problems

12. **OPEN ENDED** Draw three congruent segments, and illustrate the Transitive Property using these segments.

13. **REASONING** Choose two cities from a United States road map. Describe the distance between the cities using the Reflexive Property.

14. **CHALLENGE** Given that $\overline{LN} \cong \overline{RT}$, $\overline{RT} \cong \overline{QO}$, $\overline{LQ} \cong \overline{NO}$, $\overline{MP} \cong \overline{NO}$, S is the midpoint of $\overline{RT}$, M is the midpoint of $\overline{LN}$, and P is the midpoint of $\overline{QO}$, list three statements that you could prove using the postulates, theorems, and definitions that you have learned.



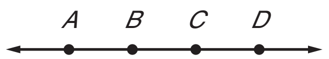
15. **Writing in Math** How can segment relationships be used for travel? Include an explanation of how a passenger can use the distances the pilot announced to find the total distance from San Diego to Dallas and an explanation of why the Segment Addition Postulate may or may not be useful when traveling.

STANDARDIZED TEST PRACTICE

16. Which reason can be used to justify Statement 5 in the proof below?

Given: $\overline{AB} \cong \overline{BC}$, $\overline{BC} \cong \overline{CD}$

Prove: $3AB = AD$



Statements	Reason
1. $\overline{AB} \cong \overline{BC}$, $\overline{BC} \cong \overline{CD}$	**1.** Given
2. $\overline{AB} \cong \overline{CD}$	**2.** ?
3. $AB = BC$, $BC = CD$	**3.** ?
4. $CD = BC$	**4.** ?
5. $AB + BC + CD = AD$	**5.** ?
6. $AB + AB + AB = AD$	**6.** Subst.
7. $3AB = AD$	**7.** Def. of Mult.

A Angle Addition Postulate

B Segment Congruence

C Segment Addition Postulate

D Midpoint Theorem

17. **REVIEW** Haru made a scale model of the park near his house. Every inch represents 5 feet. If the main sidewalk in his model is 45 inches long, how long is the actual sidewalk in the park?

F 225 ft

G 125 ft

H 15 ft

J 5 ft

18. **REVIEW** Which expression is equivalent to $\frac{12x^{-4}}{4x^{-8}}$?

A $\frac{1}{3x^4}$

B $3x^4$

C $8x^2$

D $\frac{x^4}{3}$

Spiral Review

State the property that justifies each statement. (Lesson 2-6)

19. If $m\angle P + m\angle Q = 110$ and $m\angle R = 110$, then $m\angle P + m\angle Q = m\angle R$.

20. If $x(y + z) = a$, then $xy + xz = a$.

21. If $n - 17 = 39$, then $n = 56$.

22. If $cv = md$ and $md = 15$, then $cv = 15$.

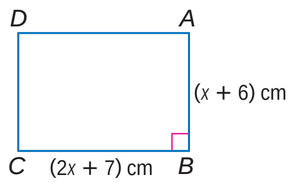
Determine whether each statement is *always*, *sometimes*, or *never* true. Explain. (Lesson 2-5)

23. A midpoint divides a segment into two noncongruent segments.

24. Three lines intersect at a single point.

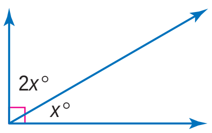
25. The intersection of two planes forms a line.

26. If the perimeter of rectangle $ABCD$ is 44 centimeters, find x and the dimensions of the rectangle. (Lesson 1-6)



PREREQUISITE SKILL **Find *x*.** (Lesson 1-5)

27.



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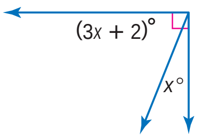
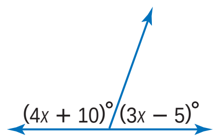


29.



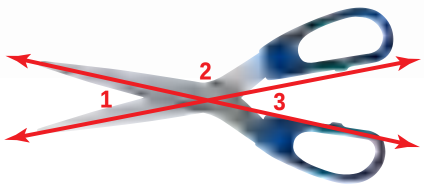
2-8 Proving Angle Relationships

Main Ideas

- Write proofs involving supplementary and complementary angles.
- Write proofs involving congruent and right angles.

GET READY for the Lesson

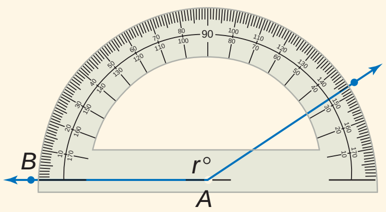
Notice that when a pair of scissors is opened, the angle formed by the two blades, ∠1, and the angle formed by a blade and a handle, ∠2, are a linear pair. Likewise, the angle formed by a blade and a handle, ∠2, and the angle formed by the two handles, ∠3, also forms a linear pair.



Supplementary and Complementary Angles Recall that when you measure angles with a protractor, you position the protractor so that one of the rays aligns with zero degrees and then determine the position of the second ray. To draw an angle of a given measure, align a ray with the zero degree mark and use the desired angle measure to position the second ray. The Protractor Postulate ensures that there is one ray you could draw with a given ray to create an angle with a given measure.

POSTULATE 2.10 ***Protractor Postulate***

Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r.

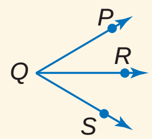


In Lesson 2-7, you learned about the Segment Addition Postulate. A similar relationship exists between the measures of angles.

POSTULATE 2.11 ***Angle Addition Postulate***

If R is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$.

If $m\angle PQR + m\angle RQS = m\angle PQS$, then R is in the interior of $\angle PQS$.



You can use the Angle Addition Postulate to solve problems involving angle measures.

Congruent and Right Angles The properties of algebra that applied to the congruence of segments and the equality of their measures also hold true for the congruence of angles and the equality of their measures.

THEOREM 2.5

Congruence of angles is reflexive, symmetric, and transitive.

Reflexive Property $\angle 1 \cong \angle 1$

Symmetric Property If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

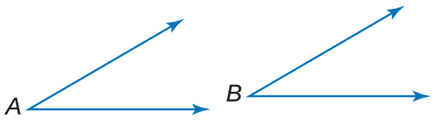
Transitive Property If $\angle 1 \cong \angle 2$, and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

You will prove the Reflexive and Transitive Properties of Angle Congruence in Exercises 18 and 19.

PROOF Symmetric Property of Congruence

Given: $\angle A \cong \angle B$

Prove: $\angle B \cong \angle A$



Method 1

Paragraph Proof:

We are given $\angle A \cong \angle B$. By the definition of congruent angles, $m\angle A = m\angle B$. Using the Symmetric Property, $m\angle B = m\angle A$. Thus, $\angle B \cong \angle A$ by the definition of congruent angles.

Method 2

Two-Column Proof:

Statements	Reasons
1. $\angle A \cong \angle B$	**1.** Given
2. $m\angle A = m\angle B$	**2.** Definition of Congruent Angles
3. $m\angle B = m\angle A$	**3.** Symmetric Property
4. $\angle B \cong \angle A$	**4.** Definition of Congruent Angles

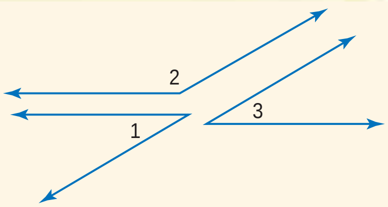
Algebraic properties can be applied to prove theorems for congruence relationships involving supplementary and complementary angles.

THEOREMS

2.6 Angles supplementary to the same angle or to congruent angles are congruent.

Abbreviation: $\measuredangle$ suppl. to same $\angle$ or $\cong$ $\measuredangle$ are $\cong$.

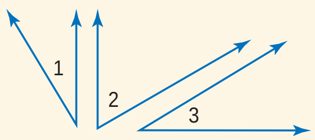
Example: If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.



2.7 Angles complementary to the same angle or to congruent angles are congruent.

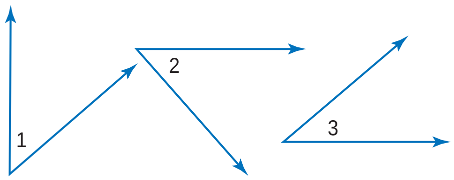
Abbreviation: $\measuredangle$ compl. to same $\angle$ or $\cong$ $\measuredangle$ are $\cong$.

Example: If $m\angle 1 + m\angle 2 = 90$ and $m\angle 2 + m\angle 3 = 90$, then $\angle 1 \cong \angle 3$.



You will prove Theorem 2.6 in Exercise 3.

PROOF Theorem 2.7



Given: $\angle 1$ and $\angle 3$ are complementary.
$\angle 2$ and $\angle 3$ are complementary.

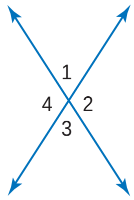
Prove: $\angle 1 \cong \angle 2$

Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 3$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	**1.** Given
2. $m\angle 1 + m\angle 3 = 90$ $m\angle 2 + m\angle 3 = 90$	**2.** Definition of complementary angles
3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	**3.** Substitution
4. $m\angle 3 = m\angle 3$	**4.** Reflexive Property
5. $m\angle 1 = m\angle 2$	**5.** Subtraction Property
6. $\angle 1 \cong \angle 2$	**6.** Definition of congruent angles

EXAMPLE Use Supplementary Angles

3 **In the figure, $\angle 1$ and $\angle 2$ form a linear pair and $\angle 2$ and $\angle 3$ form a linear pair. Prove that $\angle 1$ and $\angle 3$ are congruent.**



Given: $\angle 1$ and $\angle 2$ form a linear pair.
$\angle 2$ and $\angle 3$ form a linear pair.

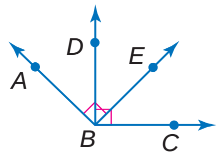
Prove: $\angle 1 \cong \angle 3$

Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ form a linear pair. $\angle 2$ and $\angle 3$ form a linear pair.	**1.** Given
2. $\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	**2.** Supplement Theorem
3. $\angle 1 \cong \angle 3$	**3.** $\angle$s suppl. to same $\angle$ or $\cong$ $\angle$s are $\cong$.

CHECK Your Progress

3. In the figure, $\angle ABE$ and $\angle DBC$ are right angles. Prove that $\angle ABD \cong \angle EBC$.



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Note that in Example 3, $\angle 1$ and $\angle 3$ are vertical angles. The conclusion in the example is a proof for the following theorem.

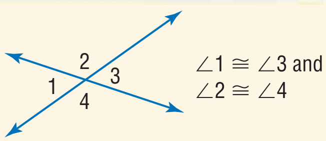
Review Vocabulary

Vertical Angles two nonadjacent angles formed by intersecting lines (Lesson 1-5)

THEOREM 2.8 *Vertical Angles Theorem*

If two angles are vertical angles, then they are congruent.

Abbreviation: Vert. $\angle$s are $\cong$.



EXAMPLE 4 Vertical Angles

If $\angle 1$ and $\angle 2$ are vertical angles and $m\angle 1 = x$ and $m\angle 2 = 228 - 3x$, find $m\angle 1$ and $m\angle 2$.

$\angle 1 \cong \angle 2$	Vertical Angles Theorem	Substitute to find the angle measures.
$m\angle 1 = m\angle 2$	Definition of congruent angles	$m\angle 1 = x$
$x = 228 - 3x$	Substitution	$= 57$
$4x = 228$	Add $3x$ to each side.	$m\angle 2 = m\angle 1$
$x = 57$	Divide each side by 4.	$= 57$

CHECK Your Progress

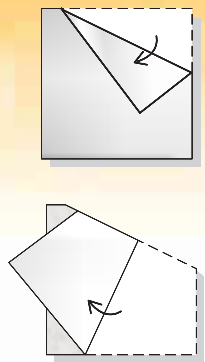
4. If $\angle 3$ and $\angle 4$ are vertical angles, $m\angle 3 = 6x + 2$, and $m\angle 4 = 8x - 14$, find $m\angle 3$ and $m\angle 4$.

You can create right angles and investigate congruent angles by paper folding.

GEOMETRY LAB

Right Angles

MAKE A MODEL



- Fold the paper so that one corner is folded downward.
- Fold along the crease so that the top edge meets the side edge.
- Unfold the paper and measure each of the angles.
- Repeat the activity three more times.

ANALYZE THE MODEL

1. What do you notice about the lines formed?

2. What do you notice about each pair of adjacent angles?

3. What are the measures of the angles formed?

4. **MAKE A CONJECTURE** What is true about perpendicular lines?

The following theorems support the conjectures you made in the Geometry Lab.

THEOREMS *Right Angle Theorems*

2.9 Perpendicular lines intersect to form four right angles.

2.10 All right angles are congruent.

2.11 Perpendicular lines form congruent adjacent angles.

2.12 If two angles are congruent and supplementary, then each angle is a right angle.

2.13 If two congruent angles form a linear pair, then they are right angles.

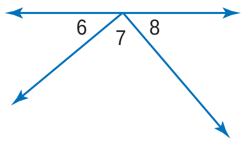
You will prove these theorems in Exercises 20–24.

CHECK Your Understanding

Find the measure of each numbered angle.

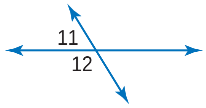
Example 1 (p. 125)

1. $\angle 6$ and $\angle 8$ are complementary, $m\angle 8 = 47$



Example 2 (p. 125)

2. $m\angle 11 = x - 4$, $m\angle 12 = 2x - 5$



Example 3 (p. 127)

3. **PROOF** Copy and complete the proof of Theorem 2.6.

Given: $\angle 1$ and $\angle 2$ are supplementary.
$\angle 3$ and $\angle 4$ are supplementary.
$\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$

Proof:

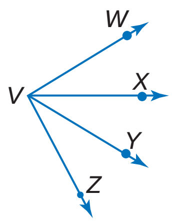
Statements	Reasons
a. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary. $\angle 1 \cong \angle 4$	**a.** ?
b. $m\angle 1 + m\angle 2 = 180$ $m\angle 3 + m\angle 4 = 180$	**b.** ?
c. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	**c.** ?
d. $m\angle 1 = m\angle 4$	**d.** ?
e. $m\angle 2 = m\angle 3$	**e.** ?
f. $\angle 2 \cong \angle 3$	**f.** ?

Example 4 (p. 128)

4. **PROOF** Write a two-column proof.

Given: $\overrightarrow{VX}$ bisects $\angle WVY$.
$\overrightarrow{VY}$ bisects $\angle XVZ$.

Prove: $\angle WVX \cong \angle YVZ$

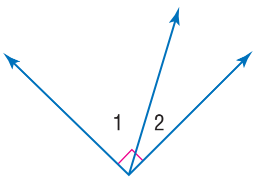


Exercises

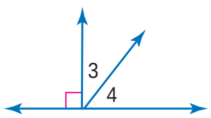
HOMEWORK HELP	
For Exercises	See Examples
5–7	1
8–10	2
14–19	3
11–13	4

Find the measure of each numbered angle.

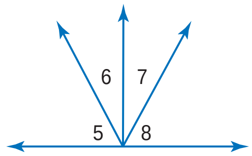
5. $m\angle 1 = 64$



6. $m\angle 3 = 38$

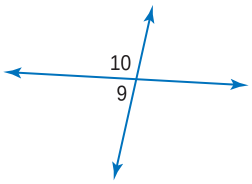


7. $\angle 7$ and $\angle 8$ are complementary. $\angle 5 \cong \angle 8$ and $m\angle 6 = 29$.

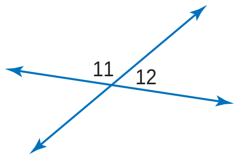


Find the measure of each numbered angle.

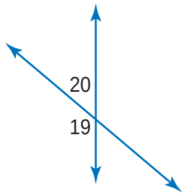
8. $m\angle 9 = 2x - 4$, $m\angle 10 = 2x + 4$



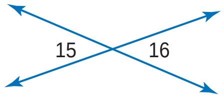
9. $m\angle 11 = 4x$, $m\angle 12 = 2x - 6$



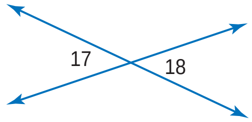
10. $m\angle 19 = 100 + 20x$, $m\angle 20 = 20x$



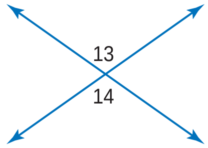
11. $m\angle 15 = x$, $m\angle 16 = 6x - 290$



12. $m\angle 17 = 2x + 7$, $m\angle 18 = x + 30$

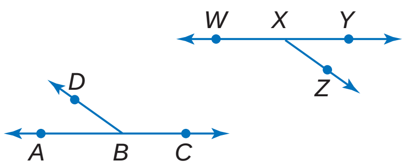


13. $m\angle 13 = 2x + 94$, $m\angle 14 = 7x + 49$

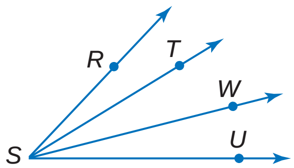


PROOF **Write a two-column proof.**

14. **Given:** $\angle ABD \cong \angle YXZ$
Prove: $\angle CBD \cong \angle WXZ$



15. **Given:** $m\angle RSW = m\angle TSU$
Prove: $m\angle RST = m\angle WSU$



Write a proof for each theorem.

16. Supplement Theorem

17. Complement Theorem

18. Reflexive Property of Angle Congruence

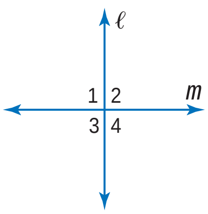
19. Transitive Property of Angle Congruence

Real-World Link

Interstate highways that run from north to south are odd-numbered with the lowest numbers in the west. East-west interstates are even-numbered and begin in the south.

Source: www.infoplease.com

PROOF **Use the figure to write a proof of each theorem.**



20. Theorem 2.9

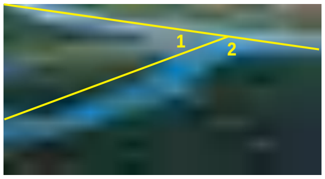
21. Theorem 2.10

22. Theorem 2.11

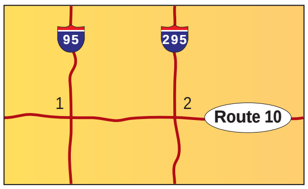
23. Theorem 2.12

24. Theorem 2.13

25. **RIVERS** Tributaries of rivers sometimes form a linear pair of angles when they meet the main river. The Yellowstone River forms the linear pair $\angle 1$ and $\angle 2$ with the Missouri River. If $m\angle 1$ is 28, find $m\angle 2$.



26. **HIGHWAYS** Near the city of Hopewell, Virginia, Route 10 runs perpendicular to Interstate 95 and Interstate 295. Show that the angles at the intersections of Route 10 with Interstate 95 and Interstate 295 are congruent.



EXTRA PRACTICE
See pages 804, 829.
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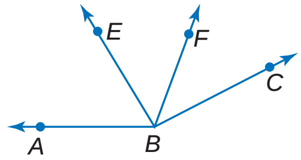
H.O.T. Problems

27. **OPEN ENDED** Draw three congruent angles. Use these angles to illustrate the Transitive Property for angle congruence.

28. **FIND THE ERROR** Tomas and Jacob wrote equations involving the angle measures shown. Who is correct? Explain your reasoning.

Tomas
$m\angle ABE + m\angle EBC = m\angle ABC$

Jacob
$m\angle ABE + m\angle FBC = m\angle ABC$

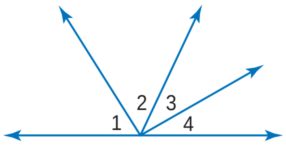


REASONING **Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.**

29. Two angles that are nonadjacent are vertical.

30. Two acute angles that are congruent are complementary to the same angle.

31. **CHALLENGE** What conclusion can you make about the sum of $m\angle 1$ and $m\angle 4$ if $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$? Explain.



32. **Writing in Math** Refer to page 124. Describe how scissors illustrate supplementary angles. Is the relationship the same for two angles complementary to the same angle?

STANDARDIZED TEST PRACTICE

33. The measures of two complementary angles are in the ratio 4:1. What is the measure of the smaller angle?

A 15

B 18

C 24

D 36

34. **REVIEW** Simplify $4(3x - 2)(2x + 4) + 3x^2 + 5x - 6$.

F $9x^2 + 3x - 14$

G $9x^2 + 13x - 14$

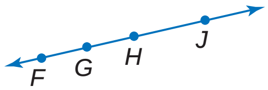
H $27x^2 + 37x - 38$

J $27x^2 + 27x - 26$

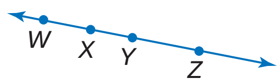
Spiral Review

PROOF **Write a two-column proof.** (Lesson 2-7)

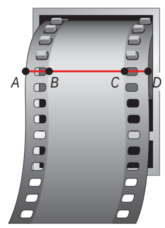
35. **Given:** G is between F and H.
H is between G and J.
Prove: $FG + GJ = FH + HJ$



36. **Given:** X is the midpoint of $\overline{WY}$.
Prove: $WX + YZ = XZ$



37. **PHOTOGRAPHY** Film is fed through a camera by gears that catch the perforation in the film. The distance from the left edge of the film, A, to the right edge of the image, C, is the same as the distance from the left edge of the image, B, to the right edge of the film, D. Show that the two perforated strips are the same width. (Lesson 2-6)



CHAPTER 2 Study Guide and Review

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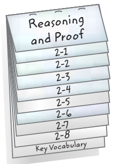
GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Inductive Reasoning and Logic (Lessons 2-1 and 2-2)

- If a statement is represented by p, then *not* p is the negation of the statement.
- A conjunction is a compound statement formed by joining two or more statements with the word *and*.
- A disjunction is a compound statement formed by joining two or more statements with the word *or*.

Conditional Statements (Lesson 2-3)

- An if-then statement is written in the form *if* p, *then* q in which p is the hypothesis, and q is the conclusion.
- The converse is formed by exchanging the hypothesis and conclusion of the conditional.
- The inverse is formed by negating both the hypothesis and conclusion of the conditional.
- The contrapositive is formed by negating both the hypothesis and conclusion of the converse statement.

Deductive Reasoning (Lesson 2-4)

- Law of Detachment: If $p \longrightarrow q$ is true and p is true, then q is also true.
- Law of Syllogism: If $p \longrightarrow q$ and $q \longrightarrow r$ are true, then $p \longrightarrow r$ is also true.

Proof (Lessons 2-5 through 2-8)

- State what is to be proven.
- List the given information.
- If possible, draw a diagram.
- State what is to be proved.
- Develop of system of deductive reasoning.

Key Vocabulary

conclusion (p. 91)
conditional statement (p. 91)
conjecture (p. 78)
conjunction (p. 84)
contrapositive (p. 93)
converse (p. 93)
counterexample (p. 79)
deductive argument (p. 111)
deductive reasoning (p. 99)
disjunction (p. 84)
hypothesis (p. 91)
if-then statement (p. 91)
inductive reasoning (p. 78)
inverse (p. 93)
negation (p. 83)
paragraph proof (p. 106)
postulate (p. 105)
proof (p. 106)
theorem (p. 106)
truth value (p. 83)
two-column proof (p. 112)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. Theorems are accepted as true.
2. A disjunction is true only when both statements in it are true.
3. In a two-column proof, the properties that justify each step are called reasons.
4. Inductive reasoning uses facts, rules, definitions, or properties to reach logical conclusions.
5. The Reflexive Property of Equality states that for every number a, $a = a$.
6. A negation is another term for axiom.
7. To show that a conjecture is false you would give a counterexample.
8. An if-then statement consists of a conjecture and a conclusion.
9. The contrapositive of a conditional is formed by exchanging the hypothesis and conclusion of the conditional statement.
10. A disjunction is formed by joining two or more sentences with the word *or*.

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Lesson-by-Lesson Review

2-1 Inductive Reasoning and Conjecture (pp. 78–82)

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

11. $\angle A$ and $\angle B$ are supplementary.

12. X, Y, and Z are collinear and $XY = YZ$.

13. **TRAFFIC** While driving on the freeway, Tonya noticed many cars ahead of her had stopped. So she immediately took the next exit. Make a conjecture about why Tonya chose to exit the freeway.

Example 1 **Given that points *P*, *Q*, and *R* are collinear, determine whether the conjecture that *Q* is between *P* and *R* is *true* or *false*. If the conjecture is false, give a counterexample.**

The figure below can be used to disprove the conjecture. In this case, R is between P and Q. Since we can find a counterexample, the conjecture is false.

2-2 Logic (pp. 83–90)

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value.

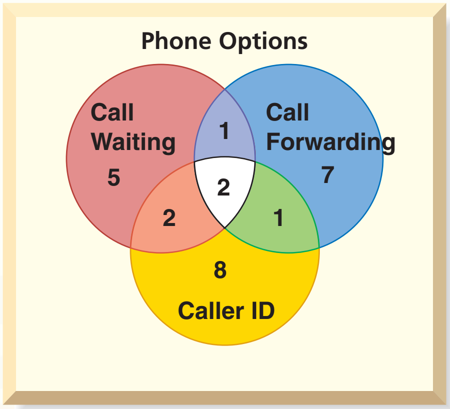
p: $-1 > 0$

q: **In a right triangle with right angle *C*,** $a^2 + b^2 = c^2$.

r: **The sum of the measures of two supplementary angles is 180°.**

14. p and $\sim q$ **15.** $\sim p \vee \sim r$

16. **PHONES** The results of a survey about phone service options are shown below. How many customers had both call waiting and caller ID?



Example 2 **Use the following statements to write a compound statement for the conjunction and disjunction below. Then find each truth value.**

p: $\sqrt{15} = 5$

q: **The measure of a right angle equals 90.**

a. ***p* and *q***

$\sqrt{15} = 5$, and the measure of a right angle equals 90.

p and q is false because p is false and q is true.

b. $p \vee q$

$\sqrt{15} = 5$, or the measure of a right angle equals 90.

$p \vee q$ is true because q is true. It does not matter that p is false.

CHAPTER 2

Study Guide and Review

2-3 Conditional Statements

(pp. 91–97)

Write the converse, inverse, and contrapositive of each conditional. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

17. March has 31 days.

18. If an ordered pair for a point has 0 for its x-coordinate, then the point lies on the y-axis.

TEMPERATURE **Find the truth value of the following statement for each set of conditions.**

Water freezes when the temperature is at most 0°C.

19. Water freezes at -10°C.

20. Water freezes at 15°C.

21. Water does not freeze at -2°C.

22. Water does not freeze at 30°C.

Example 3 **Identify the hypothesis and conclusion of the statement *The intersection of two planes is a line.* Then write the statement in if-then form.**

Hypothesis: two planes intersect

Conclusion: their intersection is a line

If two planes intersect, then their intersection is a line.

Example 4 **Write the converse of the statement *All fish live under water.* Determine whether the converse is true or false. If it is is false, find a counterexample.**

Converse: If it lives under water, it is a fish. False; dolphins live under water, but are not fish.

2-4 Deductive Reasoning

(pp. 99–104)

Determine whether statement (3) follows from statements (1) and (2) by the Laws of Detachment or Syllogism. If so, state which law was used. If not, write *invalid*.

23. (1) If a student attends North High School, then he or she has an ID number.
(2) Josh attends North High School.
(3) Josh has an ID number.

24. (1) If a rectangle has four congruent sides, then it is a square.
(2) A square has diagonals that are perpendicular.
(3) A rectangle has diagonals that are perpendicular.

Example 5 **Use the Law of Syllogism to determine whether a valid conclusion can be reached from the following statements.**

(1) If a body in our solar system is the Sun, then it is a star.

(2) Stars are in constant motion.

p: A body in our solar system is the Sun.

q: It is a star.

r: Stars are in constant motion.

Statement (1): $p \rightarrow q$

Statement (2): $q \rightarrow r$

Since the given statements are true, use the Law of Syllogism to conclude $p \rightarrow r$. That is, *If a body in our solar system is the Sun, then it is in constant motion.*

Mixed Problem Solving
For mixed problem-solving practice, see page 829.

2-5 Postulates and Paragraph Proofs (pp. 105–109)

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

25. The intersection of two different lines is a line.

26. If P is the midpoint of $\overline{XY}$, then $XP = PY$.

27. Four points determine six lines.

28. If $MX = MY$, then M is the midpoint of $\overline{XY}$.

29. HAMMOCKS Maurice has six trees in a regular hexagonal pattern in his backyard. How many different possibilities are there for tying his hammock to any two of those trees?

Example 6 **Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.**

Two points determine a line.

According to a postulate relating to points and lines, two points determine a line. Thus, the statement is *always* true.

If two angles are right angles, they are adjacent.

If two right angles form a linear pair, then they would be adjacent. This statement is *sometimes* true.

2-6 Algebraic Proof (pp. 111–117)

State the property that justifies each statement.

30. If $3(x + 2) = 6$, then $3x + 6 = 6$.

31. If $10x = 20$, then $x = 2$.

32. If $AB + 20 = 45$, then $AB = 25$.

33. If $3 = CD$ and $CD = XY$, then $3 = XY$.

Write a two-column proof.

34. If $5 = 2 - \frac{1}{2}x$, then $x = -6$.

35. If $x - 1 = \frac{x - 10}{-2}$, then $x = 4$.

36. If $AC = AB$, $AC = 4x + 1$, and $AB = 6x - 13$, then $x = 7$.

37. If $MN = PQ$ and $PQ = RS$, then $MN = RS$.

38. BIRTHDAYS Mark has the same birthday as Cami. Cami has the same birthday as Briana. Which property would show that Mark has the same birthday as Briana?

Example 7

Given: $2x + 6 = 3 + \frac{5}{3}x$

Prove: $x = -9$

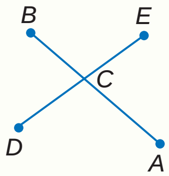
Proof:

Statements	Reasons
1. $2x + 6 = 3 + \frac{5}{3}x$	**1.** Given
2. $3(2x + 6) = 3\left(3 + \frac{5}{3}x\right)$	**2.** Multiplication Property
3. $6x + 18 = 9 + 5x$	**3.** Distributive Property
4. $6x + 18 - 5x = 9 + 5x - 5x$	**4.** Subtraction Property
5. $x + 18 = 9$	**5.** Substitution
6. $x + 18 - 18 = 9 - 18$	**6.** Subtraction Property
7. $x = -9$	**7.** Substitution

2-7 Proving Segment Relationships (pp. 118–123)

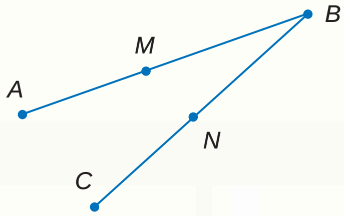
PROOF **Write a two-column proof.**

39. Given: $BC = EC$, $CA = CD$
Prove: $BA = DE$



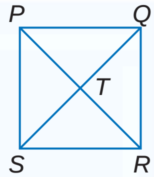
40. Given: $AB = CD$
Prove: $AC = BD$

41. KANSAS The distance from Salina to Kansas City is represented by AB, and the distance from Wichita to Kansas City is represented by CB. If $AB = CB$, M is the midpoint of AB, and N is the midpoint of CD, prove $AM = CN$.



Example 8 **Write a two-column proof.**

Given: $QT = RT$, $TS = TP$
Prove: $QS = RP$
Proof:



Statements	Reasons
1. $QT = RT$, $TS = TP$	**1.** Given
2. $QT + TS = RT + TS$	**2.** Addition Prop.
3. $QT + TS = RT + TP$	**3.** Substitution
4. $QT + TS = QS$, $RT + TP = RP$	**4.** Seg. Add. Post.
5. $QS = RP$	**5.** Substitution

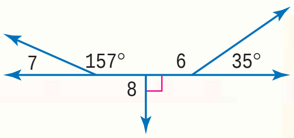
2-8 Proving Angle Relationships (pp. 124–131)

Find the measure of each angle.

42. $\angle 6$

43. $\angle 7$

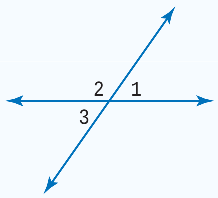
44. $\angle 8$



45. PROOF **Write a two-column proof.**

Given: $\angle 1$ and $\angle 2$ form a linear pair
$m\angle 2 = 2(m\angle 1)$
Prove: $m\angle 1 = 60$

Example 9 **Find the measure of each numbered angle if $m\angle 3 = 55$.**



$m\angle 1 = 55$, since $\angle 1$ and $\angle 3$ are vertical angles.

$\angle 2$ and $\angle 3$ form a linear pair.

$55 + m\angle 2 = 180$	Def. of suppl. $\angle$s
$m\angle 2 = 180 - 55$	Subtract.
$m\angle 2 = 125$	Simplify.

Chapter 2 Practice Test

Determine whether each conjecture is *true* or *false*. Explain your answer and give a counterexample for any false conjecture.

1. **Given:** $\angle A \cong \angle B$
   Conjecture: $\angle B \cong \angle A$
2. **Given:** y is a real number.
   Conjecture: $-y > 0$
3. **Given:** $3a^2 = 48$
   Conjecture: $a = 4$

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value.

***p*: $-3 > 2$**
***q*: $3x = 12$ when $x = 4$.**
***r*: An equilateral triangle is also equiangular.**

4. p and q
5. p or q
6. $p \vee (q \wedge r)$

7. **ADVERTISING** Identify the hypothesis and conclusion of the following statement. Then write it in if-then form.

   Hard-working people deserve a great vacation.

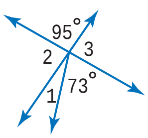
Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

8. (1) Perpendicular lines intersect.
   (2) Lines m and n are perpendicular.
   (3) Lines m and n intersect.

9. (1) If n is an integer, then n is a real number.
   (2) n is a real number.
   (3) n is an integer.

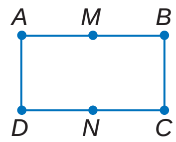
Find the measure of each numbered angle.

10. $\angle 1$
11. $\angle 2$
12. $\angle 3$



PROOF **Write the indicated type of proof.**

13. two-column
    If $y = 4x + 9$ and $x = 2$, then $y = 17$.
14. two-column
    If $2(n - 3) + 5 = 3(n - 1)$, prove that $n = 2$.
15. paragraph proof
    Given: $AM = CN$, $MB = ND$
    Prove: $AB = CD$



16. paragraph proof
    If M is the midpoint of $\overline{AB}$, and Q is the midpoint of $\overline{AM}$, then $AQ = \frac{1}{4}AB$.

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

17. Two angles that form a right angle are complementary.
18. Two angles that form a linear pair are congruent.

Identify the hypothesis and conclusion of each statement and write each statement in if-then form. Then write the converse, inverse, and contrapositive of each conditional.

19. An apple a day keeps the doctor away.
20. A rolling stone gathers no moss.

21. **MULTIPLE CHOICE** Refer to the following statements.

    p: There are 52 states in the United States.
    q: $12 + 8 = 20$
    r: A week has 8 days.

    Which compound statement is true?

    A p and q

    B p or q

    C p or r

    D q and r

CHAPTER 2

Standardized Test Practice

Cumulative, Chapters 1–2

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. *"Two lines that do not intersect are always parallel."*

Which of the following best describes a *counterexample* to the assertion above?

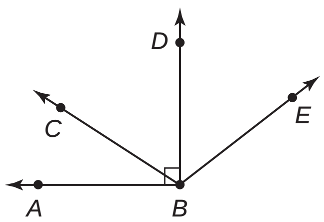
A coplanar lines

B parallel lines

C perpendicular lines

D skew lines

2. Consider the following statements about the figure shown below.



p: $\angle ABC$ is an acute angle.
q: $\angle ABC$ and $\angle CBD$ are supplementary angles.
r: $m\angle ABE$ is greater than 90°.

Which of the following compound statements is *not* true?

F $p \vee q$

G $\sim q \wedge r$

H $\sim r \wedge \sim q$

J $\sim p \vee \sim q$

3. Which of the following *best* describes an axiom?

A a conjecture made using examples

B a conjecture made using facts, rules, definition, or properties

C a statement that is accepted as true

D a statement or conjecture that has been shown to be true

4. Determine which statement follows logically from the given statements.

If it rains today, the game will be cancelled.
Cancelled games are made up on Saturdays.

F If a game is cancelled, it was because of rain.

G If it rains today, the game will be made up on Saturday.

H Some cancelled games are not made up on Saturdays.

J If it does not rain today, the game will not be made up on Saturday.

TEST-TAKING TIP

Question 4 When answering a multiple-choice question, always read every answer choice and eliminate those you decide are definitely wrong. This way, you may deduce the correct answer.

5. Which of the following statements is the contrapositive of the conditional statement: If the sum of the measures of the angles of a polygon is 180°, then the polygon is a triangle?

A If a polygon is not a triangle, then the sum of the measures of the angles of the polygon is not 180°.

B If the sum of the measures of the angles of polygon is not 180°, then the polygon is not a triangle.

C If a polygon is a triangle, then the sum of the measures of the angles of the polygon is 180°.

D If a polygon is not a triangle, then the sum of the measures of the angles of the polygon is 180°.

6. **GRIDDABLE** Samantha has 3 more trophies than Martha. Melinda has triple the number of trophies that Samantha has. Altogether the girls have 22 trophies. How many trophies does Melinda have?

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 841–856.

7. Use the proof to answer the question below.

Given: $\angle A$ is the complement of $\angle B$;
$m\angle B = 46$

Prove: $m\angle A = 44$

Statement	Reason
1. A is the complement of $\angle B$; $m\angle B = 46$	**1.** Given
2. $m\angle A + m\angle B = 90$	**2.** Def. of comp. angles
3. $m\angle A + 46 = 90$	**3.** Substitution Prop.
4. $m\angle A + 46 - 46 = 90 - 46$	**4.** ?
5. $m\angle A = 44$	**5.** Substitution Prop.

What reason can be given to justify Statement 4?

F Addition Property

G Substitution Property

H Subtraction Property

J Symmetric Property

8. **Given:** Points *A, B, C*, and *D* are collinear, with point *B* between points *A* and *C* and point *C* between points *B* and *D*. Which of the following does *not* have to be true?

A $AB + BD = AD$

B $\overline{AB} \cong \overline{CD}$

C $\overline{BC} \cong \overline{BC}$

D $BC + CD = BD$

9. A farmer needs to make a 1000-square-foot rectangular enclosure for her cows. She wants to save money by purchasing the least amount of fencing possible to enclose the area. What whole-number dimensions will require the least amount of fencing?

F 8 ft by 125 ft

G 10 ft by 100 ft

H 20 ft by 50 ft

J 25 ft by 40 ft

10. **Given:** $\angle EFG$ and $\angle GFH$ are complementary. Which of the following *must* be true?

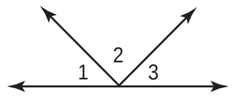
A $\overrightarrow{FE} \perp \overrightarrow{FG}$

B $\overrightarrow{FG}$ bisects $\angle EFH$.

C $m\angle EFG + m\angle GFH = 180$

D $\angle GFH$ is an acute angle.

11. In the diagram below, $\angle 1 \cong \angle 3$.



Which of the following conclusions does *not* have to be true?

F $m\angle 1 - m\angle 2 + m\angle 3 = 90$

G $m\angle 1 + m\angle 2 + m\angle 3 = 180$

H $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$

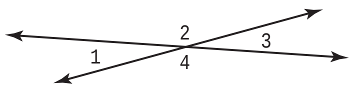
J $m\angle 2 - m\angle 1 = m\angle 2 - m\angle 3$

Pre-AP

Record your answer on a sheet of paper. Show your work.

12. **Given:** $\angle 1$ and $\angle 3$ are vertical angles.
$m\angle 1 = 3x + 5$, $m\angle 3 = 2x + 8$

Prove: $m\angle 1 = 14$



13. From a single point in her yard, Marti measures and marks distances of 18 feet and 30 feet for two sides of her garden. What length should the third side of her garden be so that it will form a right angle with the 18-foot side? If Marti decided to use the same length of fencing in a square configuration, how long would each side of the fence be? Which configuration would provide the largest area for her garden?

NEED EXTRA HELP?													
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson or Page...	2-1	2-2	2-5	2-4	2-3	781	2-6	2-7	1-6	1-6	2-8	2-8	1-3

CHAPTER 3

Parallel and Perpendicular Lines

- Identify angle relationships that occur with parallel lines and a transversal, and identify and prove lines parallel from given angle relationships.
- Use slope to analyze a line and to write its equation.
- Find the distance between a point and a line and between two parallel lines.

Key Vocabulary

parallel lines (p. 142)

transversal (p. 143)

Real-World Link

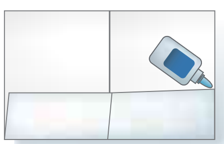
Architecture The East Building of the National Gallery of Art in Washington, D.C., designed by architect I.M. Pei, has an H-shaped façade. Viewed from above, the building appears to be made of interlocking diamonds. This design creates many parallel lines.

Parallel and Perpendicular Lines Make this Foldable to help you organize your notes. Begin with one sheet of $8\frac{1}{2}" \times 11"$ paper.

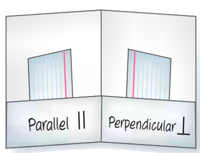
1. **Fold** in half matching the short sides.

2. **Unfold** and fold the long side up 2 inches to form a pocket.

3. **Staple** or glue the outer edges to complete the pocket.



4. **Label** each side as shown. Use index cards to record examples.



GET READY for Chapter 3

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

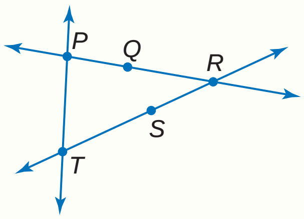
Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

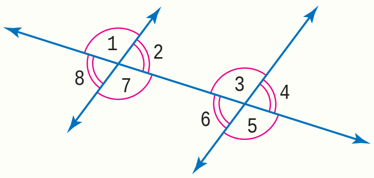
Name all of the lines that contain the given point. (Lesson 1-1)

1. Q
2. R
3. S
4. T



Name all angles congruent to the given angle. (Lessons 1-5)

5. $\angle 2$
6. $\angle 5$
7. $\angle 3$
8. $\angle 8$

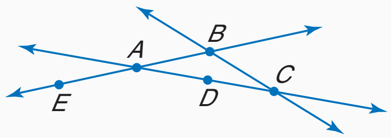


9. **MOVIES** A local movie theater is running a promotion in which a large popcorn costs \$2 with the purchase of two adult tickets. If Mr. and Mrs. Elian spent \$19 at the movie theater, write an equation to represent the cost and solve for the cost of one adult ticket. (Prerequisite Skill)

QUICKReview

EXAMPLE 1

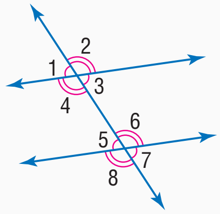
Name all of the lines that contain the point C.



Point C is the intersection point of lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$.

EXAMPLE 2

Name all angles congruent to $\angle 5$.



Look at the congruence marks. From the figure, $\angle 1$, $\angle 3$, and $\angle 7$ are each congruent to $\angle 5$.

EXAMPLE 3

Find the value of y in $2x - y = 4$ if $x = -4$.

$2x - y = 4$	Write the equation.
$-y = -2x + 4$	Subtract $2x$ from each side.
$y = 2x - 4$	Divide each side by -1.
$y = 2(-4) - 4$	Substitute -4 for x.
$y = -8 - 4$	Multiply.
$y = -12$	Simplify.

3-1 Parallel Lines and Transversals

Main Ideas

- Identify the relationships between two lines or two planes.
- Name angles formed by a pair of lines and a transversal.

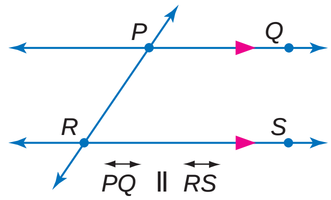
New Vocabulary

parallel lines
parallel planes
skew lines
transversal
consecutive interior angles
alternate exterior angles
alternate interior angles
corresponding angles

The Dana-Thomas House in Springfield, Illinois, is perhaps architect Frank Lloyd Wright's best preserved and most complete early "prairie" house. There are several examples of parallel lines, parallel planes, and skew lines in the design.

Relationships Between Lines and Planes Lines ℓ and m are coplanar because they lie in the same plane. If the lines were extended indefinitely, they would not intersect. Coplanar lines that do not intersect are called **parallel lines**. Segments and rays contained within parallel lines are also parallel.

The symbol $\parallel$ means *is parallel to*. Arrows are used in diagrams to indicate that lines are parallel. In the figure, the arrows indicate that $\overleftrightarrow{PQ}$ is parallel to $\overleftrightarrow{RS}$.



The symbol $\nparallel$ means *is not parallel to.*

Similarly, two planes can intersect or be parallel. In the photograph above, the front faces of the building are contained in **parallel planes**. The walls and the floor of each level lie in intersecting planes.

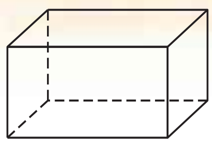
GEOMETRY LAB

Draw a Rectangular Prism

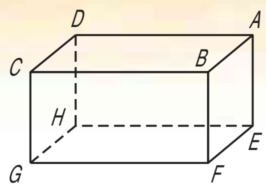
A rectangular prism can be drawn using parallel lines and parallel planes.

Step 1 Draw two parallel planes to represent the top and bottom.

Step 2 Draw the edges. Make any hidden edges dashed.



Step 3 Label the vertices.



ANALYZE

1. Identify the parallel planes in the figure.

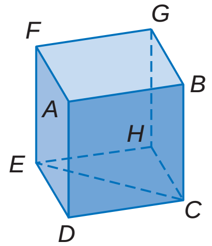
2. Name the planes that intersect plane *ABC* and name their intersections.

3. Identify all segments parallel to $\overline{BF}$.

Gibson Stock Photography

Notice that in the Geometry Lab, $\overline{AE}$ and $\overline{GF}$ do not intersect. These segments are not parallel since they do not lie in the same plane. Lines that do not intersect and are not coplanar are called **skew lines**. Segments and rays contained in skew lines are also skew.

EXAMPLE 1 Identify Relationships



a. Name all planes that are parallel to plane *ABG*.
plane *CDE*

b. Name all segments that intersect $\overline{CH}$.
$\overline{BC}$, $\overline{CD}$, $\overline{CE}$, $\overline{EH}$, and $\overline{GH}$

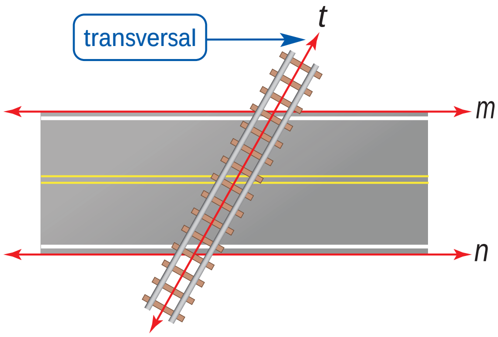
c. Name all segments that are skew to $\overline{BG}$.
$\overline{AD}$, $\overline{CD}$, $\overline{CE}$, $\overline{EF}$, and $\overline{EH}$

1. Name all segments that are parallel to $\overline{EF}$.

Study Tip

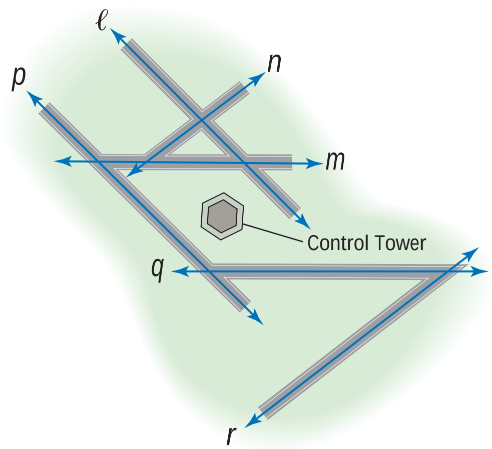
Identifying Segments
Use the segments drawn in the figure even though other segments exist.

Angle Relationships In the drawing of the railroad crossing, notice that the tracks, represented by line *t*, intersect the sides of the road, represented by lines *m* and *n*. A line that intersects two or more lines in a plane at different points is called a **transversal**.



EXAMPLE 2 Identify Transversals

AIRPORTS Some of the runways at O'Hare International Airport are shown below. Identify the sets of lines to which each given line is a transversal.



a. line *q*
If the lines are extended, line *q* intersects lines ℓ, *n*, *p*, and *r*.

b. line *m*
lines ℓ, *n*, *p*, and *r*

c. line *n*
lines ℓ, *m*, *p*, and *q*

2. line *r*

Study Tip

Transversals
The lines that the transversal intersects need not be parallel.

In the drawing of the railroad crossing above, notice that line *t* forms eight angles with lines *m* and *n*. These angles are given special names, as are specific pairings of these angles.

Study Tip

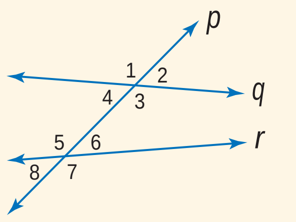
Same Side Interior Angles

Consecutive interior angles are also called *same side interior angles*.

KEY CONCEPT — Transversals and Angles

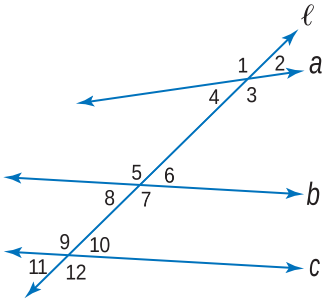
Name	Angles
exterior angles	∠1, ∠2, ∠7, ∠8
interior angles	∠3, ∠4, ∠5, ∠6
consecutive interior angles	∠3 and ∠6, ∠4 and ∠5
alternate exterior angles	∠1 and ∠7, ∠2 and ∠8
alternate interior angles	∠3 and ∠5, ∠4 and ∠6
corresponding angles	∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8

Transversal *p* intersects lines *q* and *r*.



EXAMPLE — Identify Angle Relationships

3 **Refer to the figure below. Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.**



a. **∠1 and ∠7**
alternate exterior

b. **∠2 and ∠10**
corresponding

c. **∠8 and ∠9**
consecutive interior

d. **∠3 and ∠12**
corresponding

e. **∠4 and ∠10**
alternate interior

f. **∠6 and ∠11**
alternate exterior

3A. ∠4 and ∠11

3B. ∠2 and ∠8

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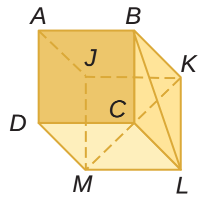
CHECK Your Understanding

Example 1 (p. 143)

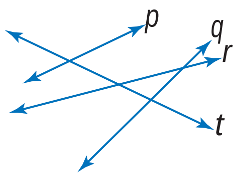
For Exercises 1–3, refer to the figure at the right.



1. Name all planes that intersect plane *ADM*.
2. Name all segments that are parallel to $\overline{CD}$.
3. Name all segments that intersect $\overline{KL}$.

Example 2 (p. 143)

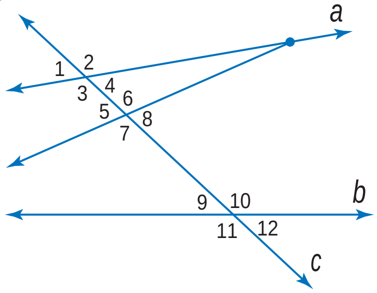
Identify the pairs of lines to which each given line is a transversal.



4. *p*
5. *r*
6. *q*
7. *t*

Example 3 (p. 144)

Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior angles*.

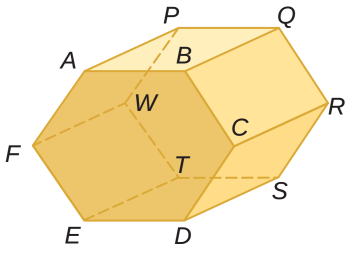


8. ∠7 and ∠10
9. ∠1 and ∠5
10. ∠4 and ∠6
11. ∠8 and ∠1

Exercises

HOMEWORK HELP	
For Exercises	See Examples
12–19	1
20–23	2
24–35	3

For Exercises 12–19, refer to the figure at the right.



12. Name all segments parallel to $\overline{AB}$.

13. Name all planes intersecting plane BCR.

14. Name all segments parallel to $\overline{TW}$.

15. Name all segments skew to $\overline{DE}$.

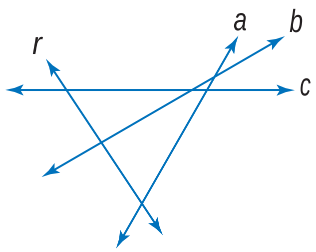
16. Name all planes intersecting plane EDS.

17. Name all segments skew to $\overline{AP}$.

18. Name all segments parallel to $\overline{DC}$.

19. Name all segments parallel to $\overline{DS}$.

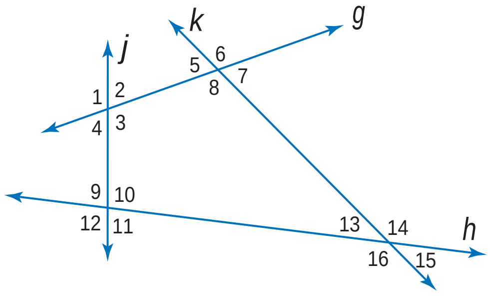
Identify the pairs of lines to which each given line is a transversal.



20. a **21.** b

22. c **23.** r

Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior angles*.



24. $\angle 2$ and $\angle 10$ **25.** $\angle 1$ and $\angle 11$

26. $\angle 5$ and $\angle 3$ **27.** $\angle 6$ and $\angle 14$

28. $\angle 5$ and $\angle 15$ **29.** $\angle 11$ and $\angle 13$

30. $\angle 8$ and $\angle 3$ **31.** $\angle 9$ and $\angle 4$

32. $\angle 6$ and $\angle 16$ **33.** $\angle 7$ and $\angle 3$

34. $\angle 10$ and $\angle 13$ **35.** $\angle 12$ and $\angle 14$

Real-World Career

Air Traffic Controller

The air traffic controller monitors and coordinates air traffic either at an airport or between airports. They monitor the position of airplanes ensuring that planes are a safe distance apart.

Math Online

For more information, go to geometryonline.com.

36. AVIATION Airplanes heading east are assigned an altitude level that is an odd number of thousands of feet. Airplanes heading west are assigned an altitude level that is an even number of thousands of feet. If one airplane is flying northwest at 34,000 feet and another airplane is flying east at 25,000 feet, describe the type of lines formed by the paths of the airplanes. Explain your reasoning.

MONUMENTS For Exercises 37–40, refer to the photograph of the Lincoln Memorial.

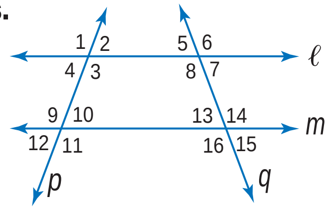
37. Describe a pair of parallel lines found on the Lincoln Memorial.

38. Find an example of parallel planes.

39. Locate a pair of skew lines.

40. Identify a transversal passing through a pair of lines.

Name the transversal that forms each pair of angles. Then identify the special name for the angle pair.



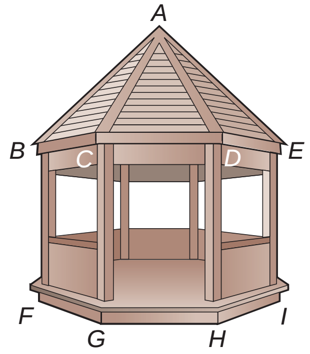
41. $\angle 3$ and $\angle 10$

42. $\angle 2$ and $\angle 12$

43. $\angle 8$ and $\angle 14$

44. $\angle 9$ and $\angle 16$

STRUCTURES **For Exercises 45-47, refer to the drawing of the gazebo at the right.**



45. Name all labeled segments parallel to $\overline{BF}$.

46. Name all labeled segments skew to $\overline{AC}$.

47. Are any of the planes on the gazebo parallel to plane *ADE*? Explain.

EXTRA PRACTICE

See pages 805, 830.

Math Online

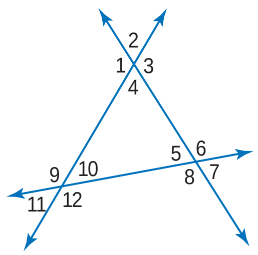
Self-Check Quiz at geometryonline.com

48. **RESEARCH** The word *parallel* describes computer processes that occur simultaneously, or devices, such as printers, that receive more than one bit of data at a time. Find two other examples for uses of the word *parallel* in other subject areas such as history, music, or sports.

H.O.T. Problems

49. **OPEN ENDED** Draw a solid figure with parallel planes. Describe which parts of the figure are parallel.

50. **FIND THE ERROR** Juanita and Eric are naming alternate interior angles in the figure at the right. One of the angles must be $\angle 4$. Who is correct? Explain your reasoning.



Juanita	Eric
$\angle 4$ and $\angle 9$	$\angle 4$ and $\angle 10$
$\angle 4$ and $\angle 6$	$\angle 4$ and $\angle 5$

CHALLENGE Suppose there is a line ℓ and a point P not on the line.

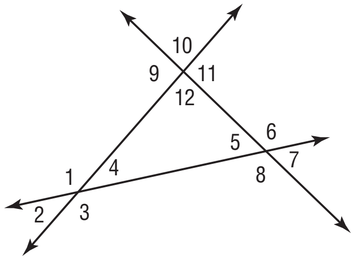
51. In space, how many lines can be drawn through P that do not intersect ℓ?

52. In space, how many lines can be drawn through P that are parallel to ℓ?

53. Writing in Math Use the information about architecture on page 142 to explain how parallel lines and planes are used in architecture. Include a description of where you might find examples of parallel lines and parallel planes, and skew lines and nonparallel planes.

STANDARDIZED TEST PRACTICE

54. Which of the following angle pairs are alternate exterior angles?



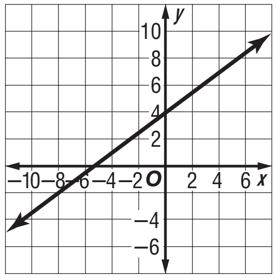
A $\angle 1$ and $\angle 5$

B $\angle 2$ and $\angle 10$

C $\angle 2$ and $\angle 6$

D $\angle 5$ and $\angle 9$

55. REVIEW Which coordinate points represent the x- and y-intercepts of the graph shown below?



F (-5.6, 0), (0, 4)

G (5.6, 0), (4, 0)

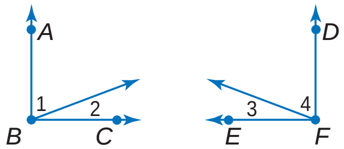
H (6, 0), (0, 4)

J (0, 4), (0, 6)

56. PROOF **Write a two-column proof.** (Lesson 2-8)

Given: $m\angle ABC = m\angle DFE$, $m\angle 1 = m\angle 4$

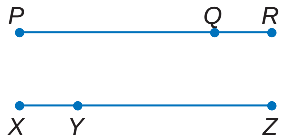
Prove: $m\angle 2 = m\angle 3$



57. PROOF **Write a paragraph proof.** (Lesson 2-7)

Given: $\overline{PQ} \cong \overline{ZY}$, $\overline{QR} \cong \overline{XY}$

Prove: $\overline{PR} \cong \overline{XZ}$



Determine whether a valid conclusion can be reached from the two true statements using the Law of Detachment or the Law of Syllogism. If a valid conclusion is possible, state it and the law that is used. If a valid conclusion does not follow, write *no conclusion*. (Lesson 2-4)

58. (1) If two angles are vertical, then they do not form a linear pair.

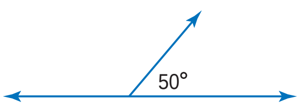
(2) If two angles form a linear pair, then they are not congruent.

59. (1) If an angle is acute, then its measure is less than 90.

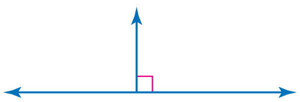
(2) $\angle EFG$ is acute.

PREREQUISITE SKILL **State the measures of linear pairs of angles in each figure.** (Lesson 2-6)

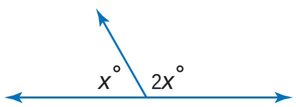
60.



61.



62.



EXPLORE 3-2

Geometry Software Lab

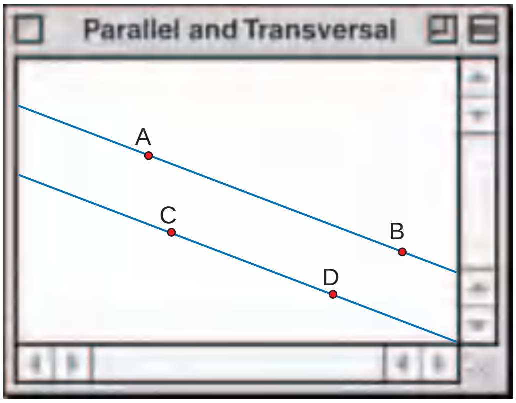
Angles and Parallel Lines

You can use The Geometer's Sketchpad® to investigate the measures of angles formed by two parallel lines and a transversal.

ACTIVITY

Step 1 Draw parallel lines.

- Construct points *A* and *B*.
- Construct a line through the points.
- Place point *C* so that it does not lie on $\overleftrightarrow{AB}$.
- Construct a line through *C* parallel to $\overleftrightarrow{AB}$.
- Place point *D* on this line.

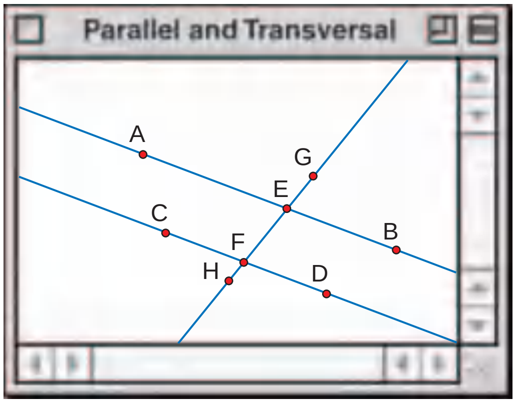


Step 2 Construct a transversal.

- Place point *E* on $\overleftrightarrow{AB}$ and point *F* on $\overleftrightarrow{CD}$.
- Construct a line through points *E* and *F*.
- Place points *G* and *H* on $\overleftrightarrow{EF}$.

Step 3 Measure angles.

- Measure each angle.



Analyze the Results

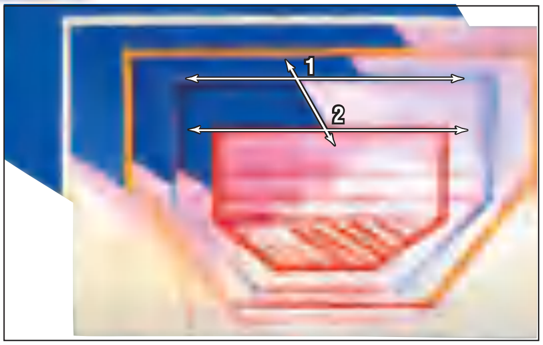
1. List pairs of angles by the special names you learned in Lesson 3-1. Which pairs have the same measure?
2. What is the relationship between consecutive interior angles?
3. **Make a conjecture** about the following pairs of angles formed by two parallel lines and a transversal. Write your conjecture in if-then form.
   a. corresponding angles
   b. alternate interior angles
   c. alternate exterior angles
   d. consecutive interior angles
4. Rotate the transversal. Are the angles with equal measures in the same relative location as the angles with equal measures in your original drawing?
5. Test your conjectures by rotating the transversal and analyzing the angles.
6. Rotate the transversal so that the measure of at least one angle is 90.
   a. What do you notice about the measures of the other angles?
   b. Make a conjecture about a transversal that is perpendicular to one of two parallel lines.

3-2 Angles and Parallel Lines

Main Ideas

- Use the properties of parallel lines to determine congruent angles.
- Use algebra to find angle measures.

In the painting, the artist uses lines and transversals to create patterns. The figure on the painting shows two parallel lines with a transversal passing through them. There is a special relationship between the angle pairs formed by these lines.

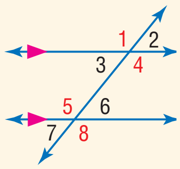


Source: *Vista* by Richard Smith

Parallel Lines and Angle Pairs In the figure above, $\angle 1$ and $\angle 2$ are corresponding angles. When the two lines are parallel, there is a special relationship between these pairs of angles.

POSTULATE 3.1 ***Corresponding Angles Postulate***

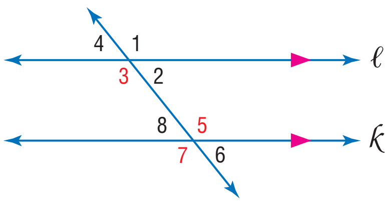
If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.



Examples: $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, $\angle 4 \cong \angle 8$

EXAMPLE **Determine Angle Measures**

1 In the figure, $m\angle 3 = 133$. Find $m\angle 5$.



$\angle 3 \cong \angle 7$	Corresponding Angles Postulate
$\angle 7 \cong \angle 5$	Vertical Angles Theorem
$\angle 3 \cong \angle 5$	Transitive Property
$m\angle 3 = m\angle 5$	Definition of congruent angles
$133 = m\angle 5$	Substitution

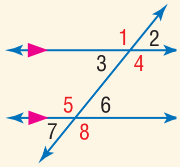
1. In the figure, $m\angle 8 = 47$. Find $m\angle 4$.

Review Vocabulary

Vertical Angles two nonadjacent angles formed by two intersecting lines (Lesson 1-6)

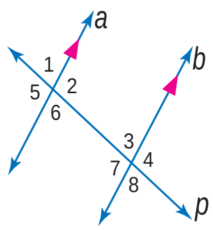
In Example 1, alternate interior angles 3 and 5 are congruent. This suggests another special relationship between angles formed by two parallel lines and a transversal. Other relationships are summarized in Theorems 3.1, 3.2, and 3.3.

THEOREM — Parallel Lines and Angle Pairs

Theorems	Examples	Model
3.1 Alternate Interior Angles If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.	$\angle 4 \cong \angle 5$ $\angle 3 \cong \angle 6$	
3.2 Consecutive Interior Angles If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.	$\angle 4$ and $\angle 6$ are supplementary. $\angle 3$ and $\angle 5$ are supplementary.	
3.3 Alternate Exterior Angles If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.	$\angle 1 \cong \angle 8$ $\angle 2 \cong \angle 7$	

You will prove Theorems 3.2 and 3.3 in Exercises 26 and 23, respectively.

Proof Theorem 3.1



Given: $a \parallel b$; p is a transversal of a and b.

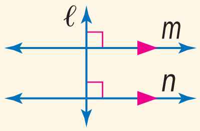
Prove: $\angle 2 \cong \angle 7$, $\angle 3 \cong \angle 6$

Paragraph Proof: We are given that $a \parallel b$ with a transversal p. By the Corresponding Angles Postulate, $\angle 2 \cong \angle 4$ and $\angle 8 \cong \angle 6$. Also, $\angle 4 \cong \angle 7$ and $\angle 3 \cong \angle 8$ because vertical angles are congruent. Therefore, $\angle 2 \cong \angle 7$ and $\angle 3 \cong \angle 6$ since congruence of angles is transitive.

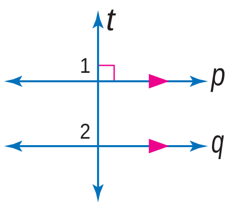
A special relationship occurs when the transversal is a perpendicular line.

THEOREM 3.4 — Perpendicular Transversal Theorem

In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.



Proof Theorem 3.4



Given: $p \parallel q$, $t \perp p$

Prove: $t \perp q$

Proof:

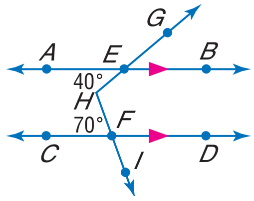
Statements	Reasons
1. $p \parallel q$, $t \perp p$	**1.** Given
2. $\angle 1$ is a right angle.	**2.** Definition of $\perp$ lines
3. $m\angle 1 = 90$	**3.** Definition of right angle
4. $\angle 1 \cong \angle 2$	**4.** Corresponding Angles Postulate
5. $m\angle 1 = m\angle 2$	**5.** Definition of congruent angles
6. $m\angle 2 = 90$	**6.** Substitution Property
7. $\angle 2$ is a right angle.	**7.** Definition of right angles
8. $t \perp q$	**8.** Definition of $\perp$ lines

STANDARDIZED TEST EXAMPLE — Use an Auxiliary Line

2 What is $m\angle GHI$?

A 50°　　**C** 130°

B 110°　　**D** 140°



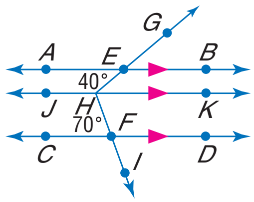
Read the Test Item

You need to find $m\angle GHI$.

Solve the Test Item

Draw $\overleftrightarrow{JK}$ through H parallel to $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

$\angle EHK \cong \angle AEH$	Alternate Interior Angles Theorem
$m\angle EHK = m\angle AEH$	Definition of congruent angles
$m\angle EHK = 40$	Substitution
$\angle FHK \cong \angle CFH$	Alternate Interior Angles Theorem
$m\angle FHK = m\angle CFH$	Definition of congruent angles
$m\angle FHK = 70$	Substitution



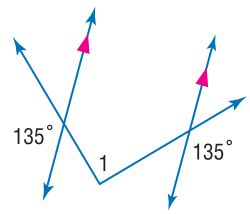
$m\angle GHI = m\angle EHK + m\angle FHK$	Angle Addition Postulate
$= 40 + 70$ or 110	$m\angle EHK = 40$, $m\angle FHK = 70$

Thus, the answer is choice B.

2. What is $m\angle 1$?

F 45°　　**H** 90°

G 65°　　**J** 135°



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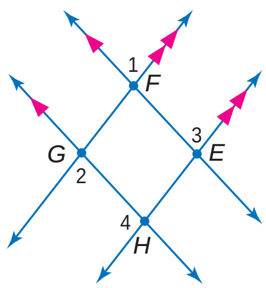
Algebra and Angle Measures Angles formed by two parallel lines and a transversal can be used to find unknown values.

EXAMPLE 3 Find Values of Variables

ALGEBRA **If $m\angle 1 = 3x + 40$ and $m\angle 3 = 2x + 70$, find x.**

Since $\overleftrightarrow{FG} \parallel \overleftrightarrow{EH}$, $\angle 1 \cong \angle 3$ by the Corresponding Angles Postulate.

$m\angle 1 = m\angle 3$	Definition of congruent angles
$3x + 40 = 2x + 70$	Substitution
$x = 30$	Subtract $2x$ and 40 from each side.



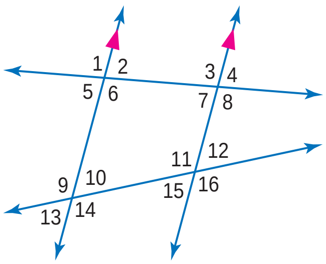
3. Refer to the figure. If $m\angle 2 = 4x + 7$ and $m\angle 3 = 5x - 13$, find $m\angle 3$.

CHECK Your Understanding

Example 1 (p. 149)

In the figure, $m\angle 3 = 110$ and $m\angle 12 = 55$. Find the measure of each angle.

1. $\angle 1$ **2.** $\angle 6$ **3.** $\angle 2$

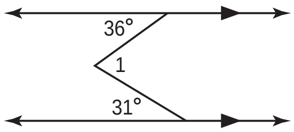


Example 2 (p. 151)

4. STANDARDIZED TEST PRACTICE What is $m\angle 1$?

A 5° **C** 36°

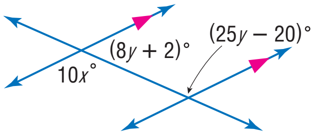
B 31° **D** 67°



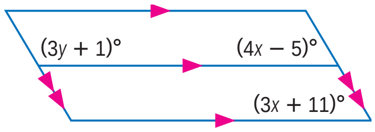
Example 3 (p. 151)

Find *x* and *y* in each figure.

5.



6.



Exercises

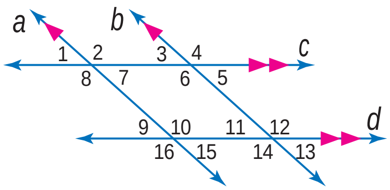
HOMEWORK HELP	
For Exercises	See Examples
7–12	1
13–18	2
19, 20	3

In the figure, $m\angle 3 = 43$. Find the measure of each angle.

7. $\angle 2$ **8.** $\angle 7$

9. $\angle 10$ **10.** $\angle 11$

11. $\angle 13$ **12.** $\angle 16$

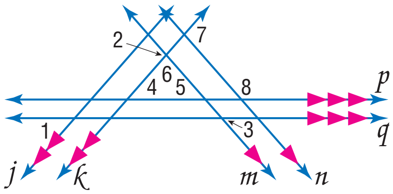


In the figure, $m\angle 1 = 50$ and $m\angle 3 = 60$. Find the measure of each angle.

13. $\angle 4$ **14.** $\angle 5$

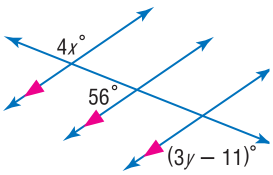
15. $\angle 2$ **16.** $\angle 6$

17. $\angle 7$ **18.** $\angle 8$

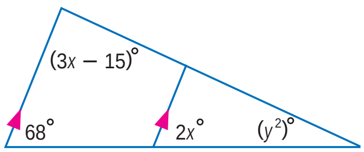


Find *x* and *y* in each figure.

19.

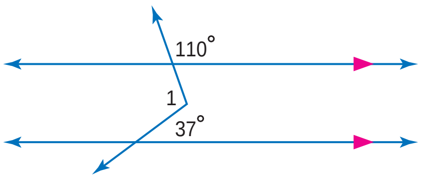


20.

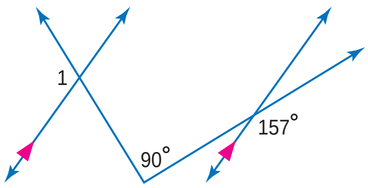


Find $m\angle 1$ in each figure.

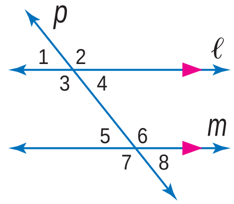
21.



22.



23. PROOF Copy and complete the proof of Theorem 3.3.



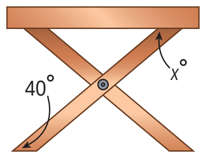
Given: $\ell \parallel m$

Prove: $\angle 1 \cong \angle 8$
$\angle 2 \cong \angle 7$

Proof:

Statements	Reasons
1. $\ell \parallel m$	**1.** ___?___
2. $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$	**2.** ___?___
3. $\angle 5 \cong \angle 8$, $\angle 6 \cong \angle 7$	**3.** ___?___
4. $\angle 1 \cong \angle 8$, $\angle 2 \cong \angle 7$	**4.** ___?___

24. CARPENTRY Anthony is building a picnic table for his patio. He cut one of the legs at an angle of 40°. At what angle should he cut the other end to ensure that the top of the table is parallel to the ground? Explain.

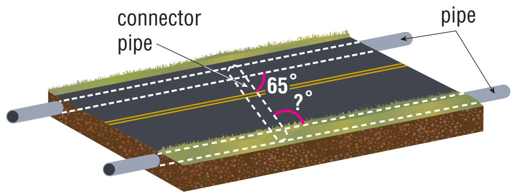


Real-World Link

In 2005, the United States budgeted about $35 billion for federal highway projects.

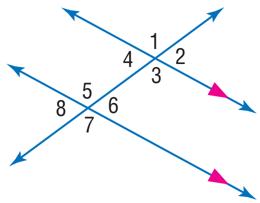
Source: U.S. Dept. of Transportation

25. CONSTRUCTION Parallel drainage pipes are connected with a third pipe. The connector pipe makes a 65° angle with a pipe as shown. What is the measure of the angle it makes with the pipe on the other side of the road? Explain.



26. PROOF Write a two-column proof of Theorem 3.2.

Refer to the figure for Exercises 27 and 28.



27. Determine whether $\angle 1$ is *always*, *sometimes*, or *never* congruent to $\angle 2$. Explain.

28. Determine the minimum number of angle measures you would have to know to find the measures of all of the angles in the figure.

EXTRA PRACTICE

See pages 805, 830.

Math Online

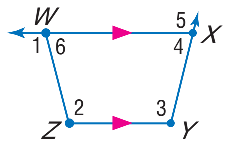
Self-Check Quiz at geometryonline.com

H.O.T. Problems

29. OPEN ENDED Use a straightedge and protractor to draw a pair of parallel lines cut by a transversal so that one pair of corresponding angles measures 35°.

30. REASONING Make a conjecture about two exterior angles on the same side of a transversal. Prove your conjecture.

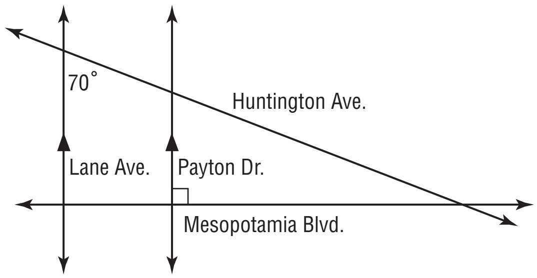
31. CHALLENGE Explain why you can conclude that $\angle 2$ and $\angle 6$ are supplementary, but you cannot state that $\angle 4$ and $\angle 6$ are necessarily supplementary.



32. Writing in Math Use the information about art from page 149 to explain how angles and lines can be used in art. Include a description of how angles and lines are used to create patterns and examples from two different artists that use lines and angles.

Keith Wood/CORBIS

33. An architect wants to design a shopping district between Huntington Avenue, Payton Drive, and Mesopotamia Boulevard.



What are the measures of the three angles of the shopping district?

A 90°, 70°, 20°

B 90°, 62°, 38°

C 90°, 60°, 30°

D 100°, 30°, 20°

34. **REVIEW** Emma has been hiring more workers for her donut shop. The table shows the number of additional workers compared to the number of donuts the shop can make in an hour.

Additional Workers	Donuts Made
0	45
1	70
2	95
3	120

Which equation best describes the relationship between *w,* the number of additional workers, and *d,* the number of donuts the shop can make in an hour?

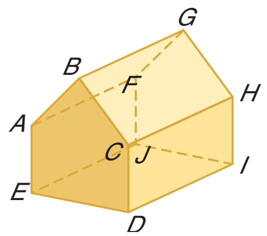
F $45w + 25 = d$

G $d - 45 = 25w$

H $d + 45 = 25w$

J $45w - 25 = d$

For Exercises 35–37, refer to the figure at the right. (Lesson 3-1)



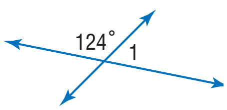
35. Name all segments parallel to $\overline{AB}$.

36. Name all segments skew to $\overline{CH}$.

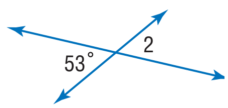
37. Name all planes parallel to *AEF*.

Find the measure of each numbered angle. (Lesson 2-8)

38.



39.



Identify the hypothesis and conclusion of each statement. (Lesson 2-3)

40. If it rains this evening, then I will mow the lawn tomorrow.

41. A balanced diet will keep you healthy.

PREREQUISITE SKILL **Simplify each expression.**

42. $\frac{7 - 9}{8 - 5}$ **43.** $\frac{-3 - 6}{2 - 8}$ **44.** $\frac{14 - 11}{23 - 15}$ **45.** $\frac{15 - 23}{14 - 11}$ **46.** $\frac{2}{9} \cdot \left(-\frac{18}{5}\right)$

EXPLORE 3-3

Graphing Calculator Lab

Investigating Slope

The rate of change of the steepness of a line is called the *slope*. Slope can be used to investigate the relationship between real-world quantities.

- Connect the data collection device to the graphing calculator. Place on a desk or table so that the data collection device can read the motion of a walker.
- Mark the floor at a distance of 1 meter and 6 meters from the device.

ACTIVITY

Step 1 Have one group member stand at the 1-meter mark. When another group member presses the button to begin collecting data, the walker begins to walk away from the device. Walk at a slow, steady pace.

Step 2 Stop collecting data when the walker passes the 6-meter mark. Save the data as Trial 1.

Step 3 Repeat the experiment, walking more quickly. Save the data as Trial 2.

Step 4 For Trial 3, repeat the experiment by walking toward the data collection device slowly.

Step 5 Repeat the experiment, walking quickly toward the device. Save the data as Trial 4.

ANALYZE THE RESULTS

1. Compare and contrast the graphs for Trials 1 and 2.
2. Use the TRACE feature of the calculator to find the coordinates of two points on each graph. Record the coordinates in a table like the one shown. Then use the points to find the slope of the line.

Trial	Point A (x_1, y_1)	Point B (x_2, y_2)	Slope $= \frac{y_2 - y_1}{x_2 - x_1}$
1			
2			

3. Compare and contrast the slopes for Trials 1 and 2.
4. The slope of a line describes the rate of change of the quantities represented by the *x*- and *y*-values. What is represented by the rate of change in this experiment?
5. **MAKE A CONJECTURE** What would the graph look like if you were to collect data while the walker was standing still? Use the data collection device to test your conjecture.

Horizons Companies

Other Calculator Keystrokes at geometryonline.com

3-3 Slopes of Lines

Main Ideas

- Find slopes of lines.
- Use slope to identify parallel and perpendicular lines.

New Vocabulary

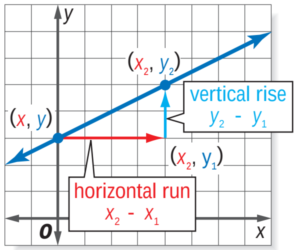
slope
rate of change

Traffic signs are often used to alert drivers to road conditions. The sign at the right indicates a hill with a 6% *grade*. This means that the road will rise or fall 6 feet vertically for every 100 horizontal feet traveled.

Slope of a Line The **slope** of a line is the ratio of its vertical rise to its horizontal run.

$$\text{slope} = \frac{\text{vertical rise}}{\text{horizontal run}}$$

You can use the coordinates of points on a line to derive a formula for slope. In a coordinate plane, the slope of a line is the ratio of the change along the *y*-axis to the change along the *x*-axis. The vertical rise is computed by finding the difference in *y*-values of the coordinates of two points on the line. Likewise, the horizontal run is defined by the difference in *x*-values of the coordinates of two points on the line.



KEY CONCEPT — *Slope Formula*

The slope m of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula

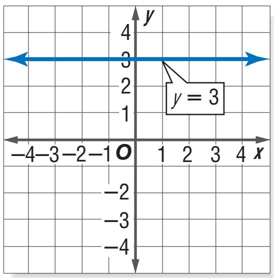
$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$

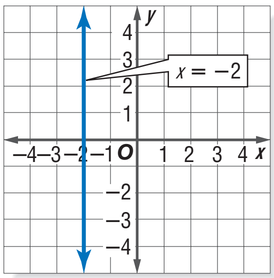
Study Tip

Slope

Lines with positive slope *rise* as you move from left to right, while lines with negative slope *fall* as you move from left to right.

The slope of a line indicates whether the line rises to the right, falls to the right, or is horizontal. The slope of a vertical line, where $x_1 = x_2$, is undefined.





EXAMPLE Find the Slope of a Line

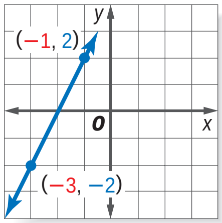
Study Tip

Common Misconception

A line with a slope of 0 is a horizontal line. The slope of a vertical line is undefined.

1 **Find the slope of each line.**

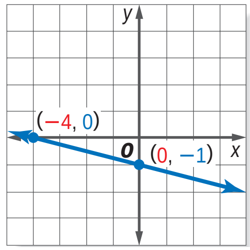
a.



From $(-3, -2)$ to $(-1, 2)$, go up 4 units and right 2 units.

$$\frac{\text{rise}}{\text{run}} = \frac{4}{2} \text{ or } 2$$

b.

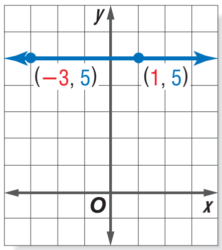


Let $(-4, 0)$ be (x_1, y_1) and $(0, -1)$ be (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}$$

$$= \frac{-1 - 0}{0 - (-4)} \text{ or } -\frac{1}{4}$$

c.

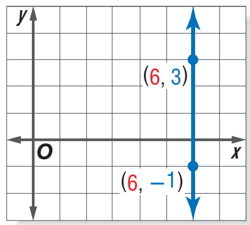


$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - 5}{-3 - 1}$$

$$= \frac{0}{-4} \text{ or } 0$$

d.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

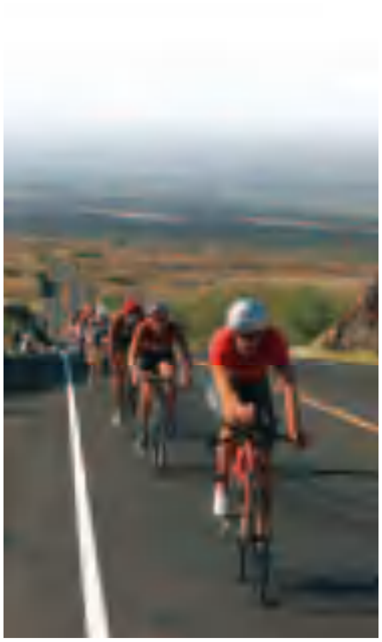
$$= \frac{3 - (-1)}{6 - 6}$$

$$= \frac{4}{0}, \text{ which is undefined}$$

1A. the line containing $(-6, -2)$ and $(3, -5)$

1B. the line containing $(8, -3)$ and $(-6, -2)$

The slope of a line can be used to identify the coordinates of any point on the line. It can also be used to describe a rate of change. The **rate of change** describes how a quantity is changing over time.



Real-World Link

Between 2000 and 2003, annual sales of exercise equipment increased by an average rate of \$314.3 million per year. In 2003, the total sales were \$4553 million.

Source: *Statistical Abstract of the United States*

EXAMPLE Use Rate of Change to Solve a Problem

2 **FITNESS** **Refer to the information at the left. If sales of fitness equipment increase at the same rate, what will the total sales be in 2010?**

Let $(x_1, y_1) = (2003, 4553)$ and $m = 314.3$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$

$$314.3 = \frac{y_2 - 4553}{2010 - 2003} \quad m = 314.3, y_1 = 4553, x_1 = 2003, \text{ and } x_2 = 2010$$

$$314.3 = \frac{y_2 - 4553}{7} \quad \text{Simplify.}$$

(continued on the next page)

$2200.1 = y_2 - 4553$ Multiply each side by 7.

$6753.1 = y_2$ Add 4553 to each side.

The coordinates of the point representing the sales for 2010 are (2010, 6753.1). Thus, the total sales in 2010 will be about \$6753.1 million.

2. **DOWNLOADS** In 2004, 200 million songs were legally downloaded from the Internet. In 2003, 20 million songs were legally downloaded. If this increases at the same rate, how many songs will be legally downloaded in 2008?

Online Personal Tutor at geometryonline.com

Parallel and Perpendicular Lines In the Geometry Lab, you will explore the slopes of parallel and perpendicular lines.

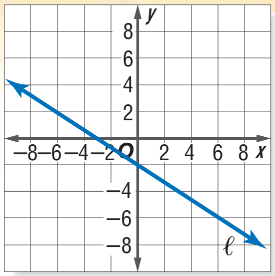
GEOMETRY LAB

Parallel and Perpendicular Lines

MAKE A MODEL

Materials: dry spaghetti, grid paper

1. Place a piece of spaghetti on the grid paper through points $(-3, 0)$ and $(2, -3)$. Label this line ℓ.
2. Place a second piece of spaghetti on the grid such that it is parallel to line ℓ. Label this line m.
3. Place a third piece of spaghetti so it is perpendicular to lines ℓ and m. Label this line n.



ANALYZE THE MODEL

1. What is the slope of line ℓ?
2. List two points that line m contains. What is the slope of line m?
3. List two points that line n contains. Determine the slope of line n.
4. Compare the slopes of line n and line m.

MAKE A CONJECTURE

5. Make a conjecture about the slopes of two parallel lines.
6. Make a conjecture about the slopes of two perpendicular lines.
7. Test your conjectures with different points.

Study Tip

Look Back

To review **if and only if statements**, see Reading Math, page 98.

The Geometry Lab suggests two important algebraic properties of parallel and perpendicular lines.

POSTULATES *Parallel and Perpendicular Lines*

3.2 Two nonvertical lines have the same slope if and only if they are parallel.

3.3 Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

EXAMPLE Determine Line Relationships

3 **Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*.**

a. $A(-2, -5), B(4, 7), C(0, 2), D(8, -2)$

Find the slopes of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

slope of $\overleftrightarrow{AB} = \frac{7 - (-5)}{4 - (-2)}$ $= \frac{12}{6}$ or 2

slope of $\overleftrightarrow{CD} = \frac{-2 - 2}{8 - 0}$ $= -\frac{4}{8}$ or $-\frac{1}{2}$

The product of the slopes is $2\left(-\frac{1}{2}\right)$ or -1. So, $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$.

b. $A(-8, -7), B(4, -4), C(-2, -5), D(1, 7)$

slope of $\overleftrightarrow{AB} = \frac{-4 - (-7)}{4 - (-8)}$ $= \frac{3}{12}$ or $\frac{1}{4}$

slope of $\overleftrightarrow{CD} = \frac{7 - (-5)}{1 - (-2)}$ $= \frac{12}{3}$ or 4

The slopes are not the same, so $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are not parallel. The product of the slopes is $4\left(\frac{1}{4}\right)$ or 1. So, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are neither parallel nor perpendicular.

Interactive Lab
geometryonline.com

3A. $A(14, 13), B(-11, 0), C(-3, 7), D(-4, -5)$

3B. $A(3, 6), B(-9, 2), C(-12, -6), D(15, 3)$

The relationships of the slopes of lines can be used to graph a line parallel or perpendicular to a given line.

EXAMPLE Use Slope to Graph a Line

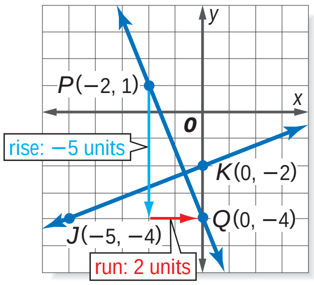
4 **Graph the line that contains $P(-2, 1)$ and is perpendicular to $\overleftrightarrow{JK}$ with $J(-5, -4)$ and $K(0, -2)$.**

First, find the slope of $\overleftrightarrow{JK}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{-2 - (-4)}{0 - (-5)}$ Substitution

$= \frac{2}{5}$ Simplify.



The product of the slopes of two perpendicular lines is -1.

Since $\frac{2}{5}\left(-\frac{5}{2}\right) = -1$, the slope of the line perpendicular to $\overleftrightarrow{JK}$ through $P(-2, 1)$ is $-\frac{5}{2}$.

Graph the line. Start at $(-2, 1)$. Move down 5 units and then move right 2 units.

Label the point Q. Draw $\overleftrightarrow{PQ}$.

Study Tip

Negative Slopes

To help determine direction with negative slopes, remember that $-\frac{5}{2} = \frac{-5}{2} = \frac{5}{-2}$.

4. Graph the line that contains $P(0, 1)$ and is perpendicular to $\overleftrightarrow{QR}$ with $Q(-6, -2)$ and $R(0, -6)$.

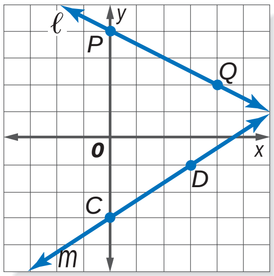
CHECK Your Understanding

Example 1 (p. 157)

Find the slope of each line.

1. ℓ

2. m



Example 2 (pp. 157–158)

MOUNTAIN BIKING For Exercises 3–5, use the following information.
A certain mountain bike trail has a section of trail with a grade of 8%.

3. What is the slope of the hill?

4. After riding on the trail, a biker is 120 meters below her original starting position. If her starting position is represented by the origin on a coordinate plane, what are possible coordinates of her current position?

5. How far has she traveled down the hill? Round to the nearest meter.

Example 3 (p. 159)

6. Determine whether $\overleftrightarrow{GH}$ and $\overleftrightarrow{RS}$ are *parallel*, *perpendicular*, or *neither* given $G(15, -9)$, $H(9, -9)$, $R(-4, -1)$, and $S(3, -1)$.

Example 4 (p. 159)

Graph the line that satisfies each condition.

7. slope $= 2$, contains $P(1, 2)$

8. contains $A(6, 4)$, perpendicular to $\overleftrightarrow{MN}$ with $M(5, 0)$ and $N(1, 2)$

Exercises

HOMEWORK HELP

For Exercises	See Examples
9–12, 17–20	1
21, 22	2
13–16, 23–28	3
29–36	4

Find the slope of each line.

9. $\overleftrightarrow{AB}$

10. $\overleftrightarrow{PQ}$

11. $\overleftrightarrow{LM}$

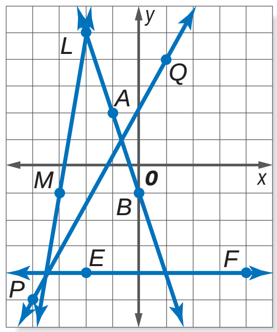
12. $\overleftrightarrow{EF}$

13. a line parallel to $\overleftrightarrow{LM}$

14. a line perpendicular to $\overleftrightarrow{PQ}$

15. a line perpendicular to $\overleftrightarrow{EF}$

16. a line parallel to $\overleftrightarrow{AB}$



Determine the slope of the line that contains the given points.

17. $A(0, 2)$, $B(7, 3)$

18. $C(-2, -3)$, $D(-6, -5)$

19. $W(3, 2)$, $X(4, -3)$

20. $Y(1, 7)$, $Z(4, 3)$

21. **RECREATION** Paintball is one of the fastest growing sports. In 2002, 1,949,000 Americans from 12–17 years old participated in paintball. In 2005, 2,209,000 participated. If participation increases at the same rate, what will the participation be in 2012 to the nearest thousand?

22. **TRAVEL** On average, the rate of travel to Canada has been increasing by 486,500 visitors per year. In 2002, 16,161,000 Americans visited Canada. Approximately how many people will visit Canada in 2010?

Determine whether $\overleftrightarrow{PQ}$ and $\overleftrightarrow{UV}$ are *parallel*, *perpendicular*, or *neither*.

23. $P(-3, -2)$, $Q(9, 1)$, $U(3, 6)$, $V(5, -2)$
24. $P(-4, 0)$, $Q(0, 3)$, $U(-4, -3)$, $V(8, 6)$
25. $P(-10, 7)$, $Q(2, 1)$, $U(4, 0)$, $V(6, 1)$
26. $P(-9, 2)$, $Q(0, 1)$, $U(-1, 8)$, $V(-2, -1)$
27. $P(1, 1)$, $Q(9, 8)$, $U(-6, 1)$, $V(2, 8)$
28. $P(5, -4)$, $Q(10, 0)$, $U(9, -8)$, $V(5, -13)$

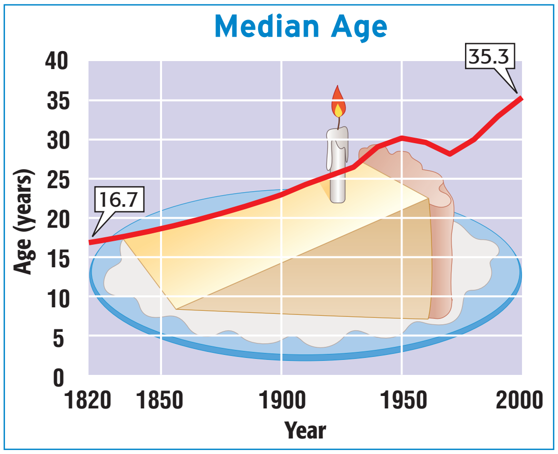
Graph the line that satisfies each condition.

29. slope $= -4$, passes through $P(-2, 1)$
30. contains $A(-1, -3)$, parallel to $\overleftrightarrow{CD}$ with $C(-1, 7)$ and $D(5, 1)$
31. contains $M(4, 1)$, perpendicular to $\overleftrightarrow{GH}$ with $G(0, 3)$ and $H(-3, 0)$
32. slope $= \frac{2}{5}$, contains $J(-7, -1)$
33. contains $Q(-2, -4)$, parallel to $\overleftrightarrow{KL}$ with $K(2, 7)$ and $L(2, -12)$
34. contains $W(6, 4)$, perpendicular to $\overleftrightarrow{DE}$ with $D(0, 2)$ and $E(5, 0)$.

35. Determine the value of x so that a line containing $(6, 2)$ and $(x, -1)$ has a slope of $-\frac{3}{7}$. Then graph the line.
36. Find the value of x so that the line containing $(4, 8)$ and $(2, -1)$ is perpendicular to the line containing $(x, 2)$ and $(-4, 5)$. Graph the lines.

POPULATION For Exercises 37–39, refer to the graph.

37. Estimate the annual rate of change of the median age from 1970 to 2000.
38. If the median age continues to increase at the same rate, what will be the median age in 2010?
39. Suppose that after 2000, the median age increases by $\frac{1}{3}$ of a year annually. In what year will the median age be 40.6?



Source: Census Bureau

EXTRA PRACTICE
See pages 804, 830.
Math Online
Self-Check Quiz at geometryonline.com

STADIUMS For Exercises 40–42 use the following information.
Monster Park is home to the San Francisco 49ers. The attendance in 2000 was 541,960, and the attendance in 2004 was 518,271.

40. What is the approximate rate of change in attendance from 2000 to 2004?
41. If this rate of change continues, predict the attendance for 2012.
42. Will the attendance continue to decrease indefinitely? Explain.

H.O.T. Problems

43. **FIND THE ERROR** Curtis and Lori calculated the slope of the line containing $A(15, 4)$ and $B(-6, -13)$. Who is correct? Explain your reasoning.

Curtis

$$m = \frac{4 - (-13)}{15 - (-6)} = \frac{17}{21}$$

Lori

$$m = \frac{4 - 13}{15 - 6} = -1$$

44. **OPEN ENDED** Give a real-world example of a line with a slope of 0 and a real-world example of a line with an undefined slope.

45. **Which One Doesn't Belong?** Identify the term that does not belong with the other three. Explain your reasoning.

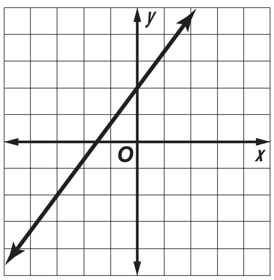
slope	rate of change	skew	steepness

46. **CHALLENGE** The line containing the point $(5 + 2t, -3 + t)$ can be described by the equations $x = 5 + 2t$ and $y = -3 + t$. Write the slope-intercept form of the equation of this line.

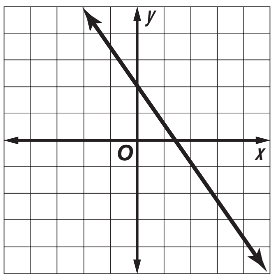
47. **Writing in Math** Use the information about grade on page 156 to explain how slope is used in transportation. Include an explanation of why it is sometimes important to display the grade of a road and an example of slope used in transportation other than roads.

48. Which graph best represents the line passing through the point at $(-2, 5)$ and perpendicular to the graph of $y = \frac{2}{3}x$?

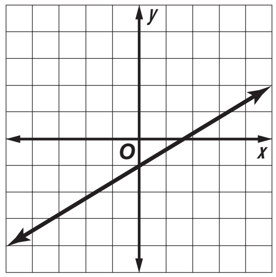
A



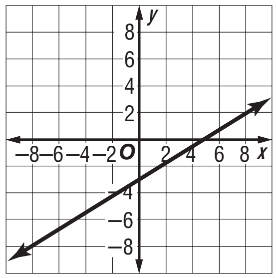
B



C



D



49. Which equation describes the line that passes through the point at $(-2, 1)$ and is perpendicular to the line $y = \frac{1}{3}x + 5$?

F $y = 3x + 7$

G $y = -3x - 5$

H $y = \frac{1}{3}x + 7$

J $y = -\frac{1}{3}x - 5$

50. **REVIEW** Which expression is equivalent to $4(x - 6) - \frac{1}{2}(x^2 + 8)$?

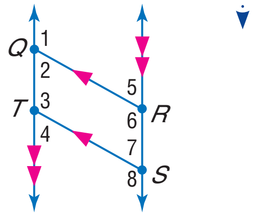
A $4x^2 + 4x - 28$

B $-\frac{1}{2}x^2 + 6x - 24$

C $-\frac{1}{2}x^2 + 4x - 28$

D $3x - 20$

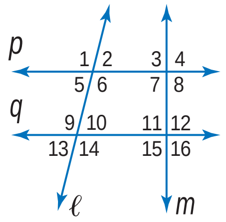
In the figure, $\overline{QR} \parallel \overline{TS}$, $\overleftrightarrow{QT} \parallel \overleftrightarrow{RS}$, and $m\angle 1 = 131$. Find the measure of each angle. (Lesson 3-2)



51. $\angle 6$ **52.** $\angle 7$ **53.** $\angle 4$

54. $\angle 2$ **55.** $\angle 5$ **56.** $\angle 8$

State the transversal that forms each pair of angles. Then identify the special name for each angle pair. (Lesson 3-1)



57. $\angle 1$ and $\angle 14$ **58.** $\angle 2$ and $\angle 10$

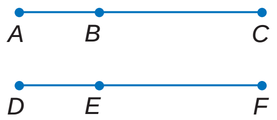
59. $\angle 3$ and $\angle 6$ **60.** $\angle 14$ and $\angle 15$

61. $\angle 7$ and $\angle 12$ **62.** $\angle 9$ and $\angle 11$

63. **PROOF** Write a two-column proof. (Lesson 2-6)

Given: $AC = DF$
$AB = DE$

Prove: $BC = EF$



Find the perimeter of $\triangle ABC$ to the nearest hundredth, given the coordinates of its vertices. (Lesson 1-6)

64. $A(10, -6)$, $B(-2, -8)$, $C(-5, -7)$ **65.** $A(-3, 2)$, $B(2, -9)$, $C(0, -10)$

DAYLIGHT SAVING TIME **All of the states in the United States observe Daylight Saving Time except for Arizona and Hawaii.** (Lesson 2-3)

66. Write a true conditional statement in if-then form for Daylight Saving Time.

67. Write the converse of the true conditional statement. State whether the statement is *true* or *false*. If false, find a counterexample.

Construct a truth table for each compound statement. (Lesson 2-2)

68. p and q

69. p or $\sim q$

70. $\sim p \wedge q$

71. $\sim p \wedge \sim q$

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture. (Lesson 2-1)

72. Points H, I, and J are each located on different sides of a triangle.

73. Collinear points X, Y, and Z; Z is between X and Y.

74. $R(3, -4)$, $S(-2, -4)$, and $T(0, -4)$

GET READY for the Next Lesson

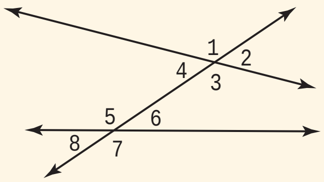
PREREQUISITE SKILL **Solve each equation for y.** (Pages 781 and 782)

75. $2x + y = 7$ **76.** $2x + 4y = 5$ **77.** $5x - 2y + 4 = 0$

CHAPTER 3 Mid-Chapter Quiz

Lessons 3-1 through 3-3

1. **MULTIPLE CHOICE** $\angle 3$ and $\angle 5$ are __?__ angles. (Lesson 3-1)



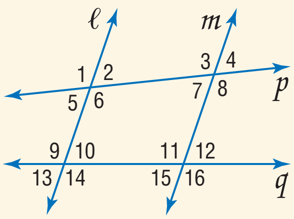
A alternate exterior

B alternate interior

C consecutive interior

D corresponding

Name the transversal that forms each pair of angles. Then identify the special name for the angle pair. (Lesson 3-1)

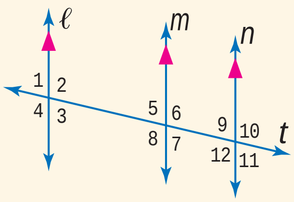


2. $\angle 1$ and $\angle 8$
3. $\angle 6$ and $\angle 10$
4. $\angle 11$ and $\angle 14$

Refer to the figure above. Find the measure of each angle if $\ell \parallel m$ and $m\angle 1 = 105$. (Lesson 3-2)

5. $\angle 6$
6. $\angle 4$

In the figure, $m\angle 9 = 75$. Find the measure of each angle. (Lesson 3-2)



7. $\angle 3$
8. $\angle 5$
9. $\angle 6$
10. $\angle 8$
11. $\angle 11$
12. $\angle 12$

13. **MULTIPLE CHOICE** Find the slope of a line perpendicular to the line containing $(-5, 1)$ and $(-3, -2)$. (Lesson 3-3)

F $-\frac{2}{3}$

G $-\frac{3}{2}$

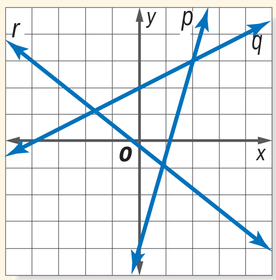
H $\frac{2}{3}$

J $\frac{3}{2}$

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*. (Lesson 3-3)

14. $A(3, -1)$, $B(6, 1)$, $C(-2, -2)$, $D(2, 4)$
15. $A(-3, -11)$, $B(3, 13)$, $C(0, -6)$, $D(8, -8)$

Find the slope of each line. (Lesson 3-3)



16. p
17. a line parallel to q
18. a line perpendicular to r

BASEBALL For Exercises 19 and 20 use the following information.

Minute Maid Ballpark in Houston is home to the Houston Astros. The average attendance per game in 2002 and 2004 are shown in the table. (Lesson 3-3)

Year	Average Attendance
2002	31,078
2004	38,122

19. What is the rate of change in average attendance per game from 2002 to 2004?
20. If this rate of change continues, predict the average attendance per game for the 2012 season.

3-4 Equations of Lines

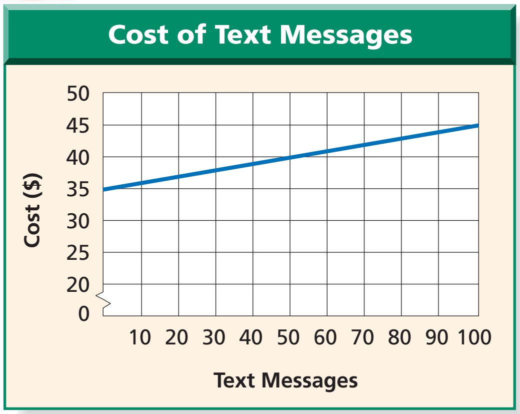
Main Ideas

- Write an equation of a line given information about its graph.
- Solve problems by writing equations.

New Vocabulary

slope-intercept form
point-slope form

Julia's cell phone plan costs \$35 per month for unlimited calls plus \$0.10 per text message. The total charge C for a month can be represented by the equation $C = 0.1t + 35$.



Write Equations of Lines You may remember from algebra that an equation of a line can be written given any of the following:

- the slope and the y-intercept,
- the slope and the coordinates of a point on the line, or
- the coordinates of two points on the line.

The graph of $C = 0.1t + 35$ has a slope of 0.1, and it intersects the y-axis at 35. These two values can be used to write an equation of the line. The **slope-intercept form** of a linear equation is $y = mx + b$, where m is the slope of the line and b is the y-intercept.

slope

$$y = mx + b \qquad C = 0.1t + 35$$

y-intercept

Study Tip

Look Back

To review **writing an equation of a line**, see pages 786–787.

EXAMPLE Slope and y-Intercept

1 **Write an equation in slope-intercept form of the line with slope of -4 and y-intercept of 1.**

$y = mx + b$ Slope-intercept form

$y = -4x + 1$ $m = -4$, $b = 1$

The slope-intercept form of the equation of the line is $y = -4x + 1$.

1. Write an equation in slope-intercept form of the line with slope of 3 and y-intercept of -8.

Extra Examples at geometryonline.com

Another method used to write an equation of a line is the point-slope form of a linear equation. The **point-slope form** is $y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of any point on the line and m is the slope of the line.

$$\overset{\text{given point } (x_1, y_1)}{y - y_1 = \underset{\text{slope}}{m}(x - x_1)}$$

EXAMPLE 2 Slope and a Point

Write an equation in point-slope form of the line with slope of $-\frac{1}{2}$ that contains $(3, -7)$.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-7) = -\frac{1}{2}(x - 3)$ $m = -\frac{1}{2}$, $(x_1, y_1) = (3, -7)$

$y + 7 = -\frac{1}{2}(x - 3)$ Simplify.

The point-slope form of the equation of the line is $y + 7 = -\frac{1}{2}(x - 3)$.

2. Write an equation in point-slope form of the line with slope of 4 that contains $(-3, -6)$.

Both the slope-intercept form and the point-slope form require the slope of a line in order to write an equation. There are occasions when the slope of a line is not given. In cases such as these, use two points on the line to calculate the slope. Then use either the slope-intercept form or the point-slope form to write an equation.

Study Tip

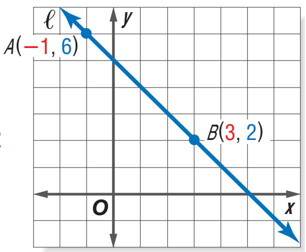
Writing Equations

Note that the point-slope form of an equation is different for each point used. However, the slope-intercept form of an equation is unique.

EXAMPLE 3 Two Points

Write an equation in slope-intercept form for line ℓ.

Find the slope of ℓ by using $A(-1, 6)$ and $B(3, 2)$.



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{2 - 6}{3 - (-1)}$ $x_1 = -1$, $x_2 = 3$, $y_1 = 6$, $y_2 = 2$

$= -\frac{4}{4}$ or -1 Simplify.

Now use the point-slope form and either point to write an equation.

Method 1 Use Point A.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 6 = -1[x - (-1)]$ $m = -1$, $(x_1, y_1) = (-1, 6)$

$y - 6 = -1(x + 1)$ Simplify.

$y - 6 = -x - 1$ Distributive Property

$y = -x + 5$ Add 6 to each side.

Method 2 Use Point B.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 2 = -1(x - 3)$ $m = -1$, $(x_1, y_1) = (3, 2)$

$y - 2 = -x + 3$ Distributive Property

$y = -x + 5$ Add 2 to each side.

The result is the same using either point.

3. Write an equation in slope-intercept form for the line that contains $(-2, 4)$ and $(8, 10)$.

EXAMPLE 4 One Point and an Equation

Write an equation in slope-intercept form for a line containing (2, 0) that is perpendicular to the line with equation $y = -x + 5$.

Since the slope of the line $y = -x + 5$ is -1, the slope of a line perpendicular to it is 1.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 0 = 1(x - 2)$ $m = 1$, $(x_1, y_1) = (2, 0)$

$y = x - 2$ Distributive Property

4. Write an equation in slope-intercept form for a line containing $(-3, 6)$ that is parallel to the graph of $y = -\frac{3}{4}x + 3$.

Write Equations to Solve Problems Many real-world situations can be modeled using linear equations. In many business applications, the slope represents a *rate*.

Write Linear Equations

5 TEXT MESSAGING **Gracia's current wireless phone plan charges $39.95 per month for unlimited calls and $0.05 per text message.**

a. Write an equation to represent the total monthly cost C for t text messages.

For each text message, the cost increases $0.05. So, the rate of change, or slope, is 0.05. The y-intercept is located where 0 messages are used, or $39.95.

$C = mt + b$ Slope-intercept form

$= 0.05t + 39.95$ $m = 0.05$, $b = 39.95$

The total monthly cost can be represented by the equation $C = 0.05t + 39.95$.

(continued on the next page)

b. Compare her current plan to the plan presented at the beginning of the lesson. If she reads or sends an average of 150 text messages each month, which plan offers the better rate?

Evaluate each equation for $t = 150$.

Current plan:	$C = 0.05t + 39.95$	
	$= 0.05(150) + 39.95$	$t = 150$
	$= 47.45$	Simplify.
Alternate plan:	$C = 0.1t + 35$	
	$= 0.1(150) + 35$	$t = 150$
	$= 50$	Simplify.

Given her average usage, Gracia's current plan offers the better rate.

5. Suppose Gracia only sends or receives 50 text messages per month. Compare each plan. Which offers a better rate? Explain.

Personal Tutor at geometryonline.com

Check Your Understanding

Example 1 (p. 165)

Write an equation in slope-intercept form of the line having the given slope and *y*-intercept.

1. $m = \frac{1}{2}$, *y*-intercept: 4
2. $m = 3$, *y*-intercept: -4
3. $m = -\frac{3}{5}$, *y*-intercept at $(0, -2)$

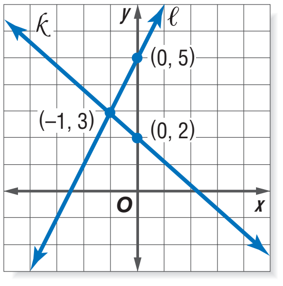
Example 2 (p. 166)

Write an equation in point-slope form of the line having the given slope that contains the given point.

4. $m = \frac{3}{2}$, $(4, -1)$
5. $m = 3$, $(7, 5)$
6. $m = 1.25$, $(20, 137.5)$

Example 3 (pp. 166–167)

Write an equation in slope-intercept form for each line in the graph.



7. ℓ
8. k

Example 4 (p. 167)

9. the line parallel to ℓ that contains (4, 4)
10. the line perpendicular to ℓ that contains $(2, -1)$

Example 5 (pp. 167–168)

MUSIC For Exercises 11 and 12, use the following information.
Justin pays \$5 per month for a subscription to an online music service. He pays \$0.79 per song that he downloads. Another online music store offers 40 downloads per month for a monthly fee of \$10.

11. Write an equation to represent the total monthly cost for each plan.
12. If Justin downloads 15 songs per month, should he keep his current plan, or change to the other plan? Explain.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
13–18	1
19–24	2
25–28	3
29–32	4
33, 34	5

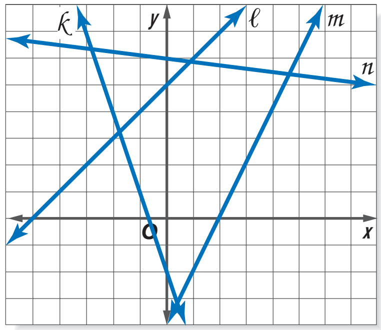
Write an equation in slope-intercept form of the line having the given slope and *y*-intercept.

13. m: 3, y-intercept: -4
14. m: 2, (0, 8)
15. m: $\frac{5}{8}$, (0, -6)
16. m: $\frac{2}{9}$, y-intercept: $\frac{1}{3}$
17. m: -1, b: -3
18. m: $-\frac{1}{12}$, b: 1

Write an equation in point-slope form of the line with the given slope that contains the given point.

19. $m = 2$, (3, 1)
20. $m = -5$, (4, 7)
21. $m = -\frac{4}{5}$, $(-12, -5)$
22. $m = \frac{1}{16}$, (3, 11)
23. $m = 0.48$, (5, 17.12)
24. $m = -1.3$, (10, 87.5)

Write an equation in slope-intercept form for each line in the graph.

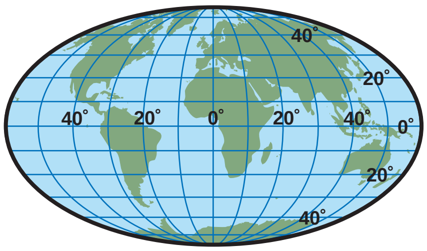


25. k
26. ℓ
27. m
28. n
29. perpendicular to line ℓ, contains $(-1, 6)$
30. parallel to line k, contains (7, 0)
31. parallel to line n, contains (0, 0)
32. perpendicular to line m, contains $(-3, -3)$

BUSINESS For Exercises 33 and 34, use the following information.
The Rainbow Paint Company sells an average of 750 gallons of paint each day.

33. The store has 10,800 gallons of paint in stock. Write an equation in slope-intercept form that describes how many gallons of paint will be on hand after x days if no new stock is added.

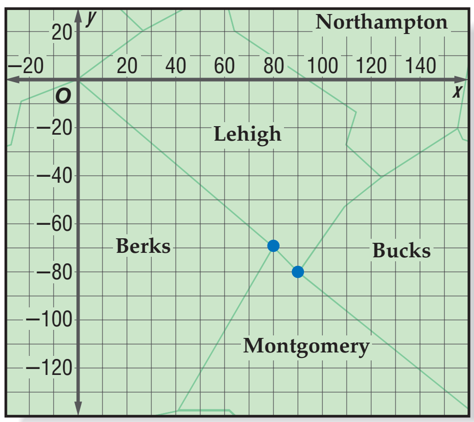
34. Draw a graph that represents the number of gallons of paint on hand at any given time.



Real-World Link
Global coordinates are usually stated as latitude, the angular distance north or south of the equator, and longitude, the angular distance east or west of the prime meridian.

Source: worldatlas.com

MAPS For Exercises 35 and 36, use the following information.
Suppose a map of Pennsylvania is placed on a coordinate plane with the western corner of Lehigh County at the origin. Berks, Montgomery, and Lehigh Counties meet at $(80, -70)$, and Montgomery, Lehigh, and Bucks Counties meet at $(90, -80)$.



35. Write an equation in slope-intercept form that models the county line between Lehigh and Montgomery Counties.

36. The line separating Lehigh and Bucks Counties runs perpendicular to the Lehigh / Montgomery County line. Write an equation in slope-intercept form of the line that contains the Lehigh / Bucks County line.

EXTRA PRACTICE
See pages 806, 830.

Math Online
Self-Check Quiz at
geometryonline.com

Write an equation in slope-intercept form for the line that satisfies the given conditions.

37. x-intercept $= 5$, y-intercept $= 3$

38. contains $(4, -1)$ and $(-2, -1)$

39. contains $(-5, -3)$ and $(10, -6)$

40. x-intercept $= 5$, y-intercept $= -1$

41. contains $(-6, 8)$ and $(-6, -4)$

42. contains $(-4, -1)$ and $(-8, -5)$

H.O.T. Problems

43. OPEN ENDED Write equations in slope-intercept form for two lines that contain $(-1, -5)$.

44. CHALLENGE The point-slope form of an equation of a line can be rewritten as $y = m(x - x_1) + y_1$. Describe how the graph of $y = m(x - x_1) + y_1$ is related to the graph of $y = mx$.

45. Writing in Math Use the information about wireless phone and text messages rates on page 165 to explain how the equation of a line can describe wireless telephone service. Include a description of how you can use equations to compare various plans.

STANDARDIZED TEST PRACTICE

46. REVIEW Jamie is collecting money to buy an \$81 gift for her teacher. She has already contributed \$24. She will collect \$3 from each contributing student. If the equation below shows this relationship, from how many students must Jamie collect?

$$3s + 24 = 81$$

A 3 students **C** 12 students

B 9 students **D** 19 students

47. The graph of which equation passes through $(-3, -2)$ and is perpendicular to the graph of $y = \frac{3}{4}x + 8$?

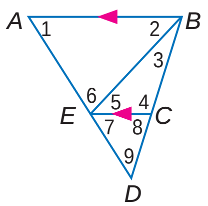
F $y = -\frac{4}{3}x - 6$

G $y = -\frac{4}{3}x + 5$

H $y = \frac{3}{4}x + \frac{1}{4}$

J $y = -\frac{3}{4}x - 5$

48. SOFTWARE In 2000, \$498 million was spent on educational software. In 2004, the sales had dropped to \$152 million. What is the rate of change between 2000 and 2004? (Lesson 3-3)



In the figure, $m\angle 1 = 58$, $m\angle 2 = 47$, and $m\angle 3 = 26$. Find the measure of each angle. (Lesson 3-2)

49. $\angle 7$

50. $\angle 5$

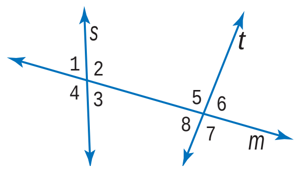
51. $\angle 6$

52. $\angle 4$

53. $\angle 8$

54. $\angle 9$

PREREQUISITE SKILL **Name the pairs of angles in the figure that meet each description.** (Lesson 3-1)



55. consecutive interior angles

56. corresponding angles

57. alternate exterior angles

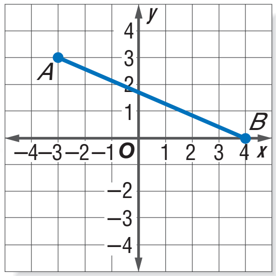
Geometry Lab

Equations of Perpendicular Bisectors

You can apply what you have learned about slope and equations of lines to geometric figures on a plane.

ACTIVITY

Find the equation of a line that is a perpendicular bisector of segment *AB* with endpoints $A(-3, 3)$ and $B(4, 0)$.



Step 1 A segment bisector contains the midpoint of the segment. Use the Midpoint Formula to find the midpoint M of $\overline{AB}$.

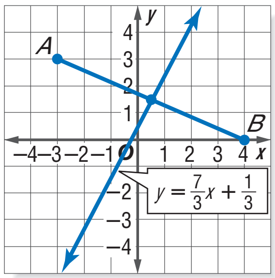
$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M\left(\frac{-3 + 4}{2}, \frac{3 + 0}{2}\right)$$
$$= M\left(\frac{1}{2}, \frac{3}{2}\right)$$

Step 2 A perpendicular bisector is perpendicular to the segment through the midpoint. To find the slope of the bisector, first find the slope of $\overline{AB}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}$$
$$= \frac{0 - 3}{4 - (-3)} \quad x_1 = -3, x_2 = 4, y_1 = 3, y_2 = 0$$
$$= -\frac{3}{7} \quad \text{Simplify.}$$

Step 3 Now use the point-slope form to write the equation of the line. The slope of the bisector is $\frac{7}{3}$ since $-\frac{3}{7}\left(\frac{7}{3}\right) = -1$.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$
$$y - \frac{3}{2} = \frac{7}{3}\left(x - \frac{1}{2}\right) \quad m = \frac{7}{3}, (x_1, y_1) = \left(\frac{1}{2}, \frac{3}{2}\right)$$
$$y - \frac{3}{2} = \frac{7}{3}x - \frac{7}{6} \quad \text{Distributive Property}$$
$$y = \frac{7}{3}x + \frac{1}{3} \quad \text{Add } \frac{3}{2} \text{ to each side.}$$



Exercises

Find the equation of the perpendicular bisector of $\overline{PQ}$ for the given endpoints.

1. $P(5, 2), Q(7, 4)$
2. $P(-3, 9), Q(-1, 5)$
3. $P(-6, -1), Q(8, 7)$
4. $P(-2, 1), Q(0, -3)$
5. $P(0, 1.6), Q(0.5, 2.1)$
6. $P(-7, 3), Q(5, 3)$

7. Extend what you have learned to find the equations of the lines that contain the sides of $\triangle XYZ$ with vertices $X(-2, 0)$, $Y(1, 3)$, and $Z(3, -1)$.

3-5 Proving Lines Parallel

Main Ideas

- Recognize angle conditions that occur with parallel lines.
- Prove that two lines are parallel based on given angle relationships.

Have you ever been in a tall building and looked down at a parking lot? The parking lot is full of line segments that appear to be parallel. The workers who paint these lines must be certain that they are parallel.

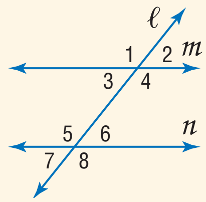
Identify Parallel Lines When each stripe of a parking space intersects the center line, the angles formed are corresponding angles. If the lines are parallel, we know that the corresponding angles are congruent. Conversely, if the corresponding angles are congruent, then the lines must be parallel.

POSTULATE 3.4

If two lines in a plane are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.

Abbreviation: *If corr.* ∠s *are* $\cong$, *then lines are* $\parallel$.

Examples: If $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, or $\angle 4 \cong \angle 8$, then $m \parallel n$.



Postulate 3.4 justifies the construction of parallel lines.

Study Tip

Look Back

To review **copying angles**, see Lesson 1-4.

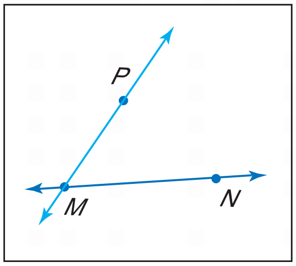
CONSTRUCTION

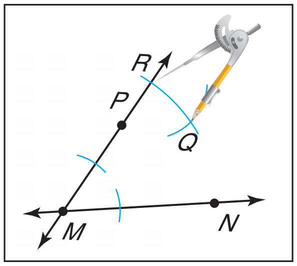
Parallel Line Through a Point Not on Line

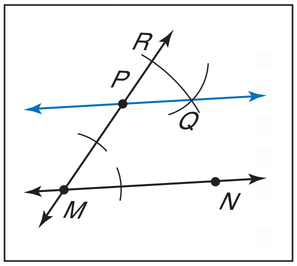
Step 1 Use a straightedge to draw $\overleftrightarrow{MN}$. Draw a point P that is not on $\overleftrightarrow{MN}$. Draw $\overleftrightarrow{PM}$.

Step 2 Copy $\angle PMN$ so that P is the vertex of the new angle. Label the intersection points Q and R.

Step 3 Draw $\overleftrightarrow{PQ}$. Because $\angle RPQ \cong \angle PMN$ by construction and they are corresponding angles, $\overleftrightarrow{PQ} \parallel \overleftrightarrow{MN}$.







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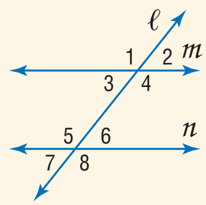
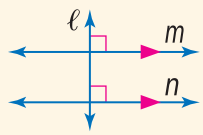
The construction establishes that there is *at least* one line through P that is parallel to $\overleftrightarrow{MN}$. In 1795, Scottish physicist and mathematician John Playfair provided the modern version of Euclid's Parallel Postulate, which states there is *exactly* one line parallel to a line through a given point not on the line.

POSTULATE 3.5 — *Parallel Postulate*

If given a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line.

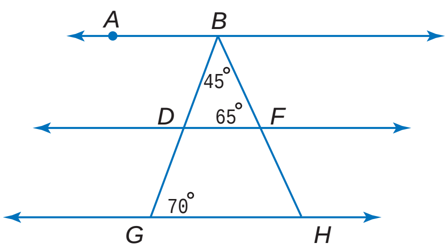
Parallel lines with a transversal create many pairs of congruent angles. Conversely, those pairs of congruent angles can determine whether a pair of lines is parallel.

THEOREM — *Proving Lines Parallel*

	Theorems	Examples	Models
3.5	If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel. **Abbreviation:** *If alt. ext. ∠s are ≅, then lines are ∥.*	If $\angle 1 \cong \angle 8$ or if $\angle 2 \cong \angle 7$, then $m \parallel n$.	
3.6	If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel. **Abbreviation:** *If cons. int. ∠s are suppl., then lines are ∥.*	If $m\angle 3 + m\angle 5$ are supplementary or if $m\angle 4$ and $m\angle 6$ are supplementary, then $m \parallel n$.	
3.7	If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel. **Abbreviation:** *If alt. int. ∠s are ≅, then lines are ∥.*	If $\angle 3 \cong \angle 6$ or if $\angle 4 \cong \angle 5$, then $m \parallel n$.	
3.8	In a plane, if two lines are perpendicular to the same line, then they are parallel. **Abbreviation:** *If 2 lines are ⊥ to the same line, then lines are ∥.*	If $\ell \perp m$ and $\ell \perp n$, then $m \parallel n$.	

You will prove Theorems 3.5, 3.6, 3.7, and 3.8 in Check Your Progress 3 and Exercises 21, 22, and 20, respectively.

EXAMPLE 1 Identify Parallel Lines

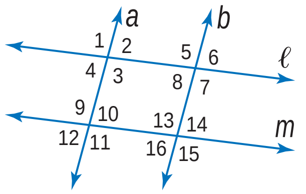


In the figure, $\overline{BG}$ bisects $\angle ABH$. Determine which lines, if any, are parallel.

- The sum of the angle measures in a triangle must be 180, so $m\angle BDF = 180 - (45 + 65)$ or 70.
- Since $\angle BDF$ and $\angle BGH$ have the same measure, they are congruent.
- Congruent corresponding angles indicate parallel lines. So, $\overleftrightarrow{DF} \parallel \overleftrightarrow{GH}$.

(continued on the next page)

- $\angle ABD \cong \angle DBF$, because $\overline{BG}$ bisects $\angle ABH$. So, $m\angle ABD = 45$.
- $\angle ABD$ and $\angle BDF$ are alternate interior angles, but they have different measures so they are not congruent.
- Thus, $\overleftrightarrow{AB}$ is not parallel to $\overleftrightarrow{DF}$ or $\overleftrightarrow{GH}$.

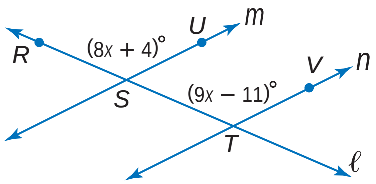


1. Given $\angle 2 \cong \angle 8$, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

Angle relationships can be used to solve problems involving unknown values.

EXAMPLE Solve Problems with Parallel Lines

2 ALGEBRA **Find x and $m\angle RSU$ so that $m \parallel n$.**



Explore From the figure, you know that $m\angle RSU = 8x + 4$ and $m\angle STV = 9x - 11$. You also know that $\angle RSU$ and $\angle STV$ are corresponding angles.

Plan For line m to be parallel to line n, the corresponding angles must be congruent. So, $m\angle RSU = m\angle STV$. Substitute the given angle measures into this equation and solve for x. Once you know the value of x, use substitution to find $m\angle RSU$.

Solve

$m\angle RSU = m\angle STV$	Corresponding angles
$8x + 4 = 9x - 11$	Substitution
$4 = x - 11$	Subtract 8x from each side.
$15 = x$	Add 11 to each side.

Now use the value of x to find $m\angle RSU$.

$m\angle RSU = 8x + 4$	Original equation
$= 8(15) + 4$	$x = 15$
$= 124$	Simplify.

Check Verify the angle measure by using the value of x to find $m\angle STV$. That is, $9x - 11 = 9(15) - 11$ or 124. Since $m\angle RSU = m\angle STV$, $\angle RSU \cong \angle STV$ and $m \parallel n$.

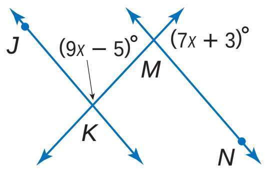
Study Tip

Proving Lines Parallel

When proving lines parallel, be sure to check for congruent corresponding angles, alternate interior angles, alternate exterior angles, or supplementary consecutive interior angles.

2A. Find x so that $\overline{JK} \parallel \overline{MN}$.

2B. Find $m\angle JKM$.



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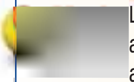
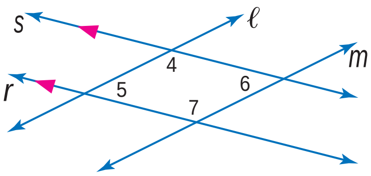
Prove Lines Parallel The angle pair relationships formed by a transversal can be used to prove that two lines are parallel.

Cross-Curricular Project



Latitude lines are parallel, and longitude lines appear parallel in certain locations on Earth. Visit geometryonline.com to continue work on your project.

EXAMPLE 3 Prove Lines Parallel



Given: $r \parallel s$; $\angle 5 \cong \angle 6$

Prove: $\ell \parallel m$

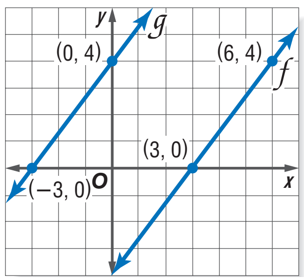
Proof:

Statements	Reasons
1. $r \parallel s$; $\angle 5 \cong \angle 6$	**1.** Given
2. $\angle 4$ and $\angle 5$ are supplementary.	**2.** Consecutive Interior Angle Theorem
3. $m\angle 4 + m\angle 5 = 180$	**3.** Definition of supplementary angles
4. $m\angle 5 = m\angle 6$	**4.** Definition of congruent angles
5. $m\angle 4 + m\angle 6 = 180$	**5.** Substitution
6. $\angle 4$ and $\angle 6$ are supplementary.	**6.** Definition of supplementary angles
7. $\ell \parallel m$	**7.** If cons. int. $\measuredangle$ are suppl., then lines are $\parallel$.

3. PROOF Write a two-column proof of Theorem 3.5.

In Lesson 3-3, you learned that parallel lines have the same slope. You can use the slopes of lines to prove that lines are parallel.

EXAMPLE 4 Slope and Parallel Lines



Determine whether $g \parallel f$.

slope of f: $m = \dfrac{4 - 0}{6 - 3}$ or $\dfrac{4}{3}$

slope of g: $m = \dfrac{4 - 0}{0 - (-3)}$ or $\dfrac{4}{3}$

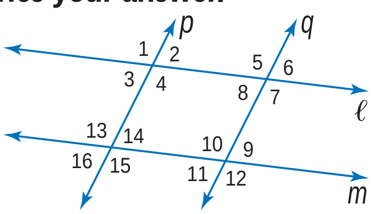
Since the slopes are the same, $g \parallel f$.

4. Line ℓ contains points at $(-5, 3)$ and $(0, 4)$. Line m contains points at $\left(2, -\frac{2}{3}\right)$ and $(12, 1)$. Determine whether $\ell \parallel m$.

CHECK Your Understanding

Example 1 (pp. 173–174)

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



1. $\angle 16 \cong \angle 3$

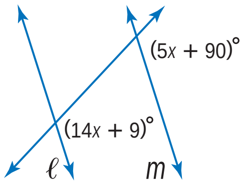
2. $\angle 4 \cong \angle 13$

3. $m\angle 14 + m\angle 10 = 180$

4. $\angle 1 \cong \angle 7$

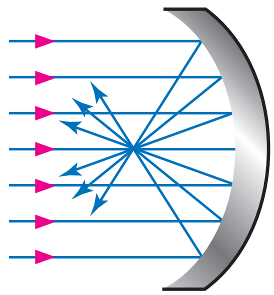
Example 2
(p. 174)

5. Find x so that $\ell \parallel m$.



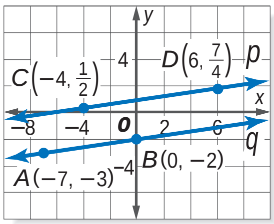
Example 3
(p. 175)

6. **PHYSICS** The Hubble Telescope gathers parallel light rays and directs them to a central focal point. Use a protractor to measure several of the angles shown in the diagram. Are the lines parallel? Explain how you know.



Example 4
(p. 175)

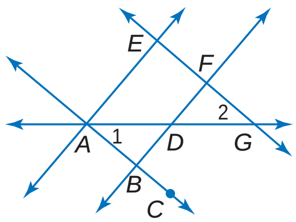
7. Determine whether $p \parallel q$.



Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
8–11	1
12–17	2
18, 19	3
20-25	4

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



8. $\angle AEF \cong \angle BFG$

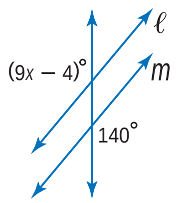
9. $\angle EAB \cong \angle DBC$

10. $\angle EFB \cong \angle CBF$

11. $m\angle GFD + m\angle CBD = 180$

Find x so that $\ell \parallel m$.

12.



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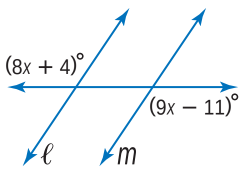

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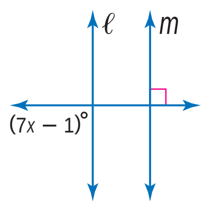

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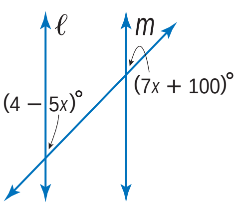

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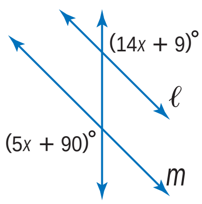
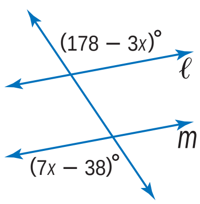
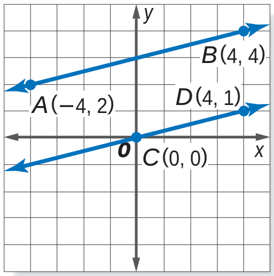


17.

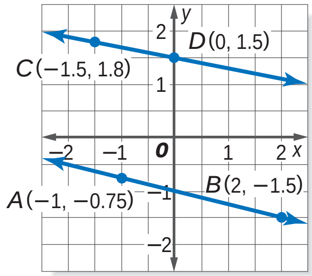


Determine whether each pair of lines is parallel. Explain why or why not.

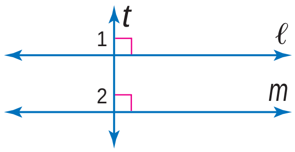
18.



19.



20. PROOF Copy and complete the proof of Theorem 3.8.



Given: $\ell \perp t$
$m \perp t$
Prove: $\ell \parallel m$

Proof:

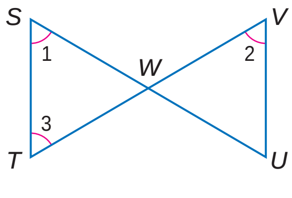
Statements	**Reasons**
1. $\ell \perp t, m \perp t$	**1.** ___?___
2. $\angle 1$ and $\angle 2$ are right angles.	**2.** ___?___
3. $\angle 1 \cong \angle 2$	**3.** ___?___
4. $\ell \parallel m$	**4.** ___?___

21. PROOF Write a two-column proof of Theorem 3.6.

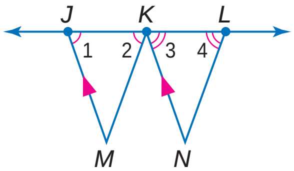
22. PROOF Write a paragraph proof of Theorem 3.7.

PROOF Write a two-column proof for each of the following.

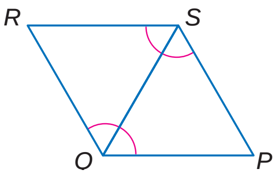
23. Given: $\angle 2 \cong \angle 1$
$\angle 1 \cong \angle 3$
Prove: $\overline{ST} \parallel \overline{UV}$



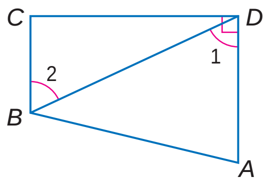
24. Given: $\overline{JM} \parallel \overline{KN}$
$\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$
Prove: $\overline{KM} \parallel \overline{LN}$



25. Given: $\angle RSP \cong \angle PQR$
$\angle QRS$ and $\angle PQR$ are supplementary.
Prove: $\overline{PS} \parallel \overline{QR}$



26. Given: $\overline{AD} \perp \overline{CD}$
$\angle 1 \cong \angle 2$
Prove: $\overline{BC} \perp \overline{CD}$



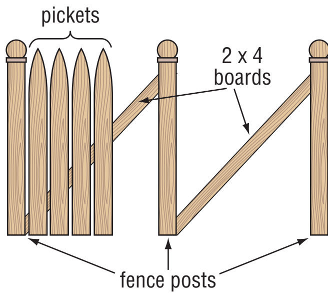
Real-World Link

In 1795, John Playfair published his version of Euclid's *Elements*. In his edition, Playfair standardized the notation used for points and figures and introduced algebraic notation for use in proofs.

Source: mathworld.wolfram.com

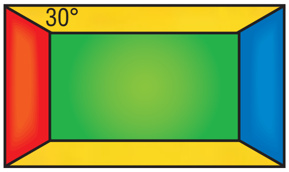
27. RESEARCH Use the Internet or other resource to find mathematicians like John Playfair who discovered new concepts and proved new theorems related to parallel lines. Briefly describe their discoveries. Include any factors that prompted their research, such as a real-world need or research in a different field.

28. HOME IMPROVEMENT To build a fence, Jim positioned the fence posts and then placed a 2×4 board at an angle between the fence posts. As he placed each picket, he made sure the angle that the picket made with the 2×4 was the same as the angle for the rest of the pickets. Why does this ensure that the pickets will be parallel?



29. FOOTBALL When striping the practice football field, Mr. Hawkinson first painted the sidelines. Next he marked off 10-yard increments on one sideline. He then constructed lines perpendicular to the sidelines at each 10-yard mark. Why does this guarantee that the 10-yard lines will be parallel?

30. CRAFTS Juan is making a stained glass piece. He cut the top and bottom pieces at a 30° angle. If the corners are right angles, explain how Juan knows that each pair of opposite sides is parallel.



EXTRA PRACTICE
See pages 806, 830.
Math Online
Self-Check Quiz at geometryonline.com

31. FRAMING Wooden picture frames are often constructed using a miter box or miter saw. These tools allow you to cut at an angle of a given size. If each of the four pieces of framing material is cut at a 45° angle, will the sides of the frame be parallel? Explain your reasoning.

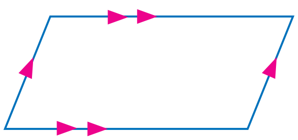
H.O.T. Problems

32. REASONING Summarize five different methods that can be used to prove that two lines are parallel.

33. REASONING Find a counterexample for the following statement.
If lines ℓ and m are cut by transversal t so that consecutive interior angles are congruent, then lines ℓ and m are parallel and t is perpendicular to both lines.

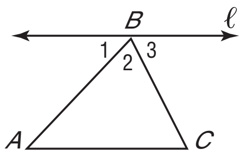
34. OPEN ENDED Describe two situations in your own life in which you encounter parallel lines. How could you verify that the lines are parallel?

35. CHALLENGE When Adeel was working on an art project, he drew a four-sided figure with two pairs of opposite parallel sides. He noticed some patterns relating to the angles in the figure. List as many patterns as you can about a 4-sided figure with two pairs of opposite parallel sides.



36. Writing in Math Use the information about parking lots on page 172 to explain how you know that the sides of a parking space are parallel. Include a comparison of the angles at which the lines forming the edges of a parking space strike the center line, and a description of the type of parking spaces that have sides that form congruent consecutive interior angles.

37. Which of the following facts would be sufficient to prove that line ℓ is parallel to $\overline{AC}$?



A $\angle 1 \cong \angle 3$

B $\angle 3 \cong \angle C$

C $\angle 1 \cong \angle C$

D $\angle 2 \cong \angle A$

38. REVIEW Kendra has at least one quarter, one dime, one nickel, and one penny. If she has three times as many pennies as nickels, the same number of nickels as dimes, and twice as many dimes as quarters, then what is the least amount of money she could have?

F \$0.41

G \$0.48

H \$0.58

J \$0.61

Write an equation in slope-intercept form for the line that satisfies the given conditions. (Lesson 3-4)

39. $m = 0.3$, y-intercept is -6

40. $m = \frac{1}{3}$, contains $(-3, -15)$

41. contains $(5, 7)$ and $(-3, 11)$

42. perpendicular to $y = \frac{1}{2}x - 4$, contains $(4, 1)$

Find the slope of each line. (Lesson 3-3)

43. $\overleftrightarrow{BD}$

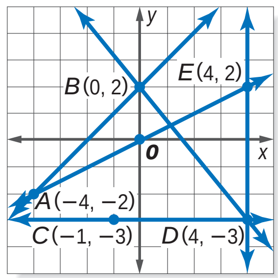
44. $\overleftrightarrow{CD}$

45. $\overleftrightarrow{AB}$

46. $\overleftrightarrow{AE}$

47. any line parallel to $\overleftrightarrow{DE}$

48. any line perpendicular to $\overleftrightarrow{BD}$



49. CARPENTRY A carpenter must cut two pieces of wood at angles so that they fit together to form the corner of a picture frame. What type of angles must he use to make sure that a 90° corner results? (Lesson 1-5)

PREREQUISITE SKILL Use the Distance Formula to find the distance between each pair of points. (Lesson 1-4)

50. $(2, 7), (7, 19)$

51. $(8, 0), (-1, 2)$

52. $(-6, -4), (-8, -2)$

EXPLORE 3-6

Graphing Calculator Lab
Points of Intersection

You can use a TI-83/84 Plus graphing calculator to determine the points of intersection of a transversal and two parallel lines.

EXAMPLE

Parallel lines ℓ and m are cut by a transversal t. The equations of ℓ, m, and t are $y = \frac{1}{2}x - 4$, $y = \frac{1}{2}x + 6$, and $y = -2x + 1$, respectively. Use a graphing calculator to determine the points of intersection of t with ℓ and m.

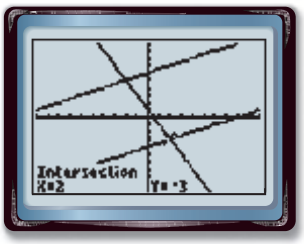
Step 1 Enter the equations in the Y= list and graph in the standard viewing window.

KEYSTROKES: [Y=] .5 [X,T,θ,n] [–] 4 [ENTER] .5 [X,T,θ,n] [+] 6 [ENTER] - 2 [X,T,θ,n] [+] 1 [Zoom] 6

Step 2 Use the CALC menu to find the points of intersection.

- Find the intersection of ℓ and t.

KEYSTROKES: [2nd] [CALC] 5 [ENTER] [▼] [ENTER] [ENTER]

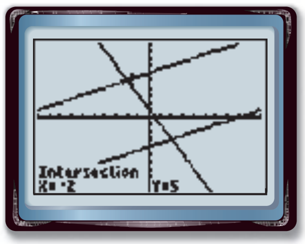


[−10, 10] scl: 1 by [−10, 10] scl: 1

Lines ℓ and t intersect at $(2, -3)$.

- Find the intersection of m and t.

KEYSTROKES: [2nd] [CALC] 5 [▼] [ENTER] [ENTER] [ENTER]



[−10, 10] scl: 1 by [−10, 10] scl: 1

Lines m and t intersect at $(-2, 5)$.

Exercises

Parallel lines a and b are cut by a transversal t. Use a graphing calculator to determine the points of intersection of t with a and b. Round to the nearest tenth.

1. a: $y = 2x - 10$
b: $y = 2x - 2$
t: $y = -\frac{1}{2}x + 4$

2. a: $y = -x - 3$
b: $y = -x + 5$
t: $y = x - 6$

3. a: $y = 6$
b: $y = 0$
t: $x = -2$

4. a: $y = -3x + 1$
b: $y = -3x - 3$
t: $y = \frac{1}{3}x + 8$

5. a: $y = \frac{4}{5}x - 2$
b: $y = \frac{4}{5}x - 7$
t: $y = -\frac{5}{4}x$

6. a: $y = -\frac{1}{6}x + \frac{2}{3}$
b: $y = -\frac{1}{6}x + \frac{5}{12}$
t: $y = 6x + 2$

Online Other Calculator Keystrokes at geometryonline.com

3-6 Perpendiculars and Distance

Main Ideas

- Find the distance between a point and a line.
- Find the distance between parallel lines.

New Vocabulary

equidistant

When installing shelves, it is important that the vertical brackets be parallel for the shelves to line up. One technique is to install the first bracket and then use a carpenter's square to measure and mark two or more points the same distance from the first bracket. You can then align the second bracket with those marks.

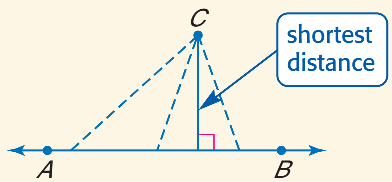
Distance from a Point to a Line In Lesson 3-5, you learned that if two lines are perpendicular to the same line, then they are parallel. The carpenter's square is used to construct a line perpendicular to each bracket. The space between each bracket is measured along the perpendicular segment. This is to ensure that the brackets are parallel. This is an example of using lines and perpendicular segments to determine distance. The shortest segment from a point to a line is the perpendicular segment from the point to the line.

Animation
geometryonline.com

KEY CONCEPT — *Distance Between a Point and a Line*

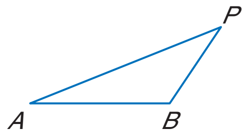
Words The distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point.

Model

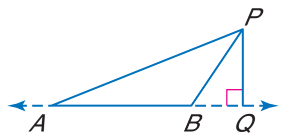


EXAMPLE — Distance from a Point to a Line

1 **Draw the segment that represents the distance from P to $\overleftrightarrow{AB}$.**

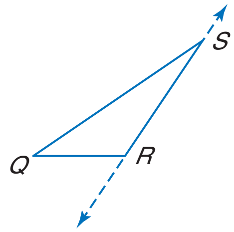


Since the distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point, extend $\overline{AB}$ and draw $\overline{PQ}$ so that $\overline{PQ} \perp \overleftrightarrow{AB}$.



Aaron Haupt

1. Draw the segment that represents the distance from Q to $\overleftrightarrow{RS}$.

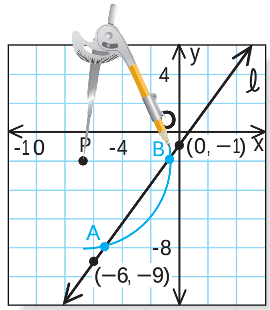


When you draw a perpendicular segment from a point to a line, you can guarantee that it is perpendicular by using the construction of a line perpendicular to a line through a point not on that line.

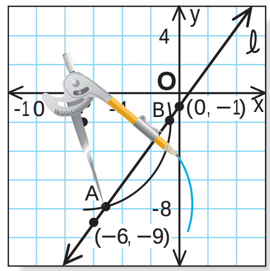
EXAMPLE Construct a Perpendicular Segment

2 COORDINATE GEOMETRY **Line ℓ contains points $(-6, -9)$ and $(0, -1)$. Construct a line perpendicular to line ℓ through $P(-7, -2)$ not on ℓ. Then find the distance from P to ℓ.**

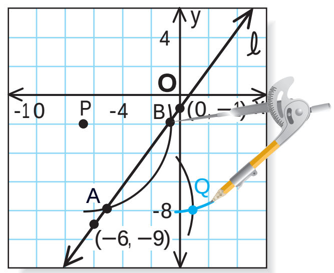
Step 1 Graph line ℓ and point P. Place the compass point at point P. Make the setting wide enough so that when an arc is drawn, it intersects ℓ in two places. Label these points of intersection A and B.



Step 2 Put the compass at point A and draw an arc below line ℓ. (*Hint:* Any compass setting greater than $\frac{1}{2}AB$ will work.)



Step 3 Using the same compass setting, put the compass at point B and draw an arc to intersect the one drawn in step 2. Label the point of intersection Q.

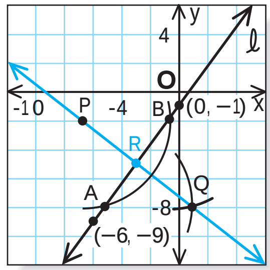


Study Tip

Distance

Note that the distance from a point to the x-axis can be determined by looking at the y-coordinate and the distance from a point to the y-axis can be determined by looking at the x-coordinate.

Step 4 Draw $\overleftrightarrow{PQ}$. $\overleftrightarrow{PQ} \perp \ell$. Label point R at the intersection of $\overleftrightarrow{PQ}$ and ℓ. *Use the slopes of $\overleftrightarrow{PQ}$ and ℓ to verify that the lines are perpendicular.*



The segment constructed from point $P(-7, -2)$ perpendicular to the line ℓ, appears to intersect line ℓ at $R(-3, -5)$. Use the Distance Formula to find the distance between point P and line ℓ.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-7 - (-3))^2 + (-2 - (-5))^2}$$

$$= \sqrt{25} \text{ or } 5$$

The distance between P and ℓ is 5 units.

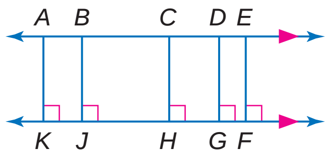
2. Line ℓ contains points (1, 2) and (5, 4). Construct a line perpendicular to ℓ through $P(1, 7)$. Then find the distance from P to ℓ.

Study Tip

Measuring the Shortest Distance

You can use tools like the corner of a piece of paper or your book to help draw a right angle.

Distance Between Parallel Lines Two lines in a plane are parallel if they are everywhere **equidistant**. Equidistant means that the distance between two lines measured along a perpendicular line to the lines is always the same. The distance between parallel lines is the length of the perpendicular segment with endpoints that lie on each of the two lines.

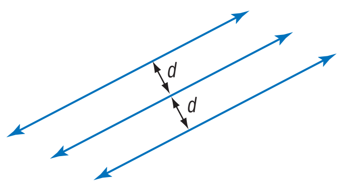


$AK = BJ = CH = DG = EF$

KEY CONCEPT *Distance Between Parallel Lines*

The distance between two parallel lines is the distance between one of the lines and any point on the other line.

Recall from Lesson 1-1 that a *locus* is the set of all points that satisfy a given condition. Parallel lines can be described as the locus of points in a plane equidistant from a given line.



THEOREM 3.9

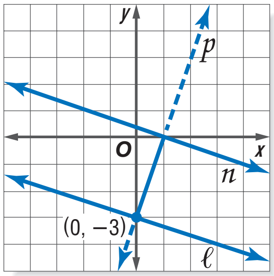
In a plane, if two lines are equidistant from a third line, then the two lines are parallel to each other.

You will prove Theorem 3.9 in Exercise 19.

EXAMPLE Distance Between Lines

3 **Find the distance between the parallel lines ℓ and n with equations $y = -\frac{1}{3}x - 3$ and $y = -\frac{1}{3}x + \frac{1}{3}$, respectively.**

You will need to solve a system of equations to find the endpoints of a segment that is perpendicular to both ℓ and n. The slope of lines ℓ and n is $-\frac{1}{3}$.



- First, write an equation of a line p perpendicular to ℓ and n. The slope of p is the opposite reciprocal of $-\frac{1}{3}$, or 3. Use the y-intercept of line ℓ, (0, −3), as one of the endpoints of the perpendicular segment.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$
$$y - (-3) = 3(x - 0) \quad x_1 = 0, y_1 = -3, m = 3$$
$$y + 3 = 3x \quad \text{Simplify.}$$
$$y = 3x - 3 \quad \text{Subtract 3 from each side.}$$

- Next, use a system of equations to determine the point of intersection of lines n and p.

n: $y = -\frac{1}{3}x + \frac{1}{3}$

p: $y = 3x - 3$

$$-\frac{1}{3}x + \frac{1}{3} = 3x - 3 \quad \text{Substitute } -\tfrac{1}{3}x + \tfrac{1}{3} \text{ for } y \text{ in the second equation.}$$
$$-\frac{1}{3}x - 3x = -3 - \frac{1}{3} \quad \text{Group like terms on each side.}$$
$$-\frac{10}{3}x = -\frac{10}{3} \quad \text{Simplify on each side.}$$
$$x = 1 \quad \text{Divide each side by } -\tfrac{10}{3}.$$

Solve for y.

$$y = 3(1) - 3 \quad \text{Substitute 1 for } x \text{ in the equation for } p.$$
$$= 0 \quad \text{Simplify.}$$

The point of intersection is (1, 0).

- Then, use the Distance Formula to determine the distance between (0, −3) and (1, 0).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(0 - 1)^2 + (-3 - 0)^2} \quad x_2 = 0, x_1 = 1, y_2 = -3, y_1 = 0$$
$$= \sqrt{10} \quad \text{Simplify.}$$

The distance between the lines is $\sqrt{10}$ or about 3.16 units.

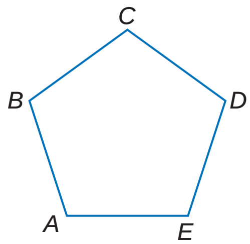
3. Find the distance between parallel lines a and b with equations $x + 3y = 6$ and $x + 3y = -14$, respectively.

Personal Tutor at geometryonline.com

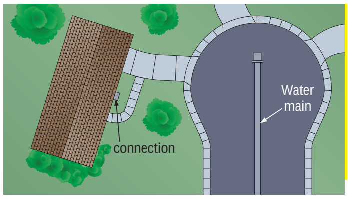
Check Your Understanding

Example 1 (p. 181)

1. Copy the figure. Draw the segment that represents the distance D to $\overleftrightarrow{AE}$.



2. **UTILITIES** Housing developers often locate the shortest distance from a house to the water main so that a minimum of pipe is required to connect the house to the water supply. Copy the diagram, and draw a possible location for the pipe.



Example 2 (pp. 182–183)

3. **COORDINATE GEOMETRY** Line ℓ contains points (0, 0) and (2, 4). Draw line ℓ. Construct a line perpendicular to ℓ through $A(2, -6)$. Then find the distance from A to ℓ.

Example 3 (p. 184)

4. Find the distance between the pair of parallel lines with the given equations.

$y = \frac{3}{4}x - 1$

$y = \frac{3}{4}x + \frac{1}{8}$

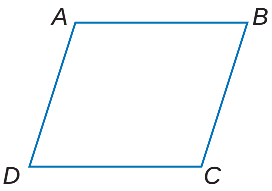
Exercises

HOMEWORK HELP

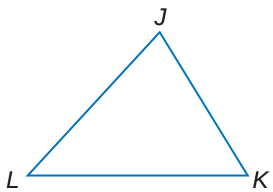
For Exercises	See Examples
5–7	1
8, 9	2
10–18	3

Copy each figure. Draw the segment that represents the distance indicated.

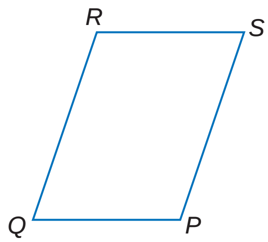
5. C to $\overleftrightarrow{AD}$



6. K to $\overleftrightarrow{JL}$



7. Q to $\overleftrightarrow{RS}$



COORDINATE GEOMETRY **Construct a line perpendicular to ℓ through P. Then find the distance from P to ℓ.**

8. Line ℓ contains points $(-3, 0)$ and $(3, 0)$. Point P has coordinates (4, 3).

9. Line ℓ contains points $(0, -2)$ and $(1, 3)$. Point P has coordinates $(-4, 4)$.

Find the distance between each pair of parallel lines with the given equations.

10. $y = -3$
$y = 1$

11. $x = 4$
$x = -2$

12. $y = 2x + 2$
$y = 2x - 3$

13. $y = \frac{1}{3}x - 3$
$y = \frac{1}{3}x + 2$

14. $x = 8.5$
$x = -12.5$

15. $y = 15$
$y = -4$

Find the distance between each pair of parallel lines with the given equations.

16. $y = 4x$
$y = 4x - 17$

17. $y = 2x - 3$
$2x - y = -4$

18. $y = -\frac{3}{4}x - 1$
$3x + 4y = 20$

19. **PROOF** Write a paragraph proof of Theorem 3.9.

Graph each line. Construct a perpendicular segment through the given point. Then find the distance from the point to the line.

20. $y = 5, (-2, 4)$

21. $y = 2x + 2, (-1, -5)$

22. $2x - 3y = -9, (2, 0)$

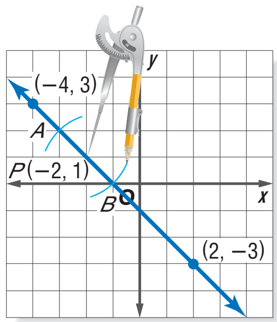
23. **CONSTRUCTION** When framing a wall during a construction project, carpenters often use a plumb line. A *plumb line* is a string with a weight called a *plumb bob* attached on one end. The plumb line is suspended from a point and then used to ensure that wall studs are vertical. How does the plumb line help to find the distance from a point to the floor?

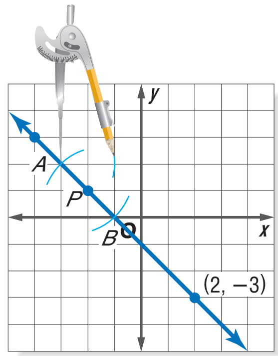
CONSTRUCTIONS **Line ℓ contains points $(-4, 3)$ and $(2, -3)$. Point P at $(-2, 1)$ is on line ℓ. Complete the following construction.**

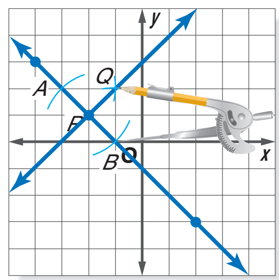
Step 1 Graph line ℓ and point P, and put the compass at point P. Using the same compass setting, draw arcs to the left and right of P. Label these points A and B.

Step 2 Open the compass to a setting greater than AP. Put the compass at point A and draw an arc above line ℓ.

Step 3 Using the same compass setting, put the compass at point B and draw an arc above line ℓ. Label the point of intersection Q. Then draw $\overleftrightarrow{PQ}$.







EXTRA PRACTICE
See pages 806, 830.
Math Online
Self-Check Quiz at
geometryonline.com

24. What is the relationship between line ℓ and $\overleftrightarrow{PQ}$? Verify your conjecture using the slopes of the two lines.

25. Repeat the construction above using a different line and point on that line.

H.O.T. Problems

26. **REASONING** Compare and contrast three different methods that you can use to show that two lines in a plane are parallel.

CHALLENGE **For Exercises 27–32, draw a diagram that represents each description.**

27. Point P is equidistant from two parallel lines.

28. Point P is equidistant from two intersecting lines.

29. Point P is equidistant from two parallel planes.

30. Point P is equidistant from two intersecting planes.

31. A line is equidistant from two parallel planes.

32. A plane is equidistant from two other planes that are parallel.

33. **Writing in Math** Refer to the information about shelving on page 181 to explain how the distance between parallel lines relates to hanging new shelves. Include an explanation of why marking several points equidistant from the first bracket will ensure that the brackets are parallel, and a description of other types of home improvement projects that require that two or more elements are parallel.

34. Segment AB is perpendicular to segment BD. Segment AB and segment CD bisect each other at point X. If $AB = 16$ and $CD = 20$, what is the measure of $\overline{BD}$?

A 6

B 8

C 10

D 18

35. **REVIEW** Pablo bought a sweater on sale for 25% off the original price and another 40% off the discounted price. If the sweater originally cost \$48, what was the final sale price of the sweater?

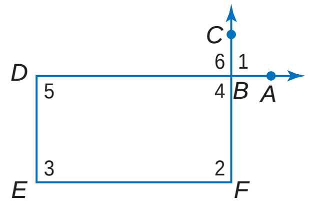
F \$14.40

G \$21.60

H \$31.20

J \$36.00

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer. (Lesson 3-5)

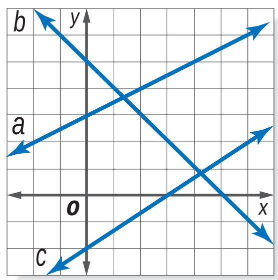


36. $\angle 5 \cong \angle 6$

37. $\angle 6 \cong \angle 2$

38. $\angle 1$ and $\angle 2$ are supplementary.

Write an equation in slope-intercept form for each line. (Lesson 3-4)



39. a **40.** b **41.** c

42. perpendicular to line a, contains $(-1, -4)$

43. parallel to line c, contains $(2, 5)$

44. **COMPUTERS** In 1999, 73% of American teenagers used the Internet. Five years later, this increased to 87%. If the rate of change is constant, estimate when 100% of American teenagers will use the Internet. (Lesson 3-3)

Cross-Curricular Project

Geometry and Earth Science

How's the Weather? It's time to complete your project. Use the information and data you have gathered about climate and locations on Earth to prepare a portfolio or Web page. Be sure to include graphs and/or tables in the presentation.

Cross-Curricular Project at geometryonline.com

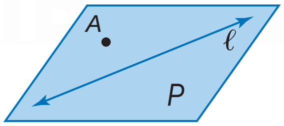
EXTEND 3-6

Geometry Lab

Non-Euclidean Geometry

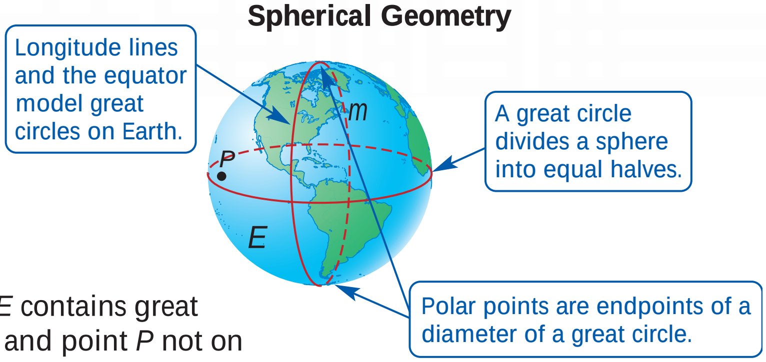
So far in this text, we have studied **plane Euclidean geometry**, which is based on a system of points, lines, and planes. **Spherical geometry** is a system of points, great circles (lines), and spheres (planes). Spherical geometry is one type of **non-Euclidean geometry**. Much of spherical geometry was developed by early Babylonians, Arabs, and Greeks. Their study was based on the astronomy of Earth and their need to be able to measure time accurately.

Plane Euclidean Geometry



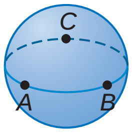
Plane P contains line ℓ and point A not on ℓ.

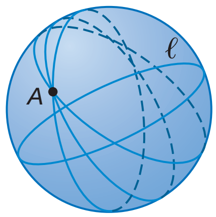
Spherical Geometry



Sphere E contains great circle m and point P not on m. m is a line on sphere E.

The table below compares and contrasts *lines* in the system of plane Euclidean geometry and *lines* (great circles) in spherical geometry.

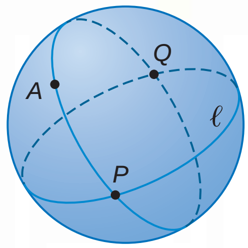
Plane Euclidean Geometry Lines on the Plane	Spherical Geometry Great Circles (Lines) on the Sphere
1. A line segment is the shortest path between two points.	**1.** An arc of a great circle is the shortest path between two points.
2. There is a unique line passing through any two points.	**2.** There is a unique great circle passing through any pair of non-polar points.
3. A line goes on infinitely in two directions.	**3.** A great circle is finite and returns to its original starting point.
4. If three points are collinear, exactly one is between the other two.	**4.** If three points are collinear, any one of the three points is between the other two.
 B is between A and C.	A is between B and C. B is between A and C. C is between A and B. 



In spherical geometry, Euclid's first four postulates and their related theorems hold true. However, theorems that depend on the parallel postulate (Postulate 5) may not be true.

In Euclidean geometry, parallel lines lie in the same plane and never intersect. In spherical geometry, the sphere is the plane, and a great circle represents a line. Every great circle containing A intersects ℓ. Thus, there exists no line through point A that is parallel to ℓ.

Every great circle of a sphere intersects all other great circles on that sphere in exactly two points. In the figure at the right, one possible line through point A intersects line ℓ at P and Q.

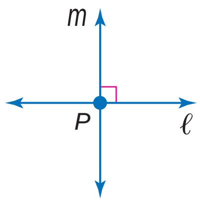


If two great circles divide a sphere into four congruent regions, the lines are perpendicular to each other at their intersection points. Each longitude circle on Earth intersects the equator at right angles.

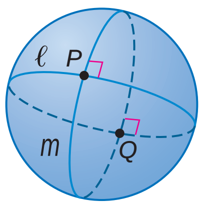
EXAMPLE Compare Plane and Spherical Geometries

For each property listed from plane Euclidean geometry, write a corresponding statement for spherical geometry.

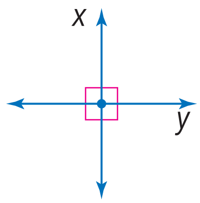
a. Perpendicular lines intersect at one point.



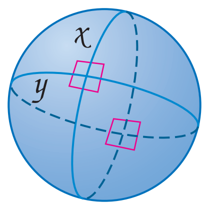
Perpendicular great circles intersect at two points.



b. Perpendicular lines form four right angles.



Perpendicular great circles form eight right angles.



Exercises

For each property from plane Euclidean geometry, write a corresponding statement for spherical geometry.

1. A line goes on infinitely in two directions.

2. A line segment is the shortest path between two points.

3. Two distinct lines with no point of intersection are parallel.

4. Parallel lines have infinitely many common perpendicular lines.

If spherical points are restricted to be nonpolar points, determine if each statement from plane Euclidean geometry is also *true* in spherical geometry. If *false*, explain your reasoning.

5. Any two distinct points determine exactly one line.

6. If three points are collinear, exactly one point is between the other two.

7. Given line ℓ and point P not on ℓ, there exists exactly one line parallel to ℓ passing through P.

READING MATH

Necessary and Sufficient Conditions

We all know that water is a *necessary* condition for fish to survive. However, it is not a *sufficient* condition. For example, fish also need food to survive.

Necessary and sufficient conditions are important in mathematics. Consider the property of having four sides. While *having four sides* is a necessary condition for something being a square, that single condition is not, by itself, a sufficient condition to guarantee that it is a square. Trapezoids are four-sided shapes that are not squares.

Condition	Definition	Examples
necessary	A condition *A* is said to be *necessary* for a condition *B*, if and only if the falsity or nonexistence of *A* guarantees the falsity or nonexistence of *B*.	Having opposite sides parallel is a necessary condition for something being a square.
sufficient	A condition *A* is said to be *sufficient* for a condition *B*, if and only if the truth or existence of *A* guarantees the truth or existence of *B*.	Being a square is a sufficient condition for something being a rectangle.

Reading to Learn

Determine whether each statement is *true* or *false*. If false, give a counterexample.

1. Being a square is a necessary condition for being a rectangle.
2. Being a rectangle is a necessary condition for being a square.
3. Being greater than 15 is a necessary condition for being less than 20.
4. Being less than 12 is a sufficient condition for being less than 20.
5. Walking on two legs is a sufficient condition for being a human being.
6. Breathing air is a necessary condition for being a human being.
7. Being an equilateral rectangle is both a necessary and sufficient condition for being a square.

Determine whether I is a *necessary* condition for II, a *sufficient* condition for II, or *both*. Explain.

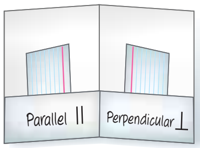
8. **I.** Two points are given.
   II. An equation of a line can be written.
9. **I.** Two planes are parallel.
   II. Two planes do not intersect.
10. **I.** Two angles are congruent.
    II. Two angles are alternate interior angles.

CHAPTER 3 Study Guide and Review

Download Vocabulary Review from geometryonline.com

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Transversals (Lessons 3-1 and 3-2)

- If two parallel lines are cut by a transversal, then each of the following is true.
  - Each pair of alternate interior angles is congruent,
  - each pair of consecutive interior angles is supplementary, and
  - each pair of alternate exterior angles is congruent.

Slope (Lessons 3-3 and 3-4)

- The slope m of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.

Proving Lines Parallel (Lesson 3-5)

- If two lines in a plane are cut by a transversal so that any of the following is true, then the two lines are parallel: a pair of alternate exterior angles is congruent, a pair of consecutive interior angles is supplementary, or a pair of alternate interior angles is congruent.
- In a plane, if two lines are perpendicular to the same line, then they are parallel.

Distance (Lesson 3-6)

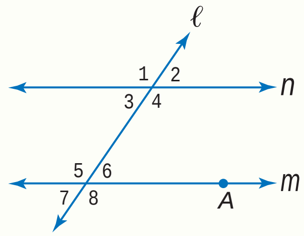
- The distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point.
- The distance between two parallel lines is the distance between one of the lines and any point on the other line.

Key Vocabulary

alternate exterior angles (p. 144)
alternate interior angles (p. 144)
consecutive interior angles (p. 144)
corresponding angles (p. 144)
equidistant (p. 183)
parallel lines (p. 142)
parallel planes (p. 142)
point-slope form (p. 166)
rate of change (p. 157)
slope (p. 156)
slope-intercept form (p. 165)
transversal (p. 143)

Vocabulary Check

Refer to the figure and choose the term that best completes each sentence.



1. Angles 4 and 5 are (consecutive, alternate) interior angles.
2. The distance from point A to line n is the length of the segment (perpendicular, parallel) to line n through A.
3. If $\angle 4$ and $\angle 6$ are supplementary, lines m and n are said to be (parallel, intersecting) lines.
4. Line ℓ is a (slope-intercept, transversal) for lines n and m.
5. $\angle 1$ and $\angle 8$ are (alternate interior, alternate exterior) angles.
6. If $n \parallel m$, $\angle 6$ and $\angle 3$ are (supplementary, congruent).
7. Angles 5 and 3 are (consecutive, alternate) interior angles.
8. If $\angle 2 \cong \angle 7$, then lines n and m are (skew, parallel) lines.

Lesson-by-Lesson Review

3-1 Parallel Lines and Transversals (pp. 142–147)

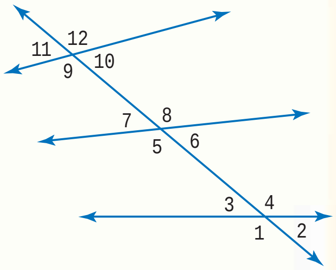
Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.

9. ∠3 and ∠6

10. ∠5 and ∠3

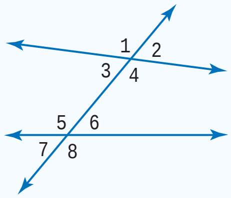
11. ∠2 and ∠7

12. ∠4 and ∠8



13. EAGLES The flight paths of two American bald eagles were tracked at an altitude of 8500 feet in a direction north to south and an altitude of 12,000 feet in a direction west to east, respectively. Describe the types of lines formed by the paths of these two eagles. Explain your reasoning.

Example 1 **Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.**



a. **∠7 and ∠3** corresponding

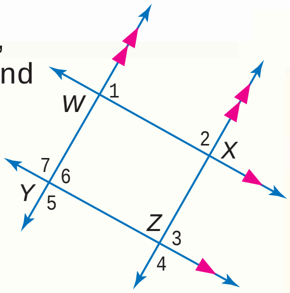
b. **∠4 and ∠6** consecutive interior

c. **∠7 and ∠2** alternate exterior

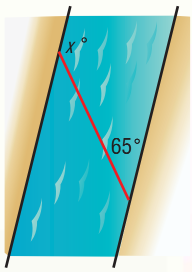
d. **∠3 and ∠6** alternate interior

3-2 Angles and Parallel Lines (pp. 149–154)

14. If $m\angle 1 = 3a + 40$, $m\angle 2 = 2a + 25$, and $m\angle 3 = 5b - 26$, find a and b.

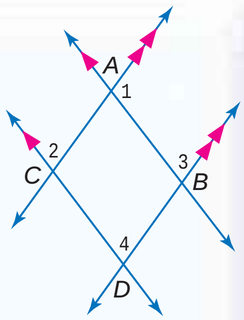


15. BOATING To cross the river safely, Georgia angles her canoe 65° from the river's edge, as shown. At what angle x will she arrive on the other side of the river?



Example 2

If $m\angle 1 = 4p + 15$, and $m\angle 3 = 3p - 10$, find p.



Since $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$, ∠1 and ∠3 are supplementary by the Consecutive Interior Angles Theorem.

$$m\angle 1 + m\angle 3 = 180 \quad \text{Def. of suppl. } \measuredangle s$$
$$(4p + 15) + (3p - 10) = 180 \quad \text{Substitution}$$
$$7p + 5 = 180 \quad \text{Simplify.}$$
$$7p = 175 \quad \text{Subtract.}$$
$$p = 25 \quad \text{Divide.}$$

Mixed Problem Solving
For mixed problem-solving practice, see page 830.

3-3 Slopes of Lines (pp. 156–163)

Graph the line that satisfies each condition.

16. contains (2, 3) and is parallel to $\overleftrightarrow{AB}$ with $A(-1, 2)$ and $B(1, 6)$

17. contains $(-2, -2)$ and is perpendicular to $\overleftrightarrow{PQ}$ with $P(5, 2)$ and $Q(3, -4)$

18. PAINTBALL During a game of paintball, Trevor and Carlos took different paths. If the field can be mapped on the coordinate plane, Trevor ran from $(-5, -3)$ to $(4, 3)$ and Carlos from $(2, -7)$ to $(-6, 5)$. Determine whether their paths are *parallel*, *perpendicular*, or *neither*.

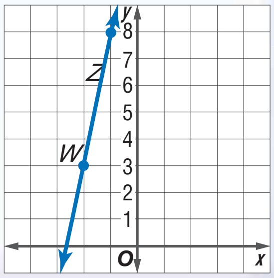
Example 3 **Graph the line that contains $W(-2, 3)$ and is parallel to $\overleftrightarrow{XY}$ with $X(3, -4)$ and $Y(5, 6)$.**

slope of $\overleftrightarrow{XY} = \frac{6-(-4)}{5-3} = \frac{10}{2} = 5$

The slope of the line parallel to $\overleftrightarrow{XY}$ through $W(-2, 3)$ is also 5, since parallel lines have the same slope.

Graph the line. Start at $(-2, 3)$. Move up 5 units and then move right 1 unit. Label the point Z.

Draw $\overleftrightarrow{WZ}$.



3-4 Equations of Lines (pp. 165–170)

Write an equation in the indicated form of the line that satisfies the given conditions.

19. $m = 2$, contains $(1, -5)$; point-slope

20. $m = -\frac{3}{2}$, contains $(2, -4)$; slope-intercept

21. contains $(-3, -7)$ and $(9, 1)$; point-slope

22. contains $(2, 5)$ and $(-2, -1)$; slope-intercept

23. DRIVING A car traveling at 30 meters per second begins to slow down or *decelerate* at a constant rate. After 2 seconds, its velocity is 16 meters per second. Write an equation that represents the car's velocity v after t seconds. Then use this equation to determine how long it will take the car to come to a complete stop.

Example 4 **Write an equation in slope-intercept form of the line that passes through $(2, -4)$ and $(-3, 1)$.**

Find the slope of the line.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope Formula
$= \frac{1-(-4)}{-3-2}$	$(x_1, y_1) = (2, -4)$, $(x_2, y_2) = (-3, 1)$
$= \frac{5}{-5}$ or -1	Simplify.

Now use the point-slope form and either point to write an equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-4) = -1(x - 2)$	$m = -1$, $(x_1, y_1) = (2, -4)$
$y + 4 = -x + 2$	Simplify.
$y = -x - 2$	Subtract 4 from each side.

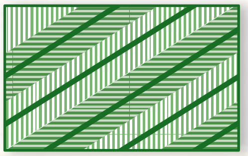
3-5 Proving Lines Parallel (pp. 172–179)

Refer to the figure at the right. Determine which lines, if any, are parallel given the following information. State the postulate or theorem that justifies your answer.

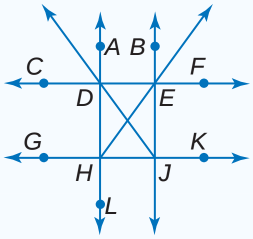
24. $\angle GHL \cong \angle EJK$

25. $m\angle ADJ + m\angle DJE = 180$

26. **OPTICAL ILLUSION** Explain how you could use a protractor to prove that the lines in the optical illusion are parallel.



Example 5 **Given that $\angle GHL \cong \angle ADE$, determine which lines, if any, are parallel.**

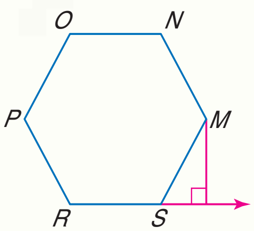


$\angle GHL$ and $\angle ADE$ are alternate exterior angles for $\overleftrightarrow{GK}$ and $\overleftrightarrow{CF}$. Since the angles are congruent, $\overleftrightarrow{GK}$ and $\overleftrightarrow{CF}$ are parallel by the Alternate Exterior Angles Theorem.

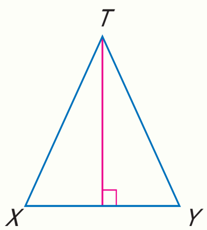
3-6 Perpendiculars and Distance (pp. 181–187)

Copy each figure. Draw the segment that represents the distance indicated.

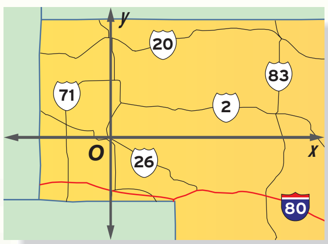
27. M to $\overleftrightarrow{RS}$



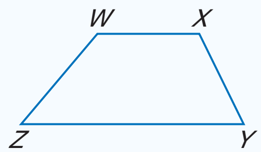
28. T to $\overleftrightarrow{XY}$



29. **NEBRASKA** The northern and southern boundaries of the Nebraska Panhandle can be represented by lines with the equations $y = 90$ and $y = -48$. Find the approximate distance across the panhandle if the units on the map are measured in miles.

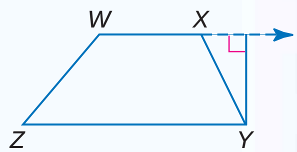


Example 6 **Copy the figure. Draw the segment that represents the distance from Y to $\overleftrightarrow{WX}$.**



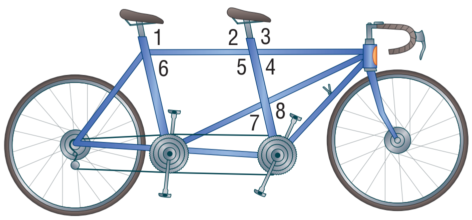
The distance from a line to a point not on the line is the length of the segment perpendicular to the line that passes through the point.

Extend $\overline{WX}$ and draw the segment perpendicular to $\overline{WX}$ from Y.



CHAPTER 3 Practice Test

1. **MULTIPLE CHOICE** The diagram shows the two posts on which seats are placed and several crossbars.



Which term *best* describes ∠6 and ∠5?

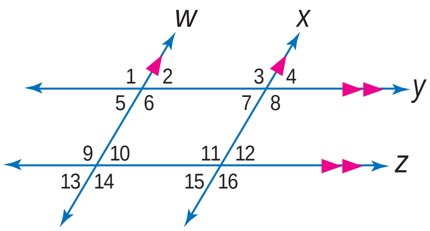
A alternate exterior angles

B alternate interior angles

C consecutive interior angles

D corresponding angles

In the figure, $m\angle 12 = 64$. Find the measure of each angle.



2. ∠8	**3.** ∠13
4. ∠7	**5.** ∠11
6. ∠3	**7.** ∠4
8. ∠9	**9.** ∠5
10. ∠16	**11.** ∠14

Graph the line that satisfies each condition.

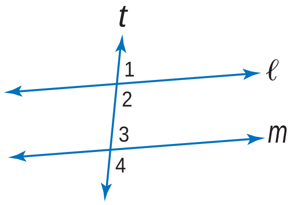
12. slope $= -1$, contains $P(-2, 1)$

13. contains $Q(-1, 3)$ and is perpendicular to $\overleftrightarrow{AB}$ with $A(-2, 0)$ and $B(4, 3)$

14. contains $M(1, -1)$ and is parallel to $\overleftrightarrow{FG}$ with $F(3, 5)$ and $G(-3, -1)$

15. slope $-\frac{4}{3}$, contains $K(3, -2)$

16. **MULTIPLE CHOICE** In the figure below, which can *not* be true if $m \parallel \ell$ and $m\angle 1 = 73$?



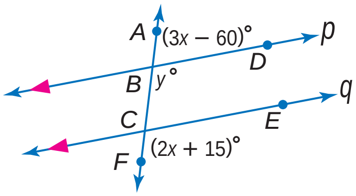
F $m\angle 4 > 73$

G $\angle 1 \cong \angle 4$

H $m\angle 2 + m\angle 3 = 180$

J $\angle 3 \cong \angle 1$

For Exercises 17–22, refer to the figure below. Find each value if $p \parallel q$.



17. x	**18.** y
19. $m\angle FCE$	**20.** $m\angle ABD$
21. $m\angle BCE$	**22.** $m\angle CBD$

Find the distance between each pair of parallel lines.

23. $y = 2x - 1$, $y = 2x + 9$

24. $y = -x + 4$, $y = -x - 2$

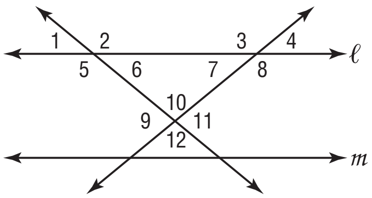
25. **COORDINATE GEOMETRY** Detroit Road starts in the center of the city, and Lorain Road starts 4 miles west of the center of the city. Both roads run southeast. If these roads are put on a coordinate plane with the center of the city at (0, 0), Lorain Road is represented by the equation $y = -x - 4$ and Detroit Road is represented by the equation $y = -x$. How far away is Lorain Road from Detroit Road?

Standardized Test Practice

Cumulative, Chapters 1–3

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. In the figure below, $\angle 3 \cong \angle 8$.



Which of the following conclusions does *not* have to be true?

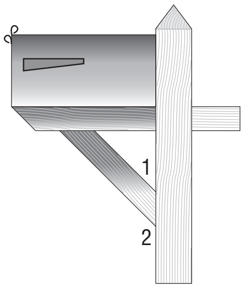
A $\angle 4 \cong \angle 8$

B $\angle 4$ and $\angle 7$ are supplementary angles.

C Line ℓ is parallel to line m.

D $\angle 5$ and $\angle 6$ are supplementary angles.

2. In the diagram below of a mailbox post, which term describes $\angle 1$ and $\angle 2$?



F alternate exterior angles

G alternate interior angles

H consecutive interior angles

J corresponding angles

3. ALGEBRA Which problem situation can *not* be described by a linear function?

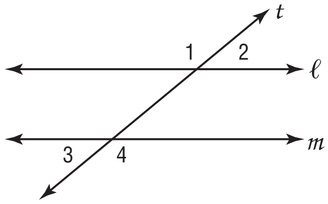
A The distance traveled at an average speed of 70 miles per hour for h hours.

B The area of an isosceles right triangle given the length of one leg.

C The amount of sales tax on a purchase if the rate is 6.5%.

D The gross weekly salary earned at an hourly rate of $5.85 for t hours.

4. In the accompanying diagram, parallel lines ℓ and m are cut by transversal t.



Which statement about angles 1 and 4 *must* be true?

F $\angle 1 \cong \angle 4$

G $\angle 1$ is the complement of $\angle 4$.

H $\angle 1$ is the supplement of $\angle 4$.

J $\angle 1$ and $\angle 4$ are acute angles.

5. What statement is needed in Step 2 to complete this proof?

Given: $\frac{4x - 6}{3} = 10$

Prove: $x = 9$

Statements	Reasons
1. $\frac{4x - 6}{3} = 10$	**1.** Given
2. ___?___	**2.** Multiplication Prop.
3. $4x - 6 = 30$	**3.** Simplify.
4. $4x = 36$	**4.** Addition Prop.
5. $x = 9$	**5.** Division Prop.

A $3\left(\frac{4x - 6}{3}\right) = 10$

B $\frac{4x - 6}{3} = 3(10)$

C $3\left(\frac{4x - 6}{3}\right) = 3(10)$

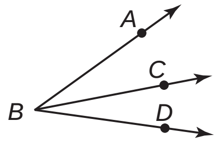
D $4x - 6 = 30$

6. GRIDDABLE Point E is the midpoint of $\overline{DF}$. If $DE = 8x - 3$ and $EF = 3x + 7$, what is x?

Preparing for Standardized Test Practice

For test-taking strategies and more practice, see pages 841-856.

7. If $\angle ABC \cong \angle CBD$, which statement *must* be true?



F $\overline{BC}$ bisects $\angle ABD$.

G $\angle ABD$ is a right angle.

H $\angle ABC$ and $\angle CBD$ are supplementary.

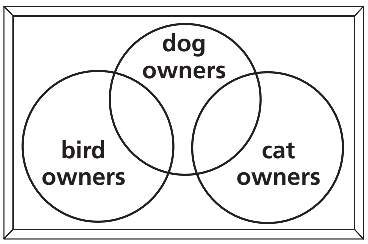
J $\overline{AB}$ and $\overline{BD}$ are perpendicular.

8. ALGEBRA Which expression is equivalent to $4y^3 8y^{-5}$?

A $32y^8$ **C** $32y^{-8}$

B $32y^{-2}$ **D** $32y^{-15}$

9. Based strictlly on this diagram, which is a valid conclusion?



F No dog owners also own cats.

G No bird owners also own dogs.

H No cat owners also own birds.

J No pet owners own more than one pet.

TEST-TAKING TIP

Question 9 Remember that overlapping regions in a Venn diagram represent common or shared elements between sets.

10. Which of the following describes the line containing the points (2, 4) and (0, –2)?

A $y = \frac{1}{3}x - 4$ **C** $y = \frac{1}{3}x - 2$

B $y = -3x + 2$ **D** $y = 3x - 2$

11. Which property could justify the first step in solving $3 \times \frac{14x + 6}{8} = 18$?

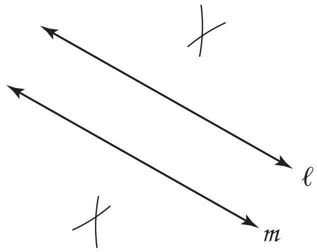
F Addition Property of Equality

G Division Property of Equality

H Substitution Property of Equality

J Transitive Property of Equality

12. If line ℓ is parallel to line m, which best describes the construction below?



A a line perpendicular to lines ℓ and m

B a line parallel to lines ℓ and m

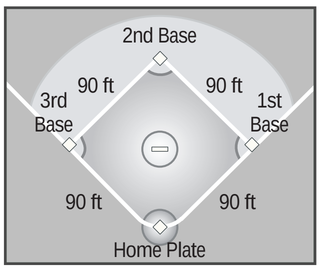
C a line intersecting line ℓ

D a line congruent to line m

Pre-AP

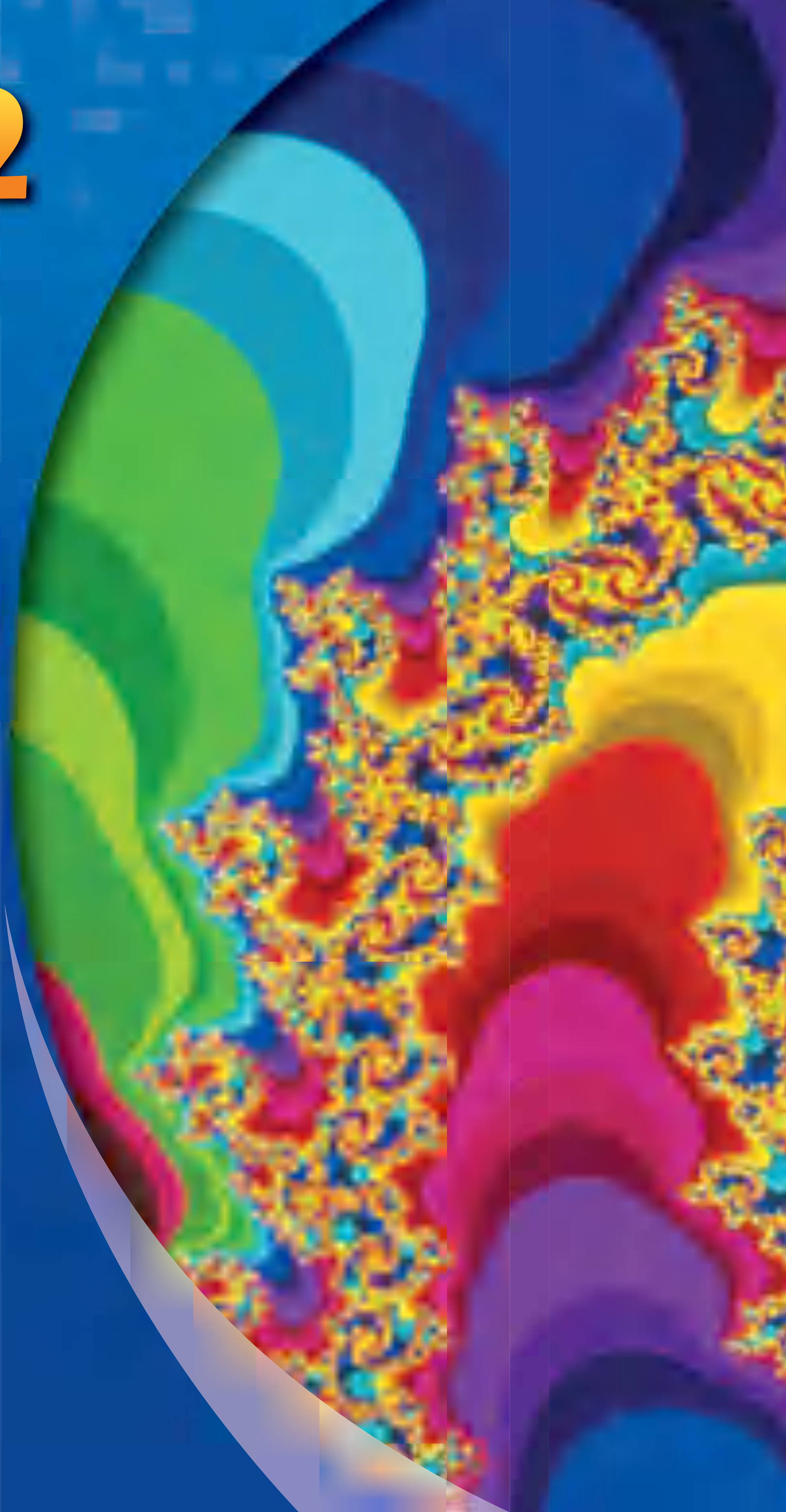
Record your answer on a sheet of paper. Show your work.

13. To get a player out who was running from third base to home, Kahlil threw the ball a distance of 120 feet, from second base toward home plate. Did the ball reach home plate? If not, how far from the plate did it land? Explain and show your calculations to justify your answer.



NEED EXTRA HELP?													
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson or Page...	3-2	3-1	786	3-2	2-4	1-5	3-5	794	2-2	3-4	1-3	2-1	1-3

UNIT 2
Congruence



Focus

Use a variety of representations, tools, and technology to solve meaningful problems by representing and transforming figures and analyzing relationships.

CHAPTER 4
Congruent Triangles

BIG Idea Analyze geometric relationships in order to make and verify conjectures involving triangles.

BIG Idea Apply the concept of congruence to justify properties of figures and solve problems.

CHAPTER 5
Relationships in Triangles

BIG Idea Use a variety of representations to describe geometric relationships and solve problems involving triangles.

CHAPTER 6
Quadrilaterals

BIG Idea Analyze properties and describe relationships in quadrilaterals.

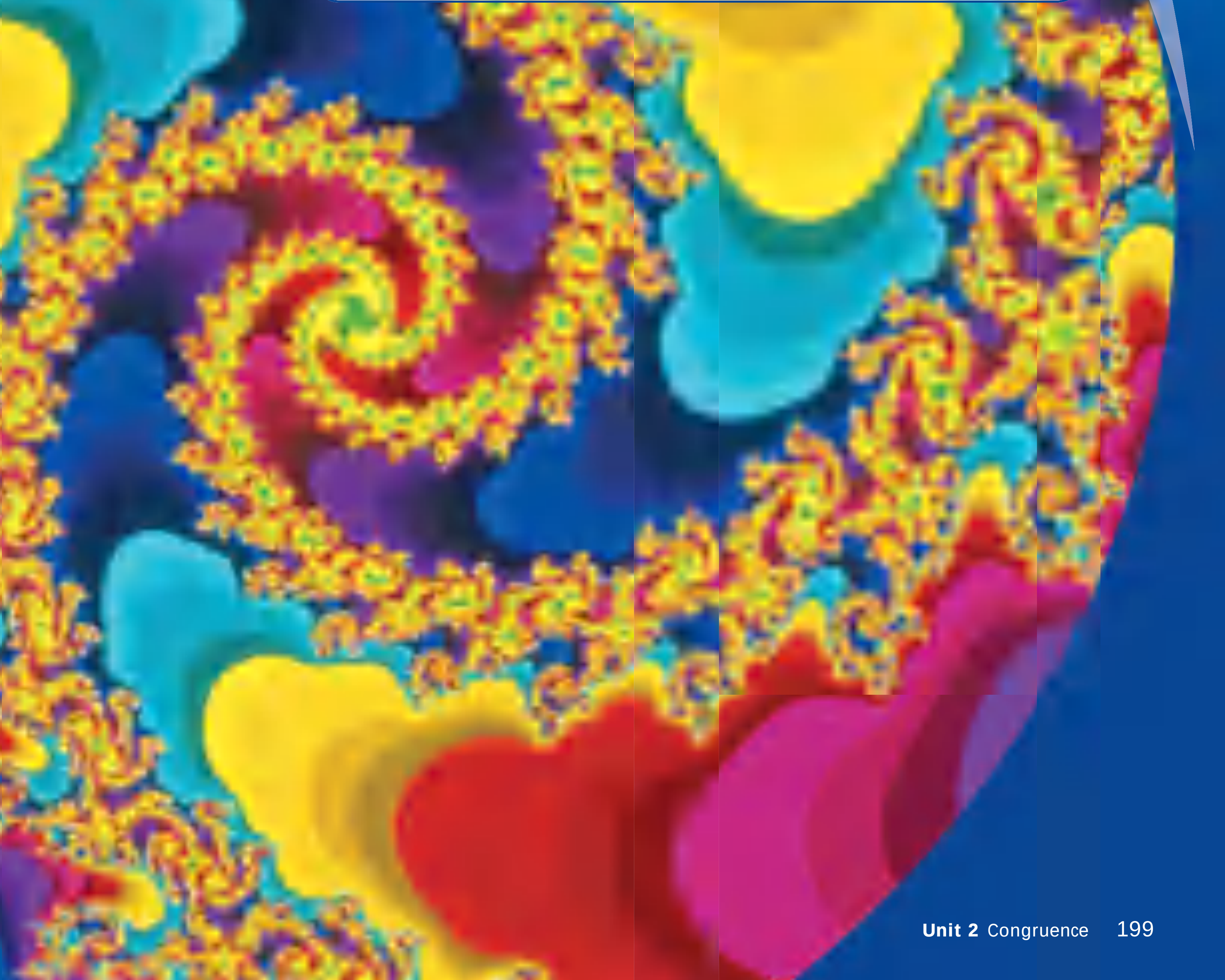
BIG Idea Apply logical reasoning to justify and prove mathematical statements involving quadrilaterals.

Gregory Sams/Photo Researchers

Geometry and History

Who is behind this geometry idea anyway? Have you ever wondered who first developed some of the ideas you are learning in your geometry class? Many ideas we study were developed many years ago, but people today are also discovering new mathematics. Mathematicians continue to study fractals that were pioneered by Benoit Mandelbrot and Gaston Julia. In this project, you will be using the Internet to research a topic in geometry. You will then prepare a portfolio or poster to display your findings.

Math Online **Log on to geometryonline.com to begin.**



CHAPTER 4

Congruent Triangles

- Classify triangles.
- Apply the Angle Sum Theorem and the Exterior Angle Theorem.
- Identify corresponding parts of congruent triangles.
- Test for triangle congruence using SSS, SAS, ASA, and AAS.
- Use properties of isosceles and equilateral triangles.
- Write coordinate proofs.

Key Vocabulary

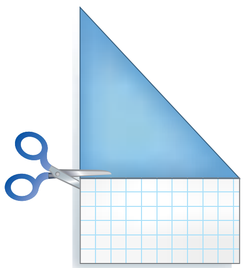
exterior angle (p. 211)
flow proof (p. 212)
corollary (p. 213)
congruent triangles (p. 217)
coordinate proof (p. 251)

Real-World Link

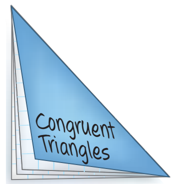
Triangles Triangles with the same size and shape can be modeled by a pair of butterfly wings.

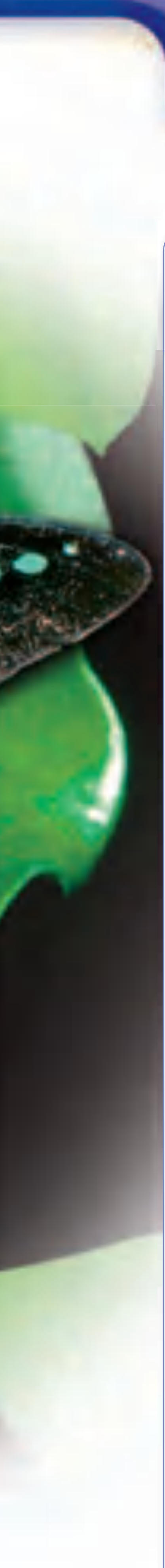
Congruent Triangles Make this Foldable to help you organize your notes. Begin with two sheets of grid paper and one sheet of construction paper.

1. **Stack** the grid paper on the construction paper. Fold diagonally to form a triangle and cut off the excess.



2. **Staple** the edge to form a booklet. Write the chapter title on the front and label each page with a lesson number and title.





GET READY for Chapter 4

Diagnose Readiness **You have two options for checking Prerequisite Skills.**

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Solve each equation. (Prerequisite Skill)

1. $2x + 18 = 5$

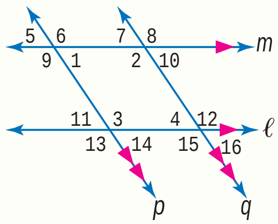
2. $3m - 16 = 12$

3. $6 = 2a + \frac{1}{2}$

4. $\frac{2}{3}b + 9 = -15$

5. **FISH** Miranda bought 4 goldfish and \$5 worth of accessories. She spent a total of \$6 at the store. Write and solve an equation to find the amount for each goldfish. (Prerequisite Skill)

Name the indicated angles or pairs of angles if $p \parallel q$ and $m \parallel \ell$. (Lesson 3-1)



6. angles congruent to $\angle 8$

7. angles supplementary to $\angle 12$

Find the distance between each pair of points. Round to the nearest tenth. (Lesson 1-3)

8. $(6, 8), (-4, 3)$

9. $(11, -8), (-3, -4)$

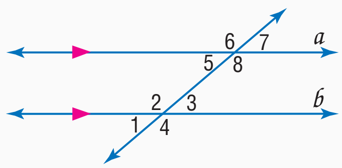
10. **MAPS** Jack laid a coordinate grid on a map where each block on the grid corresponds to a city block. If the coordinates of the football stadium are $(15, -25)$ and the coordinates of Jack's house are $(-8, 14)$, what is the distance between the stadium and Jack's house? Round to the nearest tenth. (Lesson 1-3)

QUICKReview

EXAMPLE 1 **Solve $\frac{7}{8}t + 4 = 18$.**

$\frac{7}{8}t + 4 = 18$	Write the equation.
$\frac{7}{8}t = 14$	Subtract.
$8\left(\frac{7}{8}t\right) = 14(8)$	Multiply.
$7t = 112$	Simplify.
$t = 16$	Divide each side by 7.

EXAMPLE 2 **Name the angles congruent to $\angle 6$ if $a \parallel b$.**



$\angle 8 \cong \angle 6$	Vertical Angle Theorem
$\angle 2 \cong \angle 6$	Corresponding Angles Postulate
$\angle 4 \cong \angle 6$	Alternate Exterior Angles Theorem

EXAMPLE 3 **Find the distance between $(-1, 2)$ and $(3, -4)$. Round to the nearest tenth.**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(3 - (-1))^2 + (-4 - 2)^2}$	$(x_1, y_1) = (-1, 2)$, $(x_2, y_2) = (3, -4)$
$= \sqrt{(4)^2 + (-6)^2}$	Subtract.
$= \sqrt{16 + 36}$	Simplify.
$= \sqrt{52}$	Add.
≈ 7.2	Use a calculator.

4-1 Classifying Triangles

Main Ideas

- Identify and classify triangles by angles.
- Identify and classify triangles by sides.

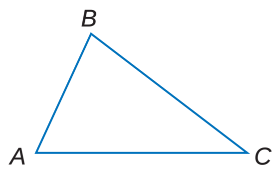
New Vocabulary

acute triangle
obtuse triangle
right triangle
equiangular triangle
scalene triangle
isosceles triangle
equilateral triangle

Many structures use triangular shapes as braces for construction. The roof sections of houses are made of triangular trusses that support the roof and the house.

Classify Triangles by Angles Triangle *ABC*, written $\triangle ABC$, has parts that are named using the letters *A*, *B*, and *C*.

- The sides of $\triangle ABC$ are $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
- The vertices are *A*, *B*, and *C*.
- The angles are $\angle ABC$ or $\angle B$, $\angle BCA$ or $\angle C$, and $\angle BAC$ or $\angle A$.



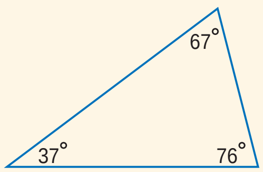
There are two ways to classify triangles. One way is by their angles. All triangles have at least two acute angles, but the third angle is used to classify the triangle.

Study Tip

Common Misconceptions
It is a common mistake to classify triangles by their angles in more than one way. These classifications are distinct groups. For example, a triangle cannot be right and acute.

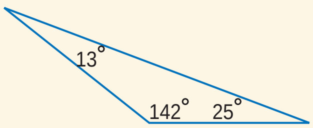
KEY CONCEPT — *Classifying Triangles by Angle*

In an **acute triangle**, all of the angles are acute.



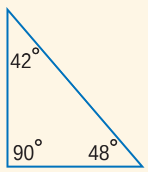
all angle measures < 90

In an **obtuse triangle**, one angle is obtuse.



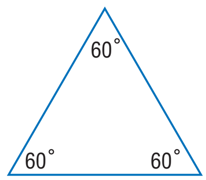
one angle measure > 90

In a **right triangle**, one angle is right.



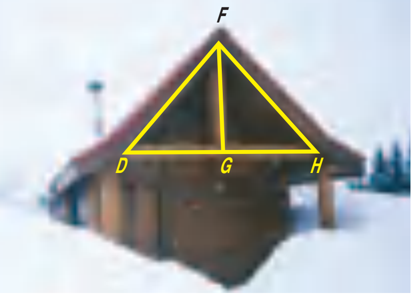
one angle measure = 90

An acute triangle with all angles congruent is an **equiangular triangle**.



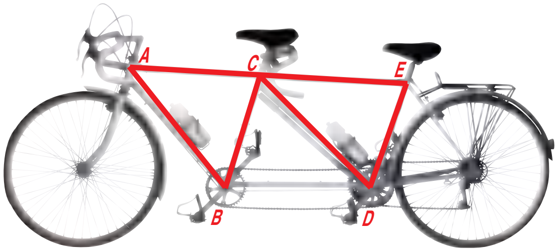
Classify Triangles by Angles

1 **ARCHITECTURE The roof of this house is made up of three different triangles. Use a protractor to classify △*DFH*, △*DFG*, and △*HFG* as *acute*, *equiangular*, *obtuse*, or *right*.**



△*DFH* has all angles with measures less than 90, so it is an acute triangle. △*DFG* and △*HFG* both have one angle with measure equal to 90. Both of these are right triangles.

1. **BICYCLES** The frame of this tandem bicycle uses triangles. Use a protractor to classify △*ABC* and △*CDE*.



Study Tip

Congruency

To indicate that sides of a triangle are congruent, an equal number of hash marks are drawn on the corresponding sides.

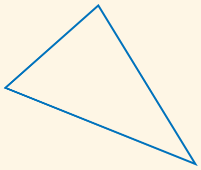
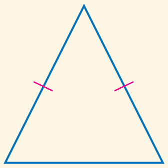
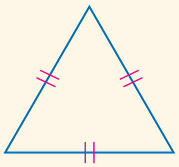
Classify Triangles by Sides Triangles can also be classified according to the number of congruent sides they have.

Study Tip

Equilateral Triangles

An equilateral triangle is a special kind of isosceles triangle.

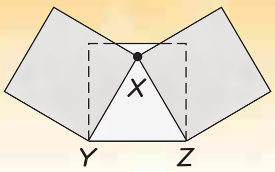
KEY CONCEPT — *Classifying Triangles by Sides*

No two sides of a **scalene triangle** are congruent.	At least two sides of an **isosceles triangle** are congruent.	All of the sides of an **equilateral triangle** are congruent.
		

GEOMETRY LAB

Equilateral Triangles

MODEL



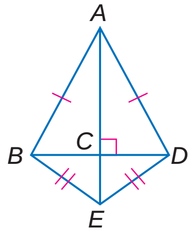
- Align three pieces of patty paper. Draw a dot at *X*.
- Fold the patty paper through *X* and *Y* and through *X* and *Z*.

ANALYZE

1. Is △*XYZ* equilateral? Explain.
2. Use three pieces of patty paper to make a triangle that is isosceles, but not equilateral.
3. Use three pieces of patty paper to make a scalene triangle.

EXAMPLE Classify Triangles by Sides

2 **Identify the indicated type of triangle in the figure.**



a. isosceles triangles

Isosceles triangles have at least two sides congruent. So, $\triangle ABD$ and $\triangle EBD$ are isosceles.

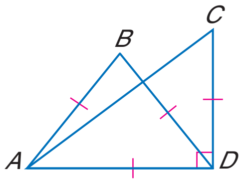
b. scalene triangles

Scalene triangles have no congruent sides. $\triangle AEB$, $\triangle AED$, $\triangle ACB$, $\triangle ACD$, $\triangle BCE$, and $\triangle DCE$ are scalene.

2 **Identify the indicated type of triangle in the figure.**

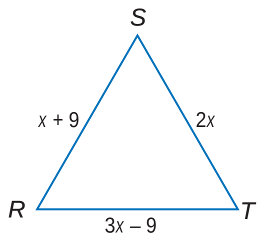
2A. equilateral

2B. isosceles



EXAMPLE Find Missing Values

3 **ALGEBRA Find *x* and the measure of each side of equilateral triangle *RST*.**



Since $\triangle RST$ is equilateral, $RS = ST$.

$x + 9 = 2x$ Substitution

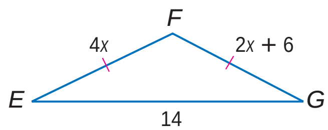
$9 = x$ Subtract x from each side.

Next, substitute to find the length of each side.

$RS = x + 9$ | $ST = 2x$ | $RT = 3x - 9$
$= 9 + 9$ or 18 | $= 2(9)$ or 18 | $= 3(9) - 9$ or 18

For $\triangle RST$, $x = 9$, and the measure of each side is 18.

3. Find x and the measure of the unknown sides of isosceles triangle *EFG*.



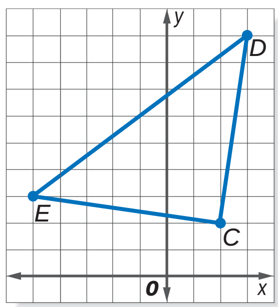
Study Tip

Look Back

To review the **Distance Formula**, see Lesson 1-3.

EXAMPLE Use the Distance Formula

4 **COORDINATE GEOMETRY Find the measures of the sides of $\triangle DEC$. Classify the triangle by sides.**



Use the Distance Formula to find the lengths of each side.

$$EC = \sqrt{(-5 - 2)^2 + (3 - 2)^2}$$
$$= \sqrt{49 + 1}$$
$$= \sqrt{50} \text{ or } 5\sqrt{2}$$

$$DC = \sqrt{(3 - 2)^2 + (9 - 2)^2} \qquad ED = \sqrt{(-5 - 3)^2 + (3 - 9)^2}$$
$$= \sqrt{1 + 49} \qquad = \sqrt{64 + 36}$$
$$= \sqrt{50} \text{ or } 5\sqrt{2} \qquad = \sqrt{100} \text{ or } 10$$

Since $\overline{EC}$ and $\overline{DC}$ have the same length, $\triangle DEC$ is isosceles.

4. Find the measures of the sides of $\triangle HIJ$ with vertices $H(-3, 1)$, $I(0, 4)$, and $J(0, 1)$. Classify the triangle by sides.

Personal Tutor at geometryonline.com

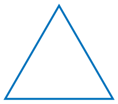
CHECK Your Understanding

Example 1 (p. 203)

Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

1.

2.

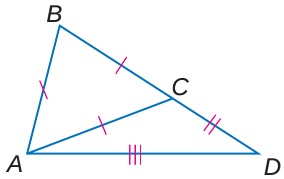


Example 2 (p. 204)

Identify the indicated type of triangle in the figure.

3. isosceles

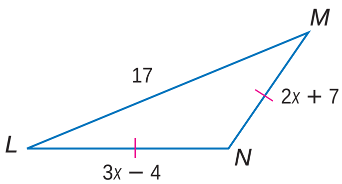
4. scalene



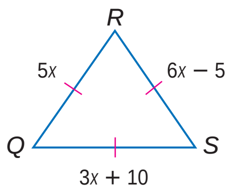
Example 3 (p. 204)

ALGEBRA Find *x* and the measures of the unknown sides of each triangle.

5.



6.



Example 4 (p. 204)

7. COORDINATE GEOMETRY Find the measures of the sides of $\triangle TWZ$ with vertices at $T(2, 6)$, $W(4, -5)$, and $Z(-3, 0)$. Classify the triangle by sides.

8. COORDINATE GEOMETRY Find the measures of the sides of $\triangle QRS$ with vertices at $Q(2, 1)$, $R(4, -3)$, and $S(-3, -2)$. Classify the triangle by sides.

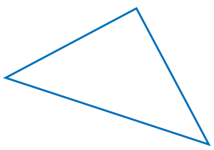
Exercises

HOMEWORK HELP

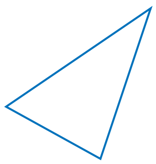
For Exercises	See Examples
9–12	1
13–14	2
15, 16	3
17–20	4

Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

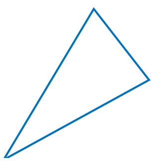
9.



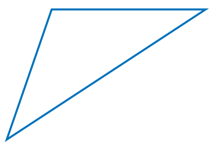
10.



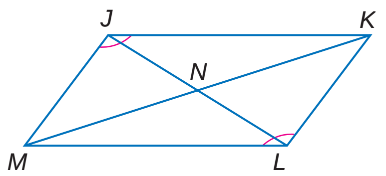
11.



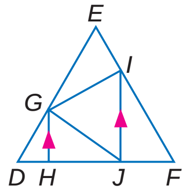
12.



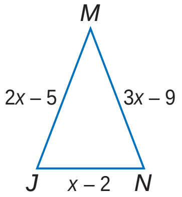
13. Identify the obtuse triangles if $\angle MJK \cong \angle KLM$, $m\angle MJK = 126$, and $m\angle JNM = 52$.



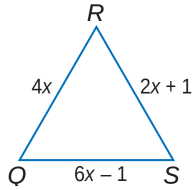
14. Identify the right triangles if $\overline{IJ} \parallel \overline{GH}$, $\overline{GH} \perp \overline{DF}$, and $\overline{GI} \perp \overline{EF}$.



15. ALGEBRA Find x, JM, MN, and JN if $\triangle JMN$ is an isosceles triangle with $\overline{JM} \cong \overline{MN}$.



16 ALGEBRA Find x, QR, RS, and QS if $\triangle QRS$ is an equilateral triangle.



COORDINATE GEOMETRY Find the measures of the sides of $\triangle ABC$ and classify each triangle by its sides.

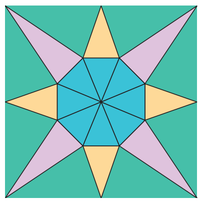
17. $A(5, 4), B(3, -1), C(7, -1)$

18. $A(-4, 1), B(5, 6), C(-3, -7)$

19. $A(-7, 9), B(-7, -1), C(4, -1)$

20. $A(-3, -1), B(2, 1), C(2, -3)$

21. QUILTING The star-shaped composite quilting square is made up of four different triangles. Use a ruler to classify the four triangles by sides.



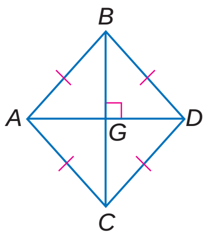
22. ARCHITECTURE The restored and decorated Victorian houses in San Francisco shown in the photograph are called the "Painted Ladies." Use a protractor to classify the triangles indicated in the photo by sides and angles.

Real-World Link

The Painted Ladies are located in Alamo Square. The area is one of 11 designated historic districts in San Francisco.

Source: www.sfvisitor.org

Identify the indicated triangles in the figure if $\overline{AB} \cong \overline{BD} \cong \overline{DC} \cong \overline{CA}$ and $\overline{BC} \perp \overline{AD}$.



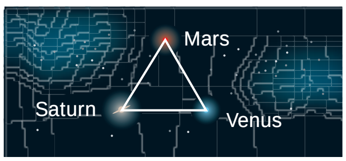
23. right

24. obtuse

25. scalene

26. isosceles

27. ASTRONOMY On May 5, 2002, Venus, Saturn, and Mars were aligned in a triangular formation. Use a protractor or ruler to classify the triangle formed by sides and angles.



28. RESEARCH Use the Internet or other resource to find out how astronomers can predict planetary alignment.

Joseph Sohm/Stock Boston

ALGEBRA Find x and the measure of each side of the triangle.

29. $\triangle GHJ$ is isosceles, with $\overline{HG} \cong \overline{JG}$, $GH = x + 7$, $GJ = 3x - 5$, and $HJ = x - 1$.

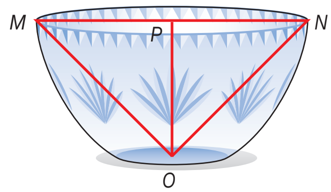
30. $\triangle MPN$ is equilateral with $MN = 3x - 6$, $MP = x + 4$, and $NP = 2x - 1$.

31. $\triangle QRS$ is equilateral. QR is two less than two times a number, RS is six more than the number, and QS is ten less than three times the number.

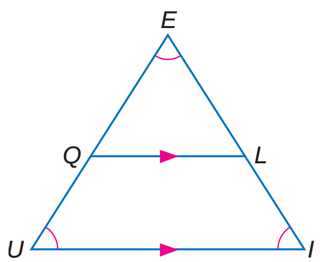
32. $\triangle JKL$ is isosceles with $\overline{KJ} \cong \overline{LJ}$. JL is five less than two times a number. JK is three more than the number. KL is one less than the number. Find the measure of each side.

33. ROAD TRIP The total distance from Charlotte to Raleigh to Winston-Salem and back to Charlotte is about 292 miles. The distance from Charlotte to Winston-Salem is 22 miles less than the distance from Raleigh to Winston-Salem. The distance from Charlotte to Raleigh is 60 miles greater than the distance from Winston-Salem to Charlotte. Classify the triangle that connects Charlotte, Raleigh, and Winston-Salem.

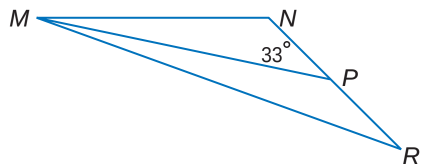
34. CRYSTAL The top of the crystal bowl pictured at the right is circular. The diameter at the top of the bowl is $\overline{MN}$. P is the midpoint of $\overline{MN}$, and $\overline{OP} \perp \overline{MN}$. If $MN = 24$ and $OP = 12$, determine whether $\triangle MPO$ and $\triangle NPO$ are equilateral.



35. PROOF Write a two-column proof to prove that $\triangle EQL$ is equiangular.

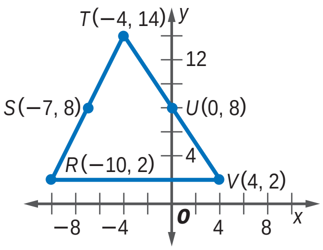


36. PROOF Write a paragraph proof to prove that $\triangle RPM$ is an obtuse triangle if $m\angle NPM = 33$.

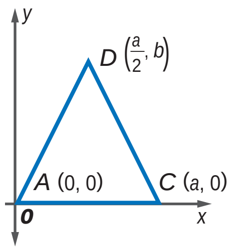


EXTRA PRACTICE
See pages 807, 831.
Math Online
Self-Check Quiz at
geometryonline.com

37. COORDINATE GEOMETRY Show that S is the midpoint of $\overline{RT}$ and U is the midpoint of $\overline{TV}$.



38. COORDINATE GEOMETRY Show that $\triangle ADC$ is isosceles.



H.O.T. Problems

39. OPEN ENDED Draw an isosceles right triangle.

REASONING Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

40. Equiangular triangles are also acute.

41. Right triangles are acute.

42. **CHALLENGE** $\overline{KL}$ is a segment representing one side of isosceles right triangle KLM with $K(2, 6)$, and $L(4, 2)$. $\angle KLM$ is a right angle, and $\overline{KL} \cong \overline{LM}$. Describe how to find the coordinates of M and name these coordinates.

43. **Writing in Math** Use the information on page 202 to explain why triangles are important in construction. Include a description of how to classify triangles and a justification of why you think one type of triangle might be used more often in architecture than other types.

44. Which type of triangle can serve as a counterexample to the conjecture below?

If two angles of a triangle are acute, then the measure of the third angle must be greater than or equal to 90.

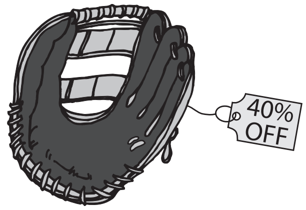
A equilateral

B obtuse

C right

D scalene

45. A baseball glove originally cost \$84.50. Jamal bought it at 40% off.



How much was deducted from the original price?

F \$50.70 **H** \$33.80

G \$44.50 **J** \$32.62

Graph each line. Construct a perpendicular segment through the given point. Then find the distance from the point to the line. (Lesson 3-6)

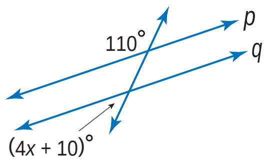
46. $y = x + 2$, $(2, -2)$

47. $x + y = 2$, $(3, 3)$

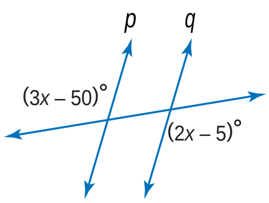
48. $y = 7$, $(6, -2)$

Find x so that $p \parallel q$. (Lesson 3-5)

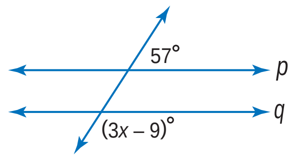
49.



50.



51.



PREREQUISITE SKILL **In the figure, $\overline{AB} \parallel \overline{RQ}$, $\overline{BC} \parallel \overline{PR}$, and $\overline{AC} \parallel \overline{PQ}$. Name the indicated angles or pairs of angles.** (Lessons 3-1 and 3-2)

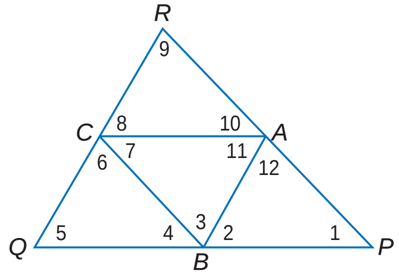
52. three pairs of alternate interior angles

53. six pairs of corresponding angles

54. all angles congruent to $\angle 3$

55. all angles congruent to $\angle 7$

56. all angles congruent to $\angle 11$

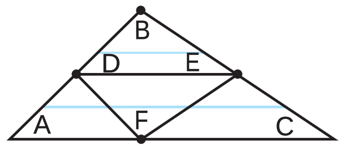


EXPLORE 4-2

Geometry Lab
Angles of Triangles

ACTIVITY 1

Find the relationship among the measures of the interior angles of a triangle.



Step 1 Draw an obtuse triangle and cut it out. Label the vertices *A*, *B*, and *C*.

Step 2 Find the midpoint of $\overline{AB}$ by matching *A* to *B*. Label this point *D*.

Step 3 Find the midpoint of $\overline{BC}$ by matching *B* to *C*. Label this point *E*.

Step 4 Draw $\overline{DE}$.

Step 5 Fold $\triangle ABC$ along $\overline{DE}$. Label the point where *B* touches $\overline{AC}$ as *F*.

Step 6 Draw $\overline{DF}$ and $\overline{FE}$. Measure each angle.

ANALYZE THE MODEL

Describe the relationship between each pair.

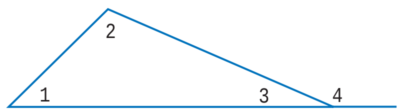
1. $\angle A$ and $\angle DFA$ **2.** $\angle B$ and $\angle DFE$ **3.** $\angle C$ and $\angle EFC$

4. What is the sum of the measures of $\angle DFA$, $\angle DFE$, and $\angle EFC$?

5. What is the sum of the measures of $\angle A$, $\angle B$, and $\angle C$?

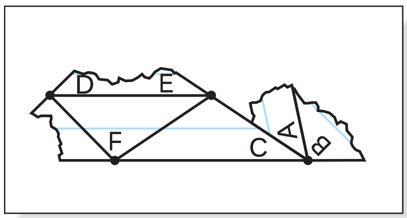
6. **Make a conjecture** about the sum of the measures of the angles of any triangle.

In the figure at the right, $\angle 4$ is called an *exterior angle* of the triangle. $\angle 1$ and $\angle 2$ are the *remote interior angles* of $\angle 4$.



ACTIVITY 2

Find the relationship among the interior and exterior angles of a triangle.



Step 1 Trace $\triangle ABC$ from Activity 1 onto a piece of paper. Label the vertices.

Step 2 Extend $\overline{AC}$ to draw an exterior angle at *C*.

Step 3 Tear $\angle A$ and $\angle B$ off the triangle from Activity 1.

Step 4 Place $\angle A$ and $\angle B$ over the exterior angle.

ANALYZE THE RESULTS

7. **Make a conjecture** about the relationship of $\angle A$, $\angle B$, and the exterior angle at *C*.

8. Repeat the steps for the exterior angles of $\angle A$ and $\angle B$.

9. Is your conjecture true for all exterior angles of a triangle?

10. Repeat Activity 2 with an acute triangle and with a right triangle.

11. **Make a conjecture** about the measure of an exterior angle and the sum of the measures of its remote interior angles.

4-2 Angles of Triangles

Main Ideas

- Apply the Angle Sum Theorem.
- Apply the Exterior Angle Theorem.

New Vocabulary

exterior angle
remote interior angles
flow proof
corollary

GET READY for the Lesson

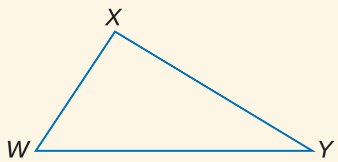
The Drachen Foundation coordinates the annual Miniature Kite Contest. In a recent year, the kite in the photograph won second place in the Most Beautiful Kite category. The overall dimensions are 10.5 centimeters by 9.5 centimeters. The wings of the beetle are triangular.

Angle Sum Theorem If the measures of two of the angles of a triangle are known, how can the measure of the third angle be determined? The Angle Sum Theorem explains that the sum of the measures of the angles of any triangle is always 180.

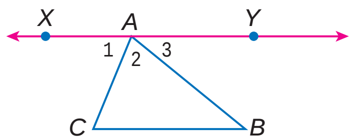
THEOREM 4.1 **Angle Sum**

The sum of the measures of the angles of a triangle is 180.

Example: $m\angle W + m\angle X + m\angle Y = 180$



PROOF **Angle Sum Theorem**



Given: $\triangle ABC$

Prove: $m\angle C + m\angle 2 + m\angle B = 180$

Proof:

Statements	Reasons
1. $\triangle ABC$	1. Given
2. Draw $\overleftrightarrow{XY}$ through A parallel to $\overline{CB}$.	2. Parallel Postulate
3. $\angle 1$ and $\angle CAY$ form a linear pair.	3. Def. of a linear pair
4. $\angle 1$ and $\angle CAY$ are supplementary.	4. If 2 ∡ form a linear pair, they are supplementary.
5. $m\angle 1 + m\angle CAY = 180$	5. Def. of suppl. ∡
6. $m\angle CAY = m\angle 2 + m\angle 3$	6. Angle Addition Postulate
7. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	7. Substitution
8. $\angle 1 \cong \angle C$, $\angle 3 \cong \angle B$	8. Alt. Int. ∡ Theorem
9. $m\angle 1 = m\angle C$, $m\angle 3 = m\angle B$	9. Def. of ≅ ∡
10. $m\angle C + m\angle 2 + m\angle B = 180$	10. Substitution

Study Tip

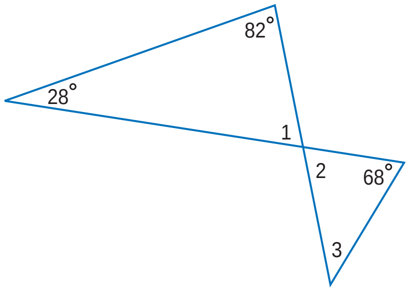
Auxiliary Lines

Recall that sometimes extra lines have to be drawn to complete a proof. These are called *auxiliary lines*.

If we know the measures of two angles of a triangle, we can find the measure of the third.

EXAMPLE Interior Angles

1 Find the missing angle measures.



Find $m\angle 1$ first because the measures of two angles of the triangle are known.

$m\angle 1 + 28 + 82 = 180$ Angle Sum Theorem

$m\angle 1 + 110 = 180$ Simplify.

$m\angle 1 = 70$ Subtract 110 from each side.

$\angle 1$ and $\angle 2$ are congruent vertical angles. So $m\angle 2 = 70$.

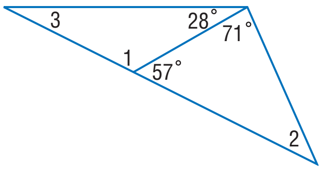
$m\angle 3 + 68 + 70 = 180$ Angle Sum Theorem

$m\angle 3 + 138 = 180$ Simplify.

$m\angle 3 = 42$ Subtract 138 from each side.

Therefore, $m\angle 1 = 70$, $m\angle 2 = 70$, and $m\angle 3 = 42$.

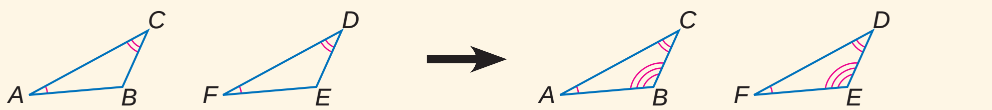
1.



The Angle Sum Theorem leads to a useful theorem about the angles in two triangles.

THEOREM 4.2 Third Angle Theorem

If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.



Example: If $\angle A \cong \angle F$ and $\angle C \cong \angle D$, then $\angle B \cong \angle E$.

You will prove this theorem in Exercise 34.

Vocabulary Link

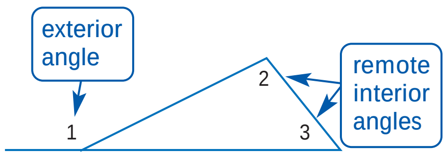
Remote

Everyday Use located far away; distant in space

Interior

Everyday Use the internal portion or area

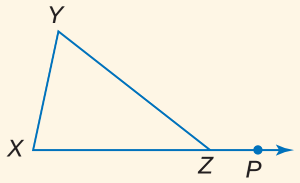
Exterior Angle Theorem Each angle of a triangle has an exterior angle. An **exterior angle** is formed by one side of a triangle and the extension of another side. The interior angles of the triangle not adjacent to a given exterior angle are called **remote interior angles** of the exterior angle.



THEOREM 4.3 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Example: $m\angle X + m\angle Y = m\angle YZP$



Study Tip

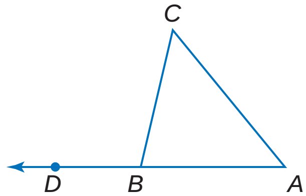
Flow Proof

Write each statement and reason on an index card. Then organize the index cards in logical order.

We will use a flow proof to prove this theorem. A **flow proof** organizes a series of statements in logical order, starting with the given statements. Each statement is written in a box with the reason verifying the statement written below the box. Arrows are used to indicate how the statements relate to each other.

PROOF Exterior Angle Theorem

Write a flow proof of the Exterior Angle Theorem.



Given: $\triangle ABC$

Prove: $m\angle CBD = m\angle A + m\angle C$

Flow Proof:

$\triangle ABC$
Given

$\angle CBD$ and $\angle ABC$ form a linear pair.
Definition of linear pair

$\angle CBD$ and $\angle ABC$ are supplementary.
If 2 ∠s form a linear pair, they are supplementary.

$m\angle A + m\angle ABC + \angle C = 180$
Angle Sum Theorem

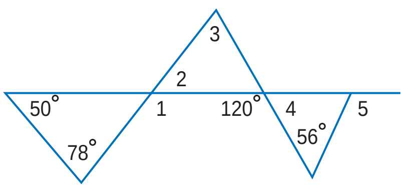
$m\angle CBD + m\angle ABC = 180$
Definition of supplementary

$m\angle A + m\angle ABC + m\angle C = m\angle CBD + m\angle ABC$
Substitution Property

$m\angle A + m\angle C = m\angle CBD$
Subtraction Property

EXAMPLE 2 Exterior Angles

Find the measure of each angle.



a. $m\angle 1$

$m\angle 1 = 50 + 78$ Exterior Angle Theorem

$= 128$ Simplify.

b. $m\angle 2$

$m\angle 1 + m\angle 2 = 180$ If 2∠s form a linear pair, they are suppl.

$128 + m\angle 2 = 180$ Substitution

$m\angle 2 = 52$ Subtract 128 from each side.

c. $m\angle 3$

$m\angle 2 + m\angle 3 = 120$ Exterior Angle Theorem

$52 + m\angle 3 = 120$ Substitution

$m\angle 3 = 68$ Subtract 52 from each side.

Therefore, $m\angle 1 = 128$, $m\angle 2 = 52$, and $m\angle 3 = 68$.

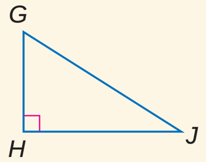
2A. $m\angle 4$ **2B.** $m\angle 5$

Personal Tutor at geometryonline.com

A statement that can be easily proved using a theorem is often called a **corollary** of that theorem. A corollary, just like a theorem, can be used as a reason in a proof.

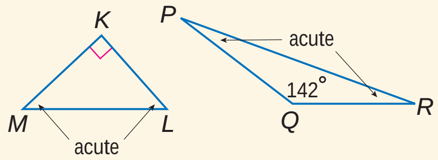
COROLLARIES

4.1 The acute angles of a right triangle are complementary.



Example: $m\angle G + m\angle J = 90$

4.2 There can be at most one right or obtuse angle in a triangle.



You will prove Corollaries 4.1 and 4.2 in Exercises 32 and 33.

Real-World EXAMPLE Right Angles

3 **SKI JUMPING** **Ski jumper Simon Ammann of Switzerland forms a right triangle with his skis and his line of sight. Find $m\angle 2$ if $m\angle 1$ is 27.**

Use Corollary 4.1 to write an equation.

$m\angle 1 + m\angle 2 = 90$

$27 + m\angle 2 = 90$ Substitution

$m\angle 2 = 63$ Subtract 27 from each side.

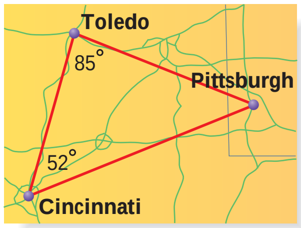
3. WIND SURFING A windsurfing sail is generally a right triangle. One of the angles that is not the right angle has a measure of 68°. What is the measure of the other nonright angle?

✓ CHECK Your Understanding

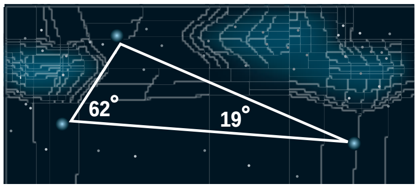
Example 1 (p. 211)

Find the missing angle measure.

1.



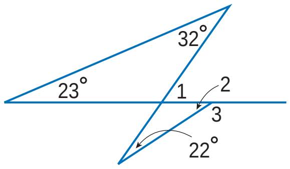
2.



Example 2 (p. 212)

Find each measure.

3. $m\angle 1$ **4.** $m\angle 2$ **5.** $m\angle 3$

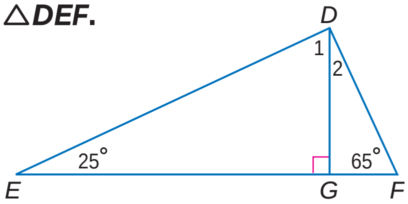


Example 3 (p. 213)

Find each measure in $\triangle DEF$.

6. $m\angle 1$

7. $m\angle 2$



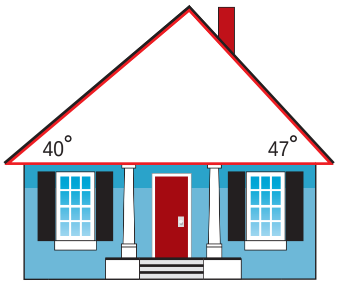
8. **SKI JUMPING** American ski jumper Jessica Jerome forms a right angle with her skis. If $m\angle 2 = 70$, find $m\angle 1$.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–12	1
13–18	2
19–22	3

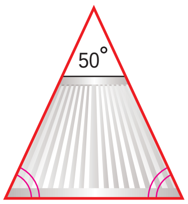
Find the missing angle measures.

9.

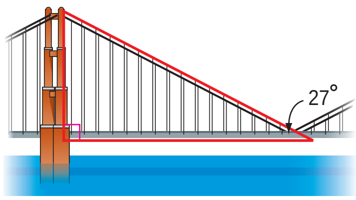


10.

11.



12.

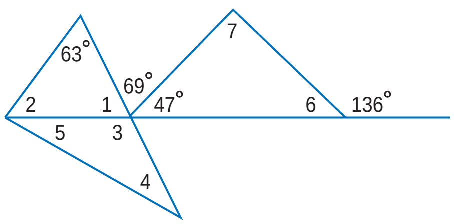


Find each measure if $m\angle 4 = m\angle 5$.

13. $m\angle 1$ **14.** $m\angle 2$

15. $m\angle 3$ **16.** $m\angle 4$

17. $m\angle 5$ **18.** $m\angle 6$



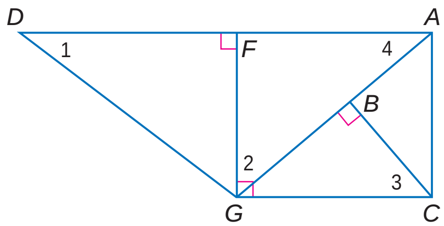
Find each measure if $m\angle DGF = 53$ and $m\angle AGC = 40$.

19. $m\angle 1$

20. $m\angle 2$

21. $m\angle 3$

22. $m\angle 4$



Real-World Link
Catriona Lemay Doan is the first Canadian to win a Gold medal in the same event in two consecutive Olympic games.

Source: catrionalemaydoan.com

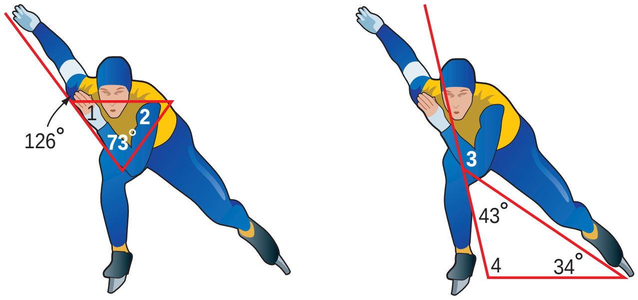
SPEED SKATING **For Exercises 23–26, use the following information.**
Speed skater Catriona Lemay Doan of Canada forms at least two sets of triangles and exterior angles as she skates. Use the measures of given angles to find each measure.

23. $m\angle 1$

24. $m\angle 2$

25. $m\angle 3$

26. $m\angle 4$

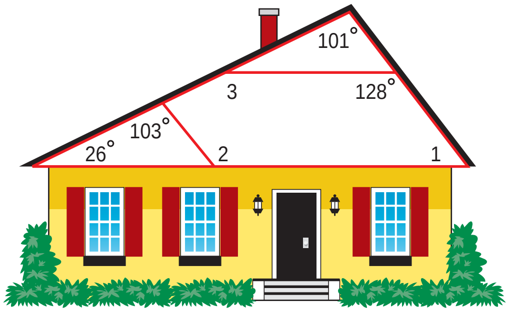


HOUSING **For Exercises 27–29, use the following information.**
The two braces for the roof of a house form triangles. Find each measure.

27. $m\angle 1$

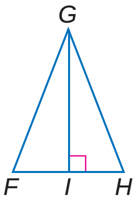
28. $m\angle 2$

29. $m\angle 3$

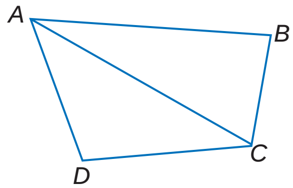


PROOF **For Exercises 30–34, write the specified type of proof.**

30. flow proof
Given: $\angle FGI \cong \angle IGH$
$\overline{GI} \perp \overline{FH}$
Prove: $\angle F \cong \angle H$



31. two-column proof
Given: $ABCD$ is a quadrilateral.
Prove: $m\angle DAB + m\angle B + m\angle BCD + m\angle D = 360$



32. flow proof of Corollary 4.1

33. paragraph proof of Corollary 4.2

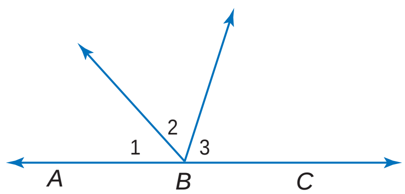
34. two-column proof of Theorem 4.2

EXTRA PRACTICE
See pages 807, 831.
Math Online
Self-Check Quiz at geometryonline.com

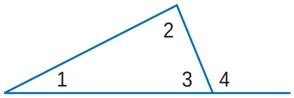
H.O.T. Problems

35. **OPEN ENDED** Draw a triangle. Label one exterior angle and its remote interior angles.

36. **CHALLENGE** $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. The measures of $\angle 1$, $\angle 2$, and $\angle 3$ are in a 4:5:6 ratio. Find the measure of each angle.



37. FIND THE ERROR Najee and Kara are discussing the Exterior Angle Theorem. Who is correct? Explain.



Najee	Kara
$m\angle 1 + m\angle 2 = m\angle 4$	$m\angle 1 + m\angle 2 + m\angle 4 = 180$

38. Writing in Math Use the information about kites provided on page 210 to explain how the angles of triangles are used to make kites. Include an explanation of how you can find the measure of a third angle if two angles of two triangles are congruent. Also include a description of the properties of two angles in a triangle if the measure of the third is 90°.

STANDARDIZED TEST PRACTICE

39. Two angles of a triangle have measures of 35° and 80°. Which of the following could *not* be a measure of an exterior angle of the triangle?

A 165°

B 145°

C 115°

D 100°

40. Which equation is equivalent to $7x - 3(2 - 5x) = 8x$?

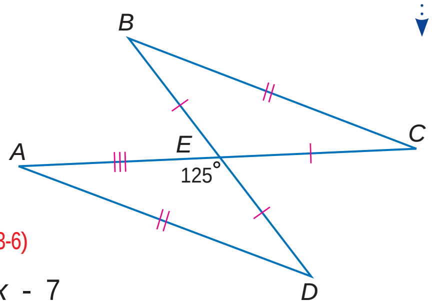
F $2x - 6 = 8x$

G $22x - 6 = 8x$

H $-8x - 6 = 8x$

J $22x + 6 = 8x$

Identify the indicated triangles if $\overline{BC} \cong \overline{AD}$, $\overline{EB} \cong \overline{EC}$, $\overline{AC}$ bisects $\overline{BD}$, and $m\angle AED = 125$. (Lesson 4-1)



41. scalene

42. obtuse

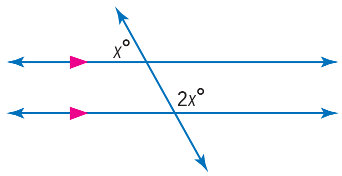
43. isosceles

Find the distance between each pair of parallel lines. (Lesson 3-6)

44. $y = x + 6$, $y = x - 10$

45. $y = -2x + 3$, $y = -2x - 7$

46. MODEL TRAINS Regan is going to set up two parallel train tracks with a third track running diagonally across the first two. To properly place a switch, she needs the angle between the diagonal and top of the second track to be twice as large as the angle between the diagonal and top of the first track. What is the value of x? (Lesson 3-2)



PREREQUISITE SKILL List the property of congruence used for each statement. (Lessons 2-5 and 2-6)

47. $\angle 1 \cong \angle 1$ and $\overline{AB} \cong \overline{AB}$.

48. If $\overline{AB} \cong \overline{XY}$, then $\overline{XY} \cong \overline{AB}$.

49. If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

50. If $\angle 2 \cong \angle 3$ and $\angle 3 \cong \angle 4$, then $\angle 2 \cong \angle 4$.

4-3 Congruent Triangles

Main Ideas

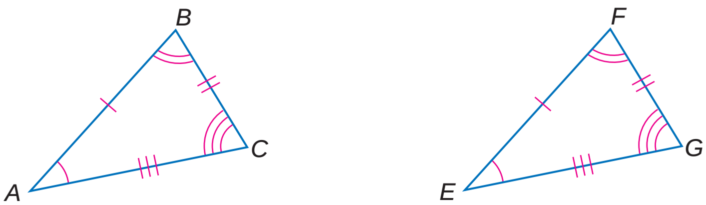
- Name and label corresponding parts of congruent triangles.
- Identify congruence transformations.

New Vocabulary

congruent triangles
congruence transformations

The Kaibab suspension bridge near Bright Angel Campground, Arizona, carries the Kaibab Trail across the Colorado River. Steel beams, stained a special color to blend in with the natural scenery of the Grand Canyon, are arranged along the side of the bridge in a triangular web. Triangles spread weight and stress evenly throughout the bridge.

Corresponding Parts of Congruent Triangles Triangles that are the same size and shape are **congruent triangles**. Each triangle has three angles and three sides. If all six of the corresponding parts of two triangles are congruent, then the triangles are congruent.



If $\triangle ABC$ is congruent to $\triangle EFG$, the vertices of the two triangles correspond in the same order as the letters naming the triangles.

$$\triangle ABC \cong \triangle EFG$$

This correspondence of vertices can be used to name the corresponding congruent sides and angles of the two triangles.

$$\angle A \cong \angle E \qquad \angle B \cong \angle F \qquad \angle C \cong \angle G$$

$$\overline{AB} \cong \overline{EF} \qquad \overline{BC} \cong \overline{FG} \qquad \overline{AC} \cong \overline{EG}$$

The corresponding sides and angles can be determined from any congruence statement by following the order of the letters.

Study Tip

Congruent Parts
In congruent triangles, congruent sides are opposite congruent angles.

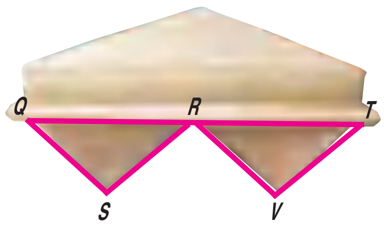
KEY CONCEPT *Definition of Congruent Triangles (CPCTC)*

Two triangles are congruent if and only if their corresponding parts are congruent.

CPCTC stands for *corresponding parts of congruent triangles are congruent*. "If and only if" is used to show that both the conditional and its converse are true.

EXAMPLE 1 Corresponding Congruent Parts

FURNITURE DESIGN **The legs of this stool form two triangles. Suppose the measures in inches are $QR = 12$, $RS = 23$, $QS = 24$, $RT = 12$, $TV = 24$, and $RV = 23$.**



a. Name the corresponding congruent angles and sides.

$\angle Q \cong \angle T$ $\angle QRS \cong \angle TRV$ $\angle S \cong \angle V$

$\overline{QR} \cong \overline{TR}$ $\overline{RS} \cong \overline{RV}$ $\overline{QS} \cong \overline{TV}$

b. Name the congruent triangles.

$\triangle QRS \cong \triangle TRV$

The measures of the sides of triangles PDQ and OEC are $PD = 5$, $DQ = 7$, $PQ = 11$; $EC = 7$, $OC = 5$, and $OE = 11$.

1A. Name the corresponding congruent angles and sides.

1B. Name the congruent triangles.

Like congruence of segments and angles, congruence of triangles is reflexive, symmetric, and transitive.

THEOREM 4.4 Properties of Triangle Congruence

Congruence of triangles is reflexive, symmetric, and transitive.

Reflexive

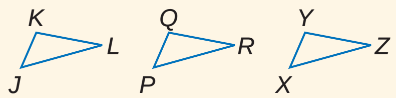
$\triangle JKL \cong \triangle JKL$

Symmetric

If $\triangle JKL \cong \triangle PQR$, then $\triangle PQR \cong \triangle JKL$.

Transitive

If $\triangle JKL \cong \triangle PQR$, and $\triangle PQR \cong \triangle XYZ$, then $\triangle JKL \cong \triangle XYZ$.



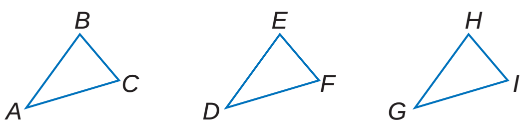
You will prove the symmetric and reflexive parts of Theorem 4.4 in Exercises 30 and 32, respectively.

Proof Theorem 4.4 (Transitive)

Given: $\triangle ABC \cong \triangle DEF$

$\triangle DEF \cong \triangle GHI$

Prove: $\triangle ABC \cong \triangle GHI$

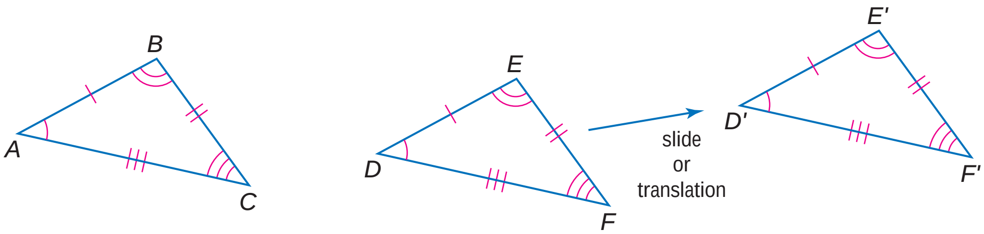


Proof: You are given that $\triangle ABC \cong \triangle DEF$. Because corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. You are also given that $\triangle DEF \cong \triangle GHI$. So $\angle D \cong \angle G$, $\angle E \cong \angle H$, $\angle F \cong \angle I$, $\overline{DE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HI}$, and $\overline{DF} \cong \overline{GI}$, by CPCTC. Therefore, $\angle A \cong \angle G$, $\angle B \cong \angle H$, $\angle C \cong \angle I$, $\overline{AB} \cong \overline{GH}$, $\overline{BC} \cong \overline{HI}$, and $\overline{AC} \cong \overline{GI}$ because congruence of angles and segments is transitive. Thus, $\triangle ABC \cong \triangle GHI$ by the definition of congruent triangles.

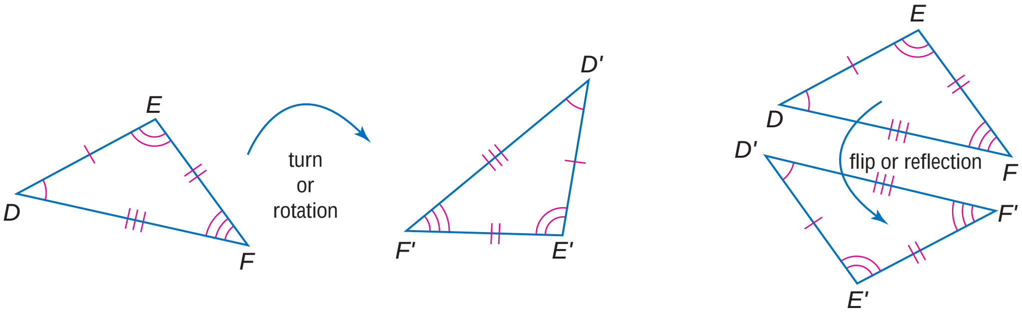
> ## Study Tip
> **Naming Congruent Triangles**
> There are six ways to name each pair of congruent triangles.

> ## Study Tip
> **Transformations**
> Not all transformations preserve congruence. Only transformations that do not change the size or shape of the figure are congruence transformations. You will learn more about transformations in Chapter 9.

Identify Congruence Transformations In the figures below, $\triangle ABC$ is congruent to $\triangle DEF$. If you *slide*, or *translate*, $\triangle DEF$ up and to the right, $\triangle DEF$ is still congruent to $\triangle ABC$.



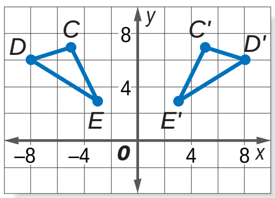
The congruency does not change whether you *turn*, or *rotate*, $\triangle DEF$ or *flip*, or *reflect*, $\triangle DEF$. $\triangle ABC$ is still congruent to $\triangle DEF$.



If you slide, flip, or turn a triangle, the size and shape do not change. These three transformations are called **congruence transformations**.

EXAMPLE 2 Transformations in the Coordinate Plane

COORDINATE GEOMETRY The vertices of $\triangle CDE$ are $C(-5, 7)$, $D(-8, 6)$, and $E(-3, 3)$. The vertices of $\triangle C'D'E'$ are $C'(5, 7)$, $D'(8, 6)$, and $E'(3, 3)$.



a. Verify that $\triangle CDE \cong \triangle C'D'E'$.

Use the Distance Formula to find the length of each side in the triangles.

$$DC = \sqrt{[-8 - (-5)]^2 + (6 - 7)^2} = \sqrt{9 + 1} \text{ or } \sqrt{10}$$

$$D'C' = \sqrt{(8 - 5)^2 + (6 - 7)^2} = \sqrt{9 + 1} \text{ or } \sqrt{10}$$

$$DE = \sqrt{[-8 - (-3)]^2 + (6 - 3)^2} = \sqrt{25 + 9} \text{ or } \sqrt{34}$$

$$D'E' = \sqrt{(8 - 3)^2 + (6 - 3)^2} = \sqrt{25 + 9} \text{ or } \sqrt{34}$$

$$CE = \sqrt{[-5 - (-3)]^2 + (7 - 3)^2} = \sqrt{4 + 16} = \sqrt{20} \text{ or } 2\sqrt{5}$$

$$C'E' = \sqrt{(5 - 3)^2 + (7 - 3)^2} = \sqrt{4 + 16} = \sqrt{20} \text{ or } 2\sqrt{5}$$

By the definition of congruence, $\overline{DC} \cong \overline{D'C'}$, $\overline{DE} \cong \overline{D'E'}$, and $\overline{CE} \cong \overline{C'E'}$. Use a protractor to measure the angles of the triangles. You will find that the measures are the same.

In conclusion, because $\overline{DC} \cong \overline{D'C'}$, $\overline{DE} \cong \overline{D'E'}$, and $\overline{CE} \cong \overline{C'E'}$, $\angle D \cong \angle D'$, $\angle C \cong \angle C'$, and $\angle E \cong \angle E'$, $\triangle CDE \cong \triangle C'D'E'$.

(continued on the next page)

b. Name the congruence transformation for $\triangle CDE$ and $\triangle C'D'E'$.

$\triangle C'D'E'$ is a flip, or reflection, of $\triangle CDE$.

COORDINATE GEOMETRY **The vertices of $\triangle LMN$ are $L(1, 1)$, $M(3, 5)$, and $N(5, 1)$. The vertices of $\triangle L'M'N'$ are $L'(-1, -1)$, $M'(-3, -5)$, and $N'(-5, -1)$.**

2A. Verify that $\triangle LMN \cong L'M'N'$.

2B. Name the congruence transformation for $\triangle LMN$ and $\triangle L'M'N'$.

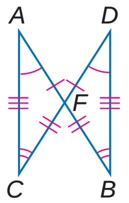
Personal Tutor at geometryonline.com

CHECK Your Understanding

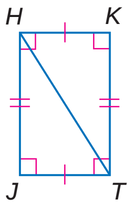
Example 1 (p. 218)

Identify the corresponding congruent angles and sides and the congruent triangles in each figure.

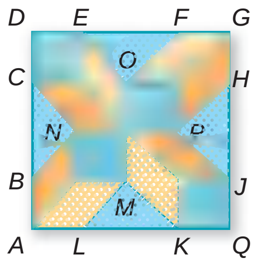
1.



2.



3. QUILTING In the quilt design, assume that angles and segments that appear to be congruent are congruent. Indicate which triangles are congruent.



Example 2 (p. 219)

4. The vertices of $\triangle SUV$ and $\triangle S'U'V'$ are $S(0, 4)$, $U(0, 0)$, $V(2, 2)$, $S'(0, -4)$, $U'(0, 0)$, and $V'(-2, -2)$. Verify that the triangles are congruent and then name the congruence transformation.

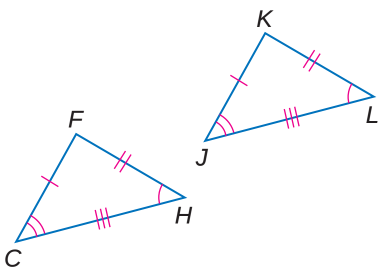
5. The vertices of $\triangle QRT$ and $\triangle Q'R'T'$ are $Q(-4, 3)$, $Q'(4, 3)$, $R(-4, -2)$, $R'(4, -2)$, $T(-1, -2)$, and $T'(1, -2)$.Verify that $\triangle QRT \cong \triangle Q'R'T'$. Then name the congruence transformation.

Exercises

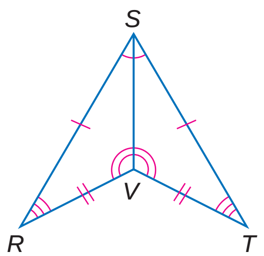
HOMEWORK HELP	
For Exercises	See Examples
6–9	1
10–13	2

Identify the congruent angles and sides and the congruent triangles in each figure.

6.

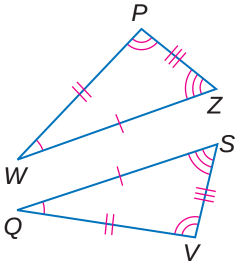


7.

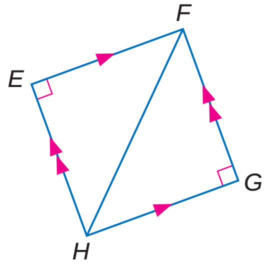


Identify the congruent angles and sides and the congruent triangles in each figure.

8.

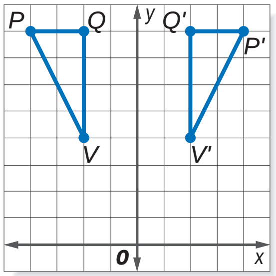


9.

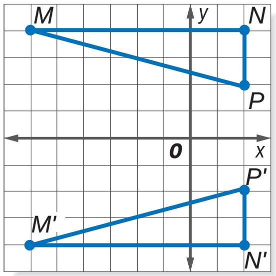


Verify each congruence and name the congruence transformation.

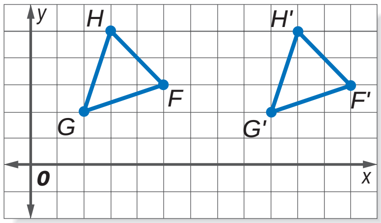
10. $\triangle PQV \cong \triangle P'Q'V'$



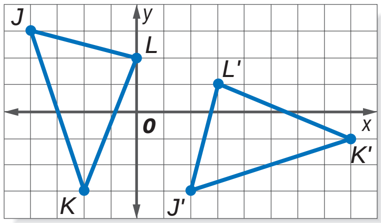
11. $\triangle MNP \cong \triangle M'N'P'$



12. $\triangle GHF \cong \triangle G'H'F'$



13. $\triangle JKL \cong \triangle J'K'L'$



Name the congruent angles and sides for each pair of congruent triangles.

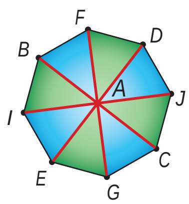
14. $\triangle TUV \cong \triangle XYZ$

15. $\triangle CDG \cong \triangle RSW$

16. $\triangle BCF \cong \triangle DGH$

17. $\triangle ADG \cong \triangle HKL$

18. UMBRELLAS Umbrellas usually have eight triangular sections with ribs of equal length. Are the statements $\triangle JAD \cong \triangle IAE$ and $\triangle JAD \cong \triangle EAI$ both correct? Explain.



Real-World Link

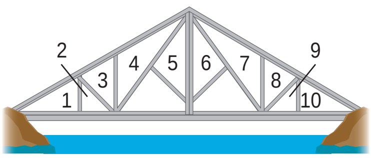
A mosaic is composed of glass, marble, or ceramic pieces often arranged in a pattern. The pieces, or *tesserae*, are set in cement. Mosaics are used to decorate walls, floors, and gardens.

Source: www.dimosaic.com

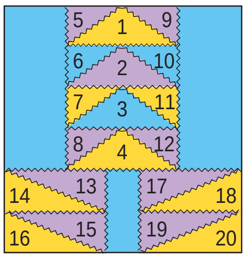
19. MOSAICS The figure at the left is the center of a Roman mosaic. If the bases of the triangles are each the same length, what else do you need to know to conclude that the four triangles surrounding the square are congruent?

Assume that segments and angles that appear to be congruent in each figure are congruent. Indicate which triangles are congruent.

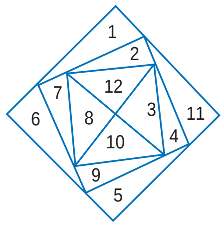
20.



21.



22.



Determine whether each statement is *true* or *false*. Draw an example or counterexample for each.

23. Two triangles with corresponding congruent angles are congruent.

24. Two triangles with angles and sides congruent are congruent.

ALGEBRA For Exercises 25 and 26, use the following information.
$\triangle QRS \cong \triangle GHJ$, $RS = 12$, $QR = 10$, $QS = 6$, and $HJ = 2x - 4$.

25. Draw and label a figure to show the congruent triangles.

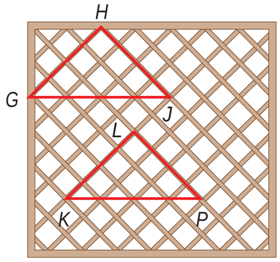
26. Find x.

ALGEBRA For Exercises 27 and 28, use the following information.
$\triangle JKL \cong \triangle DEF$, $m\angle J = 36$, $m\angle E = 64$, and $m\angle F = 3x + 52$.

27. Draw and label a figure to show the congruent triangles.

28. Find x.

29. GARDENING This garden lattice will be covered with morning glories in the summer. Malina wants to save two triangular areas for artwork. If $\triangle GHJ \cong \triangle KLP$, name the corresponding congruent angles and sides.

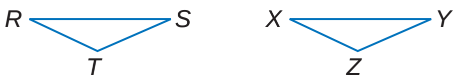


30. PROOF Put the statements used to prove the statement below in the correct order. Provide the reasons for each statement.

Congruence of triangles is symmetric.

Given: $\triangle RST \cong \triangle XYZ$

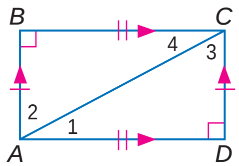
Prove: $\triangle XYZ \cong \triangle RST$



Proof:

$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$, $\overline{XY} \cong \overline{RS}$, $\overline{YZ} \cong \overline{ST}$, $\overline{XZ} \cong \overline{RT}$	$\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$	$\triangle RST \cong \triangle XYZ$	$\triangle XYZ \cong \triangle RST$
?	?	?	?

31. PROOF Copy the flow proof and provide the reasons for each statement.



Given: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$, $\overline{AD} \perp \overline{DC}$, $\overline{AB} \perp \overline{BC}$, $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \parallel \overline{CD}$

Prove: $\triangle ACD \cong \triangle CAB$

Proof:

$\overline{AB} \cong \overline{CD}$ a. ?

$\overline{AD} \cong \overline{CB}$ b. ?

$\overline{AC} \cong \overline{CA}$ c. ?

$\overline{AD} \perp \overline{DC}$ d. ? → $\angle D$ is a rt. $\angle$. e. ?

$\overline{AB} \perp \overline{BC}$ f. ? → $\angle B$ is a rt. $\angle$. g. ?

$\angle D$ is a rt. $\angle$., $\angle B$ is a rt. $\angle$. → $\angle D \cong \angle B$ h. ?

$\overline{AD} \parallel \overline{BC}$ i. ? → $\angle 1 \cong \angle 4$ j. ?

$\overline{AB} \parallel \overline{CD}$ k. ? → $\angle 2 \cong \angle 3$ l. ?

All → $\triangle ACD \cong \triangle CAB$ m. ?

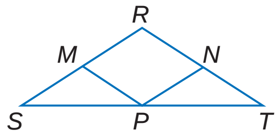
EXTRA PRACTICE
See pages 807, 831.
Math Online
Self-Check Quiz at
geometryonline.com

32. **PROOF** Write a flow proof to prove that congruence of triangles is reflexive. (Theorem 4.4)

H.O.T. Problems

33. **OPEN ENDED** Find a real-world picture of congruent triangles and explain how you know that the triangles are congruent.

34. **CHALLENGE** $\triangle RST$ is isosceles with $RS = RT$, M, N, and P are midpoints of the respective sides, $\angle S \cong \angle MPS$, and $\overline{NP} \cong \overline{MP}$. What else do you need to know to prove that $\triangle SMP \cong \triangle TNP$?



35. **Writing in Math** Use the information on page 217 to explain why triangles are used in the design and construction of bridges.

36. Triangle ABC is congruent to $\triangle HIJ$. The vertices of $\triangle ABC$ are $A(-1, 2)$, $B(0, 3)$, and $C(2, -2)$. What is the measure of side $\overline{HJ}$?

A $\sqrt{2}$ **C** 5

B 3 **D** cannot be determined

37. **REVIEW** Which is a factor of $x^2 + 19x - 42$?

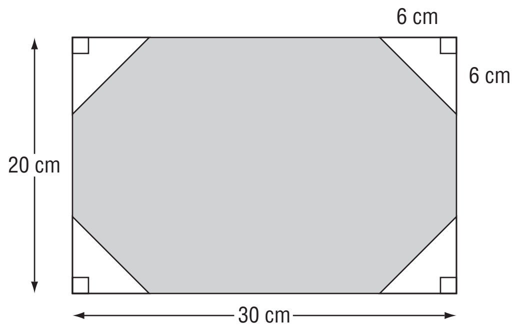
F $x + 14$

G $x + 2$

H $x - 14$

J $x - 2$

38. Bryssa cut four congruent triangles off the corners of a rectangle to make an octagon as shown below.



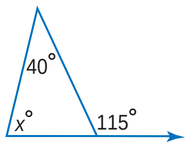
What is the area of the octagon?

A 456 cm^2 **C** 552 cm^2

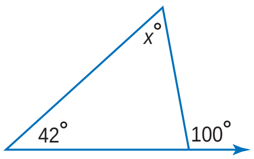
B 528 cm^2 **D** 564 cm^2

Find x. (Lesson 4-2)

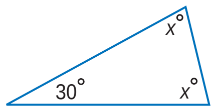
39.



40.



41.



Find x and the measure of each side of the triangle. (Lesson 4-1)

42. $\triangle BCD$ is isosceles with $\overline{BC} \cong \overline{CD}$, $BC = 2x + 4$, $BD = x + 2$ and $CD = 10$.

43. Triangle HKT is equilateral with $HK = x + 7$ and $HT = 4x - 8$.

PREREQUISITE SKILL **Find the distance between each pair of points.** (Lesson 1-3)

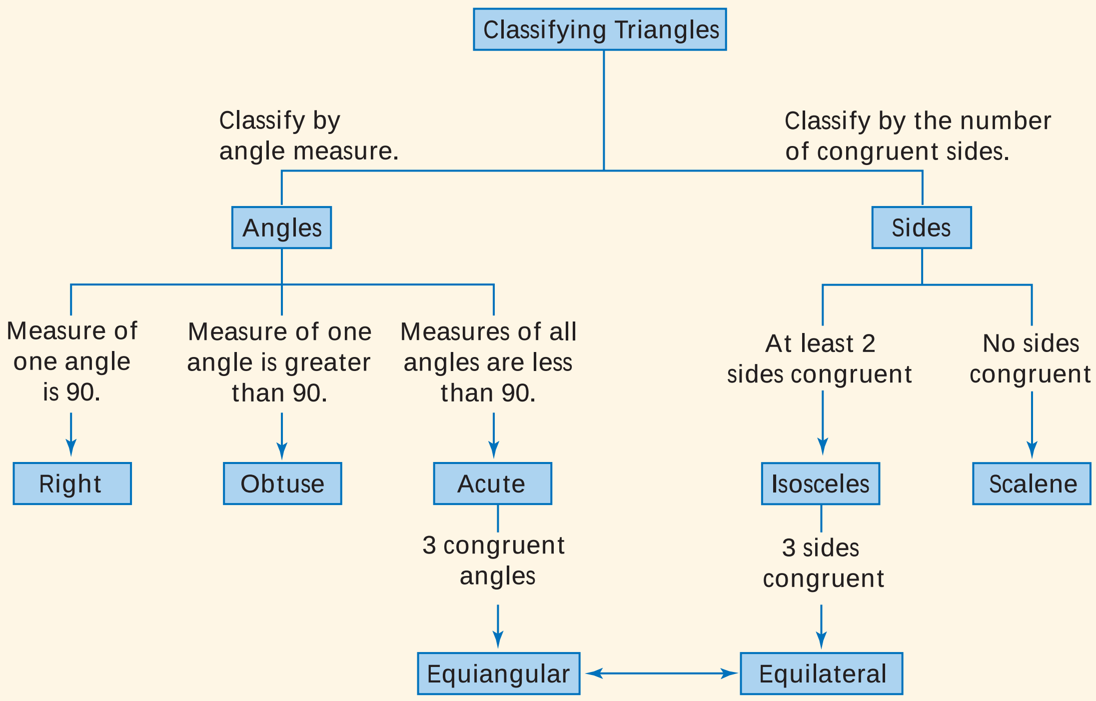
44. $(-1, 7), (1, 6)$ **45.** $(8, 2), (4, -2)$ **46.** $(3, 5), (5, 2)$ **47.** $(0, -6), (-3, -1)$

READING MATH

Making Concept Maps

When studying a chapter, it is wise to record the main topics and vocabulary you encounter. In this chapter, some of the new vocabulary words were *triangle*, *acute triangle*, *obtuse triangle*, *right triangle*, *equiangular triangle*, *scalene triangle*, *isosceles triangle*, and *equilateral triangle*. The triangles are all related by the size of the angles or the number of congruent sides.

A graphic organizer called a *concept map* is a convenient way to show these relationships. A concept map is shown below for the different types of triangles. The main ideas are in boxes. Any information that describes how to move from one box to the next is placed along the arrows.



Reading to Learn

1. Describe how to use the concept map to classify triangles by their side lengths.
2. In $\triangle ABC$, $m\angle A = 48$, $m\angle B = 41$, and $m\angle C = 91$. Use the concept map to classify $\triangle ABC$.
3. Identify the type of triangle that is linked to both classifications.

4-4 Proving Congruence—SSS, SAS

Main Ideas

- Use the SSS Postulate to test for triangle congruence.
- Use the SAS Postulate to test for triangle congruence.

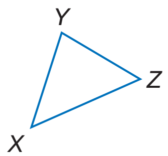
New Vocabulary

included angle

Around 120 B.C., Greek developers and land owners used the properties of geometry to accurately and precisely divide plots of land. Since that time, surveying has been used in areas such as map making and engineering. To check a measurement, land surveyors mark out a right triangle and then mark a second triangle that is congruent to the first.

SSS Postulate Is it always necessary to show that all of the corresponding parts of two triangles are congruent to prove that the triangles are congruent? In this lesson, we will explore two other methods to prove that triangles are congruent.

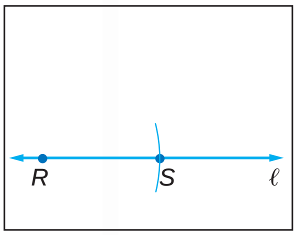
Use the following construction to construct a triangle with sides that are congruent to a given $\triangle XYZ$.



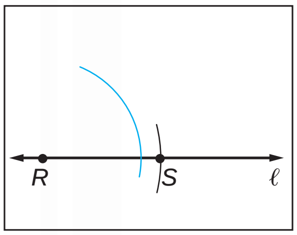
CONSTRUCTION

Congruent Triangles Using Sides

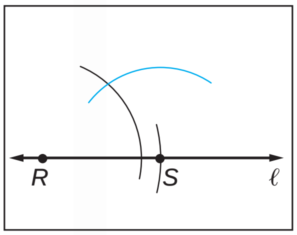
Step 1 Use a straightedge to draw any line ℓ, and select a point R. Use a compass to construct $\overline{RS}$ on ℓ, such that $\overline{RS} \cong \overline{XZ}$.



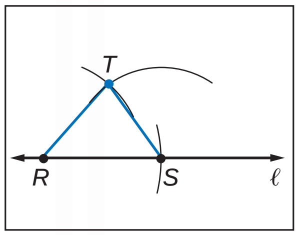
Step 2 Using R as the center, draw an arc with radius equal to XY.



Step 3 Using S as the center, draw an arc with radius equal to YZ.



Step 4 Let T be the point of intersection of the two arcs. Draw $\overline{RT}$ and $\overline{ST}$ to form $\triangle RST$.



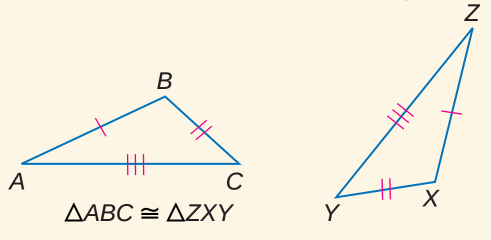
Step 5 Cut out $\triangle RST$ and place it over $\triangle XYZ$. How does $\triangle RST$ compare to $\triangle XYZ$?

If the corresponding sides of two triangles are congruent, then the triangles are congruent. This is the Side-Side-Side Postulate and is written as SSS.

POSTULATE 4.1 **Side-Side-Side Congruence**

If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

Abbreviation: *SSS*



$\triangle ABC \cong \triangle ZXY$

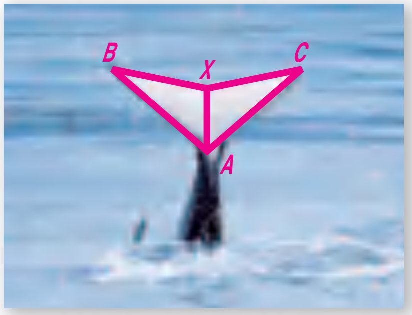
Real-World Link

Orca whales are commonly called "killer whales" because of their predatory nature. They are the largest members of the dolphin family. An average male is about 19–22 feet long and weighs between 8000 and 12,000 pounds.

Source: seaworld.org

Real-World EXAMPLE — Use SSS in Proofs

1 **MARINE BIOLOGY** **The tail of an orca whale can be viewed as two triangles that share a common side. Write a two-column proof to prove that $\triangle BXA \cong \triangle CXA$ if $\overline{AB} \cong \overline{AC}$ and $\overline{BX} \cong \overline{CX}$.**



Given: $\overline{AB} \cong \overline{AC}$; $\overline{BX} \cong \overline{CX}$

Prove: $\triangle BXA \cong \triangle CXA$

Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$; $\overline{BX} \cong \overline{CX}$	**1.** Given
2. $\overline{AX} \cong \overline{AX}$	**2.** Reflexive Property
3. $\triangle BXA \cong \triangle CXA$	**3.** SSS

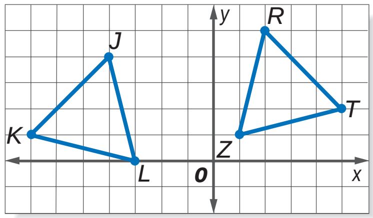
1A. A "Caution, Floor Slippery When Wet" sign is composed of three triangles. If $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{DC}$, prove that $\triangle ACB \cong \triangle ACD$.

1B. Triangle QRS is an isosceles triangle with $\overline{QR} \cong \overline{RS}$. If there exists a line $\overline{RT}$ that bisects $\angle QRS$ and $\overline{QS}$, show that $\triangle QRT \cong \triangle SRT$.

You can use the Distance Formula and postulates about triangle congruence to relate figures on the coordinate plane.

EXAMPLE SSS on the Coordinate Plane

2 COORDINATE GEOMETRY **Determine whether $\triangle RTZ \cong \triangle JKL$ for $R(2, 5)$, $Z(1, 1)$, $T(5, 2)$, $L(-3, 0)$, $K(-7, 1)$, and $J(-4, 4)$. Explain.**



Use the Distance Formula to show that the corresponding sides are congruent.

$$RT = \sqrt{(2-5)^2 + (5-2)^2} = \sqrt{9+9} = \sqrt{18} \text{ or } 3\sqrt{2}$$

$$JK = \sqrt{[-4-(-7)]^2 + (4-1)^2} = \sqrt{9+9} = \sqrt{18} \text{ or } 3\sqrt{2}$$

$$TZ = \sqrt{(5-1)^2 + (2-1)^2} = \sqrt{16+1} \text{ or } \sqrt{17}$$

$$KL = \sqrt{[-7-(-3)]^2 + (1-0)^2} = \sqrt{16+1} \text{ or } \sqrt{17}$$

$$RZ = \sqrt{(2-1)^2 + (5-1)^2} = \sqrt{1+16} \text{ or } \sqrt{17}$$

$$JL = \sqrt{[-4-(-3)]^2 + (4-0)^2} = \sqrt{1+16} \text{ or } \sqrt{17}$$

$RT = JK$, $TZ = KL$, and $RZ = JL$. By definition of congruent segments, all corresponding segments are congruent. Therefore, $\triangle RTZ \cong \triangle JKL$ by SSS.

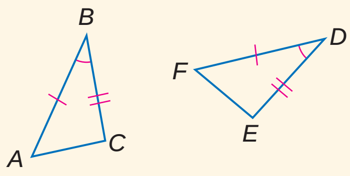
2. Determine whether triangles ABC and TDS with vertices $A(1, 1)$, $B(3, 2)$, $C(2, 5)$, $T(1, -1)$, $D(3, -3)$, and $S(2, -5)$ are congruent. Justify your reasoning.

SAS Postulate Suppose you are given the measures of two sides and the angle they form, called the **included angle**. These conditions describe a unique triangle. Two triangles in which corresponding sides and the included pairs of angles are congruent provide another way to show that triangles are congruent.

POSTULATE 4.2 *Side-Angle-Side Congruence*

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

Abbreviation: *SAS*



$\triangle ABC \cong \triangle FDE$

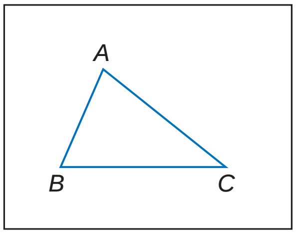
Animation
geometryonline.com

You can also construct congruent triangles given two sides and the included angle.

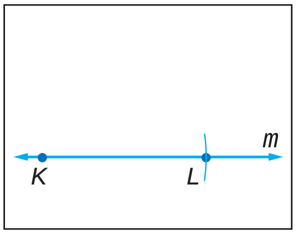
CONSTRUCTION

Congruent Triangles Using Two Sides and the Included Angle

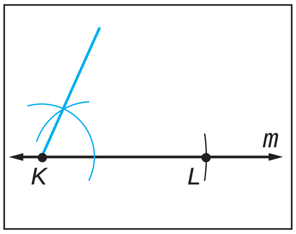
Step 1 Draw a triangle and label its vertices *A*, *B*, and *C*.



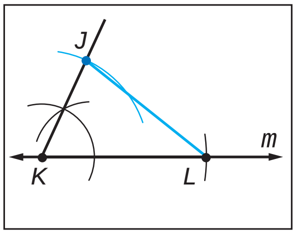
Step 2 Select a point *K* on line *m*. Use a compass to construct $\overline{KL}$ on *m* such that $\overline{KL} \cong \overline{BC}$.



Step 3 Construct an angle congruent to $\angle B$ using $\overrightarrow{KL}$ as a side of the angle and point *K* as the vertex.



Step 4 Construct $\overline{JK}$ such that $\overline{JK} \cong \overline{AB}$. Draw $\overline{JL}$ to complete $\triangle JKL$.



Step 5 Cut out $\triangle JKL$ and place it over $\triangle ABC$. How does $\triangle JKL$ compare to $\triangle ABC$?

Study Tip

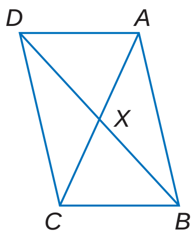
Flow Proofs
Flow proofs can be written vertically or horizontally.

EXAMPLE Use SAS in Proofs

Write a flow proof.

Given: *X* is the midpoint of $\overline{BD}$.
X is the midpoint of $\overline{AC}$.

Prove: $\triangle DXC \cong \triangle BXA$



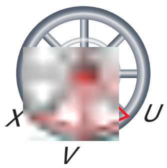
Flow Proof:

X is the midpoint of $\overline{DB}$. (Given) → $\overline{DX} \cong \overline{BX}$ (Midpoint Theorem) → $\triangle DXC \cong \triangle BXA$

X is the midpoint of $\overline{AC}$. (Given) → $\overline{CX} \cong \overline{AX}$ (Midpoint Theorem) → $\triangle DXC \cong \triangle BXA$ (SAS)

$\angle DXC \cong \angle BXA$ (Vertical ∠s are ≅.) → $\triangle DXC \cong \triangle BXA$

3. The spokes used in a captain's wheel divide the wheel into eight parts. If $\overline{TU} \cong \overline{TX}$ and $\angle XTV \cong \angle UTV$, show that $\triangle XTV \cong \triangle UTV$.



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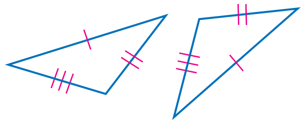
EXAMPLE 4 Identify Congruent Triangles

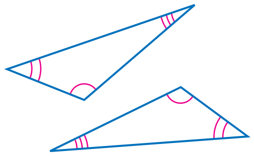
Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

a.



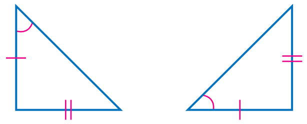
Each pair of corresponding sides are congruent. The triangles are congruent by the SSS Postulate.

b.

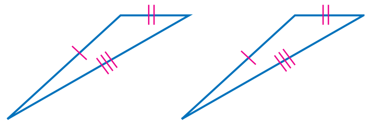


The triangles have three pairs of corresponding angles congruent. This does not match the SSS or the SAS Postulate. It is *not possible* to prove them congruent.

4A.



4B.



CHECK Your Understanding

Example 1 (p. 226)

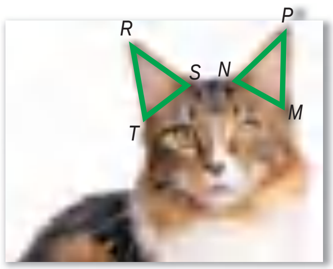
1. **JETS** The United States Navy Flight Demonstration Squadron, the Blue Angels, fly in a formation that can be viewed as two triangles with a common side. Write a two-column proof to prove that $\triangle SRT \cong \triangle QRT$ if T is the midpoint of $\overline{SQ}$ and $\overline{SR} \cong \overline{QR}$.

Example 2 (p. 227)

Determine whether $\triangle EFG \cong \triangle MNP$ given the coordinates of the vertices. Explain.

2. $E(-4, -3)$, $F(-2, 1)$, $G(-2, -3)$, $M(4, -3)$, $N(2, 1)$, $P(2, -3)$

3. $E(-2, -2)$, $F(-4, 6)$, $G(-3, 1)$, $M(2, 2)$, $N(4, 6)$, $P(3, 1)$



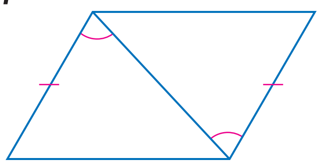
Example 3 (p. 228)

4. **CATS** A cat's ear is triangular in shape. Write a proof to prove $\triangle RST \cong \triangle PNM$ if $\overline{RS} \cong \overline{PN}$, $\overline{RT} \cong \overline{PM}$, $\angle S \cong \angle N$, and $\angle T \cong \angle M$.

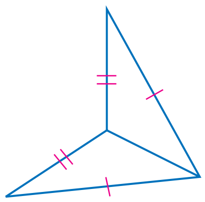
Example 4 (p. 229)

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

5.



6.

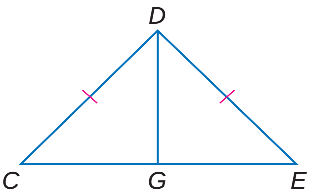


Exercises

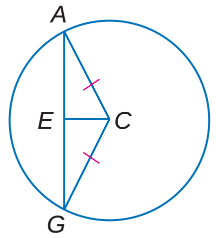
HOMEWORK HELP	
For Exercises	See Examples
7, 8	1
9–12	2
13, 14	3
15–18	4

PROOF For Exercises 7 and 8, write a two-column proof.

7. **Given:** $\triangle CDE$ is an isosceles triangle. G is the midpoint of $\overline{CE}$.
   Prove: $\triangle CDG \cong \triangle EDG$



8. **Given:** $\overline{AC} \cong \overline{GC}$
   $\overline{EC}$ bisects $\overline{AG}$.
   Prove: $\triangle GEC \cong \triangle AEC$

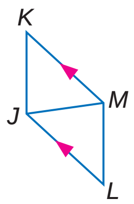


Determine whether $\triangle JKL \cong \triangle FGH$ given the coordinates of the vertices. Explain.

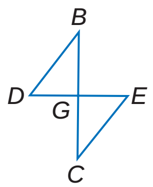
9. $J(2, 5)$, $K(5, 2)$, $L(1, 1)$, $F(-4, 4)$, $G(-7, 1)$, $H(-3, 0)$
10. $J(-1, 1)$, $K(-2, -2)$, $L(-5, -1)$, $F(2, -1)$, $G(3, -2)$, $H(2, 5)$
11. $J(-1, -1)$, $K(0, 6)$, $L(2, 3)$, $F(3, 1)$, $G(5, 3)$, $H(8, 1)$
12. $J(3, 9)$, $K(4, 6)$, $L(1, 5)$, $F(1, 7)$, $G(2, 4)$, $H(-1, 3)$

PROOF For Exercises 13 and 14, write the specified type of proof.

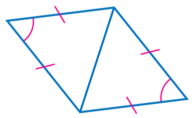
13. flow proof
    Given: $\overline{KM} \parallel \overline{LJ}$, $\overline{KM} \cong \overline{LJ}$
    Prove: $\triangle JKM \cong \triangle MLJ$

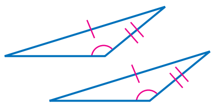


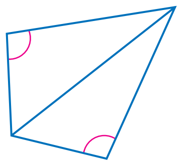
14. two-column proof
    Given: $\overline{DE}$ and $\overline{BC}$ bisect each other.
    Prove: $\triangle DGB \cong \triangle EGC$

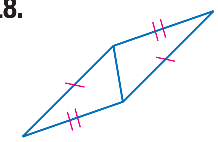


Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

15. 

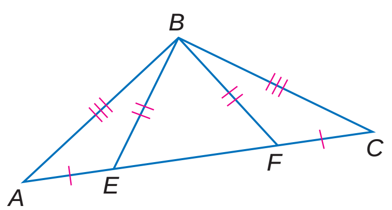
16. 

17. 

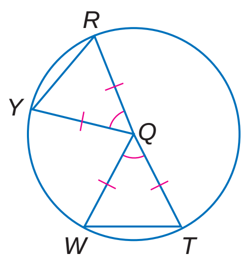
18. 

PROOF For Exercises 19 and 20, write a flow proof.

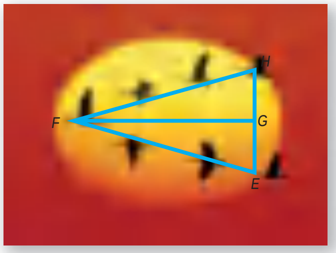
19. **Given:** $\overline{AE} \cong \overline{CF}$, $\overline{AB} \cong \overline{CB}$, $\overline{BE} \cong \overline{BF}$
    Prove: $\triangle AFB \cong \triangle CEB$



20. **Given:** $\overline{RQ} \cong \overline{TQ} \cong \overline{YQ} \cong \overline{WQ}$
    $\angle RQY \cong \angle WQT$
    Prove: $\triangle QWT \cong \triangle QYR$



21. **GEESE** A flock of geese flies in formation. Write a proof to prove that $\triangle EFG \cong \triangle HFG$ if $\overline{EF} \cong \overline{HF}$ and G is the midpoint of $\overline{EH}$.

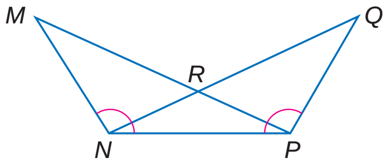


Real-World Link The infield is a square 90 feet on each side.

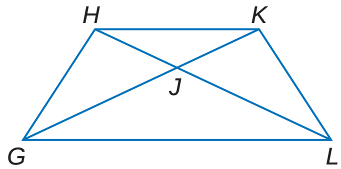
Source: mlb.com

PROOF **For Exercises 22 and 23, write a two-column proof.**

22. **Given:** $\triangle MRN \cong \triangle QRP$
$\angle MNP \cong \angle QPN$
Prove: $\triangle MNP \cong \triangle QPN$



23. **Given:** $\triangle GHJ \cong \triangle LKJ$
Prove: $\triangle GHL \cong \triangle LKG$



BASEBALL **For Exercises 24 and 25, use the following information.**
A baseball diamond is a square with four right angles and all sides congruent.

24. Write a two-column proof to prove that the distance from first base to third base is the same as the distance from home plate to second base.

25. Write a two-column proof to prove that the angle formed between second base, home plate, and third base is the same as the angle formed between second base, home plate, and first base.

EXTRA PRACTICE
See pages 818, 831.

Math Online
Self-Check Quiz at geometryonline.com

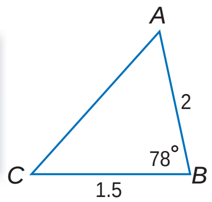
H.O.T. Problems

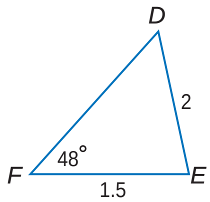
26. **REASONING** Explain how the SSS postulate can be used to prove that two triangles are congruent.

27. **OPEN ENDED** Find two triangles in a newspaper or magazine and show that they are congruent.

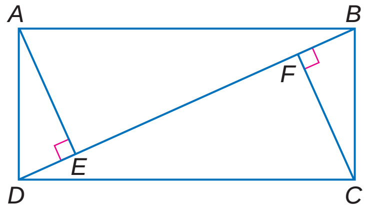
28. **FIND THE ERROR** Carmelita and Jonathan are trying to determine whether $\triangle ABC$ is congruent to $\triangle DEF$. Who is correct and why?

Carmelita	Jonathan
△ABC ≅ △DEF by SAS	Congruence cannot be determined.



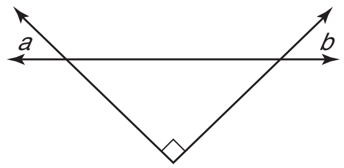


29. **CHALLENGE** Devise a plan and write a two-column proof for the following.
Given: $\overline{DE} \cong \overline{FB}$, $\overline{AE} \cong \overline{FC}$,
$\overline{AE} \perp \overline{DB}$, $\overline{CF} \perp \overline{DB}$
Prove: $\triangle ABD \cong \triangle CDB$



30. **Writing in Math** Describe two different methods that could be used to prove that two triangles are congruent.

31. Which of the following statements about the figure is true?



A $a + b < 90$

B $a + b > 90$

C $a + b = 90$

D $a + b = 45$

32. REVIEW The Murphy family just drove 300 miles to visit their grandparents. Mr. Murphy drove 70 mph for 65% of the trip and 35 mph or less for 20% of the trip that was left. Assuming that Mr. Murphy never went over 70 mph, how many miles did he travel at a speed between 35 and 70 mph?

F 195

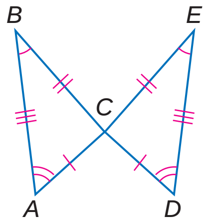
G 84

H 21

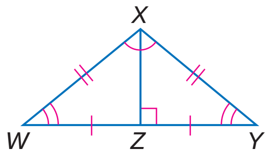
J 18

Identify the congruent triangles in each figure. (Lesson 4-3)

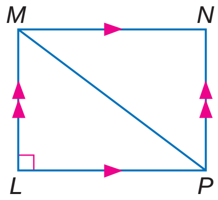
33.



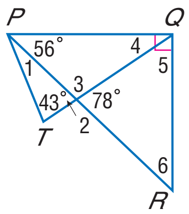
34.



35.



Find each measure if $\overline{PQ} \perp \overline{QR}$. (Lesson 4-2)



36. $m\angle 2$

37. $m\angle 3$

38. $m\angle 5$

39. $m\angle 4$

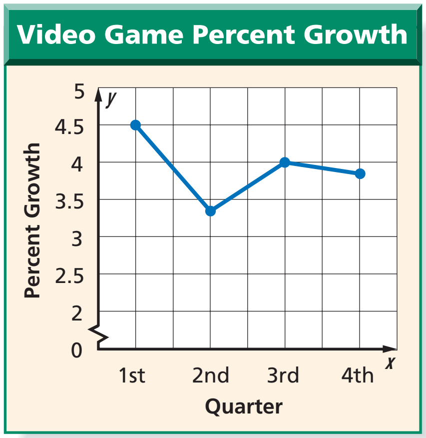
40. $m\angle 1$

41. $m\angle 6$

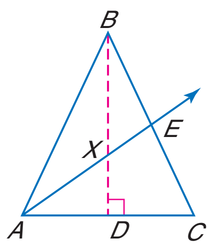
ANALYZE GRAPHS For Exercises 42 and 43, use the graph of sales of a certain video game system in a recent year. (Lesson 3-3)

42. Find the rate of change from first quarter to the second quarter.

43. Which had the greater rate of change: first quarter to second quarter, or third to fourth?



PREREQUISITE SKILL $\overline{BD}$ and $\overline{AE}$ are angle bisectors and segment bisectors. Name the indicated segments and angles. (Lessons 1-5 and 1-6)



44. segment congruent to $\overline{EC}$

45. angle congruent to $\angle ABD$

46. angle congruent to $\angle BDC$

47. segment congruent to $\overline{AD}$

48. angle congruent to $\angle BAE$

49. angle congruent to $\angle BXA$

4 Mid-Chapter Quiz

Lessons 4-1 through 4-4

1. **MULTIPLE CHOICE** Classify $\triangle ABC$ with vertices $A(-1, 1)$, $B(1, 3)$, and $C(3, -1)$. (Lesson 4-1)

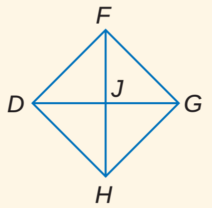
   A scalene acute

   B equilateral

   C isosceles acute

   D isosceles right

2. Identify the isosceles triangles in the figure, if $\overline{FH}$ and $\overline{DG}$ are congruent perpendicular bisectors. (Lesson 4-1)

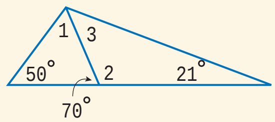


$\triangle ABC$ is equilateral with $AB = 2x$, $BC = 4x - 7$, and $AC = x + 3.5$. (Lesson 4-1)

3. Find x.
4. Find the measure of each side.

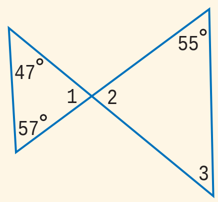
Find the measure of each angle listed below. (Lesson 4-2)

5. $m\angle 1$
6. $m\angle 2$
7. $m\angle 3$

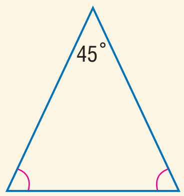


Find each measure. (Lesson 4-2)

8. $m\angle 1$
9. $m\angle 2$
10. $m\angle 3$



11. Find the missing angle measures. (Lesson 4-2)



12. If $\triangle MNP \cong \triangle JKL$, name the corresponding congruent angles and sides. (Lesson 4-3)

13. **MULTIPLE CHOICE** **Given:** $\triangle ABC \cong \triangle XYZ$. Which of the following *must* be true? (Lesson 4-3)

    F $\angle A \cong \angle Y$

    G $\overline{AC} \cong \overline{XZ}$

    H $\overline{AB} \cong \overline{YZ}$

    J $\angle Z \cong \angle B$

COORDINATE GEOMETRY **The vertices of $\triangle JKL$ are $J(7, 7)$, $K(3, 7)$, $L(7, 1)$. The vertices of $\triangle J'K'L'$ are $J'(7, -7)$, $K'(3, -7)$, $L'(7, -1)$.** (Lesson 4-3)

14. Verify that $\triangle JKL \cong \triangle J'K'L'$.
15. Name the congruence transformation for $\triangle JKL$ and $\triangle J'K'L'$.

16. Determine whether $\triangle JML \cong \triangle BDG$ given that $J(-4, 5)$, $M(-2, 6)$, $L(-1, 1)$, $B(-3, -4)$, $D(-4, -2)$, and $G(1, -1)$. (Lesson 4-4)

Determine whether $\triangle XYZ \cong \triangle TUV$ given the coordinates of the vertices. Explain. (Lesson 4-4)

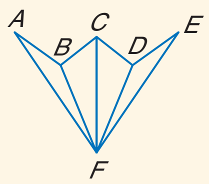
17. $X(0, 0)$, $Y(3, 3)$, $Z(0, 3)$, $T(-6, -6)$, $U(-3, -3)$, $V(-3, -6)$
18. $X(7, 0)$, $Y(5, 4)$, $Z(1, 1)$, $T(-5, -4)$, $U(-3, 4)$, $V(1,1)$
19. $X(9, 6)$, $Y(3, 7)$, $Z(9, -6)$, $T(-10, 7)$, $U(-4, 7)$, $V(-10, -7)$

Write a two-column proof. (Lesson 4-4)

20. **Given:** $\triangle ABF \cong \triangle EDF$

    $\overline{CF}$ is angle bisector of $\angle DFB$.

    Prove: $\triangle BCF \cong \triangle DCF$.



4-5 Proving Congruence—ASA, AAS

Main Ideas

- Use the ASA Postulate to test for triangle congruence.
- Use the AAS Theorem to test for triangle congruence.

New Vocabulary

included side

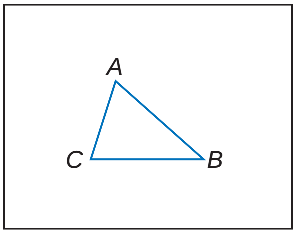
The Bank of China Tower in Hong Kong has triangular trusses for structural support. These trusses form congruent triangles. In this lesson, we will explore two additional methods of proving triangles congruent.

ASA Postulate Suppose you were given the measures of two angles of a triangle and the side between them, the **included side**. Do these measures form a unique triangle?

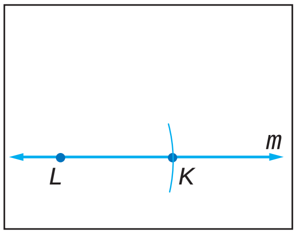
CONSTRUCTION

Congruent Triangles Using Two Angles and Included Side

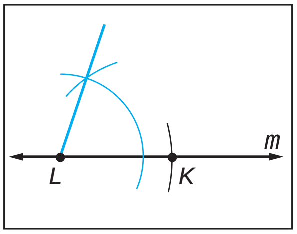
Step 1 Draw a triangle and label its vertices A, B, and C.



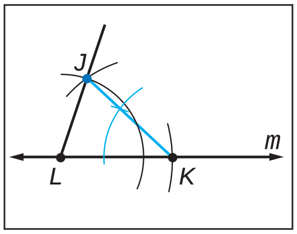
Step 2 Draw any line m and select a point L. Construct $\overline{LK}$ such that $\overline{LK} \cong \overline{CB}$.



Step 3 Construct an angle congruent to $\angle C$ at L using $\overrightarrow{LK}$ as a side of the angle.



Step 4 Construct an angle congruent to $\angle B$ at K using $\overrightarrow{LK}$ as a side of the angle. Label the point where the new sides of the angles meet J.



Step 5 Cut out $\triangle JKL$ and place it over $\triangle ABC$. How does $\triangle JKL$ compare to $\triangle ABC$?

This construction leads to the Angle-Side-Angle Postulate, written as ASA.

Sylvain Grandadam/Photo Researchers

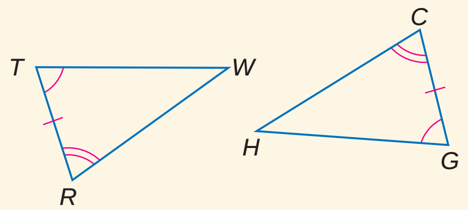
Reading Math

Included Side The *included side* refers to the side that each of the angles share.

POSTULATE 4.3 Angle-Side-Angle Congruence

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Abbreviation: *ASA*



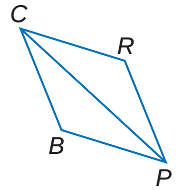
$\triangle RTW \cong \triangle CGH$

EXAMPLE Use ASA in Proofs

1 Write a paragraph proof.

Given: $\overline{CP}$ bisects $\angle BCR$ and $\angle BPR$.

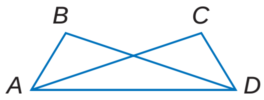
Prove: $\triangle BCP \cong \triangle RCP$



Proof: Since $\overline{CP}$ bisects $\angle BCR$ and $\angle BPR$, $\angle BCP \cong \angle RCP$ and $\angle BPC \cong \angle RPC$. $\overline{CP} \cong \overline{CP}$ by the Reflexive Property. By ASA, $\triangle BCP \cong \triangle RCP$.

1. Given: $\angle CAD \cong \angle BDA$ and $\angle CDA \cong \angle BAD$

Prove: $\triangle ABD \cong \triangle DCA$



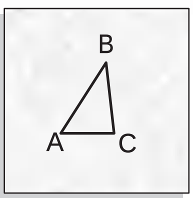
AAS Theorem Suppose you are given the measures of two angles and a nonincluded side. Is this information sufficient to prove two triangles congruent?

GEOMETRY LAB

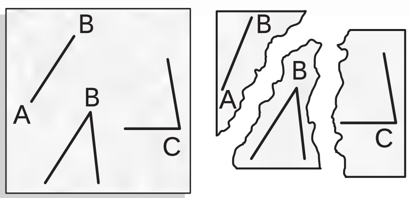
Angle-Angle-Side Congruence

MODEL

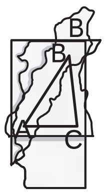
Step 1 Draw a triangle on a piece of patty paper. Label the vertices *A*, *B*, and *C*.



Step 2 Copy $\overline{AB}$, $\angle B$, and $\angle C$ on another piece of patty paper and cut them out.



Step 3 Assemble them to form a triangle in which the side is not the included side of the angles.



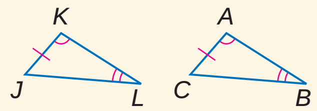
ANALYZE

1. Place the original $\triangle ABC$ over the assembled figure. How do the two triangles compare?
2. Make a conjecture about two triangles with two angles and the nonincluded side of one triangle congruent to two angles and the nonincluded side of the other triangle.

This lab leads to the Angle-Angle-Side Theorem, written as AAS.

THEOREM 4.5 ***Angle-Angle-Side Congruence***

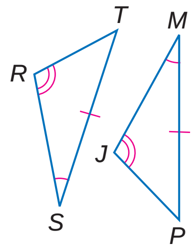
If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.



Abbreviation: *AAS*

Example: $\triangle JKL \cong \triangle CAB$

PROOF **Theorem 4.5**



Given: $\angle M \cong \angle S$, $\angle J \cong \angle R$, $\overline{MP} \cong \overline{ST}$

Prove: $\triangle JMP \cong \triangle RST$

Proof:

Statements	Reasons
1. $\angle M \cong \angle S$, $\angle J \cong \angle R$, $\overline{MP} \cong \overline{ST}$	**1.** Given
2. $\angle P \cong \angle T$	**2.** Third Angle Theorem
3. $\triangle JMP \cong \triangle RST$	**3.** ASA

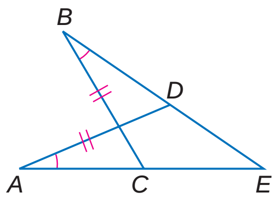
EXAMPLE **Use AAS in Proofs**

Study Tip

Overlapping Triangles

When triangles overlap, it is a good idea to draw each triangle separately and label the congruent parts.

2 **Write a flow proof.**



Given: $\angle EAD \cong \angle EBC$

$\overline{AD} \cong \overline{BC}$

Prove: $\overline{AE} \cong \overline{BE}$

Flow Proof:

$\angle EAD \cong \angle EBC$ — **Given**

$\overline{AD} \cong \overline{BC}$ — **Given**

$\angle E \cong \angle E$ — Reflexive Property

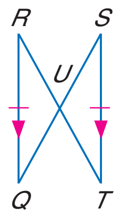
$\rightarrow$ $\triangle ADE \cong \triangle BCE$ — AAS

$\rightarrow$ $\overline{AE} \cong \overline{BE}$ — CPCTC

2. Write a flow proof.

Given: $\overline{RQ} \cong \overline{ST}$ and $\overline{RQ} \parallel \overline{ST}$

Prove: $\triangle RUQ \cong \triangle TUS$



You have learned several methods for proving triangle congruence. The Concept Summary lists ways to help you determine which method to use.

CONCEPT SUMMARY

Method	Use when. . .
Definition of Congruent Triangles	All corresponding parts of one triangle are congruent to the corresponding parts of the other triangle.
SSS	The three sides of one triangle are congruent to the three sides of the other triangle.
SAS	Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle.
ASA	Two angles and the included side of one triangle are congruent to two angles and the included side of the other triangle.
AAS	Two angles and a nonincluded side of one triangle are congruent to two angles and side of the other triangle.

Real-World Career

Architect

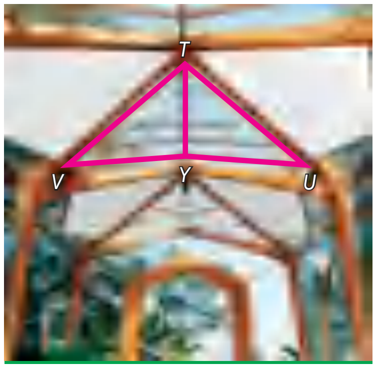
About 28% of architects are self-employed. Architects design a variety of buildings including offices, retail spaces, and schools.

Math Online

For more information, go to geometryonline.com.

Real-World EXAMPLE — Determine if Triangles Are Congruent

3 ARCHITECTURE This glass chapel was designed by Frank Lloyd Wright's son, Lloyd Wright. Suppose the redwood supports, $\overline{TU}$ and $\overline{TV}$, measure 3 feet, TY = 1.6 feet, and $m\angle U$ and $m\angle V$ are 31. Determine whether $\triangle TYU \cong \triangle TYV$. Justify your answer.

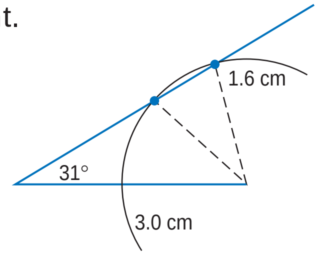


Explore We are given three measurements of each triangle. We need to determine whether the two triangles are congruent.

Plan Since $m\angle U = m\angle V$, $\angle U \cong \angle V$. Likewise, $TU = TV$ so $\overline{TU} \cong \overline{TV}$, and $TY = TY$ so $\overline{TY} \cong \overline{TY}$. Check each possibility using the five methods you know.

Solve We are given information about side-side-angle (SSA). This is not a method to prove two triangles congruent.

Check Use a compass, protractor, and ruler to draw a triangle with the given measurements. For space purposes, use centimeters instead of feet.



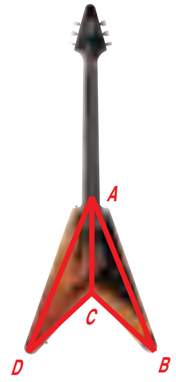
- Draw a segment 3.0 centimeters long.
- At one end, draw an angle of 31°. Extend the line longer than 3.0 centimeters.
- At the other end, draw an arc with a radius of 1.6 centimeters such that it intersects the line.

Notice that there are two possible segments that could determine the triangle. Since the given measurements do not lead to a unique triangle, we cannot show that the triangles are congruent.

(continued on the next page)

Interactive Lab
geometryonline.com

3. A flying V guitar is made up of two triangles. If $AB = 27$ inches, $AD = 27$ inches, $DC = 7$ inches, and $CB = 7$ inches, determine whether $\triangle ADC \cong \triangle ABC$. Explain.



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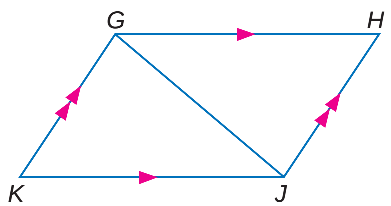
Check Your Understanding

Example 1 (p. 235)

PROOF For Exercises 1–4, write the specified type of proof.

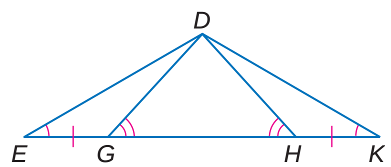
1. flow proof

Given: $\overline{GH} \parallel \overline{KJ}$, $\overline{GK} \parallel \overline{HJ}$
Prove: $\triangle GJK \cong \triangle JGH$



2. paragraph proof

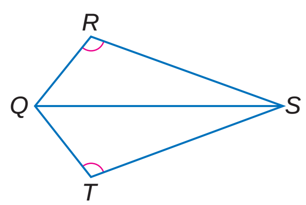
Given: $\angle E \cong \angle K$, $\angle DGH \cong \angle DHG$
$\overline{EG} \cong \overline{KH}$
Prove: $\triangle EGD \cong \triangle KHD$



Example 2 (p. 236)

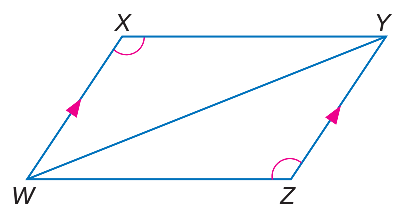
3. paragraph proof

Given: $\overline{QS}$ bisects $\angle RST$; $\angle R \cong \angle T$
Prove: $\triangle QRS \cong \triangle QTS$



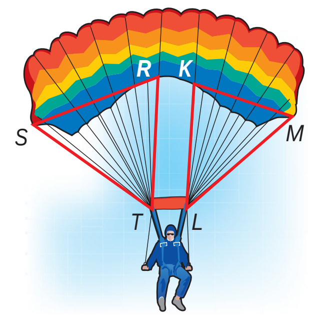
4. flow proof

Given: $\overline{XW} \parallel \overline{YZ}$, $\angle X \cong \angle Z$
Prove: $\triangle WXY \cong \triangle YZW$



Example 3 (p. 237)

5. **PARACHUTES** Suppose $\overline{ST}$ and $\overline{ML}$ each measure seven feet, $\overline{SR}$ and $\overline{MK}$ each measure 5.5 feet, and $m\angle T = m\angle L = 49$. Determine whether $\triangle SRT \cong \triangle MKL$. Justify your answer.



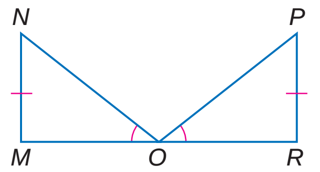
Exercises

HOMEWORK HELP	
For Exercises	See Examples
6, 7	1
8, 9	2
10, 11	3

Write a paragraph proof.

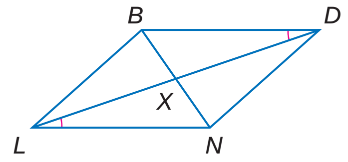
6. Given: $\angle NOM \cong \angle POR$, $\overline{NM} \perp \overline{MR}$, $\overline{PR} \perp \overline{MR}$, $\overline{NM} \cong \overline{PR}$

Prove: $\overline{MO} \cong \overline{OR}$



7. Given: $\overline{DL}$ bisects $\overline{BN}$. $\angle XLN \cong \angle XDB$

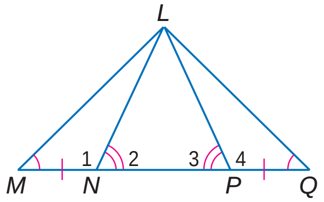
Prove: $\overline{LN} \cong \overline{DB}$



Write a flow proof.

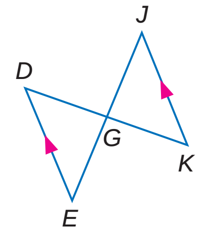
8. Given: $\overline{MN} \cong \overline{PQ}$, $\angle M \cong \angle Q$, $\angle 2 \cong \angle 3$

Prove: $\triangle MLP \cong \triangle QLN$



9. Given: $\overline{DE} \parallel \overline{JK}$, $\overline{DK}$ bisects $\overline{JE}$.

Prove: $\triangle EGD \cong \triangle JGK$



GARDENING For Exercises 10 and 11, use the following information.
Beth is planning a garden. She wants the triangular sections $\triangle CFD$ and $\triangle HFG$ to be congruent. F is the midpoint of $\overline{DG}$, and $DG = 16$ feet.

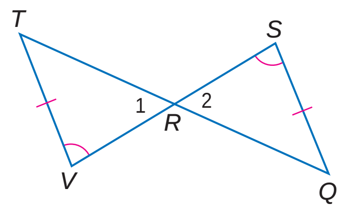
10. Suppose $\overline{CD}$ and $\overline{GH}$ each measure 4 feet and the measure of $\angle CFD$ is 29. Determine whether $\triangle CFD \cong \triangle HFG$. Justify your answer.

11. Suppose F is the midpoint of $\overline{CH}$, and $\overline{CH} \cong \overline{DG}$. Determine whether $\triangle CFD \cong \triangle HFG$. Justify your answer.

Write a flow proof.

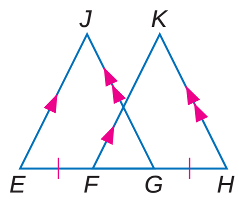
12. Given: $\angle V \cong \angle S$, $\overline{TV} \cong \overline{QS}$

Prove: $\overline{VR} \cong \overline{SR}$



13. Given: $\overline{EJ} \parallel \overline{FK}$, $\overline{JG} \parallel \overline{KH}$, $\overline{EF} \cong \overline{GH}$

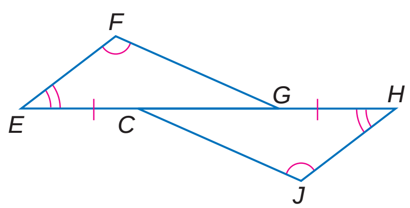
Prove: $\triangle EJG \cong \triangle FKH$



Write a paragraph proof.

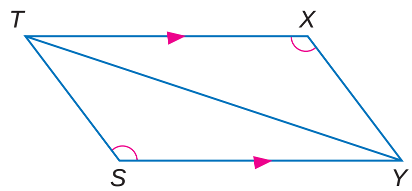
14. Given: $\angle F \cong \angle J$, $\angle E \cong \angle H$, $\overline{EC} \cong \overline{GH}$

Prove: $\overline{EF} \cong \overline{HJ}$



15. Given: $\overline{TX} \parallel \overline{SY}$, $\angle TXY \cong \angle TSY$

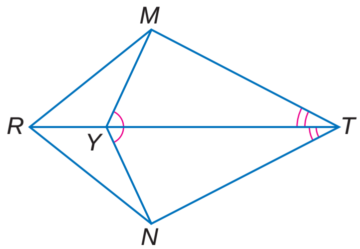
Prove: $\triangle TSY \cong \triangle YXT$



PROOF **Write a two-column proof.**

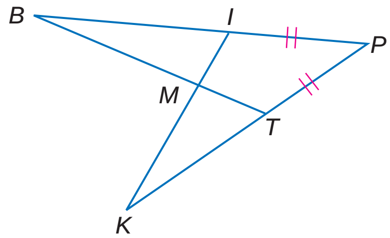
16. **Given:** $\angle MYT \cong \angle NYT$, $\angle MTY \cong \angle NTY$

Prove: $\triangle RYM \cong \triangle RYN$



17. **Given:** $\triangle BMI \cong \triangle KMT$, $\overline{IP} \cong \overline{PT}$

Prove: $\triangle IPK \cong \triangle TPB$

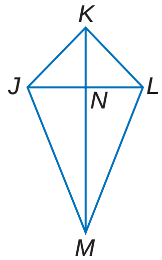


Real-World Link

The largest kite ever flown was 210 feet long and 72 feet wide.

Source: *Guinness Book of World Records*

KITES **For Exercises 18 and 19, use the following information.** Austin is making a kite. Suppose JL is two feet, JM is 2.7 feet, and the measure of $\angle NJM$ is 68.



18. If N is the midpoint of $\overline{JL}$ and $\overline{KM} \perp \overline{JL}$, determine whether $\triangle JKN \cong \triangle LKN$. Justify your answer.

19. If $\overline{JM} \cong \overline{LM}$ and $\angle NJM \cong \angle NLM$, determine whether $\triangle JNM \cong \triangle LNM$. Justify your answer.

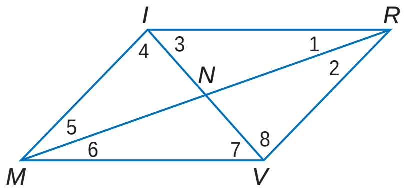
EXTRA PRACTICE

See pages 808, 831.

Math Online

Self-Check Quiz at geometryonline.com

Complete each congruence statement and the postulate or theorem that applies.



20. If $\overline{IM} \cong \overline{RV}$ and $\angle 2 \cong \angle 5$, then $\triangle INM \cong \triangle$ __?__ by __?__.

21. If $\overline{IR} \parallel \overline{MV}$ and $\overline{IR} \cong \overline{MV}$, then $\triangle IRN \cong \triangle$ __?__ by __?__.

H.O.T. Problems

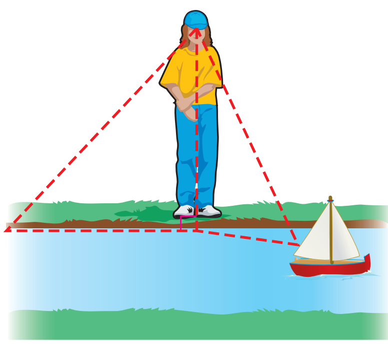
22. **Which One Doesn't Belong?** Identify the term that does not belong with the others. Explain your reasoning.

ASA	SSS	SSA	AAS

23. **REASONING** Find a counterexample to show why AAA (Angle-Angle-Angle) cannot be used to prove congruence in triangles.

24. **OPEN ENDED** Draw and label two triangles that could be proved congruent by SAS.

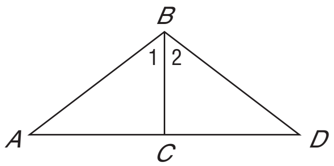
25. **CHALLENGE** Neva wants to estimate the distance between herself and a toy boat. She adjusts the visor of her cap so that it is in line with her line of sight to the toy boat. She keeps her neck stiff and turns her body to establish a line of sight to a point on the ground. Then she paces out the distance to the new point. Is the distance from the toy boat the same as the distance she just paced out? Explain your reasoning.



26. **Writing in Math** Use the information about construction on page 234 to explain how congruent triangles are used in construction. Include why it is important to use congruent triangles for support.

STANDARDIZED TEST PRACTICE

27. Given: $\overline{BC}$ is perpendicular to $\overline{AD}$; $\angle 1 \cong \angle 2$.



Which theorem or postulate could be used to prove $\triangle ABC \cong \triangle DBC$?

A AAS **C** SAS

B ASA **D** SSS

28. REVIEW Which expression can be used to find the values of $s(n)$ in the table?

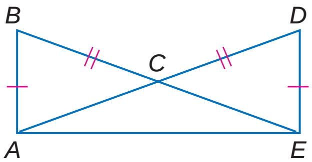
n	-8	-4	-1	0	1
$s(n)$	1.00	2.00	2.75	3.00	3.25

F $-2n + 3$ **H** $\frac{1}{4}n + 3$

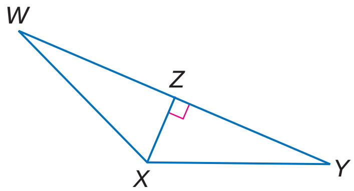
G $-n + 7$ **J** $\frac{1}{2}n + 5$

Write a flow proof. (Lesson 4-4)

29. Given: $\overline{BA} \cong \overline{DE}$, $\overline{DA} \cong \overline{BE}$
Prove: $\triangle BEA \cong \triangle DAE$

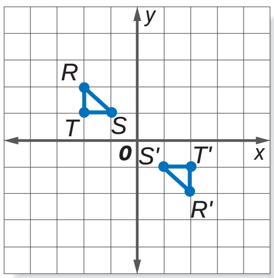


30. Given: $\overline{XZ} \perp \overline{WY}$, $\overline{XZ}$ bisects $\overline{WY}$.
Prove: $\triangle WZX \cong \triangle YZX$

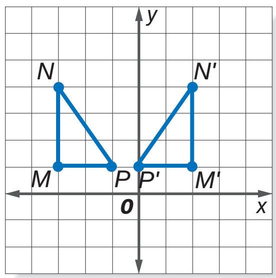


Verify congruence and name the congruence transformation. (Lesson 4-3)

31. $\triangle RTS \cong \triangle R'T'S'$



32. $\triangle MNP \cong \triangle M'N'P'$



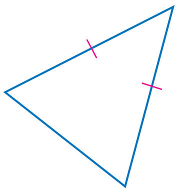
Write each statement in if-then form. (Lesson 2-3)

33. Happy people rarely correct their faults.

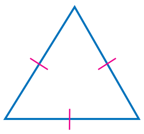
34. A champion is afraid of losing.

PREREQUISITE SKILL Classify each triangle according to its sides. (Lesson 4-1)

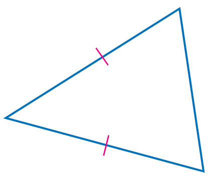
35.



36.



37.



EXTEND 4-5

Geometry Lab

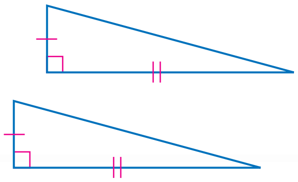
Congruence in Right Triangles

In Lessons 4-4 and 4-5, you learned theorems and postulates to prove triangles congruent. Do these theorems and postulates apply to right triangles?

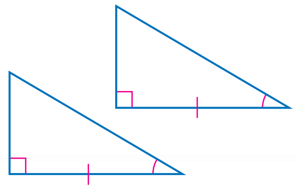
ACTIVITY 1 Triangle Congruence

Study each pair of right triangles.

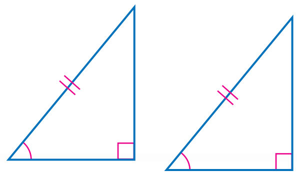
a.



b.



c.



ANALYZE THE RESULTS

1. Is each pair of triangles congruent? If so, which congruence theorem or postulate applies?
2. Rewrite the congruence rules from Exercise 1 using *leg*, (L), or *hypotenuse*, (H), to replace *side*. Omit the A for any right angle since we know that all right triangles contain a right angle and all right angles are congruent.
3. **MAKE A CONJECTURE** If you know that the corresponding legs of two right triangles are congruent, what other information do you need to declare the triangles congruent? Explain.

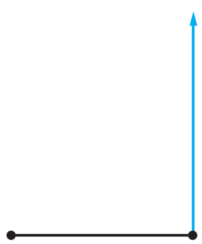
In Lesson 4-5, you learned that SSA is not a valid test for determining triangle congruence. Can SSA be used to prove right triangles congruent?

ACTIVITY 2 SSA and Right Triangles

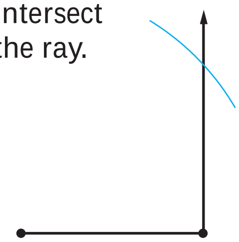
How many right triangles exist that have a hypotenuse of 10 centimeters and a leg of 7 centimeters?

Step 1
Draw $\overline{XY}$ so that $XY = 7$ centimeters.

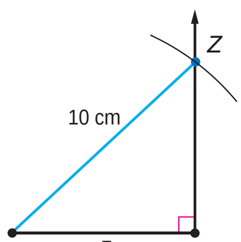
Step 2
Use a protractor to draw a ray from Y that is perpendicular to $\overline{XY}$.



Step 3
Open your compass to a width of 10 centimeters. Place the point at X and draw a long arc to intersect the ray.



Step 4
Label the intersection Z and draw $\overline{XZ}$ to complete $\triangle XYZ$.

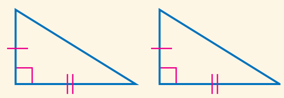
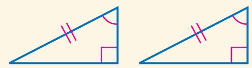
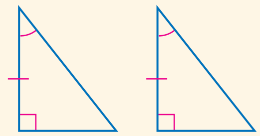
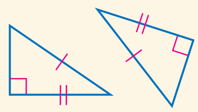


ANALYZE THE RESULTS

4. Does the model yield a unique triangle?
5. Can you use the lengths of the hypotenuse and a leg to show right triangles are congruent?
6. **Make a conjecture** about the case of SSA that exists for right triangles.

The two activities provide evidence for four ways to prove right triangles congruent.

KEY CONCEPT — *Right Triangle Congruence*

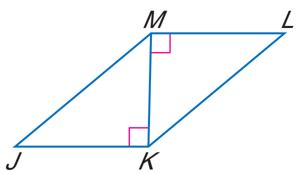
Theorems	Abbreviation	Example
4.6 Leg-Leg Congruence If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.	LL	
4.7 Hypotenuse-Angle Congruence If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the two triangles are congruent.	HA	
4.8 Leg-Angle Congruence If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.	LA	
Postulate		
4.4 Hypotenuse-Leg Congruence If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.	HL	

EXERCISES

PROOF Write a paragraph proof of each theorem.

7. Theorem 4.6
8. Theorem 4.7
9. Theorem 4.8 (*Hint*: There are two possible cases.)

Use the figure to write a two-column proof.



10. Given: $\overline{ML} \perp \overline{MK}$, $\overline{JK} \perp \overline{KM}$
$\angle J \cong \angle L$
Prove: $\overline{JM} \cong \overline{KL}$

11. Given: $\overline{JK} \perp \overline{KM}$, $\overline{JM} \cong \overline{KL}$
$\overline{ML} \parallel \overline{JK}$
Prove: $\overline{ML} \cong \overline{JK}$

4-6 Isosceles Triangles

Main Ideas

- Use properties of isosceles triangles.
- Use properties of equilateral triangles.

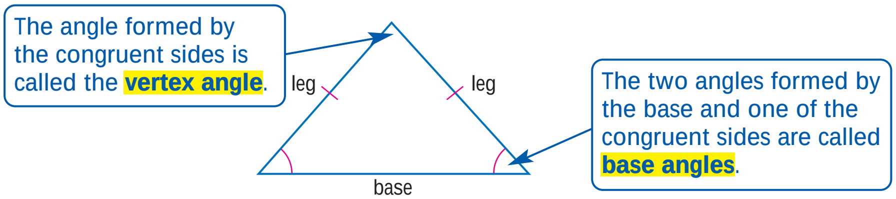
New Vocabulary

vertex angle
base angles

GET READY for the Lesson

The art of Lois Mailou Jones, a twentieth-century artist, includes paintings and textile design, as well as book illustration. Notice the isosceles triangles in this painting, *Damballah*.

Properties of Isosceles Triangles In Lesson 4-1, you learned that isosceles triangles have two congruent sides. Like the right triangle, the parts of an isosceles triangle have special names.

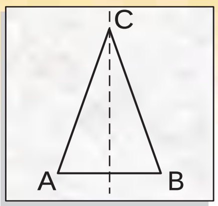


GEOMETRY LAB

Isosceles Triangles

MODEL

- Draw an acute triangle on patty paper with $\overline{AC} \cong \overline{BC}$.
- Fold the triangle through C so that A and B coincide.



ANALYZE

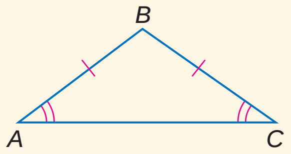
1. What do you observe about $\angle A$ and $\angle B$?
2. Draw an obtuse isosceles triangle. Compare the base angles.
3. Draw a right isosceles triangle. Compare the base angles.

The results of the Geometry Lab suggest Theorem 4.9.

THEOREM 4.9 Isosceles Triangle

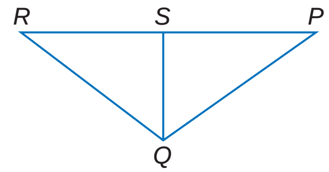
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Example: If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.



EXAMPLE 1 Proof of Theorem

Write a two-column proof of the Isosceles Triangle Theorem.



Given: $\angle PQR$, $\overline{PQ} \cong \overline{RQ}$

Prove: $\angle P \cong \angle R$

Proof:

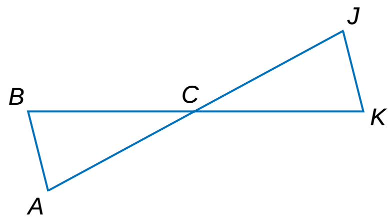
Statements	Reasons
1. Let S be the midpoint of $\overline{PR}$.	**1.** Every segment has exactly one midpoint.
2. Draw an auxiliary segment $\overline{QS}$	**2.** Two points determine a line.
3. $\overline{PS} \cong \overline{RS}$	**3.** Midpoint Theorem
4. $\overline{QS} \cong \overline{QS}$	**4.** Congruence of segments is reflexive.
5. $\overline{PQ} \cong \overline{RQ}$	**5.** Given
6. $\triangle PQS \cong \triangle RQS$	**6.** SSS
7. $\angle P \cong \angle R$	**7.** CPCTC

1. Write a two-column proof.

Given: $\overline{CA} \cong \overline{BC}$; $\overline{KC} \cong \overline{CJ}$

C is the midpoint of $\overline{BK}$.

Prove: $\triangle ABC \cong \triangle JKC$



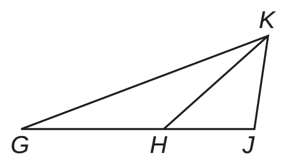
Find a Missing Angle Measure

Test-Taking Tip

Diagrams Label the diagram with the given information. Use your drawing to plan the next step in solving the problem.

If $\overline{GH} \cong \overline{HK}$, $\overline{HJ} \cong \overline{JK}$, and $m\angle GJK = 100$, what is $m\angle HGK$?

A 10 **B** 15 **C** 20 **D** 25



Read the Test Item

$\triangle GHK$ is isosceles with base $\overline{GK}$. Likewise, $\triangle HJK$ is isosceles with base $\overline{HK}$.

(continued on the next page)

Solve the Test Item

Step 1 The base angles of $\triangle HJK$ are congruent. Let $x = m\angle KHJ = m\angle HKJ$.

$$\begin{aligned} m\angle KHJ + m\angle HKJ + m\angle HJK &= 180 && \text{Angle Sum Theorem} \\ x + x + 100 &= 180 && \text{Substitution} \\ 2x + 100 &= 180 && \text{Add.} \\ 2x &= 80 && \text{Subtract 100 from each side.} \\ x &= 40 && \text{So, } m\angle KHJ = m\angle HKJ = 40. \end{aligned}$$

Step 2 $\angle GHK$ and $\angle KHJ$ form a linear pair. Solve for $m\angle GHK$.

$$\begin{aligned} m\angle KHJ + m\angle GHK &= 180 && \text{Linear pairs are supplementary.} \\ 40 + m\angle GHK &= 180 && \text{Substitution} \\ m\angle GHK &= 140 && \text{Subtract 40 from each side.} \end{aligned}$$

Step 3 The base angles of $\triangle GHK$ are congruent. Let y represent $m\angle HGK$ and $m\angle GKH$.

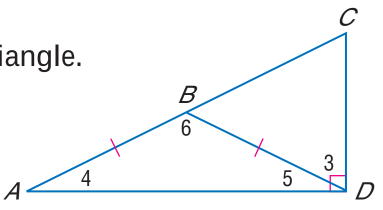
$$\begin{aligned} m\angle GHK + m\angle HGK + m\angle GKH &= 180 && \text{Angle Sum Theorem} \\ 140 + y + y &= 180 && \text{Substitution} \\ 140 + 2y &= 180 && \text{Add.} \\ 2y &= 40 && \text{Subtract 140 from each side.} \\ y &= 20 && \text{Divide each side by 2.} \end{aligned}$$

The measure of $\angle HGK$ is 20. Choice C is correct.

2. $\triangle ABD$ is isosceles, and $\triangle ACD$ is a right triangle. If $m\angle 6 = 136$, what is $m\angle 3$?

F 21 **H** 68

G 37 **J** 113



Personal Tutor at geometryonline.com

Study Tip

Look Back
You can review **converses** in Lesson 2-3.

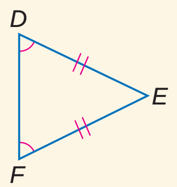
The converse of the Isosceles Triangle Theorem is also true.

THEOREM 4.10

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Abbreviation: *Conv. of Isos. △ Th.*

Example: If $\angle D \cong \angle F$, then $\overline{DE} \cong \overline{FE}$.



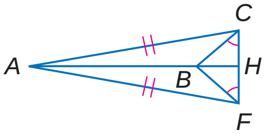
You will prove Theorem 4.10 in Exercise 13.

> **Cross-Curricular Project**
>
> You can use properties of triangles to prove Thales of Miletus' important geometric ideas. Visit geometryonline.com to continue work on your project.

EXAMPLE 3 Congruent Segments and Angles

a. Name two congruent angles.

$\angle AFC$ is opposite $\overline{AC}$ and $\angle ACF$ is opposite $\overline{AF}$, so $\angle AFC \cong \angle ACF$.

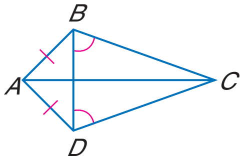


b. Name two congruent segments.

By the converse of the Isosceles Triangle Theorem, the sides opposite congruent angles are congruent. So, $\overline{BC} \cong \overline{BF}$.

3A. Name two congruent angles.

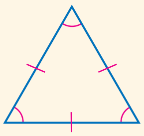
3B. Name two congruent segments.



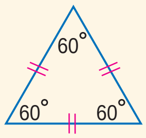
Properties of Equilateral Triangles Recall that an equilateral triangle has three congruent sides. The Isosceles Triangle Theorem leads to two corollaries about the angles of an equilateral triangle.

COROLLARIES

4.3 A triangle is equilateral if and only if it is equiangular.



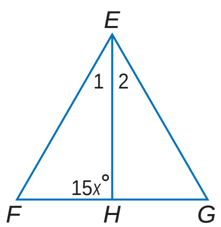
4.4 Each angle of an equilateral triangle measures 60°.



You will prove Corollaries 4.3 and 4.4 in Exercises 11 and 12.

EXAMPLE 4 Use Properties of Equilateral Triangles

$\triangle EFG$ is equilateral, and $\overline{EH}$ bisects $\angle E$.



a. Find $m\angle 1$ and $m\angle 2$.

Each angle of an equilateral triangle measures 60°. So, $m\angle 1 + m\angle 2 = 60$. Since the angle was bisected, $m\angle 1 = m\angle 2$. Thus, $m\angle 1 = m\angle 2 = 30$.

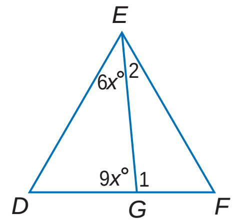
b. ALGEBRA Find x.

$m\angle EFH + m\angle 1 + m\angle EHF = 180$	Angle Sum Theorem
$60 + 30 + 15x = 180$	$m\angle EFH = 60$, $m\angle 1 = 30$, $m\angle EHF = 15x$
$90 + 15x = 180$	Add.
$15x = 90$	Subtract 90 from each side.
$x = 6$	Divide each side by 15.

$\triangle DEF$ **is equilateral.**

4A. Find x.

4B. Find $m\angle 1$ and $m\angle 2$.

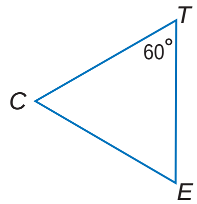


Check Your Understanding

Examples 1, 4 (pp. 245, 247)

PROOF **Write a two-column proof.**

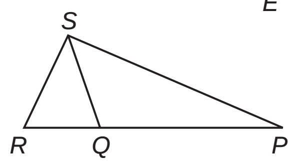
1. **Given:** $\triangle CTE$ is isosceles with vertex $\angle C$.
$m\angle T = 60$
Prove: $\triangle CTE$ is equilateral.



Example 2 (p. 246)

2. **STANDARDIZED TEST PRACTICE** If $\overline{PQ} \cong \overline{QS}$, $\overline{QR} \cong \overline{RS}$, and $m\angle PRS = 72$, what is $m\angle QPS$?

A 27 **B** 54 **C** 63 **D** 72

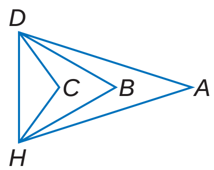


Example 3 (p. 247)

Refer to the figure.

3. If $\overline{AD} \cong \overline{AH}$, name two congruent angles.

4. If $\angle BDH \cong \angle BHD$, name two congruent segments.



Exercises

HOMEWORK HELP

For Exercises	See Examples
5–10	3
11–13	1
14, 15	4
37, 38	2

Refer to the figure for Exercises 5–10.

5. If $\overline{LT} \cong \overline{LR}$, name two congruent angles.

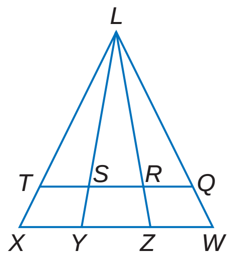
6. If $\overline{LX} \cong \overline{LW}$, name two congruent angles.

7. If $\overline{SL} \cong \overline{QL}$, name two congruent angles.

8. If $\angle LXY \cong \angle LYX$, name two congruent segments.

9. If $\angle LSR \cong \angle LRS$, name two congruent segments.

10. If $\angle LYW \cong \angle LWY$, name two congruent segments.



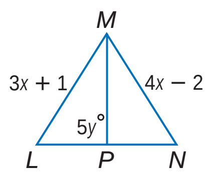
PROOF **Write a two-column proof.**

11. Corollary 4.3 **12.** Corollary 4.4 **13.** Theorem 4.10

Triangle LMN **is equilateral, and** $\overline{MP}$ **bisects** $\overline{LN}$**.**

14. Find x and y.

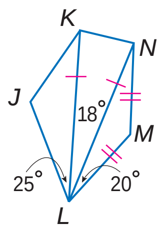
15. Find the measure of each side.



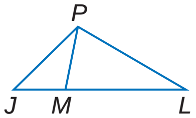
$\triangle KLN$ **and** $\triangle LMN$ **are isosceles and** $m\angle JKN = 130$**. Find each measure.**

16. $m\angle LNM$ **17.** $m\angle M$

18. $m\angle LKN$ **19.** $m\angle J$



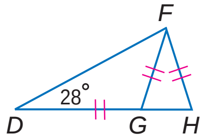
In the figure, $\overline{JM} \cong \overline{PM}$ and $\overline{ML} \cong \overline{PL}$.



20. If $m\angle PLJ = 34$, find $m\angle JPM$.

21. If $m\angle PLJ = 58$, find $m\angle PJL$.

$\triangle DFG$ and $\triangle FGH$ are isosceles, $m\angle FDH = 28$, and $\overline{DG} \cong \overline{FG} \cong \overline{FH}$. Find each measure.



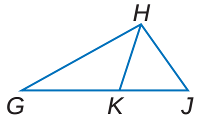
22. $m\angle DFG$

23. $m\angle DGF$

24. $m\angle FGH$

25. $m\angle GFH$

In the figure, $\overline{GK} \cong \overline{GH}$ and $\overline{HK} \cong \overline{KJ}$.



26. If $m\angle HGK = 28$, find $m\angle HJK$.

27. If $m\angle HGK = 42$, find $m\angle HKJ$.

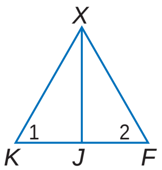
Real-World Link

Spaceship Earth is a completely spherical geodesic dome that is covered with 11,324 triangular aluminum and plastic alloy panels.

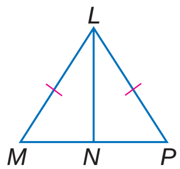
Source: disneyworld.disney.go.com

PROOF **Write a two-column proof for each of the following.**

28. Given: $\triangle XKF$ is equilateral.
$\overline{XJ}$ bisects $\angle X$.
Prove: J is the midpoint of $\overline{KF}$.



29. Given: $\triangle MLP$ is isosceles.
N is the midpoint of $\overline{MP}$.
Prove: $\overline{LN} \perp \overline{MP}$

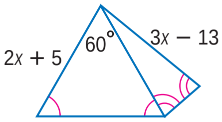


30. DESIGN The exterior of Spaceship Earth at Epcot Center in Orlando, Florida, is made up of triangles. Describe the minimum requirement to show that these triangles are equilateral.

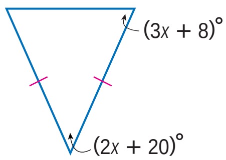
EXTRA PRACTICE
See pages 808, 831.
Math Online
Self-Check Quiz at
geometryonline.com

ALGEBRA **Find x.**

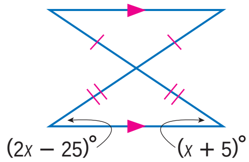
31.



32.



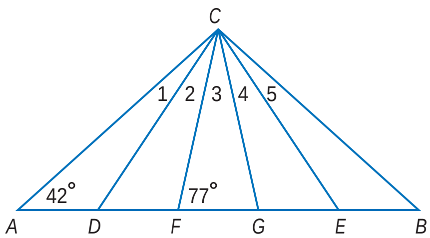
33.



H.O.T. Problems

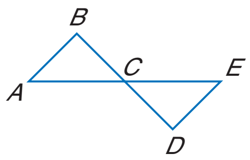
34. OPEN ENDED Describe a method to construct an equilateral triangle.

35. CHALLENGE In the figure, $\triangle ABC$ is isosceles, $\triangle DCE$ is equilateral, and $\triangle FCG$ is isosceles. Find the measures of the five numbered angles at vertex C.



36. Writing in Math Explain how triangles can be used in art. Describe at least three other geometric shapes and how they are used in art. Include an interpretation of how and why isosceles triangles are used in the painting shown at the beginning of the lesson.

37. In the figure below, $\overline{AE}$ and $\overline{BD}$ bisect each other at point C.



Which additional piece of information would be enough to prove that $\overline{CD} \cong \overline{DE}$?

A $\angle A \cong \angle C$

B $\angle B \cong \angle D$

C $\angle ACB \cong \angle EDC$

D $\angle A \cong \angle B$

38. REVIEW What quantity should be added to both sides of this equation to complete the square?

$$x^2 - 10x = 3$$

F -25

G -5

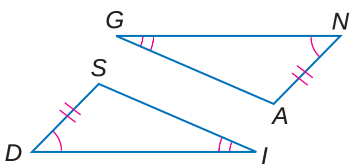
H 5

J 25

PROOF **Write a paragraph proof.** (Lesson 4-5)

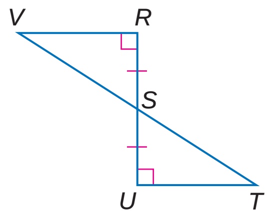
39. Given: $\angle N \cong \angle D$, $\angle G \cong \angle I$, $\overline{AN} \cong \overline{SD}$

Prove: $\triangle ANG \cong \triangle SDI$



40. Given: $\overline{VR} \perp \overline{RS}$, $\overline{UT} \perp \overline{SU}$, $\overline{RS} \cong \overline{US}$

Prove: $\triangle VRS \cong \triangle TUS$



Determine whether $\triangle QRS \cong \triangle EGH$ given the coordinates of the vertices. Explain. (Lesson 4-4)

41. $Q(-3, 1)$, $R(1, 2)$, $S(-1, -2)$, $E(6, -2)$, $G(2, -3)$, $H(4, 1)$

42. $Q(1, -5)$, $R(5, 1)$, $S(4, 0)$, $E(-4, -3)$, $G(-1, 2)$, $H(2, 1)$

43. LANDSCAPING Lucas is drawing plans for a client's backyard on graph paper. The client wants two perpendicular pathways to cross at the center of her backyard. If the center of the backyard is set at (0, 0) and the first path goes from one corner of the backyard at $(-6, 12)$ to the other corner at $(6, -12)$, at what coordinates will the second path begin and end? (Lesson 3-3)

Construct a truth table for each compound statement. (Lesson 2-2)

44. a and b

45. $\sim p$ or $\sim q$

46. k and $\sim m$

47. $\sim y$ or z

PREREQUISITE SKILL **Find the coordinates of the midpoint of the segment with endpoints that are given.** (Lesson 1-3)

48. $A(2, 15)$, $B(7, 9)$

49. $C(-4, 6)$, $D(2, -12)$

50. $E(3, 2.5)$, $F(7.5, 4)$

4-7 Triangles and Coordinate Proof

Main Ideas

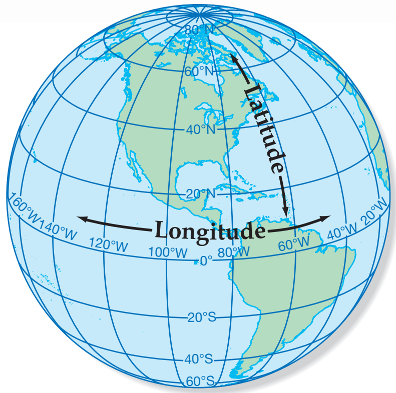
- Position and label triangles for use in coordinate proofs.
- Write coordinate proofs.

New Vocabulary

coordinate proof

GET READY for the Lesson

Navigators developed a series of circles to create a coordinate grid that allows them to determine where they are on Earth. Similar to points in coordinate geometry, locations on this grid are given two values: an east / west value (longitude) and a north / south value (latitude).



Position and Label Triangles Same as working with longitude and latitude, knowing the coordinates of points on a figure allows you to draw conclusions about it. **Coordinate proof** uses figures in the coordinate plane and algebra to prove geometric concepts. The first step in a coordinate proof is placing the figure on the coordinate plane.

Study Tip

Placement of Figures

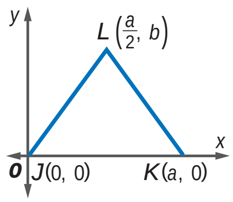
The guidelines apply to any polygon placed on the coordinate plane.

KEY CONCEPT — Placing Figures on the Coordinate Plane

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of a polygon on an axis.
3. Keep the figure within the first quadrant if possible.
4. Use coordinates that make computations as simple as possible.

EXAMPLE Position and Label a Triangle

1 **Position and label isosceles triangle *JKL* on a coordinate plane so that base $\overline{JK}$ is *a* units long.**



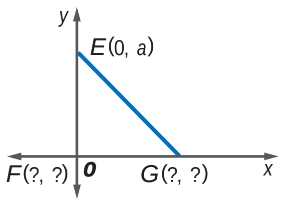
- Use the origin as vertex *J* of the triangle.
- Place the base of the triangle along the positive *x*-axis.
- Position the triangle in the first quadrant.
- Since *K* is on the *x*-axis, its *y*-coordinate is 0. Its *x*-coordinate is *a* because the base is *a* units long.
- $\triangle JKL$ is isosceles, so the *x*-coordinate of *L* is halfway between 0 and *a* or $\frac{a}{2}$. We cannot write the *y*-coordinate in terms of *a*, so call it *b*.

1. Position and label right triangle *HIJ* with legs $\overline{HI}$ and $\overline{IJ}$ on a coordinate plane so that $\overline{HI}$ is *a* units long and $\overline{IJ}$ is *b* units long.

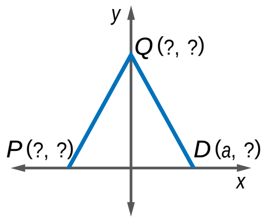
Animation
geometryonline.com

EXAMPLE 2 Find the Missing Coordinates

Name the missing coordinates of isosceles right triangle *EFG*.



Vertex *F* is positioned at the origin; its coordinates are (0, 0). Vertex *E* is on the *y*-axis, and vertex *G* is on the *x*-axis. So $\angle EFG$ is a right angle. Since $\triangle EFG$ is isosceles, $\overline{EF} \cong \overline{GF}$. *EF* is *a* units and *GF* must be the same. So, the coordinates of *G* are (*a*, 0).



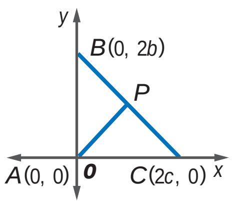
2. Name the missing coordinates of isosceles triangle *PDQ*.

Write Proofs After a figure is placed on the coordinate plane and labeled, we can coordinate proof to verify properties and to prove theorems.

EXAMPLE 3 Coordinate Proof

Write a coordinate proof to prove that the measure of the segment that joins the vertex of the right angle in a right triangle to the midpoint of the hypotenuse is one-half the measure of the hypotenuse.

Place the right angle at the origin and label it *A*. Use coordinates that are multiples of 2 because the Midpoint Formula takes half the sum of the coordinates.



Given: right $\triangle ABC$ with right $\angle BAC$
P is the midpoint of $\overline{BC}$.

Prove: $AP = \frac{1}{2}BC$

Proof:

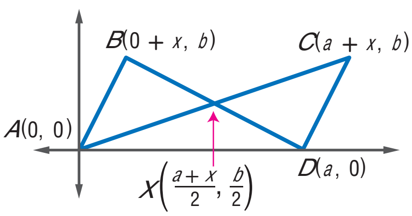
By the Midpoint Formula, the coordinates of *P* are $\left(\frac{0 + 2c}{2}, \frac{2b + 0}{2}\right)$ or (*c*, *b*). Use the Distance Formula to find *AP* and *BC*.

$$AP = \sqrt{(c - 0)^2 + (b - 0)^2}$$
$$= \sqrt{c^2 + b^2}$$

$$BC = \sqrt{(2c - 0)^2 + (0 - 2b)^2}$$
$$BC = \sqrt{4c^2 + 4b^2} \text{ or } 2\sqrt{c^2 + b^2}$$
$$\frac{1}{2}BC = \sqrt{c^2 + b^2}$$

Therefore, $AP = \frac{1}{2}BC$.

3. Use a coordinate proof to show that the triangles shown are congruent.

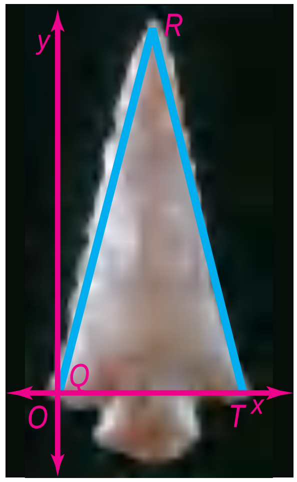


Online Personal Tutor at geometryonline.com

> **Study Tip**
>
> **Vertex Angle**
>
> Remember from the Geometry Lab on page 244 that an isosceles triangle can be folded in half. Thus, the *x*-coordinate of the vertex angle is the same as the *x*-coordinate of the midpoint of the base.

Real-World EXAMPLE **Classify Triangles**

4 **ARROWHEADS** **Write a coordinate proof to prove that this arrowhead is shaped like an isosceles triangle. The arrowhead is 3 inches long and 1.5 inches wide.**



The first step is to label the coordinates of each vertex. Q is at the origin, and T is at (1.5, 0). The y-coordinate of R is 3. The x-coordinate is halfway between 0 and 1.5 or 0.75. So, the coordinates of R are (0.75, 3).

If the legs of the triangle are the same length, it is isosceles. Use the Distance Formula to find QR and RT.

$$QR = \sqrt{(0.75 - 0)^2 + (3 - 0)^2}$$
$$= \sqrt{0.5625 + 9} \text{ or } \sqrt{9.5625}$$

$$RT = \sqrt{(1.5 - 0.75)^2 + (0 - 3)^2}$$
$$= \sqrt{0.5625 + 9} \text{ or } \sqrt{9.5625}$$

Since each leg is the same length, $\triangle QRT$ is isosceles. The arrowhead is shaped like an isosceles triangle.

CHECK Your Progress

4. Use coordinate geometry to classify a triangle with vertices located at the following coordinates $A(0, 0)$, $B(0, 6)$, and $C(3, 3)$.

CHECK Your Understanding

Example 1 (p. 251)

Position and label each triangle on the coordinate plane.

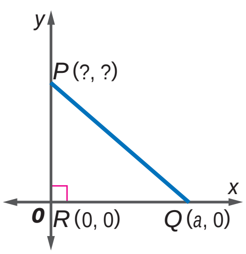
1. isosceles $\triangle FGH$ with base $\overline{FH}$ that is $2b$ units long

2. equilateral $\triangle CDE$ with sides a units long

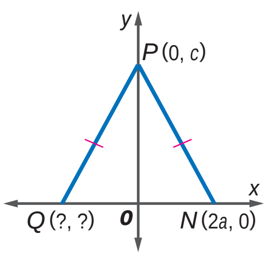
Example 2 (p. 252)

Name the missing coordinates of each triangle.

3.



4.

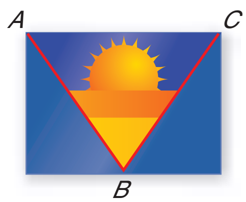


Example 3 (p. 252)

5. Write a coordinate proof for the following statement. *The midpoint of the hypotenuse of a right triangle is equidistant from each of the vertices.*

Example 4 (p. 253)

6. **FLAGS** Write a coordinate proof to prove that the large triangle in the center of the flag is isosceles. The dimensions of the flag are 4 feet by 6 feet, and point B of the triangle bisects the bottom of the flag.



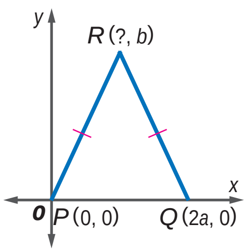
Exercises

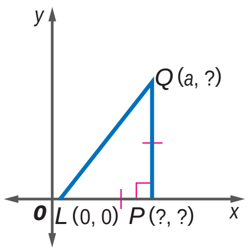
HOMEWORK HELP	
For Exercises	See Examples
7–12	1
13–18	2
19–22	3
23–26	4

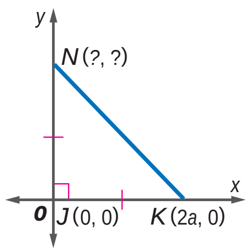
Position and label each triangle on the coordinate plane.

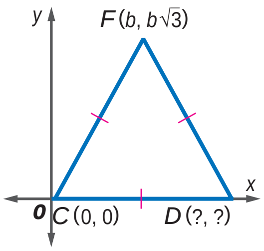
7. isosceles $\triangle QRT$ with base $\overline{QR}$ that is b units long
8. equilateral $\triangle MNP$ with sides $2a$ units long
9. isosceles right $\triangle JML$ with hypotenuse $\overline{JM}$ and legs c units long
10. equilateral $\triangle WXZ$ with sides $\frac{1}{2}b$ units long
11. isosceles $\triangle PWY$ with base $\overline{PW}(a + b)$ units long
12. right $\triangle XYZ$ with hypotenuse $\overline{XZ}$, the length of $\overline{ZY}$ is twice XY, and $\overline{XY}$ is b units long

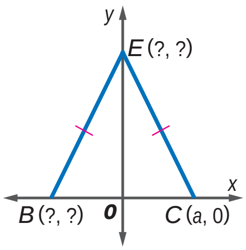
Name the missing coordinates of each triangle.

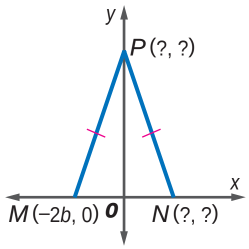
13.


14.


15.


16.


17.


18.


Write a coordinate proof for each statement.

19. The segments joining the vertices of the base angles to the midpoints of the legs of an isosceles triangle are congruent.
20. The three segments joining the midpoints of the sides of an isosceles triangle form another isosceles triangle.
21. If a line segment joins the midpoints of two sides of a triangle, then it is parallel to the third side.
22. If a line segment joins the midpoints of two sides of a triangle, then its length is equal to one-half the length of the third side.

Real-World Link

The Appalachian Trail is a 2175-mile hiking trail that stretches from Maine to Georgia. Up to 4 million people visit the trail per year.

Source: appalachiantrail.org

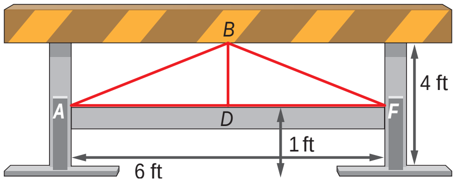
NAVIGATION For Exercises 23 and 24, use the following information.
A motor boat is located 800 yards from the port. There is a ship 800 yards to the east and another ship 800 yards to the north of the motor boat.

23. Write a coordinate proof to prove that the port, motor boat, and the ship to the north form an isosceles right triangle.
24. Write a coordinate proof to prove that the distance between the two ships is the same as the distance from the port to the northern ship.

HIKING For Exercises 25 and 26, use the following information.
Tami and Juan are hiking. Tami hikes 300 feet east of the camp and then hikes 500 feet north. Juan hikes 500 feet west of the camp and then 300 feet north.

25. Prove that Juan, Tami, and the camp form a right triangle.
26. Find the distance between Tami and Juan.

27. **STEEPLECHASE** Write a coordinate proof to prove that the triangles *ABD* and *FBD* are congruent. Suppose the hurdle is 6 feet wide and 4 feet tall, with the lower bar 1 foot off the ground.



EXTRA PRACTICE
See pages 809, 831.
Math Online
Self-Check Quiz at geometryonline.com

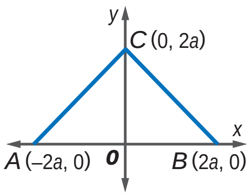
Find the coordinates of point *C* so $\triangle ABC$ is the indicated type of triangle. Point *A* has coordinates (0, 0) and *B* has coordinates (*a*, *b*).

28. right triangle **29.** isosceles triangle **30.** scalene triangle

H.O.T. Problems

31. **OPEN ENDED** Draw a scalene right triangle on the coordinate plane so it simplifies a coordinate proof. Label the coordinates of each vertex. Explain why you placed the triangle this way.

32. **CHALLENGE** Classify $\triangle ABC$ by its angles and its sides. Explain.



33. **Writing in Math** Use the information about the coordinate plane given on page 251 to explain how the coordinate plane can be used in proofs. Include a list of the different types of proof and a theorem from the chapter that could be proved using a coordinate proof.

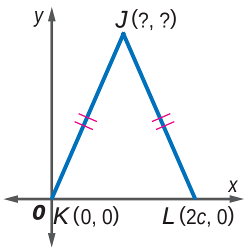
34. What are the coordinates of point *J* in the triangle below?

A $\left(\frac{c}{2}, c\right)$

B (c, b)

C $\left(\frac{b}{2}, c\right)$

D $\left(\frac{b}{2}, \frac{c}{2}\right)$



35. **REVIEW** What is the *x*-coordinate of the solution to the system of equations shown below?

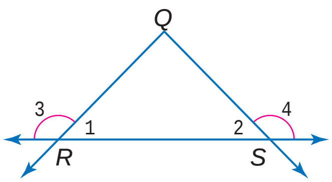
$$\begin{cases} 2x - 3y = 3 \\ -4x + 2y = -18 \end{cases}$$

F −6 **H** 3

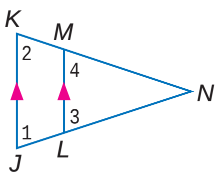
G −3 **J** 6

Write a two-column proof. (Lessons 4-5 and 4-6)

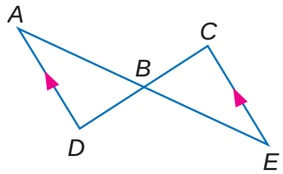
36. **Given:** $\angle 3 \cong \angle 4$
Prove: $\overline{QR} \cong \overline{QS}$



37. **Given:** isosceles triangle *JKN* with vertex $\angle N$, $\overline{JK} \parallel \overline{LM}$
Prove: $\triangle NML$ is isosceles.



38. **Given:** $\overline{AD} \cong \overline{CE}$; $\overline{AD} \parallel \overline{CE}$
Prove: $\triangle ABD \cong \triangle EBC$



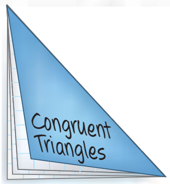
39. **JOBS** A studio engineer charges a flat fee of \$450 for equipment rental and \$42 an hour for recording and mixing time. Write the equation that shows the cost to hire the studio engineer as a function of time. How much would it cost to hire the studio engineer for 17 hours? (Lesson 3-4)

Study Guide and Review

Download Vocabulary Review from geometryonline.com

FOLDABLES GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Classifying Triangles (Lesson 4-1)

- Triangles can be classified by their angles as acute, obtuse, or right.
- Triangles can be classified by their sides as scalene, isosceles, or equilateral.

Angles of Triangles (Lesson 4-2)

- The sum of the measures of the angles of a triangle is 180°.
- The measures of an exterior angle is equal to the sum of the measures of the two remote interior angles.

Congruent Triangles (Lessons 4-3 through 4-5)

- If all of the corresponding sides of two triangles are congruent, then the triangles are congruent (SSS).
- If two corresponding sides of two triangles and the included angle are congruent, then the triangles are congruent (SAS).
- If two pairs of corresponding angles and the included sides of two triangles are congruent, then the triangles are congruent (ASA).
- If two pairs of corresponding angles and a pair of corresponding, nonincluded sides of two triangles are congruent, then the triangles are congruent (AAS).

Isosceles Triangles (Lesson 4-6)

- A triangle is equilateral if and only if it is equiangular.

Triangles and Coordinate Proof (Lesson 4-7)

- Coordinate proofs use algebra to prove geometric concepts.
- The Distance Formula, Slope Formula, and Midpoint Formula are often used in coordinate proof.

Key Vocabulary

acute triangle (p. 202)
base angles (p. 244)
congruence transformation (p. 219)
congruent triangles (p. 217)
coordinate proof (p. 251)
corollary (p. 213)
equiangular triangle (p. 202)
equilateral triangle (p. 203)
exterior angle (p. 211)
flow proof (p. 212)
included side (p. 234)
isosceles triangle (p. 203)
obtuse triangle (p. 202)
remote interior angles (p. 211)
right triangle (p. 202)
scalene triangle (p. 203)
vertex angle (p. 244)

Vocabulary Check

Select the word from the list above that best completes the following statements.

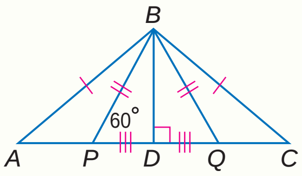
1. A triangle with an angle measure greater than 90 is a(n) ___?___.
2. A triangle with exactly two congruent sides is a(n) ___?___.
3. A triangle that has an angle with a measure of exactly 90° is a(n) ___?___.
4. An equiangular triangle is a form of a(n) ___?___.
5. A(n) ___?___ uses figures in the coordinate plane and algebra to prove geometric concepts.
6. A(n) ___?___ preserves a geometric figure's size and shape.
7. If all corresponding sides and angles of two triangles are congruent, those triangles are ___?___.

Vocabulary Review at geometryonline.com

Lesson-by-Lesson Review

4-1 Classifying Triangles (pp. 202–208)

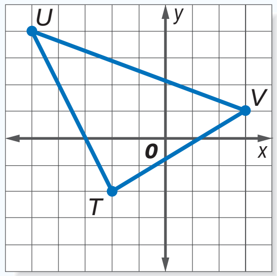
Classify each triangle by its angles and by its sides if $m\angle ABC = 100$.



8. $\triangle ABC$ **9.** $\triangle BDP$ **10.** $\triangle BPQ$

11. DISTANCE The total distance from Sufjan's to Carol's to Steven's house is 18.77 miles. The distance from Sufjan's to Steven's house is 0.81 miles longer than the distance from Sufjan's to Carol's. The distance from Sufjan's to Steven's house is 2.25 time the distance from Carol's to Steven's. Find the distance between each house. Use these lengths to classify the triangle formed by the three houses.

Example 1 **Find the measures of the sides of $\triangle TUV$. Classify the triangle by sides.**



Use the Distance Formula to find the measure of each side.

$$TU = \sqrt{[-5-(-2)]^2 + [4-(-2)]^2}$$
$$= \sqrt{9+36} \text{ or } \sqrt{45}$$
$$UV = \sqrt{[3-(-5)]^2 + (1-4)^2}$$
$$= \sqrt{64+9} \text{ or } \sqrt{73}$$
$$VT = \sqrt{(-2-3)^2 + (-2-1)^2}$$
$$= \sqrt{25+9} \text{ or } \sqrt{34}$$

Since the measures of the sides are all different, the triangle is scalene.

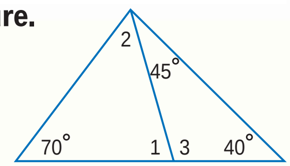
4-2 Angles of Triangles (pp. 210–216)

Find each measure.

12. $m\angle 1$

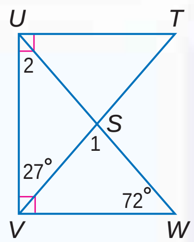
13. $m\angle 2$

14. $m\angle 3$



15. CONSTRUCTION The apex of the truss being built for Tamara's new house measures 72 degrees. If the truss is shaped like an isosceles triangle what are the measures of the other two angles?

Example 2 **If $\overline{TU} \perp \overline{UV}$ and $\overline{UV} \perp \overline{VW}$, find $m\angle 1$.**



Use the Angle Sum Theorem to write an equation.

$$m\angle 1 + 72 + m\angle TVW = 180$$
$$m\angle 1 + 72 + (90 - 27) = 180$$
$$m\angle 1 + 135 = 180$$
$$m\angle 1 = 45$$

CHAPTER 4

Study Guide and Review

4-3 Congruent Triangles (pp. 217–223)

Name the corresponding angles and sides for each pair of congruent triangles.

16. $\triangle EFG \cong \triangle DCB$ **17.** $\triangle NCK \cong \triangle KER$

18. QUILTING Meghan's mom is going to enter a quilt at the state fair. Name the congruent triangles found in the quilt block.

Example 3 **If $\triangle EFG \cong \triangle JKL$, name the corresponding congruent angles and sides.**

The letters of the triangles correspond to the congruent angles and sides. $\angle E \cong \angle J$, $\angle F \cong \angle K$, $\angle G \cong \angle L$, $\overline{EF} \cong \overline{JK}$, $\overline{FG} \cong \overline{KL}$, and $\overline{EG} \cong \overline{JL}$.

4-4 Proving Congruence—SSS, SAS (pp. 225–232)

Determine whether $\triangle MNP \cong \triangle QRS$ given the coordinates of the vertices. Explain.

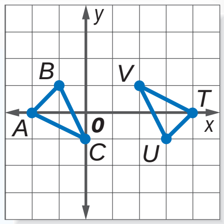
19. $M(0, 3)$, $N(-4, 3)$, $P(-4, 6)$, $Q(5, 6)$, $R(2, 6)$, $S(2, 2)$

20. $M(3, 2)$, $N(7, 4)$, $P(6, 6)$, $Q(-2, 3)$, $R(-4, 7)$, $S(-6, 6)$

21. GAMES In a game, Lupe's boats are placed at coordinates (3, 2), (0, -4), and (6, -4). Do her ships form an equilateral triangle?

22. Triangle ABC is an isosceles triangle with $\overline{AB} \cong \overline{BC}$. If there exists a line $\overline{BD}$ that bisects $\angle ABC$, show that $\triangle ABD \cong \triangle CBD$.

Example 4
Determine whether $\triangle ABC \cong \triangle TUV$. Explain.



$$AB = \sqrt{[-1-(-2)]^2 + (1-0)^2} = \sqrt{1+1} \text{ or } \sqrt{2}$$

$$BC = \sqrt{[0-(-1)]^2 + (-1-1)^2} = \sqrt{1+4} \text{ or } \sqrt{5}$$

$$CA = \sqrt{(-2-0)^2 + [0-(-1)]^2} = \sqrt{4+1} \text{ or } \sqrt{5}$$

$$TU = \sqrt{(3-4)^2 + (-1-0)^2} = \sqrt{1+1} \text{ or } \sqrt{2}$$

$$UV = \sqrt{(2-3)^2 + [1-(-1)]^2} = \sqrt{1+4} \text{ or } \sqrt{5}$$

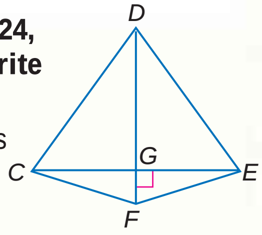
$$VT = \sqrt{(4-2)^2 + (0-1)^2} = \sqrt{4+1} \text{ or } \sqrt{5}$$

Therefore, $\triangle ABC \cong \triangle TUV$ by SSS.

Mixed Problem Solving
For mixed problem-solving practice, see page 831.

4-5 Proving Congruence—ASA, AAS (pp. 234–241)

For Exercises 23 and 24, use the figure and write a two-column proof.



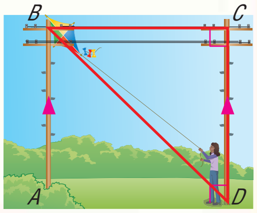
23. Given: $\overline{DF}$ bisects $\angle CDE$.
$\overline{CE} \perp \overline{DF}$

Prove: $\triangle DGC \cong \triangle DGE$

24. Given: $\triangle DGC \cong \triangle DGE$
$\triangle GCF \cong \triangle GEF$

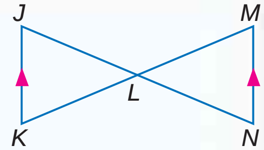
Prove: $\triangle DFC \cong \triangle DFE$

25. KITES Kyra's kite is stuck in a set of power lines. If the power lines are stretched so that they are parallel with the ground, prove that $\triangle ABD \cong \triangle CDB$.



Example 5 **Write a proof.**

Given: $\overline{JK} \parallel \overline{MN}$
L is the midpoint of $\overline{KM}$.



Prove: $\triangle JLK \cong \triangle NLM$

Flow Proof:

$\overline{JK} \parallel \overline{MN}$ — Given → $\angle JKL \cong \angle LMN$ — Alt. Int. ∠s Th.

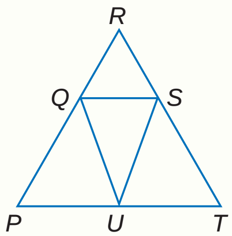
L is the midpoint of $\overline{KM}$. — Given → $\overline{KL} \cong \overline{ML}$ — Midpt. Th.

$\angle JLK \cong \angle NLM$ — Vert ∠s are ≅.

→ $\triangle JLK \cong \triangle NLM$ — ASA

4-6 Isosceles Triangles (pp. 244–250)

For Exercises 26–28, refer to the figure.

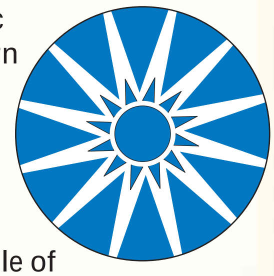


26. If $\overline{PQ} \cong \overline{UQ}$ and $m\angle P = 32$, find $m\angle PUQ$.

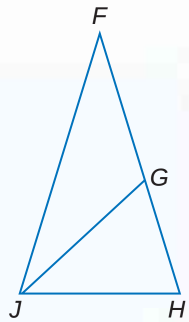
27. If $\overline{RQ} \cong \overline{RS}$ and $m\angle RQS = 75$, find $m\angle R$.

28. If $\overline{RQ} \cong \overline{RS}$, $\overline{RP} \cong \overline{RT}$, and $m\angle RQS = 80$, find $m\angle P$.

29. ART This geometric design from Western Cameroon uses approximations of isosceles triangles. Trace the figure. Identify and draw one isosceles triangle of each type from the design. Describe the similarities between the different triangles.



Example 6 **If $\overline{FG} \cong \overline{GJ}$, $\overline{GJ} \cong \overline{JH}$, $\overline{FJ} \cong \overline{FH}$, and $m\angle GJH = 40$, find $m\angle H$.**



$\triangle GHJ$ is isosceles with base $\overline{GH}$, so $\angle JGH \cong \angle H$ by the Isosceles Triangle Theorem. Thus, $m\angle JGH = m\angle H$.

$$m\angle GJH + m\angle JGH + m\angle H = 180$$
$$40 + 2(m\angle H) = 180$$
$$2 \cdot m\angle H = 140$$
$$m\angle H = 70$$

4-7 Triangle and Coordinate Proof (pp. 251–255)

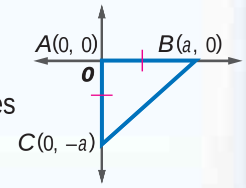
Position and label each triangle on the coordinate plane.

30. isosceles $\triangle TRI$ with base $\overline{TI}$ $4a$ units long
31. equilateral $\triangle BCD$ with side length $6m$ units long
32. right $\triangle JKL$ with leg lengths of a units and b units
33. **BOATS** A sailboat is located 400 meters to the east and 250 meters to the north of a dock. A canoe is located 400 meters to the west and 250 meters to the north of the same dock. Show that the sailboat, the canoe, and the dock all form an isosceles triangle.

Position and label isosceles right triangle $\triangle ABC$ with bases of length a units on the coordinate plane.

- Use the origin as the vertex of $\triangle ABC$ that has the right angle.
- Place each of the bases along an axis, one on the x-axis and the other on the y-axis.
- Since B is on the x-axis, its y-coordinate is 0. Its x-coordinate is a because the leg of the triangle is a units long.

Since $\triangle ABC$ is isosceles, C should also be a distance of a units from the origin. Its coordinates should be $(0, -a)$, since it is on the negative y-axis.



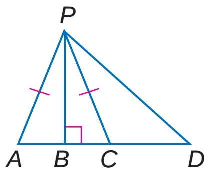
CHAPTER 4 Practice Test

Identify the indicated triangles in the figure if $\overline{PB} \perp \overline{AD}$ and $\overline{PA} \cong \overline{PC}$.

1. obtuse

2. isosceles

3. right

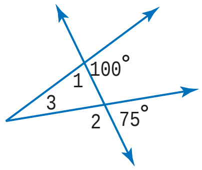


Find the measure of each angle in the figure.

4. $m\angle 1$

5. $m\angle 2$

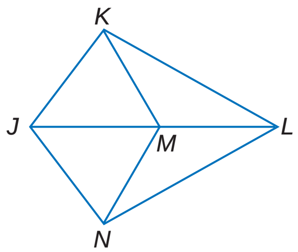
6. $m\angle 3$



7. Write a flow proof.

Given: $\triangle JKM \cong \triangle JNM$

Prove: $\triangle JKL \cong \triangle JNL$



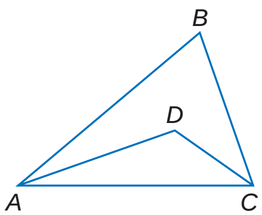
Name the corresponding angles and sides for each pair of congruent triangles.

8. $\triangle DEF \cong \triangle PQR$

9. $\triangle FMG \cong \triangle HNJ$

10. $\triangle XYZ \cong \triangle ZYX$

11. MULTIPLE CHOICE In $\triangle ABC$, $\overline{AD}$ and $\overline{DC}$ are angle bisectors and $m\angle B = 76$.



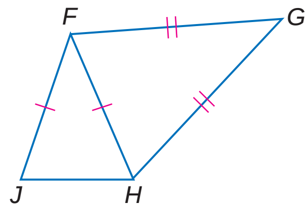
What is $m\angle ADC$?

A 26 **C** 76

B 52 **D** 128

12. Determine whether $\triangle JKL \cong \triangle MNP$ given $J(-1, -2)$, $K(2, -3)$, $L(3, 1)$, $M(-6, -7)$, $N(-2, 1)$, and $P(5, 3)$. Explain.

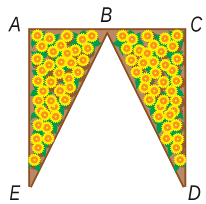
In the figure, $\overline{FJ} \cong \overline{FH}$ and $\overline{GF} \cong \overline{GH}$.



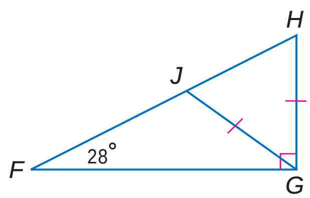
13. If $m\angle JFH = 34$, find $m\angle J$.

14. If $m\angle GHJ = 152$ and $m\angle G = 32$, find $m\angle JFH$.

15. LANDSCAPING A landscaper designed a garden shaped as shown in the figure. The landscaper has decided to place point B 22 feet east of point A, point C 44 feet east of point A, point E 36 feet south of point A, and point D 36 feet south of point C. The angles at points A and C are right angles. Prove that $\triangle ABE \cong \triangle CBD$.



16. MULTIPLE CHOICE In the figure, $\triangle FGH$ is a right triangle with hypotenuse $\overline{FH}$ and $GJ = GH$.



What is $m\angle JGH$?

F 104 **H** 56

G 62 **J** 28

Chapter Test at geometryonline.com

CHAPTER 4

Standardized Test Practice

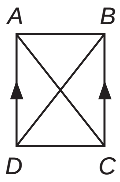
Cumulative, Chapters 1–4

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Use the proof to answer the question below.

Given: $\overline{AD} \parallel \overline{BC}$

Prove: $\triangle ABD \cong \triangle CDB$



Statements	Reasons
1. $\overline{AD} \parallel \overline{BC}$	1. Given
2. $\angle ABD \cong \angle CDB$, $\angle ADB \cong \angle CBD$	2. Alternate Interior Angles Theorem
3. $\overline{BD} \cong \overline{DB}$	3. Reflexive Property
4. $\triangle ABD \cong \triangle CDB$	4. ?

What reason can be used to prove the triangles are congruent?

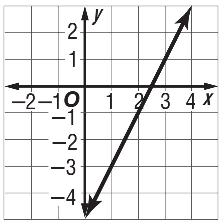
A AAS

B ASA

C SAS

D SSS

2. The graph of $y = 2x - 5$ is shown at the right. How would the graph be different if the number 2 in the equation was replaced with a 4?



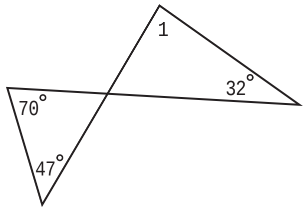
F parallel to the line shown, but shifted two units higher

G parallel to the line shown, but shifted two units lower

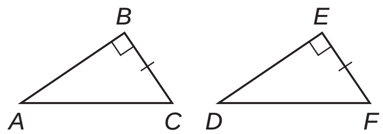
H have a steeper slope, but intercept the y-axis at the same point

J have a less steep slope, but intercept the y-axis at the same point

3. **GRIDDABLE** What is $m\angle 1$ in degrees?



4. In the figure below, $\overline{BC} \cong \overline{EF}$ and $\angle B \cong \angle E$.

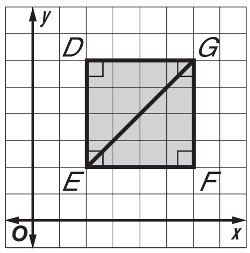


Which additional information would be enough to prove $\triangle ABC \cong \triangle DEF$?

A $\angle A \cong \angle D$ **C** $\overline{AC} \cong \overline{DF}$

B $\overline{AC} \cong \overline{BC}$ **D** $\overline{DE} \perp \overline{EF}$

5. The diagram shows square $DEFG$. Which statement could *not* be used to prove $\triangle DEG$ is a right triangle?



F $(EG)^2 = (DG)^2 + (DE)^2$

G Definition of a Square

H (slope DE)(slope DG) $= 1$

J (slope DE)(slope DG) $= -1$

6. **ALGEBRA** Which equation is equivalent to $4(y - 2) - 3(2y - 4) = 9$?

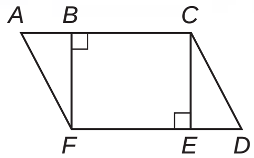
A $2y - 4 = 9$ **C** $10y - 20 = 9$

B $-2y + 4 = 9$ **D** $-2y - 4 = 9$

Standardized Test Practice at geometryonline.com

Preparing for
Standardized Tests
For test-taking strategies and more practice, see pages 846–856.

7. In the quadrilateral, which pair of segments can be established to be congruent to prove that $\overline{AC} \parallel \overline{FD}$?



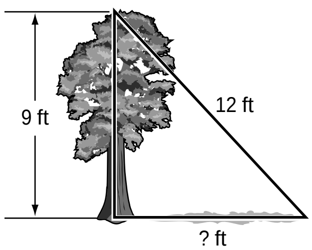
F $\overline{AC} \cong \overline{FD}$ H $\overline{BC} \cong \overline{FE}$
G $\overline{AF} \cong \overline{CD}$ J $\overline{BF} \cong \overline{CE}$

8. Which of the following is the inverse of the statement *If it is raining, then Kamika carries an umbrella?*
   A If Kamika carries an umbrella, then it is raining.
   B If Kamika does not carry an umbrella, then it is not raining.
   C If it is not raining, then Kamika carries an umbrella.
   D If it is not raining, then Kamika does not carry an umbrella.

9. **ALGEBRA** Which of the following describes the line containing the points (2, 4) and (0, -2)?

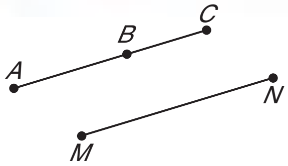
F $y = -3x + 2$ H $y = \frac{1}{3}x - 2$
G $y = -\frac{1}{3}x - 4$ J $y = -3x + 2$

10. A 9-foot tree casts a shadow on the ground. The distance from the top of the tree to the end of the shadow is 12 feet. To the nearest foot, how long is the shadow?



A 7 ft C 10 ft
B 8 ft D 12 ft

11. In the following proof, what property justifies statement 3?

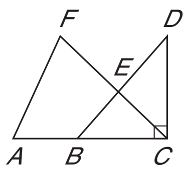


Given: $\overline{AC} \cong \overline{MN}$
Prove: $AB + BC = MN$

Statements	Reasons
1. $\overline{AC} \cong \overline{MN}$	1. Given
2. $AC = MN$	2. Def. of $\cong$ segments
3. $AC = AB + BC$	3. ?
4. $AC + BC = MN$	4. Substitution

F Definition of Midpoint
G Transitive Property
H Segment Addition Postulate
J Commutative Property

12. If $\angle ACD$ is a right angle, what is the relationship between $\angle ACF$ and $\angle DCF$?



A complementary angles
B congruent angles
C supplementary angles
D vertical angles

TEST-TAKING TIP

Question 12 When you have multiple pieces of information about a figure, make a sketch of the figure so that you can mark the information that you know.

Pre-AP

Record your answer on a sheet of paper. Show your work.

13. The measures of $\triangle ABC$ are $5x$, $4x - 1$, and $3x + 13$.
   a. Draw a figure to illustrate $\triangle ABC$ and find the measure of each angle.
   b. Prove $\triangle ABC$ is an isosceles triangle.

NEED EXTRA HELP?													
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson or Page...	4-5	3-4	4-2	4-5	3-3	782	3-6	2-2	786	1-2	2-7	1-6	4-6

CHAPTER 5

Relationships in Triangles

- Identify and use perpendicular bisectors, angle bisectors, medians, and altitudes of triangles.
- Apply properties of inequalities relating to the measures of angles and sides of triangles.
- Use indirect proof with algebra and geometry.
- Apply the Triangle Inequality Theorem and SAS and SSS inequalities.

Key Vocabulary

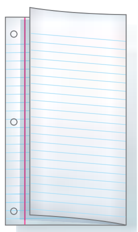
perpendicular bisector (p. 269)
median (p. 271)
altitude (p. 272)
indirect proof (p. 288)

Real-World Link

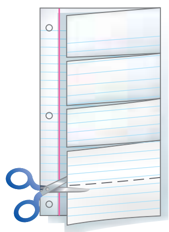
Gardening To protect a tree from heavy snow, gardeners tie a rope to each branch. The rope, the tree, and the ground form a triangle.

Relationships in Triangles Make this Foldable to help you organize your notes. Begin with one sheet of notebook paper.

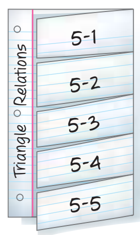
1 **Fold** lengthwise to the holes.



2 **Cut** 5 tabs.



3 **Label** the edge. Then label the tabs using lesson numbers.



GET READY for Chapter 5

Diagnose Readiness **You have two options for checking Prerequisite Skills.**

Option 2

Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

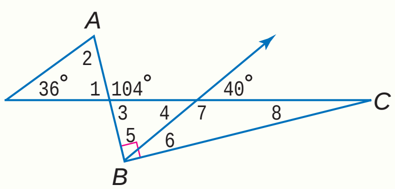
Find the coordinates of the midpoint of a segment with the given endpoints. (Lesson 1-3)

1. $A(-12, -5)$, $B(4, 15)$

2. $C(-22, -25)$, $D(10, 10)$

3. **MAPS** The coordinates of Springville are $(-15, 25)$, and the coordinates of Pickton are $(5, -16)$. Hatfield is located midway between the two cities. Find the coordinates of Hatfield. (Lesson 1-3)

Find the measure of each numbered angle if $\overline{AB} \perp \overline{BC}$. (Lesson 1-5)



4. $\angle 1$	**5.** $\angle 2$	**6.** $\angle 3$	**7.** $\angle 4$
8. $\angle 5$	**9.** $\angle 6$	**10.** $\angle 7$	**11.** $\angle 8$

Determine whether a valid conclusion can be reached from the two true statements using the Law of Detachment. If a valid conclusion is possible, state it. Otherwise, write *no conclusion*. (Lessons 4-4 and 4-5)

12. (1) If the three sides of one triangle are congruent to the three sides of a second triangle, then the triangles are congruent.
(2) $\triangle ABC$ and $\triangle PQR$ are congruent.

QUICKReview

EXAMPLE 1

Find the coordinates of the midpoint of the segment with endpoints *Y*(9, 4) and *Z*(13, 20).

Let $(x_1, y_1) = (9, 4)$ and let $(x_2, y_2) = (13, 20)$.

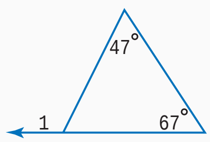
$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ Midpoint Formula

$= M\left(\frac{9 + 13}{2}, \frac{4 + 20}{2}\right)$ Substitution

$= M(11, 12)$ Simplify.

EXAMPLE 2

Find $m\angle 1$.



$m\angle 1 = 47 + 67$ Exterior Angle Theorem

$m\angle 1 = 114$ Simplify.

EXAMPLE 3

Determine whether a valid conclusion can be reached from the two true statements using the Law of Detachment. If a valid conclusion is possible, state it. Otherwise, write *no conclusion*.

(1) If two angles make a linear pair, then they are supplementary.

(2) $\angle A$ and $\angle B$ make a linear pair.

A valid conclusion can be reached from the above two statements. $\angle A$ and $\angle B$ are supplementary.

EXPLORE 5-1

Geometry Lab

Bisectors, Medians, and Altitudes of Triangles

There are four special segments in triangles. You can use the constructions you have learned for midpoints, perpendicular segments, and angle bisectors to construct the special segments in triangles.

CONSTRUCTION 1 **Perpendicular Bisector**

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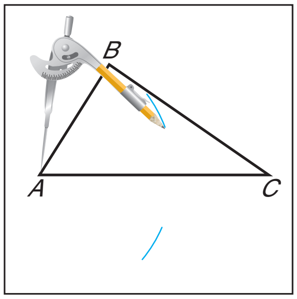
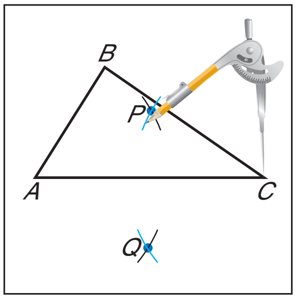
Construct the perpendicular bisector of a side of a triangle.

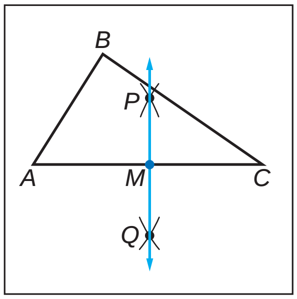
Step 1 Draw a triangle like ABC. Adjust the compass to an opening greater than $\frac{1}{2}AC$. Place the compass at vertex A, and draw an arc above and below $\overline{AC}$.



Step 2 Using the same compass settings, place the compass at vertex C. Draw an arc above and below $\overline{AC}$ Label the points of intersection of the arcs P and Q.

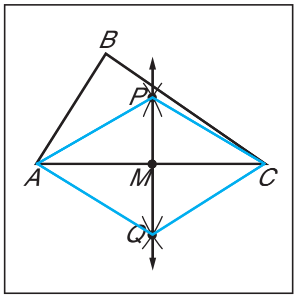


Step 3 Use a straightedge to draw $\overleftrightarrow{PQ}$. Label the point where $\overleftrightarrow{PQ}$ intersects $\overline{AC}$ as M.



This procedure works for bisecting any segment, not just a side of a triangle.

Verify the construction.



Given: $\triangle ABC$

Prove: $\overleftrightarrow{PQ}$ is the perpendicular bisector of $\overline{AC}$ at M.

Paragraph Proof: $\overline{AP} \cong \overline{CP} \cong \overline{AQ} \cong \overline{CQ}$ because the arcs were drawn with the same compass setting. $\overline{AC} \cong \overline{AC}$ by the Reflexive Property. Thus, $\triangle APC \cong \triangle AQC$ by SSS. By CPCTC, $\angle PCA \cong \angle QCA$. $\overline{MC} \cong \overline{MC}$ by the Reflexive Property. Therefore $\triangle MPC \cong \triangle MQC$ by SAS. Then $\angle PMC \cong \angle QMC$ by CPCTC. Since a linear pair of congruent angles are right angles, $\angle PMC$ and $\angle QMC$ are right angles. So $\overleftrightarrow{PQ} \perp \overline{AC}$. $\overline{PM} \cong \overline{PM}$ by the Reflexive Property. $\angle PMA \cong \angle PMC$ since perpendicular lines form four right angles and all right angles are congruent. Thus, $\triangle PMA \cong \triangle PMC$ by HL and $\overline{MA} \cong \overline{MC}$ by CPTPC. Therefore $\overleftrightarrow{PQ}$ bisects $\overline{AC}$ by the definition of bisector.

Analyze the Results

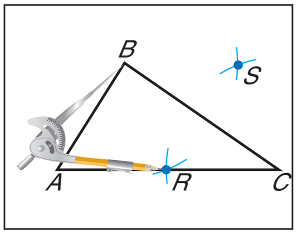
1. Construct the perpendicular bisectors for the other two sides of ABC.
2. What do you notice about the perpendicular bisectors?

A *median* of a triangle is a segment with endpoints that are a vertex of the triangle and the midpoint of the side opposite the vertex. You can construct a median of a triangle using the construction of the midpoint of a segment.

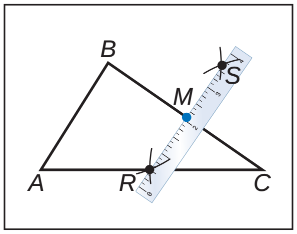
CONSTRUCTION 2 Median

Construct the median of a triangle.

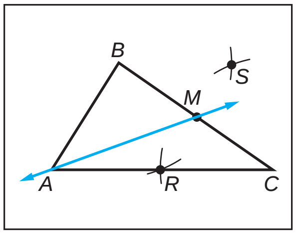
Step 1 Draw intersecting arcs above and below $\overline{BC}$. Label the points of intersection *R* and *S*.



Step 2 Use a straightedge to find the point where $\overline{RS}$ intersects $\overline{BC}$. Label the midpoint *M*.



Step 3 Draw a line through *A* and *M*. $\overline{AM}$ is a median of $\triangle ABC$.



ANALYZE THE RESULTS

3. Construct the medians of the other two sides.

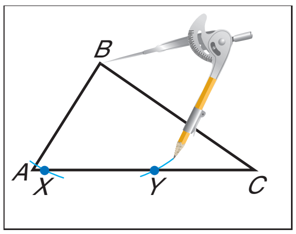
4. What do you notice about the medians of a triangle?

An *altitude* of a triangle is a segment from a vertex of the triangle to the line containing the opposite side and perpendicular to the line containing that side.

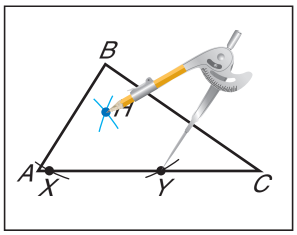
CONSTRUCTION 3 Altitude

Construct the altitude of a triangle.

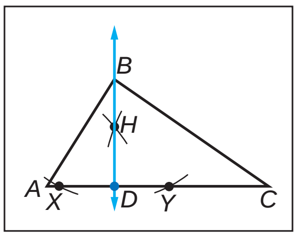
Step 1 Place the compass at vertex *B* and draw two arcs intersecting $\overleftrightarrow{AC}$. Label the points where the arcs intersect the side *X* and *Y*.



Step 2 Adjust the compass to an opening greater than $\frac{1}{2}XY$. Place the compass on point *X* and draw an arc above $\overline{AC}$. Using the same setting, place the compass on point *Y* and draw another arc above $\overline{AC}$. Label the point of intersection *H*.



Step 3 Use a straightedge to draw $\overleftrightarrow{BH}$. Label the point where $\overleftrightarrow{BH}$ intersects $\overline{AC}$ as *D*. $\overline{BD}$ is an altitude of $\triangle ABC$ and is perpendicular to $\overline{AC}$.



Analyze the Results

5. Construct the altitudes to the other two sides. (*Hint*: You may need to extend the lines containing the sides of your triangle.)
6. What observation can you make about the altitudes of your triangle?

An *angle bisector* of a triangle is a line containing a vertex of a triangle and bisecting that angle.

CONSTRUCTION 4 Angle Bisector

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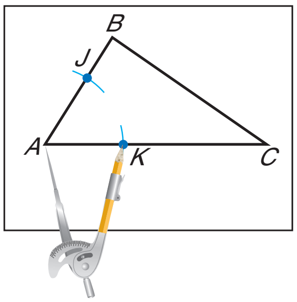
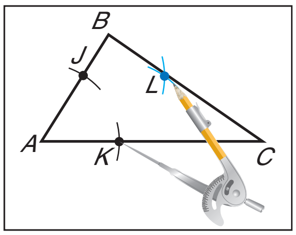
Construct an angle bisector of a triangle.

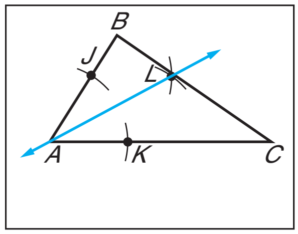
Step 1 Place the compass on vertex A, and draw an arc through $\overline{AB}$ and an arc through $\overline{AC}$. Label the points where the arcs intersect the sides as J and K.



Step 2 Place the compass on J, and draw an arc. Then place the compass on K and draw an arc intersecting the first arc. Label the intersection L.



Step 3 Use a straightedge to draw $\overleftrightarrow{AL}$. $\overleftrightarrow{AL}$ is an angle bisector of $\triangle ABC$.



Analyze the Results

7. **MAKE A CONJECTURE** Predict a relationship involving the angle bisectors of a triangle.
8. Construct the angle bisectors for the other two angles of your $\triangle ABC$. How do the results compare to your conjecture? Explain.

Extend

9. Repeat the four constructions for each type of triangle.
   - **a.** obtuse scalene
   - **b.** right scalene
   - **c.** acute isosceles
   - **d.** obtuse isosceles
   - **e.** right isosceles
   - **f.** equilateral
10. Where are the points of intersection of the lines for an acute triangle?
11. In an obtuse triangle, where are the points of intersection of the lines?
12. Where are the points of intersection of the lines for a right triangle?
13. Under what circumstances do the special lines of triangles coincide with each other?

5-1 Bisectors, Medians, and Altitudes

Main Ideas

- Identify and use perpendicular bisectors and angle bisectors in triangles.
- Identify and use medians and altitudes in triangles.

New Vocabulary

perpendicular bisector
concurrent lines
point of concurrency
circumcenter
incenter
median
centroid
altitude
orthocenter

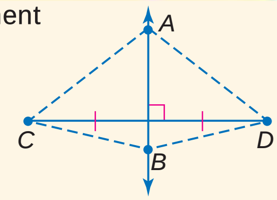
Acrobats and jugglers often balance objects when performing. These skilled artists need to find the center of gravity for each object or body position in order to keep balanced. The center of gravity for any triangle can be found by drawing the *medians* of a triangle and locating the point where they intersect.

Perpendicular Bisectors and Angle Bisectors The first construction you made in the Geometry Lab on pages 266–268 was the perpendicular bisector of a side of a triangle. A **perpendicular bisector** of a side of a triangle is a line, segment, or ray that passes through the midpoint of the side and is perpendicular to that side. Perpendicular bisectors of segments have some special properties. Two of those properties are stated in Theorems 5.1 and 5.2.

THEOREMS — *Points on Perpendicular Bisectors*

5.1 Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

Example: If $\overline{AB} \perp \overline{CD}$ and $\overline{AB}$ bisects $\overline{CD}$, then $AC = AD$ and $BC = BD$.



5.2 Any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.

Example: If $AC = AD$, then A lies on the perpendicular bisector of $\overline{CD}$.
If $BC = BD$, then B lies on the perpendicular bisector of $\overline{CD}$.

You will prove Theorems 5.1 and 5.2 in Check Your Progress 1 and Exercise 23, respectively.

Recall that a locus is the set of all points that satisfy a given condition. A perpendicular bisector of a given segment can be described as the locus of points in a plane equidistant from the endpoints of the given segment.

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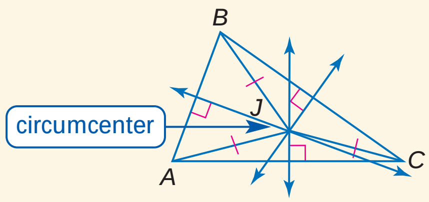
Since a triangle has three sides, there are three perpendicular bisectors in a triangle. The perpendicular bisectors of a triangle intersect at a common point. When three or more lines intersect at a common point, the lines are called **concurrent lines**, and their point of intersection is called the **point of concurrency**. The point of concurrency of the perpendicular bisectors of a triangle is called the **circumcenter**.

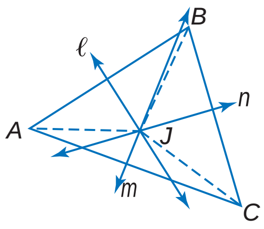
THEOREM 5.3 ***Circumcenter Theorem***

The circumcenter of a triangle is equidistant from the vertices of the triangle.

Example: If J is the circumcenter of $\triangle ABC$, then $AJ = BJ = CJ$.



Proof Theorem 5.3



Given: ℓ, m, and n are perpendicular bisectors of $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$, respectively.

Prove: $AJ = BJ = CJ$

Paragraph Proof:

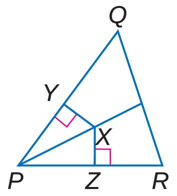
Since J lies on the perpendicular bisector of $\overline{AB}$, it is equidistant from A and B. By the definition of equidistant, $AJ = BJ$. The perpendicular bisector of $\overline{BC}$ also contains J. Thus, $BJ = CJ$. By the Transitive Property of Equality, $AJ = CJ$. Thus, $AJ = BJ = CJ$.

Another special line, segment, or ray in triangles is an angle bisector.

Review Vocabulary

Angle Bisector a ray that divides an angle into two congruent angles (Lesson 1-4)

EXAMPLE 1 Use Angle Bisectors



Given: $\overrightarrow{PX}$ bisects $\angle QPR$, $\overline{XY} \perp \overline{PQ}$, and $\overline{XZ} \perp \overline{PR}$.

Prove: $\overline{XY} \cong \overline{XZ}$

Proof:

Statements	Reasons
1. $\overrightarrow{PX}$ bisects $\angle QPR$, $\overline{XY} \perp \overline{PQ}$, and $\overline{XZ} \perp \overline{PR}$.	**1.** Given
2. $\angle YPX \cong \angle ZPX$	**2.** Definition of angle bisector
3. $\angle PYX$ and $\angle PZX$ are right angles.	**3.** Definition of perpendicular
4. $\angle PYX \cong \angle PZX$	**4.** Right angles are congruent.
5. $\overline{PX} \cong \overline{PX}$	**5.** Reflexive Property
6. $\triangle PYX \cong \triangle PZX$	**6.** AAS
7. $\overline{XY} \cong \overline{XZ}$	**7.** CPCTC

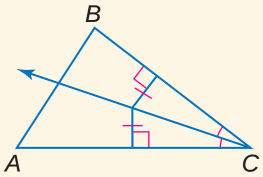
1. **PROOF** Write a paragraph proof of Theorem 5.1.

In Example 1, XY and XZ are lengths representing the distance from X to each side of $\angle QPR$. So, Example 1 is a proof of Theorem 5.4.

THEOREMS — *Points on Angle Bisectors*

5.4 Any point on the angle bisector is equidistant from the sides of the angle.

5.5 Any point equidistant from the sides of an angle lies on the angle bisector.



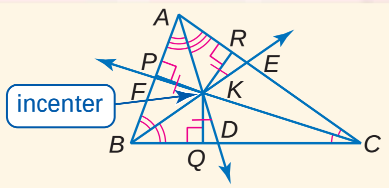
You will prove Theorem 5.5 in Exercise 24.

As with perpendicular bisectors, there are three angle bisectors in any triangle. The angle bisectors of a triangle are concurrent, and their point of concurrency is called the **incenter** of a triangle.

THEOREM 5.6 — *Incenter Theorem*

The incenter of a triangle is equidistant from each side of the triangle.

Example: If K is the incenter of $\triangle ABC$, then $KP = KQ = KR$.



You will prove Theorem 5.6 in Exercise 25.

Study Tip

Medians as Bisectors

Because the median contains the midpoint, it is also a bisector of the side of the triangle.

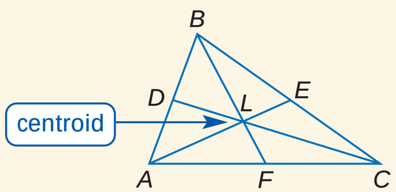
Medians and Altitudes A **median** is a segment whose endpoints are a vertex of a triangle and the midpoint of the side opposite the vertex. Every triangle has three medians.

The medians of a triangle also intersect at a common point. The point of concurrency for the medians of a triangle is called a **centroid**. The centroid is the point of balance for any triangle.

THEOREM 5.7 — *Centroid Theorem*

The centroid of a triangle is located two thirds of the distance from a vertex to the midpoint of the side opposite the vertex on a median.

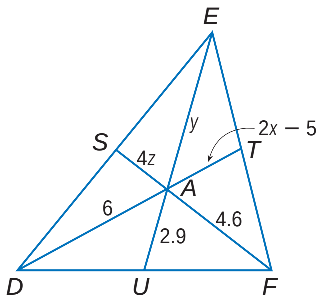
Example: If L is the centroid of $\triangle ABC$, $AL = \frac{2}{3}AE$, $BL = \frac{2}{3}BF$, and $CL = \frac{2}{3}CD$.



You can use the theorems about special segments of triangles to solve problems involving measures in triangles.

EXAMPLE 2 Segment Measures

ALGEBRA Points S, T, and U are the midpoints of $\overline{DE}$, $\overline{EF}$, and $\overline{DF}$, respectively. Find x, y, and z.



- Find x.

$DT = DA + AT$	Segment Addition Postulate
$= 6 + (2x - 5)$	Substitution
$= 2x + 1$	Simplify.
$DA = \frac{2}{3}DT$	Centroid Theorem
$6 = \frac{2}{3}[2x + 1]$	$DA = 6$, $DT = 2x + 1$
$18 = 4x + 2$	Multiply each side by 3 and simplify.
$16 = 4x$	Subtract 2 from each side.
$4 = x$	Divide each side by 4.

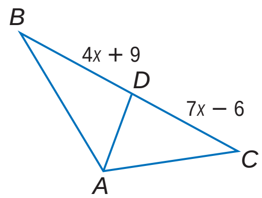
- Find y.

$EA = \frac{2}{3}EU$	Centroid Theorem
$y = \frac{2}{3}(y + 2.9)$	$EA = y$, $EU = y + 2.9$
$3y = 2y + 5.8$	Multiply each side by 3 and simplify.
$y = 5.8$	Subtract $2y$ from each side.

- Find z.

$FA = \frac{2}{3}FS$	Centroid Theorem
$4.6 = \frac{2}{3}(4.6 + 4z)$	$FA = 4.6$, $FS = 4.6 + 4z$
$13.8 = 9.2 + 8z$	Multiply each side by 3 and simplify.
$4.6 = 8z$	Subtract 9.2 from each side.
$0.575 = z$	Divide each side by 8.

2. ALGEBRA Find x if $\overline{AD}$ is a median of $\triangle ABC$.

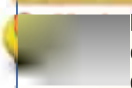


Study Tip

Eliminating Fractions

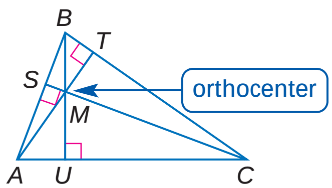
You could also multiply the equation $DA = \frac{2}{3}DT$ by 3 to eliminate the denominator.

Cross-Curricular Project



Finding the orthocenter can be used to help you construct your own nine-point circle. Visit geometryonline.com to continue work on your project.

An **altitude** of a triangle is a segment from a vertex to the line containing the opposite side and perpendicular to the line containing that side. Every triangle has three altitudes. The intersection point of the altitudes of a triangle is called the **orthocenter**.



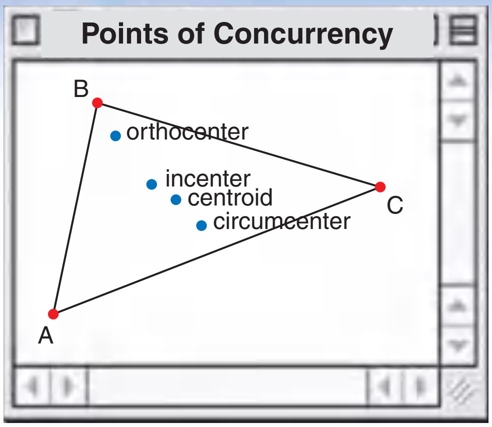
If the vertices of a triangle are located on a coordinate plane, you can use a system of equations to find the coordinates of the orthocenter.

GEOMETRY SOFTWARE LAB

Points of Concurrency

CONSTRUCT A FIGURE

- Use The Geometer's Sketchpad to construct acute scalene $\triangle ABC$.
- Construct and label the circumcenter, incenter, centroid, and orthocenter of $\triangle ABC$.

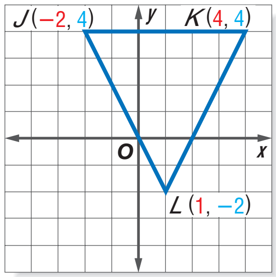


ANALYZE THE FIGURE

1. Drag the vertices of $\triangle ABC$ such that $\triangle ABC$ is a right triangle. Describe the position of each of the points of concurrency.
2. Drag a vertex of $\triangle ABC$ such that $\triangle ABC$ is an obtuse scalene triangle. Describe the position of each of the points of concurrency.
3. Explain your findings.

EXAMPLE Use a System of Equations to Find a Point

3 **COORDINATE GEOMETRY** **The vertices of $\triangle JKL$ are $J(-2, 4)$, $K(4, 4)$, and $L(1, -2)$. Find the coordinates of the orthocenter of $\triangle JKL$.**



Find an equation of the altitude from J to $\overline{KL}$. The slope of $\overline{KL}$ is 2, so the slope of the altitude is $-\frac{1}{2}$.

$$(y - y_1) = m(x - x_1) \quad \text{Point-slope form}$$

$$(y - 4) = -\frac{1}{2}(x - (-2)) \quad (x_1, y_1) = (-2, 4)$$

$$y - 4 = -\frac{1}{2}x - 1 \quad \text{Simplify.}$$

$$y = -\frac{1}{2}x + 3 \quad \text{Add 4 to each side.}$$

Find an equation of the altitude from K to $\overline{JL}$. The slope of $\overline{JL}$ is -2, so the slope of the altitude is $\frac{1}{2}$.

$$(y - y_1) = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 4 = \frac{1}{2}(x - 4) \quad (x_1, y_1) = (4, 4)$$

$$y - 4 = \frac{1}{2}x - 2 \quad \text{Simplify.}$$

$$y = \frac{1}{2}x + 2 \quad \text{Add 4 to each side.}$$

Solve a system of equations to find the point of intersection of the altitudes.

(continued on the next page)

> **Study Tip**
>
> **Graphing Calculator**
>
> Once you have two equations, you can graph the two lines and use the Intersect option on the Calc menu to determine where the two lines meet.

Study Tip

Simultaneous Equations

Systems of equations are also known as *simultaneous equations*, because a solution consists of values for the variables that satisfy all of the equations at the same time, or *simultaneously*.

Add to eliminate x.

	$y = -\frac{1}{2}x + 3$	Equation of altitude from J
	$(+)\ y = \frac{1}{2}x + 2$	Equation of altitude from K
	$2y = 5$	Add.
	$y = \frac{5}{2}$ or $2\frac{1}{2}$	Divide each side by 2.

Then replace y with $\frac{5}{2}$ in either equation to find x.

	$y = \frac{1}{2}x + 2$	
	$\frac{5}{2} = \frac{1}{2}x + 2$	$y = \frac{5}{2}$
	$\frac{1}{2} = \frac{1}{2}x$	Subtract 2 from each side.
	$1 = x$	Divide each side by $\frac{1}{2}$.

The coordinates of the orthocenter of $\triangle JKL$ are $\left(1, 2\frac{1}{2}\right)$. To check reasonableness, draw the altitudes of each side of the triangle on the coordinate grid. The intersection is the orthocenter.

3. Find the circumcenter of $\triangle JKL$.

Personal Tutor at geometryonline.com

You can also use systems of equations to find the coordinates of the circumcenter and the centroid of a triangle graphed on a coordinate plane.

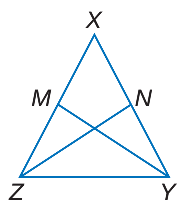
CONCEPT SUMMARY — *Special Segments in Triangles*

Name	Type	Point of Concurrency
perpendicular bisector	line, segment, or ray	circumcenter
angle bisector	line, segment, or ray	incenter
median	segment	centroid
altitude	segment	orthocenter

CHECK Your Understanding

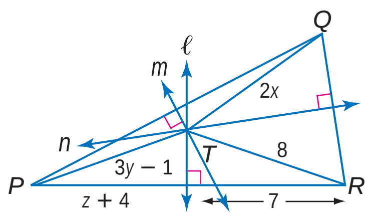
Example 1 (p. 270)

1. PROOF **Write a two-column proof.**
Given: $\overline{XY} \cong \overline{XZ}$
$\overline{YM}$ and $\overline{ZN}$ are medians.
Prove: $\overline{YM} \cong \overline{ZN}$



Example 2 (p. 272)

2. ALGEBRA Lines ℓ, m, and n are perpendicular bisectors of $\triangle PQR$ and meet at T. If $TQ = 2x$, $PT = 3y - 1$, and $TR = 8$, find x, y, and z.



Example 3 (pp. 273–274)

3. COORDINATE GEOMETRY The vertices of $\triangle ABC$ are $A(-3, 3)$, $B(3, 2)$, and $C(1, -4)$. Find the coordinates of the circumcenter.

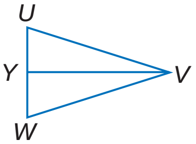
Exercises

HOMEWORK HELP	
For Exercises	See Examples
4–5, 23–25	1
6–15	2
16–22	3

PROOF Write a two-column proof.

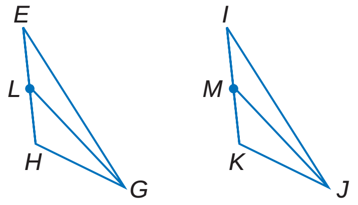
4. Given: $\triangle UVW$ is isosceles with vertex angle UVW. $\overline{YV}$ is the bisector of $\angle UVW$.

Prove: $\overline{YV}$ is a median.

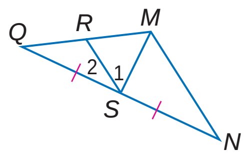


5. Given: $\overline{GL}$ is a median of $\triangle EGH$. $\overline{JM}$ is a median of $\triangle IJK$. $\triangle EGH \cong \triangle IJK$

Prove: $\overline{GL} \cong \overline{JM}$



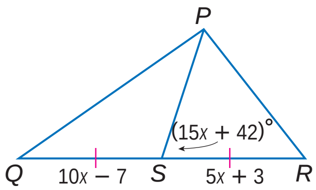
For Exercises 6 and 7, refer to $\triangle MNQ$ at the right.



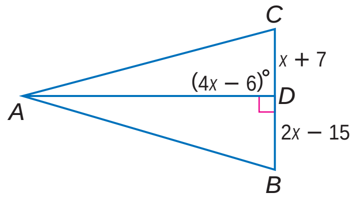
6. ALGEBRA Find x and $m\angle 2$ if $\overline{MS}$ is an altitude of $\triangle MNQ$, $m\angle 1 = 3x + 11$, and $m\angle 2 = 7x + 9$.

7. ALGEBRA If $\overline{MS}$ is a median of $\triangle MNQ$, $QS = 3a - 14$, $SN = 2a + 1$, and $m\angle MSQ = 7a + 1$, find the value of a. Is $\overline{MS}$ also an altitude of $\triangle MNQ$? Explain.

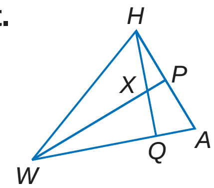
8. ALGEBRA Find x if $\overline{PS}$ is a median of $\triangle PQR$.



9. ALGEBRA Find x if $\overline{AD}$ is an altitude of $\triangle ABC$.



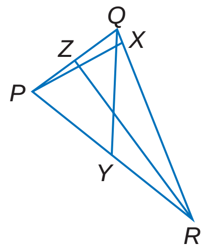
ALGEBRA For Exercises 10 and 11, refer to $\triangle WHA$ at the right.



10. If $\overline{WP}$ is a median and an angle bisector, $AP = 3y + 11$, $PH = 7y - 5$, $m\angle HWP = x + 12$, $m\angle PAW = 3x - 2$, and $m\angle HWA = 4x - 16$, find x and y. Is $\overline{WP}$ also an altitude? Explain.

11. If $\overline{WP}$ is a perpendicular bisector, $m\angle WHA = 8q + 17$, $m\angle HWP = 10 + q$, $AP = 6r + 4$, and $PH = 22 + 3r$, find r, q, and $m\angle HWP$.

ALGEBRA For Exercises 12–15, use the following information.



In $\triangle PQR$, $ZQ = 3a - 11$, $ZP = a + 5$, $PY = 2c - 1$, $YR = 4c - 11$, $m\angle PRZ = 4b - 17$, $m\angle ZRQ = 3b - 4$, $m\angle QYR = 7b + 6$, and $m\angle PXR = 2a + 10$.

12. $\overline{PX}$ is an altitude of $\triangle PQR$. Find a.

13. If $\overline{RZ}$ is an angle bisector, find $m\angle PRZ$.

14. Find PR if $\overline{QY}$ is a median.

15. If $\overleftrightarrow{QY}$ is a perpendicular bisector of $\overline{PR}$, find b.

COORDINATE GEOMETRY The vertices of $\triangle DEF$ are $D(4, 0)$, $E(-2, 4)$, and $F(0, 6)$. Find the coordinates of the points of concurrency of $\triangle DEF$.

16. centroid **17.** orthocenter **18.** circumcenter

COORDINATE GEOMETRY **For Exercises 19–22, use the following information.**

$R(3, 3)$, $S(-1, 6)$, and $T(1, 8)$ are the vertices of $\triangle RST$, and $\overline{RX}$ is a median.

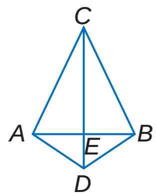
19. What are the coordinates of X?

20. Find RX.

21. Determine the slope of $\overleftrightarrow{RX}$. Then find the equation of the line.

22. Is $\overline{RX}$ an altitude of $\triangle RST$? Explain.

PROOF **Write a two-column proof for each theorem.**



23. Theorem 5.2

Given: $\overline{CA} \cong \overline{CB}$, $\overline{AD} \cong \overline{BD}$

Prove: C and D are on the perpendicular bisector of $\overline{AB}$.

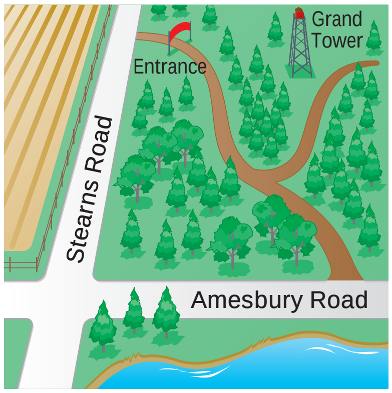
24. Theorem 5.5

25. Theorem 5.6

26. ORIENTEERING Orienteering is a competitive sport, originating in Sweden, that tests the skills of map reading and cross-country running. Competitors race through an unknown area to find various checkpoints using only a compass and topographical map. On an amateur course, clues are given to locate the first flag.

- The flag is as far from the Grand Tower as it is from the park entrance.
- If you run straight from Stearns Road to the flag or from Amesbury Road to the flag, you would run the same distance.

Describe how to find the first flag.

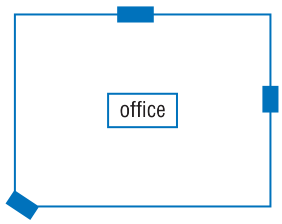


Real-World Link

The International Orienteering Federation World Cup consists of a series of nine races held throughout the world in which the runners compete for points based on their completion times.

Source: orienteering.org

27. ARCHITECTURE An architect is designing a high school building. Describe how to position the central office so it is equidistant from each of the three entrances to the school.



STATISTICS **For Exercises 28–31, use the following information.**

The *mean* of a set of data is an average value of the data. Suppose $\triangle ABC$ has vertices $A(16, 8)$, $B(2, 4)$, and $C(-6, 12)$.

28. Find the mean of the x-coordinates of the vertices.

29. Find the mean of the y-coordinates of the vertices.

30. Graph $\triangle ABC$ and its medians.

31. **Make a conjecture** about the centroid and the means of the coordinates of the vertices.

State whether each sentence is *always*, *sometimes*, or *never* true. Justify your reasoning.

32. The three medians of a triangle intersect at a point inside the triangle.

33. The three altitudes of a triangle intersect at a vertex of the triangle.

34. The three angle bisectors of a triangle intersect at a point in the exterior of the triangle.

35. The three perpendicular bisectors of a triangle intersect at a point in the exterior of the triangle.

EXTRA PRACTICE

See pages 809, 832.

Math Online

Self-Check Quiz at geometryonline.com

H.O.T. Problems

36. **REASONING** Compare and contrast a perpendicular bisector and a median of a triangle.

37. **REASONING** Find a counterexample to the statement *An altitude and an angle bisector of a triangle are never the same segment*.

38. **OPEN ENDED** Draw a triangle in which the circumcenter lies outside the triangle.

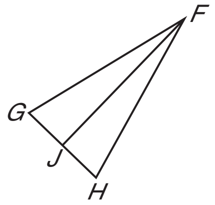
39. **Which One Doesn't Belong?** Identify the term that does not belong with the other three. Explain your reasoning.

orthocenter	point of concurrency	altitude	circumcenter

40. **CHALLENGE** Draw any $\triangle XYZ$ with median $\overline{XN}$ and altitude $\overline{XO}$. Recall that the area of a triangle is one-half the product of the measures of the base and the altitude. What conclusion can you make about the relationship between the areas of $\triangle XYN$ and $\triangle XZN$?

41. **Writing in Math** Explain how to balance a paper triangle on a pencil point. Is it possible for the incenter of a triangle to be the center of gravity?

42. In the figure below, $\overline{GJ} \cong \overline{HJ}$.



Which statement about $\overline{FJ}$ *must* be true?

A $\overline{FJ}$ is an angle bisector of $\triangle FGH$.

B $\overline{FJ}$ is a perpendicular bisector of $\triangle FGH$.

C $\overline{FJ}$ is a median of $\triangle FGH$.

D $\overline{FJ}$ is an altitude of $\triangle FGH$.

43. **REVIEW** An object that is projected straight upward with initial velocity v meters per second travels an estimated distance of $s = -vt + 10t^2$, where t = time in seconds. If Sherise is standing at the edge of a balcony 54 meters above the ground and throws a ball straight up with an initial velocity of 12 meters per second, after how many seconds will it hit the ground?

F 3 seconds

G 4 seconds

H 6 seconds

J 9 seconds

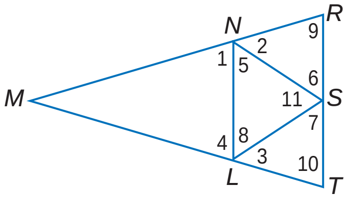
Position and label each triangle on the coordinate plane. (Lesson 4-7)

44. equilateral $\triangle ABC$ with base $\overline{AB}$ that is n units long

45. isosceles $\triangle DEF$ with congruent sides $2a$ units long and base a units long

46. right $\triangle GHI$ with hypotenuse $\overline{GI}$, HI is three times GH, and $\overline{GH}$ is x units long

For Exercises 47–50, refer to the figure at the right. (Lesson 4-6)



47. If $\angle 9 \cong \angle 10$, name two congruent segments.

48. If $\overline{NL} \cong \overline{SL}$, name two congruent angles.

49. If $\overline{LT} \cong \overline{LS}$, name two congruent angles.

50. If $\angle 1 \cong \angle 4$, name two congruent segments.

51. INTERIOR DESIGN Stacey is installing a curtain rod on the wall above the window. To ensure that the rod is parallel to the ceiling, she measures and marks 6 inches below the ceiling in several places. If she installs the rod at these markings centered over the window, how does she know the curtain rod will be parallel to the ceiling? (Lesson 3-6)

Determine the slope of the line that contains the given points. (Lesson 3-3)

52. $A(0, 6), B(4, 0)$ **53.** $G(8, 1), H(8, -6)$ **54.** $E(6, 3), F(-6, 3)$

55. Copy and complete the truth table. (Lesson 2-2)

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture. (Lesson 2-1)

56. Given: x is an integer.
Conjecture: $-x$ is negative.

57. Given: $WXYZ$ is a rectangle.
Conjecture: $WX = YZ$ and $WZ = XY$

58. $\angle L$ and $\angle M$ are complementary angles. $\angle N$ and $\angle P$ are complementary angles. If $m\angle L = y - 2$, $m\angle M = 2x + 3$, $m\angle N = 2x - y$, and $m\angle P = x - 1$, find the values of x, y, $m\angle L$, $m\angle M$, $m\angle N$, and $m\angle P$. (Lesson 1-5)

PREREQUISITE SKILL **Replace each ● with < or > to make each sentence true.**

59. $\frac{3}{8}$ ● $\frac{5}{16}$ **60.** 2.7 ● $\frac{5}{3}$ **61.** -4.25 ● $-\frac{19}{4}$ **62.** $-\frac{18}{25}$ ● $-\frac{19}{27}$

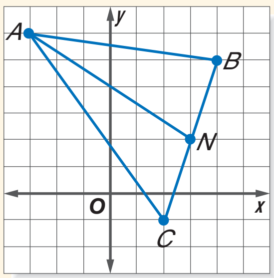
READING MATH

Writing Explanations

Often in mathematics, simply providing an answer is not sufficient. You must be able to show understanding by explaining your answers or justifying your reasoning.

EXAMPLE

Is $\overline{AN}$ an altitude of $\triangle ABC$? Justify your reasoning.



It is not enough to say that $\overline{AN}$ is not an altitude of $\triangle ABC$ because "it does not look like it." You must support your reasoning.

$$\text{slope of } \overline{AN} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - 6}{3 - (-3)} \quad (x_1, y_1) = (-3, 6), (x_2, y_2) = (3, 2)$$

$$= -\frac{2}{3} \quad \text{Simplify.}$$

$$\text{slope of } \overline{BC} = \frac{y_2 - y_1}{x_2 - x_1}$$

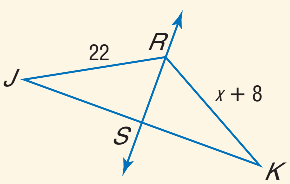
$$= \frac{-1 - 5}{2 - 4} \quad (x_1, y_1) = (4, 5), (x_2, y_2) = (2, -1)$$

$$= 3 \quad \text{Simplify.}$$

Complete Answer:
The product of the slopes of $\overline{AN}$ and $\overline{BC}$ is not -1, so the segments are not perpendicular. Therefore, $\overline{AN}$ is not an altitude of $\triangle ABC$.

Reading to Learn

1. Describe some ways that you can explain your answer or justify your reasoning in mathematics.
2. Refer to the graph of $\triangle ABC$ above. Is $\overline{AN}$ a median of $\triangle ABC$? Justify your reasoning.
3. Refer to $\triangle RKJ$ shown at the right. $\overleftrightarrow{RS}$ is a perpendicular bisector of $\overline{JK}$. What is the value of x? Explain.



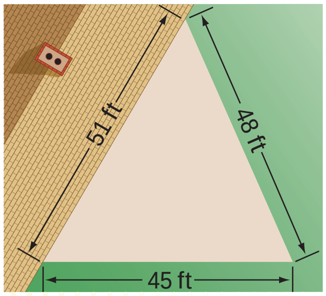
4. In $\triangle XYZ$, $XY = 15$ centimeters, $YZ = 12$ centimeters, and $ZX = 23$ centimeters. List the angles from greatest to least measure. Explain your reasoning.
5. How is writing explanations and justifications useful in making decisions and critical judgments in problem situations?

5-2 Inequalities and Triangles

Main Ideas

- Recognize and apply properties of inequalities to the measures of angles of a triangle.
- Recognize and apply properties of inequalities to the relationships between angles and sides of a triangle.

Bryan is delivering a potted tree for a patio. The tree is to be placed in the largest corner of the patio. All Bryan has is a diagram of the triangular patio that shows the measurements. Bryan can find the *largest corner* because the measures of the angles of a triangle are related to the measures of the sides opposite them.



Angle Inequalities In algebra, you learned about the inequality relationship between two real numbers. This relationship is often used in proofs.

KEY CONCEPT *Definition of Inequality*

For any real numbers a and b, $a > b$ if and only if there is a positive number c such that $a = b + c$.

Example: If $6 = 4 + 2$, $6 > 4$ and $6 > 2$.

The table below lists several properties of inequalities you studied in algebra. These properties can be applied to the measures of angles and segments since these are real numbers.

Properties of Inequalities for Real Numbers	
	For all numbers a, b, and c
Comparison Property	$a < b$, $a = b$, or $a > b$
Transitive Property	**1.** If $a < b$ and $b < c$, then $a < c$. **2.** If $a > b$ and $b > c$, then $a > c$.
Addition and Subtraction Properties	**1.** If $a > b$, then $a + c > b + c$ and $a - c > b - c$. **2.** If $a < b$, then $a + c < b + c$ and $a - c < b - c$.
Multiplication and Division Properties	**1.** If $c > 0$ and $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$. **2.** If $c > 0$ and $a > b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$. **3.** If $c < 0$ and $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$. **4.** If $c < 0$ and $a > b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

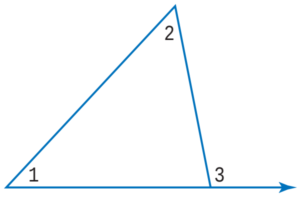
Study Tip

Symbols for Angles and Inequalities

The symbol for angle ($\angle$) looks similar to the symbol for less than ($<$), especially when handwritten. Be careful to write the symbols correctly in situations where both are used.

EXAMPLE 1 Compare Angle Measures

Determine which angle has the greatest measure.



Explore Compare the measure of $\angle 3$ to the measures of $\angle 1$ and $\angle 2$.

Plan Use properties and theorems of real numbers to compare the angle measures.

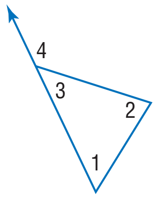
Solve Compare $m\angle 1$ to $m\angle 3$.
By the Exterior Angle Theorem, $m\angle 3 = m\angle 1 + m\angle 2$. Since angle measures are positive numbers and from the definition of inequality, $m\angle 3 > m\angle 1$.

Compare $m\angle 2$ to $m\angle 3$.

Again, by the Exterior Angle Theorem, $m\angle 3 = m\angle 1 + m\angle 2$. The definition of inequality states that if $m\angle 3 = m\angle 1 + m\angle 2$, then $m\angle 3 > m\angle 2$.

Check $m\angle 3$ is greater than $m\angle 1$ and $m\angle 2$. Therefore, $\angle 3$ has the greatest measure.

1. Determine which angle has the greatest measure.



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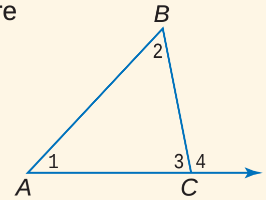
The results from Example 1 suggest that the measure of an exterior angle is always greater than either of the measures of the remote interior angles.

THEOREM 5.8 *Exterior Angle Inequality*

If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its corresponding remote interior angles.

Example: $m\angle 4 > m\angle 1$

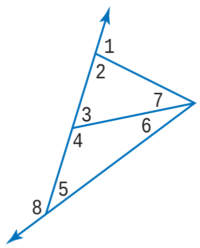
$m\angle 4 > m\angle 2$



The proof of Theorem 5.8 is in Lesson 5-3.

EXAMPLE 2 Exterior Angles

Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.



a. measures less than $m\angle 8$

By the Exterior Angle Inequality Theorem, $m\angle 8 > m\angle 4$, $m\angle 8 > m\angle 6$, $m\angle 8 > m\angle 2$, and $m\angle 8 > m\angle 6 + m\angle 7$. Thus, the measures of $\angle 4$, $\angle 6$, $\angle 2$, and $\angle 7$ are all less than $m\angle 8$.

(continued on the next page)

b. measures greater than $m\angle 2$

By the Exterior Angle Inequality Theorem, $m\angle 8 > m\angle 2$ and $m\angle 4 > m\angle 2$. Thus, the measures of $\angle 4$ and $\angle 8$ are greater than $m\angle 2$.

2. measures less than $\angle 3$

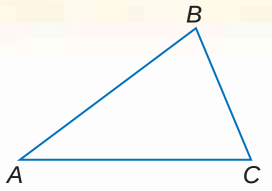
Angle-Side Relationships Recall that if two sides of a triangle are congruent, then the angles opposite those sides are congruent. In the following Geometry Activity, you will investigate the relationship between sides and angles when they are not congruent.

GEOMETRY LAB

Inequalities for Sides and Angles of Triangles

MODEL

Step 1 **Draw an acute scalene triangle, and label the vertices *A*, *B*, and *C*.**



Step 2 **Measure each side of the triangle. Record the measures in a table.**

Side	Measure
$\overline{BC}$	
$\overline{AC}$	
$\overline{AB}$	

Step 3 **Measure each angle of the triangle. Record each measure in a table.**

Angle	Measure
$\angle A$	
$\angle B$	
$\angle C$	

ANALYZE

1. Describe the measure of the angle opposite the longest side in terms of the other angles.

2. Describe the measure of the angle opposite the shortest side in terms of the other angles.

3. Repeat the activity using other triangles.

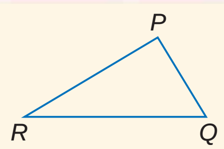
MAKE A CONJECTURE

4. What can you conclude about the relationship between the measures of sides and angles of a triangle?

The Geometry Lab suggests the following theorem.

THEOREM 5.9

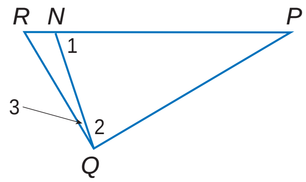
If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.



PROOF Theorem 5.9

Given: $\triangle PQR$
$PQ < PR$
$\overline{PN} \cong \overline{PQ}$

Prove: $m\angle R < m\angle PQR$



Proof:

Statements	Reasons
1. $\triangle PQR$, $PQ < PR$, $\overline{PN} \cong \overline{PQ}$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Isosceles Triangle Theorem
3. $m\angle 1 = m\angle 2$	**3.** Definition of congruent angles
4. $m\angle R < m\angle 1$	**4.** Exterior Angle Inequality Theorem
5. $m\angle 2 + m\angle 3 = m\angle PQR$	**5.** Angle Addition Postulate
6. $m\angle 2 < m\angle PQR$	**6.** Definition of inequality
7. $m\angle 1 < m\angle PQR$	**7.** Substitution Property of Equality
8. $m\angle R < m\angle PQR$	**8.** Transitive Property of Inequality

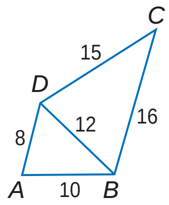
Study Tip

Theorem 5.9

The longest side in a triangle is opposite the largest angle in that triangle.

EXAMPLE Side-Angle Relationships

3 **Determine the relationship between the measures of the given angles.**



a. $\angle ADB$, $\angle DBA$

The side opposite $\angle ADB$ is longer than the side opposite $\angle DBA$, so $m\angle ADB > m\angle DBA$.

b. $\angle CDA$, $\angle CBA$

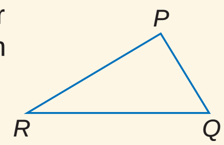
$$m\angle DBA < m\angle ADB$$
$$m\angle CBD < m\angle CDB$$
$$m\angle DBA + m\angle CBD < m\angle ADB + m\angle CDB$$
$$m\angle CBA < m\angle CDA$$

3. $\angle CBD$, $\angle CDB$

The converse of Theorem 5.9 is also true.

THEOREM 5.10

If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.



You will prove Theorem 5.10 in Lesson 5-3, Exercise 21.

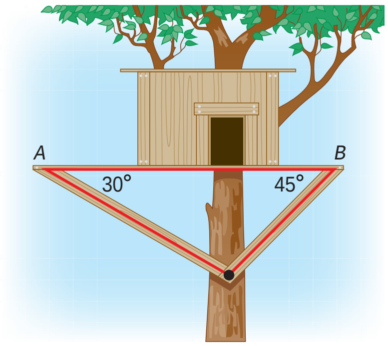
Real-World Link

The strength of the tree is the most important concern when building a treehouse. It is important to look for a tree that has thick, strong branches.

Source: www.treehouses.com

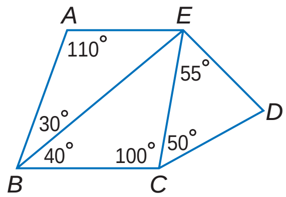
EXAMPLE Angle-Side Relationships

4 **TREEHOUSES** **Mrs. Sanchez is constructing the framework for part of a treehouse for her daughter. She plans to install braces at the ends of a certain floor support as shown. Which brace will be longer— the brace attached to *A* or to *B*?**



Theorem 5.10 states that if one angle of a triangle has a greater measure, then the side opposite that angle is longer than the side opposite the other angle. Therefore, the brace attached to the end marked *A* will be longer than the brace attached to the end marked *B*.

4. Determine the relationship between *BC* and *EC*.



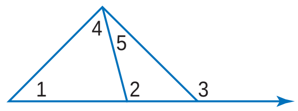
Check Your Understanding

Example 1 (p. 281) **Determine which angle has the greatest measure.**

1. ∠1, ∠2, ∠4

2. ∠2, ∠3, ∠5

3. ∠1, ∠2, ∠3, ∠4, ∠5

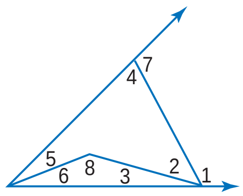


Example 2 (pp. 281–282) **Use the Exterior Angle Inequality Theorem to list all angles that satisfy the stated condition.**

4. measures less than $m\angle 1$

5. measures greater than $m\angle 6$

6. measures less than $m\angle 7$

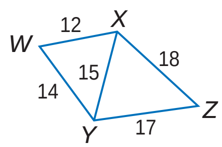


Example 3 (p. 283) **Determine the relationship between the measures of the given angles.**

7. ∠*WXY*, ∠*XYW*

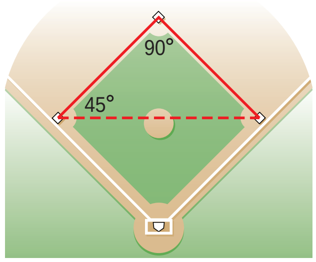
8. ∠*XZY*, ∠*XYZ*

9. ∠*WYX*, ∠*XWY*



Example 4 (p. 284)

10. **BASEBALL** During a baseball game, the batter hits the ball to the third baseman and begins to run toward first base. At the same time, the runner on first base runs toward second base. If the third baseman wants to throw the ball to the nearest base, to which base should he throw? Explain.



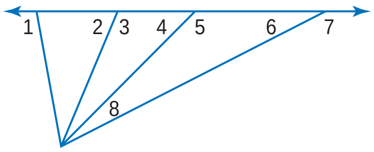
Tony Freeman/PhotoEdit

Exercises

For Exercises	See Examples
11–16	1
17–20	2
21–26	3
27–32	4

HOMEWORK HELP

Determine which angle has the greatest measure.

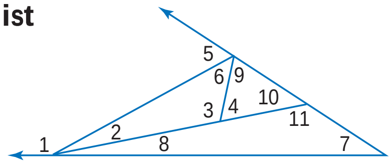


11. $\angle 1, \angle 2, \angle 4$ **12.** $\angle 2, \angle 4, \angle 6$

13. $\angle 3, \angle 5, \angle 7$ **14.** $\angle 1, \angle 2, \angle 6$

15. $\angle 5, \angle 7, \angle 8$ **16.** $\angle 2, \angle 6, \angle 8$

Use the Exterior Angle Inequality Theorem to list all angles that satisfy the stated condition.



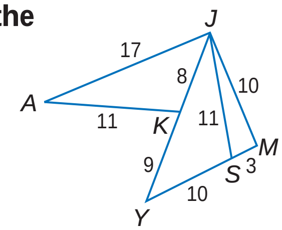
17. measures less than $m\angle 5$

18. measures greater than $m\angle 6$

19. measures greater than $m\angle 10$

20. measures less than $m\angle 11$

Determine the relationship between the measures of the given angles.

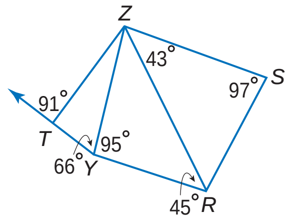


21. $\angle KAJ, \angle AJK$ **22.** $\angle MJY, \angle JYM$

23. $\angle SMJ, \angle MJS$ **24.** $\angle AKJ, \angle JAK$

25. $\angle MYJ, \angle JMY$ **26.** $\angle JSY, \angle JYS$

Determine the relationship between the lengths of the given sides.



27. $\overline{ZY}, \overline{YR}$ **28.** $\overline{SR}, \overline{ZS}$

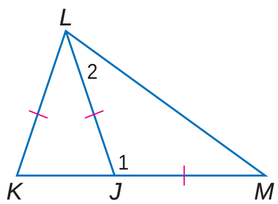
29. $\overline{RZ}, \overline{SR}$ **30.** $\overline{ZY}, \overline{RZ}$

31. $\overline{TY}, \overline{ZY}$ **32.** $\overline{TY}, \overline{ZT}$

PROOF **Write a two-column proof.**

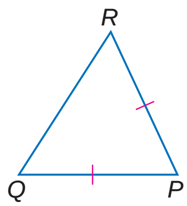
33. Given: $\overline{JM} \cong \overline{JL}$
$\overline{JL} \cong \overline{KL}$

Prove: $m\angle 1 > m\angle 2$



34. Given: $\overline{PR} \cong \overline{PQ}$
$QR > QP$

Prove: $m\angle P > m\angle Q$



Real-World Link

One sixth of adult Americans have never flown in a commercial aircraft.

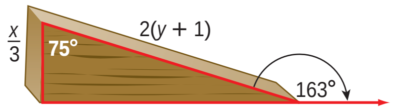
Source: U.S. Bureau of Transportation Statistics

35. TRAVEL A plane travels from Chicago to Atlanta, on to Austin, and then completes the trip directly back to Chicago as shown in the diagram. Name the legs of the trip in order from longest to shortest.

36. **COORDINATE GEOMETRY** Triangle KLM has vertices $K(3, 2)$, $L(-1, 5)$, and $M(-3, -7)$. List the angles in order from the least to the greatest measure.

37. If $AB > AC > BC$ in $\triangle ABC$ and $\overline{AM}$, $\overline{BN}$, and $\overline{CO}$ are the medians of the triangle, list AM, BN, and CO in order from least to greatest.

38. **SKATEBOARDING** The wedge at the right represents a skateboard ramp. The values of x and y are in inches. Write an inequality relating x and y. Then solve the inequality for y in terms of x.



ALGEBRA Find the value of n. List the sides of $\triangle PQR$ in order from shortest to longest for the given angle measures.

39. $m\angle P = 9n + 29$, $m\angle Q = 93 - 5n$, $m\angle R = 10n + 2$
40. $m\angle P = 12n - 9$, $m\angle Q = 62 - 3n$, $m\angle R = 16n + 2$
41. $m\angle P = 9n - 4$, $m\angle Q = 4n - 16$, $m\angle R = 68 - 2n$
42. $m\angle P = 3n + 20$, $m\angle Q = 2n + 37$, $m\angle R = 4n + 15$
43. $m\angle P = 4n + 61$, $m\angle Q = 67 - 3n$, $m\angle R = n + 74$

EXTRA PRACTICE
See pages 809, 832.
Math Online
Self-Check Quiz at geometryonline.com

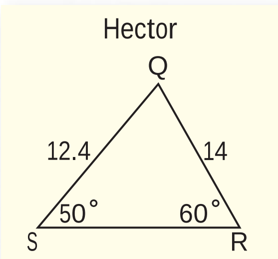
44. **PROOF** Write a paragraph proof for the following statement.
If a triangle is not isosceles, then the measure of the median to any side of the triangle is greater than the measure of the altitude to that side.

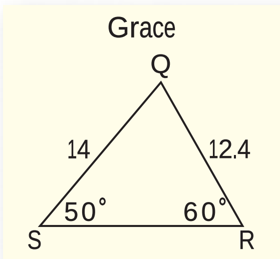
H.O.T. Problems

45. **REASONING** Is the following statement *always*, *sometimes*, or *never* true? Justify your answer.
In $\triangle JKL$ with right angle J, if $m\angle J$ is twice $m\angle K$, then the side opposite $\angle J$ is twice the length of the side opposite $\angle K$.

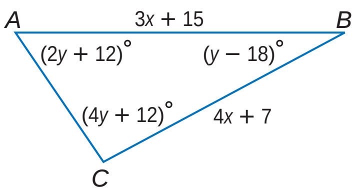
46. **OPEN ENDED** Draw $\triangle ABC$ such that $m\angle A > m\angle B > m\angle C$. Do not measure the angles. Explain how you know the greatest and least angle measures.

47. **FIND THE ERROR** Hector and Grace each labeled $\triangle QRS$. Who is correct? Explain.





48. **CHALLENGE** Write and solve an inequality for x.



49. **Writing in Math** Refer to the diagram on page 280. How can you tell which corner is largest? Include the name of the theorem or postulate that lets you determine the comparison of the angle measures and which angles in the diagram are the largest.

50. Two angles of a triangle have measures 45° and 92°. What type of triangle is it?

A obtuse scalene

B obtuse isosceles

C acute scalene

D acute isosceles

51. What is the x-intercept of the graph of $4x - 6y = 12$?

F -3 **H** 2

G -2 **J** 3

52. REVIEW The chart below describes the speed of four students folding letters to be mailed to local businesses.

Student	Description
Neiva	Folds 1 page every 3 seconds
Sarah	Folds 2 pages every 5 seconds
Quin	Folds 100 pages per minute
Deron	Folds 180 pages in 2 minutes

Which student is the *fastest*?

A Sarah **C** Neiva

B Quin **D** Deron

ALGEBRA For Exercises 53–55, use the following information. (Lesson 5-1)

Two vertices of $\triangle ABC$ are $A(3, 8)$ and $B(9, 12)$. $\overline{AD}$ is a median with D at (12, 3).

53. What are the coordinates of C?

54. Is $\overline{AD}$ an altitude of $\angle ABC$? Explain.

55. The graph of point E is at (6, 6). $\overline{EF}$ intersects $\overline{BD}$ at F. If F is at $\left(10\frac{1}{2}, 7\frac{1}{2}\right)$, is $\overline{EF}$ a perpendicular bisector of $\overline{BD}$? Explain.

56. AMUSEMENT PARK Miguel and his friends are at the Ferris wheel. They head 50 feet east to the snack hut. Then Miguel and a friend head north 25 feet to wait in line for a roller coaster ride. The rest of their group continues walking east 50 feet to the water park. Write a coordinate proof to prove that the Ferris wheel, the end of the line for the roller coaster, and the water park form an isosceles triangle. (Lesson 4-7)

Name the corresponding congruent angles and sides for each pair of congruent triangles. (Lesson 4-3)

57. $\triangle TUV \cong \triangle XYZ$ **58.** $\triangle CDG \cong \triangle RSW$ **59.** $\triangle BCF \cong \triangle DGH$

60. Find the value of x so that the line containing points at $(x, 2)$ and $(-4, 5)$ is perpendicular to the line containing points at (4, 8) and $(2, -1)$. (Lesson 3-3)

PREREQUISITE SKILL Determine whether each equation is *true* or *false* if $a = 2$, $b = 5$, and $c = 6$.

61. $2ab = 20$ **62.** $c(b - a) = 15$ **63.** $a + c > a + b$

5-3 Indirect Proof

Main Ideas

- Use indirect proof with algebra.
- Use indirect proof with geometry.

New Vocabulary

indirect reasoning
indirect proof
proof by contradiction

In *The Adventure of the Blanched Soldier*, Sherlock Holmes describes his detective technique, stating, "That process starts upon the supposition that when you have eliminated all which is impossible, then whatever remains, . . . must be the truth." The method Sherlock Holmes uses is an example of *indirect reasoning*.

Indirect Proof with Algebra The proofs you have written so far use direct reasoning, in which you start with a true hypothesis and prove that the conclusion is true. When using **indirect reasoning**, you assume that the conclusion is false and then show that this assumption leads to a contradiction of the hypothesis, or some other accepted fact, such as a definition, postulate, theorem, or corollary. Since all other steps in the proof are logically correct, the assumption has been proven false, so the original conclusion must be true. A proof of this type is called an **indirect proof** or a **proof by contradiction**. The following steps summarize the process of an indirect proof.

Study Tip

Truth Value of a Statement
Recall that a statement must be either true or false. To review **truth values**, see Lesson 2-2.

KEY CONCEPT — *Writing an Indirect Proof*

1. Assume that the conclusion is false.
2. Show that this assumption leads to a contradiction of the hypothesis, or some other fact, such as a definition, postulate, theorem, or corollary.
3. Point out that because the false conclusion leads to an incorrect statement, the original conclusion must be true.

EXAMPLE 1 State Assumptions

State the assumption you would make to start an indirect proof of each statement.

a. $AB \neq MN$

$AB = MN$

b. **$\triangle PQR$ is an isosceles triangle.**

$\triangle PQR$ is not an isosceles triangle.

c. **If 9 is a factor of n, then 3 is a factor of n.**

The conclusion of the conditional statement is *3 is a factor of n*. The negation of the conclusion is *3 is not a factor of n*.

1A. $x < 4$

1B. $\angle 3$ is an obtuse angle.

Joshua Ets-Hokin/Getty Images

Indirect proofs can be used to prove algebraic concepts.

EXAMPLE 2 **Algebraic Proof**

Given: $2x - 3 > 7$

Prove: $x > 5$

Indirect Proof:

Step 1 Assume that $x \leq 5$. That is, assume that $x < 5$ or $x = 5$.

Step 2 Make a table with several possibilities for x given that $x < 5$ or $x = 5$. This is a contradiction because when $x < 5$ or $x = 5$, $2x - 3 \leq 7$.

x	$2x - 3$
1	-1
2	1
3	3
4	5
5	7

Step 3 In both cases, the assumption leads to the contradiction of a known fact. Therefore, the assumption that $x \leq 5$ must be false, which means that $x > 5$ must be true.

2. If $7x < 56$, then $x < 8$.

Indirect reasoning and proof can be used in everyday situations.

3 **SHOPPING** **Lawanda bought two skirts for just over \$60, before tax. A few weeks later, her friend Tiffany asked her how much each skirt cost. Lawanda could not remember the individual prices. Use indirect reasoning to show that at least one of the skirts cost more than \$30.**

Given: The two skirts cost more than \$60.

Prove: At least one of the skirts cost more than \$30.

That is, if $x + y > 60$, then either $x > 30$ or $y > 30$.

Indirect Proof:

Step 1 Assume that neither skirt costs more than \$30. That is, $x \leq 30$ and $y \leq 30$.

Step 2 If $x \leq 30$ and $y \leq 30$, then $x + y \leq 60$. This is a contradiction because we know that the two skirts cost more than \$60.

Step 3 The assumption leads to the contradiction of a known fact. Therefore, the assumption that $x \leq 30$ and $y \leq 30$ must be false. Thus, at least one of the skirts had to have cost more than \$30.

3. Ben traveled over 360 miles and made one stop. Use indirect reasoning to prove that he traveled more than 180 miles on one part of his trip.

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Real-World Link

The West Edmonton Mall in Edmonton, Alberta, Canada, is the world's largest entertainment and shopping center, with an area of 5.3 million square feet. The mall houses an amusement park, water park, ice rink, and aquarium, along with over 800 stores and services.

Source: westedmall.com

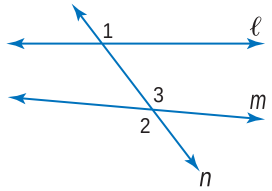
Indirect Proof with Geometry

Indirect reasoning can be used to prove statements in geometry.

EXAMPLE 4 Geometry Proof

Given: $\ell \nparallel m$

Prove: $\angle 1 \not\cong \angle 3$



Indirect Proof:

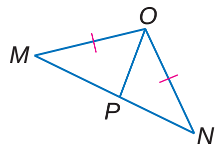
Step 1 Assume that $\angle 1 \cong \angle 3$.

Step 2 $\angle 1$ and $\angle 3$ are corresponding angles. If two lines are cut by a transversal so that corresponding angles are congruent, the lines are parallel. This means that $\ell \parallel m$. However, this contradicts the given statement.

Step 3 Since the assumption leads to a contradiction, the assumption must be false. Therefore, $\angle 1 \not\cong \angle 3$.

4. Given: $\overline{MO} \cong \overline{ON}$, $\overline{MP} \not\cong \overline{NP}$

Prove: $\angle MOP \not\cong \angle NOP$

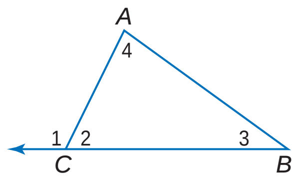


Indirect proofs can also be used to prove theorems.

Proof Exterior Angle Inequality Theorem

Given: $\angle 1$ is an exterior angle of $\triangle ABC$.

Prove: $m\angle 1 > m\angle 3$ and $m\angle 1 > m\angle 4$



Indirect Proof:

Step 1 Make the assumption that $m\angle 1 \ngtr m\angle 3$ or $m\angle 1 \ngtr m\angle 4$. In other words, $m\angle 1 \leq m\angle 3$ or $m\angle 1 \leq m\angle 4$.

Step 2 You only need to show that the assumption $m\angle 1 \leq m\angle 3$ leads to a contradiction as the argument for $m\angle 1 \leq m\angle 4$ follows the same reasoning.

$m\angle 1 \leq m\angle 3$ means that either $m\angle 1 = m\angle 3$ or $m\angle 1 < m\angle 3$.

Case 1: $m\angle 1 = m\angle 3$

$m\angle 1 = m\angle 3 + m\angle 4$ Exterior Angle Theorem

$m\angle 3 = m\angle 3 + m\angle 4$ Substitution

$0 = m\angle 4$ Subtract $m\angle 3$ from each side.

This contradicts the fact that the measure of an angle is greater than 0, so $m\angle 1 \neq m\angle 3$.

Case 2: $m\angle 1 < m\angle 3$

By the Exterior Angle Theorem, $m\angle 1 = m\angle 3 + m\angle 4$. Since angle measures are positive, the definition of inequality implies $m\angle 1 > m\angle 3$ and $m\angle 1 > m\angle 4$. This contradicts the assumption.

Step 3 In both cases, the assumption leads to the contradiction of a theorem or definition. Therefore, the assumption that $m\angle 1 > m\angle 3$ and $m\angle 1 > m\angle 4$ must be true.

Study Tip

Inequalities

Notice that the opposite of $m\angle 1 > m\angle 3$ is $m\angle 1 \leq m\angle 3$, not $m\angle 1 < m\angle 3$.

Interactive Lab
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✓ CHECK Your Understanding

Example 1 (p. 288)

Write the assumption you would make to start an indirect proof of each statement.

1. If $5x < 25$, then $x < 5$.

2. Two lines that are cut by a transversal so that alternate interior angles are congruent are parallel.

3. If the alternate interior angles formed by two lines and a transversal are congruent, the lines are parallel.

Example 2 (p. 289)

PROOF Write an indirect proof.

4. Given: $a > 0$
Prove: $\frac{1}{a} > 0$

5. Given: n is odd.
Prove: n^2 is odd.

Example 3 (p. 289)

6. BICYCLING The Tour de France bicycle race takes place over several weeks in various stages throughout France. During the first two stages of the 2005 Tour de France, riders raced for just over 200 kilometers. Prove that at least one of the stages was longer than 100 kilometers.

Example 4 (p. 290)

7. PROOF Use an indirect proof to show that the hypotenuse of a right triangle is the longest side.

Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
8–13	1
14, 15	2
16–21	3, 4

Write the assumption you would make to start an indirect proof of each statement.

8. $\overline{PQ} \cong \overline{ST}$

9. If $3x > 12$, then $x > 4$.

10. If a rational number is any number that can be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$, 6 is a rational number.

11. A median of an isosceles triangle is also an altitude.

12. Points P, Q, and R are collinear.

13. The angle bisector of the vertex angle of an isosceles triangle is also an altitude of the triangle.

PROOF For Exercises 14–19, write an indirect proof.

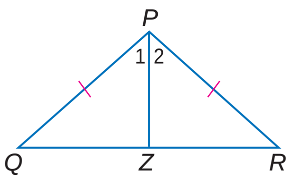
14. Given: $\frac{1}{a} < 0$
Prove: a is negative.

15. Given: n^2 is even.
Prove: n^2 is divisible by 4.

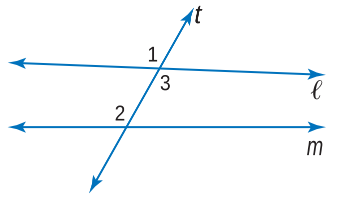
16. If $a > 0$, $b > 0$, and $a > b$, then $\frac{a}{b} > 1$.

17. If two sides of a triangle are not congruent, then the angles opposite those sides are not congruent.

18. Given: $\overline{PQ} \cong \overline{PR}$
$\angle 1 \not\cong \angle 2$
Prove: $\overline{PZ}$ is not a median of $\triangle PQR$.



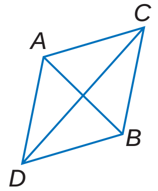
19. Given: $m\angle 2 \neq m\angle 1$
Prove: $\ell \not\parallel m$



PROOF **For Exercises 20 and 21, write an indirect proof.**

20. Given: $\triangle ABC$ and $\triangle ABD$ are equilateral. $\triangle ACD$ is not equilateral.

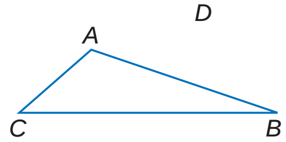
Prove: $\triangle BCD$ is not equilateral.



21. Theorem 5.10

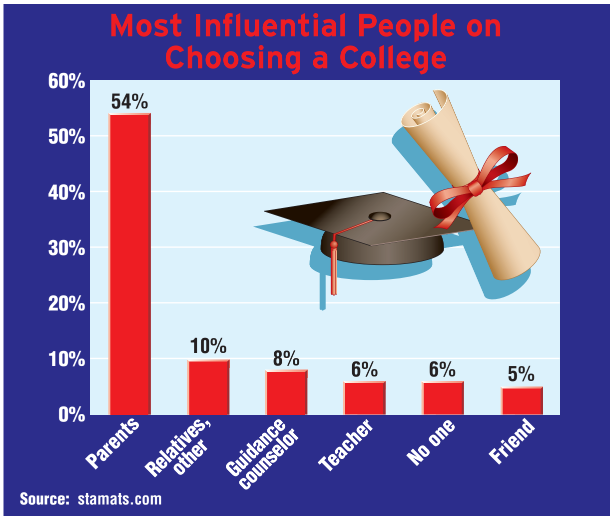
Given: $m\angle A > m\angle ABC$

Prove: $BC > AC$



22. BASKETBALL Ramon scored 85 points for his high school basketball team during the last six games. Prove that his average points per game were less than 15.

COLLEGE **For Exercises 23–25, refer to the graphic.**



23. Prove the following statement. *The majority of college-bound seniors stated that their parents were the most influential people in their choice of a college.*

24. If 1500 seniors were polled for this survey, verify that 75 said a friend influenced their decision most.

25. Were more seniors most influenced by their guidance counselors or by their teachers and friends? Explain.

26. LAW During the opening arguments of a trial, a defense attorney stated, "My client is innocent. The police report states that the crime was committed on November 6 at approximately 10:15 A.M. in San Diego. I can prove that my client was on vacation in Chicago with his family at this time. A verdict of not guilty is the only possible verdict." Explain whether this is an example of indirect reasoning.

27. GAMES Use indirect reasoning and a chart to solve this problem. A computer game involves a knight on a quest for treasure. At the end of the journey, the knight approaches two doors. A sign on the door on the right reads *Behind this door is a treasure chest and behind the other door is a ferocious dragon.* The door on the left has a sign that reads *One of these doors leads to a treasure chest and the other leads to a ferocious dragon.* A servant tells the knight that one of the signs is true and the other is false. Which door should the knight choose? Explain your reasoning.

EXTRA PRACTICE

See pages 810, 832.

Math Online

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H.O.T. Problems

28. REASONING Compare and contrast indirect proof and direct proof.

29. OPEN ENDED State a conjecture. Then write an indirect proof to prove your conjecture.

30. **CHALLENGE** Recall that a rational number is any number that can be expressed in the form $\frac{a}{b}$, where a and b are integers with no common factors and $b \neq 0$, or as a terminating or repeating decimal. Use indirect reasoning to prove that $\sqrt{2}$ is not a rational number.

31. **Writing in Math** Refer to the information on page 288. Explain how Sherlock Holmes used indirect proof, and include an example of indirect proof used every day.

32. **Theorem:** Angles supplementary to the same angle are congruent.

Dia is proving the theorem above by contradiction. She began by assuming that $\angle A$ and $\angle B$ are supplementary to $\angle C$ and $\angle A \ncong \angle B$. Which of the following reasons will Dia use to reach a contradiction?

A If two angles form a linear pair, then they are supplementary angles.

B If two supplementary angles are equal, the angles each measure 90.

C The sum of the measures of the angles in a triangle is 180.

D If two angles are supplementary, the sum of their measures is 180.

33. **REVIEW** At a five-star restaurant, a waiter's total earnings t in dollars for working h hours in which he receives \$198 in tips is given by the following equation.

$$t = 2.5h + 198$$

If the waiter earned a total of \$213, how many hours did he work?

F 2 **H** 6

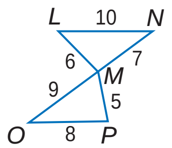
G 4 **J** 8

34. **REVIEW** Which expression has the *least* value?

A $|-28|$ **C** $|45|$

B $|15|$ **D** $|-39|$

For Exercises 35 and 36, refer to the figure at the right. (Lesson 5-2)



35 Which angle in $\triangle MOP$ has the greatest measure?

36. Name the angle with the least measure in $\triangle LMN$.

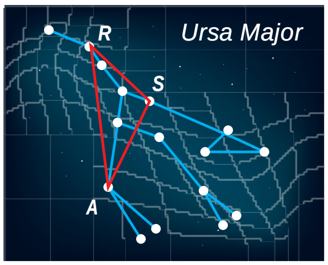
PROOF **Write a two-column proof.** (Lesson 5-1)

37. If an angle bisector of a triangle is also an altitude of the triangle, then the triangle is isosceles.

38. The median to the base of an isosceles triangle bisects the vertex angle.

39. Corresponding angle bisectors of congruent triangles are congruent.

40. **ASTRONOMY** Constellations were studied by astronomers to develop time-keeping systems. The Big Dipper is a part of the larger constellation Ursa Major. Three of the brighter stars in the constellation form $\triangle RSA$. If $m\angle R = 41$ and $m\angle S = 109$, find $m\angle A$. (Lesson 4-2)



PREREQUISITE SKILL **Determine whether each inequality is *true* or *false*.**

41. $19 - 10 < 11$

42. $31 - 17 < 12$

43. $38 + 76 > 109$

CHAPTER 5

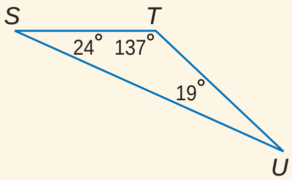
Mid-Chapter Quiz

Lessons 5-1 through 5-3

State whether each statement is *always*, *sometimes*, or *never* true. (Lesson 5-1)

1. The medians of a triangle intersect at one of the vertices of the triangle.
2. The angle bisectors of a triangle intersect at a point in the interior of the triangle.
3. The altitudes of a triangle intersect at a point in the exterior of the triangle.
4. The perpendicular bisectors of a triangle intersect at a point on the triangle.
5. Describe a triangle in which the angle bisectors all intersect in a point outside the triangle. If no triangle exists, write *no triangle*. (Lesson 5-1)

6. **MULTIPLE CHOICE** Which list gives the sides of $\triangle STU$ in order from longest to shortest? (Lesson 5-2)



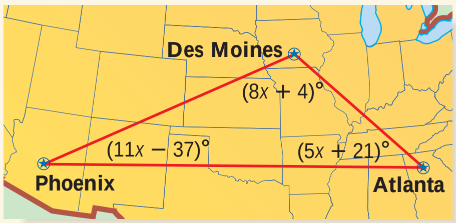
A $\overline{TU}, \overline{ST}, \overline{SU}$ **C** $\overline{SU}, \overline{ST}, \overline{TU}$

B $\overline{SU}, \overline{TU}, \overline{ST}$ **D** $\overline{ST}, \overline{TU}, \overline{SU}$

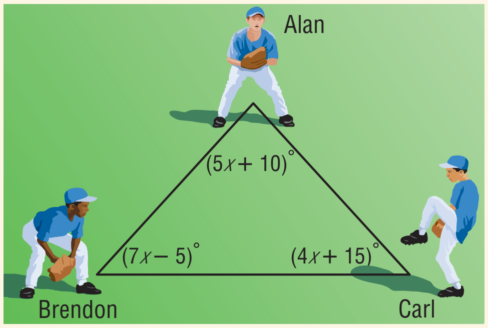
In $\triangle QRS$, $m\angle Q = x + 15$, $m\angle R = 2x + 10$, and $m\angle S = 4x + 15$. (Lesson 5-2)

7. Determine the measure of each angle.
8. List the sides in order from shortest to longest.

9. **TRAVEL** A plane travels from Des Moines to Phoenix, on to Atlanta, and then completes the trip directly back to Des Moines, as shown in the diagram. Write the lengths of the legs of the trip in order from greatest to least. (Lesson 5-2)



10. **BASEBALL** Alan, Brendon, and Carl were standing in the triangular shape shown below, throwing a baseball to warm up for a game. Between which two players was the throw the longest? (Lesson 5-2)

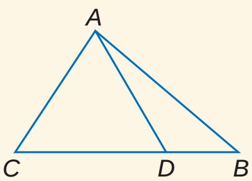


Write the assumption you would make to start an indirect proof of each statement. (Lesson 5-3)

11. The number 117 is divisible by 13.
12. $m\angle C < m\angle D$
13. n^3 is odd.
14. In a right triangle, $a^2 + b^2 = c^2$.
15. $\angle JKL \cong \angle WXY$
16. If n is an odd number, then $2n$ is an even number.
17. If $2x = 18$, then $x = 9$.

Write an indirect proof. (Lesson 5-3)

18. **Given:** $\triangle ABC$
    Prove: There can be no more than one obtuse angle in $\triangle ABC$.
19. **Given:** For lines m and n in plane $\mathcal{K}$, $m \nparallel n$.
    Prove: Lines m and n intersect at exactly one point.
20. **Given:** $m\angle ADC \neq m\angle ADB$
    Prove: $\overline{AD}$ is not an altitude of $\triangle ABC$.



EXPLORE 5-4

Graphing Calculator Lab
The Triangle Inequality

You can use the Cabri Junior application on a TI-83/84 Plus graphing calculator to discover properties of triangles.

ACTIVITY

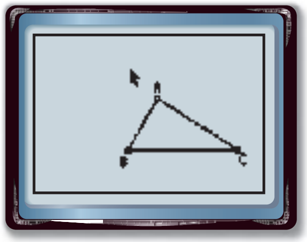
Construct a triangle. Observe the relationship between the sum of the lengths of two sides and the length of the other side.

Step 1 Construct a triangle using the triangle tool on the F2 menu. Then use the Alph-Num tool on the F5 menu to label the vertices as A, B, and C.

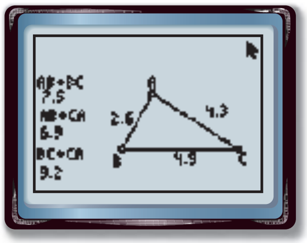
Step 2 Access the distance & length tool, shown as D. & Length, under Measure on the F5 menu. Use the tool to measure each side of the triangle.

Step 3 Display $AB + BC$, $AB + CA$, and $BC + CA$ by using the Calculate tool on the F5 menu. Label the measures.

Step 4 Click and drag the vertices to change the shape of the triangle.



Step 1



Steps 2 and 3

ANALYZE THE RESULTS

1. Replace each ● with <, >, or = to make a true statement.

$AB + BC$ ● CA $\quad$ $AB + CA$ ● BC $\quad$ $BC + CA$ ● AB

2. Click and drag the vertices to change the shape of the triangle. Then review your answers to Exercise 1. What do you observe?

3. Click on point A and drag it to lie on line BC. What do you observe about AB, BC, and CA? Are A, B, and C the vertices of a triangle? Explain.

4. **Make a conjecture** about the sum of the lengths of two sides of a triangle and the length of the third side.

5. Replace each ● with <, >, or = to make a true statement.

$|AB - BC|$ ● CA $\quad$ $|AB - CA|$ ● BC $\quad$ $|BC - CA|$ ● AB

Then click and drag the vertices to change the shape of the triangle and review your answers. What do you observe?

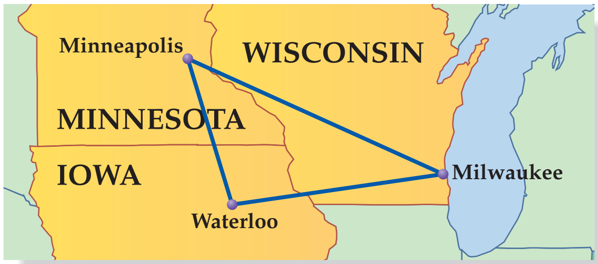
6. How could you use your observations to determine the possible lengths of the third side of a triangle if you are given the lengths of the other two sides?

5-4 The Triangle Inequality

Main Ideas

- Apply the Triangle Inequality Theorem.
- Determine the shortest distance between a point and a line.

Chuck Noland travels between Minneapolis, Waterloo, and Milwaukee as part of his job. Mr. Noland lives in Minneapolis and needs to get to Milwaukee as soon as possible. Should he take a flight that goes from Minneapolis to Milwaukee, or a flight that goes from Minneapolis to Waterloo, then to Milwaukee?

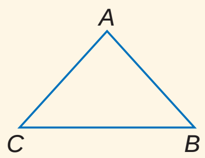


The Triangle Inequality If you think Mr. Noland should fly directly from Minneapolis to Milwaukee, you probably reasoned that a straight route is shorter. This is an example of the Triangle Inequality Theorem.

THEOREM 5.11 *Triangle Inequality Theorem*

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Examples:
$AB + BC > AC$
$BC + AC > AB$
$AC + AB > BC$



You will prove Theorem 5.11 in Exercise 21.

The Triangle Inequality Theorem can be used to determine whether three segments can form a triangle.

Study Tip

Inequality
If the sum of the least number and the middle number is greater than the greatest number, then each combination of inequalities is true.

EXAMPLE Identify Sides of a Triangle

1 **Determine whether the given measures can be the lengths of the sides of a triangle. 2, 4, 5**

Check each inequality.

$2 + 4 \stackrel{?}{>} 5$ | $2 + 5 \stackrel{?}{>} 4$ | $4 + 5 \stackrel{?}{>} 2$
$6 > 5$ ✓ | $7 > 4$ ✓ | $9 > 2$ ✓

All of the inequalities are true, so 2, 4, and 5 can be the lengths of the sides of a triangle.

1A. 6, 8, 14 **1B.** 8, 15, 17

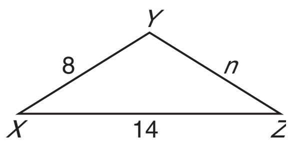
When you know the lengths of two sides of a triangle, you can determine the range of possible lengths for the third side.

STANDARDIZED TEST EXAMPLE — **Determine Possible Side Length**

1 Which of the following could *not* be the value of n?

A 6 **C** 14

B 10 **D** 18



Read the Test Item

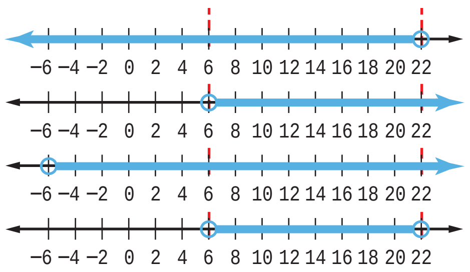
You need to determine which value is not valid.

Solve the Test Item

Solve each inequality to determine the range of values for YZ.

$XY + XZ > YZ$	$XY + YZ > XZ$	$YZ + XZ > XY$
$8 + 14 > n$	$8 + n > 14$	$n + 14 > 8$
$22 > n$ or $n < 22$	$n > 6$	$n > -6$

Graph the inequalities on the same number line.



Graph $n < 22$.

Graph $n > 6$.

Graph $n > -6$.

Find the intersection.

The range of values that fit all three inequalities is $6 < n < 22$.

Examine the answer choices. The only value that does not satisfy the compound inequality is 6 since $6 = 6$. Thus, the answer is choice A.

2. If the measures of two sides of a triangle are 57 and 32, what is the *least* possible measure of the third side if the measure is an integer?

F 25 **G** 26 **H** 88 **J** 89

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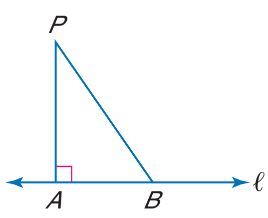
Test-Taking Tip

Testing Choices If you are short on time, you can test each choice to find the correct answer and eliminate any remaining choices.

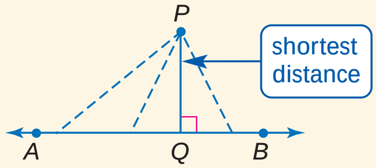
Distance Between a Point and a Line Recall that the distance between point P and line ℓ is measured along a perpendicular segment from the point to the line. It was accepted without proof that $\overline{PA}$ was the shortest segment from P to ℓ. The theorems involving the relationships between the angles and sides of a triangle can now be used to prove that a perpendicular segment is the shortest distance between a point and a line.



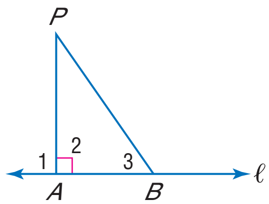
THEOREM 5.12

The perpendicular segment from a point to a line is the shortest segment from the point to the line.

Example: $\overline{PQ}$ is the shortest segment from P to $\overleftrightarrow{AB}$.



EXAMPLE 3 Prove Theorem 5.12



Given: $\overline{PA} \perp \ell$

$\overline{PB}$ is any nonperpendicular segment from P to ℓ.

Prove: $PB > PA$

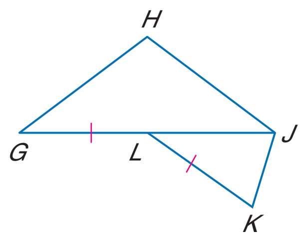
Proof:

Statements	Reasons
1. $PA \perp \ell$	**1.** Given
2. $\angle 1$ and $\angle 2$ are right angles.	**2.** $\perp$ lines form right angles.
3. $\angle 1 \cong \angle 2$	**3.** All right angles are congruent.
4. $m\angle 1 = m\angle 2$	**4.** Definition of congruent angles
5. $m\angle 1 > m\angle 3$	**5.** Exterior Angle Inequality Theorem
6. $m\angle 2 > m\angle 3$	**6.** Substitution Property
7. $PB > PA$	**7.** If an angle of a triangle is greater than a second angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

3. Write a two-column proof.

Given: $GL = LK$

Prove: $JH + GH > JK$



Study Tip

Shortest Distance to a Line

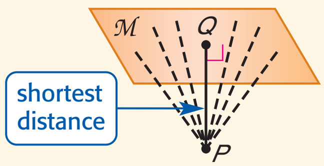
If a line is horizontal, the shortest distance from a point to that line will be along a vertical line. Likewise, the shortest distance from a point to a vertical line lies along a horizontal line.

Corollary 5.1 follows directly from Theorem 5.12.

COROLLARY 5.1

The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

Example: $\overline{QP}$ is the shortest segment from P to Plane $\mathcal{M}$.



You will prove Corollary 5.1 in Exercise 6.

Check Your Understanding

Example 1 (p. 296)

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.

1. 5, 4, 3

2. 5, 15, 10

3. 30.1, 0.8, 31

4. 5.6, 10.1, 5.2

Example 2 (p. 297)

5. MULTIPLE CHOICE An isosceles triangle has a base 10 units long. If the congruent sides have whole number measures, what is the *least* possible length of the sides?

A 5 **B** 6 **C** 17 **D** 21

Example 3 (p. 298)

6. PROOF Write a proof for Corollary 5.1.
Given: $\overline{PQ} \perp$ plane $\mathcal{M}$
Prove: $\overline{PQ}$ is the shortest segment from P to plane $\mathcal{M}$.

Exercises

HOMEWORK HELP

For Exercises	See Examples
7–12	1
13–18	2
19–20	3

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.

7. 1, 2, 3

8. 2, 6, 11

9. 8, 8, 15

10. 13, 16, 29

11. 18, 32, 21

12. 9, 21, 20

Find the range for the measure of the third side of a triangle given the measures of two sides.

13. 5 and 11

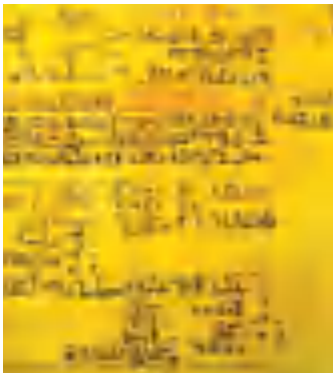
14. 7 and 9

15. 10 and 15

16. 12 and 18

17. 21 and 47

18. 32 and 61



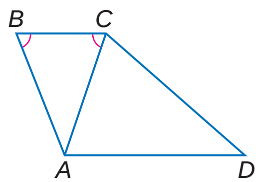
Real-World Link

Ancient Egyptians used pieces of flattened, dried papyrus reed as paper. From surviving examples like the Rhind Papyrus and the Moscow Papyrus, we have learned a bit about Egyptian mathematics.

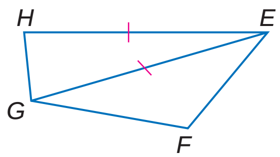
Source: aldokkan.com

PROOF **Write a two-column proof.**

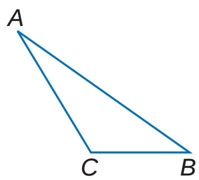
19. Given: $\angle B \cong \angle ACB$
Prove: $AD + AB > CD$



20. Given: $\overline{HE} \cong \overline{EG}$
Prove: $HE + FG > EF$



21. Given: $\triangle ABC$
Prove: $AC + BC > AB$ (Triangle Inequality Theorem)
(*Hint:* Draw auxiliary segment $\overline{CD}$, so that C is between B and D and $\overline{CD} \cong \overline{AC}$.)



22. HISTORY The early Egyptians used to make triangles by using a rope with knots tied at equal intervals. Each vertex of the triangle had to occur at a knot. How many different triangles can be formed using the rope below?

ALGEBRA **Determine whether the given coordinates are the vertices of a triangle. Explain.**

23. $A(5, 8), B(2, -4), C(-3, -1)$

24. $L(-24, -19), M(-22, 20), N(-5, -7)$

25. $X(0, -8), Y(16, -12), Z(28, -15)$

26. $R(1, -4), S(-3, -20), T(5, 12)$

SCRAPBOOKING **For Exercises 27 and 28, use the following information.** Carlota has several strips of trim she wishes to use as a triangular border for a spread in her scrapbook. The strips measure 3 centimeters, 4 centimeters, 5 centimeters, 6 centimeters, and 12 centimeters.

27. How many different triangles could Carlota make with the strips?

28. How many different triangles could Carlota make that have a perimeter that is divisible by 3?

PROBABILITY **For Exercises 29 and 30, use the following information.** One side of a triangle is 2 feet long. Let m represent the measure of the second side and n represent the measure of the third side. Suppose m and n are whole numbers and that $14 < m < 17$ and $13 < n < 17$.

29. List the measures of the sides of the triangles that are possible.

30. What is the probability that a randomly chosen triangle that satisfies the given conditions will be isosceles?

EXTRA PRACTICE
See pages 810, 832.

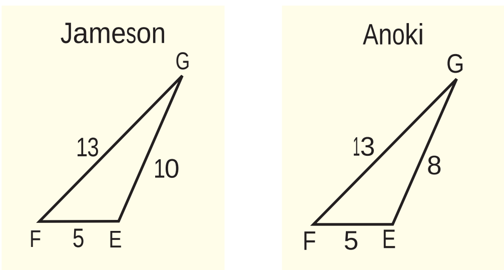
Math Online
Self-Check Quiz at
geometryonline.com

H.O.T. Problems

31. **REASONING** Explain why the distance between two nonhorizontal parallel lines on a coordinate plane cannot be found using the distance between their y-intercepts.

32. **OPEN ENDED** Find three numbers that can be the lengths of the sides of a triangle and three numbers that cannot be the lengths of the sides of a triangle. Justify your reasoning with a drawing.

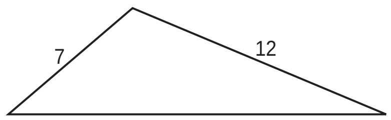
33. **FIND THE ERROR** Jameson and Anoki drew $\triangle EFG$ with $FG = 13$ and $EF = 5$. Each chose a possible measure for $\overline{GE}$. Who is correct? Explain.



34. **CHALLENGE** State and prove a theorem that compares the measures of each side of a triangle with the differences of the measures of the other two sides.

35. **Writing in Math** Refer to the information on page 296. Explain why it is not always possible to apply the Triangle Inequality Theorem when traveling.

36. If two sides of a triangle measure 12 and 7, which of the following can *not* be the perimeter of the triangle?



A 29 **C** 37

B 34 **D** 38

37. REVIEW Which equation describes the line that passes through the point (5, 3) and is parallel to the line represented by the equation $-2x + y = -4$?

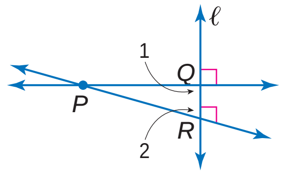
F $y = \frac{1}{2}x + 5.5$

G $y = 2x - 7$

H $y = -2x + 13$

J $y = \frac{2}{3}x + 15$

38. PROOF Write an indirect proof. (Lesson 5-3)
Given: P is a point not on line ℓ.
Prove: $\overline{PQ}$ is the only line through P perpendicular to ℓ.



39. TRAVEL Maddie drove 175 miles from Seattle, Washington, to Portland, Oregon. It took her three hours to complete the trip. Prove that her average driving speed was less than 60 miles per hour. (Lesson 5-3)

ALGEBRA List the sides of $\triangle PQR$ in order from longest to shortest if the angles of $\triangle PQR$ have the given measures. (Lesson 5-2)

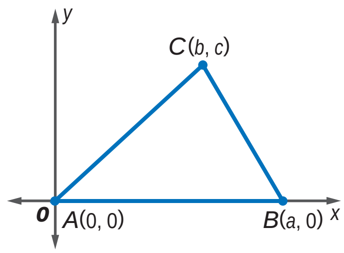
40. $m\angle P = 7x + 8$, $m\angle Q = 8x - 10$, $m\angle R = 7x + 6$

41. $m\angle P = 3x + 44$, $m\angle Q = 68 - 3x$, $m\angle R = x + 61$

For Exercises 42 and 43, refer to the figure. (Lesson 4-7)

42. Find the coordinates of D if the x-coordinate of D is the mean of the x-coordinates of the vertices of $\triangle ABC$ and the y-coordinate is the mean of the y-coordinates of the vertices of $\triangle ABC$.

43. Prove that D is the intersection of the medians of $\triangle ABC$.



Determine whether $\triangle JKL \cong \triangle PQR$ given the coordinates of the vertices. Explain. (Lesson 4-4)

44. $J(0, 5)$, $K(0, 0)$, $L(-2, 0)$, $P(4, 8)$, $Q(4, 3)$, $R(6, 3)$

45. $J(6, 4)$, $K(1, -6)$, $L(-9, 5)$, $P(0, 7)$, $Q(5, -3)$, $R(15, 8)$

46. $J(-6, -3)$, $K(1, 5)$, $L(2, -2)$, $P(2, -11)$, $Q(5, -4)$, $R(10, -10)$

PREREQUISITE SKILL Solve each inequality. (Pages 783–784)

47. $3x + 54 < 90$

48. $8x - 14 < 3x + 19$

49. $4x + 7 < 180$

5-5 Inequalities Involving Two Triangles

Main Ideas

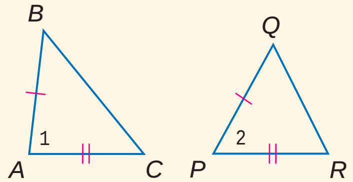
- Apply the SAS Inequality.
- Apply the SSS Inequality.

Many objects have a fixed arm connected with a joint or hinge to a second arm or stand. This thrill ride at Cedar Point in Sandusky, Ohio, sends riders into the sky in a pendulum motion. As the pendulum rises, the angle between the arm and the legs of the stand decreases until the arm moves past the stand. Then the angle increases. The distance between the riders and the docking station changes as the angle changes.

SAS Inequality The relationship of the arms and the angle between them illustrates the following theorem.

THEOREM 5.13 — SAS Inequality/Hinge Theorem

Two sides of a triangle are congruent to two sides of another triangle. If the included angle in the first triangle has a greater measure than the included angle in the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.

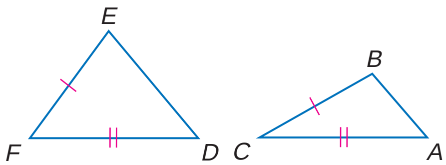


Example: Given $\overline{AB} \cong \overline{PQ}$, $\overline{AC} \cong \overline{PR}$, if $m\angle 1 > m\angle 2$, then $BC > QR$.

PROOF SAS Inequality Theorem

Given: $\triangle ABC$ and $\triangle DEF$
$\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$
$m\angle F > m\angle C$

Prove: $DE > AB$



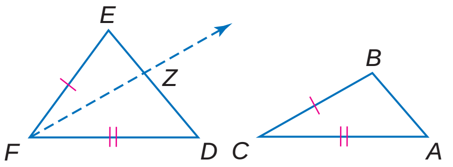
Proof:

We are given that $\overline{AC} \cong \overline{DF}$ and $\overline{BC} \cong \overline{EF}$. We also know that $m\angle F > m\angle C$. Draw auxiliary ray FZ such that $m\angle DFZ = m\angle C$ and that $\overline{ZF} \cong \overline{BC}$. This leads to two cases.

Study Tip

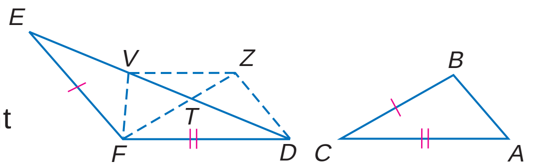
SAS Inequality
The SAS Inequality Theorem is also called the *Hinge Theorem.*

Case 1: If Z lies on $\overline{DE}$, then $\triangle FZD \cong \triangle CBA$ by SAS. Thus, $ZD = BA$ by CPCTC and the definition of congruent segments.



By the Segment Addition Postulate, $DE = EZ + ZD$. Also, $DE > ZD$ by the definition of inequality. Therefore, $DE > AB$ by the Substitution Property.

Case 2: If Z does not lie on $\overline{DE}$, then let the intersection of $\overline{FZ}$ and $\overline{ED}$ be point T. Now draw another auxiliary segment $\overline{FV}$ such that V is on $\overline{DE}$ and $\angle EFV \cong \angle VFZ$.

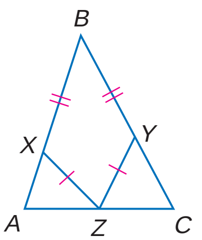


Since $\overline{FZ} \cong \overline{BC}$ and $\overline{BC} \cong \overline{EF}$, we have $\overline{FZ} \cong \overline{EF}$ by the Transitive Property. Also $\overline{VF}$ is congruent to itself by the Reflexive Property. Thus, $\triangle EFV \cong \triangle ZFV$ by SAS. By CPCTC, $\overline{EV} \cong \overline{ZV}$ or $EV = ZV$. Also, $\triangle FZD \cong \triangle CBA$ by SAS. So, $\overline{ZD} \cong \overline{BA}$ by CPCTC or $ZD = BA$.

In $\triangle VZD$, $VD + ZV > ZD$ by the Triangle Inequality Theorem. By substitution, $VD + EV > ZD$. Since $ED = VD + EV$ by the Segment Addition Postulate, $ED > ZD$. Using substitution, $ED > BA$ or $DE > AB$.

EXAMPLE 1 Use SAS Inequality in a Proof

Write a two-column proof.



Given: $\overline{YZ} \cong \overline{XZ}$
Z is the midpoint of $\overline{AC}$.
$m\angle CZY > m\angle AZX$
$\overline{BY} \cong \overline{BX}$

Prove: $BC > AB$

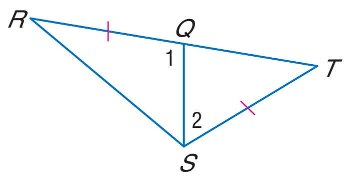
Proof:

Statements	Reasons
1. $\overline{YZ} \cong \overline{XZ}$; Z is the midpoint of $\overline{AC}$.; $m\angle CZY > m\angle AZX$; $\overline{BY} \cong \overline{BX}$	**1.** Given
2. $CZ = AZ$	**2.** Definition of midpoint
3. $CY > AX$	**3.** SAS Inequality
4. $BY = BX$	**4.** Definition of congruent segments
5. $CY + BY > AX + BX$	**5.** Addition Property
6. $BC = CY + BY$; $AB = AX + BX$	**6.** Segment Addition Postulate
7. $BC > AB$	**7.** Substitution Property

1. Write a two-column proof.

Given: $\overline{RQ} \cong \overline{ST}$

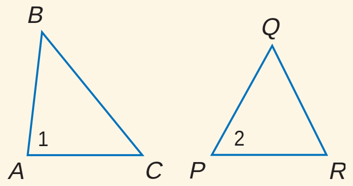
Prove: $RS > TQ$



SSS Inequality The converse of the SAS Inequality Theorem is the SSS Inequality Theorem.

THEOREM 5.14 *SSS Inequality Theorem*

If two sides of a triangle are congruent to two sides of another triangle and the third side in one triangle is longer than the third side in the other, then the angle between the pair of congruent sides in the first triangle is greater than the corresponding angle in the second triangle.



Example: Given $\overline{AB} \cong \overline{PQ}$, $\overline{AC} \cong \overline{PR}$, if $BC > QR$, then $m\angle 1 > m\angle 2$.

You will prove Theorem 5.14 in Exercise 24.

Study Tip

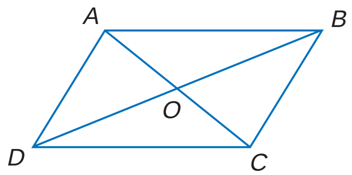
Proofs

Check each step in your proof. Make sure that each statement has a reason. Each statement should follow logically from the previous or given statements.

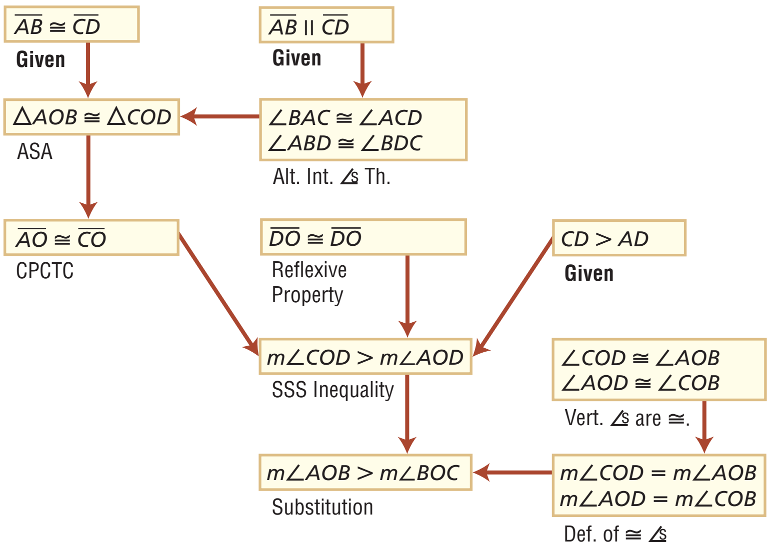
EXAMPLE 2 Prove Triangle Relationships

Given: $\overline{AB} \cong \overline{CD}$
$\overline{AB} \parallel \overline{CD}$
$CD > AD$

Prove: $m\angle AOB > m\angle BOC$



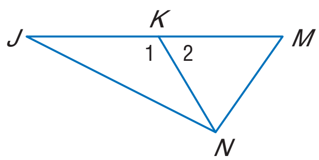
Flow Proof:



2. Write a two-column proof.

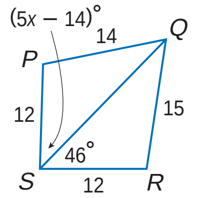
Given: $\overline{NK}$ is a median of $\triangle JMN$.
$JN > NM$

Prove: $m\angle 1 > m\angle 2$



EXAMPLE Relationships Between Two Triangles

3 **ALGEBRA Write an inequality using the information in the figure.**



a. Compare $m\angle QSR$ and $m\angle QSP$.

In $\triangle PQS$ and $\triangle RQS$, $\overline{PS} \cong \overline{RS}$, $\overline{QS} \cong \overline{QS}$, and $QR > QP$. The SAS Inequality allows us to conclude that $m\angle QSR > m\angle QSP$.

b. Find the range of values containing x.

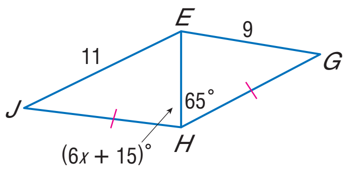
By the SSS Inequality, $m\angle QSR > m\angle QSP$, or $m\angle QSP < m\angle QSR$.

$m\angle QSP < m\angle QSR$	SSS Inequality
$5x - 14 < 46$	Substitution
$5x < 60$	Add 14 to each side.
$x < 12$	Divide each side by 5.

Also, recall that the measure of any angle is always greater than 0.

$5x - 14 > 0$	
$5x > 14$	Add 14 to each side.
$x > \frac{14}{5}$ or 2.8	Divide each side by 5.

The two inequalities can be written as the compound inequality $2.8 < x < 12$.



3A. Write an inequality to compare $m\angle JHE$ and $m\angle GHE$.

3B. Find the range of values containing x.

Personal Tutor at geometryonline.com

Real-World Link

Physical therapists help their patients regain range of motion after an illness or injury.

Source: www.apta.org

EXAMPLE Use Triangle Inequalities

4 **HEALTH *Range of motion* describes how much a limb can be moved from a straight position. To determine the range of motion of a person's arm, determine the distance from the wrist to the shoulder when the elbow is bent as far as possible.**

Jessica can bend her left arm so her left wrist is 5 inches from her shoulder and her right arm so her right wrist is 3 inches from her shoulder. Which arm has the greater range of motion? Explain.

The distance between the wrist and shoulder is smaller on her right arm. Assuming that both arms have the same measurements, the SSS inequality tells us that the angle formed at the elbow is smaller on the right arm. This means that the right arm has a greater range of motion.

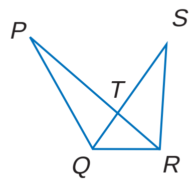
4. After physical therapy, Jessica can bend her left arm so her left wrist is 2 inches from her shoulder. She can bend her right arm so her right wrist is $2\frac{1}{2}$ inches from her shoulder. Which arm has the better range of motion now? Explain.

✓ CHECK Your Understanding

PROOF **Write a two-column proof.**

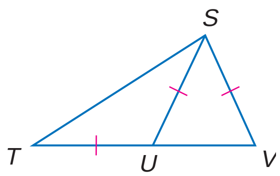
Example 1 (p. 303)

1. **Given:** $\overline{PQ} \cong \overline{SQ}$
   Prove: $PR > SR$



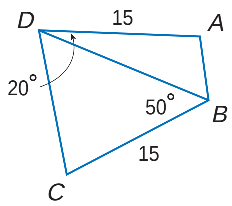
Example 2 (p. 304)

2. **Given:** $\overline{TU} \cong \overline{US}$
   $\overline{US} \cong \overline{SV}$
   Prove: $ST > UV$

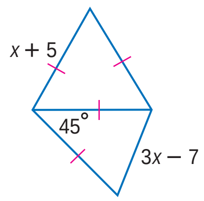


Example 3 (p. 305)

3. Write an inequality relating AB and CD.

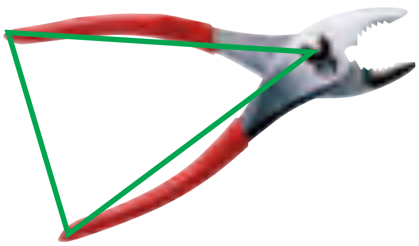


4. Write an inequality to describe the possible values of x.



Example 4 (p. 305)

5. **PHYSICAL SCIENCE** A lever is used to multiply the force applied to an object. One example of a lever is a pair of pliers. Use the SAS or SSS Inequality to explain how to use a pair of pliers.

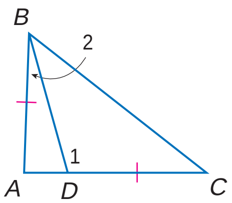


Exercises

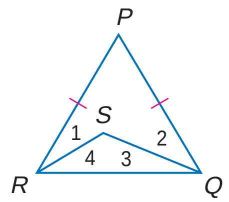
HOMEWORK HELP	
For Exercises	See Examples
6, 7	1
8, 9	2
10–15	3
16, 17	4

PROOF **Write a two-column proof.**

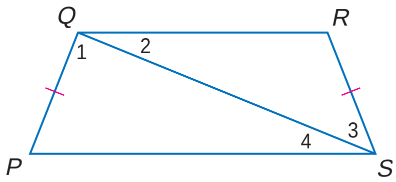
6. **Given:** $\triangle ABC$
   $\overline{AB} \cong \overline{CD}$
   Prove: $BC > AD$



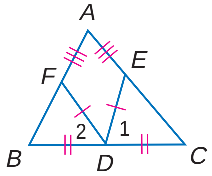
7. **Given:** $\overline{PR} \cong \overline{PQ}$
   $SQ > SR$
   Prove: $m\angle 1 < m\angle 2$



8. Given: $\overline{PQ} \cong \overline{RS}$
$QR < PS$
Prove: $m\angle 3 < m\angle 1$



9. Given: $\overline{ED} \cong \overline{DF}$
$m\angle 1 > m\angle 2$
D is the midpoint of $\overline{CB}$.
$\overline{AE} \cong \overline{AF}$
Prove: $AC > AB$



Real-World Career
Landscape Architect
Landscape architects design the settings of buildings and parks. By arranging the locations of the buildings and the plants, they make the site functional, beautiful, and environmentally friendly.

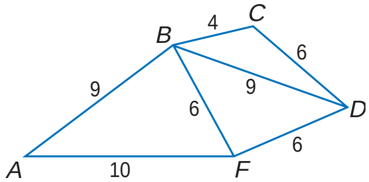
Math Online
For more information, go to geometryonline.com.

Write an inequality relating the given pair of angles or segment measures.

10. AB, FD

11. $m\angle BDC$, $m\angle FDB$

12. $m\angle FBA$, $m\angle DBF$

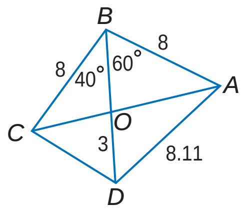


Write an inequality relating the given pair of angles or segment measures.

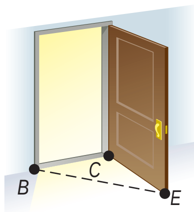
13. AD, DC

14. OC, OA

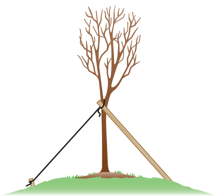
15. $m\angle AOD$, $m\angle AOB$



16. DOORS Open a door slightly. With the door open, measure the angle made by the door and the door frame. Measure the distance from the end of the door to the door frame. Open the door wider, and measure again. How do the measures compare?

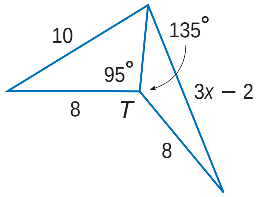


17. LANDSCAPING When landscapers plant new trees, they usually brace the tree using a stake tied to the trunk of the tree. Use the SAS or SSS Inequality to explain why this is an effective method for keeping a newly planted tree perpendicular to the ground .

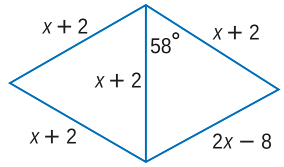


Write an inequality to describe the possible values of x.

18.



19.



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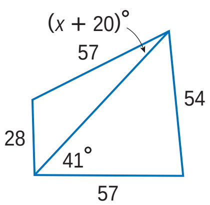
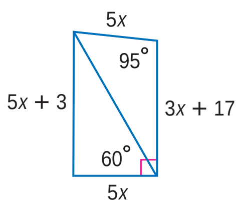


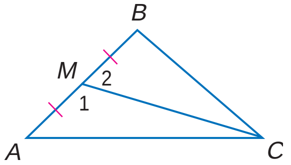
21.



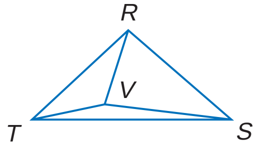
Rachel Epstein/PhotoEdit

Write an inequality to describe the possible values of *x*.

22. In the figure, $\overline{AM} \cong \overline{MB}$, $AC > BC$, $m\angle 1 = 5x + 20$ and $m\angle 2 = 8x - 100$.



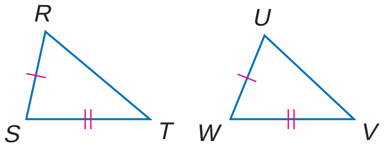
23. In the figure, $m\angle RVS = 15 + 5x$, $m\angle SVT = 10x - 20$, $RS < ST$, and $\angle RTV \cong \angle TRV$.



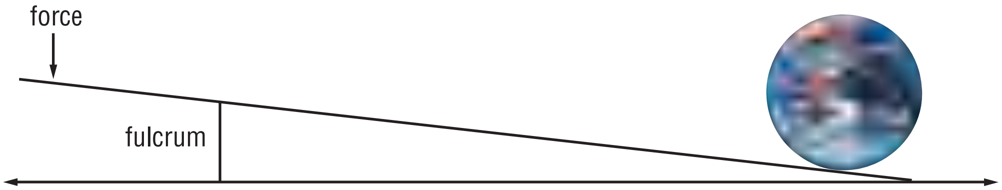
24. PROOF Use an indirect proof to prove the SSS Inequality Theorem (Theorem 5.14).

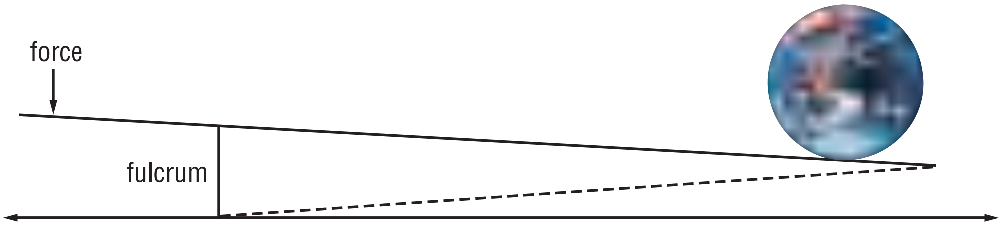
Given: $\overline{RS} \cong \overline{UW}$
$\overline{ST} \cong \overline{WV}$
$RT > UV$

Prove: $m\angle S > m\angle W$



25. HISTORY When force is applied to a lever that is balanced on a fulcrum, you can lift a heavy object. In the third century, Archimedes said, "Give me a place to stand and a lever long enough, and I will move the Earth." Write a description of how the triangle formed from the height of the fulcrum and the length of the lever from the fulcrum to Earth applies the SAS Inequality Theorem.





EXTRA PRACTICE
See pages 810, 832.
Math Online
Self-Check Quiz at
geometryonline.com

H.O.T. Problems

26. OPEN ENDED Describe a real-world object that illustrates either the SAS or the SSS inequality.

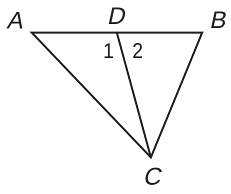
27. REASONING Compare and contrast the SSS Inequality Theorem to the SSS Postulate for triangle congruence.

28. CHALLENGE The SAS Inequality states that the base of an isosceles triangle gets longer as the measure of the vertex angle increases. Describe the effect of changing the measure of the vertex angle on the measure of the altitude. Justify your answer.

29. Writing in Math Refer to the information on page 302. Write a description of the angle between the arm and the stand as the ride operator lifts and then lowers the pendulum. Include an explanation of how the distance between the ends of the arm and stand is related to the angle between them.

STANDARDIZED TEST PRACTICE

30. If $\overline{DC}$ is a median of $\triangle ABC$ and $m\angle 1 > m\angle 2$, which of the following statements is *not* true?



A $AD = BD$

B $m\angle ADC = m\angle BDC$

C $AC > BC$

D $m\angle 1 > m\angle B$

31. REVIEW The weight of an object on Jupiter varies directly with its weight on Earth. If an object that weighs 5 pounds on Earth weighs 11.5 pounds on Jupiter, how much will a 7-pound object weigh on Jupiter?

F 9.3 pounds

G 13.5 pounds

H 16.1 pounds

J 80.5 pounds

Spiral Review

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain. (Lesson 5-4)

32. 25, 1, 21 **33.** 16, 6, 19 **34.** 8, 7, 15

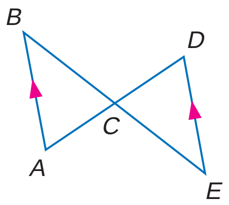
Write the assumption you would make to start an indirect proof of each statement. (Lesson 5-3)

35. $\overline{AD}$ is a median of $\triangle ABC$.

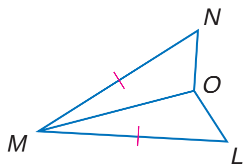
36. If two altitudes of a triangle are congruent, then the triangle is isosceles.

PROOF Write a two-column proof. (Lesson 4-5)

37. Given: $\overline{AD}$ bisects $\overline{BE}$.
$\overline{AB} \parallel \overline{DE}$
Prove: $\triangle ABC \cong \triangle DEC$



38. Given: $\overline{OM}$ bisects $\angle LMN$.
$\overline{LM} \cong \overline{MN}$
Prove: $\triangle MOL \cong \triangle MON$



Find the measures of the sides of $\triangle EFG$ with the given vertices and classify each triangle by its sides. (Lesson 4-1)

39. $E(4, 6)$, $F(4, 11)$, $G(9, 6)$

40. $E(-7, 10)$, $F(15, 0)$, $G(-2, -1)$

41. $E(16, 14)$, $F(7, 6)$, $G(-5, -14)$

42. $E(9, 9)$, $F(12, 14)$, $G(14, 6)$

Write an equation in point-slope form of the line having the given slope that contains the given point. (Lesson 3-4)

43. $m = 2$, $(4, 3)$ **44.** $m = -3$, $(2, -2)$ **45.** $m = 11$, $(-4, -9)$

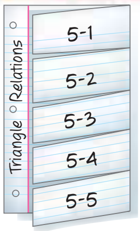
46. ADVERTISING An ad for Wildflowers Gift Boutique says *When it has to be special, it has to be Wildflowers.* Catalina needs a special gift. Does it follow that she should go to Wildflowers? Explain. (Lesson 2-4)

CHAPTER 5 Study Guide and Review

Download Vocabulary Review from geometryonline.com

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Special Segments in Triangles (Lesson 5-1)

- The special segments of triangles are perpendicular bisectors, angle bisectors, medians, and altitudes.
- The intersection points of each of the special segments of a triangle are called the *points of concurrency.*
- The points of concurrency for a triangle are the circumcenter, incenter, centroid, and orthocenter.

Indirect Proof (Lesson 5-3)

- Writing an Indirect Proof:
  1. Assume that the conclusion is false.
  2. Show that this assumption leads to a contradiction.
  3. Since the false conclusion leads to an incorrect statement, the original conclusion must be true.

Triangle Inequalities (Lessons 5-2, 5-4, 5-5)

- The largest angle in a triangle is opposite the longest side, and the smallest angle is opposite the shortest side.
- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- SAS Inequality (Hinge Theorem): In two triangles, if two sides are congruent, then the measure of the included angle determines which triangle has the longer third side.
- SSS Inequality: In two triangles, if two corresponding sides of each triangle are congruent, then the length of the third side determines which triangle has the included angle with the greater measure.

Key Vocabulary

altitude (p. 272)
centroid (p. 271)
circumcenter (p. 270)
concurrent lines (p. 270)
incenter (p. 271)
indirect proof (p. 288)
indirect reasoning (p. 288)
median (p. 271)
orthocenter (p. 272)
perpendicular bisector (p. 269)
point of concurrency (p. 270)
proof by contradiction (p. 288)

Vocabulary Check

Choose the correct term to complete each sentence.

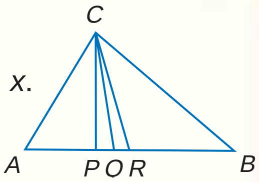
1. All of the angle bisectors of a triangle meet at the (incenter, circumcenter).
2. In $\triangle RST$, if point P is the midpoint of $\overline{RS}$, then $\overline{PT}$ is a(n) (angle bisector, median).
3. The theorem that the sum of the lengths of two sides of a triangle is greater than the length of the third side is the (Triangle Inequality Theorem, SSS Inequality).
4. The three medians of a triangle intersect at the (centroid, orthocenter).
5. In $\triangle JKL$, if point H is equidistant from $\overrightarrow{KJ}$ and $\overrightarrow{KL}$, then $\overleftrightarrow{HK}$ is an (angle bisector, altitude).
6. The circumcenter of a triangle is the point where all three (perpendicular bisectors, medians) of the sides of the triangle intersect.
7. In $\triangle ABC$, if $\overleftrightarrow{AK} \perp \overleftrightarrow{BC}$, $\overleftrightarrow{BK} \perp \overleftrightarrow{AC}$, and $\overleftrightarrow{CK} \perp \overleftrightarrow{AB}$, then K is the (orthocenter, incenter) of $\triangle ABC$.
8. In writing an indirect proof, begin by assuming that the (hypothesis, conclusion) is false.

Mixed Problem Solving
For mixed problem-solving practice, see page 832.

Lesson-by-Lesson Review

5-1 Bisectors, Medians, and Altitudes (pp. 269–278)

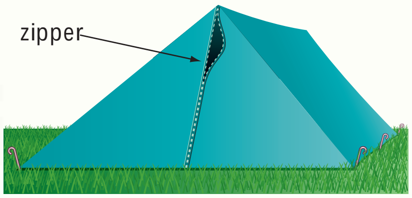
In the figure, $\overline{CP}$ is an altitude, $\overline{CQ}$ is the angle bisector of $\angle ACB$, and R is the midpoint of $\overline{AB}$.



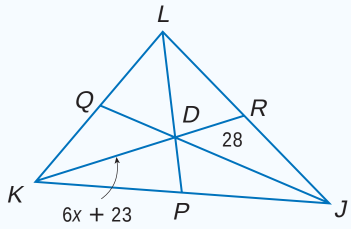
9. Find $m\angle ACQ$ if $m\angle ACB = 123 - x$ and $m\angle QCB = 42 + x$.

10. Find AB if $AR = 3x + 6$ and $RB = 5x - 14$.

11. TENT DESIGN Kame created a design for a new tent. She placed a zipper that extended from the midpoint of the base of one triangular face of the tent all the way to the top of the tent, as shown. Which special segment of triangles could represent this zipper?



Example 1 **Points P, Q, and R are the midpoints of $\overline{JK}$, $\overline{KL}$, and $\overline{JL}$, respectively. Find x.**



$KD = \frac{2}{3}(KR)$	Centroid Theorem
$6x + 23 = \frac{2}{3}(6x + 51)$	Substitution
$6x + 23 = 4x + 34$	Simplify.
$2x = 11$	Subtract $4x + 23$ from each side.
$x = \frac{11}{2}$	Divide each side by 2.

5-2 Inequalities and Triangles (pp. 280–287)

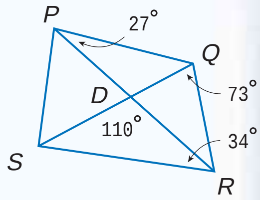
Use the figure in Example 2 to determine the relationship between the lengths of the given sides.

12. $\overline{SR}$, $\overline{SD}$ **13.** $\overline{DQ}$, $\overline{DR}$

14. $\overline{PQ}$, $\overline{QR}$ **15.** $\overline{SR}$, $\overline{SQ}$

16. COORDINATE GEOMETRY Triangle WXY has vertices $W(2, 1)$, $X(-1, -2)$, and $Y(3, -4)$. List the angles in order from the least to the greatest measure.

Example 2 **Determine the relationship between the lengths of $\overline{SD}$ and $\overline{QD}$.**



$\overline{SD}$ is opposite $\angle SRD$.
$\overline{QD}$ is opposite $\angle QRD$.
Since $m\angle QDR = 70$ by the Supplement Theorem, and $m\angle QRD = 37$ by the Angle Sum Theorem, then $m\angle SRD < m\angle QRD$. Therefore, $SD < QD$.

CHAPTER 5

Study Guide and Review

5-3 Indirect Proof (pp. 288–293)

17. FOOTBALL Gabriel plays quarterback for his high school team. This year, he completed 101 passes in the five games in which he played. Prove that, in at least one game, Gabriel completed more than 20 passes.

Example 3 State the assumption you would make to start an indirect proof of the statement *If 3x + 1 > 10, then x > 3.*

The conclusion of the conditional statement is $x > 3$. The negation of the conclusion is $x \leq 3$.

5-4 The Triangle Inequality (pp. 296–301)

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.

18. 7, 20, 5 **19.** 16, 20, 5

20. 18, 20, 6 **21.** 19, 19, 41

22. GARDENING James has three garden timbers that measure 2 feet, 3 feet, and 6 feet long. Could these be used to enclose a triangular garden? Explain.

Example 4 Determine whether 7, 6, and 14 can be the lengths of the sides of a triangle.

Check each inequality.

$7 + 6 \overset{?}{>} 14$
$13 \ngtr 14$ False

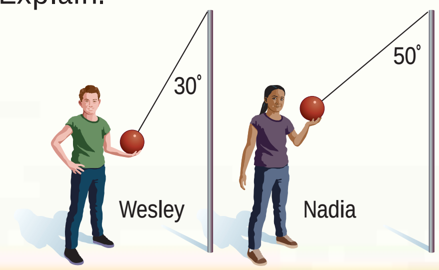
$7 + 14 \overset{?}{>} 6$
$21 > 6$ True

$6 + 14 \overset{?}{>} 7$
$20 > 7$ True

Because the inequalities are not true in all cases, the sides cannot form a triangle.

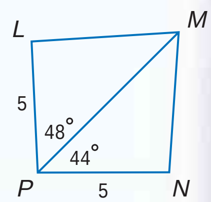
5-5 Inequalities Involving Two Triangles (pp. 302–309)

23. SPORTS Wesley and Nadia are playing tetherball. The photo shows them at two different points in the game. Who was standing closer to the pole? Explain.



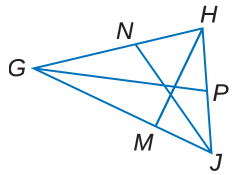
Example 5 Write an inequality relating *LM* and *MN*.

In $\triangle LMP$ and $\triangle NMP$, $\overline{LP} \cong \overline{NP}$, $\overline{PM} \cong \overline{PM}$, and $m\angle LPM > m\angle NPM$. The SAS Inequality allows us to conclude that $LM > MN$.



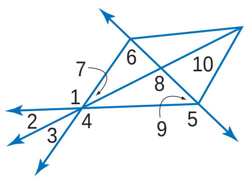
CHAPTER 5 Practice Test

In $\triangle GHJ$, $HP = 5x - 16$, $PJ = 3x + 8$, $m\angle GJN = 6y - 3$, $m\angle NJH = 4y + 23$, and $m\angle HMG = 4z + 14$.



1. $\overline{GP}$ is a median of $\triangle GHJ$. Find HJ.
2. Find $m\angle GJH$ if $\overline{JN}$ is an angle bisector.
3. If $\overline{HM}$ is an altitude of $\triangle GHJ$, find the value of z.

Refer to the figure below. Determine which angle in each set has the greatest measure.



4. $\angle 8$, $\angle 5$, $\angle 7$
5. $\angle 6$, $\angle 7$, $\angle 8$
6. $\angle 1$, $\angle 6$, $\angle 9$

Write the assumption you would make to start an indirect proof of each statement.

7. If n is a natural number, then $2n + 1$ is odd.
8. Alternate interior angles are congruent.

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.

9. 7, 24, 25
10. 25, 35, 60
11. 20, 3, 18
12. 5, 10, 6

13. **DESIGN** A landscape architect is making a model of a site. If the lengths of rods are 4 inches, 6 inches, and 8 inches, can these rods form a triangle? Explain.

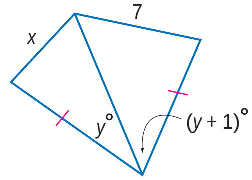
14. **BUSINESS** Over the course of three days, Marcus spent one and a half hours in a teleconference for his marketing firm. Use indirect reasoning to show that, on at least one day, Marcus spent at least a half hour in a teleconference.

Find the range for the measure of the third side of a triangle given the measures of two sides.

15. 1 and 14
16. 14 and 11

Write an inequality for the possible values of x.

17.



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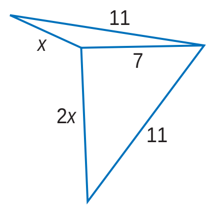

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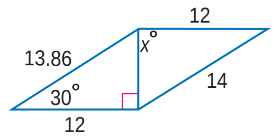
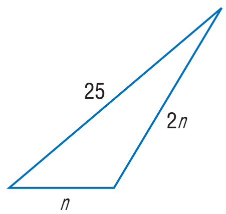


20. **MULTIPLE CHOICE** In the figure below, n is a whole number. What is the *least* possible value for n?



A 8 **C** 11
B 9 **D** 24

CHAPTER 5

Standardized Test Practice

Cumulative, Chapters 1–5

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which of the following is a logical conclusion based on the statement and its converse below?

Statement: If the measure of an angle is 50°, then the angle is an acute angle.

Converse: If an angle is an acute angle, then the measure of the angle is 50°.

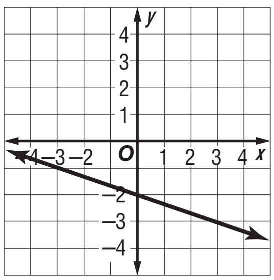
A The statement and its converse are both true.

B The statement and its converse are both false.

C The statement is true, but its converse is false.

D The statement is false, but its converse is true.

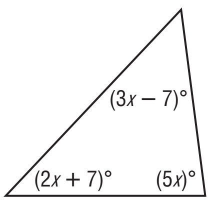
2. **ALGEBRA** Which linear function best describes the graph shown below?



F $y = -\frac{1}{3}x - 2$ **H** $y = \frac{1}{3}x + 2$

G $y = \frac{1}{3}x - 2$ **J** $y = -\frac{1}{3}x + 2$

3. Which of the following *best* describes this triangle?



A acute isosceles

B right isosceles

C acute scalene

D right scalene

4. If $\triangle ABC$ is isosceles and $m\angle A = 94°$, which of the following *must* be true?

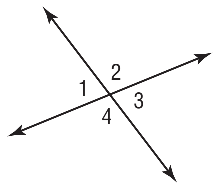
F $\angle B = 94°$

G $\angle B = 47°$

H $AB = AC$

J $AB = BC$

5. **Theorem:** If two angles are vertical angles, then they are congruent.



Tamara is proving the theorem above by contradiction. She began by assuming that vertical angles $\angle 1$ and $\angle 3$ in the diagram above are not congruent. Which theorem will Tamara use to reach a contradiction?

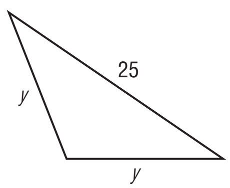
A If two angles are complementary to the same angle, the angles are congruent.

B If two angles are supplementary to the same angle, the angles are congruent.

C All right angles are congruent.

D If two angles are supplementary, the sum of their measures is 180.

6. **GRIDDABLE** In the figure below, *y* is a whole number. What is the *least* possible value for *y*?



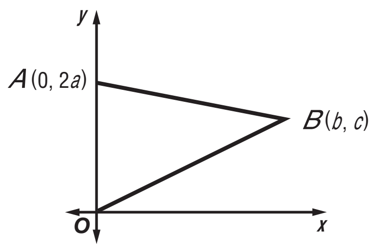
7. Which of the following could be the dimensions of a triangle in units?

F 1.9, 3.2, 4 **H** 3, 7.2, 7.5

G 1.6, 3, 3.4 **J** 2.6, 4.5, 6

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 841–856.

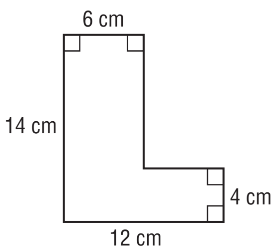
8. The diagram shows $\triangle OAB$.



What is the slope of the line that contains the altitude through vertex B of $\triangle OAB$?

A $\frac{c - a}{b}$
B undefined
C 0
D $\frac{b}{c - a}$

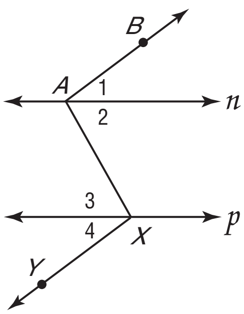
9. **GRIDDABLE** What is the perimeter of the figure in centimeters?



TEST-TAKING TIP

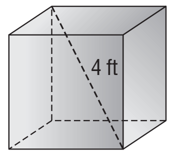
Question 9 When finding the perimeter of a figure, look for sides with measures that are missing. Find the missing measures before calculating the perimeter.

10. If line n is parallel to line p, which information would be enough to prove that $\overline{AB} \parallel \overline{XY}$?



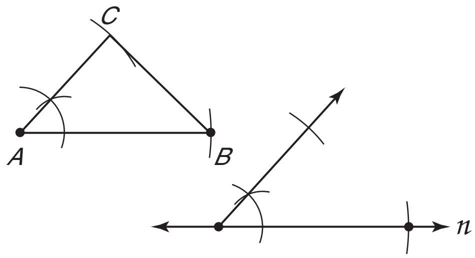
F $m\angle 1 = m\angle 2$
G $m\angle 1 = m\angle 3$
H $m\angle 1 = m\angle 4$
J $m\angle 3 = m\angle 4$

11. What is the surface area of a cube with a 4-foot diagonal?



A $\frac{4\sqrt{3}}{3}$ ft^2
B 8 ft^2
C 32 ft^2
D 60 ft^2

12. Karl is using a straightedge and compass to do the construction shown below.



Which best describes the construction Karl is doing?

F a triangle congruent to $\triangle ABC$ using three sides
G a triangle congruent to $\triangle ABC$ using two sides and the included angle
H a triangle congruent to $\triangle ABC$ using two angles and the included angle side
J a triangle congruent to $\triangle ABC$ using two angles

Pre-AP

Record your answer on a sheet of paper. Show your work.

13. The vertices of $\triangle ABC$ are $A(-3, 1)$, $B(0, 2)$, and $C(3, 4)$. Graph $\triangle ABC$. Find the measure of each side to the nearest tenth.

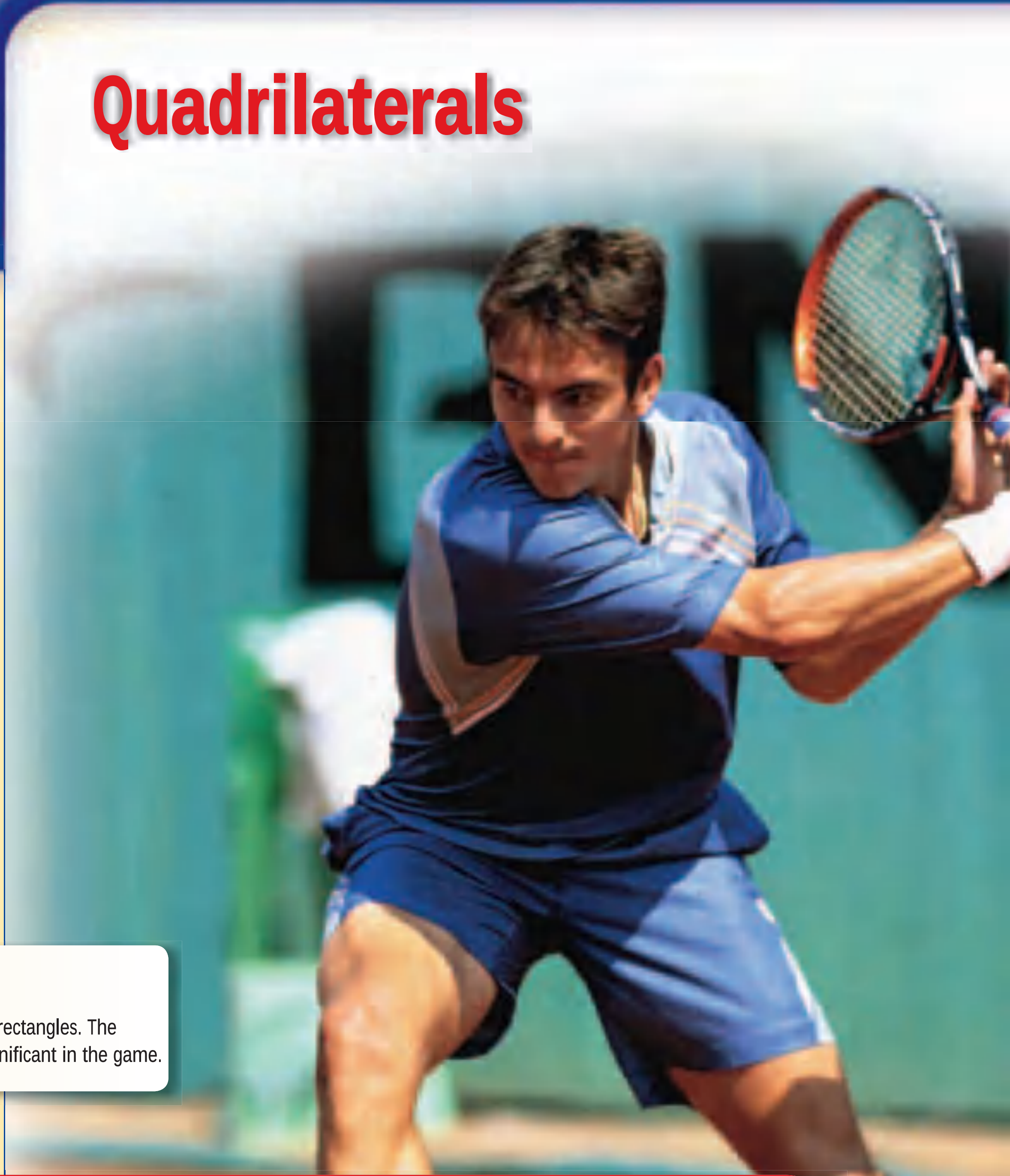
a. What type of triangle is $\triangle ABC$? How do you know?

b. Describe the relationship between $m\angle A$ and $m\angle B$, $m\angle A$ and $m\angle C$, and $m\angle B$ and $m\angle C$. Explain.

NEED EXTRA HELP?													
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson or Page...	2-3	786	5-4	5-3	4-6	1-6	5-4	5-1	4-1	4-4	1-7	3-5	5-2

Chapter 6

Quadrilaterals



- Investigate interior and exterior angles of polygons.
- Recognize and apply the properties of parallelograms, rectangles, rhombi, squares, and trapezoids.
- Position quadrilaterals for use in coordinate proof.

Key Vocabulary

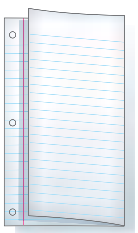
parallelogram (p. 325)
rectangle (p. 340)
rhombus (p. 348)
square (p. 349)
trapezoid (p. 356)

Real-World Link

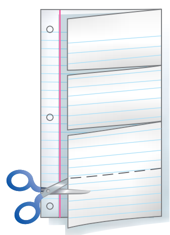
Tennis A tennis court is made up of rectangles. The boundaries of these rectangles are significant in the game.

Quadrilaterals Make this Foldable to help you organize your notes. Begin with a sheet of notebook paper.

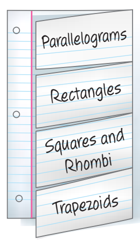
1. **Fold** lengthwise to the left margin.



2. **Cut** 4 tabs.



3. **Label** the tabs using the lesson concepts.



Michael Newman/PhotoEdit

GET READY for Chapter 6

Diagnose Readiness **You have two options for checking Prerequisite Skills.**

Option 2

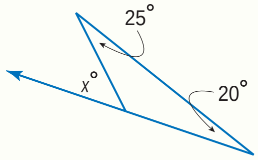
Take the Online Readiness Quiz at geometryonline.com.

Option 1

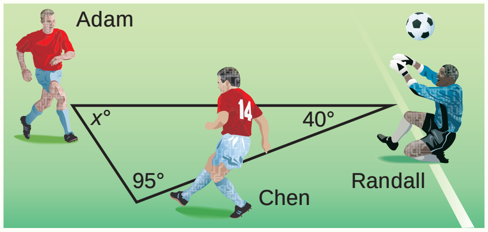
Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

1. Find ***x***. (Lesson 4-2)



2. **SOCCER** During a soccer game, Chen passed the ball to Adam who scored a goal. What is the angle formed by Chen, Adam, and Randall? (Lesson 4-2)

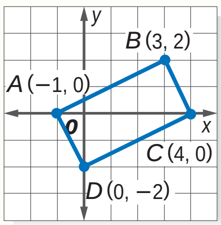


Find the slopes of $\overline{RS}$ and $\overline{TS}$ for the given points, *R*, *T*, and *S*. Determine whether $\overline{RS}$ and $\overline{TS}$ are *perpendicular* or *not perpendicular*. (Lesson 3-3)

3. $R(4, 3)$, $S(-1, 10)$, $T(13, 20)$

4. $R(-9, 6)$, $S(3, 8)$, $T(1, 20)$

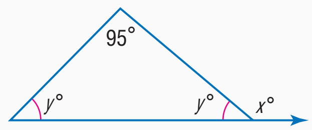
5. **FRAMES** Determine whether the corners of the frame are right angles. (Lesson 3-3)



QUICKReview

EXAMPLE 1

Find *x*.



$95 + y + y = 180$ Angle Sum Theorem

$95 + 2y = 180$ Combine like terms.

$2y = 85$ Subtract 95 from each side.

$y = 42.5$ Divide each side by 2.

$x + y = 180$ Supplement Theorem

$x + 42.5 = 180$ Substitution

$x = 137.5$ Subtract 42.5 from each side.

EXAMPLE 2

Find the slopes of $\overline{RS}$ and $\overline{TS}$ for the given points, *R*, *T*, and *S* with coordinates $R(0, 0)$, $S(2, 3)$, $T(-1, 5)$. Determine whether $\overline{RS}$ and $\overline{TS}$ are *perpendicular* or *not perpendicular*.

First, find the slope $\overline{RS}$.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3 - 0}{2 - 0} \qquad (x_1, y_1) = (0, 0), (x_2, y_2) = (2, 3)$$

$$= \frac{3}{2} \qquad \text{Simplify.}$$

Next, find the slope of $\overline{TS}$. Let $(x_1, y_1) = (-1, 5)$ and $(x_2, y_2) = (2, 3)$.

$$\text{slope} = \frac{3 - 5}{2 - (-1)} \text{ or } \frac{-2}{3}$$

Since the product of the slopes is -1, $\overline{RS} \perp \overline{TS}$.

6-1 Angles of Polygons

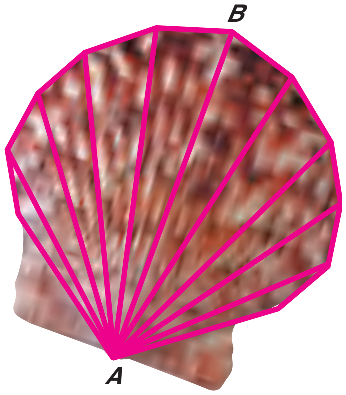
Main Ideas

- Find the sum of the measures of the interior angles of a polygon to classify figures and solve problems.
- Find the sum of the measures of the exterior angles of a polygon to classify figures and solve problems.

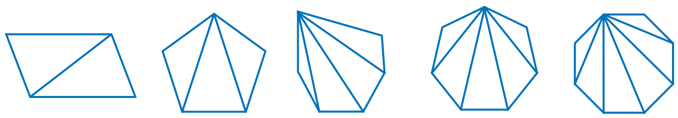
New Vocabulary

diagonal

This scallop shell resembles a 12-sided polygon with diagonals drawn from one of the vertices. A **diagonal** of a polygon is a segment that connects any two nonconsecutive vertices. For example, $\overline{AB}$ is one of the diagonals of this polygon.



Sum of Measures of Interior Angles Polygons with more than three sides have diagonals. The polygons below show all of the possible diagonals drawn from one vertex.



quadrilateral pentagon hexagon heptagon octagon

In each case, the polygon is separated into triangles. The sum of the angle measures of each polygon is the sum of the angle measures of the triangles. Since the sum of the angle measures of a triangle is 180, we can make a table to find the sum of the angle measures for several convex polygons.

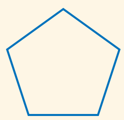
Convex Polygon	Number of Sides	Number of Triangles	Sum of Angle Measures
triangle	3	1	$(1 \cdot 180)$ or 180
quadrilateral	4	2	$(2 \cdot 180)$ or 360
pentagon	5	3	$(3 \cdot 180)$ or 540
hexagon	6	4	$(4 \cdot 180)$ or 720
heptagon	7	5	$(5 \cdot 180)$ or 900
octagon	8	6	$(6 \cdot 180)$ or 1080

Look for a pattern in the sum of the angle measures.

THEOREM 6.1 *Interior Angle Sum*

If a convex polygon has n sides and S is the sum of the measures of its interior angles, then $S = 180(n - 2)$.

Example:



$n = 5$
$S = 180(n - 2)$
$= 180(5 - 2)$ or 540

Real-World EXAMPLE Interior Angles of Regular Polygons

1 **CONSTRUCTION** **The Paddington family is assembling a hexagonal sandbox. What is the sum of the measures of the interior angles of the hexagon?**

$S = 180(n - 2)$ Interior Angle Sum Theorem

$= 180(6 - 2)$ $n = 6$

$= 180(4)$ or 720 The sum of the measures of the interior angles is 720.

CHECK Your Progress

1. Find the sum of the measures of the interior angles of a nonagon.

EXAMPLE Sides of a Polygon

Review Vocabulary

A **regular polygon** is a convex polygon in which all of the sides are congruent and all of the angles are congruent. (Lesson 1-6)

2 **The measure of an interior angle of a regular polygon is 108. Find the number of sides in the polygon.**

$S = 180(n - 2)$ Interior Angle Sum Theorem

$(108)n = 180(n - 2)$ $S = 108n$

$108n = 180n - 360$ Distributive Property

$0 = 72n - 360$ Subtract $108n$ from each side.

$360 = 72n$ Add 360 to each side.

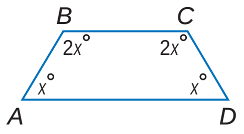
$5 = n$ Divide each side by 72. The polygon has 5 sides.

CHECK Your Progress

2. The measure of an interior angle of a regular polygon is 135. Find the number of sides in the polygon.

EXAMPLE Interior Angles of Nonregular Polygons

3 **ALGEBRA** **Find the measure of each interior angle.**



Since $n = 4$, the sum of the measures of the interior angles is $180(4 - 2)$ or 360.

$360 = m\angle A + m\angle B + m\angle C + m\angle D$ Sum of measures of interior angles

$360 = x + 2x + 2x + x$ Substitution

$360 = 6x$ Combine like terms.

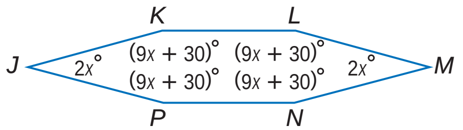
$60 = x$ Divide each side by 6.

Use the value of x to find the measure of each angle.

$m\angle A = 60$, $m\angle B = 2 \cdot 60$ or 120, $m\angle C = 2 \cdot 60$ or 120, and $m\angle D = 60$.

CHECK Your Progress

3.



Personal Tutor at geometryonline.com

Review Vocabulary

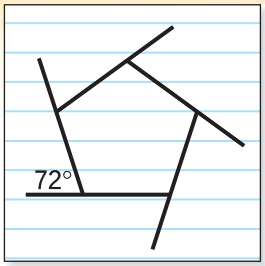
An **exterior angle** is an angle formed by one side of a polygon and the extension of another side. (Lesson 4-2)

Sum of Measures of Exterior Angles Is there a relationship among the exterior angles of a convex polygon?

GEOMETRY LAB

Sum of the Exterior Angles of a Polygon

COLLECT DATA



- Draw a triangle, a convex quadrilateral, a convex pentagon, a convex hexagon, and a convex heptagon.
- Extend the sides of each polygon to form exactly one exterior angle at each vertex.
- Use a protractor to measure each exterior angle of each polygon and record it on your drawing.

ANALYZE THE DATA

1. Copy and complete the table.

Polygon	triangle	quadrilateral	pentagon	hexagon	heptagon
Number of Exterior Angles					
Sum of Measures of Exterior Angles					

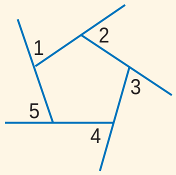
2. What conjecture can you make?

The Geometry Lab suggests Theorem 6.2.

THEOREM 6.2 *Exterior Angle Sum*

If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360.

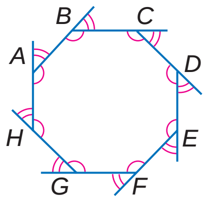
Example: $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360$



You will prove Theorem 6.2 in Exercise 30.

EXAMPLE Exterior Angles

4 **Find the measures of an exterior angle and an interior angle of convex regular octagon *ABCDEFGH*.**



$8n = 360$ n = measure of each exterior angle

$n = 45$ Divide each side by 8.

The measure of each exterior angle is 45. Since each exterior angle and its corresponding interior angle form a linear pair, the measure of the interior angle is 180 - 45 or 135.

4. Find the measures of an exterior angle and an interior angle of a convex regular dodecagon.

Check Your Understanding

Example 1 (p. 319)

1. **AQUARIUMS** The regular polygon at the right is the base of a fish tank. Find the sum of the measures of the interior angles of the pentagon.

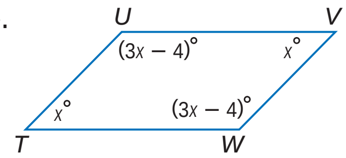
Example 2 (p. 319)

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

2. 60
3. 90

Example 3 (p. 319)

4. **ALGEBRA** Find the measure of each interior angle.



Find the measures of an exterior angle and an interior angle given the number of sides of each regular polygon.

Example 4 (p. 320)

5. 6
6. 18

Exercises

HOMEWORK HELP	
For Exercises	See Examples
7–14	1
15–18	2
19–22	3
23–26	4

Find the sum of the measures of the interior angles of each convex polygon.

7. 32-gon
8. 18-gon
9. 19-gon
10. 27-gon
11. $4y$-gon
12. $2x$-gon

13. **GARDENING** Carlotta is designing a garden for her backyard. She wants a flower bed shaped like a regular octagon. Find the sum of the measures of the interior angles of the octagon.

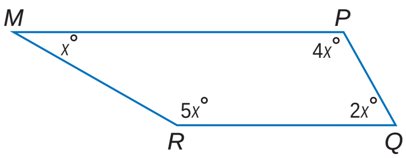
14. **GAZEBOS** A company is building regular hexagonal gazebos. Find the sum of the measures of the interior angles of the hexagon.

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

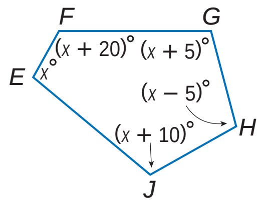
15. 140
16. 170
17. 160
18. 176.4

ALGEBRA Find the measure of each interior angle.

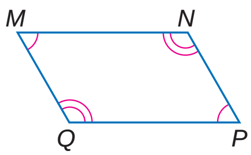
19.



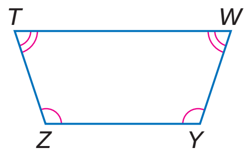
20.



21. parallelogram $MNPQ$ with $m\angle M = 10x$ and $m\angle N = 20x$



22. isosceles trapezoid $TWYZ$ with $\angle Z \cong \angle Y$, $m\angle Z = 30x$, $\angle T \cong \angle W$, and $m\angle T = 20x$



Find the measures of each exterior angle and each interior angle for each regular polygon.

23. decagon **24.** hexagon **25.** nonagon **26.** octagon

Find the measures of an interior angle and an exterior angle given the number of sides of each regular polygon. Round to the nearest tenth if necessary.

27. 11 **28.** 7 **29.** 12

30. **PROOF** Use algebra to prove the Exterior Angle Sum Theorem.

Real-World Link

Thomas Jefferson's home, Monticello, features a dome on an octagonal base. The architectural elements on either side of the dome were based on a regular octagon.

Source: www.monticello.org

31. **ARCHITECTURE** The Pentagon building in Washington, D.C., was designed to resemble a regular pentagon. Find the measure of an interior angle and an exterior angle of the courtyard.

32. **ARCHITECTURE** Use the information at the left to compare the dome to the architectural elements on each side of the dome. Are the interior and exterior angles the same? Find the measures of the interior and exterior angles.

EXTRA PRACTICE
See pages 811, 833.
Math Online
Self-Check Quiz at geometryonline.com

ALGEBRA **Find the measure of each interior angle using the given information.**

33. decagon in which the measures of the interior angles are $x + 5$, $x + 10$, $x + 20$, $x + 30$, $x + 35$, $x + 40$, $x + 60$, $x + 70$, $x + 80$, and $x + 90$

34. polygon *ABCDE* with the interior angle measures shown in the table

Angle	Measure (°)
A	$6x$
B	$4x + 13$
C	$x + 9$
D	$2x - 8$
E	$4x - 1$

H.O.T. Problems

35. **REASONING** Explain why the Interior Angle Sum Theorem and the Exterior Angle Sum Theorem apply only to convex polygons.

36. **OPEN ENDED** Draw a regular convex polygon and a convex polygon that is not regular with the same number of sides. Compare the sum of the interior angles for each.

37. **CHALLENGE** Two formulas can be used to find the measure of an interior angle of a regular polygon: $s = \frac{180(n - 2)}{n}$ and $s = 180 - \frac{360}{n}$. Show that these are equivalent.

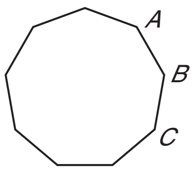
38. **Writing in Math** Explain how triangles are related to the Interior Angle Sum Theorem.

39. The sum of the interior angles of a polygon is twice the sum of its exterior angles. What type of polygon is it?

A pentagon **C** octagon

B hexagon **D** decagon

40. If the polygon shown is regular, what is $m\angle ABC$?



F 140°

G 144°

H 162°

J 180°

41. **REVIEW** If x is subtracted from x^2, the difference is 72. Which of the following could be a value of x?

A -36

B -9

C -8

D 72

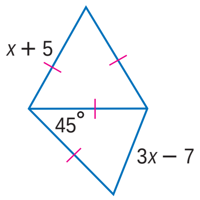
42. **REVIEW** $\dfrac{3^2 \cdot 4^5 \cdot 5^3}{5^3 \cdot 3^3 \cdot 4^6} =$

F $\dfrac{1}{60}$ **H** $\dfrac{3}{4}$

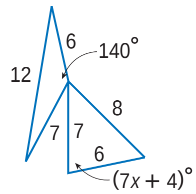
G $\dfrac{1}{12}$ **J** 12

Write an inequality to describe the possible values of *x*. (Lesson 5-5)

43.



44.



Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain. (Lesson 5-4)

45. 5, 17, 9

46. 17, 30, 30

47. 8.4, 7.2, 3.5

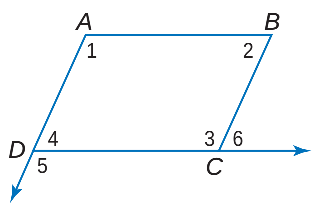
48. 4, 0.9, 4.1

49. 14.3, 12, 2.2

50. 0.18, 0.21, 0.52

51. **GARDENING** A landscape designer is putting black plastic edging around a rectangular flower garden that has length 5.7 meters and width 3.8 meters. The edging is sold in 5-meter lengths. Find the perimeter of the garden and determine how much edging the designer should buy. (Lesson 1-6)

PREREQUISITE SKILL **In the figure, $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. Name all pairs of angles for each type indicated.** (Lesson 3-1)



52. consecutive interior angles

53. alternate interior angles

EXTEND 6-1

Spreadsheet Lab

Angles of Polygons

It is possible to find the interior and exterior measurements along with the sum of the interior angles of any regular polygon with *n* number of sides using a spreadsheet.

ACTIVITY

Design a spreadsheet using the following steps.

- Label the columns as shown in the spreadsheet below.
- Enter the digits 3–10 in the first column.
- The number of triangles formed by diagonals from the same vertex in a polygon is 2 less than the number of sides. Write a formula for Cell B2 to subtract 2 from each number in Cell A2.
- Enter a formula for Cell C2 so the spreadsheet will find the sum of the measures of the interior angles. Remember that the formula is $S = (n - 2)180$.
- Continue to enter formulas so that the indicated computation is performed. Then, copy each formula through Row 9. The final spreadsheet will appear as below.

Polygons and Angles.xls

	A	B	C	D	E	F
1	Number of Sides	Number of Triangles	Sum of Measures of Interior Angles	Measure of Each Interior Angle	Measure of Each Exterior Angle	Sum of Measures of Exterior Angles
2	3	1	180	60	120	360
3	4	2	360	90	90	360
4	5	3	540	108	72	360
5	6	4	720	120	60	360
6	7	5	900	128.57	51.43	360
7	8	6	1080	135	45	360
8	9	7	1260	140	40	360
9	10	8	1440	144	36	360
10						

Sheet 1 Sheet 2 Sheet 3

ANALYZE THE RESULTS

1. Write the formula to find the measure of each interior angle in the polygon.
2. Write the formula to find the sum of the measures of the exterior angles.
3. What is the measure of each interior angle if the number of sides is 1? 2?
4. Is it possible to have values of 1 and 2 for the number of sides? Explain.

For Exercises 5–7, use the spreadsheet.

5. How many triangles are in a polygon with 15 sides?
6. Find the measure of an exterior angle of a polygon with 15 sides.
7. Find the measure of an interior angle of a polygon with 110 sides.

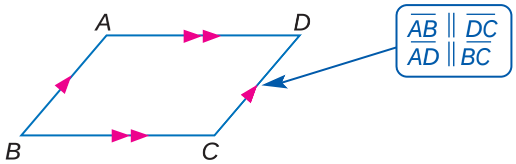
6-2 Parallelograms

Main Ideas

- Recognize and apply properties of the sides and angles of parallelograms.
- Recognize and apply properties of the diagonals of parallelograms.

To chart a course, sailors use a parallel ruler. One edge of the ruler is placed at the starting position. Then the other ruler is moved until its edge reaches the compass rose printed on the chart. Reading the compass determines which direction to travel. Each pair of opposite sides of the ruler are parallel.

Sides and Angles of Parallelograms A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.



▱*ABCD*

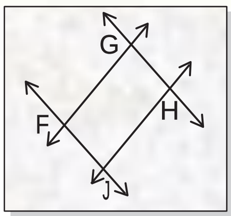
This lab will help you make conjectures about the sides and angles of a parallelogram.

GEOMETRY LAB

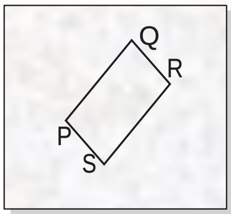
Properties of Parallelograms

MAKE A MODEL

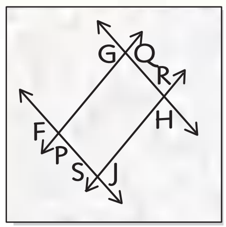
Step 1 Draw two sets of intersecting parallel lines on patty paper. Label the vertices *FGHJ*.



Step 2 Trace *FGHJ*. Label the second parallelogram *PQRS* so $\angle F$ and $\angle P$ are congruent.



Step 3 Rotate *PQRS* on *FGHJ* to compare sides and angles.



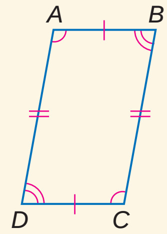
ANALYZE

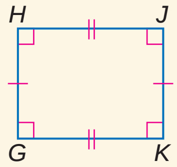
1. List all of the segments that are congruent.
2. List all of the angles that are congruent.
3. **Make a conjecture** about the angle relationships you observed.
4. Test your conjecture.

The Geometry Lab leads to four properties of parallelograms.

THEOREMS

Theorem		Examples
6.3	Opposite sides of a parallelogram are congruent. **Abbreviation:** *Opp. sides of ▱ are ≅.*	$\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$
6.4	Opposite angles in a parallelogram are congruent. **Abbreviation:** *Opp. ∠s of ▱ are ≅.*	$\angle A \cong \angle C$, $\angle B \cong \angle D$
6.5	Consecutive angles in a parallelogram are supplementary. **Abbreviation:** *Cons. ∠s in ▱ are suppl.*	$m\angle A + m\angle B = 180$, $m\angle B + m\angle C = 180$, $m\angle C + m\angle D = 180$, $m\angle D + m\angle A = 180$
6.6	If a parallelogram has one right angle, it has four right angles. **Abbreviation:** *If ▱ has 1 rt. ∠, it has 4 rt. ∠s.*	$m\angle G = 90$, $m\angle H = 90$, $m\angle J = 90$, $m\angle K = 90$

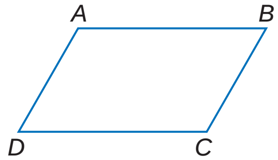




You will prove Theorems 6.3 and 6.5 in Exercises 34 and 35, respectively.

EXAMPLE 1 Proof of Theorem 6.4

Write a two-column proof of Theorem 6.4.



Given: ▱*ABCD*

Prove: $\angle A \cong \angle C$, $\angle D \cong \angle B$

Proof:

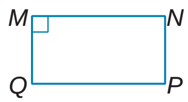
Statements	Reasons
1. ▱*ABCD*	1. Given
2. $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$	2. Definition of parallelogram
3. $\angle A$ and $\angle D$ are supplementary. $\angle D$ and $\angle C$ are supplementary. $\angle C$ and $\angle B$ are supplementary.	3. If parallel lines are cut by a transversal, consecutive interior angles are supplementary.
4. $\angle A \cong \angle C$, $\angle D \cong \angle B$	4. Supplements of the same angles are congruent.

Study Tip

Including a Figure

Theorems are presented in general terms. In a proof, you must include a drawing so that you can refer to segments and angles specifically.

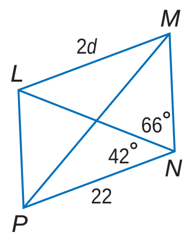
1. **PROOF** Write a paragraph proof of Theorem 6.6.



Given: ▱*MNPQ*
$\angle M$ is a right angle.

Prove: $\angle N$, $\angle P$, and $\angle Q$ are right angles.

EXAMPLE 2 Properties of Parallelograms



ADVERTISING **Quadrilateral *LMNP* is a parallelogram designed to be part of a new company logo. Find *m∠PLM*, *m∠LMN*, and *d*.**

$m\angle MNP = 66 + 42$ or 108 Angle Addition Theorem

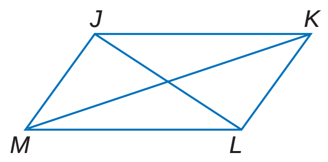
$\angle PLM \cong \angle MNP$ — Opp. $\angle$s of $\square$ are $\cong$.
$m\angle PLM = m\angle MNP$ — Definition of congruent angles
$m\angle PLM = 108$ — Substitution

$m\angle PLM + m\angle LMN = 180$ — Cons. $\angle$s of $\square$ are suppl.
$108 + m\angle LMN = 180$ — Substitution
$m\angle LMN = 72$ — Subtract 108 from each side.

$\overline{LM} \cong \overline{PN}$ — Opp. sides of $\square$ are $\cong$.
$LM = PN$ — Definition of congruent segments
$2d = 22$ — Substitution
$d = 11$ — Divide each side by 2.

2. Refer to $\square LMNP$. If the perimeter of the parallelogram is 74 units, find MN.

Diagonals of Parallelograms In parallelogram $JKLM$, $\overline{JL}$ and $\overline{KM}$ are diagonals. Theorem 6.7 states the relationship between diagonals of a parallelogram.

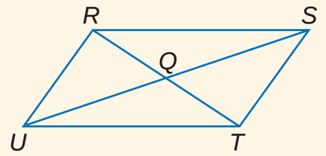


THEOREM 6.7

The diagonals of a parallelogram bisect each other.

Abbreviation: *Diag. of $\square$ bisect each other.*

Example: $\overline{RQ} \cong \overline{QT}$ and $\overline{SQ} \cong \overline{QU}$



You will prove Theorem 6.7 in Exercise 36.

Diagonals of a Parallelogram

What are the coordinates of the intersection of the diagonals of parallelogram $ABCD$ with vertices $A(2, 5)$, $B(6, 6)$, $C(4, 0)$, and $D(0, -1)$?

A (4, 2)　　**B** (4.5, 2)　　**C** $\left(\frac{7}{6}, -\frac{5}{2}\right)$　　**D** (3, 2.5)

Read the Test Item

Since the diagonals of a parallelogram bisect each other, the intersection point is the midpoint of $\overline{AC}$ and $\overline{BD}$.

Solve the Test Item

Find the midpoint of $\overline{AC}$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 4}{2}, \frac{5 + 0}{2}\right) \quad \text{Midpoint Formula}$$

$$= (3, 2.5) \quad \text{Simplify.}$$

The coordinates of the intersection of the diagonals of parallelogram $ABCD$ are (3, 2.5). The answer is D.

Test-Taking Tip

Check Answers Always check your answer. To check the answer to this problem, find the coordinates of the midpoint of $\overline{BD}$.

3. **COORDINATE GEOMETRY** Determine the coordinates of the intersection of the diagonals of ▱*RSTU* with vertices $R(-8, -2)$, $S(-6, 7)$, $T(6, 7)$, and $U(4, -2)$.

F $(-1, 2.5)$ **G** $(1, -4)$ **H** $(5, 4.5)$ **J** $(-1.5, -2, 5)$

Personal Tutor at geometryonline.com

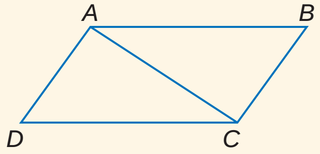
Theorem 6.8 describes another characteristic of the diagonals of a parallelogram.

THEOREM 6.8

Each diagonal of a parallelogram separates the parallelogram into two congruent triangles.

Abbreviation: *Diag. separates ▱ into 2 ≅ △s.*

Example: $\triangle ACD \cong \triangle CAB$



You will prove Theorem 6.8 in Exercise 37.

CHECK Your Understanding

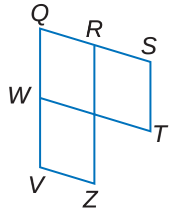
Example 1 (p. 326)

PROOF **Write the indicated type of proof.**

1. two-column

Given: ▱*VZRQ* and ▱*WQST*

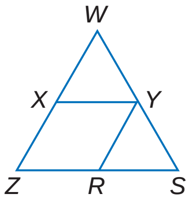
Prove: $\angle Z \cong \angle T$



2. paragraph

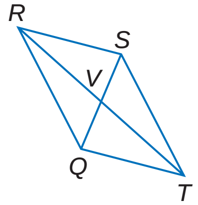
Given: ▱*XYRZ*, $\overline{WZ} \cong \overline{WS}$

Prove: $\angle XYR \cong \angle S$



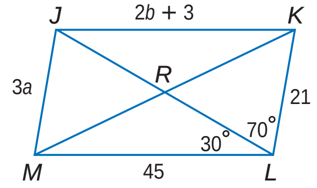
Example 2 (p. 326)

Complete each statement about ▱*QRST*. Justify your answer.



3. $\overline{SV} \cong$ __?__.
4. $\triangle VRS \cong$ __?__.
5. $\angle TSR$ is supplementary to __?__.

Use ▱*JKLM* to find each measure or value.



6. $m\angle MJK$
7. $m\angle JML$
8. $m\angle JKL$
9. $m\angle KJL$
10. a
11. b

Example 3 (p. 327)

12. **STANDARDIZED TEST PRACTICE** Parallelogram *GHJK* has vertices $G(-3, 4)$, $H(1, 1)$, and $J(3, -5)$. Which are possible coordinates for vertex *K*?

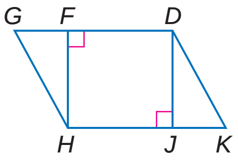
A $(-1, 1)$ **B** $(-2, 0)$ **C** $(-1, -2)$ **D** $(-2, -1)$

Exercises

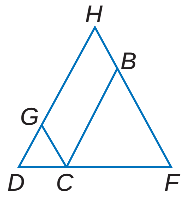
HOMEWORK HELP	
For Exercises	**See Examples**
13, 14, 34–37	1
15–30	2
31–33	3

PROOF **Write a two-column proof.**

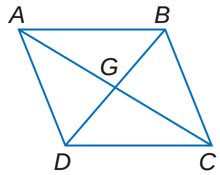
13. Given: $\square DGHK$, $\overline{FH} \perp \overline{GD}$, $\overline{DJ} \perp \overline{HK}$
Prove: $\triangle DJK \cong \triangle HFG$



14. Given: $\square BCGH$, $\overline{HD} \cong \overline{FD}$
Prove: $\angle F \cong \angle GCB$



Complete each statement about $\square ABCD$. Justify your answer.



15. $\angle DAB \cong$ __?__.

16. $\angle ABD \cong$ __?__.

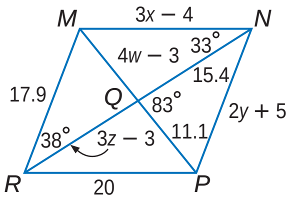
17. $\overline{AB} \parallel$ __?__.

18. $\overline{BG} \cong$ __?__.

19. $\triangle ABD \cong$ __?__.

20. $\angle ACD \cong$ __?__.

ALGEBRA **Use $\square MNPR$ to find each measure or value. Round to the nearest tenth if necessary.**



21. $m\angle MNP$

22. $m\angle NRP$

23. $m\angle RNP$

24. $m\angle RMN$

25. $m\angle MQN$

26. $m\angle MQR$

27. x

28. y

29. w

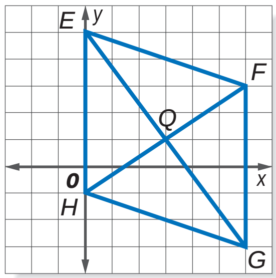
30. z

Real-World Link

Shiro Kuramata designed furniture that was functional and aesthetically pleasing. His style is surreal and minimalist.

Source: designboom.com

COORDINATE GEOMETRY **For Exercises 31–33, refer to $\square EFGH$.**



31. Use the Distance Formula to verify that the diagonals bisect each other.

32. Determine whether the diagonals of this parallelogram are congruent.

33. Find the slopes of $\overline{EH}$ and $\overline{EF}$. Are the consecutive sides perpendicular? Explain.

Write the indicated type of proof.

34. two-column proof of Theorem 6.3

35. two-column proof of Theorem 6.5

36. paragraph proof of Theorem 6.7

37. two-column proof of Theorem 6.8

38. **DESIGN** The chest of drawers shown at the left is called *Side 2*. It was designed by Shiro Kuramata. Describe the properties of parallelograms the artist may have used to place each drawer pull.

39. **ALGEBRA** Parallelogram $ABCD$ has diagonals $\overline{AC}$ and $\overline{DB}$ that intersect at P. If $AP = 3a + 18$, $AC = 12a$, $PB = a + 2b$, and $PD = 3b + 1$, find a, b, and DB.

40. **ALGEBRA** In parallelogram $ABCD$, $AB = 2x + 5$, $m\angle BAC = 2y$, $m\angle B = 120$, $m\angle CAD = 21$, and $CD = 21$. Find x and y.

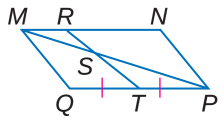
EXTRA PRACTICE
See pages 813, 835.

Math Online
Self-Check Quiz at geometryonline.com

H.O.T. Problems

41. OPEN ENDED Draw a parallelogram with one side twice as long as another side.

42. CHALLENGE Compare the corresponding angles of $\triangle MSR$ and $\triangle PST$, given that $MNPQ$ is a parallelogram with $MR = \frac{1}{4}MN$. What can you conclude about these triangles?



43. Writing in Math Describe the characteristics of the sides and angles of a parallelogram and the properties of the diagonals of a parallelogram.

STANDARDIZED TEST PRACTICE

44. Two consecutive angles of a parallelogram measure $(3x + 42)°$ and $(9x - 18)°$. What are the measures of the angles?

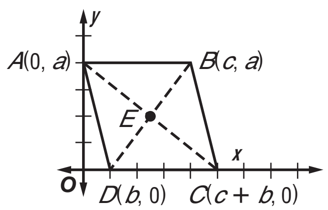
A 13, 167

B 58.5, 31.5

C 39, 141

D 81, 99

45. Figure $ABCD$ is a parallelogram.



What are the coordinates of point E?

F $\left(\frac{a}{c}, \frac{c}{2}\right)$

H $\left(\frac{a+c}{2}, \frac{b}{2}\right)$

G $\left(\frac{c+b}{2}, \frac{a+b}{2}\right)$

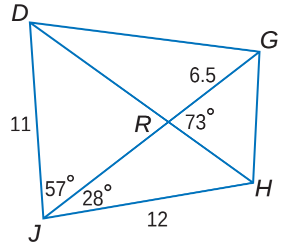
J $\left(\frac{c+b}{2}, \frac{a}{2}\right)$

Spiral Review

Find the sum of the measures of the interior angles of each convex polygon. (Lesson 6-1)

46. 14-gon **47.** 22-gon **48.** 17-gon **49.** 36-gon

Write an inequality relating the given pair of angles or segment measures. (Lesson 5-5)



50. $m\angle DRJ$, $m\angle HRJ$

51. DG, GH

52. $m\angle JDH$, $m\angle DHJ$

53. JOBS Jamie works at a gift shop after school. She is paid \$10 per hour plus a 15% commission on merchandise that she sells. Write an equation that represents her earnings in a week if she sold \$550 worth of merchandise. (Lesson 3-4)

PREREQUISITE SKILL The vertices of a quadrilateral are $A(-5, -2)$ $B(-2, 5)$, $C(2, -2)$, and $D(-1, -9)$. Determine whether each segment is a side or a diagonal of the quadrilateral, and find the slope of each segment. (Lesson 3-3)

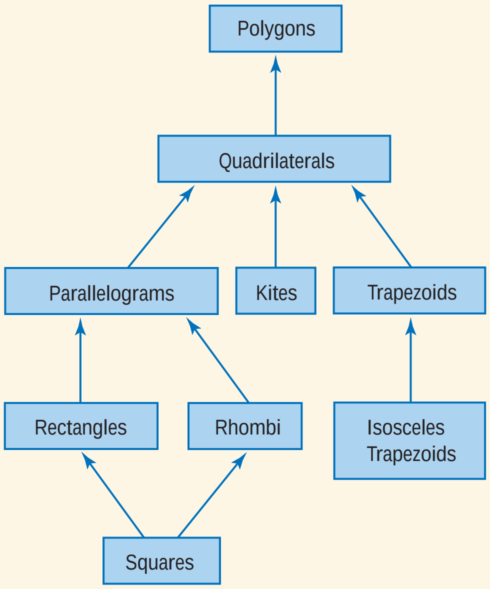
54. $\overline{AB}$ **55.** $\overline{BD}$ **56.** $\overline{CD}$

READING MATH

Hierarchy of Polygons

A *hierarchy* is a ranking of classes or sets of things. Examples of some classes of polygons are rectangles, rhombi, trapezoids, parallelograms, squares, and quadrilaterals. These classes are arranged in the hierarchy at the right.

You will study rectangles, squares, rhombi, trapezoids, and kites in the remaining lessons of Chapter 6.

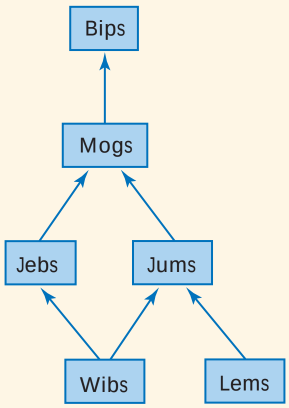


Use the following information to help read the hierarchy diagram.

- The class that is the broadest is listed first, followed by the other classes in order. For example, *polygons* is the broadest class in the hierarchy diagram above, and *squares* is a very specific class.
- Each class is contained within any class linked above it in the hierarchy. For example, *all* squares are also rhombi, rectangles, parallelograms, quadrilaterals, and polygons. However, an isosceles trapezoid is not a square or a kite.
- Some, but not all, elements of each class are contained within lower classes in the hierarchy. For example, some trapezoids are isosceles trapezoids, and some rectangles are squares.

Reading to Learn

Refer to the hierarchy diagram at the right. Write *true, false,* or *not enough information* for each statement.



1. All mogs are jums.

2. Some jebs are jums.

3. All lems are jums.

4. Some wibs are jums.

5. All mogs are bips.

6. Draw a hierarchy diagram to show these classes: equilateral triangles, polygons, isosceles triangles, triangles, and scalene triangles.

EXPLORE 6-3

Graphing Calculator Lab

Parallelograms

You can use the Cabri Junior application on a TI-83/84 Plus graphing calculator to discover properties of parallelograms.

ACTIVITY

Construct a quadrilateral with one pair of sides that are both parallel and congruent.

Step 1 Construct a segment using the Segment tool on the F2 menu. Label the segment $\overline{AB}$. This is one side of the quadrilateral.

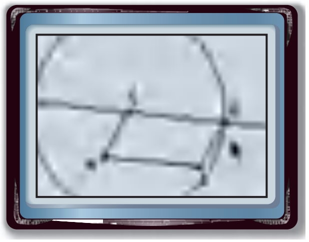
Step 2 Use the Parallel tool on the F3 menu to construct a line parallel to the segment. Pressing ENTER will draw the line and a point on the line. Label the point C.

Step 3 Access the Compass tool on the F3 menu. Set the compass to the length of $\overline{AB}$ by selecting one endpoint of the segment and then the other. Construct a circle centered at C.

Step 4 Use the Point Intersection tool on the F2 menu to draw a point at the intersection of the line and the circle. Label the point D. Then use the Segment tool on the F2 menu to draw $\overline{AC}$ and $\overline{BD}$.

Step 5 Use the Hide/Show tool on the F5 menu to hide the circle. Then access the Slope tool under Measure on the F5 menu. Display the slopes of $\overline{AB}$, $\overline{BD}$, $\overline{CD}$, and $\overline{AC}$.

Steps 1 and 2



Steps 3 and 4

Step 5

Analyze the Results

1. What is the relationship between sides $\overline{AB}$ and $\overline{CD}$? Explain how you know.
2. What do you observe about the slopes of opposite sides of the quadrilateral? What type of quadrilateral is $ABDC$? Explain.
3. Click on point A and drag it to change the shape of $ABDC$. What do you observe?
4. **Make a conjecture** about a quadrilateral with a pair of opposite sides that are both congruent and parallel.
5. Use the graphing calculator to construct a quadrilateral with both pairs of opposite sides congruent. Then analyze the slopes of the sides of the quadrilateral. **Make a conjecture** based on your observations.

Other Calculator Keystrokes at geometryonline.com

6-3 Tests for Parallelograms

Main Ideas

- Recognize the conditions that ensure a quadrilateral is a parallelogram.
- Prove that a set of points forms a parallelogram in the coordinate plane.

The roof of the covered bridge appears to be a parallelogram. Each pair of opposite sides looks as if they are the same length. How can we know for sure if this shape is really a parallelogram?

Conditions for a Parallelogram By definition, the opposite sides of a parallelogram are parallel. So, if a quadrilateral has each pair of opposite sides parallel, then it is a parallelogram. Other tests can be used to determine if a quadrilateral is a parallelogram.

GEOMETRY LAB

Testing for a Parallelogram

MODEL

- Cut two straws to one length and two other straws to a different length.
- Connect the straws by inserting a pipe cleaner in one end of each size of straw to form a quadrilateral like the one shown at the right.
- Shift the sides to form quadrilaterals of different shapes.

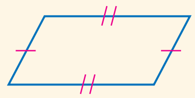
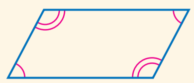
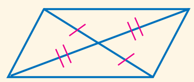
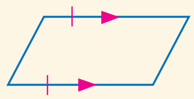
ANALYZE

1. Measure the distance between the opposite sides of the quadrilateral in at least three places. Repeat this process for several figures. What can you conclude about opposite sides?
2. Classify the quadrilaterals that you formed.
3. Compare the measures of pairs of opposite sides.
4. Measure the four angles in several of the quadrilaterals. What relationships do you find?

MAKE A CONJECTURE

5. What conditions are necessary to verify that a quadrilateral is a parallelogram?

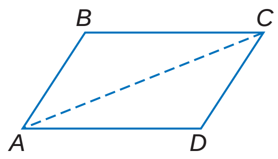
THEOREMS — *Proving Parallelograms*

		Example
6.9	If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. Abbreviation: *If both pairs of opp. sides are ≅, then quad. is ▱.*	
6.10	If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. Abbreviation: *If both pairs of opp. ∠s are ≅, then quad. is ▱.*	
6.11	If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. Abbreviation: *If diag. bisect each other, then quad. is ▱.*	
6.12	If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram. Abbreviation: *If one pair of opp. sides is ∥ and ≅, then the quad. is a ▱.*	

You will prove Theorems 6.9 and 6.11 in Exercises 18 and 19, respectively.

EXAMPLE 1 Write a Proof

Proof **Write a paragraph proof of Theorem 6.10.**



Given: $\angle A \cong \angle C$, $\angle B \cong \angle D$

Prove: $ABCD$ is a parallelogram.

Paragraph Proof:

Because two points determine a line, we can draw $\overline{AC}$. We now have two triangles. We know the sum of the angle measures of a triangle is 180, so the sum of the angle measures of two triangles is 360. Therefore, $m\angle A + m\angle B + m\angle C + m\angle D = 360$.

Since $\angle A \cong \angle C$ and $\angle B \cong \angle D$, $m\angle A = m\angle C$ and $m\angle B = m\angle D$. Substitute to find that $m\angle A + m\angle A + m\angle B + m\angle B = 360$, or $2(m\angle A) + 2(m\angle B) = 360$. Dividing each side of the equation by 2 yields $m\angle A + m\angle B = 180$. This means that consecutive angles are supplementary and $\overline{AD} \parallel \overline{BC}$.

Likewise, $2m\angle A + 2m\angle D = 360$, or $m\angle A + m\angle D = 180$. These consecutive supplementary angles verify that $\overline{AB} \parallel \overline{DC}$. Opposite sides are parallel, so $ABCD$ is a parallelogram.

1. **PROOF** Write a two-column proof of Theorem 6.12.

Real-World Link

Ellsworth Kelly created *Sculpture for a Large Wall* in 1957. The sculpture is made of 104 aluminum panels. The piece is over 65 feet long, 11 feet high, and 2 feet deep.

Source: www.moma.org

Properties of Parallelograms

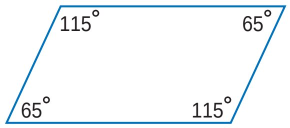
2 **ART Some panels in the sculpture appear to be parallelograms. Describe the information needed to determine whether these panels are parallelograms.**

A panel is a parallelogram if both pairs of opposite sides are congruent, or if one pair of opposite sides is congruent and parallel. If the diagonals bisect each other, or if both pairs of opposite angles are congruent, then the panel is a parallelogram.

2. **ART** Tiffany has several pieces of tile that she is planning to make into a mosaic. How can she tell if the quadrilaterals are parallelograms?

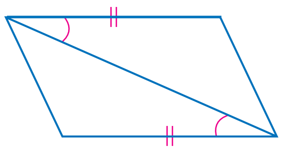
EXAMPLE Properties of Parallelograms

3 **Determine whether the quadrilateral is a parallelogram. Justify your answer.**



Each pair of opposite angles has the same measure. Therefore, they are congruent. If both pairs of opposite angles are congruent, the quadrilateral is a parallelogram.

3.



A quadrilateral is a parallelogram if any one of the following is true.

CONCEPT SUMMARY *Tests for a Parallelogram*

1. Both pairs of opposite sides are parallel. (Definition)
2. Both pairs of opposite sides are congruent. (Theorem 6.9)
3. Both pairs of opposite angles are congruent. (Theorem 6.10)
4. Diagonals bisect each other. (Theorem 6.11)
5. A pair of opposite sides is both parallel and congruent. (Theorem 6.12)

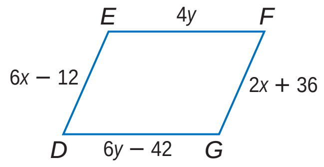
Study Tip

Common Misconceptions

If a quadrilateral meets one of the five tests, it is a parallelogram. All of the properties of parallelograms need not be shown.

EXAMPLE 4 Find Measures

ALGEBRA Find x and y so that the quadrilateral is a parallelogram.

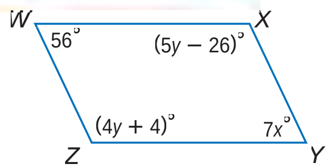


Opposite sides of a parallelogram are congruent.

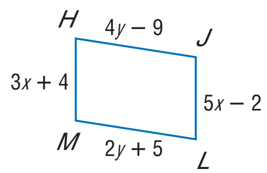
$\overline{EF} \cong \overline{DG}$	Opp. sides of ▱ are ≅.	$\overline{DE} \cong \overline{FG}$	Opp. sides of ▱ are ≅.
$EF = DG$	Def. of ≅ segments	$DE = FG$	Def. of ≅ segments
$4y = 6y - 42$	Substitution	$6x - 12 = 2x + 36$	Substitution
$-2y = -42$	Subtract $6y$.	$4x = 48$	Subtract $2x$ and add 12.
$y = 21$	Divide by -2.	$x = 12$	Divide by 4.

So, when x is 12 and y is 21, $DEFG$ is a parallelogram.

4A.



4B.



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Parallelograms on the Coordinate Plane We can use the Distance Formula and the Slope Formula to determine if a quadrilateral is a parallelogram in the coordinate plane.

EXAMPLE 5 Use Slope and Distance

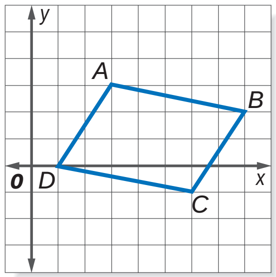
COORDINATE GEOMETRY Determine whether the figure with vertices $A(3, 3)$, $B(8, 2)$, $C(6, -1)$, $D(1, 0)$ is a parallelogram. Use the Slope Formula.

Study Tip

Coordinate Geometry

The Midpoint Formula can also be used to show that a quadrilateral is a parallelogram by Theorem 6.11.

If the opposite sides of a quadrilateral are parallel, then it is a parallelogram.



slope of $\overline{AB} = \frac{2 - 3}{8 - 3}$ or $\frac{-1}{5}$

slope of $\overline{DC} = \frac{-1 - 0}{6 - 1}$ or $\frac{-1}{5}$

slope of $\overline{AD} = \frac{3 - 0}{3 - 1}$ or $\frac{3}{2}$

slope of $\overline{BC} = \frac{-1 - 2}{6 - 8}$ or $\frac{3}{2}$

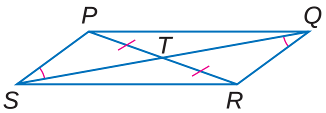
Since opposite sides have the same slope, $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. Therefore, $ABCD$ is a parallelogram by definition.

5. $F(-2, 4)$, $G(4, 2)$, $H(4, -2)$, $J(-2, -1)$; Midpoint Formula

Check Your Understanding

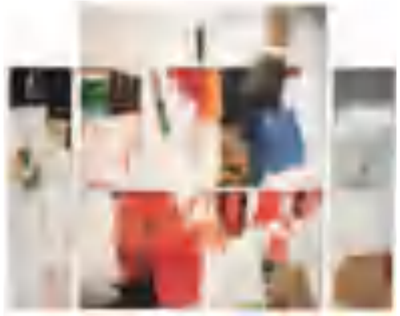
Example 1 (p. 334)

1. PROOF Write a two-column proof to prove that $PQRS$ is a parallelogram given that $\overline{PT} \cong \overline{TR}$ and $\angle TSP \cong \angle TQR$.



Example 2 (p. 335)

2. ART Texas artist Robert Rauschenberg created *Trophy II (for Teeny and Marcel Duchamp)* in 1960. The piece is a combination of several canvases. Describe one method to determine if the panels are parallelograms.

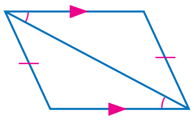


Robert Rauschenberg. *Trophy II (for Teeny and Marcel Duchamp)*, 1960. Oil, charcoal, paper, fabric, metal on canvas, drinking glass, metal chain, spoon, necktie. Collection Walker Art Center, Minneapolis. Gift of the T.B. Walker Foundation, 1970. Art © Robert Rauschenberg/Licensed by VAGA, New York, NY

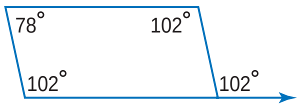
Example 3 (p. 335)

Determine whether each quadrilateral is a parallelogram. Justify your answer.

3.



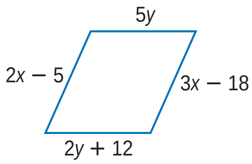
4.



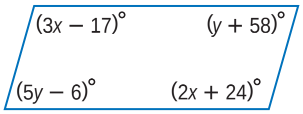
Example 4 (p. 336)

ALGEBRA Find *x* and *y* so that each quadrilateral is a parallelogram.

5.



6.



Example 5 (p. 336)

COORDINATE GEOMETRY Determine whether the figure with the given vertices is a parallelogram. Use the method indicated.

7. $B(0, 0)$, $C(4, 1)$, $D(6, 5)$, $E(2, 4)$; Slope Formula

8. $E(-4, -3)$, $F(4, -1)$, $G(2, 3)$, $H(-6, 2)$; Midpoint Formula

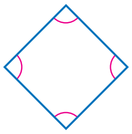
Exercises

HOMEWORK HELP

For Exercises	See Examples
9–14	3
15–17	2
18, 19	1
20–25	4
26–29	5

Determine whether each quadrilateral is a parallelogram. Justify your answer.

9.



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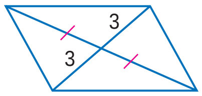
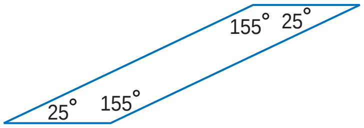


11.



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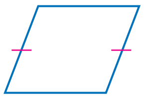
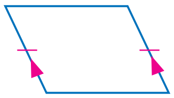
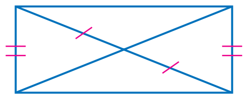


13.

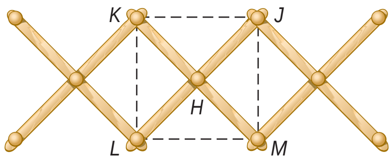


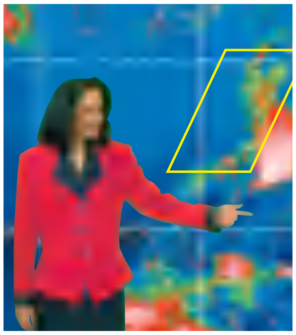
14.



15. TANGRAMS A tangram set consists of seven pieces: a small square, two small congruent right triangles, two large congruent right triangles, a medium-sized right triangle, and a quadrilateral. How can you determine the shape of the quadrilateral? Explain.

16. STORAGE Songan purchased an expandable hat rack that has 11 pegs. In the figure, H is the midpoint of $\overline{KM}$ and $\overline{JL}$. What type of figure is $JKLM$? Explain.





Real-World Career

Atmospheric Scientist

An atmospheric scientist, or meteorologist, uses math to study weather patterns. They can work for private companies, the Federal Government, or television stations.

Math Online For more information, go to geometryonline.com.

17. METEOROLOGY To show the center of a storm, television stations superimpose a "watchbox" over the weather map. Describe how you can tell whether the watchbox is a parallelogram.

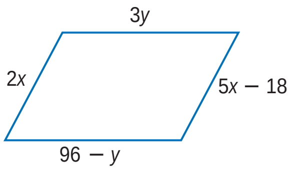
PROOF **Write a two-column proof of each theorem.**

18. Theorem 6.9

19. Theorem 6.11

ALGEBRA **Find x and y so that each quadrilateral is a parallelogram.**

20.



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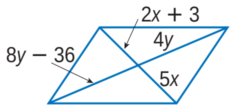

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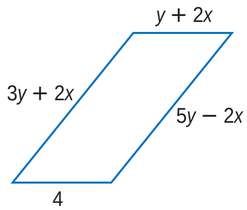

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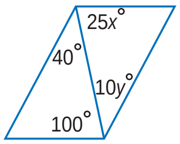

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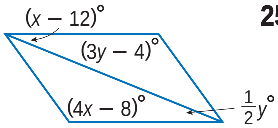

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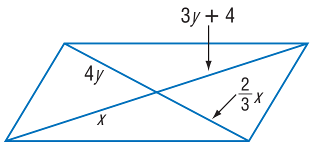


COORDINATE GEOMETRY **Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.**

26. $B(-6, -3)$, $C(2, -3)$, $E(4, 4)$, $G(-4, 4)$; Midpoint Formula

27. $H(5, 6)$, $J(9, 0)$, $K(8, -5)$, $L(3, -2)$; Distance Formula

28. $C(-7, 3)$, $D(-3, 2)$, $F(0, -4)$, $G(-4, -3)$; Distance and Slope Formulas

29. $G(-2, 8)$, $H(4, 4)$, $J(6, -3)$, $K(-1, -7)$; Distance and Slope Formulas

30. Quadrilateral $MNPR$ has vertices $M(-6, 6)$, $N(-1, -1)$, $P(-2, -4)$, and $R(-5, -2)$. Determine how to move one vertex to make $MNPR$ a parallelogram.

31. Quadrilateral $QSTW$ has vertices $Q(-3, 3)$, $S(4, 1)$, $T(-1, -2)$, and $W(-5, -1)$. Determine how to move one vertex to make $QSTW$ a parallelogram.

EXTRA PRACTICE

See pages 811, 833.

Math Online Self-Check Quiz at geometryonline.com

COORDINATE GEOMETRY **The coordinates of three of the vertices of a parallelogram are given. Find the possible coordinates for the fourth vertex.**

32. $A(1, 4)$, $B(7, 5)$, and $C(4, -1)$

33. $Q(-2, 2)$, $R(1, 1)$, and $S(-1, -1)$

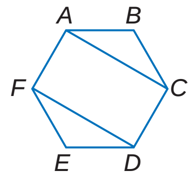
H.O.T. Problems

34. REASONING Felisha claims she discovered a new geometry theorem: a diagonal of a parallelogram bisects its angles. Determine whether this theorem is true. Find an example or counterexample.

35. OPEN ENDED Draw a parallelogram. Label the congruent angles. Explain how you determined it was a parallelogram.

36. **FIND THE ERROR** Carter and Shaniqua are describing ways to show that a quadrilateral is a parallelogram. Who is correct? Explain your reasoning.

Carter	Shaniqua
A quadrilateral is a parallelogram if one pair of sides is congruent and one pair of opposite sides is parallel.	A quadrilateral is a parallelogram if one pair of opposite sides is congruent and parallel.



37. **CHALLENGE** Write a proof to prove that *FDCA* is a parallelogram if *ABCDEF* is a regular hexagon.

38. **Writing in Math** Describe the information needed to prove that a quadrilateral is a parallelogram.

39. If sides $\overline{AB}$ and $\overline{DC}$ of quadrilateral *ABCD* are parallel, which additional information would be sufficient to prove that quadrilateral *ABCD* is a parallelogram?

A $\overline{AB} \cong \overline{AC}$ **C** $\overline{AC} \cong \overline{BD}$

B $\overline{AB} \cong \overline{DC}$ **D** $\overline{AD} \cong \overline{BC}$

40. **REVIEW** Jarod's average driving speed for a 5-hour trip was 58 miles per hour. During the first 3 hours, he drove 50 miles per hour. What was his average speed in miles per hour for the last 2 hours of his trip?

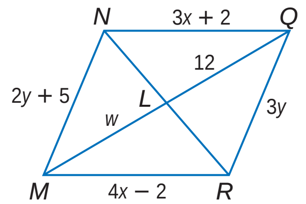
F 70 **H** 60

G 66 **J** 54

Use ▱*NQRM* to find each measure or value. (Lesson 6–2)

41. *w* 42. *x*

43. *NQ* 44. *QR*



The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon. (Lesson 6–1)

45. 135 46. 144 47. 168 48. 162

49. **ATHLETICS** Maddox was at the gym for just over two hours. He swam laps in the pool and lifted weights. Prove that he did one of these activities for more than an hour. (Lesson 5–3)

PREREQUISITE SKILL **Use slope to determine whether $\overline{AB}$ and $\overline{BC}$ are *perpendicular* or *not perpendicular*.** (Lesson 3–3)

50. $A(2, 5)$, $B(6, 3)$, $C(8, 7)$ 51. $A(-1, 2)$, $B(0, 7)$, $C(4, 1)$

6-4 Rectangles

Main Ideas

- Recognize and apply properties of rectangles.
- Determine whether parallelograms are rectangles.

New Vocabulary

rectangle

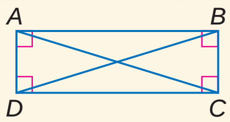
Many sports are played on fields marked by parallel lines. A tennis court has parallel serving lines for each player. Parallel lines divide the court for singles and doubles play. The service box is marked by perpendicular lines.

Properties of Rectangles A **rectangle** is a quadrilateral with four right angles. Since both pairs of opposite angles are congruent, it follows that it is a special type of parallelogram. Thus, a rectangle has all the properties of a parallelogram. In addition, the diagonals of a rectangle are also congruent.

THEOREM 6.13

If a parallelogram is a rectangle, then the diagonals are congruent.

Abbreviation: *If ▱ is rectangle, diag. are ≅.*



$\overline{AC} \cong \overline{BD}$

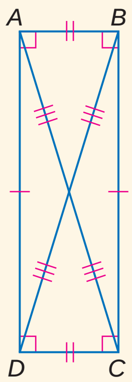
You will prove Theorem 6.13 in Exercise 33.

If a quadrilateral is a rectangle, then the following properties are true.

KEY CONCEPT — Rectangle

Words A rectangle is a quadrilateral with four right angles.

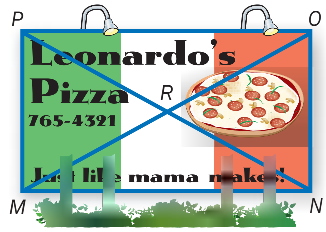
Properties	Examples
1. Opposite sides are congruent and parallel.	$\overline{AB} \cong \overline{DC}$ $\overline{AB} \parallel \overline{DC}$ $\overline{BC} \cong \overline{AD}$ $\overline{BC} \parallel \overline{AD}$
2. Opposite angles are congruent.	$\angle A \cong \angle C$ $\angle B \cong \angle D$
3. Consecutive angles are supplementary.	$m\angle A + m\angle B = 180$ $m\angle B + m\angle C = 180$ $m\angle C + m\angle D = 180$ $m\angle D + m\angle A = 180$
4. Diagonals are congruent and bisect each other.	$\overline{AC} \cong \overline{BD}$ $\overline{AC}$ and $\overline{BD}$ bisect each other.
5. All four angles are right angles.	$m\angle DAB = m\angle BCD =$ $m\angle ABC = m\angle ADC = 90$



Simon Bruty/Getty Images

Diagonals of a Rectangle

1 **ALGEBRA** **Quadrilateral *MNOP* is a billboard in the shape of a rectangle. If $MO = 6x + 14$ and $PN = 9x + 5$, find *x*. Then find *NR*.**



$\overline{MO} \cong \overline{PN}$	Diagonals of a rectangle are $\cong$.
$MO = PN$	Definition of congruent segments
$6x + 14 = 9x + 5$	Substitution
$14 = 3x + 5$	Subtract 6*x* from each side.
$9 = 3x$	Subtract 5 from each side.
$3 = x$	Divide each side by 3.

$NR = \frac{1}{2}PN$	Diagonals bisect each other.
$= \frac{1}{2}(9x + 5)$	Substitution
$= \frac{1}{2}(9 \cdot 3 + 5)$	Substitute 3 for *x*.
$= \frac{1}{2}(27 + 5)$	
$= \frac{1}{2}(32)$	
$= 16$	

1. Refer to rectangle *MNOP*. If $MO = 4y + 12$ and $PR = 3y - 5$, find *y*.

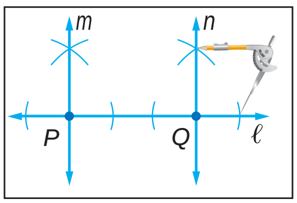
Animation
geometryonline.com

Rectangles can be constructed using perpendicular lines.

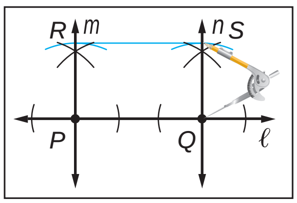
CONSTRUCTION

Rectangle

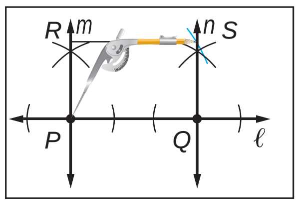
Step 1 Use a straightedge to draw line ℓ. Label points *P* and *Q* on ℓ. Now construct lines perpendicular to ℓ through *P* and through *Q*. Label them *m* and *n*.



Step 2 Place the compass point at *P* and mark off a segment on *m*. Using the same compass setting, place the compass at *Q* and mark a segment on *n*. Label these points *R* and *S*. Draw $\overline{RS}$.

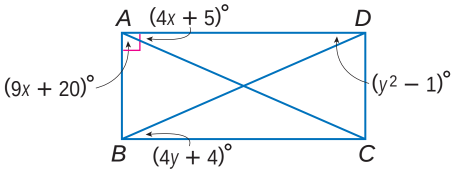


Step 3 Locate the compass setting that represents *PS* and compare to the setting for *QR*. The measures should be the same.



EXAMPLE 2 Angles of a Rectangle

ALGEBRA Quadrilateral *ABCD* is a rectangle. Find *y*.



Since a rectangle is a parallelogram, opposite sides are parallel. So, alternate interior angles are congruent.

$\angle ADB \cong \angle CBD$	Alternate Interior Angles Theorem
$m\angle ADB = m\angle CBD$	Definition of $\cong$ angles
$y^2 - 1 = 4y + 4$	Substitution
$y^2 - 4y - 5 = 0$	Subtract $4y$ and 4 from each side.
$(y - 5)(y + 1) = 0$	Factor.
$y - 5 = 0 \qquad y + 1 = 0$	
$y = 5 \qquad y = -1$	Disregard $y = -1$ because it yields angle measures of 0.

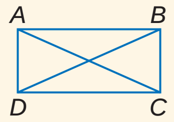
2. Refer to rectangle *ABCD*. Find *x*.

Prove That Parallelograms Are Rectangles The converse of Theorem 6.13 is also true.

THEOREM 6.14

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Abbreviation: *If diagonals of ▱ are ≅, ▱ is a rectangle.*



$\overline{AC} \cong \overline{BD}$

You will prove Theorem 6.14 in Exercise 34.

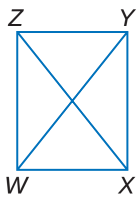
Real-World Link

It is important to square a window frame because over time the opening may have become "out-of-square." If the window is not properly situated in the framed opening, air and moisture can leak through cracks.

Source: www.supersealwindows.com/guide/measurement

Diagonals of a Parallelogram

3 WINDOWS Trent is building a tree house for his younger brother. He has measured the window opening to be sure that the opposite sides are congruent. He measures the diagonals to make sure that they are congruent. This is called *squaring* the frame. How does he know that the corners are 90° angles?



First draw a diagram and label the vertices. We know that $\overline{WX} \cong \overline{ZY}$, $\overline{XY} \cong \overline{WZ}$, and $\overline{WY} \cong \overline{XZ}$.

Because $\overline{WX} \cong \overline{ZY}$ and $\overline{XY} \cong \overline{WZ}$, *WXYZ* is a parallelogram.

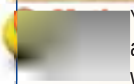
$\overline{XZ}$ and $\overline{WY}$ are diagonals and they are congruent. A parallelogram with congruent diagonals is a rectangle. So, the corners are 90° angles.

3. CRAFTS Antonia is making her own picture frame. How can she determine if the measure of each corner is 90°?

Study Tip

Rectangles and Parallelograms
A rectangle is a parallelogram, but a parallelogram is not necessarily a rectangle.

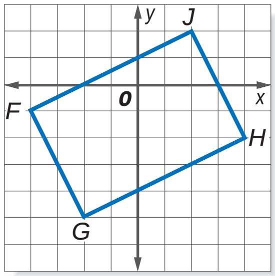
Cross-Curricular Project



You can use a rectangle with special dimensions to discover the golden mean. Visit geometryonline.com.

EXAMPLE Rectangle on a Coordinate Plane

4 **COORDINATE GEOMETRY** **Quadrilateral *FGHJ* has vertices *F*(−4, −1), *G*(−2, −5), *H*(4, −2), and *J*(2, 2). Determine whether *FGHJ* is a rectangle.**



Method 1 Use the Slope Formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to see if consecutive sides are perpendicular.

slope of $\overline{FJ} = \frac{2 - (-1)}{2 - (-4)}$ or $\frac{1}{2}$

slope of $\overline{GH} = \frac{-2 - (-5)}{4 - (-2)}$ or $\frac{1}{2}$

slope of $\overline{FG} = \frac{-5 - (-1)}{-2 - (-4)}$ or -2

slope of $\overline{JH} = \frac{-2 - 2}{4 - 2}$ or -2

Because $\overline{FJ} \parallel \overline{GH}$ and $\overline{FG} \parallel \overline{JH}$, quadrilateral *FGHJ* is a parallelogram.

The product of the slopes of consecutive sides is −1. This means that $\overline{FJ} \perp \overline{FG}$, $\overline{FJ} \perp \overline{JH}$, $\overline{JH} \perp \overline{GH}$, and $\overline{FG} \perp \overline{GH}$. The perpendicular segments create four right angles. Therefore, by definition *FGHJ* is a rectangle.

Method 2 Use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, to determine whether opposite sides are congruent.

First, we must show that quadrilateral *FGHJ* is a parallelogram.

$$FJ = \sqrt{(-4 - 2)^2 + (-1 - 2)^2} = \sqrt{36 + 9} = \sqrt{45} \text{ or } 3\sqrt{5}$$

$$GH = \sqrt{(-2 - 4)^2 + [-5 - (-2)]^2} = \sqrt{36 + 9} = \sqrt{45} \text{ or } 3\sqrt{5}$$

$$FG = \sqrt{[-4 - (-2)]^2 + [-1 - (-5)]^2} = \sqrt{4 + 16} = \sqrt{20} \text{ or } 2\sqrt{5}$$

$$JH = \sqrt{(2 - 4)^2 + [2 - (-2)]^2} = \sqrt{4 + 16} = \sqrt{20} \text{ or } 2\sqrt{5}$$

Since each pair of opposite sides of the quadrilateral have the same measure, they are congruent. Quadrilateral *FGHJ* is a parallelogram.

$$FH = \sqrt{(-4 - 4)^2 + [-1 - (-2)]^2} = \sqrt{64 + 1} = \sqrt{65}$$

$$GJ = \sqrt{(-2 - 2)^2 + (-5 - 2)^2} = \sqrt{16 + 49} = \sqrt{65}$$

The length of each diagonal is $\sqrt{65}$. Since the diagonals are congruent, *FGHJ* is a rectangle by Theorem 6.14.

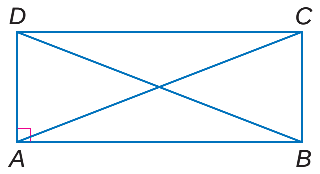
4. **COORDINATE GEOMETRY** Quadrilateral *JKLM* has vertices *J*(−10, 2), *K*(−8, −6), *L*(5, −3), and *M*(2, 5). Determine whether *JKLM* is a rectangle. Justify your answer.

Personal Tutor at geometryonline.com

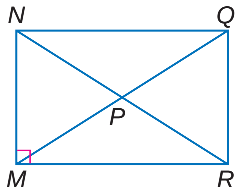
CHECK Your Understanding

Example 1 (p. 341)

1. **ALGEBRA** *ABCD* is a rectangle. If $AC = 30 - x$ and $BD = 4x - 60$, find x.

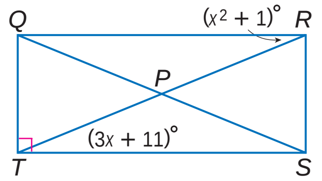


2. **ALGEBRA** *MNQR* is a rectangle. If $NR = 2x + 10$ and $NP = 2x - 30$, find MP.



Example 2 (p. 342)

ALGEBRA Quadrilateral *QRST* is a rectangle. Find each value or measure.



3. x
4. $m\angle RPS$

Example 3 (p. 342)

5. **FRAMING** Mrs. Walker has a rectangular picture that is 12 inches by 48 inches. Because this is not a standard size, a special frame must be built. What can the framer do to guarantee that the frame is a rectangle? Justify your reasoning.

Example 4 (p. 343)

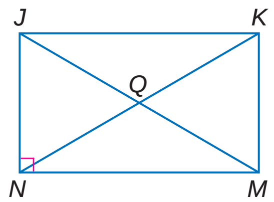
6. **COORDINATE GEOMETRY** Quadrilateral *EFGH* has vertices $E(-4, -3)$, $F(3, -1)$, $G(2, 3)$, and $H(-5, 1)$. Determine whether *EFGH* is a rectangle.

Exercises

HOMEWORK HELP

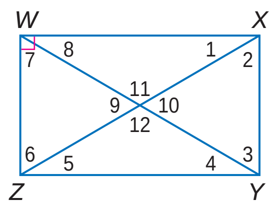
For Exercises	See Examples
7–12	1
13–21	2
22, 23	3
24–31	4

ALGEBRA Quadrilateral *JKMN* is a rectangle.



7. If $NQ = 5x - 3$ and $QM = 4x + 6$, find NK.
8. If $NQ = 2x + 3$ and $QK = 5x - 9$, find JQ.
9. If $NM = 8x - 14$ and $JK = x^2 + 1$, find JK.
10. If $m\angle NJM = 2x - 3$ and $m\angle KJM = x + 5$, find x.
11. If $m\angle NKM = x^2 + 4$ and $m\angle KNM = x + 30$, find $m\angle JKN$.
12. If $m\angle JKN = 2x^2 + 2$ and $m\angle NKM = 14x$, find x.

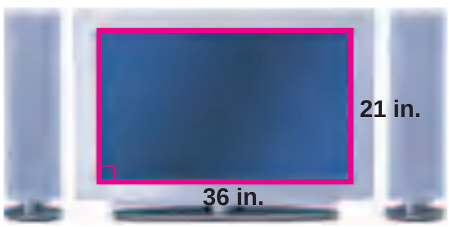
***WXYZ* is a rectangle. Find each measure if $m\angle 1 = 30$.**



13. $m\angle 2$
14. $m\angle 3$
15. $m\angle 4$
16. $m\angle 5$
17. $m\angle 6$
18. $m\angle 7$
19. $m\angle 8$
20. $m\angle 9$
21. $m\angle 12$

22. **PATIOS** A contractor has been hired to pour a rectangular concrete patio. How can he be sure that the frame in which he will pour the concrete is rectangular?

23. **TELEVISION** Television screens are measured on the diagonal. What is the measure of the diagonal of this screen?



Zenith Electronics Corp./AP/Wide World Photos

Real-World Link

Myrtle Beach, South Carolina, has 45 miniature golf courses within 20 miles of the Grand Strand, the region that is home to Myrtle Beach and several other towns.

Source: U.S. ProMini Golf Association

COORDINATE GEOMETRY **Determine whether *DFGH* is a rectangle given each set of vertices. Justify your answer.**

24. $D(9, -1)$, $F(9, 5)$, $G(-6, 5)$, $H(-6, 1)$

25. $D(6, 2)$, $F(8, -1)$, $G(10, 6)$, $H(12, 3)$

26. $D(-4, -3)$, $F(-5, 8)$, $G(6, 9)$, $H(7, -2)$

COORDINATE GEOMETRY **The vertices of *WXYZ* are $W(2, 4)$, $X(-2, 0)$, $Y(-1, -7)$, and $Z(9, 3)$.**

27. Find *WY* and *XZ*.

28. Find the coordinates of the midpoints of $\overline{WY}$ and $\overline{XZ}$.

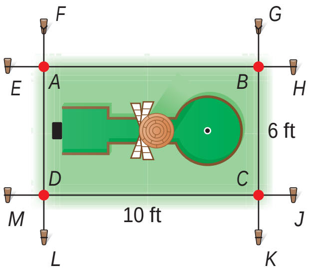
29. Is *WXYZ* a rectangle? Explain.

COORDINATE GEOMETRY **The vertices of parallelogram *ABCD* are $A(-4, -4)$, $B(2, -1)$, $C(0, 3)$, and $D(-6, 0)$.**

30. Determine whether *ABCD* is a rectangle.

31. If *ABCD* is a rectangle and *E*, *F*, *G*, and *H* are midpoints of its sides, what can you conclude about *EFGH*?

32. **MINIATURE GOLF** The windmill section of a miniature golf course will be a rectangle 10 feet long and 6 feet wide. Suppose the contractor placed stakes and strings to mark the boundaries with the corners at *A*, *B*, *C*, and *D*. The contractor measured *BD* and *AC* and found that $AC > BD$. Describe where to move the stakes *L* and *K* to make *ABCD* a rectangle. Explain.

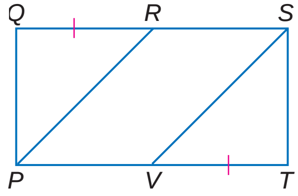


PROOF **Write a two-column proof.**

33. Theorem 6.13

34. Theorem 6.14

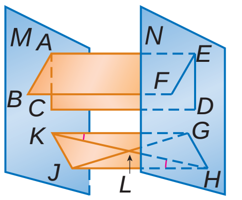
35. **Given:** *PQST* is a rectangle.
$\overline{QR} \cong \overline{VT}$
Prove: $\overline{PR} \cong \overline{VS}$



EXTRA PRACTICE
See pages 812, 833.

Math Online
Self-Check Quiz at
geometryonline.com

36. **Given:** *DEAC* and *FEAB* are rectangles.
$\angle GKH \cong \angle JHK$
$\overline{GJ}$ and $\overline{HK}$ intersect at *L*.
Prove: *GHJK* is a parallelogram.



Study Tip

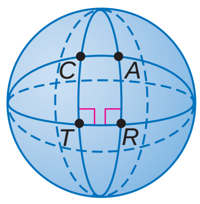
Look Back
To review **Non-Euclidean geometry**, refer to Extend Lesson 3-6.

NON-EUCLIDEAN GEOMETRY **The figure shows a *Saccheri quadrilateral* on a sphere. Note that it has four sides with $\overline{CT} \perp \overline{TR}$, $\overline{AR} \perp \overline{TR}$, and $\overline{CT} \cong \overline{AR}$.**

37. Is $\overline{CT}$ parallel to $\overline{AR}$? Explain.

38. How does *AC* compare to *TR*?

39. Can a rectangle exist in non-Euclidean geometry? Explain.



40. **RESEARCH** Use the Internet or another source to investigate the similarities and differences between non-Euclidean geometry and Euclidean geometry.

H.O.T. Problems

41. **REASONING** Draw a counterexample to the statement *If the diagonals are congruent, the quadrilateral is a rectangle.*

42. **OPEN ENDED** Draw two congruent right triangles with a common hypotenuse. Do the legs form a rectangle? Justify your answer.

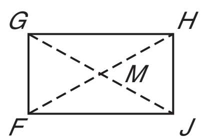
43. **FIND THE ERROR** McKenna and Consuelo are defining a rectangle for an assignment. Who is correct? Explain.

McKenna	Consuelo
A rectangle is a parallelogram with one right angle.	A rectangle has a pair of parallel opposite sides and a right angle.

44. **CHALLENGE** Using four of the twelve points as corners, how many rectangles can be drawn?

45. **Writing in Math** How can you determine whether a parallelogram is a rectangle? Explain your reasoning.

46. If $FJ = -3x + 5y$, $FM = 3x + y$, $GH = 11$, and $GM = 13$, what values of x and y make parallelogram $FGHJ$ a rectangle?



A $x = 3, y = 4$ **C** $x = 7, y = 8$

B $x = 4, y = 3$ **D** $x = 8, y = 7$

47. **REVIEW** A rectangular playground is surrounded by an 80-foot fence. One side of the playground is 10 feet longer than the other. Which of the following equations could be used to find s, the shorter side of the playground?

F $10s + s = 80$

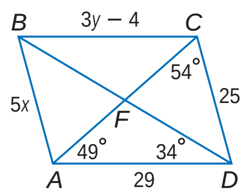
G $4s + 10 = 80$

H $s(s + 10) = 80$

J $2(s + 10) + 2s = 80$

48. **OPTIC ART** Victor Vasarely created art in the op art style. This piece *AMBIGU-B*, consists of multi-colored parallelograms. Describe one method to ensure that the shapes are parallelograms. (Lesson 6-3)

For Exercises 49–54, use ▱*ABCD*. Find each measure or value. (Lesson 6–2)



49. $m\angle AFD$ 50. $m\angle CDF$

51. y 52. x

PREREQUISITE SKILL Find the distance between each pair of points. (Lesson 1–4)

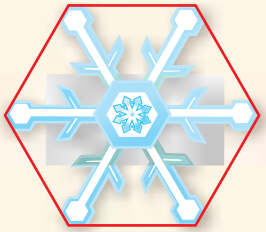
53. $(1, -2), (-3, 1)$ 54. $(-5, 9), (5, 12)$ 55. $(1, 4), (22, 24)$

CHAPTER 6

Mid-Chapter Quiz

Lessons 6-1 through 6-4

1. **SNOW** The snowflake pictured is a regular hexagon. Find the sum of the measures of the interior angles of the hexagon. (Lesson 6-1)



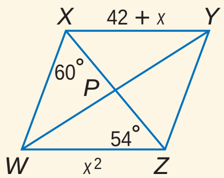
2. The measure of an interior angle of a regular polygon is $147\frac{3}{11}$. Find the number of sides in the polygon. (Lesson 6-1)

3. How many degrees are there in the sum of the exterior angles of a dodecagon? (Lesson 6-1)

4. Find the measure of each exterior angle of a regular pentagon. (Lesson 6-1)

5. If each exterior angle of a regular polygon measures 40°, how many sides does the polygon have? (Lesson 6-1)

Use ▱*WXYZ* to find each measure. (Lesson 6-2)



6. WZ = ___?___

7. $m\angle XYZ$ = ___?___

8. **MULTIPLE CHOICE** Two opposite angles of a parallelogram measure $(5x - 25)°$ and $(3x + 5)°$. Find the measures of the angles. (Lesson 6-2)

A 50, 50

B 55, 125

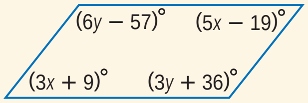
C 90, 90

D 109, 71

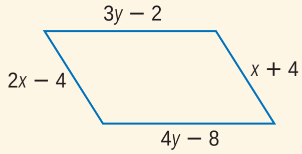
9. Parallelogram *JKLM* has vertices $J(0, 7)$, $K(9, 7)$, and $L(6, 0)$. Find the coordinates of M. (Lesson 6-2)

ALGEBRA Find *x* and *y* so that each quadrilateral is a parallelogram. (Lesson 6-3)

10.



11.

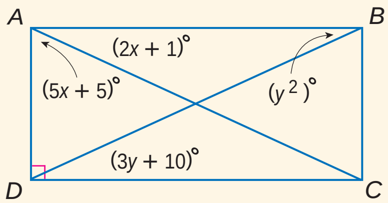


COORDINATE GEOMETRY Determine whether a figure with the given vertices is a parallelogram. Use the method indicated. (Lesson 6-3)

12. $Q(-3, -6)$, $R(2, 2)$, $S(-1, 6)$, $T(-5, 2)$; Distance and Slope formulas

13. $W(-6, -5)$, $X(-1, -4)$, $Y(0, -1)$, $Z(-5, -2)$; Midpoint formula

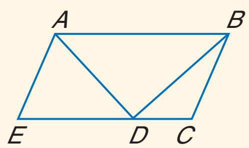
Quadrilateral *ABCD* is a rectangle. (Lesson 6-4)



14. Find x.

15. Find y.

16. **MULTIPLE CHOICE** In the figure, quadrilateral *ABCE* is a parallelogram. If $\angle ADE \cong \angle BDC$, which of the following *must* be true? (Lesson 6-4)



F $\overline{AD} \cong \overline{DB}$

G $\overline{ED} \cong \overline{AD}$

H $\overline{ED} \cong \overline{DC}$

J $\overline{AE} \cong \overline{DC}$

6-5 Rhombi and Squares

Main Ideas

- Recognize and apply the properties of rhombi.
- Recognize and apply the properties of squares.

New Vocabulary

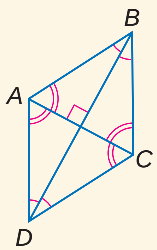
rhombus
square

Professor Stan Wagon at Macalester College in St. Paul, Minnesota, developed a bicycle with square wheels. There are two front wheels so the rider can balance without turning the handlebars. Riding over a specially curved road ensures a smooth ride.

Properties of Rhombi A square is a special type of parallelogram called a rhombus. A **rhombus** is a quadrilateral with all four sides congruent. All of the properties of parallelograms can be applied to rhombi. There are three other characteristics of rhombi described in the following theorems.

THEOREMS *Rhombus*

	Examples
6.15 The diagonals of a rhombus are perpendicular.	$\overline{AC} \perp \overline{BD}$
6.16 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (Converse of Theorem 6.15)	If $\overline{BD} \perp \overline{AC}$ then $\square ABCD$ is a rhombus.
6.17 Each diagonal of a rhombus bisects a pair of opposite angles.	$\angle DAC \cong \angle BAC \cong \angle DCA \cong \angle BCA$ $\angle ABD \cong \angle CBD \cong \angle ADB \cong \angle CDB$

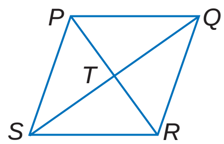


You will prove Theorems 6.16 and 6.17 in Exercises 9 and 10, respectively.

EXAMPLE 1 **Proof of Theorem 6.15**

Given: $PQRS$ is a rhombus.

Prove: $\overline{PR} \perp \overline{SQ}$



Paragraph Proof:

By the definition of a rhombus, $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{PS}$.

Study Tip

Proof

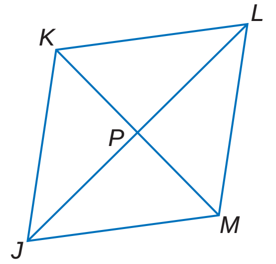
Since a rhombus has four congruent sides, one diagonal separates the rhombus into two congruent isosceles triangles. Drawing two diagonals separates the rhombus into four congruent right triangles.

A rhombus is a parallelogram and the diagonals of a parallelogram bisect each other, so $\overline{QS}$ bisects $\overline{PR}$ at T. Thus, $\overline{PT} \cong \overline{RT}$. $\overline{QT} \cong \overline{QT}$ because congruence of segments is reflexive. Thus, $\triangle PQT \cong \triangle RQT$ by SSS. $\angle QTP \cong \angle QTR$ by CPCTC. $\angle QTP$ and $\angle QTR$ also form a linear pair. Two congruent angles that form a linear pair are right angles. $\angle QTP$ is a right angle, so $\overline{PR} \perp \overline{SQ}$ by the definition of perpendicular lines.

1. PROOF Write a paragraph proof.

Given: $JKLM$ is a parallelogram.
$\triangle JKL$ is isosceles.

Prove: $JKLM$ is a rhombus.

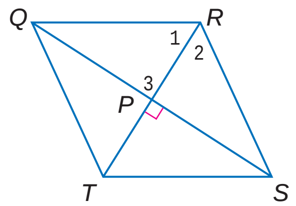


Reading Math

Rhombi The plural form of rhombus is *rhombi*, pronounced ROM-bye.

EXAMPLE 2 Measures of a Rhombus

ALGEBRA Use rhombus *QRST* and the given information to find the value of each variable.



a. Find y if $m\angle 3 = y^2 - 31$.

$m\angle 3 = 90$ The diagonals of a rhombus are perpendicular.

$y^2 - 31 = 90$ Substitution

$y^2 = 121$ Add 31 to each side.

$y = \pm 11$ Take the square root of each side.

The value of y can be 11 or -11.

b. Find $m\angle TQS$ if $m\angle RST = 56$.

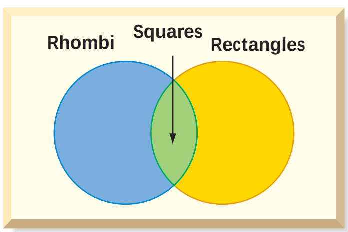
$m\angle TQR = m\angle RST$ Opposite angles are congruent.

$m\angle TQR = 56$ Substitution

The diagonals of a rhombus bisect the angles. So, $m\angle TQS$ is $\frac{1}{2}(56)$ or 28.

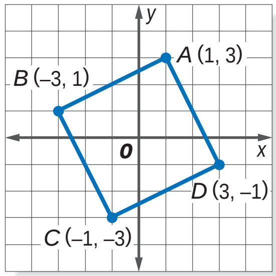
2. ALGEBRA Use rhombus $QRST$ to find $m\angle QTS$ if $m\angle 2 = 57$.

Properties of Squares If a quadrilateral is both a rhombus and a rectangle, then it is a **square**. All of the properties of parallelograms and rectangles can be applied to squares.



EXAMPLE Squares

3 COORDINATE GEOMETRY Determine whether parallelogram *ABCD* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.



Explore Plot the vertices on a coordinate plane.

Plan If the diagonals are perpendicular, then *ABCD* is either a rhombus or a square. The diagonals of a rectangle are congruent. If the diagonals are congruent and perpendicular, then *ABCD* is a square.

Solve Use the Distance Formula to compare the lengths of the diagonals.

$$DB = \sqrt{[3-(-3)]^2 + (-1-1)^2} = \sqrt{36+4} = \sqrt{40} \text{ or } 2\sqrt{10}$$

$$AC = \sqrt{[1-(-1)]^2 + [3-(-3)]^2} = \sqrt{4+36} = \sqrt{40} \text{ or } 2\sqrt{10}$$

Use slope to determine whether the diagonals are perpendicular.

$$\text{slope of } \overline{DB} = \frac{1-(-1)}{-3-3} \text{ or } -\frac{1}{3} \qquad \text{slope of } \overline{AC} = \frac{-3-3}{-1-1} \text{ or } 3$$

Since the slope of $\overline{AC}$ is the negative reciprocal of the slope of $\overline{DB}$, the diagonals are perpendicular. $\overline{DB}$ and $\overline{AC}$ have the same measure, so the diagonals are congruent. *ABCD* is a rhombus, a rectangle, and a square.

Check You can verify that *ABCD* is a square by finding the measure and slope of each side. All four sides are congruent and consecutive sides are perpendicular.

3. COORDINATE GEOMETRY Given the vertices $J(5, 0)$, $K(8, -11)$, $L(-3, -14)$, $M(-6, -3)$, determine whether parallelogram *JKLM* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.

Animation
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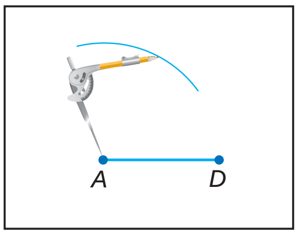
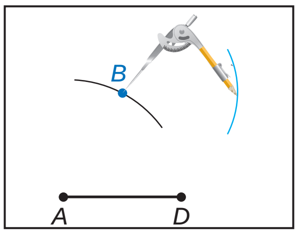
CONSTRUCTION

Rhombus

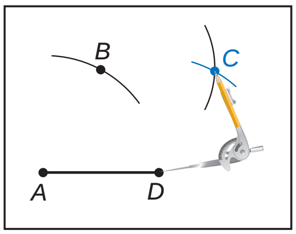
Step 1 Draw any segment $\overline{AD}$. Place the compass point at *A*, open to the width of *AD*, and draw an arc above $\overline{AD}$.



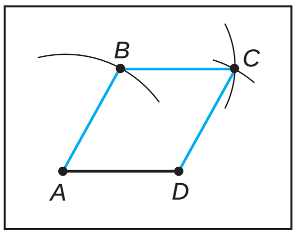
Step 2 Label any point on the arc as *B*. Using the same setting, place the compass at *B*, and draw an arc to the right of *B*.



Step 3 Place the compass at *D*, and draw an arc to intersect the arc drawn from *B*. Label the point of intersection *C*.



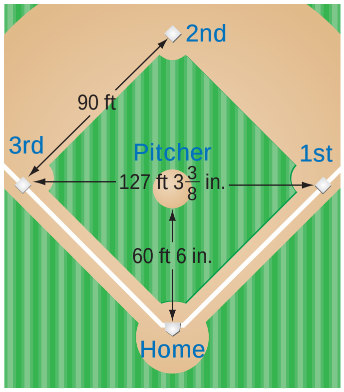
Step 4 Use a straightedge to draw $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$.



Conclusion: Since all of the sides are congruent, quadrilateral *ABCD* is a rhombus.

Diagonals of a Square

4 **BASEBALL** **The infield of a baseball diamond is a square, as shown at the right. Is the pitcher's mound located in the center of the infield? Explain.**



Since a square is a parallelogram, the diagonals bisect each other. Since a square is a rhombus, the diagonals are congruent. Therefore, the distance from first base to third base is equal to the distance between home plate and second base.

Thus, the distance from home plate to the center of the infield is 127 feet $3\frac{3}{8}$ inches divided by 2 or 63 feet $7\frac{11}{16}$ inches. This distance is longer than the distance from home plate to the pitcher's mound so the pitcher's mound is not located in the center of the field. It is about 3 feet closer to home.

4. **STAINED GLASS** Kathey is designing a stained glass piece with rhombus-shaped tiles. Describe how she can determine if the tiles are rhombi.

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If a quadrilateral is a rhombus or a square, then the following properties are true.

Study Tip

Square and Rhombus

A square is a rhombus, but a rhombus is not necessarily a square.

CONCEPT SUMMARY *Properties of Rhombi and Squares*

Rhombi	Squares
1. A rhombus has all the properties of a parallelogram.	1. A square has all the properties of a parallelogram.
2. All sides are congruent.	2. A square has all the properties of a rectangle.
3. Diagonals are perpendicular.	3. A square has all the properties of a rhombus.
4. Diagonals bisect the angles of the rhombus.	

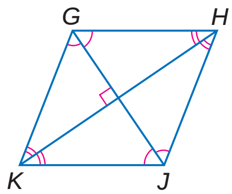
Check Your Understanding

Example 1 (p. 349)

1. **PROOF** Write a two-column proof.

Given: $\triangle KGH$, $\triangle HJK$, $\triangle GHJ$, and $\triangle JKG$ are isosceles.

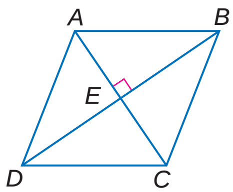
Prove: $GHJK$ is a rhombus.



Example 2 (p. 349)

ALGEBRA **In rhombus $ABCD$, $AB = 2x + 3$ and $BC = 5x$.**

2. Find x.
3. Find AD.
4. Find $m\angle AEB$.
5. Find $m\angle BCD$ if $m\angle ABC = 83.2$.



Example 3 (p. 350)

COORDINATE GEOMETRY **Given each set of vertices, determine whether ▱*MNPQ* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain your reasoning.**

6. $M(0, 3)$, $N(-3, 0)$, $P(0, -3)$, $Q(3, 0)$

7. $M(-4, 0)$, $N(-3, 3)$, $P(2, 2)$, $Q(1, -1)$

Example 4 (p. 351)

8. **REMODELING** The Steiner family is remodeling their kitchen. Each side of the floor measures 10 feet. What other measurements should be made to determine whether the floor is a square?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–14	1
15–18	2
19–22	3
23-24	4

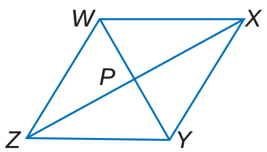
PROOF **Write a paragraph proof for each theorem.**

9. Theorem 6.16

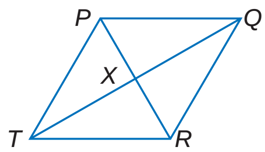
10. Theorem 6.17

PROOF **Write a two-column proof.**

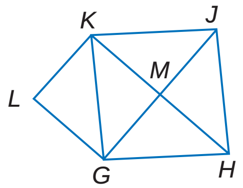
11. Given: $\triangle WZY \cong \triangle WXY$, $\triangle WZY$ and $\triangle XYZ$ are isosceles.
Prove: *WXYZ* is a rhombus.



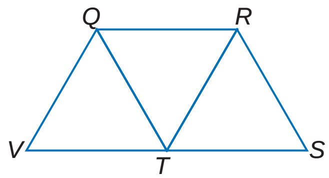
12. Given: $\triangle TPX \cong \triangle QPX \cong \triangle QRX \cong \triangle TRX$
Prove: *TPQR* is a rhombus.



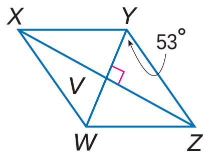
13. Given: $\triangle LGK \cong \triangle MJK$
GHJK is a parallelogram.
Prove: *GHJK* is a rhombus.



14. Given: *QRST* and *QRTV* are rhombi.
Prove: $\triangle QRT$ is equilateral.



ALGEBRA **Use rhombus *XYZW* with $m\angle WYZ = 53$, $VW = 3$, $XV = 2a - 2$, and $ZV = \frac{5a + 1}{4}$.**



15. Find $m\angle YZV$.

16. Find $m\angle XYW$.

17. Find XZ.

18. Find XW.

COORDINATE GEOMETRY **Given each set of vertices, determine whether ▱*EFGH* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain your reasoning.**

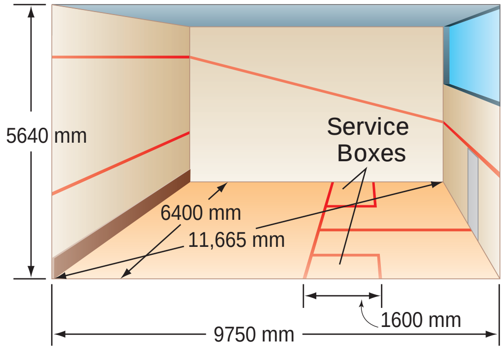
19. $E(1, 10)$, $F(-4, 0)$, $G(7, 2)$, $H(12, 12)$

20. $E(-7, 3)$, $F(-2, 3)$, $G(1, 7)$, $H(-4, 7)$

21. $E(1, 5)$, $F(6, 5)$, $G(6, 10)$, $H(1, 10)$

22. $E(-2, -1)$, $F(-4, 3)$, $G(1, 5)$, $H(3, 1)$

SQUASH For Exercises 23 and 24, use the diagram of the court for squash, a game similar to racquetball and tennis.



23. The diagram labels the diagonal as 11,665 millimeters. Is this correct? Explain.

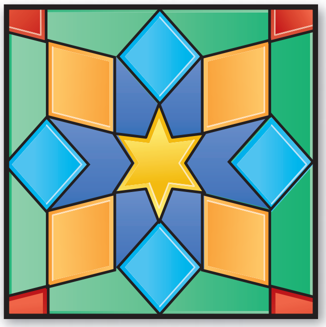
24. The service boxes are squares. Find the length of the diagonal.

Construct each figure using a compass and ruler.

25. a square with one side 3 centimeters long

26. a square with a diagonal 5 centimeters long

27. **MOSAIC** This pattern is composed of repeating shapes. Use a ruler or a protractor to determine which type of quadrilateral best represents the brown shapes.



28. **DESIGN** Otto Prutscher designed the plant stand at the left in 1903. The base is a square, and the base of each of the five boxes is also a square. Suppose each smaller box is one half as wide as the base. Use the information at the left to find the dimensions of the base of one of the smaller boxes.

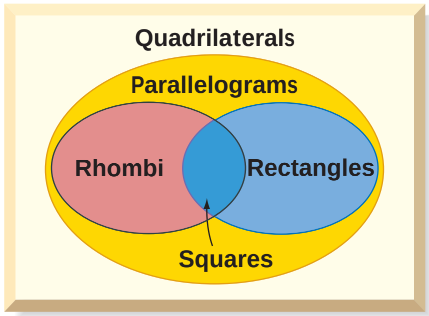
Real-World Link

The overall dimensions of the plant stand are $36\frac{1}{2}$ inches tall by $15\frac{3}{4}$ inches wide.

Source: www.metmuseum.org

29. **PERIMETER** The diagonals of a rhombus are 12 centimeters and 16 centimeters long. Find the perimeter of the rhombus.

Use the Venn diagram to determine whether each statement is *always*, *sometimes*, or *never* true.



30. A parallelogram is a square.

31. A square is a rhombus.

32. A rectangle is a parallelogram.

33. A rhombus is a rectangle but not a square.

34. A rhombus is a square.

EXTRA PRACTICE

See pages 812, 833.

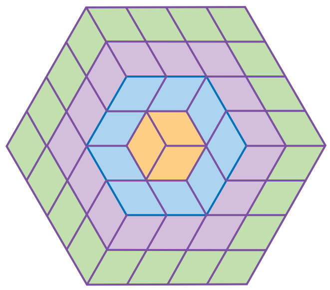
Math Online

Self-Check Quiz at geometryonline.com

35. *True* or *false*? A quadrilateral is a square only if it is also a rectangle. Explain your reasoning.

H.O.T. Problems

36. **CHALLENGE** The pattern at the right is a series of rhombi that continue to form hexagons that increase in size. Copy and complete the table.



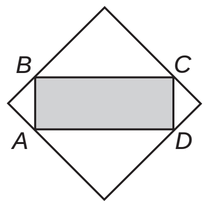
Hexagon	Number of Rhombi
1	3
2	12
3	27
4	48
5	
6	
x	

37. **CHALLENGE** State the converse of Theorem 6.17. Then write a paragraph proof of this converse.

38. **OPEN ENDED** Find the vertices of a square with diagonals that are contained in the lines $y = x$ and $y = -x + 6$. Justify your reasoning.

39. **Writing in Math** Refer to the information on page 348. Explain the difference between squares and rhombi, and describe how nonsquare rhombus-shaped wheels would work with the curved road.

40. Points A, B, C, and D are on a square. The area of the square is 36 square units. What is the perimeter of rectangle $ABCD$?



A 24 units

B $12\sqrt{2}$ units

C 12 units

D $6\sqrt{2}$

41. **REVIEW** If the equation below has no real solutions, then which of the following could *not* be the value of a?

$$ax^2 - 6x + 2 = 0$$

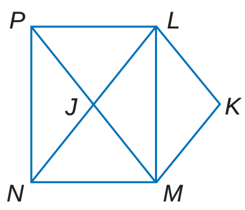
F 3

G 4

H 5

J 6

ALGEBRA **Use rectangle *LMNP*, parallelogram *LKMJ*, and the given information to solve each problem.** (Lesson 6-4)



42. If $LN = 10$, $LJ = 2x + 1$, and $PJ = 3x - 1$, find x.

43. If $m\angle PLK = 110$, find $m\angle LKM$.

44. If $m\angle MJN = 35$, find $m\angle MPN$.

COORDINATE GEOMETRY **Determine whether the points are the vertices of a parallelogram. Use the method indicated.** (Lesson 6-3)

45. $P(0, 2)$, $Q(6, 4)$, $R(4, 0)$, $S(-2, -2)$; Distance Formula

46. $K(-3, -7)$, $L(3, 2)$, $M(1, 7)$, $N(-3, 1)$; Slope Formula

47. **GEOGRAPHY** The distance between San Jose, California, and Las Vegas, Nevada, is about 375 miles. The distance from Las Vegas to Carlsbad, California, is about 243 miles. Use the Triangle Inequality Theorem to find the possible distance between San Jose and Carlsbad. (Lesson 5-4)

PREREQUISITE SKILL **Solve each equation.** (Pages 781 and 782)

48. $\frac{1}{2}(8x - 6x - 7) = 5$

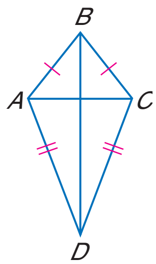
49. $\frac{1}{2}(7x + 3x + 1) = 12.5$

50. $\frac{1}{2}(4x + 6 + 2x + 13) = 15.5$

EXTEND 6-5

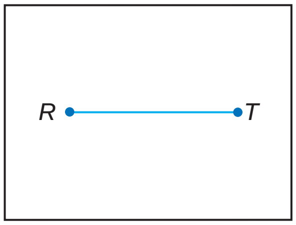
Geometry Lab
Kites

A **kite** is a quadrilateral with exactly two distinct pairs of adjacent congruent sides. In kite $ABCD$, diagonal $\overline{BD}$ separates the kite into two congruent triangles (SSS). Diagonal $\overline{AC}$ separates the kite into two noncongruent isosceles triangles.

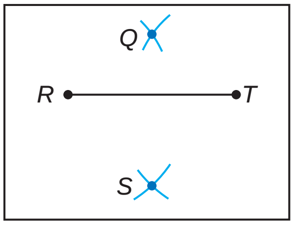


ACTIVITY Construct a kite $QRST$.

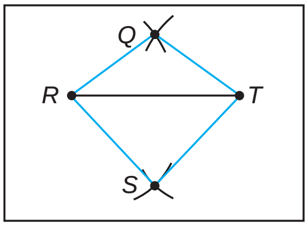
Step 1 Draw $\overline{RT}$.



Step 2 Choose a compass setting greater than $\frac{1}{2}RT$. Place the compass at point R and draw an arc above $\overline{RT}$. Then without changing the compass setting, move the compass to point T and draw an arc that intersects the first one. Label the intersection point Q. Increase the compass setting. Place the compass at R and draw an arc below $\overline{RT}$. Then, without changing the compass setting, draw an arc from point T to intersect the other arc. Label the intersection point S.



Step 3 Draw $QRST$.



Model

1. Draw $\overline{QS}$ in kite $QRST$. Use a protractor to measure the angles formed by the intersection of $\overline{QS}$ and $\overline{RT}$.
2. Measure the interior angles of kite $QRST$. Are any congruent?
3. Label the intersection of $\overline{QS}$ and $\overline{RT}$ as point N. Find the lengths of $\overline{QN}$, $\overline{NS}$, $\overline{TN}$, and $\overline{NR}$. How are they related?
4. How many pairs of congruent triangles can be found in kite $QRST$?
5. Construct another kite $JKLM$. Repeat Exercises 1–4.
6. Make conjectures about angles, sides, and diagonals of kites.
7. Determine whether the lines with equations $y = 4x - 3$, $y = 7x - 60$, $x - 4y = -3$, and $x - 7y = -60$ determine the sides of a kite. Justify your reasoning.

6-6 Trapezoids

Main Ideas

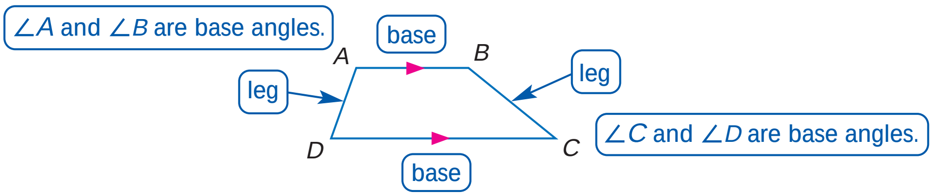
- Recognize and apply the properties of trapezoids.
- Solve problems involving the medians of trapezoids.

New Vocabulary

trapezoid
isosceles trapezoid
median

Cleopatra's Needle in New York City's Central Park was given to the United States in the late 19th century by the Egyptian government. The width of the base is longer than the width at the top. Each face of the monument is an example of a trapezoid.

Properties of Trapezoids A **trapezoid** is a quadrilateral with exactly one pair of parallel sides called *bases.* There are two pairs of *base angles* formed by one base and the legs. The nonparallel sides are called *legs.* If the legs are congruent, then the trapezoid is an **isosceles trapezoid**.



THEOREMS *Isosceles Trapezoid*

6.18 Each pair of base angles of an isosceles trapezoid are congruent.

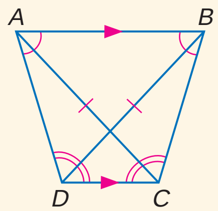
6.19 The diagonals of an isosceles trapezoid are congruent.

Example:

$\angle DAB \cong \angle CBA$

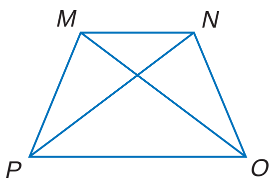
$\angle ADC \cong \angle BCD$

$\overline{AC} \cong \overline{BD}$



EXAMPLE 1 **Proof of Theorem 6.19**

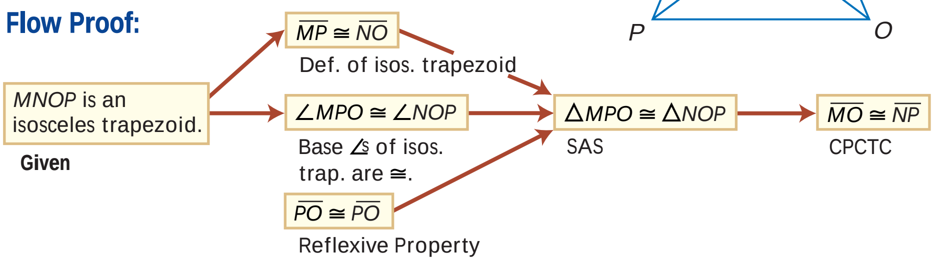
Write a flow proof of Theorem 6.19.



Given: *MNOP* is an isosceles trapezoid.

Prove: $\overline{MO} \cong \overline{NP}$

Flow Proof:



Rudi Von Briel/Index Stock Imagery

1. PROOF Write a paragraph proof of Theorem 6.18.

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Identify Isosceles Trapezoids

Real-World Link

Barnett Newman designed this sculpture to be 50% larger. This piece was designed for an exhibition in Japan but it could not be built as large as the artist wanted because of size limitations on cargo from New York to Japan.

Source: www.sfmoma.org

2 **ART** **The sculpture pictured is *Zim Zum I* by Barnett Newman. The walls are connected at right angles. In perspective, the rectangular panels appear to be trapezoids. Use a ruler and protractor to determine if the images of the front panels are isosceles trapezoids. Explain.**

The panel on the left is an isosceles trapezoid. The bases are parallel and are different lengths. The legs are not parallel, and they are the same length.

The panel on the right is not an isosceles trapezoid. Each side is a different length.

2. Use a compass and ruler to construct an equilateral triangle. Draw a segment with endpoints that are the midpoints of two sides. Use a protractor and a ruler to determine if this segment separates the triangle into an equilateral triangle and an isosceles trapezoid.

EXAMPLE Identify Trapezoids

3 **COORDINATE GEOMETRY** **Quadrilateral *JKLM* has vertices *J*(−18, −1), *K*(−6, 8), *L*(18, 1), and *M*(−18, −26).**

a. Verify that *JKLM* is a trapezoid.

A quadrilateral is a trapezoid if exactly one pair of opposite sides are parallel. Use the Slope Formula.

$$\text{slope of } \overline{JK} = \frac{8-(-1)}{-6-(-18)} = \frac{9}{12} \text{ or } \frac{3}{4} \qquad \text{slope of } \overline{ML} = \frac{1-(-26)}{18-(-18)} = \frac{27}{36} \text{ or } \frac{3}{4}$$

$$\text{slope of } \overline{JM} = \frac{-26-(-1)}{-18-(-18)} = \frac{-25}{0} \text{ or undefined} \qquad \text{slope of } \overline{KL} = \frac{1-8}{18-(-6)} = \frac{-7}{24}$$

Since $\overline{JK} \parallel \overline{ML}$, *JKLM* is a trapezoid.

b. Determine whether *JKLM* is an isosceles trapezoid. Explain.

First use the Distance Formula to show that the legs are congruent.

$$JM = \sqrt{[-18-(-18)]^2 + [-1-(-26)]^2} = \sqrt{0+625} = \sqrt{625} \text{ or } 25$$

$$KL = \sqrt{(-6-18)^2 + (8-1)^2} = \sqrt{576+49} = \sqrt{625} \text{ or } 25$$

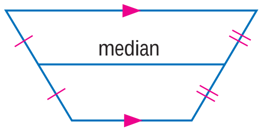
Since the legs are congruent, *JKLM* is an isosceles trapezoid.

3. Quadrilateral $QRST$ has vertices $Q(-8, -4)$, $R(0, 8)$, $S(6, 8)$, and $T(-6, -10)$. Verify that $QRST$ is a trapezoid and determine whether $QRST$ is an isosceles trapezoid.

Study Tip

Median
The median of a trapezoid can also be called a *midsegment.*

Medians of Trapezoids The segment that joins the midpoints of the legs of a trapezoid is called the **median**. It is parallel to and equidistant from each base. You can construct the median of a trapezoid using a compass and a straightedge.



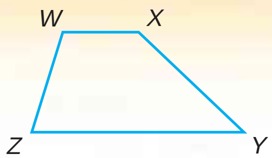
Vocabulary Link
Median
Everyday Use a strip dividing a highway
Math Use a segment dividing the legs of a trapezoid in half

GEOMETRY LAB

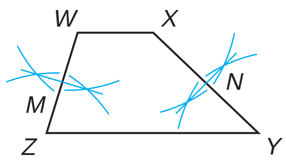
Median of a Trapezoid

MODEL

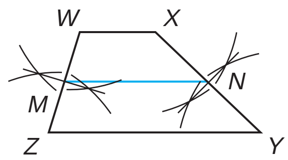
Step 1 Draw a trapezoid $WXYZ$ with legs $\overline{XY}$ and $\overline{WZ}$.



Step 2 Construct the perpendicular bisectors of $\overline{WZ}$ and $\overline{XY}$. Label the midpoints M and N.



Step 3 Draw $\overline{MN}$.



ANALYZE

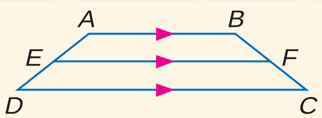
1. Measure $\overline{WX}$, $\overline{ZY}$, and $\overline{MN}$ to the nearest millimeter.
2. **Make a conjecture** based on your observations.
3. Draw an isosceles trapezoid $WXYZ$. Repeat Steps 1, 2, and 3. Is your conjecture valid? Explain.

The results of the Geometry Lab suggest Theorem 6.20.

THEOREM 6.20

The median of a trapezoid is parallel to the bases, and its measure is one-half the sum of the measures of the bases.

Example: $EF = \frac{1}{2}(AB + DC)$



You will prove Theorem 6.20 in Exercise 26 of Lesson 6-7.

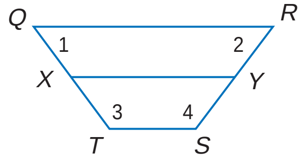
Study Tip

Isosceles Trapezoid

If you extend the legs of an isosceles trapezoid until they meet, you will have an isosceles triangle. Recall that the base angles of an isosceles triangle are congruent.

EXAMPLE Median of a Trapezoid

4 **ALGEBRA** **In the diagram, *QRST* represents an outdoor eating area in the shape of an isosceles trapezoid. The median $\overline{XY}$ represents the sidewalk through the area.**



a. Find *TS* if *QR* = 22 and *XY* = 15.

$XY = \frac{1}{2}(QR + TS)$	Theorem 6.20
$15 = \frac{1}{2}(22 + TS)$	Substitution
$30 = 22 + TS$	Multiply each side by 2.
$8 = TS$	Subtract 22 from each side.

b. Find $m\angle1$, $m\angle2$, $m\angle3$, and $m\angle4$ if $m\angle1 = 4a - 10$ and $m\angle3 = 3a + 32.5$.

Since $\overline{QR} \parallel \overline{TS}$, $\angle1$ and $\angle3$ are supplementary. Because this is an isosceles trapezoid, $\angle1 \cong \angle2$ and $\angle3 \cong \angle4$.

$m\angle1 + m\angle3 = 180$	Consecutive Interior Angles Theorem
$4a - 10 + 3a + 32.5 = 180$	Substitution
$7a + 22.5 = 180$	Combine like terms.
$7a = 157.5$	Subtract 22.5 from each side.
$a = 22.5$	Divide each side by 7.

If $a = 22.5$, then $m\angle1 = 80$ and $m\angle3 = 100$.
Because $\angle1 \cong \angle2$ and $\angle3 \cong \angle4$, $m\angle2 = 80$ and $m\angle4 = 100$.

Interactive Lab
geometryonline.com

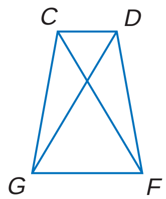
4A. **ALGEBRA** *JKLM* is an isosceles trapezoid with $\overline{JK} \parallel \overline{LM}$ and median $\overline{RP}$. Find *RP* if $JK = 2(x + 3)$, $RP = 5 + x$, and $ML = \frac{1}{2}x - 1$.

4B. Find the measure of each base angle of *JKLM* if $m\angle L = x$ and $m\angle J = 3x + 12$.

CHECK Your Understanding

Example 1 (p. 356)

1. **PROOF** *CDFG* is an isosceles trapezoid with bases $\overline{CD}$ and $\overline{FG}$. Write a flow proof to prove $\angle DGF \cong \angle CFG$.



Example 2 (p. 357)

2. **PHOTOGRAPHY** Photographs can show a building in a perspective that makes it appear to be a different shape. Identify the types of quadrilaterals in the photograph.

Example 3 (p. 357)

COORDINATE GEOMETRY **Quadrilateral *QRST* has vertices $Q(-3, 2)$, $R(-1, 6)$, $S(4, 6)$, and $T(6, 2)$.**

3. Verify that *QRST* is a trapezoid.

4. Determine whether *QRST* is an isosceles trapezoid. Explain.

Example 4 (p. 359)

5. **ALGEBRA** $EFGH$ is an isosceles trapezoid with bases $\overline{EF}$ and $\overline{GH}$ and median $\overline{YZ}$. If $EF = 3x + 8$, $GH = 4x - 10$, and $YZ = 13$, find x.
6. **ALGEBRA** Find the measure of each base angle of $EFGH$ if $m\angle E = 7x$ and $m\angle G = 16x - 4$.

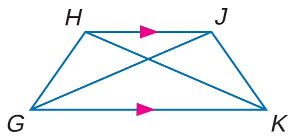
Exercises

HOMEWORK HELP

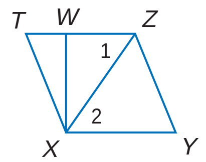
For Exercises	See Examples
7–10	1
11–12	2
13–16	3
17–20	4

PROOF **Write a flow proof.**

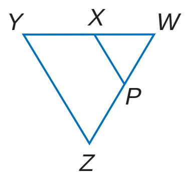
7. **Given:** $\overline{HJ} \parallel \overline{GK}$, $\triangle HGK \cong \triangle JKG$, $\overline{HG} \nparallel \overline{JK}$
   Prove: $GHJK$ is an isosceles trapezoid.



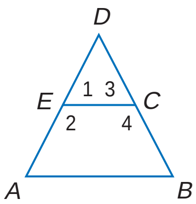
8. **Given:** $\triangle TZX \cong \triangle YXZ$, $\overline{WX} \nparallel \overline{ZY}$
   Prove: $XYZW$ is a trapezoid.



9. **Given:** $ZYXP$ is an isosceles trapezoid.
   Prove: $\triangle PWX$ is isosceles.



10. **Given:** E and C are midpoints of $\overline{AD}$ and $\overline{DB}$; $\overline{AD} \cong \overline{DB}$ and $\angle A \cong \angle 1$.
    Prove: $ABCE$ is an isosceles trapezoid.

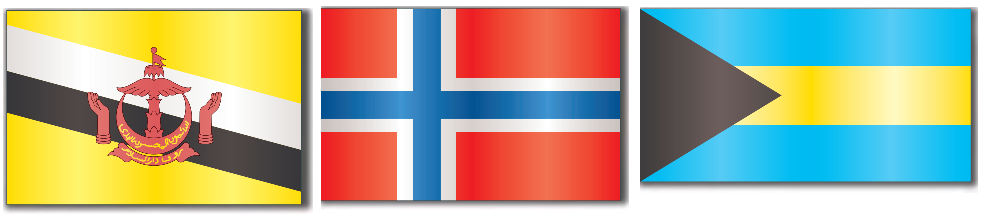


Real-World Link

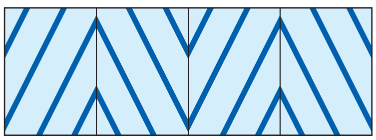
Ohio is the only state not to have a rectangular flag. The swallowtail design is properly called the Ohio burgee.

Source: 50states.com

11. **FLAGS** Study the flags shown below. Use a ruler and protractor to determine if any of the flags contain parallelograms, rectangles, rhombi, squares, or trapezoids.



12. **INTERIOR DESIGN** Peta is making a valance for a window treatment. She is using striped fabric cut on the bias, or diagonal, to create a chevron pattern. Identify the polygons formed in the fabric.

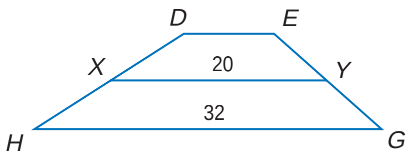


COORDINATE GEOMETRY **For each quadrilateral with the vertices given, a. verify that the quadrilateral is a trapezoid, and b. determine whether the figure is an isosceles trapezoid.**

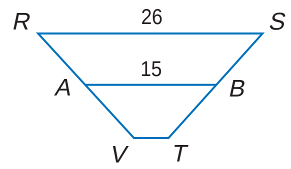
13. $A(-3, 3)$, $B(-4, -1)$, $C(5, -1)$, $D(2, 3)$
14. $G(-5, -4)$, $H(5, 4)$, $J(0, 5)$, $K(-5, 1)$
15. $C(-1, 1)$, $D(-5, -3)$, $E(-4, -10)$, $F(6, 0)$
16. $Q(-12, 1)$, $R(-9, 4)$, $S(-4, 3)$, $T(-11, -4)$

ALGEBRA **Find the missing value for the given trapezoid.**

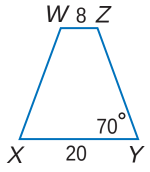
17. For trapezoid *DEGH*, *X* and *Y* are midpoints of the legs. Find *DE*.



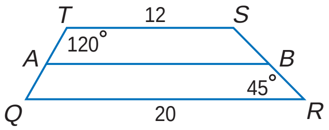
18. For trapezoid *RSTV*, *A* and *B* are midpoints of the legs. Find *VT*.



19. For isosceles trapezoid *XYZW*, find the length of the median, $m\angle W$, and $m\angle Z$.



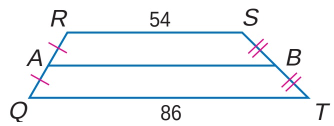
20. For trapezoid *QRST*, *A* and *B* are midpoints of the legs. Find *AB*, $m\angle Q$, and $m\angle S$.



For Exercises 21 and 22, use trapezoid *QRST*.

21. Let $\overline{GH}$ be the median of *RSBA*. Find *GH*.

22. Let $\overline{JK}$ be the median of *ABTQ*. Find *JK*.



CONSTRUCTION **Use a compass and ruler to construct each figure.**

23. an isosceles trapezoid

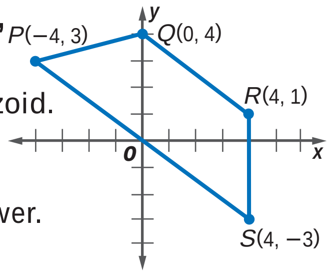
24. trapezoid with a median 2 centimeters long

COORDINATE GEOMETRY **Determine whether each figure is a *trapezoid*, a *parallelogram*, a *square*, a *rhombus*, or a *quadrilateral* given the coordinates of the vertices. Choose the most specific term. Explain.**

25. $B(1, 2)$, $C(4, 4)$, $D(5, 1)$, $E(2, -1)$

26. $G(-2, 2)$, $H(4, 2)$, $J(6, -1)$, $K(-4, -1)$

COORDINATE GEOMETRY **For Exercises 27–29, refer to quadrilateral *PQRS*.**



27. Determine whether the figure is a trapezoid. If so, is it isosceles? Explain.

28. Is the median contained in the line with equation $y = -\frac{3}{4}x + 1$? Justify your answer.

29. Find the length of the median.

EXTRA PRACTICE
See pages 812, 833.

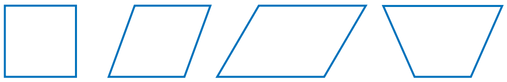
Math Online
Self-Check Quiz at
geometryonline.com

H.O.T. Problems

30. OPEN ENDED Draw an isosceles trapezoid and a trapezoid that is not isosceles. Draw the median for each. Is the median parallel to the bases in both trapezoids? Justify your answer.

31. CHALLENGE State the converse of Theorem 6.19. Then write a paragraph proof of this converse.

32. Which One Doesn't Belong? Identify the figure that does not belong with the other three. Explain.



33. **Writing in Math** Describe the characteristics of a trapezoid. List the minimum requirements to show that a quadrilateral is a trapezoid.

STANDARDIZED TEST PRACTICE

34. Which figure can serve as a counterexample to the conjecture below?

> If the diagonals of a quadrilateral are congruent, then the quadrilateral is a rectangle.

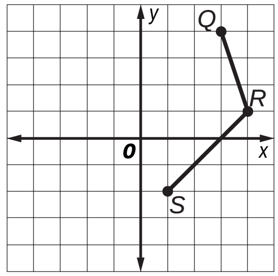
A square

B rhombus

C parallelogram

D isosceles trapezoid

35. **REVIEW** A portion of isosceles trapezoid *QRST* is shown.

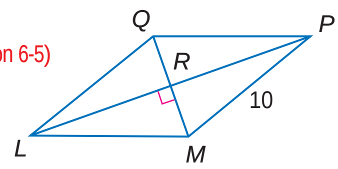


At what coordinates should vertex *T* be placed so that $\overline{TQ} \parallel \overline{SR}$ in order to complete *QRST*?

F $(0, 1)$ **H** $(-2, -1)$

G $(-1, 0)$ **J** $(-2, 0)$

ALGEBRA **In rhombus *LMPQ*, $m\angle QLM = 2x^2 - 10$, $m\angle QPM = 8x$, and $MP = 10$. Find the indicated measures.** (Lesson 6-5)



36. $m\angle LPQ$ **37.** QL

38. $m\angle LQP$ **39.** $m\angle LQM$

COORDINATE GEOMETRY **For Exercises 40–42, refer to quadrilateral *RSTV* with vertices $R(-7, -3)$, $S(0, 4)$, $T(3, 1)$, and $V(-4, -7)$.** (Lesson 6-4)

40. Find *RS* and *TV*.

41. Find the coordinates of the midpoints of $\overline{RT}$ and $\overline{SV}$.

42. Is *RSTV* a rectangle? Explain.

43. **RECREATION** The table below shows the number of visitors to areas in the United States National Park system in millions. What is the average rate of change of the number of visitors per year? (Lesson 3-3)

Year	1999	2002
Visitors (millions)	287.1	277.3

Source: *Statistical Abstract of the United States*

PREREQUISITE SKILL **Write an expression for the slope of the segment given the coordinates of the endpoints.** (Lesson 3-3)

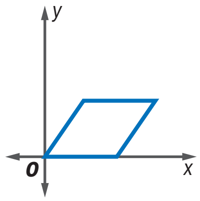
44. $(0, a), (-a, 2a)$ **45.** $(-a, b), (a, b)$ **46.** $(c, c), (c, d)$

6-7 Coordinate Proof with Quadrilaterals

Main Ideas

- Position and label quadrilaterals for use in coordinate proofs.
- Prove theorems using coordinate proofs.

In Chapter 4, you learned that variable coordinates can be assigned to the vertices of triangles. Then the Distance and Midpoint Formulas and coordinate proofs were used to prove theorems. The same can be done with quadrilaterals.



Study Tip

Look Back

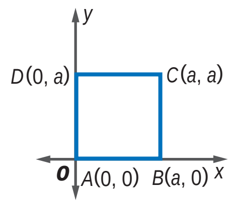
To review **placing a figure on a coordinate plane,** see Lesson 4-7.

Position Figures The first step to using a coordinate proof is to place the figure on the coordinate plane. The placement of the figure can simplify the steps of the proof.

EXAMPLE 1 **Positioning a Square**

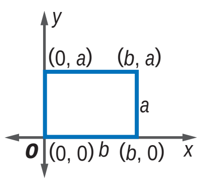
Position and label a square with sides *a* units long on the coordinate plane.

- Let *A*, *B*, *C*, and *D* be vertices of a square with sides *a* units long.
- Place the square with vertex *A* at the origin, $\overline{AB}$ along the positive *x*-axis, and $\overline{AD}$ along the *y*-axis. Label the vertices *A*, *B*, *C*, and *D*.
- The *y*-coordinate of *B* is 0 because the vertex is on the *x*-axis. Since the side length is *a*, the *x*-coordinate is *a*.
- *D* is on the *y*-axis so the *x*-coordinate is 0. The *y*-coordinate is 0 + *a* or *a*.
- The *x*-coordinate of *C* is also *a*. The *y*-coordinate is 0 + *a* or *a* because the side $\overline{BC}$ is *a* units long.

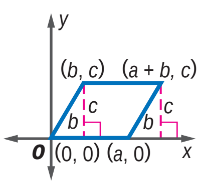


1. Position and label a rectangle with a length of 2*a* units and a width of *a* units.

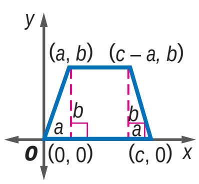
Some examples of quadrilaterals placed on the coordinate plane are given below. Notice how the figures have been placed so the coordinates of the vertices are as simple as possible.



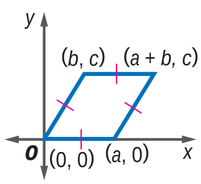
rectangle



parallelogram



isosceles trapezoid



rhombus

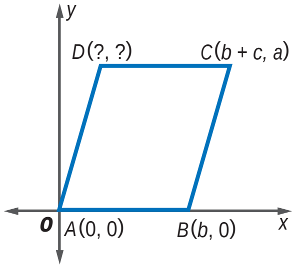
EXAMPLE 2 Find Missing Coordinates

Name the missing coordinates for the parallelogram.

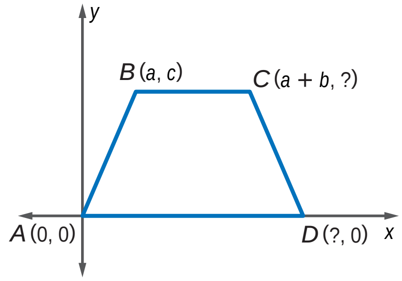
Opposite sides of a parallelogram are congruent and parallel. So, the y-coordinate of D is a.

The length of $\overline{AB}$ is b, and the length of $\overline{DC}$ is b. So, the x-coordinate of D is $(b + c) - b$ or c.

The coordinates of D are (c, a).



2. Name the missing coordinates for the isosceles trapezoid.



Prove Theorems Once a figure has been placed on the coordinate plane, we can prove theorems using the Slope, Midpoint, and Distance Formulas.

Study Tip

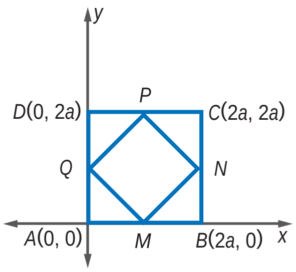
Problem Solving

To prove that a quadrilateral is a square, you can also show that all sides are congruent and that the diagonals bisect each other.

EXAMPLE 3 Coordinate Proof

Place a square on a coordinate plane. Label the midpoints of the sides, *M*, *N*, *P*, and *Q*. Write a coordinate proof to prove that *MNPQ* is a square.

The first step is to position a square on the coordinate plane. Label the vertices to make computations as simple as possible.



Given: $ABCD$ is a square.
M, N, P, and Q are midpoints.

Prove: $MNPQ$ is a square.

Coordinate Proof:

By the Midpoint Formula, the coordinates of M, N, P, and Q are as follows.

$$M\left(\frac{2a + 0}{2}, \frac{0 + 0}{2}\right) = (a, 0)$$

$$N\left(\frac{2a + 2a}{2}, \frac{2a + 0}{2}\right) = (2a, a)$$

$$P\left(\frac{0 + 2a}{2}, \frac{2a + 2a}{2}\right) = (a, 2a)$$

$$Q\left(\frac{0 + 0}{2}, \frac{0 + 2a}{2}\right) = (0, a)$$

Find the slopes of $\overline{QP}$, $\overline{MN}$, $\overline{QM}$, and $\overline{PN}$.

slope of $\overline{QP} = \frac{2a - a}{a - 0}$ or 1

slope of $\overline{MN} = \frac{a - 0}{2a - a}$ or 1

slope of $\overline{QM} = \frac{0 - a}{a - 0}$ or -1

slope of $\overline{PN} = \frac{a - 2a}{2a - a}$ or -1

Each pair of opposite sides have the same slope, so they are parallel. Consecutive sides form right angles because their slopes are negative reciprocals.

Use the Distance Formula to find the lengths of $\overline{QP}$ and $\overline{QM}$.

$$QP = \sqrt{(0 - a)^2 + (a - 2a)^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2} \text{ or } a\sqrt{2}$$

$$QM = \sqrt{(0 - a)^2 + (a - 0)^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2} \text{ or } a\sqrt{2}$$

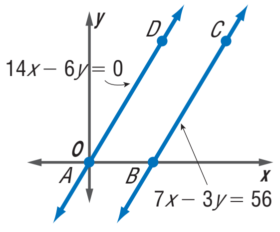
MNPQ is a square because each pair of opposite sides is parallel, and consecutive sides form right angles and are congruent.

3. Write a coordinate proof for the statement: *If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.*

Personal Tutor at geometryonline.com

Properties of Quadrilaterals

4 PARKING Write a coordinate proof to prove that the sides of the parking space are parallel.



Given: $14x - 6y = 0$; $7x - 3y = 56$

Prove: $\overline{AD} \parallel \overline{BC}$

Proof: Rewrite both equations in slope-intercept form.

$14x - 6y = 0$

$$\frac{-6y}{-6} = \frac{-14x}{-6}$$

$$y = \frac{7}{3}x$$

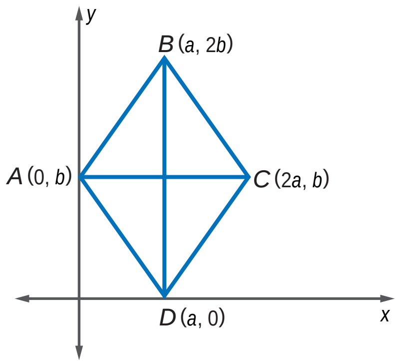
$7x - 3y = 56$

$$\frac{-3y}{-3} = \frac{-7x + 56}{-3}$$

$$y = \frac{7}{3}x - \frac{56}{3}$$

Since $\overline{AD}$ and $\overline{BC}$ have the same slope, they are parallel.

4. Write a coordinate proof to prove that the crossbars of a rhombus-shaped window are perpendicular.



CHECK Your Understanding

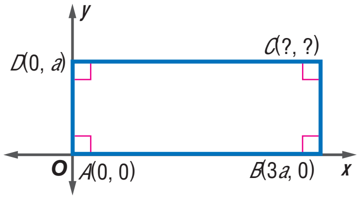
Example 1 (p. 363)

1. Position and label a rectangle with length a units and height $a + b$ units on the coordinate plane.

Name the missing coordinates for each quadrilateral.

Example 2 (p. 364)

2.



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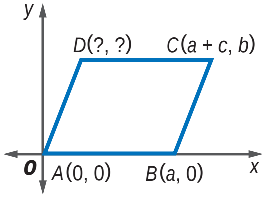


Write a coordinate proof for each statement.

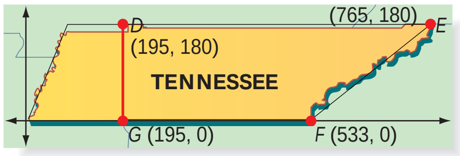
Example 3 (p. 364)

4. The diagonals of a parallelogram bisect each other.

5. The diagonals of a square are perpendicular.

Example 4 (p. 365)

6. STATES The state of Tennessee can be separated into two shapes that resemble quadrilaterals. Write a coordinate proof to prove that *DEFG* is a trapezoid. All measures are approximate and given in kilometers.



Exercises

HOMEWORK HELP	
For Exercises	See Examples
7, 8	1
9–14	2
15–20	3
21–23	4

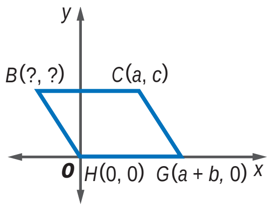
Position and label each quadrilateral on the coordinate plane.

7. isosceles trapezoid with height c units, bases a units and $a + 2b$ units

8. parallelogram with side length c units and height b units

Name the missing coordinates for each parallelogram or trapezoid.

9.



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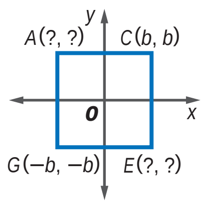

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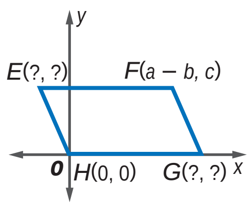

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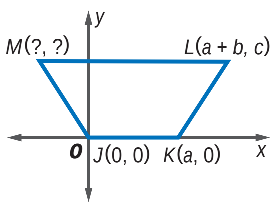

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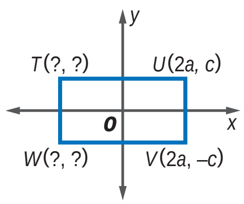

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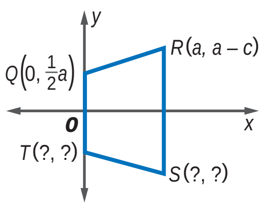


EXTRA PRACTICE
See pages 813, 833.

Math Online
Self-Check Quiz at
geometryonline.com

Real-World Link

The Leaning Tower of Pisa is sinking. In 1838, the foundation was excavated to reveal the bases of the columns.

Source: torre.duomo.pisa.it

Position and label each figure on the coordinate plane. Then write a coordinate proof for each of the following.

15. The diagonals of a rectangle are congruent.

16. If the diagonals of a parallelogram are congruent, then it is a rectangle.

17. The diagonals of an isosceles trapezoid are congruent.

18. The median of an isosceles trapezoid is parallel to the bases.

19. The segments joining the midpoints of the sides of a rectangle form a rhombus.

20. The segments joining the midpoints of the sides of a quadrilateral form a parallelogram.

ARCHITECTURE **For Exercises 21–23, use the following information.**
The Leaning Tower of Pisa is approximately 60 meters tall, from base to belfry. The tower leans about 5.5° so the top right corner is 4.5 meters to the right of the bottom right corner.

21. Position and label the tower on a coordinate plane.

22. Is it possible to write a coordinate proof to prove that the sides of the tower are parallel? Explain.

23. From the given information, what conclusion can be drawn?

H.O.T. Problems

24. **REASONING** Explain how to position a quadrilateral to simplify the steps of the proof.

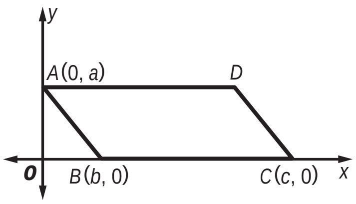
25. **OPEN ENDED** Position and label a trapezoid with two vertices on the *y*-axis.

26. **CHALLENGE** Position and label a trapezoid that is not isosceles on the coordinate plane. Then write a coordinate proof to prove Theorem 6.20 on page 358.

27. **Writing in Math** Describe how the coordinate plane can be used in proofs. Include guidelines for placing a figure on a coordinate grid in your answer.

STANDARDIZED TEST PRACTICE

28. In the figure, *ABCD* is a parallelogram. What are the coordinates of point *D*?



A $(a, c + b)$

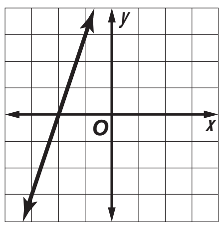
B $(c + b, a)$

C $(b - c, a)$

D $(c - b, a)$

29. **REVIEW** Which *best* represents the graph of $-3x + y = -2$?

F



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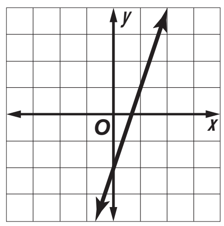
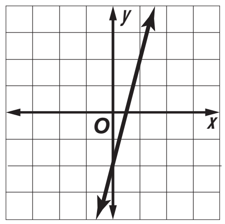


G



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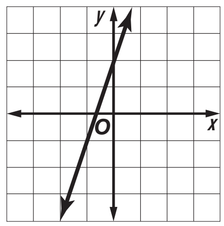


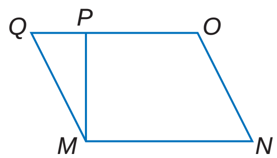
Paul Trummer/Getty Images

Spiral Review

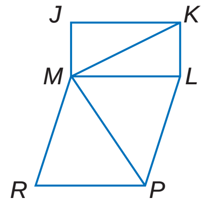
30. **PROOF** Write a two-column proof. (Lesson 6-6)

Given: $MNOP$ is a trapezoid with bases $\overline{MN}$ and $\overline{OP}$.
$\overline{MN} \cong \overline{QO}$

Prove: $MNOQ$ is a parallelogram.



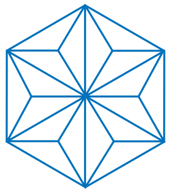
***JKLM* is a rectangle. *MLPR* is a rhombus. $\angle JMK \cong \angle RMP$, $m\angle JMK = 55$, and $m\angle MRP = 70$.** (Lesson 6-5)



31. Find $m\angle MPR$.
32. Find $m\angle KML$.
33. Find $m\angle KLP$.

34. **COORDINATE GEOMETRY** Given $\triangle STU$ with vertices $S(0, 5)$, $T(0, 0)$, and $U(-2, 0)$, and $\triangle XYZ$ with vertices $X(4, 8)$, $Y(4, 3)$, and $Z(6, 3)$, show that $\triangle STU \cong \triangle XYZ$. (Lesson 4-4)

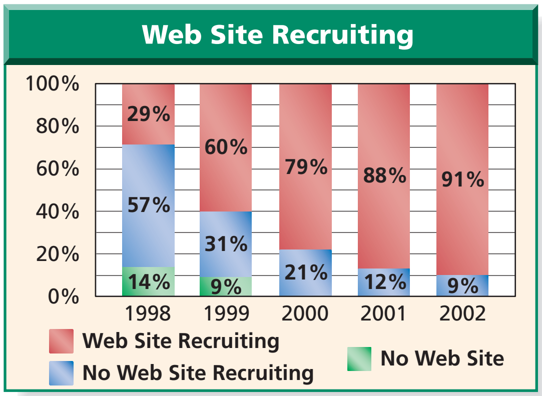
ARCHITECTURE For Exercises 35 and 36, use the following information.
The geodesic dome was developed by Buckminster Fuller in the 1940s as an energy-efficient building. The figure at the right shows the basic structure of one geodesic dome. (Lesson 4-1)



35. How many equilateral triangles are in the figure?
36. How many obtuse triangles are in the figure?

JOBS For Exercises 37–39, refer to the graph at the right. (Lesson 3-3)

37. What was the rate of change for companies that did not use Web sites to recruit employees from 1998 to 2002?
38. What was the rate of change for companies that did use Web sites to recruit employees from 1998 to 2002?
39. Predict the year in which 100% of companies will use Web sites for recruitment. Justify your answer.

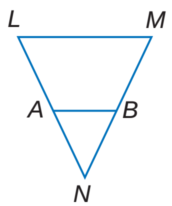


Source: iLogos Research

40. **PROOF Write a two-column proof.** (Lesson 2-7)

Given: $NL = NM$
$AL = BM$

Prove: $NA = NB$



Cross-Curricular Project

Geometry and History

Who is behind this geometry idea anyway? It is time to complete your project. Use the information and data you have gathered about your research topic, two mathematicians, and a geometry problem to prepare a portfolio or Web page. Be sure to include illustrations and/or tables in the presentation.

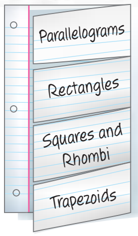
Math Online Cross-Curricular Project at geometryonline.com

Study Guide and Review

Download Vocabulary Review from geometryonline.com

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Angles of Polygons (Lesson 6-1)

- The sum of the measures of the interior angles of a polygon is given by the formula $S = 180(n - 2)$.
- The sum of the measures of the exterior angles of a convex polygon is 360.

Properties of Parallelograms (Lesson 6-2)

- Opposite sides are congruent and parallel.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- If a parallelogram has one right angle, it has four right angles.
- Diagonals bisect each other.

Tests for Parallelograms (Lesson 6-3)

- If a quadrilateral has the properties of a parallelogram, then it is a parallelogram.

Properties of Rectangles, Rhombi, Squares, and Trapezoids (Lessons 6-4 to 6-6)

- A rectangle has all the properties of a parallelogram. Diagonals are congruent and bisect each other. All four angles are right angles.
- A rhombus has all the properties of a parallelogram. All sides are congruent. Diagonals are perpendicular. Each diagonal bisects a pair of opposite angles.
- A square has all the properties of a parallelogram, a rectangle, and a rhombus.
- In an isosceles trapezoid, both pairs of base angles are congruent and the diagonals are congruent.

Key Vocabulary

diagonal (p. 318)
isosceles trapezoid (p. 356)
kite (p. 355)
median (p. 358)
parallelogram (p. 325)
rectangle (p. 340)
rhombus (p. 348)
square (p. 349)
trapezoid (p. 356)

Vocabulary Check

State whether each sentence is *true* or *false.* If *false,* replace the underlined word or number to make a true sentence.

1. The diagonals of a <u>rhombus</u> are perpendicular.
2. A <u>trapezoid</u> has all the properties of a parallelogram, a rectangle, and a rhombus.
3. If a parallelogram is a <u>rhombus</u>, then the diagonals are congruent.
4. Every <u>parallelogram</u> is a quadrilateral.
5. A(n) <u>rhombus</u> is a quadrilateral with exactly one pair of parallel sides.
6. Each diagonal of a <u>rectangle</u> bisects a pair of opposite angles.
7. If a quadrilateral is both a rhombus and a rectangle, then it is a <u>square</u>.
8. Both pairs of base angles in a(n) <u>isosceles trapezoid</u> are congruent.
9. All <u>squares</u> are rectangles.
10. If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a <u>rhombus</u>.

Lesson-by-Lesson Review

6–1 Angles of Polygons (pp. 318–321)

11. ARCHITECTURE The schoolhouse below was built in 1924 in Essex County, New York. If its floor is in the shape of a regular polygon and the measure of an interior angle is 135, find the number of sides the schoolhouse has.

Example 1 Find the sum of the measures of the interior angles and the measure of an interior angle of a regular decagon.

$S = 180(n - 2)$ Interior Angle Sum Theorem

$= 180(10 - 2)$ $n = 10$

$= 180(8)$ or 1440 Simplify.

The sum of the measures of the interior angles is 1440. The measure of each interior angle is $1440 \div 10$ or 144.

6–2 Parallelograms (pp. 323–329)

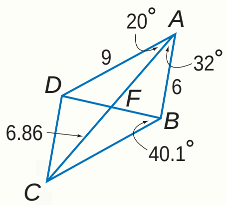
Use ▱*ABCD* to find each measure.

12. $m\angle BCD$

13. AF

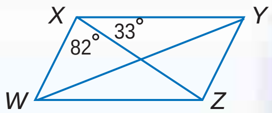
14. $m\angle BDC$

15. BC



16. ART One way to draw a cube is to draw three parallelograms. State which properties of a parallelogram an artist might use to draw a cube.

Example 2 *WXYZ* is a parallelogram. Find $m\angle YZW$ and $m\angle XWZ$.



$m\angle YZW = m\angle WXY$

$= 82 + 33$ or 115

$m\angle XWZ + m\angle WXY = 180$

$m\angle XWZ + (82 + 33) = 180$

$m\angle XWZ + 115 = 180$

$m\angle XWZ = 65$

6–3 Tests for Parallelograms (pp. 331–337)

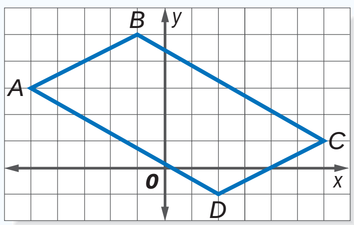
Determine whether the figure with the given vertices is a parallelogram. Use the method indicated.

17. $A(-2, 5)$, $B(4, 4)$, $C(6, -3)$, and $D(-1, -2)$; Distance Formula

18. $H(0, 4)$, $J(-4, 6)$, $K(5, 6)$, and $L(9, 4)$; Midpoint Formula

19. $S(-2, -1)$, $T(2, 5)$, $V(-10, 13)$, and $W(-14, 7)$; Slope Formula

Example 3 Determine whether the figure below is a parallelogram. Use the Distance and Slope Formulas.



Adirondack Architectural Heritage

Mixed Problem Solving
For mixed problem-solving practice, see page 833.

20. GEOGRAPHY Describe how you could tell whether a map of the state of Colorado is a parallelogram.

$AB = \sqrt{[-5-(-1)]^2 + (3-5)^2}$

$= \sqrt{(-4)^2 + (-2)^2} = \sqrt{20}$ or $2\sqrt{5}$

$CD = \sqrt{(6-2)^2 + [1-(-1)]^2}$

$= \sqrt{4^2 + 2^2} = \sqrt{20}$ or $2\sqrt{5}$

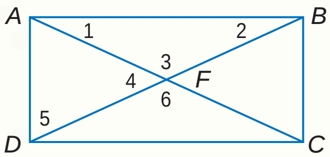
slope of $\overline{AB} = \frac{5-3}{-1-(-5)}$ or $\frac{1}{2}$

slope of $\overline{CD} = \frac{-1-1}{2-6}$ or $\frac{1}{2}$

Since one pair of opposite sides is congruent and parallel, $ABCD$ is a parallelogram.

6–4 Rectangles (pp. 338–344)

21. If $m\angle 1 = 12x + 4$ and $m\angle 2 = 16x - 12$ in rectangle $ABCD$, find $m\angle 2$.



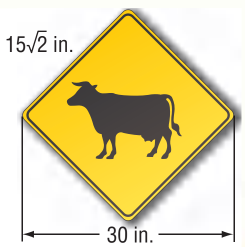
22. QUILTS Mrs. Diller is making a quilt. She has cut several possible rectangles out of fabric. If Mrs. Diller does not own a protractor, how can she be sure that the pieces she has cut are rectangles?

Example 4 **Refer to rectangle *ABCD*. If $CF = 4x + 1$ and $DF = x + 13$, find x.**

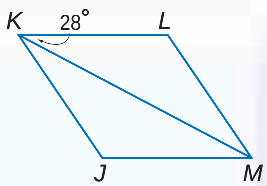
$\overline{CF} \cong \overline{DF}$	Diag. bisect each other.
$CF = DF$	Def. of $\cong$ segments
$4x + 1 = x + 13$	Substitution
$3x + 1 = 13$	Subtract x from each side.
$3x = 12$	Subtract 1 from each side.
$x = 4$	Divide each side by 3.

6–5 Rhombi and Squares (pp. 346–352)

23. SIGNS This sign is a parallelogram. Determine if it is also a square. Explain.



Example 5 **Find $m\angle JMK$.**



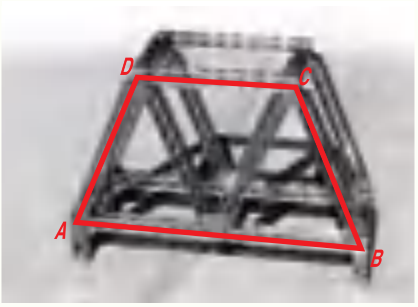
Opposite sides of a rhombus are parallel, so $\overline{KL} \parallel \overline{JM}$.

$\angle JMK \cong \angle LKM$ by the Alternate Interior Angle Theorem. By substitution, $m\angle JMK = 28$.

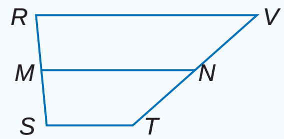
6–6 Trapezoids (pp. 354–361)

24. Trapezoid $JKLM$ has median XY. Find a if $JK = 28$, $XY = 4a - 4.5$, and $ML = 3a - 2$.

25. ART Artist Chris Burden created the sculpture *Trapezoid Bridge* shown below. State how you could determine whether the bridge is an isosceles trapezoid.



Example 6 **Trapezoid $RSTV$ has median $\overline{MN}$. Find x if $MN = 60$, $ST = 4x - 1$, and $RV = 6x + 11$.**



$MN = \frac{1}{2}(ST + RV)$ — Median of a trapezoid

$60 = \frac{1}{2}[(4x - 1) + (6x + 11)]$ — Substitution

$120 = 4x - 1 + 6x + 11$ — Multiply.

$120 = 10x + 10$ — Simplify.

$110 = 10x$ — Subtract 10 from each side.

$11 = x$ — Divide each side by 10.

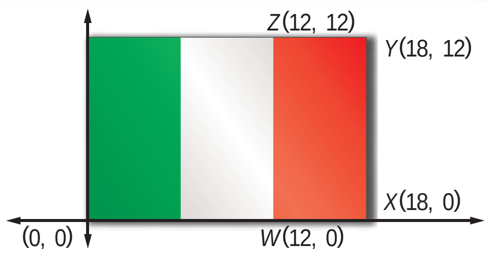
6–7 Coordinate Proof with Quadrilaterals (pp. 363–368)

Position and label each figure on the coordinate plane. Then write a coordinate proof for each of the following.

26. The diagonals of a square are perpendicular.

27. A diagonal separates a parallelogram into two congruent triangles.

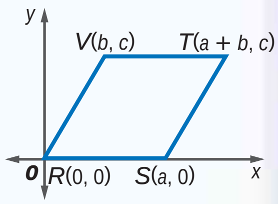
28. FLAGS An Italian flag is 12 inches by 18 inches and is made up of three quadrilaterals. Write a coordinate proof to prove that $WXYZ$ is a rectangle.



Example 7 **Write a coordinate proof to prove that each pair of opposite sides of rhombus $RSTV$ is parallel.**

Given: $RSTV$ is a rhombus.

Prove: $\overline{RV} \parallel \overline{ST}$, $\overline{RS} \parallel \overline{VT}$



Coordinate Proof:

slope of $\overline{RV} = \frac{c - 0}{b - 0}$ or $\frac{c}{b}$

slope of $\overline{RS} = \frac{0 - 0}{a - 0}$ or 0

slope of $\overline{ST} = \frac{c - 0}{(a + b) - a}$ or $\frac{c}{b}$

slope of $\overline{VT} = \frac{c - c}{(a + b) - b}$ or 0

$\overline{RV}$ and $\overline{ST}$ have the same slope, so $\overline{RV} \parallel \overline{ST}$. $\overline{RS}$ and $\overline{VT}$ have the same slope, and $\overline{RS} \parallel \overline{VT}$.

Chapter 6 Practice Test

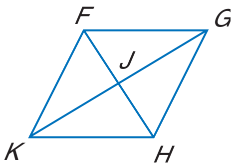
1. What is the measure of one exterior angle of a regular decagon?

2. Find the sum of the measures of the interior angles of a nine-sided polygon.

3. Each interior angle of a regular polygon measures 162°. How many sides does the polygon have?

Complete each statement about quadrilateral *FGHK*. Justify your answer.

4. $\overline{HK} \cong$ __?__
5. $\angle FKH \cong$ __?__
6. $\angle FKJ \cong$ __?__
7. $\overline{GH} \parallel$ __?__

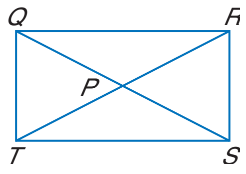


Determine whether the figure with the given vertices is a parallelogram. Justify your answer.

8. $A(4, 3)$, $B(6, 0)$, $C(4, -8)$, $D(2, -5)$
9. $S(-2, 6)$, $T(2, 11)$, $V(3, 8)$, $W(-1, 3)$
10. $F(7, -3)$, $G(4, -2)$, $H(6, 4)$, $J(12, 2)$
11. $W(-4, 2)$, $X(-3, 6)$, $Y(2, 7)$, $Z(1, 3)$

ALGEBRA ***QRST* is a rectangle.**

12. If $QP = 3x + 11$ and $PS = 4x + 8$, find QS.
13. If $m\angle QTR = 2x^2 + 7$ and $m\angle SRT = x^2 + 18$, find $m\angle QTR$.

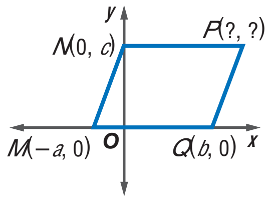


COORDINATE GEOMETRY **Determine whether parallelogram *ABCD* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain your reasoning.**

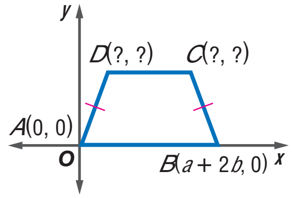
14. $A(12, 0)$, $B(6, -6)$, $C(0, 0)$, $D(6, 6)$
15. $A(-2, 4)$, $B(5, 6)$, $C(12, 4)$, $D(5, 2)$

Name the missing coordinates for each parallelogram or trapezoid.

16.

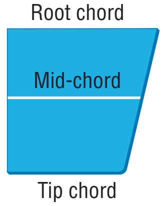


17.



18. Position and label an isosceles trapezoid on the coordinate plane. Write a coordinate proof to prove that the median is parallel to each base.

19. **SAILING** Many large sailboats have a *keel* to keep the boat stable in high winds. A keel is shaped like a trapezoid with its top and bottom parallel. If the root chord is 9.8 feet and the tip chord is 7.4 feet, find the length of the mid-chord.



20. **MULTIPLE CHOICE** If the measure of an interior angle of a regular polygon is 108, what type of polygon is it?

A octagon **C** pentagon

B hexagon **D** triangle

6 Standardized Test Practice

Cumulative, Chapters 1–6

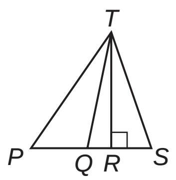
Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which figure can serve as a counterexample to the conjecture below?

> If all the angles of a quadrilateral are right angles, then the quadrilateral is a square.

A parallelogram
B rectangle
C rhombus
D trapezoid

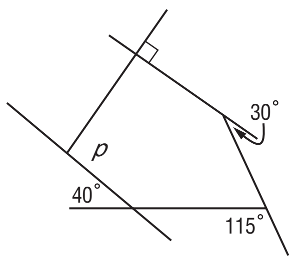
2. In the figure below, $\overline{TR}$ is an altitude of $\triangle PST$.



If we assume that $\overline{TQ}$ is the shortest segment from T to $\overline{PS}$, then it follows that $\overline{TQ}$ is an altitude of $\triangle PST$. Since $\triangle PST$ can have only one altitude from vertex T, this contradicts the given statement. What conclusion can be drawn from this contradiction?

F $TQ > TP$ **H** $TQ < TP$
G $TQ > TR$ **J** $TQ < TR$

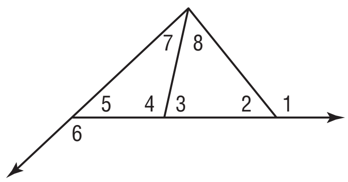
3. GRIDDABLE What is $m\angle p$ in degrees?



4. ALGEBRA If x is subtracted from x^2, the sum is 72. Which of the following could be the value of x?

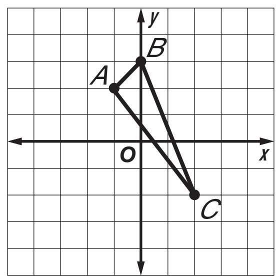
A -9
B -8
C 18
D 72

5. Which lists contains all of the angles with measures that *must* be less than $m\angle 6$?



F $\angle 1, \angle 2, \angle 4, \angle 7, \angle 8$
G $\angle 2, \angle 3, \angle 4, \angle 5$
H $\angle 2, \angle 4, \angle 6, \angle 7, \angle 8$
J $\angle 2, \angle 4, \angle 7, \angle 8$

6. GRIDDABLE Triangle ABC is congruent to $\triangle HIJ$. What is the measure of side $\overline{HJ}$?



TEST-TAKING TIP

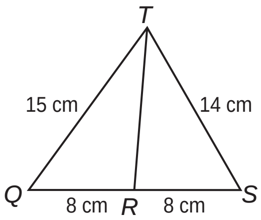
Question 6 Review any terms and formulas that you have learned before you take the test. Remember that the Distance Formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Standardized Test Practice at geometryonline.com

Preparing for Standardized Tests

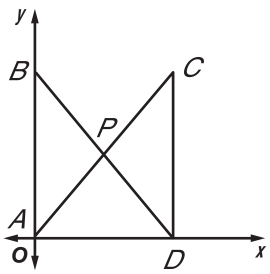
For test-taking strategies and more practice, see pages 841–856.

7. Which postulate or theorem can be used to prove the measure of $\angle QRT$ is greater than the measure of $\angle SRT$?



A AAS Inequality
B ASA Inequality
C SAS Inequality
D SSS Inequality

8. Which statement(s) would prove that $\triangle ABP \cong \triangle CDP$?



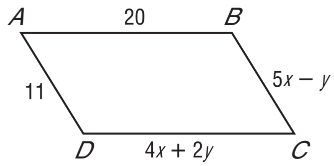
F slope $\overline{AB}$ = slope $\overline{CD}$, and the distance from A to C = distance from B to D

G (slope $\overline{AB}$)(slope $\overline{CD}$) = -1, and the distance from A to C = distance from B to D

H slope $\overline{AB}$ = slope $\overline{CD}$, and the distance from B to P = distance from D to P

J (slope $\overline{AB}$)(slope $\overline{CD}$) = 1, and the distance from A to B = distance from D to C

9. What values of x and y make quadrilateral $ABCD$ a parallelogram?



A $x = 4, y = 3$
B $x = \frac{31}{9}, y = \frac{11}{9}$
C $x = 3, y = 4$
D $x = \frac{11}{9}, y = \frac{31}{9}$

10. Which is the *converse* of the statement "If I am in La Quinta, then I am in Riverside County"?

F If I am not in Riverside County, then I am not in La Quinta.

G If I am not in La Quinta, then I am not in Riverside County.

H If I am in Riverside County, then I am in La Quinta.

J If I am in Riverside County, then I am not in La Quinta.

Pre-AP

Record your answer on a sheet of paper. Show your work.

11. Quadrilateral $ABCD$ has vertices with coordinates $A(0, 0)$, $B(a, 0)$, $C(a + b, c)$, and $D(b, c)$.

a. Position and label $ABCD$ in the coordinate plane.

b. Prove that $ABCD$ is a parallelogram.

c. If $a^2 = b^2 + c^2$, determine classify parallelogram $ABCD$. Justify your answer using coordinate geometry.

NEED EXTRA HELP?											
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11
Go to Lesson or Page...	6-6	5-4	6-1	796	5-2	4-3	5-5	4-7	6-2	2-3	6-7

UNIT 3
Similarity

Focus

Explore proportional relationships between similar triangles, the relationships among the angles and sides of right triangles, and transformations in the coordinate plane.

CHAPTER 7
Proportions and Similarity

BIG Idea Prove basic theorems involving similarity and that triangles are similiar.

BIG Idea Determine how changes in dimensions affect the perimeter and area of common geometric figures.

CHAPTER 8
Right Triangles and Trigonometry

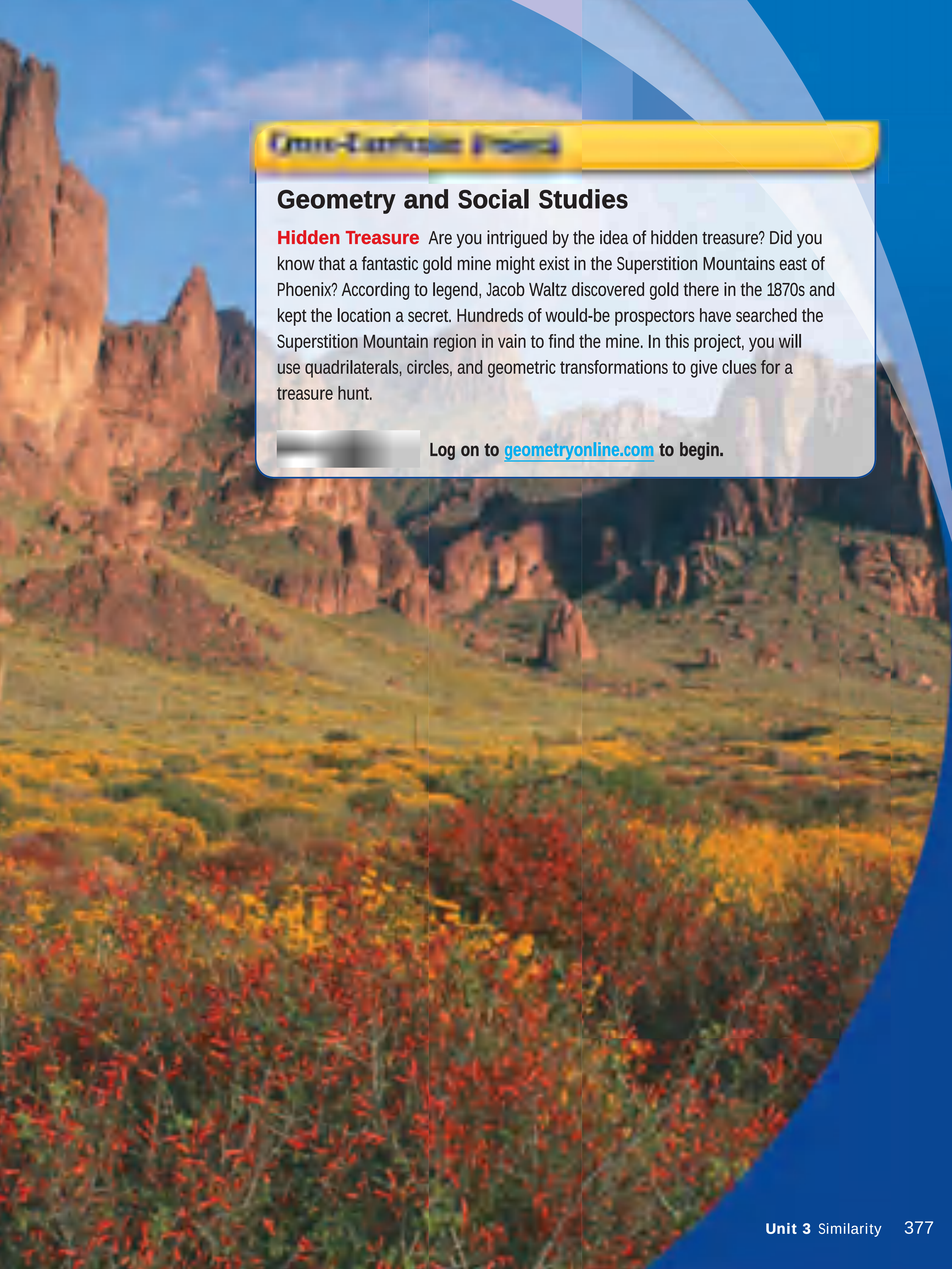
BIG Idea Prove the Pythagorean Theorem, use it to determine distance, and find missing right triangle lengths.

BIG Idea Know and use the definitions of the basic trigonometric functions defined by the angles of a right triangle.

BIG Idea Know and use angle and side relationships in problems with special right triangles.

CHAPTER 9
Transformations

BIG Idea Know the effect of rigid motions on figures in the coordinate plane, including rotations, translations, and reflections.



Geometry and Social Studies

Hidden Treasure Are you intrigued by the idea of hidden treasure? Did you know that a fantastic gold mine might exist in the Superstition Mountains east of Phoenix? According to legend, Jacob Waltz discovered gold there in the 1870s and kept the location a secret. Hundreds of would-be prospectors have searched the Superstition Mountain region in vain to find the mine. In this project, you will use quadrilaterals, circles, and geometric transformations to give clues for a treasure hunt.

Log on to geometryonline.com to begin.

CHAPTER 7

Proportions and Similarity

- Identify similar polygons and use ratios and proportions to solve problems.
- Recognize and use proportional parts, corresponding perimeters, altitudes, angle bisectors, and medians of similar triangles to solve problems.

Key Vocabulary

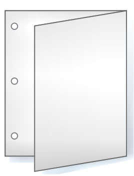
proportion (p. 381)
cross products (p. 381)
similar polygons (p. 388)
scale factor (p. 389)
midsegment (p. 406)

Real-World Link

Proportion The seven-story tall Longaberger Home Office in Newark, Ohio, is a replica of a Longaberger Medium Market Basket, reproduced on a 1:160 scale.

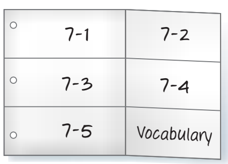
Proportions and Similarity Make this Foldable to help you organize your notes. Begin with one sheet of 11" × 17" paper.

1 **Fold** widthwise. Leave space to punch holes so it can be placed in your binder.

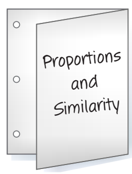


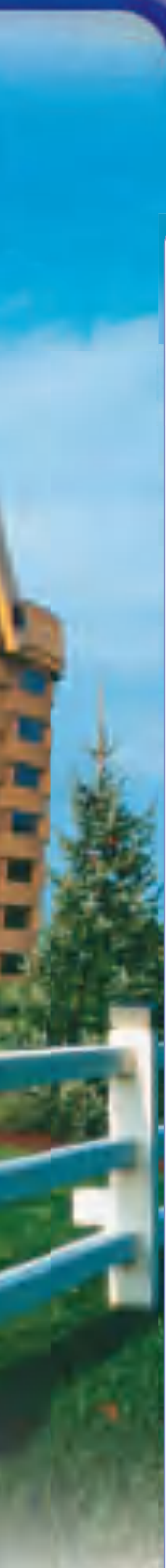
2 **Open** the flap and draw lines to divide the inside into six equal parts.

3 **Label** each part using the lesson numbers.



4 **Write** the name of the chapter on the front.





GET READY for Chapter 7

Diagnose Readiness **You have two options for checking Prerequisite Skills.**

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Solve each equation. (Prerequisite Skill)

1. $\frac{2}{3}y - 4 = 6$

2. $\frac{5}{6} = \frac{x - 4}{12}$

3. $\frac{4}{3} = \frac{y + 2}{y - 1}$

4. $\frac{2y}{4} = \frac{32}{y}$

5. BICYCLING Randy rode his bicycle 15 miles in 2 hours. At this rate, how far can he ride in 5 hours? (Prerequisite Skill)

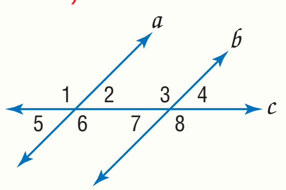
Find the slope of the line given the coordinates of two points on the line. (Lesson 3-3)

6. $(-6, -3)$ and $(2, -3)$

7. $(-3, 4)$ and $(2, -2)$

8. SPACE The budget for space research was \$7215 million in 2003 and \$7550 million in 2004. What is the rate of change? (Lesson 3-3)

Given the following information, determine whether *a* || *b*. State the postulate or theorem that justifies your answer. (Lesson 3-5)



9. $\angle 1 \cong \angle 8$

10. $\angle 3 \cong \angle 6$

11. $\angle 5 \cong \angle 3$

12. $\angle 2 \cong \angle 4$

QUICKReview

EXAMPLE 1

Solve the equation $\frac{5n + 2}{n - 1} = 2$.

$$\frac{5n + 2}{n - 1} = 2$$

$$(n - 1)\left(\frac{5n + 2}{n - 1}\right) = 2(n - 1) \quad \text{Multiply.}$$

$$5n + 2 = 2n - 2 \quad \text{Simplify.}$$

$$3n = -4 \quad \text{Combine like terms.}$$

$$n = -\frac{4}{3} \quad \text{Divide each side by 3.}$$

EXAMPLE 2

Find the slope of the line that contains the points $(-41, 17)$ and $(31, 29)$.

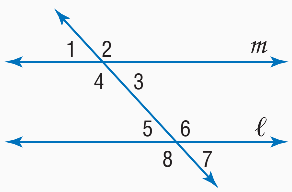
$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$

$$= \frac{29 - 17}{31 - (-41)} \quad \text{Substitution}$$

$$= \frac{12}{72} \text{ or } \frac{1}{6} \quad \text{Simplify.}$$

EXAMPLE 3

Determine whether $m \parallel \ell$ if $\angle 4 \cong \angle 6$. Justify your answer.



These angles are alternate interior angles. If alternate interior angles are congruent, then the lines are parallel.

7-1 Proportions

Main Ideas

- Write ratios.
- Use properties of proportions.

New Vocabulary

ratio
proportion
cross products
extremes
means

Stained-glass artist Louis Comfort Tiffany used geometric shapes in his designs. In a portion of *Clematis Skylight* shown at the right, rectangular shapes are used as the background for the flowers and vines. Tiffany also used ratio and proportion in the design of this piece.

Write Ratios A **ratio** is a comparison of two quantities using division. The ratio of a to b can be expressed as $\frac{a}{b}$, where b is not zero. This ratio can also be written as a:b.

EXAMPLE Write a Ratio

1 SOCCER **The U.S. Census Bureau surveyed 9490 schools nationally about their girls' soccer programs. They found that 309,032 girls participated in high school soccer programs in the 2003–2004 school year. Find the ratio of girl soccer players per school rounded to the nearest person.**

Divide the number of girl soccer players by the number of schools.

$$\frac{\text{number of girl soccer players}}{\text{number of schools}} = \frac{309{,}032}{9490} \text{ or about } \frac{33}{1}$$

A ratio in which the denominator is 1 is called a *unit ratio.*

The ratio was 33 girl soccer players per school.

1. The ratio of football players to high schools in Montgomery County is 546:26. What is the ratio of football players to high schools written as a unit ratio?

Real-World Link

Mia Hamm is considered to be the greatest female soccer player. She has scored over 149 goals in international soccer.

Source: kidzworld.com

Extended ratios can be used to compare three or more numbers. The expression a:b:c means that the ratio of the first two numbers is a:b, the ratio of the last two numbers is b:c, and the ratio of the first and last numbers is a:c.

Study Tip

Writing Equations

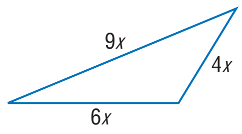
Extended ratio problems require a variable to be the common factor among the terms of the ratio. This will enable you to write an equation to solve the problem.

EXAMPLE Extended Ratios in Triangles

2 **In a triangle, the ratio of the measures of three sides is 4:6:9, and its perimeter is 190 inches. Find the length of the longest side of the triangle.**

Explore You are asked to apply the ratio to the three sides of the triangle and the perimeter to find the longest side.

Plan Recall that equivalent fractions can be found by multiplying the numerator and the denominator by the same number. So, $2{:}3 = \frac{2}{3} \cdot \frac{x}{x}$ or $\frac{2x}{3x}$. We can rewrite 4:6:9 as $4x{:}6x{:}9x$ and use those measures for the triangle's sides.



Solve

$4x + 6x + 9x = 190$	Perimeter
$19x = 190$	Combine like terms.
$x = 10$	Divide each side by 19.

Now find the measures of the sides: $4x = 4(10)$ or 40, $6x = 6(10)$ or 60, and $9x = 9(10)$ or 90. The longest side is 90 inches.

Check To check the reasonableness of this result, add the lengths of the sides to make sure that the perimeter is 190. $40 + 60 + 90 = 190$ ✓

2. In a triangle, the ratio of the measures of three sides is 3:3:8, and its perimeter is 392 inches. Find the length of the longest side of the triangle.

Personal Tutor at geometryonline.com

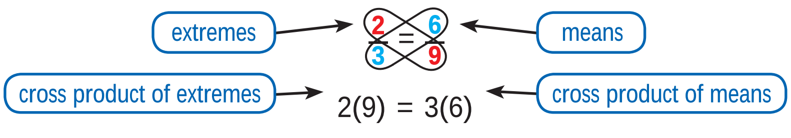
Reading Math

Proportions When a proportion is written using colons, it is read using the word *to* for the colon. For example, 2:3 is read *2 to 3*. The means are the inside numbers, and the extremes are the outside numbers.

$$\overbrace{2{:}\underbrace{3 = 6}_{\text{means}}{:}9}^{\text{extremes}}$$

Use Properties of Proportions An equation stating that two ratios are equal is called a **proportion**. Equivalent fractions set equal to each other form a proportion. Since $\frac{2}{3}$ and $\frac{6}{9}$ are equivalent fractions, $\frac{2}{3} = \frac{6}{9}$ is a proportion.

Every proportion has two **cross products**. The cross products in $\frac{2}{3} = \frac{6}{9}$ are 2 times 9 and 3 times 6. The **extremes** of the proportion are 2 and 9. The **means** are 3 and 6.



$$18 = 18$$

In a proportion, the product of the means equals the product of the extremes.

$\frac{a}{b} = \frac{c}{d}$	$b \neq 0, d \neq 0$
$(bd)\frac{a}{b} = (bd)\frac{c}{d}$	Multiply each side by the common denominator, *bd*.
$da = bc$	Simplify.
$ad = bc$	Commutative Property

KEY CONCEPT — *Property of Proportions*

Words For any numbers a and c and any nonzero numbers b and d, $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.

Example $\frac{4}{5} = \frac{12}{15}$ if and only if $4 \cdot 15 = 5 \cdot 12$.

To *solve a proportion* means to find the value of the variable that makes the proportion true.

EXAMPLE 3 Solve Proportions by Using Cross Products

Solve each proportion.

a. $\frac{3}{5} = \frac{x}{75}$

$$\frac{3}{5} = \frac{x}{75} \quad \text{Original proportion}$$
$$3(75) = 5x \quad \text{Cross products}$$
$$225 = 5x \quad \text{Multiply.}$$
$$45 = x \quad \text{Divide each side by 5.}$$

b. $\frac{3x - 5}{4} = \frac{-13}{2}$

$$\frac{3x - 5}{4} = \frac{-13}{2} \quad \text{Original proportion}$$
$$(3x - 5)2 = 4(-13) \quad \text{Cross products}$$
$$6x - 10 = -52 \quad \text{Simplify.}$$
$$6x = -42 \quad \text{Add 10 to each side.}$$
$$x = -7 \quad \text{Divide each side by 6.}$$

3A. $\frac{x}{4} = \frac{11}{-6}$

3B. $\frac{-4}{7} = \frac{6}{2x + 5}$

Proportions can be used to solve problems involving two objects that are said to be *in proportion.* This means that for ratios comparing the measures of all parts of one object with the measures of comparable parts of the other object, a true proportion always exists.

EXAMPLE 4 Solve Problems Using Proportions

AVIATION **A twinjet airplane has a length of 78 meters and a wingspan of 90 meters. A toy model is made in proportion to the real airplane. If the wingspan of the toy is 36 centimeters, find the length of the toy.**

$$\frac{\text{plane's length (m)}}{\text{model's length (cm)}} = \frac{\text{plane's wingspan (m)}}{\text{model's wingspan (cm)}}$$
$$\frac{78}{x} = \frac{90}{36} \quad \text{Substitution}$$
$$(78)(36) = x \cdot 90 \quad \text{Cross products}$$
$$2808 = 90x \quad \text{Multiply.}$$
$$31.2 = x \quad \text{Divide each side by 90.}$$

The length of the model is 31.2 centimeters.

4. The scale on a map shows that 1.5 centimeters represents 100 miles. If the distance on the map from Atlanta to Los Angeles is 29.2 centimeters, approximately how many miles apart are the two cities?

Study Tip

Common Misconception

The proportion shown in Example 4 is not the only correct proportion. There are four equivalent proportions:

$\frac{a}{b} = \frac{c}{d}$, $\frac{a}{d} = \frac{c}{b}$,

$\frac{b}{a} = \frac{d}{c}$, and $\frac{b}{c} = \frac{d}{a}$.

All of these have identical cross products.

CHECK Your Understanding

Example 1 (p. 380)

1. **CURRENCY** In a recent month, 107 South African rands was equivalent to 18 United States dollars. Find the ratio of rands to dollars.

Example 2 (p. 381)

2. The ratio of the measures of three sides of a triangle is 9:8:7, and its perimeter is 144 units. Find the measure of each side of the triangle.

3. The ratios of the measures of three angles of a triangle are 5:7:8. Find the measure of each angle of the triangle.

Example 3 (p. 382)

Solve each proportion.

4. $\frac{3}{x} = \frac{21}{6}$

5. $\frac{2.3}{4} = \frac{x}{3.7}$

6. $\frac{x - 2}{2} = \frac{4}{5}$

Example 4 (p. 382)

7. **MAPS** The scale on a map indicates that 1.5 centimeters represents 200 miles. If the distance on the map between Norfolk, Virginia, and Chapel Hill, North Carolina, measures 1.2 centimeters, approximately how many miles apart are the cities?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–11	1
12–17	2
18–25	3
26–29	4

8. **PETS** Out of a survey of 1000 households, 460 had at least one dog or cat as a pet. What is the ratio of pet owners to households?

9. **BASKETBALL** During tryouts for the basketball team, 30 students tried out for 15 spots on the team. What is the ratio of open spots to the number of students competing?

10. **EDUCATION** In the 2003–2004 school year, Arizona State University had 58,156 students and 2165 full-time faculty members. What was the ratio of the students per faculty member rounded to the nearest person?

11. **SCULPTURE** A replica of *The Thinker* is 10 inches tall. A statue of *The Thinker,* located in front of Grawemeyer Hall on the Belnap Campus of the University of Louisville, is 10 feet tall. What is the ratio of the replica to the statue in Louisville?

Find the measures of the sides of each triangle.

12. The ratio of the measures of three sides of a triangle is 8:7:5. Its perimeter is 240 feet.
13. The ratio of the measures of the sides of a triangle is 3:4:5. Its perimeter is 72 inches.
14. The ratio of the measures of three sides of a triangle are $\frac{1}{4}:\frac{1}{3}:\frac{1}{6}$, and its perimeter is 31.5 inches. Find the measure of each side of the triangle.
15. The ratio of the measures of three sides of a triangle are $\frac{1}{2}:\frac{1}{3}:\frac{1}{5}$, and its perimeter is 6.2 centimeters. Find the measure of each side of the triangle.

Find the measures of the angles of each triangle.

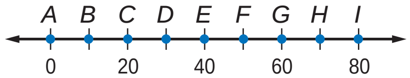
16. The ratio of the measures of the three angles is 2:5:3.
17. The ratio of the measures of the three angles is 6:9:10.

Solve each proportion.

18. $\frac{3}{8} = \frac{x}{5}$ **19.** $\frac{w}{6.4} = \frac{1}{2}$ **20.** $\frac{4x}{24} = \frac{56}{112}$ **21.** $\frac{11}{20} = \frac{55}{20x}$

22. $\frac{2x - 13}{28} = \frac{-4}{7}$ **23.** $\frac{4x + 3}{12} = \frac{5}{4}$ **24.** $\frac{b + 1}{b - 1} = \frac{5}{6}$ **25.** $\frac{3x - 1}{2} = \frac{-2}{x + 2}$

26. Use the number line at the right to determine the ratio of AC to BH.



27. A cable that is 42 feet long is divided into lengths in the ratio of 3:4. What are the two lengths into which the cable is divided?

LITERATURE For Exercises 28 and 29, use the following information.
Throughout Lewis Carroll's book *Alice's Adventures in Wonderland,* Alice's size changes. Her normal height was about 50 inches tall. She came across a door, about 15 inches high, that led to a garden. Alice's height changes to 10 inches so she can visit the garden.

28. Find the ratio of the height of the door to Alice's height in Wonderland.

29. How tall would the door have been in Alice's normal world?

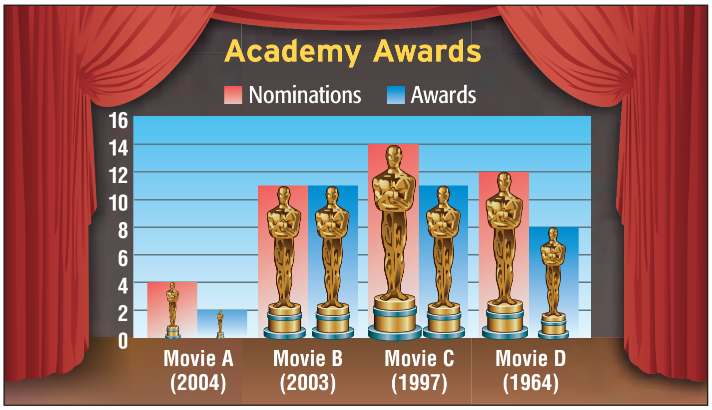
Real-World Link

The attendance at movie theaters in 1970 was 921 million. In 2004 it was 1536 million.

Source: *Statistical Abstract of the United States*

30. ENTERTAINMENT The Great Moments with Mr. Lincoln presentation at Disneyland in California features a life-size animatronic figure of Abraham Lincoln. Before actual construction of the exhibit, Walt Disney and his design company built models that were in proportion to the displays they planned to build. In the model, Lincoln is 8 inches tall. Mr. Lincoln's actual height was 6 feet 4 inches. What is the ratio of the height of the model of Mr. Lincoln compared to his actual height?

MOVIES For Exercises 31 and 32, refer to the graphic.



Source: oscars.org

31. Of the films listed, which had the greatest ratio of Academy awards to number of nominations?

32. Which film listed had the lowest ratio of awards to nominations?

FOOD For Exercises 33 and 34, use the following information.
There were approximately 295,346,288 people in the United States in a recent year. According to figures from the United States Census, they consumed about 1.4 billion gallons of ice cream that year.

33. Find the approximate consumption of ice cream per person.

34. If there were 8,186,268 people in North Carolina, about how much ice cream might they have been expected to consume?

Bob Daemmrich/PhotoEdit

YEARBOOKS **For Exercises 35 and 36, use the following information.**
Damon resized a photograph that is 8 inches by 10 inches so that it would fit in a 4-inch by 4-inch area on a yearbook page.

35. Find the maximum dimensions of the reduced photograph.

36. What is the percent of reduction of the length?

GOLDEN RECTANGLES **For Exercises 37–39, use the following information.**
Many artists have used golden rectangles in their work. In a golden rectangle, the ratio of the length to the width is about 1.618. This ratio is known as the *golden ratio.*

37. A rectangle has dimensions of 19.42 feet and 12.01 feet. Determine if the rectangle is a golden rectangle. Then find the length of the diagonal.

EXTRA PRACTICE
See pages 813, 834.
Math Online
Self-Check Quiz at geometryonline.com

38. **TELEVISION** The *aspect ratio*, or the ratio of the width to the height, of a widescreen television set is 16:9. The aspect ratio of a fullscreen television set is 4:3. Compare these ratios to the golden ratio. Are either television screens golden rectangles? Explain.

39. **RESEARCH** Use the Internet or other sources to find examples of golden rectangles.

H.O.T. Problems

40. **OPEN ENDED** Write two proportions with extremes 5 and 8.

41. **REASONING** Explain how you would solve $\frac{28}{48} = \frac{21}{x}$.

42. **Which One Doesn't Belong?** Identify the proportion that doesn't belong with the other three. Explain your reasoning.

$\frac{3}{8} = \frac{8.4}{22.4}$	$\frac{2}{3} = \frac{5}{7.5}$	$\frac{5}{6} = \frac{14}{16.8}$	$\frac{7}{9} = \frac{19.6}{25.2}$

CHALLENGE **The ratios of the sides of three polygons are given. Make a conjecture about the type of each polygon described.**

43. 2:2:3 **44.** 3:3:3:3 **45.** 4:5:4:5

46. **Writing in Math** Refer to page 380. Describe how Louis Comfort Tiffany used ratios. Include four rectangles from the photo that appear to be in proportion, and an estimate in inches of the ratio of the width of the skylight to the length of the skylight given that the dimensions of the rectangle in the bottom left corner are approximately 3.5 inches by 5.5 inches.

STANDARDIZED TEST PRACTICE

47. A breakfast cereal contains wheat, rice, and oats in the ratio 3:1:2. If the manufacturer makes a mixture using 120 pounds of oats, how many pounds of wheat will be used?

A 60 lb **C** 120 lb

B 80 lb **D** 180 lb

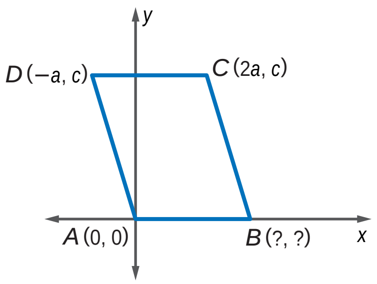
48. **REVIEW** The base of a triangle is 6 centimeters less than twice its height. The area of the triangle is 270 square centimeters. What is the height of the triangle?

F 12 cm **H** 18 cm

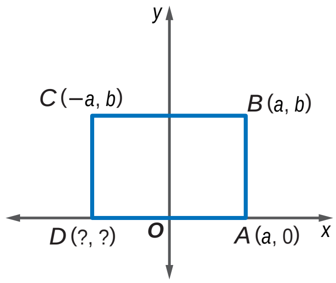
G 15 cm **J** 21 cm

Name the missing coordinates for each parallelogram or rectangle. (Lesson 6-7)

49.

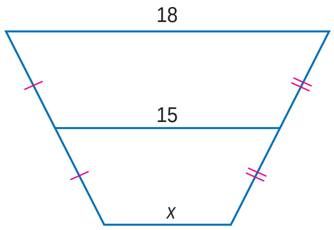


50.

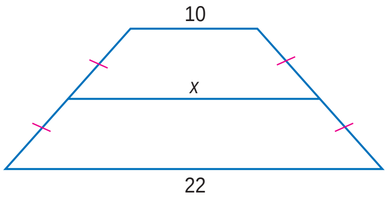


ALGEBRA **Find the missing measure for each trapezoid.** (Lesson 6-6)

51.



52.

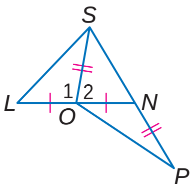


In the figure, $\overline{SO}$ is a median of $\triangle SLN$, $\overline{OS} \cong \overline{NP}$, $m\angle 1 = 3x - 50$, and $m\angle 2 = x + 30$. Determine whether each statement is *always, sometimes,* or *never* true. (Lesson 5-5)

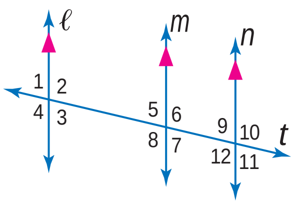
53. $LS > SN$

54. $SN < OP$

55. $x = 45$



In the figure, $m\angle 9 = 75$. Find the measure of each angle. (Lesson 3-2)



56. $\angle 3$

57. $\angle 5$

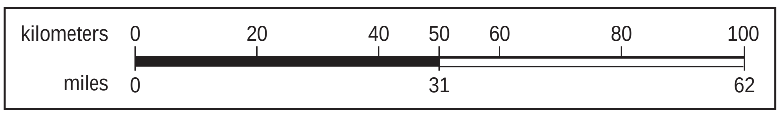
58. $\angle 6$

59. $\angle 8$

60. $\angle 11$

61. $\angle 12$

62. **MAPS** On a U.S. map, there is a scale that lists kilometers on the top and miles on the bottom.



Suppose $\overline{AB}$ and $\overline{CD}$ are segments on this map. If $AB = 100$ kilometers and $CD = 62$ miles, is $\overline{AB} \cong \overline{CD}$? Explain. (Lesson 2-7)

PREREQUISITE SKILL **Find the distance between each pair of points to the nearest tenth.** (Lesson 1-3)

63. $A(12, 3), B(-8, 3)$

64. $C(0, 0), D(5, 12)$

65. $E\left(\frac{4}{5}, -1\right), F\left(2, \frac{-1}{2}\right)$

66. $G\left(3, \frac{3}{7}\right), H\left(4, -\frac{2}{7}\right)$

EXTEND 7-1

Graphing Calculator Lab

Fibonacci Sequence and Ratios

Leonardo Pisano (c. 1170–c. 1250), or Fibonacci, was born in Italy but educated in North Africa. As a result, his work is similar to that of other North African authors of that time. His book, *Liber abaci*, published in 1202, introduced what is now called the Fibonacci sequence, in which each term after the first two terms is the sum of the two numbers before it.

Term	1	2	3	4	5	6	7
Fibonacci Number	1	1	2	3	5	8	13
			↑ 1 + 1	↑ 1 + 2	↑ 2 + 3	↑ 3 + 5	↑ 5 + 8

ACTIVITY

You can use CellSheet on a TI-83/84 Plus graphing calculator to create terms of the Fibonacci sequence. Then compare each term with its preceding term.

Step 1 Access the CellSheet application by pressing the APPS key. Choose the number for CellSheet and press ENTER.

Step 2 Enter the column headings in row 1. Use the ALPHA key to enter letters and press ["] at the beginning of each label.

Step 3 Enter 1 into cell A2. Then insert the formula =A2+1 in cell A3. Press STO▸ to insert the = in the formula. Then use F3 to copy this formula and use F4 to paste it in each cell in the column.

Step 4 In column B, we will record the Fibonacci numbers. Enter 1 in cells B2 and B3 since you do not have two previous terms to add. Then insert the formula =B2+B3 in cell B4. Copy this formula down the column.

Step 5 In column C, we will find the ratio of each term to its preceding term. Enter 1 in cell C2 since there is no preceding term. Then enter =B3/B2 in cell C3. Copy this formula down the column. *The screens show the results for terms 1 through 10.*

Analyze the Results

1. What happens to the Fibonacci number as the number of the term increases?
2. What pattern of odd and even numbers do you notice in the Fibonacci sequence?
3. As the number of terms gets greater, what pattern do you notice in the ratio column?
4. Extend the spreadsheet to calculate fifty terms of the Fibonacci sequence. Describe any differences in the patterns you described in Exercises 1–3.
5. **MAKE A CONJECTURE** How might the Fibonacci sequence relate to the golden ratio?

7-2 Similar Polygons

Main Ideas

- Identify similar figures.
- Solve problems involving scale factors.

New Vocabulary

similar polygons

scale factor

M.C. Escher (1898–1972) was a Dutch graphic artist known for drawing impossible structures, spatial illusions, and repeating interlocking geometric patterns. The image at the right is a print of Escher's *Circle Limit IV.* It includes winged images that have the same shape, but are different in size. Also note that there are similar dark images and similar light images.

Circle Limit IV
M.C. Escher (1960)

Identify Similar Figures When polygons have the same shape but may be different in size, they are called **similar polygons**.

Study Tip

Similarity and Congruence

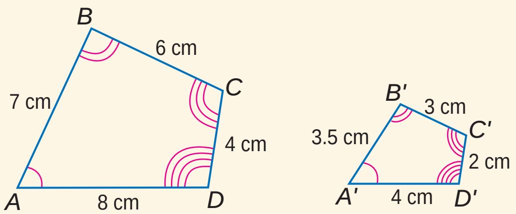
If two polygons are congruent, they are also similar. All of the corresponding angles are congruent, and the lengths of the corresponding sides have a ratio of 1:1.

KEY CONCEPT — Similar Polygons

Words Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.

Symbol $\sim$ is read *is similar to*

Example



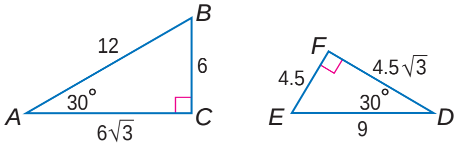
The order of the vertices in a similarity statement is important. It identifies the corresponding angles and the corresponding sides.

similarity statement	congruent angles	corresponding sides
$ABCD \sim EFGH$	$\angle A \cong \angle E$ $\angle B \cong \angle F$ $\angle C \cong \angle G$ $\angle D \cong \angle H$	$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$

Like congruent polygons, similar polygons may be repositioned so that corresponding parts are easy to identify.

EXAMPLE 1 Similar Polygons

Determine whether the pair of triangles is similar. Justify your answer.



All right angles are congruent, so $\angle C \cong \angle F$. Since $m\angle A = m\angle D$, $\angle A \cong \angle D$. By the Third Angle Theorem, $\angle B \cong \angle E$. Thus, all corresponding angles are congruent.

Now determine whether corresponding sides are proportional.

Sides opposite 90° angle	Sides opposite 30° angle	Sides opposite 60° angle
$\frac{AB}{DE} = \frac{12}{9}$ or $1.\overline{3}$	$\frac{BC}{EF} = \frac{6}{4.5}$ or $1.\overline{3}$	$\frac{AC}{DF} = \frac{6\sqrt{3}}{4.5\sqrt{3}}$ or $1.\overline{3}$

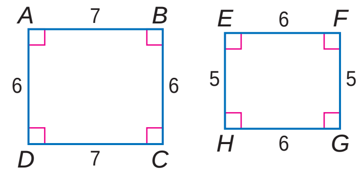
The ratios of the measures of the corresponding sides are equal, and the corresponding angles are congruent, so $\triangle ABC \sim \triangle DEF$.

Study Tip

Common Misconception

When two figures have vertices that are in alphabetical order, this does not mean that the corresponding vertices in the similarity statement will follow alphabetical order.

1.



Scale Factors When you compare the lengths of corresponding sides of similar figures, you usually get a numerical ratio. This ratio is called the **scale factor** for the two figures.

2 **MODEL CARS** **This is a miniature replica of a 1923 Checker Cab. The length of the model is 6.5 inches long. If the length of the car is 13 feet, what is the scale factor of the model compared to the car?**

Study Tip

Checking Results

When solving word problems, analyze your answer to make sure it makes sense and that you answered the question asked.

Both measurements need to have the same unit of measure.

$13(12) = 156$ inches — Convert feet to inches.

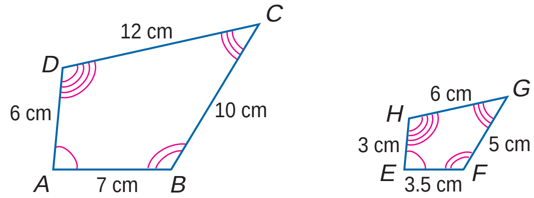
$\frac{\text{length of model}}{\text{length of real car}} = \frac{6.5 \cancel{\text{ inches}}}{156 \cancel{\text{ inches}}}$ or $\frac{1}{24}$ — Write a proportion and simplify.

The scale factor is $\frac{1}{24}$. So the model is $\frac{1}{24}$ the length of the real car.

2. SCALE MODELS The height of the Soldiers' National Monument in Gettysburg, Pennsylvania, is 60 feet. The height of a model is 10 inches. What is the scale factor of the model compared to the original?

Animation
geometryonline.com

When finding the scale factor for two similar polygons, the scale factor will depend on the order of comparison.

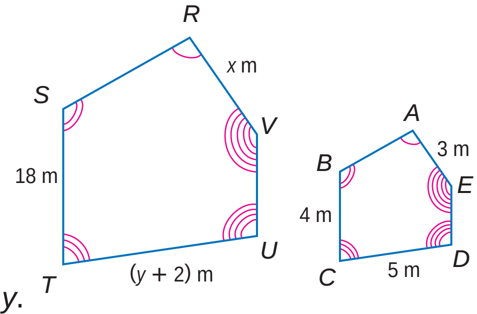


- The scale factor of quadrilateral *ABCD* to quadrilateral *EFGH* is 2.
- The scale factor of quadrilateral *EFGH* to quadrilateral *ABCD* is $\frac{1}{2}$.

EXAMPLE 3 Proportional Parts and Scale Factor

The two polygons are similar.

a. Write a similarity statement. Then find *x*, *y*, and *UT*.



Use the congruent angles to write the corresponding vertices in order.

polygon *RSTUV* ~ polygon *ABCDE*

Now write proportions to find *x* and *y*.

To find *x*:

$\frac{ST}{BC} = \frac{VR}{EA}$	Similarity proportion
$\frac{18}{4} = \frac{x}{3}$	$ST = 18$, $BC = 4$ $VR = x$, $EA = 3$
$18(3) = 4(x)$	Cross products
$54 = 4x$	Multiply.
$13.5 = x$	Divide each side by 4.

To find *y*:

$\frac{ST}{BC} = \frac{UT}{DC}$	Similarity proportion
$\frac{18}{4} = \frac{y + 2}{5}$	$ST = 18$, $BC = 4$ $UT = y + 2$, $DC = 5$
$18(5) = 4(y + 2)$	Cross products
$90 = 4y + 8$	Multiply.
$82 = 4y$	Subtract 8 from each side.
$20.5 = y$	Divide each side by 4.

$UT = y + 2$, so $UT = 20.5 + 2$ or 22.5.

b. Find the scale factor of polygon *RSTUV* to polygon *ABCDE*.

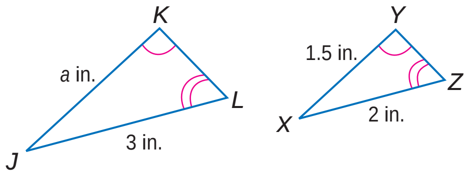
The ratio of the lengths of any two corresponding sides is $\frac{ST}{BC} = \frac{18}{4}$ or $\frac{9}{2}$.

Study Tip

Checking Solutions

To verify the scale factor, find the ratio of two other corresponding sides.

3. Write a similarity statement. Then find *a* and the scale factor of $\triangle JKL$ to $\triangle XYZ$.



EXAMPLE 4 Enlargement or Reduction of a Figure

Triangle *ABC* is similar to $\triangle XYZ$ with a scale factor of $\frac{2}{3}$. If the sides of $\triangle ABC$ are 6, 8, and 10 inches, what are the lengths of the sides of $\triangle XYZ$?

Write proportions for finding side measures.

$$\frac{\triangle ABC \rightarrow 6}{\triangle XYZ \rightarrow x} = \frac{2}{3} \qquad \frac{\triangle ABC \rightarrow 8}{\triangle XYZ \rightarrow y} = \frac{2}{3} \qquad \frac{\triangle ABC \rightarrow 10}{\triangle XYZ \rightarrow z} = \frac{2}{3}$$

$$18 = 2x \qquad 24 = 2y \qquad 30 = 2z$$

$$9 = x \qquad 12 = y \qquad 15 = z$$

The lengths of the sides of $\triangle XYZ$ are 9, 12, and 15 inches.

CHECK Your Progress

4. Rectangle $QRST$ is similar to rectangle $JKLM$ with a scale factor of $\frac{4}{5}$. If the lengths of the sides of rectangle $QRST$ are 5 centimeters and 12 centimeters, what are the lengths of the sides of rectangle $JKLM$?

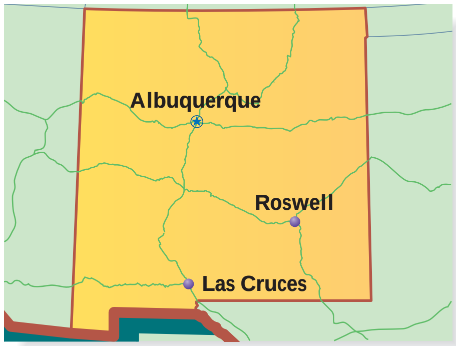
Real-World Link

The earliest mapmakers placed grid lines over a map to locate places. This is an early example of coordinate geometry. In modern cartography, latitude and longitude are used to determine the position of places on Earth.

Source: *History of Cartography*

EXAMPLE 5 Scales on Maps

MAPS The scale on the map of New Mexico is 2 centimeters = 160 miles. The width of New Mexico through Albuquerque on the map is 4.1 centimeters. How long would it take to drive across New Mexico if you drove at an average of 60 miles per hour?



Explore Every 2 centimeters represents 160 miles. The distance across the map is 4.1 centimeters.

Plan Create a proportion relating the measurements to the scale to find the distance in miles. Then use the formula $d = rt$ to find the time.

Solve

$$\begin{array}{rl} \text{centimeters} \rightarrow & \dfrac{2}{160} = \dfrac{4.1}{x} \quad \begin{array}{l}\leftarrow \text{centimeters}\\ \leftarrow \text{miles}\end{array} \\ \text{miles} \rightarrow & \end{array}$$

$2x = 656$ Cross products

$x = 328$ Divide each side by 2.

The indicated distance is 328 miles.

$d = rt$

$328 = 60t$ $\quad d = 328$ and $r = 60$

$\frac{328}{60} = t$ Divide each side by 60.

$5\frac{7}{15} = t$ Simplify.

It would take $5\frac{7}{15}$ hours or 5 hours and 28 minutes to drive across New Mexico at an average of 60 miles per hour.

Check Reexamine the scale. The map is about 4 centimeters wide, so the distance across New Mexico is about 320 miles. The answer is about 5.5 hours and at 60 miles per hour, the trip would be 330 miles. The two distances are close estimates, so the answer is reasonable.

Study Tip

Units of Time

Remember that there are 60 minutes in an hour. When rewriting $\frac{328}{60}$ as a mixed number, you could also write $5\frac{28}{60}$, which means 5 hours 28 minutes.

5. The distance on the map from Las Cruces to Roswell is 1.8 centimeters. How long would it take to drive if you drove an average of 55 miles per hour?

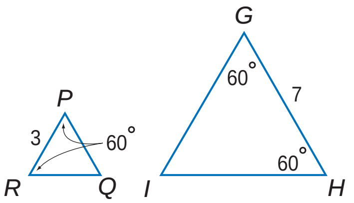
Online Personal Tutor at geometryonline.com

Check Your Understanding

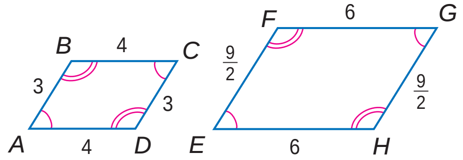
Example 1 (p. 389)

Determine whether each pair of figures is similar. Justify your answer.

1.



2.



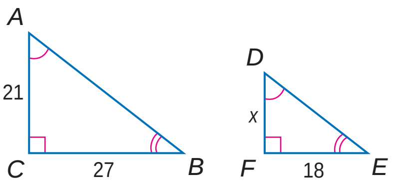
Example 2 (p. 389)

3. **MODELS** Suki made a scale model of a local bridge. If the span of the bridge was 50 feet and the span of the model was 6 inches, what scale factor did Suki use to build her model?

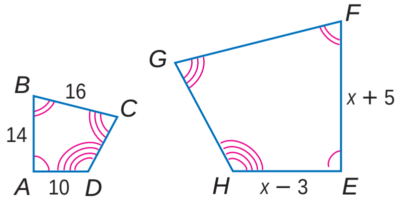
Example 3 (p. 390)

Each pair of polygons is similar. Write a similarity statement, and find *x*, the measure(s) of the indicated side(s), and the scale factor.

4. $\overline{DF}$



5. $\overline{FE}$, $\overline{EH}$, and $\overline{GF}$



Example 4 (p. 390)

6. Triangle *JKL* is similar to $\triangle TUV$ with a scale factor of $\frac{3}{4}$. If the lengths of the sides of $\triangle TUV$ are 4, 6, and 8 centimeters, what are the lengths of the sides of $\triangle JKL$?

Example 5 (p. 391)

7. **MAPS** Refer to Example 5 on page 391. The distance on the map from Albuquerque to Roswell is 1.9 centimeters. How long would it take to drive if you drove at an average of 65 miles per hour?

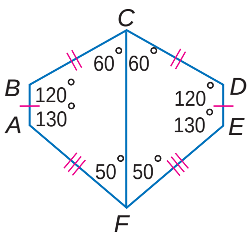
Exercises

HOMEWORK HELP

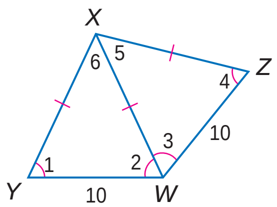
For Exercises	See Examples
8–11	1
12, 13	2
14–19	3
22, 23	4
20, 21	5

Determine whether each pair of figures is similar. Justify your answer.

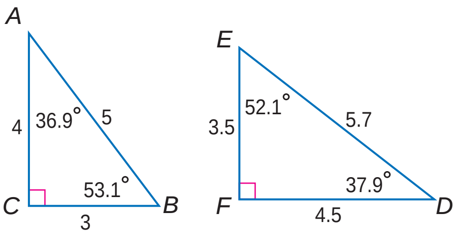
8.



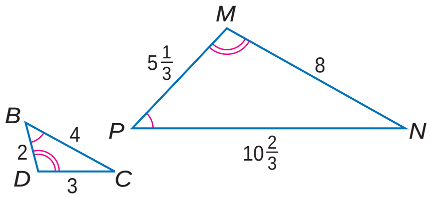
9.



10.



11.



Real-World Link

Scale replicas of the Statue of Liberty are also found in Alabama, Colorado, and Georgia.

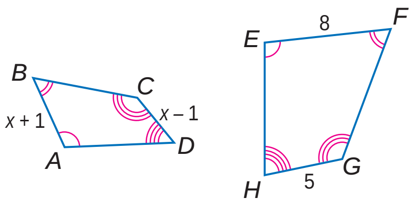
Source: worldweb.com

12. PHOTOCOPYING Mrs. Barojas walked to a copier in her office, made a copy of her proposal, and sent the original to one of her customers. When Mrs. Barojas looked at her copy before filing it, she saw that the copy had been made at an 80% reduction. She needs her filing copy to be the same size as the original. What enlargement scale factor must she use on the first copy to make a second copy the same size as the original?

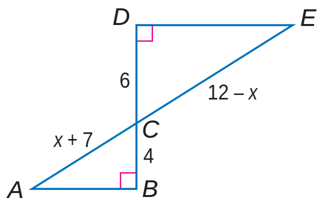
13. ARCHITECTURE A replica of the Statue of Liberty in Austin, Texas, is $16\frac{3}{4}$ feet tall. The statue in New York Harbor is 151 feet. What is the scale factor comparing the actual statue to the replica?

Each pair of polygons is similar. Write a similarity statement, and find *x*, the measures of the indicated sides, and the scale factor.

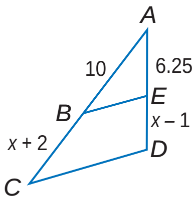
14. $\overline{AB}$ and $\overline{CD}$



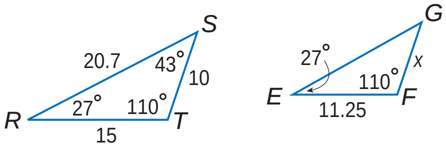
15. $\overline{AC}$ and $\overline{CE}$



16. $\overline{BC}$ and $\overline{ED}$

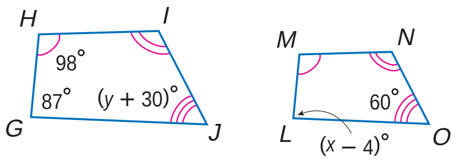


17. $\overline{GF}$ and $\overline{EG}$

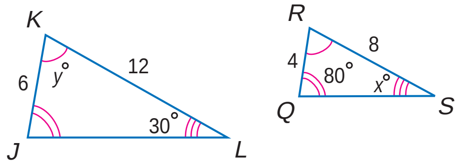


Each pair of polygons is similar. Find *x* and *y*. Round to the nearest hundredth if necessary.

18.



19.

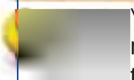


MAPS For Exercises 20 and 21 use the following information.
The scale on the map of Georgia is 1 in. = 40 miles.

20. The width of Georgia from Columbus to Dublin is $2\frac{3}{4}$ inches on the map. How long would it take to drive this distance if you drove at an average of 55 miles per hour?

21. The distance from Atlanta to Savannah is $5\frac{3}{4}$ inches on the map. How long would it take to drive from Atlanta to Savannah if you drove at an average of 65 miles per hour?

Cross-Curricular Project



You can use a map to begin to find the hidden treasure. Visit geometryonline.com.

22. A triangle has side lengths of 3 meters, 5 meters, and 4 meters. The triangle is enlarged so that the larger triangle is similar to the original and the scale factor is 5. Find the perimeter of the larger triangle.

23. A rectangle with length 60 centimeters and height 40 centimeters is reduced so that the new rectangle is similar to the original and the scale factor is $\frac{1}{4}$. Find the length and width of the new rectangle.

Real-World Link

Crew Stadium in Columbus, Ohio, was specifically built for Major League Soccer. The dimensions of the field are about 69 meters by 105 meters.

Source: MLSnet.com

SPORTS **Make a scale drawing of each playing field using the given scale.**

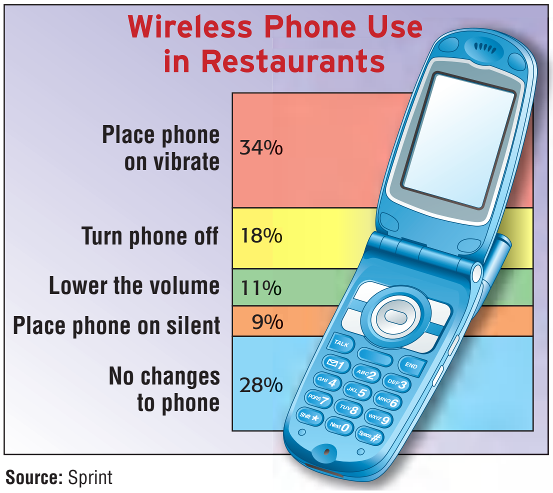
24. A baseball diamond is a square 90 feet on each side. Use the scale $\frac{1}{4}$ in. = 9 ft.

25. A high school football field is a rectangle with length 300 feet and width 160 feet. Use the scale $\frac{1}{16}$ in. = 5 ft.

ANALYZE GRAPHS **For Exercises 26 and 27, refer to the graphic, which uses rectangles to represent percents.**

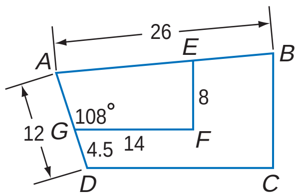
26. Are the rectangles representing 34% and 18% similar? Explain.

27. What is the ratio of the areas of the rectangles representing 18% and 9%? Compare the ratio of the areas to the ratio of the percents.



Source: Sprint

For Exercises 28–35, use the following information to find each measure.
Polygon *ABCD* ~ polygon *AEFG*, $m\angle AGF = 108$, $GF = 14$, $AD = 12$, $DG = 4.5$, $EF = 8$, and $AB = 26$.



28. scale factor of trapezoid *ABCD* to trapezoid *AEFG*

29. *AG*

30. *DC*

31. $m\angle ADC$

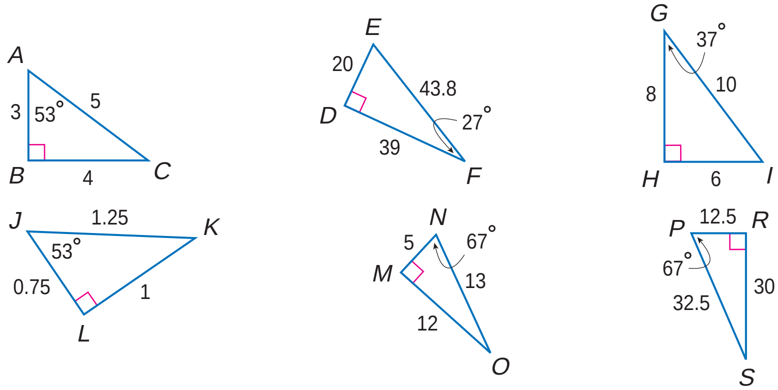
32. *BC*

33. perimeter of trapezoid *ABCD*

34. perimeter of trapezoid *AEFG*

35. ratio of the perimeter of polygon *ABCD* to the perimeter of polygon *AEFG*

36. Determine which of the following right triangles are similar. Justify your answer.



Determine whether each statement is *always*, *sometimes*, or *never* true.

37. Two congruent triangles are similar.

38. Two squares are similar.

39. Two isosceles triangles are similar.

40. Two obtuse triangles are similar.

41. Two equilateral triangles are similar.

COORDINATE GEOMETRY For Exercises 42–47, use the following information.
Scale factors can be used to produce similar figures. The resulting figure is an enlargement or reduction of the original figure depending on the scale factor. Triangle *ABC* has vertices *A*(0, 0), *B*(8, 0), and *C*(2, 7). Suppose the coordinates of each vertex are multiplied by 2 to create the similar triangle *A'B'C'*.

42. Find the coordinates of the vertices of $\triangle A'B'C'$.

43. Graph $\triangle ABC$ and $\triangle A'B'C'$.

44. Use the Distance Formula to find the measures of the sides of each triangle.

45. Find the ratios of the sides that appear to correspond.

46. How could you use slope to determine if angles are congruent?

47. Is $\triangle ABC \sim \triangle A'B'C'$? Explain your reasoning.

EXTRA PRACTICE
See pages 814, 834.

Math Online
Self-Check Quiz at
geometryonline.com

H.O.T. Problems

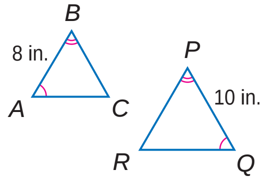
48. FIND THE ERROR Roberto and Garrett have calculated the scale factor for two similar triangles. Who is correct? Explain your reasoning.

Roberto

$$\frac{AB}{QP} = \frac{8}{10} = \frac{4}{5}$$

Garrett

$$\frac{QP}{AB} = \frac{10}{8} = \frac{5}{4}$$



49. OPEN ENDED Find a counterexample for the statement *All rectangles are similar*.

CHALLENGE For Exercises 50–52, use the following information.
Rectangle *ABCD* is similar to rectangle *WXYZ* with sides in a ratio of 4:1.

50. What is the ratio of the areas of the two rectangles?

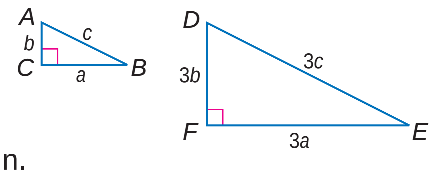
51. Suppose the dimension of each rectangle is tripled. What is the new ratio of the sides of the rectangles?

52. What is the ratio of the areas of these larger rectangles?

CHALLENGE For Exercises 53 and 54, $\triangle ABC \sim \triangle DEF$.

53. Show that the perimeters of $\triangle ABC$ and $\triangle DEF$ have the same ratio as their corresponding sides.

54. If 6 units are added to the lengths of each side, are the new triangles similar? Explain.



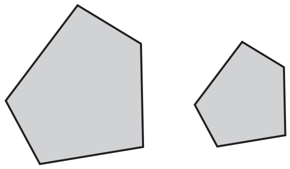
Writing in Math Refer to *Circle Limit IV* on page 388.

55. Describe how M.C. Escher used similar figures to create the artwork.

56. RESEARCH This art print is a model of a non-Euclidean geometry called *hyperbolic geometry*. Hyperbolic geometry is a two-dimensional space. In this geometry system, lines are arcs with ends that are perpendicular to the edge of the disk. Use the Internet or other source to research hyperbolic geometry. Compare and contrast this geometry system with Euclidean geometry.

STANDARDIZED TEST PRACTICE

57. A scale factor of $\frac{2}{3}$ was used to produce the smaller pentagon from the larger one.



How does the perimeter of the smaller pentagon compare to the perimeter of the larger pentagon?

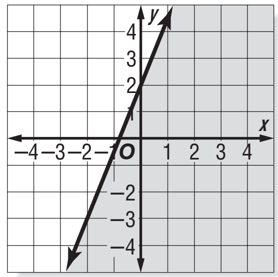
A The perimeter is $\frac{2}{3}$ as large.

B The perimeter is $\frac{4}{9}$ as large.

C The perimeter is $\frac{8}{27}$ as large.

D The perimeter is $\frac{1}{3}$ as large.

58. **REVIEW** Which inequality *best* represents the graph below?



F $2y - 5x < 4$

G $2y + 5x \geq 2$

H $2y + 5x < 4$

J $2y - 5x > 2$

Solve each proportion. (Lesson 7-1)

59. $\frac{b}{7.8} = \frac{2}{3}$

60. $\frac{c - 2}{c + 3} = \frac{5}{4}$

61. $\frac{2}{4y + 5} = \frac{-4}{y}$

62. $\frac{2x + 3}{x - 1} = \frac{-4}{5}$

63. $\frac{2d - 8}{6} = \frac{3d + 4}{-2}$

64. $\frac{-5}{3k + 1} = \frac{-3}{2k - 6}$

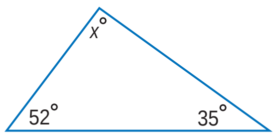
Position and label each quadrilateral on a coordinate plane. (Lesson 6-7)

65. parallelogram with height *c* and width *b*

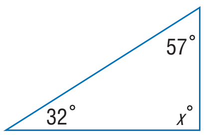
66. rectangle with width 2*a* and height *b*

Find *x*. (Lesson 4-2)

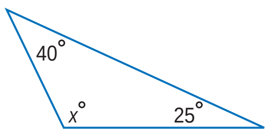
67.



68.

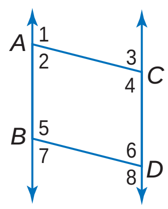


69.



70. Suppose two parallel lines are cut by a transversal and $\angle 1$ and $\angle 2$ are alternate interior angles. Find $m\angle 1$ and $m\angle 2$ if $m\angle 1 = 10x - 9$ and $m\angle 2 = 9x + 3$. (Lesson 3-2)

PREREQUISITE SKILL **In the figure, $\overline{AB} \parallel \overline{CD}$, $\overline{AC} \parallel \overline{BD}$, and $m\angle 4 = 118$. Find the measure of each angle.** (Lesson 3-2)



71. $\angle 1$

72. $\angle 2$

73. $\angle 3$

74. $\angle 5$

75. $\angle 6$

76. $\angle 8$

7-3 Similar Triangles

Main Ideas

- Identify similar triangles.
- Use similar triangles to solve problems.

The Eiffel Tower was built in Paris for the 1889 world exhibition by Gustave Eiffel. Eiffel (1832–1923) was a French engineer who specialized in revolutionary steel constructions. He used thousands of triangles, some the same shape but different in size, to build the Eiffel Tower because triangular shapes result in rigid construction.

Identify Similar Triangles In Chapter 4, you learned several tests to determine whether two triangles are congruent. There are also tests to determine whether two triangles are similar.

GEOMETRY LAB

Similar Triangles

- Draw $\triangle DEF$ with $m\angle D = 35$, $m\angle F = 80$, and $DF = 4$ centimeters.
- Draw $\triangle RST$ with $m\angle T = 35$, $m\angle S = 80$, and $ST = 7$ centimeters.
- Measure $\overline{EF}$, $\overline{ED}$, $\overline{RS}$, and $\overline{RT}$.
- Calculate the ratios $\frac{FD}{ST}$, $\frac{EF}{RS}$, and $\frac{ED}{RT}$.

ANALYZE THE RESULTS

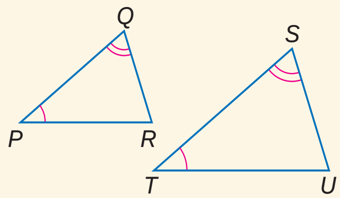
1. What can you conclude about all of the ratios?
2. Repeat the activity with two more triangles with the same angle measures but different side measures. Then repeat the activity with a third pair of triangles. Are all of the triangles similar? Explain.
3. What are the minimum requirements for two triangles to be similar?

The previous lab leads to the following postulate.

POSTULATE 7.1 *Angle-Angle (AA) Similarity*

If the two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Example: $\angle P \cong \angle T$ and $\angle Q \cong \angle S$, so $\triangle PQR \sim \triangle TSU$.

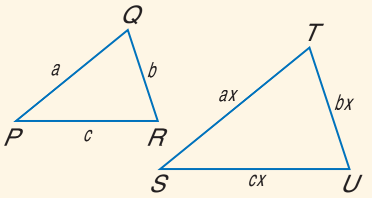


You can use the AA Similarity Postulate to prove two theorems that also verify triangle similarity.

THEOREMS 7.1–7.2

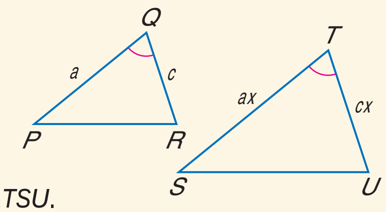
7.1 Side-Side-Side (SSS) Similarity If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.

Example: $\frac{PQ}{ST} = \frac{QR}{SU} = \frac{RP}{UT}$, so $\triangle PQR \sim \triangle TSU$.



7.2 Side-Angle-Side (SAS) Similarity If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

Example: $\frac{PQ}{ST} = \frac{QR}{SU}$ and $\angle Q \cong \angle S$, so $\triangle PQR \sim \triangle TSU$.



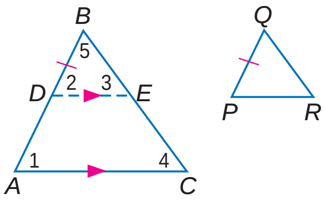
You will prove Theorem 7.2 in Exercise 23.

PROOF Theorem 7.1

Given: $\frac{PQ}{AB} = \frac{QR}{BC} = \frac{RP}{CA}$

Prove: $\triangle BAC \sim \triangle QPR$

Locate D on $\overline{AB}$ so that $\overline{DB} \cong \overline{PQ}$ and draw $\overline{DE}$ so that $\overline{DE} \parallel \overline{AC}$.



Paragraph Proof:

Since $\overline{DE} \parallel \overline{AC}$, $\angle 2$ and $\angle 1$ and $\angle 3$ and $\angle 4$ are corresponding angles. Therefore, $\angle 2 \cong \angle 1$ and $\angle 3 \cong \angle 4$. By AA Similarity, $\triangle BDE \sim \triangle BAC$.

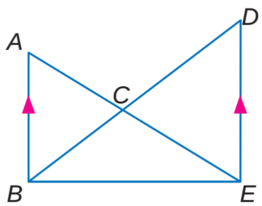
Since $\overline{DB} \cong \overline{PQ}$, $DB = PQ$. By substitution, $\frac{PQ}{AB} = \frac{QR}{BC} = \frac{RP}{CA}$ becomes $\frac{DB}{AB} = \frac{QR}{BC} = \frac{RP}{CA}$.

By the definition of similar polygons, $\frac{DB}{AB} = \frac{BE}{BC} = \frac{ED}{CA}$.

By substitution, $\frac{QR}{BC} = \frac{BE}{BC}$ and $\frac{RP}{CA} = \frac{ED}{CA}$. This means that $QR = BE$ and $RP = ED$ or $\overline{QR} \cong \overline{BE}$ and $\overline{RP} \cong \overline{ED}$. With these congruences and $\overline{DB} \cong \overline{PQ}$, $\triangle BDE \cong \triangle QPR$ by SSS. By CPCTC, $\angle B \cong \angle Q$ and $\angle 2 \cong \angle P$. But $\angle 2 \cong \angle A$, so $\angle A \cong \angle P$. By AA Similarity, $\triangle BAC \sim \triangle QPR$.

STANDARDIZED TEST EXAMPLE Are Triangles Similar?

1 **In the figure, $\overline{AB} \parallel \overline{DE}$. Which theorem or postulate can be used to prove $\triangle ACB \sim \triangle ECD$?**



A ASA **B** SSS **C** AA **D** SAS

Read the Test Item You are asked to identify which theorem or postulate can be used to prove that $\triangle ACB$ is similar to $\triangle ECD$.

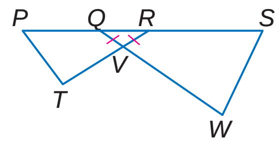
Solve the Test Item Since $\overline{AB} \parallel \overline{DE}$, $\angle BAE \cong \angle DEA$ by the Alternate Interior Angles Theorem. $\angle ACB \cong \angle ECD$ by the Vertical Angle Theorem. So, by AA Similarity, $\triangle ACB \sim \triangle ECD$. The answer is C.

Test-Taking Tip

Overlapping Triangles

When two triangles overlap, you may wish to draw them separately so the corresponding parts are in the same position on the paper. Then write the corresponding angles and sides.

1. In the figure, $\overline{QV} \cong \overline{RV}$, $PR = 9$, $QS = 15$, $TR = 12$, and $QW = 20$. Which statement *must* be true?



F $\triangle PTR \sim \triangle QWS$ **H** $\triangle PTR \sim \triangle QVR$

G $\triangle QVR \sim \triangle SWQ$ **J** $\triangle PTR \sim \triangle SWQ$

Like the congruence of triangles, similarity of triangles is reflexive, symmetric, and transitive.

THEOREM 7.3

Similarity of triangles is reflexive, symmetric, and transitive.

Examples:

Reflexive: $\triangle ABC \sim \triangle ABC$ **Symmetric:** If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$.

Transitive: If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle GHI$, then $\triangle ABC \sim \triangle GHI$.

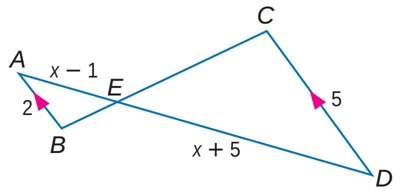
You will prove Theorem 7.3 in Exercise 25.

Use Similar Triangles

Similar triangles can be used to solve problems.

EXAMPLE 2 Parts of Similar Triangles

ALGEBRA Find *AE* and *DE*.



Since $\overline{AB} \parallel \overline{CD}$, $\angle BAE \cong \angle CDE$ and $\angle ABE \cong \angle DCE$ because they are the alternate interior angles. By AA Similarity, $\triangle ABE \sim \triangle DCE$. Using the definition of similar polygons, $\frac{AB}{DC} = \frac{AE}{DE}$.

$$\frac{AB}{DC} = \frac{AE}{DE}$$

$$\frac{2}{5} = \frac{x-1}{x+5} \quad \text{Substitution}$$

$$2(x+5) = 5(x-1) \quad \text{Cross products}$$

$$2x + 10 = 5x - 5 \quad \text{Distributive Property}$$

$$-3x = -15 \quad \text{Subtract } 5x \text{ and 10 from each side.}$$

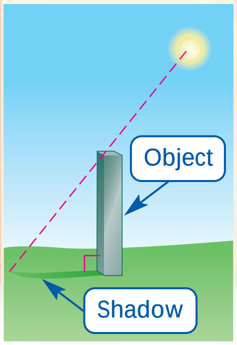
$$x = 5 \quad \text{Divide each side by } -3.$$

Now find *AE* and *ED*. $AE = x - 1 = 5 - 1$ or 4 $ED = x + 5 = 5 + 5$ or 10

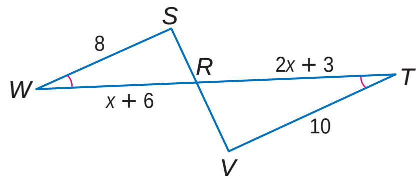
Study Tip

Shadow Problems

In shadow problems, we assume that a right triangle is formed by the Sun's ray from the top of the object to the end of the shadow.



2. Find *WR* and *RT*.



Similar triangles can be used to find measurements indirectly.

Indirect Measurement

3 **ROLLER COASTERS** **For a school project, Hallie needs to determine the height of the Superman roller coaster in Mitchellville, Maryland. She is 5 feet tall and her shadow is 2 feet 9 inches long. If the length of the shadow of the roller coaster is 110 feet, how tall is the roller coaster?**

Assuming that the Sun's rays form similar triangles, the following proportion can be written.

$$\frac{\text{height of the roller coaster}}{\text{height of Hallie}} = \frac{\text{roller coaster shadow length}}{\text{Hallie's shadow length}}$$

Now, substitute the known values and let x be the height of the roller coaster.

$\frac{x}{5} = \frac{110}{2.75}$ Substitution

$x \cdot 2.75 = 5(110)$ Cross products

$2.75x = 550$ Simplify.

$x = 200$ Divide each side by 2.75.

The roller coaster is 200 feet tall.

3. Alex is standing next to the Palmetto Building in Columbia, South Carolina. He is 6 feet tall and the length of his shadow is 9 feet. If the length of the shadow of the tower is 322.5 feet, how tall is the tower?

Interactive Lab
geometryonline.com

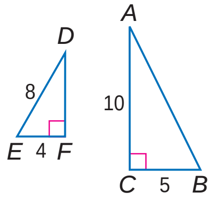
Personal Tutor at geometryonline.com

Check Your Understanding

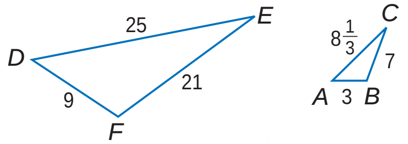
Example 1 (p. 398)

Determine whether each pair of triangles is similar. Justify your answer.

1.



2.



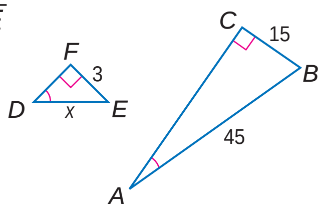
3. **MULTIPLE CHOICE** If $\triangle ABC$ and $\triangle FGH$ are two triangles such that $\angle A \cong \angle F$, which of the following would be sufficient to prove the triangles are similar?

A $\frac{BC}{GH} = \frac{AC}{FH}$ **B** $\frac{AC}{FH} = \frac{AB}{FG}$ **C** $\frac{AB}{FG} = \frac{BC}{GH}$ **D** $\frac{AB}{BC} = \frac{FG}{GH}$

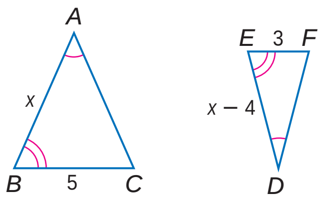
Example 2 (p. 399)

ALGEBRA **Identify the similar triangles. Find x and the measures of the indicated sides.**

4. $\overline{DE}$



5. $\overline{AB}$ and $\overline{DE}$



Example 3 (p. 400)

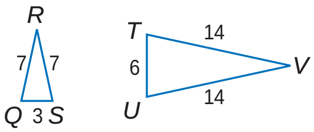
6. **COMMUNICATION** A cell phone tower casts a 100-foot shadow. At the same time, a 4 foot 6 inch post near the tower casts a shadow of 3 feet 4 inches. Find the height of the tower. (*Hint:* Make a drawing.)

Exercises

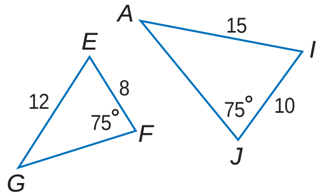
HOMEWORK HELP	
For Exercises	See Examples
7–10	1
11–13	3
14–17	2

Determine whether each pair of triangles is similar. Justify your answer.

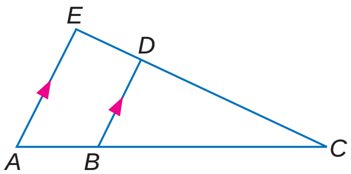
7.



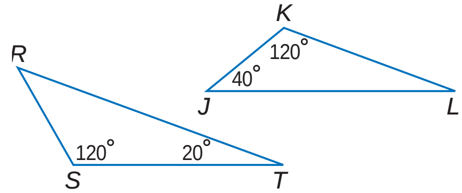
8.



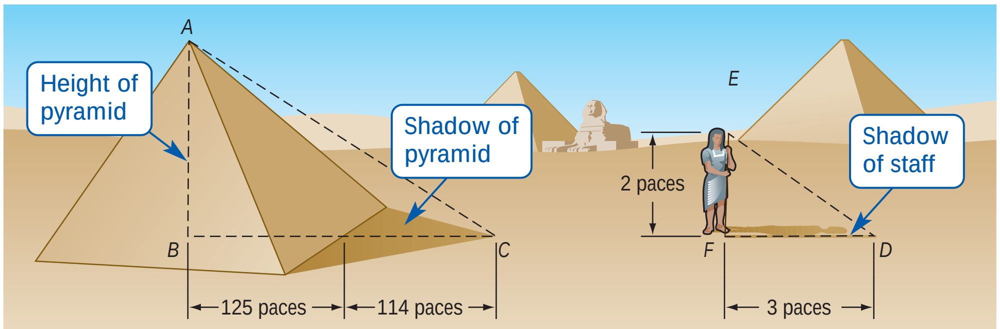
9.



10.

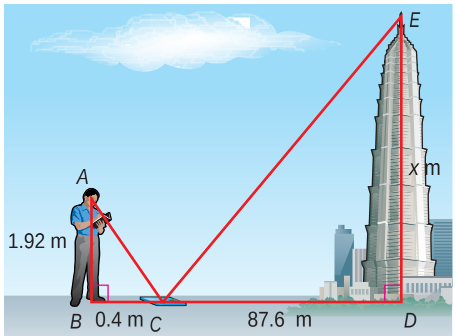


11. **HISTORY** The Greek mathematician Thales was the first to measure the height of a pyramid by using geometry. He showed that the ratio of a pyramid to a staff was equal to the ratio of one shadow to the other. If a pace is about 3 feet, approximately how tall was the pyramid at that time?



TOWERS For Exercises 12 and 13, use the following information.

To estimate the height of the Jin Mao Tower in Shanghai, a tourist sights the top of the tower in a mirror that is on the ground and faces upward.

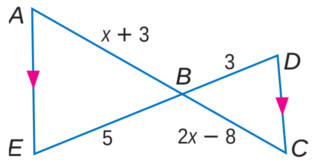


12. How tall is the tower?

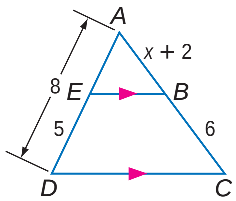
13. Why is the mirror reflection a better way to indirectly measure the tower than by using shadows in this situation?

ALGEBRA Identify the similar triangles, and find *x* and the measures of the indicated sides.

14. $\overline{AB}$ and $\overline{BC}$

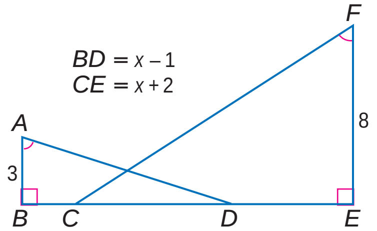


15. $\overline{AB}$ and $\overline{AC}$

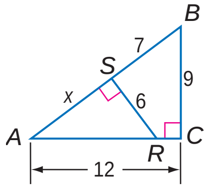


ALGEBRA Identify the similar triangles, and find x and the measures of the indicated sides.

16. $\overline{BD}$ and $\overline{EC}$



17. $\overline{AB}$ and $\overline{AS}$



COORDINATE GEOMETRY Triangles *ABC* and *TBS* have vertices $A(-2, -8)$, $B(4, 4)$, $C(-2, 7)$, $T(0, -4)$, and $S(0, 6)$.

18. Graph the triangles and prove that $\triangle ABC \sim \triangle TBS$.

19. Find the ratio of the perimeters of the two triangles.

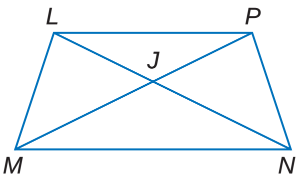
20. The lengths of the sides of triangle *ABC* are 6 centimeters, 4 centimeters, and 9 centimeters. Triangle *DEF* is similar to triangle *ABC*. The length of one side of triangle *DEF* is 36 centimeters. What is the greatest perimeter possible for triangle *DEF*? Explain.

PROOF For Exercises 21–25, write the type of proof specified.

21. Two-column proof

Given: $\overline{LP} \parallel \overline{MN}$

Prove: $\frac{LJ}{JN} = \frac{PJ}{JM}$

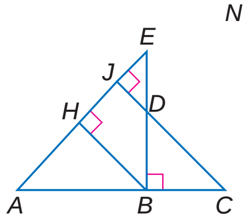


22. Paragraph proof

Given: $\overline{EB} \perp \overline{AC}$, $\overline{BH} \perp \overline{AE}$, $\overline{CJ} \perp \overline{AE}$

Prove: **a.** $\triangle ABH \sim \triangle DCB$

b. $\frac{BC}{BE} = \frac{BD}{BA}$



23. Two-column proof: If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar. (Theorem 7.2)

24. Two-column proof: If the measures of the legs of two right triangles are proportional, the triangles are similar.

25. Two-column proof: Similarity of triangles is reflexive, symmetric, and transitive. (Theorem 7.3)

EXTRA PRACTICE
See pages 814, 834.
Math Online
Self-Check Quiz at
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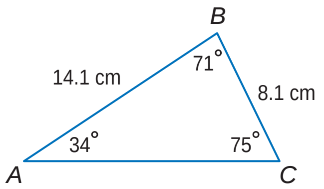
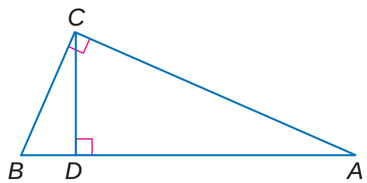
H.O.T. Problems

26. OPEN ENDED Draw a triangle that is similar to $\triangle ABC$. Explain how you know that it is similar.



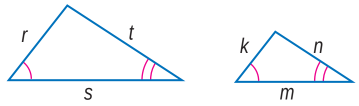
27. REASONING Is it possible that $\triangle ABC$ is not similar to $\triangle RST$ and that $\triangle RST$ is not similar to $\triangle EFG$, but that $\triangle ABC$ is similar to $\triangle EFG$? Explain.

28. CHALLENGE Triangle *ABC* is similar to the two triangles formed by altitude $\overline{CD}$, and these two triangles are similar to each other. Write three similarity statements about these triangles. Why are the triangles similar to each other?



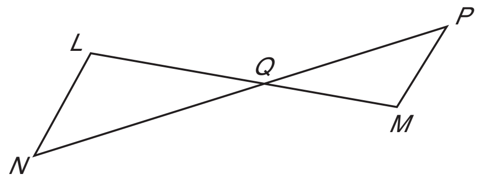
29. **FIND THE ERROR** Alicia and Jason were writing proportions for the similar triangles shown at the right. Who is correct? Explain your reasoning.

Alicia	Jason
$\frac{r}{k} = \frac{s}{m}$	$\frac{r}{k} = \frac{m}{s}$
$rm = ks$	$rs = km$



30. **Writing in Math** Compare and contrast the tests to prove triangles similar with the tests to prove triangles congruent.

31. In the figure below, $\overline{LM}$ intersects $\overline{NP}$ at point Q.



Which additional information would be enough to prove that $\triangle LNQ \sim \triangle MPQ$?

A $\overline{LQ}$ and $\overline{MQ}$ are congruent.

B $\angle QMP$ is a right angle.

C $\overline{LN}$ and $\overline{PM}$ are parallel.

D $\angle NLQ$ and $\angle PQM$ are congruent.

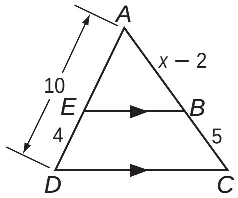
32. If $\overline{EB} \parallel \overline{DC}$, find the value of x.

F 9.5

G 5

H 4

J 2



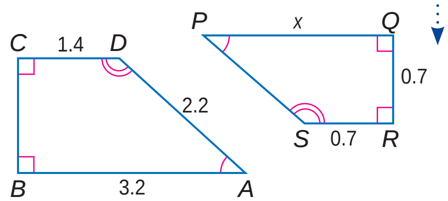
33. **REVIEW** What is the y-coordinate of the solution of the system of linear equations below?

$$5x + 3y = 1$$
$$-3x - 2y = -2$$

A -5 **C** 4

B -4 **D** 7

34. The pair of polygons is similar. Write a similarity statement, find x, BC, PS, and the scale factor. (Lesson 7-2)



Solve each proportion. (Lesson 7-1)

35. $\frac{1}{y} = \frac{3}{15}$

36. $\frac{6}{8} = \frac{7}{b}$

37. $\frac{20}{28} = \frac{m}{21}$

38. $\frac{16}{7} = \frac{9}{s}$

39. **ROLLER COASTERS** The sign in front of the Electric Storm roller coaster states ALL riders must be at least 54 inches tall to ride. If Adam is 5 feet 8 inches tall, can he ride the Electric Storm? Which law of logic leads you to this conclusion? (Lesson 2-4)

PREREQUISITE SKILL **Simplify.** (Pages 790–791)

40. $\sqrt{\frac{24}{64}}$

41. $\sqrt{\frac{75}{81}}$

42. $\sqrt{\frac{72}{144}}$

43. $\sqrt{\frac{32}{108}}$

Mid-Chapter Quiz

Lessons 7-1 through 7-3

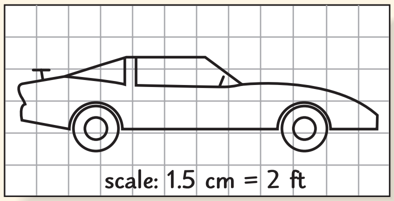
Solve each proportion. (Lesson 7-1)

1. $\frac{3}{4} = \frac{x}{12}$
2. $\frac{7}{3} = \frac{28}{z}$
3. $\frac{z}{40} = \frac{5}{8}$
4. $\frac{x + 2}{5} = \frac{14}{10}$
5. $\frac{3}{7} = \frac{7}{y - 3}$
6. $\frac{4 - x}{3 + x} = \frac{16}{25}$

7. **MAPS** The scale on a map shows that 1.5 centimeters represents 100 miles. If the distance on the map from Seattle, Washington, to Indianapolis, Indiana, is 28.1 centimeters, approximately how many miles apart are the two cities? (Lesson 7-1)

8. **BASEBALL** A player's slugging percentage is the ratio of the number of total bases from hits to the number of total at-bats. The ratio is converted to a decimal (rounded to three places) by dividing. If a professional baseball player has 281 total bases in 432 at-bats, what is his slugging percentage? (Lesson 7-1)

9. **MULTIPLE CHOICE** Miguel is using centimeter grid paper to make a scale drawing of his favorite car. The width of the drawing is 11.25 centimeters. How many feet long is the actual car? (Lesson 7-1)



A 15.0 ft

B 18.75 ft

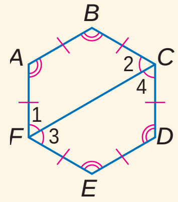
C 22.5 ft

D 33.0 ft

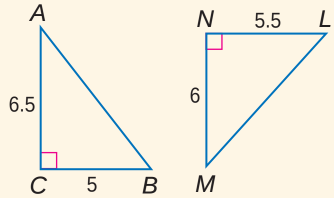
10. A 108-inch-long board is cut into two pieces that have lengths in the ratio 2:7. How long is each new piece? (Lesson 7-1)

Determine whether each pair of figures is similar. Justify your answer. (Lesson 7-2)

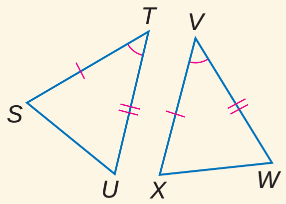
11.



12.



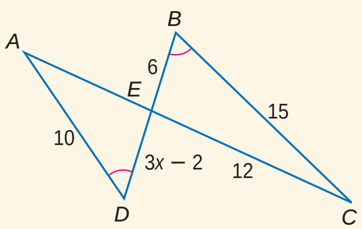
13.



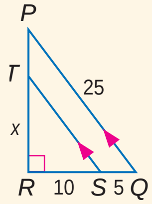
14. **ARCHITECTURE** The replica of the Eiffel Tower at an amusement park is $350\frac{2}{3}$ feet tall. The actual Eiffel Tower is 1052 feet tall. What is the scale factor comparing the amusement park tower to the actual tower? (Lesson 7-2)

Identify the similar triangles. Find *x* and the measures of the indicated sides. (Lesson 7-3)

15. $\overline{AE}$, $\overline{DE}$



16. $\overline{PT}$, $\overline{ST}$



7-4 Parallel Lines and Proportional Parts

Main Ideas

- Use proportional parts of triangles.
- Divide a segment into parts.

New Vocabulary

midsegment

GET READY for the Lesson

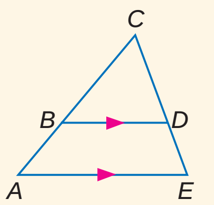
Street maps frequently have parallel and perpendicular lines. In Chicago, because of Lake Michigan, Lake Shore Drive runs at an angle between Oak Street and Ontario Street. City planners need to take this angle into account when determining dimensions of available land along Lake Shore Drive.

Proportional Parts of Triangles Nonparallel transversals that intersect parallel lines can be extended to form similar triangles. So the sides of the triangles are proportional.

THEOREM 7.4 *Triangle Proportionality Theorem*

If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional lengths.

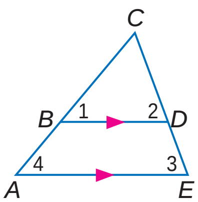
Example: If $\overline{BD} \parallel \overline{AE}$, $\frac{BA}{CB} = \frac{DE}{CD}$.



PROOF Theorem 7.4

Given: $\overline{BD} \parallel \overline{AE}$

Prove: $\frac{BA}{CB} = \frac{DE}{CD}$



Paragraph Proof:

Since $\overline{BD} \parallel \overline{AE}$, $\angle 4 \cong \angle 1$ and $\angle 3 \cong \angle 2$ because they are corresponding angles. Then, by AA Similarity, $\triangle ACE \sim \triangle BCD$. From the definition of similar polygons, $\frac{CA}{CB} = \frac{CE}{CD}$. By the Segment Addition Postulate, $CA = BA + CB$ and $CE = DE + CD$.

Substituting for CA and CE in the ratio, we get the following proportion.

$$\frac{BA + CB}{CB} = \frac{DE + CD}{CD}$$

$$\frac{BA}{CB} + \frac{CB}{CB} = \frac{DE}{CD} + \frac{CD}{CD} \quad \text{Rewrite as a sum.}$$

$$\frac{BA}{CB} + 1 = \frac{DE}{CD} + 1 \quad \frac{CB}{CB} = 1 \text{ and } \frac{CD}{CD} = 1$$

$$\frac{BA}{CB} = \frac{DE}{CD} \quad \text{Subtract 1 from each side.}$$

Study Tip

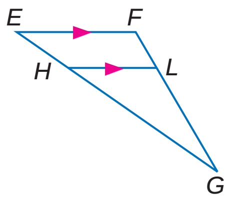
Overlapping Triangles

Trace two copies of $\triangle ACE$. Cut along $\overline{BD}$ to form $\triangle BCD$. Now $\triangle ACE$ and $\triangle BCD$ are no longer overlapping. Place the triangles side-by-side to compare corresponding angles and sides.

EXAMPLE Find the Length of a Side

1 **In $\triangle EFG$, $\overline{HL} \parallel \overline{EF}$, $EH = 9$, $HG = 21$, and $FL = 6$. Find LG.**

From the Triangle Proportionality Theorem, $\frac{EH}{HG} = \frac{FL}{LG}$.



Substitute the known measures.

$$\frac{9}{21} = \frac{6}{LG}$$

$9(LG) = (21)6$ Cross products

$9(LG) = 126$ Multiply.

$LG = 14$ Divide each side by 9.

1. In $\triangle EFG$, if $EH = 6$, $FL = 4$, and $LG = 18$, find HG.

Study Tip

Using Fractions

You can also rewrite $\frac{9}{21}$ as $\frac{3}{7}$. Then use your knowledge of fractions to find the missing denominator.

× 2

$$\frac{3}{7} = \frac{6}{?}$$

× 2

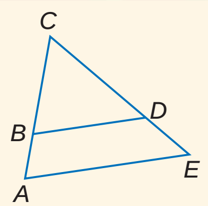
The correct denominator is 14.

Proportional parts of a triangle can also be used to prove the converse of Theorem 7.4.

THEOREM 7.5 Converse of the Triangle Proportionality Theorem

If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.

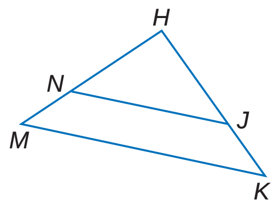
Example: If $\frac{BA}{CB} = \frac{DE}{CD}$, then $\overline{BD} \parallel \overline{AE}$.



You will prove Theorem 7.5 in Exercise 42.

EXAMPLE Determine Parallel Lines

2 **In $\triangle HKM$, $HM = 15$, $HN = 10$, and $\overline{HJ}$ is twice the length of $\overline{JK}$. Determine whether $\overline{NJ} \parallel \overline{MK}$. Explain.**



$HM = HN + NM$ Segment Addition Postulate

$15 = 10 + NM$ $HM = 15$, $HN = 10$

$5 = NM$ Subtract 10 from each side.

In order to show $\overline{NJ} \parallel \overline{MK}$, we must show that $\frac{HN}{NM} = \frac{HJ}{JK}$.

$\frac{HN}{NM} = \frac{10}{5}$ or 2. Let $JK = x$. Then $HJ = 2x$. So, $\frac{HJ}{JK} = \frac{2x}{x}$ or 2.

Thus, $\frac{HN}{NM} = \frac{HJ}{JK} = 2$. Since the sides have proportional lengths, $\overline{NJ} \parallel \overline{MK}$.

2. In $\triangle HKM$, $\overline{NM}$ is half the length of $\overline{NH}$, $HJ = 10$, and $JK = 6$. Determine whether $\overline{NJ} \parallel \overline{MK}$.

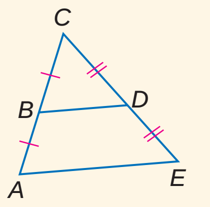
A **midsegment** of a triangle is a segment with endpoints that are the midpoints of two sides of the triangle.

THEOREM 7.6 Triangle Midsegment Theorem

A midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side.

Example: If B and D are midpoints of $\overline{AC}$ and $\overline{EC}$, respectively,

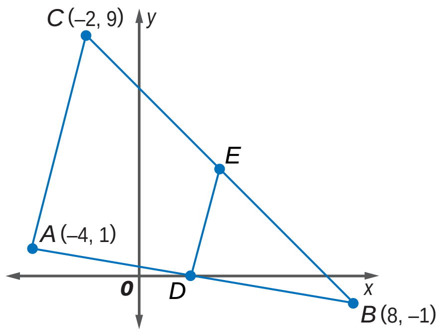
$\overline{BD} \parallel \overline{AE}$ and $BD = \frac{1}{2}AE$.



You will prove Theorem 7.6 in Exercise 43.

EXAMPLE Midsegment of a Triangle

3 **Triangle *ABC* has vertices *A*(-4, 1), *B*(8, -1), and *C*(-2, 9). $\overline{DE}$ is a midsegment of △*ABC*.**



a. Find the coordinates of *D* and *E*.

Use the Midpoint Formula to find the midpoints of $\overline{AB}$ and $\overline{CB}$.

$$D\left(\frac{-4+8}{2}, \frac{1+(-1)}{2}\right) = D(2, 0)$$

$$E\left(\frac{-2+8}{2}, \frac{9+(-1)}{2}\right) = E(3, 4)$$

b. Verify that $\overline{AC}$ is parallel to $\overline{DE}$.

If the slopes of $\overline{AC}$ and $\overline{DE}$ are equal, $\overline{AC} \parallel \overline{DE}$.

slope of $\overline{AC} = \frac{9-1}{-2-(-4)}$ or 4 slope of $\overline{DE} = \frac{4-0}{3-2}$ or 4

Because the slopes of $\overline{AC}$ and $\overline{DE}$ are equal, $\overline{AC} \parallel \overline{DE}$.

c. Verify that $DE = \frac{1}{2}AC$.

First, use the Distance Formula to find AC and DE.

$$AC = \sqrt{[-2-(-4)]^2 + (9-1)^2} \qquad DE = \sqrt{(3-2)^2 + (4-0)^2}$$

$$= \sqrt{4+64} \text{ or } \sqrt{68} \qquad = \sqrt{1+16} \text{ or } \sqrt{17}$$

$$\frac{DE}{AC} = \frac{\sqrt{17}}{\sqrt{68}}$$

$$= \sqrt{\frac{17}{68}} \text{ or } \frac{1}{2} \qquad \text{If } \frac{DE}{AC} = \frac{1}{2}, \text{ then } DE = \frac{1}{2}AC.$$

Study Tip

Look Back

To review the **Distance and Midpoint Formulas**, see Lesson 1-3.

Triangle *JKL* has vertices *J*(2, 5), *K*(-4, -1), and *L*(6, -3). $\overline{MN}$ is the midsegment of *JKL* and is parallel to $\overline{KL}$.

3A. Find the coordinates of M and N.

3B. Verify that $\overline{KL} \parallel \overline{MN}$.

3C. Verify that $MN = \frac{1}{2}KL$.

Divide Segments Proportionally We have seen that parallel lines cut the sides of a triangle into proportional parts. Three or more parallel lines also separate transversals into proportional parts. If the ratio is 1, they separate the transversals into congruent parts.

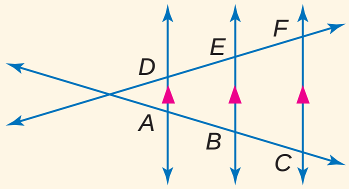
Study Tip

Three Parallel Lines

Corollary 7.1 is a special case of Theorem 7.4. In some drawings, the transversals are not shown to intersect. But, if extended, they will intersect and therefore, form triangles with each parallel line and the transversals.

COROLLARIES

7.1 If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.



Example: If $\overleftrightarrow{DA} \parallel \overleftrightarrow{EB} \parallel \overleftrightarrow{FC}$, then $\frac{AB}{BC} = \frac{DE}{EF}$, $\frac{AC}{DF} = \frac{BC}{EF}$, and $\frac{AC}{BC} = \frac{DF}{EF}$.

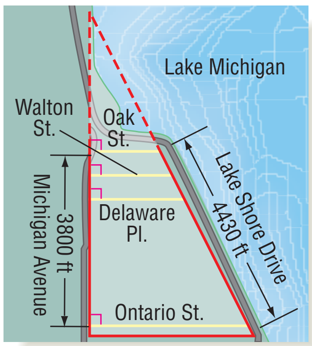
7.2 If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

Example: If $\overline{AB} \cong \overline{BC}$, then $\overline{DE} \cong \overline{EF}$.

You will prove Corollaries 7.1 and 7.2 in Exercises 40 and 41, respectively.

EXAMPLE 4 Proportional Segments

MAPS Refer to the map at the beginning of the lesson. The streets from Oak Street to Ontario Street are all parallel to each other. If the distance from Delaware Place to Walton Street along Michigan Avenue is about 411 feet, what is the distance between those streets along Lake Shore Drive?



Notice that the streets form the bottom portion of a triangle that is cut by parallel lines. So you can use the Triangle Proportionality Theorem.

$$\frac{\text{Delaware to Walton}}{\text{Oak to Ontario}}_{\text{(Michigan Avenue)}} = \frac{\text{Delaware to Walton}}{\text{Oak to Ontario}}_{\text{(Lake Shore Drive)}} \quad \text{Triangle Proportionality Theorem}$$

$$\frac{411}{3800} = \frac{x}{4430} \quad \text{Substitution}$$

$$3800 \cdot x = 411(4430) \quad \text{Cross products}$$

$$3800x = 1{,}820{,}730 \quad \text{Multiply.}$$

$$x = 479 \quad \text{Divide each side by 3800.}$$

The distance from Delaware Place to Walton Street along Lake Shore Drive is about 479 feet.

4. The distance from Delaware Place to Ontario Street along Lake Shore Drive is 2555 feet. What is the distance between these streets along Michigan Avenue?

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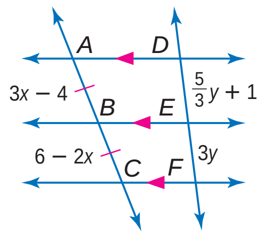
EXAMPLE Congruent Segments

5 **Find *x* and *y*.**



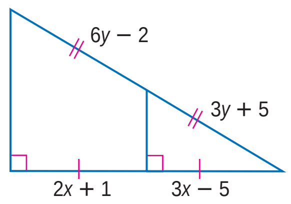
To find *x*:

$AB = BC$	Given
$3x - 4 = 6 - 2x$	Substitution
$5x - 4 = 6$	Add $2x$ to each side.
$5x = 10$	Add 4 to each side.
$x = 2$	Divide each side by 5.

To find *y*:

$\overline{DE} \cong \overline{EF}$	Parallel lines that cut off congruent segments on one transversal cut off congruent segments on every transversal.
$DE = EF$	Definition of congruent segments
$3y = \frac{5}{3}y + 1$	Substitution
$9y = 5y + 3$	Multiply each side by 3 to eliminate the denominator.
$4y = 3$	Subtract $5y$ from each side.
$y = \frac{3}{4}$	Divide each side by 4.

5.



Animation
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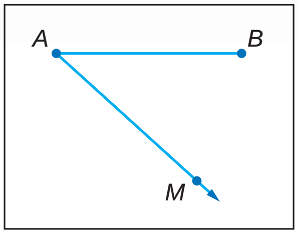
A segment cannot be separated into three congruent parts by constructing perpendicular bisectors. To do this, you must use parallel lines and the similarity theorems from this lesson. This technique can be used to separate a segment into any number of congruent parts.

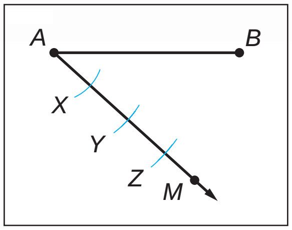
CONSTRUCTION

Trisect a Segment

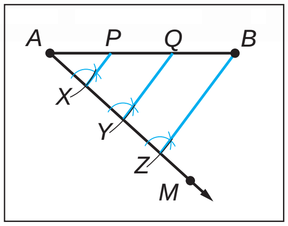
Step 1 Draw $\overline{AB}$ to be trisected. Then draw $\overline{AM}$.



Step 2 With the compass at *A*, mark off an arc that intersects $\overrightarrow{AM}$ at *X*. Use the same compass setting to construct $\overline{XY}$ and $\overline{YZ}$ congruent to $\overline{AX}$.



Step 3 Draw $\overline{ZB}$. Then construct lines through *Y* and *X* that are parallel to $\overline{ZB}$. Label the intersection points on $\overline{AB}$ as *P* and *Q*.

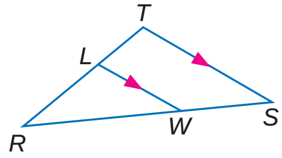


Conclusion: Since parallel lines cut off congruent segments on transversals, $\overline{AP} \cong \overline{PQ} \cong \overline{QB}$.

✓ Check Your Understanding

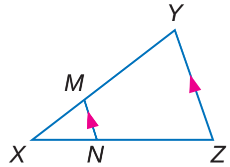
Example 1 (p. 406)

For Exercises 1 and 2, refer to $\triangle RST$.



1. If $RL = 5$, $RT = 9$, and $WS = 6$, find RW.
2. If $TR = 8$, $LR = 3$, and $RW = 6$, find WS.

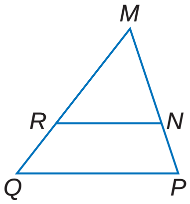
For Exercises 3 and 4, refer to $\triangle XYZ$.



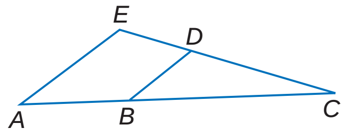
3. If $XM = 4$, $XN = 6$, and $NZ = 9$, find XY.
4. If $XN = t - 2$, $NZ = t + 1$, $XM = 2$, and $XY = 10$, solve for t.

Example 2 (p. 406)

5. In $\triangle MQP$, $MP = 25$, $MN = 9$, $MR = 4.5$, and $MQ = 12.5$. Determine whether $\overline{RN} \parallel \overline{QP}$. Justify your answer.



6. In $\triangle ACE$, $ED = 8$, $DC = 20$, $BC = 25$, and $AB = 12$. Determine whether $\overline{AE} \parallel \overline{BD}$. Justify your answer.



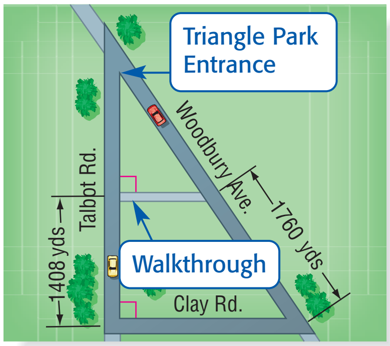
Example 3 (p. 407)

COORDINATE GEOMETRY For Exercises 7–9, use the following information. Triangle ABC has vertices $A(-2, 6)$, $B(-4, 0)$, and $C(10, 0)$. $\overline{DE}$ is a midsegment parallel to $\overline{BC}$.

7. Find the coordinates of D and E.
8. Verify that $\overline{DE}$ is parallel to $\overline{BC}$.
9. Verify that $DE = \frac{1}{2}BC$.

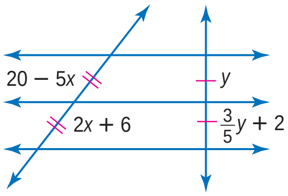
Example 4 (p. 408)

10. **MAPS** The distance along Talbot Road from the Triangle Park entrance to the Walkthrough is 880 yards. If the Walkthrough is parallel to Clay Road, find the distance from the entrance to the Walkthrough along Woodbury.

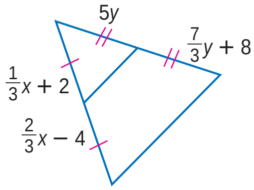


Example 5 (p. 409)

11. Find x and y.



12. Find x and y.

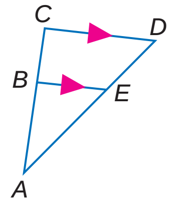


Exercises

HOMEWORK HELP

For Exercises	See Examples
13–19	1
20–23	2
24–27	3
28–31	4
32, 33	5

For Exercises 13–15, refer to $\triangle ACD$.

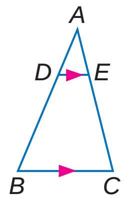


13. Find ED if $AB = 6$, $BC = 4$, and $AE = 9$.

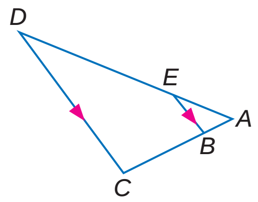
14. Find AE if $AB = 12$, $AC = 16$, and $ED = 5$.

15. Find CD if $AE = 8$, $ED = 4$, and $BE = 6$.

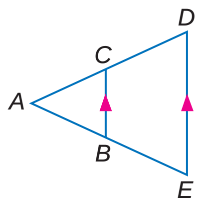
16. If $DB = 24$, $AE = 3$, and $EC = 18$, find AD.



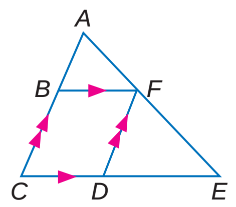
17. Find x and ED if $AE = 3$, $AB = 2$, $BC = 6$, and $ED = 2x - 3$.



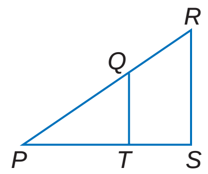
18. Find x, AC, and CD if $AC = x - 3$, $BE = 20$, $AB = 16$, and $CD = x + 5$.



19. Find BC, FE, CD, and DE if $AB = 6$, $AF = 8$, $BC = x$, $CD = y$, $DE = 2y - 3$, and $FE = x + \frac{10}{3}$.



Determine whether $\overline{QT} \parallel \overline{RS}$. Justify your answer.



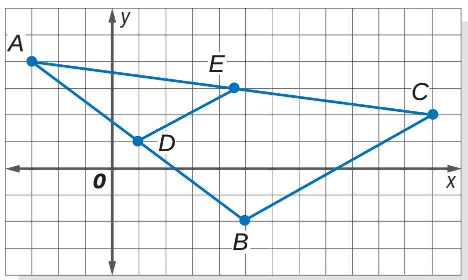
20. $PR = 30$, $PQ = 9$, $PT = 12$, and $PS = 18$

21. $QR = 22$, $RP = 65$, and SP is 3 times TS.

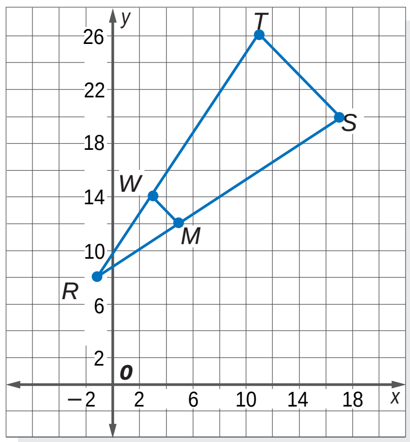
22. $TS = 8.6$, $PS = 12.9$, and PQ is half RQ.

23. $PQ = 34.88$, $RQ = 18.32$, $PS = 33.25$, and $TS = 11.45$

24. COORDINATE GEOMETRY Find the length of $\overline{BC}$ if $\overline{BC} \parallel \overline{DE}$ and $\overline{DE}$ is a midsegment of $\triangle ABC$.

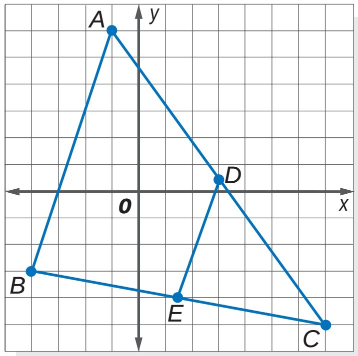


25. COORDINATE GEOMETRY Show that $\overline{WM} \parallel \overline{TS}$ and determine whether $\overline{WM}$ is a midsegment.



COORDINATE GEOMETRY **For Exercises 26 and 27, use the following information.**

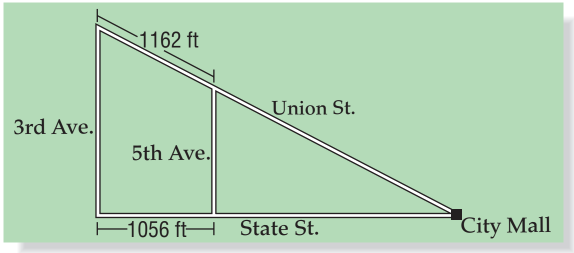
Triangle ABC has vertices $A(-1, 6)$, $B(-4, -3)$, and $C(7, -5)$. $\overline{DE}$ is a midsegment.

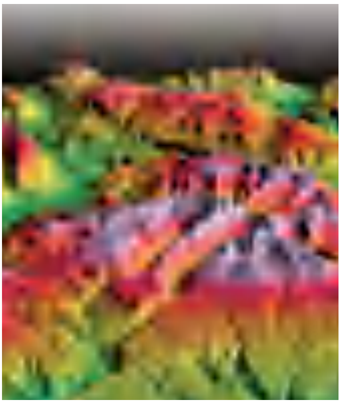


26. Verify that $\overline{DE}$ is parallel to $\overline{AB}$.

27. Verify that $DE = \frac{1}{2}AB$.

28. **MAPS** Refer to the map at the right. Third Avenue and 5th Avenue are parallel. If the distance from 3rd Avenue to City Mall along State Street is 3201 feet, find the distance between 5th Avenue and City Mall along Union Street. Round to the nearest tenth.





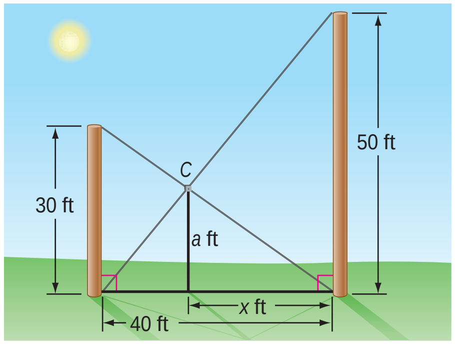
Real-World Link

Modern map-making techniques use images taken from space to produce accurate representations on paper. In February 2000, the crew of the space shuttle *Endeavor* collected a trillion radar images of 46 million square miles of Earth.

Source: www.nima.mil

CONSTRUCTION **For Exercises 29–31, use the following information and diagram.**

Two poles, 30 feet and 50 feet tall, are 40 feet apart and perpendicular to the ground. The poles are supported by wires attached from the top of each pole to the bottom of the other, as in the figure. A coupling is placed at C where the two wires cross.



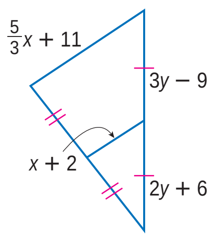
29. Find x, the distance from C to the taller pole.

30. How high above the ground is the coupling?

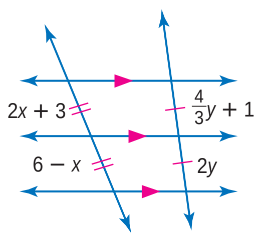
31. How far down the wire from the smaller pole is the coupling?

Find x and y.

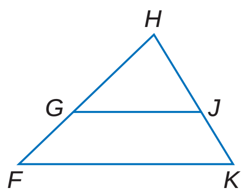
32.



33.



Find x so that $\overline{GJ} \parallel \overline{EK}$.



34. $GE = 12$, $HG = 6$, $HJ = 8$, $JK = x - 4$

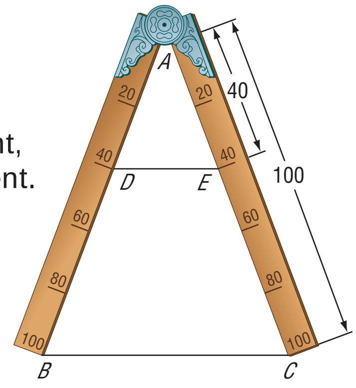
35. $HJ = x - 5$, $JK = 15$, $EG = 18$, $HG = x - 4$

36. $GH = x + 3.5$, $HJ = x - 8.5$, $EH = 21$, $HK = 7$

37. **COORDINATE GEOMETRY** Given $A(2, 12)$ and $B(5, 0)$, find the coordinates of P such that P separates $\overline{AB}$ into two parts with lengths in a ratio of 2 to 1.

38. **COORDINATE GEOMETRY** In $\triangle LMN$, $\overline{PR}$ divides $\overline{NL}$ and $\overline{MN}$ proportionally. If the vertices are $N(8, 20)$, $P(11, 16)$, and $R(3, 8)$ and $\frac{LP}{PN} = \frac{2}{1}$, find the coordinates of L and M.

39. **MATH HISTORY** The sector compass was a tool perfected by Galileo in the sixteenth century for measurement and calculation. To draw a segment two-fifths the length of a given segment, align the ends of the arms with the given segment. Then draw a segment at the 40 mark. Write a justification that explains why the sector compass works for proportional measurement.



PROOF **Write a paragraph proof for each corollary.**

40. Corollary 7.1

41. Corollary 7.2

EXTRA PRACTICE
See pages 814, 834.
Math Online
Self-Check Quiz at
geometryonline.com

PROOF **Write a two-column proof of each theorem.**

42. Theorem 7.5

43. Theorem 7.6

CONSTRUCTION **Construct each segment as directed.**

44. a segment 8 centimeters long, separated into three congruent segments

45. a segment separated into four congruent segments

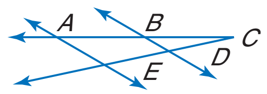
46. a segment separated into two segments in which their lengths have a ratio of 1 to 4

H.O.T. Problems

47. **REASONING** Explain how you would know if a line that intersects two sides of a triangle is parallel to the third side.

48. **OPEN ENDED** Draw two segments that are intersected by three lines so that the parts are proportional. Then draw a counterexample.

49. **PROOF** Write a two-column proof.
Given: $AB = 4$ and $BC = 4$, $CD = DE$
Prove: $\overline{BD} \parallel \overline{AE}$



50. **CHALLENGE** Copy the figure that accompanies Corollary 7.1 on page 408. Draw $\overline{DC}$. Let G be the intersection point of $\overline{DC}$ and $\overline{BE}$. Using that segment, explain how you could prove $\frac{AB}{BC} = \frac{DE}{EF}$.

CHALLENGE **Draw any quadrilateral *ABCD* on a coordinate plane. Points *E*, *F*, *G*, and *H* are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively.**

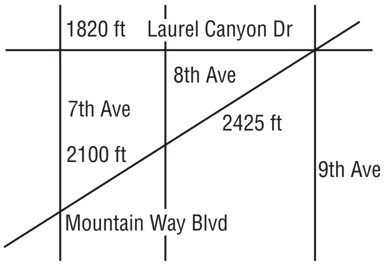
51. Connect the midpoints to form quadrilateral $EFGH$. Describe what you know about the sides of quadrilateral $EFGH$.

52. Will the same reasoning work with five-sided polygons? Explain.

53. **Writing in Math** Refer to the information on city planning on page 405. Describe the geometry facts a city planner needs to know to explain why the block between Chestnut and Pearson is longer on Lake Shore Drive than on Michigan Avenue.

STANDARDIZED TEST PRACTICE

54. The streets 7th Avenue, 8th Avenue, and 9th Avenue are parallel. They all intersect Laurel Canyon Drive and Mountain Way Boulevard.

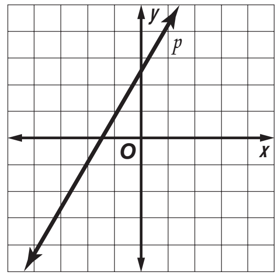


If all these streets are straight line segments, how long is Laurel Canyon Drive between 7th Avenue and 9th Avenue?

A 2101.7 ft **C** 3921.7 ft

B 2145 ft **D** 4436 ft

55. **REVIEW** What will happen to the slope of line p if the line is shifted so that the y-intercept stays the same and the x-intercept increases?



F The slope will change from negative to positive.

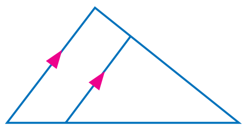
G The slope will become zero.

H The slope will decrease.

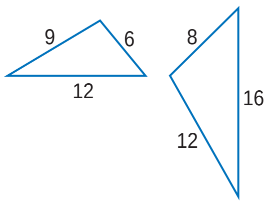
J The slope will increase.

Determine whether each pair of triangles is similar. Justify your answer. (Lesson 7-3)

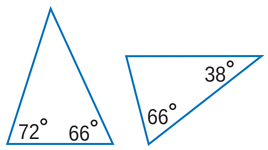
56.



57.

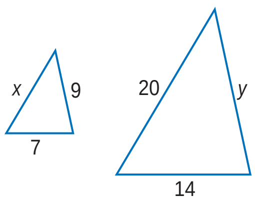


58.

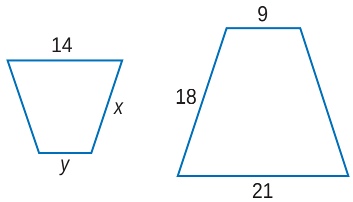


Each pair of polygons is similar. Find x and y. (Lesson 7-2)

59.



60.



61. **ALGEBRA** Quadrilateral $ABCD$ has a perimeter of 95 centimeters. Find the length of each side if $AB = 3a + 2$, $BC = 2(a - 1)$, $CD = 6a + 4$, and $AD = 5a - 5$. (Lesson 1-6)

PREREQUISITE SKILL **Write all the pairs of corresponding parts for each pair of congruent triangles.** (Lesson 4-3)

62. $\triangle ABC \cong \triangle DEF$

63. $\triangle RST \cong \triangle XYZ$

64. $\triangle PQR \cong \triangle KLM$

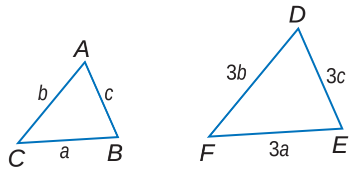
7-5 Parts of Similar Triangles

Main Ideas

- Recognize and use proportional relationships of corresponding perimeters of similar triangles.
- Recognize and use proportional relationships of corresponding angle bisectors, altitudes, and medians of similar triangles.

Professional photographers often use 35-millimeter film cameras for clear images. The camera lens was 6.16 meters from this Dale Chihuly glass sculpture when the photographer took this photograph. The image on the film is 35 millimeters tall. Similar triangles enable us to find the height of the actual sculpture.

Perimeters Triangle ABC is similar to $\triangle DEF$ with a scale factor of 1:3. You can use variables and the scale factor to compare their perimeters. Let the measures of the sides of $\triangle ABC$ be a, b, and c. The measures of the corresponding sides of $\triangle DEF$ would be $3a$, $3b$, and $3c$.



$$\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{a + b + c}{3a + 3b + 3c} = \frac{1(a + b + c)}{3(a + b + c)} \text{ or } \frac{1}{3}$$

The perimeters are in the same proportion as the side measures of the two similar figures. This suggests Theorem 7.7, the Proportional Perimeters Theorem.

THEOREM 7.7 *Proportional Perimeters Theorem*

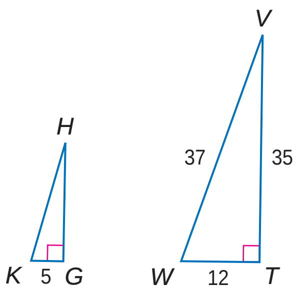
If two triangles are similar, then the perimeters are proportional to the measures of corresponding sides.

You will prove Theorem 7.7 in Exercise 14.

EXAMPLE Perimeters of Similar Triangles

1 **If $\triangle GHK \sim \triangle TVW$, $TV = 35$, $VW = 37$, $WT = 12$, and $KG = 5$, find the perimeter of $\triangle GHK$.**

The perimeter of $\triangle TVW = 35 + 37 + 12$ or 84.
Use a proportion to find the perimeter of $\triangle GHK$.
Let x represent the perimeter of $\triangle GHK$.



(continued on the next page)

$$\frac{KG}{WT} = \frac{\text{perimeter of } \triangle GHK}{\text{perimeter of } \triangle TVW}$$ Proportional Perimeter Theorem

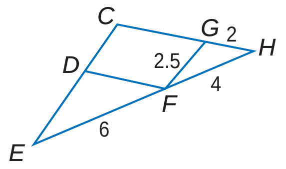
$$\frac{5}{12} = \frac{x}{84}$$ Substitution

$12x = 420$ Cross products

$x = 35$ Divide each side by 12.

The perimeter of $\triangle GHK$ is 35 units.

1. If $\triangle DEF \sim \triangle GFH$, find the perimeter of $\triangle DEF$.

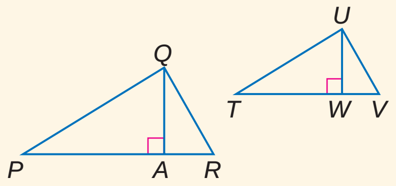


Special Segments of Similar Triangles Think about a triangle drawn on a piece of paper being placed on a copy machine and either enlarged or reduced. The copy is similar to the original triangle. Now suppose you drew in special segments of a triangle, such as the altitudes, medians, or angle bisectors, on the original. When you enlarge or reduce that original triangle, all of those segments are enlarged or reduced at the same rate. This conjecture is formally stated in Theorems 7.8, 7.9, and 7.10.

THEOREMS — Special Segments of Similar Triangles

7.8 If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides.

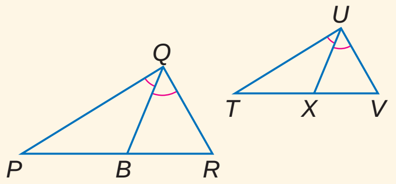
Abbreviation: $\sim \triangle$s *have corr. altitudes proportional to the corr. sides.*



$$\frac{QA}{UW} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

7.9 If two triangles are similar, then the measures of the corresponding angle bisectors of the triangles are proportional to the measures of the corresponding sides.

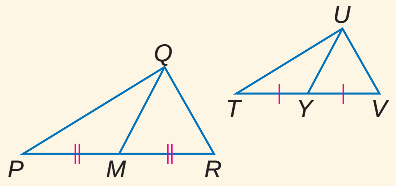
Abbreviation: $\sim \triangle$s *have corr.* $\angle$ *bisectors proportional to the corr. sides.*



$$\frac{QB}{UX} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

7.10 If two triangles are similar, then the measures of the corresponding medians are proportional to the measures of the corresponding sides.

Abbreviation: $\sim \triangle$s *have corr. medians proportional to the corr. sides.*



$$\frac{QM}{UY} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

You will prove Theorems 7.8 and 7.10 in Check Your Progress 2 and Exercise 3, respectively.

Study Tip

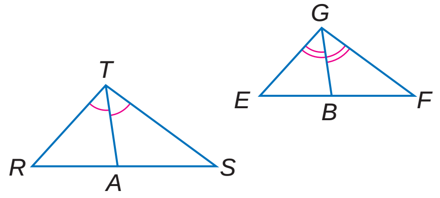
Points of Concurrency

In a triangle that is enlarged or reduced proportionally, the points of concurrency remain in the same position.

EXAMPLE Write a Proof

2 **Write a paragraph proof of Theorem 7.9.**

Since the corresponding angles to be bisected are chosen at random, we need not prove this for every pair of bisectors.



Given: $\triangle RTS \sim \triangle EGF$

$\overline{TA}$ and $\overline{GB}$ are angle bisectors.

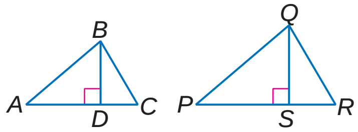
Prove: $\frac{TA}{GB} = \frac{RT}{EG}$

Paragraph Proof: Because corresponding angles of similar triangles are congruent, $\angle R \cong \angle E$ and $\angle RTS \cong \angle EGF$. Since $\angle RTS$ and $\angle EGF$ are bisected, we know that $\frac{1}{2}m\angle RTS = \frac{1}{2}m\angle EGF$ or $m\angle RTA = m\angle EGB$. This makes $\angle RTA \cong \angle EGB$ and $\triangle RTA \sim \triangle EGB$ by AA Similarity. Thus, $\frac{TA}{GB} = \frac{RT}{EG}$.

2. Write a paragraph proof of Theorem 7.8.

Given: $\triangle ABC \sim \triangle PQR$

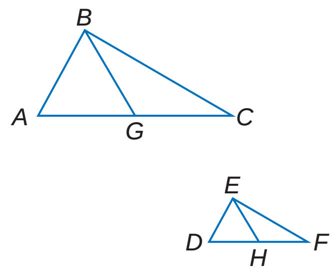
Prove: $\frac{BD}{QS} = \frac{BA}{QP}$



The medians of similar triangles are also proportional.

EXAMPLE Medians of Similar Triangles

3 **In the figure, $\triangle ABC \sim \triangle DEF$. $\overline{BG}$ is a median of $\triangle ABC$, and $\overline{EH}$ is a median of $\triangle DEF$. Find EH if $BC = 30$, $BG = 15$, and $EF = 15$.**



Let x represent EH.

$\frac{BG}{EH} = \frac{BC}{EF}$ Write a proportion.

$\frac{15}{x} = \frac{30}{15}$ $BG = 15$, $EH = x$, $BC = 30$, and $EF = 15$

$30x = 225$ Cross products

$x = 7.5$ Divide each side by 30.

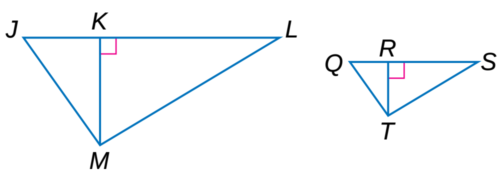
Thus, $EH = 7.5$.

Study Tip

Proportions

This proportion can be written in different ways. The proportion $\frac{BG}{BC} = \frac{EH}{EF}$ gives the same result.

3. In the figure, $\triangle JLM \sim \triangle QST$. $\overline{KM}$ is an altitude of $\triangle JLM$, and $\overline{RT}$ is an altitude of $\triangle QST$. Find RT if $JL = 12$, $QS = 8$, and $KM = 5$.



The theorems about the relationships of special segments in similar triangles can be used to solve real-life problems.

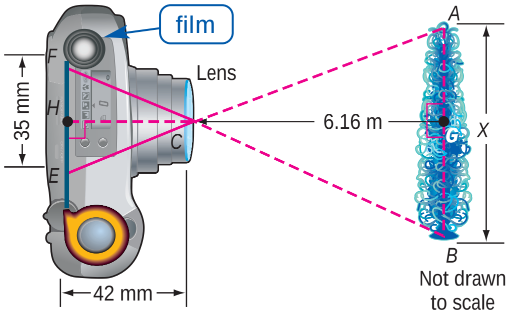
Real-World Link

The first consumer-oriented digital cameras were produced for sale in 1994 with a 640 × 480 pixel resolution. In 2005, a 12.8-megapixel camera could take a picture with 4368 × 2912 pixel resolution, which is a sharper picture than most computer monitors can display.

Source: www.howstuffworks.com

EXAMPLE Solve Problems with Similar Triangles

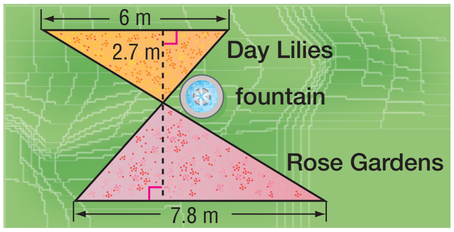
4 **PHOTOGRAPHY** **Refer to the application at the beginning of the lesson. The drawing below illustrates the position of the camera and the distance from the lens of the camera to the film. Find the height of the sculpture.**



$\triangle ABC$ and $\triangle EFC$ are similar. $\overline{CG}$ and $\overline{CH}$ are altitudes of $\triangle ABC$ and $\triangle EFC$, respectively. If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides. This leads to the proportion $\frac{AB}{EF} = \frac{GC}{HC}$.

$\frac{AB}{EF} = \frac{GC}{HC}$	Write the proportion.
$\frac{x \text{ m}}{35 \text{ mm}} = \frac{6.16 \text{ m}}{42 \text{ mm}}$	$AB = x$ m, $EF = 35$ mm, $GC = 6.16$ m, $HC = 42$ mm
$x \cdot 42 = 35(6.16)$	Cross products
$42x = 215.6$	Simplify.
$x \approx 5.13$	The sculpture is about 5.13 meters tall.

4. LANDSCAPING The landscaping team at a botanical garden is planning to add sidewalks around the fountain. The gardens form two similar triangles. Find the distance from the fountain to the rose gardens.



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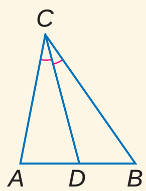
An angle bisector also divides the side of the triangle opposite the angle proportionally.

THEOREM 7.11 *Angle Bisector Theorem*

An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides.

Example: $\frac{AD}{DB} = \frac{AC}{BC}$ ← segments with vertex *A* ← segments with vertex *B*



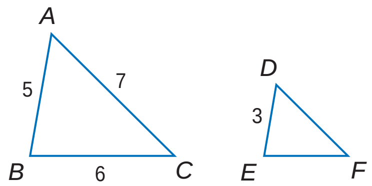
You will prove Theorem 7.11 in Exercise 15.

CHECK Your Understanding

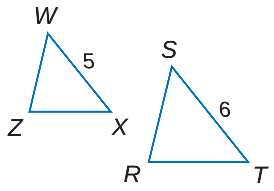
Example 1 (p. 415)

Find the perimeter of the given triangle.

1. $\triangle DEF$, if $\triangle ABC \sim \triangle DEF$, $AB = 5$, $BC = 6$, $AC = 7$, and $DE = 3$



2. $\triangle WZX$, if $\triangle WZX \sim \triangle SRT$, $ST = 6$, $WX = 5$, and the perimeter of $\triangle SRT = 15$



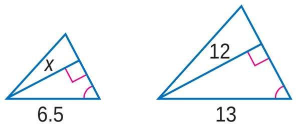
Example 2 (p. 417)

3. Write a two-column proof of Theorem 7.10.

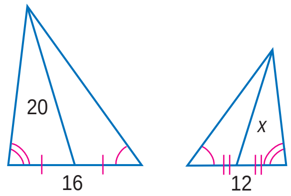
Example 3 (p. 417)

Find *x*.

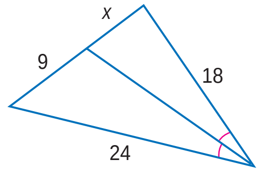
4.



5.



6.



Example 4 (p. 418)

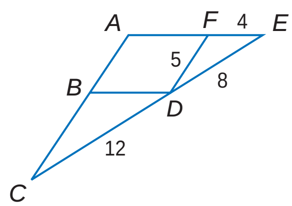
7. **PHOTOGRAPHY** The distance from the film to the lens in a camera is 10 centimeters. The film image is 3 centimeters high. Tamika is 165 centimeters tall. How far should she be from the camera in order for the photographer to take a full-length picture?

Exercises

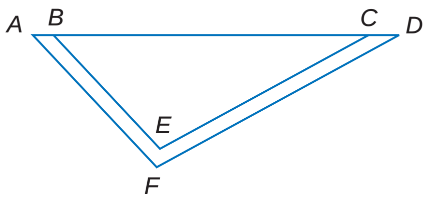
HOMEWORK HELP	
For Exercises	See Examples
8–13	1
14–18	2
19–22	3
23, 24	4

Find the perimeter of the given triangle.

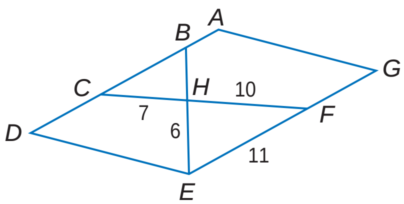
8. $\triangle BCD$, if $\triangle BCD \sim \triangle FDE$, $CD = 12$, $FD = 5$, $FE = 4$, and $DE = 8$



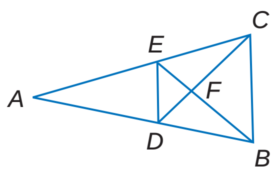
9. $\triangle ADF$, if $\triangle ADF \sim \triangle BCE$, $BC = 24$, $EB = 12$, $CE = 18$, and $DF = 21$



10. $\triangle CBH$, if $\triangle CBH \sim \triangle FEH$, $ADEG$ is a parallelogram, $CH = 7$, $FH = 10$, $FE = 11$, and $EH = 6$

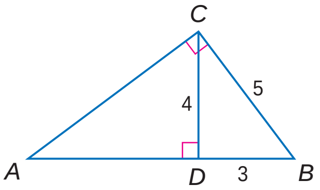


11. $\triangle DEF$, if $\triangle DEF \sim \triangle CBF$, perimeter of $\triangle CBF = 27$, $DF = 6$, $FC = 8$

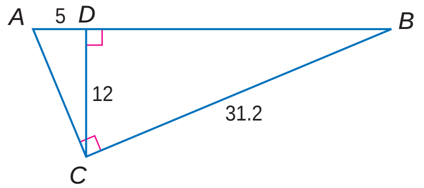


Find the perimeter of the given triangle.

12. $\triangle ABC$, if $\triangle ABC \sim \triangle CBD$, $CD = 4$, $DB = 3$, and $CB = 5$



13. $\triangle ABC$, if $\triangle ABC \sim \triangle CBD$, $AD = 5$, $CD = 12$, $BC = 31.2$

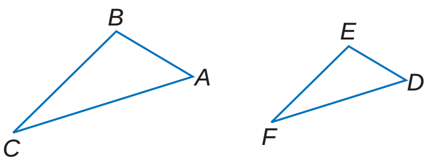


PROOF **For Exercises 14–18, write the indicated type of proof.**

14. Write a paragraph proof of Theorem 7.7.

Given: $\triangle ABC \sim \triangle DEF$ and $\frac{AB}{DE} = \frac{m}{n}$

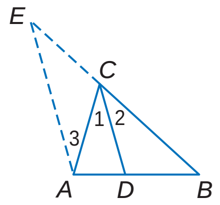
Prove: $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{m}{n}$



15. Two-column proof of the Angle Bisector Theorem (Theorem 7.11)

Given: $\overline{CD}$ bisects $\angle ACB$. By construction, $\overline{AE} \parallel \overline{CD}$.

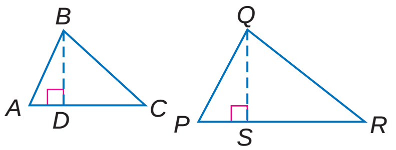
Prove: $\frac{AD}{AC} = \frac{BD}{BC}$



16. Paragraph proof

Given: $\triangle ABC \sim \triangle PQR$
$\overline{BD}$ is an altitude of $\triangle ABC$.
$\overline{QS}$ is an altitude of $\triangle PQR$.

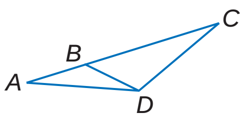
Prove: $\frac{QP}{BA} = \frac{QS}{BD}$



17. Flow proof

Given: $\angle C \cong \angle BDA$

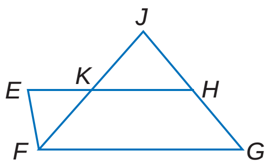
Prove: $\frac{AC}{DA} = \frac{AD}{BA}$



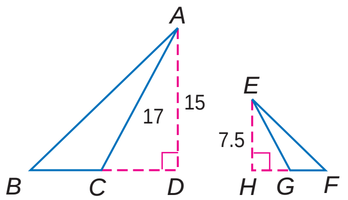
18. Two-column proof

Given: $\overline{JF}$ bisects $\angle EFG$.
$\overline{EH} \parallel \overline{FG}$, $\overline{EF} \parallel \overline{HG}$

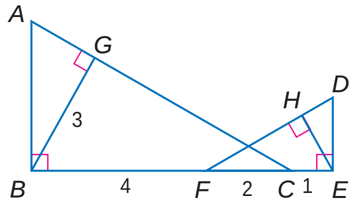
Prove: $\frac{EK}{KF} = \frac{GJ}{JF}$



19. Find EG if $\triangle ACB \sim \triangle EGF$, $\overline{AD}$ is an altitude of $\triangle ACB$, $\overline{EH}$ is an altitude of $\triangle EGF$, $AC = 17$, $AD = 15$, and $EH = 7.5$.

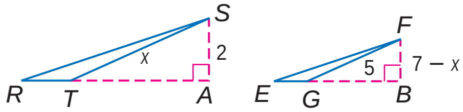


20. Find EH if $\triangle ABC \sim \triangle DEF$, $\overline{BG}$ is an altitude of $\triangle ABC$, $\overline{EH}$ is an altitude of $\triangle DEF$, $BG = 3$, $BF = 4$, $FC = 2$, and $CE = 1$.

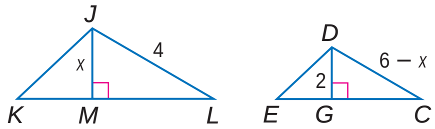


EXTRA PRACTICE
See pages 814, 834.
Math Online
Self-Check Quiz at
geometryonline.com

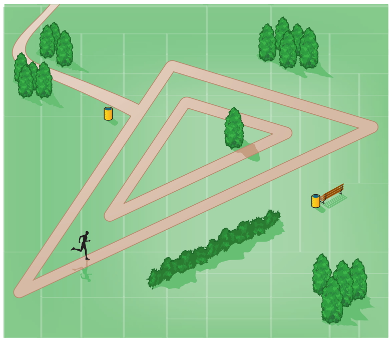
21. Find FB if $\overline{SA}$ and $\overline{FB}$ are altitudes and $\triangle RST \sim \triangle EFG$.



22. Find DC if $\overline{DG}$ and $\overline{JM}$ are altitudes and $\triangle KJL \sim \triangle EDC$.



23. PHYSICAL FITNESS A park has two similar triangular jogging paths as shown. The dimensions of the inner path are 300 meters, 350 meters, and 550 meters. The shortest side of the outer path is 600 meters. Will a jogger on the inner path run half as far as one on the outer path? Explain.



Real-World Career

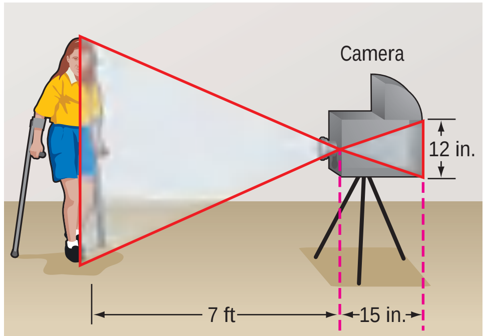
Photographer

Photographers need a good eye for composition. They are also proficient in developing film. Photographers who own their own business must be good at math and other business skills.

Math Online

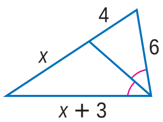
For more information, go to geometryonline.com.

24. PHOTOGRAPHY One of the first cameras invented was called a *camera obscura.* Light entered an opening in the front, and an image was reflected in the back of the camera, upside down, forming similar triangles. If the image of the person on the back of the camera is 12 inches, the distance from the opening to the person is 7 feet, and the camera itself is 15 inches long, how tall is the person being photographed?

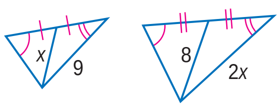


Find x.

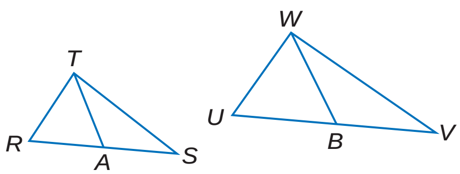
25.



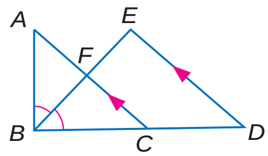
26.



27. Find UB if $\triangle RST \sim \triangle UVW$, $\overline{TA}$ and $\overline{WB}$ are medians, $TA = 8$, $RA = 3$, $WB = 3x - 6$, and $UB = x + 2$.

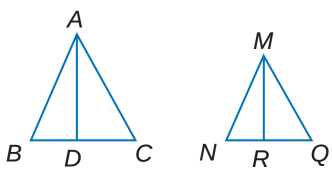


28. Find CF and BD if $\overline{BF}$ bisects $\angle ABC$ and $\overline{AC} \parallel \overline{ED}$, $BA = 6$, $BC = 7.5$, $AC = 9$, and $DE = 9$.

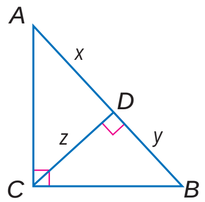


H.O.T. Problems

29. REASONING Explain what must be true about $\triangle ABC$ and $\triangle MNQ$ before you can conclude that $\frac{AD}{MR} = \frac{BA}{NM}$.



30. **CHALLENGE** $\overline{CD}$ is an altitude to the hypotenuse $\overline{AB}$. Make a conjecture about x, y, and z. Justify your reasoning.

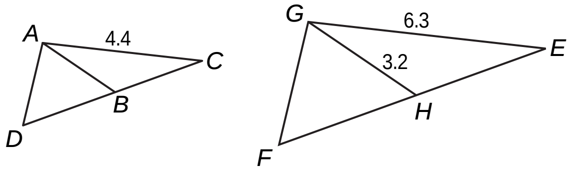


31. **OPEN ENDED** The perimeter of one triangle is 24 centimeters, and the perimeter of a second triangle is 36 centimeters. If the length of one side of the smaller triangle is 6, find possible lengths of the other sides of the triangles so that they are similar.

32. **Writing in Math** Explain how geometry is related to photography. Include a sketch of how a camera works showing the image and the film, and why the two isosceles triangles are similar.

STANDARDIZED TEST PRACTICE

33. In the figures below, $\overline{DB} \cong \overline{BC}$ and $\overline{FH} \cong \overline{HE}$.



If $\triangle ACD \sim \triangle GEF$, find the approximate length of $\overline{AB}$.

A 2.2
C 8.7
B 4.6
D 11.1

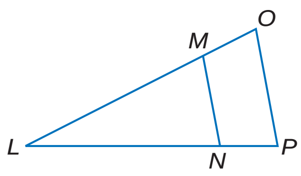
34. **REVIEW** Which shows 0.00234 written in scientific notation?

F 2.34×10^5
H 2.34×10^{-2}
G 2.34×10^3
J 2.34×10^{-3}

35. **REVIEW** The sum of three numbers is 180. Two of the numbers are the same, and each of them is one third of the greatest number. What is the least number?

A 30
C 45
B 36
D 72

Determine whether $\overline{MN} \parallel \overline{OP}$. Justify your answer. (Lesson 7-4)



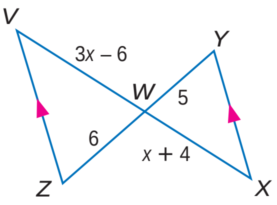
36. $LM = 7$, $LN = 9$, $LO = 14$, $LP = 16$

37. $LM = 6$, $MN = 4$, $LO = 9$, $OP = 6$

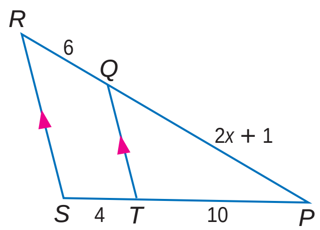
38. $LN = 12$, $NP = 4$, $LM = 15$, $MO = 5$

Identify the similar triangles. Find x and the measure(s) of the indicated side(s). (Lesson 7-3)

39. $\overline{VW}$ and $\overline{WX}$



40. $\overline{PQ}$



41. **BUSINESS** Elisa charges $5 to paint mailboxes and $4 per hour to mow lawns. Write an equation to represent the amount of money Elisa can earn from each homeowner. (Lesson 3-4)

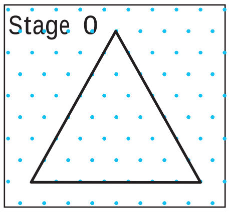
EXTEND 7-5

Geometry Lab
Sierpinski Triangle

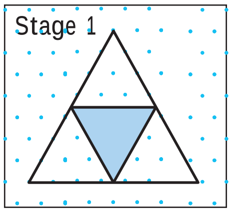
A **fractal** is a geometric figure that is created using iteration. **Iteration** is a process of repeating the same pattern over and over again. Fractals are **self-similar**, which means that the smaller and smaller details of the shape have the same geometric characteristics as the original form.

ACTIVITY

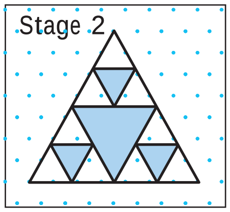
Stage 0 On isometric dot paper, draw an equilateral triangle in which each side is 8 units long.



Stage 1 Connect the midpoints of each side to form another triangle. Shade the center triangle.



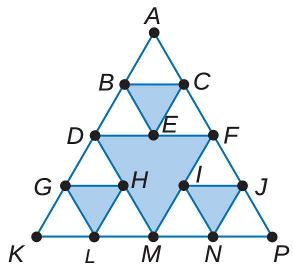
Stage 2 Repeat the process using the three nonshaded triangles. Connect the midpoints of each side to form other triangles.



If you repeat this process indefinitely, the figure that results is called the **Sierpinski Triangle**.

Analyze the Results

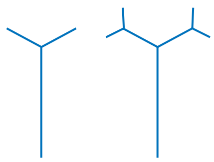
1. Continue the process through Stage 4. How many nonshaded triangles do you have at Stage 4?
2. What is the perimeter of a nonshaded triangle in Stage 0 through Stage 4?
3. If you continue the process indefinitely, describe what will happen to the perimeter of each nonshaded triangle.



4. Study $\triangle DFM$ in Stage 2 of the Sierpinski Triangle shown at the right. Is this an equilateral triangle? Are $\triangle BCE$, $\triangle GHL$, or $\triangle IJN$ equilateral?
5. Is $\triangle BCE \sim \triangle DFM$? Explain your answer.
6. How many Stage 1 Sierpinski triangles are there in Stage 2?

For Exercises 7 and 8, use the following information.

A *fractal tree* can be drawn by making two new branches from the endpoint of each original branch, each one-third as long as the previous branch.



Stage 1 **Stage 2**

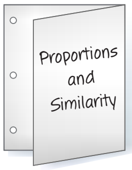
7. Draw Stages 3 and 4 of a fractal tree. How many total branches do you have in Stages 1 through 4? (Do not count the stems.)
8. Find a pattern to predict the number of branches at each stage.

CHAPTER 7 Study Guide and Review

Download Vocabulary Review from geometryonline.com

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Proportions (Lesson 7-1)

- For any numbers a and c and any nonzero numbers b and d, $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.

Similar Polygons and Triangles (Lessons 7-2 and 7-3)

- Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.
- Two triangles are similar if:
  AA: the two angles of one triangle are congruent to two angles of another triangle

  SSS: the measures of the corresponding sides of two triangles are proportional

  SAS: the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent

Proportional Parts (Lesson 7-4)

- If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional length.
- A midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side.

Parts of Similar Triangles (Lesson 7-5)

- If two triangles are similar, then each of the following are proportional: the perimeters, the measures of the corresponding altitudes, the measures of the corresponding angle bisectors of the triangles, and the measures of the corresponding medians.

Key Vocabulary

cross products (p. 381)
extremes (p. 381)
means (p. 381)
midsegment (p. 406)
proportion (p. 381)
ratio (p. 380)
scale factor (p. 389)
similar polygons (p. 388)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. The symbol $\sim$ means "is congruent to."
2. A midsegment of a triangle is a segment with endpoints that are the midpoints of two sides of the triangle.
3. Two polygons are similar if and only if their corresponding angles are congruent and the measures of the corresponding sides are equal.
4. AA (Angle-Angle) is a congruence postulate.
5. A proportion is a comparison of two quantities by division.
6. If two triangles are similar, then the perimeters are proportional to the measures of the corresponding angles.
7. A midsegment of a triangle is parallel to one side of the triangle, and its length is twice the length of that side.
8. If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional length, then the line is one-half the length of the third side.

Vocabulary Review at geometryonline.com

Mixed Problem Solving
For mixed problem-solving practice, see page 834.

Lesson-by-Lesson Review

7-1 Proportions (pp. 380–386)

Solve each proportion.

9. $\frac{x - 12}{6} = \frac{x + 7}{-4}$

10. $\frac{18}{7w + 5} = \frac{9}{4w - 1}$

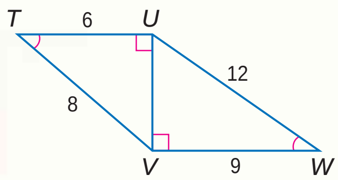
11. **BABIES** The average length and weight of a newborn is 20.16 inches and 7.63 pounds, respectively. If length and weight remained proportional over time, what would be the average weight for an adult who is 71 inches tall? Do length and weight remain proportional as children grow? Explain.

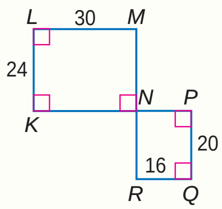
Example 1 **Solve $\frac{m - 13}{m + 13} = \frac{21}{34}$.**

$\frac{m - 13}{m + 13} = \frac{21}{34}$	Original proportion
$34(m - 13) = 21(m + 13)$	Cross Products
$34m - 442 = 21m + 273$	Distributive Property
$13m - 442 = 273$	Subtract.
$13m = 715$	Add.
$m = 55$	Divide.

7-2 Similar Polygons (pp. 388–396)

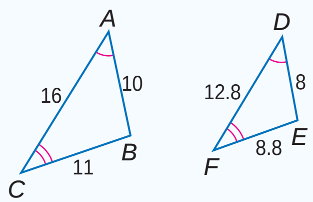
Determine whether each pair of figures is similar. Justify your answer.

12. 

13. 

14. **SOLAR SYSTEM** In creating an accurate scale model of our solar system, Lana placed Earth 1 foot from the Sun. The actual distance from Earth to the Sun is 93,000,000 miles. If the actual distance from Pluto to the Sun is 3,695,950,000 miles, how far from the Sun would Lana need to place Pluto in her model?

Example 2 **Determine whether the pair of triangles is similar. Justify your answer.**



$\angle A \cong \angle D$ and $\angle C \cong \angle F$, so by the Third Angle Theorem, $\angle B \cong \angle E$. All of the corresponding angles are congruent.

Next check the corresponding sides.

$\frac{AB}{DE} = \frac{10}{8} = \frac{5}{4}$ or 1.25 $\quad \frac{BC}{EF} = \frac{11}{8.8} = \frac{5}{4}$ or 1.25

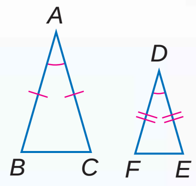
$\frac{CA}{FD} = \frac{16}{12.8} = \frac{5}{4}$ or 1.25

Since the corresponding angles are congruent and the ratios of the measures of the corresponding sides are equal, $\triangle ABC \sim \triangle DEF$.

7-3 Similar Triangles (pp. 397–403)

15. INDIRECT MEASUREMENT To estimate the height of a flagpole, Sonia sights the top of the pole in a mirror on the ground that is facing upward 21 feet from the pole. Sonia is 3 feet from the mirror, and the distance from her eyes to the ground is 5.8 feet. How tall is the flagpole?

Example 3 Determine whether the pair of triangles is similar. Justify your answer.



$\triangle ABC \sim \triangle DFE$ by SAS Similarity.

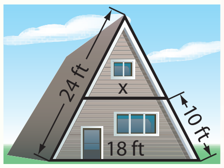
7-4 Parallel Lines and Proportional Parts (pp. 405–414)

Use the figure in Example 4 to determine whether $\overline{MN} \parallel \overline{SR}$. Justify your answer.

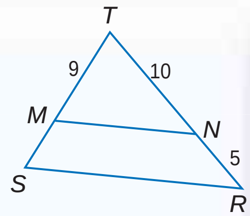
16. $TM = 21$, $MS = 14$, $RN = 9$, $NT = 15$

17. $SM = 10$, $MT = 35$, $TN = 28$, $TR = 36$

18. HOUSES In an A-frame house, the roof slopes to the ground. Find the width x of the second floor.



Example 4 In $\triangle TRS$, $TS = 12$. Determine whether $\overline{MN} \parallel \overline{SR}$.



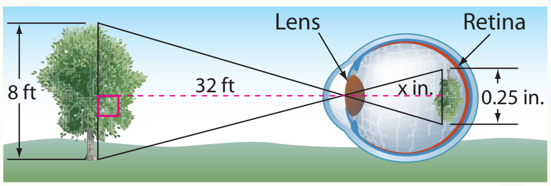
If $TS = 12$, then $MS = 12 - 9$ or 3.
Compare the measures of the segments.

$\frac{TM}{MS} = \frac{9}{3} = 3$ $\qquad$ $\frac{TN}{NR} = \frac{10}{5} = 2$

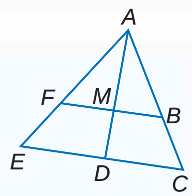
Since $\frac{TM}{MS} \neq \frac{TN}{NR}$, $\overline{MN} \nparallel \overline{SR}$.

7-5 Parts of Similar Triangles (pp. 415–422)

19. HUMAN EYE The human eye uses similar triangles to invert and reduce an object as it passes through the lens onto the retina. What is the length from your lens to your retina?



Example 5 If $\overline{FB} \parallel \overline{ED}$, $\overline{AD}$ is an angle bisector of $\angle A$, $BF = 6$, $CE = 10$, and $AD = 5$, find AM.



Since $\angle ACE \cong \angle ABF$ and $\angle EAC \cong \angle FAB$, $\triangle ABF \sim \triangle ACE$ by AA Similarity.

$\frac{AM}{AD} = \frac{BF}{CE}$ $\qquad$ $\sim \triangle$s have corr. $\angle$ bisectors proportional to the corr. sides.

$\frac{x}{5} = \frac{6}{10}$ $\qquad$ $AD = 5$, $BF = 6$, $CE = 4$, $AM = x$

$10x = 30$ $\qquad$ Cross products

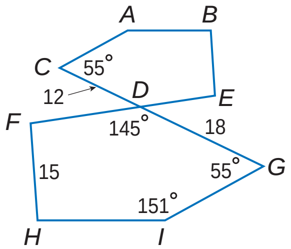
$x = 3$ $\qquad$ Divide. Thus, $AM = 3$.

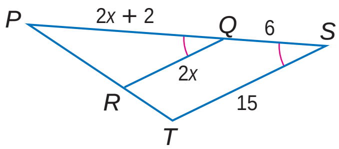
CHAPTER 7 Practice Test

Solve each proportion.

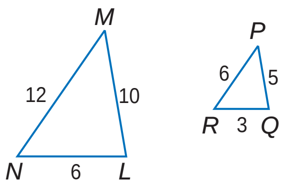
1. $\frac{x}{14} = \frac{1}{2}$
2. $\frac{4x}{3} = \frac{108}{x}$
3. $\frac{k+2}{7} = \frac{k-2}{3}$

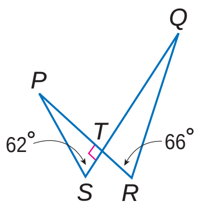
Each pair of polygons is similar. Write a similarity statement and find the scale factor.

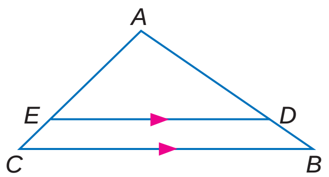
4. 

5. 

Determine whether each pair of triangles is similar. Justify your answer.

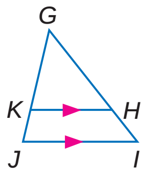
6. 

7. 

8. 

9. **BASKETBALL** Terry wants to measure the height of the top of the backboard of his basketball hoop. At 4:00, the shadow of a 4-foot fence is 20 inches, and the shadow of the backboard is 65 inches. What is the height of the top of the backboard?

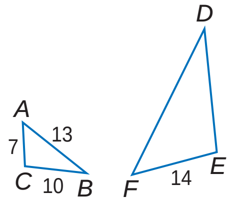
Refer to the figure below.



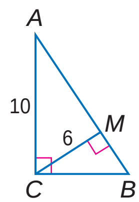
10. Find KJ if $GJ = 8$, $GH = 12$, and $HI = 4$.
11. Find GK if $GI = 14$, $GH = 7$, and $KJ = 6$.
12. Find GI if $GH = 9$, $GK = 6$, and $KJ = 4$.

Find the perimeter of the given triangle.

13. $\triangle DEF$, if $\triangle DEF \sim \triangle ACB$



14. $\triangle ABC$



15. **MULTIPLE CHOICE** Joely builds a corkboard that is 45 inches tall and 63 inches wide. She wants to build a smaller corkboard with a similar shape for the kitchen. Which could be the dimensions of that corkboard?

    A 4 in. by 3 in.

    B 7 in. by 5 in.

    C 12 in. by 5 in.

    D 21 in. by 14 in.

CHAPTER 7

Standardized Test Practice

Cumulative, Chapters 1–7

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which triangles are *not* necessarily similar?

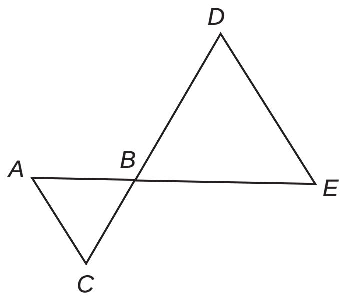
A two right triangles with one angle measuring 30°

B two right triangles with one angle measuring 45°

C two isosceles triangles

D two equilateral triangles

2. Which of the following facts would be sufficient to prove that triangles *ABC* and *EBD* are similar?



F $\overline{AB} \cong \overline{EB}$

G $\overline{AC} \parallel \overline{DE}$

H $\overline{CB} \cong \overline{DB}$

J $\angle D \cong \angle E$

3. **GRIDDABLE** In a quadrilateral, the ratio of the measures of the sides is 2:3:5:9, and its longest side is 13.5 cm. Find the perimeter of the quadrilateral in centimeters.

4. **Given:** *MNOP* is an isosceles trapezoid with diagonals $\overline{MO}$ and $\overline{NP}$. Which of the following is *not* true?

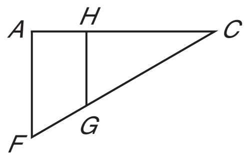
A $\overline{MO} \cong \overline{NP}$

B $\overline{MO}$ bisects $\overline{NP}$.

C $\angle M \cong \angle N$

D $\angle O \cong \angle P$

5. Which of the following facts would *not* be sufficient to prove that triangles *ACF* and *HCG* are similar?



F $\overline{AF} \parallel \overline{HG}$

G $\dfrac{AC}{HC} = \dfrac{FC}{GC}$

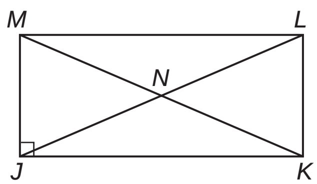
H $\dfrac{CG}{CF} = \dfrac{1}{2}$

J $\angle FAH$ and $\angle CHG$ are right angles.

TEST-TAKING TIP

Question 5 In similar triangles, corresponding angles are congruent and corresponding sides are proportional. When you set up a proportion, be sure that it compares corresponding sides.

6. **GRIDDABLE** In rectangle *JKLM* shown below, $\overline{JL}$ and $\overline{MK}$ are diagonals. If $JL = 2x + 5$, and $KM = 4x - 11$, what is x?



7. If the measure of each exterior angle of a regular polygon is less than 50°, which of the following could *not* be the polygon?

A decagon

B octagon

C heptagon

D pentagon

Standardized Test Practice at geometryonline.com

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 841–856.

8. ALGEBRA Which equation describes the line that passes through $(-2, 3)$ and is perpendicular to $2x - y = 3$?

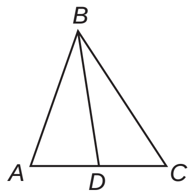
F $y = 2x - 2$

G $y = \frac{1}{2}x - 4$

H $y = -\frac{1}{2}x + 2$

J $y = -2x + 4$

9. In $\triangle ABC$, $\overline{BD}$ is a median. If $AD = 3x + 5$, and $CD = 5x - 1$, find AC.



A 3

B 11

C 14

D 28

10. At the International Science Fair, a Canadian student recorded temperatures in degrees Celsius. A student from the United States recorded the same temperatures in degrees Fahrenheit. They used their data to plot a graph of Celsius versus Fahrenheit. What is the slope of their graph?

F $\frac{5}{9}$

G 1

H $\frac{9}{5}$

J 2

11. Quadrilateral *HJKL* is a parallelogram. If the diagonals are perpendicular, which statement must be true?

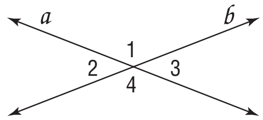
A Quadrilateral *HJKL* is a square.

B Quadrilateral *HJKL* is a rhombus.

C Quadrilateral *HJKL* is a rectangle.

D Quadrilateral *HJKL* is an isosceles trapezoid.

12. If $\angle 4$ and $\angle 3$ are supplementary, which reason could you use as the first step in proving that $\angle 1$ and $\angle 2$ are supplementary?



F Definition of a vertical angle

G Definition of similar angles

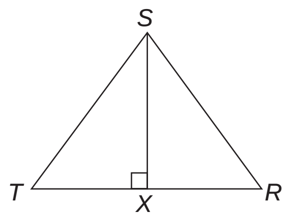
H Definition of perpendicular lines

J Division Property

Pre-AP

Record your answer on a sheet of paper. Show your work.

13. Toby, Rani, and Sasha are practicing for a double Dutch rope-jumping tournament. Toby and Rani are standing at points *T* and *R* and are turning the ropes. Sasha is standing at *S*, equidistant from both Toby and Rani. Sasha will jump into the middle of the turning rope to point *X*. Prove that when Sasha jumps into the rope, she will be at the midpoint between Toby and Rani.



NEED EXTRA HELP?													
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson or Page...	7-3	7-4	7-1	6-6	7-3	6-4	6-1	786	5-1	3-3	6-5	2-7	4-6

CHAPTER 8

Right Triangles and Trigonometry

- Solve problems using the geometric mean, the Pythagorean Theorem, and its converse.
- Use trigonometric ratios to solve right triangle problems.
- Solve triangles using the Law of Sines and the Law of Cosines.

Key Vocabulary

trigonometric ratio (p. 456)

Law of Sines (p. 471)

Law of Cosines (p. 479)

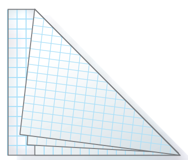
Real-World Link

Bridges The William H. Natcher bridge across the Ohio River has a cable-stayed design. The cables form right triangles with the supports and the length of the bridge.

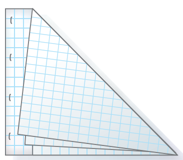
Right Triangles and Trigonometry Make this Foldable to help you organize your notes. Begin with seven sheets of grid paper.

1. **Stack** the sheets. Fold the top right corner to the bottom edge to form a square.

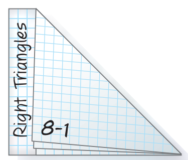
2. **Fold** the rectangular part in half.



3. **Staple** the sheets along the fold in four places.



4. **Label** each sheet with a lesson number and the rectangular part with the chapter title.



David Sailors/CORBIS

GET READY for Chapter 8

Diagnose Readiness **You have two options for checking Prerequisite Skills.**

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Solve each proportion. Round to the nearest hundredth. (Lesson 7-1)

1. $\frac{3}{4} = \frac{12}{a}$

2. $\frac{c}{5} = \frac{8}{3}$

3. $\frac{d}{20} = \frac{6}{5} = \frac{f}{10}$

4. $\frac{4}{3} = \frac{6}{y} = \frac{1}{z}$

5. **MINIATURES** The proportion $\frac{1 \text{ in.}}{12 \text{ in.}} = \frac{3.5}{x}$ relates the height of a miniature chair to the height of a real chair. Solve the proportion. (Lesson 7-1)

Find the measure of the hypotenuse of each right triangle having legs with the given measures. Round to the nearest hundredth. (Extend 1-3)

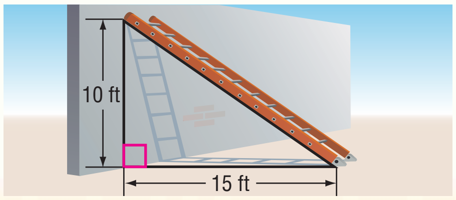
6. 5 and 12

7. 6 and 8

8. 15 and 15

9. 14 and 27

10. **PAINTING** A ladder is propped against a wall as shown. To the nearest tenth, what is the length of the ladder? (Extend 1-3)



11. The measure of one angle in a right triangle is three times the measure of the second angle. Find the measures of each angle of the triangle. Find x. (Lesson 4-2)

QuickReview

EXAMPLE 1

Solve the proportion $\frac{a}{30} = \frac{31}{5}$. Round to the nearest hundredth if necessary.

$\frac{a}{30} = \frac{31}{5}$	Write the proportion.
$5a = 30(31)$	Find the cross products.
$5a = 930$	Simplify.
$a = 186$	Divide each side by 5.

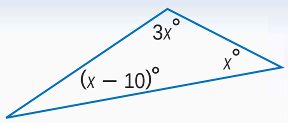
EXAMPLE 2

Find the measure of the hypotenuse of the right triangle having legs with the measures 10 and 24. Round to the nearest hundredth if necessary.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$10^2 + 24^2 = c^2$	Substitution
$100 + 576 = c^2$	Evaluate the exponents.
$676 = c^2$	Simplify.
$\sqrt{676} = \sqrt{c^2}$	Take the square root of each side.
$26 = c$	Simplify.

EXAMPLE 3

Find *x*.



$x + 3x + x - 10 = 180$

$5x = 190$

$x = 38$

8-1 Geometric Mean

Main Ideas

- Find the geometric mean between two numbers.
- Solve problems involving relationships between parts of a right triangle and the altitude to its hypotenuse.

New Vocabulary

geometric mean

When you look at a painting, you should stand at a distance that allows you to see all of the details in the painting. The distance that creates the best view is the geometric mean of the distance from the top of the painting to eye level and the distance from the bottom of the painting to eye level.

Geometric Mean The **geometric mean** between two numbers is the positive square root of their product.

Study Tip

You may wish to review **square roots** and **simplifying radicals** on pp. 790–791.

KEY CONCEPT *Geometric Mean*

For two positive numbers a and b, the geometric mean is the positive number x where the proportion $a : x = x : b$ is true. This proportion can be written using fractions as $\frac{a}{x} = \frac{x}{b}$ or with cross products as $x^2 = ab$ or $x = \sqrt{ab}$.

EXAMPLE Geometric Mean

1 **Find the geometric mean between each pair of numbers.**

a. 4 and 9

$\frac{4}{x} = \frac{x}{9}$ Definition of geometric mean

$x^2 = 36$ Cross products

$x = \sqrt{36}$ Take the positive square root of each side.

$x = 6$ Simplify.

b. 6 and 15

$\frac{6}{x} = \frac{x}{15}$ Definition of geometric mean

$x^2 = 90$ Cross products

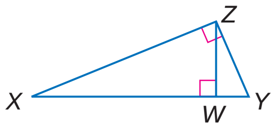
$x = \sqrt{90}$ Take the positive square root of each side.

$x = 3\sqrt{10}$ Simplify.

$x \approx 9.5$ Use a calculator.

1A. 5 and 45 **1B.** 8 and 10

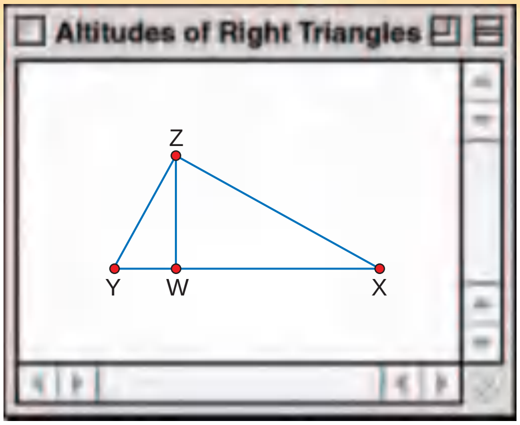
Altitude of a Triangle Consider right triangle XYZ with altitude $\overline{WZ}$ drawn from the right angle Z to the hypotenuse $\overline{XY}$. A special relationship exists for the three right triangles, $\triangle XYZ$, $\triangle XZW$, and $\triangle ZYW$.



GEOMETRY SOFTWARE LAB

Right Triangles Formed by the Altitude

Use The Geometer's Sketchpad to draw a right triangle XYZ with right angle Z. Draw the altitude $\overline{ZW}$ from the right angle to the hypotenuse.



THINK AND DISCUSS

1. Find the measures of $\angle X$, $\angle XZY$, $\angle Y$, $\angle XWZ$, $\angle XZW$, $\angle YWZ$, and $\angle YZW$.
2. What is the relationship between $m\angle X$ and $m\angle YZW$? between $m\angle Y$ and $m\angle XZW$?
3. Drag point Z to another position. Describe the relationship between the measures of $\angle X$ and $\angle YZW$ and between $m\angle Y$ and $m\angle XZW$.

MAKE A CONJECTURE

4. How are $\triangle XYZ$, $\triangle XZW$, and $\triangle ZYW$ related?

The Geometry Software Lab suggests the following theorem.

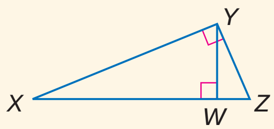
Study Tip

Altitudes of a Right Triangle
The altitude drawn to the hypotenuse originates from the right angle. The other two altitudes of a right triangle are the legs.

THEOREM 8.1

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and to each other.

Example: $\triangle XYZ \sim \triangle XWY \sim \triangle YWZ$



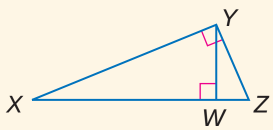
You will prove Theorem 8.1 in Exercise 38.

By Theorem 8.1, since $\triangle XWY \sim \triangle YWZ$, the corresponding sides are proportional. Thus, $\frac{XW}{YW} = \frac{YW}{ZW}$. Notice that $\overline{XW}$ and $\overline{ZW}$ are segments of the hypotenuse of the largest triangle.

THEOREM 8.2

The measure of an altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse.

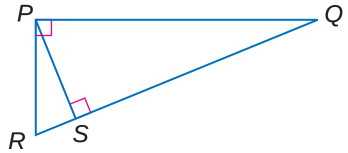
Example: YW is the geometric mean of XW and ZW.



You will prove Theorem 8.2 in Exercise 39.

EXAMPLE Altitude and Segments of the Hypotenuse

2 **In $\triangle PQR$, $RS = 3$ and $QS = 14$. Find PS.**



$\frac{RS}{PS} = \frac{PS}{QS}$ Theorem 8.2

$\frac{3}{x} = \frac{x}{14}$ $RS = 3$, $QS = 14$, and $PS = x$

$x^2 = 42$ Cross products

$x \approx 6.5$ Use a calculator to take the positive square root of each side.

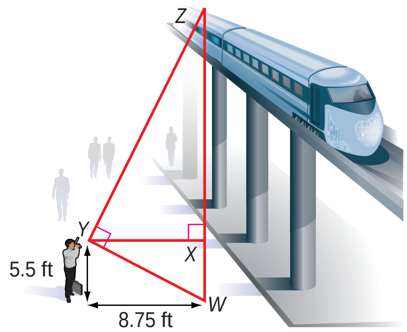
Study Tip

Square Roots
Since these numbers represent measures, you can ignore the negative square root value.

2. Refer to $\triangle PQR$ above. If $RS = 0.8$ and $QS = 2.2$, find PS.

3 **ARCHITECTURE** **Mr. Martinez is designing a walkway to pass over a train. To find the train height, he holds a carpenter's square at eye level and sights along the edges from the street to the top of the train. If Mr. Martinez's eye level is 5.5 feet above the street and he is 8.75 feet from the train, find the train's height. Round to the nearest tenth.**

Draw a diagram. Let $\overline{YX}$ be the altitude drawn from the right angle of $\triangle WYZ$.



$\frac{WX}{YX} = \frac{YX}{ZX}$ Theorem 8.2

$\frac{5.5}{8.75} = \frac{8.75}{ZX}$ $WX = 5.5$ and $YX = 8.75$

$5.5ZX = 76.5625$ Cross products

$ZX \approx 13.9$ Divide each side by 5.5.

The elevated train is $5.5 + 13.9$ or about 19.4 feet high.

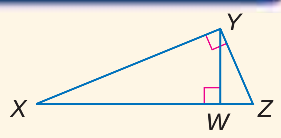
3. Makayla is using a carpenter's square to sight the top of a waterfall. If her eye level is 5 feet from the ground and she is a horizontal distance of 28 feet from the waterfall, find the height of the waterfall to the nearest tenth.

Personal Tutor at geometryonline.com

The altitude to the hypotenuse of a right triangle determines another relationship between the segments.

THEOREM 8.3

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the measure of a leg of the triangle is the geometric mean between the measures of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

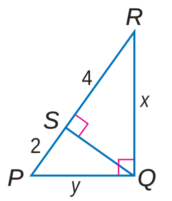


Example: $\frac{XZ}{XY} = \frac{XY}{XW}$ and $\frac{XZ}{YZ} = \frac{YZ}{WZ}$

You will prove Theorem 8.3 in Exercise 40.

EXAMPLE 4 Hypotenuse and Segment of Hypotenuse

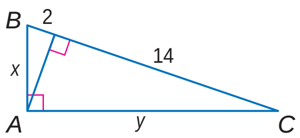
Find x and y in $\triangle PQR$.



$\overline{PQ}$ and $\overline{RQ}$ are legs of right triangle PQR. Use Theorem 8.3 to write a proportion for each leg and then solve.

$\frac{PR}{PQ} = \frac{PQ}{PS}$		$\frac{PR}{RQ} = \frac{RQ}{SR}$	
$\frac{6}{y} = \frac{y}{2}$	$PS = 2$, $PQ = y$, $PR = 6$	$\frac{6}{x} = \frac{x}{4}$	$RS = 4$, $RQ = x$, $PR = 6$
$y^2 = 12$	Cross products	$x^2 = 24$	Cross products
$y = \sqrt{12}$	Take the square root.	$x = \sqrt{24}$	Take the square root.
$y = 2\sqrt{3}$	Simplify.	$x = 2\sqrt{6}$	Simplify.
$y \approx 3.5$	Use a calculator.	$x \approx 4.9$	Use a calculator.

4. Find x and y in $\triangle ABC$.



Check Your Understanding

Example 1 (p. 432)

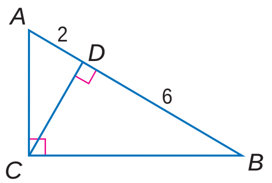
Find the geometric mean between each pair of numbers.

1. 9 and 4 **2.** 36 and 49 **3.** 6 and 8 **4.** $2\sqrt{2}$ and $3\sqrt{2}$

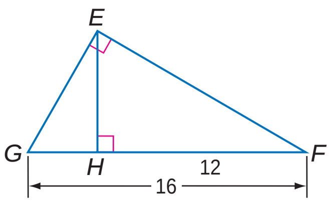
Example 2 (p. 434)

Find the measure of the altitude drawn to the hypotenuse.

5.

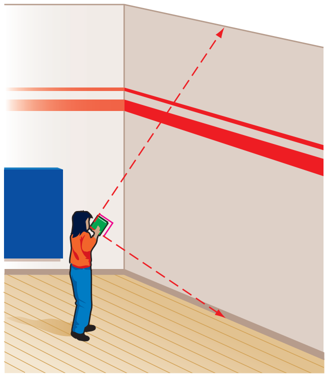


6.



Example 3 (p. 434)

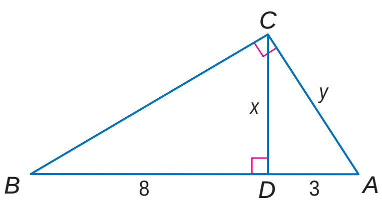
7. DANCES Danielle is making a banner for the dance committee. The banner is to be as high as the wall of the gymnasium. To find the height of the wall, Danielle held a book up to her eyes so that the top and bottom of the wall were in line with the bottom edge and binding of the cover. If Danielle's eye level is 5 feet off the ground and she is standing 12 feet from the wall, how high is the wall?



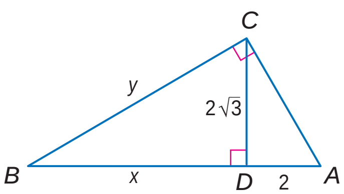
Example 4 (p. 435)

Find x and y.

8.



9.



Exercises

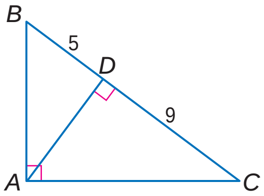
HOMEWORK HELP	
For Exercises	See Examples
10–17	1
18–23	2
24–25	3
26–31	4

Find the geometric mean between each pair of numbers.

10. 5 and 6 **11.** 24 and 25 **12.** $\sqrt{45}$ and $\sqrt{80}$ **13.** $\sqrt{28}$ and $\sqrt{1372}$

14. $\frac{3}{5}$ and 1 **15.** $\frac{8\sqrt{3}}{5}$ and $\frac{6\sqrt{3}}{5}$ **16.** $\frac{2\sqrt{2}}{6}$ and $\frac{5\sqrt{2}}{6}$ **17.** $\frac{13}{7}$ and $\frac{5}{7}$

Find the measure of the altitude drawn to the hypotenuse.

18.


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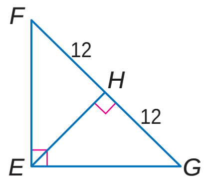

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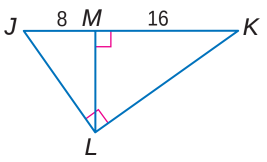

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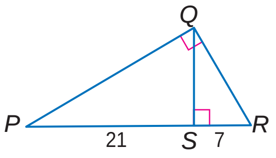

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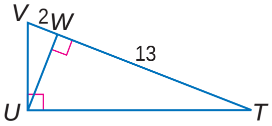
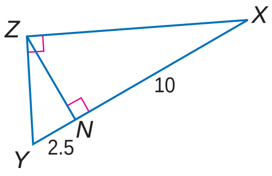
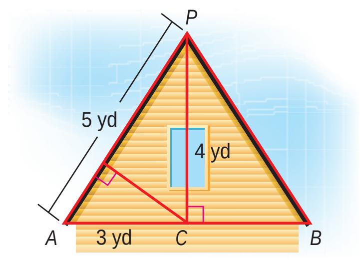
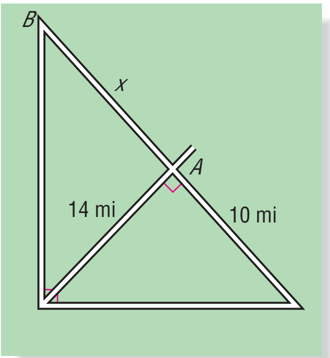


23.


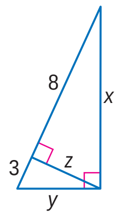
24. CONSTRUCTION The slope of the roof shown below is $\frac{4}{3}$. A builder wants to put a support brace from point C perpendicular to $\overline{AP}$. Find the length of the brace.



25. ROADS City planners want to build a road to connect points A and B. Find out how long this road will need to be.



Find x, y, and z.

26.


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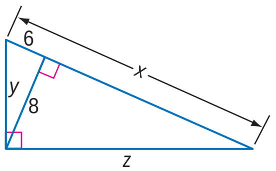

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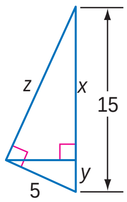

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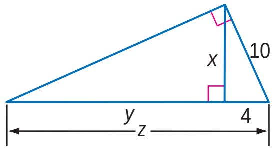
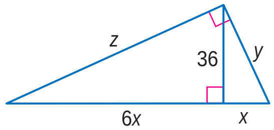
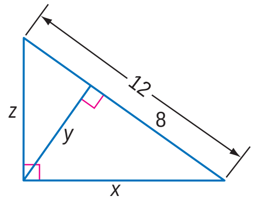


30.


31.


The geometric mean and one extreme are given. Find the other extreme.

32. $\sqrt{17}$ is the geometric mean between a and b. Find b if $a = 7$.

33. $\sqrt{12}$ is the geometric mean between x and y. Find x if $y = \sqrt{3}$.

Determine whether each statement is *always*, *sometimes*, or *never* true.

34. The geometric mean for consecutive positive integers is the average of the two numbers.

35. The geometric mean for two perfect squares is a positive integer.

36. The geometric mean for two positive integers is another integer.

37. The measure of the altitude of a triangle is the geometric mean between the measures of the segments of the side opposite the initial vertex.

PROOF **Write a proof for each theorem.**

38. Theorem 8.1 **39.** Theorem 8.2 **40.** Theorem 8.3

41. **RESEARCH** Use the Internet or other resource to write a brief description of the golden ratio, which is also known as the divine proportion, golden mean, or golden section.

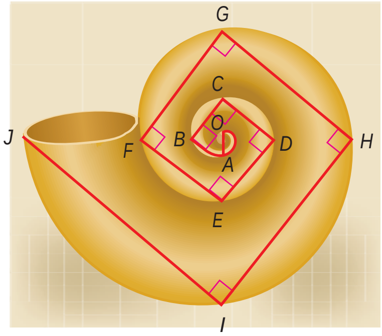
EXTRA PRACTICE

See pages 815, 835.

Math Online

Self-Check Quiz at geometryonline.com

42. **PATTERNS** The spiral of the state shell of Texas, the lightning whelk, can be modeled by a geometric mean. Consider the sequence of segments $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, $\overline{OD}$, $\overline{OE}$, $\overline{OF}$, $\overline{OG}$, $\overline{OH}$, $\overline{OI}$, and $\overline{OJ}$. The length of each of these segments is the geometric mean between the lengths of the preceding segment and the succeeding segment. Explain this relationship. (*Hint*: Consider $\triangle FGH$.)



H.O.T. Problems

43. **OPEN ENDED** Find two pairs of numbers with a geometric mean of 12.

44. **REASONING** Draw and label a right triangle with an altitude drawn from the right angle. From your drawing, explain the meaning of *the hypotenuse and the segment of the hypotenuse adjacent to that leg* in Theorem 8.3.

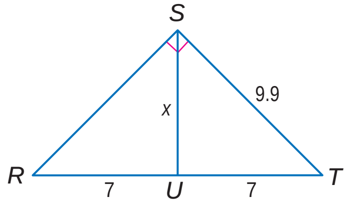
45. **FIND THE ERROR** $\triangle RST$ is a right isosceles triangle. Holly and Ian are finding the measure of altitude $\overline{SU}$. Who is correct? Explain your reasoning.

Holly

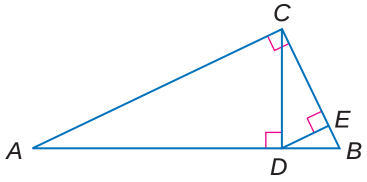
$$\frac{RS}{SU} = \frac{SU}{RT}$$
$$\frac{9.9}{x} = \frac{x}{14}$$
$$x^2 = 138.5$$
$$x = \sqrt{138.5}$$
$$x = 11.8$$

Ian

$$\frac{RU}{SU} = \frac{SU}{TU}$$
$$\frac{7}{x} = \frac{x}{7}$$
$$x = 49$$
$$x = 7$$

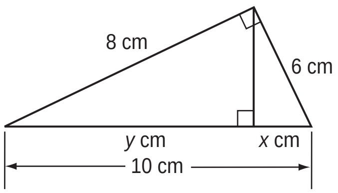


46. **CHALLENGE** Find the exact value of DE, given $AD = 12$ and $BD = 4$.



47. **Writing in Math** Describe how the geometric mean can be used to view paintings. Include an explanation of what happens when you are too far or too close to a painting.

48. What are the values of x and y?



A 4 and 6

B 2.5 and 7.5

C 3.6 and 6.4

D 3 and 7

49. **REVIEW** What are the solutions for the quadratic equation $x^2 + 9x = 36$?

F $-3, -12$ H $3, -12$

G $3, 12$ J $-3, 12$

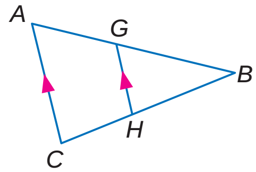
50. **REVIEW** Tulia borrowed \$300 at 15% simple interest for two years. If she makes no payments either year, how much interest will she owe at the end of the two-year period?

A \$90.00 C \$30.00

B \$45.00 D \$22.50

51. The measures of the sides of a triangle are 20, 24, and 30. Find the measures of the segments formed where the bisector of the smallest angle meets the opposite side. (Lesson 7-5)

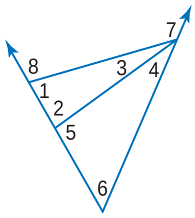
For Exercises 52 and 53, use $\triangle ABC$. (Lesson 7-4)



52. If $AG = 4$, $GB = 6$, and $BH = 8$, find BC.

53. If $AB = 12$, $BC = 14$, and $HC = 4$, find AG.

Use the Exterior Angle Inequality Theorem to list all angles that satisfy the stated condition. (Lesson 5-2)



54. measures less than $m\angle 8$

55. measures greater than $m\angle 1$

56. measures less than $m\angle 7$

57. measures greater than $m\angle 6$

Write an equation in slope-intercept form for the line that satisfies the given conditions. (Lesson 3-4)

58. $m = 2$, y-intercept $= 4$

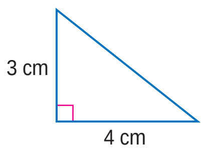
59. passes through (2, 6) and $(-1, 0)$

60. $m = -4$, passes through $(-2, -3)$

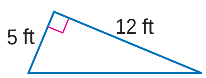
61. x-intercept is 2, y-intercept $= -8$

PREREQUISITE SKILL **Use the Pythagorean Theorem to find the length of the hypotenuse of each right triangle.** (Lesson 1-4)

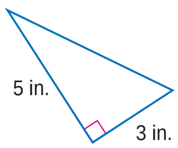
62.



63.



64.



EXPLORE 8-2

Geometry Lab

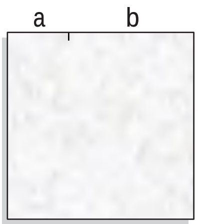
The Pythagorean Theorem

In Chapter 1, you learned that the Pythagorean Theorem relates the measures of the legs and the hypotenuse of a right triangle. Ancient cultures used the Pythagorean Theorem before it was officially named in 1909.

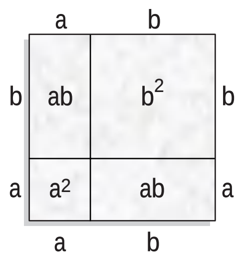
ACTIVITY

Use paper folding to develop the Pythagorean Theorem.

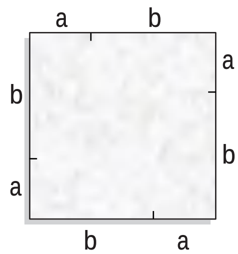
Step 1 On a piece of patty paper, make a mark along one side so that the two resulting segments are not congruent. Label one as a and the other as b.



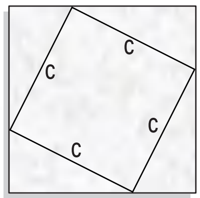
Step 2 Copy these measures on the other sides in the order shown at the right. Fold the paper to divide the square into four sections. Label the area of each section.



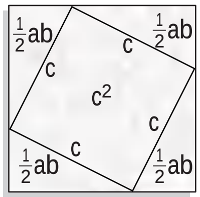
Step 3 On another sheet of patty paper, mark the same lengths a and b on the sides in the different pattern shown at the right.



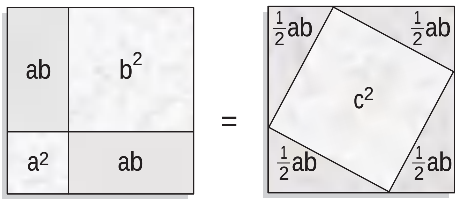
Step 4 Use your straightedge and pencil to connect the marks as shown at the right. Let c represent the length of each hypotenuse.



Step 5 Label the area of each section, which is $\frac{1}{2}ab$ for each triangle and c^2 for the square.



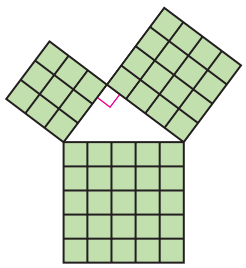
Step 6 Place the squares side by side and color the corresponding regions that have the same area. For example, $ab = \frac{1}{2}ab + \frac{1}{2}ab$.



The parts that are not shaded tell us that $a^2 + b^2 = c^2$.

Analyze the Results

1. Use a ruler to find actual measures for a, b, and c. Do these measures confirm that $a^2 + b^2 = c^2$?
2. Repeat the activity with different a and b values. What do you notice?
3. Explain why the drawing at the right is an illustration of the Pythagorean Theorem.



4. **CHALLENGE** Use a geometric diagram to show that for any positive numbers a and b, $a + b > \sqrt{a^2 + b^2}$.

8-2 The Pythagorean Theorem and Its Converse

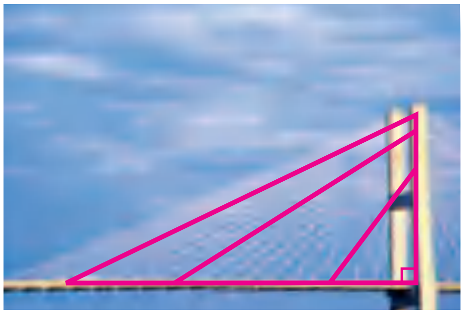
Main Ideas

- Use the Pythagorean Theorem.
- Use the converse of the Pythagorean Theorem.

New Vocabulary

Pythagorean triple

The Talmadge Memorial Bridge over the Savannah River, in Georgia, has two soaring towers of suspension cables. Note the right triangles being formed by the roadway, the perpendicular tower, and the suspension cables. The Pythagorean Theorem can be used to find measures in any right triangle.

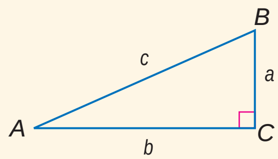


The Pythagorean Theorem In Lesson 1-3, you used the Pythagorean Theorem to find the distance between two points by finding the length of the hypotenuse when given the lengths of the two legs of a right triangle. You can also find the measure of any side of a right triangle given the other two measures.

THEOREM 8.4 ***Pythagorean Theorem***

In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

Symbols: $a^2 + b^2 = c^2$

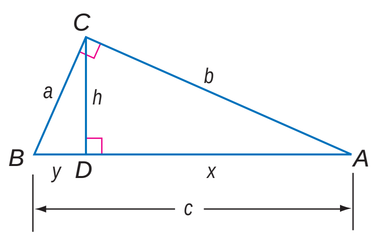


The geometric mean can be used to prove the Pythagorean Theorem.

Proof **Pythagorean Theorem**

Given: $\triangle ABC$ with right angle at C

Prove: $a^2 + b^2 = c^2$



Proof:

Draw right triangle ABC so C is the right angle. Then draw the altitude from C to $\overline{AB}$. Let $AB = c$, $AC = b$, $BC = a$, $AD = x$, $DB = y$, and $CD = h$.

Alexandra Michaels/Getty Images

Two geometric means now exist.

$\frac{c}{a} = \frac{a}{y}$ and $\frac{c}{b} = \frac{b}{x}$

$a^2 = cy$ and $b^2 = cx$ Cross products

Add the equations.

$a^2 + b^2 = cy + cx$

$a^2 + b^2 = c(y + x)$ Factor.

$a^2 + b^2 = c^2$ Since $c = y + x$, substitute c for $(y + x)$.

You can use the Pythagorean Theorem to find the length of the hypotenuse or a leg of a right triangle if the other two sides are known.

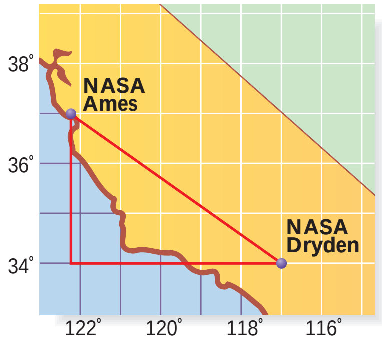
Real-World Link

Due to the curvature of Earth, the distance between two points is often expressed as *degree distance* using latitude and longitude. This measurement closely approximates the distance on a plane.

Source: NASA

EXAMPLE 1 Find the Length of the Hypotenuse

GEOGRAPHY California's NASA Dryden is located at about 117 degrees longitude and 34 degrees latitude. NASA Ames, also in California, is located at about 122 degrees longitude and 37 degrees latitude. Use the lines of longitude and latitude to find the degree distance to the nearest tenth between NASA Dryden and NASA Ames.



The change in longitude between the two locations is |117 - 122| or 5 degrees. Let this distance be *a*.

The change in latitude is |37 - 34| or 3 degrees latitude. Let this distance be *b*.

Use the Pythagorean Theorem to find the distance in degrees from NASA Dryden to NASA Ames, represented by *c*.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$5^2 + 3^2 = c^2$ $a = 5, b = 3$

$25 + 9 = c^2$ Simplify.

$34 = c^2$ Add.

$\sqrt{34} = c$ Take the positive square root of each side.

$5.8 \approx c$ Use a calculator.

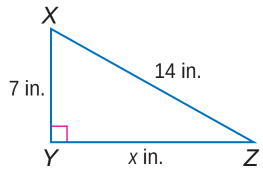
The degree distance between NASA Dryden and NASA Ames is about 5.8 degrees.

1. **GEOGRAPHY** Houston, Texas, is located at about 30 degrees latitude and about 95 degrees longitude. Raleigh, North Carolina, is located at about 36 degrees latitude and about 79 degrees longitude. Find the degree distance to the nearest tenth.

Personal Tutor at geometryonline.com

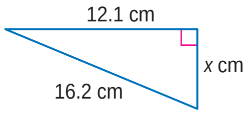
EXAMPLE 2 Find the Length of a Leg

Find x.



$$(XY)^2 + (YZ)^2 = (XZ)^2 \quad \text{Pythagorean Theorem}$$
$$7^2 + x^2 = 14^2 \quad XY = 7,\ XZ = 14$$
$$49 + x^2 = 196 \quad \text{Simplify.}$$
$$x^2 = 147 \quad \text{Subtract 49 from each side.}$$
$$x = \sqrt{147} \quad \text{Take the square root of each side.}$$
$$x = 7\sqrt{3} \quad \text{Simplify.}$$
$$x \approx 12.1 \quad \text{Use a calculator.}$$

2. Find x.

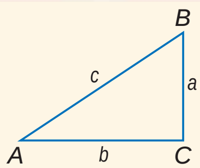


Converse of the Pythagorean Theorem The converse of the Pythagorean Theorem can help you determine whether three measures of the sides of a triangle are those of a right triangle.

THEOREM 8.5 Converse of the Pythagorean Theorem

If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

Symbols: If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle.



You will prove Theorem 8.5 in Exercise 30.

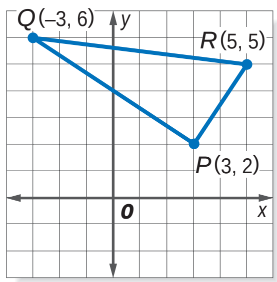
Study Tip

Distance Formula

When using the Distance Formula, be sure to follow the order of operations carefully. Perform the operation inside the parentheses first, square each term, and then add.

EXAMPLE 3 Verify a Triangle is a Right Triangle

COORDINATE GEOMETRY **Verify that $\triangle PQR$ is a right triangle.**



Use the Distance Formula to determine the lengths of the sides.

$$PQ = \sqrt{(-3-3)^2 + (6-2)^2} \quad x_1 = 3, y_1 = 2, x_2 = -3, y_2 = 6$$
$$= \sqrt{(-6)^2 + 4^2} \quad \text{Subtract.}$$
$$= \sqrt{52} \quad \text{Simplify.}$$

$$QR = \sqrt{[5-(-3)]^2 + (5-6)^2} \quad x_1 = -3, y_1 = 6, x_2 = 5, y_2 = 5$$
$$= \sqrt{8^2 + (-1)^2} \quad \text{Subtract.}$$
$$= \sqrt{65} \quad \text{Simplify.}$$

$$PR = \sqrt{(5-3)^2 + (5-2)^2} \quad x_1 = 3, y_1 = 2, x_2 = 5, y_2 = 5$$
$$= \sqrt{2^2 + 3^2} \quad \text{Subtract.}$$
$$= \sqrt{13} \quad \text{Simplify.}$$

By the converse of the Pythagorean Theorem, if the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

$$PQ^2 + PR^2 = QR^2 \quad \text{Converse of the Pythagorean Theorem}$$
$$(\sqrt{52})^2 + (\sqrt{13})^2 \stackrel{?}{=} (\sqrt{65})^2 \quad PQ = \sqrt{52},\ PR = \sqrt{13},\ QR = \sqrt{65}$$
$$52 + 13 \stackrel{?}{=} 65 \quad \text{Simplify.}$$
$$65 = 65 \quad \text{Add.}$$

Since the sum of the squares of two sides equals the square of the longest side, $\triangle PQR$ is a right triangle.

3. Verify that $\triangle ABC$ with vertices $A(2, -3)$, $B(3, 0)$, and $C(5, -1)$ is a right triangle.

A **Pythagorean triple** is three whole numbers that satisfy the equation $a^2 + b^2 = c^2$, where c is the greatest number. One common Pythagorean triple is 3-4-5. If the measures of the sides of any right triangle are whole numbers, the measures form a Pythagorean triple.

EXAMPLE Pythagorean Triples

4 **Determine whether each set of measures can be the sides of a right triangle. Then state whether they form a Pythagorean triple.**

a. 8, 15, 16

Since the measure of the longest side is 16, 16 must be c, and a or b are 8 and 15, respectively.

$$a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem}$$
$$8^2 + 15^2 \stackrel{?}{=} 16^2 \quad a = 8,\ b = 15,\ c = 16$$
$$64 + 225 \stackrel{?}{=} 256 \quad \text{Simplify.}$$
$$289 \neq 256 \quad \text{Add.}$$

Since $289 \neq 256$, segments with these measures cannot form a right triangle. Therefore, they do not form a Pythagorean triple.

b. $\frac{\sqrt{3}}{5}, \frac{\sqrt{6}}{5}$, **and** $\frac{3}{5}$

$$a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem}$$
$$\left(\frac{\sqrt{3}}{5}\right)^2 + \left(\frac{\sqrt{6}}{5}\right)^2 \stackrel{?}{=} \left(\frac{3}{5}\right)^2 \quad a = \frac{\sqrt{3}}{5},\ b = \frac{\sqrt{6}}{5},\ c = \frac{3}{5}$$
$$\frac{3}{25} + \frac{6}{25} \stackrel{?}{=} \frac{9}{25} \quad \text{Simplify.}$$
$$\frac{9}{25} = \frac{9}{25}\ \checkmark \quad \text{Add.}$$

Since $\frac{9}{25} = \frac{9}{25}$, segments with these measures form a right triangle. However, the three numbers are not whole numbers. Therefore, they do not form a Pythagorean triple.

Study Tip

Comparing Numbers

If you cannot quickly identify the greatest number, use a calculator to find decimal values for each number and compare.

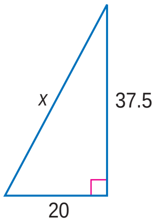
4A. 20, 48, and 52

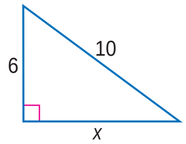
4B. $\frac{\sqrt{2}}{7}, \frac{\sqrt{3}}{7}$, and $\frac{\sqrt{5}}{7}$

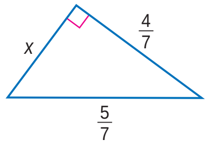
✓ CHECK Your Understanding

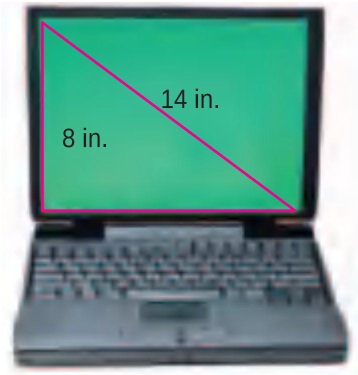
Examples 1 and 2 (pp. 441–442)

Find x.

1.


2.


3.




4. COMPUTERS Computer displays are usually measured along the diagonal of the screen. A 14-inch display has a diagonal that measures 14 inches. If the height of the screen is 8 inches, how wide is the screen?

Example 3 (p. 442)

5. COORDINATE GEOMETRY Determine whether $\triangle JKL$ with vertices $J(-2, 2)$, $K(-1, 6)$, and $L(3, 5)$ is a right triangle. Explain.

Example 4 (p. 443)

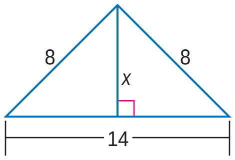
Determine whether each set of numbers can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

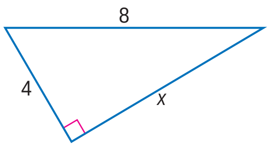
6. 15, 36, 39 **7.** $\sqrt{40}$, 20, 21 **8.** $\sqrt{44}$, 8, $\sqrt{108}$

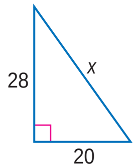
Exercises

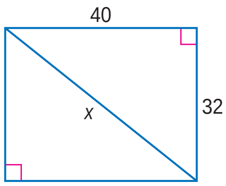
HOMEWORK HELP	
For Exercises	See Examples
9–14	1, 2
15–18	3
19–26	4

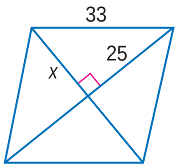
Find x.

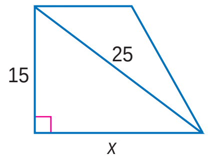
9.


10.


11.


12.


13.


14.


COORDINATE GEOMETRY Determine whether $\triangle QRS$ is a right triangle for the given vertices. Explain.

15. $Q(1, 0)$, $R(1, 6)$, $S(9, 0)$ **16.** $Q(3, 2)$, $R(0, 6)$, $S(6, 6)$

17. $Q(-4, 6)$, $R(2, 11)$, $S(4, -1)$ **18.** $Q(-9, -2)$, $R(-4, -4)$, $S(-6, -9)$

Determine whether each set of numbers can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

19. 8, 15, 17 **20.** 7, 24, 25 **21.** 20, 21, 31 **22.** 37, 12, 34

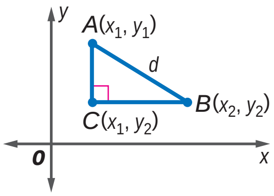
23. $\frac{1}{5}, \frac{1}{7}, \frac{\sqrt{74}}{35}$ **24.** $\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{3}, \frac{35}{36}$ **25.** $\frac{3}{4}, \frac{4}{5}, 1$ **26.** $\frac{6}{7}, \frac{8}{7}, \frac{10}{7}$

27. GARDENING Scott wants to plant flowers in a triangular plot. He has three lengths of plastic garden edging that measure 20 inches, 21 inches, and 29 inches. Discuss whether these pieces form a right triangle. Explain.

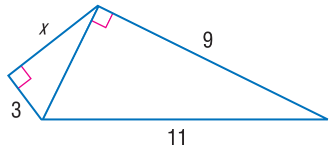
28. **NAVIGATION** A fishing trawler off the coast of Alaska was ordered by the U.S. Coast Guard to change course. They were to travel 6 miles west and then sail 12 miles south to miss a large iceberg before continuing on the original course. How many miles out of the way did the trawler travel?

29. **PROOF** Use the Pythagorean Theorem and the figure at the right to prove the Distance Formula.

30. **PROOF** Write a paragraph proof of Theorem 8.5.



31. Find the value of x in the figure shown.



Real-World Career

Military

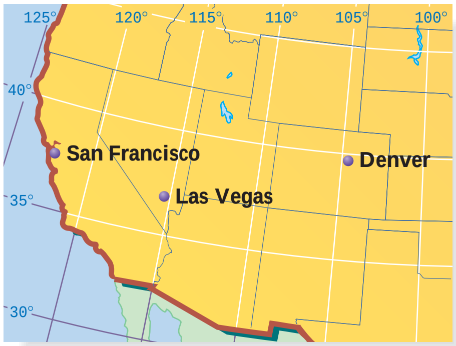
All branches of the military use navigation. Some of the jobs using navigation include radar/sonar operators, boat operators, airplane navigators, and space operations officers.

Math Online

For more information, go to geometryonline.com.

GEOGRAPHY For Exercises 32 and 33, use the following information.

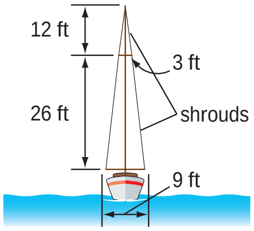
Denver is located at about 105° longitude and 40° latitude. San Francisco is located at about 122° longitude and 38° latitude. Las Vegas is located at about 115° longitude and 36° latitude. Using the lines of longitude and latitude, find each degree distance.



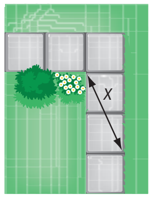
32. San Francisco to Denver

33. Las Vegas to Denver

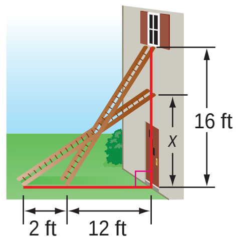
34. **SAILING** The mast of a sailboat is supported by wires called *shrouds*. What is the total length of wire needed to form these shrouds?



35. **LANDSCAPING** Six congruent square stones are arranged in an L-shaped walkway through a garden. If $x = 15$ inches, then find the area of the L-shaped walkway.



36. **PAINTING** A painter sets a ladder up to reach the bottom of a second-story window 16 feet above the ground. The base of the ladder is 12 feet from the house. While the painter mixes the paint, a neighbor's dog bumps the ladder, which moves the base 2 feet farther away from the house. How far up the side of the house does the ladder reach?



EXTRA PRACTICE

See pages 815, 835.

Math Online

Self-Check Quiz at geometryonline.com

H.O.T. Problems

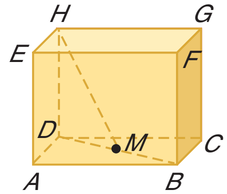
37. **FIND THE ERROR** Maria and Colin are determining whether 5-12-13 is a Pythagorean triple. Who is correct? Explain your reasoning.

Colin	Maria
$13^2 + 5^2 \stackrel{?}{=} 12^2$	$5^2 + 12^2 = 13^2$
$169 + 25 \stackrel{?}{=} 144$	$25 + 144 = 169$
$193 \neq 144$	$169 = 169$
no	yes

38. **OPEN ENDED** Draw a pair of similar right triangles. Are the measures of the sides of each triangle a Pythagorean triple? Explain.

39. **REASONING** *True* or *false*? Any two right triangles with the same hypotenuse have the same area. Explain your reasoning.

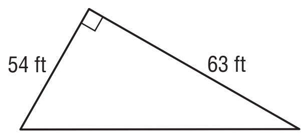
40. **CHALLENGE** The figure at the right is a rectangular prism with $AB = 8$, $BC = 6$, and $BF = 8$. Find HB.



41. **Writing in Math** Explain how right triangles are used to build suspension bridges. Which parts of the right triangle are formed by the cables?

STANDARDIZED TEST PRACTICE

42. Miko is going to rope off an area of the park for an upcoming concert. He is going to place a plastic flag for every three feet of rope.



About how many flags is Miko going to place?

A 82 **B** 75 **C** 67 **D** 45

43. A rectangle has an area of 25 square inches. If the dimensions of the rectangle are doubled, what will be the area of the new rectangle?

F 12.5 in^2 **H** 100 in^2

G 50 in^2 **J** 625 in^2

44. **REVIEW** Which equation is equivalent to $5(3 - 2x) = 7 - 2(1 - 4x)$?

A $18x = 10$ **C** $2x = 10$

B $2x = -10$ **D** $10x = -10$

Find the geometric mean between each pair of numbers. (Lesson 8-1)

45. 3 and 12 **46.** 9 and 12 **47.** 11 and 7 **48.** 6 and 9

49. **GARDENS** A park has a garden plot shaped like a triangle. It is bordered by a path. The triangle formed by the outside edge of the path is similar to the triangular garden. The perimeter of the outside edge of the path is 53 feet, the longest edge is 20 feet. The longest edge of the garden plot is 12 feet. What is the perimeter of the garden? (Lesson 7-5)

50. Could the sides of a triangle have the lengths 12, 13, and 25? Explain. (Lesson 5-4)

PREREQUISITE SKILL **Simplify each expression by rationalizing the denominator.** (Pages 790–791)

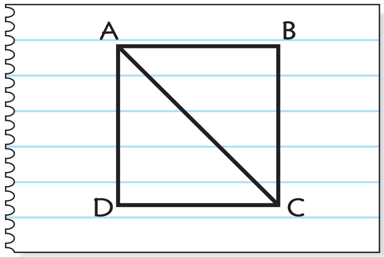
51. $\frac{7}{\sqrt{3}}$ **52.** $\frac{18}{\sqrt{2}}$ **53.** $\frac{\sqrt{14}}{\sqrt{2}}$ **54.** $\frac{3\sqrt{11}}{\sqrt{3}}$ **55.** $\frac{24}{\sqrt{2}}$

EXPLORE 8-3

Geometry Lab
Patterns in Special Right Triangles

Triangles with angles 45°-45°-90° measuring or 30°-60°-90° are called *special right triangles*. There are patterns in the measures of the sides of these triangles.

ACTIVITY 1 **Identify patterns in 45°-45°-90° triangles.**



Step 1 Draw a square with sides 4 centimeters long. Label the vertices A, B, C, and D.

Step 2 Draw the diagonal $\overline{AC}$.

Step 3 Use a protractor to measure $\angle CAB$ and $\angle ACB$.

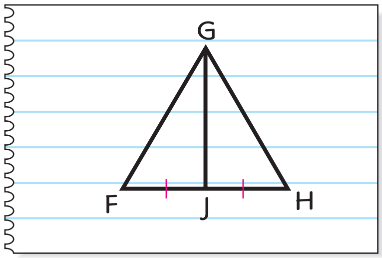
Step 4 Use the Pythagorean Theorem to find AC. Write in simplest form.

Analyze the Results

1. Repeat the activity for squares with sides 6 centimeters long and 8 centimeters long.

2. **MAKE A CONJECTURE** What is the length of the hypotenuse of a 45°-45°-90° triangle with legs that are n units long?

ACTIVITY 2 **Identify patterns in 30°-60°-90° triangles.**



Step 1 Construct an equilateral triangle with sides 2 inches long. Label the vertices F, G, and H.

Step 2 Find the midpoint of $\overline{FH}$ and label it J. Draw median $\overline{GJ}$.

Step 3 Use a protractor to measure $\angle FGJ$, $\angle F$, and $\angle GJF$.

Step 4 Use the Pythagorean Theorem to find GJ. Write in simplest form.

Analyze the Results

3. Repeat the activity to complete a table like the one at the right.

FG	*FJ*	*GJ*
2 in.		
4 in.		
5 in.		

4. **MAKE A CONJECTURE** What are the lengths of the long leg and the hypotenuse of a 30°-60°-90° triangle with a short leg n units long?

8-3 Special Right Triangles

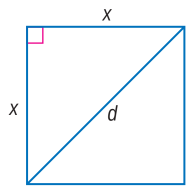
Main Ideas

- Use properties of 45°-45°-90° triangles.
- Use properties of 30°-60°-90° triangles.

Many quilt patterns use *half square triangles* to create a design. The pinwheel design was created with eight half square triangles rotated around the center. The measures of the angles in the half square triangles are 45°, 45°, and 90°.

Properties of 45°-45°-90° Triangles Facts about 45°-45°-90° triangles are used to solve many geometry problems. The Pythagorean Theorem allows us to discover special relationships that exist among the sides of a 45°-45°-90° triangle.

Draw a diagonal of a square. The two triangles formed are isosceles right triangles. Let x represent the measure of each side and let d represent the measure of the hypotenuse.

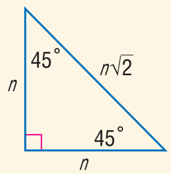


$d^2 = x^2 + x^2$	Pythagorean Theorem
$d^2 = 2x^2$	Add.
$d = \sqrt{2x^2}$	Take the positive square root of each side.
$d = \sqrt{2} \cdot \sqrt{x^2}$	Factor.
$d = x\sqrt{2}$	Simplify.

This algebraic proof verifies that the length of the hypotenuse of any 45°-45°-90° triangle is $\sqrt{2}$ times the length of its leg. The ratio of the sides is $1 : 1 : \sqrt{2}$.

THEOREM 8.6

In a 45°-45°-90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

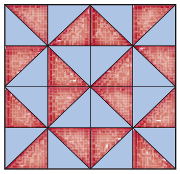


You can use this relationship to find the measure of the hypotenuse of a 45°-45°-90° triangle given the measure of a leg of the triangle.

Pin Wheel Quilt. American, 19th century. Private Collection/Bridgeman Art Library

EXAMPLE Find the Measure of the Hypotenuse

1 **WALLPAPER TILING Assume that the length of one of the legs of the 45°-45°-90° triangles in the wallpaper in the figure is 4 inches. What is the length of the diagonal of the entire wallpaper square?**



The length of each leg of the 45°-45°-90° triangle is 4 inches.

The length of the hypotenuse is $\sqrt{2}$ times as long as a leg. So, the length of the hypotenuse of one of the triangles is $4\sqrt{2}$.

There are four 45°-45°-90° triangles along the diagonal of the square. So, the length of the diagonal of the square is $4\left(4\sqrt{2}\right)$ or $16\sqrt{2}$ inches.

1. The length of the leg of a 45°-45°-90° triangle is 7 centimeters. What is the length of the hypotenuse?

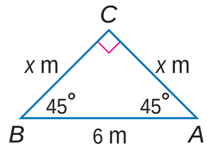
EXAMPLE Find the Measure of the Legs

Study Tip

Rationalizing Denominators

To rationalize a denominator, multiply the fraction by 1 in the form of a radical over itself so that the product in the denominator is a rational number.

2 **Find x.**



The length of the hypotenuse of a 45°-45°-90° triangle is $\sqrt{2}$ times the length of a leg of the triangle.

$$AB = (AC)\sqrt{2}$$

$$6 = x\sqrt{2} \qquad AB = 6,\ AC = x$$

$$\frac{6}{\sqrt{2}} = x \qquad \text{Divide each side by } \sqrt{2}.$$

$$\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = x \qquad \text{Rationalize the denominator.}$$

$$\frac{6\sqrt{2}}{2} = x \qquad \text{Multiply.}$$

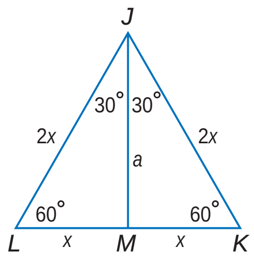
$$3\sqrt{2} = x \qquad \text{Divide.}$$

$$4.24 \approx x \qquad \text{Use a calculator.}$$

2. Refer to $\triangle ABC$. Suppose $BA = 5m$. Find x.

Properties of 30°-60°-90° Triangles There is also a special relationship among the measures of the sides of a 30°-60°-90° triangle.

When an altitude is drawn from any vertex of an equilateral triangle, two congruent 30°-60°-90° triangles are formed. $\overline{LM}$ and $\overline{KM}$ are congruent segments, so let $LM = x$ and $KM = x$. By the Segment Addition Postulate, $LM + KM = KL$. Thus, $KL = 2x$. Since $\triangle JKL$ is an equilateral triangle, $KL = JL = JK$. Therefore, $JL = 2x$ and $JK = 2x$.



Let a represent the measure of the altitude. Use the Pythagorean Theorem to find a.

$(JM)^2 + (LM)^2 = (JL)^2$	Pythagorean Theorem
$a^2 + x^2 = (2x)^2$	$JM = a$, $LM = x$, $JL = 2x$
$a^2 + x^2 = 4x^2$	Simplify.
$a^2 = 3x^2$	Subtract x^2 from each side.
$a = \sqrt{3x^2}$	Take the positive square root of each side.
$a = \sqrt{3} \cdot \sqrt{x^2}$	Factor.
$a = x\sqrt{3}$	Simplify.

So, in a 30°-60°-90° triangle, the measures of the sides are x, $x\sqrt{3}$, and $2x$. The ratio of the sides is $1:\sqrt{3}:2$.

The relationship of the side measures leads to Theorem 8.7.

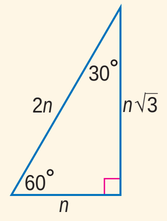
Study Tip

30°-60°-90° Triangle

The shorter leg is opposite the 30° angle, and the longer leg is opposite the 60° angle.

THEOREM 8.7

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

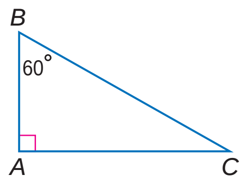


EXAMPLE 3 30°-60°-90° Triangles

Find the missing measures.

a. If *BC* = 14 inches, find *AC*.

$\overline{AC}$ is the longer leg, $\overline{AB}$ is the shorter leg, and $\overline{BC}$ is the hypotenuse.



$AB = \frac{1}{2}(BC)$

$= \frac{1}{2}(14)$ or 7 $\quad$ $BC = 14$

$AC = \sqrt{3}(AB)$

$= \sqrt{3}(7)$ or $7\sqrt{3}$ $\quad$ $AB = 7$

≈ 12.12 $\quad$ AC is $7\sqrt{3}$ or about 12.12 inches.

b. If *AC* = 8 inches, find *BC*.

$AC = \sqrt{3}(AB)$

$8 = \sqrt{3}(AB)$

$\frac{8}{\sqrt{3}} = AB$

$\frac{8\sqrt{3}}{3} = AB$

$BC = 2AB$

$= 2\left(\frac{8\sqrt{3}}{3}\right)$

$= \frac{16\sqrt{3}}{3}$

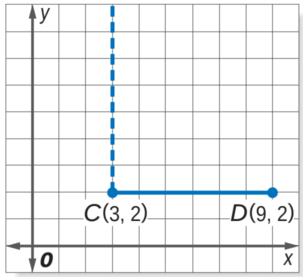
≈ 9.24

BC is $\frac{16\sqrt{3}}{3}$ or about 9.24 inches.

3. Refer to $\triangle ABC$. Suppose $AC = 12$ in. Find BC.

EXAMPLE 4 Special Triangles in a Coordinate Plane

COORDINATE GEOMETRY **Triangle *PCD* is a 30°-60°-90° triangle with right angle *C*. $\overline{CD}$ is the longer leg with endpoints *C*(3, 2) and *D*(9, 2). Locate point *P* in Quadrant I.**

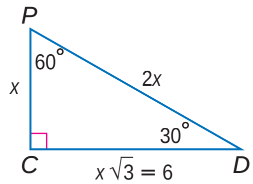


$\overline{CD}$ lies on a horizontal gridline. Since $\overline{PC}$ will be perpendicular to $\overline{CD}$, it lies on a vertical gridline. Find the length of $\overline{CD}$.

$CD = |9 - 3| = 6$

$\overline{CD}$ is the longer leg. $\overline{PC}$ is the shorter leg.

So, $CD = \sqrt{3}(PC)$. Use *CD* to find *PC*.



$$CD = \sqrt{3}(PC)$$

$$6 = \sqrt{3}(PC) \quad CD = 6$$

$$\frac{6}{\sqrt{3}} = PC \quad \text{Divide each side by } \sqrt{3}.$$

$$\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = PC \quad \text{Rationalize the denominator.}$$

$$\frac{6\sqrt{3}}{3} = PC \quad \text{Multiply.}$$

$$2\sqrt{3} = PC \quad \text{Simplify.}$$

Point *P* has the same *x*-coordinate as *C*. *P* is located $2\sqrt{3}$ units above *C*. So, the coordinates of *P* are $(3, 2 + 2\sqrt{3})$ or about (3, 5.46).

4. Triangle *RST* is a 30°-60°-90° triangle with right angle *RST*. $\overline{ST}$ is the shorter leg with endpoints *S*(1, 1) and *T*(4, 1). Locate point *R* in Quadrant I.

Personal Tutor at geometryonline.com

Study Tip

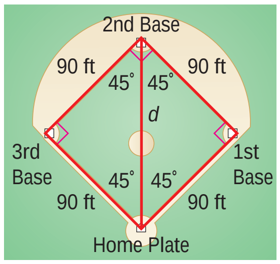
Checking Reasonableness of Results

To check the coordinates of *P* in Example 4, use a protractor to draw $\overline{DP}$ such that $m\angle CDP = 30$. Then from the graph, you can estimate the coordinates of *P*.

Check Your Understanding

Example 1 (p. 449)

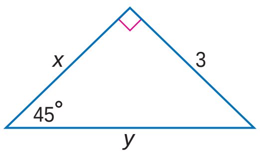
1. SOFTBALL Find the distance from home plate to second base if the bases are 90 feet apart.



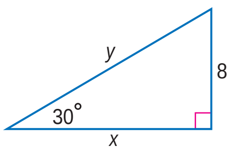
Example 2 (p. 449)

Find *x* and *y*.

2.



3.

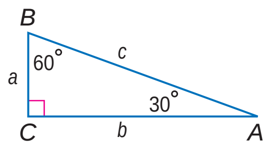


Example 3 (p. 450)

Find the missing measures.

4. If $c = 8$, find *a* and *b*.

5. If $b = 18$, find *a* and *c*.



Example 4 (p. 451)

Triangle *ABD* is a 30°-60°-90° triangle with right angle *B* and with $\overline{AB}$ as the shorter leg. Graph *A* and *B*, and locate point *D* in Quadrant I.

6. $A(8, 0)$, $B(8, 3)$

7. $A(6, 6)$, $B(2, 6)$

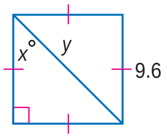
Exercises

HOMEWORK HELP

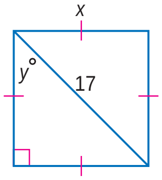
For Exercises	See Examples
8–10, 18, 20	2
11–17, 19, 21–23	3
24–27	4
28–32	1

Find *x* and *y*.

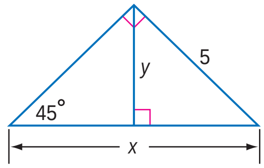
8.



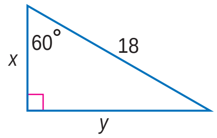
9.



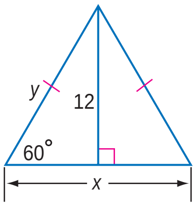
10.



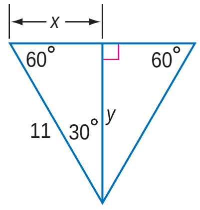
11.



12.



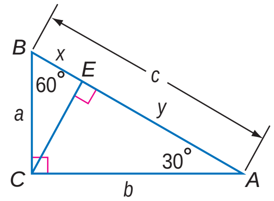
13.



For Exercises 14 and 15, use the figure at the right.

14. If $a = 10\sqrt{3}$, find *CE* and *y*.

15. If $x = 7\sqrt{3}$, find *a*, *CE*, *y*, and *b*.



16. The length of an altitude of an equilateral triangle is 12 feet. Find the length of a side of the triangle.

17. The perimeter of an equilateral triangle is 45 centimeters. Find the length of an altitude of the triangle.

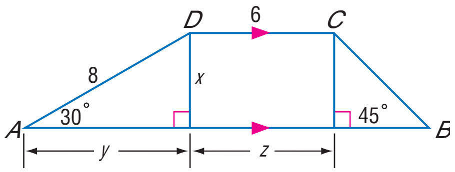
18. The length of a diagonal of a square is $22\sqrt{2}$ millimeters. Find the perimeter of the square.

19. The altitude of an equilateral triangle is 7.4 meters long. Find the perimeter of the triangle.

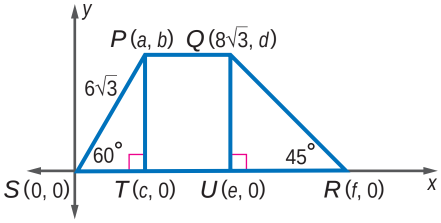
20. The diagonals of a rectangle are 12 inches long and intersect at an angle of 60°. Find the perimeter of the rectangle.

21. The sum of the squares of the measures of the sides of a square is 256. Find the measure of a diagonal of the square.

22. Find *x*, *y*, *z*, and the perimeter of trapezoid *ABCD*.



23. If $\overline{PQ} \parallel \overline{SR}$, find *a*, *b*, *c*, and *d*.



24. $\triangle PAB$ is a 45°-45°-90° triangle with right angle *B*. Find the coordinates of *P* in Quadrant I for $A(-3, 1)$ and $B(4, 1)$.

25. $\triangle PGH$ is a 45°-45°-90° triangle with $m\angle P = 90°$. Find the coordinates of *P* in Quadrant I for $G(4, -1)$ and $H(4, 5)$.

26. $\triangle PCD$ is a 30°-60°-90° triangle with right angle C and $\overline{CD}$ the longer leg. Find the coordinates of P in Quadrant III for $C(-3, -6)$ and $D(-3, 7)$.

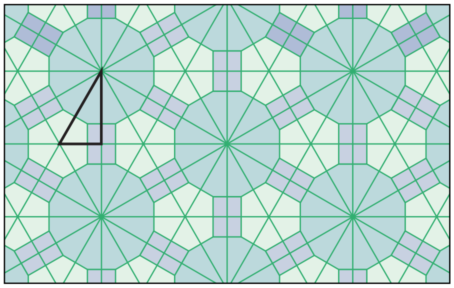
27. $\triangle PCD$ is a 30°-60°-90° triangle with $m\angle C = 30$ and hypotenuse $\overline{CD}$. Find the coordinates of P for $C(2, -5)$ and $D(10, -5)$ if P lies above $\overline{CD}$.

Real-World Link

Triangle Tiling Buildings in Federation Square in Melbourne, Australia, feature a tiling pattern called a pinwheel tiling. The sides of each right triangle are in the ratio $1 : 2 : \sqrt{5}$.

Source: www.federationsquare.com.au

TRIANGLE TILING **For Exercises 28–31, use the following information.**
Triangle tiling refers to the process of taking many copies of a single triangle and laying them next to each other to fill an area. For example, the pattern shown is composed of tiles like the one outlined.



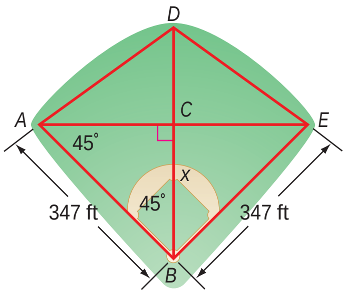
28. How many 30°-60°-90° triangles are used to create the basic pattern, which resembles a circle?

29. Which angle of the 30°-60°-90° triangle is being rotated to make the basic shape?

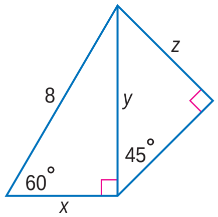
30. Explain why there are no gaps in the basic pattern.

31. Use grid paper to cut out 30°-60°-90° triangles. Color the same pattern on each triangle. Create one basic figure that would be part of a wallpaper tiling.

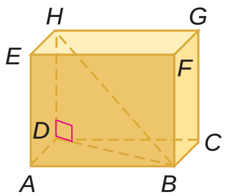
32. **BASEBALL** The diagram at the right shows some dimensions of U.S. Cellular Field in Chicago, Illinois. $\overline{BD}$ is a segment from home plate to dead center field, and $\overline{AE}$ is a segment from the left field foul-ball pole to the right field foul-ball pole. If the center fielder is standing at C, how far is he from home plate?



33. Find x, y, and z.

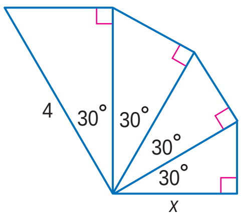


34. If $BD = 8\sqrt{3}$ and $m\angle DHB = 60°$, find BH.

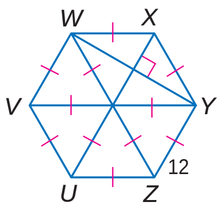


EXTRA PRACTICE
See pages 815, 835.
Math Online
Self-Check Quiz at geometryonline.com

35. Each triangle in the figure is a 30°-60°-90° triangle. Find x.



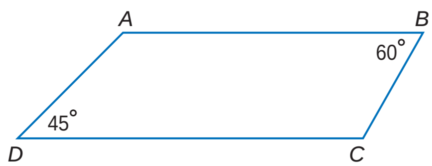
36. In regular hexagon $UVWXYZ$, each side is 12 centimeters long. Find WY.



H.O.T. Problems

37. **OPEN ENDED** Draw a rectangle that has a diagonal twice as long as its width. Then write an equation to find the length of the rectangle.

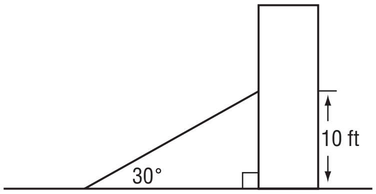
38. **CHALLENGE** Given figure $ABCD$, with $\overline{AB} \parallel \overline{DC}$, $m\angle B = 60°$, $m\angle D = 45°$, $BC = 8$, and $AB = 24$, find the perimeter.



39. **Writing in Math** Refer to the information about quilting on page 448. Describe why quilters use the term *half square triangles* to describe 45°-45°-90° triangles. Explain why 45°-45°-90° triangles are used in this pattern instead of 30°-60°-90° triangles.

STANDARDIZED TEST PRACTICE

40. A ladder is propped against a building at a 30° angle.

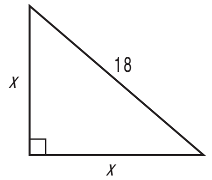


What is the length of the ladder?

A 5 ft **C** $10\sqrt{3}$ ft

B 10 ft **D** 20 ft

41. Look at the right triangle below. Which of the following could be the triangle's dimensions?



F 9 **H** $18\sqrt{2}$

G $9\sqrt{2}$ **J** 36

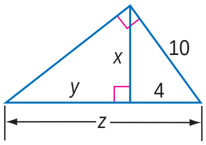
Determine whether each set of measures contains the sides of a right triangle. Then state whether they form a Pythagorean triple. (Lesson 7-2)

42. 3, 4, 5 **43.** 9, 40, 41 **44.** 20, 21, 31

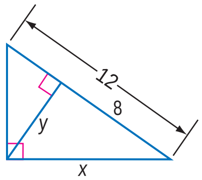
45. 20, 48, 52 **46.** 7, 24, 25 **47.** 12, 34, 37

Find *x*, *y*, and *z*. (Lesson 7-1)

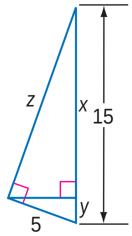
48.



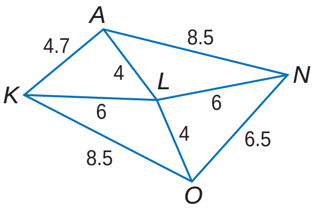
49.



50.



Write an inequality relating each pair of angles. (Lesson 5-5)



51. $m\angle ALK$, $m\angle ALN$

52. $m\angle ALK$, $m\angle NLO$

53. $m\angle OLK$, $m\angle NLO$

54. $m\angle KLO$, $m\angle ALN$

55. **SCALE MODELS** Taipa wants to build a scale model of the Canadian Horseshoe Falls at Niagara Falls. The height is 52 meters. If she wants the model to be 80 centimeters tall, what scale factor will she use? (Lesson 7-1)

PREREQUISITE SKILL **Solve each equation.** (Pages 781–782)

56. $5 = \frac{x}{3}$ **57.** $\frac{x}{9} = 0.14$ **58.** $0.5 = \frac{10}{k}$ **59.** $0.2 = \frac{13}{g}$

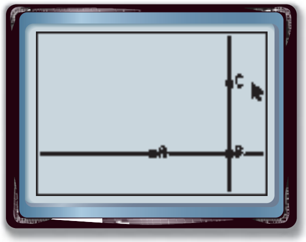
60. $\frac{7}{n} = 0.25$ **61.** $9 = \frac{m}{0.8}$ **62.** $\frac{24}{x} = 0.4$ **63.** $\frac{35}{y} = 0.07$

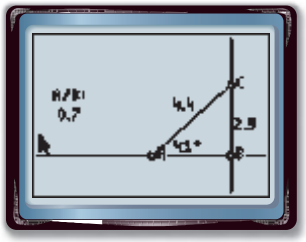
EXPLORE 8-4

Graphing Calculator Lab
Trigonometry

You have investigated the patterns in the measures of special right triangles. The study of the patterns in all right triangles is called *trigonometry*. You can use the Cabri Junior application on a TI-83/84 Plus to investigate these patterns.

ACTIVITY





Step 1 Use the line tool on the F2 menu to draw a line. Label the points on the line A and B.

Step 2 Press F3 and choose the Perpendicular tool to create a perpendicular line through point B. Draw and label a point C on the perpendicular.

Step 3 Use the segment tool on the F2 menu to draw $\overline{AC}$.

Step 4 Find and label the measures of $\overline{BC}$ and $\overline{AC}$ using the Distance and Length tool under Measure on the F5 menu. Use the Angle tool for the measure of $\angle A$.

Step 6 Calculate and display the ratio $\frac{BC}{AC}$ using the Calculate tool on the F5 menu. Label the ratio as A/B.

Step 7 Press CLEAR. Then use the arrow keys to move the cursor close to point B. When the arrow is clear, press and hold the ALPHA key. Drag B and observe the ratio.

ANALYZE THE RESULTS

1. Discuss the effect of dragging point B on BC, AC, $m\angle A$, and the ratio $\frac{BC}{AC}$.

2. Use the calculate tool to find the ratios $\frac{AB}{AC}$ and $\frac{BC}{AB}$. Then drag B and observe the ratios.

3. **MAKE A CONJECTURE** The *sine*, *cosine*, and *tangent* functions are trigonometric functions based on angle measures. Make a note of $m\angle A$. Exit Cabri Jr. and use SIN, COS and TAN on the calculator to find *sine*, *cosine* and *tangent* for $m\angle A$. Compare the results to the ratios you found in the activity. Make a conjecture about the definitions of sine, cosine, and tangent.

8-4 Trigonometry

Main Ideas

- Find trigonometric ratios using right triangles.
- Solve problems using trigonometric ratios.

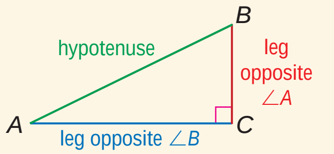
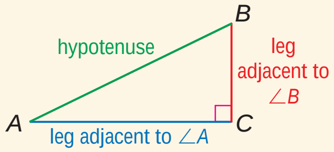
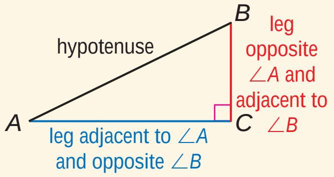
New Vocabulary

trigonometry
trigonometric ratio
sine
cosine
tangent

The branch of mathematics known as *trigonometry* was developed for use by astronomers and surveyors. Surveyors use an instrument called a theodolite (thee AH duh lite) to measure angles. It consists of a telescope mounted on a vertical axis and a horizontal axis. After measuring the angles, surveyors apply trigonometry to calculate distance or height.

Trigonometric Ratios The word **trigonometry** comes from two Greek terms, *trigon*, meaning triangle, and *metron*, meaning measure. The study of trigonometry involves triangle measurement. A ratio of the lengths of sides of a right triangle is called a **trigonometric ratio**. The three most common trigonometric ratios are **sine**, **cosine**, and **tangent**.

KEY CONCEPT *Trigonometric Ratios*

Words	Symbols	Models
sine of $\angle A = \dfrac{\text{leg opposite } \angle A}{\text{hypotenuse}}$ sine of $\angle B = \dfrac{\text{leg opposite } \angle B}{\text{hypotenuse}}$	$\sin A = \dfrac{BC}{AB}$ $\sin B = \dfrac{AC}{AB}$	
cosine of $\angle A = \dfrac{\text{leg adjacent to } \angle A}{\text{hypotenuse}}$ cosine of $\angle B = \dfrac{\text{leg adjacent to } \angle B}{\text{hypotenuse}}$	$\cos A = \dfrac{AC}{AB}$ $\cos B = \dfrac{BC}{AB}$	
tangent of $\angle A = \dfrac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A}$ tangent of $\angle B = \dfrac{\text{leg opposite } \angle B}{\text{leg adjacent to } \angle B}$	$\tan A = \dfrac{BC}{AC}$ $\tan B = \dfrac{AC}{BC}$	

Trigonometric ratios are related to the acute angles of a right triangle, not the right angle.

Reading Math

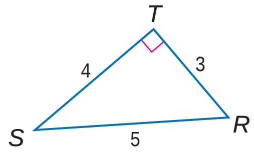
Memory Hint SOH-CAH-TOA is a mnemonic device for learning the ratios for sine, cosine, and tangent using the first letter of each word in the ratios.

$\sin A = \frac{\text{opp}}{\text{hyp}}$

$\cos A = \frac{\text{adj}}{\text{hyp}}$

$\tan A = \frac{\text{opp}}{\text{adj}}$

EXAMPLE Find Sine, Cosine, and Tangent Ratios



1 **Find sin *R*, cos *R*, tan *R*, sin *S*, cos *S*, and tan *S*. Express each ratio as a fraction and as a decimal.**

$\sin R = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{ST}{RS} = \frac{4}{5} \text{ or } 0.8$

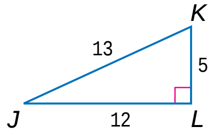
$\cos R = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{RT}{RS} = \frac{3}{5} \text{ or } 0.6$

$\tan R = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{ST}{RT} = \frac{4}{3} \text{ or } 1.\overline{3}$

$\sin S = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{RT}{RS} = \frac{3}{5} \text{ or } 0.6$

$\cos S = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{ST}{RS} = \frac{4}{5} \text{ or } 0.8$

$\tan S = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{RT}{ST} = \frac{3}{4} \text{ or } 0.75$

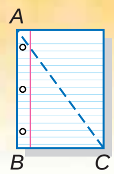


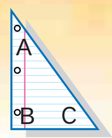
1. Find sin *J*, cos *J*, tan *J*, sin *K*, cos *K*, and tan *K*. Express each ratio as a fraction and as a decimal.

You can use paper folding to investigate trigonometric ratios in similar right triangles.

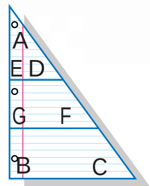
GEOMETRY LAB

Trigonometric Ratios





- Fold a rectangular piece of paper along a diagonal from *A* to *C*. Then cut along the fold to form right triangle *ABC*. Write the name of each angle on the inside of the triangle.



- Fold the triangle so that there are two segments perpendicular to $\overline{BA}$. Label points *D*, *E*, *F*, and *G* as shown. Use a ruler to measure $\overline{AC}$, $\overline{AB}$, $\overline{BC}$, $\overline{AF}$, $\overline{AG}$, $\overline{FG}$, $\overline{AD}$, $\overline{AE}$, and $\overline{DE}$ to the nearest millimeter.

ANALYZE THE RESULTS

1. What is true of $\triangle AED$, $\triangle AGF$, and $\triangle ABC$?

2. Copy the table. Write the ratio of the side lengths for each ratio. Then calculate a value for each ratio to the nearest ten-thousandth.

	In $\triangle AED$	In $\triangle AGF$	In $\triangle ABC$
sin *A*			
cos *A*			
tan *A*			

3. Study the table. Write a sentence about the patterns you observe.

4. What is true about $m\angle A$ in each triangle?

As the Geometry Lab suggests, the value of a trigonometric ratio depends *only* on the measure of the angle. It does not depend on the size of the triangle.

Study Tip

Graphing Calculator

Be sure your calculator is in degree mode rather than radian mode. Your calculator may require you to input the angle *before* using the trigonometric key.

EXAMPLE 2 Use a Calculator to Evaluate Expressions

Use a calculator to find cos 39° to the nearest ten-thousandth.

KEYSTROKES: [COS] 39 [ENTER]

$\cos 39° \approx 0.7771$

2. $\sin 67°$

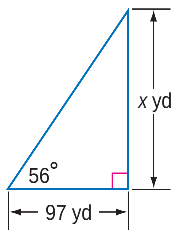
Use Trigonometric Ratios

You can use trigonometric ratios to find the missing measures of a right triangle if you know the measures of two sides of a triangle or the measure of one side and one acute angle.

EXAMPLE 3 Use Trigonometric Ratios to Find a Length

SURVEYING **Dakota is standing on the ground 97 yards from the base of a cliff. Using a theodolite, he noted that the angle formed by the ground and the line of sight to the top of the cliff is 56°. Find the height of the cliff to the nearest yard.**

Let x be the height of the cliff in yards.



$\tan 56° = \frac{x}{97}$ $\quad \tan = \frac{\text{leg opposite}}{\text{leg adjacent}}$

$97 \tan 56° = x$ $\quad$ Multiply each side by 97.

Use a calculator to find x.

KEYSTROKES: 97 [TAN] 56 [ENTER] 143.8084139

The cliff is about 144 yards high.

3. **MEASUREMENT** Jonathan is standing 15 yards from a roller coaster. The angle formed by the ground to the top of the roller coaster is 71°. How tall is the roller coaster?

When solving equations like $3x = -27$, you use the inverse of multiplication to find x. In trigonometry, you can find the measure of the angle by using the inverse of sine, cosine, or tangent.

Given equation	To find the angle	Read as
$\sin A = x$	$A = \sin^{-1}(x)$	*A* equals *the inverse sine of x.*
$\cos A = y$	$A = \cos^{-1}(y)$	*A* equals *the inverse cosine of y.*
$\tan A = z$	$A = \tan^{-1}(z)$	*A* equals *the inverse tangent of z.*

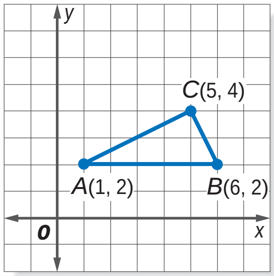
Study Tip

Calculators

The second functions of the SIN, COS, and TAN keys are usually the inverses.

EXAMPLE 4 Use Trigonometric Ratios to Find an Angle Measure

COORDINATE GEOMETRY **Find $m\angle A$ in right triangle ABC for $A(1, 2)$, $B(6, 2)$, and $C(5, 4)$.**



Explore You know the coordinates of the vertices of a right triangle and that $\angle C$ is the right angle. You need to find the measure of one of the angles.

Plan Use the Distance Formula to find the measure of each side. Then use one of the trigonometric ratios to write an equation. Use the inverse to find $m\angle A$.

Solve

$$AB = \sqrt{(6-1)^2 + (2-2)^2} = \sqrt{25 + 0} \text{ or } 5$$

$$BC = \sqrt{(5-6)^2 + (4-2)^2} = \sqrt{1 + 4} \text{ or } \sqrt{5}$$

$$AC = \sqrt{(5-1)^2 + (4-2)^2} = \sqrt{16 + 4} = \sqrt{20} \text{ or } 2\sqrt{5}$$

Use the cosine ratio.

$\cos A = \frac{AC}{AB}$ — $\cos = \frac{\text{leg adjacent}}{\text{hypotenuse}}$

$\cos A = \frac{2\sqrt{5}}{5}$ — $AC = 2\sqrt{5}$ and $AB = 5$

$A = \cos^{-1}\left(\frac{2\sqrt{5}}{5}\right)$ — Solve for A.

Use a calculator to find $m\angle A$.

KEYSTROKES: 2nd [COS^{-1}] 2 2nd [√] 5) ÷ 5 ENTER

$m\angle A \approx 26.56505118$

The measure of $\angle A$ is about 26.6.

Check Use the sine ratio to check the answer.

$\sin A = \frac{BC}{AB}$ — $\sin = \frac{\text{leg opposite}}{\text{hypotenuse}}$

$\sin A = \frac{\sqrt{5}}{5}$ — $BC = \sqrt{5}$ and $AB = 5$

KEYSTROKES: 2nd [SIN^{-1}] 2nd [√] 5) ÷ 5 ENTER

$m\angle A \approx 26.56505118$

The answer is correct.

Cross-Curricular Project

You can use trigonometry to help you come closer to locating the hidden treasure. Visit geometryonline.com.

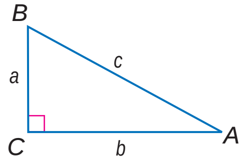
4. Find $m\angle P$ in right $\triangle PQR$ for $P(2, -1)$, $Q(4, 3)$, and $R(8, 1)$.

Personal Tutor at geometryonline.com

CHECK Your Understanding

Example 1 (p. 457)

Use $\triangle ABC$ to find sin A, cos A, tan A, sin B, cos B, and tan B. Express each ratio as a fraction and as a decimal to the nearest hundredth.



1. $a = 14$, $b = 48$, and $c = 50$
2. $a = 8$, $b = 15$, and $c = 17$

Example 2 (p. 458)

Use a calculator to find each value. Round to the nearest ten-thousandth.

3. sin 57°
4. cos 60°
5. cos 33°
6. tan 30°
7. tan 45°
8. sin 85°

Example 3 (p. 458)

9. **SURVEYING** Maureen is standing on horizontal ground level with the base of the CN Tower in Toronto, Ontario. The angle formed by the ground and the line segment from her position to the top of the tower is 31.2°. She knows that the height of the tower to the top of the antennae is about 1815 feet. Find her distance from the CN Tower to the nearest foot.

Find the measure of each angle to the nearest tenth of a degree.

10. tan $A = 1.4176$
11. sin $B = 0.6307$

Example 4 (p. 459)

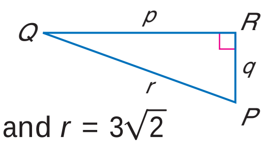
COORDINATE GEOMETRY **Find the measure of the angle to the nearest tenth in each right triangle *ABC*.**

12. $\angle A$ in $\triangle ABC$, for $A(6, 0)$, $B(-4, 2)$, and $C(0, 6)$
13. $\angle B$ in $\triangle ABC$, for $A(3, -3)$, $B(7, 5)$, and $C(7, -3)$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
14–17	1
18–23	2
24, 25	3
26–28	4

Use $\triangle PQR$ with right angle R to find sin P, cos P, tan P, sin Q, cos Q, and tan Q. Express each ratio as a fraction, and as a decimal to the nearest hundredth.

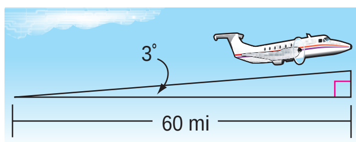


14. $p = 12$, $q = 35$, and $r = 37$
15. $p = \sqrt{6}$, $q = 2\sqrt{3}$, and $r = 3\sqrt{2}$
16. $p = \frac{3}{2}$, $q = \frac{3\sqrt{3}}{2}$, and $r = 3$
17. $p = 2\sqrt{3}$, $q = \sqrt{15}$, and $r = 3\sqrt{3}$

Use a calculator to find each value. Round to the nearest ten-thousandth.

18. sin 6°
19. tan 42.8°
20. cos 77°
21. sin 85.9°
22. tan 12.7°
23. cos 22.5°

24. **AVIATION** A plane is one mile above sea level when it begins to climb at a constant angle of 3° for the next 60 ground miles. About how far above sea level is the plane after its climb?



David R. Frazier/Photo Researchers

Real-World Link

The Jefferson Davis Monument in Fairview, Kentucky, is the fourth tallest monument in the United States. The walls are seven feet thick at the base, tapering to two feet thick at the top.

Source: parks.ky.gov

25. **MONUMENTS** At 351 feet tall, the Jefferson Davis Monument in Fairview, Kentucky, is the largest concrete obelisk in the world. Pedro is looking at the top of the monument at an angle of 75°. How far away from the monument is he standing?

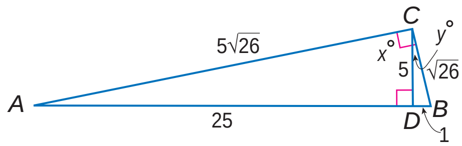
COORDINATE GEOMETRY **Find the measure of each angle to the nearest tenth in each right triangle.**

26. $\angle J$ in $\triangle JCL$ for $J(2, 2)$, $C(2, -2)$, and $L(7, -2)$

27. $\angle C$ in $\triangle BCD$ for $B(-1, -5)$, $C(-6, -5)$, and $D(-1, 2)$

28. $\angle X$ in $\triangle XYZ$ for $X(-5, 0)$, $Y(7, 0)$, and $Z(0, \sqrt{35})$

Use the figure to find each trigonometric ratio. Express answers as a fraction and as a decimal rounded to the nearest ten-thousandth.



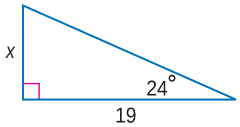
29. sin A	**30.** tan B	**31.** cos A
32. sin $x°$	**33.** cos $x°$	**34.** tan A
35. cos B	**36.** sin $y°$	**37.** tan $x°$

Find the measure of each angle to the nearest tenth of a degree.

38. $\sin B = 0.7245$	**39.** $\cos C = 0.2493$	**40.** $\tan E = 9.4618$
41. $\sin A = 0.4567$	**42.** $\cos D = 0.1212$	**43.** $\tan F = 0.4279$

Find *x*. Round to the nearest tenth.

44.



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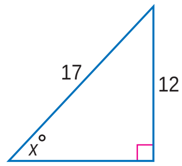

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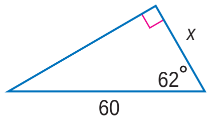

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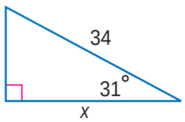
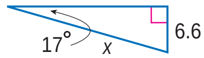


48.



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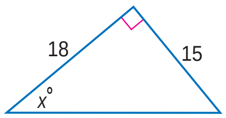
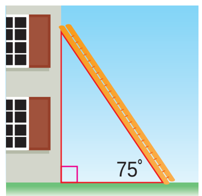


SAFETY **For Exercises 50 and 51, use the following information.**

To guard against a fall, a ladder should make an angle of 75° or less with the ground.

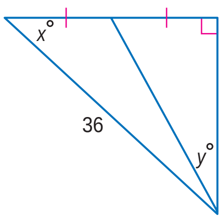


50. What is the maximum height that a 20-foot ladder can reach safely?

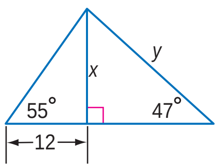
51. How far from the building is the base of the ladder at the maximum height?

Find *x* and *y*. Round to the nearest tenth.

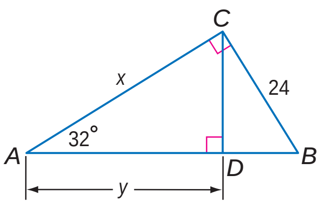
52.



53.



54.

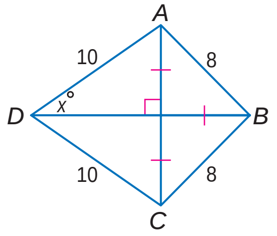


EXTRA PRACTICE
See pages 816, 835.
Math Online
Self-Check Quiz at
geometryonline.com

H.O.T. Problems

55. OPEN ENDED Draw a right triangle and label the measure of one acute angle and the measure of the side opposite that angle. Then solve for the remaining measures.

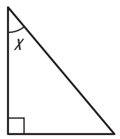
56. CHALLENGE Use the figure at the right to find $\sin x°$.



57. REASONING Explain the difference between $\tan A = \frac{x}{y}$ and $\tan^{-1}\left(\frac{x}{y}\right) = A$.

58. Writing in Math Refer to the information on theodolites on page 456. Explain how surveyors determine angle measures. Include the kind of information one obtains from a theodolite.

59. In the figure, if $\cos x = \frac{20}{29}$, what are $\sin x$ and $\tan x$?



A $\sin x = \frac{29}{21}$ and $\tan x = \frac{29}{21}$

B $\sin x = \frac{21}{29}$ and $\tan x = \frac{20}{21}$

C $\sin x = \frac{29}{21}$ and $\tan x = \frac{21}{20}$

D $\sin x = \frac{21}{29}$ and $\tan x = \frac{21}{20}$

60. REVIEW What is the solution set of the quadratic equation $x^2 + 4x - 2 = 0$?

F $\{-2, 2\}$

G $\{-2 + \sqrt{6}, -2 - \sqrt{6}\}$

H $\{-2 + \sqrt{2}, -2 - \sqrt{2}\}$

J no real solution

61. REVIEW Which of the following has the same value as $9^{-15} \times 9^3$?

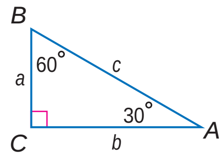
A 9^{-45} **C** 9^{-12}

B 9^{-18} **D** 9^{-5}

Find each measure. (Lesson 8-3)

62. If $a = 4$, find b and c.

63. If $b = 3$, find a and c.



Determine whether each set of measures can be the sides of a right triangle. Then state whether they form a Pythagorean triple. (Lesson 8-2)

64. 4, 5, 6 **65.** 5, 12, 13 **66.** 9, 12, 15 **67.** 8, 12, 16

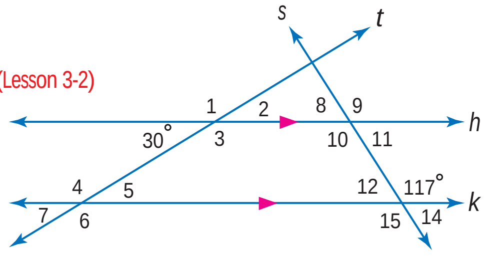
68. TELEVISION During a 30-minute television program, the ratio of minutes of commercials to minutes of the actual show is 4 : 11. How many minutes are spent on commercials? (Lesson 7-1)

PREREQUISITE SKILL **Find each angle measure if $h \parallel k$.** (Lesson 3-2)

69. $m\angle 15$ **70.** $m\angle 7$

71. $m\angle 3$ **72.** $m\angle 12$

73. $m\angle 11$ **74.** $m\angle 4$

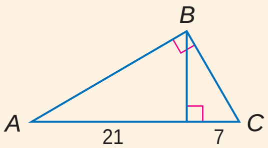


CHAPTER 8 Mid-Chapter Quiz

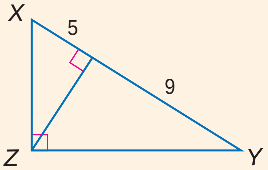
Lessons 8-1 through 8-4

Find the measure of the altitude drawn to the hypotenuse. (Lesson 8-1)

1.

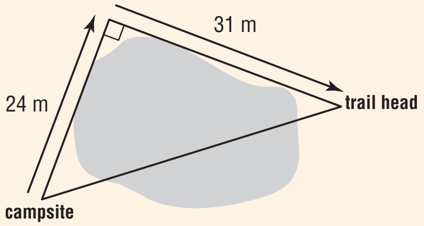


2.



3. Determine whether $\triangle ABC$ with vertices $A(2, 1)$, $B(4, 0)$, and $C(5, 7)$ is a right triangle. Explain. (Lesson 8-2)

4. MULTIPLE CHOICE To get from your campsite to a trail head, you must take the path shown below to avoid walking through a pond.

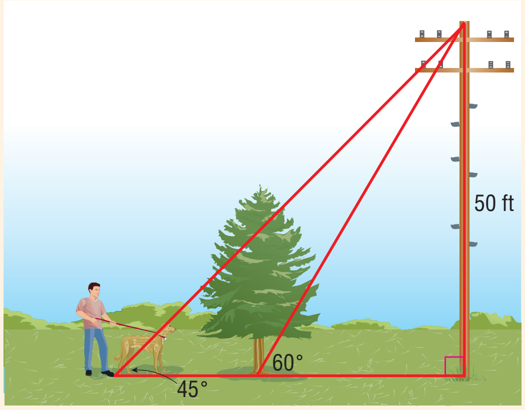


About how many meters would be saved if it were possible to walk through the pond? (Lesson 8-2)

A 55.0 **C** 24.7

B 39.2 **D** 15.8

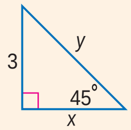
5. DOG WALKING A man is walking his dog on level ground in a straight line with the dog's favorite tree. The angle from the man's present position to the top of a nearby telephone pole is 45º. The angle from the tree to the top of the telephone pole is 60º. If the telephone pole is 50 feet tall, about how far is the man with the dog from the tree? (Lesson 8-3)



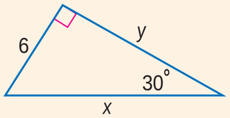
6. WOODWORKING Ginger made a small square table for her workshop with a diagonal that measures 55 inches. What are the measures of the sides? Recall that a square has right angles at the corners and congruent sides. (Lesson 8-3)

Find *x* and *y*. (Lesson 8-3)

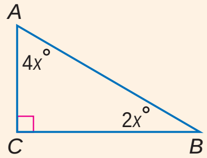
7.



8.



9. MULTIPLE CHOICE In the right triangle, what is AB if $BC = 6$? (Lesson 8-3)

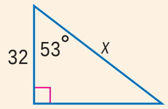


F 12 units **H** $4\sqrt{3}$ units

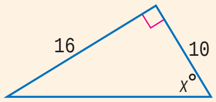
G $6\sqrt{2}$ units **J** $2\sqrt{3}$ units

Find *x* to the nearest tenth. (Lesson 8-4)

10.



11.



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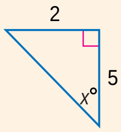
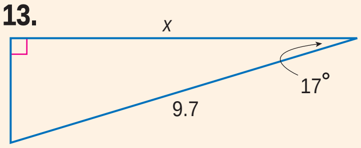


13.



14. GARDENING The lengths of the sides of a triangular garden are 32 feet, 24 feet, and 40 feet. What are the measures of the angles formed on each side of the garden? (Lesson 8-4)

Find the measure of each angle to the nearest tenth of a degree. (Lesson 8-4)

15. $\sin T = 0.5299$

16. $\cos W = 0.0175$

8-5 Angles of Elevation and Depression

Main Ideas

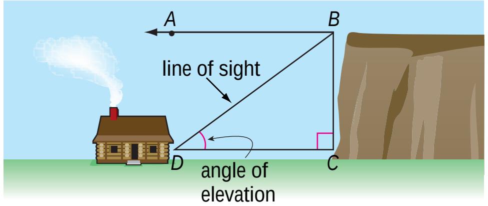
- Solve problems involving angles of elevation.
- Solve problems involving angles of depression.

New Vocabulary

angle of elevation
angle of depression

A pilot is getting ready to take off from Mountain Valley airport. She looks up at the peak of a mountain immediately in front of her. The pilot must estimate the speed needed and the angle formed by a line along the runway and a line from the plane to the peak of the mountain to clear the mountain.

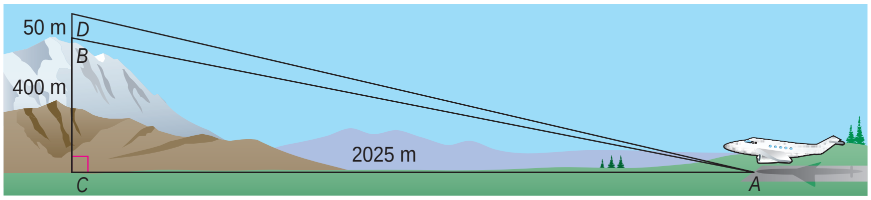
Angles of Elevation An **angle of elevation** is the angle between the line of sight and the horizontal when an observer looks upward.



EXAMPLE 1 Angle of Elevation

AVIATION The peak of Goose Bay Mountain is 400 meters higher than the end of a local airstrip. The peak rises above a point 2025 meters from the end of the airstrip. A plane takes off from the end of the runway in the direction of the mountain at an angle that is kept constant until the peak has been cleared. If the pilot wants to clear the mountain by 50 meters, what should the angle of elevation be for the takeoff to the nearest tenth of a degree?

Make a drawing.



Since *CB* is 400 meters and *BD* is 50 meters, *CD* is 450 meters. Let x represent $m\angle DAC$.

$\tan x° = \frac{CD}{AC}$ — $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$\tan x° = \frac{450}{2025}$ — $CD = 450$, $AC = 2025$

$x = \tan^{-1}\left(\frac{450}{2025}\right)$ — Solve for x.

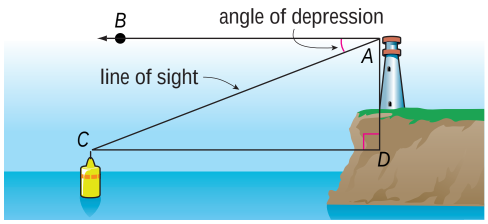
$x \approx 12.5$ — Use a calculator.

The angle of elevation for the takeoff should be more than 12.5°.

1. **SHADOWS** Find the angle of elevation of the Sun when a 7.6-meter flagpole casts a 18.2-meter shadow. Round to the nearest tenth of a degree.

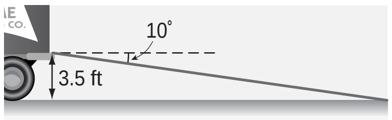
Extra Examples at geometryonline.com

Angles of Depression An **angle of depression** is the angle between the line of sight when an observer looks downward and the horizontal.



STANDARDIZED TEST EXAMPLE **Angle of Depression**

2 The tailgate of a moving van is 3.5 feet above the ground. A loading ramp is attached to the rear of the van at an incline of 10°. Which is closest to the length of the ramp?



$\sin 10° \approx 0.17$
$\cos 10° \approx 0.98$
$\tan 10° \approx 0.18$

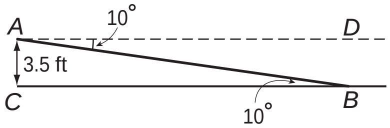
A 3.6 ft **C** 19.8 ft

B 12.2 ft **D** 20.2 ft

Read the Test Item

The angle of depression between the ramp and the horizontal is 10°. Use trigonometry to find the length of the ramp.

Solve the Test Item



The ground and the horizontal level with the back of the van are parallel. Therefore, $m\angle DAB = m\angle ABC$ since they are alternate interior angles.

$$\sin 10° = \frac{3.5}{AB} \qquad \sin = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$AB \sin 10° = 3.5 \qquad \text{Multiply each side by } AB.$$

$$AB = \frac{3.5}{\sin 10°} \qquad \text{Divide each side by } \sin 10°.$$

$$AB \approx \frac{3.5}{0.17} \qquad \sin 10° \approx 0.17$$

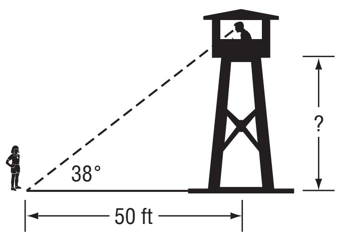
$$AB \approx 20.2 \qquad \text{Divide.}$$

The ramp is about 20.2 feet long. So the correct answer is choice D.

Test-Taking Tip

Check Results Before moving on to the next question, check the reasonableness of your answer. Analyze your result to determine that it makes sense.

2. HIKING Ayana is hiking in a national park. A forest ranger is standing in a fire tower that overlooks a meadow. She sees Ayana at an angle of depression measuring 38°. If Ayana is 50 feet away from the base of the tower, which is closest to the height of the fire tower?



$\sin 38° \approx 0.62$
$\cos 38° \approx 0.79$
$\tan 38° \approx 0.78$

F 30.8 ft **H** 39.4 ft

G 39.1 ft **J** 63.5 ft

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Angles of elevation or depression to two different objects can be used to find the distance between those objects.

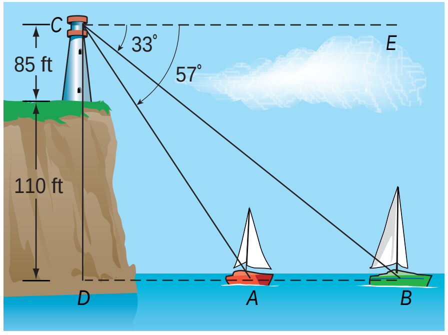
Study Tip

Common Misconception

The angle of depression is often not an angle of the triangle but the complement to an angle of the triangle. In $\triangle DBC$, the angle of depression is $\angle BCE$, not $\angle DCB$.

EXAMPLE 3 Indirect Measurement

Olivia works in a lighthouse on a cliff. She observes two sailboats due east of the lighthouse. The angles of depression to the two boats are 33° and 57°. Find the distance between the two sailboats to the nearest foot.



$\triangle CDA$ and $\triangle CDB$ are right triangles, and $CD = 110 + 85$ or 195. The distance between the boats is AB or $DB - DA$. Use the right triangles to find these two lengths.

Because $\overline{CE}$ and $\overline{DB}$ are horizontal lines, they are parallel. Thus, $\angle ECB \cong \angle CBD$ and $\angle ECA \cong \angle CAD$ because they are alternate interior angles. This means that $m\angle CBD = 33$ and $m\angle CAD = 57$.

Use the measures of $\triangle CBD$ to find DB.

$\tan 33° = \dfrac{195}{DB}$	$\tan = \dfrac{\text{opposite}}{\text{adjacent}}$; $m\angle CBD = 33$
$DB \tan 33° = 195$	Multiply each side by DB.
$DB = \dfrac{195}{\tan 33°}$	Divide each side by tan 33°.
$DB \approx 300.27$	Use a calculator.

Use the measures of $\triangle CAD$ to find DA.

$\tan 57° = \dfrac{195}{DA}$	$\tan = \dfrac{\text{opposite}}{\text{adjacent}}$; $m\angle CAD = 57$
$DA \tan 57° = 195$	Multiply each side by DA.
$DA = \dfrac{195}{\tan 57°}$	Divide each side by tan 57°.
$DA \approx 126.63$	Use a calculator.

The distance between the boats is $DB - DA$.

$DB - DA \approx 300.27 - 126.63$ or about 174 feet

3. **BOATING** Two boats are observed by a parasailer 75 meters above a lake. The angles of depression are 12.5° and 7°. How far apart are the boats?

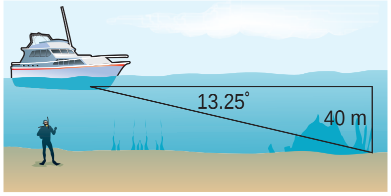
Check Your Understanding

Example 1 (p. 464)

1. **AVIATION** A pilot is flying at 10,000 feet and wants to take the plane up to 20,000 feet over the next 50 miles. What should be his angle of elevation to the nearest tenth? (*Hint:* There are 5280 feet in a mile.)

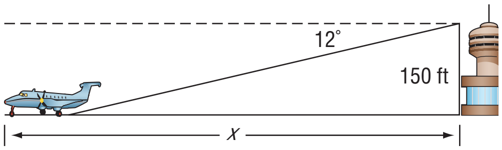
Example 2
(p. 465)

2. OCEAN ARCHAEOLOGY A salvage ship uses sonar to determine the angle of depression to a wreck on the ocean floor that is 40 meters below the surface. How far must a diver, lowered from the salvage ship, walk along the ocean floor to reach the wreck?



Example 3
(p. 466)

3. STANDARDIZED TEST EXAMPLE From the top of a 150-foot high tower, an air traffic controller observes an airplane on the runway. Which equation would be used to find the distance from the base of the tower to the airplane?

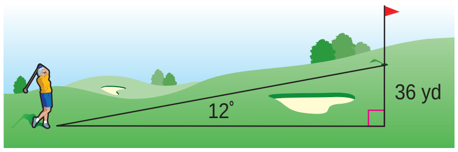


A $x = 150 \tan 12°$ **B** $x = \frac{150}{\cos 12°}$ **C** $x = \frac{150}{\tan 12°}$ **D** $x = \frac{150}{\sin 12°}$

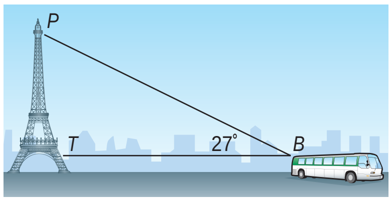
Exercises

HOMEWORK HELP	
For Exercises	See Examples
4–11	1
12, 13	2
14–17	3

4. GOLF A golfer is standing at the tee, looking up to the green on a hill. If the tee is 36 yards lower than the green and the angle of elevation from the tee to the hole is 12°, find the distance from the tee to the hole.



5. TOURISM Crystal is on a bus in France with her family. She sees the Eiffel Tower at an angle of 27°. If the tower is 986 feet tall, how far away is the bus? Round to the nearest tenth.



CIVIL ENGINEERING For Exercises 6 and 7, use the following information.
The percent grade of a highway is the ratio of the vertical rise or fall over a horizontal distance expressed to the nearest whole percent. Suppose a highway has a vertical rise of 140 feet for every 2000 feet of horizontal distance.

6. Calculate the percent grade of the highway.

7. Find the angle of elevation that the highway makes with the horizontal.

8. SKIING A ski run has an angle of elevation of 24.4° and a vertical drop of 1100 feet. To the nearest foot, how long is the ski run?

GEYSERS For Exercises 9 and 10, use the following information.
Kirk visits Yellowstone Park and Old Faithful on a perfect day. His eyes are 6 feet from the ground, and the geyser can reach heights ranging from 90 feet to 184 feet.

9. If Kirk stands 200 feet from the geyser and the eruption rises 175 feet in the air, what is the angle of elevation to the top of the spray to the nearest tenth?

10. In the afternoon, Kirk returns and observes the geyser's spray reach a height of 123 feet when the angle of elevation is 37°. How far from the geyser is Kirk standing to the nearest tenth of a foot?

11. **RAILROADS** Refer to the information at the left. Determine the incline of the Monongahela Incline.

Real-World Link

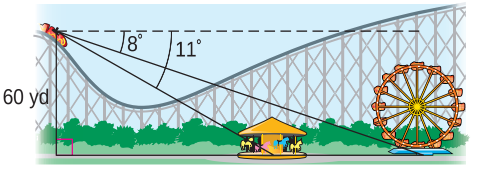
The Monongahela Incline, in Pittsburgh, Pennsylvania, is 635 feet long with a vertical rise of 369.39 feet. Although opened on May 28, 1870, it is still used by commuters to and from Pittsburgh.

Source: www.portauthority.org

12. **AVIATION** After flying at an altitude of 500 meters, a helicopter starts to descend when its ground distance from the landing pad is 11 kilometers. What is the angle of depression for this part of the flight?

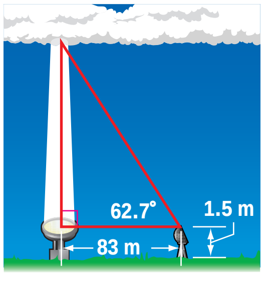
13. **SLEDDING** A sledding run is 300 yards long with a vertical drop of 27.6 yards. Find the angle of depression of the run.

14. **AMUSEMENT PARKS** From the top of a roller coaster, 60 yards above the ground, a rider looks down and sees the merry-go-round and the Ferris wheel. If the angles of depression are 11° and 8°, respectively, how far apart are the merry-go-round and the Ferris wheel?



15. **BIRD WATCHING** Two observers are 200 feet apart, in line with a tree containing a bird's nest. The angles of elevation to the bird's nest are 30° and 60°. How far is each observer from the base of the tree?

16. **METEOROLOGY** The altitude of the base of a cloud formation is called the ceiling. To find the ceiling one night, a meteorologist directed a spotlight vertically at the clouds. Using a theodolite placed 83 meters from the spotlight and 1.5 meters above the ground, he found the angle of elevation to be 62.7°. How high was the ceiling?



17. **TRAVEL** Kwan-Yong uses a theodolite to measure the angle of elevation from the ground to the top of Ayers Rock to be 15.85°. He walks half a kilometer closer and measures the angle of elevation to be 25.6°. How high is Ayers Rock to the nearest meter?

18. **PHOTOGRAPHY** A digital camera with a panoramic lens is described as having a view with an angle of elevation of 38º. If the camera is on a 3-foot tripod aimed directly at a 124-foot monument, how far from the monument should you place the tripod to see the entire monument in your photograph?

MEDICINE For Exercises 19–21, use the following information.
A doctor is using a treadmill to assess the strength of a patient's heart. At the beginning of the exam, the 48-inch long treadmill is set at an incline of 10°.

19. How far off the horizontal is the raised end of the treadmill at the beginning of the exam?

20. During one stage of the exam, the end of the treadmill is 10 inches above the horizontal. What is the incline of the treadmill to the nearest degree?

21. Suppose the exam is divided into five stages and the incline of the treadmill is increased 2° for each stage. Does the end of the treadmill rise the same distance between each stage?

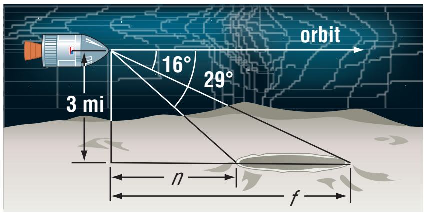
EXTRA PRACTICE

See pages 816, 835.

Math Online

Self-Check Quiz at geometryonline.com

22. **AEROSPACE** On July 20, 1969, Neil Armstrong became the first human to walk on the Moon. During this mission, Apollo 11 orbited the Moon three miles above the surface. At one point in the orbit, the onboard guidance system measured the angles of depression to the far and near edges of a large crater. The angles measured 16° and 29°, respectively. Find the distance across the crater.



H.O.T. Problems

23. **OPEN ENDED** Find a real-life example of an angle of depression. Draw a diagram and identify the angle of depression.

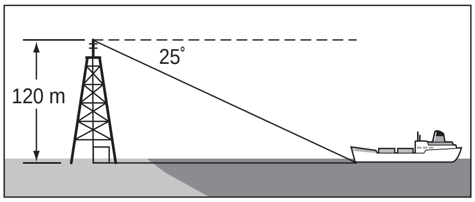
24. **REASONING** Explain why an angle of elevation is given that name.

25. **CHALLENGE** Two weather observation stations are 7 miles apart. A weather balloon is located between the stations. From Station 1, the angle of elevation to the weather balloon is 33°. From Station 2, the angle of elevation to the balloon is 52°. Find the altitude of the balloon to the nearest tenth of a mile.

26. **Writing in Math** Describe how an airline pilot would use angles of elevation and depression. Make a diagram and label the angles of elevation and depression. Then describe the difference between the two.

STANDARDIZED TEST PRACTICE

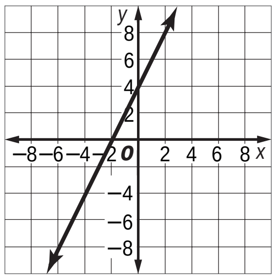
27. The top of a signal tower is 120 meters above sea level. The angle of depression from the top of the tower to a passing ship is 25°. Which is closest to the distance from the foot of the tower to the ship?



sin 25° ≈ 0.42
cos 25° ≈ 0.91
tan 25° ≈ 0.47

A 283.9 m
B 257.3 m
C 132.4 m
D 56.0 m

28. **REVIEW** What will happen to the slope of line p if the line is shifted so that the y-intercept decreases and the x-intercept remains the same?



F The slope will change from negative to positive.

G The slope will become undefined.

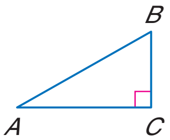
H The slope will decrease.

J The slope will increase.

Find the measure of each angle to the nearest tenth of a degree. (Lesson 8-4)

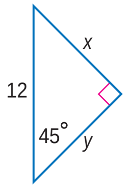
29. $\cos A = 0.6717$ **30.** $\sin B = 0.5127$ **31.** $\tan C = 2.1758$

32. If $\cos B = \frac{1}{4}$, find $\tan B$. (Lesson 8-4)



Find x and y. (Lesson 8-3)

33.



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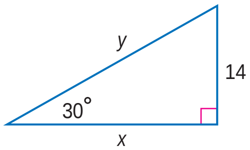

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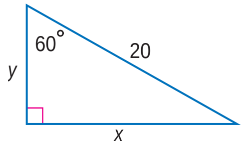


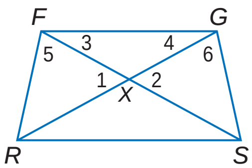
36. LANDSCAPING Paulo is designing two gardens shaped like similar triangles. One garden has a perimeter of 53.5 feet, and the longest side is 25 feet. He wants the second garden to have a perimeter of 32.1 feet. Find the length of the longest side of this garden. (Lesson 7-5)

37. MODEL AIRPLANES A twin-engine airplane used for medium-range flights has a length of 78 meters and a wingspan of 90 meters. If a scale model is made with a wingspan of 36 centimeters, find its length. (Lesson 6-2)

38. Copy and complete the flow proof. (Lesson 4-6)

Given: $\angle 5 \cong \angle 6$
$\overline{FR} \cong \overline{GS}$

Prove: $\angle 4 \cong \angle 3$



Proof:

$\angle 5 \cong \angle 6$
Given

$\overline{FR} \cong \overline{GS}$
Given

a. ___?___
Vert. ∠s are ≅.

→ $\triangle FXR \cong \triangle GXS$
b. ___?___

→ **c.** ___?___
d. ___?___

→ **e.** ___?___
f. ___?___

Determine the truth value of the following statement for each set of conditions.
If you have a fever, then you are sick. (Lesson 2-3)

39. You do not have a fever, and you are sick.

40. You have a fever, and you are not sick.

41. You do not have a fever, and you are not sick.

42. You have a fever, and you are sick.

PREREQUISITE SKILL Solve each proportion. (Lesson 7-1)

43. $\frac{x}{6} = \frac{35}{42}$ **44.** $\frac{3}{x} = \frac{5}{45}$ **45.** $\frac{12}{17} = \frac{24}{x}$ **46.** $\frac{24}{36} = \frac{x}{15}$

8-6 The Law of Sines

Main Ideas

- Use the Law of Sines to solve triangles.
- Solve problems by using the Law of Sines.

New Vocabulary

Law of Sines
solving a triangle

The Statue of Liberty was designed by Frederic-Auguste Bartholdi between 1865 and 1875. Copper sheets were hammered and fastened to an interior skeletal framework, which was designed by Alexandre-Gustave Eiffel. The skeleton is 94 feet high and composed of wrought iron bars. These bars are arranged in triangular shapes, many of which are not right triangles.

The Law of Sines In trigonometry, the **Law of Sines** can be used to find missing parts of triangles that are not right triangles.

Study Tip

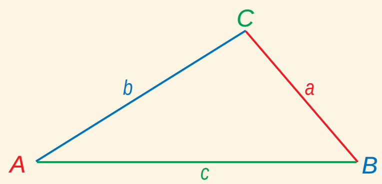
Obtuse Angles

There are also values for sin A, cos A, and tan A, when $A \geq 90°$. Values of the ratios for these angles will be found using the trigonometric functions on your calculator.

THEOREM 8.8 — Law of Sines

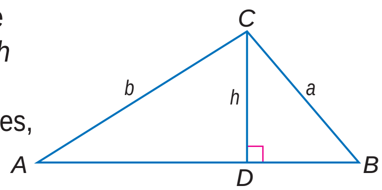
Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$



PROOF Law of Sines

$\triangle ABC$ is a triangle with an altitude from C that intersects $\overline{AB}$ at D. Let h represent the measure of $\overline{CD}$. Since $\triangle ADC$ and $\triangle BDC$ are right triangles, we can find sin A and sin B.



$$\sin A = \frac{h}{b} \qquad \sin B = \frac{h}{a}$$ Definition of sine

$$b \sin A = h \qquad a \sin B = h$$ Cross products

$$b \sin A = a \sin B$$ Substitution

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$ Divide each side by ab.

The proof can be completed by using a similar technique with the other altitudes to show that $\frac{\sin A}{a} = \frac{\sin C}{c}$ and $\frac{\sin B}{b} = \frac{\sin C}{c}$.

EXAMPLE Use the Law of Sines

1 **Given measures of $\triangle ABC$, find the indicated measure. Round angle measures to the nearest degree and side measures to the nearest tenth.**

a. If $m\angle A = 37$, $m\angle B = 68$, and $a = 3$, find b.

Use the Law of Sines to write a proportion.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{Law of Sines}$$

$$\frac{\sin 37°}{3} = \frac{\sin 68°}{b} \quad m\angle A = 37,\ a = 3,\ m\angle B = 68$$

$$b \sin 37° = 3 \sin 68° \quad \text{Cross products}$$

$$b = \frac{3 \sin 68°}{\sin 37°} \quad \text{Divide each side by } \sin 37°.$$

$$b \approx 4.6 \quad \text{Use a calculator.}$$

Study Tip

Rounding

If you round before the final answer, your results may differ from results in which rounding was not done until the final answer.

b. If $b = 17$, $c = 14$, and $m\angle B = 92$, find $m\angle C$.

Write a proportion relating $\angle B$, $\angle C$, b, and c.

$$\frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{Law of Sines}$$

$$\frac{\sin 92°}{17} = \frac{\sin C}{14} \quad m\angle B = 92,\ b = 17,\ c = 14$$

$$14 \sin 92° = 17 \sin C \quad \text{Cross products}$$

$$\frac{14 \sin 92°}{17} = \sin C \quad \text{Divide each side by 17.}$$

$$\sin^{-1}\left(\frac{14 \sin 92°}{17}\right) = C \quad \text{Solve for } C.$$

$$55° \approx C \quad \text{Use a calculator.}$$

So, $m\angle C \approx 55$.

1A. If $m\angle B = 32$, $m\angle C = 51$, $c = 12$, find a.

1B. If $a = 22$, $b = 18$, $m\angle A = 25$, find $m\angle B$.

The Law of Sines can be used to solve a triangle. **Solving a triangle** means finding the measures of all of the angles and sides of a triangle.

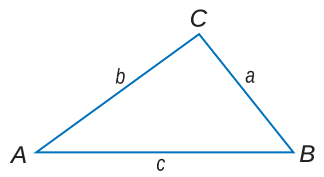
Study Tip

Look Back

To review the **Angle Sum Theorem**, see Lesson 4-2.

EXAMPLE Solve Triangles

2 **a. Solve $\triangle ABC$ if $m\angle A = 33$, $m\angle B = 47$, and $b = 14$. Round angle measures to the nearest degree and side measures to the nearest tenth.**



We know the measures of two angles of the triangle. Use the Angle Sum Theorem to find $m\angle C$.

Extra Examples at geometryonline.com

Study Tip

An Equivalent Proportion

The Law of Sines may also be written as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. You may wish to use this form when finding the length of a side.

$$
\begin{aligned}
m\angle A + m\angle B + m\angle C &= 180 && \text{Angle Sum Theorem}\\
33 + 47 + m\angle C &= 180 && m\angle A = 33,\ m\angle B = 47\\
80 + m\angle C &= 180 && \text{Add.}\\
m\angle C &= 100 && \text{Subtract 80 from each side.}
\end{aligned}
$$

Since we know $m\angle B$ and b, use proportions involving $\frac{\sin B}{b}$.

To find a:		To find c:
$\frac{\sin B}{b} = \frac{\sin A}{a}$	Law of Sines	$\frac{\sin B}{b} = \frac{\sin C}{c}$
$\frac{\sin 47°}{14} = \frac{\sin 33°}{a}$	Substitute.	$\frac{\sin 47°}{14} = \frac{\sin 100°}{c}$
$a \sin 47° = 14 \sin 33°$	Cross products	$c \sin 47° = 14 \sin 100°$
$a = \frac{14 \sin 33°}{\sin 47°}$	Divide each side by sin 47°.	$c = \frac{14 \sin 100°}{\sin 47°}$
$a \approx 10.4$	Use a calculator.	$c \approx 18.9$

Therefore, $m\angle C = 100$, $a \approx 10.4$, and $c \approx 18.9$.

b. Solve $\triangle ABC$ if $m\angle C = 98$, $b = 14$, and $c = 20$. Round angle measures to the nearest degree and side measures to the nearest tenth.

$$
\begin{aligned}
\frac{\sin B}{b} &= \frac{\sin C}{c} && \text{Law of Sines}\\
\frac{\sin B}{14} &= \frac{\sin 98°}{20} && m\angle C = 98,\ b = 14, \text{ and } c = 20\\
20 \sin B &= 14 \sin 98° && \text{Cross products}\\
\sin B &= \frac{14 \sin 98°}{20} && \text{Divide each side by 20.}\\
B &= \sin^{-1}\left(\frac{14 \sin 98°}{20}\right) && \text{Solve for } B.\\
B &\approx 44° && \text{Use a calculator.}
\end{aligned}
$$

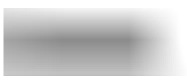
$$
\begin{aligned}
m\angle A + m\angle B + m\angle C &= 180 && \text{Angle Sum Theorem}\\
m\angle A + 44 + 98 &= 180 && m\angle B = 44 \text{ and } m\angle C = 98\\
m\angle A + 142 &= 180 && \text{Add.}\\
m\angle A &= 38 && \text{Subtract 142 from each side.}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\sin A}{a} &= \frac{\sin C}{c} && \text{Law of Sines}\\
\frac{\sin 38°}{a} &= \frac{\sin 98°}{20} && m\angle A = 38,\ m\angle C = 98, \text{ and } c = 20\\
20 \sin 38° &= a \sin 98° && \text{Cross products}\\
\frac{20 \sin 38°}{\sin 98°} &= a && \text{Divide each side by sin 98°.}\\
12.4 &\approx a && \text{Use a calculator.}
\end{aligned}
$$

Therefore, $A \approx 38°$, $B \approx 44°$, and $a \approx 12.4$.

Find the missing angles and sides of $\triangle PQR$. Round angle measures to the nearest degree and side measures to the nearest tenth.

2A. $m\angle R = 66$, $m\angle Q = 59$, $p = 72$ **2B.** $p = 32$, $r = 11$, $m\angle P = 105$



Interactive Lab
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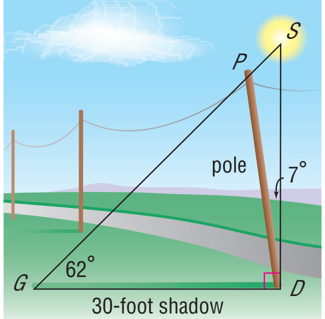
Use the Law of Sines to Solve Problems

The Law of Sines is very useful in solving direct and indirect measurement applications.

Example 3 Indirect Measurement

ENGINEERING When the angle of elevation to the Sun is 62°, a telephone pole tilted at an angle of 7° from the vertical casts a shadow 30 feet long on the ground. Find the length of the telephone pole to the nearest tenth of a foot.

Draw a diagram.



Draw $\overline{SD} \perp \overline{GD}$. Then find $m\angle GDP$ and $m\angle GPD$.

$m\angle GDP = 90 - 7$ or 83

$m\angle GPD + 62 + 83 = 180$ or $m\angle GPD = 35$

Since you know the measures of two angles of the triangle, $m\angle GDP$ and $m\angle GPD$, and the length of a side opposite one of the angles ($\overline{GD}$ is opposite $\angle GPD$) you can use the Law of Sines to find the length of the pole.

$\frac{PD}{\sin \angle DGP} = \frac{GD}{\sin \angle GPD}$ Law of Sines

$\frac{PD}{\sin 62°} = \frac{30}{\sin 35°}$ $m\angle DGP = 62$, $m\angle GPD = 35$, and $GD = 30$

$PD \sin 35° = 30 \sin 62°$ Cross products

$PD = \frac{30 \sin 62°}{\sin 35°}$ Divide each side by sin 35°.

$PD \approx 46.2$ Use a calculator.

The telephone pole is about 46.2 feet long.

3. **AVIATION** Two radar stations that are 35 miles apart located a plane at the same time. The first station indicated that the position of the plane made an angle of 37° with the line between the stations. The second station indicated that it made an angle of 54° with the same line. How far is each station from the plane?

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Study Tip

Law of Sines
Case 2 of the Law of Sines can lead to two different triangles. This is called the *ambiguous case* of the Law of Sines.

KEY CONCEPT — *Law of Sines*

The Law of Sines can be used to solve a triangle in the following cases.

Case 1 You know the measures of two angles and any side of a triangle. (AAS or ASA)

Case 2 You know the measures of two sides and an angle opposite one of these sides of the triangle. (SSA)

Check Your Understanding

Example 1 (p. 472)

Find each measure using the given measures of $\triangle XYZ$. Round angle measures to the nearest degree and side measures to the nearest tenth.

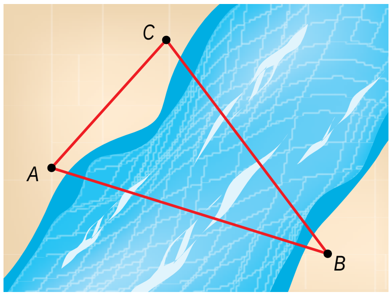
1. If $x = 3$, $m\angle X = 37$, and $m\angle Y = 68$, find y.

2. If $y = 12.1$, $m\angle X = 57$, and $m\angle Z = 72$, find x.

3. If $y = 7$, $z = 11$, and $m\angle Z = 37$, find $m\angle Y$.

4. If $y = 17$, $z = 14$, and $m\angle Y = 92$, find $m\angle Z$.

5. **SURVEYING** To find the distance between two points A and B that are on opposite sides of a river, a surveyor measures the distance to point C on the same side of the river as point A. The distance from A to C is 240 feet. He then measures the angle across from A to B as 62° and measures the angle across from C to B as 55°. Find the distance from A to B.



Example 2 (p. 472)

Solve each $\triangle PQR$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth.

6. $m\angle P = 33$, $m\angle R = 58$, $q = 22$

7. $p = 28$, $q = 22$, $m\angle P = 120$

8. $m\angle P = 50$, $m\angle Q = 65$, $p = 12$

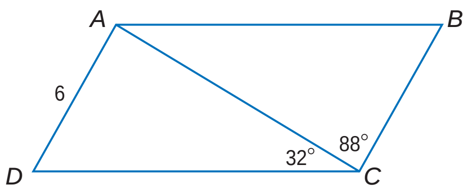
9. $q = 17.2$, $r = 9.8$, $m\angle Q = 110.7$

10. $m\angle P = 49$, $m\angle R = 57$, $p = 8$

11. $m\angle P = 40$, $m\angle Q = 60$, $r = 20$

Example 3 (p. 474)

12. Find the perimeter of parallelogram $ABCD$ to the nearest tenth.



Exercises

HOMEWORK HELP

For Exercises	See Examples
13–18	1
19–26	2
27, 28	3

Find each measure using the given measures of $\triangle KLM$. Round angle measures to the nearest degree and side measures to the nearest tenth.

13. If $k = 3.2$, $m\angle L = 52$, and $m\angle K = 70$, find ℓ.

14. If $m = 10.5$, $k = 18.2$, and $m\angle K = 73$, find $m\angle M$.

15. If $k = 10$, $m = 4.8$, and $m\angle K = 96$, find $m\angle M$.

16. If $m\angle M = 59$, $\ell = 8.3$, and $m = 14.8$, find $m\angle L$.

17. If $m\angle L = 45$, $m\angle M = 72$, and $\ell = 22$, find k.

18. If $m\angle M = 61$, $m\angle K = 31$, and $m = 5.4$, find ℓ.

Solve each $\triangle WXY$ described below. Round measures to the nearest tenth.

19. $m\angle Y = 71$, $y = 7.4$, $m\angle X = 41$

20. $x = 10.3$, $y = 23.7$, $m\angle Y = 96$

21. $m\angle X = 25$, $m\angle W = 52$, $y = 15.6$

22. $m\angle Y = 112$, $x = 20$, $y = 56$

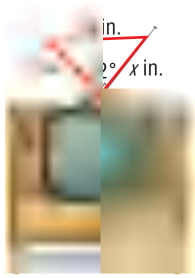
23. $m\angle W = 38$, $m\angle Y = 115$, $w = 8.5$

24. $m\angle W = 36$, $m\angle Y = 62$, $w = 3.1$

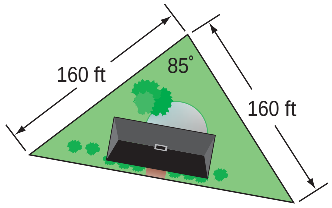
25. $w = 30$, $y = 9.5$, $m\angle W = 107$

26. $x = 16$, $w = 21$, $m\angle W = 88$

27. **TELEVISIONS** To gain better reception on his antique TV, Mr. Ramirez positioned the two antennae 13 inches apart with an angle between them of approximately 82°. If one antenna is 5 inches long, about how long is the other antenna?

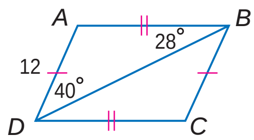


28. **REAL ESTATE** A house is built on a triangular plot of land. Two sides of the plot are 160 feet long, and they meet at an angle of 85°. If a fence is to be placed along the perimeter of the property, how much fencing material is needed?



29. An isosceles triangle has a base of 46 centimeters and a vertex angle of 44°. Find the perimeter.

30. Find the perimeter of quadrilateral *ABCD* to the nearest tenth.



31. **SURVEYING** Maria Lopez is a surveyor who must determine the distance across a section of the Rio Grande Gorge in New Mexico. On one side of the ridge, she measures the angle formed by the edge of the ridge and the line of sight to a tree on the other side of the ridge. She then walks along the ridge 315 feet, passing the tree and measures the angle formed by the edge of the ridge and the new line of sight to the same tree. If the first angle is 80° and the second angle is 85°, find the distance across the gorge.

EXTRA PRACTICE

See pages 816, 835.

Math Online
Self-Check Quiz at geometryonline.com

HIKING **For Exercises 32 and 33, use the following information.**
Kayla, Jenna, and Paige are hiking at a state park and they get separated. Kayla and Jenna are 120 feet apart. Paige sends up a signal. Jenna turns 95° in the direction of the signal and Kayla rotates 60°.

32. To the nearest foot, how far apart are Kayla and Paige?

33. To the nearest foot, how far apart are Jenna and Paige?

H.O.T. Problems

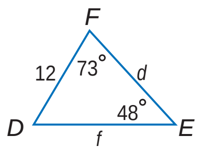
34. **FIND THE ERROR** Makayla and Felipe are trying to find *d* in $\triangle DEF$. Who is correct? Explain your reasoning.

Makayla

$\sin 59° = \frac{d}{12}$

Felipe

$\frac{\sin 59°}{d} = \frac{\sin 48°}{12}$

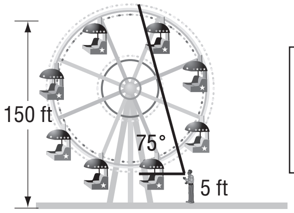


35. **OPEN ENDED** Draw an acute triangle and label the measures of two angles and the length of one side. Explain how to solve the triangle.

36. **CHALLENGE** Does the Law of Sines apply to the acute angles of a right triangle? Explain your answer.

37. **Writing in Math** Refer to the information on the Statue of Liberty on page 471. Describe how triangles are used in structural support.

38. Soledad is looking at the top of a 150-foot tall Ferris wheel at an angle of 75°.



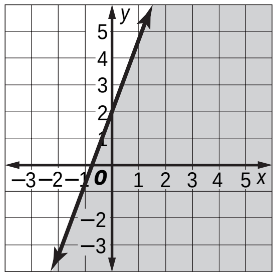
sin 75° ≈ 0.97
cos 75° ≈ 0.26
tan 75° ≈ 3.73

If she is 5 feet tall, how far is Soledad from the Ferris wheel?

A 15.0 ft **C** 75.8 ft

B 38.9 ft **D** 541.1 ft

39. REVIEW Which inequality *best* describes the graph below?



F $y \geq -x + 2$

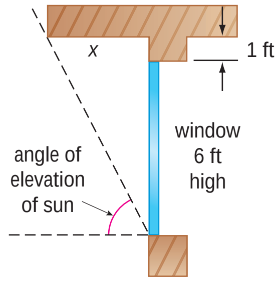
G $y \leq x + 2$

H $y \geq -3x + 2$

J $y \leq 3x + 2$

ARCHITECTURE For Exercises 40 and 41, use the following information.

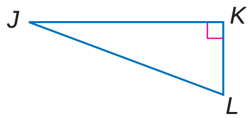
Mr. Martinez is an architect who designs houses so that the windows receive minimum Sun in the summer and maximum Sun in the winter. For Columbus, Ohio, the angle of elevation of the Sun at noon on the longest day is 73.5° and on the shortest day is 26.5°. Suppose a house is designed with a south-facing window that is 6 feet tall. The top of the window is to be installed 1 foot below the overhang. (Lesson 8-5)



40. How long should the architect make the overhang so that the window gets no direct sunlight at noon on the longest day?

41. Using the overhang from Exercise 40, how much of the window will get direct sunlight at noon on the shortest day?

Use △*JKL* to find sin *J*, cos *J*, tan *J*, sin *L*, cos *L*, and tan *L*. Express each ratio as a fraction and as a decimal to the nearest hundredth. (Lesson 8-4)



42. $j = 8, k = 17, l = 15$

43. $j = 20, k = 29, l = 21$

44. $j = 12, k = 24, l = 12\sqrt{3}$

45. $j = 7\sqrt{2}, k = 14, l = 7\sqrt{2}$

PREREQUISITE SKILL Evaluate $\frac{c^2 - a^2 - b^2}{-2ab}$ for the given values of *a*, *b*, and *c*. (Page 780)

46. $a = 7, b = 8, c = 10$

47. $a = 4, b = 9, c = 6$

48. $a = 5, b = 8, c = 10$

49. $a = 16, b = 4, c = 13$

50. $a = 3, b = 10, c = 9$

51. $a = 5, b = 7, c = 11$

EXTEND 8-6

Geometry Software Lab

The Ambiguous Case of the Law of Sines

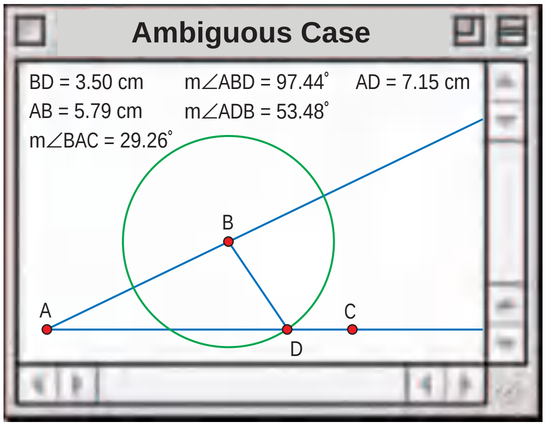
In Lesson 8-6, you learned that you could solve a triangle using the Law of Sines if you know the measures of two angles and any side of the triangle (AAS or ASA). You can also solve a triangle by the Law of Sines if you know the measures of two sides and an angle opposite one of the sides (SSA). When you use SSA to solve a triangle, and the given angle is acute, sometimes it is possible to find two different triangles. You can use The Geometer's Sketchpad to explore this case, called the **ambiguous case**, of the Law of Sines.

ACTIVITY

Step 1 Construct $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Construct a circle whose center is B so that it intersects $\overrightarrow{AC}$ at two points. Then, construct any radius $\overline{BD}$.

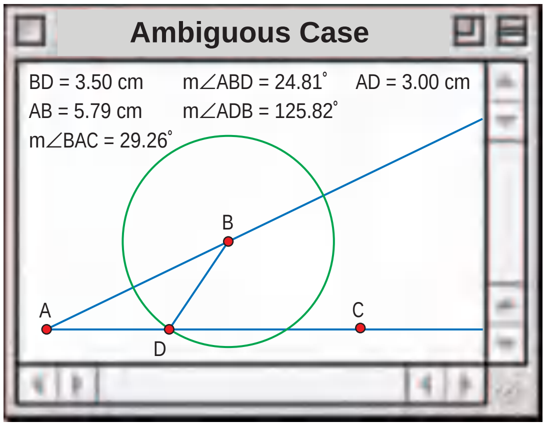
Step 2 Find the measures of $\overline{BD}$, $\overline{AB}$, and $\angle A$.

Step 3 Use the rotate tool to move D so that it lies on one of the intersection points of circle B and $\overrightarrow{AC}$. In $\triangle ABD$, find the measures of $\angle ABD$, $\angle BDA$, and $\overline{AD}$.



Step 4 Using the rotate tool, move D to the other intersection point of circle B and $\overrightarrow{AC}$.

Step 5 Note the measures of $\angle ABD$, $\angle BDA$, and $\overline{AD}$ in $\triangle ABD$.



ANALYZE THE RESULTS

1. Which measures are the same in both triangles?
2. Repeat the activity using different measures for $\angle A$, $\overline{BD}$, and $\overline{AB}$. How do the results compare to the earlier results?
3. Compare your results with those of your classmates. How do the results compare?
4. What would have to be true about circle B in order for there to be one unique solution? Test your conjecture by repeating the activity.
5. Is it possible, given the measures of $\overline{BD}$, $\overline{AB}$, and $\angle A$, to have no solution? Test your conjecture and explain.

8-7 The Law of Cosines

German architect Ludwig Mies van der Rohe entered the design at the right in the Friedrichstrasses Skyscraper Competition in Berlin in 1921. The skyscraper was to be built on a triangular plot of land. In order to maximize space, the design called for three towers in a triangular shape. However, the skyscraper was never built.

Main Ideas

- Use the Law of Cosines to solve triangles.
- Solve problems by using the Law of Cosines.

New Vocabulary

Law of Cosines

The Law of Cosines Suppose you know the lengths of the sides of the triangular building and want to solve the triangle. The **Law of Cosines** allows us to solve a triangle when the Law of Sines cannot be used.

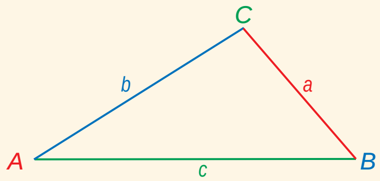
THEOREM 8.9 — Law of Cosines

Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite angles A, B, and C, respectively. Then the following equations are true.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Study Tip

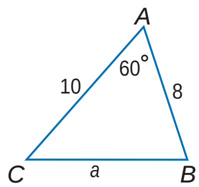
Side and Angle

Note that the letter of the side length on the left-hand side of each equation corresponds to the angle measure used with the cosine.

The Law of Cosines can be used to find missing measures in a triangle if you know the measures of two sides and their included angle.

EXAMPLE Two Sides and the Included Angle

1 Find a if $c = 8$, $b = 10$, and $m\angle A = 60$.



Use the Law of Cosines since the measures of two sides and the included are known.

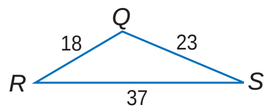
$a^2 = b^2 + c^2 - 2bc \cos A$	Law of Cosines
$a^2 = 10^2 + 8^2 - 2(10)(8) \cos 60°$	$b = 10$, $c = 8$, and $m\angle A = 60$
$a^2 = 164 - 160 \cos 60°$	Simplify.
$a = \sqrt{164 - 160 \cos 60°}$	Take the square root of each side.
$a \approx 9.2$	Use a calculator.

1. In $\triangle DEF$, $e = 19$, $f = 28$, and $m\angle D = 49$. Find d.

Extra Examples at geometryonline.com

You can also use the Law of Cosines to find the measures of angles of a triangle when you know the measures of the three sides.

EXAMPLE Three Sides



2 **Find $m\angle R$.**

$r^2 = q^2 + s^2 - 2qs \cos R$	Law of Cosines
$23^2 = 37^2 + 18^2 - 2(37)(18) \cos R$	$r = 23$, $q = 37$, $s = 18$
$529 = 1693 - 1332 \cos R$	Simplify.
$-1164 = -1332 \cos R$	Subtract 1693 from each side.
$\frac{-1164}{-1332} = \cos R$	Divide each side by -1332.
$R = \cos^{-1}\left(\frac{1164}{1332}\right)$	Solve for R.
$R \approx 29.1°$	Use a calculator.

2. In $\triangle TVW$, $v = 18$, $t = 24$, and $w = 30$. Find $m\angle W$.

Use the Law of Cosines to Solve Problems Most problems can be solved using more than one method. Choosing the most efficient way to solve a problem is sometimes not obvious.

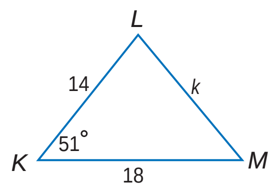
When solving right triangles, you can use sine, cosine, or tangent ratios. When solving other triangles, you can use the Law of Sines or the Law of Cosines. You must decide how to solve each problem depending on the given information.

CONCEPT SUMMARY — *Solving a Triangle*

To solve	Given	Begin by using
Right triangle	two legs	tangent
	leg and hypotenuse	sine or cosine
	angle and hypotenuse	sine or cosine
	angle and a leg	sine, cosine, or tangent
Any triangle	two angles and any side	Law of Sines
	two sides and the angle opposite one of them	Law of Sines
	two sides and the included angle	Law of Cosines
	three sides	Law of Cosines

EXAMPLE Select a Strategy

3 **Solve $\triangle KLM$. Round angle measures to the nearest degree and side measures to the nearest tenth.**



We do not know whether $\triangle KLM$ is a right triangle, so we must use the Law of Cosines or the Law of Sines. We know the measures of two sides and the included angle. This is SAS, so use the Law of Cosines.

$k^2 = \ell^2 + m^2 - 2\ell m \cos K$ — Law of Cosines

$k^2 = 18^2 + 14^2 - 2(18)(14) \cos 51°$ — $\ell = 18$, $m = 14$, and $m\angle K = 51$

$k = \sqrt{18^2 + 14^2 - 2(18)(14) \cos 51°}$ — Take the square root of each side.

$k \approx 14.2$ — Use a calculator.

Study Tip

Law of Cosines

If you use the Law of Cosines to find another measure, your answer may differ slightly from one found using the Law of Sines. This is due to rounding.

Next, we can find $m\angle L$ or $m\angle M$. If we decide to find $m\angle L$, we can use either the Law of Sines or the Law of Cosines to find this value. In this case, we will use the Law of Sines.

$\frac{\sin L}{\ell} = \frac{\sin K}{k}$ — Law of Sines

$\frac{\sin L}{18} \approx \frac{\sin 51°}{14.2}$ — $\ell = 18$, $k \approx 14.2$, and $m\angle K = 51$

$14.2 \sin L \approx 18 \sin 51°$ — Cross products

$\sin L \approx \frac{18 \sin 51°}{14.2}$ — Divide each side by 14.2.

$L \approx \sin^{-1}\left(\frac{18 \sin 51°}{14.2}\right)$ — Take the inverse sine of each side.

$L \approx 80°$ — Use a calculator.

Use the Angle Sum Theorem to find $m\angle M$.

$m\angle K + m\angle L + m\angle M = 180$ — Angle Sum Theorem

$51 + 80 + m\angle M \approx 180$ — $m\angle K = 51$ and $m\angle L \approx 80$

$m\angle M \approx 49$ — Subtract 131 from each side.

Therefore, $k \approx 14.2$, $m\angle K \approx 80$, and $m\angle M \approx 49$.

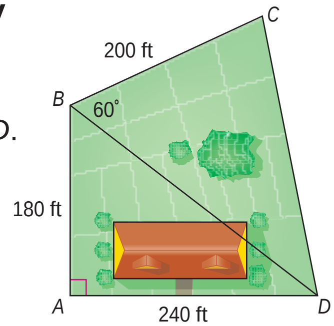
CHECK Your Progress

3. Solve $\triangle XYZ$ for $x = 10$, $y = 11$, and $z = 12$.

Personal Tutor at ca.geometryonline.com

Real-World EXAMPLE — **Use the Law of Cosines**

4 **REAL ESTATE** **Ms. Jenkins is buying some property that is shaped like quadrilateral *ABCD*. Find the perimeter of the property.**



Use the Pythagorean Theorem to find BD in $\triangle ABD$.

$(AB)^2 + (AD)^2 = (BD)^2$ — Pythagorean Theorem

$180^2 + 240^2 = (BD)^2$ — $AB = 180$, $AD = 240$

$90{,}000 = (BD)^2$ — Simplify.

$300 = BD$ — Take the square root of each side.

Next, use the Law of Cosines to find CD in $\triangle BCD$.

$(CD)^2 = (BC)^2 + (BD)^2 - 2(BC)(BD) \cos \angle CBD$ — Law of Cosines

$(CD)^2 = 200^2 + 300^2 - 2(200)(300) \cos 60°$ — $BC = 200$, $BD = 300$, $m\angle CBD = 60$

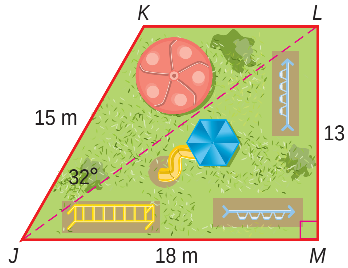
$(CD)^2 = 130{,}000 - 120{,}000 \cos 60°$ — Simplify.

$CD = \sqrt{130{,}000 - 120{,}000 \cos 60°}$ — Take the square root of each side.

$CD \approx 264.6$ — Use a calculator.

The perimeter is $180 + 200 + 264.6 + 240$ or about 884.6 feet.

4. ARCHITECTURE An architect is designing a playground in the shape of a quadrilateral. Find the perimeter of the playground to the nearest tenth.



Check Your Understanding

Example 1 (p. 479)

In $\triangle BCD$, given the following measures, find the measure of the missing side.

1. $c = \sqrt{2}$, $d = 5$, $m\angle B = 45$
2. $b = 107$, $c = 94$, $m\angle D = 105$

Example 2 (p. 480)

In $\triangle RST$, given the lengths of the sides, find the measure of the stated angle to the nearest degree.

3. $r = 33$, $s = 65$, $t = 56$; $m\angle S$
4. $r = 2.2$, $s = 1.3$, $t = 1.6$; $m\angle R$

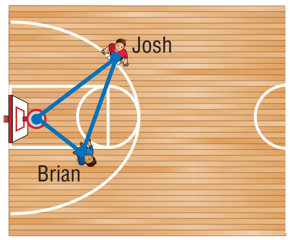
Example 3 (p. 480)

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

5. $\triangle XYZ$: $x = 5$, $y = 10$, $z = 13$
6. $\triangle JKL$: $j = 20$, $\ell = 24$, $m\angle K = 47$

Example 4 (p. 481)

7. **BASKETBALL** Josh and Brian are playing basketball. Josh passes the ball to Brian, who takes a shot. Josh is 12 feet from the hoop and 10 feet from Brian. The angle formed by the hoop, Josh, and Brian is 34°. Find the distance Brian is from the hoop.



Exercises

HOMEWORK HELP

For Exercises	See Examples
8–11	1
12–15	2
16–22, 25–32	3
23, 24	4

In $\triangle TUV$, given the following measures, find the measure of the missing side.

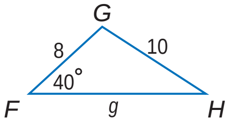
8. $t = 9.1$, $v = 8.3$, $m\angle U = 32$
9. $t = 11$, $u = 17$, $m\angle V = 78$
10. $u = 11$, $v = 17$, $m\angle T = 105$
11. $v = 11$, $u = 17$, $m\angle T = 59$

In $\triangle EFG$, given the lengths of the sides, find the measure of the stated angle to the nearest degree.

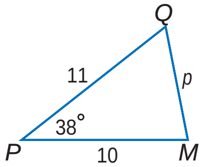
12. $e = 9.1$, $f = 8.3$, $g = 16.7$; $m\angle F$
13. $e = 14$, $f = 19$, $g = 32$; $m\angle E$
14. $e = 325$, $f = 198$, $g = 208$; $m\angle F$
15. $e = 21.9$, $f = 18.9$, $g = 10$; $m\angle G$

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

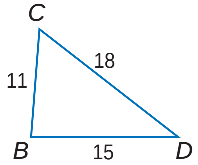
16.



17.



18.



Real-World Link

The Swissôtel in Chicago, Illinois, is built in the shape of a triangular prism. The lengths of the sides of the triangle are 180 feet, 186 feet, and 174 feet.

Source: Swissôtel

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

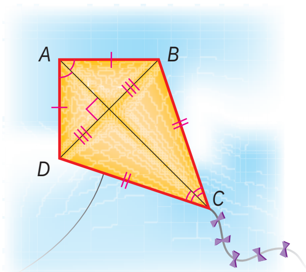
19. $\triangle ABC$: $m\angle A = 42$, $m\angle C = 77$, $c = 6$

20. $\triangle ABC$: $a = 10.3$, $b = 9.5$, $m\angle C = 37$

21. $\triangle ABC$: $a = 15$, $b = 19$, $c = 28$

22. $\triangle ABC$: $m\angle A = 53$, $m\angle C = 28$, $c = 14.9$

23. **KITES** Beth is building a kite like the one at the right. If $\overline{AB}$ is 5 feet long, $\overline{BC}$ is 8 feet long, and $\overline{BD}$ is $7\frac{2}{3}$ feet long, find the measures of the angle between the short sides and the angle between the long sides to the nearest degree.



24. **BUILDINGS** Refer to the information at the left. Find the measures of the angles of the triangular building to the nearest tenth.

Solve each $\triangle LMN$ described below. Round measures to the nearest tenth.

25. $m = 44$, $\ell = 54$, $m\angle L = 23$

26. $m\angle M = 46$, $m\angle L = 55$, $n = 16$

27. $m = 256$, $\ell = 423$, $n = 288$

28. $m\angle M = 55$, $\ell = 6.3$, $n = 6.7$

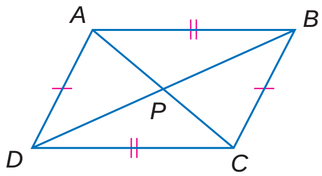
29. $m\angle M = 27$, $\ell = 5$, $n = 10$

30. $n = 17$, $m = 20$, $\ell = 14$

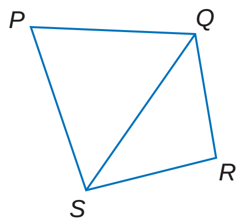
31. $\ell = 14$, $m = 15$, $n = 16$

32. $m\angle L = 51$, $\ell = 40$, $n = 35$

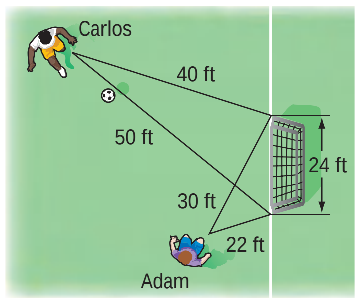
33. In quadrilateral $ABCD$, $AC = 188$, $BD = 214$, $m\angle BPC = 70$, and P is the midpoint of $\overline{AC}$ and $\overline{BD}$. Find the perimeter of $ABCD$.



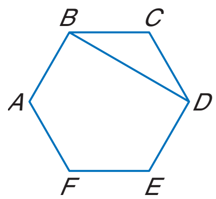
34. In quadrilateral $PQRS$, $PQ = 721$, $QR = 547$, $RS = 593$, $PS = 756$, and $m\angle P = 58$. Find QS, $m\angle PQS$, and $m\angle R$.



35. **SOCCER** Carlos and Adam are playing soccer. Carlos is standing 40 feet from one post of the goal and 50 feet from the other post. Adam is standing 30 feet from one post of the goal and 22 feet from the other post. If the goal is 24 feet wide, which player has a greater angle to make a shot on goal?



36. Each side of regular hexagon $ABCDEF$ is 18 feet long. What is the length of the diagonal $\overline{BD}$? Explain your reasoning.

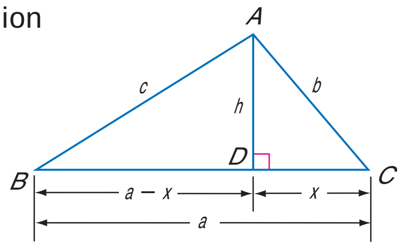


EXTRA PRACTICE
See pages 816, 835.
Math Online
Self-Check Quiz at
geometryonline.com

37. PROOF Justify each statement for the derivation of the Law of Cosines.

Given: $\overline{AD}$ is an altitude of $\triangle ABC$.

Prove: $c^2 = a^2 + b^2 - 2ab \cos C$



Proof:

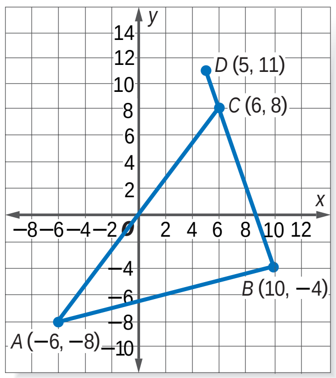
Statement	**Reasons**
a. $c^2 = (a - x)^2 + h^2$	**a.** ___?___
b. $c^2 = a^2 - 2ax + x^2 + h^2$	**b.** ___?___
c. $x^2 + h^2 = b^2$	**c.** ___?___
d. $c^2 = a^2 - 2ax + b^2$	**d.** ___?___
e. $\cos C = \frac{x}{b}$	**e.** ___?___
f. $b \cos C = x$	**f.** ___?___
g. $c^2 = a^2 - 2a(b \cos C) + b^2$	**g.** ___?___
h. $c^2 = a^2 + b^2 - 2ab \cos C$	**h.** ___?___

H.O.T. Problems

38. OPEN ENDED Draw and label one acute and one obtuse triangle, illustrating when you can use the Law of Cosines to find the missing measures.

39. REASONING Find a counterexample for the following statement.
The Law of Cosines can be used to find the length of a missing side in any triangle.

40. CHALLENGE Graph $A(-6, -8)$, $B(10, -4)$, $C(6, 8)$, and $D(5, 11)$ on the coordinate plane. Find the measure of interior angle ABC and the measure of exterior angle DCA.



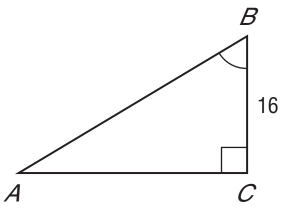
41. Which One Doesn't Belong? Analyze the four terms and determine which does not belong with the others.

Pythagorean triple	Pythagorean Theorem
Law of Cosines	cosine

42. Writing in Math Refer to the information about the Friedrichstrasses Skyscraper Competition on page 479. Describe how triangles were used in van der Rohe's design. Explain why the Law of Cosines could not be used to solve the triangle.

Standardized Test Practice

43. In the figure below, $\cos B = 0.8$.



What is the length of $\overline{AB}$?

A 12.8

B 16.8

C 20.0

D 28.8

44. **REVIEW** Which of the following shows $2x^2 - 24xy - 72y^2$ factored completely?

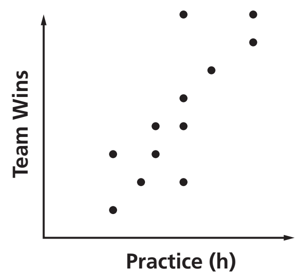
F $(2x - 18y)(x + 4y)$

G $2(x - 6y)(x + 6y)$

H $(2x - 8y)(x - 9)$

J $2(x - 6y)(x + 18y)$

45. **REVIEW** The scatter plot shows the responses of swim coaches to a survey about the hours of swim team practice and the number of team wins.



Which statement *best* describes the relationship between the two quantities?

A As the number of practice hours increases, the number of team wins increases.

B As the number of practice hours increases, the number of team wins decreases.

C As the number of practice hours increases, the number of team wins at first decreases, then increases.

D There is no relationship between the number of practice hours and the number of team wins.

Spiral Review

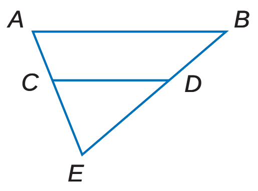
Find each measure using the given measures from $\triangle XYZ$. Round angle measure to the nearest degree and side measure to the nearest tenth. (Lesson 8-6)

46. If $y = 4.7$, $m\angle X = 22$, and $m\angle Y = 49$, find x.

47. If $y = 10$, $x = 14$, and $m\angle X = 50$, find $m\angle Y$.

48. **SURVEYING** A surveyor is 100 meters from a building and finds that the angle of elevation to the top of the building is 23°. If the surveyor's eye level is 1.55 meters above the ground, find the height of the building. (Lesson 8-5)

For Exercises 49–51, determine whether $\overline{AB} \parallel \overline{CD}$. (Lesson 7-4)



49. $AC = 8.4$, $BD = 6.3$, $DE = 4.5$, and $CE = 6$

50. $AC = 7$, $BD = 10.5$, $BE = 22.5$, and $AE = 15$

51. $AB = 8$, $AE = 9$, $CD = 4$, and $CE = 4$

COORDINATE GEOMETRY **The vertices of $\triangle XYZ$ are $X(8, 0)$, $Y(-4, 8)$, and $Z(0, 12)$. Find the coordinates of the points of concurrency of $\triangle XYZ$ to the nearest tenth.** (Lesson 5-1)

52. orthocenter

53. centroid

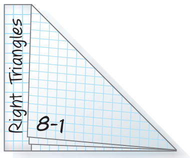
54. circumcenter

CHAPTER 8 Study Guide and Review

Download Vocabulary Review from geometryonline.com

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Geometric Mean (Lesson 8-1)

- For two positive numbers a and b, the geometric mean is the positive number x where the proportion $a : x = x : b$ is true. This proportion can be written using fractions as $\frac{a}{x} = \frac{x}{b}$ or with cross products as $x^2 = ab$ or $x = \sqrt{ab}$.

Pythagorean Theorem (Lesson 8-2)

- In a right triangle, the sum of the squares of the measures of the legs equals the square of the hypotenuse.

Special Right Triangles (Lesson 8-3)

- The measures of the sides of a 45°–45°–90° triangle are x, x, and $x\sqrt{2}$.
- The measures of the sides of a 30°–60°–90° triangle are x, $x\sqrt{3}$, and $2x$.

Trigonometry (Lesson 8-4)

- Trigonometric Ratios:

$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

Laws of Sines and Cosines (Lessons 8-6 and 8-7)

Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively.

- Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Law of Cosines: $a^2 = b^2 + c^2 - 2bc \cos A$
  $b^2 = a^2 + c^2 - 2ac \cos B$
  $c^2 = a^2 + b^2 - 2ab \cos C$

Key Vocabulary

angle of depression (p. 465)
angle of elevation (p. 464)
cosine (p. 456)
geometric mean (p. 432)
Pythagorean triple (p. 443)
sine (p. 456)
solving a triangle (p. 472)
tangent (p. 456)
trigonometric ratio (p. 456)
trigonometry (p. 456)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a *true* sentence.

1. To solve a triangle means to find the measures of all its sides and angles.
2. The Law of Sines can be applied if you know the measures of two sides and an angle opposite one of these sides of the triangle.
3. In any triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.
4. An angle of depression is the angle between the line of sight and the horizontal when an observer looks upward.
5. The geometric mean between two numbers is the positive square root of their product.
6. A 30°-60°-90° triangle is isosceles.
7. Looking at a city while flying in a plane is an example that uses an angle of elevation.
8. The numbers 3, 4, and 5 form a Pythagorean identity.

Lesson-by-Lesson Review

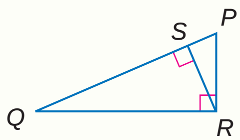
8-1 Geometric Mean (pp. 432–438)

Find the geometric mean between each pair of numbers.

9. 4 and 16 **10.** 4 and 81

11. 20 and 35 **12.** 18 and 44

13. In PQR, $PS = 8$, and $QS = 14$. Find RS.



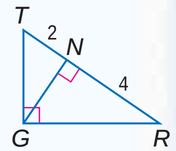
14. INDIRECT MEASUREMENT To estimate the height of the Space Needle in Seattle, Washington, James held a book up to his eyes so that the top and bottom of the building were in line with the bottom edge and binding of the cover. If James' eye level is 6 feet from the ground and he is standing 60 feet from the tower, how tall is the tower?

Example 1 **Find the geometric mean between 10 and 30.**

$\frac{10}{x} = \frac{x}{30}$	Definition of geometric mean
$x^2 = 300$	Cross products
$x = \sqrt{300}$ or $10\sqrt{3}$	Simplify.

Example 2 **Find NG in $\triangle TGR$.**

The measure of the altitude is the geometric mean between the measures of the two hypotenuse segments.

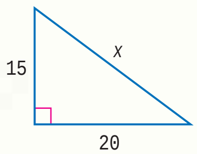


$\frac{TN}{GN} = \frac{GN}{RN}$	Definition of geometric mean
$\frac{2}{GN} = \frac{GN}{4}$	$TN = 2$, $RN = 4$
$8 = (GN)^2$	Cross products
$\sqrt{8}$ or $2\sqrt{2} = GN$	Take the square root of each side.

8-2 The Pythagorean Theorem and Its Converse (pp. 440–446)

Find x.

15.



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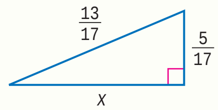
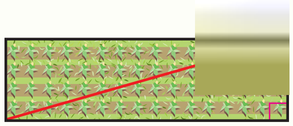
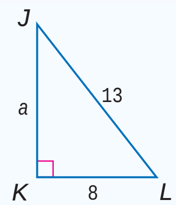


17. FARMING A farmer wishes to create a maze in his corn field. He cuts a path 625 feet across the diagonal of the rectangular field. Did the farmer create two right triangles? Explain.



Example 3 **Use $\triangle JKL$ to find a.**



$a^2 + (LK)^2 = (JL)^2$	Pythagorean Theorem
$a^2 + 8^2 = 13^2$	$LK = 8$ and $JL = 13$
$a^2 + 64 = 169$	Simplify.
$a^2 = 105$	Subtract 64 from each side.
$a = \sqrt{105}$	Take the square root of each side.
$a \approx 10.2$	Use a calculator.

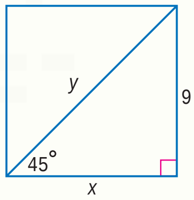
CHAPTER 8

Study Guide and Review

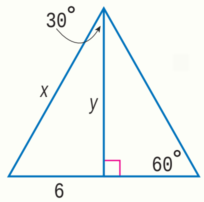
8-3 Special Right Triangles (pp. 448–454)

Find x and y.

18.



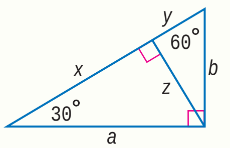
19.



For Exercises 20 and 21, use the figure.

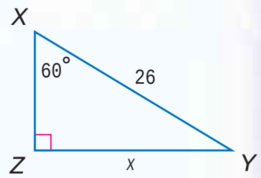
20. If $y = 18$, find z and a.

21. If $x = 14$, find a, z, b, and y.



22. **ORIGAMI** To create a bird, Michelle first folded a square piece of origami paper along one of the diagonals. If the diagonal measured 8 centimeters, find the length of one side of the square.

Example 4 **Find x.**

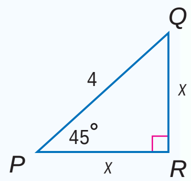


The shorter leg, $\overline{XZ}$, of $\triangle XYZ$ is half the measure of the hypotenuse $\overline{XY}$.

Therefore, $XZ = \frac{1}{2}(26)$ or 13. The longer leg is $\sqrt{3}$ times the measure of the shorter leg.

So, $x = 13\sqrt{3}$.

Example 5 **Find x.**



The hypotenuse of a 45°-45°-90° triangle is $\sqrt{2}$ times the length of a leg.

$$x\sqrt{2} = 4$$

$$x = \frac{4}{\sqrt{2}}$$

$$x = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \text{ or } 2\sqrt{2}$$

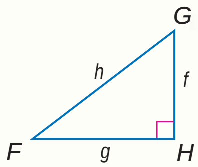
8-4 Trigonometry (pp. 456–462)

Use $\triangle FGH$ to find sin F, cos F, tan F, sin G, cos G, and tan G. Express each ratio as a fraction and as a decimal to the nearest hundredth.

23. $f = 9, g = 12, h = 15$

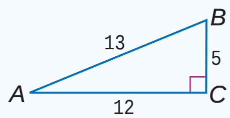
24. $f = 7, g = 24, h = 25$

25. $f = 9, g = 40, h = 41$



26. **SPACE FLIGHT** A space shuttle is directed towards the Moon but drifts 0.8° from its calculated path. If the distance from Earth to the Moon is 240,000 miles, how far has the space shuttle drifted from its path when it reaches the Moon?

Example 6 **Find sin A, cos A, and tan A. Express as a fraction and as a decimal.**



$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{5}{13} \text{ or about } 0.38$$

$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{12}{13} \text{ or about } 0.92$$

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC} = \frac{5}{12} \text{ or about } 0.42$$

Mixed Problem Solving
For mixed problem-solving practice, see page 835.

8-5 Angles of Elevation and Depression (pp. 464–470)

Determine the angles of elevation or depression in each situation.

27. Upon takeoff, an airplane must clear a 60-foot pole at the end of a runway 500 yards long.

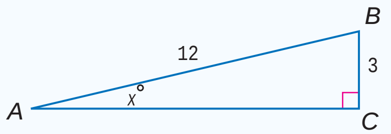
28. An escalator descends 100 feet for each horizontal distance of 240 feet.

29. A hot-air balloon ascends 50 feet for every 1000 feet traveled horizontally.

30. **EAGLES** An eagle, 1350 feet in the air, notices a rabbit on the ground. If the horizontal distance between the eagle and the rabbit is 700 feet, at what angle of depression must the eagle swoop down to catch the rabbit and fly in a straight path?

Example 7 **The ramp of a loading dock measures 12 feet and has a height of 3 feet. What is the angle of elevation?**

Make a drawing.



Let x represent $m\angle BAC$.

$\sin x^\circ = \frac{BC}{AB}$	$\sin x = \frac{\text{opposite leg}}{\text{hypotenuse}}$
$\sin x^\circ = \frac{3}{12}$	$BC = 3$ and $AB = 12$
$x = \sin^{-1}\left(\frac{3}{12}\right)$	Find the inverse.
$x \approx 14.5$	Use a calculator.

The angle of elevation for the ramp is about 14.5°.

8-6 The Law of Sines (pp. 471–477)

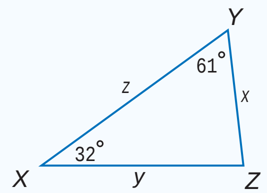
Find each measure using the given measures of $\triangle FGH$. Round angle measures to the nearest degree and side measures to the nearest tenth.

31. Find f if $g = 16$, $m\angle G = 48$, and $m\angle F = 82$.

32. Find $m\angle H$ if $h = 10.5$, $g = 13$, and $m\angle G = 65$.

33. **GARDENING** Elena is planning a triangular garden. She wants to build a fence around the garden to keep out the deer. The length of one side of the garden is 26 feet. If the angles at the end of this side are 78° and 44°, find the length of fence needed to enclose the garden.

Example 8 **Find x if $y = 15$. Round to the nearest tenth.**



To find x and z, use proportions involving $\sin Y$ and y.

$\frac{\sin Y}{y} = \frac{\sin X}{x}$	Law of Sines
$\frac{\sin 61^\circ}{15} = \frac{\sin 32^\circ}{x}$	Substitute.
$x \sin 61^\circ = 15 \sin 32^\circ$	Cross Products
$x = \frac{15 \sin 32^\circ}{\sin 61^\circ}$	Divide.
$x \approx 9.1$	Use a calculator.

8-7 The Law of Cosines (pp. 479–485)

In $\triangle XYZ$, given the following measures, find the measures of the missing side.

34. $x = 7.6$, $y = 5.4$, $m\angle Z = 51$

35. $x = 21$, $m\angle Y = 73$, $z = 16$

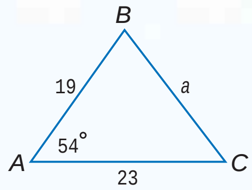
Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

36. $c = 18$, $b = 13$, $m\angle A = 64$

37. $b = 5.2$, $m\angle C = 53$, $c = 6.7$

38. ART Adelina is creating a piece of art that is in the shape of a parallelogram. Its dimensions are 35 inches by 28 inches and one angle is 80°. Find the lengths of both diagonals.

Example 9 **Find *a*.**



$a^2 = b^2 + c^2 - 2bc \cos A$ Law of Cosines

$a^2 = 23^2 + 19^2 - 2(23)(19) \cos 54°$ $b = 23$, $c = 19$, and $m\angle A = 54$

$a^2 = 890 - 874 \cos 54°$ Simplify.

$a = \sqrt{890 - 874 \cos 54°}$ Take the square root of each side.

$a \approx 19.4$ Use a calculator.

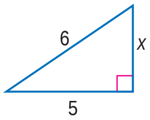
CHAPTER 8 Practice Test

Find the geometric mean between each pair of numbers.

1. 7 and 63 **2.** 6 and 24 **3.** 10 and 50

Find the missing measures.

4.



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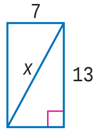

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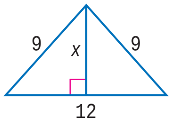

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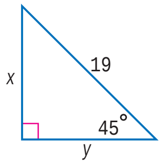

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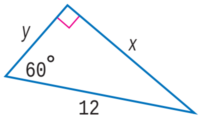
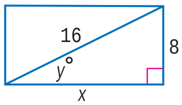
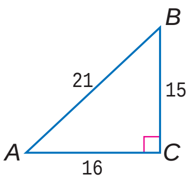


9.



Use the figure to find each trigonometric ratio. Express answers as a fraction.



10. cos B

11. tan A

12. sin A

Find each measure using the given measures from $\triangle FGH$. Round to the nearest tenth.

13. Find g if $m\angle F = 59$, $f = 13$, and $m\angle G = 71$.

14. Find $m\angle H$ if $m\angle F = 52$, $f = 10$, and $h = 12.5$.

15. Find f if $g = 15$, $h = 13$, and $m\angle F = 48$.

16. Find h if $f = 13.7$, $g = 16.8$, and $m\angle H = 71$.

Solve each triangle. Round each angle measure to the nearest degree and each side measure to the nearest tenth.

17. $a = 15$, $b = 17$, $m\angle C = 45$

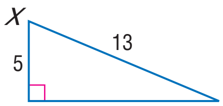
18. $a = 12.2$, $b = 10.9$, $m\angle B = 48$

19. $a = 19$, $b = 23.2$, $c = 21$

20. **TRAVEL** From an airplane, Janara looked down to see a city. If she looked down at an angle of 9° and the airplane was half a mile above the ground, what was the horizontal distance to the city?

21. **CIVIL ENGINEERING** A section of freeway has a steady incline of 10°. If the horizontal distance from the beginning of the incline to the end is 5 miles, how high does the incline reach?

22. **MULTIPLE CHOICE** Find tan X.

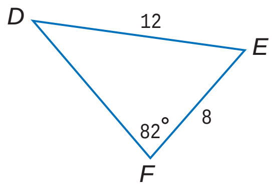


A $\frac{5}{12}$ **C** $\frac{17}{12}$

B $\frac{12}{13}$ **D** $\frac{12}{5}$

23. **COMMUNICATIONS** To secure a 500-foot radio tower against high winds, guy wires are attached to the tower 5 feet from the top. The wires form a 15° angle with the tower. Find the distance from the centerline of the tower to the anchor point of the wires.

24. Solve $\triangle DEF$.



25. **MULTIPLE CHOICE** The top of the Boone Island Lighthouse in Boone Island, Maine, is 137 feet above sea level. The angle of depression from the light on the top of the tower to a passing ferry is 37°. How many feet from the foot of the lighthouse is the ferry?

F 181.8 ft **H** 109.4 ft

G 171.5 ft **J** 103.2 ft

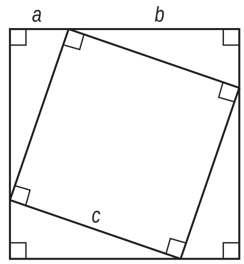
Chapter Test at geometryonline.com

Chapter 8 Standardized Test Practice

Cumulative, Chapters 1–8

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A diagram from a proof of the Pythagorean Theorem is pictured below. Which statement would be used in the proof of the Pythagorean Theorem?



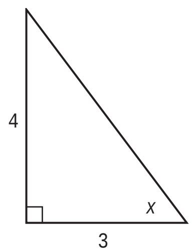
A The area of the larger square equals $(a + b)^2$.

B The area of the inner square is equal to half of the area of the larger square.

C The area of the larger square is equal to the sum of the areas of the smaller square and the four congruent triangles.

D The four right triangles are similar.

2. In the figure below, if $\tan x = \frac{4}{3}$, what are $\cos x$ and $\sin x$?



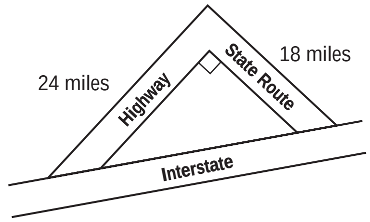
F $\cos x = \frac{3}{4}, \sin x = \frac{4}{5}$

G $\cos x = \frac{3}{4}, \sin x = \frac{5}{4}$

H $\cos x = \frac{3}{5}, \sin x = \frac{4}{5}$

J $\cos x = \frac{3}{5}, \sin x = \frac{5}{4}$

3. A detour has been set up on the interstate due to a gas leak. The diagram below shows the detour route. How many extra miles will drivers have to travel due to the detour?



A 12 miles
B 30 miles
C 42 miles
D 80 miles

4. A right triangle has legs of length $\sqrt{39}$ and 8. What is the length of the hypotenuse?

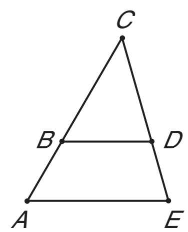
F 10   **H** 11

G $\sqrt{103}$ **J** $4\sqrt{26}$

TEST-TAKING TIP

Question 4 If a standardized test question involves trigonometric ratios, draw a diagram that represents the problem. Use a calculator (if allowed) or the table of trigonometric values provided to help you find the answer.

5. Given: $\overline{BD} \parallel \overline{AE}$



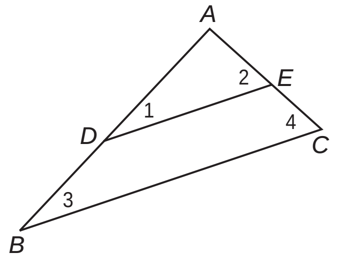
What theorem or postulate can be used to prove $\triangle ACE \sim \triangle BCD$?

A SSS   **C** ASA

B SAS   **D** AA

Standardized Test Practice at geometryonline.com

6. In $\triangle ABC$, D is the midpoint of $\overline{AB}$, and E is the midpoint of $\overline{AC}$.



Which of the following is *not* true?

F $\angle 1 \cong \angle 4$ **H** $\overline{DE} \parallel \overline{BC}$

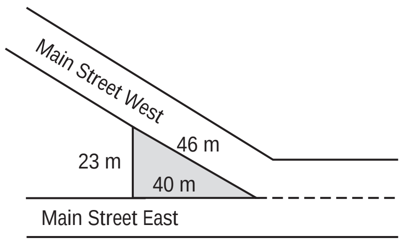
G $\triangle ABC \sim \triangle ADE$ **J** $\frac{AD}{DB} = \frac{AE}{EC}$

7. If the sum of the measures of the interior angles of a polygon is 900, how many sides does the polygon have?

A 5 **C** 8

B 7 **D** 10

8. GRIDDABLE A city planner designs a triangular traffic median on Main Street to provide more green space in the downtown area. The planner builds a model so that the section of the median facing Main Street East measures 20 centimeters. What is the perimeter, in centimeters, of the model of the traffic median?

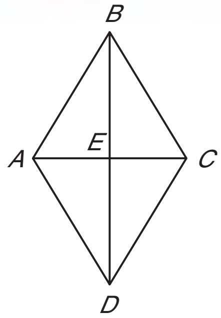


9. ALGEBRA Find $(x^2 + 2x - 24) \div (x - 4)$.

F $x - 8$ **H** $x - 6$

G $x + 8$ **J** $x + 6$

10. Rhombus $ABCD$ is shown. Which pair of triangles can be aestablished to be congruent to prove that $\overline{AC}$ bisects $\overline{BD}$?



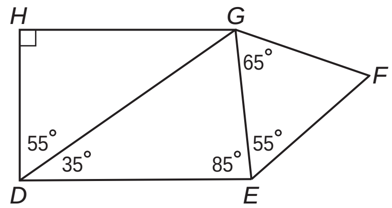
A $\triangle ABD$ and $\triangle CBD$

B $\triangle ACD$ and $\triangle ACB$

C $\triangle AEB$ and $\triangle BEC$

D $\triangle AEB$ and $\triangle CED$

11. What is the shortest side of quadrilateral $DEFG$?



F $\overline{GF}$ **H** $\overline{DG}$

G $\overline{FE}$ **J** $\overline{DE}$

Pre-AP

Record your answer on a sheet of paper. Show your work.

12. An extension ladder leans against the side of a house while gutters are being cleaned. The base of the ladder is 12 feet from the house, and the top of the ladder rests 16 feet up the side of the house.

a. Draw a figure representing this situation. What is the length of the ladder?

b. For safety, a ladder should have a climbing angle of no less than 75°. Is the climbing angle of this ladder safe?

c. If not, what distance from the house should the ladder be placed so that it still rests 16 feet up the side of the house at a 75° climbing angle and to what new length will the ladder need to be adjusted?

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson or Page...	8-2	8-4	8-2	8-2	7-3	7-4	6-1	7-5	794	6-3	5-3	8-7

CHAPTER 9

Transformations

- Name, draw, and recognize figures that have been reflected, translated, rotated, or dilated.
- Identify and create different types of tessellations.
- Find the magnitude and direction of vectors and perform operations on vectors.

Key Vocabulary

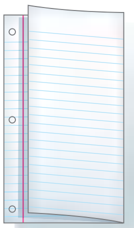
reflection (p. 497)
translation (p. 504)
rotation (p. 510)
tessellation (p. 519)
dilation (p. 525)
vector (p. 534)

Real-World Link

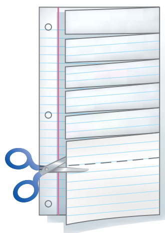
The patterns in these quilts are made by repeating a basic pattern called a block. This repetition of a basic pattern across a plane is called a tessellation.

Transformations Make this Foldable to help you organize your notes. Begin with one sheet of notebook paper.

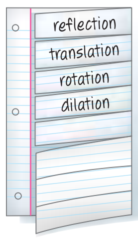
1 **Fold** a sheet of notebook paper in half lengthwise.



2 **Cut** on every third line to create 8 tabs.



3 **Label** each tab with a vocabulary word from this chapter.



William A. Bake/CORBIS

GET READY for Chapter 9

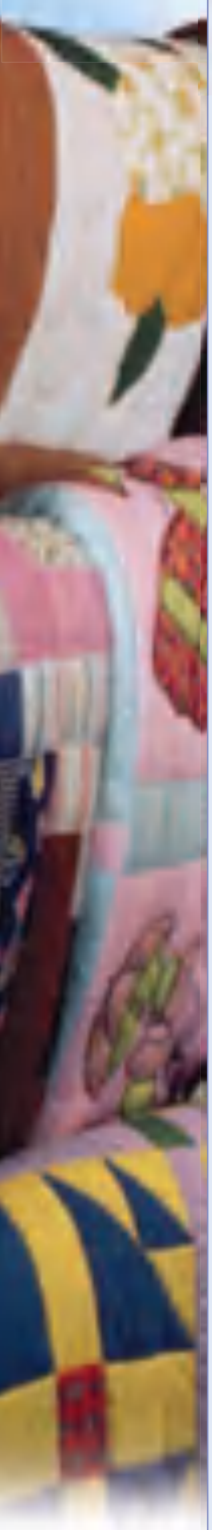
Diagnose Readiness **You have two options for checking Prerequisite Skills.**

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.



QUICKCheck

Graph each pair of points. (Prerequisite Skills, pp. 774–775)

1. $A(1, 3)$, $B(-1, 3)$
2. $C(-3, 2)$, $D(-3, -2)$
3. $E(-2, 1)$, $F(-1, -2)$
4. $G(2, 5)$, $H(5, -2)$
5. $J(-7, 10)$, $K(-6, 7)$

MAPS **For Exercises 6–8, refer to the map.** (Prerequisite Skills, pp. 774–775)

6. Where is Pittsburgh located?
7. Where is Harrisburg located?
8. Which city is located at (10, G)?

Find $m\angle A$. Round to the nearest tenth. (Lesson 8-4)

9. $\tan A = \frac{3}{4}$
10. $\tan A = \frac{5}{8}$
11. $\sin A = \frac{2}{3}$
12. $\sin A = \frac{4}{5}$
13. $\cos A = \frac{9}{12}$
14. $\cos A = \frac{15}{17}$
15. **INDIRECT MEASUREMENT** Michelle is 12 feet away from a statue. She sights the top of the statue at an 18° angle. If she is 5 feet 4 inches tall, how tall is the statue? Round to the nearest tenth. (Lesson 8-4)

QUICKReview

EXAMPLE 1

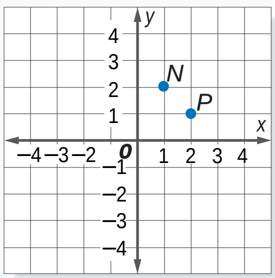
Graph the pair of points $N(1, 2)$, $P(2, 1)$.

Point *N*:

Start at the origin. Move 1 unit to the right, since the *x*-coordinate is 1. From there, move 2 units up, since the *y*-coordinate is 2. Draw a dot, and label it *N*.

Point *P*:

Start at the origin. Move 2 units to the right, since the *x*-coordinate is 2. From there, move 1 unit up, since the *y*-coordinate is 1. Draw a dot, and label it *P*.



EXAMPLE 2

Find $m\angle A$ if $\cos A = \frac{1}{2}$. Round to the nearest tenth.

$\cos A = \frac{1}{2}$

$A = \cos^{-1}\left(\frac{1}{2}\right)$ Inverse cosine

$A = 60°$ Use a calculator.

EXPLORE 9-1

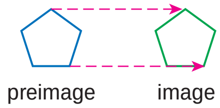
Geometry Lab
Transformations

In a plane, you can slide, flip, turn, enlarge, or reduce figures to create new figures. These corresponding figures are frequently designed into wallpaper borders, mosaics, and artwork. Each figure that you see will correspond to another figure. These corresponding figures are formed using transformations.

A **transformation** maps an initial figure, called a *preimage,* onto a final figure, called an *image.* Below are some of the types of transformations. The red lines show some corresponding points.

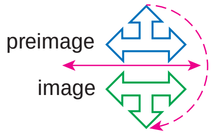
translation

A figure can be slid in any direction.



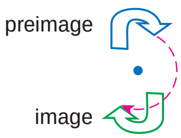
reflection

A figure can be flipped over a line.



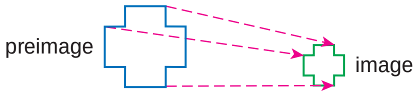
rotation

A figure can be turned around a point.



dilation

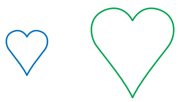
A figure can be enlarged or reduced.

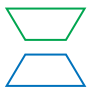


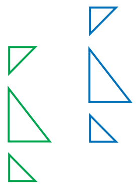
Exercises

Identify the following transformations. The blue figure is the preimage.

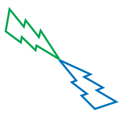
1. 

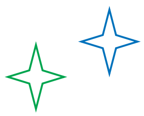
2. 

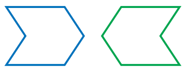
3. 

4. 

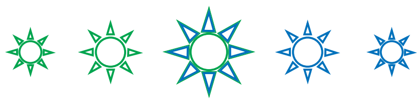
5. 

6. 

7. 

8. 

9. 

10. 

Reading Math

Rigid Motions
Isometries are also known as *rigid motions* because the preimage undergoing the transformation does not change in size or shape. It remains rigid.

Analyze the Results

11. **MAKE A CONJECTURE** An *isometry* is a transformation in which the resulting image is congruent to the preimage. Which transformations are isometries?

9-1 Reflections

Main Ideas

- Draw reflected images.
- Recognize and draw lines of symmetry and points of symmetry.

New Vocabulary

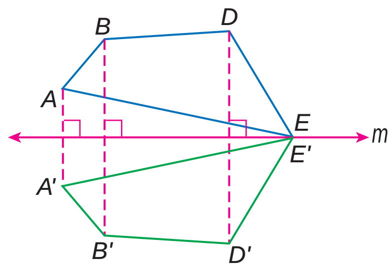
reflection
line of reflection
isometry
line of symmetry
point of symmetry

GET READY for the Lesson

On clear, bright days glacial-fed lakes provide vivid reflections of the surrounding vistas. Note that each point above the water line has a corresponding point in the image in the lake. The distance that a point lies above the water line appears the same as the distance its image lies below the water.

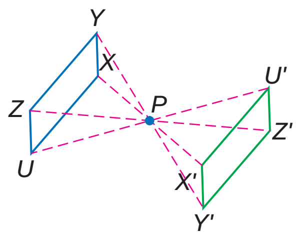
Draw Reflections A **reflection** is a transformation representing a flip of a figure. Figures may be reflected in a point, a line, or a plane.

The figure shows a reflection of *ABDE* in line *m*. Note that the segment connecting a point and its image is perpendicular to line *m* and is bisected by line *m*. Line *m* is called the **line of reflection** for *ABDE* and its image *A'B'D'E'*. Because *E* lies on the line of reflection, its preimage and image are the same point.



A', A'', A''', and so on, name corresponding points for one or more transformations.

It is possible to reflect a preimage in a point. In the figure below, polygon *UXYZ* is reflected in point *P*.



Note that *P* is the midpoint of each segment connecting a point with its image.

$\overline{UP} \cong \overline{U'P}$, $\overline{XP} \cong \overline{X'P}$, $\overline{YP} \cong \overline{Y'P}$, $\overline{ZP} \cong \overline{Z'P}$

Review Vocabulary

Congruence Transformation
A mapping for which a geometric figure and its image are congruent. (Lesson 4-3)

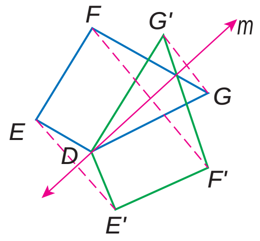
When reflecting a figure in a line or in a point, the image is congruent to the preimage. Thus, a reflection is a *congruence transformation*, or an **isometry**. That is, reflections preserve distance, angle measure, betweenness of points, and collinearity. In the figure above, polygon *UXYZ* $\cong$ polygon *U'X'Y'Z'*.

Corresponding Sides	Corresponding Angles
$\overline{XU} \cong \overline{X'U'}$	$\angle YXU \cong \angle Y'X'U'$
$\overline{XY} \cong \overline{X'Y'}$	$\angle XYZ \cong \angle X'Y'Z'$
$\overline{YZ} \cong \overline{Y'Z'}$	$\angle YZU \cong \angle Y'Z'U'$
$\overline{UZ} \cong \overline{U'Z'}$	$\angle ZUX \cong \angle Z'U'X'$

Robert Glusic/Getty Images

EXAMPLE Reflecting a Figure in a Line

1 **Draw the reflected image of quadrilateral *DEFG* in line *m*.**



Step 1 Since *D* is on line *m*, *D* is its own reflection. Draw segments perpendicular to line *m* from *E*, *F*, and *G*.

Step 2 Locate *E*', *F*', and *G*' so that line *m* is the perpendicular bisector of *EE*', *FF*', and *GG*'. Points *E*', *F*', and *G*' are the respective images of *E*, *F*, and *G*.

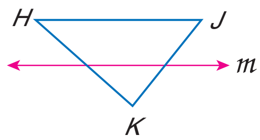
Step 3 Connect vertices *D*, *E*', *F*', and *G*'.

Step 4 Check points to make sure they are images of *D*, *E*, *F*, and *G*.

Since points *D*, *E*', *F*', and *G*' are the images of points *D*, *E*, *F*, and *G* under reflection in line *m*, then quadrilateral *DE*'*F*'*G*' is the reflection of quadrilateral *DEFG* in line *m*.

CHECK Your Progress

1. Draw the reflected image of triangle *HJK* in line *m*.



EXAMPLE Reflection on a Coordinate Plane

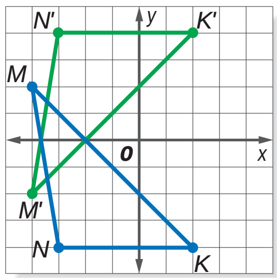
Reading Math

Shorthand The expression *K*(2, −4) → *K'*(2, 4) can be read as *point K is mapped to new location K'*. This means that point *K'* in the image corresponds to point *K* in the preimage.

2 COORDINATE GEOMETRY **Triangle *KMN* has vertices *K*(2, −4), *M*(−4, 2), and *N*(−3, −4).**

a. Graph *KMN* and its image under reflection in the *x*-axis. Compare the coordinates of each vertex with the coordinates of its image.

Use the vertical grid lines to find a corresponding point for each vertex so that the *x*-axis is equidistant from each vertex and its image.



K(2, −4) → *K'*(2, 4)

M(−4, 2) → *M'*(−4, −2)

N(−3, −4) → *N'*(−3, 4)

Plot the reflected vertices and connect to form the image *K'M'N'*. The *x*-coordinates stay the same, but the *y*-coordinates are opposites. That is, (*a*, *b*) → (*a*, −*b*).

b. Graph *KMN* and its image under reflection in the origin. Compare the coordinates of each vertex with the coordinates of its image.

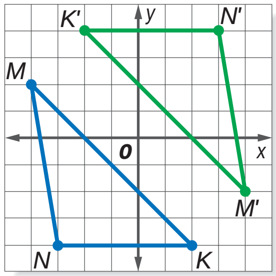
Since $\overline{KK'}$ passes through the origin, use the horizontal and vertical distances from *K* to the origin to find the coordinates of *K*'. From *K* to the origin is 4 units up and 2 units left. *K*' is located by repeating that pattern from the origin. Four units up and 2 units left yields *K*'(−2, 4).

$K(2, -4) \rightarrow K'(-2, 4)$

$M(-4, 2) \rightarrow M'(4, -2)$

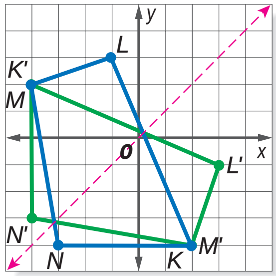
$N(-3, -4) \rightarrow N'(3, 4)$

Plot the reflected vertices and connect to form the image $K'M'N'$. Comparing coordinates shows that $(a, b) \rightarrow (-a, -b)$.



c. Graph *KMN* and its image under reflection in the line $y = x$. Compare the coordinates of each vertex with the coordinates of its image.

The slope of $y = x$ is 1. $\overline{KK'}$ is perpendicular to $y = x$, so its slope is -1. From K to the line $y = x$, move up three units and left three units. From the line $y = x$, move up three units and left three to $K'(-4, 2)$.



$K(2, -4) \rightarrow K'(-4, 2)$

$M(-4, 2) \rightarrow M'(2, -4)$

$N(-3, -4) \rightarrow N'(-4, -3)$

Plot the reflected vertices and connect to form the image $K'M'N'$. Comparing coordinates shows that $(a, b) \rightarrow (b, a)$.

Cross-Curricular Project

A reflection on the map will help you make the final step in locating the hidden treasure. Visit geometryonline.com to continue your work.

CHECK Your Progress

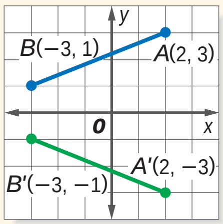
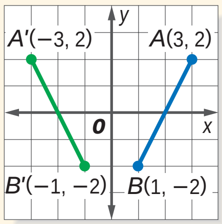
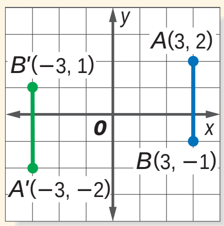
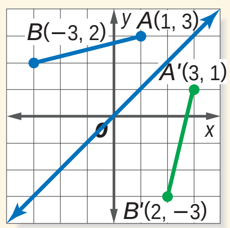
2A. Quadrilateral *RUDV* has vertices $R(-2, 2)$, $U(3, 1)$, $D(4, -1)$, and $V(-2, -2)$ and is reflected in the y-axis. Graph *RUDV* and its image. Compare the coordinates of each vertex with the coordinates of its image.

2B. Quadrilateral *RUDV* is reflected in the origin. Graph *RUDV* and its image under reflection in the origin.

2C. Quadrilateral *RUDV* is reflected in the equation $y = x$. Graph *RUDV* and its image under reflection in the origin.

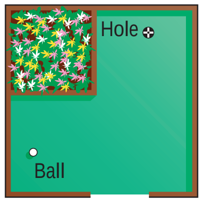
Personal Tutor at geometryonline.com

CONCEPT SUMMARY — *Reflections in the Coordinate Plane*

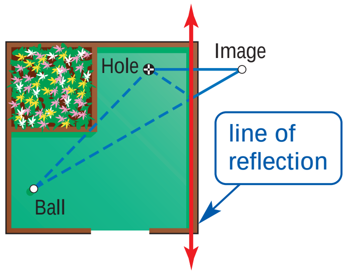
Reflection	x-axis	y-axis	origin	$y = x$
Preimage to Image	$(a, b) \rightarrow (a, -b)$	$(a, b) \rightarrow (-a, b)$	$(a, b) \rightarrow (-a, -b)$	$(a, b) \rightarrow (b, a)$
How to find coordinates	Multiply the y-coordinate by -1.	Multiply the x-coordinate by -1.	Multiply both coordinates by -1.	Interchange the x- and y-coordinates.
Example				

Real-World EXAMPLE **Use Reflections**

3 **GOLF** **Adeel and Natalie are playing miniature golf. Adeel says that reflections can help make a hole-in-one on most miniature golf holes. Describe how he should putt the ball to make a hole-in-one.**



If Adeel tries to putt the ball directly to the hole, he will strike the border as indicated by the blue line. So, he can mentally reflect the hole in the line that contains the right border. If he putts the ball at the reflected image of the hole, the ball will strike the border, and it will rebound on a path toward the hole.



CHECK Your Progress

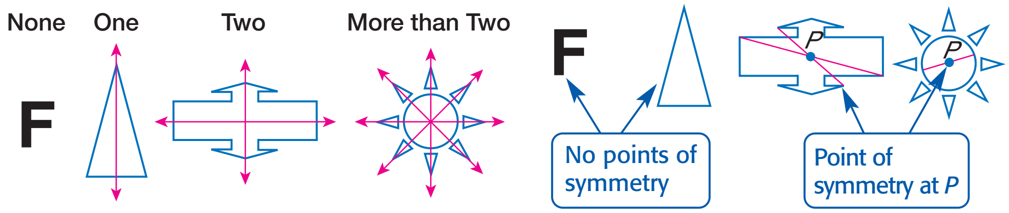
3. Tony wants to bounce pass a basketball to Jamal. Describe how Tony could use a reflection to discover where to bounce the ball so that Jamal can catch it at waist level.

Study Tip

A Point of Symmetry
A point of symmetry is the midpoint of all the segments joining a preimage to an image. Each point on the figure must have an image on the figure for a point of symmetry to exist.

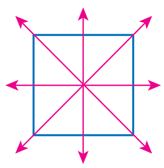
Lines and Points of Symmetry Some figures can be folded so that the two halves match exactly. The fold is a line of reflection called a **line of symmetry**. For some figures, a point can be found that is a common point of reflection for all points on a figure. This common point of reflection is called a **point of symmetry**.

Lines of Symmetry **Points of Symmetry**

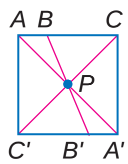


EXAMPLE **Draw Lines of Symmetry**

4 **Determine how many lines of symmetry a square has. Then determine whether a square has point symmetry.**



A square has four lines of symmetry.



P is the point of symmetry such that $AP = PA'$, $BP = PB'$, $CP = PC'$, and so on.

CHECK Your Progress

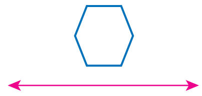
4. Determine how many lines of symmetry a rectangle that is not a square has. Does the rectangle have point symmetry?

CHECK Your Understanding

Example 1 (p. 498)

Draw the reflected image of the polygons in line m and line ℓ.

1. 

2. 

Example 2 (p. 498)

COORDINATE GEOMETRY **Graph each figure and its image under the given reflection.**

3. $\triangle XYZ$ with vertices $X(0, 0)$, $Y(3, 0)$, and $Z(0, 3)$ reflected in the x-axis

4. $\triangle ABC$ with vertices $A(-1, 4)$, $B(4, -2)$, and $C(0, -3)$ reflected in the y-axis

5. $\triangle DEF$ with vertices $D(-1, -3)$, $E(3, -2)$, and $F(1, 1)$ reflected in the origin

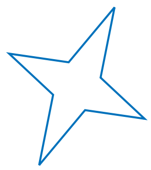
6. $\square GHIJ$ with vertices $G(-1, 2)$, $H(2, 3)$, $I(6, 1)$, and $J(3, 0)$ reflected in the line $y = x$

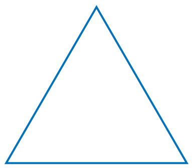
Example 3 (p. 500)

7. TENNIS Tanya is serving a tennis ball. Describe how she could use a reflection to discover where to serve the ball so that it arrives below her opponent's waist.

Example 4 (p. 500)

Determine how many lines of symmetry each figure has. Then determine whether the figure has point symmetry.

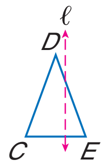
8. 

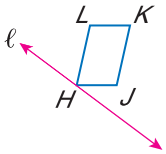
9. 

Exercises

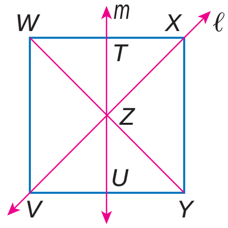
HOMEWORK HELP	
For Exercises	See Examples
10–20	1
21–24	4
25–32	2
33, 34	3

Copy each figure. Draw the image of each figure under a reflection in line ℓ.

10. 

11. 

For Exercises 12–20, refer to the figure at the right.



Name the image of each figure under a reflection in line ℓ.

12. $\overline{WX}$ **13.** $\overline{WZ}$ **14.** $\angle XZY$

Name the image of each figure under a reflection in line m.

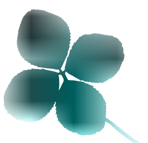
15. T **16.** $\overline{UY}$ **17.** $\triangle YVW$

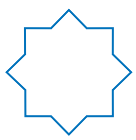
Name the image of each figure under a reflection in point Z.

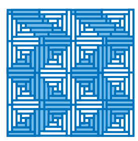
18. U **19.** $\angle TXZ$ **20.** $\triangle YUZ$

Determine how many lines of symmetry each object has. Then determine whether each object has point symmetry.

21. 

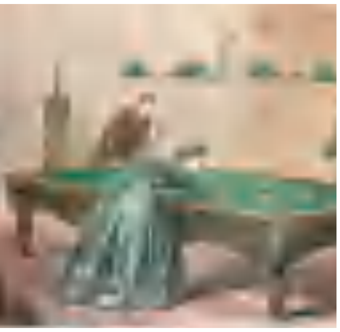
22. 

23. 

24. 

COORDINATE GEOMETRY Graph each figure and its image under the given reflection.

25. $\overline{AB}$ with endpoints $A(2, 4)$ and $B(-3, -3)$ in the x-axis
26. square $QRST$ with vertices $Q(-1, 4)$, $R(2, 5)$, $S(3, 2)$, and $T(0, 1)$ in the x-axis
27. $\overline{DJ}$ with endpoints $D(4, 4)$ and $J(-3, 2)$ in the y-axis
28. trapezoid with vertices $D(4, 0)$, $E(-2, 4)$, $F(-2, -1)$, and $G(4, -3)$ in the y-axis
29. rectangle $MNQP$ with vertices $M(2, 3)$, $N(-2, 3)$, $Q(-2, -3)$, and $P(2, -3)$ in the origin
30. quadrilateral $GHIJ$ with vertices $G(-2, -2)$, $H(2, 0)$, $I(3, 3)$, and $J(-2, 4)$ in the origin
31. $\triangle ABC$ with vertices $A(-3, -1)$, $B(0, 2)$, and $C(3, -2)$ in the line $y = x$
32. $\triangle KLM$ with vertices $K(4, 0)$, $L(-2, 4)$, and $M(-2, 1)$ in the line $y = -x$

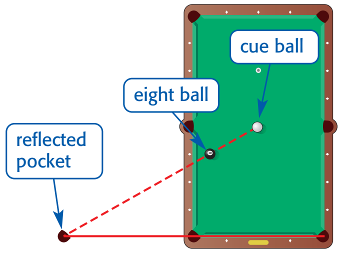


Real-World Link

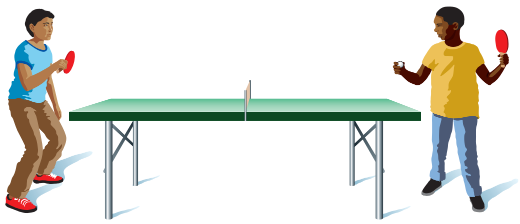
Billiards in its present form was popular in the early 1800s, but games similar to billiards appeared as early as the 14th century. There are three types of billiards: carom billiards, pocket billiards (pool), and snooker.

Source: www.infoplease.com

33. **BILLIARDS** Tanya is playing billiards. She wants to pocket the eight ball in the lower right pocket using the white cue ball. Copy the diagram and sketch the path the eight ball must travel after being struck by the cue ball.



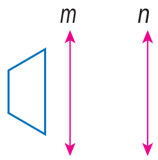
34. **TABLE TENNIS** Martin wants to hit the ball so that when it reaches Mao it is at about elbow height. Copy the diagram and sketch the path the ball must travel after being struck by Martin's paddle.



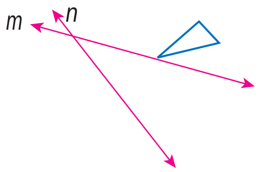
35. **COORDINATE GEOMETRY** Triangle ABC has been reflected in the x-axis, then in the y-axis, and then in the origin. The result has vertices at $A'''(4, 7)$, $B'''(10, -3)$, and $C'''(-6, -8)$. Find the coordinates of A, B, and C.

Copy each figure and then reflect the figure in line *m* first and then reflect that image in line *n*. Compare the preimage with the final image.

36.

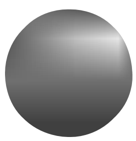


37.

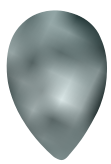


DIAMONDS For Exercises 38–41, use the following information.
Diamond jewelers offer a variety of cuts. For each top view, identify any lines or points of symmetry.

38. round cut

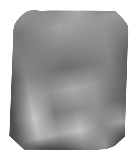


39. pear cut



40. heart cut

41. emerald cut



EXTRA PRACTICE
See pages 817, 836.
Math Online
Self-Check Quiz at geometryonline.com

H.O.T. Problems

42. **OPEN ENDED** Draw a figure on the coordinate plane with an image that when reflected in an axis, looks exactly like the original figure. What general type of figures share this characteristic?

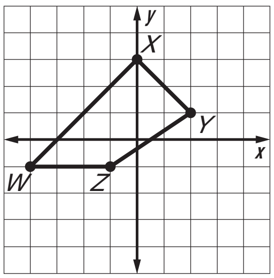
43. **REASONING** Find a counterexample for the statement *The intersection of 2 or more lines of symmetry for a plane figure is a point of symmetry.*

44. **CHALLENGE** Show that the image of a point upon reflection in the origin is the same image obtained when reflecting a point in the x-axis and then the y-axis.

45. **Writing in Math** Explain where reflections can be found in nature. Include in your answer two examples from nature that have line symmetry. Refer to page 497 and give an explanation of how the distance from each point above the water line relates to the image in the water.

STANDARDIZED TEST PRACTICE

46. If quadrilateral $WXYZ$ is reflected across the y-axis to become quadrilateral $W'X'Y'Z'$, what are the coordinates of X'?



A (0, -3) C (-3, 0)

B (0, 3) D (3, 0)

47. **REVIEW** Over the next four days, Diondre plans to drive 160 miles, 235 miles, 185 miles, and 220 miles. If his car gets an average of 32 miles per gallon of gas, how many gallons of gas should he expect to use in all?

F 20 gallons H 32 gallons

G 25 gallons J 50 gallons

48. **REVIEW** What is the solution set of the inequality $3z + 4 < 6 + 7z$?

A $\{z \mid z > -0.5\}$ C $\{z \mid z < -0.5\}$

B $\{z \mid z > -2\}$ D $\{z \mid z < -2\}$

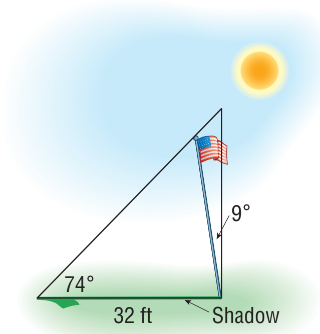
Spiral Review

In $\triangle ABC$, given the lengths of the sides, find the measure of the given angle to the nearest tenth. (Lesson 8-7)

49. $a = 6$, $b = 9$, $c = 11$; $m\angle C$

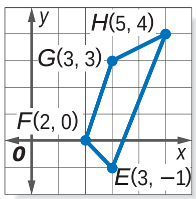
50. $a = 15.5$, $b = 23.6$, $c = 25.1$; $m\angle B$

51. **INDIRECT MEASUREMENT** When the angle of elevation to the Sun is 74°, a flagpole tilted at an angle of 9° from the vertical casts a shadow 32 feet long on the ground. Find the length of the flagpole to the nearest tenth of a foot. (Lesson 8-6)



GET READY for the Next Lesson

PREREQUISITE SKILL Find the exact length of each side of quadrilateral *EFGH*. (Lesson 1-3)



52. $\overline{EF}$

53. $\overline{FG}$

54. $\overline{GH}$

55. $\overline{HE}$

9-2 Translations

Main Ideas

- Draw translated images using coordinates.
- Draw translated images by using repeated reflections.

New Vocabulary

translation

composition

The sights and pageantry of a marching band performance can add to the excitement of a sporting event.

The movements of each band member as they progress through the show are examples of *translations.*

Translations Using Coordinates A **translation** is a transformation that moves all points of a figure the same distance in the same direction. Translations on the coordinate plane can be drawn if you know the direction and how far the figure is moving horizontally and / or vertically. For the fixed values of a and b, a translation moves every point $P(x, y)$ of a plane figure to an image $P'(x + a, y + b)$, or $(x, y) \rightarrow (x + a, y + b)$.

STANDARDIZED TEST EXAMPLE — Translations in Coordinate Plane

1 **Triangle QRS has vertices $Q(-4, 2)$, $R(3, 0)$, and $S(4, 3)$. If $\triangle QRS$ is translated 4 units down and 6 units right to create $\triangle Q'R'S'$, what are the coordinates of the vertices of $\triangle Q'R'S'$?**

A $Q'(-8, 8)$, $R'(-1, 6)$, $S'(0, 9)$ **C** $Q'(-1, -2)$, $R'(-3, -4)$, $S'(-2, 9)$

B $Q'(0, 8)$, $R'(7, 6)$, $S'(8, 9)$ **D** $Q'(2, -2)$, $R'(9, -4)$, $S'(10, -1)$

Test-Taking Tip

Read Carefully
Be sure to read problems carefully. In the example shown, the vertical translation (y-coordinate) is given *before* the horizontal translation (x-coordinate). Rearrange the information to prevent errors.

Read the Test Item

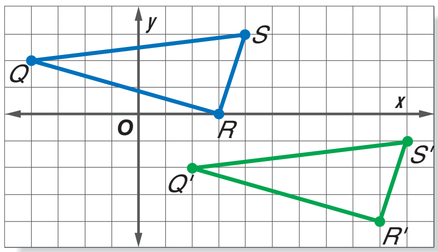
You are asked to find the coordinates of the image of $\triangle QRS$ after a translation 4 units down and 6 units right, or $(x, y) \rightarrow (x + 6, y - 4)$.

Solve the Test Item

$Q(-4, 2) \rightarrow Q'(-4 + 6, 2 - 4)$ or $Q'(2, -2)$

$R(3, 0) \rightarrow R'(3 + 6, 0 - 4)$ or $R'(9, -4)$

$S(4, 3) \rightarrow S'(4 + 6, 3 - 4)$ or $S'(10, -1)$



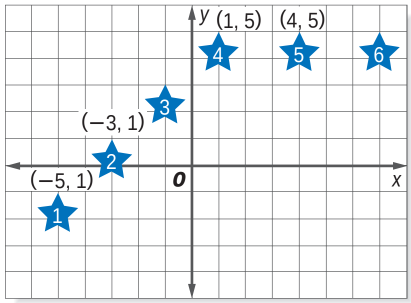
To check your answer, graph and compare $\triangle QRS$ and its image $\triangle Q'R'S'$. Each image vertex is 6 units right and 4 units down from each preimage vertex. Choice D is correct.

1. Quadrilateral $HJLK$ has vertices $H(1, 0)$, $J(0, 4)$, $L(3, 1)$ and $K(2, 5)$. If $HJLK$ is translated 3 units left and 5 units down, what are the coordinates of the vertices of point K'?

F $K'(-6, -3)$ **G** $K'(-1, 0)$ **H** $K'(5, 10)$ **J** $K'(-5, -10)$

Repeated Translations

2 **ANIMATION Computers are often used to create animation. The graph shows repeated translations that result in animation of the star. Find the translation that moves star 1 to star 2.**



To find the translations, use the coordinates at the top of each star.

$(x, y) \rightarrow (x + a, y + b)$ Translation formula

$(-5, -1) \rightarrow (-3, 1)$ Use coordinates $(-5, -1)$ and $(-3, 1)$.

$x + a = -3$ $y + b = 1$

$-5 + a = -3$ $x = -5$ $-1 + b = 1$ $y = -1$

$a = 2$ Add 5 to each side. $b = 2$ Add 1 to each side.

The translation is $(x, y) \rightarrow (x + 2, y + 2)$ from star 1 to star 2.

2. Find the translation that was used to move star 4 to star 5.

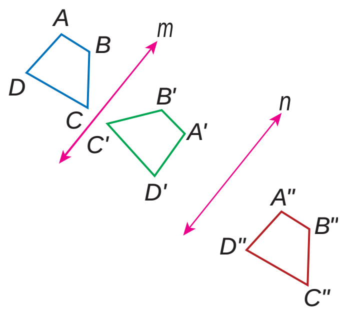
Reading Math

Translation A translation is also called a *slide*, a *shift*, or a *glide*.

Translations By Repeated Reflections Another way to find a translation is to perform a reflection in the first of two parallel lines and then reflect the image in the other parallel line. A transformation made up of successive transformations is called a **composition**.

EXAMPLE Find a Translation Using Reflections

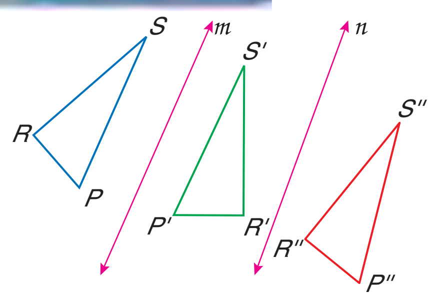
3 **In the figure, lines m and n are parallel. Determine whether the red figure is a translation image of the blue preimage.**



Reflect quadrilateral $ABCD$ in line m. The result is the green image, quadrilateral $A'B'C'D'$. Then reflect the green image, quadrilateral $A'B'C'D'$ in line n. The red image, quadrilateral $A''B''C''D''$, has the same orientation as quadrilateral $ABCD$.

Quadrilateral $A''B''C''D''$ is the translation image of quadrilateral $ABCD$.

3.



Study Tip

Isometries
Since translations are compositions of two reflections, all translations are isometries. Thus, all properties preserved by reflections are preserved by translations. These properties include betweenness of points, collinearity, and angle and distance measure.

Check Your Understanding

Example 1 (p. 504)

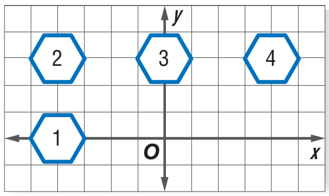
1. **COORDINATE GEOMETRY** Graph $\overline{DE}$ with endpoints $D(-3, -4)$ and $E(4, 2)$ under the translation $(x, y) \rightarrow (x + 1, y + 3)$.

2. **STANDARDIZED TEST PRACTICE** If $\triangle KLM$ with vertices $K(5, -2)$, $L(-3, -1)$, and $M(0, 5)$ is translated 4 units down and 3 units left to create $\triangle XYZ$, what are the coordinates of the vertices of $\triangle XYZ$?

   A $X(2, -6)$, $Y(-6, -5)$, $Z(-3, 1)$
   C $X(1, -5)$, $Y(-7, -4)$, $Z(-4, 2)$
   B $X(-2, -6)$, $Y(-6, -5)$, $Z(-3, -1)$
   D $X(-1, -5)$, $Y(-7, -4)$, $Z(-4, -2)$

Example 2 (p. 505)

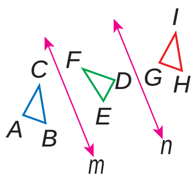
3. **ANIMATION** Find the translations that move hexagon 1 to hexagon 2 and the translation that moves hexagon 3 to hexagon 4.



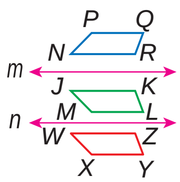
Example 3 (p. 505)

In each figure, lines m and n are parallel. Determine whether the red figure is a translation image of the blue figure. Write *yes* or *no*. Explain your answer.

4.



5.



Exercises

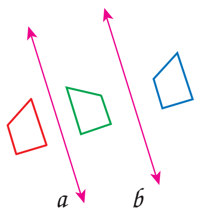
HOMEWORK HELP	
For Exercises	See Examples
6–11	1
12–17	3
18, 19	2

COORDINATE GEOMETRY **Graph each figure and the image under the given translation.**

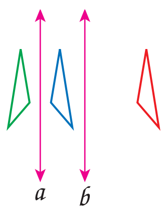
6. $\overline{PQ}$ with endpoints $P(2, -4)$ and $Q(4, 2)$ translated left 3 units and up 4 units
7. $\overline{AB}$ with endpoints $A(-3, 7)$ and $B(-6, -6)$ translated 4 units to the right and down 2 units
8. $\triangle MJP$ with vertices $M(-2, -2)$, $J(-5, 2)$, and $P(0, 4)$ translated by $(x, y) \rightarrow (x + 1, y + 4)$
9. $\triangle EFG$ with vertices $E(0, -4)$, $F(-4, -4)$, and $G(0, 2)$ translated by $(x, y) \rightarrow (x + 2, y - 1)$
10. quadrilateral $PQRS$ with vertices $P(1, 4)$, $Q(-1, 4)$, $R(-2, -4)$, and $S(2, -4)$ translated by $(x, y) \rightarrow (x - 5, y + 3)$
11. pentagon $VWXYZ$ with vertices $V(-3, 0)$, $W(-3, 2)$, $X(-2, 3)$, $Y(0, 2)$, and $Z(-1, 0)$ translated by $(x, y) \rightarrow (x + 4, y - 3)$

In each figure, $a \parallel b$. Determine whether the red figure is a translation image of the blue figure. Write *yes* or *no*. Explain your answer.

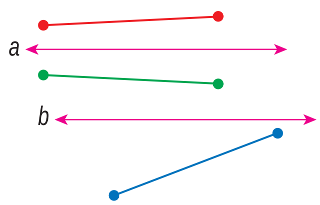
12.



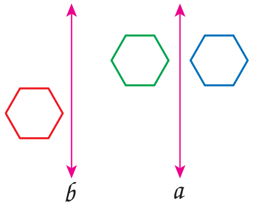
13.

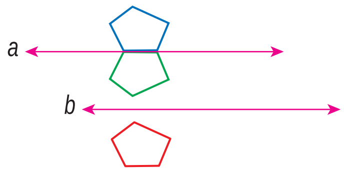


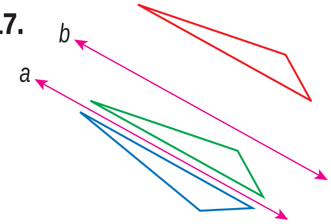
14.



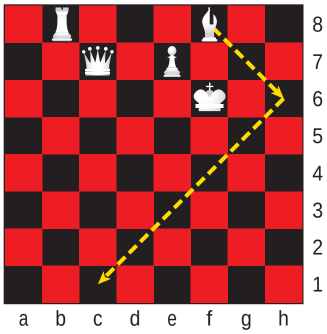
In each figure, $a \parallel b$. Determine whether the red figure is a translation image of the blue figure. Write *yes* or *no*. Explain your answer.

15. 

16. 

17. 

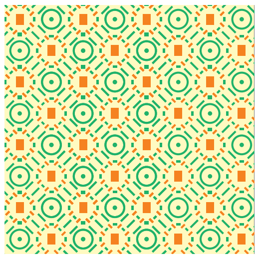
18. CHESS The bishop shown in square f8 can only move diagonally along dark squares. If the bishop is in c1 after two moves, describe the translation.



19. CHESS Describe the translation that moves the queen from c7 to c1 to take the bishop from the previous problem.

20. RESEARCH Use the Internet or other resource to write a possible translation for each chess piece for a single move.

21. DECORATION A wallpaper pattern is composed of various repeated figures. Is it possible to divide the pattern into 2 halves by drawing a line so that the reflection of one half matches the other half exactly? Explain.



Real-World Link

Wallpaper can be traced back to 200 B.C. when the Chinese pasted rice paper on their walls. Modern-style wallpaper, with block designs in continuous patterns, was developed in 1675 by the French engraver Jean Papillon.

Source: Wikipedia

A translation maps $A(-4, 3)$ onto A'. Find the coordinates of point B', the image of $B(-1, -2)$, under the same translation for each set of coordinates.

22. $A'(2, 5)$

23. $A(-1, 1)$

COORDINATE GEOMETRY Graph each figure and the image under the given translation.

24. $\triangle PQR$ with vertices $P(-3, -2)$, $Q(-1, 4)$, and $R(2, -2)$ translated by $(x, y) \rightarrow (x + 2, y - 4)$

25. $\triangle RST$ with vertices $R(-4, -1)$, $S(-1, 3)$, and $T(-1, 1)$ reflected in $y = 2$ and then reflected in $y = -2$

26. Under $(x, y) \rightarrow (x - 4, y + 5)$, $\triangle ABC$ has translated vertices $A'(-8, 5)$, $B'(2, 7)$, and $C'(3, 1)$. Find the coordinates of A, B, and C.

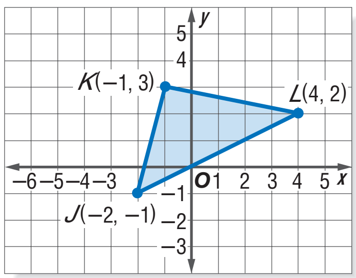
27. Triangle FGH is translated to $\triangle MNP$. Given $F(3, 9)$, $G(-1, 4)$, $M(4, 2)$, and $P(6, -3)$, find the coordinates of H and N. Then write the coordinate form of the translation.

The coordinates of the vertices of $\triangle JKL$ are $J(-3, 4)$, $K(0, 5)$, and $L(5, 10)$.

28. Measure each angle using a protractor.

29. Graph the image of $\triangle JKL$ after a reflection in $x = 2$ and one in $x = 6$.

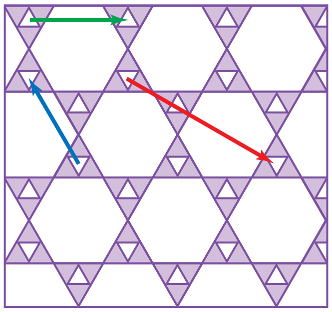
30. Measure $\angle J'$, $\angle K'$, and $\angle L'$. Compare to the angle measures of $\triangle JKL$. Which angles, if any, are congruent? Justify your answer.



EXTRA PRACTICE
See pages 817, 836.
Math Online
Self-Check Quiz at
geometryonline.com

G. Schuster/zefa/CORBIS

MOSAICS For Exercises 31–33, use the following information.



The mosaic tiling shown at the right is a thirteenth-century Roman inlaid marble tiling. Suppose the length of a side of the small white equilateral triangle is 12 inches. All triangles and hexagons are regular. Describe the translations in inches represented by each line.

31. green line **32.** blue line **33.** red line

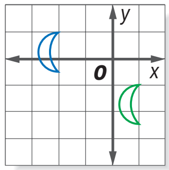
H.O.T. Problems

34. OPEN ENDED Choose integer coordinates for any two points *A* and *B* on the coordinate plane. Then describe how you could count to find the translation of point *A* to point *B*.

35. REASONING Explain which properties are preserved in a translation and why they are preserved.

36. FIND THE ERROR Allie and Tyrone are describing the transformation in the drawing. Who is correct? Explain your reasoning.

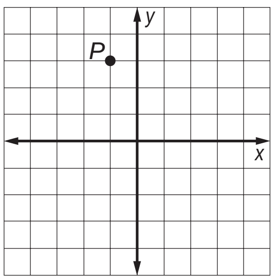
Allie	Tyrone
This is a translation right 3 units and down 2 units.	This is a reflection in the y-axis and then the x-axis.



37. CHALLENGE Triangle *TWY* has vertices *T*(3, −7), *W*(7, −4), and *Y*(9, −8). Triangle *BDG* has vertices *B*(3, 3), *D*(7, 6), and *G*(9, 2). If △*BDG* is the translation image of △*TWY* with respect to two parallel lines, find the equations that represent two possible parallel lines.

38. Writing in Math Use the information about marching bands on page 504 to explain how translations are used in marching band shows. Include the types of movements used by band members that are translations and a description of a simple pattern for a band member.

39. Identify the location of point *P* under translation $(x + 3, y + 1)$.



A (0, 6) **C** (2, −4)

B (0, 3) **D** (2, 4)

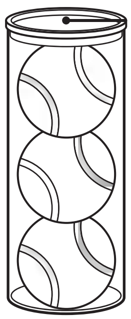
40. REVIEW Three tennis balls are packaged in a cylinder, and each ball has a circumference of about 7 inches. About how tall is the cylinder?
$C = \pi d$, $\pi \approx 3.14$

F 3.5 in.

G 7 in.

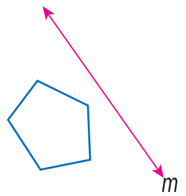
H 10 in.

J 14 in.



Copy each figure. Draw the reflected image of each figure in line *m*. (Lesson 9-1)

41.



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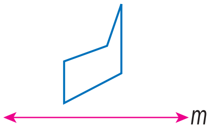

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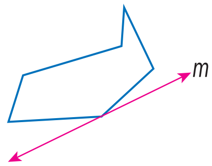
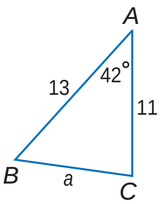
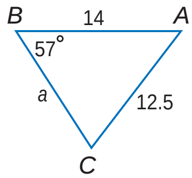


Determine whether the *Law of Sines* or the *Law of Cosines* should be used to solve each triangle. Then solve each triangle. Round to the nearest tenth. (Lessons 8-6 and 8-7)

44.



45.



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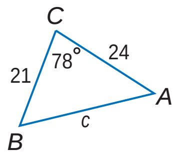
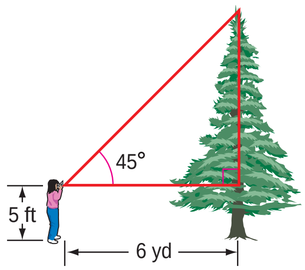
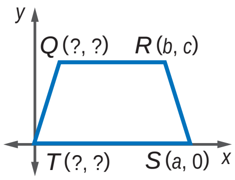


47. **LANDSCAPING** Juanna needs to determine the height of a tree. Holding a drafter's 45° triangle so that one leg is horizontal, she sights the top of the tree along the hypotenuse, as shown at the right. If she is 6 yards from the tree and her eyes are 5 feet from the ground, find the height of the tree. (Lessons 8-3)

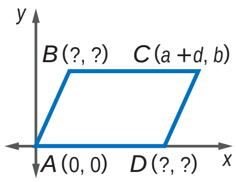


Name the missing coordinates for each quadrilateral. (Lessons 6-7)

48. *QRST* is an isosceles trapezoid.



49. *ABCD* is a parallelogram.



State the assumption you would make to start an indirect proof of each statement. (Lesson 5-3)

50. Every shopper who walks through the door is greeted by a salesperson.

51. If you get a job, you have filled out an application.

52. If $4y + 17 = 41$, then $y = 6$.

53. If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the two lines are parallel.

Find the distance between each pair of parallel lines. (Lesson 3-6)

54. $x = -2$, $x = 5$

55. $y = -6$, $y = -1$

56. $y = 2x + 3$, $y = 2x - 7$

57. $y = x + 2$, $y = x - 4$

PREREQUISITE SKILL **Use a protractor and draw an angle for each degree measure.** (Lesson 1-4)

58. 30 **59.** 45 **60.** 52 **61.** 60 **62.** 105 **63.** 150

9-3 Rotations

Main Ideas

- Draw rotated images using the angle of rotation.
- Identify figures with rotational symmetry.

New Vocabulary

rotation
center of rotation
angle of rotation
rotational symmetry
invariant points
direct isometry
indirect isometry

In 1926, Herbert Sellner invented the Tilt-A-Whirl. Today, no carnival is complete without these cars that send riders tipping and spinning around a circular track.

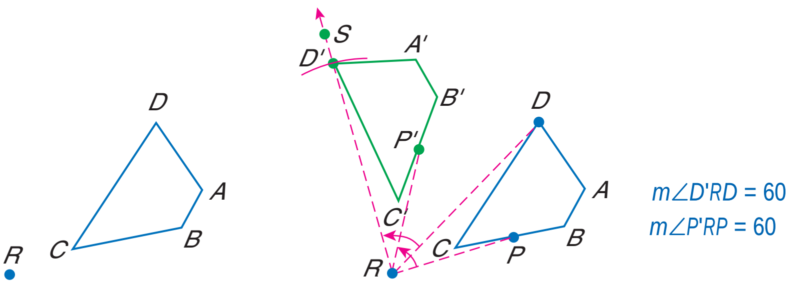
The Tilt-A-Whirl is an example of rotation.

Draw Rotations A **rotation** is a transformation that turns every point of a preimage through a specified angle and direction about a fixed point. The fixed point is called the **center of rotation**.

In the figure in Example 1, *R* is the center of rotation for the preimage *ABCD*. The measures of angles *ARA*', *BRB*', *CRC*', and *DRD*' are equal. Any point *P* on the preimage *ABCD* has an image *P*' on *A*'*B*'*C*'*D*' such that the measure of ∠*PRP*' is a constant measure. This is called the **angle of rotation**. A rotation exhibits all of the properties of isometries, including preservation of distance and angle measure. Therefore, it is an isometry.

EXAMPLE 1 Draw a Rotation

a. Rotate ▱*ABCD* 60° counterclockwise about point *R*.

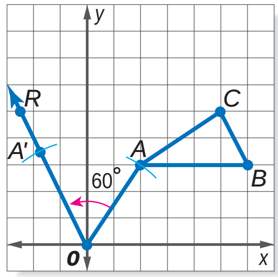


- Draw a segment from point *R* to point *D*.
- Use a protractor to measure a 60° angle counterclockwise with $\overline{RD}$ as one side.
- Label the other side of this angle as $\overrightarrow{RS}$.
- Use a compass to copy $\overline{RD}$ onto $\overrightarrow{RS}$. Name the endpoint *D*'.
- Repeat this process for points *A*, *B*, and *C*.
- Connect points *A*'*B*'*C*'*D*'.

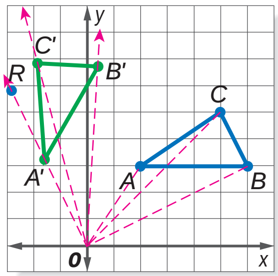
Study Tip

Turns
A rotation, sometimes called a *turn,* is generally measured as a counterclockwise turn. A half-turn is 180° and a full turn is 360°.

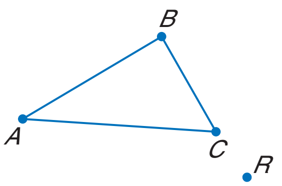
b. Triangle *ABC* has vertices *A*(2, 3), *B*(6, 3), and *C*(5, 5). Draw the image of △*ABC* under a rotation of 60° counterclockwise about the origin.



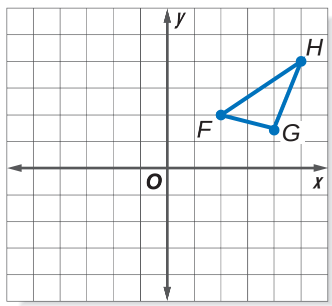
- First graph △*ABC*.
- Draw a segment from the origin *O*, to point *A*.
- Use a protractor to measure a 60° angle counterclockwise with $\overline{OA}$ as one side.
- Draw $\overrightarrow{OR}$.
- Use a compass to copy $\overline{OA}$ onto $\overrightarrow{OR}$. Name the segment $\overline{OA'}$.
- Repeat with points *B* and *C*. △*A'B'C'* is the image of △*ABC* under a 60° counterclockwise rotation about the origin.



1A. Copy triangle *ABC*. Then rotate the triangle 120° counterclockwise around the point *R*.



1B. Triangle *FGH* has vertices *F*(2, 2), *G*$\left(4, 1\frac{1}{2}\right)$, and *H*(5, 4). Draw the image of △*FGH* under a rotation of 90° counterclockwise about the origin.



Another way to perform rotations is by reflecting a figure successively in two intersecting lines. Reflecting a figure once and then reflecting the image in a second line is another example of a composition of reflections.

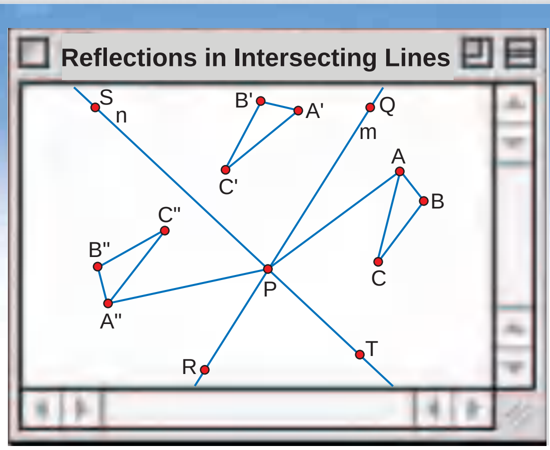
GEOMETRY SOFTWARE LAB

Reflections in Intersecting Lines

CONSTRUCT A FIGURE

- Use The Geometer's Sketchpad to construct scalene triangle *ABC*.
- Construct lines *m* and *n* so that they intersect outside △*ABC*.
- Label the point of intersection *P*.

(continued on the next page)



ANALYZE

1. Reflect $\triangle ABC$ in line m. Then, reflect $\triangle A'B'C'$ in line n.

2. Describe the transformation of $\triangle ABC$ to $\triangle A''B''C''$.

3. Measure the acute angle formed by m and n.

4. Construct a segment from A to P and from A'' to P. Find the measure of the angle of rotation, $\angle APA''$. Then find $m\angle BPB''$ and $m\angle CPC''$.

MAKE A CONJECTURE

5. What is the relationship between the measures of the angles of rotation and the measure of the acute angle formed by m and n?

When rotating a figure by reflecting it in two intersecting lines, there is a relationship between the angle of rotation and the angle formed by the intersecting lines.

THEOREMS

Theorem 9.1	In a given rotation, if A is the preimage, A'' is the image, and P is the center of rotation, then the measure of the angle of rotation $\angle APA''$ is twice the measure of the acute or right angle formed by the intersecting lines of reflection.
Corollary 9.1	Reflecting an image successively in two perpendicular lines results in a 180° rotation.

You will prove Theorem 9.1 and Corollary 9.1 in Exercises 32 and 33, respectively.

Study Tip

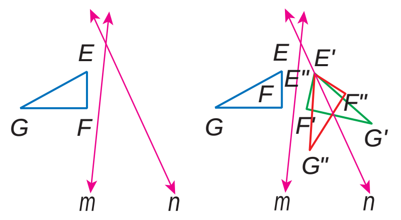
Common Misconception

The order in which you reflect a figure in two nonperpendicular intersecting lines produces rotations of the same degree measure, but one is clockwise and the other is counterclockwise.

Every isometry of the coordinate plane is a translation, the composition of a translation and a rotation, or the composition of a translation, a rotation, and a reflection.

EXAMPLE 2 Reflections in Intersecting Lines

Find the image of triangle $\triangle EFG$ under reflections in line m and then line n.

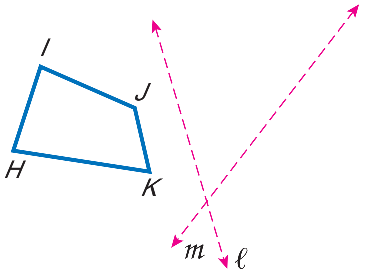


First reflect $\triangle EFG$ in line m.
Then label the image $\triangle E'F'G'$.

Next, reflect the image in line n.

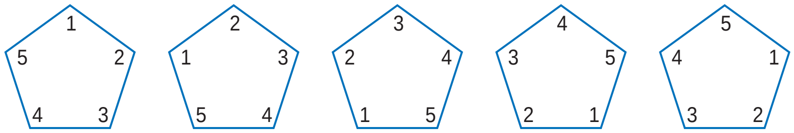
$\triangle E''F''G''$ is the image of $\triangle EFG$ under reflections in lines m and n. How can you transform $\triangle EFG$ directly to $\triangle E''F''G''$ by using a rotation?

2. Find the image of quadrilateral $HIJK$ under reflections in line ℓ and then line m.



Personal Tutor at geometryonline.com

Rotational Symmetry Some objects have rotational symmetry. If a figure can be rotated less than 360 degrees about a point so that the image and the preimage are indistinguishable, then the figure has **rotational symmetry**.



In the figure, the pentagon has rotational symmetry of *order* 5 because there are 5 rotations of less than 360° (including 0 degrees) that produce an image indistinguishable from the original. The rotational symmetry has a *magnitude* of 72° because 360 degrees divided by the order, in this case 5, produces the magnitude of the symmetry.

EXAMPLE 3 Identifying Rotational Symmetry

QUILTS Identify the order and magnitude of the symmetry in each part of the award-winning quilt made by Judy Mathieson of Sebastopol, California.

a. large star in center of quilt

The large star in the center of the quilt has rotational symmetry of order 20 and magnitude of 18°.

b. entire quilt

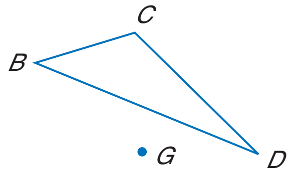
The entire quilt has rotational symmetry of order 4 and magnitude of 90°.

3. Identify the order and magnitude of the rotational symmetry in a regular octagon.

Check Your Understanding

Example 1 (p. 510)

1. Copy $\triangle BCD$ and rotate the triangle 60° counterclockwise about point G.

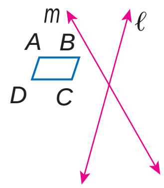


2. Quadrilateral $WRST$ has coordinates $W(0, 1)$, $R(0, 2)$, $S(1, 2)$, and $T(4, 0)$. Draw the image of quadrilateral $WRST$ under a rotation of 45° clockwise about the origin.

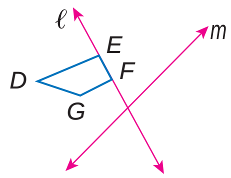
Example 2 (p. 512)

Copy each figure. Use a composition of reflections to find the rotation image with respect to lines ℓ and m.

3.



4.



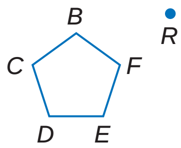
Example 3 (p. 513)

5. MUSIC A five-disc CD changer rotates as each CD is played. Identify the magnitude of the rotational symmetry from one CD to another.

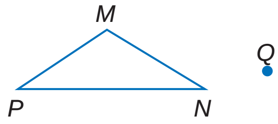
Exercises

HOMEWORK HELP	
For Exercises	See Examples
6–9	1
10–12	2
12, 14	3

6. Copy pentagon *BCDEF*. Then rotate the pentagon 110° counterclockwise about point *R*.



7. Copy $\triangle MNP$. Then rotate the triangle 180° counterclockwise around point *Q*.

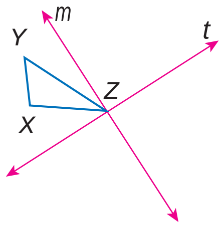


8. $\overline{XY}$ has endpoints $X(-5, 8)$ and $Y(0, 3)$. Draw the image of $\overline{XY}$ under a rotation of 45° clockwise about the origin.

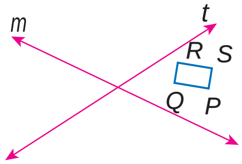
9. $\triangle PQR$ has vertices $P(-1, 8)$, $Q(4, -2)$, and $R(-7, -4)$. Draw the image of $\triangle PQR$ under a rotation of 90° counterclockwise about the origin.

Copy each figure. Use a composition of reflections to find the rotation image with respect to lines *m* and *t*.

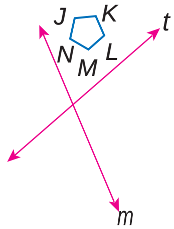
10.



11.



12.

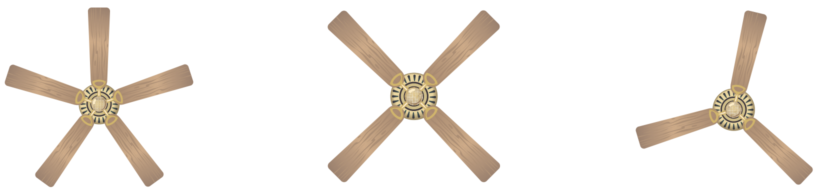


Real-World Link

Fans have been used by different societies and cultures for centuries. In traditional Asian societies, a person's gender and status dictated what type of fan they would use.

Source: Wikipedia

13. FANS The blades of a fan exhibit rotational symmetry. Identify the order and magnitude of the symmetry of the blades of each fan below.



14. STATE FLAGS The New Mexico state flag is an example of a rotation. Identify the order and magnitude of the symmetry of the figure on the flag.

COORDINATE GEOMETRY **Draw the rotation image of each figure 90° in the given direction about the center point and label the vertices.**

15. $\triangle XYZ$ with vertices $X(0, -1)$, $Y(3, 1)$, and $Z(1, 5)$ counterclockwise about the point $P(-1, 1)$

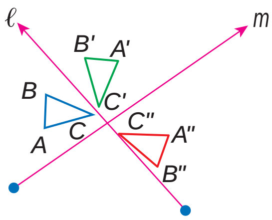
16. $\triangle RST$ with vertices $R(0, 1)$, $S(5, 1)$, and $T(2, 5)$ clockwise about the point $P(-2, 5)$

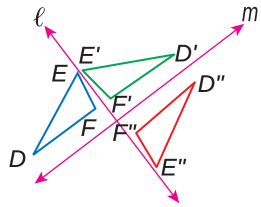
COORDINATE GEOMETRY **Draw the rotation image of each triangle by reflecting the triangles in the given lines. State the coordinates of the rotation image and the angle of rotation.**

17. $\triangle TUV$ with vertices $T(0, 4)$, $U(2, 3)$, and $V(1, 2)$, reflected in the *y*-axis and then the *x*-axis

18. $\triangle KLM$ with vertices $K(0, 5)$, $L(2, 4)$, and $M(-2, 4)$, reflected in the line $y = x$ and then the *x*-axis

Determine whether the indicated composition of reflections is a rotation. Explain.

19. 

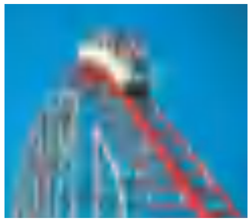
20. 

AMUSEMENT RIDES **For each ride, determine whether the rider undergoes a rotation. Write *yes* or *no*.**

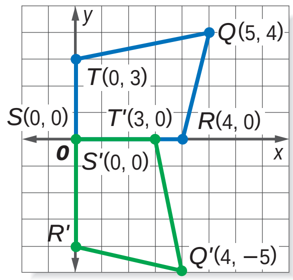
21. spinning teacups

22. scrambler

23. roller coaster loop



24. **COORDINATE GEOMETRY** Quadrilateral *QRST* is rotated 90° clockwise about the origin. Describe the transformation using coordinate notation.



25. If a rotation is performed on a coordinate plane, what angles of rotation would make the rotations easier? Explain.

26. Explain two techniques that can be used to rotate a figure.

For Exercises 27–30, use the following information.

A **direct isometry** is one in which the image of a figure is found by moving the figure intact within the plane. An **indirect isometry** cannot be performed by maintaining the orientation of the points as in a direct isometry.

27. Copy and complete the table below. Determine whether each transformation preserves the given property. Write *yes* or *no*.

Transformation	Angle Measure	Betweenness of Points	Orientation	Collinearity	Distance Measure
reflection					
translation					
rotation					

Identify each type of transformation as a direct isometry or an indirect isometry.

28. reflection

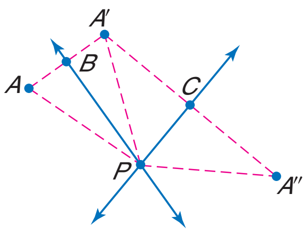
29. translation

30. rotation

31. **ALPHABET** Which capital letters of the alphabet produce the same letter after being rotated 180°?

EXTRA PRACTICE
See pages 818, 836.
Math Online
Self-Check Quiz at
geometryonline.com

PROOF **In the diagram at the right, point A' is the image of point A after a reflection in $\overleftrightarrow{BP}$. A'' is the image of point A' after a reflection in $\overleftrightarrow{PC}$. Points A, B, and A' are collinear. Points A', C, and A'' are collinear. Use the diagram to write a paragraph proof of each of the following.**



32. Theorem 9.1

33. Corollary 9.1

H.O.T. Problems

34. **OPEN ENDED** Draw a figure on the coordinate plane in Quadrant I. Rotate the figure clockwise 90 degrees about the origin. Then rotate the figure 90 degrees counterclockwise. Describe the results using the coordinates.

CHALLENGE **For Exercises 35–37, use the following information.**

Points that do not change under a transformation are called **invariant points.** For each of the following transformations, identify any invariant points.

35. reflection in a line

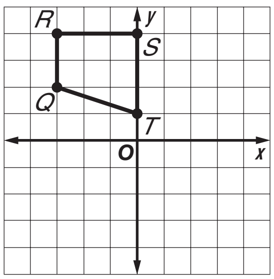
36. a rotation of $x°$ $(0 < x < 360)$ about point P

37. $(x, y) \rightarrow (x + a, y + b)$, where a and b are not 0

38. Writing in Math Describe how certain amusement rides display rotations. Include a description of how the Tilt-A-Whirl (pictured on p. 510) actually rotates in two ways. What are some other amusement rides that use rotation? Explain how they use rotation.

STANDARDIZED TEST PRACTICE

39. Figure *QRST* is shown on the coordinate plane.



What rotation creates an image with point R' at (4, 3)?

A 270° counterclockwise about the point T

B 185° counterclockwise about the point T

C 180° clockwise about the origin

D 90° clockwise about the origin

40. **REVIEW** The table shows the population and area in square miles of some Kansas counties.

County	Population	Area
Logan	2827	1073
Mitchell	6564	700
Sedgwick	463,802	999
Seward	23,237	640
Shawnee	171,716	550

Source: quickfacts.census.gov

Which statement *best* describes the relationship between the population and the area of a state?

F The larger a county's area, the larger its population.

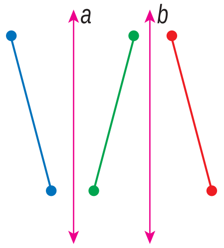
G No relationship can be determined from the data in the table.

H Shawnee County has the smallest population because it has the smallest area.

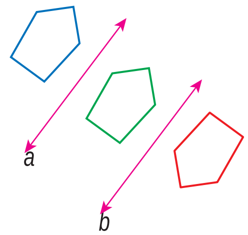
J Sedgwick is the largest county in Kansas.

In each figure, $a \parallel b$. Determine whether the blue figure is a translation image of the red figure. Write *yes* or *no*. Explain your answer. (Lesson 9-2)

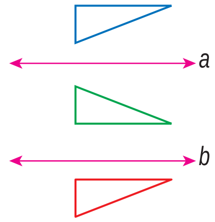
41.



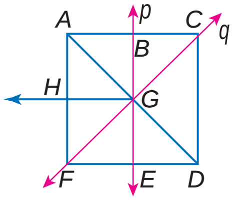
42.



43.



Refer to the figure at the right. Name the reflected image of each image. (Lesson 9-1)



44. $\overline{AG}$ in line p

45. F in point G

46. $\overline{GE}$ in line q

47. $\angle CGD$ in line p

COORDINATE GEOMETRY **For Exercises 48–51, use the following information.** The vertices of quadrilateral $PQRS$ are $P(5, 2)$, $Q(1, 6)$, $R(-3, 2)$, and $S(1, -2)$. (Lessons 3-3 and 1-3)

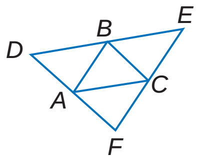
48. Show that the opposite sides of quadrilateral $PQRS$ are parallel.

49. Show that the adjacent sides of quadrilateral $PQRS$ are perpendicular.

50. Determine the length of each side of quadrilateral $PQRS$.

51. What type of figure is quadrilateral $PQRS$?

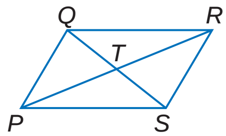
***A*, *B*, and *C* are the midpoints of $\overline{DF}$, $\overline{DE}$, and $\overline{EF}$, respectively.** (Lesson 7-4)



52. If $BC = 11$, $AC = 13$, and $AB = 15$, find the perimeter of $\triangle DEF$.

53. If $DE = 18$, $DA = 10$, and $FC = 7$, find AB, BC, and AC.

Complete each statement about parallelogram *PQRS*. Justify your answer. (Lesson 6-2)



54. $\overline{QR} \parallel$ __?__

55. $\overline{PT} \cong$ __?__

56. $\angle SQR \cong$ __?__

57. $\angle QPS \cong$ __?__

58. **MEASUREMENT** Jeralyn says that her backyard is shaped like a triangle and that the length of its sides are 22 feet, 23 feet, and 45 feet. Do you think these measurements are correct? Explain your reasoning. (Lesson 5-4)

PREREQUISITE SKILL **Find whole-number values for the variable so each equation is true.** (Pages 781–782)

59. $180a = 360$

60. $180a + 90b = 360$

61. $135a + 45b = 360$

62. $120a + 30b = 360$

63. $180a + 60b = 360$

64. $180a + 30b = 360$

CHAPTER 9

Mid-Chapter Quiz

Lessons 9-1 through 9-3

Graph each figure and the image in the given reflection. (Lesson 9-1)

1. $\triangle DEF$ with vertices $D(-1, 1)$, $E(1, 4)$, and $F(3, 2)$ in the origin
2. quadrilateral $ABCD$ with vertices $A(0, 2)$, $B(2, 2)$, $C(3, 0)$, and $D(-1, 1)$ in the line $y = x$

3. **MULTIPLE CHOICE** The image of $A(-2, 5)$ under a reflection is $A'(2, -5)$. Which reflection or group of reflections was used? (Lesson 9-1)

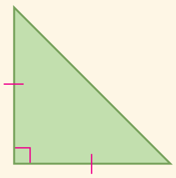
   I. reflected in the x-axis

   II. reflected in the y-axis

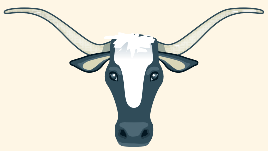
   III. reflected in the origin

   A I **B** II **C** III **D** II and III

For Exercises 4–6, identify any lines or points of symmetry each figure has. (Lesson 9-1)

4. 

5. 

6. 

Graph each figure and the image under the given translation. (Lesson 9-2)

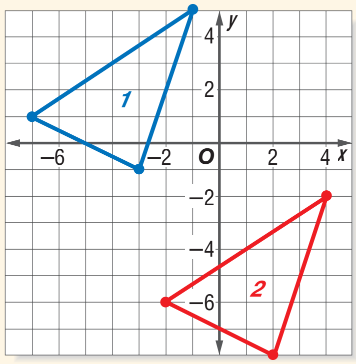
7. $\overline{PQ}$ with endpoints $P(1, -4)$ and $Q(4, -1)$ under the translation left 3 units and up 4 units
8. $\triangle KLM$ with vertices $K(-2, 0)$, $L(-4, 2)$, and $M(0, 4)$ under the translation $(x, y) \rightarrow (x + 1, y - 4)$

9. **MULTIPLE CHOICE** Triangle XYZ with vertices $X(5, 4)$, $Y(3, -1)$, and $Z(0, 2)$ is translated so that X' is at $(3, 1)$. What are the coordinates of Y' and Z'? (Lesson 9-2)

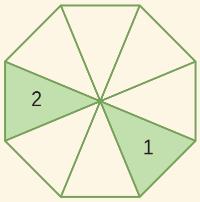
   F $Y'(5, 2)$, $Z'(2, 5)$ **H** $Y'(0, -3)$, $Z'(-3, 0)$

   G $Y'(11, 4)$, $Z'(-2, -1)$ **J** $Y'(1, -4)$, $Z'(-2, -1)$

10. Find the translation that moves $\triangle 1$ to $\triangle 2$. (Lesson 9-2)

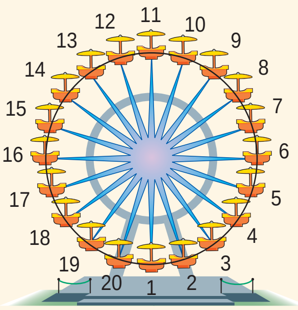


11. In the figure, describe the rotation that moves triangle 1 to triangle 2. (Lesson 9-3)



RECREATION For Exercises 12–14, use the following information. (Lesson 9-3)

A Ferris wheel's motion is an example of a rotation. The Ferris wheel shown has 20 cars.



12. Identify the order and magnitude of the symmetry of a 20-seat Ferris wheel.
13. What is the measure of the angle of rotation if seat 1 of a 20-seat Ferris wheel is moved to the seat 5 position?
14. If seat 1 of a 20-seat Ferris wheel is rotated 144°, find the original seat number of the position it now occupies.

Draw the rotation image of each figure 90° in the clockwise direction about the origin and label the vertices. (Lesson 9-3)

15. $\triangle TUV$ with vertices $T(5, 5)$, $U(7, 3)$, and $V(1, 2)$
16. $\triangle QRS$ with vertices $Q(-9, -4)$, $R(-3, -2)$, and $S(0, 0)$

9-4 Tessellations

Main Ideas

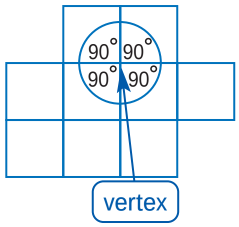
- Identify regular tessellations.
- Create tessellations with specific attributes.

New Vocabulary

tessellation
regular tessellation
uniform
semi-regular tessellation

Tessellating began as the Greeks decorated their walls, floors, and ceilings with tile mosaics. However, it was the Dutch artist M.C. Escher (1898–1972) who showed that they are not just visually appealing, but that their characteristics appear in areas such as mathematics, physics, geology, chemistry, and psychology. In the picture, figures can be reduced to basic regular polygons. Equilateral triangles and regular hexagons are prominent in the repeating patterns.

Regular Tessellations A pattern that covers a plane by transforming the same figure or set of figures so that there are no overlapping or empty spaces is called a **tessellation**.



In a tessellation, the sum of the measures of the angles of the polygons surrounding a point (at a vertex) is 360.

GEOMETRY LAB

Tessellations of Regular Polygons

MODEL AND ANALYZE

Study a set of pattern blocks to determine which shapes are regular. Make a tessellation with each type of regular polygon.

1. Which shapes in the set are regular?
2. Write an expression for the sum of the angles at each vertex of the tessellation.
3. Copy and complete the table below.

Regular Polygon	triangle	square	pentagon	hexagon	heptagon	octagon
Measure of One Interior Angle						
Does it tessellate?						

4. **MAKE A CONJECTURE** What must be true of the angle measure of a regular polygon for a regular tessellation to occur?

The tessellations you formed in the Geometry Lab are regular tessellations. A **regular tessellation** is a tessellation formed by only one type of regular polygon. In the activity, you found that if a regular polygon has an interior angle with a measure that is a factor of 360, then the polygon will tessellate the plane.

Study Tip

Look Back

To review finding the measure of an interior angle of a regular polygon, see Lesson 6-1.

EXAMPLE 1 Regular Polygons

Determine whether a regular 24-gon tessellates the plane. Explain.

Let $\angle 1$ represent one interior angle of a regular 24-gon.

$$m\angle 1 = \frac{180(n-2)}{n} \quad \text{Interior Angle Formula}$$

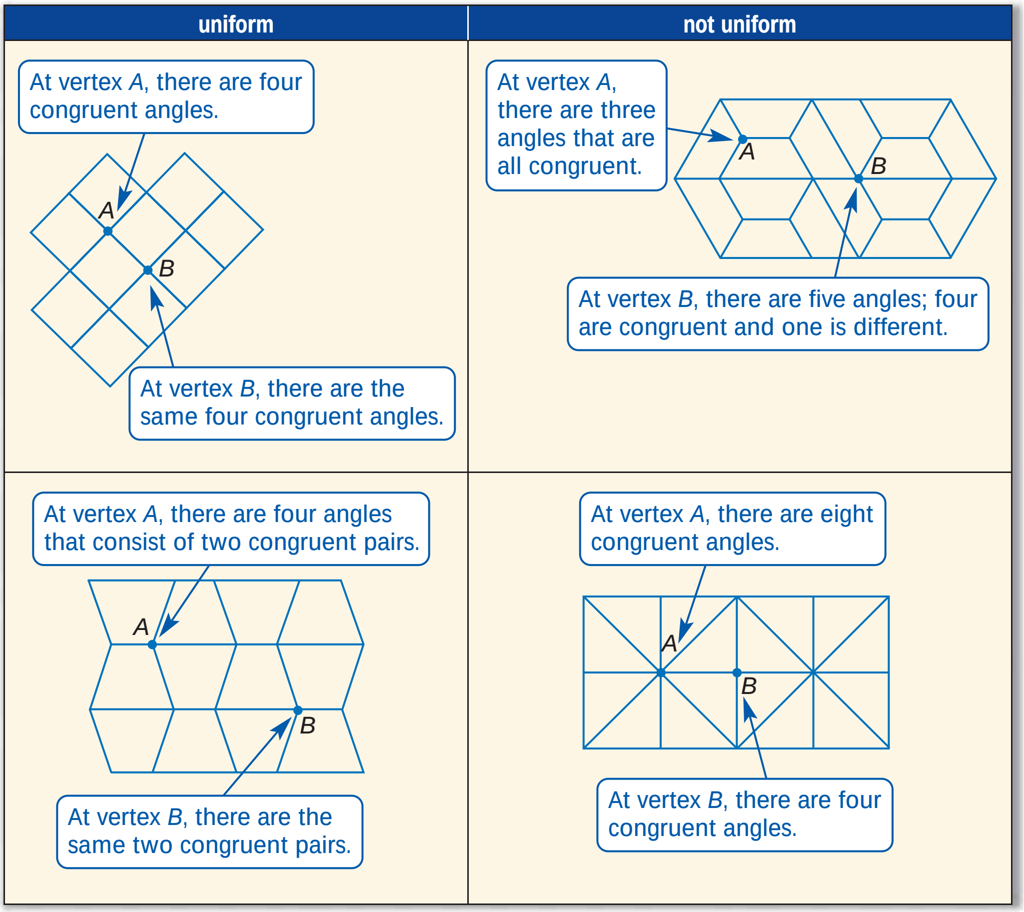
$$= \frac{180(24-2)}{24} \quad \text{Substitution}$$

$$= 165 \quad \text{Simplify.}$$

Since 165 is not a factor of 360, a 24-gon will not tessellate the plane.

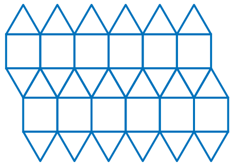
1. Determine whether a regular 18-gon tessellates the plane. Explain.

Tessellations with Specific Attributes A tessellation pattern can contain any type of polygon. Tessellations containing the same arrangement of shapes and angles at each vertex are called **uniform**.



Animation
geometryonline.com

Tessellations can be formed using more than one type of polygon. A uniform tessellation formed using two or more regular polygons is called a **semi-regular tessellation**.



EXAMPLE Semi-Regular Tessellation

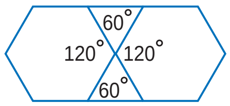
2 **Determine whether a semi-regular tessellation can be created from regular hexagons and equilateral triangles, all having sides 1 unit long.**

Study Tip

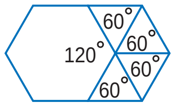
Drawing
When creating your own tessellation, it is sometimes helpful to complete one pattern piece, cut it out, and trace it for the other units.

Method 1 Make a model.
Two semi-regular models are shown. You will notice that the spaces at each vertex can be filled in with equilateral triangles. Model 1 has two hexagons and two equilateral triangles arranged in an alternating pattern around each vertex. Model 2 has one hexagon and four equilateral triangles at each vertex.

Model 1



Model 2



Method 2 Solve algebraically.
Each interior angle of a regular hexagon measures $\frac{180(6 - 2)}{6}$ or 120°.

Each angle of an equilateral triangle measures 60°. Find whole-number values for h and t so that $120h + 60t = 360$.

Let $h = 1$.		Let $h = 2$.
$120(1) + 60t = 360$	Substitution	$120(2) + 60t = 360$
$120 + 60t = 360$	Simplify.	$240 + 60t = 360$
$60t = 240$	Subtract from each side.	$60t = 120$
$t = 4$	Divide each side by 60.	$t = 2$

When $h = 1$ and $t = 4$, there is one hexagon with four equilateral triangles at each vertex. (Model 2)

When $h = 2$ and $t = 2$, there are two hexagons and two equilateral triangles. (Model 1)

Note if $h = 0$ and $t = 6$ or $h = 3$ and $t = 0$, then the tessellations are regular because there would be only one regular polygon.

2. Determine whether a semi-regular tessellation can be created from squares and equilateral triangles having sides 1 unit in length.

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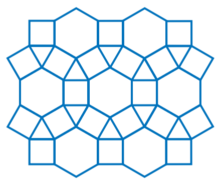
Classify Tessellation

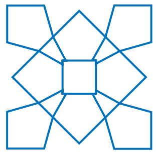
3 **FLOORING** **Tile flooring comes in many shapes and patterns. Determine whether the pattern is a tessellation. If so, describe it as *uniform, not uniform, regular,* or *semi-regular.***



The pattern is a tessellation because at the different vertices the sum of the angles is 360°.

The tessellation is uniform because at each vertex there are two squares, a triangle, and a hexagon arranged in the same order. The tessellation is also semi-regular since more than one regular polygon is used.

3. Determine whether the pattern is a tessellation. If so, describe it as *uniform*, *not uniform*, *regular*, or *semi-regular*.



CHECK Your Understanding

Example 1 (p. 520)

Determine whether each regular polygon tessellates the plane. Explain.

1. decagon

2. 30-gon

Example 2 (p. 521)

Determine whether a semi-regular tessellation can be created from each figure. Assume that each figure has side length of 1 unit.

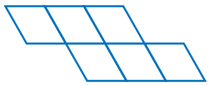
3. a regular pentagon and a triangle

4. a regular octagon and a square

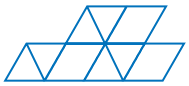
Example 3 (p. 521)

Determine whether each pattern is a tessellation. If so, describe it as *uniform, not uniform, regular,* or *semi-regular.*

5.



6.



7. QUILTING The "Postage Stamp" pattern is used in quilting. Explain why this is a tessellation and what kind.

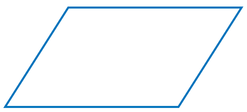
Exercises

HOMEWORK HELP

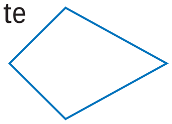
For Exercises	See Examples
8–11	3
12–17	1
18–21	2

Determine whether each polygon tessellates the plane. If so, describe the tessellation as *uniform, not uniform, regular,* or *semi-regular.*

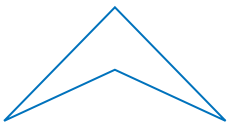
8. parallelogram



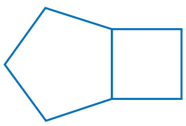
9. kite



10. quadrilateral



11. pentagon and square



Real-World Career

Bricklayer

Bricklayers arrange and secure bricks and concrete blocks to construct or repair walls, fireplaces, and other structures. They must understand how the pieces tessellate to complete the structures.

Math Online

For more information, go to geometryonline.com.

Determine whether each regular polygon tessellates the plane. Explain.

12. nonagon **13.** hexagon **14.** equilateral triangle

15. dodecagon **16.** 23-gon **17.** 36-gon

Determine whether a semi-regular tessellation can be created from each set of figures. Assume that each figure has side length of 1 unit.

18. regular octagons and nonsquare rhombi

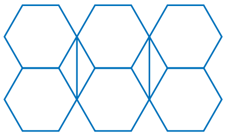
19. regular dodecagons and equilateral triangles

20. regular dodecagons, squares, and equilateral triangles

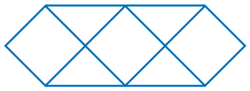
21. regular heptagons, squares, and equilateral triangles

Determine whether each pattern is a tessellation. If so, describe it as *uniform, not uniform, regular,* or *semi-regular.*

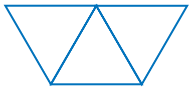
22.



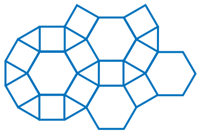
23.



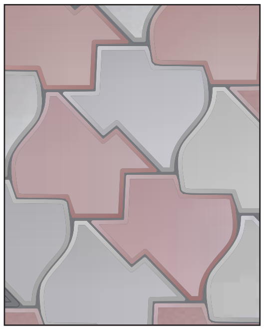
24.



25.



26. BRICKWORK A popular patio brick, these tiles can be found in the backyards of many Texan homes. Describe the tessellation shown. Is it *uniform, not uniform, regular, semi-regular,* or something else? Explain your response.



Determine whether each statement is *always, sometimes,* or *never* true. Justify your answers.

27. Any triangle will tessellate the plane.

28. Every quadrilateral will tessellate the plane.

29. Regular 16-gons will tessellate the plane.

EXTRA PRACTICE

See pages 818, 836.

Math Online

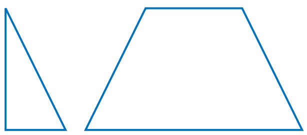
Self-Check Quiz at geometryonline.com

30. BEES A honeycomb is composed of hexagonal cells made of wax in which bees store honey. Determine whether this pattern is a tessellation. If so, describe it as *uniform, not uniform, regular,* or *semi-regular.*

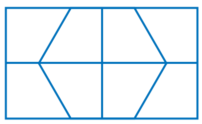
H.O.T. Problems

31. Which One Doesn't Belong? Which word doesn't belong in a group of words that could be used to describe the tessellation on a checkerboard: *regular, uniform,* or *semi-regular?*

32. OPEN ENDED Use these pattern blocks to create a tessellation.



33. REASONING Explain why the tessellation is *not* a regular tessellation.

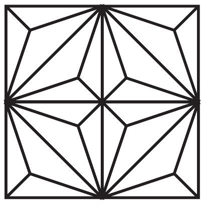


34. CHALLENGE What could be the measures of the interior angles in a pentagon that tessellate a plane? Is this tessellation regular? uniform?

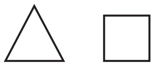
35. Writing in Math Explain how tessellations are used in art. Include an explanation of how equilateral triangles and regular hexagons form a tessellation. Also include a list of other geometric figures that can be found in the picture on page 519.

STANDARDIZED TEST PRACTICE

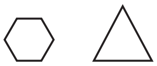
36. The tessellation shown can be made using which of the following shapes?



A 

B 

C 

D 

37. REVIEW A computer company ships computers in wooden crates that by themselves each weigh 45 pounds. If each computer weighs no more than 13 pounds, which inequality *best* describes the total weight in pounds w of a crate of computers that contains c computers?

F $c \leq 13 + 45w$

G $c \geq 13w + 45$

H $w \leq 13c + 45$

J $w \geq 13c + 45$

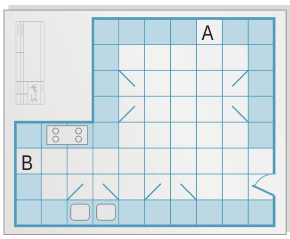
COORDINATE GEOMETRY **Draw the rotation image of each figure 90° in the given direction about the center point and label the coordinates.** (Lesson 9-3)

38. $\triangle ABC$ with $A(8, 1)$, $B(2, -6)$, and $C(-4, -2)$ counterclockwise about $P(-2, 2)$

39. $\triangle DEF$ with $D(6, 2)$, $E(6, -3)$, and $F(2, 3)$ clockwise about $P(3, -2)$

40. $\square GHIJ$ with $G(-1, 2)$, $H(-3, -3)$, $I(-5, -6)$, and $J(-3, -1)$ counterclockwise about $P(-2, -3)$

41. $\square KLMN$ with $K(-3, -5)$, $L(3, 3)$, $M(7, 0)$, and $N(1, -8)$ counterclockwise about $P(-2, 0)$



42. REMODELING The diagram at the right shows the floor plan of Justin's kitchen. Each square on the diagram represents a 3-foot by 3-foot area. While remodeling his kitchen, Justin moved his refrigerator from square A to square B. Describe the move. (Lesson 9-2)

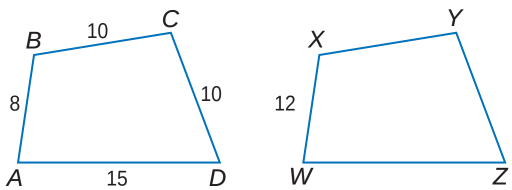
PREREQUISITE SKILL **If *ABCD* ~ *WXYZ*, find each of the following.** (Lesson 7-2)

43. scale factor of *ABCD* to *WXYZ*

44. *XY*

45. *YZ*

46. *WZ*



9-5 Dilations

Main Ideas

- Determine whether a dilation is an enlargement, a reduction, or a congruence transformation.
- Determine the scale factor for a given dilation.

New Vocabulary

dilation

similarity transformation

Have you ever tried to paste an image into an electronic document and the picture was too large? Many software programs allow you to scale the size of the picture so that you can fit it in your document. Scaling a picture is an example of a dilation.

Classify Dilations All of the transformations you have studied so far in this chapter produce images that are congruent to the original figure. A **dilation** is another type of transformation. However, it may change the size of a figure.

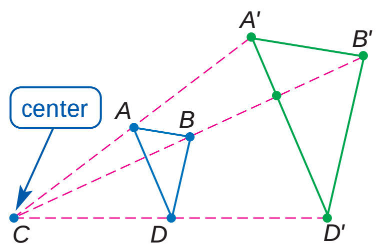
A dilation requires a center point and a scale factor. The letter r usually represents the scale factor. The figures show how dilations can result in a larger figure and a smaller figure than the original.

Study Tip

Scale Factor

When discussing dilations, scale factor has the same meaning as with proportions.

$r = 2$



Triangle $A'B'D'$ is a dilation of $\triangle ABD$.

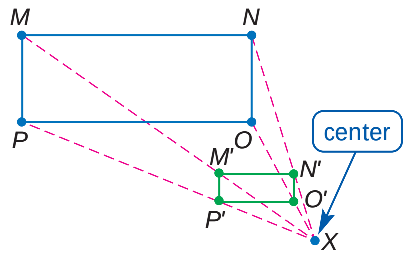
$CA' = 2(CA)$

$CB' = 2(CB)$

$CD' = 2(CD)$

$\triangle A'B'D'$ is larger than $\triangle ABD$.

$r = \frac{1}{3}$



Rectangle $M'N'O'P'$ is a dilation of rectangle $MNOP$.

$XM' = \frac{1}{3}(XM)$ $XN' = \frac{1}{3}(XN)$

$XO' = \frac{1}{3}(XO)$ $XP' = \frac{1}{3}(XP)$

Rectangle $M'N'O'P'$ is smaller than rectangle $MNOP$.

The value of r determines whether the dilation is an enlargement or a reduction.

KEY CONCEPT — Dilation

If $|r| > 1$, the dilation is an enlargement.

If $0 < |r| < 1$, the dilation is a reduction.

If $|r| = 1$, the dilation is a congruence transformation.

Study Tip

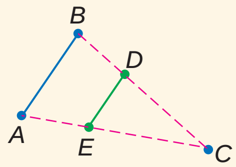
Isometry Dilation

A dilation with a scale factor of 1 produces an image that coincides with the preimage. The two are congruent.

Dilations preserve angle measure, betweenness of points, and collinearity, but do not preserve distance. That is, dilations produce similar figures. Therefore, a dilation is a **similarity transformation**. This means that in the figures on the previous page $\triangle ABD \sim \triangle A'B'D'$ and $\square MNOP \sim \square M'N'O'P'$. This implies that $\frac{A'B'}{AB} = \frac{B'D'}{BD} = \frac{A'D'}{AD}$ and $\frac{M'N'}{MN} = \frac{N'O'}{NO} = \frac{O'P'}{OP} = \frac{M'P'}{MP}$. The ratios of measures of the corresponding parts is equal to the absolute value scale factor of the dilation, $|r|$. So, $|r|$ determines the size of the image as compared to the size of the preimage.

THEOREM 9.2

If a dilation with center C and a scale factor of r transforms A to E and B to D, then $ED = |r|(AB)$.



You will prove Theorem 9.2 in Exercise 37.

EXAMPLE 1 Determine Measures Under Dilations

Find the measure of the dilation image $\overline{A'B'}$ or the preimage $\overline{AB}$ using the given scale factor.

a. $AB = 12, r = 2$

$A'B' = |r|(AB)$

$= 2(12)$ — $|r| = 2, AB = 12$

$= 24$ — Multiply.

b. $A'B' = 36, r = \frac{1}{4}$

$A'B' = |r|(AB)$

$36 = \frac{1}{4}(AB)$ — $A'B' = 36,\ |r| = \frac{1}{4}$

$144 = AB$ — Multiply each side by 4.

1A. $A'B' = 5, r = \frac{1}{4}$

1B. $AB = 3, r = 3$

When the scale factor is negative, the image falls on the opposite side of the center than the preimage.

Animation
geometryonline.com

KEY CONCEPT — Dilations

If $r > 0$, P' lies on $\overrightarrow{CP}$, and $CP' = r \cdot CP$.

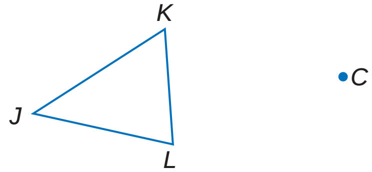
If $r < 0$, P' lies on $\overrightarrow{CP'}$ the ray opposite $\overrightarrow{CP}$, and $CP' = |r| \cdot CP$.

The center of a dilation is always its own image.

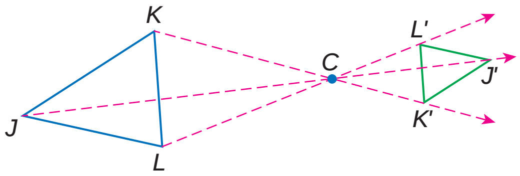
EXAMPLE 2 Draw a Dilation

Draw the dilation image of $\triangle JKL$ with center C and $r = -\frac{1}{2}$.

Since $0 < |r| < 1$, the dilation is a reduction of $\triangle JKL$.

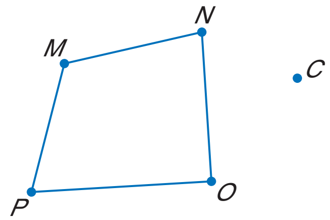


Draw $\overline{CJ}$, $\overline{CK}$, and $\overline{CL}$. Since r is negative, J', K', and L' will lie on $\overrightarrow{CJ'}$, $\overrightarrow{CK'}$, and $\overrightarrow{CL'}$, respectively. Locate J', K', and L' so that $CJ' = \frac{1}{2}(CJ)$, $CK' = \frac{1}{2}(CK)$, and $CL' = \frac{1}{2}(CL)$.



Draw $\triangle J'K'L'$.

2. Draw the dilation image of quadrilateral *MNOP* with center *C* and $r = \frac{3}{4}$.



In the coordinate plane, you can use the scale factor to determine the coordinates of the image of dilations centered at the origin.

THEOREM 9.3

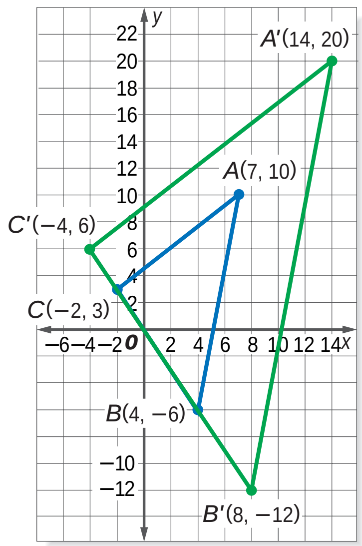
If $P(x, y)$ is the preimage of a dilation centered at the origin with a scale factor r, then the image is $P'(rx, ry)$.

You will prove Theorem 9.3 in Exercise 38.

EXAMPLE Dilations in the Coordinate Plane

3 **COORDINATE GEOMETRY** **Triangle *ABC* has vertices $A(7, 10)$, $B(4, -6)$, and $C(-2, 3)$. Find the image of $\triangle ABC$ after a dilation centered at the origin with a scale factor of 2. Sketch the preimage and the image.**

Preimage (x, y)	Image $(2x, 2y)$
$A(7, 10)$	$A'(14, 20)$
$B(4, -6)$	$B'(8, -12)$
$C(-2, 3)$	$C'(-4, 6)$



3. Quadrilateral *DEFG* has vertices $D(-1, 3)$, $E(2, 0)$, $F(-2, -1)$, and $G(-3, 1)$. Find the image of quadrilateral *DEFG* after a dilation centered at the origin with a scale factor of $\frac{3}{2}$. Sketch the preimage and the image.

Identify the Scale Factor In Chapter 7, you found scale factors of similar figures. If you know the measurement of a figure and its dilated image, you can determine the scale factor.

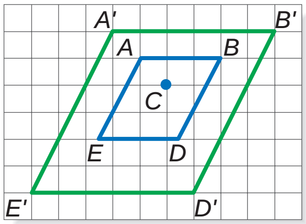
EXAMPLE Identify Scale Factor

4 **Determine the scale factor for each dilation with center *C*. Then determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*.**

Study Tip

Look Back
To review **scale factor**, see Lesson 7-2.

a.



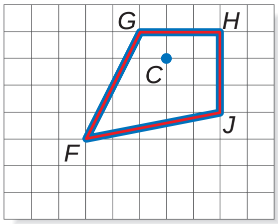
$$\text{scale factor} = \frac{\text{image length}}{\text{preimage length}}$$

$$= \frac{6 \text{ units}}{3 \text{ units}}$$ ← image length, ← preimage length

$= 2$ Simplify.

Since the scale factor is greater than 1, the dilation is an enlargement.

b.



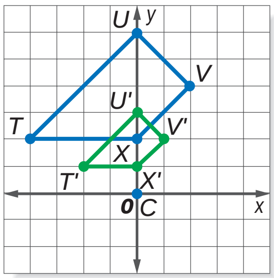
$$\text{scale factor} = \frac{\text{image length}}{\text{preimage length}}$$

$$= \frac{4 \text{ units}}{4 \text{ units}}$$ ← image length, ← preimage length

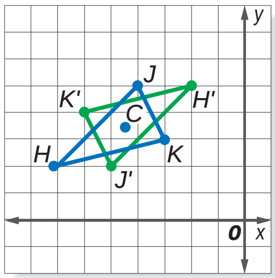
$= 1$ Simplify.

Since the scale factor is 1, the dilation is a congruence transformation.

4A.



4B.



Scale Drawing

5 **ART Jacob wants to make a scale drawing of a painting in an art museum. The painting is 4 feet wide and 8 feet long. Jacob decides on a dilation reduction factor of $\frac{1}{6}$. What size paper should he use, $8\frac{1}{2}$ inches by 11 inches, 11 inches by 14 inches, or 11 inches by 17 inches?**

The painting's dimensions are given in feet, and the paper choices are in inches. You need to convert from feet to inches in the problem.

Step 1 Convert feet to inches. 4 feet = 4(12) or 48 inches

8 feet = 8(12) or 96 inches

Step 2 Use the scale factor to find the image dimensions.

$w = \frac{1}{6}(48)$ or 8 $\qquad \ell = \frac{1}{6}(96)$ or 16

Step 3 The dimensions are 8 inches by 16 inches. He should use an 11-inch by 17-inch piece of paper to make his drawing.

5. Jamie is using a photo editing program to reduce posters that are 1 foot by 1.5 feet to 4 inch by 6 inch. What scale factor did he use?

Online Personal Tutor at geometryonline.com

CHECK Your Understanding

Example 1 (p. 526)

Find the measure of the dilation image $\overline{A'B'}$ or the preimage of $\overline{AB}$ using the given scale factor.

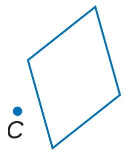
1. $AB = 3, r = 4$

2. $A'B' = 8, r = -\frac{2}{5}$

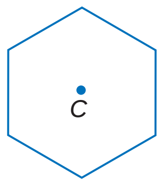
Example 2 (p. 526)

Draw the dilation image of each figure with center *C* and the given scale factor.

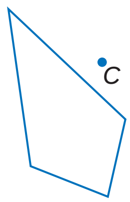
3. $r = 4$



4. $r = \frac{1}{5}$



5. $r = -2$



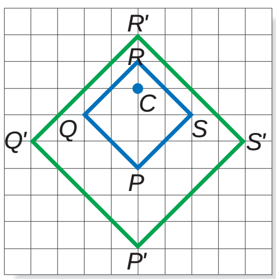
Example 3 (p. 527)

6. $\overline{PQ}$ has endpoints $P(9, 0)$ and $Q(0, 6)$. Find the image of $\overline{PQ}$ after a dilation centered at the origin with a scale factor $r = \frac{1}{3}$. Sketch the preimage and the image.

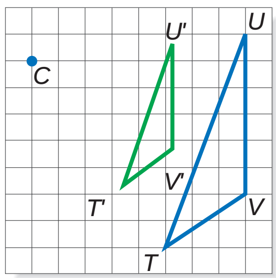
Example 4 (p. 528)

Determine the scale factor used for each dilation with center *C*. Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*.

7.



8.



Example 5 (p. 528-529)

9. GARDENING Alexis made a scale drawing of the plan for her spring garden. It will be a rectangle measuring 18 feet by 12 feet. On the drawing, it measures 8 inches on the longer sides. What is the measure, in inches, of the shorter sides?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
10–15	1
16–19	3
20–25	2
26–31	4
48–50	5

Find the measure of the dilation image or the preimage of $\overline{ST}$ using the given scale factor.

10. $ST = 6, r = -1$

11. $ST = \frac{4}{5}, r = \frac{3}{4}$

12. $S'T' = 12, r = \frac{2}{3}$

13. $S'T' = \frac{12}{5}, r = \frac{-3}{5}$

14. $ST = 32, r = \frac{-5}{4}$

15. $ST = 2.25, r = 0.4$

COORDINATE GEOMETRY **Find the image of each polygon, given the vertices, after a dilation centered at the origin with a scale factor of 2. Then graph a dilation centered at the origin with a scale factor of $\frac{1}{2}$.**

16. $F(3, 4), G(6, 10), H(-3, 5)$

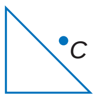
17. $X(1, -2), Y(4, -3), Z(6, -1)$

18. $P(1, 2), Q(3, 3), R(3, 5), S(1, 4)$

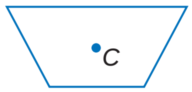
19. $K(4, 2), L(-4, 6), M(-6, -8), N(6, -10)$

Draw the dilation image of each figure with center *C* and given scale factor.

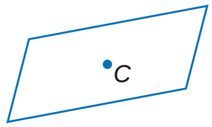
20. $r = 3$



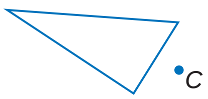
21. $r = 2$



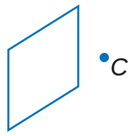
22. $r = \frac{1}{2}$



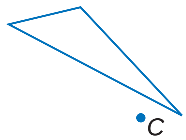
23. $r = \frac{2}{5}$



24. $r = -1$

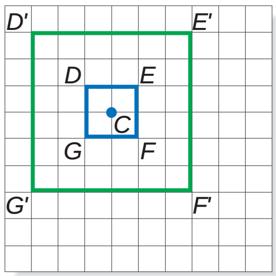


25. $r = -\frac{1}{4}$

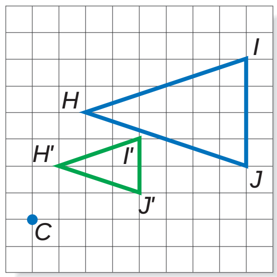


Determine the scale factor for each dilation with center *C*. Determine whether the dilation is an *enlargement, reduction,* or *congruence transformation*.

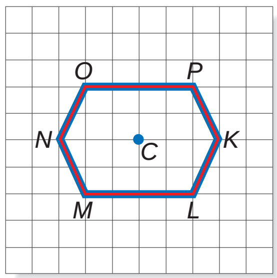
26.



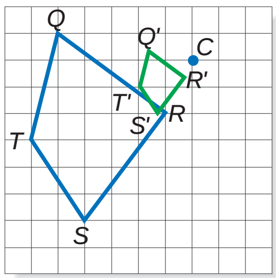
27.



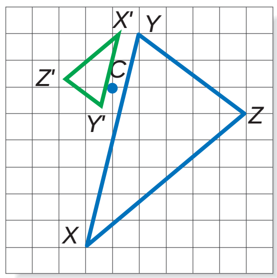
28.



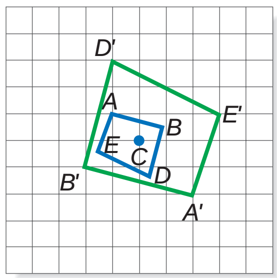
29.



30.



31.



PHOTOCOPY **For Exercises 32 and 33, use the following information.**

A 10-inch by 14-inch rectangular design is being reduced on a photocopier by a factor of 75%.

32. What are the new dimensions of the design?

33. How has the area of the preimage changed?

Real-World Link

The SR-71 Blackbird is 107 feet 5 inches long with a wingspan of 55 feet 7 inches and can fly at speeds over 2200 miles per hour. It can fly nonstop from Los Angeles to Washington, D.C., in just over an hour, while a standard commercial jet takes about five hours to complete the trip.

Source: NASA

34. MODELS Etay is building a model of the SR-71 Blackbird. If the wingspan of his model is 14 inches, what is the scale factor of the model?

35. DESKTOP PUBLISHING Grace is creating a template for the class newsletter. She has a photograph that is 10 centimeters by 12 centimeters, but the maximum space available for the photograph is 6 centimeters by 8 centimeters. She wants the photograph to be as large as possible on the page. When she uses a scanner to save the photograph, at what percent of the original photograph's size should she save the image file?

36. COORDINATE GEOMETRY Triangle *ABC* has vertices $A(12, 4)$, $B(4, 8)$, and $C(8, -8)$. After two successive dilations centered at the origin with the same scale factor, the final image has vertices $A''(3, 1)$, $B''(1, 2)$, and $C''(2, -2)$. Determine the scale factor r of the dilation from $\triangle ABC$ to $\triangle A''B''C''$.

PROOF Write a paragraph proof of each of the following.

37. Theorem 9.2

38. Theorem 9.3

DIGITAL PHOTOGRAPHY For Exercises 39–41, use the following information.

Dinah is editing a digital photograph that is 640 pixels wide and 480 pixels high on her monitor.

39. If Dinah zooms the image on her monitor 150%, what are the dimensions of the image?

40. Suppose that Dinah wishes to use the photograph on a Web page and wants the image to be 32 pixels wide. What scale factor should she use?

41. Dinah resizes the photograph so that it is 600 pixels high. What scale factor did she use?

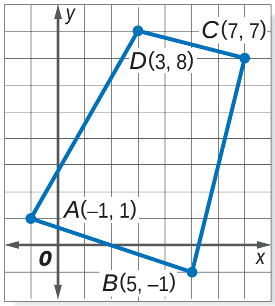
EXTRA PRACTICE

See pages 818, 836.

Math Online

Self-Check Quiz at geometryonline.com

For Exercises 42–44, use quadrilateral *ABCD*.



42. Find the perimeter of quadrilateral *ABCD*.

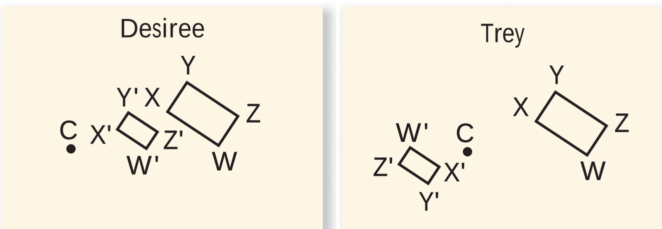
43. Graph the image of quadrilateral *ABCD* after a dilation centered at the origin with scale factor -2.

44. Find the perimeter of quadrilateral *A'B'C'D'* and compare it to the perimeter of quadrilateral *ABCD*.

H.O.T. Problems

45. REASONING It is *sometimes*, *always*, or *never* true that dilations preserve a figure's angle measures? Explain.

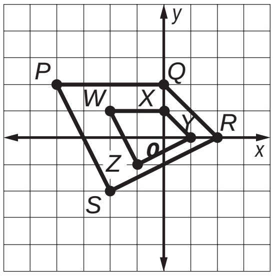
46. FIND THE ERROR Desiree and Trey are trying to describe the effect of a negative *r* value for a dilation of quadrilateral *WXYZ*. Who is correct? Explain your reasoning.



47. Writing in Math Use the information about computers on page 525 to explain how dilations can be used when working with computers. In addition to other examples, include an explanation of how a "cut and paste" in word processing may be an example of a dilation.

STANDARDIZED TEST PRACTICE

48. Quadrilateral *PQRS* was dilated to form quadrilateral *WXYZ*.

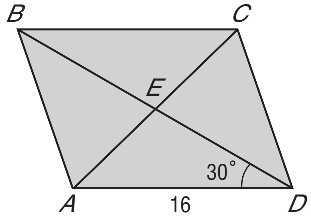


Which number *best* represents the scale factor used to change quadrilateral *PQRS* into quadrilateral *WXYZ*?

A -2 **C** $-\frac{1}{2}$

B 2 **D** $\frac{1}{2}$

49. If *ABCD* is a rhombus, what is the area in square units of $\triangle AED$?



F $16\sqrt{2}$

G 64

H $32\sqrt{3}$

J $64\sqrt{3}$

50. REVIEW How many ounces of pure water must a pharmacist add to 50 ounces of a 15% saline solution to make a saline solution that is 10% salt?

A 25 ounces **C** 15 ounces

B 20 ounces **D** 5 ounces

Determine whether a semi-regular tessellation can be created from each figure. Assume that each figure is regular and has a side length of 1 unit. (Lesson 9-4)

51. a triangle and a pentagon

52. an octagon and a hexagon

53. a square and a triangle

54. a hexagon and a dodecagon

COORDINATE GEOMETRY Draw the rotation image of each figure 90° in the given direction about the center point and label the coordinates. (Lesson 9-3)

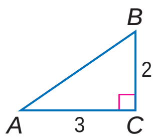
55. $\triangle ABC$ with $A(7, -1)$, $B(5, 0)$, and $C(1, 6)$ counterclockwise about $P(-1, 4)$

56. ▱$DEFG$ with $D(-4, -2)$, $E(-3, 3)$, $F(3, 1)$, and $G(2, -4)$ clockwise about $P(-4, -6)$

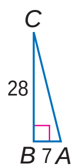
57. CONSTRUCTION The Vanamans are building an addition to their house. Ms. Vanaman is cutting an opening for a new window. If she measures to see that the opposite sides are the same length and that the diagonal measures are the same, can Ms. Vanaman be sure that the window opening is rectangular? Explain. (Lesson 6-4)

PREREQUISITE SKILL Find $m\angle A$ to the nearest tenth. (Lesson 8-4)

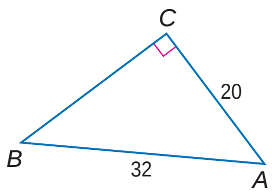
58.



59.



60.



READING MATH

Scalars and Vectors

Many quantities in nature and in everyday life can be thought of as vectors. The science of physics involves many vector quantities. In reading about applications of mathematics, ask yourself whether the quantities involve only magnitude or both magnitude and direction. The first kind of quantity is called **scalar**. The second kind is a **vector**.

A scalar is a measurement that involves only a number. Scalars are used to measure things, such as size, length, or speed. A vector is a measurement that involves a number and a direction. Vectors are used to measure things like force or acceleration.

Example of a Scalar

A Chihuahua weighs an average of 4 pounds.

Example of a Vector

After a free kick, a soccer ball travels 40 mph east.

Reading to Learn

Classify each of the following. Write *scalar* or *vector*.

1. the mass of a book
2. a car traveling north at 55 miles per hour
3. a balloon rising 24 feet per minute
4. the length of a gymnasium
5. a room temperature of 22 degrees Celsius
6. a force of 35 pounds acting on an object at a 20° angle
7. a west wind of 15 mph
8. the batting average of a baseball player
9. the force of Earth's gravity acting on a moving satellite
10. the area of a CD rotating in a CD player
11. a rock falling at 10 mph
12. the length of a vector in the coordinate plane

(l)Dave King/Dorling Kindersley/Getty Images, (r)Joe McBride/CORBIS

9-6 Vectors

Main Ideas

- Find magnitudes and directions of vectors.
- Perform translations with vectors.

New Vocabulary

vector
magnitude
direction
standard position
component form
equal vectors
parallel vectors
resultant
scalar
scalar multiplication

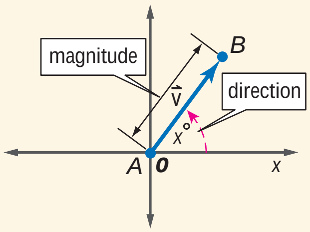
While the concept of vectors can be traced to the philosopher and mathematician Bernard Bolzano, it was Edwin Wilson who researched many of their practical applications in aviation. Commercial pilots must submit flight plans prior to departure. These flight plans take into account the speed and direction of the plane as well as the speed and direction of the wind.

Magnitude and Direction The speed and direction of a plane and the wind can be represented by vectors.

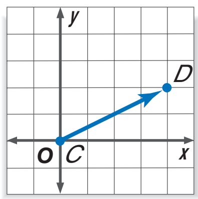
KEY CONCEPT — Vectors

Words A **vector** is a quantity that has both **magnitude**, or length, and **direction**, and is represented by a directed segment.

Symbols $\vec{v}$ $\overrightarrow{AB}$, where A is the initial point and B is the endpoint



A vector in **standard position** has its initial point at the origin. In the diagram, $\overrightarrow{CD}$ is in standard position and can be represented by the ordered pair $\langle 4, 2\rangle$.



A vector can also be drawn anywhere in the coordinate plane. To write such a vector as an ordered pair, find the change in the x-values and the change in y-values, $\langle$change in x, change in $y\rangle$, from the tip to the tail of the directed segment. The ordered pair representation is the **component form** of the vector.

EXAMPLE 1 Write Vectors in Component Form

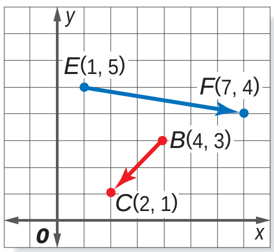
Write the component form of $\overrightarrow{EF}$.

Find the change in x-values and the corresponding change in y-values.

$\overrightarrow{EF} = \langle x_2 - x_1, y_2 - y_1\rangle$ Component form of vector

$= \langle 7 - 1, 4 - 5\rangle$ $x_1 = 1, y_1 = 5, x_2 = 7, y_2 = 4$

$= \langle 6, -1\rangle$ Simplify.



CHECK Your Progress

1. Write the component form of $\overrightarrow{BC}$.

Common Misconception

The notation for a vector from C to D, $\overrightarrow{CD}$, is similar to the notation for a ray from C to D, $\overrightarrow{CD}$. Be sure to use the correct arrow above the letters when writing each.

The **Distance Formula** can be used to find the magnitude of a vector. The symbol for the magnitude of $\overrightarrow{AB}$ is $|\overrightarrow{AB}|$. **The *direction* of a vector** is the measure of the angle that the vector forms with the positive x-axis or any other horizontal line. You can use the trigonometric ratios to find the direction of a vector.

EXAMPLE Magnitude and Direction of a Vector

2 **Find the magnitude and direction of $\overrightarrow{PQ}$ for $P(3, 8)$ and $Q(-4, 2)$.**

Find the magnitude.

$$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$= \sqrt{(-4 - 3)^2 + (2 - 8)^2} \quad x_1 = 3, y_1 = 8, x_2 = -4, y_2 = 2$$

$$= \sqrt{85} \quad \text{Simplify.}$$

$$\approx 9.2 \quad \text{Use a calculator.}$$

Graph $\overrightarrow{PQ}$ to determine how to find the direction. Draw a right triangle that has $\overrightarrow{PQ}$ as its hypotenuse and an acute angle at P.

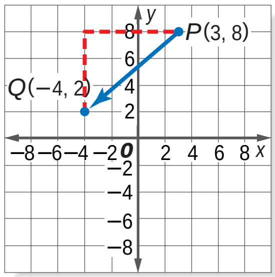
$$\tan P = \frac{y_2 - y_1}{x_2 - x_1} \quad \tan = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$= \frac{2 - 8}{-4 - 3} \quad \text{Substitution}$$

$$= \frac{6}{7} \quad \text{Simplify.}$$

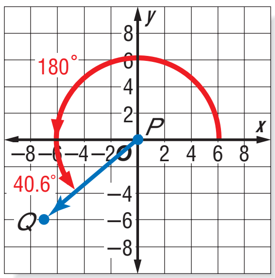
$$m\angle P = \tan^{-1}\frac{6}{7}$$

$$\approx 40.6 \quad \text{Use a calculator.}$$



A vector in standard position that is equal to $\overrightarrow{PQ}$ lies in the third quadrant and forms an angle with the negative x-axis that has a measure equal to $m\angle P$. The x-axis is a straight angle with a measure that is 180. So, the direction of $\overrightarrow{PQ}$ is $m\angle P + 180$ or about 220.6°.

Thus, $\overrightarrow{PQ}$ has a magnitude of about 9.2 units and a direction of about 220.6°.



2. Find the magnitude and direction of $\overrightarrow{RT}$ for $R(3, 1)$ and $T(-1, 3)$.

Using Slope

Even though slope is not associated with vectors, you can use the concept of slope to determine if vectors are parallel before actually comparing their directions.

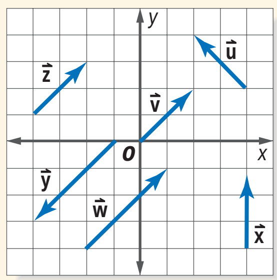
KEY CONCEPT

Equal Vectors Two vectors are equal if and only if they have the same magnitude and direction.

Example $\vec{v} = \vec{z}$ **Nonexample** $\vec{v} \neq \vec{u}$

Parallel Vectors Two vectors are parallel if and only if they have the same or opposite direction.

Example $\vec{v} \parallel \vec{w}$ **Nonexample** $\vec{v} \nparallel \vec{x}$



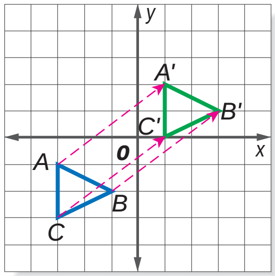
Translations with Vectors

Vectors can be used to describe translations.

EXAMPLE 3 Translations with Vectors

Graph the image of $\triangle ABC$ with vertices $A(-3, -1)$, $B(-1, -2)$, and $C(-3, -3)$ under the translation $\vec{v} = \langle 4, 3 \rangle$.

First, graph $\triangle ABC$. Next, translate each vertex by $\vec{v}$, 4 units right and 3 units up. Connect the vertices to form $\triangle A'B'C'$.



3. Graph the image of $\square BCDF$ with vertices $B(2, 0)$, $C(3, 2)$, $D(4, 3)$, and $F(4, 0)$ under the translation $\vec{v} = \langle 0, -2 \rangle$.

Vectors can be combined to perform a composition of translations by adding the vectors. The sum of two vectors is called the **resultant**.

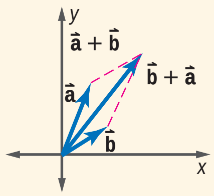
KEY CONCEPT — Vector Addition

Words To add two vectors, add the corresponding components.

Symbols If $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$,

then $\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2 \rangle$,

and $\vec{b} + \vec{a} = \langle b_1 + a_1, b_2 + a_2 \rangle$.



EXAMPLE 4 Add Vectors

Graph the image of $\square QRST$ with vertices $Q(-4, 4)$, $R(-1, 4)$, $S(-2, 2)$, and $T(-5, 2)$ under the translation $\vec{m} = \langle 5, -1 \rangle$ and $\vec{n} = \langle -2, -6 \rangle$.

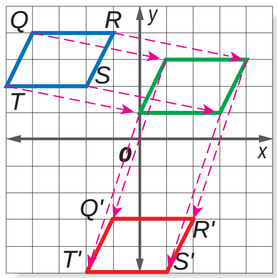
Method 1 Translate two times.

Translate $\square QRST$ by $\vec{m}$. Then translate the image of $\square QRST$ by $\vec{n}$.

Translate each vertex 5 units right and 1 unit down.

Then translate each vertex 2 units left and 6 units down.

Label the image $\square Q'R'S'T'$.

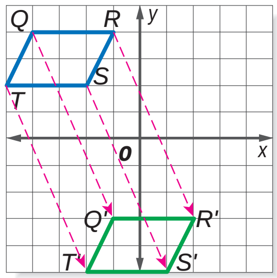


Method 2 Find the resultant, and then translate.

Add $\vec{m}$ and $\vec{n}$.

$$\vec{m} + \vec{n} = \langle 5 - 2, -1 - 6 \rangle$$
$$= \langle 3, -7 \rangle$$

Translate each vertex 3 units right and 7 units down.
Notice that the image is the same for either method.



4. Graph the image of $\square FGHJ$ with vertices $F(2, 0)$, $G(3, 2)$, $H(4, 3)$, and $J(4, 0)$ under the translations $\vec{v} = \langle 0, -2 \rangle$ and $\vec{t} = \langle 3, -1 \rangle$.

Interactive Lab
geometryonline.com

GEOMETRY LAB

Comparing Magnitude and Components of Vectors

MODEL AND ANALYZE

- Draw $\vec{a}$ in standard position.
- Draw $\vec{b}$ in standard position with the same direction as $\vec{a}$, but with a magnitude twice the magnitude of $\vec{a}$.

1. Write $\vec{a}$ and $\vec{b}$ in component form.

2. What do you notice about the components of $\vec{a}$ and $\vec{b}$?

3. Draw $\vec{b}$ so that its magnitude is three times that of $\vec{a}$. How do the components of $\vec{a}$ and $\vec{b}$ compare?

MAKE A CONJECTURE

4. Describe the vector magnitude and direction of a vector $\langle x, y\rangle$ after the components are multiplied by n.

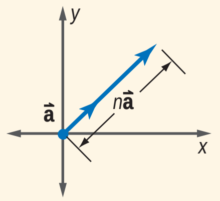
In the Geometry Lab, you found that a vector can be multiplied by a positive constant, called a **scalar**, that will change the magnitude of the vector, but not affect its direction. Multiplying a vector by a positive scalar is called **scalar multiplication**.

KEY CONCEPT — *Scalar Multiplication*

Words To multiply a vector by a scalar, multiply each component by the scalar.

Symbols If $\vec{a} = \langle a_1, a_2\rangle$ has a magnitude $|\vec{a}|$ and direction d, then $n\vec{a} = n\langle a_1, a_2\rangle = \langle na_1, na_2\rangle$, where n is a positive real number, the magnitude is $|n\vec{a}|$, and its direction is d.

Model



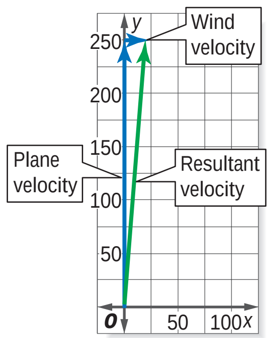
Real-World Link

A tailwind will allow a plane to arrive faster than anticipated without the tailwind. A headwind will cause the plane to take more time to travel than without the headwind.

EXAMPLE 5 Solve Problems Using Vectors

AVIATION Suppose a pilot begins a flight along a path due north flying at 250 miles per hour. If the wind is blowing due east at 20 miles per hour, what is the resultant velocity and direction of the plane?

- The initial path of the plane is due north, so a vector representing the path lies on the positive y-axis 250 units long.
- The wind is blowing due east, so a vector representing the wind will be parallel to the positive x-axis 20 units long.
- The resultant path can be represented by a vector from the initial point of the vector representing the plane to the terminal point of the vector representing the wind.



(continued on the next page)

Use the Pythagorean Theorem.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 250^2 + 20^2$ $a = 250, b = 20$

$c^2 = 62{,}900$ Simplify.

$c = \sqrt{62{,}900}$ Take the square root of each side.

$c \approx 250.8$

The resultant speed of the plane is about 250.8 miles per hour.

Reading Math

Symbols The Greek letter theta, θ, is used to represent the unknown measure of an angle.

Use the tangent ratio to find the direction of the plane.

$\tan \theta = \frac{20}{250}$ side opposite $= 20$, side adjacent $= 250$

$\theta = \tan^{-1} \frac{20}{250}$ Solve for θ.

$\theta \approx 4.6$ Use a calculator.

The resultant direction of the plane is about 4.6° east of due north. Therefore, the resultant vector is 250.8 miles per hour at 4.6° east of due north.

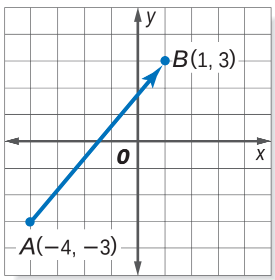
5. If the wind velocity doubles, what is the resultant path and velocity of the plane?

Online **Personal Tutor at** geometryonline.com

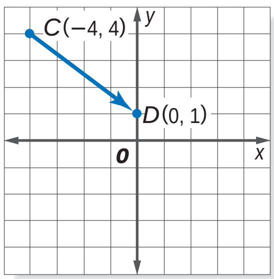
Check Your Understanding

Example 1 (p. 534) **Write the component form of each vector.**

1.



2.



Example 2 (p. 535) **Find the magnitude and direction of $\overrightarrow{AB}$ for the given coordinates.**

3. $A(2, 7)$, $B(-3, 3)$

4. $A(-6, 0)$, $B(-12, -4)$

Graph the image of each figure under a translation by the given vector.

Example 3 (p. 536)

5. $\triangle JKL$ with vertices $J(2, -1)$, $K(-7, -2)$, $L(-2, 8)$; $\vec{t} = \langle -1, 9 \rangle$

6. trapezoid $PQRS$ with vertices $P(1, 2)$, $Q(7, 3)$, $R(15, 1)$, $S(3, -1)$; $\vec{u} = \langle 3, -3 \rangle$

Example 4 (p. 536)

7. Graph the image of $\square WXYZ$ with vertices $W(6, -6)$, $X(3, -8)$, $Y(-4, -4)$, and $Z(-1, -2)$ under the translation $\vec{e} = \langle -1, 6 \rangle$ and $\vec{f} = \langle 8, -5 \rangle$.

Example 5 (p. 537)

8. **BOATING** Raphael sails his boat due east at a rate of 10 knots. If there is a current of 3 knots moving 30° south of east, what is the resultant speed and direction of the boat?

Find the magnitude and direction of each resultant for the given vectors.

9. $\vec{g} = \langle 4, 0 \rangle$, $\vec{h} = \langle 0, 6 \rangle$

10. $\vec{t} = \langle 0, -9 \rangle$, $\vec{u} = \langle 12, -9 \rangle$

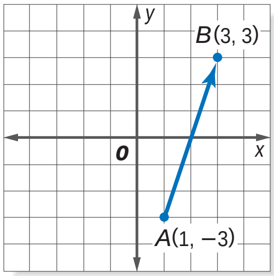
Exercises

HOMEWORK HELP

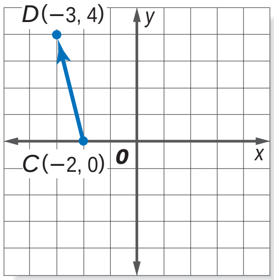
For Exercises	See Examples
11–16	1
17–22	2
23–28	3
29–32	4
33, 34	5

Write the component form of each vector.

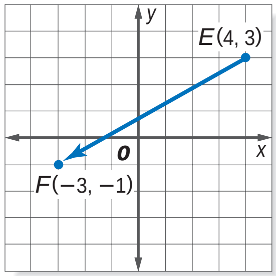
11.



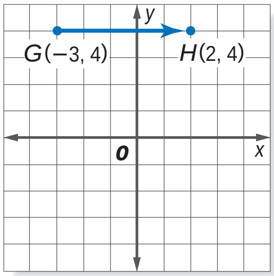
12.



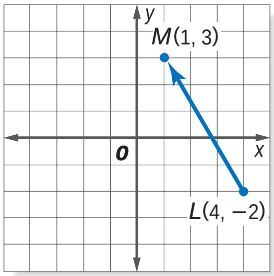
13.



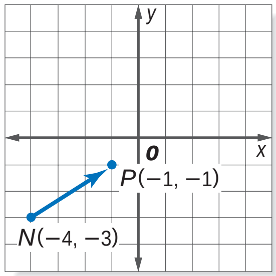
14.



15.



16.



Find the magnitude and direction of $\overrightarrow{MN}$ for the given coordinates.

17. $M(-3, 3), N(-9, 9)$ **18.** $M(8, 1), N(2, 5)$ **19.** $M(0, 2), N(-12, -2)$

20. $M(-1, 7), N(6, -8)$ **21.** $M(-1, 10), N(1, -12)$ **22.** $M(-4, 0), N(-6, -4)$

Graph the image of each figure under a translation by the given vector.

23. $\triangle ABC$ with vertices $A(3, 6)$, $B(3, -7)$, $C(-6, 1)$; $\vec{\mathbf{a}} = \langle 0, -6 \rangle$

24. $\triangle DEF$ with vertices $D(-12, 6)$, $E(7, 6)$, $F(7, -3)$; $\vec{\mathbf{b}} = \langle -3, -9 \rangle$

25. square $GHIJ$ with vertices $G(-1, 0)$, $H(-6, -3)$, $I(-9, 2)$, $J(-4, 5)$; $\vec{\mathbf{c}} = \langle 3, -8 \rangle$

26. quadrilateral $KLMN$ with vertices $K(0, 8)$, $L(4, 6)$, $M(3, -3)$, $N(-4, 8)$; $\vec{\mathbf{x}} = \langle -10, 2 \rangle$

27. pentagon $OPQRS$ with vertices $O(5, 3)$, $P(5, -3)$, $Q(0, -4)$, $R(-5, 0)$, $S(0, 4)$; $\vec{\mathbf{y}} = \langle -5, 11 \rangle$

28. hexagon $TUVWXY$ with vertices $T(4, -2)$, $U(3, 3)$, $V(6, 4)$, $W(9, 3)$, $X(8, -2)$, $Y(6, -5)$; $\vec{\mathbf{z}} = \langle -18, 12 \rangle$

Graph the image of each figure under a translation by the given vectors.

29. $\square ABCD$ with vertices $A(-1, -6)$, $B(4, -8)$, $C(-3, -11)$, $D(-8, -9)$; $\vec{\mathbf{p}} = \langle 11, 6 \rangle$, $\vec{\mathbf{q}} = \langle -9, -3 \rangle$

30. $\triangle XYZ$ with vertices $X(3, -5)$, $Y(9, 4)$, $Z(12, -2)$; $\vec{\mathbf{p}} = \langle 2, 2 \rangle$, $\vec{\mathbf{q}} = \langle -4, -7 \rangle$

31. quadrilateral $EFGH$ with vertices $E(-7, -2)$, $F(-3, 8)$, $G(4, 15)$, $H(5, -1)$; $\vec{\mathbf{p}} = \langle -6, 10 \rangle$, $\vec{\mathbf{q}} = \langle 1, -8 \rangle$

32. pentagon $STUVW$ with vertices $S(1, 4)$, $T(3, 8)$, $U(6, 8)$, $V(6, 6)$, $W(4, 4)$; $\vec{\mathbf{p}} = \langle -4, 5 \rangle$, $\vec{\mathbf{q}} = \langle 12, 11 \rangle$

Real-World Link

The Congo River is one of the fastest rivers in the world. It has no dry season because it has tributaries both north and south of the Equator. The river flows so quickly that it doesn't form a delta where it ends in the Atlantic like most rivers do when they enter an ocean.

Source: *Compton's Encyclopedia*

33. SHIPPING A freighter has to go around an oil spill in the Pacific Ocean. The captain sails due east for 35 miles. Then he turns the ship and heads due south for 28 miles. What is the distance and direction of the ship from its original point of course correction?

34. RIVERS Suppose a section of the Minnesota River has a current of 2 miles per hour. If a swimmer can swim at a rate of 4.5 miles per hour, how does the current in the Minnesota River affect the speed and direction of the swimmer as she tries to swim directly across the river?

Find the magnitude and direction of $\overrightarrow{CD}$ for the given coordinates.

35. $C(4, 2), D(9, 2)$
36. $C(-2, 1), D(2, 5)$
37. $C(-5, 10), D(-3, 6)$
38. $C(0, -7), D(-2, -4)$
39. $C(-8, -7), D(6, 0)$
40. $C(10, -3), D(-2, -2)$

41. What is the magnitude and direction of $\vec{t} = \langle 7, 24 \rangle$?

42. Find the magnitude and direction of $\vec{u} = \langle -12, 15 \rangle$.

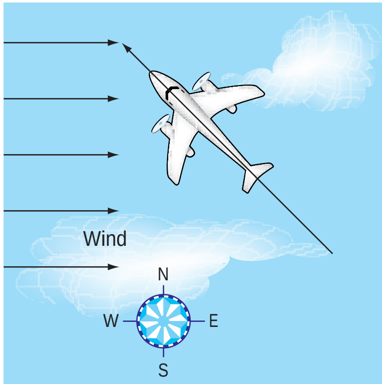
43. What is the magnitude and direction of $\vec{v} = \langle -25, -20 \rangle$?

44. Find the magnitude and direction of $\vec{w} = \langle 36, -15 \rangle$.

Find the magnitude and direction of each resultant for the given vectors.

45. $\vec{a} = \langle 5, 0 \rangle, \vec{b} = \langle 0, 12 \rangle$
46. $\vec{c} = \langle 0, -8 \rangle, \vec{d} = \langle -8, 0 \rangle$
47. $\vec{e} = \langle -4, 0 \rangle, \vec{f} = \langle 7, -4 \rangle$
48. $\vec{u} = \langle 12, 6 \rangle, \vec{v} = \langle 0, 6 \rangle$
49. $\vec{w} = \langle 5, 6 \rangle, \vec{x} = \langle -1, -4 \rangle$
50. $\vec{y} = \langle 9, -10 \rangle, \vec{z} = \langle -10, -2 \rangle$

AVIATION For Exercises 51–53, use the following information.
A jet is flying northwest, and its velocity is represented by $\langle -450, 450 \rangle$ miles per hour. The wind is from the west, and its velocity is represented by $\langle 100, 0 \rangle$ miles per hour.



51. Find the resultant vector for the jet in component form.

52. Find the magnitude of the resultant.

53. Find the direction of the resultant.

EXTRA PRACTICE
See pages 819, 836.
Math Online
Self-Check Quiz at geometryonline.com

54. BIKING Shanté is riding her bike south at a velocity of 13 miles per hour. The wind is blowing 2 miles per hour in the opposite direction. What is the resultant velocity and direction of Shanté's bike?

H.O.T. Problems

55. OPEN ENDED Draw a pair of vectors on a coordinate plane. Label each vector in component form and then find their sum.

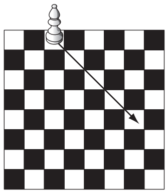
56. REASONING Discuss the similarity of using vectors to translate a figure and using an ordered pair.

57. CHALLENGE If two vectors have opposite directions but the same magnitude, the resultant is $\langle 0, 0 \rangle$ when they are added. Find three vectors of equal magnitude, each with its tail at the origin, the sum of which is $\langle 0, 0 \rangle$.

58. **Writing in Math** Use the information about aviation on page 534 to explain how vectors help a pilot plan a flight. Include an explanation of how westerly winds affect the overall velocity of a plane traveling east.

Standardized Test Practice

59. A chess player moves his bishop as shown. What is the magnitude and direction of the resultant vector?



A 4; 45°

B $4\sqrt{2}$; 45°

C $4\sqrt{2}$; 315°

D 4; 315°

60. **REVIEW** Simplify $\frac{18x^2 - 2}{3x^2 - 5x - 2}$ to lowest terms.

F $\frac{18}{3x + 1}$

G $\frac{2(3x + 1)}{x - 2}$

H $\frac{2(3x - 1)}{x - 2}$

J $2(3x - 1)$

Spiral Review

Find the measure of the dilation image or the preimage of $\overline{AB}$ with the given scale factor. (Lesson 9-5)

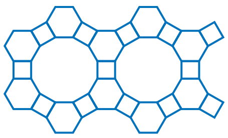
61. $AB = 8$, $r = 2$

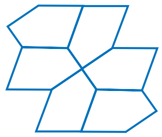
62. $AB = 12$, $r = \frac{1}{2}$

63. $A'B' = 15$, $r = 3$

64. $A'B' = 12$, $r = \frac{1}{4}$

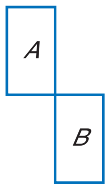
Determine whether each pattern is a tessellation. If so, describe it as *uniform, not uniform, regular,* or *semi-regular.* (Lesson 9-4)

65. 

66. 

Determine whether each phrase completes the statement below to form a true statement. (Lessons 9-1 through 9-3)

Rectangle B cannot be obtained from rectangle A by means of __________.



67. a rotation about the common vertex

68. a translation

69. a reflection about a line through the common vertex

70. a translation followed by a reflection

71. Each side of a rhombus is 30 centimeters long. One diagonal makes a 25° angle with a side. What is the length of each diagonal to the nearest tenth? (Lesson 8-4)

Cross-Curricular Project

Geometry and Social Studies

Hidden Treasure It's time to complete your project. Use the information and data you have gathered about a treasure hunt to prepare a portfolio or Web page. Be sure to include illustrations and/or tables in the presentation.

Math Online Cross-Curricular Project at geometryonline.com

EXTEND 9-6

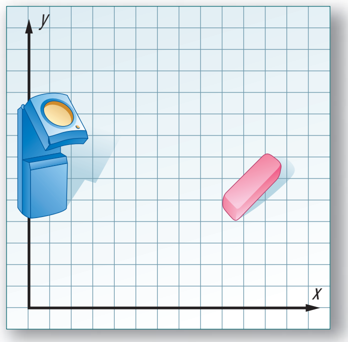
Graphing Calculator Lab
Using Vectors

Biologists use animal radio tags to locate study animals in the field and to transmit information such as body temperature or heart rate about wild or captive animals. This allows for study of the species, tracking of herds, and other scientific study. You can use a data collection device to simulate an animal tracking study.

- Tape a large piece of paper to the floor. Draw a 1-meter by 1-meter square on the paper. Label the sides of the square as a coordinate system with gridlines 10 centimeters apart.
- Attach the motion sensor to the data collection device.

ACTIVITY

Step 1 Place an object in the square to represent an animal. Mark its position on the paper.



Step 2 Position the motion detector on the *y*-axis aligned with the object, as shown. Use the data collection device to measure the distance to the object. This is the *x*-coordinate of the object's position.

Step 3 Repeat Step 2, placing the motion detector on the *x*-axis. Use the device to find the *y*-coordinate of the object's position.

Step 4 Reposition the object and find its coordinates 4 more times. Each time, mark the position on the paper. Then connect consecutive positions with a vector.

Step 5 Use the graphing calculator to create a line graph of the data.

KEYSTROKES: 2nd [STAT PLOT] ENTER ENTER ▼ ▶ ENTER ▼ 2nd [L1] ENTER ▼ 2nd [L2] Zoom 9 Zoom 5

Analyze the Results

1. Compare and contrast the calculator graph and the paper model.
2. Examine the graph to determine between which two positions the animal moved the most and the least.
3. Find the magnitude and direction of each vector. Do the results verify your answer to Exercise 2? Explain.
4. **RESEARCH** Research radio tag studies. What types of animals are tracked in this way? What kind of information is gathered?

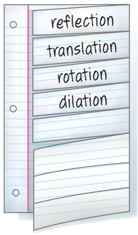
Joel W. Rogers/CORBIS

CHAPTER 9 Study Guide and Review

Download Vocabulary Review from geometryonline.com

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Reflections, Translations, and Rotations (Lesson 9-1 through 9-3)

- The line of symmetry in a figure is a line where the figure could be folded in half so that the two halves match exactly.
- A translation moves the all points of a figure the same distance in the same direction.
- A translation can be represented as a composition of reflections.
- A rotation turns each point in a figure through the same angle about a fixed point.
- An object has rotational symmetry when you can rotate it less than 360° and the preimage and image are indistinguishable.

Tessellations (Lesson 9-4)

- A tessellation is a repetitious pattern that covers a plane without any overlap.
- A regular tessellation contains the same combination of shapes and angles at every vertex.

Dilations (Lesson 9-5)

- Dilations can be enlargements, reductions, or congruence transformations.

Vectors (Lesson 9-6)

- A vector is a quantity with both magnitude and direction.
- Vectors can be used to translate figures on the coordinate plane.

Key Vocabulary

angle of rotation (p. 510)
center of rotation (p. 510)
component form (p. 534)
composition (p. 505)
dilation (p. 525)
direction (p. 534)
invariant points (p. 516)
isometry (p. 497)
line of reflection (p. 497)
line of symmetry (p. 500)
magnitude (p. 534)
point of symmetry (p. 500)
reflection (p. 497)
regular tessellation (p. 520)
resultant (p. 536)
rotation (p. 510)
rotational symmetry (p. 512)
scalar (p. 537)
scalar multiplication (p. 537)
semi-regular tessellation (p. 521)
similarity transformation (p. 526)
standard position (p. 534)
tessellation (p. 519)
translation (p. 504)
uniform (p. 520)
vector (p. 534)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. A dilation can change the distance between each point on the figure and the given line of symmetry.
2. A tessellation is uniform if the same combination of shapes and angles is present at every vertex.
3. Two vectors can be added easily if you know their magnitude.
4. Scalar multiplication affects only the direction of a vector.
5. In a rotation, the figure is turned about the point of symmetry.
6. A reflection is a transformation determined by a figure and a line.

CHAPTER 9

Study Guide and Review

Lesson-by-Lesson Review

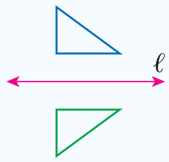
9–1 Reflections (pp. 497–503)

Graph each figure and its image under the given reflection.

7. triangle ABC with $A(2, 1)$, $B(5, 1)$, and $C(2, 3)$ in the x-axis
8. parallelogram $WXYZ$ with $W(-4, 5)$, $X(-1, 5)$, $Y(-3, 3)$, and $Z(-6, 3)$ in the line $y = x$
9. rectangle $EFGH$ with $E(-4, -2)$, $F(0, -2)$, $G(0, -4)$, and $H(-4, -4)$ in the line $x = 1$

10. **ANTS** 12 ants are walking on a mirror. Each ant has 6 legs. How many legs can be seen during this journey?

Example 1 Copy the figure. Draw the image of the figure under a reflection in line ℓ.



The green triangle is the reflected image of the blue triangle.

9–2 Translations (pp. 504–509)

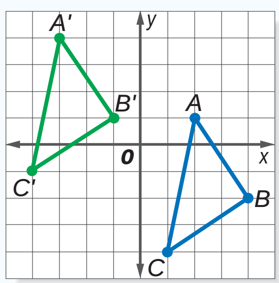
Graph each figure and the image under the given translation.

11. quadrilateral $EFGH$ with $E(2, 2)$, $F(6, 2)$, $G(4, -2)$, $H(1, -1)$ under the translation $(x, y) \rightarrow (x - 4, y - 4)$
12. $\overline{ST}$ with endpoints $S(-3, -5)$, $T(-1, -1)$ under the translation $(x, y) \rightarrow (x + 2, y + 4)$
13. $\triangle XYZ$ with $X(2, 5)$, $Y(1, 1)$, $Z(5, 1)$ under the translation $(x, y) \rightarrow (x + 1, y - 3)$

14. **CLASSROOM** A classroom has a total of 30 desks. Six rows across the front of the room and 5 rows back. If the teacher moves Jimmy's seat from the first seat on the right in the second row to the last seat on the left in the last row, describe the translation.

Example 2 COORDINATE GEOMETRY Triangle ABC has vertices $A(2, 1)$, $B(4, -2)$, and $C(1, -4)$. Graph $\triangle ABC$ and its image for the translation $(x, y) \rightarrow (x - 5, y + 3)$.

(x, y)	$(x - 5, y + 3)$
(2, 1)	(−3, 4)
(4, −2)	(−1, 1)
(1, −4)	(−4, −1)



This translation moved every point of the preimage 5 units left and 3 units up.

Mixed Problem Solving
For mixed problem-solving practice, see page 836.

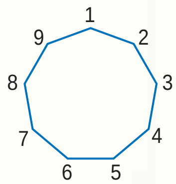
9–3 Rotations (pp. 510–517)

Draw the rotation image of each triangle by reflecting the triangles in the given lines. State the coordinates of the rotation image and the angle of rotation.

15. $\triangle BCD$ with vertices $B(-3, 5)$, $C(-3, 3)$, and $D(-5, 3)$ reflected in the x-axis and then the y-axis

16. FGH with vertices $F(0, 3)$, $G(-1, 0)$, $H(-4, 1)$ reflected in the line $y = x$ and then the line $y = -x$

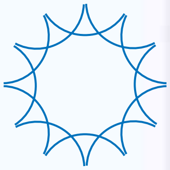
STEAMBOATS **The figure below is a diagram of a paddle wheel on a steamboat. The paddle wheel consists of nine evenly spaced paddles.**



17. Identify the order and magnitude of the symmetry

18. What is the measure of the angle of rotation if paddle 2 is moved counterclockwise to the current position of paddle 6?

Example 4 **Identify the order and magnitude of the rotational symmetry in the figure.**



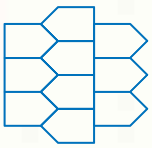
The figure has rotational symmetry of order 12 because there are 12 rotations of less than 360° (including 0°) that produce an image indistinguishable from the original.

The magnitude is $360° \div 12$ or $30°$.

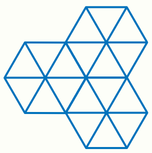
9–4 Tessellations (pp. 519–524)

Determine whether each pattern is a tessellation. If so, describe it as *uniform, not uniform, regular,* or *semi-regular.*

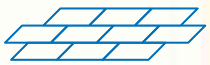
19.



20.



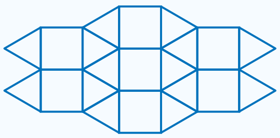
21.



INTERIOR DESIGN **Determine whether each regular polygon tile will tessellate the bathroom floor. Explain.**

22. pentagon **23.** triangle **24.** decagon

Example 5 **Classify the tessellation below.**



The tessellation is uniform, because at each vertex there are two squares and three equilateral triangles. Both the square and equilateral triangle are regular polygons.

Since there is more than one regular polygon in the tessellation, it is a semi-regular tessellation.

9–5 Dilations (pp. 525–532)

Find the measure of the dilation image $\overline{C'D'}$ or preimage $\overline{CD}$ using the given scale factor.

25. $CD = 8, r = 3$

26. $CD = \frac{2}{3}, r = -6$

27. $C'D' = 24, r = 6$

28. $C'D' = 60, r = \frac{10}{3}$

29. $CD = 12, r = -\frac{5}{6}$

30. $C'D' = \frac{55}{2}, r = \frac{5}{4}$

Find the image of each polygon, given the vertices, after a dilation centered at the origin with a scale factor of -2.

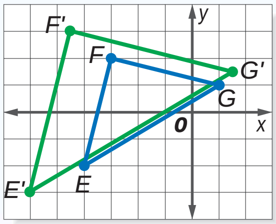
31. $P(-1, 3), Q(2, 2), R(1, -1)$

32. $E(-3, 2), F(1, 2), G(1, -2), H(-3, -2)$

33. **PHOTOGRAPHY** A man is 6 feet tall. If the same man in a photograph is $1\frac{1}{2}$ inches tall, what is the approximate scale factor of the photo?

Example 6 **Triangle *EFG* has vertices $E(-4, -2)$, $F(-3, 2)$, and $G(1, 1)$. Find the image of $\triangle EFG$ after a dilation centered at the origin with a scale factor of $\frac{3}{2}$.**

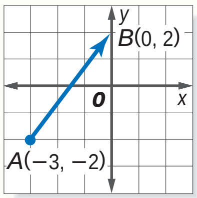
Preimage (x, y)	$\left(\frac{3}{2}x, \frac{3}{2}y\right)$
$E(-4, -2)$	$E'(-6, -3)$
$F(-3, 2)$	$F'\left(-\frac{9}{2}, 3\right)$
$G(1, 1)$	$G'\left(\frac{3}{2}, \frac{3}{2}\right)$



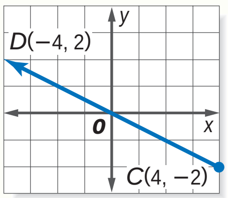
9–6 Vectors (pp. 534–541)

Write the component form of each vector.

34.



35.



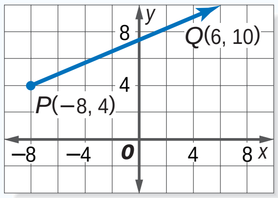
Find the magnitude and direction of $\overrightarrow{AB}$ for the given coordinates.

36. $A(-6, 4), B(-9, -3)$

37. $A(-14, 2), B(15, -5)$

38. **CANOEING** Jessica is trying to canoe directly across a river with a current of 3 miles per hour. If Jessica can canoe at a rate of 7 miles per hour, how does the current of the river affect her speed and direction?

Example 7 **Find the magnitude and direction of $\overrightarrow{PQ}$ for $P(-8, 4)$ and $Q(6, 10)$.**



Find the magnitude.

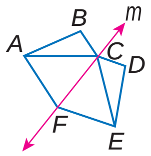
$$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(6 + 8)^2 + (10 - 4)^2}$$
$$= \sqrt{232} \text{ or about } 15.3$$

Find the direction.

$$\tan P = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{10 - 4}{6 + 8} \text{ or } \frac{3}{7}$$
$$m\angle P = \tan^{-1}\frac{3}{7}$$
$$\approx 23.2$$

Chapter 9 Practice Test

Name the reflected image of each figure under a reflection in line *m*.



1. A
2. $\overline{BC}$
3. $\triangle DCE$

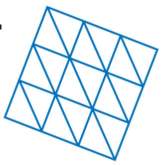
COORDINATE GEOMETRY Graph each figure and its image under the given translation.

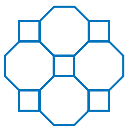
4. $\triangle PQR$ with $P(-3, 5)$, $Q(-2, 1)$, and $R(-4, 2)$ under the translation right 3 units and up 1 unit
5. Parallelogram $WXYZ$ with $W(-2, -5)$, $X(1, -5)$, $Y(2, -2)$, and $Z(-1, -2)$ under the translation up 5 units and left 3 units
6. with $F(3, 5)$ and $G(6, -1)$ under the translation $(x, y) \rightarrow (x - 4, y - 1)$

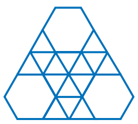
Draw the rotation image of each triangle by reflecting the triangles in the given lines. State the coordinates of the rotation image and the angle of rotation.

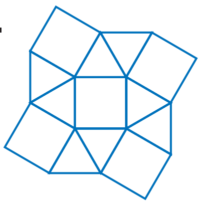
7. $\triangle JKL$ with $J(-1, -2)$, $K(-3, -4)$, $L(1, -4)$ reflected in the y-axis and then the x-axis
8. $\triangle ABC$ with $A(-3, -2)$, $B(-1, 1)$, $C(3, -1)$ reflected in the line $y = x$ and then the line $y = -x$
9. $\triangle RST$ with $R(1, 6)$, $S(1, 1)$, $T(3, -2)$ reflected in the y-axis and then the line $y = x$

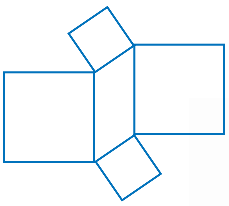
Determine whether each pattern is a tessellation. If so, describe it as *uniform, not uniform, regular,* or *semi-regular.*

10.


11.


12.


13.


14.


Find the measure of the dilation image $\overline{M'N'}$ or preimage of $\overline{MN}$ using the given scale factor.

15. $MN = 5, r = 4$
16. $MN = 8, r = \frac{1}{4}$
17. $M'N' = 36, r = 3$
18. $MN = 9, r = -\frac{1}{5}$
19. $M'N' = 20, r = \frac{2}{3}$
20. $M'N' = \frac{29}{5}, r = -\frac{3}{5}$
21. $MN = 35, r = \frac{2}{7}$
22. $M'N' = 14, r = -7$

Find the magnitude and direction of each vector.

23. $\vec{v} = \langle -3, 2 \rangle$
24. $\vec{w} = \langle -6, -8 \rangle$

25. **CYCLING** Suppose Lynette rides her bicycle due south at a rate of 16 miles per hour. If the wind is blowing due west at 4 miles per hour, what is the resultant velocity and direction of the bicycle?

26. **TRAVEL** In trying to calculate how far she must travel for an appointment, Gunja measured the distance between Richmond, Virginia, and Charlotte, North Carolina, on a map. The distance on the map was 2.25 inches, and the scale factor was 1 inch equals 150 miles. How far must she travel?

27. **MULTIPLE CHOICE** What reflections could be used to create the image $(3, 4)$ from $(3, -4)$?

    I. reflection in the x-axis
    II. reflection in the y-axis
    III. reflection in the origin

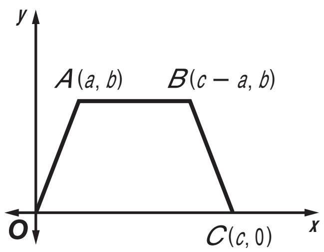
    A I only
    B III only
    C I and III
    D I and II

Standardized Test Practice

Cumulative, Chapters 1–9

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Figure *ABCO* is an isosceles trapezoid.



Which of the following are the coordinates of an endpoint of the median of *ABCO*?

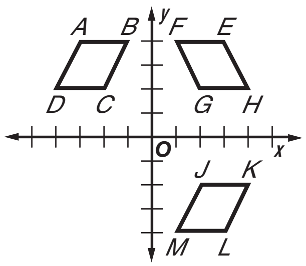
A $\left(\frac{a+b}{2}, \frac{a+b}{2}\right)$

B $\left(\frac{2c-a}{2}, \frac{b}{2}\right)$

C $\left(\frac{c}{2}, 0\right)$

D $\left(\frac{a}{2}, \frac{b}{2}\right)$

2. Which of the following statements about the figures below is true?



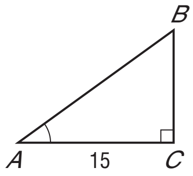
F Parallelogram *JKLM* is a reflection image of ▱*ABCD*.

G Parallelogram *EFGH* is a translation image of ▱*ABCD*.

H Parallelogram *JKLM* is a translation image of ▱*EFGH*.

J Parallelogram *JKLM* is a translation image of ▱*ABCD*.

3. **GRIDDABLE** In the figure below, $\tan A = 0.6$.



What is the length of $\overline{BC}$?

4. The vertices of $\triangle JKL$ are $J(-5, 3)$, $K(1, 4)$, and $L(-3, -2)$. If $\triangle JKL$ is reflected across the *x*-axis to create $\triangle MPQ$, what are the coordinates of the vertices of $\triangle MPQ$?

A $M(-5, -3)$, $P(1, -4)$, $Q(-3, 2)$

B $M(-3, 5)$, $P(-4, -1)$, $Q(2, 3)$

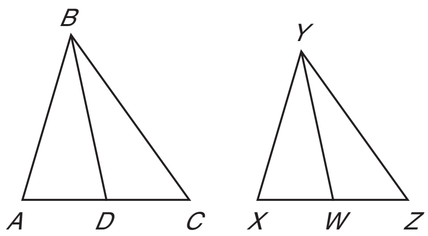
C $M(3, -5)$, $P(4, 1)$, $Q(-2, -3)$

D $M(5, 3)$, $P(-1, 4)$, $Q(3, -2)$

TEST-TAKING TIP

Question 4 To check your answer, remember the following rule. In a reflection over the *x*-axis, the *x*-coordinate must remain the same, and the *y*-coordinate changes its sign.

5. In the figure below, $\triangle ABC \sim \triangle XYZ$.



Which additional information would *not* be enough to prove that $\frac{BD}{YW} = \frac{AC}{XZ}$?

F $\overline{BD}$ and $\overline{YZ}$ are angle bisectors of triangles *ABC* and *XYZ*, respectively.

G $\overline{BD}$ and $\overline{YZ}$ are medians of triangles *ABC* and *XYZ*, respectively.

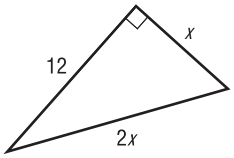
H $XZ = \frac{1}{3}AC$

J $\angle BDC$ and $\angle YWZ$ are right angles.

Standardized Test Practice at geometryonline.com

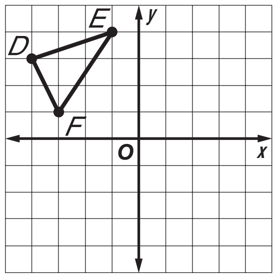
Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 841-856.

6. What is the value of x in the triangle shown?



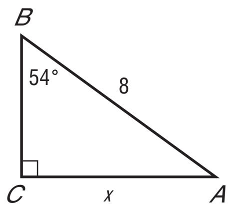
A $4\sqrt{3}$
B $6\sqrt{2}$
C $12\sqrt{3}$
D 24

7. If $\triangle DEF$ is rotated 90 degrees about the origin, what are the coordinates of D'?



F (3, 4) **H** (-4, -3)
G (-3, -4) **J** (4, -3)

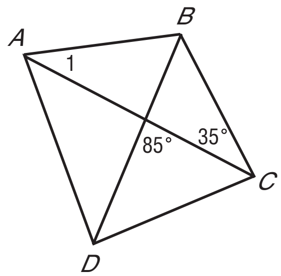
8. In the accompanying diagram, $m\angle B = 54°$ and $AB = 8$. Which equation could be used to find x in $\triangle ABC$?



A $x = \dfrac{8}{\cos 54°}$
B $x = 8 \cos 54°$
C $x = 8 \sin 54°$
D $x = \dfrac{8}{\sin 54°}$

9. **ALGEBRA** Which is a factor of $x^2 - 3x - 40$?
F $x + 2$
G $x - 2$
J $x - 5$
H $x + 5$

10. If $\overline{BD}$ bisects $\angle ABC$, what is $m\angle 1$?
A 35
B 45
C 50
D 120



Pre-AP

Record your answer on a sheet of paper. Show your work.

11. Paul is studying to become a landscape architect. He drew a map view of a park using the following vertices: $Q(2, 2)$, $R(-2, 4)$, $S(-3, -3)$, and $T(3, -4)$. Paul then noticed that his original drawing was oriented incorrectly, with north at the bottom of the graph instead of at the top.

a. What transformation should Paul use to reorient his drawing with north at the top of his graph?

b. Is this the only transformation that would work? Explain.

c. Graph and label the coordinates of quadrilateral $QRST$.

d. Graph and label the coordinates of its reoriented image $Q'R'S'T'$.

e. Explain how Paul could determine the coordinates of the vertices of $Q'R'S'T'$ without using a coordinate plane.

NEED EXTRA HELP?											
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11
Go to Lesson or Page...	6-7	9-2	9-1	8-4	7-5	8-3	9-3	8-4	796	4-2	9-3

UNIT 4

Two- and Three-Dimensional Measurement

Focus

Calculate measures in two- and three-dimensions and use the properties of circles.

CHAPTER 10
Circles

BIG Idea Prove and use theorems involving the properties of circles and the relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.

CHAPTER 11
Areas of Polygons and Circles

BIG Idea Students derive formulas and solve problems involving the areas of circles and polygons.

CHAPTER 12
Extending Surface Area

BIG Idea Derive formulas and solve problems involving the lateral area and surface area of solids.

CHAPTER 13
Extending Volume

BIG Idea Derive formulas and solve problems involving the volumes of three-dimensional figures.

BIG Idea Determine how changes in dimensions affect the volume of solids.

BIG Idea Investigate the effect of rigid motions on figures in the space.

Photodisc/Getty Images

Geometry and Architecture

Memorials Help Us Pay Tribute Have you ever visited a memorial dedicated to the people who lost their lives defending our country or its principles? The Vietnam War was fought from 1961 to 1973. During that time, over 58,000 Americans lost their lives. Americans have memorials in memory of the service of these people in the Vietnam War in 32 states and in Washington, D.C. In this project, you will use circles, polygons, surface area, and volume to design a memorial to honor war veterans.

Math Online Log on to geometryonline.com to begin.

CHAPTER 10

Circles

BIG Ideas

- Identify parts of a circle and solve problems involving circumference.
- Find arc and angle measures in a circle.
- Find measures of segments in a circle.
- Write the equation of a circle.

Key Vocabulary

chord (p. 554)
circumference (p. 556)
arc (p. 564)
tangent (p. 588)
secant (p. 599)

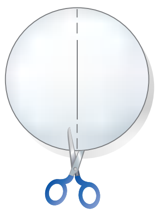
Real-World Link

Ferris Wheels Modeled after the very first Ferris wheel built for the 1893 World Colombian Exposition, the Navy Pier Ferris wheel in Chicago, Illinois, is 150 feet high. It has 40 gondolas that each seat six passengers, and its 40 spokes span a diameter of 140 feet.

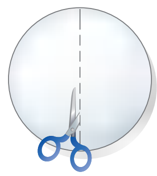
FOLDABLES

Circles Make this Foldable to help you organize your notes. Begin with five sheets of plain 8½" × 11" paper, and cut out five large circles that are the same size.

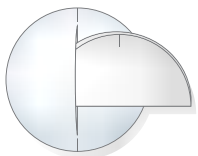
1. **Fold** two of the circles in half and cut one-inch slits at each end of the folds.



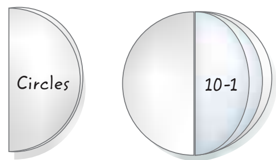
2. **Fold** the remaining three circles in half and cut a slit in the middle of the fold.



3. **Slide** the two circles with slits on the ends through the large slit of the other circles.



4. **Fold** to make a booklet. Label the cover with the title of the chapter and each sheet with a lesson number.



GET READY for Chapter 10

Diagnose Readiness **You have two options for checking Prerequisite Skills.**

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Solve each equation for the given variable. (Prerequisite Skill)

1. $\frac{4}{9}p = 72$ for p

2. $6.3p = 15.75$

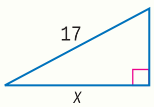
3. $3x + 12 = 8x$ for x

4. $7(x + 2) = 3(x - 6)$

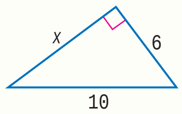
5. The circumference of a circle is given by the formula $C = 2\pi r$. Solve for r.

Find x to the nearest tenth unit. (Lesson 8-2)

6.



7.



8. The lengths of the legs of an isosceles right triangle are 72 inches. Find the length of the hypotenuse. (Lesson 8-2)

Solve each equation by using the Quadratic Formula. Round to the nearest tenth. (Prerequisite Skill)

9. $x^2 - 4x = 10$

10. $3x^2 - 2x - 4 = 0$

11. $x^2 = x + 15$

12. **PHYSICS** A rocket is launched vertically up in the air from ground level. The distance in feet from the ground d after t seconds is given by the equation $d = 96t - 16t^2$. Find the values of t when $d = 102$ feet. Round to the nearest tenth. (Prerequisite Skill)

QUICKReview

EXAMPLE 1

Solve the equation $5 - y = 13(y + 2)$ for y.

$5 - y = 13(y + 2)$

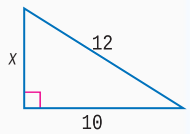
$5 - y = 13y + 26$ Distributive Property

$-21 = 14y$ Combine like terms.

$-1.5 = y$ Divide.

EXAMPLE 2

Find x. Round to the nearest tenth if necessary.



$a^2 + b^2 = c^2$ Pythagorean Theorem

$x^2 + 10^2 = 12^2$ Substitution

$x^2 + 100 = 144$ Simplify.

$x^2 = 44$ Subtract 100 from each side.

$x = \sqrt{44}$ Take the square root of each side.

$x \approx 6.6$ Use a calculator.

EXAMPLE 3

Solve $x^2 + 3x - 10 = 0$ by using the Quadratic Formula. Round to the nearest tenth.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-10)}}{2(1)}$ $a = 1, b = 3, c = -10$

$= \frac{-3 \pm \sqrt{9 + 40}}{2}$ Simplify.

$= \frac{-3 \pm 7}{2}$ Simplify.

$x = \frac{-3 + 7}{2}$ or 2 $\quad x = \frac{-3 - 7}{2}$ or -5

10-1 Circles and Circumference

Main Ideas

- Identify and use parts of circles.
- Solve problems involving the circumference of a circle.

New Vocabulary

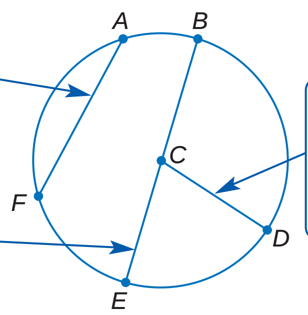
circle
center
chord
radius
diameter
circumference
pi (π)

The largest carousel in the world still in operation is located in Spring Green, Wisconsin. It weighs 35 tons and contains 260 animals, none of which is a horse! The rim of the carousel base is a circle. The width, or diameter, of the circle is 80 feet. The distance that an animal on the outer edge travels can be determined by special segments in a circle.

Parts of Circles A **circle** is the locus of all points in a plane equidistant from a given point called the **center** of the circle. A circle is usually named by its center point. The figure below shows circle C, which can be written as $\odot C$. Several special segments in circle C are also shown.

Any segment with both endpoints that are on the circle is a **chord** of the circle. $\overline{AF}$ and $\overline{BE}$ are chords.

A chord that passes through the center is a **diameter** of the circle. $\overline{BE}$ is a diameter.



Any segment with endpoints that are the center and a point on the circle is a **radius**. $\overline{CD}$, $\overline{CB}$, and $\overline{CE}$ are radii of the circle.

The plural of radius is *radii,* pronounced RAY-dee-eye. The term *radius* can mean a segment or the measure of that segment. This is also true of the term *diameter*.

Note that diameter $\overline{BE}$ is made up of collinear radii $\overline{CB}$ and $\overline{CE}$.

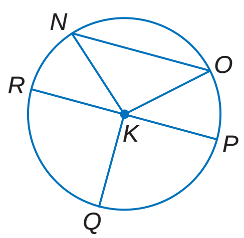
EXAMPLE 1 Identify Parts of a Circle

a. Name the circle.

The circle has its center at K, so it is named circle K, or $\odot K$.

b. Name a radius of the circle.

Five radii are shown: $\overline{KN}$, $\overline{KO}$, $\overline{KP}$, $\overline{KQ}$, and $\overline{KR}$.



Study Tip

Centers of Circles In this text, the center of the circle will often be shown in the figure with a dot.

c. Name a chord of the circle.

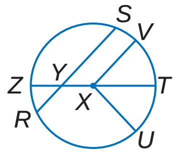
Two chords are shown: $\overline{NO}$ and $\overline{RP}$.

d. Name a diameter of the circle.

$\overline{RP}$ is the only chord that goes through the center, so $\overline{RP}$ is a diameter.

CHECK Your Progress

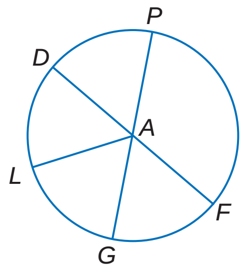
1. Name the circle, a radius, a chord, and a diameter of the circle.



By definition, the distance from the center to any point on a circle is always the same. Therefore, all radii r are congruent. A diameter d is composed of two radii, so all diameters are congruent. Thus, $d = 2r$ and $r = \frac{d}{2}$ or $\frac{1}{2}d$.

EXAMPLE 2 Find Radius and Diameter

Circle *A* has diameters $\overline{DF}$ and $\overline{PG}$.



a. If $DF = 10$, find DA.

$r = \frac{1}{2}d$ Formula for radius

$= \frac{1}{2}(10)$ or 5 Substitute and simplify.

b. If $AG = 12$, find LA.

Since all radii are congruent, $LA = AG$. So, $LA = 12$.

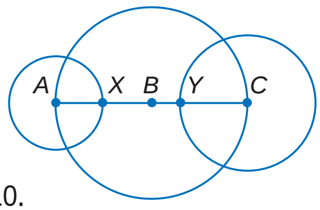
2A. If $PA = 7$, find PG.

2B. If $PG = 15$, find DF.

The segment connecting the centers of the two intersecting circles contains a radius of each circle.

EXAMPLE 3 Find Measures in Intersecting Circles

The diameters of $\odot A$, $\odot B$, and $\odot C$ are 10 inches, 20 inches, and 14 inches, respectively. Find XB.



Since the diameter of $\odot A$ is 10, $AX = 5$.

Since the diameter of $\odot B$ is 20, $AB = 10$ and $BC = 10$.

$\overline{XB}$ is part of radius $\overline{AB}$.

$AX + XB = AB$ Segment Addition Postulate

$5 + XB = 10$ Substitution

$XB = 5$ Subtract 5 from each side.

3. Find BY.

Study Tip

Congruent Circles

The circles shown in Example 3 have different radii. They are *not* congruent circles. For two circles to be *congruent circles,* they must have congruent radii or congruent diameters.

Circumference The **circumference** of a circle is the distance around the circle. Circumference is most often represented by the letter C.

GEOMETRY LAB

Circumference Ratio

A special relationship exists between the circumference of a circle and its diameter.

GATHER DATA AND ANALYZE

Collect ten round objects.

1. Measure the circumference and diameter of each object in millimeters. Record the measures in a table.

2. Compute the value of $\frac{C}{d}$ to the nearest hundredth for each object. Record the result in the fourth column of the table.

Object	C	d	$\frac{C}{d}$
1			
2			
3			
⋮			
10			

3. **MAKE A CONJECTURE** What seems to be the relationship between the circumference and the diameter of the circle?

The Geometry Lab suggests that the circumference of any circle can be found by multiplying the diameter by a number slightly larger than 3. By definition, the ratio $\frac{C}{d}$ is an irrational number called **pi**, symbolized by the Greek letter **π**. Two formulas for the circumference can be derived using this definition.

$\frac{C}{d} = \pi$ Definition of pi

$C = \pi d$ Multiply each side by d.

$C = \pi d$

$C = \pi(2r)$ $d = 2r$

$C = 2\pi r$ Simplify.

Study Tip

Radii and Diameters

There are an infinite number of radii in each circle. Likewise, there are an infinite number of diameters.

KEY CONCEPT *Circumference*

For a circumference of C units and a diameter of d units or a radius of r units, $C = \pi d$ or $C = 2\pi r$.

EXAMPLE 4 Find Circumference, Diameter, and Radius

a. Find C if $r = 7$ centimeters.

$C = 2\pi r$ Circumference formula

$= 2\pi(7)$ Substitution

$= 14\pi$ or about 43.98 cm

b. Find C if $d = 12.5$ inches.

$C = \pi d$ Circumference formula

$= \pi(12.5)$ Substitution

$= 12.5\pi$ or 39.27 in.

c. Find d and r to the nearest hundredth if $C = 136.9$ meters.

$C = \pi d$	Circumference formula	$r = \frac{1}{2}d$	Radius formula
$136.9 = \pi d$	Substitution	$\approx \frac{1}{2}(43.58)$	$d \approx 43.58$
$\frac{136.9}{\pi} = d$	Divide each side by π.	≈ 21.79 m	Use a calculator.
$43.58 \text{ m} \approx d$	Use a calculator.		

4A. Find C if $r = 12$ inches.

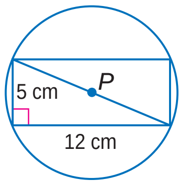
4B. Find C if $d = 7.25$ meters.

4C. Find d and r to the nearest hundredth if $C = 77.8$ centimeters.

You can also use other geometric figures to help you find the circumference of a circle.

EXAMPLE Use Other Figures to Find Circumference

5 Find the exact circumference of $\odot P$.



The diameter of the circle is the same as the hypotenuse of the right triangle.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$5^2 + 12^2 = c^2$	Substitution
$169 = c^2$	Simplify.
$13 = c$	Take the square root of each side.

So the diameter of the circle is 13 centimeters. To find the circumference, substitute 13 for d in $C = \pi d$. The *exact* circumference is 13π.

5. A square with a side length of 8 inches is inscribed in $\odot N$. Find the exact circumference of $\odot N$.

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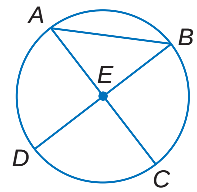
Your Understanding

Examples 1, 2 (pp. 554–555)

For Exercises 1–6, refer to the circle at the right.



1. Name the circle.

2. Name a radius.

3. Name a chord.

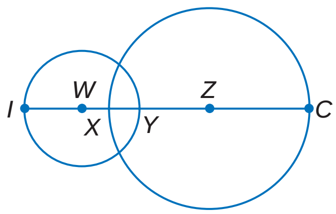
4. Name a diameter.

5. Suppose $BD = 12$ millimeters. Find the radius of the circle.

6. Suppose $CE = 5.2$ inches. What is the diameter of the circle?

Example 3 (p. 555)

Circle W has a radius of 4 units, $\odot Z$ has a radius of 7 units, and $XY = 2$. Find each measure.



7. YZ

8. IX

9. IC

Example 4 (p. 556)

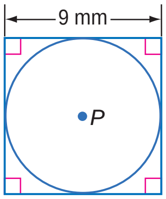
The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

10. $r = 5$ m, $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

11. $C = 2368$ ft, $d = \underline{\ ?\ }$, $r = \underline{\ ?\ }$

Example 5 (p. 557)

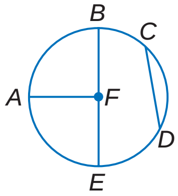
12. Find the exact circumference of the circle.



Exercises

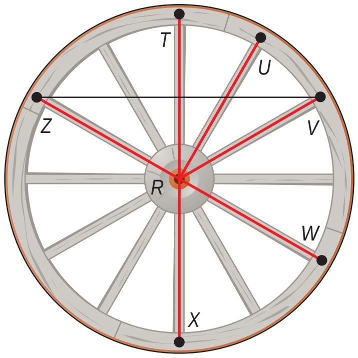
HOMEWORK HELP	
For Exercises	See Examples
13–22	1
23–28	2
29–34	3
35–42	4
43–46	5

For Exercises 13–17, refer to the circle at the right.



13. Name the circle.
14. Name a radius.
15. Name a chord.
16. Name a diameter.
17. Name a radius not contained in a diameter.

HISTORY For Exercises 18–28, refer to the model of the Conestoga wagon wheel.

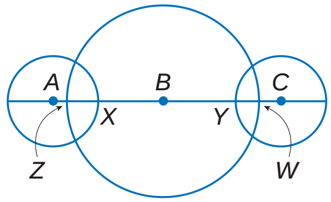


18. Name the circle.
19. Name a radius of the circle.
20. Name a chord of the circle.
21. Name a diameter of the circle.
22. Name a radius not contained in a diameter.
23. Suppose the radius of the circle is 2 feet. Find the diameter.
24. The larger wheel of a wagon was often 5 or more feet tall. What is the radius of a 5-foot wheel?
25. If $TX = 120$ centimeters, find TR.
26. If $RZ = 32$ inches, find ZW.
27. If $UR = 18$ inches, find RV.
28. If $XT = 1.2$ meters, find UR.

Real-World Link

The Conestoga Wagon, used from about 1717 to the 1820s, began as a farm wagon that was adapted for use in the hilly countryside of Pennsylvania. The wheels were larger to enable the wagon to go over stones and roots in the road.

The diameters of $\odot A$, $\odot B$, and $\odot C$ are 10, 30, and 10 units, respectively. Find each measure if $\overline{AZ} \cong \overline{CW}$ and $CW = 2$.



29. AZ **30.** ZX **31.** BX

32. BY **33.** YW **34.** AC

The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

35. $r = 7$ mm, $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

36. $d = 26.8$ cm, $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

37. $C = 26\pi$ mi, $d = \underline{\ ?\ }$, $r = \underline{\ ?\ }$

38. $C = 76.4$ m, $d = \underline{\ ?\ }$, $r = \underline{\ ?\ }$

39. $d = 12\frac{1}{2}$ yd, $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

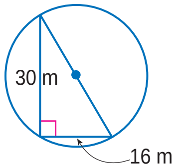
40. $r = 6\frac{3}{4}$ in., $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

41. $d = 2a$, $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

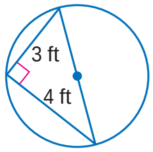
42. $r = \frac{a}{6}$, $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

Find the exact circumference of each circle.

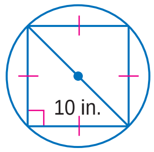
43.



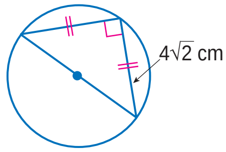
44.



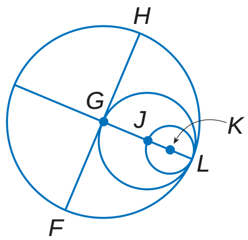
45.



46.



Circles *G*, *J*, and *K* all intersect at *L*. If *GH* = 10, find each measure.



47. *FG*

48. *FH*

49. *GL*

50. *GJ*

51. *JL*

52. *JK*

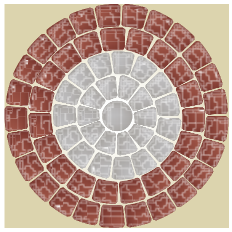
53. PROBABILITY Find the probability that a segment whose endpoints are the center of the circle and a point on the circle is a radius. Explain.

54. PROBABILITY Find the probability that a chord that does not contain the center of a circle is the longest chord of the circle.

Cross-Curricular Project

Drawing a radius and circle will help you begin to design your memorial. Visit geometryonline.com to continue work on your project.

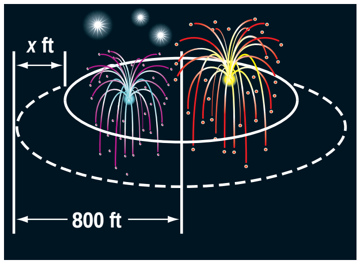
PATIO For Exercises 55 and 56, use the following information.
Mr. Hintz is going to build a patio as shown at the right.



55. If the radius of the entire patio is six feet, what is the approximate circumference?

56. If Mr. Hintz wants the inner circle to have a circumference of approximately 19 feet, what should the radius of the circle be to the nearest foot?

FIREWORKS For Exercises 57–59, use the following information.
Every July 4th, Boston puts on a gala with the Boston Pops Orchestra, followed by a huge fireworks display. The fireworks are shot from a barge in the river. There is an explosion circle inside which all of the fireworks will explode. Spectators sit outside a safety circle that is 800 feet from the center of the fireworks display.



57. Find the approximate circumference of the safety circle.

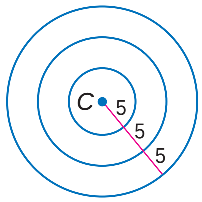
58. If the safety circle is 200 to 300 feet farther from the center than the explosion circle, find the range of values for the radius of the explosion circle.

59. Find the least and maximum circumference of the explosion circle to the nearest foot.

EXTRA PRACTICE
See pages 819, 837.
Math Online
Self-Check Quiz at geometryonline.com

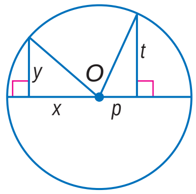
H.O.T. Problems

60. REASONING Circles that have the same center, but different radii, are called **concentric circles**. Use the figure at the right to find the exact circumference of each circle. List the circumferences in order from least to greatest.



61. OPEN ENDED Draw a circle with circumference between 8 and 12 centimeters. What is the radius of the circle? Explain.

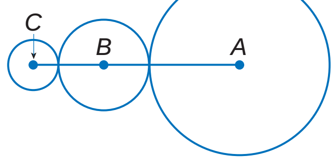
62. CHALLENGE In the figure, O is the center of the circle, and $x^2 + y^2 + p^2 + t^2 = 288$. What is the exact circumference of $\odot O$?



63. Which One Doesn't Belong? A circle has diameter d, radius r, circumference C, and area A. Which ratio does *not* belong?

$\frac{C}{2r}$ $\frac{A}{r^2}$ $\frac{C}{r}$ $\frac{C}{d}$

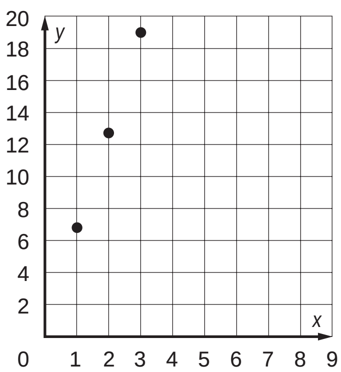
64. CHALLENGE In the figure, the radius of $\odot A$ is twice the radius of $\odot B$ and four times the radius of $\odot C$. If the sum of the circumferences of the three circles is 42π, find the measure of $\overline{AC}$.



65. Writing in Math Use the information about carousels on page 554 to explain how far a carousel horse will travel in one rotation. Describe how the circumference of a circle relates to the distance traveled by the horse and whether a horse located one foot from the outside edge of the carousel travels a mile when it makes 22 rotations for each ride.

STANDARDIZED TEST PRACTICE

66. REVIEW Which of the following is *best* represented by the data in the graph?



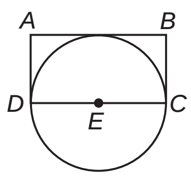
A comparing the length of a side of a square to the width of that square

B comparing the length of a side of a cube to the cube's surface area

C comparing a circle's radius to its circumference

D comparing a circle's radius to its diameter

67. Compare the circumference of $\odot E$ with the perimeter of rectangle $ABCD$. Which statement is true?



F The perimeter of $ABCD$ is greater than the circumference of circle E.

G The circumference of circle E is greater than the perimeter of $ABCD$.

H The perimeter of $ABCD$ equals the circumference of circle E.

J There is not enough information to determine this comparison.

Find the magnitude to the nearest tenth and direction to the nearest degree of each vector. (Lesson 9-6)

68. $\overrightarrow{AB} = \langle 1, 4 \rangle$

69. $\vec{v} = \langle 4, 9 \rangle$

70. $\overrightarrow{AB}$ if $A(4, 2)$ and $B(7, 22)$

71. $\overrightarrow{CD}$ if $C(0, -20)$ and $D(40, 0)$

Find the measure of the dilation image of $\overline{AB}$ for each scale factor k. (Lesson 9-5)

72. $AB = 5, k = 6$

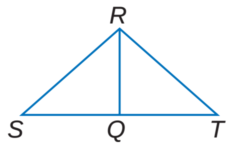
73. $AB = 16, k = 1.5$

74. $AB = \frac{2}{3}, k = -\frac{1}{2}$

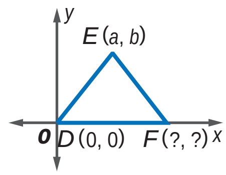
75. PROOF Write a two-column proof. (Lesson 5-2)

Given: $\overline{RQ}$ bisects $\angle SRT$.

Prove: $m\angle SQR > m\angle SRQ$



76. COORDINATE GEOMETRY Name the missing coordinates if $\triangle DEF$ is isosceles with vertex angle E. (Lesson 4-3)

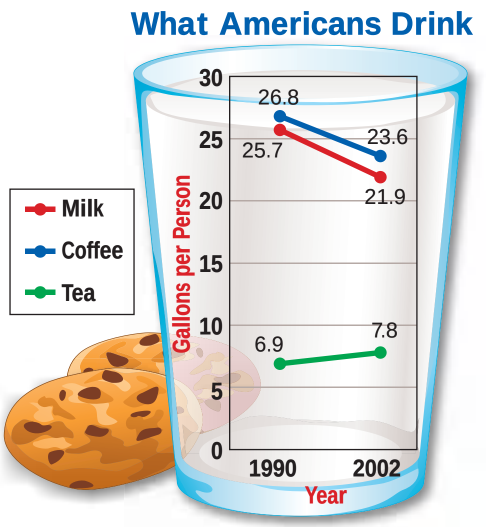


POPULATION For Exercises 77–79, refer to the graph. (Lesson 3-3)

77. Estimate the annual rate of change for gallons of tea consumed from 1990 to 2002.

78. If the trend continues in consumption of coffee, how many gallons of coffee will each American drink in 2010?

79. If the consumption of milk continues to decrease at the same rate, in what year will each American drink about 18 gallons of milk?

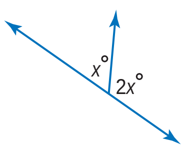


Source: *Statistical Abstract of the United States*

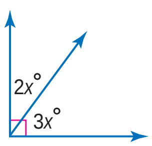
GET READY for the Next Lesson

PREREQUISITE SKILL **Find x.** (Lesson 1-4)

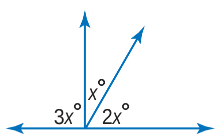
80.



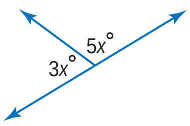
81.



82.



83.



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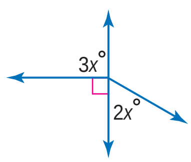

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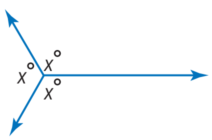


READING MATH

Everyday Expressions Containing Math Words

Have you ever heard of someone “going around in circles”? Does the mathematical meaning of *circle* have anything to do with the meaning of the expression? Expressions used in everyday conversations often are related to the math meaning of the words they contain.

Expression	Everyday Meaning	Math Meaning
an acute pain	sharp pain	an angle that measures less than 90
the degree of her involvement	the measure or scope of an action	a unit of measure used in measuring angles and arcs
dietary supplements	something added to fulfill nutritional requirements	two supplementary angles have measures that have a sum of 180
going to extremes	to the greatest possible extent	in $a:b = c:d$, the numbers a and d

Notice that in all of these cases, the math meaning of the underlined word is related to the everyday meaning of the expression.

Reading to Learn

Describe the everyday meaning of each expression and how, if at all, it is related to the mathematical meaning of the word it contains.

1. going around in circles

2. going off on a tangent

3. striking a chord

4. getting to the point

5. having no proof

6. How is the mathematical definition of *arc* related to an *arcade*?

7. **RESEARCH** Use the Internet or another resource to describe the everyday meaning and the related mathematical meaning of each term.

  a. inscribed

  b. intercepted

10-2 Measuring Angles and Arcs

Main Ideas

- Recognize major arcs, minor arcs, semicircles, and central angles and their measures.
- Find arc length.

New Vocabulary

central angle
arc
minor arc
major arc
semicircle

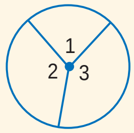
Most clocks on electronic devices are digital, showing the time as numerals. Analog clocks are often used in decorative furnishings and wrist watches. An analog clock has moving hands that indicate the hour, minute, and sometimes the second. This clock face is a circle. The three hands form three central angles of the circle.

Angles and Arcs In Chapter 1, you learned that a degree is $\frac{1}{360}$ of the circular rotation about a point. This means that the sum of the measures of the angles about the center of the clock above is 360. Each of the angles formed by the clock hands is called a central angle. A **central angle** has the center of the circle as its vertex, and its sides contain two radii of the circle.

KEY CONCEPT — *Sum of Central Angles*

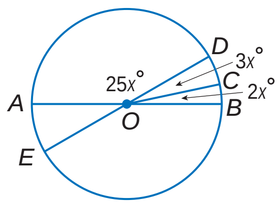
Words The sum of the measures of the central angles of a circle with no interior points in common is 360.

Example: $m\angle 1 + m\angle 2 + m\angle 3 = 360$



EXAMPLE 1 Measures of Central Angles

ALGEBRA Refer to ⊙*O*.



Find $m\angle AOD$.

$\angle AOD$ and $\angle DOB$ are a linear pair, and the angles of a linear pair are supplementary.

$$m\angle AOD + m\angle DOB = 180$$

$$m\angle AOD + m\angle DOC + m\angle COB = 180 \quad \text{Angle Sum Theorem}$$

$$25x + 3x + 2x = 180 \quad \text{Substitution}$$

$$30x = 180 \quad \text{Simplify.}$$

$$x = 6 \quad \text{Divide each side by 30.}$$

Use the value of x to find $m\angle AOD$.

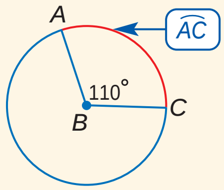
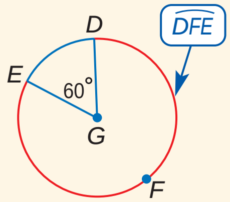
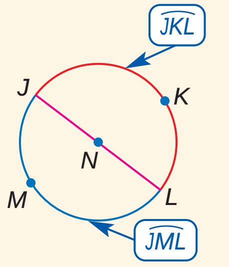
$$m\angle AOD = 25x \quad \text{Given}$$

$$= 25(6) \text{ or } 150 \quad \text{Substitution}$$

1. Find $m\angle AOE$.

A central angle separates the circle into two parts, each of which is an **arc**. The measure of each arc is related to the measure of its central angle.

KEY CONCEPT — *Arcs of a Circle*

Type of Arc:	**minor arc**	**major arc**	**semicircle**
Definition:	an arc that measures less than 180°	an arc that measures greater than 180°	an arc that measures 180°
Example:			
Named:	usually by the letters of the two endpoints $\widehat{AC}$	by the letters of the two endpoints and another point on the arc $\widehat{DFE}$	by the letters of the two endpoints and another point on the arc $m\widehat{JML}$ and $\widehat{JKL}$
Arc Degree Measure Equals:	the measure of the central angle $m\angle ABC = 110$, so $m\widehat{AC} = 110$	360 minus the measure of the minor arc with the same endpoints $m\widehat{DFE} = 360 - m\widehat{DE}$ $m\widehat{DFE} = 360 - 60$ or 300	$360 \div 2$ or 180 $m\widehat{JML} = 180$ $m\widehat{JKL} = 180$

Study Tip

Naming Arcs

Do not assume that because an arc is named by three letters that it is a semicircle or major arc. You can also correctly name a minor arc using three letters.

Animation
geometryonline.com

Arcs with the same measure in the same circle or in congruent circles are congruent.

THEOREM 10.1

In the same or in congruent circles, two arcs are congruent if and only if their corresponding central angles are congruent.

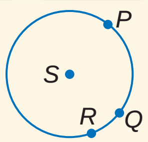
You will prove Theorem 10.1 in Exercise 50.

Arcs of a circle that have exactly one point in common are *adjacent arcs.* Like adjacent angles, the measures of adjacent arcs can be added.

POSTULATE 10.1 — *Arc Addition Postulate*

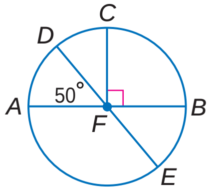
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Example: In $\odot S$, $m\widehat{PQ} + m\widehat{QR} = m\widehat{PQR}$.



EXAMPLE Measures of Arcs

2 **In $\odot F$, $m\angle DFA = 50$ and $\overline{CF} \perp \overline{FB}$. Find each measure.**



a. $m\overset{\frown}{BE}$

$\overset{\frown}{BE}$ is a minor arc, so $m\overset{\frown}{BE} = m\angle BFE$.

$\angle BFE \cong \angle DFA$	Vertical angles are congruent.
$m\angle BFE = m\angle DFA$	Definition of congruent angles
$m\overset{\frown}{BE} = m\angle DFA$	Transitive Property
$m\overset{\frown}{BE} = 50$	Substitution

b. $m\overset{\frown}{CBE}$

$\overset{\frown}{CBE}$ is composed of adjacent arcs, $\overset{\frown}{CB}$ and $\overset{\frown}{BE}$.

$m\overset{\frown}{CB} = m\angle CFB$	
$= 90$	$\angle CFB$ is a right angle.
$m\overset{\frown}{CBE} = m\overset{\frown}{CB} + m\overset{\frown}{BE}$	Arc Addition Postulate
$m\overset{\frown}{CBE} = 90 + 50$ or 140	Substitution

c. $m\overset{\frown}{ACE}$

One way to find $m\overset{\frown}{ACE}$ is by using $\overset{\frown}{ACB}$ and $\overset{\frown}{BE}$. $\overset{\frown}{ACB}$ is a semicircle.

$m\overset{\frown}{ACE} = m\overset{\frown}{ACB} + \overset{\frown}{BE}$	Arc Addition Postulate
$m\overset{\frown}{ACE} = 180 + 50$ or 230	Substitution

Find each measure.

2A. $m\overset{\frown}{CD}$ **2B.** $m\overset{\frown}{DCB}$ **2C.** $m\overset{\frown}{CAE}$

Personal Tutor at geometryonline.com

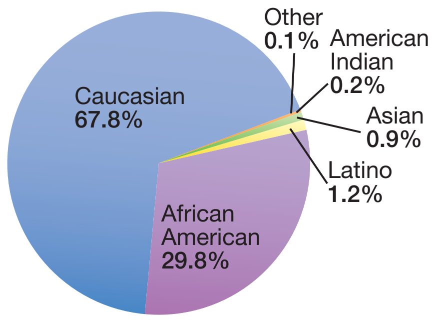
In a circle graph, the central angles divide a circle into wedges to represent data, often expressed as a percent. The size of the angle is proportional to the percent.

Circle Graphs

3 **POPULATION Refer to the graphic. Find the measurement of the central angle for each category.** The sum of the percents is 100% and represents the whole. Use the percents to determine what part of the whole circle (360°) each central angle contains.

$$67.8\%(360°) = 244.08°$$
$$29.8\%(360°) = 107.28°$$
$$1.2\%(360°) = 4.32°$$
$$0.9\%(360°) = 3.24°$$
$$0.2\%(360°) = 0.72°$$
$$0.1\%(360°) = 0.36°$$

South Carolina Population by Race



Source: U.S. Census Bureau

SPORTS **Refer to the table, which shows the seven most popular sports for females by percentage of participation.**

3A. If you were to construct a circle graph of this information, how many degrees would be needed for each category?

3B. Do any of the sports have congruent arcs? Why or why not? What is the measurement of the arcs for volleyball and soccer together?

Female Participation in Sports	
basketball	20%
track & field	18%
volleyball	18%
softball (fast pitch)	16%
soccer	14%
cross country	7%
tennis	7%

Source: NFHS

Arc Length Another way to measure an arc is by its length. An arc is part of the circle, so the length of an arc is a part of the circumference.

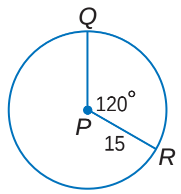
Study Tip

Look Back

To review **proportions**, see Lesson 7-1.

EXAMPLE Arc Length

4 **In $\odot P$, $PR = 15$ and $m\angle QPR = 120$. Find the length of $\widehat{QR}$.**



In $\odot P$, $r = 15$, so $C = 2\pi(15)$ or 30π and $m\widehat{QR} = m\angle QPR$ or 120. Write a proportion to compare each part to its whole.

$$\text{degree measure of arc} \rightarrow \frac{120}{360} = \frac{\ell}{30\pi} \leftarrow \text{arc length}$$
$$\text{degree measure of whole circle} \rightarrow \qquad \leftarrow \text{circumference}$$

Now solve the proportion for ℓ.

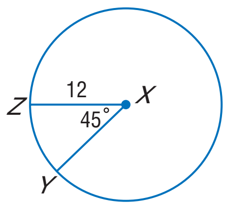
$$\frac{120}{360} = \frac{\ell}{30\pi}$$

$$\frac{120}{360}(30\pi) = \ell \quad \text{Multiply each side by } 30\pi.$$

$$10\pi = \ell \quad \text{Simplify.}$$

The length of $\widehat{QR}$ is 10π units or about 31.42 units.

4. Find the length of $\widehat{ZY}$.



The proportion used to find the arc length in Example 4 can be adapted to find the arc length in any circle.

KEY CONCEPT ***Arc Length***

$$\text{degree measure of arc} \rightarrow \frac{A}{360} = \frac{\ell}{2\pi r} \leftarrow \text{arc length}$$
$$\text{degree measure of whole circle} \rightarrow \qquad \leftarrow \text{circumference}$$

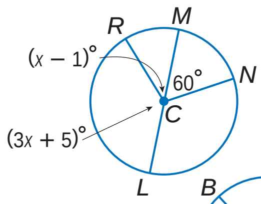
This can also be expressed as $\frac{A}{360} \cdot C = \ell$.

✓ CHECK Your Understanding

Example 1 (p. 563)

ALGEBRA **Find each measure.**

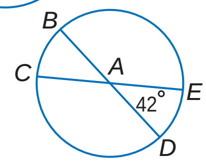
1. $m\angle NCL$
2. $m\angle RCL$
3. $m\angle RCM$
4. $m\angle RCN$



Example 2 (p. 565)

In $\odot A$, $m\angle EAD = 42$. Find each measure.

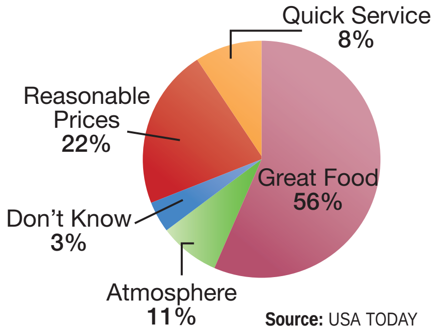
5. $m\widehat{BC}$
6. $m\widehat{CBE}$
7. $m\widehat{EDB}$
8. $m\widehat{CD}$



Example 3 (p. 565)

9. **RESTAURANTS** The graph shows the results of a survey taken by diners relating what is most important about the restaurants where they eat. Determine the measurement of each angle of the graph. Round to the nearest degree.

What Diners Want



Source: USA TODAY

Example 4 (p. 566)

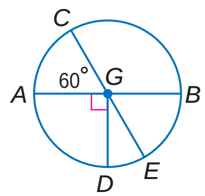
10. Points T and R lie on $\odot W$ so that $WR = 12$ and $m\angle TWR = 60$. Find the length of $\widehat{TR}$.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
11–20	1
21–28	2
29–31	3
32–35	4

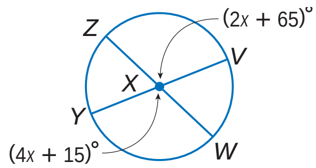
Find each measure.

11. $m\angle CGB$
12. $m\angle BGE$
13. $m\angle AGD$
14. $m\angle DGE$
15. $m\angle CGD$
16. $m\angle AGE$



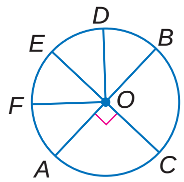
ALGEBRA **Find each measure.**

17. $m\angle ZXV$
18. $m\angle YXW$
19. $m\angle ZXY$
20. $m\angle VXW$



In $\odot O$, $\overline{EC}$ and $\overline{AB}$ are diameters, and $\angle BOD \cong \angle DOE \cong \angle EOF \cong \angle FOA$. Find each measure.

21. $m\widehat{BC}$
22. $m\widehat{AC}$
23. $m\widehat{AE}$
24. $m\widehat{EB}$
25. $m\widehat{ACB}$
26. $m\widehat{AD}$
27. $m\widehat{CBF}$
28. $m\widehat{ADC}$



FOOD **For Exercises 29–31, refer to the table and use the following information.**
A recent survey asked Americans how long food could be on the floor and still be safe to eat. The results are shown in the table.

Dropped Food	
Do you eat food dropped on the floor?	
Not safe to eat	78%
Three-second rule*	10%
Five-second rule*	8%
Ten-second rule*	4%

Source: American Diabetic Association

* The length of time the food is on the floor.

29. If you were to construct a circle graph of this information, how many degrees would be needed for each category?

30. Describe the kind of arc associated with each category.

31. Construct a circle graph for these data.

Real-World Link

In the Great Plains of the United States, farmers use center-pivot irrigation systems to water crops. New low-energy spray systems water circles of land that are thousands of feet in diameter with minimal water loss to evaporation from the spray.

Source: U.S. Geological Survey

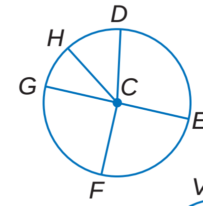
EXTRA PRACTICE

See pages 819, 837.

Math Online

Self-Check Quiz at geometryonline.com

The diameter of ⊙C is 32 units long. Find the length of each arc for the given angle measure.



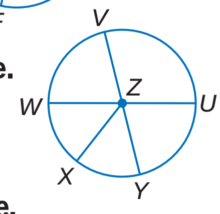
32. $\widehat{DE}$ if $m\angle DE = 100$

33. $\widehat{DHE}$ if $m\angle DCE = 90$

34. $\widehat{HDF}$ if $m\angle HCF = 125$

35. $\widehat{HD}$ if $m\angle DCH = 45$

ALGEBRA In ⊙Z, $\angle WZX \cong \angle XZY$, $m\angle VZU = 4x$, $m\angle UZY = 2x + 24$, and $\overline{VY}$ and $\overline{WU}$ are diameters. Find each measure.



36. $m\widehat{UY}$ **37.** $m\widehat{WV}$ **38.** $m\widehat{WX}$ **39.** $m\widehat{XY}$

40. $m\widehat{WUY}$ **41.** $m\widehat{YVW}$ **42.** $m\widehat{XVY}$ **43.** $m\widehat{WUX}$

Determine whether each statement is *sometimes, always,* or *never* true.

44. The measure of a major arc is greater than 180.

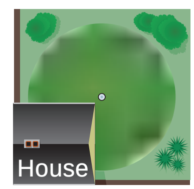
45. The central angle of a minor arc is an acute angle.

46. The sum of the measures of the central angles of a circle depends on the measure of the radius.

47. The semicircles of two congruent circles are congruent.

48. CLOCKS The hands of a clock form the same angle at various times of the day. For example, the angle formed at 2:00 is congruent to the angle formed at 10:00. If a clock has a diameter of 1 foot, what is the distance along the edge of the clock from the minute hand to the hour hand at 2:00?

49. IRRIGATION Some irrigation systems spray water in a circular pattern. You can adjust the nozzle to spray in certain directions. The nozzle in the diagram is set so it does not spray on the house. If the spray has a radius of 12 feet, what is the approximate length of the arc that the spray creates?



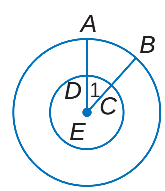
H.O.T. Problems

50. PROOF Write a proof of Theorem 10.1.

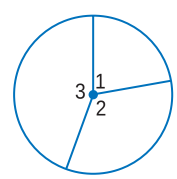
51. REASONING Compare and contrast *concentric* circles and *congruent* circles.

52. OPEN ENDED Draw a circle and locate three points on the circle. Name all of the arcs determined by the three points and use a protractor to find the measure of each arc.

53. CHALLENGE The circles at the right are concentric circles that both have point E as their center. If $m\angle 1 = 42$, determine whether $\widehat{AB} \cong \widehat{CD}$. Explain.

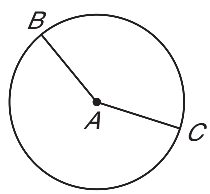


54. CHALLENGE Central angles 1, 2, and 3 have measures in the ratio 2:3:4. Find the measure of each angle.



55. Writing in Math Use the information about clocks on page 563 to explain what kinds of angles the hands on a clock form. Include the kind of angle formed by the hands of a clock, and describe several times of day when these angles are congruent.

56. In the figure, $\overline{AB}$ is a radius of circle A, and $\widehat{BC}$ is a minor arc.



If $AB = 5$ inches and the length of $\widehat{BC}$ is 4π inches, what is $m\angle BAC$?

A 150° **C** 120°

B 144° **D** 72°

57. **REVIEW** Lupe received a 4% raise at her job. If she was earning x dollars before, which expression represents how much money she is earning now?

F $x + 0.4x$

G $x + 0.4$

H $x + 0.04x$

J $x + 0.04$

The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth. (Lesson 10-1)

58. $r = 10$, $d =$ ___?___, $C =$ ___?___

59. $d = 13$, $r =$ ___?___, $C =$ ___?___

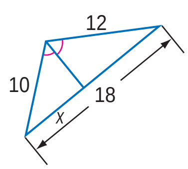
60. $C = 28\pi$, $d =$ ___?___, $r =$ ___?___

61. $C = 75.4$, $d =$ ___?___, $r =$ ___?___

62. **SOCCER** Two soccer players kick the ball at the same time. One exerts a force of 72 newtons east. The other exerts a force of 45 newtons north. What is the magnitude to the nearest tenth and direction to the nearest degree of the resultant force on the soccer ball? (Lesson 9-6)

ALGEBRA Find *x*. (Lesson 7-5)

63.



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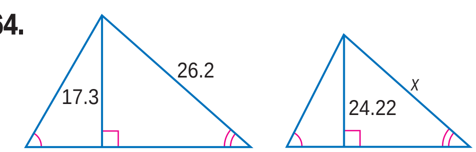


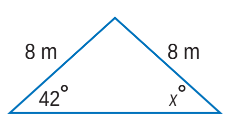
Find the exact distance between each point and line or pair of lines. (Lesson 3-6)

65. point $Q(6, -2)$ and the line with equation $y - 7 = 0$

66. parallel lines with equations $y = x + 3$ and $y = x - 4$

PREREQUISITE SKILL **Find *x*.** (Lesson 4-6)

67.



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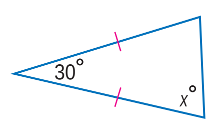

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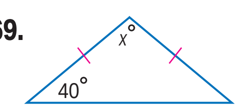

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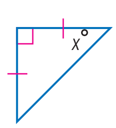
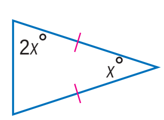


71.



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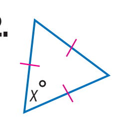


10-3 Arcs and Chords

Main Ideas

- Recognize and use relationships between arcs and chords.
- Recognize and use relationships between chords and diameters.

New Vocabulary

inscribed
circumscribed

GET READY for the Lesson

Waffle irons have grooves in each heated plate that result in the waffle pattern when the batter is cooked. One model of a Belgian waffle iron is round, and each groove is a chord of the circle.

Arcs and Chords The endpoints of a chord are also endpoints of an arc. If you trace the waffle pattern on patty paper and fold along the diameter, $\overline{AB}$ and $\overline{CD}$ match exactly, as well as $\widehat{AB}$ and $\widehat{CD}$. This suggests the following theorem.

Reading Math

If and only if Remember that the phrase *if and only if* means that the conclusion and the hypothesis can be switched and the statement is still true.

THEOREM 10.2

In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

Abbreviations:

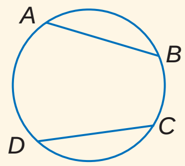
In ⊙, 2 minor arcs are ≅, corr. chords are ≅.

In ⊙, 2 chords are ≅, corr. minor arcs are ≅.

Examples:

If $\overline{AB} \cong \overline{CD}$, $\widehat{AB} \cong \widehat{CD}$.

If $\widehat{AB} \cong \widehat{CD}$, $\overline{AB} \cong \overline{CD}$.



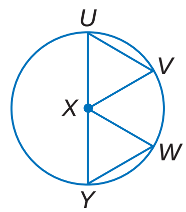
You will prove part 2 of Theorem 10.2 in Exercise 1.

EXAMPLE 1 Prove Theorem 10.2

Theorem 10.2 (part 1)

Given: $\odot X$, $\widehat{UV} \cong \widehat{YW}$

Prove: $\overline{UV} \cong \overline{YW}$



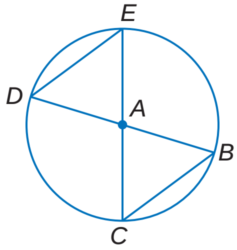
Proof:

Statements	Reasons
1. $\odot X$, $\widehat{UV} \cong \widehat{YW}$	**1.** Given
2. $\angle UXV \cong \angle WXY$	**2.** If arcs are ≅, their corresponding central ∠s are ≅.
3. $\overline{UX} \cong \overline{XV} \cong \overline{WX} \cong \overline{XY}$	**3.** All radii of a circle are congruent.
4. $\triangle UXV \cong \triangle WXY$	**4.** SAS
5. $\overline{UV} \cong \overline{YW}$	**5.** CPCTC

KS Studios

1. **Given:** $\odot A$, $\widehat{BC} \cong \widehat{DE}$

Prove: $\overline{BC} \cong \overline{DE}$

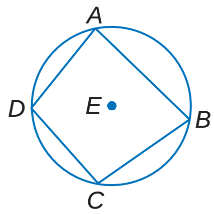


Study Tip

Circumcircle

The *circumcircle* of a polygon is a circle that passes through all of the vertices of a polygon.

The chords of adjacent arcs can form a polygon. Quadrilateral *ABCD* is an **inscribed** polygon because all of its vertices lie on the circle. Circle *E* is **circumscribed** about the polygon because it contains all the vertices of the polygon.

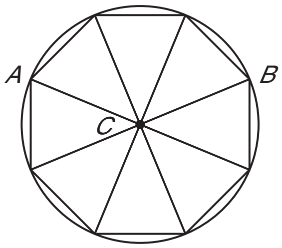


STANDARDIZED TEST EXAMPLE

2 A regular octagon is inscribed in a circle as part of a stained glass art piece. If opposite vertices are connected by line segments, what is the measure of angle *ACB*?

A 108 **C** 135

B 120 **D** 150



All the central angles of a regular polygon are congruent. The measure of each angle of a rectangular octagon is 360 ÷ 8 or 45. Angle *ACB* is made up of three central angles, so its measure is 3(45) or 135. The correct answer is C.

Real-World Link

The Pentagon, in Washington, D.C., houses the Department of Defense. Construction was finished on January 15, 1943. About 23,000 employees work at the Pentagon.

Source: defenselink.mil

2. A circle is circumscribed about a regular pentagon. What is the measure of the arc between each pair of consecutive vertices?

F 60

G 72

H 36

J 30

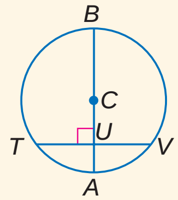
Personal Tutor at geometryonline.com

Diameters and Chords Diameters that are perpendicular to chords create special segment and arc relationships. Suppose you draw circle *C* and one of its chords $\overline{WX}$ on a piece of patty paper and fold the paper to construct the perpendicular bisector. You will find that the bisector also cuts $\widehat{WX}$ in half and passes through the center of the circle, making it contain a diameter.

THEOREM 10.3

In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.

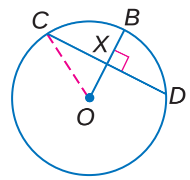
Example: If $\overline{BA} \perp \overline{TV}$, then $\overline{UT} \cong \overline{UV}$ and $\widehat{AT} \cong \widehat{AV}$.



You will prove Theorem 10.3 in Exercise 8.

EXAMPLE Radius Perpendicular to a Chord

3 **Circle O has a radius of 13 inches. Radius $\overline{OB}$ is perpendicular to chord $\overline{CD}$, which is 24 inches long.**



a. If $m\widehat{CD} = 134$, find $m\widehat{CB}$.

$\overline{OB}$ bisects $\widehat{CD}$, so $m\widehat{CB} = \frac{1}{2}m\widehat{CD}$.

$m\widehat{CB} = \frac{1}{2}m\widehat{CD}$ — Definition of arc bisector

$= \frac{1}{2}(134)$ or 67 — $m\widehat{CD} = 134$

b. Find OX.

Draw radius $\overline{OC}$. $\triangle CXO$ is a right triangle.

$CO = 13$ — $r = 13$

$\overline{OB}$ bisects $\overline{CD}$. — A radius perpendicular to a chord bisects it.

$CX = \frac{1}{2}(CD)$ — Definition of segment bisector

$= \frac{1}{2}(24)$ or 12 — $CD = 24$

Use the Pythagorean Theorem to find OX.

$(CX)^2 + (OX)^2 = (CO)^2$ — Pythagorean Theorem

$12^2 + (OX)^2 = 13^2$ — $CX = 12$, $CO = 13$

$144 + (OX)^2 = 169$ — Simplify.

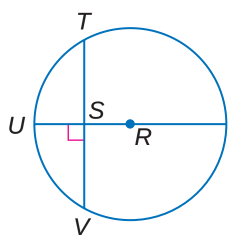
$(OX)^2 = 25$ — Subtract 144 from each side.

$OX = 5$ — Take the square root of each side.

Circle R has a radius of 16 centimeters. Radius $\overline{RU}$ is perpendicular to chord $\overline{TV}$, which is 22 centimeters long.

3A. If $m\widehat{TV} = 110$, find $m\widehat{UV}$.

3B. Find RS.



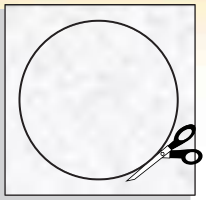
In the next lab, you will discover another property of congruent chords.

GEOMETRY LAB

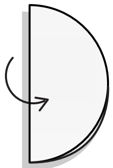
Congruent Chords and Distance

MODEL

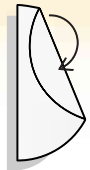
Step 1 Use a compass to draw a large circle on patty paper. Cut out the circle.



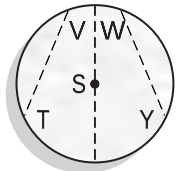
Step 2 Fold the circle in half.



Step 3 Without opening the circle, fold the edge of the circle so it does not intersect the first fold.



Step 4 Unfold the circle and label as shown.



Step 5 Fold the circle, laying point V onto T to bisect the chord. Open the circle and fold again to bisect $\overline{WY}$. Label the intersection points U and X as shown.

ANALYZE

1. What is the relationship between $\overline{SU}$ and $\overline{VT}$? $\overline{SX}$ and $\overline{WY}$?

2. Use a centimeter ruler to measure $\overline{VT}$, $\overline{WY}$, $\overline{SU}$, and $\overline{SX}$. What do you find?

3. **Make a conjecture** about the distance that two chords are from the center when they are congruent.

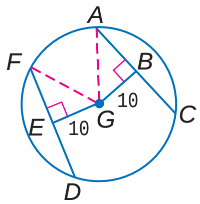
THEOREM 10.4

In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

You will prove Theorem 10.4 in Exercises 34 and 35.

EXAMPLE Chords Equidistant from Center

4 **Chords $\overline{AC}$ and $\overline{DF}$ are equidistant from the center. If the radius of $\odot G$ is 26, find AC and DE.**



$\overline{AC}$ and $\overline{DF}$ are equidistant from G, so $\overline{AC} \cong \overline{DF}$.

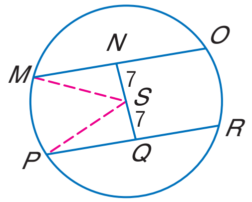
Draw $\overline{AG}$ and $\overline{GF}$ to form two right triangles.

$(AB)^2 + (BG)^2 = (AG)^2$	Pythagorean Theorem
$(AB)^2 + 10^2 = 26^2$	$BG = 10$, $AG = 26$
$(AB)^2 + 100 = 676$	Simplify.
$(AB)^2 = 576$	Subtract 100 from each side.
$AB = 24$	Take the square root of each side.

$AB = \frac{1}{2}(AC)$, so $AC = 2(24)$ or 48.

$\overline{AC} \cong \overline{DF}$, so DF also equals 48. $DE = \frac{1}{2}DF$, so $DE = \frac{1}{2}(48)$ or 24.

4. Chords $\overline{MO}$ and $\overline{PR}$ are equidistant from the center. If the radius of $\odot S$ is 15, find MO and PQ.



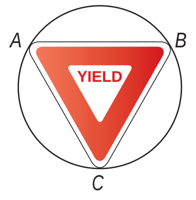
CHECK Your Understanding

Example 1 (p. 570)

1. **PROOF** (Part 2 of Theorem 10.2) Given $\odot X$ and $\overline{UV} \cong \overline{YW}$, prove $\overset{\frown}{UV} \cong \overset{\frown}{YW}$. (Use the figure from part 1 of Theorem 10.2.)

Example 2 (p. 571)

2. **STANDARDIZED TEST PRACTICE** A yield sign, an equilateral triangle, isinscribed in a circle. What is the measure of $\overset{\frown}{ABC}$?



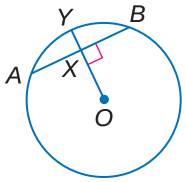
A 60 **C** 180

B 120 **D** 240

Example 3 (p. 572)

Circle O has a radius of 10, $AB = 10$, and $m\overset{\frown}{AB} = 60$. Find each measure.

3. $m\overset{\frown}{AY}$ 4. AX 5. OX

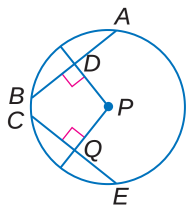


Exercises 3–5

Example 4 (p. 573)

In $\odot P$, $PD = 10$, $PQ = 10$, and $QE = 20$. Find each measure.

6. AB 7. PE



Exercises 6–7

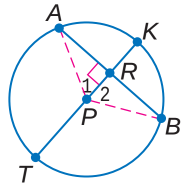
Exercises

HOMEWORK HELP	
For Exercises	See Examples
8	1
9–11	2
12–19	3
20–27	4

8. **PROOF** **Write a two-column proof of Theorem 10.3.**

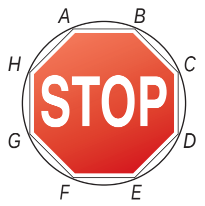
Given: $\odot P$, $\overline{AB} \perp \overline{TK}$

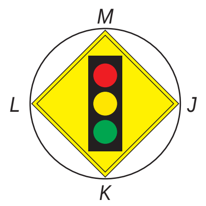
Prove: $\overline{AR} \cong \overline{BR}$, $\overset{\frown}{AK} \cong \overset{\frown}{BK}$

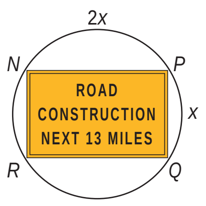


TRAFFIC SIGNS **Determine the measure for each arc of the circle circumscribed about the traffic sign.**

9. regular octagon 10. square 11. rectangle







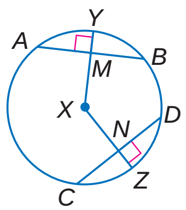
In $\odot X$, $AB = 30$, $CD = 30$, and $m\overset{\frown}{CZ} = 40$. Find each measure.

12. AM 13. MB

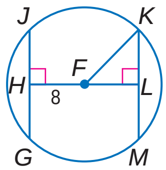
14. CN 15. ND

16. $m\overset{\frown}{DZ}$ 17. $m\overset{\frown}{CD}$

18. $m\overset{\frown}{AB}$ 19. $m\overset{\frown}{YB}$



In $\odot F$, $\overline{FH} \cong \overline{FL}$ and $FK = 17$. Find each measure.



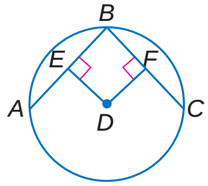
20. LK

21. KM

22. JG

23. JH

In $\odot D$, $CF = 8$, $DE = FD$, and $DC = 10$. Find each measure.



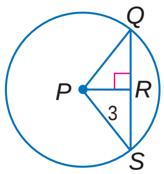
24. FB

25. BC

26. AB

27. ED

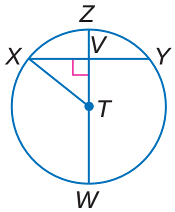
The radius of $\odot P$ is 5 and $PR = 3$. Find each measure.



28. QR

29. QS

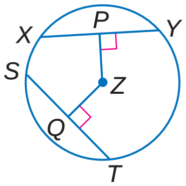
In $\odot T$, $ZV = 1$, and $TW = 13$. Find each measure.



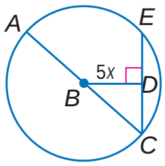
30. XV

31. XY

32. ALGEBRA In $\odot Z$, $PZ = ZQ$, $XY = 4a - 5$, and $ST = -5a + 13$. Find SQ.



33. ALGEBRA In $\odot B$, the diameter is 20 units long, and $m\angle ACE = 45$. Find x.



Real-World Link

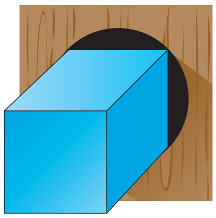
Many everyday sayings or expressions have a historical origin. The square peg comment is attributed to Sydney Smith (1771–1845), a witty British lecturer, who said "Trying to get those two together is like trying to put a square peg in a round hole."

PROOF **Write a proof for each part of Theorem 10.4.**

34. In a circle, if two chords are equidistant from the center, then they are congruent.

35. In a circle, if two chords are congruent, then they are equidistant from the center.

36. SAYINGS An old adage states that "You can't fit a square peg in a round hole." Actually, you can; it just won't fill the hole. If a hole is 4 inches in diameter, what is the approximate width of the largest square peg that fits in the round hole?



For Exercises 37–39, draw and label a figure. Then solve.

37. The radius of a circle is 34 meters long, and a chord of the circle is 60 meters long. How far is the chord from the center of the circle?

38. The diameter of a circle is 60 inches, and a chord of the circle is 48 inches long. How far is the chord from the center of the circle?

39. A chord of a circle is 48 centimeters long and is 10 centimeters from the center of the circle. Find the radius.

Study Tip

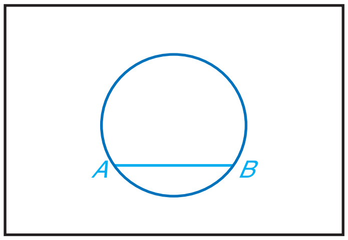
Finding the Center of a Circle

The process Mr. Ortega used can be done by construction and is often called *locating the center of a circle.*

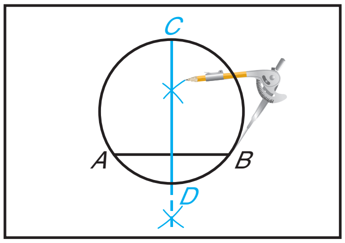
40. CARPENTRY Mr. Ortega wants to drill a hole in the center of a round picnic table for an umbrella pole. To locate the center of the circle, he draws two chords of the circle and uses a ruler to find the midpoint for each chord. Then he uses a framing square to draw a line perpendicular to each chord at its midpoint. Explain how this process locates the center of the tabletop.

CONSTRUCTION Consider the following construction for Exercises 41–43.

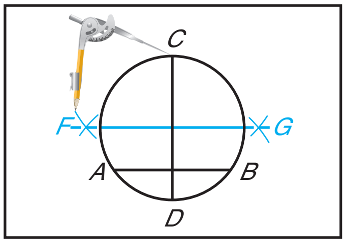
Step 1 Trace the bottom of a circular object and draw a chord $\overline{AB}$.



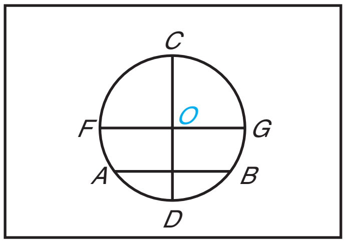
Step 2 Construct the perpendicular bisector of $\overline{AB}$. Label it $\overline{CD}$.



Step 3 Construct the perpendicular bisector of $\overline{CD}$. Label it $\overline{FG}$.



Step 4 Label the point of intersection of the two perpendicular bisectors O.



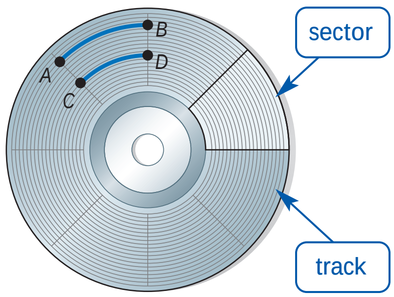
41. Repeat this construction using a different circular object.

42. Use an indirect proof to show that $\overline{CD}$ passes through the center of the circle by assuming that the center of the circle is *not* on $\overline{CD}$.

43. Prove that O is the center of the circle.

COMPUTERS For Exercises 44 and 45, use the following information.

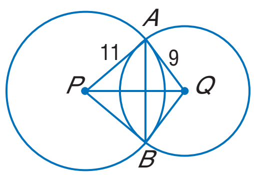
A computer hard drive contains platters divided into tracks, which are defined by concentric circles, and sectors, defined by radii of the circles.



44. In the diagram at the right, what is the relationship between $m\widehat{AB}$ and $m\widehat{CD}$?

45. Are $\widehat{AB}$ and $\widehat{CD}$ congruent? Explain.

46. The common chord $\overline{AB}$ between $\odot P$ and $\odot Q$ is perpendicular to the segment connecting the centers of the circles. If $AB = 10$, what is the length of $\overline{PQ}$? Explain your reasoning.



EXTRA PRACTICE
See pages 820, 837.

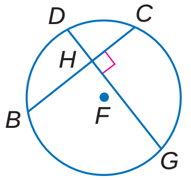
Math Online
Self-Check Quiz at geometryonline.com

Aaron Haupt

H.O.T. Problems

47. OPEN ENDED Construct a circle and inscribe any polygon. Draw the radii and use a protractor to determine whether any sides of the polygon are congruent. Describe a situation in which this would be important.

48. FIND THE ERROR Lucinda and Tokei are writing conclusions about the chords in ⊙F. Who is correct? Explain your reasoning.



Lucinda
Because $\overline{DG} \perp \overline{BC}$, $\angle DHB \cong \angle DHC \cong \angle CHG \cong \angle BHG$, and $\overline{DG}$ bisects $\overline{BC}$.

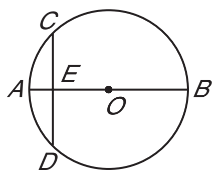
Tokei
$\overline{DG} \perp \overline{BC}$, but $\overline{DG}$ does not bisect $\overline{BC}$ because it is not a diameter.

49. CHALLENGE A diameter of ⊙P has endpoints A and B. Radius $\overline{PQ} \perp \overline{AB}$. Chord $\overline{DE}$ bisects $\overline{PQ}$ and is parallel to $\overline{AB}$. Does $DE = \frac{1}{2}(AB)$? Explain.

50. Writing in Math Refer to the information about Belgian waffles on page 570. Explain how the grooves in a Belgian waffle iron model segments in a circle. Include a description of how you might find the length of a groove without directly measuring it.

STANDARDIZED TEST PRACTICE

51. $\overline{AB}$ is a diameter of circle O and intersects chord $\overline{CD}$ at point E.



If $AE = 2$ and $OB = 10$, what is the length of $\overline{CD}$?

A 4 **C** 8

B 6 **D** 9

52. REVIEW

Solve: $-4y > 18 - 2(y + 8)$

Step 1: $-4y > 18 - 2y - 16$

Step 2: $-4y > -2y + 2$

Step 3: $-2y > 2$

Step 4: $y > -1$

Which is the first *incorrect* step in the solution shown above?

F Step 1 **H** Step 3

G Step 2 **J** Step 4

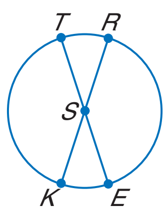
In ⊙S, $m\angle TSR = 42$. Find each measure. (Lesson 10-2)

53. $m\overparen{KT}$ **54.** $\overparen{ERT}$ **55.** $\overparen{KRT}$

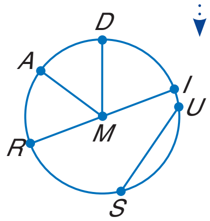
Refer to ⊙M. (Lesson 10-1)

56. Name a chord that is not a diameter.

57. If $MD = 7$, find RI.



Exercises 53–55



Exercises 56–57

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation. (Pages 781 and 782)

58. $\frac{1}{2}x = 120$ **59.** $2x = \frac{1}{2}(45 + 35)$ **60.** $3x = \frac{1}{2}(120 - 60)$ **61.** $45 = \frac{1}{2}(4x + 30)$

10-4 Inscribed Angles

Main Ideas

- Find measures of inscribed angles.
- Find measures of angles of inscribed polygons.

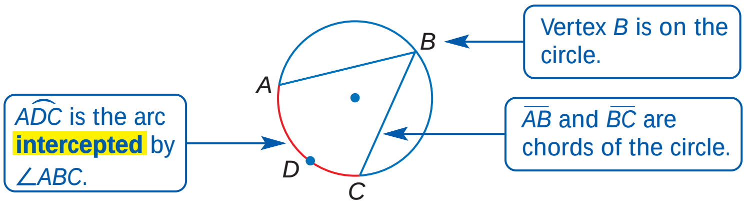
New Vocabulary

intercepted

GET READY for the Lesson

A socket is a tool that comes in varying diameters. It is used to tighten or unscrew nuts or bolts. The "hole" in the socket is a hexagon cast in a metal cylinder.

Inscribed Angles In Lesson 10-3, you learned that a polygon that has its vertices on a circle is called an inscribed polygon. Likewise, an *inscribed angle* is an angle that has its vertex on the circle and its sides contained in chords of the circle.

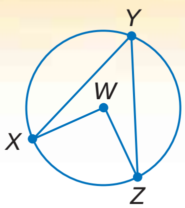


GEOMETRY LAB

Measure of Inscribed Angles

MODEL

- Use a compass to draw $\odot W$.
- Draw an inscribed angle and label it XYZ.
- Draw $\overline{WX}$ and $\overline{WZ}$.



ANALYZE

1. Measure $\angle XYZ$ and $\angle XWZ$.

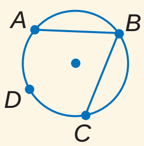
2. Find $m\widehat{XZ}$ and compare it with $m\angle XYZ$.

3. Make a conjecture about the relationship of the measure of an inscribed angle and the measure of its intercepted arc.

THEOREM 10.5 — *Inscribed Angle Theorem*

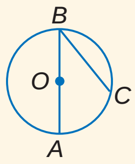
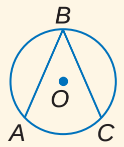
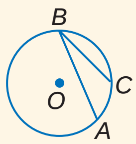
If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc (or the measure of the intercepted arc is twice the measure of the inscribed angle).

Example: $m\angle ABC = \frac{1}{2}(m\widehat{ADC})$ or $2(m\angle ABC) = m\widehat{ADC}$

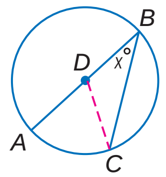


Aaron Haupt

To prove Theorem 10.5, you must consider three cases.

	Case 1	Case 2	Case 3
Model of Angle Inscribed in $\odot O$			
Location of center of circle	on a side of the angle	in the interior of the angle	in the exterior of the angle

PROOF **Theorem 10.5 (Case 1)**



Given: $\angle ABC$ inscribed in $\odot D$ and $\overline{AB}$ is a diameter.

Prove: $m\angle ABC = \frac{1}{2}m\widehat{AC}$

Draw $\overline{DC}$ and let $m\angle B = x$.

Proof:

Since $\overline{DB}$ and $\overline{DC}$ are congruent radii, $\triangle BDC$ is isosceles and $\angle B \cong \angle C$. Thus, $m\angle B = m\angle C = x$. By the Exterior Angle Theorem, $m\angle ADC = m\angle B + m\angle C$. So $m\angle ADC = 2x$. From the definition of arc measure, we know that $m\widehat{AC} = m\angle ADC$ or $2x$. Comparing $m\widehat{AC}$ and $m\angle ABC$, we see that $m\widehat{AC} = 2(m\angle ABC)$ or that $m\angle ABC = \frac{1}{2}m\widehat{AC}$.

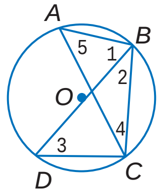
You will prove Cases 2 and 3 of Theorem 10.5 in Exercises 33 and 34.

Study Tip

Using Variables

You can also assign a variable to an unknown measure. So, if you let $m\widehat{AD} = x$, the second equation becomes $140 + 100 + x + x = 360$, or $240 + 2x = 360$. This last equation may seem simpler to solve.

EXAMPLE 1 **Measures of Inscribed Angles**



In $\odot O$, $m\widehat{AB} = 140$, $m\widehat{BC} = 100$, and $m\widehat{AD} = m\widehat{DC}$. Find the measures of $\angle 1$, $\angle 2$, and $\angle 3$.

First determine $m\widehat{DC}$ and $m\widehat{AD}$.

$m\widehat{AB} + m\widehat{BC} + m\widehat{DC} + m\widehat{AD} = 360$	Arc Addition Theorem
$140 + 100 + m\widehat{DC} + m\widehat{DC} = 360$	$m\widehat{AB} = 140$, $m\widehat{BC} = 100$, $m\widehat{DC} = m\widehat{AD}$
$240 + 2(m\widehat{DC}) = 360$	Simplify.
$2(m\widehat{DC}) = 120$	Subtract 240 from each side.
$m\widehat{DC} = 60$	Divide each side by 2.

So, $m\widehat{DC} = 60$ and $m\widehat{AD} = 60$.

$m\angle 1 = \frac{1}{2}m\widehat{AD} = \frac{1}{2}(60)$ or 30

$m\angle 2 = \frac{1}{2}m\widehat{DC} = \frac{1}{2}(60)$ or 30

$m\angle 3 = \frac{1}{2}m\widehat{BC} = \frac{1}{2}(100)$ or 50

1A. Find $m\angle 4$.

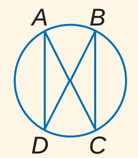
1B. Find $m\angle 5$.

In Example 1, note that $\angle 3$ and $\angle 5$ intercept the same arc and are congruent.

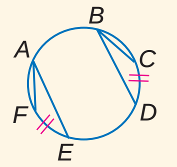
THEOREM 10.6

If two inscribed angles of a circle (or congruent circles) intercept congruent arcs or the same arc, then the angles are congruent.

Abbreviations:
Inscribed ⦣ of ≅ arcs are ≅.
Inscribed ⦣ of same arc are ≅.



$\angle DAC \cong \angle DBC$



$\angle FAE \cong \angle CBD$

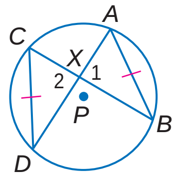
You will prove Theorem 10.6 in Exercise 35.

EXAMPLE 2 Proof with Inscribed Angles

Given: $\odot P$ with $\overline{CD} \cong \overline{AB}$

Prove: $\triangle AXB \cong \triangle CXD$

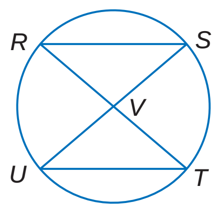
Proof:



Statements	Reasons
1. $\angle DAB$ intercepts $\overset{\frown}{DB}$. $\angle DCB$ intercepts $\overset{\frown}{DB}$.	**1.** Definition of intercepted arc
2. $\angle DAB \cong \angle DCB$	**2.** Inscribed ⦣ of same arc are ≅.
3. $\angle 1 \cong \angle 2$	**3.** Vertical ⦣ are ≅.
4. $\overline{CD} \cong \overline{AB}$	**4.** Given
5. $\triangle AXB \cong \triangle CXD$	**5.** AAS

2. Given: $\overline{RT}$ bisects $\overline{SU}$; $\overline{RV} \cong \overline{SV}$

Prove: $\triangle RVS \cong \triangle UVT$



Study Tip

Eliminate the Possibilities

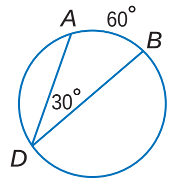
Think about what would be true if D was on minor arc $\overset{\frown}{AB}$. Then $\angle ADB$ would intercept the major arc. Thus, $m\angle ADB$ would be half of 300, or 150. This is not the desired angle measure in the problem, so you can eliminate the possibility that D can lie on $\overset{\frown}{AB}$.

You can also use the measure of an inscribed angle to determine probability of a point lying on an arc.

EXAMPLE 3 Inscribed Arcs and Probability

PROBABILITY Points A and B are on a circle so that $m\overset{\frown}{AB} = 60$. Suppose point D is randomly located on the same circle so that it does not coincide with A or B. What is the probability that $m\angle ADB = 30$?

Since the angle measure is half the arc measure, inscribed $\angle ADB$ must intercept $\overset{\frown}{AB}$, so D must lie on major arc AB. Draw a figure and label any information you know.



$m\overset{\frown}{BDA} = 360 - m\overset{\frown}{AB}$

$= 360 - 60$ or 300

Since $\angle ADB$ must intercept $\widehat{AB}$, the probability that $m\angle ADB = 30$ is the same as the probability of D being contained in $\widehat{BDA}$.

The probability that D is located on $\widehat{ADB}$ is $\frac{300}{360}$ or $\frac{5}{6}$. So, the probability that $m\angle ADB = 30$ is also $\frac{5}{6}$.

3. Points X and Y are on a circle so that $m\widehat{XY} = 90$. Suppose point Z is randomly located on the same circle so that it does not coincide with X or Y. What is the probability that $m\angle XZY = 45$?

Study Tip

Inscribed Polygons

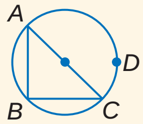
Remember that for a polygon to be an inscribed polygon, *all* of its vertices must lie on the circle.

Angles of Inscribed Polygons An inscribed triangle with a side that is a diameter is a special type of triangle.

THEOREM 10.7

If the inscribed angle of a triangle intercepts a semicircle, the angle is a right angle.

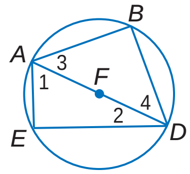
Example: $\widehat{ADC}$ is a semicircle, so $m\angle ABC = 90$.



You will prove Theorem 10.7 in Exercise 36.

EXAMPLE Angles of an Inscribed Triangle

4 **ALGEBRA Triangles ABD and ADE are inscribed in $\odot F$ with $\widehat{AB} \cong \widehat{BD}$. Find the measures of $\angle 1$ and $\angle 2$ if $m\angle 1 = 12x - 8$ and $m\angle 2 = 3x + 8$.**



AED is a right angle because $\widehat{AED}$ is a semicircle.

$m\angle 1 + m\angle 2 + m\angle AED = 180$	Angle Sum Theorem
$(12x - 8) + (3x + 8) + 90 = 180$	$m\angle 1 = 12x - 8$, $m\angle 2 = 3x + 8$, $m\angle AED = 90$
$15x + 90 = 180$	Simplify.
$15x = 90$	Subtract 90 from each side.
$x = 6$	Divide each side by 15.

Use the value of x to find the measures of $\angle 1$ and $\angle 2$.

$m\angle 1 = 12x - 8$	Given	$m\angle 2 = 3x + 8$	Given
$= 12(6) - 8$	$x = 6$	$= 3(6) + 8$	$x = 6$
$= 64$	Simplify.	$= 26$	Simplify.

CHECK $90 + 64 + 26 = 180$

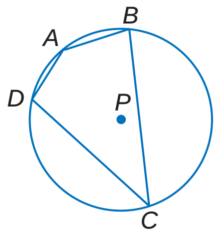
$180 = 180$ ✓

4A. Find $m\angle 3$.

4B. Find $m\angle 4$.

EXAMPLE 5 Angles of an Inscribed Quadrilateral

Quadrilateral *ABCD* is inscribed in ⊙*P*. If $m\angle B = 80$ and $m\angle C = 40$, find $m\angle A$ and $m\angle D$.



To find $m\angle A$, we need to know $m\widehat{BCD}$.

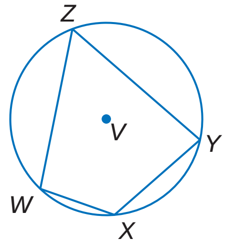
To find $m\widehat{BCD}$, first find $m\widehat{DAB}$.

$m\widehat{DAB} = 2(m\angle C)$	Inscribed Angle Theorem
$= 2(40)$ or 80	$m\angle C = 40$
$m\widehat{BCD} + m\widehat{DAB} = 360$	Sum of angles in circle $= 360$
$m\widehat{BCD} + 80 = 360$	$m\widehat{DAB} = 80$
$m\widehat{BCD} = 280$	Subtract 80 from each side.
$m\widehat{BCD} = 2(m\angle A)$	Inscribed Angle Theorem
$280 = 2(m\angle A)$	Substitution
$140 = m\angle A$	Divide each side by 2.

To find $m\angle D$, we need to know $m\widehat{ABC}$, but first we must find $m\widehat{ADC}$.

$m\widehat{ADC} = 2(m\angle B)$	Inscribed Angle Theorem
$m\widehat{ADC} = 2(80)$ or 160	$m\angle B = 80$
$m\widehat{ABC} + m\widehat{ADC} = 360$	Sum of angles in circle $= 360$
$m\widehat{ABC} + 160 = 360$	$m\widehat{ADC} = 160$
$m\widehat{ABC} = 200$	Subtract 160 from each side.
$m\widehat{ABC} = 2(m\angle D)$	Inscribed Angle Theorem
$200 = 2(m\angle D)$	Substitution
$100 = m\angle D$	Divide each side by 2.

CHECK Your Progress



5. Quadrilateral *WXYZ* is inscribed in ⊙*V*. If $m\angle W = 95$ and $m\angle Z = 60$, find $m\angle X$ and $m\angle Y$.

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In Example 5, note that the opposite angles of the quadrilateral are supplementary.

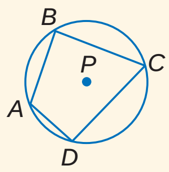
Study Tip

Quadrilaterals

Theorem 10.8 can be verified by considering that the arcs intercepted by opposite angles of an inscribed quadrilateral form a circle.

THEOREM 10.8

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.



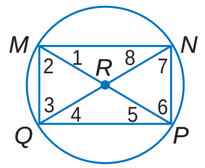
Example: Quadrilateral *ABCD* is inscribed in ⊙*P*.
$\angle A$ and $\angle C$ are supplementary.
$\angle B$ and $\angle D$ are supplementary.

You will prove Theorem 10.8 in Exercise 37.

Check Your Understanding

Example 1 (p. 579)

1. In $\odot R$, $m\overset{\frown}{MN} = 120$ and $m\overset{\frown}{MQ} = 60$. Find the measure of each numbered angle.



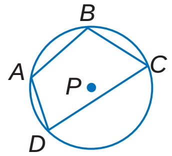
Example 2 (p. 580)

2. PROOF Write a paragraph proof.

Given: Quadrilateral $ABCD$ is inscribed in $\odot P$.

$m\angle C = \frac{1}{2} m\angle B$

Prove: $m\overset{\frown}{CDA} = 2(m\overset{\frown}{DAB})$

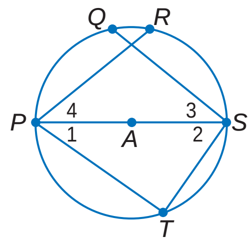


Example 3 (p. 580)

3. PROBABILITY Points X and Y are endpoints of a diameter of $\odot W$. Point Z is another point on the circle. Find the probability that $\angle XZY$ is a right angle.

Example 4 (p. 581)

4. ALGEBRA In $\odot A$ at the right, $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$. Find the measure of each numbered angle if $m\angle 1 = 6x + 11$, $m\angle 2 = 9x + 19$, $m\angle 3 = 4y - 25$, and $m\angle 4 = 3y - 9$.



Example 5 (p. 582)

5. Quadrilateral $VWXY$ is inscribed in $\odot C$. If $m\angle X = 28$ and $m\angle W = 110$, find $m\angle V$ and $m\angle Y$.

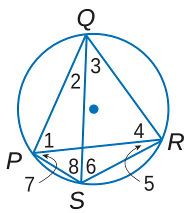
Exercises

HOMEWORK HELP

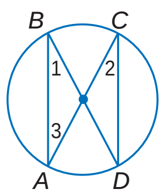
For Exercises	See Examples
6–8	1
9–10	2
11–14	3
15–19	4
20–23	5

Find the measure of each numbered angle for each figure.

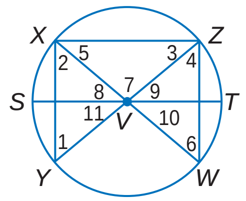
6. $\overline{PQ} \cong \overline{RQ}$, $m\overset{\frown}{PS} = 45$, and $m\overset{\frown}{SR} = 75$



7. $m\angle BDC = 25$, $m\overset{\frown}{AB} = 120$, and $m\overset{\frown}{CD} = 130$



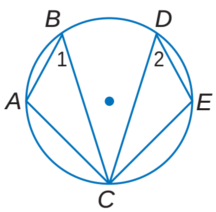
8. $m\overset{\frown}{XZ} = 100$, $\overline{XY} \perp \overline{ST}$, and $\overline{ZW} \perp \overline{ST}$



PROOF **Write a two-column proof.**

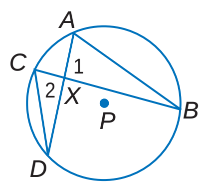
9. Given: $\overset{\frown}{AB} \cong \overset{\frown}{DE}$, $\overset{\frown}{AC} \cong \overset{\frown}{CE}$

Prove: $\triangle ABC \cong \triangle EDC$

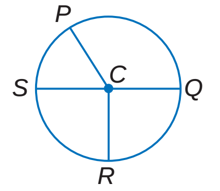


10. Given: $\odot P$

Prove: $\triangle AXB \sim \triangle CXD$



PROBABILITY **Use the following information for Exercises 11–14.**
In $\odot C$, point T is randomly selected so that it does not coincide with points P, Q, R, or S. $\overline{SQ}$ is a diameter of $\odot C$.



11. Find the probability that $m\angle PTS = 20$ if $m\widehat{PS} = 40$.

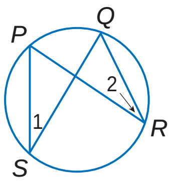
12. Find the probability that $m\angle PTR = 55$ if $m\widehat{PSR} = 110$.

13. Find the probability that $m\angle STQ = 90$.

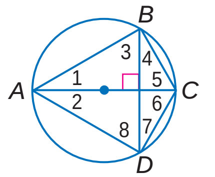
14. Find the probability that $m\angle PTQ = 180$.

ALGEBRA **Find the measure of each numbered angle for each figure.**

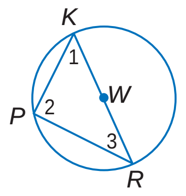
15. $m\angle 1 = x$,
$m\angle 2 = 2x - 30$



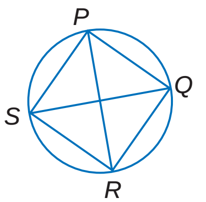
16. $m\widehat{AB} = 120$



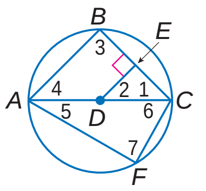
17. $m\angle R = \frac{1}{3}x + 5$,
$m\angle K = \frac{1}{2}x$



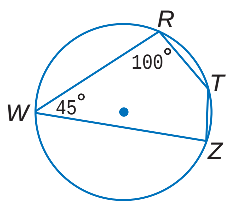
18. $PQRS$ is a rhombus inscribed in a circle. Find $m\angle QRP$ and $m\widehat{SP}$.



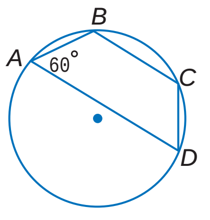
19. In $\odot D$, $\overline{DE} \cong \overline{EC}$, $m\widehat{CF} = 60$, and $\overline{DE} \perp \overline{EC}$. Find $m\angle 4$, $m\angle 5$, and $m\widehat{AF}$.



20. Quadrilateral $WRTZ$ is inscribed in a circle. Find $m\angle T$ and $m\angle Z$.



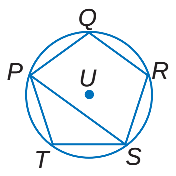
21. Trapezoid $ABCD$ is inscribed in a circle. Find $m\angle B$, $m\angle C$, and $m\angle D$.



22. Rectangle $PDQT$ is inscribed in a circle. What can you conclude about $\overline{PQ}$?

23. Square $EDFG$ is inscribed in a circle. What can you conclude about $\overline{EF}$?

Regular pentagon *PQRST* is inscribed in $\odot U$.
Find each measure.



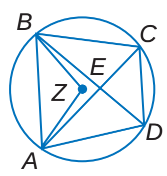
24. $m\widehat{QR}$

25. $m\angle PSR$

26. $m\angle PQR$

27. $m\widehat{PTS}$

Quadrilateral *ABCD* is inscribed in $\odot E$ such that $m\angle BZA = 104$, $m\widehat{CB} = 94$, and $\overline{AB} \parallel \overline{DC}$.
Find each measure.



28. $m\widehat{BA}$

29. $m\widehat{ADC}$

30. $m\angle BDA$

31. $m\angle ZAC$

Real-World Link Many companies that sell school rings also offer schools and individuals the option to design their own ring.

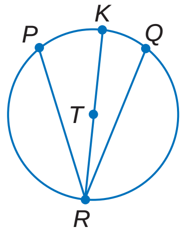
32. SCHOOL RINGS Some designs of class rings involve adding gold or silver to the surface of the round stone. The design at the right includes two inscribed angles. If $m\angle ABC = 50$ and $m\widehat{DBF} = 128$, find $m\widehat{AC}$ and $m\angle DEF$.

PROOF **Write the indicated type of proof for each theorem.**

33. two-column proof: Case 2 of Theorem 10.5

Given: T lies inside $\angle PRQ$. $\overline{RK}$ is a diameter.

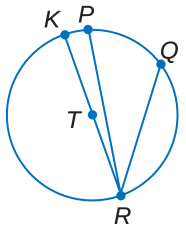
Prove: $m\angle PRQ = \frac{1}{2}m\widehat{PQ}$



34. two-column proof: Case 3 of Theorem 10.5

Given: T lies outside $\angle PRQ$. $\overline{RK}$ is a diameter.

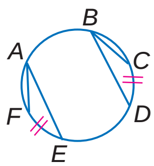
Prove: $m\angle PRQ = \frac{1}{2}m\widehat{PQ}$



35. two-column proof: Theorem 10.6

Given: $\widehat{DC} \cong \widehat{EF}$

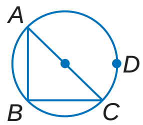
Prove: $\angle FAE \cong \angle CBD$



36. paragraph proof: Theorem 10.7

Given: $\widehat{ABC}$ is a semicircle.

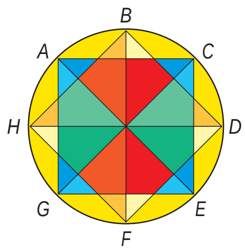
Prove: $\angle ABC$ is a right angle.



EXTRA PRACTICE
See pages 820, 837.
Math Online
Self-Check Quiz at geometryonline.com

37. Write a paragraph proof for Theorem 10.8, which states: If a quadrilateral is inscribed in a circle, then opposite angles are supplementary.

STAINED GLASS In the stained glass window design, all of the small arcs around the circle are congruent. Suppose the center of the circle is point O.



38. What is the measure of each of the small arcs?

39. What kind of figure is $\triangle AOC$? Explain.

40. What kind of figure is quadrilateral $BDFH$? Explain.

41. What kind of figure is quadrilateral $ACEG$? Explain.

H.O.T. Problems

42. REASONING Compare and contrast an inscribed angle and a central angle that intercepts the same arc.

43. OPEN ENDED Find a real-world logo with an inscribed polygon.

44. CHALLENGE A trapezoid $ABCD$ is inscribed in $\odot O$. Explain how you can verify that $ABCD$ must be an isosceles trapezoid.

45. Writing in Math Use the information about sockets on page 578 and the definition of an inscribed polygon to explain how a socket is like an inscribed polygon. Expain how you would find the length of a regular hexagon inscribed in a circle with a diameter of $\frac{3}{4}$ inch.

STANDARDIZED TEST PRACTICE

46. A square is inscribed in a circle. What is the ratio of the area of the circle to the area of the square?

A $\frac{1}{4}$

B $\frac{1}{2}$

C $\frac{\pi}{2}$

D $\frac{\pi}{4}$

47. **REVIEW** Simplify $4(3x - 2)(2x + 4) + 3x^2 + 5x - 6$.

F $9x^2 + 3x - 14$

G $9x^2 + 13x - 14$

H $27x^2 + 37x - 38$

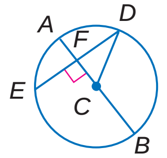
J $27x^2 + 27x - 26$

Find each measure. (Lesson 10-3)

48. If $AB = 60$ and $DE = 48$, find CF.

49. If $AB = 32$ and $FC = 11$, find FE.

50. If $DE = 60$ and $FC = 16$, find AB.

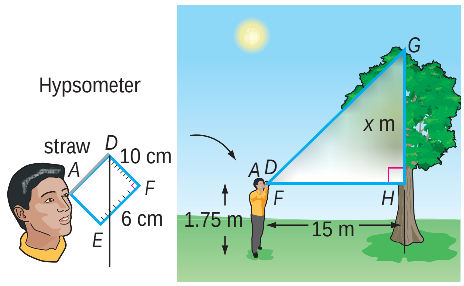


Points Q and R lie on $\odot P$. Find the length of $\widehat{QR}$ for the given radius and angle measure. (Lesson 10-2)

51. $PR = 12$, and $m\angle QPR = 60$

52. $m\angle QPR = 90$, $PR = 16$

53. **FORESTRY** A hypsometer as shown can be used to estimate the height of a tree. Bartolo looks through the straw to the top of the tree and obtains the readings given. Find the height of the tree. (Lesson 7-3)



Complete each sentence with *sometimes*, *always*, or *never*. (Lesson 4-1)

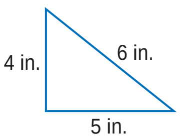
54. Equilateral triangles are __?__ isosceles.

55. Acute triangles are __?__ equilateral.

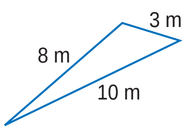
56. Obtuse triangles are __?__ scalene.

PREREQUISITE SKILL **Determine whether each figure is a right triangle.** (Lesson 8-2)

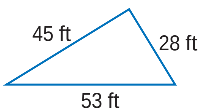
57.



58.



59.



CHAPTER 10

Mid-Chapter Quiz

Lessons 10-1 through 10-4

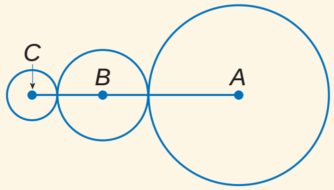
1. **MULTIPLE CHOICE** In the figure, the radius of circle A is twice the radius of circle B and four times the radius of circle C. If the sum of the circumferences of the three circles is 42π, find the measure of $\overline{AC}$.

   A 22

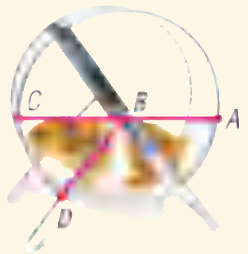
   B 27

   C 30

   D 34



For Exercises 2–8, refer to the front circular edge of the hamster wheel shown below. (Lessons 10-1 and 10-2)



2. Name the circle.
3. Name three radii of the wheel.
4. If $BD = 3x$ and $CB = 7x - 3$, find AC.
5. If $m\angle CBD = 85$, find $m\overset{\frown}{AD}$.
6. If $r = 3$ inches, find the circumference of circle B to the nearest tenth of an inch.
7. There are 40 equally spaced rungs on the wheel. What is the degree measure of an arc connecting two consecutive rungs?
8. What is the length of $\overset{\frown}{CAD}$ to the nearest tenth if $m\angle ABD = 150$ and $r = 3$?

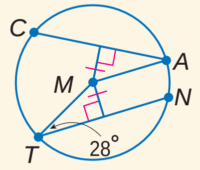
ENTERTAINMENT **For Exercises 9–11, refer to the table, which shows the number of movies students at West Lake High School see in the theater each week.** (Lesson 10-2)

Movies	
No movies	17%
1 movie	53%
2 movies	23%
3 or more movies	7%

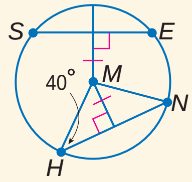
9. If you were to construct a circle graph of the data, how many degrees would be allotted to each category?
10. Describe the arcs for each category.
11. Construct a circle graph for these data.

Find each measure. (Lesson 10-3)

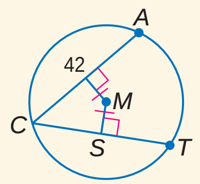
12. $m\angle CAM$



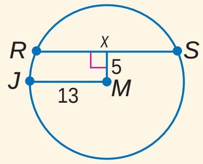
13. $m\overset{\frown}{ES}$



14. SC



15. x



16. **MULTIPLE CHOICE** The diameter of a circle is 30 inches, and a chord of the circle is 24 inches long. How far is the chord from the center of the circle? (Lesson 10-3)

    F 5 inches

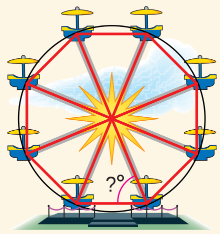
    G 7 inches

    H 9 inches

    J 11 inches

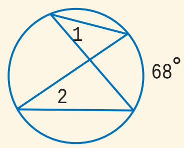
17. Quadrilateral $WXYZ$ is inscribed in a circle. If $m\angle X = 50$ and $m\angle Y = 70$, find $m\angle W$ and $m\angle Z$. (Lesson 10-4)

18. **AMUSEMENT RIDES** A Ferris wheel is shown. If the distances between the seat axles are the same, what is the measure of an angle formed by the braces attaching consecutive seats? (Lesson 10-4)



19. **PROBABILITY** In $\odot A$, point X is randomly located so that it does not coincide with points P or Q. If $m\overset{\frown}{PQ} = 160$, what is the probability that $m\angle PXQ = 80$? (Lesson 10-4)

20. Find the measure of each numbered angle. (Lesson 10-4)



10-5 Tangents

Main Ideas

- Use properties of tangents.
- Solve problems involving circumscribed polygons.

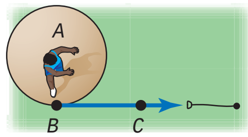
New Vocabulary

tangent
point of tangency

GET READY for the Lesson

In April 2004, Yipsi Moreno of Cuba set the hammer throw record for North America, Central America, and the Carribean with a throw of 75.18 meters in La Habana, Cuba. The hammer is a metal ball, usually weighing 16 pounds, attached to a steel wire at the end of which is a grip. The ball is spun around by the thrower and then released, with the greatest distance thrown winning the event.

Tangents The figure at the right models the hammer throw event. Circle A represents the circular area containing the spinning thrower. Ray BC represents the path the hammer takes when released. $\overrightarrow{BC}$ is **tangent** to $\odot A$, because the line containing $\overrightarrow{BC}$ intersects the circle in exactly one point. This point is called the **point of tangency**.

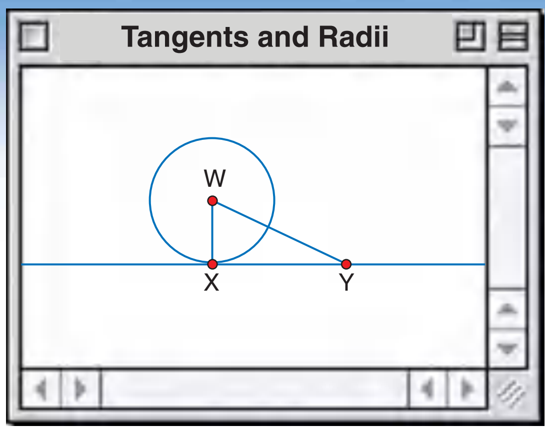


GEOMETRY SOFTWARE LAB

Tangents and Radii

MODEL

- Use The Geometer's Sketchpad to draw a circle with center W. Then draw a segment tangent to $\odot W$. Label the point of tangency as X.
- Choose another point on the tangent and name it Y. Draw $\overline{WY}$.



THINK AND DISCUSS

1. What is $\overline{WX}$ in relation to the circle?
2. Measure $\overline{WY}$ and $\overline{WX}$. Write a statement to relate WX and WY.
3. Move point Y. How does the location of Y affect the statement you wrote in Exercise 2?
4. Measure $\angle WXY$. What conclusion can you make?
5. **Make a conjecture** about the shortest distance from the center of the circle to a tangent of the circle.

Andy Lyons/Getty Images

The lab suggests that the shortest distance from a tangent to the center of a circle is the radius drawn to the point of tangency. Since the shortest distance from a point to a line is a perpendicular, the radius and the tangent must be perpendicular.

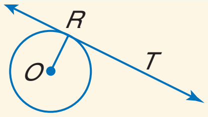
Study Tip

Tangent Lines
All of the theorems applying to tangent lines also apply to parts of the line that are tangent to the circle.

THEOREM 10.9

If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

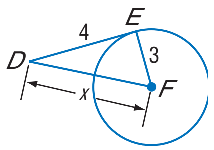
Example: If $\overleftrightarrow{RT}$ is a tangent, $\overline{OR} \perp \overleftrightarrow{RT}$.



You will prove Theorem 10.9 in Exercise 24.

EXAMPLE Find Lengths

1 **ALGEBRA** **$\overline{ED}$ is tangent to $\odot F$ at point E. Find x.**



Because the radius is perpendicular to the tangent at the point of tangency, $\overline{EF} \perp \overline{DE}$. This makes $\angle DEF$ a right angle and $\triangle DEF$ a right triangle. Use the Pythagorean Theorem to find x.

$(EF)^2 + (DE)^2 = (DF)^2$ — Pythagorean Theorem

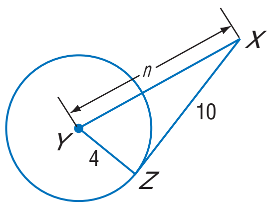
$3^2 + 4^2 = x^2$ — $EF = 3, DE = 4, DF = x$

$25 = x^2$ — Simplify.

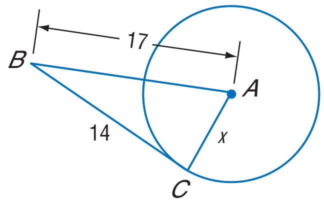
$\pm 5 = x$ — Take the square root of each side.

Because x is the length of $\overline{DF}$, ignore the negative result. Thus, $x = 5$.

1A. $\overline{XZ}$ is tangent to $\odot Y$ at point Z. Find n.



1B. $\overline{BC}$ is tangent to $\odot A$ at point C. Find x.

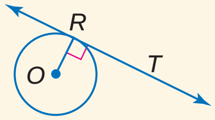


The converse of Theorem 10.9 is also true.

THEOREM 10.10

If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

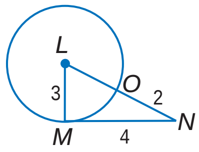
Example: If $\overline{OR} \perp \overleftrightarrow{RT}$, $\overleftrightarrow{RT}$ is a tangent.



You will prove this theorem in Exercise 25.

EXAMPLE 2 Identify Tangents

a. Determine whether $\overline{MN}$ is tangent to $\odot L$. Justify your reasoning.



First determine whether $\triangle LMN$ is a right triangle by using the converse of the Pythagorean Theorem.

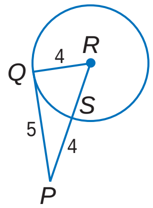
$(LM)^2 + (MN)^2 \stackrel{?}{=} (LN)^2$ Converse of Pythagorean Theorem

$3^2 + 4^2 \stackrel{?}{=} 5^2$ $LM = 3$, $MN = 4$, $LN = 3 + 2$ or 5

$25 = 25 \checkmark$ Simplify.

Because $3^2 + 4^2 = 5^2$, the converse of the Pythagorean Theorem allows us to conclude that $\triangle LMN$ is a right triangle and $\angle LMN$ is a right angle. Thus, $\overline{LM} \perp \overline{MN}$, making $\overline{MN}$ a tangent to $\odot L$.

b. Determine whether $\overline{PQ}$ is tangent to $\odot R$. Justify your reasoning.



Since $RQ = RS$, $RP = 4 + 4$ or 8 units.

$(RQ)^2 + (PQ)^2 \stackrel{?}{=} (RP)^2$ Converse of Pythagorean Theorem

$4^2 + 5^2 \stackrel{?}{=} 8^2$ $RQ = 4$, $PQ = 5$, $RP = 8$

$41 \neq 64$ Simplify.

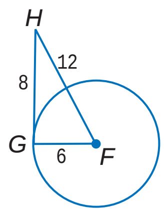
Because $RQ^2 + PQ^2 \neq RP^2$, $\triangle RQP$ is not a right triangle. So, $\overline{PQ}$ is not tangent to $\odot R$.

Study Tip

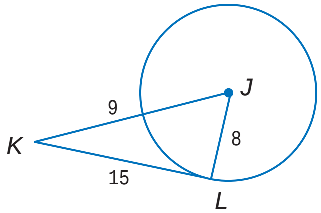
Identifying Tangents

Never assume that a segment is tangent to a circle by appearance unless told otherwise. The figure must either have a right angle symbol or include the measurements that confirm a right angle.

2A. Determine whether $\overline{GH}$ is tangent to $\odot F$. Justify your reasoning.



2B. Determine whether $\overline{KL}$ is tangent to $\odot J$. Justify your reasoning.



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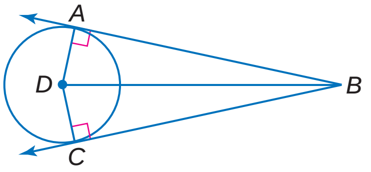
More than one line can be tangent to the same circle. In the figure, $\overline{AB}$ and $\overline{BC}$ are tangent to $\odot D$. So, $(AB)^2 + (AD)^2 = (DB)^2$ and $(BC)^2 + (CD)^2 = (DB)^2$.



$(AB)^2 + (AD)^2 = (BC)^2 + (CD)^2$ Substitution

$(AB)^2 + (AD)^2 = (BC)^2 + (AD)^2$ $AD = CD$

$(AB)^2 = (BC)^2$ Subtract $(AD)^2$ from each side.

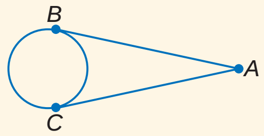
$AB = BC$ Take the square root of each side.

The last statement implies that $\overline{AB} \cong \overline{BC}$. This is a proof of Theorem 10.11.

THEOREM 10.11

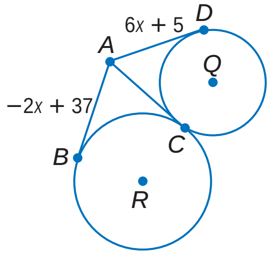
If two segments from the same exterior point are tangent to a circle, then they are congruent.

Example: $\overline{AB} \cong \overline{AC}$



EXAMPLE Congruent Tangents

3 ALGEBRA **Find x. Assume that segments that appear tangent to circles are tangent.**



$\overline{AD}$ and $\overline{AC}$ are drawn from the same exterior point and are tangent to $\odot Q$, so $\overline{AD} \cong \overline{AC}$. $\overline{AC}$ and $\overline{AB}$ are drawn from the same exterior point and are tangent to $\odot R$, so $\overline{AC} \cong \overline{AB}$. By the Transitive Property, $\overline{AD} \cong \overline{AB}$.

$AD = AB$ Definition of congruent segments

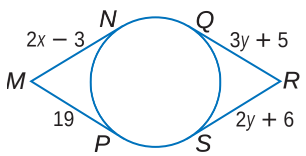
$6x + 5 = -2x + 37$ Substitution

$8x + 5 = 37$ Add $2x$ to each side.

$8x = 32$ Subtract 5 from each side.

$x = 4$ Divide each side by 8.

3. Find x and y.

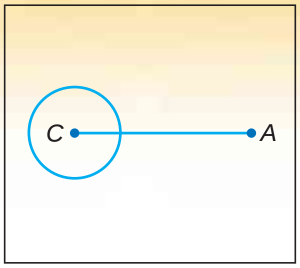


In the construction that follows, you will learn how to construct a line tangent to a circle through a point exterior to the circle.

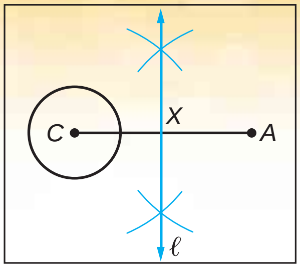
CONSTRUCTION

Line Tangent to a Circle Through a Point Exterior to the Circle

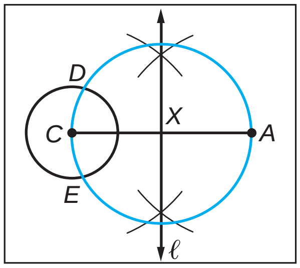
Step 1 Construct a circle. Label the center *C*. Draw a point outside ⊙*C*. Then draw $\overline{CA}$.



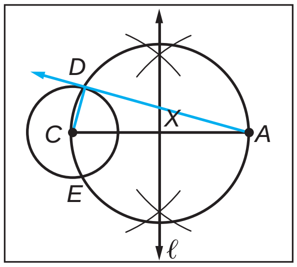
Step 2 Construct the perpendicular bisector of $\overline{CA}$ and label it line ℓ. Label the intersection of ℓ and $\overline{CA}$ as point *X*.



Step 3 Construct circle *X* with radius $\overline{XC}$. Label the points where the circles intersect as *D* and *E*.

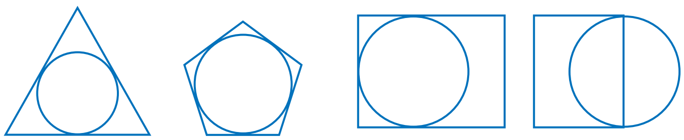


Step 4 Draw $\overleftrightarrow{AD}$. △*ADC* is inscribed in a semicircle. So ∠*ADC* is a right angle, and $\overleftrightarrow{AD}$ is a tangent.



You will construct a line tangent to a circle through a point on the circle in Exercise 23.

Circumscribed Polygons In Lesson 10-3, you learned that circles can be circumscribed about a polygon. Likewise, polygons can be circumscribed about a circle, or the circle is inscribed in the polygon. Notice that the vertices of the polygon *do not* lie on the circle, but every side of the polygon is tangent to the circle.



Polygons are circumscribed.

Polygons are *not* circumscribed.

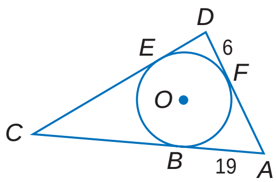
Study Tip

Common Misconceptions

Just because the circle is tangent to one or more of the sides of a polygon does not mean that the polygon is circumscribed about the circle, as shown in the second pair of figures.

EXAMPLE 4 Triangles Circumscribed About a Circle

Triangle *ADC* is circumscribed about ⊙*O*. Find the perimeter of △*ADC* if *EC* = *DE* + *AF*.



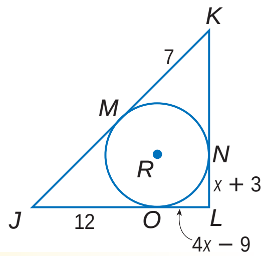
Use Theorem 10.10 to determine the equal measures:
$AB = AF = 19$, $FD = DE = 6$, and $EC = CB$.
We are given that $EC = DE + AF$, so $EC = 6 + 19$ or 25.

$P = AB + BC + EC + DE + FD + AF$ Definition of perimeter

$= 19 + 25 + 25 + 6 + 6 + 19$ or 100 Substitution

The perimeter of △*ADC* is 100 units.

4. Triangle *JKL* is circumscribed about ⊙*R*. Find *x* and the perimeter of △*JKL*.



Interactive Lab
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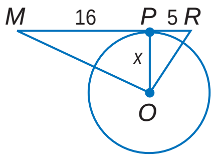
CHECK Your Understanding

Examples 1 and 2 (pp. 589–590)

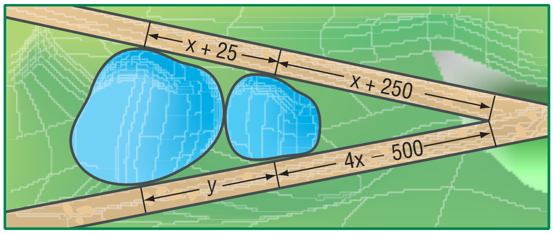
For Exercises 1 and 2, use the figure at the right.



1. Tangent $\overline{MP}$ is drawn to $\odot O$. Find x if $MO = 20$.
2. If $RO = 13$, determine whether $\overline{PR}$ is tangent to $\odot O$.

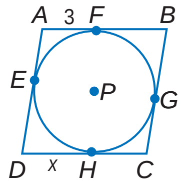
Example 3 (p. 591)

3. **LANDSCAPE ARCHITECT** A landscape architect is planning to pave two walking paths beside two ponds, as shown. Find the values of x and y. What is the total length of the walking paths?



Example 4 (p. 592)

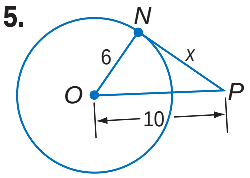
4. Rhombus $ABCD$ is circumscribed about $\odot P$ and has a perimeter of 32 units. Find x.

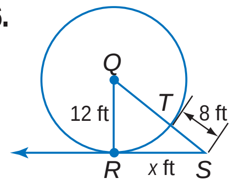


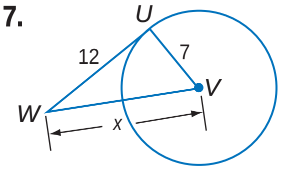
Exercises

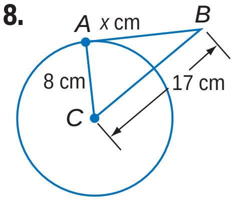
HOMEWORK HELP	
For Exercises	See Examples
5–10	1
11–14	2
15–16	3
17–22	4

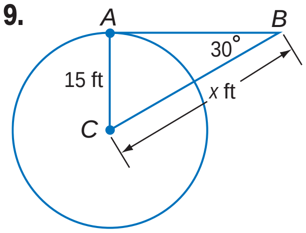
Find x. Assume that segments that appear to be tangent are tangent.

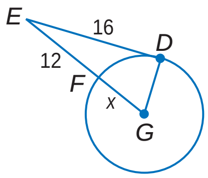
5. 

6. 

7. 

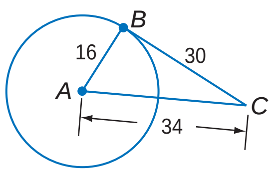
8. 

9. 

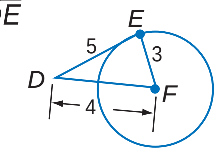
10. 

Determine whether each segment is tangent to the given circle.

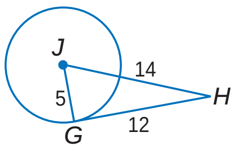
11. $\overline{BC}$



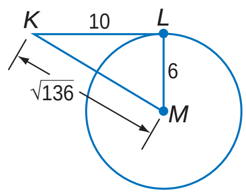
12. $\overline{DE}$



13. $\overline{GH}$

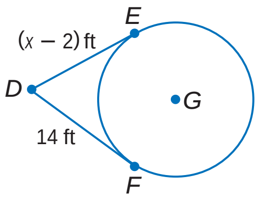


14. $\overline{KL}$

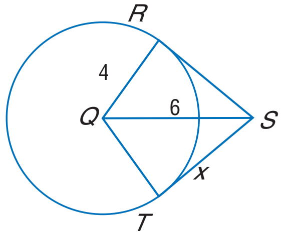


Find x. Assume that segments that appear to be tangent are tangent.

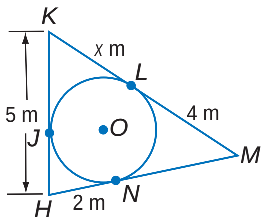
15.



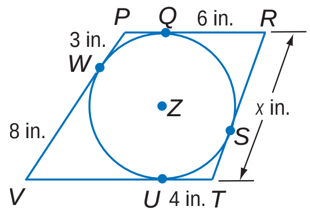
16.



17.

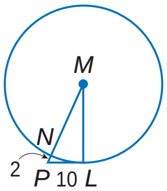


18.

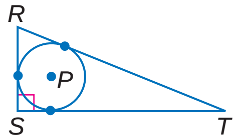


Find the perimeter of each polygon for the given information.

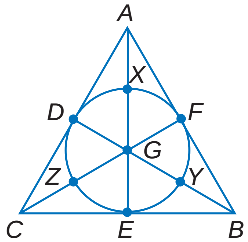
19.



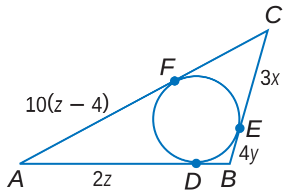
20. $ST = 18$, radius of $\odot P = 5$



21. $BY = CZ = AX = 2$
radius of $\odot G = 3$



22. $CF = 6(3 - x)$, $DB = 12y - 4$



23. CONSTRUCTION Construct a line tangent to a circle through a point on the circle following these steps.

- Construct a circle with center T and locate a point P on $\odot T$ and draw $\overrightarrow{TP}$.
- Construct a perpendicular to $\overrightarrow{TP}$ through point T.

Study Tip

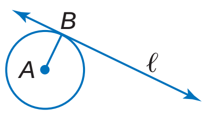
Look Back

To review **constructing perpendiculars to a line**, see Lesson 3-6.

24. PROOF Write an indirect proof of Theorem 10.9 by assuming that ℓ is not perpendicular to $\overline{AB}$.

Given: ℓ is tangent to $\odot A$ at B, $\overline{AB}$ is a radius of $\odot A$.

Prove: Line ℓ is perpendicular to $\overline{AB}$.



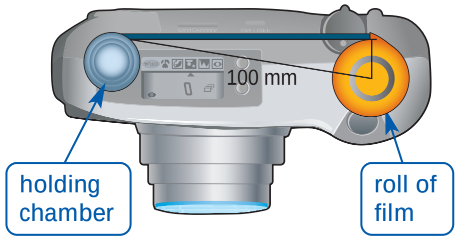
25. PROOF Write an indirect proof of Theorem 10.10 by assuming that ℓ is not tangent to $\odot A$.

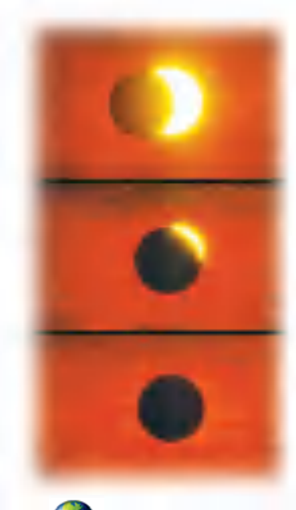
Given: $\ell \perp \overline{AB}$, $\overline{AB}$ is a radius of $\odot A$.

Prove: Line ℓ is tangent to $\odot A$.

26. PROOF Write a two-column proof to show that if a quadrilateral is circumscribed about a circle, then the sum of the measures of the two opposite sides is equal to the sum of the measures of the two remaining sides.

27. **PHOTOGRAPHY** The film in a 35-mm camera unrolls from a cylinder, travels across an opening for exposure, and then goes into another circular chamber as each photograph is taken. The roll of film has a diameter of 25 millimeters, and the distance from the center of the roll to the intake of the chamber is 100 millimeters. To the nearest millimeter, how much of the film would be exposed if the camera were opened before the roll had been used up?



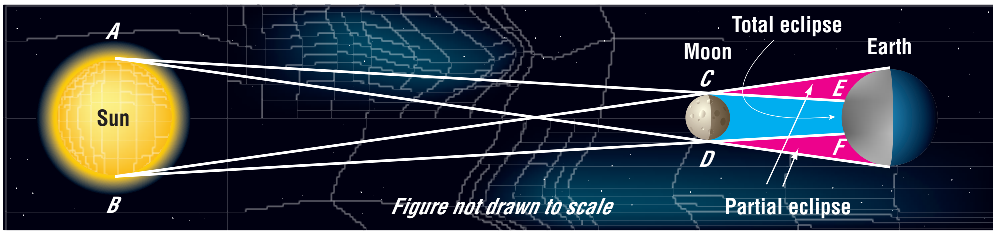


Real-World Link

During the 20th century, there were 78 total solar eclipses, but only 15 of these affected parts of the United States. The next total solar eclipse visible in the U.S. will be in 2017.

Source: *World Almanac*

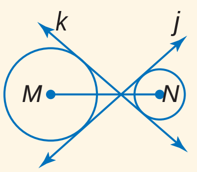
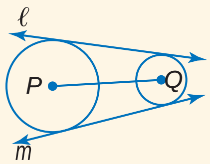
ASTRONOMY **For Exercises 28 and 29, use the following information.**
A solar eclipse occurs when the Moon blocks the Sun's rays from hitting Earth. Some areas of the world will experience a total eclipse, others a partial eclipse, and some no eclipse at all, as shown in the diagram below.



28. The blue section denotes a total eclipse on that part of Earth. Which tangents define the blue area?

29. The pink section denotes the area that will have a partial eclipse. Which tangents define the northern and southern boundaries of the partial eclipse?

COMMON TANGENTS **A line that is tangent to two circles in the same plane is called a *common tangent*.**

Common internal tangents intersect the segment connecting the centers.	*Common external tangents* do not intersect the segment connecting the centers.
 Lines k and j are common internal tangents.	 Lines ℓ and m are common external tangents.

Refer to the diagram of the eclipse above.

30. Name two common internal tangents.

31. Name two common external tangents.

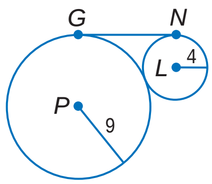
EXTRA PRACTICE
See pages 820, 837.
Math Online
Self-Check Quiz at geometryonline.com

H.O.T. Problems

32. **REASONING** Determine the number of tangents that can be drawn to a circle for each point. Explain your reasoning.
   a. containing a point outside the circle
   b. containing a point inside the circle
   c. containing a point on the circle

33. **OPEN ENDED** Draw an example of a circumscribed polygon and an example of an inscribed polygon, and give real-life examples of each.

34. **CHALLENGE** Find the measure of tangent $\overline{GN}$. Explain.

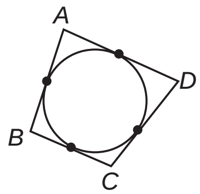


35. **REASONING** Write an argument to support this statement or provide a counterexample. *If two lines are tangent to the same circle, the lines intersect.*

36. **Writing in Math** Using the information about tangents and track and field on page 588, explain how the hammer throw models a tangent. Determine the distance the hammer landed from Moreno if the wire and handle are 1.2 meters long and her arm is 0.8 meter long.

STANDARDIZED TEST PRACTICE

37. Quadrilateral $ABCD$ is circumscribed about a circle. If $AB = 19$, $BC = 6$, and $CD = 14$, what is the measure of $\overline{AD}$?



A 11 **C** 25

B 20 **D** 27

38. **REVIEW** A paper company ships reams of paper in a box that weighs 1.3 pounds. Each ream of paper weighs 4.4 pounds, and a box can carry no more than 12 reams of paper. Which inequality best describes the total weight in pounds w to be shipped in terms of the number of reams of paper r in each box?

F $w \geq 1.3 + 4.4r$, $r \geq 12$

G $w = 1.3 + 4.4r$, $r \leq 12$

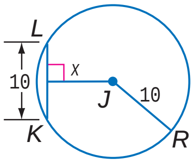
H $w \leq 1.3 + 4.4r$, $r \leq 12$

J $w = 1.3 + 4.4r$, $r \geq 12$

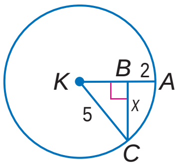
39. **ADVERTISING** Circles are often used in logos for commercial products. The logo at the right shows two inscribed angles and two central angles. If $\overparen{AC} \cong \overparen{BD}$, $m\overparen{AF} = 90$, $m\overparen{FE} = 45$, and $m\overparen{ED} = 90$, find $m\angle AFC$ and $m\angle BED$. (Lesson 10-4)

Find each measure to the nearest tenth. (Lesson 10-3)

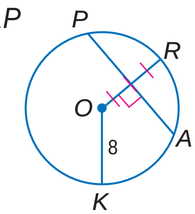
40. x



41. BC



42. AP



PREREQUISITE SKILL **Solve each equation.** (pages 781 and 782)

43. $x + 3 = \frac{1}{2}[(4x + 6) - 10]$

44. $2x - 5 = \frac{1}{2}[(3x + 16) - 20]$

45. $2x + 4 = \frac{1}{2}[(x + 20) - 10]$

46. $x + 3 = \frac{1}{2}[(4x + 10) - 45]$

EXTEND 10-5

Geometry Lab

Inscribed and Circumscribed Triangles

In Lesson 5-1, you learned that there are special points of concurrency in a triangle. Two of these will be used in these activities.

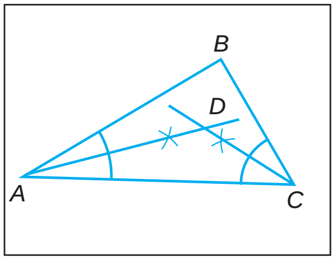
- The *incenter* is the point at which the angle bisectors meet. It is equidistant from the sides of the triangle.
- The *circumcenter* is the point at which the perpendicular bisectors of the sides intersect. It is equidistant from the vertices of the triangle.

Animation geometryonline.com

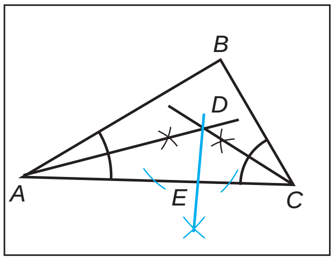
ACTIVITY 1 Construct a circle inscribed in a triangle.

The triangle is circumscribed about the circle.

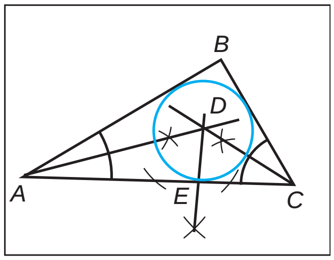
Step 1 Draw a triangle and label its vertices A, B, and C. Construct two angle bisectors of the triangle to locate the incenter. Label it D.



Step 2 Construct a segment perpendicular to a side of $\triangle ABC$ through the incenter. Label the intersection E.



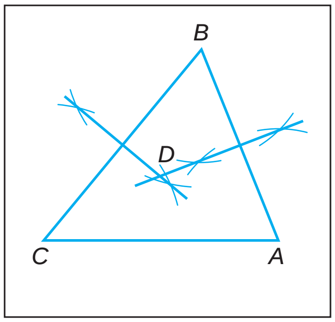
Step 3 Use the compass to measure DE. Then put the point of the compass on D, and draw a circle with that radius.



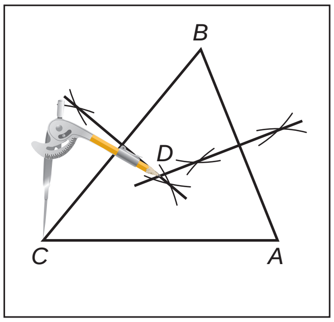
ACTIVITY 2 Construct a circle through any three noncollinear points.

This construction may be referred to as circumscribing a circle about a triangle.

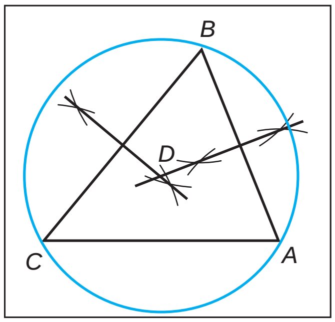
Step 1 Draw a triangle and label its vertices A, B, and C. Construct perpendicular bisectors of two sides of the triangle to locate the circumcenter. Label it D.



Step 2 Use the compass to measure the distance from the circumcenter D to any of the three vertices.



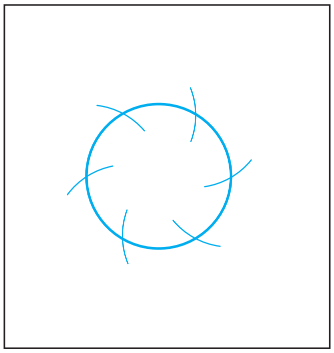
Step 3 Using that setting, place the compass point at D, and draw a circle about the triangle.



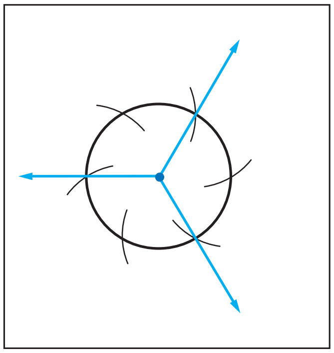
For the next activity, refer to the construction of an inscribed regular hexagon on page 576.

ACTIVITY 3 **Construct an equilateral triangle circumscribed about a circle.**

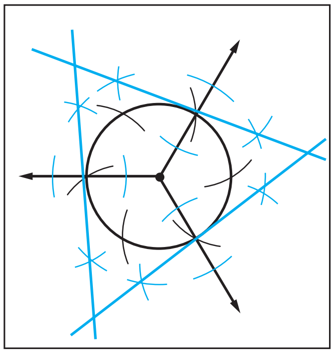
Step 1 Construct a circle and divide it into six congruent arcs.



Step 2 Place a point at every other arc. Draw rays from the center through these points.



Step 3 Construct a line perpendicular to each of the rays through the points.



ANALYZE THE RESULTS

1. Draw an obtuse triangle and inscribe a circle in it.
2. Draw a right triangle and circumscribe a circle about it.
3. Draw a circle of any size and circumscribe an equilateral triangle about it.

Refer to Activity 1.

4. Why do you only have to construct the perpendicular to one side of the triangle?
5. How can you use the Incenter Theorem to explain why this construction is valid?

Refer to Activity 2.

6. Why do you only have to measure the distance from the circumcenter to any one vertex?
7. How can you use the Circumcenter Theorem to explain why this construction is valid?

Refer to Activity 3.

8. What is the measure of each of the six congruent arcs?
9. Write a convincing argument as to why the lines constructed in Step 3 form an equilateral triangle.
10. Why do you think the terms *incenter* and *circumcenter* are good choices for the points they define?

10-6 Secants, Tangents, and Angle Measures

Main Ideas

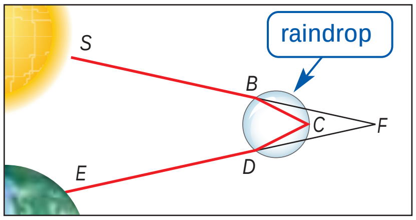
- Find measures of angles formed by lines intersecting on or inside a circle.
- Find measures of angles formed by lines intersecting outside the circle.

New Vocabulary

secant

GET READY for the Lesson

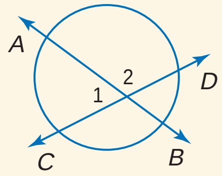
Droplets of water in the air refract or bend sunlight as it passes through them, creating a rainbow. The various angles of refraction result in an arch of colors. In the figure, the sunlight from point *S* enters the raindrop at *B* and is bent. The light proceeds to the back of the raindrop, and is reflected at *C* to leave the raindrop at point *D* heading to Earth. Angle *F* represents the measure of how the resulting ray of light deviates from its original path.



Intersections on or Inside a Circle A line that intersects a circle in exactly two points is called a **secant**. In the figure above, $\overline{SF}$ and $\overline{EF}$ are secants of the circle. When two secants intersect inside a circle, the angles formed are related to the arcs they intercept.

THEOREM 10.12

If two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.



Examples: $m\angle 1 = \frac{1}{2}(m\overset{\frown}{AC} + m\overset{\frown}{BD})$

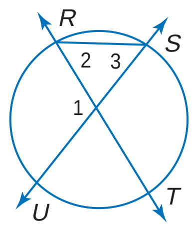
$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AD} + m\overset{\frown}{BC})$

PROOF Theorem 10.12

Given: secants $\overleftrightarrow{RT}$ and $\overleftrightarrow{SU}$

Prove: $m\angle 1 = \frac{1}{2}(m\overset{\frown}{ST} + m\overset{\frown}{RU})$

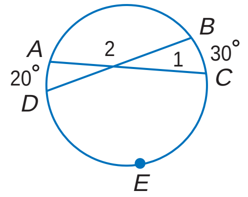
Proof:



Statements	Reasons
1. $m\angle 1 = m\angle 2 + m\angle 3$	**1.** Exterior Angle Theorem
2. $m\angle 2 = \frac{1}{2}m\overset{\frown}{ST}$, $m\angle 3 = \frac{1}{2}m\overset{\frown}{RU}$	**2.** The measure of the inscribed $\angle$ = half the measure of the intercepted arc.
3. $m\angle 1 = \frac{1}{2}m\overset{\frown}{ST} + \frac{1}{2}m\overset{\frown}{RU}$	**3.** Substitution
4. $m\angle 1 = \frac{1}{2}(m\overset{\frown}{ST} + m\overset{\frown}{RU})$	**4.** Distributive Property

EXAMPLE 1 Secant-Secant Angle

Find $m\angle 2$ if $m\widehat{BC} = 30$ and $m\widehat{AD} = 20$.



Method 1 Find $m\angle 1$.

$m\angle 1 = \frac{1}{2}(m\widehat{BC} + m\widehat{AD})$ Theorem 10.12

$= \frac{1}{2}(30 + 20)$ or 25 Substitution

$m\angle 2 = 180 - m\angle 1$

$= 180 - 25$ or 155

Method 2 Find $m\widehat{AB}$ and $m\widehat{DEC}$ first.

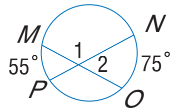
$m\angle 2 = \frac{1}{2}(m\widehat{AB} + m\widehat{DEC})$ Theorem 10.12

$= \frac{1}{2}[360 - (m\widehat{BC} + m\widehat{AD})]$ $m\widehat{AB} + m\widehat{DEC} = 360 - (m\widehat{BC} + m\widehat{AD})$

$= \frac{1}{2}[360 - (30 + 20)]$ Substitution

$= \frac{1}{2}(310)$ or 155 Simplify.

1. Find $m\angle 1$.

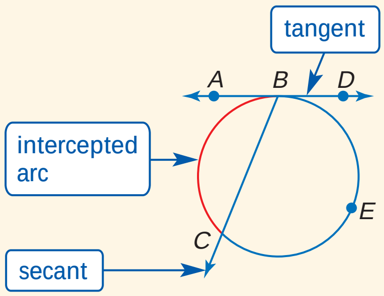


THEOREM 10.13

If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

Examples: $m\angle ABC = \frac{1}{2}m\widehat{BC}$

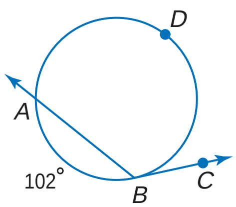
$m\angle DBC = \frac{1}{2}m\widehat{BEC}$



You will prove Theorem 10.13 in Exercise 41.

EXAMPLE 2 Secant-Tangent Angle

Find $m\angle ABC$ if $m\widehat{AB} = 102$.



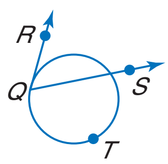
$m\widehat{ADB} = 360 - m\widehat{AB}$

$= 360 - 102$ or 258

$m\angle ABC = \frac{1}{2}m\widehat{ADC}$

$= \frac{1}{2}(258)$ or 129

2. Find $m\angle RQS$ if $m\widehat{QTS} = 238$.

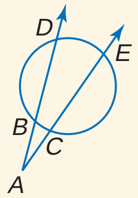
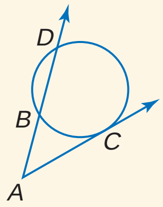
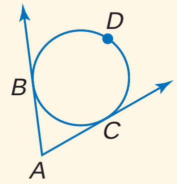


Intersections Outside a Circle Secants and tangents can also meet outside a circle. The measure of the angle formed also involves half of the measures of the arcs they intercept.

> **Study Tip**
>
> **Absolute Value**
> The measure of each $\angle A$ can also be expressed as one-half the absolute value of the difference of the arc measures. In this way, the order of the arc measures does not affect the outcome of the calculation.

THEOREM 10.14

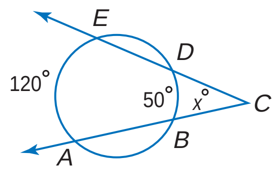
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.

Two Secants	Secant-Tangent	Two Tangents
		
$m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$	$m\angle A = \frac{1}{2}(m\widehat{DC} - m\widehat{BC})$	$m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$

You will prove Theorem 10.14 in Exercise 40.

EXAMPLE Secant-Secant Angle

3 **Find x.**

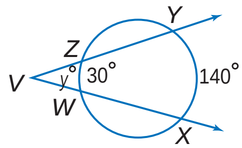


$m\angle C = \frac{1}{2}(m\widehat{EA} - m\widehat{DB})$

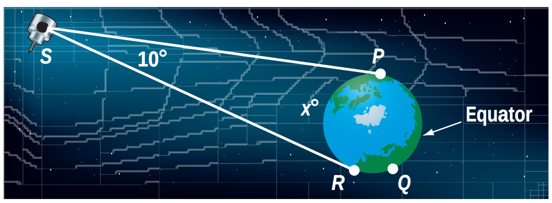
$x = \frac{1}{2}(120 - 50)$ Substitution

$x = \frac{1}{2}(70)$ or 35 Simplify.

3. Find y.



4 **SATELLITES** **Suppose a satellite S orbits above Earth rotating so that it appears to hover directly over the equator. Use the figure to determine the arc measure on the equator visible to this satellite.**



$\widehat{PR}$ represents the arc along the equator visible to the satellite S. If $x = m\widehat{PR}$, then $m\widehat{PQR} = 360 - x$. Use the measure of the given angle to find $m\widehat{PR}$.

$$m\angle S = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$$

$10 = \frac{1}{2}[(360 - x) - x]$ Substitution

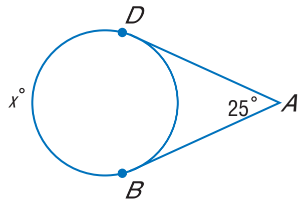
$20 = 360 - 2x$ Multiply each side by 2 and simplify.

$-340 = -2x$ Subtract 360 from each side.

$170 = x$ Divide each side by -2.

The measure of the arc on Earth visible to the satellite is 170.

4. Find x.



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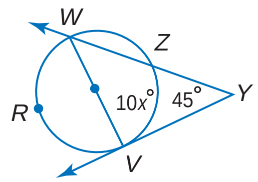
EXAMPLE Secant-Tangent Angle

5 **Find x.**



$\widehat{WRV}$ is a semicircle because $\overline{WV}$ is a diameter. So, $m\widehat{WRV} = 180$.

$$m\angle Y = \frac{1}{2}(m\widehat{WV} - m\widehat{ZV})$$

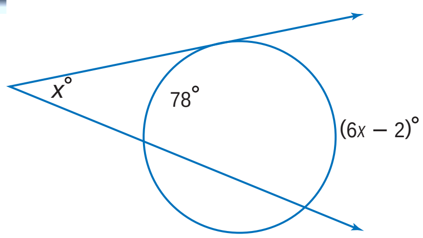
$45 = \frac{1}{2}(180 - 10x)$ Substitution

$90 = 180 - 10x$ Multiply each side by 2.

$-90 = -10x$ Subtract 180 from each side.

$9 = x$ Divide each side by -10.

5. Find x.

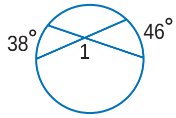


Check Your Understanding

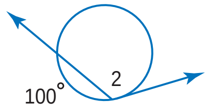
Examples 1, 2 (p. 600)

Find each measure.

1. $m\angle 1$



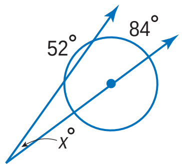
2. $m\angle 2$



Examples 3 to 5
(pp. 601, 602)

Find *x*.

3.



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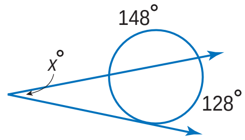
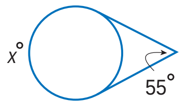


5.



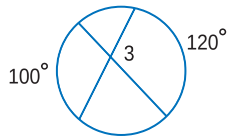
Exercises

HOMEWORK HELP

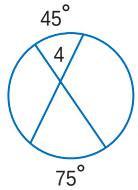
For Exercises	See Examples
6–9, 14	1
10–13	2
15–18	3
19–20	4
21–24	5

Find each measure.

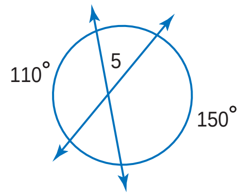
6. $m\angle 3$



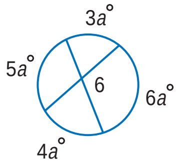
7. $m\angle 4$



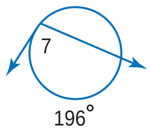
8. $m\angle 5$



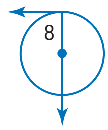
9. $m\angle 6$



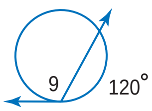
10. $m\angle 7$



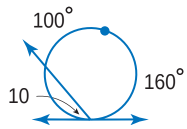
11. $m\angle 8$



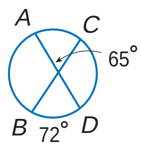
12. $m\angle 9$



13. $m\angle 10$

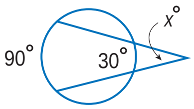


14. $m\widehat{AC}$



Find *x*. Assume that any segment that appears to be tangent is tangent.

15.



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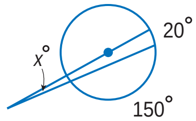
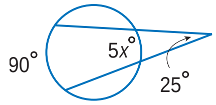


17.



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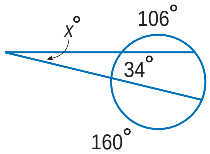

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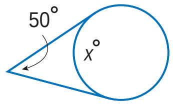

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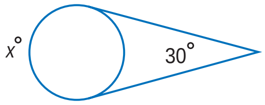

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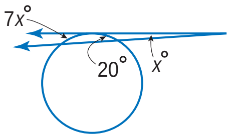

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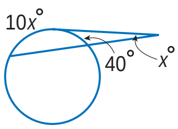

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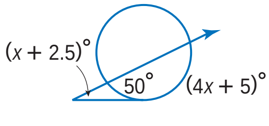

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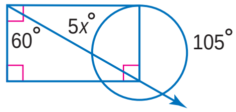

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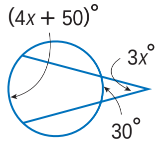

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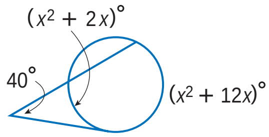
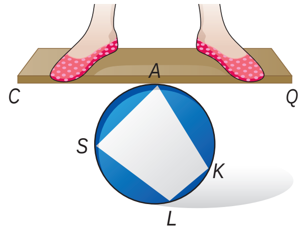


CIRCUS **For Exercises 27–30, refer to the figure and the information below.**

One of the acrobatic acts in the circus requires the artist to balance on a board that is placed on a round drum, as shown at the right. Find each measure if $\overline{SA} \parallel \overline{LK}$, $m\angle SLK = 78$, and $m\overset{\frown}{SA} = 46$.



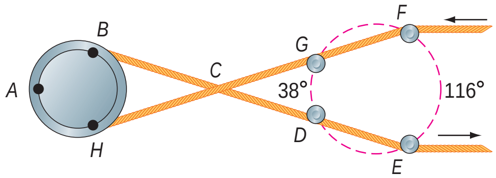
27. $m\angle CAS$

28. $m\angle QAK$

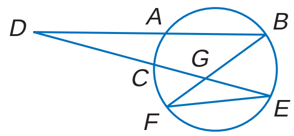
29. $m\overset{\frown}{KL}$

30. $m\overset{\frown}{SL}$

31. **WEAVING** Once yarn is woven from wool fibers, it is often dyed and then threaded along a path of pulleys to dry. One set of pulleys is shown below. Note that the yarn appears to intersect itself at C, but in reality it does not. Use the information from the diagram to find $m\overset{\frown}{BH}$.



Find each measure if $m\overset{\frown}{FE} = 118$, $m\overset{\frown}{AB} = 108$, $m\angle EGB = 52$, and $m\angle EFB = 30$.



32. $m\overset{\frown}{AC}$

33. $m\overset{\frown}{CF}$

34. $m\angle EDB$

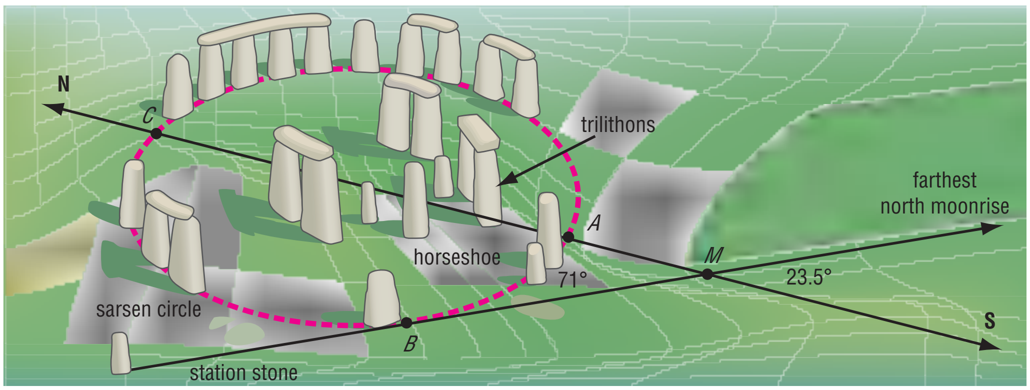
Real-World Link

Stonehenge is located in southern England near Salisbury. In its final form, Stonehenge included 30 upright stones about 18 feet tall by 7 feet thick.

Source: *World Book Encyclopedia*

LANDMARKS **For Exercises 35–37, use the following information.**

Stonehenge is a British landmark made of huge stones arranged in a circular pattern that reflects the movements of Earth and the moon. The diagram shows that the angle formed by the north / south axis and the line aligned from the station stone to the northmost moonrise position measures 23.5°.



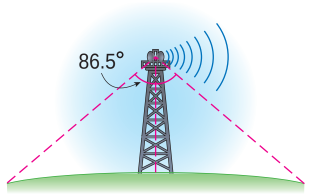
35. Find $m\overset{\frown}{BC}$.

36. Is $\overset{\frown}{ABC}$ a semicircle? Explain.

37. If the circle measures about 100 feet across, approximately how far would you walk around the circle from point B to point C?

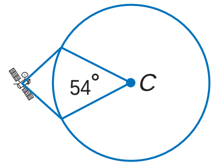
file photo

38. TELECOMMUNICATIONS The signal from a telecommunication tower follows a ray that has its endpoint on the tower and is tangent to Earth. Suppose a tower is located at sea level, as shown in the figure. Determine the measure of the arc intercepted by the two tangents.



Note: Art not drawn to scale

39. SATELLITES A satellite is orbiting so that it maintains a constant altitude above the equator. The camera on the satellite can detect an arc of 6000 kilometers on Earth's surface. This arc measures 54°. What is the measure of the angle of view of the camera located on the satellite?

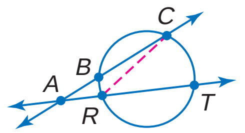


40. PROOF Write a two-column proof of Theorem 10.14. Consider each case.

a. Case 1: two secants

Given: $\overleftrightarrow{AC}$ and $\overleftrightarrow{AT}$ are secants to the circle.

Prove: $m\angle CAT = \frac{1}{2}(m\overset{\frown}{CT} - m\overset{\frown}{BR})$

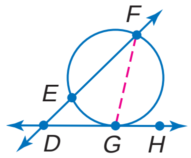


b. Case 2: secant and a tangent

Given: $\overleftrightarrow{DG}$ is a tangent to the circle.

$\overleftrightarrow{DF}$ is a secant to the circle.

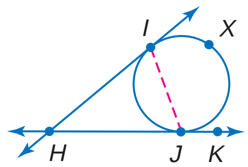
Prove: $m\angle FDG = \frac{1}{2}(m\overset{\frown}{FG} - m\overset{\frown}{GE})$



c. Case 3: two tangents

Given: $\overleftrightarrow{HI}$ and $\overleftrightarrow{HJ}$ are tangents to the circle.

Prove: $m\angle IHJ = \frac{1}{2}(m\overset{\frown}{IXJ} - m\overset{\frown}{IJ})$



41. PROOF Write a paragraph proof of Theorem 10.13.

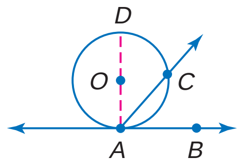
a. Given: $\overleftrightarrow{AB}$ is a tangent of $\odot O$.

$\overrightarrow{AC}$ is a secant of $\odot O$.

$\angle CAB$ is acute.

Prove: $m\angle CAB = \frac{1}{2}m\overset{\frown}{CA}$

b. Prove Theorem 10.13 if the angle in part **a** is obtuse.



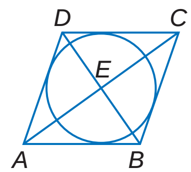
EXTRA PRACTICE

See pages 821, 837.

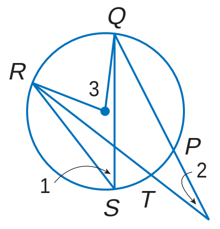
H.O.T. Problems

42. OPEN ENDED Draw a circle and one of its diameters. Call the diameter $\overline{AC}$. Draw a line tangent to the circle at A. What type of angle is formed by the tangent and the diameter? Explain.

43. CHALLENGE Circle E is inscribed in rhombus $ABCD$. The diagonals of the rhombus are 10 centimeters and 24 centimeters long. To the nearest tenth centimeter, how long is the radius of circle E? (*Hint:* Draw an altitude from E.)



44. CHALLENGE In the figure, $\angle 3$ is a central angle. List the numbered angles in order from greatest measure to least measure. Explain your reasoning.



45. Writing in Math Refer to the information on page 599 to explain how you would calculate the angle representing how the light deviates from its original path. Include in your description the types of segments represented in the figure on page 599.

STANDARDIZED TEST PRACTICE

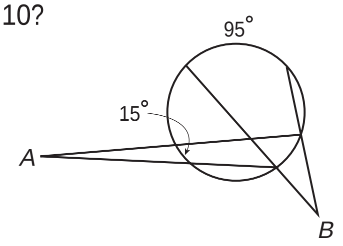
46. What is the measure of $\angle B$ if $m\angle A = 10$?

A 30

B 35

C 47.5

D 90



47. REVIEW Larry's Fish Food comes in tubes that have a radius of 2 centimeters and a height of 7 centimeters. Odell bought a full tube, but he thinks he's used about $\frac{1}{4}$ of it. About how much is left?

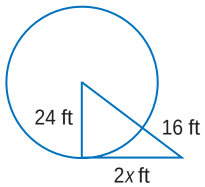
F 88 cm^3 **H** 53 cm^3

G 66 cm^3 **J** 41 cm^3

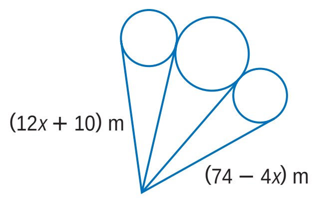
Spiral Review

Find *x*. Assume that segments that appear to be tangent are tangent. (Lesson 10-5)

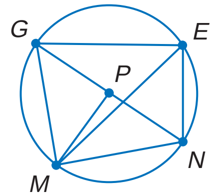
48.



48.



In $\odot P$, $m\widehat{EN} = 66$ and $m\angle GPM = 89$. Find each measure. (Lesson 10-4)



50. $m\angle EGN$ **51.** $m\angle GME$ **52.** $m\angle GNM$

RAMPS For Exercises 53 and 54, use the following information.
The Americans with Disabilities Act requires that wheelchair ramps have at least a 12-inch run for each rise of 1 inch. (Lesson 3-3)

53. Determine the slope represented by this requirement.

54. The maximum length the law allows for a ramp is 30 feet. How many inches tall is the highest point of this ramp?

GET READY for the Next Lesson

PREREQUISITE SKILL Use the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $ax^2 + bx + c = 0$, to solve each equation to the nearest tenth.

55. $x^2 + 6x - 40 = 0$ **56.** $2x^2 + 7x - 30 = 0$ **57.** $3x^2 - 24x + 45 = 0$

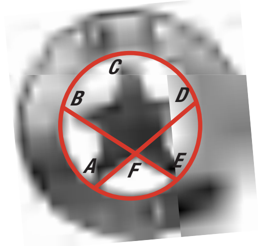
10-7 Special Segments in a Circle

Main Ideas

- Find measures of segments that intersect in the interior of a circle.
- Find measures of segments that intersect in the exterior of a circle.

Get Ready for the Lesson

The U.S. Marshals Service is the nation's oldest federal law enforcement agency, serving the country since 1789. Appointed by the President, there are 94 U.S. Marshals, one for each federal court district in the country. The "Eagle Top" badge, introduced in 1941, was the first uniform U.S. Marshals badge.



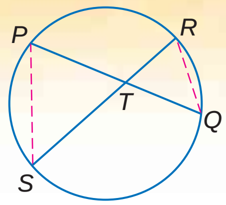
Segments Intersecting Inside a Circle In Lesson 10-2, you learned how to find lengths of parts of a chord that is intersected by the perpendicular diameter. But how do you find lengths for other intersecting chords?

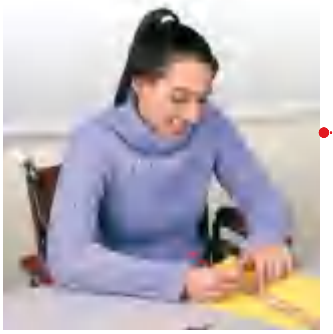
GEOMETRY LAB

Intersecting Chords

MAKE A MODEL

- Draw a circle and two intersecting chords.
- Name the chords $\overline{PQ}$ and $\overline{RS}$ intersecting at T.
- Draw $\overline{PS}$ and $\overline{RQ}$.





ANALYZE

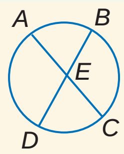
1. Name pairs of congruent angles. Explain your reasoning.
2. How are $\triangle PTS$ and $\triangle RTQ$ related? Why? Cut out both triangles, move them, and verify your conjecture.
3. Make a conjecture about the relationship of $\overline{PT}$, $\overline{TQ}$, $\overline{RT}$, and $\overline{ST}$.
4. Measure each angle and verify your conjecture.

The results of the lab suggest a proof for Theorem 10.15.

THEOREM 10.15

If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal.

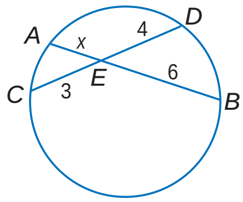
Example: $AE \cdot EC = BE \cdot ED$



You will prove Theorem 10.15 in Exercise 16.

EXAMPLE Intersection of Two Chords

1 **Find *x*.**



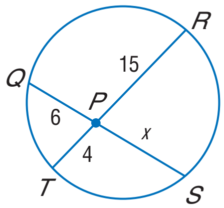
$AE \cdot EB = CE \cdot ED$

$x \cdot 6 = 3 \cdot 4$ Substitution

$6x = 12$ Multiply.

$x = 2$ Divide each side by 6.

1. Find *x*.



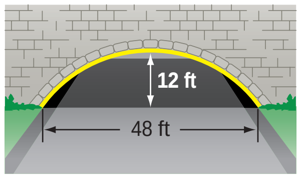
Intersecting chords can also be used to measure arcs.

Real-World Link

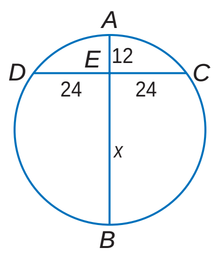
The Astrodome in Houston was the first ballpark to be built with a roof over the playing field. It originally had real grass and clear panels to allow sunlight in. This setup made it difficult to see the ball in the air, so they painted the ceiling and replaced the grass with carpet, which came to be known as "astro-turf."

Source: ballparks.com

2 **TUNNELS Tunnels are constructed to allow roadways to pass through mountains. What is the radius of the circle containing the arc if the opening is not a semicircle?**



Draw a model using a circle. Let *x* represent the unknown measure of the segment of diameter $\overline{AB}$. Use the products of the lengths of the intersecting chords to find the length of the diameter.



$AE \cdot EB = DE \cdot EC$ Segment products

$12x = 24 \cdot 24$ Substitution

$x = 48$ Divide each side by 12.

$AB = AE + EB$ Segment Addition Postulate

$AB = 12 + 48$ or 60 Substitution and addition

Since the diameter is 60, $r = 30$.

2. ASTRODOME The highest point, or apex, of the Astrodome is 208 feet high, and the diameter of the circle containing the arc is 710 feet. How long is the stadium from one side to the other?

Personal Tutor at geometryonline.com

Segments Intersecting Outside a Circle Nonparallel chords of a circle that do not intersect inside the circle can be extended to form secants that intersect in the exterior of a circle. The special relationship among secant segments excludes the chord.

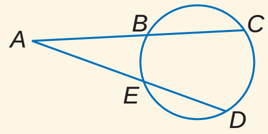
Study Tip

Helping You Remember

To remember this concept, the wording of Theorem 10.16 can be simplified by saying that each side of the equation is the product of the exterior part and the whole segment.

THEOREM 10.16

If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.

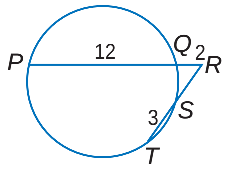


Example: $AB \cdot AC = AE \cdot AD$

You will prove Theorem 10.16 in Exercise 25.

EXAMPLE 3 Intersection of Two Secants

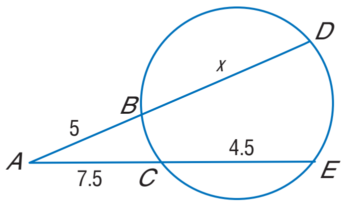
Find RS if $PQ = 12$, $QR = 2$, and $TS = 3$.



Let $RS = x$.

$QR \cdot PR = RS \cdot RT$	Secant Segment Products
$2 \cdot (12 + 2) = x \cdot (x + 3)$	Substitution
$28 = x^2 + 3x$	Distributive Property
$0 = x^2 + 3x - 28$	Subtract 28 from each side.
$0 = (x + 7)(x - 4)$	Factor.
$x + 7 = 0 \quad x - 4 = 0$	
$x = -7 \quad x = 4$	Disregard negative value.

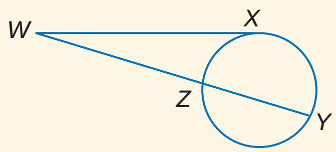
3. Find x.



The same secant segment product can be used with a secant segment and a tangent. In this case, the tangent is both the exterior part and the whole segment. This is stated in Theorem 10.17.

THEOREM 10.17

If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.

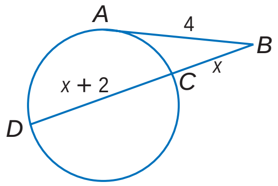


Example: $WX \cdot WX = WZ \cdot WY$

You will prove Theorem 10.17 in Exercise 26.

EXAMPLE Intersection of a Secant and a Tangent

4 **Find x. Assume that segments that appear to be tangent are tangent.**



$(AB)^2 = BC \cdot BD$

$4^2 = x\,(x + x + 2)$

$16 = x\,(2x + 2)$

$16 = 2x^2 + 2x$

$0 = 2x^2 + 2x - 16$

$0 = x^2 + x - 8$

This expression is not factorable. Use the Quadratic Formula.

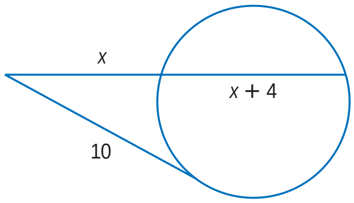
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-8)}}{2(1)}$ $a = 1, b = 1, c = -8$

$= \dfrac{-1 + \sqrt{33}}{2}$ or $x = \dfrac{-1 - \sqrt{33}}{2}$ Disregard the negative solution.

≈ 2.37 Use a calculator.

4. Find x.

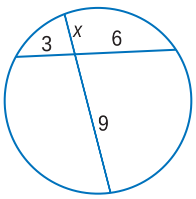


CHECK Your Understanding

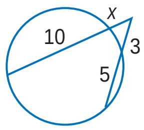
Examples 1, 3, 4 (pp. 608, 609, 610)

Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.

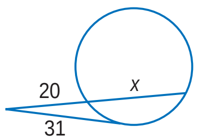
1.



2.

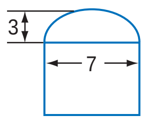


3.



Example 2 (p. 608)

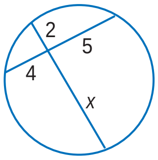
4. **HISTORY** The Roman Coliseum has many “entrances” in the shape of a door with an arched top. The ratio of the arch width to the arch height is 7:3. Find the ratio of the arch width to the radius of the circle that contains the arch.

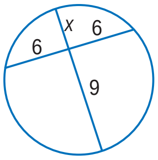


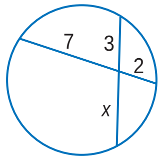
Exercises

HOMEWORK HELP	
For Exercises	See Examples
5–7	1
8–9	2
10–12	3
13–15	4

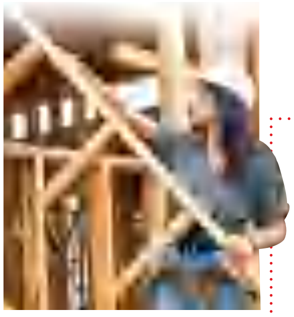
Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.

5. 

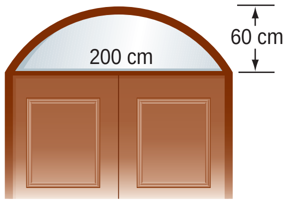
6. 

7. 

8. KNOBS If you remove a knob from a kitchen appliance, you will notice that the hole is not completely round. Suppose the flat edge is 4 millimeters long and the distance from the curved edge to the flat edge is about 4.25 millimeters. Find the radius of the circle containing the hole.



9. ARCHITECTURE An arch over a courtroom door is 60 centimeters high and 200 centimeters wide. Find the radius of the circle containing the arc of the arch.



Real-World Career

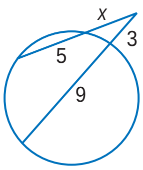
Construction Worker

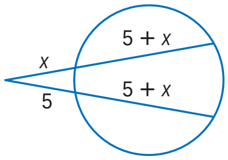
Construction workers must know how to measure and fit shapes together to make a sound building that will last for years to come. These workers also must master using machines to cut wood and metal to certain specifications that are based on geometry.

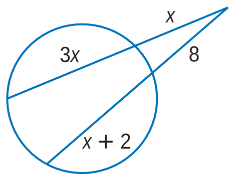
Math Online

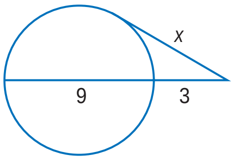
For more information, go to geometryonline.com.

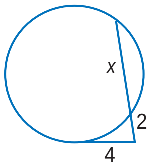
Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.

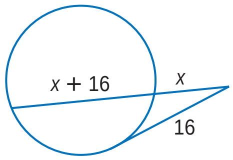
10. 

11. 

12. 

13. 

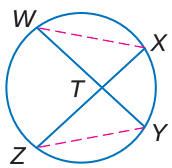
14. 

15. 

16. PROOF Copy and complete the proof of Theorem 10.15.

Given: $\overline{WY}$ and $\overline{ZX}$ intersect at T.

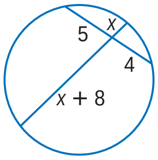
Prove: $WT \cdot TY = ZT \cdot TX$



Statements	Reasons
a. $\overline{WY}$ and $\overline{ZX}$ intersect at T	**a.** Given
b. $\angle W \cong \angle Z$, $\angle X \cong \angle Y$	**b.** ___?___
c. ___?___	**c.** AA Similarity
d. $\frac{WT}{ZT} = \frac{TX}{TY}$	**d.** ___?___
e. ___?___	**e.** Cross products

Find each variable to the nearest tenth.

17.



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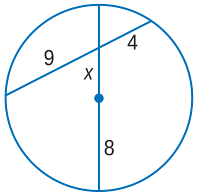

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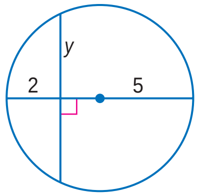

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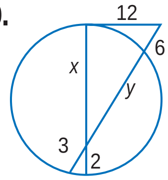

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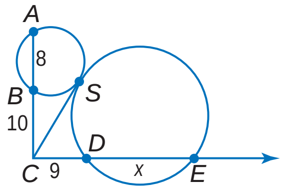

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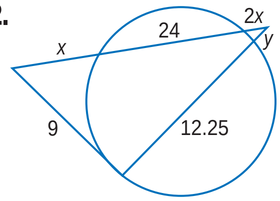

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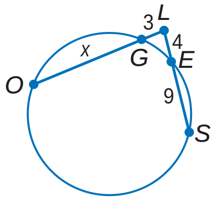
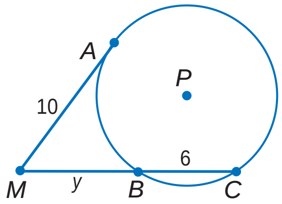


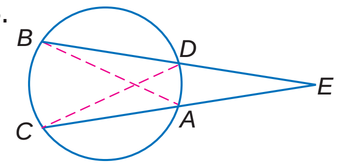
24.



25. PROOF Write a paragraph proof of Theorem 10.16.

Given: secants $\overline{EC}$ and $\overline{EB}$

Prove: $EA \cdot EC = ED \cdot EB$



EXTRA PRACTICE

See pages 821, 837.

Math Online

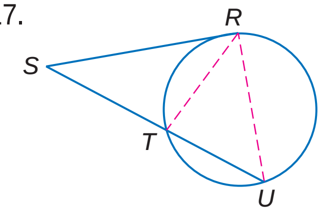
Self-Check Quiz at

geometryonline.com

26. PROOF Write a two-column proof of Theorem 10.17.

Given: tangent $\overline{SR}$,
secant $\overline{SU}$

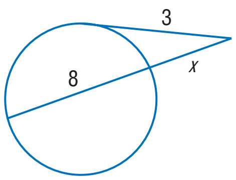
Prove: $(SR)^2 = ST \cdot SU$



H.O.T. Problems

27. REASONING Explain how the products for secant segments are similar to the products for a tangent and a secant segment.

28. FIND THE ERROR Becky and Latisha are writing products to find x. Who is correct? Explain your reasoning.



Becky

$3^2 = x \cdot 8$

$9 = 8x$

$\frac{9}{8} = x$

Latisha

$3^2 = x(x + 8)$

$9 = x^2 + 8x$

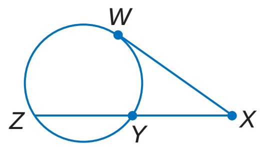
$0 = x^2 + 8x - 9$

$0 = (x + 9)(x - 1)$

$x = 1$

29. OPEN ENDED Draw a circle with two secant segments and one tangent segment that intersect at the same point. Give a real-life object that could be modeled by this drawing.

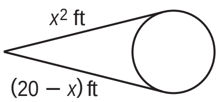
30. CHALLENGE In the figure, Y is the midpoint of $\overline{XZ}$. Find WX. Explain how you know this.



31. Writing in Math Use the figure on page 607 to explain how the lengths of intersecting chords are related. Describe the segments that are formed by the intersecting segments, $\overline{AD}$ and $\overline{EF}$, and the relationship among these segments.

STANDARDIZED TEST PRACTICE

32. Find two possible values for x from the information in the figure.



A $-4, -5$ **C** $4, 5$

B $-4, 5$ **D** $4, -5$

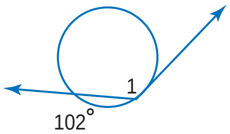
33. REVIEW In the system of equations $4x + 3y = 6$ and $-5x + 2y = 13$, which expression can be substituted for x in the equation $4x + 3y = 6$?

F $\frac{3}{2} - \frac{3}{4}y$ **H** $-\frac{13}{5} + \frac{2}{5}y$

G $6 - 3y$ **J** $13 - 2y$

Find the measure of each numbered angle. Assume that segments that appear tangent are tangent. (Lesson 10-6)

34.



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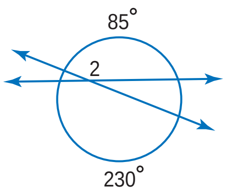

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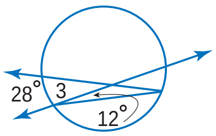
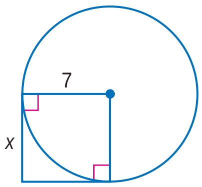


Find x. Assume that segments that appear to be tangent are tangent. (Lesson 10-5)

37.



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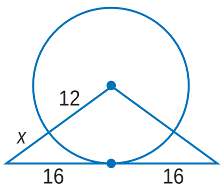
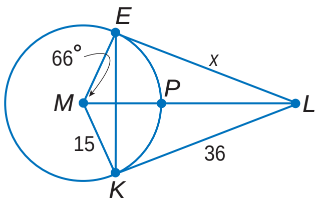


39.



40. INDIRECT MEASUREMENT Joseph is measuring the width of a stream to build a bridge over it. He picks out a rock across the stream as landmark A and places a stone on his side as point B. Then he measures 5 feet at a right angle from $\overline{AB}$ and marks this C. From C, he sights a line to point A on the other side of the stream and measures the angle to be about 67º. How far is it across the stream rounded to the nearest whole foot? (Lesson 8-5)

GET READY for the Next Lesson

PREREQUISITE SKILL **Find the distance between each pair of points.** (Lesson 1-3)

41. $C(-2, 7), D(10, 12)$ **42.** $E(1, 7), F(3, 4)$ **43.** $G(9, -4), H(15, -2)$

10-8 Equations of Circles

Main Ideas

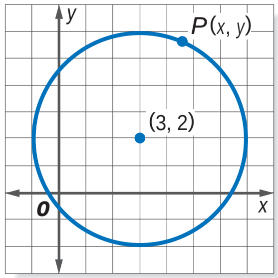
- Write the equation of a circle.
- Graph a circle on the coordinate plane.

GET READY for the Lesson

When a rock enters the water, ripples move out from the center forming concentric circles. If the rock is assigned coordinates, each ripple can be modeled by an equation of a circle.

Equation of a Circle The fact that a circle is the *locus* of points in a plane equidistant from a given point creates an equation for any circle.

Suppose the center is at (3, 2) and the radius is 4. The radius is the distance from the center. Let $P(x, y)$ be the endpoint of any radius.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

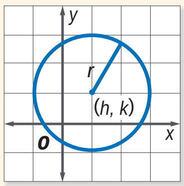
$$4 = \sqrt{(x - 3)^2 + (y - 2)^2} \quad d = 4, (x_1, y_1) = (3, 2)$$

$$16 = (x - 3)^2 + (y - 2)^2 \quad \text{Square each side.}$$

Applying this same procedure to an unknown center (h, k) and radius r yields a general equation for any circle.

KEY CONCEPT — *Standard Equation of a Circle*

An equation for a circle with center at (h, k) and radius of r units is $(x - h)^2 + (y - k)^2 = r^2$.



Study Tip

Equations of Circles

Note that the equation of a circle is kept in the form shown above. The terms being squared are not expanded.

EXAMPLE Equation of a Circle

1 **Write an equation for the circle with center at (-2, 4), $d = 4$.**

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{Equation of a circle}$$

$$[x - (-2)]^2 + [y - 4]^2 = 2^2 \quad (h, k) = (-2, 4), \text{ If } d = 4, r = 2.$$

$$(x + 2)^2 + (y - 4)^2 = 4 \quad \text{Simplify.}$$

Write an equation for each circle described below.

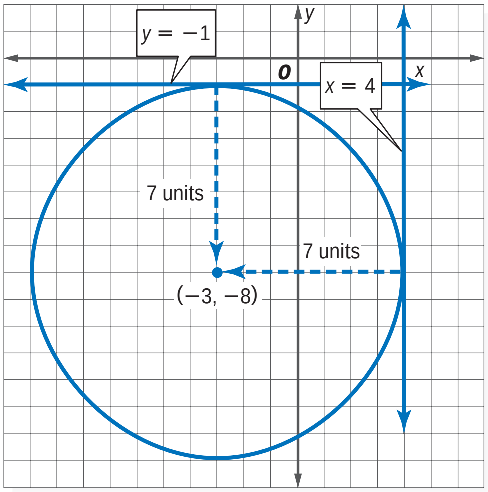
1A. center at (3, -2), $d = 10$ **1B.** center at origin, $r = 6$

Other information about a circle can be used to find the equation of the circle.

EXAMPLE **Use Characteristics of Circles**

2 **A circle with a diameter of 14 has its center in the third quadrant. The lines $y = -1$ and $x = 4$ are tangent to the circle. Write an equation of the circle.**

Sketch a drawing of the two tangent lines.



Since $d = 14$, $r = 7$. The line $x = 4$ is perpendicular to a radius. Since $x = 4$ is a vertical line, the radius lies on a horizontal line. Count 7 units to the left from $x = 4$. Find the value of h.

$$h = 4 - 7 \text{ or } -3$$

Likewise, the radius perpendicular to the line $y = -1$ lies on a vertical line. The value of k is 7 units down from -1.

$$k = -1 - 7 \text{ or } -8$$

The center is at $(-3, -8)$, and the radius is 7. An equation for the circle is $(x + 3)^2 + (y + 8)^2 = 49$.

2. A circle with center at (5, 4) has a radius with endpoint at $(-3, 4)$. Write an equation of the circle.

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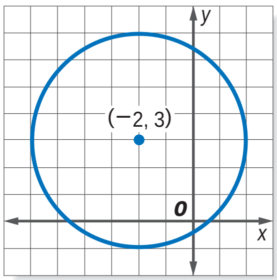
Graph Circles You can analyze the equation of a circle to find information that will help you graph the circle on a coordinate plane.

EXAMPLE **Graph a Circle**

3 **Graph $(x + 2)^2 + (y - 3)^2 = 16$.**

Compare each expression in the equation to the standard form.



$(x - h)^2 = (x + 2)^2$	$(y - k)^2 = (y - 3)^2$
$x - h = x + 2$	$y - k = y - 3$
$-h = 2$	$-k = -3$
$h = -2$	$k = 3$

$r^2 = 16$, so $r = 4$.

The center is at $(-2, 3)$, and the radius is 4.

Graph the center. Use a compass set at a width of 4 grid squares to draw the circle.

3A. $(x - 4)^2 + (y + 1)^2 = 9$ **3B.** $x^2 + y^2 = 25$

Study Tip

Graphing Calculator

To use the center and radius to graph a circle, select a suitable window that contains the center of the circle. For a TI-83/84 Plus, press ZOOM 5.

Then use **9: Circle (** on the **Draw** menu. Put in the coordinates of the center and then the radius so that the screen shows "Circle (-2, 3, 4)."

Then press ENTER.

If you know three points on the circle, you can find the center and radius of the circle and write its equation.

Study Tip

Locus

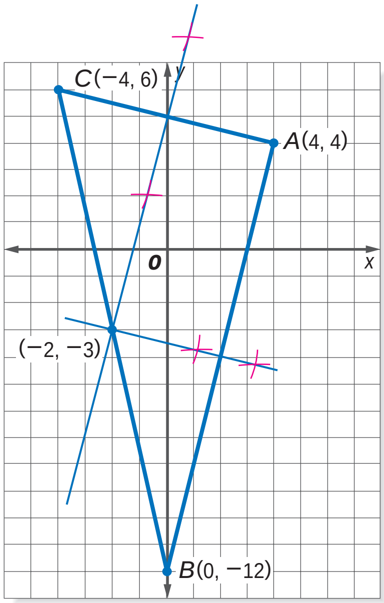
The center of the circle is the locus of points equidistant from the three given points. This is a **compound locus** because the point satisfies more than one condition.

4 **CELL PHONES** **Cell phones work by the transfer of phone signals from one tower to another via satellite. Cell phone companies try to locate towers so that they service multiple communities. Suppose three large metropolitan areas are modeled by the points $A(4, 4)$, $B(0, -12)$, and $C(-4, 6)$, and each unit equals 100 miles. Determine the location of a tower equidistant from all three cities, and write an equation for the circle.**

Explore You are given three points that lie on a circle.

Plan Graph $\triangle ABC$. Construct the perpendicular bisectors of two sides to locate the center, which is the location of the tower. Find the length of a radius. Use the center and radius to write an equation.

Solve Graph $\triangle ABC$ and construct the perpendicular bisectors of two sides. The center appears to be at $(-2, -3)$. This is the location of the tower.



Find r by using the Distance Formula with the center and any of the three points.

$$r = \sqrt{[-2 - 4]^2 + [-3 - 4]^2}$$

$$= \sqrt{85}$$

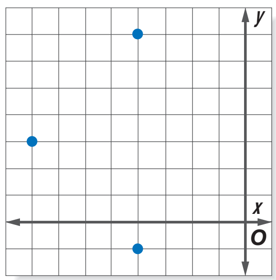
Write an equation.

$$[x - (-2)]^2 + [y - (-3)]^2 = (\sqrt{85})^2$$

$$(x + 2)^2 + (y + 3)^2 = 85$$

Check You can verify the location of the center by finding the equations of the two bisectors and solving a system of equations. You can verify the radius by finding the distance between the center and another of the three points on the circle.

4. Three tornado sirens are placed strategically on a circle around a town so that they can be heard by all of the people. Write the equation of the circle on which they are placed.



Check Your Understanding

Example 1 (p. 614)

1. **WEATHER** Meteorologists track severe storms using Doppler radar. A polar grid is used to measure distances as the storms progress. If the center of the radar screen is the origin and each ring is 10 miles farther from the center, what is the equation of the fourth ring?

Example 2 (p. 615)

Write an equation for each circle described below.

2. center at $(-3, 5)$, $r = 10$

3. center at origin, $r = \sqrt{7}$

4. diameter with endpoints at $(2, 7)$ and $(-6, 15)$

5. diameter with endpoints at $(-7, -2)$ and $(-15, 6)$

Example 3 (p. 615)

Graph each equation.

6. $(x + 5)^2 + (y - 2)^2 = 9$ **7.** $(x - 3)^2 + y^2 = 16$

Example 4 (p. 616)

8. Write an equation of a circle that contains $M(-2, -2)$, $N(2, -2)$, and $Q(2, 2)$. Then graph the circle.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–16	1
17–21	2
22–27	3
28–29	4

Write an equation for each circle described below.

9. center at origin, $r = 3$

10. center at $(-2, -8)$, $r = 5$

11. center at $(1, -4)$, $r = \sqrt{17}$

12. center at $(0, 0)$, $d = 12$

13. center at $(5, 10)$, $r = 7$

14. center at $(0, 5)$, $d = 20$

15. center at $(-8, 8)$, $d = 16$

16. center at $(-3, -10)$, $d = 24$

17. a circle with center at $(-3, 6)$ and a radius with endpoint at $(0, 6)$.

18. a circle whose diameter has endpoints at $(2, 2)$ and $(-2, 2)$

19. a circle with center at $(-2, 1)$ and a radius with endpoint at $(1, 0)$

20. a circle with $d = 12$ and a center translated 18 units left and 7 units down from the origin

21. a circle with its center in quadrant I, radius of 5 units, and tangents $x = 2$ and $y = 3$

Graph each equation.

22. $x^2 + y^2 = 25$

23. $x^2 + y^2 = 36$

24. $x^2 + y^2 - 1 = 0$

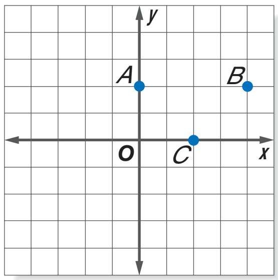
25. $x^2 + y^2 - 49 = 0$

26. $(x - 2)^2 + (y - 1)^2 = 4$

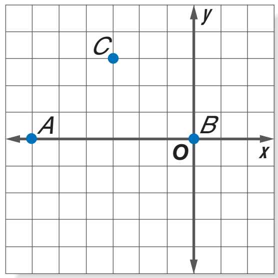
27. $(x + 1)^2 + (y + 2)^2 = 9$

Write an equation of the circle containing each set of points. Copy and complete the graph of the circle.

28.



29.



NOAA

30. Find the radius of a circle that has equation $(x - 2)^2 + (y - 3)^2 = r^2$ and contains (2, 5).

31. Find the radius of a circle that has equation $(x - 5)^2 + (y - 3)^2 = r^2$ and contains (5, 1).

32. **COORDINATE GEOMETRY** Refer to the Check part of Example 4. Verify the coordinates of the center by solving a system of equations that represent the perpendicular bisectors.

MODEL ROCKETS **For Exercises 33–35, use the following information.**

Different sized engines will launch model rockets to different altitudes. The higher the rocket goes, the larger the circle of possible landing sites becomes. Under normal wind conditions, the landing radius is three times the altitude of the rocket.

33. Write the equation of the landing circle for a rocket that travels 300 feet in the air.

34. What type of circles are modeled by the landing areas for engines that take the rocket to different altitudes?

35. What would the radius of the landing circle be for a rocket that travels 1000 feet in the air?

36. The equation of a circle is $(x - 6)^2 + (y + 2)^2 = 36$. Determine whether the line $y = 2x - 2$ is a secant, a tangent, or neither of the circle. Explain.

37. The equation of a circle is $x^2 - 4x + y^2 + 8y = 16$. Find the center and radius of the circle.

38. **WEATHER** The geographic center of Tennessee is near Murfreesboro. The closest Doppler weather radar is in Nashville. If Murfreesboro is designated as the origin, then Nashville has coordinates $(-58, 55)$, where each unit is one mile. If the radar has a radius of 80 miles, write an equation for the circle that represents the radar coverage from Nashville.

Real-World Link

The Apollo program was designed to successfully land a man on the moon. The first landing was July 20, 1969. There were a total of six landings on the moon during 1969–1972.

Source: infoplease.com

39. **SPACE TRAVEL** Apollo 8 was the first manned spacecraft to orbit the Moon at an average altitude of 185 kilometers above the Moon's surface. Write an equation to model a single circular orbit of the command module if the radius of the Moon is 1740 kilometers. Let the center of the Moon be at the origin.

EXTRA PRACTICE

See pages 821, 837.

Math Online

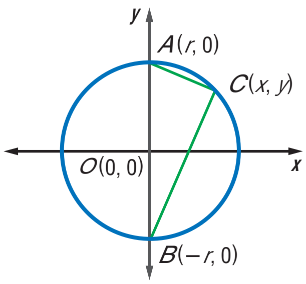
Self-Check Quiz at geometryonline.com

40. **RESEARCH** Use the Internet or other materials to find the closest Doppler radar to your home. Write an equation of the circle for the radar coverage if your home is the center.

H.O.T. Problems

41. **OPEN ENDED** Draw an obtuse triangle on a coordinate plane and construct the circle that circumscribes it.

42. **CHALLENGE** Write a coordinate proof to show that if an inscribed angle intercepts the diameter of a circle, as shown the angle is a right angle.



43. **REASONING** Explain how the definition of a circle leads to its equation.

44. **Writing in Math** Refer to the information on page 614 to describe the kinds of equations used to describe the ripples of a splash. Include the general form of the equation of a circle in your answer. Then produce the equations of five ripples if each ripple is 3 inches farther from the center.

STANDARDIZED TEST PRACTICE

45. Which of the following is an equation of a circle with center at $(-2, 7)$ and a diameter of 18?

A $x^2 + y^2 - 4x + 14y + 53 = 324$

B $x^2 + y^2 + 4x - 14y + 53 = 81$

C $x^2 + y^2 - 4x + 14y + 53 = 18$

D $x^2 + y^2 + 4x - 14y + 53 = 3$

46. **REVIEW** A rectangle has an area of 180 square feet and a perimeter of 54 feet. What are the dimensions of the rectangle?

F 13 ft and 13 ft

G 13 ft and 14 ft

H 15 ft and 12 ft

J 16 ft and 9 ft

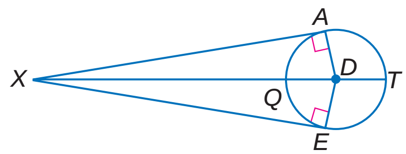
Find each measure if $EX = 24$ and $DE = 7$. (Lesson 10-7)

47. AX

48. DX

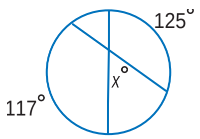
49. QX

50. TX

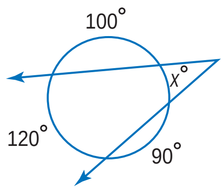


Find x. (Lesson 10-6)

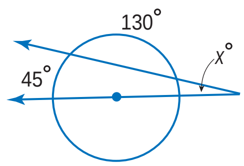
51.



52.



53.

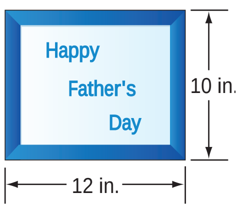


Use the following information for Exercises 54 and 55.
Triangle ABC has vertices $A(-3, 2)$, $B(4, -1)$, and $C(0, -4)$.

54. What are the coordinates of the image after moving $\triangle ABC$ 3 units left and 4 units up? (Lesson 9-2)

55. What are the coordinates of the image of $\triangle ABC$ after a reflection in the y-axis? (Lesson 9-1)

56. **CRAFTS** For a Father's Day present, a kindergarten class is making foam plaques. The edge of each plaque is covered with felt ribbon all the way around with 1 inch overlap. There are 25 children in the class. How much ribbon does the teacher need for all 25 children to complete this craft? (Lesson 1-6)



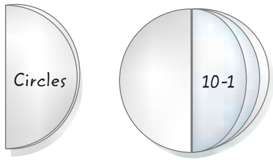
Study Guide and Review

Download Vocabulary Review from geometryonline.com

FOLDABLES™ Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Circles and Circumference (Lesson 10-1)

- Circumference: $C = \pi d$ or $C = 2\pi r$

Angles, Arcs, Chords, and Inscribed Angles

(Lessons 10-2 to 10-4)

- The sum of the measures of the central angles of a circle is 360. The measure of each arc is related to the measure of its central angle.
- The length of an arc is proportional to the length of the circumference.
- Diameters perpendicular to chords bisect chords and intercepted arcs.
- The measure of the inscribed angle is half the measure of its intercepted arc.

Tangents, Secants, and Angle Measures

(Lessons 10-5 and 10-6)

- A line that is tangent to a circle intersects the circle in exactly one point and is perpendicular to a radius.
- Two segments tangent to a circle from the same exterior point are congruent.
- The measure of an angle formed by two secant lines is half the positive difference of its intercepted arcs.
- The measure of an angle formed by a secant and tangent line is half its intercepted arc.

Special Segments and Equation of a Circle

(Lessons 10-7 and 10-8)

- The lengths of intersecting chords in a circle can be found by using the products of the measures of the segments.
- The equation of a circle with center (h, k) is $(x - h)^2 + (y - k)^2 = r^2$.

Key Vocabulary

arc (p. 564)	intercepted (p. 578)
center (p. 554)	major arc (p. 564)
central angle (p. 563)	minor arc (p. 564)
chord (p. 554)	pi (π) (p. 556)
circle (p. 554)	point of tangency (p. 588)
circumference (p. 556)	radius (p. 554)
circumscribed (p. 571)	secant (p. 599)
diameter (p. 554)	semicircle (p. 564)
inscribed (p. 571)	tangent (p. 588)

Vocabulary Check

Choose the term that best matches each phrase. Choose from the list above.

1. a line that intersects a circle in exactly one point
2. a polygon with all of its vertices on the circle
3. an angle with a vertex that is at the center of the circle
4. a segment that has its endpoints on the circle
5. a line that intersects a circle in exactly two points
6. the distance around a circle
7. a chord that passes through the center of a circle
8. an irrational number that is the ratio of $\frac{C}{d}$
9. an arc that measures greater than 180
10. a point where a circle meets a tangent
11. locus of all points in a plane equidistant from a given point
12. a central angle separates the circle into two of these

Vocabulary Review at geometryonline.com

Lesson-by-Lesson Review

10–1 Circles and Circumference (pp. 554–561)

The radius, diameter, or circumference of a circle is given. Find the missing measures. Round to the nearest hundredth if necessary.

13. $d = 15$ in., $r = \underline{?}$, $C = \underline{?}$

14. $C = 68$ yd, $r = \underline{?}$, $d = \underline{?}$

15. $r = 11$ mm, $d = \underline{?}$, $C = \underline{?}$

16. BICYCLES If the circumference of the bicycle tire is 81.7 inches, how long is one spoke?

Example 1 **Find r to the nearest hundredth if $C = 76.2$ feet.**

$C = 2\pi r$	Circumference formula
$76.2 = 2\pi r$	Substitution
$\frac{76.2}{2\pi} = r$	Divide each side by 2π.
$12.13 \approx r$	Use a calculator.

10–2 Measuring Angles and Arcs (pp. 563–569)

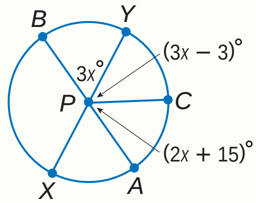
Find each measure.

17. $m\widehat{YC}$

18. $m\widehat{BC}$

19. $m\widehat{BX}$

20. $m\widehat{BCA}$

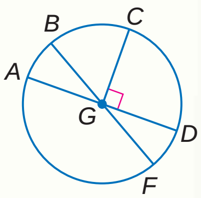


In $\odot G$, $m\angle AGB = 30$ and $\overline{CG} \perp \overline{GD}$. Find each measure.

21. $m\widehat{AB}$ **22.** $m\widehat{BC}$

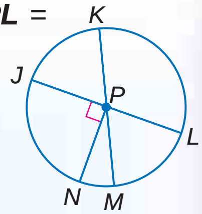
23. $m\widehat{FD}$ **24.** $m\widehat{CDF}$

25. $m\widehat{BCD}$ **26.** $m\widehat{FAB}$



27. CLOCKS If a clock has a diameter of 6 inches, what is the distance along the edge of the clock from the minute hand to the hour hand at 5:00?

Example 2 **In $\odot P$, $m\angle MPL = 65$ and $\overline{NP} \perp \overline{PL}$.**



a. Find $m\widehat{NM}$.

$\widehat{NM}$ is a minor arc, so $m\widehat{NM} = m\angle NPM$.
$\angle JPN$ is a right angle and $m\angle MPL = 65$, so $m\angle NPM = 25$.
$m\widehat{NM} = 25$

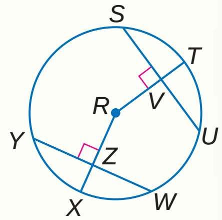
b. Find $m\widehat{NJK}$.

$\widehat{NJK}$ is composed of adjacent arcs $\widehat{NJ}$ and $\widehat{JK}$. $\angle MPL \cong \angle JPK$, so $m\angle JPK = 65$.

$m\widehat{NJ} = m\angle NPJ$ or 90	$\angle NPJ$ is a right angle.
$m\widehat{NJK} = m\widehat{NJ} + m\widehat{JK}$	Arc Addition Postulate
$m\widehat{NJK} = 90 + 65$ or 155	Substitution

10–3 Arcs and Chords (pp. 570–577)

In $\odot R$, $SU = 20$, $YW = 20$, and $m\widehat{YX} = 45$. Find each measure.



28. SV

29. WZ

30. UV

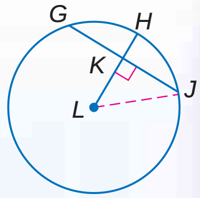
31. $m\widehat{YW}$

32. $m\widehat{ST}$

33. $m\widehat{SU}$

34. **ART** Leonardo DaVinci saw the ideal proportions for man as being measured in relation to two geometric shapes: the circle and the square. The square inscribed in a circle and the circle inscribed in a square are useful to artists, architects, engineers, and designers. Find the measure of each arc of the circle circumscribed about the square.

Example 4 **Circle L has a radius of 32. Find LK if $GJ = 40$.**



Draw radius $\overline{LJ}$. $LJ = 32$ and $\triangle LKJ$ is a right triangle. Since $\overline{LH} \perp \overline{GJ}$, $\overline{LH}$ bisects $\overline{GJ}$.

$KJ = \frac{1}{2}(GJ)$ Definition of segment bisector

$= \frac{1}{2}(40)$ or 20 $GJ = 40$

Use the Pythagorean Theorem to find LK.

$(LK)^2 + (KJ)^2 = (LJ)^2$ Pythagorean Theorem

$(LK)^2 + 20^2 = 32^2$ $KJ = 20$, $LJ = 32$

$(LK)^2 + 400 = 1024$ Simplify.

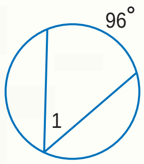
$(LK)^2 = 624$ Subtract.

$LK = \sqrt{624}$ or about 24.98

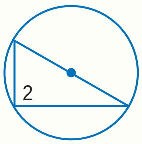
10–4 Inscribed Angles (pp. 578–586)

Find the measure of each numbered angle.

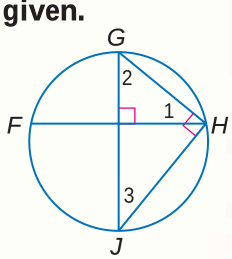
35.



36.



Find the measure of each numbered angle for each situation given.



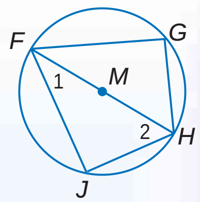
37. $m\widehat{GH} = 78$

38. $m\angle 2 = 2x$, $m\angle 3 = x$

39. $m\widehat{JH} = 114$

40. **ICE SKATING** Sara skates on a circular ice rink and inscribes quadrilateral $ABCD$ in the circle. If $m\angle A = 120$ and $m\angle B = 66$, find $m\angle C$ and $m\angle D$.

Example 5 **Triangle FGH and FHJ are inscribed in $\odot M$ with $FG \cong FJ$. Find x if $m\angle 1 = 6x - 5$, and $m\angle 2 = 7x + 4$.**



$\angle FJH$ is a right angle because $\widehat{FJH}$ is a semicircle.

$m\angle 1 + m\angle 2 + m\angle FJH = 180$ Angle Sum Th.

$(6x - 5) + (7x + 4) + 90 = 180$ Substitution

$13x + 89 = 180$ Simplify.

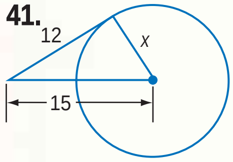
$x = 7$ Solve for x.

Mixed Problem Solving
For mixed problem-solving practice, see page 835.

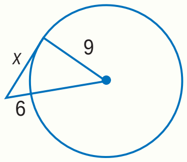
10–5 Tangents (pp. 588–596)

Find x. Assume that segments that appear to be tangent are tangent.

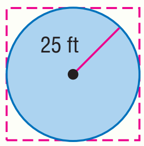
41.



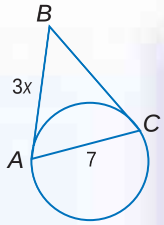
42.



43. SPRINKLER A sprinkler waters a circular section of lawn that is surrounded by a fenced-in square field. If the spray extends to a distance of 25 feet, what is the total length of the fence around the field?



Example 6 **Given that the perimeter of $\triangle ABC = 25$, find x. Assume that segments that appear to be tangent to circles are tangent.**



In the figure, $\overline{AB}$ and $\overline{BC}$ are drawn from the same exterior point and are tangent to $\odot Q$. So, $\overline{AB} \cong \overline{BC}$.

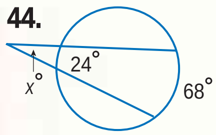
The perimeter of the triangle, $AB + BC + AC$, is 25.

$AB + BC + AC = 25$	Definition of perimeter
$3x + 3x + 7 = 25$	$AB = BC = 3x, AC = 7$
$6x + 7 = 25$	Simplify.
$6x = 18$	Subtract 7 from each side.
$x = 3$	Divide each side by 6.

10–6 Secants, Tangents, and Angle Measures (pp. 599–606)

Find x.

44.



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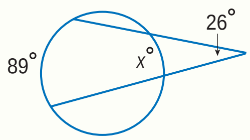
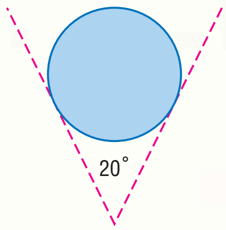
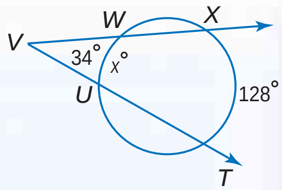


46. JEWELRY Mary has a circular pendant hanging from a chain around her neck. The chain is tangent to the pendant and then forms an angle of 20° below the pendant. Find the measure of the arc at the bottom of the pendant.



Example 7 **Find x.**



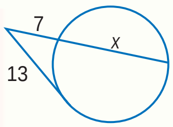
$m\angle V = \frac{1}{2}(m\widehat{XT} - m\widehat{WU})$	
$34 = \frac{1}{2}(128 - x)$	Substitution
$-30 = -\frac{1}{2}x$	Simplify.
$x = 60$	Multiply each side by -2.

Study Guide and Review

10–7 Special Segments in a Circle (pp. 607–613)

Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.

47.



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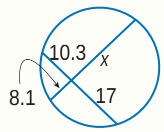
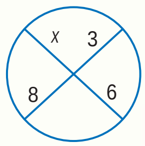
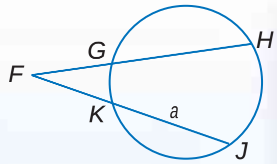


49. LAMPSHADE The top of a lampshade is a circle with two intersecting chords. Use the figure to find x.



Example 8 **Find a, if $FG = 18$, $GH = 42$, and $FK = 15$.**



Let $KJ = a$.

$FK \cdot FJ = FG \cdot FH$	Secant Segment Products
$15(a + 15) = 18(18 + 42)$	Substitution
$15a + 225 = 1080$	Distributive Property
$15a = 855$	Subtract.
$a = 57$	Divide each side by 15.

10–8 Equations of Circles (pp. 614–619)

Write an equation for each circle.

50. center at (0, 0), $r = \sqrt{5}$

51. center at $(-4, 8)$, $d = 6$

52. center at $(-1, 4)$ and is tangent to $x = 1$

Graph each equation.

53. $x^2 + y^2 = 2.25$

54. $(x - 4)^2 + (y + 1)^2 = 9$

For Exercises 55 and 56, use the following information.

A circle graphed on a coordinate plane contains $A(0, 6)$, $B(6, 0)$, $C(6, 6)$.

55. Write an equation of the circle.

56. Graph the circle.

57. PIZZA A pizza parlor is located at the coordinates (7, 3) on a coordinate grid. The pizza parlor's delivery service extends for 15 miles. Write the equation of the circle which represents the outer edge of the pizza delivery service area.

Example 9 **Write an equation of a circle with center $(-1, 4)$ and radius 3.**

$(x - h)^2 + (y - k)^2 = r^2$	Equation of a circle
$[x - (-1)]^2 + (y - 4)^2 = 3^2$	$h = -1, k = 4, r = 3$
$(x + 1)^2 + (y - 4)^2 = 9$	Simplify.

Example 10
Graph $(x - 2)^2 + (y + 3)^2 = 6.25$.

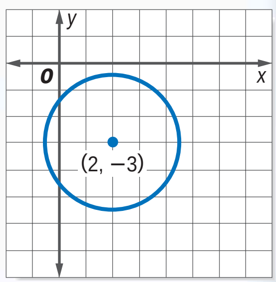
Identify the values of h, k, and r by writing the equation in standard form.

$$(x - 2)^2 + (y + 3)^2 = 6.25$$

$$(x - 2)^2 + [y - (-3)]^2 = 2.5^2$$

$$h = 2,\ k = -3, \text{ and } r = 2.5$$

Graph the center (2, −3) and use a compass to construct a circle with radius 2.5 units.

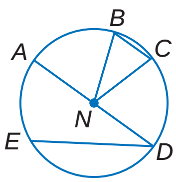


CHAPTER 10

Practice Test

1. Determine the radius of a circle with circumference 25π units. Round to the nearest tenth.

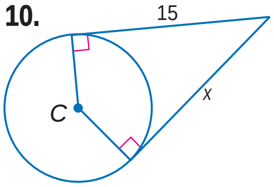
For Questions 2–9, refer to $\odot N$.



2. Name the radii of $\odot N$.
3. If $AD = 24$, find CN.
4. Is $ED > AD$? Explain.
5. If AN is 5 meters long, find the exact circumference of $\odot N$.
6. If $m\angle BNC = 20$, find $m\widehat{BC}$.
7. If $\overline{BE} \cong \overline{ED}$ and $m\widehat{ED} = 120$, find $m\widehat{BE}$.
8. If $m\widehat{BC} = 30$ and $\widehat{AB} \cong \widehat{CD}$, find $m\widehat{AB}$.
9. If $\widehat{AE} = 75$, find $m\angle ADE$.

Find x. Assume that segments that appear to be tangent are tangent.

10.



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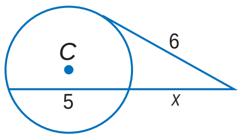

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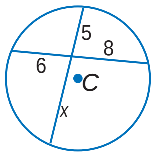

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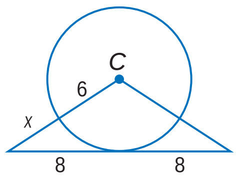

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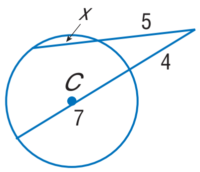

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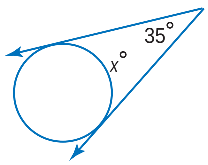

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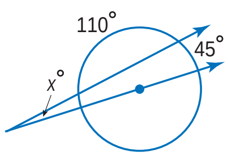

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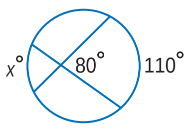


18. **AMUSEMENT RIDES** Suppose a Ferris wheel is 50 feet wide. Approximately how far does a rider travel in one rotation of the wheel?

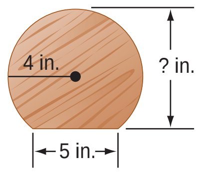
19. Write an equation of a circle with center at $(-2, 5)$ and a diameter of 50.

20. **EARTHQUAKES** When an earthquake strikes, it releases seismic waves that travel in concentric circles from the epicenter of the earthquake. Suppose a seismograph station determines that the epicenter of an earthquake is located 63 miles from the station. If the station is located at the origin, write an equation for the circle that represents a possible epicenter of the earthquake.

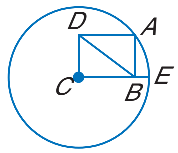
21. Graph $(x - 1)^2 + (y + 2)^2 = 4$.

22. **PROOF** Write a two-column proof.
Given: $\odot X$ with diameters $\overline{RS}$ and $\overline{TV}$
Prove: $\widehat{RT} \cong \widehat{VS}$

23. **CRAFTS** Takita is making bookends out of circular wood pieces, as shown at the right. What is the height of the cut piece of wood?



24. **MULTIPLE CHOICE** Circle C has radius r and $ABCD$ is a rectangle. Find DB.



A r
B $r\frac{\sqrt{2}}{2}$
C $r\sqrt{3}$
D $r\frac{\sqrt{3}}{2}$

25. **CONSTRUCTION** An arch over a doorway is 2 feet high and 7 feet wide. Find the radius of the circle containing the arch.

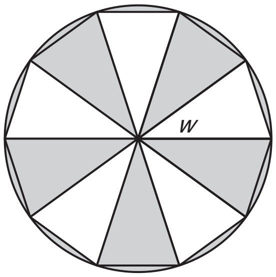
CHAPTER 10

Standardized Test Practice

Cumulative, Chapters 1–10

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A regular decagon is drawn in a circle as a design on the cover of a school yearbook. Opposite vertices are connected by a line segment.



What is the measure of $\angle w$?

A 45° **C** 60°

B 50° **D** 90°

2. The vertices of $\triangle EFG$ are $E(3, 1)$, $F(4, 5)$, $G(1, -2)$. If $\triangle EFG$ is translated 2 units down and 3 units to the right to create $\triangle MNP$, what are the coordinates of the vertices of $\triangle MNP$?

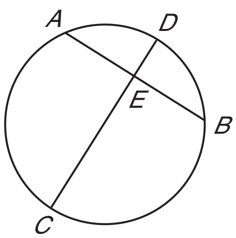
F $M(1, 4)$, $N(3, 8)$, $P(-1, 1)$

G $M(5, 4)$, $N(6, 8)$, $P(3, 1)$

H $M(6, -1)$, $N(7, 3)$, $P(4, -4)$

J $M(6, 3)$, $N(7, 7)$, $P(4, 0)$

3. **GRIDDABLE** In the circle below, $\overline{AB}$ and $\overline{CD}$ are chords intersecting at E.



If $AE = 8$, $DE = 4$, and $EB = 9$, what is the length of $\overline{EC}$?

4. **ALGEBRA** Which inequality is equivalent to $7x < 9x - 3(2x + 5)$?

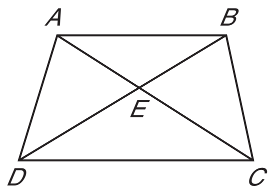
A $4x < 15$

B $4x > -15$

C $4x < -15$

D $4x > 15$

5. Trapezoid $ABCD$ is shown below.



Which pair of triangles can be established as similar to prove that $\frac{AE}{AB} = \frac{EC}{DC}$?

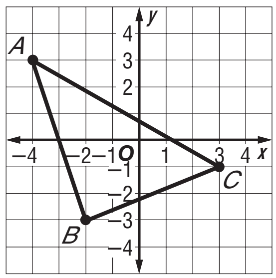
F $\triangle ADB$ and $\triangle BCA$

G $\triangle AEB$ and $\triangle CED$

H $\triangle ADC$ and $\triangle BCD$

J $\triangle AED$ and $\triangle BEC$

6. If $\triangle ABC$ is reflected across the y-axis, what are the coordinates of C'?



A (3, 1)

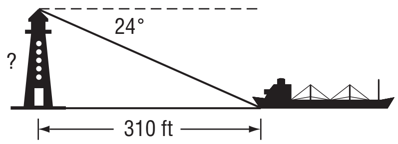
B $(-1, 3)$

C $(-3, -1)$

D $(-1, -3)$

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 841–856.

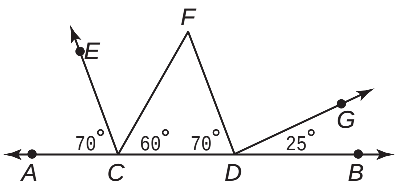
7. **GRIDDABLE** A passing ship is 310 feet from the base of a lighthouse. The angle of depression from the top of the lighthouse to the ship is 24°.



sin 24° ≈ 0.41
cos 24° ≈ 0.91
tan 24° ≈ 0.45

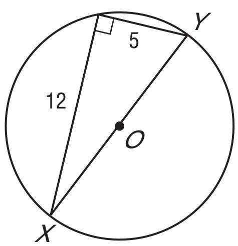
What is the height of the lighthouse in feet to the nearest tenth?

8. Which of the following statements is true?



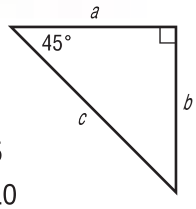
F $\overline{CE} \cong \overline{DF}$
G $\overline{CF} \parallel \overline{DG}$
H $\overline{CF} \cong \overline{DF}$
J $\overline{CE} \parallel \overline{DF}$

9. If $\overline{XY}$ is a diameter of circle O, what is the circumference of circle O?



A 5π **C** 7.5π
B 7π **D** 13π

10. If $a = 5\sqrt{2}$ in the right triangle below, what is the value of c?

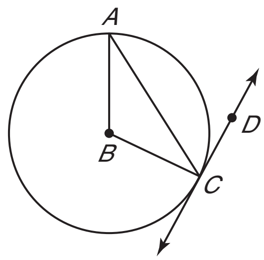


F $5\sqrt{6}$ **H** 5
G $10\sqrt{2}$ **J** 10

TEST-TAKING TIP

Question 10 Question 10 is a multi-step problem. First, identify the figure shown as a 45°-45°-90° right triangle. Next, use the relationship between the sides in this kind of triangle to determine that $c = \sqrt{2}a$. Finally, recall from algebra that $\sqrt{m} \cdot \sqrt{n} = \sqrt{m \cdot n}$.

11. **GRIDDABLE** $\overleftrightarrow{CD}$ is tangent at point C to a circle, with B at its center. $\overline{BC}$ is a radius. If $m\angle ACB = 35°$, what is $m\angle ACD$?



12. A triangle is dilated so that the ratio between the areas of the triangle and its image is 9 to 8. What is the ratio between the perimeters of the two triangles?

A 3 to 22 **C** 18 to 16
B 4.5 to 4 **D** 81 to 64

Pre-Ap

Record your answers on a sheet of paper. Show your work.

13. The segment with endpoints $A(1, -2)$ and $B(1, 6)$ is the diameter of a circle.

a. Graph the points and draw the circle.
b. What is the circumference of the circle?
c. What is the equation of the circle?

NEED EXTRA HELP?													
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson or Page...	10-4	9-2	10-7	783	7-3	8-5	9-1	3-5	10-1	8-3	10-5	9-5	10-8

CHAPTER 11

Areas of Polygons and Circles

BIG Ideas

- Find areas of parallelograms, triangles, rhombi, trapezoids, regular polygons, and circles.
- Find areas of composite figures.
- Find geometric probability and areas of sectors and segments of circles.

Key Vocabulary

apothem (p. 649)

composite figure (p. 658)

geometric probability (p. 665)

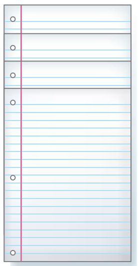
sector (p. 666)

Real-World Link

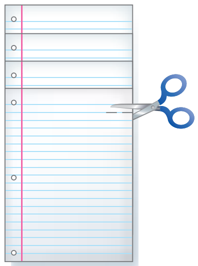
Hang Gliding The shape of the nylon parachute of the hang glider comprises two triangular wings.

Areas of Polygons and Circles Make this Foldable to help you organize your notes. Begin with five sheets of notebook paper.

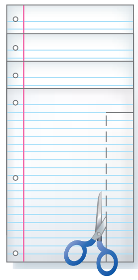
1. **Stack** 4 of the 5 sheets of notebook paper as illustrated.



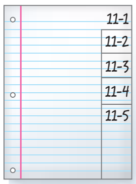
2. **Cut** in about 1 inch along the heading line on the top sheet of paper.



3. **Cut** the margins off along the right edge.



4. **Stack** in order of cuts, placing the uncut fifth sheet at the back. Label the tabs as shown. Staple edge to form a book.



Steve Satushek/The Image Bank/Getty Images

GET READY for Chapter 11

Diagnose Readiness **You have two options for checking Prerequisite Skills.**

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

The area and width of a rectangle are given. Find the length of the rectangle. (Lesson 1-6)

1. $A = 150, w = 15$ **2.** $A = 38, w = 19$

3. $A = 21.16, w = 4.6$ **4.** $A = 2000, w = 32$

5. $A = 450, w = 25$ **6.** $A = 256, w = 20$

7. **GARDENS** The area of a rectangular garden is 115 square feet. If the width is 11 feet, what is the length? Round to the nearest tenth. (Lesson 1-6)

Evaluate each expression for $a = 6$, $b = 8$, $c = 10$, and $d = 11$. (Prerequisite Skills)

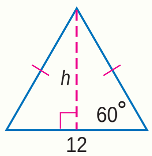
8. $\frac{1}{2}a(b + c)$ **9.** $\frac{1}{2}ab$

10. $\frac{1}{2}(2b + c)$ **11.** $\frac{1}{2}d(a + c)$

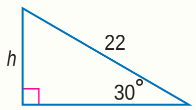
12. $\frac{1}{2}(b + c)$ **13.** $\frac{1}{2}cd$

Find h in each triangle. (Lesson 8-3)

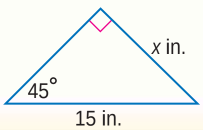
14.



15.



16. **WINDOWS** Miss Valdez has a triangular window pane above the door of her house, as shown. Find the length of each of the legs of the triangle. (Lesson 8-3)



QUICKReview

EXAMPLE 1

The area of a rectangle is 81 square units and the width is 3 units. Find the length.

$A = \ell w$		Definition of Area
$81 = \ell(3)$		Substitution
$27 = \ell$		Divide each side by 3.

A rectangle with an area of 81 square units and a width of 3 units has a length of 27 units.

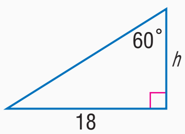
EXAMPLE 2

Evaluate $\frac{1}{2}(2x + y)$ for $x = 5$ and $y = 18$.

$$\begin{aligned} \tfrac{1}{2}(2x + y) &= \tfrac{1}{2}(2(5) + 18) && \text{Substitution} \\ &= \tfrac{1}{2}(10 + 18) && \text{Multiply.} \\ &= \tfrac{1}{2}(28) \text{ or } 14 && \text{Simplify.} \end{aligned}$$

EXAMPLE 3

Find h in the triangle.



By the Angle Sum Theorem, the third angle is 30°. The sides of a 30°-60°-90° triangle are in the ratio $x{:}x\sqrt{3}{:}2x$.

$$x\sqrt{3} = 18$$

$$x = \frac{18}{\sqrt{3}} \text{ or } 6\sqrt{3}$$

Since h is opposite the 30° angle, $h = 6\sqrt{3}$.

11-1 Areas of Parallelograms

Main Ideas

- Find perimeters and areas of parallelograms.
- Determine whether points on a coordinate plane define a parallelogram.

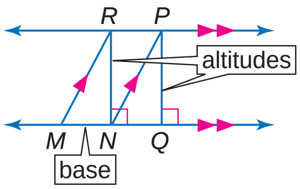
New Vocabulary

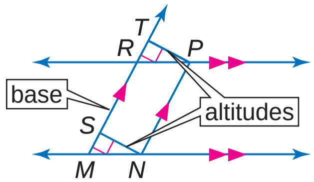
height of a parallelogram

GET READY for the Lesson

Land is usually measured in acres. Acre is a historic Saxon term that means "field." An acre was a unit of measure that represented a field that could be plowed in one day. Unlike other units of measure for area, an acre is not a square, but a rectangle 22 yards by 220 yards.

Areas of Parallelograms Recall that a *parallelogram* is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called a base. For each base, any segment that is perpendicular to the base is an altitude. The length of an altitude is called the **height of the parallelogram**.



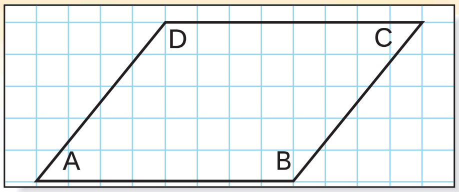


GEOMETRY LAB

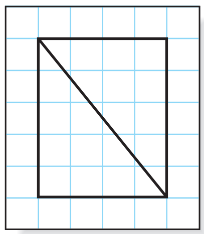
Area of a Parallelogram

MODEL

- Draw parallelogram *ABCD* on grid paper. Label the vertices on the interior of the angles with letters *A*, *B*, *C*, and *D*.
- Fold ▱*ABCD* so that *A* lies on *B* and *C* lies on *D*, forming a rectangle.



ANALYZE



1. What is the area of the rectangle?

2. How many rectangles form the parallelogram?

3. What is the area of the parallelogram?

4. How do the base and altitude of the parallelogram relate to the length and width of the rectangle?

5. **MAKE A CONJECTURE** Use what you observed to write a formula for the area of a parallelogram.

Study Tip

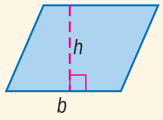
Units

Length is measured in linear units, and area is measured in square units.

The Geometry Lab leads to the formula for the area of a parallelogram.

KEY CONCEPT — Area of a Parallelogram

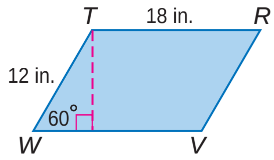
Words If a parallelogram has an area of A square units, a base of b units, and a height of h units, then area equals the product of the base and the height.



Symbols $A = bh$

EXAMPLE Perimeter and Area of a Parallelogram

1 **Find the perimeter and area of $\square TRVW$.**



Bases and Sides: Each pair of opposite sides of a parallelogram has the same measure. Each base is 18 inches long, and each side is 12 inches long.

Perimeter: The perimeter of a polygon is the sum of the measures of its sides. So, the perimeter of $\square TRVW$ is $2(18) + 2(12)$ or 60 inches.

Height: Use a 30°-60°-90° triangle to find the height. Recall that if the measure of the leg opposite the 30° angle is x, then the length of the hypotenuse is $2x$, and the length of the leg opposite the 60° angle is $x\sqrt{3}$.

$12 = 2x$ Substitute 12 for the hypotenuse.

$6 = x$ Divide each side by 2.

So, the height of the parallelogram is $x\sqrt{3}$ or $6\sqrt{3}$ inches.

Area: $A = bh$ Area of a parallelogram

$= 18\left(6\sqrt{3}\right)$ $b = 18, h = 6\sqrt{3}$

$= 108\sqrt{3}$ or about 187.1

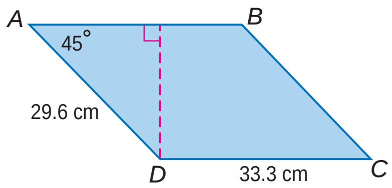
The perimeter of $\square TRVW$ is 60 inches, and the area is about 187.1 square inches.

Study Tip

Look Back

To review **special right triangles**, see Lesson 8-3.

1. Find the perimeter and area of $\square ABCD$.



2 **INTERIOR DESIGN** **The Navarros are painting a room in their house. The rectangular room is 14 feet long and 12 feet wide. The walls are 10 feet high. Find the area of the walls to be painted.**

The dimensions of the floor and the height of the walls are given. The area of the walls to be painted is the sum of the area of each wall in the room.

Area of each long wall

$A = bh$ Area of a rectangle

$= (14)(10)$ $b = 14$ ft, $h = 10$ ft

$= 140$ Multiply.

Area of each short wall

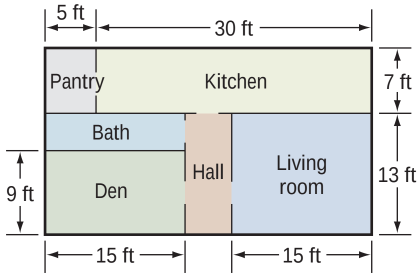
$A = bh$ Area of a rectangle

$= (12)(10)$ $b = 12$ ft, $h = 10$ ft

$= 120$ Multiply.

Since the room is rectangular, the total area is $2(140) + 2(120)$ or 520 square feet.

2. **INTERIOR DESIGN** The Waroners are planning to carpet part of their house. The carpet they plan to buy is sold by the square yard. Find the amount of carpeting needed to cover the living room, den, and hall if all are rectangular rooms.



Parallelograms on the Coordinate Plane Recall the properties of quadrilaterals that you studied in Chapter 6. Using these properties as well as the formula for slope and the Distance Formula, you can find the perimeters and areas of quadrilaterals on the coordinate plane.

Study Tip

Look Back

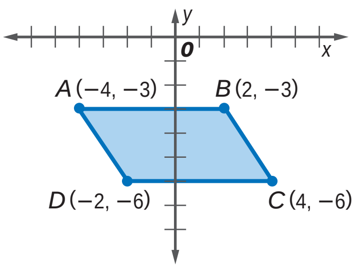
To review properties of **parallelograms, rectangles,** and **squares,** see Lessons 6-3, 6-4, and 6-5.

EXAMPLE Perimeter and Area on the Coordinate Plane

3 **COORDINATE GEOMETRY** **The vertices of a quadrilateral are $A(-4, -3)$, $B(2, -3)$, $C(4, -6)$, and $D(-2, -6)$.**

a. Determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*.

First graph each point and draw the quadrilateral. Then determine the slope of each side.



$$\text{slope of } \overline{AB} = \frac{-3-(-3)}{-4-2} = \frac{0}{-6} \text{ or } 0$$

$$\text{slope of } \overline{CD} = \frac{-6-(-6)}{4-(-2)} = \frac{0}{6} \text{ or } 0$$

slope of $\overline{BC} = \frac{-3-(-6)}{2-4}$
$= \frac{3}{-2}$

slope of $\overline{AD} = \frac{-3-(-6)}{-4-(-2)}$
$= \frac{3}{-2}$

Opposite sides have the same slope, so they are parallel. *ABCD* is a parallelogram. The slopes of the consecutive sides are *not* negative reciprocals of each other, so the sides are not perpendicular. Thus, the parallelogram is neither a square nor a rectangle.

b. Find the perimeter of quadrilateral *ABCD*.

Since $\overline{AB}$ and $\overline{CD}$ are parallel to the *x*-axis, you can subtract the *x*-coordinates of the endpoints to find their measures.

$AB = 2 - (-4)$
$= |6|$ or 6

$CD = 4 - (-2)$
$= |6|$ or 6

Use the Distance Formula to find *BC* and *AD*.

$BC = \sqrt{(4-2)^2 + (-6-(-3))^2}$
$= \sqrt{2^2 + (-3)^2}$
$= \sqrt{13}$

$AD = \sqrt{(-2-(-4))^2 + (-6-(-3))^2}$
$= \sqrt{2^2 + (-3)^2}$
$= \sqrt{13}$

Now add to find the perimeter.

perimeter of $ABCD = AB + BC + CD + AD$	Definition of perimeter
$= 6 + \sqrt{13} + 6 + \sqrt{13}$	Substitution
$= 12 + 2\sqrt{13}$	Add like terms.

The perimeter of quadrilateral *ABCD* is $12 + 2\sqrt{13}$ or about 19.21 units.

Study Tip

Alternative Method

It was already proved that *ABCD* is a parallelogram. So since opposite sides of a parallelogram are congruent, it could be assumed that $AB = CD$ and $BC = AD$.

c. Find the area of quadrilateral *ABCD*.

Base: From Part b, $CD = 6$.

Height: Since $\overline{AB}$ and $\overline{CD}$ are horizontal segments, the distance between them, or the height, can be measured on any vertical segment. Reading from the graph, the height is 3.

$A = bh$	Area formula
$= 6(3)$	$b = 6, h = 3$
$= 18$	Simplify.

The area of ▱*ABCD* is 18 square units.

COORDINATE GEOMETRY **The vertices of a quadrilateral are $J(0, -3)$, $K(-3, 1)$, $L(-15, -8)$, and $M(-12, -12)$.**

3A. Determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*.

3B. Find the perimeter of quadrilateral *JKLM*.

3C. Find the area of quadrilateral *JKLM*.

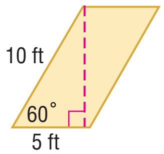
Personal Tutor at geometryonline.com

CHECK Your Understanding

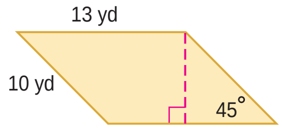
Example 1 (p. 631)

Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

1.



2.



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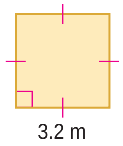
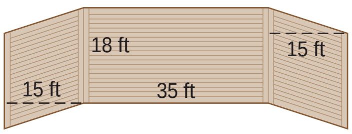


Example 2 (p. 632)

4. HOME IMPROVEMENT Mr. Esperanza is planning to stain his deck. To know how much stain to buy, he needs to find the area of the deck. What is the area?



Example 3 (p. 632)

Given the coordinates of the vertices of quadrilateral *TVXY*, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the perimeter and area of *TVXY*.

5. $T(0, 0)$, $V(2, 6)$, $X(6, 6)$, $Y(4, 0)$

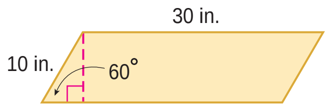
6. $T(10, 16)$, $V(2, 18)$, $X(-3, -2)$, $Y(5, -4)$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
7–12	1
13–17	2
18–21	3

Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

7.



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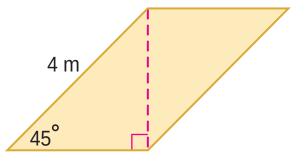

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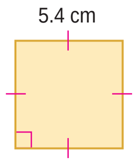
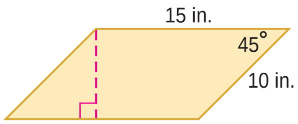
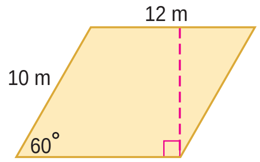


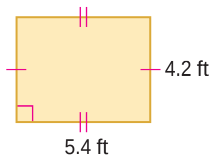
10.



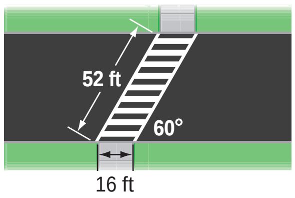
11.



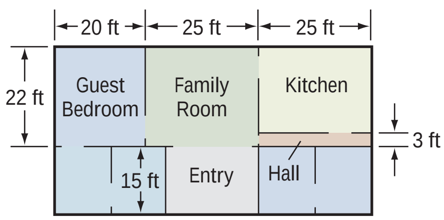
12.



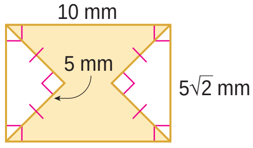
13. ROADS A crosswalk with two stripes, each 52 feet long, is at a 60° angle to the curb. The width of the crosswalk at the curb is 16 feet. Find the perpendicular distance between the stripes of the crosswalk.

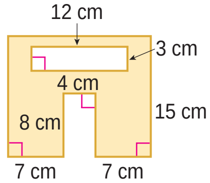


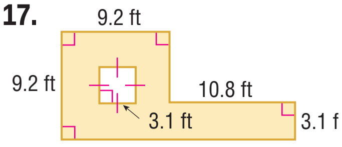
14. INTERIOR DESIGN The Bessos are planning to have new carpet installed in their guest bedroom, family room, and hallway. Find the number of square yards of carpet they should order if all rooms are rectangular.



Find the area of each shaded region. Round to the nearest tenth if necessary.

15. 

16. 

17. 

COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the perimeter and area of the quadrilateral.**

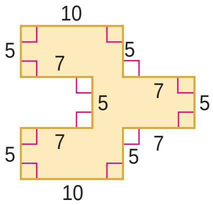
18. $A(0, 0)$, $B(4, 0)$, $C(5, 5)$, $D(1, 5)$

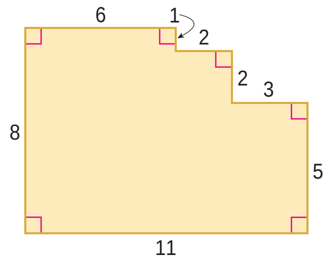
19. $E(-5, -3)$, $F(3, -3)$, $G(5, 4)$, $H(-3, 4)$

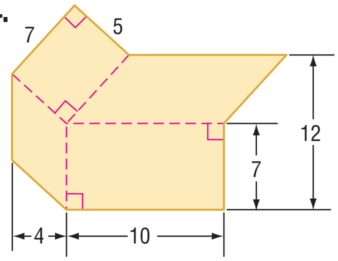
20. $R(-2, 4)$, $S(8, 4)$, $T(8, -3)$, $U(-2, -3)$

21. $V(1, 10)$, $W(4, 8)$, $X(2, 5)$, $Y(-1, 7)$

Find the area of each figure.

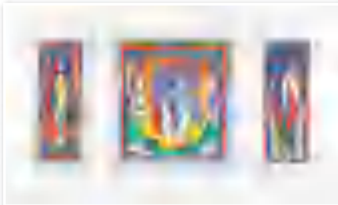
22. 

23. 

24. 

ART **For Exercises 25 and 26, use the following information.**

A triptych painting is a series of three pieces with a similar theme displayed together. Suppose the center panel is a 12-inch square and the panels on either side are 12 inches by 5 inches. The panels are 2 inches apart with a 3 inch wide border around the edges.



25. Determine whether the triptych will fit a 45-inch by 20-inch frame. Explain.

26. Find the area of the artwork, including the border.

CHANGING DIMENSIONS **For Exercises 27–29, use the following information.**

A parallelogram has a base of 8 meters, sides of 11 meters, and a height of 10 meters.

27. Find the perimeter and area of the parallelogram.

28. Suppose the base of the parallelogram was divided in half. Find the new perimeter and area. Compare to the perimeter and area of the original parallelogram.

29. Suppose the original dimensions of the parallelogram were divided in half. Find the perimeter and the area. Compare the perimeter and area of the parallelogram with the original.

EXTRA PRACTICE
See pages 822, 838.
Math Online
Self-Check Quiz at
geometryonline.com

H.O.T. Problems

30. **OPEN ENDED** Make and label a scale drawing of your bedroom. Then find its area in square yards.

31. **REASONING** Given a parallelogram of base b and height h, determine an expression for the area of a parallelogram with each dimension halved. Determine the formulas for the area and perimeter. Compare to the original formulas. **Make a conjecture** about the area and perimeter of a parallelogram in which each dimension was divided in half.

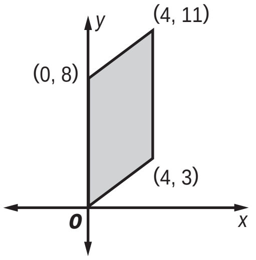
32. REASONING Determine the formulas for the area and perimeter of a parallelogram in which one dimension was divided in half. Compare to the original formula. **Make a conjecture** about the area of a parallelogram in which one dimension was divided in half.

33. CHALLENGE A piece of twine 48 inches long is cut into two lengths. Each length is then used to form a square. The sum of the areas of the two squares is 74 square inches. Find the length of each side of the smaller square and the larger square.

34. Writing in Math Refer to the information on acres on page 630. Explain how area is related to acres.

STANDARDIZED TEST PRACTICE

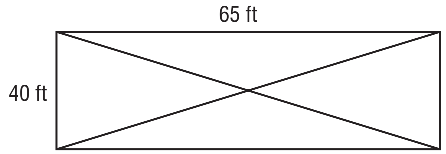
35. What is the area, in square units, of the parallelogram shown?



A 12 **C** 24

B 20 **D** 40

36. REVIEW Tia is going to spray paint a rectangle and its two diagonals in a field for a game. If each can of spray paint covers about 100 feet, how many cans of spray paint should Tia buy?



F 3 **H** 5

G 4 **J** 6

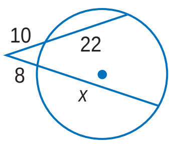
Determine the coordinates of the center and the measure of the radius for each circle with the given equation. (Lesson 10-8)

37. $(x - 5)^2 + (y - 2)^2 = 49$

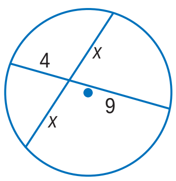
38. $(x + 3)^2 + (y + 9)^2 - 81 = 0$

Find *x*. Assume that segments that appear to be tangent are tangent. (Lesson 10-7)

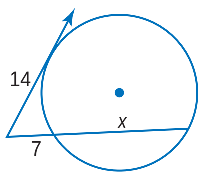
39.



40.



41.



42. BIKES Tariq is making a ramp for bike jumps. The ramp support forms a right angle. The base is 12 feet long, and the height is 5 feet. What length of plywood does Tariq need for the ramp? (Lesson 8-2)

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate each expression if $w = 8$, $x = 4$, $y = 2$, and $z = 5$. (Page 780)

43. $\frac{1}{2}(7y)$

44. $\frac{1}{2}wx$

45. $\frac{1}{2}z(x + y)$

46. $\frac{1}{2}x(y + w)$

EXPLORE 11-2

Graphing Calculator Lab

Investigating Area

ACTIVITY 1

Step 1 Construct a line using the line tool on the F2 menu. Label two points on the line as A and B.

Step 2 Use the Parallel tool on the F3 menu to construct a line parallel to the first line. Pressing ENTER will draw the line and a point on the line. Label the point C.

Step 3 Construct triangle ABC using the Triangle tool on the F2 menu.

Step 4 Access the Area tool under Measure on the F5 menu. Display the area of $\triangle ABC$. Then display the measure of $\overline{AB}$ and the distance from C to $\overleftrightarrow{AB}$.

Step 5 Click on point C and drag it along the line to change the shape of $\triangle ABC$.

1A. What do you observe about the base and height of $\triangle ABC$?

1B. What do you observe about the area of $\triangle ABC$?

1C. Use what you know about the formula for the area of a rectangle to write a conjecture about the formula for the area of a triangle.

ACTIVITY 2

Step 1 Construct a line using the line tool on the F2 menu. Label two points on the line as W and X. Then use the Parallel tool on the F3 menu to construct a line parallel. Label points on this line as Y and Z.

Step 2 Access the Quadrilateral tool on the F2 menu. Construct quadrilateral $WXYZ$.

Step 3 Use the Area tool under Measure on the F5 menu to display the area of $WXYZ$. Then display the measures of $\overline{WX}$ and $\overline{YZ}$, and find the distance from $\overleftrightarrow{WX}$ to $\overleftrightarrow{YZ}$.

Step 4 Click on point W and drag it along the line.

2A. What kind of quadrilateral is $WXYZ$? Explain.

2B. Use what you know about the formula for the area of a rectangle to write a conjecture about the formula for the area of this type of quadrilateral. Verify your conjecture.

ANALYZE THE RESULTS

The area of a rhombus is dependent upon the measures of the diagonals. Use Cabri Jr. to draw a rhombus and make a conjecture about the formula for the area of a rhombus.

11-2 Areas of Triangles, Trapezoids, and Rhombi

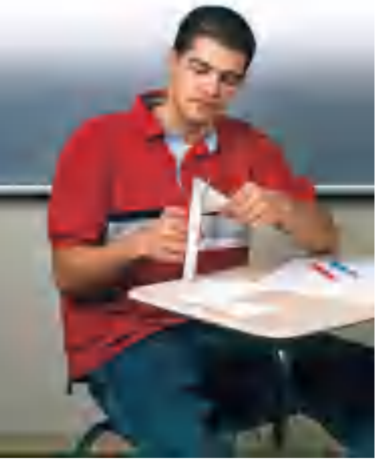
Main Ideas

- Find areas of triangles.
- Find areas of trapezoids and rhombi.

GET READY for the Lesson

Umbrellas can protect you from rain, wind, and sun. The umbrella shown at the right is made of triangular panels. To cover the umbrella frame with canvas panels, you need to know the area of each panel.

Areas of Triangles You have learned how to find the areas of squares, rectangles, and parallelograms. The formula for the area of a triangle is related to these formulas.

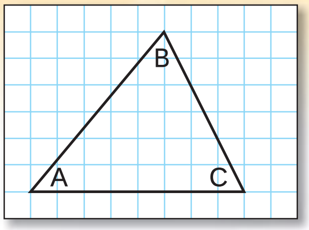


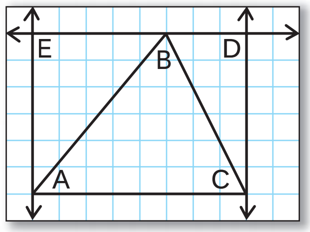
GEOMETRY LAB

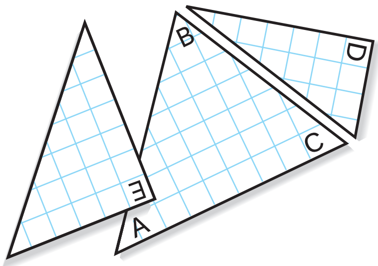
Area of a Triangle

MODEL

- Draw a triangle on grid paper so that one edge is along a horizontal line. Label the vertices on the interior of the angles of the triangle as A, B, and C.
- Draw a line perpendicular to $\overline{AC}$ through A.
- Draw a line perpendicular to $\overline{AC}$ through C.
- Draw a line parallel to $\overline{AC}$ through B.
- Label the points of intersection of the lines drawn as D and E as shown.
- Find the area of rectangle $ACDE$ in square units.
- Cut out rectangle $ACDE$. Then cut out $\triangle ABC$. Place the two smaller pieces over $\triangle ABC$ to completely cover the triangle.







ANALYZE THE RESULTS

1. What do you observe about the two smaller triangles and $\triangle ABC$?

2. What fraction of rectangle $ACDE$ is $\triangle ABC$?

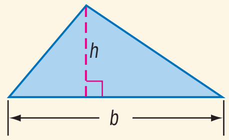
3. Derive a formula that could be used to find the area of $\triangle ABC$.

(l)Matt Meadows, (r)Paul Baron/CORBIS

The Geometry Lab suggests the formula for finding the area of a triangle.

KEY CONCEPT — ***Area of a Triangle***

Words If a triangle has an area of A square units, a base of b units, and a corresponding height of h units, then the area equals one half the product of the base and the height.



Symbols $A = \frac{1}{2}bh$

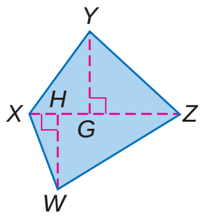
Study Tip

Look Back
To review the **height and altitude of a triangle**, see Lesson 5-1.

EXAMPLE Areas of Triangles

1 **Find the area of quadrilateral *XYZW* if $XZ = 39$, $HW = 20$, and $YG = 21$.**

The area of the quadrilateral is equal to the sum of the areas of $\triangle XWZ$ and $\triangle XYZ$.



area of $XYZW$ = area of $\triangle XYZ$ + area of $\triangle XWZ$

$= \frac{1}{2}bh_1 + \frac{1}{2}bh_2$

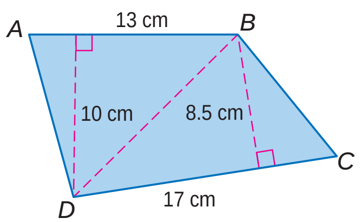
$= \frac{1}{2}(39)(21) + \frac{1}{2}(39)(20)$ Substitution

$= 409.5 + 390$ Simplify.

$= 799.5$

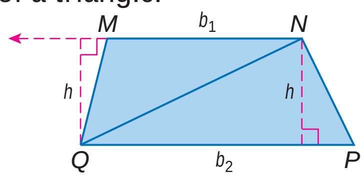
The area of quadrilateral $XYZW$ is 799.5 square units.

1. Find the area of quadrilateral $ABCD$.



Areas of Trapezoids and Rhombi The formulas for the areas of trapezoids and rhombi are related to the formula for the area of a triangle.

The diagonal $\overline{QN}$ separates trapezoid $MNPQ$ into two triangles.



area of		area of		area of
trapezoid $MNPQ$	=	$\triangle MNQ$	+	$\triangle NPQ$

$A = \frac{1}{2}(b_1)h + \frac{1}{2}(b_2)h$ Let $MN = b_1$, and $QP = b_2$.

$= \frac{1}{2}(b_1 + b_2)h$ Factor.

$= \frac{1}{2}h(b_1 + b_2)$ Commutative Property

This is the formula for the area of any trapezoid.

Study Tip

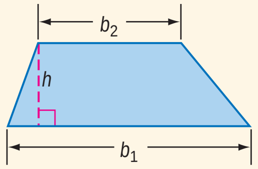
Alternate Method

Notice that the formula for the area of a trapezoid can also be expressed as a product of the height and the mean of the lengths of the bases.

KEY CONCEPT — Area of a Trapezoid

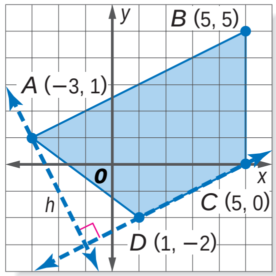
Words If a trapezoid has an area of A square units, bases of b_1 units and b_2 units, and a height of h units, then the area equals the product of one half the height and the sum of the lengths of each base.

Symbols $A = \frac{1}{2}h(b_1 + b_2)$



EXAMPLE Area of a Trapezoid on a Coordinate Plane

2 **COORDINATE GEOMETRY** **Find the area of trapezoid *ABCD* with vertices $A(-3, 1)$, $B(5, 5)$, $C(5, 0)$, and $D(1, -2)$.**



Height: To find the height, extend the line that passes through D and C.

The slope of this line is $\frac{1}{2}$.

Next, graph the line perpendicular to the bases that passes through A. From the graph, you can determine that the coordinates of the point of intersection are $(-1, -3)$.

The height of the trapezoid is the distance between $(-3, 1)$ and $(-1, -3)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$h = \sqrt{(-1 - (-3))^2 + (-3 - 1)^2}$ $(x_1, y_1) = (-3, 1)$, $(x_2, y_2) = (-1, -3)$

$= \sqrt{2^2 + (-4)^2}$ Subtract.

$= \sqrt{4 + 16}$ or $\sqrt{20}$ Simplify.

Bases: Use the Distance Formula to determine the length of each base.

$\overline{AB}$: $A(-3, 1)$ and $B(5, 5)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-3 - 5)^2 + (1 - 5)^2}$

$= \sqrt{(-8)^2 + (-4)^2}$

$= \sqrt{64 + 16}$ or $\sqrt{80}$

$\overline{DC}$: $D(1, -2)$ and $C(5, 0)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1 - 5)^2 + (-2 - 0)^2}$

$= \sqrt{(-4)^2 + (-2)^2}$

$= \sqrt{16 + 4}$ or $\sqrt{20}$

Area: $A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$= \frac{1}{2}(\sqrt{20})(\sqrt{80} + \sqrt{20})$ $h = \sqrt{20}$, $b_1 = \sqrt{80}$, $b_2 = \sqrt{20}$

$= \frac{1}{2}\sqrt{1600} + \frac{1}{2}\sqrt{400}$ Distributive Property

$= \frac{1}{2}(40) + \frac{1}{2}(20)$ or 30 The area of the trapezoid is 30 square units.

2. **COORDINATE GEOMETRY** Find the area of trapezoid *ABCD* with vertices $A(-10, 5)$, $B(13, 5)$, $C(7, -3)$, $D(-8, -3)$.

Online **Personal Tutor at** geometryonline.com

The formula for the area of a triangle can also be used to derive the formula for the area of a rhombus.

Study Tip

Area of a Rhombus

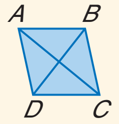
Because a rhombus is also a parallelogram, you can also use the formula $A = bh$ to determine the area.

KEY CONCEPT — *Area of a Rhombus*

Words If a rhombus has an area of A square units and diagonals of d_1 and d_2 units, then area equals one half the product of the length of each diagonal.

Symbols $A = \frac{1}{2}d_1d_2$

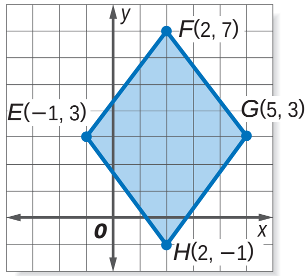
Example $A = \frac{1}{2}(AC)(BD)$



You will derive this formula in Exercise 41.

EXAMPLE Area of a Rhombus on the Coordinate Plane

3 **COORDINATE GEOMETRY Find the area of rhombus *EFGH*.**



Explore To find the area of the rhombus, we need to know the lengths of each diagonal.

Plan Use coordinate geometry to find the length of each diagonal. Use the formula to find the area of rhombus *EFGH*.

Solve Let $\overline{EG}$ be d_1 and $\overline{FH}$ be d_2.

Subtract the x-coordinates of E and G to find that d_1 is 6. Subtract the y-coordinates of F and H to find that d_2 is 8.

$A = \frac{1}{2}d_1d_2$ — Area of a rhombus

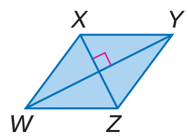
$= \frac{1}{2}(6)(8)$ or 24 — $d_1 = 6, d_2 = 8$

Check The area of rhombus *EFGH* is 24 square units.

3. COORDINATE GEOMETRY Find the area of rhombus $JKLM$ with vertices $J(0, 2), K(2, 6), L(4, 2), M(2, -2)$.

EXAMPLE Find Missing Measures

4 **ALGEBRA Rhombus *WXYZ* has an area of 100 square meters. Find *WY* if *XZ* = 10 meters.**



Use the formula for the area of a rhombus and solve for d_2.

$A = \frac{1}{2}d_1d_2$ — Area of a rhombus

$100 = \frac{1}{2}(10)(d_2)$ — $A = 100, d_1 = 10$

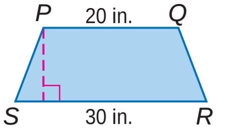
$100 = 5d_2$ — Simplify.

$20 = d_2$ — Divide.

WY is 20 meters.

CHECK Your Progress

4. Trapezoid *PQRS* has an area of 250 square inches. Find the height of *PQRS*.



Since the dimensions of congruent figures are equal, the areas of congruent figures are also equal.

POSTULATE 11.1

Congruent figures have equal areas.

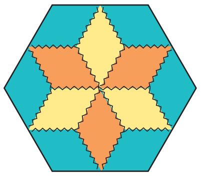
Study Tip

Look Back

To review **the properties of rhombi and trapezoids**, see Lessons 6-5 and 6-6.

EXAMPLE 5 Area of Congruent Figures

QUILTING This quilt block is composed of twelve congruent rhombi arranged in a regular hexagon. The height of the hexagon is 8 inches. If the total area of the rhombi is 48 square inches, find the lengths of each diagonal and the area of one rhombus.



Step 1 Use the area formula to find the length of the other diagonal.

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

$4 = \frac{1}{2}(4)d_2$ $A = 4, d_1 = 4$

$2 = d_2$ Solve for d_2.

Step 2 Find the length of one diagonal. The height of the hexagon is equal to the sum of the long diagonals of two rhombi. Since the rhombi are congruent, the long diagonals must be congruent. So, the long diagonal is equal to 8 ÷ 2, or 4 inches.

Step 3 Find the area of one rhombus. From Postulate 11.1, the area of each rhombus is the same. So, the area of each rhombus is 48 ÷ 12 or 4 square inches.

Each rhombus in the pattern has an area of 4 square inches and diagonals 3 inches and 2 inches long.

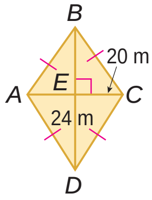
5A. **RECREATION** Rodrigo wants to cover a kite frame with decorative paper. If the length of one diagonal is 20 inches and the other diagonal measures 25 inches, find the area of the surface of the kite.

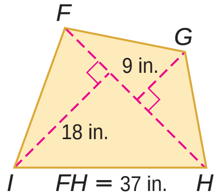
5B. **GARDENS** Clara has enough topsoil to cover 200 square feet. Her garden is shaped like a rhombus with one diagonal that is 25 feet. If she uses all of the topsoil on the garden, what is the length of the other diagonal?

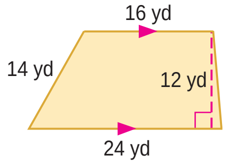
✓ CHECK Your Understanding

Examples 1–3 (pp. 639–641)

Find the area of each quadrilateral. Round to the nearest tenth.

1. 

2. 

3. 

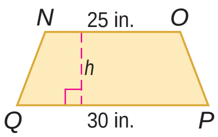
COORDINATE GEOMETRY **Find the area of each figure given the coordinates of the vertices.**

4. $\triangle ABC$ with $A(2, -3)$, $B(-5, -3)$, and $C(-1, 3)$
5. trapezoid $FGHJ$ with $F(-1, 8)$, $G(5, 8)$, $H(3, 4)$, and $J(1, 4)$
6. rhombus $LMPQ$ with $L(-4, 3)$, $M(-2, 4)$, $P(0, 3)$, and $Q(-2, 2)$

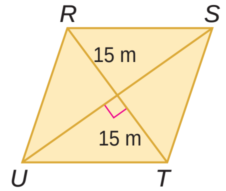
Example 4 (p. 641)

ALGEBRA **Find the missing measure for each quadrilateral.**

7. Trapezoid *NOPQ* has an area of 302.5 square inches. Find the height of *NOPQ*.

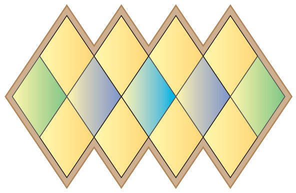


8. Rhombus *RSTU* has an area of 675 square meters. Find *SU*.



Example 5 (p. 642)

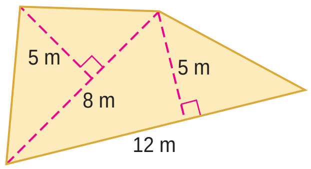
9. **INTERIOR DESIGN** Jacques is designing a window hanging composed of 13 congruent rhombi. The total width of the window hanging is 15 inches, and the total area is 82 square inches. Find the length of each diagonal and the area of one rhombus.

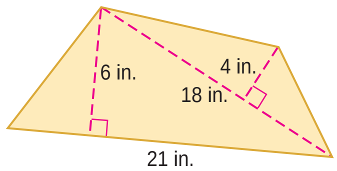


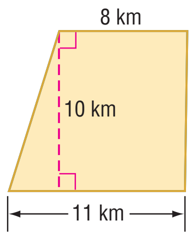
Exercises

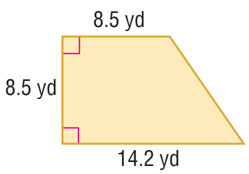
HOMEWORK HELP	
For Exercises	See Examples
10, 11	1
12, 13, 16–19	2
14, 15 20–23	3
24–29	4
30–33	5

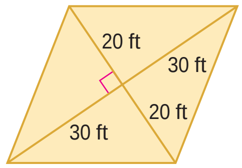
Find the area of each figure. Round to the nearest tenth if necessary.

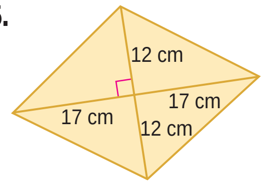
10. 

11. 

12. 

13. 

14. 

15. 

COORDINATE GEOMETRY Find the area of trapezoid *PQRT* given the coordinates of the vertices.

16. $P(0, 3), Q(3, 7), R(5, 7), T(6, 3)$

17. $P(-4, -5), Q(-2, -5), R(4, 6), T(-4, 6)$

18. $P(0, 3), Q(3, 1), R(2, -7), T(-7, -1)$

19. $P(-5, 2), Q(10, 7), R(6, -1), T(0, -3)$

COORDINATE GEOMETRY Find the area of rhombus *JKLM* given the coordinates of the vertices.

20. $J(2, 1), K(7, 4), L(12, 1), M(7, -2)$

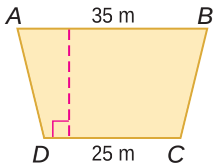
21. $J(-1, 2), K(1, 7), L(3, 2), M(1, -3)$

22. $J(-1, -4), K(2, 2), L(5, -4), M(2, -10)$

23. $J(2, 4), K(6, 6), L(10, 4), M(6, 2)$

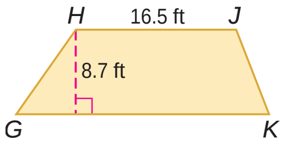
ALGEBRA Find the missing measure for each figure.

24. Find the height of trapezoid *ABCD*.



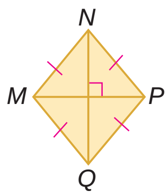
$A = 750 \text{ m}^2$

25. If *HJ* is 16.5 feet, find *GK*.



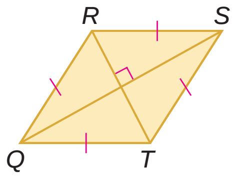
$A = 188.35 \text{ ft}^2$

26. If *MP* is 25 inches, find *NQ*.



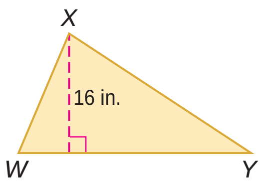
$A = 375 \text{ in}^2$

27. If *RT* is 12.2 meters, find *QS*.



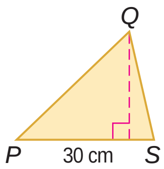
$A = 137.9 \text{ m}^2$

28. Find the length of the base.



$A = 248 \text{ in}^2$

29. Find the height.



$A = 300 \text{ cm}^2$

Real-World Career

Real Estate Agent

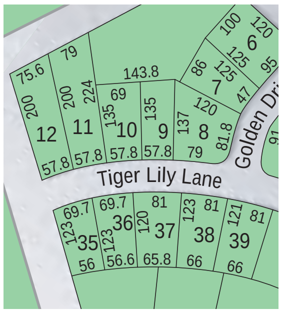
Real estate agents must have a broad knowledge of the neighborhoods in their community. About two-thirds of real estate agents are self-employed. Success is dependent on selling properties.

Source: www.bls.gov

Math Online For more information, go to geometryonline.com.

REAL ESTATE For Exercises 30 and 31, use the following information.

The map shows the layout and dimensions of several lot parcels in Aztec Falls. Suppose Lots 35 and 12 are trapezoids.

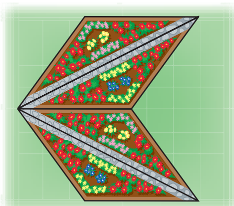


30. If the height of Lot 35 is 122.81 feet, find the area of this lot.

31. If the height of Lot 12 is 199.8 feet, find the area of this lot.

GARDENS For Exercises 32 and 33, use the following information.

Keisha designed a garden that is shaped like two congruent rhombi. She wants the long diagonals lined with a stone walkway. The total area of the garden is 150 square feet, and the shorter diagonals are each 12 feet long.



32. Find the length of each stone walkway.

33. Find the length of each side of the garden.

Find the area of each figure.

34. rhombus with a perimeter of 20 meters and a diagonal of 8 meters

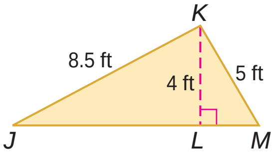
35. rhombus with a perimeter of 52 inches and a diagonal of 24 inches

36. isosceles trapezoid with a perimeter of 52 yards; the measure of one base is 10 yards greater than the other base, the measure of each leg is 3 yards less than twice the length of the shorter base

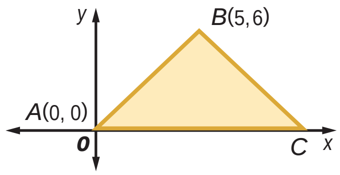
37. equilateral triangle with a perimeter of 15 inches

38. scalene triangle with sides that measure 34.0 meters, 81.6 meters, and 88.4 meters

39. Find the area of $\triangle JKM$.



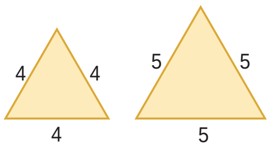
40. In the figure, if point B lies on the perpendicular bisector of $\overline{AC}$, what is the area of $\triangle ABC$?



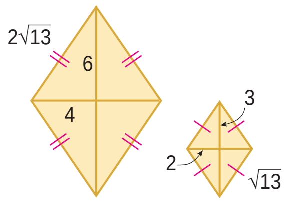
41. Derive the formula for the area of a rhombus using the formula for the area of a triangle.

CHANGING DIMENSIONS **Each pair of figures is similar. Find the area and perimeter of each figure. Describe how changing the dimensions affects the perimeter and area.**

42.

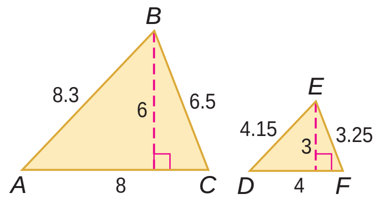


43.



SIMILAR FIGURES **For Exercises 44–49, use the following information.**

Triangle ABC is similar to triangle DEF.



44. Find the scale factor of $\triangle ABC$ to $\triangle DEF$.

45. Find the perimeter of each triangle. Compare the ratio of the perimeters of the triangles to the scale factor.

Study Tip

Look Back

To review **scale factor**, see Lesson 7–2.

46. Find the area of each triangle. Compare the ratio of the areas of the triangles to the scale factor.

47. Compare the ratio of the areas of the triangles to the ratio of the perimeters of the triangles.

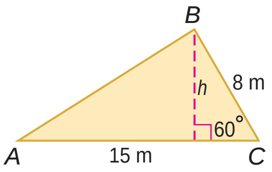
48. **Make a conjecture** about the ratios of the areas of similar triangles as compared to the scale factor.

49. **CHANGING DIMENSIONS** Suppose in $\triangle DEF$ the altitude stays the same, but the base is changed to twice its current measure. The new leg measures are 6 and 4.2 units. How do the perimeter and area of new $\triangle DEF$ compare to those of $\triangle DEF$?

EXTRA PRACTICE
See pages 822, 838.

Math Online
Self-Check Quiz at
geometryonline.com

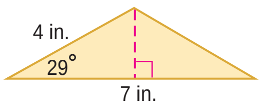
TRIGONOMETRY AND AREA For Exercises 50–53, use the triangle at the right.

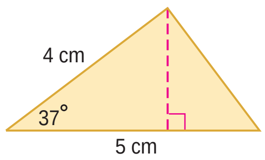


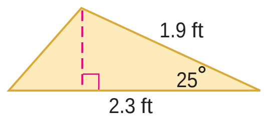
The area of any triangle can be found given the measures of two sides of the triangle and the measure of the included angle. Suppose we are given $AC = 15$, $BC = 8$, and $m\angle C = 60$. To find the height of the triangle, use the sine ratio, $\sin A = \frac{h}{BC}$. Then use the value of h in the formula for the area of a triangle. So, the area is $\frac{1}{2}(15)(8 \sin 60°)$ or 52.0 square units.

50. Derive a formula to find the area of any triangle, given the measures of two sides of the triangle and their included angle.

Find the area of each triangle.

51.


52.


53.


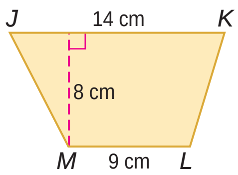
H.O.T. Problems

54. REASONING Determine whether the statement *Two triangles that have the same area also have the same perimeter* is *true* or *false*. Give an example or counterexample.

55. REASONING Determine whether it is *always*, *sometimes*, or *never* true that rhombi with the same area have the same diagonal lengths. Explain your reasoning.

56. OPEN ENDED Draw an isosceles trapezoid that contains at least one isosceles triangle. Then find the area of the trapezoid.

57. FIND THE ERROR Robert and Kiku are finding the area of trapezoid $JKLM$. Who is correct? Explain your reasoning.



Robert

$A = \frac{1}{2}(8)(14 + 9)$

$= \frac{1}{2}(8)(14) + 9$

$= 56 + 9$

$= 65 \text{ cm}^2$

Kiku

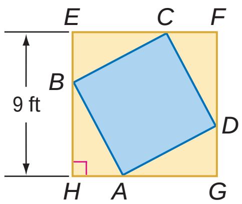
$A = \frac{1}{2}(8)(14 + 9)$

$= \frac{1}{2}(8)(23)$

$= 4(23)$

$= 92 \text{ cm}^2$

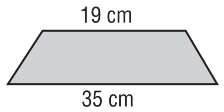
58. CHALLENGE In the figure, the vertices of quadrilateral *ABCD* intersect the square *EFGH* and divide its sides into segments with measures that have a ratio of 1:2. Find the area of *ABCD*. Describe the relationship between the areas of *ABCD* and *EFGH*.



59. Writing in Math Describe how to find the area of a triangle. Explain how the area of a triangle can help you find the areas of rhombi and trapezoids.

STANDARDIZED TEST PRACTICE

60. The lengths of the bases of an isosceles trapezoid are shown below.



If the perimeter is 74 centimeters, what is its area?

A 162 cm^2

B 270 cm^2

C 332.5 cm^2

D 342.25 cm^2

61. **REVIEW** What is the effect on the graph of the equation $y = \frac{1}{2}x$ when the equation is changed to $y = -2x$?

F The graph is moved 1 unit down.

G The graph is moved 1 unit up.

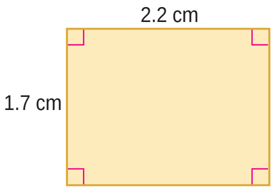
H The graph is rotated 45° about the origin.

J The graph is rotated 90° about the origin.

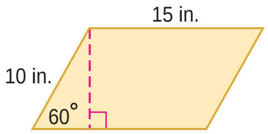
Spiral Review

Find the area of each figure. Round to the nearest tenth. (Lesson 11-1)

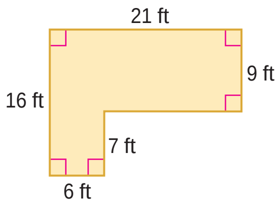
62.



63.



64.



Write an equation of circle *R* based on the given information. (Lesson 10-8)

65. center: $R(1, 2)$
radius: 7

66. center: $R\left(-4, \frac{1}{2}\right)$
radius: $\frac{11}{2}$

67. center: $R(-1.3, 5.6)$
radius: 3.5

68. **CRAFTS** Andria created a pattern to sew flowers onto a quilt by first drawing a regular pentagon that was 3.5 inches long on each side. Then she added a semicircle onto each side of the pentagon to create the appearance of five petals. How many inches of gold trim does she need to edge 10 flowers? (Lesson 10-1)

Given the magnitude and direction of a vector, find the component form with values rounded to the nearest tenth. (Lesson 9-6)

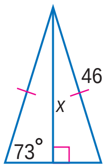
69. magnitude of 136 at a direction of 25 degrees with the positive x-axis

70. magnitude of 280 at a direction of 52 degrees with the positive x-axis

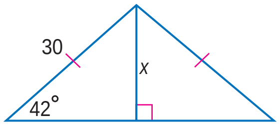
GET READY for the Next Lesson

PREREQUISITE SKILL **Find x. Round to the nearest tenth.** (Lesson 8-4)

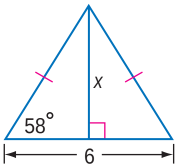
71.



72.



73.



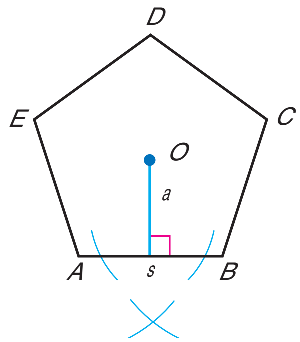
EXPLORE 11-3

Geometry Lab

Investigating Areas of Regular Polygons

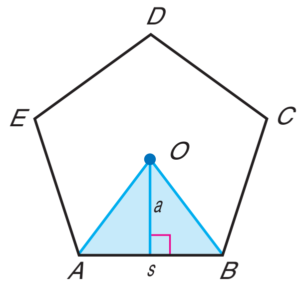
The point in the interior of a regular polygon that is equidistant from all of the vertices is the *center* of the polygon. A segment from the center to a side of the polygon that is perpendicular to the side is an **apothem**.

ACTIVITY



Step 1 Copy regular pentagon *ABCDE* and its center *O*.

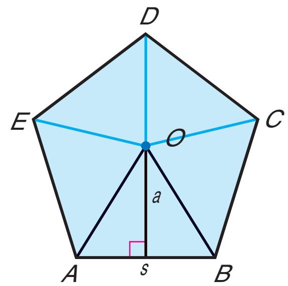
Step 2 Draw the apothem from *O* to side $\overline{AB}$ by constructing the perpendicular bisector of $\overline{AB}$. Label the apothem measure as *a*. Label the measure of $\overline{AB}$ as *s*.



Step 3 Use a straightedge to draw $\overline{OA}$ and $\overline{OB}$.

Step 4 What measure in $\triangle AOB$ represents the base of the triangle? What measure represents the height?

Step 5 Find the area of $\triangle AOB$ in terms of *s* and *a*.



Step 6 Draw $\overline{OC}$, $\overline{OD}$, and $\overline{OE}$. What is true of the five small triangles formed?

Step 7 How do the areas of the five triangles compare?

ANALYZE THE RESULTS

1. The area of a pentagon *ABCDE* can be found by adding the areas of the given triangles that make up the pentagonal region.
   $A = \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa$
   $A = \frac{1}{2}(sa + sa + sa + sa + sa)$ or $\frac{1}{2}(5sa)$
   What does $5s$ represent?

2. Write a formula for the area of a pentagon in terms of perimeter *P*.

11-3 Areas of Regular Polygons and Circles

Main Ideas

- Find areas of regular polygons.
- Find areas of circles.

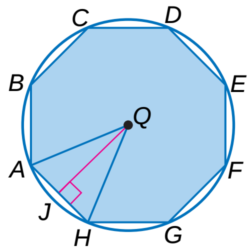
New Vocabulary

apothem

GET READY for the Lesson

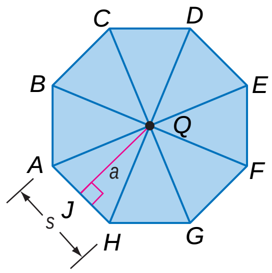
Connecticut's Mystic Seaport is a private maritime museum and home to one of the largest collections of boats in the world. The Village Green is the location of a 50-foot long model of the Mystic River area and a gazebo. The base of the gazebo is an octagon. Suppose the caretakers want to replace the gazebo flooring. How can they determine the area of the floor?

Areas of Regular Polygons In regular octagon *ABCDEFGH* inscribed in circle Q, $\overline{QA}$ and $\overline{QH}$ are radii from the center of the circle Q to two vertices of the octagon. $\overline{QJ}$ is drawn from the center of the regular polygon perpendicular to a side of the polygon. This segment is called an **apothem**.



Triangle QAH is an isosceles triangle, since the radii are congruent. If all of the radii were drawn, they would separate the octagon into 8 nonoverlapping congruent isosceles triangles.

The area of the octagon can be determined by adding the areas of the triangles. Since $\overline{QJ}$ is perpendicular to $\overline{AH}$, it is an altitude of $\triangle QAH$. Let a represent the length of $\overline{QJ}$ and let s represent the length of a side of the octagon.



$$\text{Area of } \triangle QAH = \frac{1}{2}bh$$
$$= \frac{1}{2}sa$$

The area of one triangle is $\frac{1}{2}sa$ square units. So the area of the octagon is $8\left(\frac{1}{2}sa\right)$ square units. Notice that the perimeter P of the octagon is $8s$ units. We can substitute P for $8s$ in the area formula.

$$\text{Area of octagon} = 8\left(\frac{1}{2}sa\right)$$
$$= 8s\left(\frac{1}{2}a\right) \quad \text{Commutative and Associative Properties}$$
$$= P\left(\frac{1}{2}a\right) \quad \text{Substitution}$$
$$= \frac{1}{2}Pa \quad \text{Commutative Property}$$

This formula can be used for the area of any regular polygon.

KEY CONCEPT — **Area of a Regular Polygon**

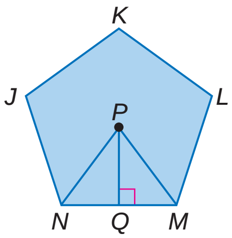
Words If a regular polygon has an area of A square units, a perimeter of P units, and an apothem of a units, then the area is one-half the product of the perimeter and the apothem.

Symbols $A = \frac{1}{2}Pa$

EXAMPLE 1 Area of a Regular Polygon

Find the area of a regular pentagon with a perimeter of 40 centimeters.

Draw a diagram of the pentagon. To find the area, you must first find the apothem.



Apothem: The central angles of a regular pentagon are all congruent. Therefore, the measure of each angle is $\frac{360}{5}$ or 72. $\overline{PQ}$ is an apothem of pentagon $JKLMN$. It bisects $\angle NPM$ and is a perpendicular bisector of $\overline{NM}$. So, $m\angle MPQ = \frac{1}{2}(72)$ or 36.

Since the perimeter is 40 centimeters, each side is 8 centimeters and $QM = 4$ centimeters.

Write a trigonometric ratio to find the length of $\overline{PQ}$.

$$\tan \angle MPQ = \frac{QM}{PQ} \qquad \tan\theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$\tan 36° = \frac{4}{PQ} \qquad m\angle MPQ = 36, QM = 4$$

$$(PQ) \tan 36° = 4 \qquad \text{Multiply each side by } PQ.$$

$$PQ = \frac{4}{\tan 36°} \qquad \text{Divide each side by } \tan 36°.$$

$$PQ \approx 5.5 \qquad \text{Use a calculator.}$$

Area:

$$A = \frac{1}{2}Pa \qquad \text{Area of a regular polygon}$$

$$\approx \frac{1}{2}(40)(5.5) \qquad P = 40, a \approx 5.5$$

$$\approx 110 \qquad \text{Simplify.}$$

So, the area of the pentagon is about 110 square centimeters.

1A. Find the area of a regular octagon with a perimeter of 124 inches.

1B. Find the area of a square with apothem length of 2.5 meters.

1C. Find the area of a regular hexagon with apothem length of 18 inches.

Study Tip

Problem Solving

There is another method for finding the apothem of a regular polygon. You can use the Interior Angle Sum Theorem to find $m\angle PMQ$ and then write a trigonometric ratio to find PQ.

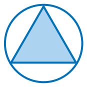
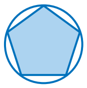
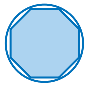
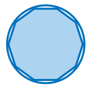
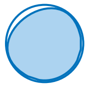
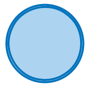
Areas of Circles You can use a calculator to help derive the formula for the area of a circle from the areas of regular polygons.

GEOMETRY LAB

Area of a Circle

Suppose each regular polygon is inscribed in a circle of radius *r*.

1. Copy and complete the following table. Round to the nearest hundredth.

Inscribed Polygon						
Number of Sides	3	5	8	10	20	50
Measure of a Side	$1.73r$	$1.18r$	$0.77r$	$0.62r$	$0.31r$	$0.126r$
Measure of Apothem	$0.5r$	$0.81r$	$0.92r$	$0.95r$	$0.99r$	$0.998r$
Area						

ANALYZE THE RESULTS

2. What happens to the appearance of the polygon as the number of sides increases?

3. What happens to the measures of the apothems and the areas as the number of sides increases?

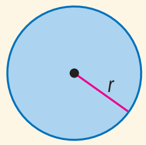
4. **Make a conjecture** about the formula for the area of a circle.

You can see from the Geometry Lab that the more sides a regular polygon has, the more closely it resembles a circle.

KEY CONCEPT — *Area of a Circle*

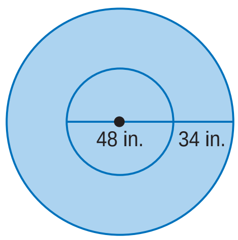
Words If a circle has an area of A square units and a radius of r units, then the area is the product of π and the square of the radius.

Symbols $A = \pi r^2$



2 **SEWING A caterer has a 48-inch diameter table that is 34 inches tall. She wants a tablecloth that will touch the floor. Find the area of the tablecloth in square yards.**

The diameter of the table is 48 inches, and the tablecloth must extend 34 inches in each direction. So the diameter of the tablecloth is 34 + 48 + 34 or 116 inches. Divide by 2 to find that the radius is 58 inches.



$A = \pi r^2$ Area of a circle

$= \pi(58)^2$ Substitution

$\approx 10{,}568.3$ Use a calculator.

The area of the tablecloth is 10,568.3 square inches. To convert to square yards, divide by 1296. The area of the tablecloth is 8.2 square yards to the nearest tenth.

Study Tip

Square Yards

A square yard measures 36 inches by 36 inches or 1296 square inches.

2. Susana is planning to paint the circular frame for a mirror. She needs to know the area of the frame in order to purchase enough paint. If the diameter of the frame is 18 inches and the diameter of the mirror is 10 inches, what is the area of the frame?

Review Vocabulary

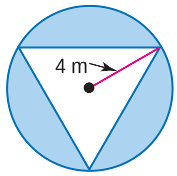
inscribed polygon
a polygon in which each vertex lies on a circle

circumscribed polygon
a polygon that contains a circle (Lesson 10-3)

You can use the properties of circles and regular polygons to find the areas of inscribed and circumscribed polygons.

EXAMPLE Area of an Inscribed Polygon

3 **Find the area of the shaded region. Assume that the triangle is equilateral.**



The area of the shaded region is the difference between the area of the circle and the area of the triangle.

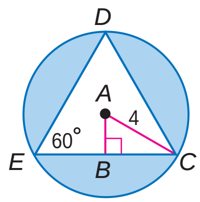
Step 1 Find the area of the circle.

$A = \pi r^2$ Area of a circle

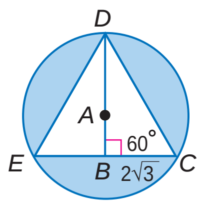
$= \pi(4)^2$ Substitution

≈ 50.3 Use a calculator.

Step 2 Find the area of the triangle. Use properties of 30°-60°-90° triangles. First, find the length of the base. The hypotenuse of $\triangle ABC$ is 4, so BC is $2\sqrt{3}$. Since $EC = 2(BC)$, $EC = 4\sqrt{3}$.



Next, find the height of the triangle, DB. Since $m\angle DCB$ is 60, $DB = 2\sqrt{3}(\sqrt{3})$ or 6.



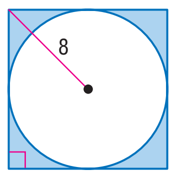
$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(4\sqrt{3})(6)$ $b = 4\sqrt{3}$, $h = 6$

≈ 20.8 Use a calculator.

Step 3 The area of the shaded region is 50.3 - 20.8 or 29.5 square meters to the nearest tenth.

3A. Find the area of the shaded region. Assume that the quadrilateral is a square. Round to the nearest tenth.



3B. An equilateral triangle is circumscribed around a circle with a radius of 5 units. Find the area of the region between the triangle and the circle.

Online Personal Tutor at geometryonline.com

CHECK Your Understanding

Example 1 (p. 650)

Find the area of each polygon. Round to the nearest tenth.

1. a regular hexagon with a perimeter of 42 yards
2. a regular nonagon with a perimeter of 108 meters

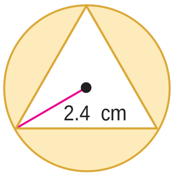
Example 2 (p. 651)

3. **FURNITURE DESIGN** Tyra wants to cover the cushions of her papasan chair with a different fabric. If there are seven circular cushions that are the same size with a diameter of 12 inches, around a center cushion with a diameter of 20 inches, find the area of fabric in square yards that she will need to cover both sides of the cushions. Allow an extra 3 inches of fabric around each cushion.

Example 3 (p. 652)

Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.

4.



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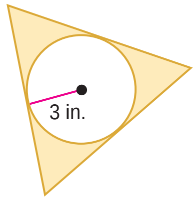


Exercises

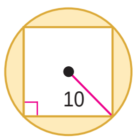
HOMEWORK HELP	
For Exercises	See Examples
6–11	1
12–20	3
21–24	2

Find the area of each polygon. Round to the nearest tenth.

6. a regular octagon with a perimeter of 72 inches
7. a square with a perimeter of $84\sqrt{2}$ meters
8. a square with apothem length of 12 centimeters
9. a regular hexagon with apothem length of 24 inches
10. a regular triangle with side length of 15.5 inches
11. a regular octagon with side length of 10 kilometers

Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.

12.



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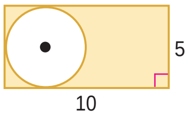

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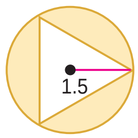

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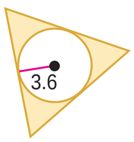

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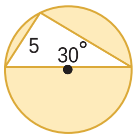

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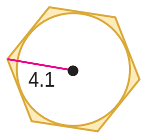

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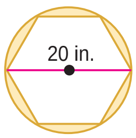

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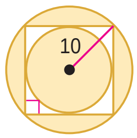
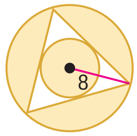


20.



21. **PIZZA** A pizza shop sells 8-inch pizzas for \$5 and 16-inch pizzas for \$10. Which would give you more pizza, two 8-inch pizzas or one 16-inch pizza? Explain.

22. **ALGEBRA** A circle is inscribed in a square, which is circumscribed by another circle. If the diagonal of the square is $2x$, find the ratio of the area of the large circle to the area of the small circle.

23. **ALGEBRA** A circle with radius $3x$ is circumscribed about a square. Find the area of the square.

24. **CAKE** A bakery sells single-layer mini-cakes that are 3 inches in diameter for \$4 each. They also have a cake with a 9-inch diameter for \$15. If both cakes are the same thickness, which option gives you more cake for the money, nine mini-cakes or one 9-inch cake? Explain.

COORDINATE GEOMETRY The coordinates of the vertices of a regular polygon are given. Find the area of each polygon to the nearest tenth.

25. $T(0, 0), U(-7, -7), V(0, -14), W(7, -7)$

26. $G(-12, 0), H\left(0, 4\sqrt{3}\right), J\left(0, -4\sqrt{3}\right)$

Find the area of each circle given the measure of its circumference. Round to the nearest tenth.

27. 34π

28. 17π

29. 54.8

30. 91.4

GAMING For Exercises 31–33, refer to the following information.

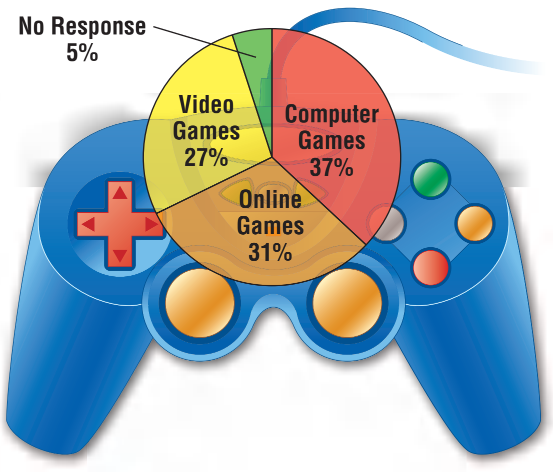
Students were surveyed about which type of game they play at least once per week. The results are shown in the circle graph.

31. Suppose the radius of the circle on the graph is 1.3 centimeters. Find the area of the circle on the graph.

32. Francesca wants to use this circle graph for a presentation. She wants the circle to use as much space on a 22" by 28" sheet of poster board as possible. Find the area of the circle.

33. **Make a conjecture** about how you could determine the area of the region representing students who play computer games.

Types of Games



Source: Pew Internet & American Life Project

Study Tip

Make a Drawing

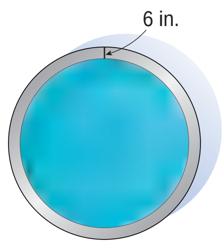
When an exercise does not provide a figure, it is helpful to draw one and label it with the given information.

SWIMMING POOL For Exercises 34 and 35, use the following information.

The area of a circular in-ground pool is approximately 7850 square feet. The owner wants to replace the tiling at the edge of the pool.

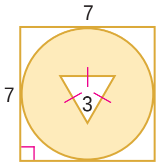
34. The edging is 6 inches wide, so she plans to use 6-inch square tiles to form a continuous inner edge. How many tiles will she need to purchase?

35. Once the square tiles are in place around the pool, there will be extra space between the tiles. What shape of tile will best fill this space? How many tiles of this shape should she purchase?

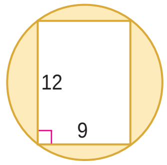


Find the area of each shaded region. Round to the nearest tenth.

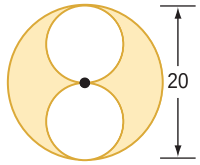
36.



37.



38.



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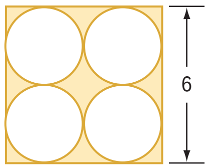

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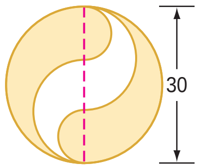

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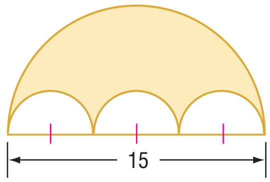
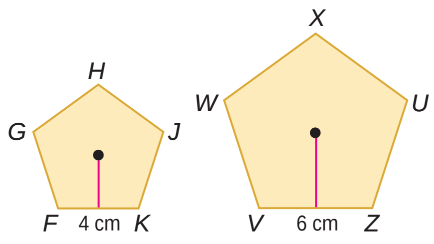


42. A square is inscribed in a circle of area 18π square units. Find the length of a side of the square.

SIMILAR FIGURES **For Exercises 43–47, use the following information.**
Polygons *FGHJK* and *VWXUZ* are similar regular pentagons.

43. Find the scale factor.

44. Find the perimeter of each pentagon. Compare the ratio of the perimeters of the pentagons to the scale factor.



45. Find the area of each pentagon. Compare the ratio of the areas of the pentagons to the scale factor.

46. Compare the ratio of the areas of the pentagons to the ratio of the perimeters of the pentagons.

47. Determine whether the relationship between the ratio of the areas of the pentagons to the scale factor is applicable to all similar polygons. Explain.

EXTRA PRACTICE
See pages 822, 838.
Math Online
Self-Check Quiz at
geometryonline.com

H.O.T. Problems

48. **REASONING** Explain how to derive the formula for the area of a regular polygon.

49. **REASONING** Describe the effect on the area and circumference of a circle when the length of the radius is doubled.

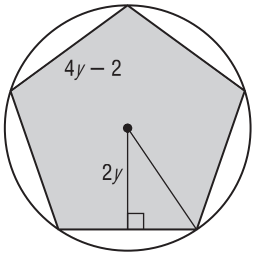
50. **OPEN ENDED** Draw a polygon inscribed in a circle. Find the area of the space in the interior of the circle and the exterior of the polygon.

51. **CHALLENGE** A circle inscribes one regular hexagon and circumscribes another. If the radius of the circle is 10 units long, find the ratio of the area of the smaller hexagon to the area of the larger hexagon.

52. **Writing in Math** Refer to the Geometry Lab on page 651. What shape does a regular polygon approximate when the number of sides is increased infinitely? Explain how the formula for the area of a regular polygon can approximate the formula for the area of a circle.

STANDARDIZED TEST PRACTICE

53. Which polynomial best represents the area of the regular pentagon shown at the right?



A $10y^2 - 5$

B $10y^2 + 5y$

C $20y^2 + 10$

D $20y^2 - 10y$

54. REVIEW In the system of equations $-\frac{5}{4}x + \frac{1}{3}y = 7$ and $2x - 6y = 8$, which expression can be correctly substituted for y in the equation $2x - 6y = 8$?

F $21 + \frac{15}{4}x$

G $7 - 3y$

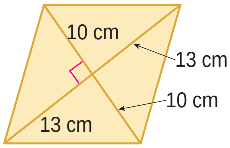
H $\frac{4}{3} + \frac{1}{3}x$

J $4 + 3y$

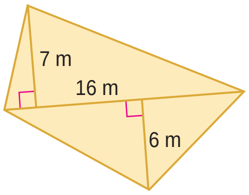
Spiral Review

Find the area of each quadrilateral. (Lesson 11-2)

55.



56.



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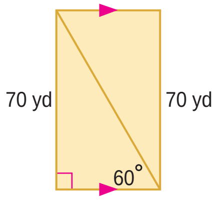


COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral. (Lesson 11-1)

58. $A(-3, 2)$, $B(4, 2)$, $C(2, -1)$, $D(-5, -1)$

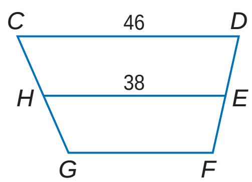
59. $F(4, 1)$, $G(4, -5)$, $H(-2, -5)$, $J(-2, 1)$

COORDINATE GEOMETRY Draw the rotation image of each triangle by reflecting the triangles in the given lines. State the coordinates of the rotation image and the angle of rotation. (Lesson 9-3)

60. $\triangle ABC$ with vertices $A(-1, 3)$, $B(-4, 6)$, and $C(-5, 1)$, reflected in y-axis and then in x-axis

61. $\triangle FGH$ with vertices $F(0, 4)$, $G(-2, 2)$, and $H(2, 2)$, reflected in $y = x$ and then in y-axis

Refer to trapezoid *CDFG* with median $\overline{HE}$. (Lesson 6-6)



62. Find GF.

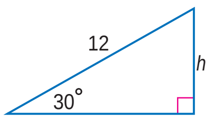
63. Let $\overline{WX}$ be the median of $CDEH$. Find WX.

64. Let $\overline{YZ}$ be the median of $HEFG$. Find YZ.

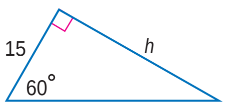
GET READY for the Next Lesson

PREREQUISITE SKILL Find *h*. (Lesson 8-3)

65.



66.



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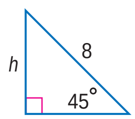


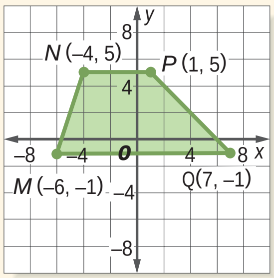
CHAPTER 11 Mid-Chapter Quiz

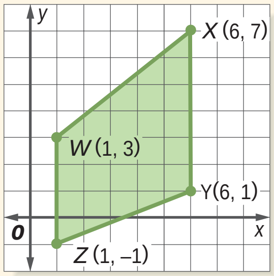
Lessons 11-1 through 11-3

The coordinates of the vertices of quadrilateral *JKLM* are *J*(−8, 4), *K*(−4, 0), *L*(0, 4), and *M*(−4, 8). (Lesson 11-1)

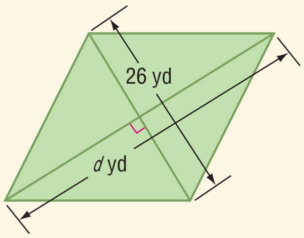
1. Determine whether *JKLM* is a square, a rectangle, or a parallelogram.
2. Find the area of *JKLM*.

Find the area of each trapezoid. (Lesson 11-2)

3. 

4. 

5. **MULTIPLE CHOICE** The area of the rhombus is 546 square yards. What is *d*? (Lesson 11-2)

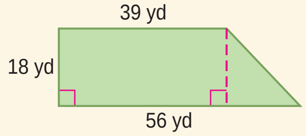


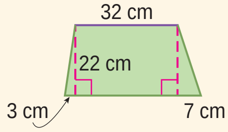
A 21 **C** 42

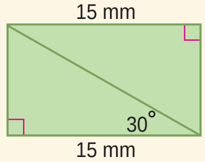
B 26 **D** 52

6. Find the area of an isosceles trapezoid that has a perimeter of 90 meters. The longer base is 5 meters less than twice the length of the shorter base. The length of each leg is 3 meters less than the length of the shorter base. (Lesson 11-2)

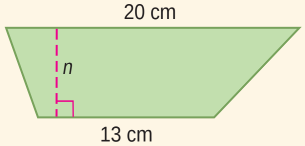
Find the area of each figure. Round to the nearest tenth if necessary. (Lesson 11-2)

7. 

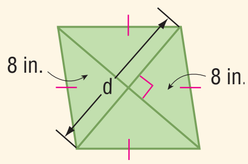
8. 

9. 

Find the missing measure for each quadrilateral. (Lesson 11-2)

10. 

$A = 99 \text{ cm}^2$

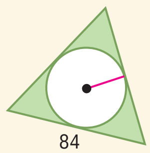
11. 

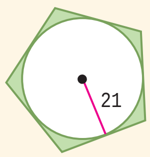
$A = 104 \text{ in}^2$

Find the area of each polygon. Round to the nearest tenth. (Lesson 11-3)

12. regular hexagon with apothem length of 14 millimeters
13. regular octagon with a perimeter of 72 inches

Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth. (Lesson 11-3)

14. 

15. 

16. **CRAFTS** Lori is making a circular pillow. She wants the diameter of the finished pillow to be 12 inches. When cutting the fabric, she allows a $1\frac{1}{2}$ inch border for sewing. What is the total area of fabric needed for one pillow? (Lesson 11-3)

11-4 Areas of Composite Figures

Main Ideas

- Find areas of composite figures.
- Find areas of composite figures on the coordinate plane.

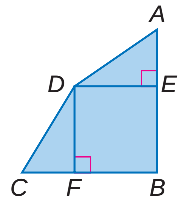
New Vocabulary

composite figure

GET READY for the Lesson

The sail for a windsurf board cannot be classified as a triangle or a parallelogram. However, it can be separated into figures that can be identified, such as trapezoids and a triangle.

Composite Figures A **composite figure** is a figure that can be separated into regions that are basic figures. To find the area of a composite figure, separate the figure into basic figures of which we can find the area. The sum of the areas of the basic figures is the area of the composite figure.



Auxiliary lines are drawn in quadrilateral $ABCD$. $\overline{DE}$ and $\overline{DF}$ separate the figure into $\triangle ADE$, $\triangle CDF$, and rectangle $BEDF$.

POSTULATE 11.2

The area of a region is the sum of the areas of all of its nonoverlapping parts.

STANDARDIZED TEST EXAMPLE — **Area of a Composite Figure**

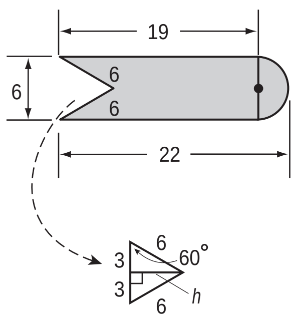
1 Which is closest to the area of this composite figure?

A 112.5 units^2

B 116.4 units^2

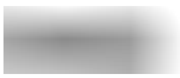
C 126.7 units^2

D 132.0 units^2



Read the Test Item

The figure can be separated into a rectangle with dimensions 6 units by 19 units, an equilateral triangle with sides each measuring 6 units, and a semicircle with a radius of 3 units.



Animation
geometryonline.com

Solve the Test Item

Use 30°-60°-90° relationships to find that the height of the triangle is $3\sqrt{3}$.

area of composite figure = area of rectangle - area of triangle + area of semicircle

$= \ell w - \frac{1}{2}bh + \frac{1}{2}\pi r^2$ Area formulas

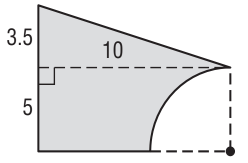
$= 19 \cdot 6 - \frac{1}{2}(6)(3\sqrt{3}) + \frac{1}{2}\pi(3^2)$ Substitution

$= 114 - 9\sqrt{3} + \frac{9}{2}\pi$ Simplify.

≈ 112.5 Use a calculator.

The area of the composite figure is 112.5 square units to the nearest tenth. The correct answer is A.

1. Which is closest to the area of the composite figure?



F 45.7 units^2 H 67.5 units^2

G 47.9 units^2 J 87.1 units^2

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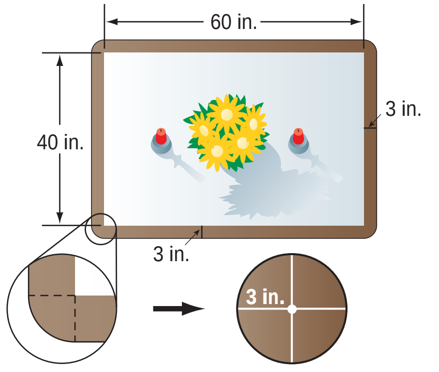
EXAMPLE Find the Area of a Composite Figure to Solve a Problem

2 **FURNITURE Melissa's dining room table has hardwood around the outside. Find the area of wood around the edge of the table.**



First, draw auxiliary lines to separate the figure into regions. The table can be separated into four rectangles and four corners.

The four corners of the table form a circle with radius 3 inches.

area of wood edge = area of rectangles + area of circle

$= 2\ell w + 2\ell w + \pi r^2$ Area formulas

$= 2(3)(60) + 2(3)(40) + \pi(3^2)$ Substitution

$= 360 + 240 + 9\pi$ Simplify.

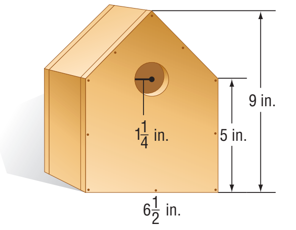
≈ 628.3 Use a calculator.

The area of the wood edge is 628.3 square inches to the nearest tenth.

Cross-Curricular Project Identifying the polygons forming a region such as a tessellation will help you determine the type of tessellation. Visit geometryonline.com to continue work on your project.

CHECK Your Progress

2. **BIRDHOUSES** Ramon is building a birdhouse. He is going to paint the front side. What is the area to be painted? Round to the nearest tenth.



Composite Figures on the Coordinate Plane To find the area of a composite figure on the coordinate plane, separate the figure into basic figures, the areas of which can be determined.

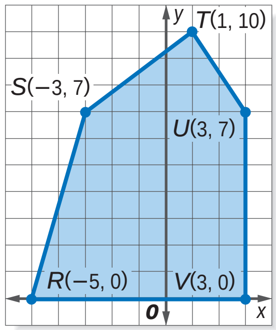
Study Tip

Estimation

Estimate the area of a composite figure on the coordinate plane by counting the unit squares. Use the estimate to determine if your answer is reasonable.

EXAMPLE Coordinate Plane

3 **COORDINATE GEOMETRY** **Find the area of the composite figure.**



First, separate the figure into regions. Draw an auxiliary line from S to U. This divides the figure into triangle STU and trapezoid $RSUV$.

Find the difference between x-coordinates to find the length of the base of the triangle and the lengths of the bases of the trapezoid. Find the difference between y-coordinates to find the heights of the triangle and trapezoid.

area of $RSTUV$ = area of $\triangle STU$ + area of trapezoid $RSUV$

$= \frac{1}{2}bh + \frac{1}{2}h(b_1 + b_2)$ Area formulas

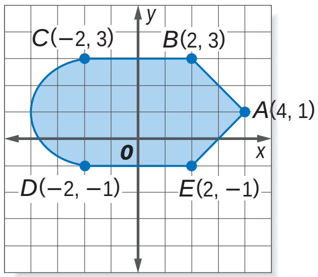
$= \frac{1}{2}(6)(3) + \frac{1}{2}(7)(8 + 6)$ Substitution

$= 9 + 49$ Multiply.

$= 58$ Simplify.

The area of $RSTUV$ is 58 square units.

3. **COORDINATE GEOMETRY** Find the area of the composite figure.



Interactive Lab
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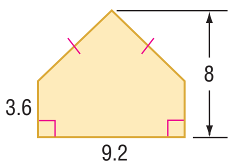
Check Your Understanding

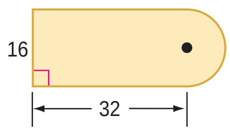
Example 1 (p. 658)

Find the area of each figure. Round to the nearest tenth if necessary.

1.

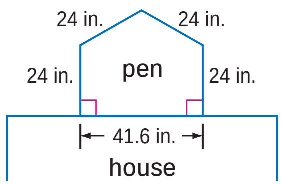


2.



Example 2 (p. 659)

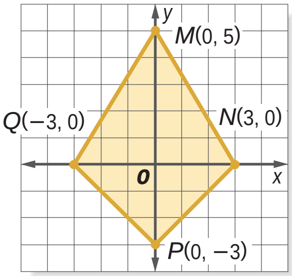
3. **DOGS** Owen's family has a system of interlocking gates that attach to the wall of the house to form a pen for their dog. Find the area enclosed by the wall and gates.



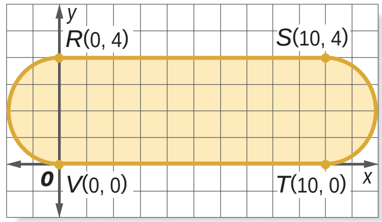
Example 3
(p. 660)

COORDINATE GEOMETRY **Find the area of each figure.**

4.



5.

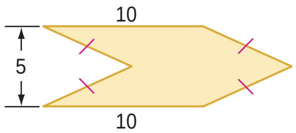


Exercises

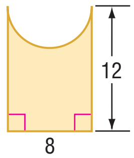
HOMEWORK HELP	
For Exercises	See Examples
6–11	1
12, 13	2
14–20	3

Find the area of each figure. Round to the nearest tenth if necessary.

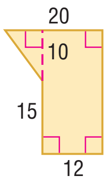
6.



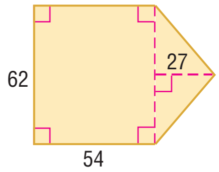
7.



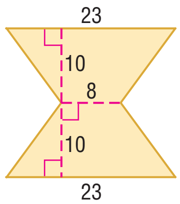
8.



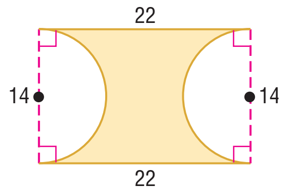
9.



10.



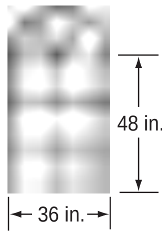
11.



WINDOWS **For Exercises 12 and 13, use the following information.**
Mr. Frazier needs to replace this window in his house. The window panes are rectangles and sectors.

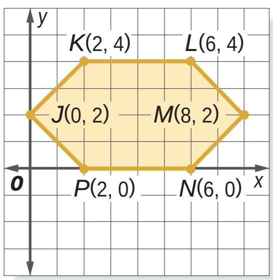
12. Find the perimeter of the window.

13. Find the area of the window.

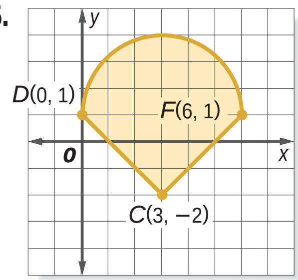


COORDINATE GEOMETRY **Find the area of each figure. Round to the nearest tenth if necessary.**

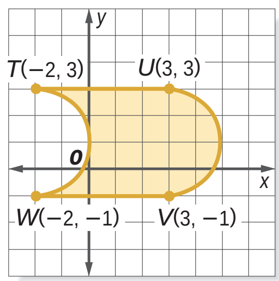
14.



15.



16.



COORDINATE GEOMETRY **The vertices of a composite figure are given. Find the area of each figure.**

17. $M(-4, 0)$, $N(0, 3)$, $P(5, 3)$, $Q(5, 0)$

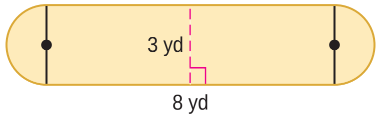
18. $T(-4, -2)$, $U(-2, 2)$, $V(3, 4)$, $W(3, -2)$

19. $G(-3, -1)$, $H(-3, 1)$, $I(2, 4)$, $J(5, -1)$, $K(1, -3)$

20. $P(-8, 7)$, $Q(3, 7)$, $R(3, -2)$, $S(-1, 3)$, $T(-11, 1)$

PAINTING **For Exercises 21 and 22, use the following information.**

The senior class of Westwood High School wants to paint the entrance hallway floor of their school as shown at the right.

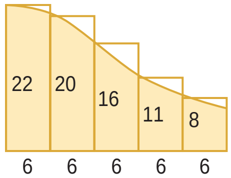


21. Find the area of the floor to be painted.

22. Paint costs $20 per gallon. Five gallons of paint covers 2000 square feet. How much will paint cost if the students use four coats of paint?

CALCULUS **For Exercises 23–25, use the following information.**

In the branch of mathematics called *calculus*, you can find the area of an *irregular* shape by approximating the shape with rectangles of equal width. This is called a *Riemann sum*.

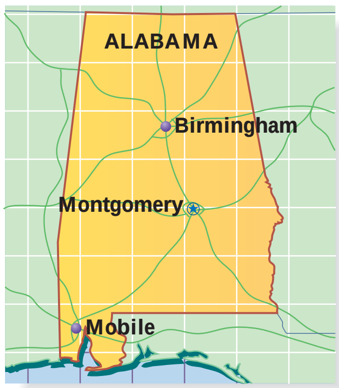


23. Use the rectangles to approximate the area of the region.

24. Analyze the estimate. Do you think the actual area is larger or smaller than your estimate? Explain.

25. How could the irregular region be separated to give an estimate of the area that is more accurate?

26. **GEOGRAPHY** Estimate the area of the state of Alabama. Each square on the grid represents 2500 square miles.



27. **RESEARCH** Find a map of your state or a state of your choice. Estimate the area. Then use the Internet or other source to check the accuracy of your estimate.

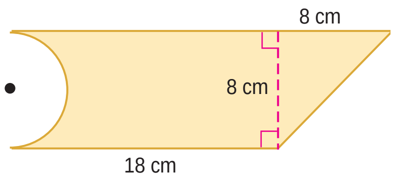
EXTRA PRACTICE
See pages 822, 838.
Math Online
Self-Check Quiz at geometryonline.com

H.O.T. Problems

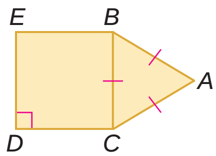
28. **OPEN ENDED** Sketch a composite polygon on a coordinate plane and find its area.

29. **RESEARCH** Use a dictionary or other Internet resource to find the definition of *composite*. Describe below how the definition of composite relates to composite figures.

30. **REASONING** Describe two different methods to find the area of the composite figure at right. Then find the area of the figure. Round to the nearest tenth.



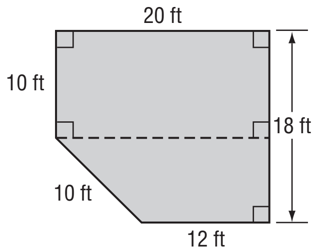
31. **CHALLENGE** Find the ratio of the area of $\triangle ABC$ to the area of square $BCDE$.



32. **Writing in Math** Describe how to find the area of a composite figure.

STANDARDIZED TEST PRACTICE

33. A landscape architect gives the diagram of a yard to a fencing company.



What is the area of the yard to be fenced, in square feet?

A 70

B 264

C 328

D 360

34. REVIEW Tammy borrowed money from her parents to pay for a trip. The data in the table show the remaining balance b of Tammy's loan after each payment p.

Number of Payments	1	2	3	4	5
Loan Balance ($)	2142	1989	1836	1683	1530

If payments were graphed on the horizontal axis and balances were graphed on the vertical axis, what would be the equation of a line that fits the data?

F $b = 1530 + 153p$

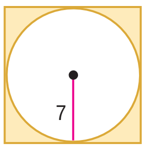
G $b = 2142 - 153p$

H $b = 2295 - 153p$

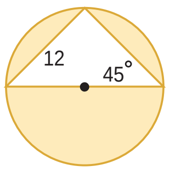
J $b = 2448 + 153p$

Find the area of each shaded region. Round to the nearest tenth. (Lesson 11-3)

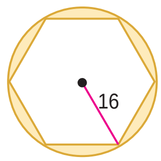
35.



36.



37.



Find the area of each figure. Round to the nearest tenth if necessary. (Lesson 11-2)

38. equilateral triangle with perimeter of 57 feet

39. rhombus with a perimeter of 40 yards and a diagonal of 12 yards

40. COORDINATE GEOMETRY The point (6, 0) is rotated 45° clockwise about the origin. Find the exact coordinates of its image. (Lesson 9-3)

Find the range for the measure of the third side of a triangle given the measures of two sides. (Lesson 5-4)

41. 16 and 31 **42.** 26 and 40 **43.** 11 and 23

GET READY for the Next Lesson

PREREQUISITE SKILL **Express each fraction as a decimal to the nearest hundredth.**

44. $\frac{5}{8}$ **45.** $\frac{13}{16}$ **46.** $\frac{9}{47}$ **47.** $\frac{10}{21}$

READING MATH

Prefixes

Many of the words used in mathematics use the same prefixes as other everyday words. Understanding the meaning of the prefixes can help you understand the terminology better.

Prefix	Meaning	Everyday Words	Meaning
bi-	2	bicycle	a 2-wheeled vehicle
		bipartisan	involving members of 2 political parties
tri-	3	triangle	closed figure with 3 sides
		tricycle	a 3-wheeled vehicle
		triplet	one of 3 children born at the same time
quad-	4	quadrilateral	closed figure with 4 sides
		quadriceps	muscles with 4 parts
		quadruple	four times as many
penta-	5	pentagon	closed figure with 5 sides
		pentathlon	athletic contest with 5 events
hexa-	6	hexagon	closed figure with 6 sides
hept-	7	heptagon	closed figure with 7 sides
oct-	8	octagon	closed figure with 8 sides
		octopus	animal with 8 legs
dec-	10	decagon	closed figure with 10 sides
		decade	a period of 10 years
		decathlon	athletic contest with 10 events

Several pairs of words in the chart have different prefixes, but the same root word. *Pentathlon* and *decathlon* are both athletic contests. *Heptagon* and *octagon* are both closed figures. Knowing the meaning of the root of the term as well as the prefix can help you learn vocabulary.

Reading to Learn

Use a dictionary to find the meanings of the prefix and root for each term. Then write a definition of the term.

1. bisector
2. polygon
3. equilateral
4. concentric
5. circumscribe
6. collinear

7. **RESEARCH** Use a dictionary to find the meanings of the prefix and root of *circumference*.

8. **RESEARCH** Use a dictionary or the Internet to find as many words as you can with the prefix *poly-* and the definition of each.

11-5 Geometric Probability and Areas of Sectors

Main Ideas

- Solve problems involving geometric probability.
- Solve problems involving sectors and segments of circles.

New Vocabulary

geometric probability
sector
segment

GET READY for the Lesson

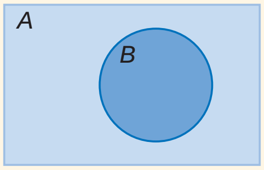
To win at darts, you have to throw a dart at either the center or the part of the dartboard that earns the most points. In games, probability can sometimes be used to determine chances of winning. Probability that involves a geometric measure such as length or area is called **geometric probability**.

Geometric Probability In Chapter 1, you learned that the probability that a point lies on a part of a segment can be found by comparing the length of the part to the length of the whole segment. Similarly, you can find the probability that a point lies in a part of a two-dimensional figure by comparing the area of the part to the area of the whole figure.

KEY CONCEPT — Probability and Area

If a point in region A is chosen at random, then the probability $P(B)$ that the point is in region B, which is in the interior of region A, is

$$P(B) = \frac{\text{area of region } B}{\text{area of region } A}$$



When determining geometric probability with targets, we assume
- that the object lands within the target area, and
- it is equally likely that the object will land anywhere in the region.

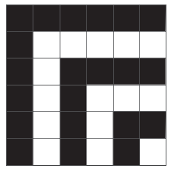
EXAMPLE Probability with Area

1 **A square game board has black and white stripes of equal width, as shown. What is the chance that a dart that strikes the board will land on a white stripe?**

You want to find the probability of landing on a white stripe, not a black stripe.

(continued on the next page)

We need to divide the area of the white stripes by the total area of the game board. Extend the sides of each stripe. This separates the square into 36 small unit squares.



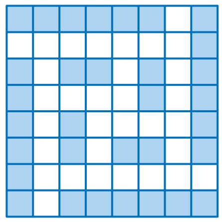
The white stripes have an area of 15 square units. The total area is 36 square units.

The probability of tossing a chip onto the white stripes is $\frac{15}{36}$ or $\frac{5}{12}$.

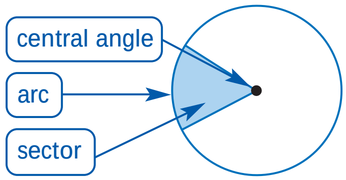
Study Tip

Probability
The probability of an event can be expressed as a decimal, a fraction, or a percent.

1. Find the probability that a point chosen at random from the figure lies in the shaded region.



Sectors and Segments of Circles Sometimes you need to know the area of a sector of a circle in order to find a geometric probability. A **sector** of a circle is a region of a circle bounded by a central angle and its intercepted arc.

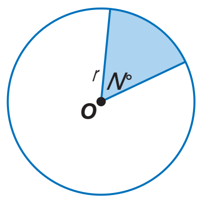


Proportional reasoning can be used to derive the formula for the area of a sector.

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{N^\circ}{360^\circ} \quad \begin{matrix} \leftarrow \text{Degrees in the sector} \\ \leftarrow \text{Degrees in the circle} \end{matrix}$$

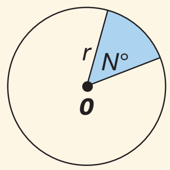
$$\text{area of sector} = \frac{N \cdot \text{area of circle}}{360^\circ} \quad \text{Multiply.}$$

$$\text{area of sector} = \frac{N}{360}\pi r^2 \quad \text{area of circle} = \pi r^2$$



KEY CONCEPT — *Area of a Sector*

If a sector of a circle has an area of A square units, a central angle measuring $N°$, and a radius of r units, then $A = \frac{N}{360}\pi r^2$.

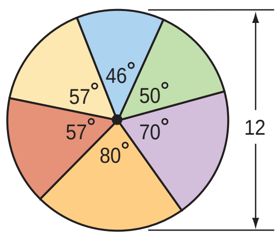


EXAMPLE 2 **Probability with Sectors**

a. Find the area of the blue sector.

Use the formula to find the area of the sector.

$$\begin{aligned} A &= \frac{N}{360}\pi r^2 && \text{Area of a sector} \\ &= \frac{46}{360}\pi(6^2) && N = 46, r = 6 \\ &= 4.6\pi && \text{Simplify.} \end{aligned}$$



b. Find the probability that a point chosen at random lies in the blue region.

To find the probability, divide the area of the sector by the area of the circle. The area of the circle is πr^2 with a radius of 6.

$P(\text{blue}) = \dfrac{\text{area of sector}}{\text{area of circle}}$ Geometric probability formula

$= \dfrac{4.6\pi}{\pi \cdot 6^2}$ Area of sector $= 4.6\pi$, area of circle $= \pi \cdot 6^2$

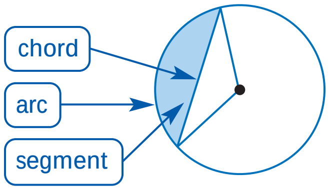
≈ 0.13 Use a calculator.

The probability that a random point is in the blue sector is about 0.13 or 13%.

2A. Find the area of the orange sector.

2B. Find the probability that a point chosen at random lies in the orange region.

The region of a circle bounded by an arc and a chord is called a **segment** of a circle. To find the area of a segment, subtract the area of the triangle formed by the radii and the chord from the area of the sector containing the segment.



EXAMPLE 3 Probability with Segments

A regular hexagon is inscribed in a circle with a diameter of 14.

a. Find the area of the red segment.

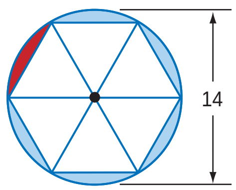
Area of the sector:

$A = \dfrac{N}{360}\pi r^2$ Area of a sector

$= \dfrac{60}{360}\pi(7^2)$ $N = 60, r = 7$

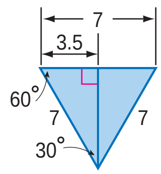
$= \dfrac{49}{6}\pi$ Simplify.

≈ 25.66 Use a calculator.



Area of the triangle:

Since the hexagon was inscribed in the circle, the triangle is equilateral, with each side 7 units long. Use properties of 30°-60°-90° triangles to find the apothem. The value of x is 3.5, the apothem is $x\sqrt{3}$ or $3.5\sqrt{3}$ which is approximately 6.06.



$A = \dfrac{1}{2}bh$ Area of a triangle

$= \dfrac{1}{2}(7)(6.06)$ $b = 7, h = 6.06$

≈ 21.22 Simplify.

(continued on the next page)

Study Tip

Look Back

To review the **properties of special right triangles**, see Lesson 8-3.

Area of the segment:

$$\text{area of segment} = \text{area of sector} - \text{area of triangle}$$

$$\approx 25.66 - 21.22 \quad \text{Substitution}$$

$$\approx 4.44 \quad \text{Simplify.}$$

b. Find the probability that a point chosen at random lies in the red region.

Divide the area of the sector by the area of the circle to find the probability. First, find the area of the circle. The radius is 7, so the area is $\pi(7^2)$ or about 153.94 square units.

$$P(\text{red}) = \frac{\text{area of segment}}{\text{area of circle}} \quad \text{Geometric probability formula}$$

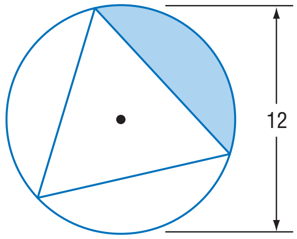
$$\approx \frac{4.44}{153.94} \quad \text{Substitution}$$

$$\approx 0.03 \quad \text{Use a calculator.}$$

The probability that a random point is on the red segment is about 0.03 or 3%.

3A. Find the area of the shaded region.

3B. Find the probability that a point chosen at random will be in the shaded region.



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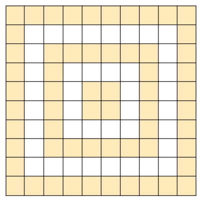
CHECK Your Understanding

Example 1 (p. 665)

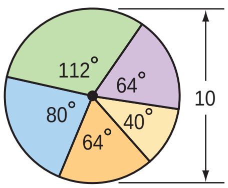
1. Find the probability that a point chosen at random lies in the shaded region.



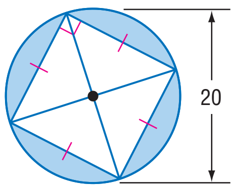
Examples 2 and 3 (pp. 666–667)

Find the area of the blue region. Then find the probability that a point chosen at random will be in the blue region.

2.



3.

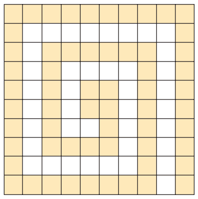


Exercises

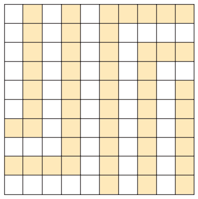
HOMEWORK HELP	
For Exercises	**See Examples**
4-7	1
8–13	2
14–16	3

Find the probability that a point chosen at random lies in the shaded region.

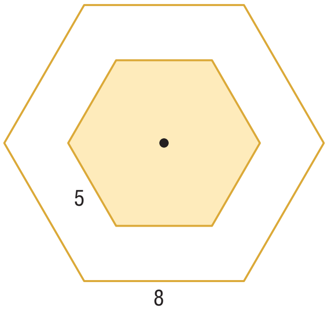
4.



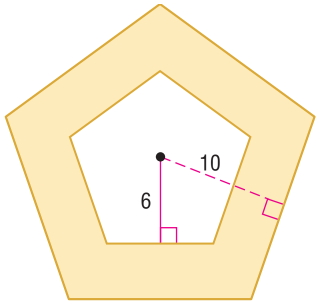
5.



6.

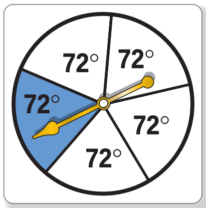


7.

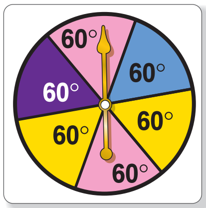


Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of each spinner is 15 centimeters.

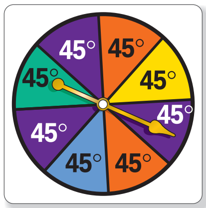
8. blue



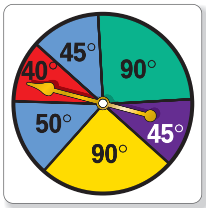
9. pink



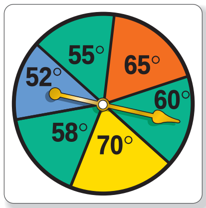
10. purple



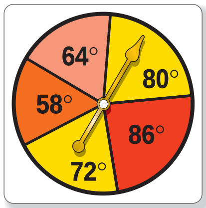
11. red



12. green

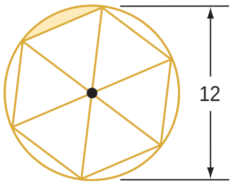


13. yellow

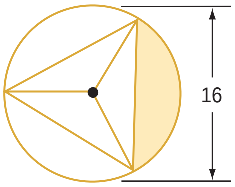


Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume all inscribed polygons are regular.

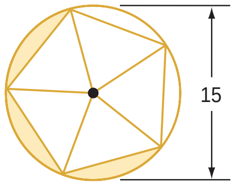
14.



15.



16.

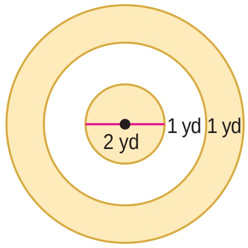


17. GEOGRAPHY The land area of the state of Alaska is 571,951 square miles. The land area of the United States is 3,537,438 square miles. If a point is chosen at random in the United States, what is the probability that it is in Alaska?

EXTRA PRACTICE
See pages 823, 838.

Math Online
Self-Check Quiz at
geometryonline.com

18. **PARACHUTES** A skydiver must land on a target of three concentric circles. The diameter of the center circle is 2 yards, and the circles are spaced 1 yard apart. Find the probability that she will land on the shaded area.



Real-World Link

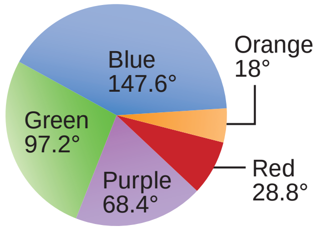
In tennis, the linesperson determines whether the hit ball is in or out. The umpire may only overrule the linesperson if he or she immediately thinks the call was wrong without a doubt and never as a result of a player's protest.

Source: www.usta.com

SURVEYS For Exercises 19–22, use the following information.

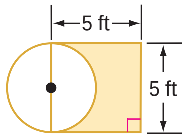
The circle graph shows the results of a survey of high school students about favorite colors. The measurement of each central angle is shown. If a person is chosen at random from the school, find the probability of each response.

What's Your Favorite Color?



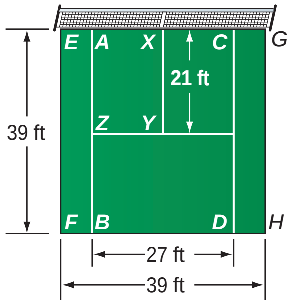
19. Favorite color is red.
20. Favorite color is blue or green.
21. Favorite color is not red or blue.
22. Favorite color is not orange or green.

23. One side of a square is a diameter of a circle. The length of one side of the square is 5 feet. To the nearest hundredth, what is the probability that a point chosen at random is in the shaded region?



TENNIS For Exercises 24 and 25, use the following information.

A tennis court has stripes dividing it into rectangular regions. For singles play, the inbound region is defined by segments $\overline{AB}$ and $\overline{CD}$. The doubles court is bound by the segments $\overline{EF}$ and $\overline{GH}$.



24. Find the probability that a ball in a singles game will land inside the court but out of bounds.
25. When serving, the ball must land within $AXYZ$, the service box. Find the probability that a ball will land in the service box, relative to the court.

H.O.T. Problems

26. **OPEN ENDED** List three games that involve geometric probability.

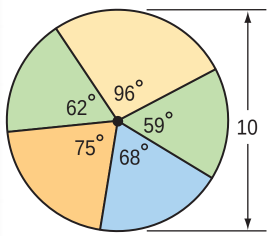
27. **FIND THE ERROR** Rachel and Taimi are finding the probability that a point chosen at random lies in the green region. Who is correct? Explain your answer.

Rachel

$$A = \frac{N}{360}\pi r^2$$
$$= \frac{59 + 62}{360}\pi(5^2)$$
$$\approx 26.4$$
$$P(\text{green}) \approx \frac{26.4}{25\pi} \approx 0.34$$

Taimi

$$A = \frac{N}{360}\pi r^2$$
$$= \frac{59}{360}\pi(5^2) + \frac{62}{360}$$
$$\approx 13.0$$
$$P(\text{green}) \approx \frac{13.0}{25\pi} \approx 0.17$$



28. **REASONING** Explain how to find the area of a sector of a circle.

Stu Forster/Getty Images

29. **Which One Doesn't Belong?** Identify the term that does not belong with the others. Explain your answer.

chord	radius	apothem	segment

CHALLENGE **Study each spinner in Exercises 11–13.**

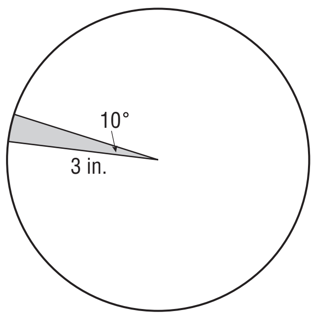
30. Are the chances of landing on each color equal? Explain.

31. Would this be considered a fair spinner to use in a game? Explain.

32. **Writing in Math** Explain how geometric probability can help a person design a dartboard and assign values to spaces.

STANDARDIZED TEST PRACTICE

33. A grocery store is slicing a wheel of cheese into slivers for free samples.



What is the area, in square inches, of one sliver?

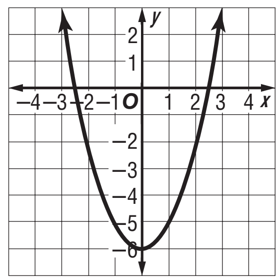
A $\frac{9\pi}{10}$

B $\frac{3\pi}{5}$

C $\frac{\pi}{4}$

D $\frac{\pi}{6}$

34. **REVIEW** Which is the equation of the function graphed below?



F $y = x^2 - 2.5$

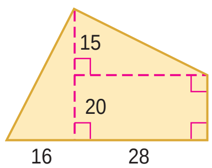
G $y = -x^2 + 2.5$

H $y = x^2 - 6$

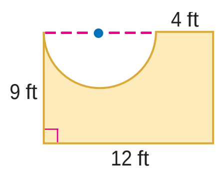
J $y = -x^2 - 6$

Find the area of each figure. Round to the nearest tenth if necessary. (Lesson 11-4)

35.



36.



Find the area of each polygon. Round to the nearest tenth if necessary. (Lesson 11-3)

37. a regular triangle with a perimeter of 48 feet

38. a square with a side length of 21 centimeters

39. a regular hexagon with an apothem length of 8 inches

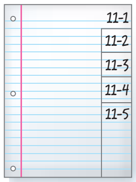
CHAPTER 11

Study Guide and Review

Download Vocabulary Review from geometryonline.com

FOLDABLES™ Study Organizer — GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Area of Parallelograms (Lesson 11-1)

- The area of a parallelogram is the product of the base and the height.

Areas of Triangles, Rhombi, and Trapezoids (Lesson 11-2)

- The formula for the area of a triangle can be used to find the areas of many different figures.
- Congruent figures have equal areas.

Areas of Regular Polygons and Circles (Lessons 11-3)

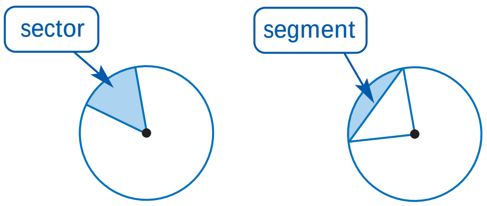
- A regular n-gon is made up of n congruent isosceles triangles.
- The area of a circle of radius r units is πr^2 square units.

Areas of Composite Figures (Lessons 11-4)

- The area of a composite figure is the sum of the areas of its nonoverlapping parts.

Geometric Probability and Areas of Sectors (Lessons 11-5)

- To find a geometric probability, divide the area of a part of a figure by the total area.
- A sector is a region of a circle bounded by a central angle and its intercepted arc.
- The area of a sector is given by the formula, $A = \frac{N}{360}\pi r^2$.
- A segment of a circle is a region bounded by an arc and a chord.



Key Vocabulary

apothem (p. 649)
composite figure (p. 658)
geometric probability (p. 665)
sector (p. 666)
segment (p. 667)

Vocabulary Check

Choose the term that best matches each phrase. Choose from the list above.

1. A figure that cannot be classified as a single polygon is a(n) ____________.
2. The region of a circle bounded by an arc and a chord is called a(n) ____________ of a circle.
3. ____________ uses the principles of length and area to find the probability of an event.
4. A(n) ____________ is a segment that is drawn from the center of a regular polygon perpendicular to a side of the polygon.
5. A(n) ____________ of a circle is a region of a circle bounded by a central angle and its intercepted arc.
6. To find the ____________, divide the area of a part of a figure by the total area.
7. A circle graph is separated into ____________(s).
8. To find the area of a(n)____________, find the area of the triangle and subtract it from the area of the sector.
9. The area of a rectangular polygon is one-half the product of the perimeter and the ____________.
10. A(n)____________ can be separated into basic shapes.

Mixed Problem Solving
For mixed problem-solving practice, see page 838.

Lesson-by-Lesson Review

11-1 Area of Parallelograms (pp. 630–636)

COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.

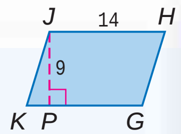
11. $A(-6, 1)$, $B(1, 1)$, $C(1, -6)$, $D(-6, -6)$

12. $E(7, -2)$, $F(1, -2)$, $G(2, 2)$, $H(8, 2)$

13. $J(-1, -4)$, $K(-5, 0)$, $L(-5, 5)$, $M(-1, 1)$

14. $P(-7, -1)$, $Q(-3, 3)$, $R(-1, 1)$, $S(-5, -3)$

Example 1 **Find the area of parallelogram *GHJK*.**



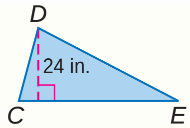
$A = bh$ Area of a parallelogram

$= 14(9)$ or 126 $b = 14, h = 9$

The area of *GHJK* is 126 square units.

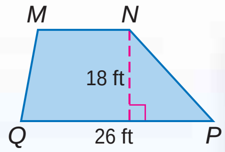
11-2 Areas of Triangles, Rhombi, and Trapezoids (pp. 638–647)

15. Triangle *CDE* has an area of 336 square inches. Find *CE*.



16. **FUND-RAISER** The school marching band is selling pennants. Each pennant is cut in the shape of a triangle 3 feet long and 1 foot high. How many square feet of fabric are needed to make 150 pennants, assuming no waste?

Example 2 **Trapezoid *MNPQ* has an area of 360 square feet. Find the length of $\overline{MN}$.**



$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$360 = \frac{1}{2}(18)(b_1 + 26)$ $A = 360, h = 18, b_2 = 26$

$360 = 9b_1 + 234$ Multiply.

$14 = b_1$ Solve for b_1.

The length of $\overline{MN}$ is 14 feet.

11-3 Areas of Regular Polygons and Circles (pp. 649–656)

Find the area of each polygon. Round to the nearest tenth.

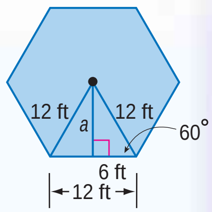
17. a regular pentagon with perimeter of 100 inches

18. an octagon with a perimeter of 96 feet

19. **BAKING** Todd wants to make a cheesecake for a birthday party. The recipe calls for a 9 inch diameter round pan. Todd only has square pans. He has an 8 inch square pan, a 9 inch square pan, and a 10 inch square pan. Which pan comes closest in area to the one that the recipe suggests?

Example 3 **Find the area of the regular hexagon.**

The central angle of a hexagon is 60°. Use the properties of 30°-60°-90° triangles to find that the apothem is $6\sqrt{3}$ feet.



$A = \frac{1}{2}Pa$ Area of a regular polygon

$= \frac{1}{2}(72)\left(6\sqrt{3}\right)$ $P = 72, a = 6\sqrt{3}$

$= 216\sqrt{3} \approx 374.1$

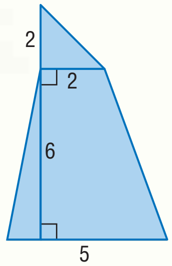
The area of the regular hexagon is 374.1 square feet to the nearest tenth.

Study Guide and Review

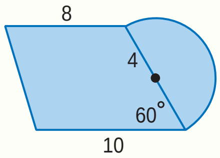
11-4 Areas of Composite Figures (pp. 658–663)

Find the area of each figure to the nearest tenth.

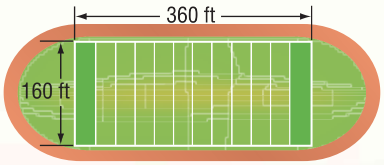
20.



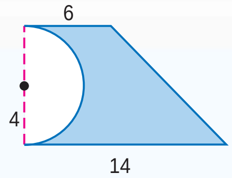
21.



22. RECREATION The football field in the back of the high school is surrounded by a track. The football field has dimensions 160 feet by 360 feet. Find the area of the figure inside the track to the nearest tenth.



Example 4 **Find the area of the figure.**



Separate the figure into a rectangle and a triangle.

Area of composite figure = area of rectangle - area of semicircle + area of triangle.

$= \ell w - \frac{1}{2}\pi r^2 + \frac{1}{2}bh$

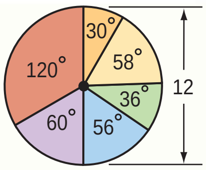
$= (6)(8) - \frac{1}{2}\pi(4^2) + \frac{1}{2}(8)(8)$

$= 48 - 8\pi + 32$ or about 54.9

The area of the composite figure is 54.9 square units to the nearest tenth.

11-5 Geometric Probability and Areas of Sectors (pp. 665–671)

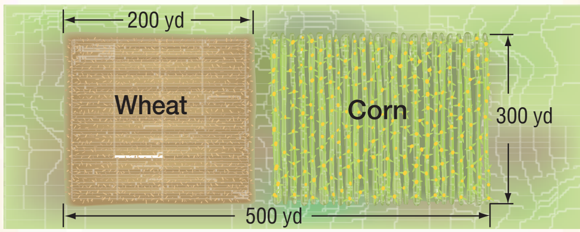
Find the probability that a point chosen at random will be in the sector of the given color.



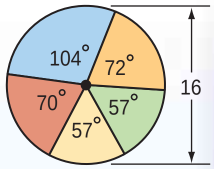
23. red

24. purple or green

25. FARMING A farmer grows corn and wheat in a field shown below. What is the probability that a lightning bolt that strikes will hit the wheat field?



Example 5 **Find the probability that a point chosen at random will be in the blue sector.**



First find the area of the blue sector.

$A = \frac{N}{360}\pi r^2$ Area of a sector

$= \frac{104}{360}\pi(8^2)$ or about 58.08 Substitute and simplify.

To find the probability, divide the area of the sector by the area of the circle.

$P(\text{blue}) = \frac{\text{area of sector}}{\text{area of circle}}$ Geometric probability formula

$= \frac{58.08}{\pi(8^2)}$ or about 0.29

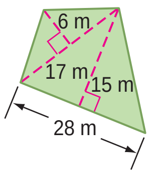
The probability is about 0.29 or 29%.

CHAPTER 11 Practice Test

COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.**

1. $R(-6, 8)$, $S(-1, 5)$, $T(-1, 1)$, $U(-6, 4)$
2. $R(7, -1)$, $S(9, 3)$, $T(5, 5)$, $U(3, 1)$
3. $R(2, 0)$, $S(4, 5)$, $T(7, 5)$, $U(5, 0)$
4. $R(3, -6)$, $S(9, 3)$, $T(12, 1)$, $U(6, -8)$

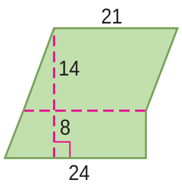
Find the area of each figure. Round to the nearest tenth if necessary.

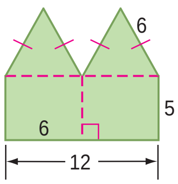
5. 

6. a regular octagon with apothem length of 3 ft
7. a regular pentagon with a perimeter of 115 cm

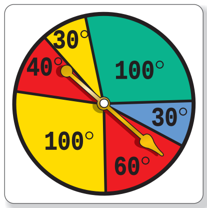
8. **SOCCER BALLS** The surface of a soccer ball is made of a pattern of regular pentagons and hexagons. If each hexagon on a soccer ball has a perimeter of 9 inches, what is the area of a hexagon?

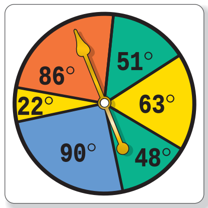
Find the area of each figure. Round to the nearest tenth.

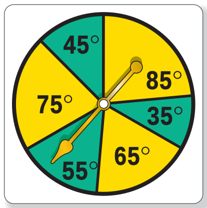
9. 

10. 

Each spinner has a diameter of 12 inches. Find the probability of spinning the indicated color.

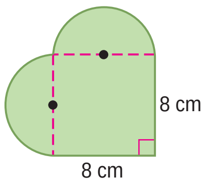
11. red 

12. orange 

13. green 

14. **COORDINATE GEOMETRY** Find the area of *CDGHJ* with vertices $C(-3, -2)$, $D(1, 3)$, $G(5, 5)$, $H(8, 3)$, and $J(5, -2)$.

15. **MULTIPLE CHOICE** What is the area of the figure in square centimeters?



A $64 + 64\pi$

B 80π

C 64π

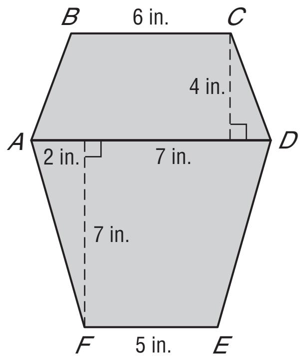
D $64 + 16\pi$

Standardized Test Practice

Cumulative, Chapters 1–11

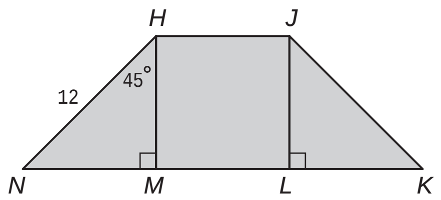
Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What is the area of figure *ABCDEF* in square inches?



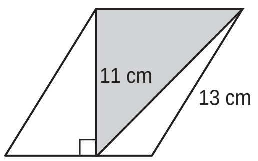
A 158 in^2 **C** 79 in^2
B 100 in^2 **D** 68 in^2

2. If *HJKN* is an isosceles trapezoid, what is the area of $\triangle JKL$?

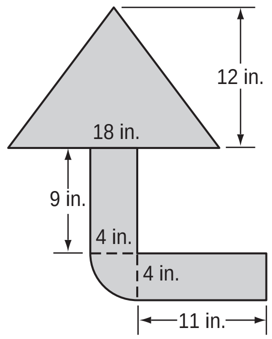


F $144\sqrt{2}$ **H** $72\sqrt{2}$
G 144 **J** 36

3. **GRIDDABLE** The rhombus below has side length of 13 centimeters and height of 11 centimeters. If the shaded area is removed, what is the area of the remaining figures in square centimeters?



4. Henry is painting directional arrows in the school parking lot. He needs to know the area of each arrow in order to calculate the amount of paint he needs to buy. Find the area using the diagram below.



A $188 + 4\pi$ in^2 **C** $188 + 16\pi$ in^2
B $188 + 8\pi$ in^2 **D** $296 + 4\pi$ in^2

5. Which statement is *always* true?

F When an angle is inscribed in a circle, the angle's measure equals one-half of the measure of the intercepted arc.

G In a circle, an inscribed quadrilateral will have consecutive angles that are supplementary.

H In a circle, an inscribed angle that intercepts a semicircle is obtuse.

J If two inscribed angles of a circle intercept congruent arcs, then the angles are complementary.

6. **ALGEBRA** The width of a parallelogram can be represented using the expression $\frac{x^2 + 2x - 48}{x + 8}$, where the numerator represents the area and the denominator represents the length. What is the width of the parallelogram?

A $x - 4$
B $x + 4$
C $x - 6$
D $x + 6$

Standardized Test Practice at geometryonline.com

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 841–856.

7. Which of the segments described could be a secant of a circle?

F has its endpoints on a circle
G intersects exactly one point on a circle
H intersects exactly two points on a circle
J one endpoint at the center of the circle

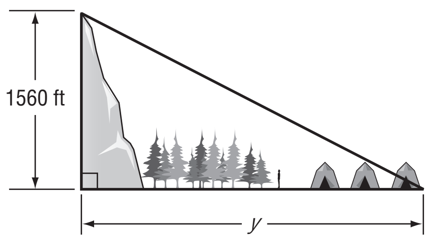
8. Two triangles are drawn on a coordinate grid. One has vertices at (0, 1), (0, 7), and (6, 4). The other has vertices at (7, 7), (10, 7), and (8.5, 10). What scale factor can be used to compare the smaller triangle to the larger?

A 2.5
B 2
C 1.5
D 0.5

TEST-TAKING TIP

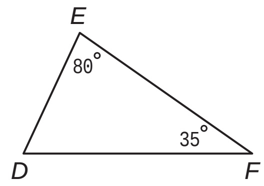
Question 8 If a question does not provide you with a figure that represents the problem, draw one yourself. By recording the information that you know, the problem becomes more understandable.

9. Lori and her family are camping near a mountain. Their campground is in a clearing next to a stretch of forest. The angle of elevation from the far edge of the campground to the top of the mountain is 35°. Find the distance y from the base of the mountain to the far edge of the campground.



F 2719 ft
G 2228 ft
H 1904 ft
J 1092 ft

10. Which of the following lists the sides of $\triangle DEF$ from greatest to least length?



A $\overline{DE}, \overline{EF}, \overline{DF}$
B $\overline{DE}, \overline{DF}, \overline{EF}$
C $\overline{DF}, \overline{EF}, \overline{DE}$
D $\overline{DF}, \overline{DE}, \overline{EF}$

11. Ms. Lee told her students, "If you do not get enough rest, you will be tired. If you are tired, you will not be able to concentrate." Which of the following is a logical conclusion that could follow Ms. Lee's statements?

F If you get enough rest, you will be tired.
G If you are tired, you will be able to concentrate.
H If you do not get enough rest, you will be able to concentrate.
J If you do not get enough rest, you will not be able to concentrate.

Pre-AP

Record your answer on a sheet of paper. Show your work.

12. Quadrilateral *ABCD* has vertices $A(1, 2)$, $B(5, 5)$, and $D(5, 0)$.

a. Find the coordinates of point C such that *ABCD* is a parallelogram and plot the parallelogram on a coordinate plane.
b. Using the plot you created, find the midpoint of $\overline{CD}$ and label it M. Draw a segment from point B to M and from point A to M. Find the area of triangle AMB.
c. What is the area of each of the other triangles formed by the construction?

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson or Page...	11-4	11-2	11-1	11-4	10-4	796	10-6	9-5	8-5	5-3	2-4	11-2

CHAPTER 12

Extending Surface Area

BIG Ideas

- Find the lateral and surface areas of prisms, cylinders, pyramids, and cones.
- Find the surface areas of spheres and hemispheres.

Key Vocabulary

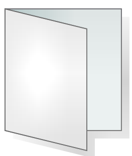
cross section (p. 681)
lateral area (p. 686)
slant height (p. 699)

Real-World Link

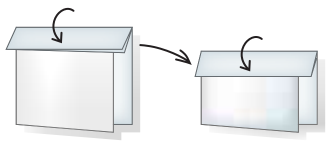
Gemology Diamonds and other gemstones are cut to enhance the beauty of the stones. The surface of each cut is a *facet.* Mathematics is used in the cutting of the stones so the stone will reflect and refract the most light.

Surface Area Make this Foldable to help you organize your notes. Begin with a sheet of 11" × 17" paper.

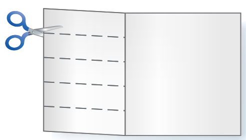
1 **Fold** lengthwise leaving a two-inch tab.



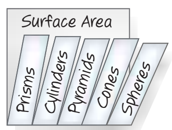
2 **Fold** the paper into five sections.



3 **Open.** Cut along each fold to make five tabs.



4 **Label** as shown.



Richard T. Nowitz/Photo Researchers

GET READY for Chapter 12

Diagnose Readiness **You have two options for checking Prerequisite Skills.**

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

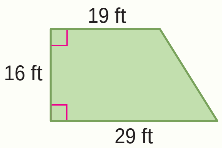
Take the Quick Check below. Refer to the Quick Review for help.

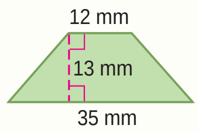
QUICKCheck

Refer to the figure in Example 1. Determine whether each statement is *true*, *false*, or *cannot be determined*. (Lesson 1-1)

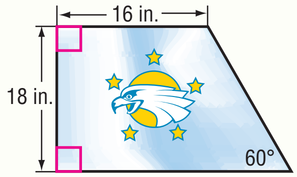
1. $\triangle ADC$ lies in plane N.
2. $\triangle ABC$ lies in plane K.
3. The line containing $\overline{AB}$ is parallel to plane K.
4. The line containing $\overline{AC}$ lies in plane K.

Find the area of each figure. Round to the nearest tenth if necessary. (Lesson 11-2)

5. 

6. 

7. **FLAGS** Find the area of fabric needed to make the flag shown. Round to the nearest tenth. (Lesson 11-2)



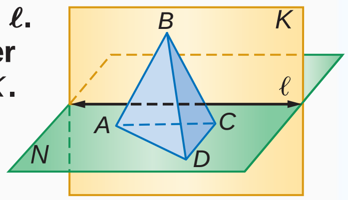
Find the area of each circle with the given radius or diameter to the nearest tenth. (Lesson 11-3)

8. $d = 19.0$ cm
9. $r = 1.5$ yd
10. **MOSAICS** A museum wants to install a circular mosaic in the floor of the lobby. If the diameter of the mosaic is 9.5 feet, what is the area of the mosaic? Round to the nearest tenth.

QUICKReview

EXAMPLE 1

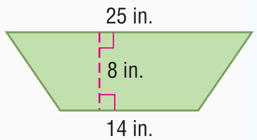
In the figure, $\overline{AC} \parallel \ell$. Determine whether plane $N \perp$ plane K.



Although it is known that $\overline{AC} \parallel \ell$, it cannot be determined whether plane $N \perp$ plane K.

EXAMPLE 2

Find the area of the figure. Round to the nearest tenth if necessary.



$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$= \frac{1}{2}(8)(25 + 14)$ Substitution

$= \frac{1}{2}(8)(39)$ Simplify.

$= 156$ Add.

The area of the trapezoid is 156 square inches.

EXAMPLE 3

Find the area of the circle with a radius of 4 inches. Round to the nearest tenth.

$A = \pi r^2$ Area of a circle

$= \pi(4)^2$ Substitution

$= 16\pi$ $4^2 = 16$

$\approx 50.3 \text{ in}^2$ Simplify.

12-1 Representations of Three-Dimensional Figures

Main Ideas

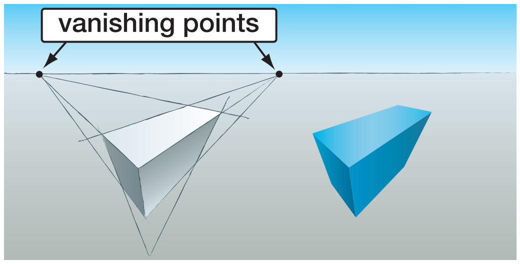
- Draw isometric views of three-dimensional figures.
- Investigate cross sections of three-dimensional figures.

New Vocabulary

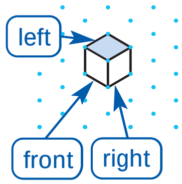
corner view
perspective view
cross section
reflection symmetry

GET READY for the Lesson

Artists use three-point perspective to draw three-dimensional objects with a high degree of realism. Each vanishing point is aligned with the height, width, and length of a box.



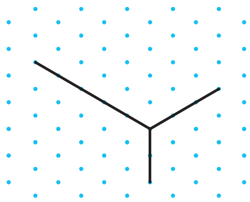
Isometric View The view of a figure from a corner is called the **corner view** or **perspective view**. You can use isometric dot paper to draw the corner view of a solid figure. In this lesson, isometric dot paper will be used to draw and construct two-dimensional models of geometric solids.



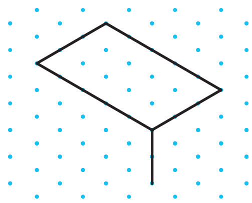
EXAMPLE Draw a Solid

1 **Sketch a rectangular prism 2 units high, 5 units long, and 3 units wide using isometric dot paper.**

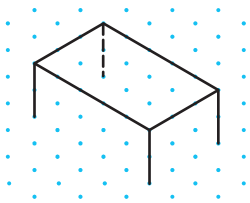
Step 1 Mark the corner of the solid. Then draw 2 units down, 5 units to the left, and 3 units to the right.



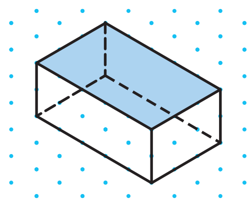
Step 2 Draw a parallelogram for the top of the solid.



Step 3 Draw segments 2 units down from each vertex for the vertical edges.



Step 4 Connect the corresponding vertices. Use dashed lines for the hidden edges. Shade the top of the solid.



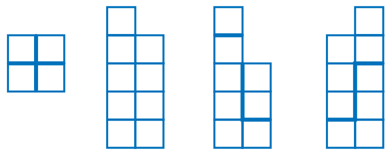
1. Sketch a triangular prism 3 units high with a base that is a right triangle with legs that are 2 units and 4 units.

Review Vocabulary

Orthographic Drawing
This is a two-dimensional drawing that shows the top view, left view, front view, and right view of a three-dimensional object. (Lesson 1-7B)

EXAMPLE Orthographic Drawings

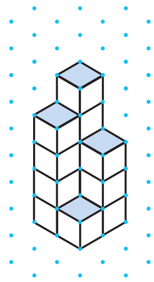
2 **Draw a corner view of the figure given its orthographic drawing.**



top view left view front view right view

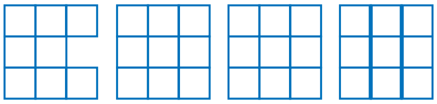
- The top view indicates two rows and two columns of different heights.
- The front view indicates that the left side is 5 blocks high and the right side is 3 blocks high. The dark segments indicate breaks in the surface.
- The right view indicates that the right front column is only one block high. The left front column is 4 blocks high. The right back column is 3 blocks high.

The lowest columns should be in front so the difference in height between the columns is visible. Connect the dots on the isometric dot paper to represent the edges of the solid. Shade the tops of each column.



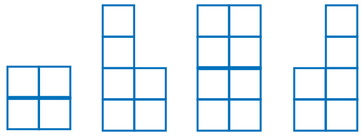
CHECK Your Progress

2A.



top view left view front view right view

2B.



top view left view front view right view

Personal Tutor at geometryonline.com

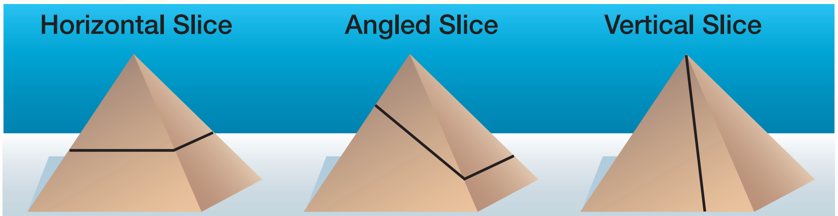
Interesting shapes occur when a plane intersects, or slices, a solid figure. The intersection of the solid and a plane is called a **cross section** of the solid.

GEOMETRY LAB

Cross Sections of Solids

MODEL

Use modeling clay to form a square pyramid. Use dental floss to slice through the pyramid as shown below. Trace the cut surface onto a piece of paper. Identify the shape determined by each slice.



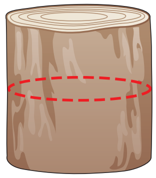
ANALYZE

1. Describe and draw the shape created by each slice.

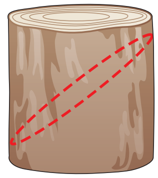
2. Create another solid figure. What shapes are determined by horizontal, angled, and vertical slices?

EXAMPLE

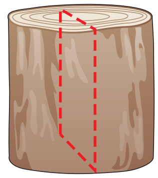
3 **CARPENTRY** **A carpenter wants to cut a cylindrical tree trunk into a circle, an oval, and a rectangle. How could the tree trunk be cut to get each shape?**



If the blade of the saw was placed parallel to the bases, the cross section would be a circle.



If the blade was placed at an angle to the bases of the tree trunk, the slice would be an oval shape, or an ellipse.



To cut a rectangle from the cylinder, place the blade perpendicular to the bases. The slice is a rectangle.

3. **CAKE DECORATING** Carolyn has a cake pan shaped like half of a sphere. Describe the shape of the cross sections of cake baked in this pan if they are cut horizontally and vertically on its base.

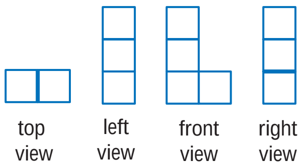
CHECK Your Understanding

Example 1 (p. 680)

1. Sketch a rectangular prism 4 units high, 2 units long, and 3 units wide using isometric dot paper.

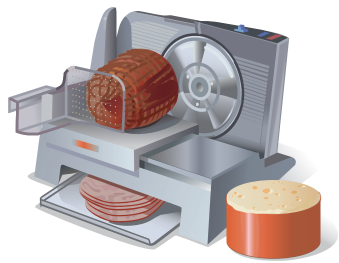
Example 2 (p. 681)

2. Draw a corner view of the figure given its orthographic drawing.



Example 3 (p. 682)

3. **DELICATESSEN** A deli slicer is used to cut cylindrical blocks of cheese for sandwiches. Suppose a customer wants slices of cheese that are round and slices that are rectangular. How can the cheese be placed on the slicer to get each shape?



Exercises

HOMEWORK HELP

For Exercises	See Examples
4–9	1
10–13	2
14–26	3

Sketch each solid using isometric dot paper.

4. rectangular prism 3 units high, 4 units long, and 5 units wide

5. cube 5 units on each edge

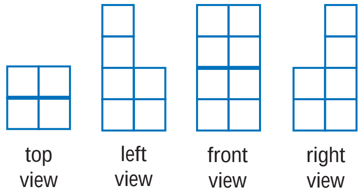
6. cube 4 units on each edge

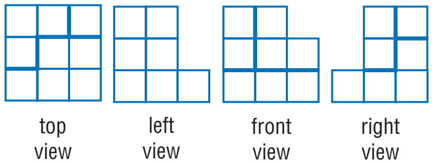
7. rectangular prism 6 units high, 6 units long, and 3 units wide

8. triangular prism 4 units high, with bases that are right triangles with legs 5 units and 4 units long

9. triangular prism 2 units high, with bases that are right triangles with legs 3 units and 7 units long

Draw a corner view of a figure given each orthographic drawing.

10.


11.


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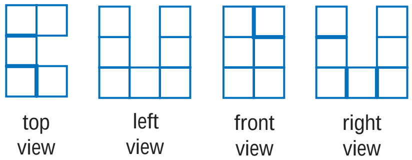
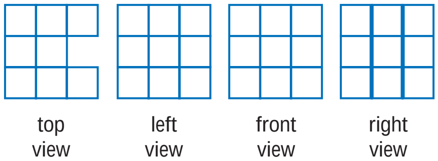
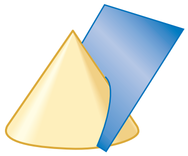
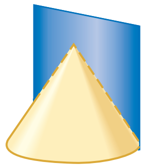
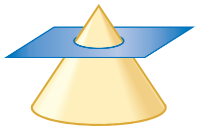


13.


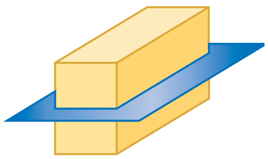
Determine the shape resulting from each cross section of the cone.

14.


15.


16.


Determine the shape resulting from each cross section of the rectangular prism.

17.


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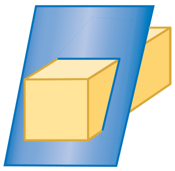
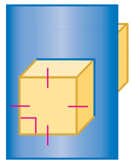


19.


Draw a diagram and describe how a plane can slice a tetrahedron to form the following shapes.

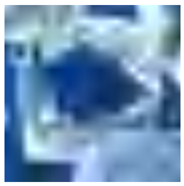
20. equilateral triangle **21.** isosceles triangle **22.** quadrilateral

Real-World Link

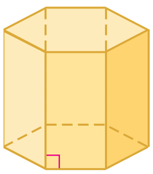
There are 32 different classes of crystals, each with a different type of symmetry.

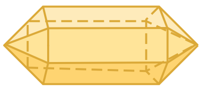
Source: infoplease.com

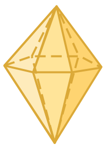
23. GEMOLOGY A well-cut diamond enhances the natural beauty of the stone. These cuts are called *facets*. Describe the shapes seen in an uncut diamond.

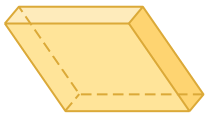


GEOLOGY For Exercises 24–27, use the following information.
Many minerals have a crystalline structure. The forms of three minerals are shown below. Describe the cross sections from a horizontal and vertical slice of each crystal.

24.


25.


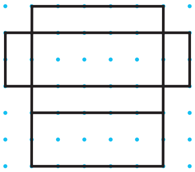
26.


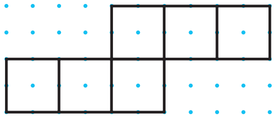
27.


Review Vocabulary

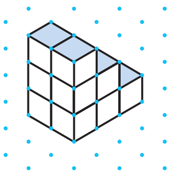
Net a two-dimensional figure that when folded, forms a three-dimensional solid (Lesson 1-7B)

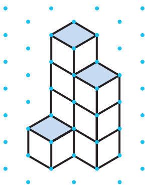
Given the net of a solid, use isometric dot paper to draw the solid.

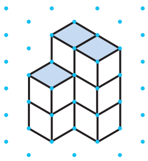
28. 

29. 

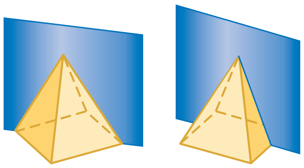
Given the corner view of a figure, sketch the orthographic drawing.

30. 

31. 

32. 

SYMMETRY For Exercises 33–35, use the following information.
In a two-dimensional plane, figures are symmetric with respect to a line or a point. In three-dimensional space, solids are symmetric with respect to a plane. This is called **reflection symmetry**. A square pyramid has four planes of symmetry. Two pass through the altitude and one pair of opposite vertices of the base. Two pass through the altitude and the midpoint of one pair of opposite edges of the base.



For each solid, determine the number of planes of symmetry and describe them.

33. tetrahedron

34. cylinder

35. sphere

EXTRA PRACTICE
See pages 823, 839.
Math Online
Self-Check Quiz at geometryonline.com

Three-dimensional solids exhibit rotational symmetry in the same way as two-dimensional figures. Identify the order and magnitude of the rotational symmetry of each solid with respect to its base.

36. regular pentagonal prism

37. tetrahedron

H.O.T. Problems

38. **CHALLENGE** The cross section of a solid is an octagon. Name the solids that have octagonal cross sections.

39. **REASONING** Of the two-dimensional representations that you have studied, which would you choose to represent plans for a new skyscraper? Explain.

40. **OPEN ENDED** Select three different three-dimensional figures. Draw a net for each on cardboard. Cut out the cardboard and use tape to construct each model.

41. **Which One Doesn't Belong?** Identify the term that does not belong with the other three. Explain your reasoning.

isometric drawing	net	construction	orthographic drawing

42. **Writing in Math** Analyze the relationship between a three-dimensional solid and a two-dimensional representation. Include a description of orthographic drawings and isometric drawings.

STANDARDIZED TEST PRACTICE

43. Which of the following *cannot* be formed by the intersection of a cube and a plane?

A a triangle

B a rectangle

C a point

D a circle

44. REVIEW How many centimeters are in 35 millimeters?

F 0.35 centimeters

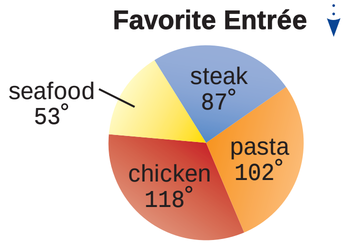
G 3.5 centimeters

H 350 centimeters

J 35,000 centimeters

Spiral Review

SURVEYS For Exercises 45–48, use the following information.
The results of a restaurant survey are shown in the circle graph with the measurement of each central angle. Each customer was asked to choose a favorite entrée. If a customer is chosen at random, find the probability of each response. (Lesson 11-5)



45. steak

46. not seafood

47. either pasta or chicken

48. neither pasta nor steak

COORDINATE GEOMETRY The coordinates of the vertices of a composite figure are given. Find the area of each figure. (Lesson 11-4)

49. $A(1, 4)$, $B(4, 1)$, $C(1, -2)$, $D(-3, 1)$

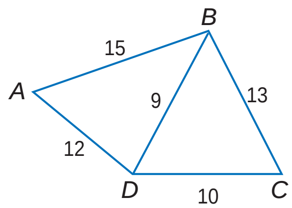
50. $F(-2, -4)$, $G(-2, -1)$, $H(1, 1)$, $J(4, 1)$, $K(6, -4)$

Solve each $\triangle ABC$ described below. Round to the nearest tenth if necessary. (Lesson 7-7)

51. $m\angle A = 54$, $b = 6.3$, $c = 7.1$

52. $m\angle B = 47$, $m\angle C = 69$, $a = 15$

Determine the relationship between the measures of the given angles. (Lesson 5-2)



53. $\angle ADB$, $\angle ABD$

54. $\angle ABD$, $\angle BAD$

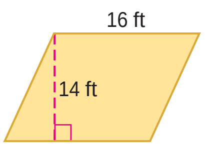
55. $\angle BCD$, $\angle CDB$

56. $\angle CBD$, $\angle BCD$

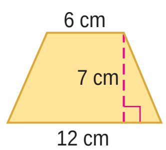
GET READY for the Next Lesson

PREREQUISITE SKILL Find the area of each figure. (Lessons 11-1 and 11-2)

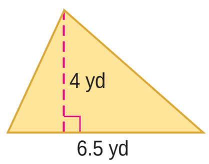
57.



58.



59.



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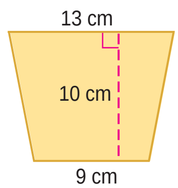


12-2 Surface Areas of Prisms

Main Ideas

- Find lateral areas of prisms.
- Find surface areas of prisms.

New Vocabulary

lateral faces
lateral edges
right prism
lateral area

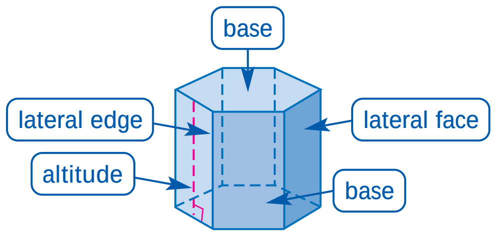
GET READY for the Lesson

In 1901, architect Daniel Burnham designed the first modern triangular building, the Flatiron Building of New York City. The architect designed the building in the shape of a triangular prism to best use the plot of land formed by the intersection of Broadway and 5th Avenue. He needed to know the lateral area of the building to estimate the amount of materials for the outside.

Lateral Areas of Prisms Most buildings are prisms or combinations of prisms. Prisms have the following characteristics.

Vocabulary Link
Lateral
Everyday Use extending from side to side
Math Use related to a side, not a base, of a prism

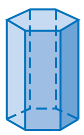
- The bases are congruent faces in parallel planes.
- The rectangular faces that are not bases are called **lateral faces**.
- The lateral faces intersect at the **lateral edges**. Lateral edges are parallel segments.

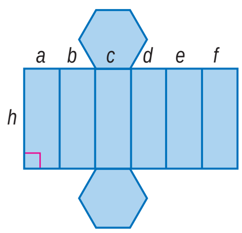


right hexagonal prism

- A segment perpendicular to the bases, with an endpoint in each plane, is called an *altitude* of the prism. The height of a prism is the length of the altitude.
- A prism with lateral edges that are also altitudes is called a **right prism**.

The **lateral area** L is the sum of the areas of the lateral faces.





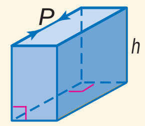
$$L = ah + bh + ch + dh + eh + fh$$
$$= h(a + b + c + d + e + f) \quad \text{Distributive Property}$$
$$= Ph \quad P = a + b + c + d + e + f$$

Reading Math

Solids From this point in the text, you can assume that solids are right solids. If a solid is oblique, it will be clearly stated.

KEY CONCEPT — *Lateral Area of a Prism*

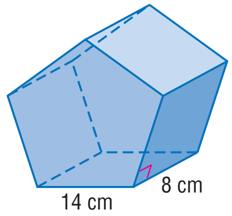
If a right prism has a lateral area of L square units, a height of h units, and each base has a perimeter of P units, then $L = Ph$.



EXAMPLE Lateral Area of a Pentagonal Prism

1 **Find the lateral area of the regular pentagonal prism.**

The bases are regular pentagons. So the perimeter of one base is 5(14) or 70 centimeters.



$L = Ph$	Lateral area of a prism
$= (70)(8)$	$P = 70$, $h = 8$
$= 560$	Multiply.

The lateral area is 560 square centimeters.

1. The length of each side of the base of a regular octagonal prism is 6 inches, and the height is 11 inches. Find the lateral area.

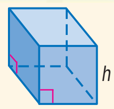
Study Tip

Right Prisms
The bases of a right prism are congruent, but the faces are not always congruent.

Surface Areas of Prisms The surface area of a prism is the lateral area plus the areas of the bases. The bases are congruent, so the areas are equal.

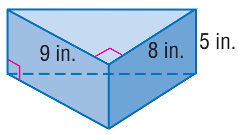
KEY CONCEPT — *Surface Area of a Prism*

If the surface area of a right prism is T square units, its lateral area is L square units, and each base has an area of B square units, then $T = L + 2B$.



EXAMPLE Surface Area of a Triangular Prism

2 **Find the surface area of the triangular prism.**

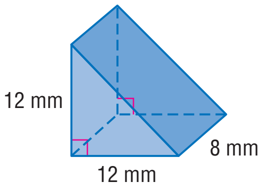


First, find the measure of the third side of the triangular base.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$c^2 = 8^2 + 9^2$	Substitution
$c^2 = 145$	Simplify.
$c = \sqrt{145}$	Take the square root of each side.
$T = L + 2B$	Surface area of a prism
$= Ph + 2B$	$L = Ph$
$= (8 + 9 + \sqrt{145})5 + 2\left[\frac{1}{2}(8 \cdot 9)\right]$	Substitution
≈ 217.2	Use a calculator.

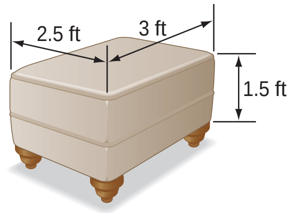
The surface area is approximately 217.2 square inches.

2. Find the surface area of the prism.



EXAMPLE 3

FURNITURE Nicolás wants to have an ottoman reupholstered. Find the surface area that will be reupholstered.

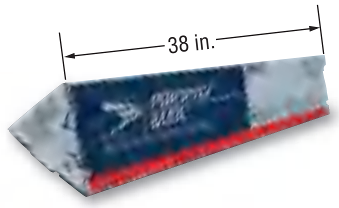


The ottoman is shaped like a rectangular prism. Since the bottom of the ottoman is not covered with fabric, find the lateral area and then add the area of one base. The perimeter of a base is 2(3) + 2(2.5) or 11 feet. The area of a base is 3(2.5) or 7.5 square feet.

$T = L + B$	Formula for surface area
$= (11)(1.5) + 7.5$	$P = 11$, $h = 1.5$, and $B = 7.5$
$= 24$	Simplify.

The total area that will be reupholstered is 24 square feet.

3. The United States Postal Service offers a mailer for posters or artwork that is a triangular prism. The base is an equilateral triangle with sides that measure 6 inches. Find the surface area of the mailer to the nearest tenth.



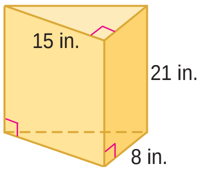
Online **Personal Tutor at** geometryonline.com

CHECK Your Understanding

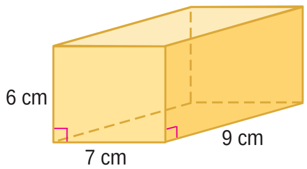
Example 1 (p. 687)

Find the lateral area of each prism.

1.



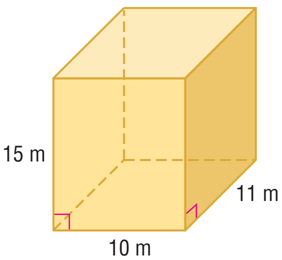
2.



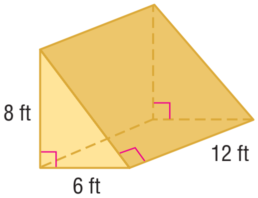
Example 2 (p. 687)

Find the surface area of each prism.

3.



4.



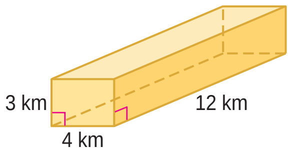
Example 3
(p. 688)

5. PAINTING Eva and Casey are planning to paint the walls and ceiling of their living room. The room is 20 feet long, 15 feet wide, and 12 feet high. Find the surface area to be painted.

Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
6–13	1
14–21	2
22–24	3

Find the lateral area of each prism or solid. Round to the nearest tenth if necessary.

6.


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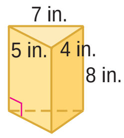

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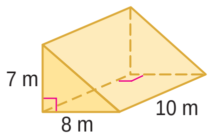

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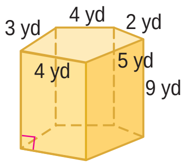

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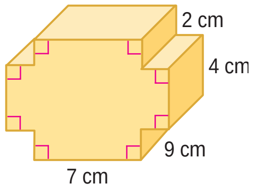

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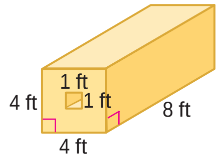


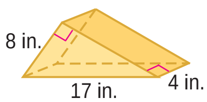
12. Find the lateral area of a rectangular prism with a length of 25 centimeters, a width of 18 centimeters, and a height of 12 centimeters.

13. Find the lateral area of a triangular prism with a base that is a right triangle, with legs that measure 9 inches and 12 inches, and a height of 6 inches.

14. Find the surface area of a triangular prism with a base that is a right triangle, with legs 16 centimeters and 30 centimeters, and a height of 14 centimeters.

15. Find the surface area of a rectangular prism with a length of 4 feet, a width of 8 feet, and a height of 12 feet.

Find the surface area of each prism. Round to the nearest tenth if necessary.

16.


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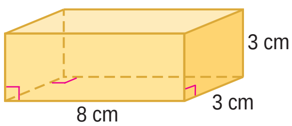

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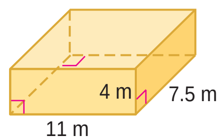

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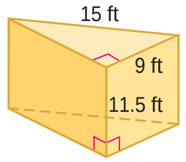

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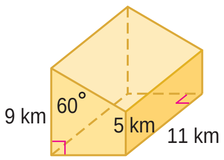

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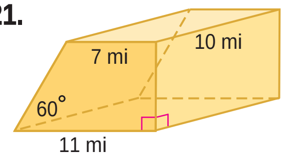


Real-World Link

It takes 60 gallons of paint for the fountain at the University at Albany, State University of New York. The cost of paint and thinner is about $1400.

Source: albany.edu

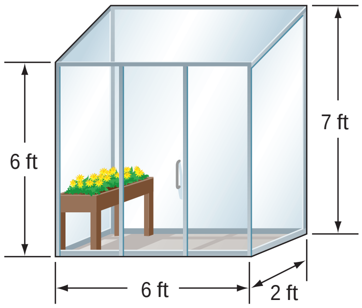
University at Albany

PAINTING For Exercises 22 and 23, use the following information.
Suppose a gallon of paint costs $16 and covers 400 square feet. Two coats of paint are recommended for even coverage. The room to be painted is 10 feet high, 15 feet long, and 15 feet wide.

22. The homeowner has $1\frac{1}{2}$ gallons of paint left from another project. Is this enough paint for the walls of the room? Explain.

23. If all new paint is purchased, how much will it cost to paint the walls and ceiling? Explain.

24. **GARDENING** This greenhouse is designed for a home gardener. The frame on the back of the greenhouse attaches to one wall of the house. The outside of the greenhouse is covered with tempered safety glass. Find the area of the glass covering the greenhouse.



25. The surface area of a cube is 864 square inches. Find the length of the lateral edge of the cube.

26. The surface area of a triangular prism is 540 square centimeters. The bases are right triangles with legs measuring 12 centimeters and 5 centimeters. Find the height.

27. The lateral area of a rectangular prism is 156 square inches. What are the possible whole-number dimensions of the prism if the height is 13 inches?

28. The lateral area of a rectangular prism is 96 square meters. What are the possible whole-number dimensions of the prism if the height is 4 meters?

CHANGING DIMENSIONS For Exercises 29–33, use prisms A, B, and C.

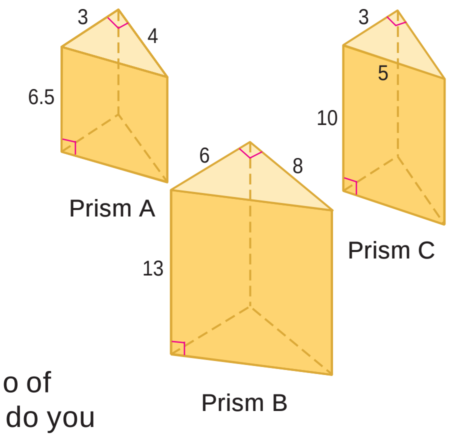
29. Compare the bases of each prism.

30. Write three ratios to compare the perimeters of the bases of the prisms.

31. Write three ratios to compare the areas of the bases of the prisms.

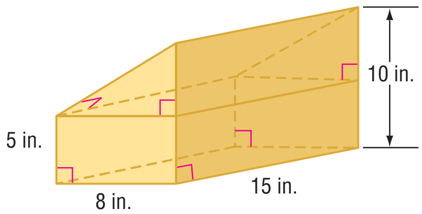
32. Write three ratios to compare the surface areas of the prisms.

33. Which pairs of prisms have the same ratio of base areas as ratio of surface areas? Why do you think this is so?



A composite solid is a three-dimensional figure that is composed of simpler figures. Find the surface area of each composite solid. Round to the nearest tenth if necessary.

34.



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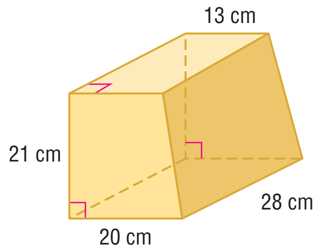


EXTRA PRACTICE
See pages 824, 839.

Math Online
Self-Check Quiz at geometryonline.com

H.O.T. Problems

36. **OPEN ENDED** Draw a prism with a surface area of 24 square units. Label the bases, lateral faces, and lateral edges.

37. **CHALLENGE** Suppose the lateral area of a right rectangular prism is 144 square centimeters. If the length is three times the width and the height is twice the width, find the surface area.

38. **Writing in Math** Suppose a rectangular prism and a triangular prism are the same height. The base of the triangular prism is an isosceles triangle, the altitude of which is the same as the height of the base of the rectangular prism. Compare the lateral areas.

STANDARDIZED TEST PRACTICE

39. Lucita needs to figure out how much it will cost to repaint the walls in her bedroom. She knows the cost of paint per gallon g, how many square feet a gallon will cover f, and the length ℓ, width w, and height h of her room. Which formula should Lucita use to calculate the total cost c of painting her room?

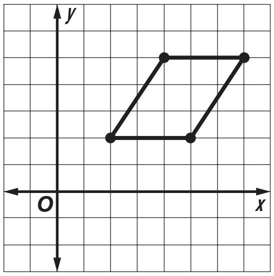
A $g = \dfrac{w\ell h + c}{f}$

B $c = \dfrac{(wh + \ell h)}{f} \cdot g$

C $c = \dfrac{2(wh + \ell h)}{f} \cdot g$

D $c = \dfrac{2(wh + \ell h + \ell w)}{g} \cdot f$

40. **REVIEW** A parallelogram is graphed on the coordinate grid.



Which function describes a line that would include an edge of the parallelogram?

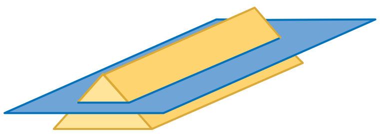
F $y = \frac{3}{2}x$

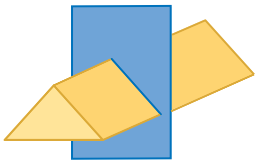
G $y = \frac{2}{3}x$

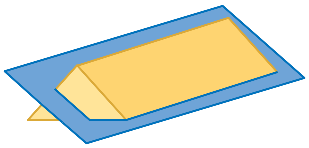
H $y = \frac{2}{3}x + 1$

J $y = \frac{3}{2}x - 1$

Determine the shape resulting from each slice of the triangular prism. (Lesson 12-1)

41. 

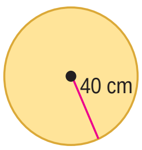
42. 

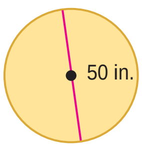
43. 

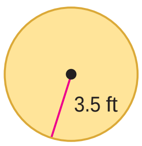
44. **PROBABILITY** A rectangular garden is 100 feet long and 200 feet wide and includes a square flower bed that is 20 feet on each side. Find the probability that a butterfly in the garden is somewhere in the flower bed. (Lesson 11-5)

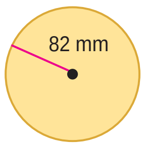
GET READY for the Next Lesson

PREREQUISITE SKILL **Find the area of each circle. Round to the nearest hundredth.** (Lesson 11-3)

45. 

46. 

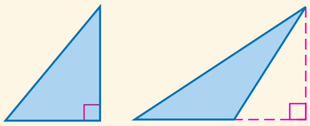
47. 

48. 

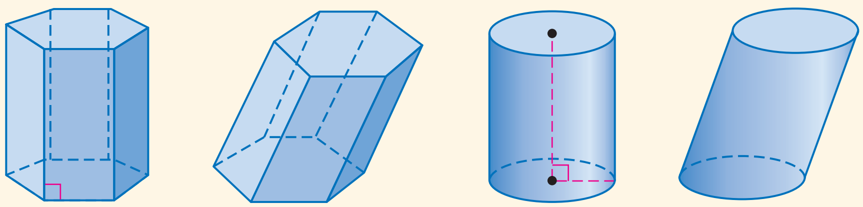
READING MATH

Right Solids and Oblique Solids

You know that in a right triangle one of the sides is an altitude. However, in an obtuse triangle, the altitude is outside of the triangle. This same concept can be applied to solids.



A prism with lateral edges that are also altitudes is called a *right prism.* If the lateral edges are not perpendicular to the bases, it is an **oblique prism**. Similarly, if the axis of a cylinder is also the altitude, then the cylinder is called a *right cylinder*. Otherwise, the cylinder is an **oblique cylinder**.



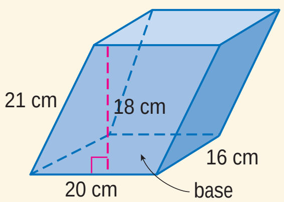
The altitude of an oblique prism is not the length of a lateral edge. For an oblique rectangular prism, the bases are rectangles, two faces are rectangles, and two faces are parallelograms. To find the lateral area and the surface area, you can apply the definitions of each.

Reading to Learn

1. Explain the difference between a right prism and an oblique prism.
2. Make a sketch of an oblique rectangular prism. Describe the shapes of its bases and lateral faces.
3. Compare and contrast the net of a right cylinder and the net of an oblique cylinder.
4. **RESEARCH** Use the Internet or another resource to find the meaning of the term *oblique*. How is the everyday meaning related to the mathematical meaning?

Find the lateral area and surface area of each oblique prism.

5.



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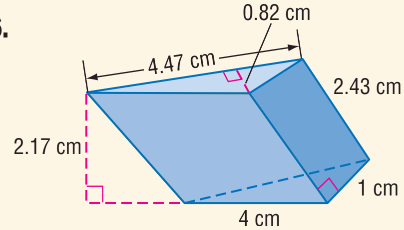


12-3 Surface Areas of Cylinders

GET READY for the Lesson

Skaters commonly use a ramp called a half-pipe. Some skate parks also feature a ramp called the full-pipe, shown at the right. The full-pipe is a concrete cylinder. As skaters skate along the interior surface, they build momentum. This allows them to skate higher.

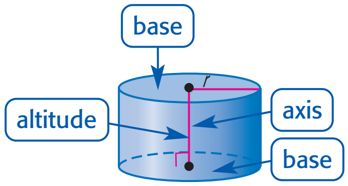
Main Ideas

- Find lateral areas of cylinders.
- Find surface areas of cylinders.

New Vocabulary

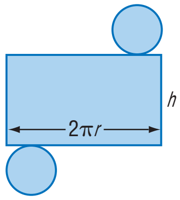
axis
right cylinder

Lateral Areas of Cylinders The **axis** of the cylinder is the segment with endpoints that are centers of the circular bases. If the axis is also the altitude, then the cylinder is called a **right cylinder**.



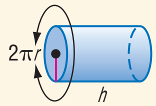
right cylinder

The net of a cylinder is composed of two congruent circles and a rectangle. The area of this rectangle is the lateral area. The length of the rectangle is the same as the circumference of the base, $2\pi r$. So, the lateral area of a right cylinder is $2\pi rh$.



KEY CONCEPT — Lateral Area of a Cylinder

If a right cylinder has a lateral area of L square units, a height of h units, and the bases have radii of r units, then $L = 2\pi rh$.



Study Tip

Formulas

An alternate formula for the lateral area of a cylinder is $L = \pi dh$, with πd as the circumference of a circle.

1 **MANUFACTURING** **An office has recycling barrels that are cylindrical with cardboard sides and plastic lids and bases. Each barrel is 3 feet tall, and the diameter is 30 inches. How many square feet of cardboard are used to make each barrel?**

The cardboard part represents the lateral area of a cylinder.

$L = 2\pi rh$		Lateral area of a cylinder
	$= 2\pi(15)(36)$	$r = \frac{30}{2}$ or 15, $h = 3 \cdot 12$ or 36
	≈ 3392.9	Use a calculator.

Each barrel uses approximately 3393 square inches of cardboard. Because 144 square inches equal one square foot, there are $3393 \div 144$ or about 23.6 square feet of cardboard per barrel.

Extra Examples at geometryonline.com

Study Tip

Estimation

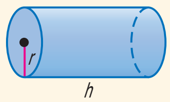
Before solving for the lateral area, use mental math to estimate the answer. To estimate the lateral area, mulitply the diameter by 3 (to appoximate π) by the height of the cylinder.

1. **CARS** Matt is buying new tire rims that are 14 inches in diameter and 6 inches wide. Determine the lateral area of each rim.

Surface Areas of Cylinders To find the surface area of a cylinder, first find the lateral area and then add the areas of the bases.

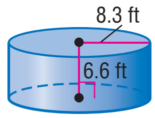
KEY CONCEPT — *Surface Area of a Cylinder*

If a right cylinder has a surface area of T square units, a height of h units, and the bases have radii of r units, then $T = 2\pi rh + 2\pi r^2$.



EXAMPLE 2 Surface Area of a Cylinder

Find the surface area of the cylinder.



$T = 2\pi rh + 2\pi r^2$	Surface area of a cylinder
$= 2\pi(8.3)(6.6) + 2\pi(8.3)^2$	$r = 8.3, h = 6.6$
≈ 777.0	Use a calculator.

The surface area is approximately 777.0 square feet.

Study Tip

Making Connections

The formula for the surface area of a right cylinder is like that of a prism, $T = L + 2B$.

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

2A. $d = 6$ cm, $h = 11$ cm

2B. $r = 5$ in., $h = 9$ in.

Personal Tutor at geometryonline.com

EXAMPLE 3 Find Missing Dimensions

Find the radius of the base of a right cylinder if the surface area is 128π square centimeters and the height is 12 centimeters.

Use the formula for surface area to write and solve an equation for the radius.

$T = 2\pi rh + 2\pi r^2$	Surface area of a cylinder
$128\pi = 2\pi(12)r + 2\pi r^2$	Substitution
$128\pi = 24\pi r + 2\pi r^2$	Simplify.
$64 = 12r + r^2$	Divide each side by 2π.
$0 = r^2 + 12r - 64$	Subtract 64 from each side.
$0 = (r - 4)(r + 16)$	Factor.
$r = 4$ or -16	

Since the radius of a circle cannot have a negative value, -16 is eliminated. So, the radius of the base is 4 centimeters.

3. Find the diameter of a base of a right cylinder if the surface area is 464π square inches and the height is 21 inches.

Check Your Understanding

Example 1 (p. 693)

1. **ART PROJECTS** Mrs. Fairway's class is collecting labels from soup cans for an art project. The students collected labels from 3258 cans. If the cans are 4 inches high with a diameter of 2.5 inches, find the total area of the labels.

Example 2 (p. 694)

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

2. $r = 4$ ft, $h = 6$ ft

3. $d = 22$ m, $h = 11$ m

Example 3 (p. 694)

4. The surface area of a cylinder is 96π square centimeters, and its height is 8 centimeters. Find its radius.

5. The surface area of a cylinder is 140π square feet, and its height is 9 feet. Find its diameter.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
6–13	2
14–17	3
18–22	1

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

6. $r = 13$ m, $h = 15.8$ m

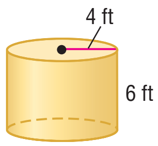
7. $d = 13.6$ ft, $h = 1.9$ ft

8. $d = 14.2$ in., $h = 4.5$ in.

9. $r = 14$ mm, $h = 14$ mm

Find the surface area of each cylinder. Round to the nearest tenth.

10.



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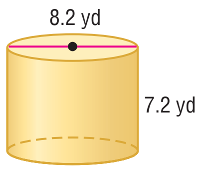
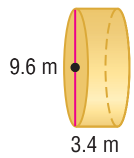


12.

13.



Find the radius of the base of each cylinder.

14. The surface area is 48π square centimeters, and the height is 5 centimeters.

15. The surface area is 340π square inches, and the height is 7 inches.

Find the diameter of the base of each cylinder.

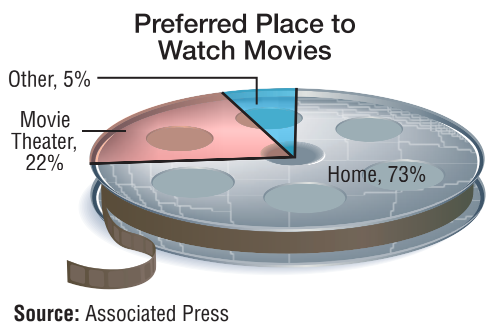
16. The surface area is 320π square meters, and the height is 12 meters.

17. The surface area is 425.1 square feet, and the height is 6.8 feet.

ENTERTAINMENT **For Exercises 18 and 19, use the graphic at the right.**

18. Suppose the film can in the graphic is a cylinder. Explain how to find the surface area of the portion that represents people who prefer to watch movies at home.

19. If the film can is 12 inches in diameter and 3 inches tall, find the surface area of the portion in Exercise 18.



Source: Associated Press

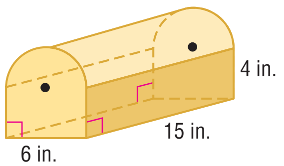
20. **LAMPS** This lamp shade is a cylinder of height 18 inches with a diameter of $6\frac{3}{4}$ inches. What is the lateral area of the shade to the nearest tenth?

21. **WORLD RECORDS** The largest beverage can made was displayed in Taiwan in 2002. The can was a cylinder with a height of 4.67 meters and a diameter of 2.32 meters. What was the surface area of the can to the nearest tenth?

22. **KITCHENS** Raul purchased a set of cylindrical canisters with diameters of 5 inches and heights of 9 inches, 6 inches, and 3 inches. Make a conjecture about the relationship between the heights of the canisters and their lateral areas. Check your conjecture.

Find the surface areas of the composite solids. Round to the nearest tenth.

23.



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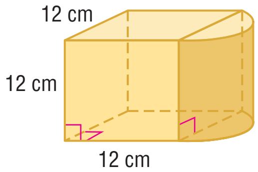
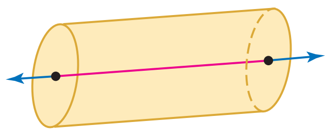


EXTRA PRACTICE
See pages 824, 839.
Math Online
Self-Check Quiz at
geometryonline.com

LOCUS A cylinder can be defined in terms of locus. The locus of points in space a given distance from a line is the lateral surface of a cylinder. Draw a figure and describe the locus of all points in space that satisfy each set of conditions.



25. 5 units from a given line

26. equidistant from two opposite vertices of a face of a cube

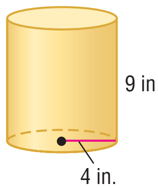
H.O.T. Problems

27. **REASONING** Compare and contrast finding the surface areas of a prism and a cylinder.

28. **OPEN ENDED** Draw a net of a cylinder that is different from the one on page 693.

29. **FIND THE ERROR** Jamie and Dwayne are finding the surface area of a cylinder with one base. Who is correct? Explain.

Jamie	Dwayne
$T = 2\pi(4)(9) + \pi(4^2)$	$T = 2\pi(4)(9) + 2\pi(4^2)$
$= 72\pi + 16\pi$	$= 72\pi + 32\pi$
$= 88\pi \text{ in}^2$	$= 104\pi \text{ in}^2$

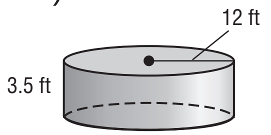


30. **CHALLENGE** Some pencils are cylindrical, and others are hexagonal prisms. If the diameter of the cylinder is the same length as the longest diagonal of the hexagon, which has the greater surface area? Explain. Assume that each pencil is 11 inches long and unsharpened.

31. **Writing in Math** Refer to the information on skateboarding on page 693. Explain how to find the lateral area of the interior of the full-pipe.

STANDARDIZED TEST PRACTICE

32. Isabel has a swimming pool that is shaped like a cylinder. She wants to get a plastic sheet to keep the side of the pool from getting scratched. What is the area of the sheet? (Use 3.14 for π.)



A 263.76 ft^2

B 527.52 ft^2

C 1334.76 ft^2

D 1582.56 ft^2

33. REVIEW For the band concert, student tickets cost \$2, and adult tickets cost \$5. A total of 200 tickets were sold. If the total sales were more than \$500, what was the minimum number of adult tickets sold?

F 30

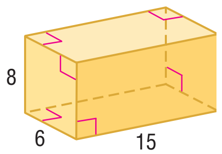
G 33

H 34

J 40

Find the lateral area of each prism. (Lesson 12-2)

34.



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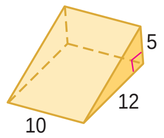

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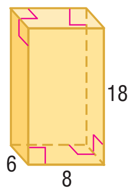
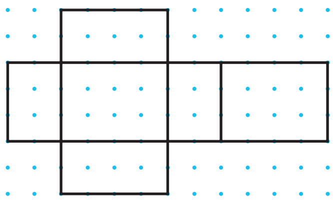


Given the net of a solid, use isometric dot paper to draw the solid. (Lesson 12-1)

37.



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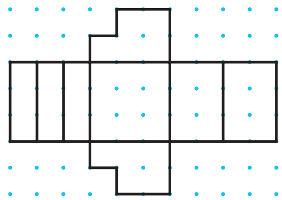
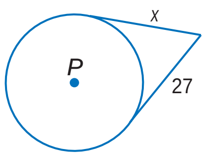


Find x. Assume that segments that appear to be tangent are tangent. (Lesson 10-5)

39.



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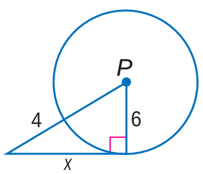

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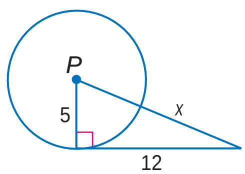
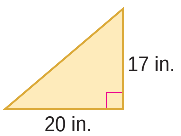


42. ART Kiernan drew a sketch of a house. If the height of the house in her drawing was 5.5 inches and the actual height of the house was 33 feet, find the scale factor of the drawing. (Lesson 7-1)

GET READY for the Next Lesson

PREREQUISITE SKILL **Find the area of each figure.** (Lesson 11-2)

43.



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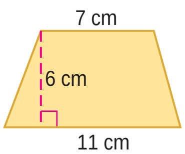
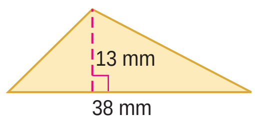


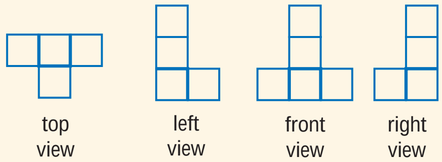
45.



CHAPTER 12 Mid-Chapter Quiz

Lessons 12-1 through 12-3

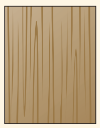
1. Draw a corner view of the figure given the orthographic drawing (Lesson 12-1)

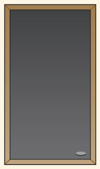


2. Sketch a rectangular prism 2 units wide, 3 units long, and 2 units high using isometric dot paper. (Lesson 12-1)

For Exercises 3 and 4, use the following information. (Lesson 12-1)

The top and front views of a speaker for a stereo system are shown.


Top View


Front View

3. Is it possible to determine the shape of the speaker? Explain.
4. Describe possible shapes for the speaker. Draw the left and right views of one of the possible shapes.

TOURISM For Exercises 5–7, use the following information. (Lesson 12-2)

The World's Only Corn Palace is located in Mitchell, South Dakota. The sides of the building are covered with huge murals made from corn and other grains.

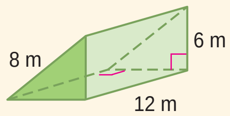
5. Estimate the area of the Corn Palace to be covered if its base is 310 by 185 feet and it is 45 feet tall, not including the turrets.
6. Suppose a bushel of grain can cover 15 square feet. How many bushels of grain does it take to cover the Corn Palace?
7. Will the actual amount of grain needed be higher or lower than the estimate? Explain.

8. **MULTIPLE CHOICE** The surface area of a cube is 121.5 square meters. What is the length of each edge? (Lesson 12-2)

A 4.05 m **C** 4.95 m

B 4.5 m **D** 5 m

For Exercises 9 and 10, use the prism pictured below.

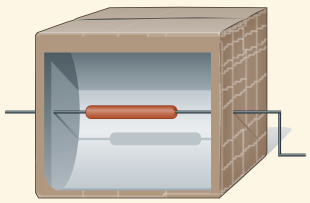


9. Find the lateral area of the prism. Round to the nearest tenth. (Lesson 12-2)
10. Find the surface area of the prism. Round to the nearest tenth. (Lesson 12-2)

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth. (Lesson 12-3)

11. $r = 11$ cm, $h = 9.5$ cm
12. $d = 8.3$ ft, $h = 4.5$ ft
13. $r = 5.7$ m, $h = 3.6$ m
14. $d = 10.1$ in., $h = 12.2$ in.

15. **MULTIPLE CHOICE** Campers can use a solar cooker to cook food. You can make a solar cooker from supplies you have on hand. The reflector in the cooker shown is half of a cardboard cylinder covered with aluminum foil. (Lesson 12-3)



The reflector is 18 inches long and has a diameter of $5\frac{1}{2}$ inches. How much aluminum foil was needed to cover the inside of the reflector? Round to the nearest tenth.

F 155.9 in^2 **H** 170.8 in^2

G 163.4 in^2 **J** 179.3 in^2

12-4 Surface Areas of Pyramids

Main Ideas

- Find lateral areas of regular pyramids.
- Find surface areas of regular pyramids.

New Vocabulary

regular pyramid
slant height

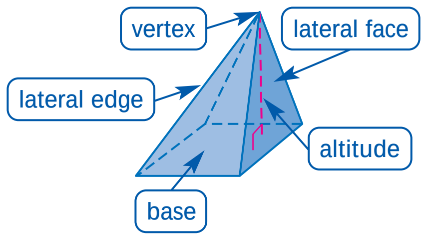
GET READY for the Lesson

In 1989, a new entrance was completed in the courtyard of the Louvre museum in Paris, France. Visitors can enter the museum through a glass pyramid that stands 71 feet tall. The pyramid is glass with a structural system of steel rods and cables.

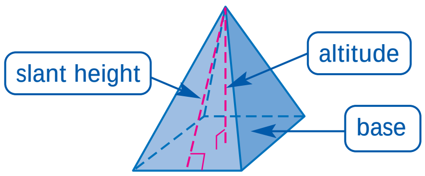
Lateral Areas of Regular Pyramids Pyramids have the following characteristics.

- All of the faces, except the base, intersect at one point called the *vertex*.
- The base is always a polygon.
- The faces that intersect at the vertex are called *lateral faces* and are triangles. The edges of the lateral faces that have the vertex as an endpoint are called *lateral edges*.
- The *altitude* is the segment from the vertex perpendicular to the base.

If the base of a pyramid is a regular polygon and the segment with endpoints that are the center of the base and the vertex is perpendicular to the base, then the pyramid is called a **regular pyramid**. They have specific characteristics. The altitude is the segment with endpoints that are the center of the base and the vertex. All of the lateral faces are congruent isosceles triangles. The height of each lateral face is called the **slant height** ℓ of the pyramid.

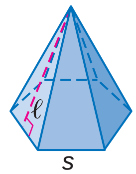


square pyramid



regular square pyramid

The figure at the right is a regular hexagonal pyramid. Its lateral area L can be found by adding the areas of all its congruent triangular faces as shown in its net.

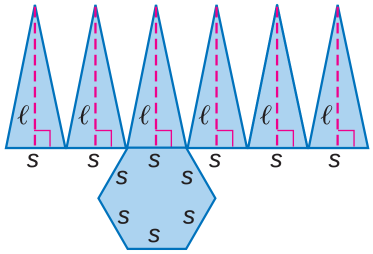


Study Tip

Right Pyramid

In a *right pyramid,* the altitude is the segment with endpoints that are the center of the base and the vertex. But the base is not always a regular polygon.

First Image



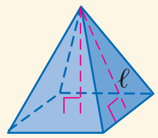
$$L = \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell$$ Sum of the areas of the lateral faces

$$= \frac{1}{2}\ell(s + s + s + s + s + s)$$ Distributive Property

$$= \frac{1}{2}P\ell$$ $P = s + s + s + s + s + s$

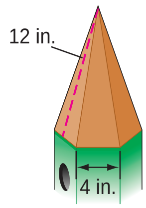
KEY CONCEPT *Lateral Area of a Regular Pyramid*

If a regular pyramid has a lateral area of L square units, a slant height of ℓ units, and its base has a perimeter of P units, then $L = \frac{1}{2}P\ell$.



EXAMPLE Find Lateral Area

1 **BIRDHOUSES The roof of a birdhouse is a regular hexagonal pyramid. The base of the pyramid has sides of 4 inches, and the slant height of the roof is 12 inches. If the roof is made of copper, find the amount of copper used for the roof.**



We need to find the lateral area of the hexagonal pyramid. The sides of the base measure 4 inches, so the perimeter is 6(4) or 24 inches.

$L = \frac{1}{2}P\ell$ Lateral area of a regular pyramid

$= \frac{1}{2}(24)(12)$ $P = 24, \ell = 12$

$= 144$ Multiply.

So, 144 square inches of copper are used to cover the roof of the birdhouse.

1. **ARCHITECTURE** Find the lateral area of a pyramid-shaped building that has a slant height of 210 feet and a square base with dimensions 332 feet by 322 feet. Round to the nearest tenth.

Study Tip

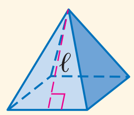
Making Connections

The total surface area for a pyramid is $L + B$, because there is only one base to consider.

Surface Areas of Regular Pyramids The surface area of a regular pyramid is the sum of the lateral area and the area of the base.

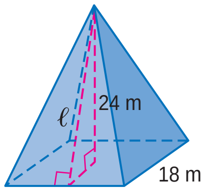
KEY CONCEPT *Surface Area of a Regular Pyramid*

If a regular pyramid has a surface area of T square units, a slant height of ℓ units, and its base has a perimeter of P units and an area of B square units, then $T = \frac{1}{2}P\ell + B$.



EXAMPLE Surface Area of a Square Pyramid

2 **Find the surface area of the square pyramid.**



To find the surface area, first find the slant height of the pyramid. The slant height is the hypotenuse of a right triangle with legs that are the altitude and a segment with a length that is one-half the side measure of the base.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$\ell^2 = 9^2 + 24^2$	$a = 9, b = 24, c = \ell$
$\ell = \sqrt{657}$	Simplify.

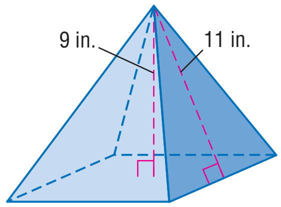
Now find the surface area of a regular pyramid. The perimeter of the base is 4(18) or 72 meters, and the area of the base is 18^2 or 324 square meters.

$T = \frac{1}{2}P\ell + B$	Surface area of a regular pyramid
$= \frac{1}{2}(72)\sqrt{657} + 324$	$P = 72, \ell = \sqrt{657}, B = 324$
≈ 1246.8	Use a calculator.

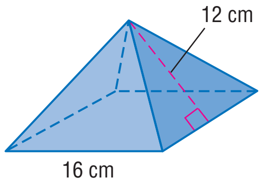
The surface area is 1246.8 square meters to the nearest tenth.

Find the surface area of each square pyramid.

2A.

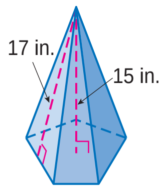


2B.



EXAMPLE Surface Area of a Regular Pyramid

3 **Find the surface area of the regular pyramid.**



The altitude, slant height, and apothem form a right triangle. Use the Pythagorean Theorem to find the apothem. Let a represent the length of the apothem.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$(17)^2 = a^2 + 15^2$	$b = 15, c = 17$
$8 = a$	Simplify.

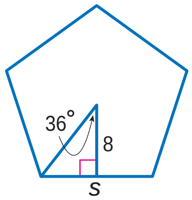
Now find the length of the sides of the base. The central angle of the pentagon measures $\frac{360°}{5}$ or 72°. Let x represent the measure of the angle formed by a radius and the apothem. Then, $x = \frac{72}{2}$ or 36.

(continued on the next page)

Cross-Curricular Project

Making a sketch of a pyramid can help you find its slant height, lateral area, and base area. Visit geometryonline.com to continue work on your project.

Use trigonometry to find the length of the sides.



$\tan 36° = \dfrac{\frac{1}{2}s}{8}$ $\quad \tan x° = \dfrac{\text{opposite}}{\text{adjacent}}$

$8(\tan 36°) = \frac{1}{2}s$ $\quad$ Multiply each side by 8.

$16(\tan 36°) = s$ $\quad$ Multiply each side by 2.

$11.6 \approx s$ $\quad$ Use a calculator.

Next, find the perimeter and area of the base.

$P = 5s$ $\quad$ Perimeter of a regular pentagon

$\approx 5(11.6)$ or 58 $\quad s \approx 11.6$

$B = \frac{1}{2}Pa$ $\quad$ Area of a regular pentagon

$\approx \frac{1}{2}(58)(8)$ or 232 $\quad P = 58, a = 8$

Study Tip

Look Back

To review **finding the areas of regular polygons,** see Lesson 11-3. To review **trigonometric ratios,** see Lesson 7-4.

Finally, find the surface area.

$T = \frac{1}{2}P\ell + B$ $\quad$ Surface area of a regular pyramid

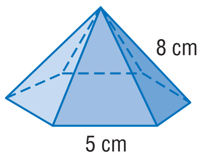
$\approx \frac{1}{2}(58)(17) + 232$ $\quad P \approx 58, \ell = 17, B \approx 232$

≈ 726.5 $\quad$ Simplify.

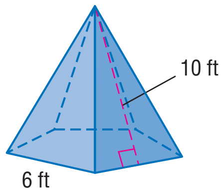
The surface area is approximately 726.5 square inches.

Find the surface area of each regular pyramid.

3A.



3B.



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Check Your Understanding

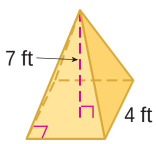
Example 1 (p. 700)

1. DECORATIONS Kata purchased 3 decorative three-dimensional stars. Each star is composed of 6 congruent square pyramids with faces of paper and a base of cardboard. If the base is 2 inches on each side and the slant height is 4 inches, find the amount of paper used for one star.

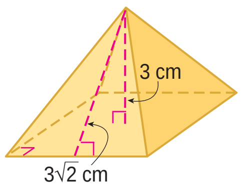
Examples 2 and 3 (pp. 701–702)

Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

2.



3.



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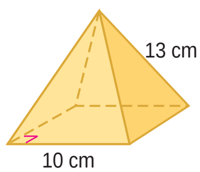
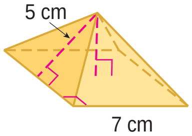
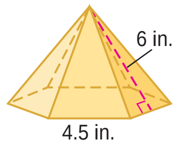


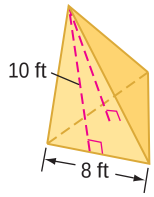
Exercises

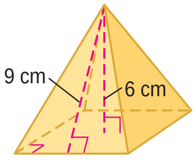
HOMEWORK HELP	
For Exercises	See Examples
5, 7, 8, 12, 13	2
6, 9–11	3
14–17	1

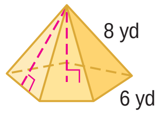
Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

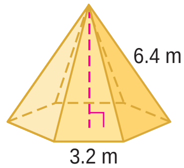
5. 

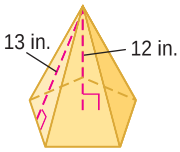
6. 

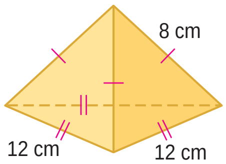
7. 

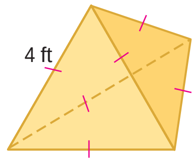
8. 

9. 

10. 

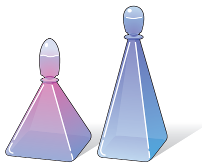
11. 

12. 

13. 

14. **CONSTRUCTION** The roof on a building is a square pyramid with no base. If the altitude of the pyramid measures 5 feet and the slant height measures 20 feet, find the area of the roof.

15. **PERFUME BOTTLES** Some perfumes are packaged in square pyramidal containers. The base of one bottle is 3 inches square, and the slant height is 4 inches. A second bottle has the same surface area, but the slant height is 6 inches long. Find the dimensions of the base of the second bottle.



16. **STADIUMS** The Pyramid Arena in Memphis, Tennessee, is the third largest pyramid in the world. The base is 360,000 square feet, and the pyramid is 321 feet tall. Find the lateral area of the pyramid. (Assume that the base is a square.)

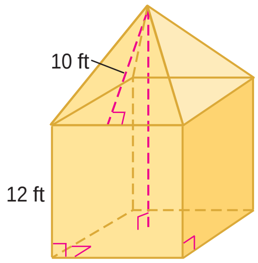
17. **HISTORY** Each side of the square base of Khafre's Pyramid in Egypt is 214.5 meters. The sides rise at an angle of about 53°. Find the lateral area of the pyramid.

Real-World Link

Egyptologists believe that the Great Pyramids of Egypt were originally covered with white limestone that has worn away or been removed.

Source: www.pbs.org

For Exercises 18–21, use the following information.
This solid is a composite of a cube and square pyramid. The base of the solid is the base of the cube. Find the indicated measurements for the solid.



18. Find the height.

19. Find the lateral area.

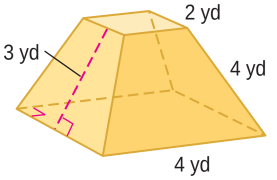
20. Find the surface area.

21. Which has the greater lateral area: the pyramid or the cube? Explain.

EXTRA PRACTICE
See pages 824, 839.

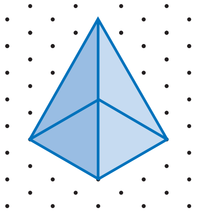
Math Online
Self-Check Quiz at
geometryonline.com

22. A **frustum** is the part of a solid that remains after the top portion has been cut by a plane parallel to the base. Find the lateral area of the frustum of a regular pyramid.



H.O.T. Problems

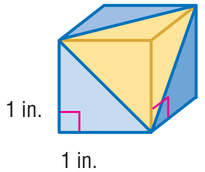
23. **REASONING** Refer to the isometric view of a square pyramid shown at the right. Draw a net of the square pyramid, and then make a concrete model. Find the surface area of your model.



24. **REASONING** Explain whether a regular pyramid can also be a regular polyhedron.

25. **OPEN ENDED** Draw a regular pyramid and a pyramid that is not regular. Explain the difference between the two.

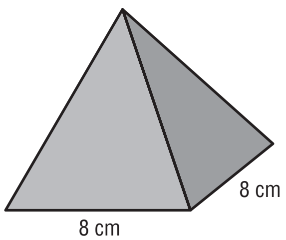
26. **CHALLENGE** This square prism measures 1 inch on each side. The corner of the cube is cut off, or truncated as shown. Does this change the surface area of the cube? Include the surface area of the original cube and that of the truncated cube in your answer.



27. **Writing in Math** Explain the information needed to find the lateral area and surface area of a pyramid. Include another example of pyramidal shapes used in architecture.

STANDARDIZED TEST PRACTICE

28. At a party, guests will be given 8-centimeter tall, pyramid-shaped boxes like the one below.



Ignoring overlap, what is the amount of cardboard needed to create each box, in square centimeters?

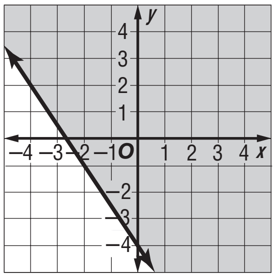
A $64 + 16\sqrt{5}$

B $64 + 64\sqrt{5}$

C $64 + 32\sqrt{5}$

D $64 + 128\sqrt{3}$

29. **REVIEW** Which inequality *best* describes the graph shown below?



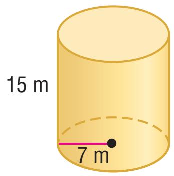
F $y \geq -4x - 2\frac{1}{2}$

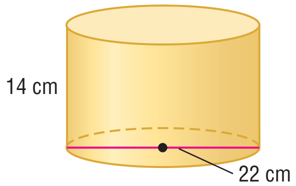
G $y \leq -\frac{3}{2}x - 4$

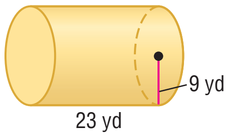
H $y \geq -\frac{3}{2}x - 4$

J $y \leq 4x + 2\frac{1}{2}$

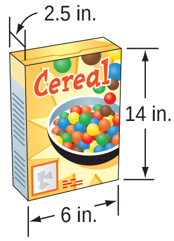
Find the surface area of each cylinder. Round to the nearest tenth. (Lesson 12-3)

30. 

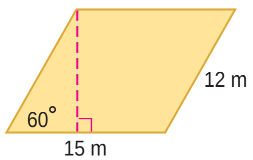
31. 

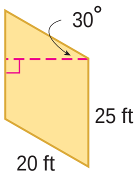
32. 

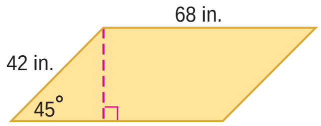
33. FOOD Most cereals are packaged in cardboard boxes. If a box of cereal is 14 inches high, 6 inches wide, and 2.5 inches deep, find the surface area of the box. (Lesson 12-2)



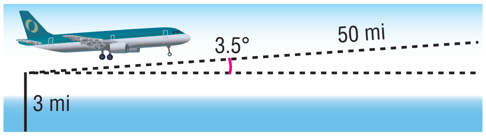
Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary. (Lesson 11-1)

34. 

35. 

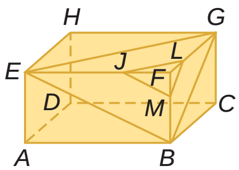
36. 

37. NAVIGATION An airplane is three miles above sea level when it begins to climb at a 3.5° angle. If this angle is constant, how far above sea level is the airplane after flying 50 miles? (Lesson 8-4)



Use the figure at the right to write a paragraph proof. (Lesson 7-3)

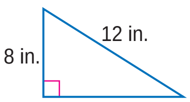
38. Given: $\triangle JFM \sim \triangle EFB$
$\triangle LFM \sim \triangle GFB$
Prove: $\triangle JFL \sim \triangle EFG$

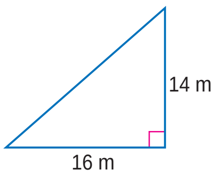


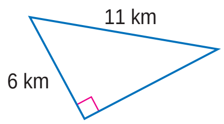
39. Given: $\overline{JM} \parallel \overline{EB}$
$\overline{LM} \parallel \overline{GB}$
Prove: $\overline{JL} \parallel \overline{EG}$

GET READY for the Next Lesson

PREREQUISITE SKILL **Solve for the missing length in each triangle. Round to the nearest tenth.** (Lesson 8-2)

40. 

41. 

42. 

12-5 Surface Areas of Cones

Main Ideas

- Find lateral areas of cones.
- Find surface areas of cones.

New Vocabulary

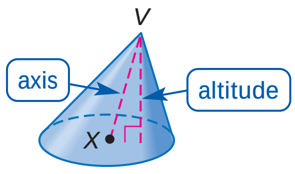
circular cone
right cone
oblique cone

GET READY for the Lesson

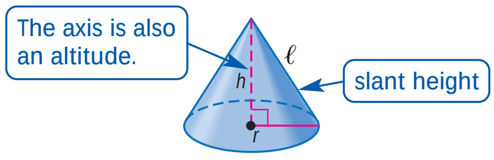
Native American tribes on the Great Plains typically lived in tepees, or tipis (TEE peez). Tent poles were arranged in a conical shape, and animal skins were stretched over the frame for shelter. The top of a tepee was left open for smoke to escape.

Lateral Areas of Cones The shape of a tepee suggests a **circular cone**. Cones have the following characteristics.

- The base is a circle and the vertex is the point V.
- The *axis* is the segment with endpoints that are the vertex and the center of the base.
- The segment that has the vertex as one endpoint and is perpendicular to the base is called the *altitude* of the cone.



oblique cone



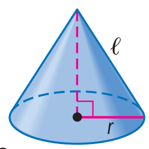
right cone

Reading Math

Cones From this point in the text, you can assume that cones are right circular cones. If the cone is oblique, it will be clearly stated.

A cone with an axis that is also an altitude is called a **right cone**. Otherwise, it is called an **oblique cone**. The measure of any segment joining the vertex of a right cone to the edge of the circular base is called the *slant height*, ℓ. The measure of the altitude is the height h of the cone.

We can use the net for a cone to derive the formula for the lateral area of a cone. The lateral region of the cone is a sector of a circle with radius ℓ, the slant height of the cone. The arc length of the sector is the same as the circumference of the base, or $2\pi r$. The circumference of the circle containing the sector is $2\pi\ell$. The area of the sector is proportional to the area of the circle.

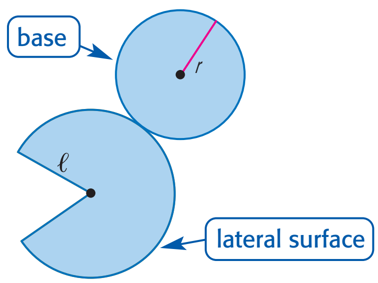


$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{measure of arc}}{\text{circumference of circle}}$$

$$\frac{\text{area of sector}}{\pi\ell^2} = \frac{2\pi r}{2\pi\ell}$$

$$\text{area of sector} = \frac{(\pi\ell^2)(2\pi r)}{2\pi\ell}$$

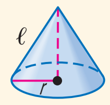
$$\text{area of sector} = \pi r\ell$$



This derivation leads to the formula for the lateral area of a right circular cone.

KEY CONCEPT *Lateral Area of a Cone*

If a right circular cone has a lateral area of L square units, a slant height of ℓ units, and the radius of the base is r units, then $L = \pi r \ell$.



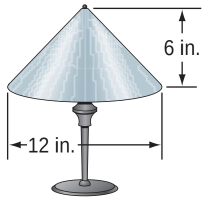
Study Tip

Storing Values in Calculator Memory

You can store the calculated value of ℓ by [√] 72 [STO▸] [ALPHA] [L]. To find the lateral area, use [2nd] [π] [×] 6 [×] [ALPHA] [L] [ENTER].

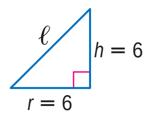
EXAMPLE **Lateral Area of a Cone**

1 LAMPS **Diego has a conical lampshade with an altitude of 6 inches and a diameter of 12 inches. Find the lateral area of the lampshade.**



Explore We are given the altitude and the diameter of the base. We need to find the slant height of the cone.

Plan The radius of the base, height, and slant height form a right triangle. Use the Pythagorean Theorem to solve for the slant height. Then use the formula for the lateral area of a right circular cone.



Solve Write an equation and solve for ℓ.

$\ell^2 = 6^2 + 6^2$ Pythagorean Theorem

$\ell^2 = 72$ Simplify.

$\ell = \sqrt{72}$ or $6\sqrt{2}$ Take the square root of each side.

Next, use the formula for the lateral area of a right circular cone.

$L = \pi r \ell$ Lateral area of a cone

$\approx \pi(6)(6\sqrt{2})$ $r = 6$, $\ell = 6\sqrt{2}$

≈ 159.9 Use a calculator.

The lateral area is approximately 159.9 square inches.

Check Use estimation to check the reasonableness of this result. The lateral area is approximately $3 \cdot 6 \cdot 9$ or 162 square inches. Compared to the estimate, the answer is reasonable.

1. ICE CREAM An ice cream shop makes their own waffle cones. If a cone is 5.5 inches tall and the diameter of the base is 2.5 inches, find the lateral area of the cone.

Personal Tutor at geometryonline.com

Surface Areas of Cones To find the surface area of a cone, add the area of the base to the lateral area.

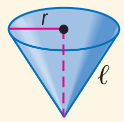
Study Tip

Making Connections

The surface area of a cone is like the surface area of a pyramid, $T = L + B$.

KEY CONCEPT — *Surface Area of a Cone*

If a right circular cone has a surface area of T square units, a slant height of ℓ units, and the radius of the base is r units, then $T = \pi r\ell + \pi r^2$.



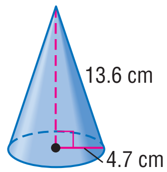
EXAMPLE Surface Area of a Cone

2 **Find the surface area of the cone.**

$T = \pi r\ell + \pi r^2$ — Surface area of a cone

$= \pi(4.7)(13.6) + \pi(4.7)^2$ — $r = 4.7$, $\ell = 13.6$

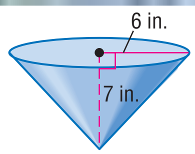
≈ 270.2 — Use a calculator.



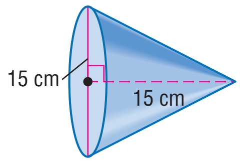
The surface area is approximately 270.2 square centimeters.

Find the surface area of each cone.

2A.



2B.

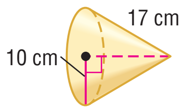


CHECK Your Understanding

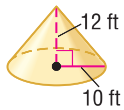
Example 1 (p. 707)

Find the surface area of each cone. Round to the nearest tenth.

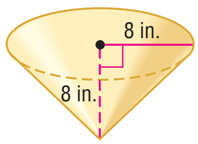
1.



2.



3.



Example 2 (p. 708)

4. **ROAD SALT** Many states use a cone structure to store salt used to melt snow on highways and roads. Find the lateral area of one of these cone structures if the building measures 24 feet tall and the diameter of the base is 45 feet.

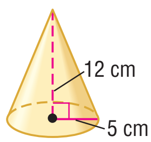
Exercises

HOMEWORK HELP

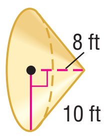
For Exercises	See Examples
5–14	2
15–17	1

Find the surface area of each cone. Round to the nearest tenth.

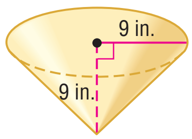
5.



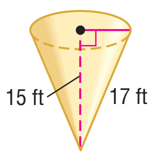
6.



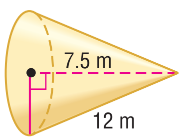
7.



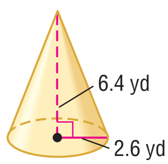
8.



9.



10.



For Exercises 11–14, round to the nearest tenth.

11. Find the surface area of a cone if the height is 16 inches and the slant height is 18 inches.

12. Find the surface area of a cone if the height is 8.7 meters and the slant height is 19.1 meters.

13. The surface area of a cone is 1020 square meters and the radius is 14.5 meters. Find the slant height.

14. The surface area of a cone is 293.2 square feet and the radius is 6.1 feet. Find the slant height.

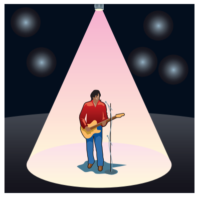
Real-World Link

The Saamis Tepee in Medicine Hat, Alberta, Canada, is the world's largest tepee. The frame is made of steel, instead of wood. It weighs 200 tons.

Source: www.medicinehatchamber.com

15. PARTY HATS Shelley plans to make eight conical party hats for her niece's birthday. If each hat is to be 18 inches tall and the bases of each to be 22 inches in circumference, how much material will she use to make the hats?

16. SPOTLIGHTS A spotlight was positioned directly above a performer. If the lateral area of the cone of light was approximately 500 square feet and the slant height was 20 feet, find the diameter of light on stage.



17. TEPEES A rectangular piece of canvas 50 feet by 60 feet is available to cover a tepee. The diameter of the base is 42 feet, and the slant height is 47.9 feet. Is there enough canvas to cover the tepee? Explain.

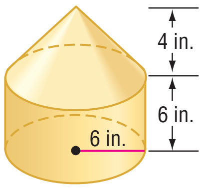
Find the radius of a cone given the surface area and slant height. Round to the nearest tenth.

18. $T = 359 \text{ ft}^2$, $\ell = 15$ ft

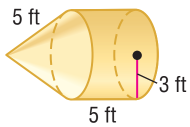
19. $T = 523 \text{ m}^2$, $\ell = 12.1$ m

Find the surface area of each composite solid. Round to the nearest tenth.

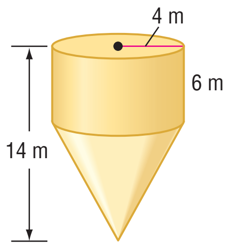
20.



21.



22.



The height of a cone is 7 inches, and the radius is 4 inches. Round final answers to the nearest ten-thousandth.

23. Find the lateral area of the cone using the store feature of a calculator.

24. Round the slant height to the nearest tenth and then calculate the lateral area of the cone.

25. Round the slant height to the nearest hundredth and then calculate the lateral area of the cone.

26. Compare the lateral areas for Exercises 23–25. Which is most accurate? Explain.

Determine whether each statement is *sometimes, always,* or *never* true. Explain.

27. If the diagonal of the base of a square pyramid is equal to the diameter of the base of a cone and the heights of both solids are equal, then the pyramid and cone have equal lateral areas.

28. The ratio of the radii of the bases of two cones is equal to the ratio of the surface areas of the cones.

EXTRA PRACTICE

See pages 825, 839.

Math Online

Self-Check Quiz at geometryonline.com

H.O.T. Problems

29. OPEN ENDED Draw an oblique cone with a base area greater than 10 square centimeters. Mark the vertex and the center of the base.

30. REASONING Explain why the formula for the lateral area of a right circular cone does not apply to oblique cones.

31. CHALLENGE If you were to move the vertex of a right cone down the axis toward the center of the base, explain what would happen to the lateral area and surface area of the cone.

32. Writing in Math Explain why the lateral area of a cone is used to cover tepees. Include information needed to find the lateral area of the canvas covering, and how the open top of a tepee affects the lateral area of the canvas covering it.

STANDARDIZED TEST PRACTICE

33. The Fun Times For All Company is constructing a conical tent for a festival. If the radius of the base is 6 feet and the slant height is 10 feet, what is the lateral area of the cone?

A 48π **C** $12\pi\sqrt{34}$

B 60π **D** 384π

34. REVIEW What is the x-coordinate of the solution of the system of equations below?

$$-4x + 6y = 24$$
$$3x - \frac{7}{5}y = -\frac{5}{2}$$

F 12.4 **H** 5

G 6 **J** 1.5

Spiral Review

35. ARCHITECTURE The Transamerica Tower in San Francisco is a regular pyramid with a square base that is 149 feet on each side and a height of 853 feet. Find its lateral area. (Lesson 12-4)

Find the radius of the base of the right cylinder. Round to the nearest tenth. (Lesson 12-3)

36. The surface area is 563 square feet, and the height is 9.5 feet.

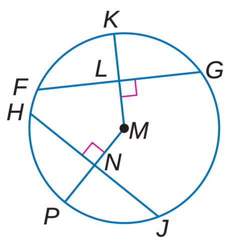
37. The surface area is 185 square meters, and the height is 11 meters.

In $\odot M$, $FL = 24$, $HJ = 48$, and $m\overset{\frown}{HP} = 45$. Find each measure. (Lesson 10-3)

38. FG **39.** NJ

40. HN **41.** LG

42. $m\overset{\frown}{PJ}$ **43.** $m\overset{\frown}{HJ}$



GET READY for the Next Lesson

PREREQUISITE SKILL **Find the circumference of each circle given the radius or the diameter. Round to the nearest tenth.** (Lesson 10-1)

44. $r = 6$ **45.** $d = 8$ **46.** $d = 18$ **47.** $r = 8.2$

12-6 Surface Areas of Spheres

Main Ideas

- Recognize and define basic properties of spheres.
- Find surface areas of spheres.

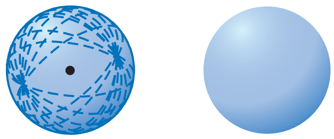
New Vocabulary

great circle
hemisphere

GET READY for the Lesson

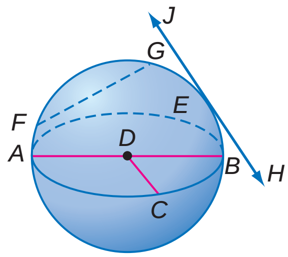
This soccer ball globe was designed and constructed for the 2006 World Cup soccer tournament. It is 66 feet in diameter. During the day the structure looks like a soccer ball. Through colored lighting effects, the structure looks like a globe at night.

Properties of Spheres To visualize a sphere, such as a soccer ball, consider infinitely many congruent circles in space, all with the same point for their center. Considered together, these circles form a sphere. In space, a sphere is the locus of all points that are a given distance from a given point called its *center*.



There are several special segments and lines related to spheres.

- A segment with endpoints that are the center of the sphere and a point on the sphere is a **radius** of the sphere. In the figure, $\overline{DC}$, $\overline{DA}$, and $\overline{DB}$ are radii.
- A **chord** of a sphere is a segment with endpoints that are points on the sphere. In the figure, $\overline{GF}$ and $\overline{AB}$ are chords.
- A chord that contains the center of the sphere is a **diameter** of the sphere. In the figure, $\overline{AB}$ is a diameter.
- A **tangent** to a sphere is a line that intersects the sphere in exactly one point. In the figure, $\overleftrightarrow{JH}$ is tangent to the sphere at E.

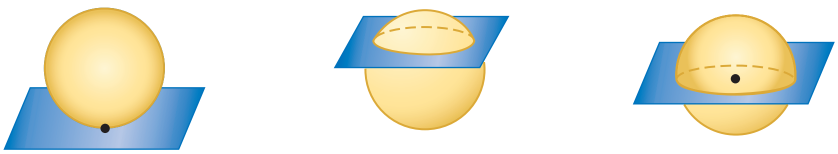


Study Tip

Circles and Spheres

The shortest distance between any two points on a sphere is the length of the arc of a great circle passing through those two points.

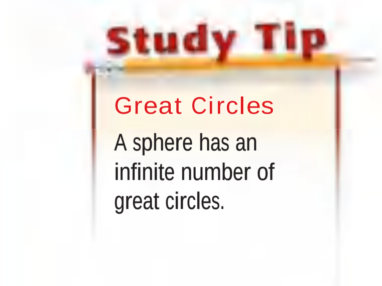
The intersection of a plane and a sphere can be a point or a circle. When a plane intersects a sphere so that it contains the center of the sphere, the intersection is called a **great circle**. A great circle has the same center as the sphere, and its radii are also radii of the sphere.



a point

a circle

a great circle

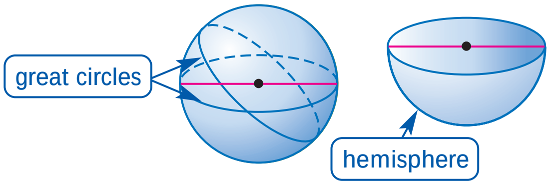


Study Tip

Great Circles

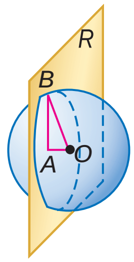
A sphere has an infinite number of great circles.

Each great circle separates a sphere into two congruent halves, each called a **hemisphere**. Note that a hemisphere has a circular base.



EXAMPLE Spheres and Circles

1 **In the figure, *O* is the center of the sphere, and plane R intersects the sphere in circle *A*. If *AO* = 3 centimeters and *OB* = 10 centimeters, find *AB*.**



The radius of circle A is the segment $\overline{AB}$, B is a point on circle A and on sphere O. Use the Pythagorean Theorem for right triangle ABO to solve for AB.

$OB^2 = AB^2 + AO^2$	Pythagorean Theorem
$10^2 = AB^2 + 3^2$	$OB = 10$, $AO = 3$
$100 = AB^2 + 9$	Simplify.
$91 = AB^2$	Subtract 9 from each side.
$9.5 \approx AB$	Use a calculator.

AB is approximately 9.5 centimeters.

1. If the radius of the sphere in Example 1 is 18 inches and the radius of circle A is 16 inches, find AO.

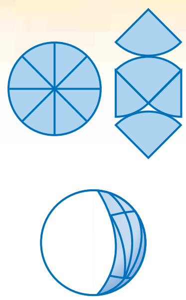
Surface Areas of Spheres You will investigate the surface area of a sphere in the Geometry Lab.

GEOMETRY LAB

Surface Area of a Sphere

MODEL

- Cut a polystyrene ball along a great circle. Trace the great circle onto a piece of paper. Then cut out the circle.
- Fold the circle into eight sectors. Then unfold and cut the pieces apart. Tape the pieces back together in the pattern shown at the right.
- Use tape or glue to put the two pieces of the ball together. Tape the paper pattern to the sphere.



Aaron Haupt

ANALYZE THE RESULTS

1. Approximately what fraction of the surface of the sphere is covered by the pattern?

2. What is the area of the pattern in terms of r, the radius of the sphere?

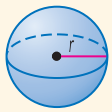
MAKE A CONJECTURE

3. Make a conjecture about the formula for the surface area of a sphere.

The lab leads us to the formula for the surface area of a sphere.

KEY CONCEPT — *Surface Area of a Sphere*

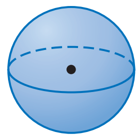
If a sphere has a surface area of T square units and a radius of r units, then $T = 4\pi r^2$.



EXAMPLE 2 Surface Area

a. Find the surface area of the sphere given the area of the great circle.

From the lab, we find that the surface area of a sphere is four times the area of the great circle.

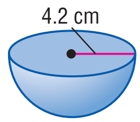


$T = 4\pi r^2$	Surface area of a sphere
$\approx 4(201.1)$	$\pi r^2 \approx 201.1$
≈ 804.4	Multiply.

The surface area is approximately 804.4 square inches.

b. Find the surface area of the hemisphere.

A hemisphere is half of a sphere. To find the surface area, find half of the surface area of the sphere and add the area of the great circle.



$\text{surface area} = \frac{1}{2}(4\pi r^2) + \pi r^2$	Surface area of a hemisphere
$= \frac{1}{2}[4\pi(4.2)^2] + \pi(4.2)^2$	Substitution
≈ 166.3	Use a calculator.

The surface area is approximately 166.3 square centimeters.

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

2A. sphere with the circumference of a great circle 5π centimeters

2B. hemisphere with the circumference of a great circle 3π inches

STANDARDIZED TEST EXAMPLE

3 A baseball is a sphere with a circumference of 9 inches. What is the surface area of the ball?

A $\frac{81}{\pi}$ in^2 **B** $\frac{81}{4\pi}$ in^2 **C** $\frac{81\pi}{4}$ in^2 **D** $81\pi^2$ in^2

Test-Taking Tip

Multi-Step Problems Many standardized test problems require multiple steps to find the solution. It is a good idea to write out the measures that you need to solve in order to answer the question.

Read the Test Item

We are asked to find the surface area of a sphere given the circumference.

Solve the Test Item

In order to find the surface area of the sphere, we first need to find the radius.

$C = 2\pi r$	Circumference of a circle
$9 = 2\pi r$	$C = 9$
$\frac{9}{2\pi} = r$	Divide each side by 2π.

Next, find the surface area of a sphere.

$T = 4\pi r^2$	Surface area of a sphere
$\approx 4\pi\left(\frac{9}{2\pi}\right)^2$	$r = \frac{9}{2\pi}$
$\approx 4\pi\left(\frac{81}{4\pi^2}\right)$ or $\frac{81}{\pi}$	Simplify.

The surface area is $\frac{81}{\pi}$. The correct answer is Choice A.

3. What is the surface area of felt that covers a tennis ball with a diameter of $2\frac{1}{2}$ inches?

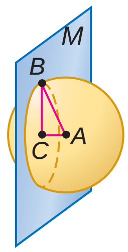
F $\frac{25}{16}\pi$ in^2 **G** $\frac{25}{4}\pi$ in^2 **H** 10π in^2 **J** 100π in^2

Online Personal Tutor at geometryonline.com

CHECK Your Understanding

Example 1 (p. 712)

In the figure, *A* is the center of the sphere, and plane M intersects the sphere in circle *C*. Round to the nearest tenth if necessary.



1. If $AC = 9$ and $BC = 12$, find AB.

2. If the radius of the sphere is 15 units and the radius of the circle is 10 units, find AC.

3. If Q is a point on $\odot C$ and $AB = 18$, find AQ.

Example 2 (p. 713)

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

4. a sphere with radius 6.8 inches

5. a hemisphere with the circumference of a great circle 8π centimeters

6. a sphere with the area of a great circle approximately 18.1 square meters

Example 3 (p. 714)

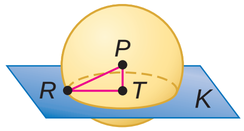
7. **STANDARDIZED TEST PRACTICE** An NCAA (National Collegiate Athletic Association) basketball has a radius of $4\frac{3}{4}$ inches. What is its surface area?

A $\frac{361\pi}{16}$ in^2 **B** $\frac{361\pi}{4}$ in^2 **C** 19π in^2 **D** 361π in^2

Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–13	1
15–22	2
14, 23, 24	3

In the figure, *P* is the center of the sphere, and plane K intersects the sphere in circle *T*. Round to the nearest tenth if necessary.



8. If $PT = 4$ and $RT = 3$, find PR.

9. If $PT = 3$ and $RT = 8$, find PR.

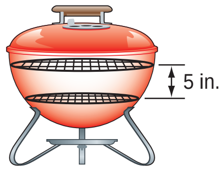
10. If the radius of the sphere is 13 units and the radius of $\odot T$ is 12 units, find PT.

11. If the radius of the sphere is 17 units and the radius of $\odot T$ is 15 units, find PT.

12. If X is a point on $\odot T$ and $PR = 9.4$, find PX.

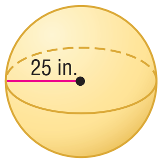
13. If Y is a point on $\odot T$ and $PR = 12.8$, find PY.

14. GRILLS A hemispherical barbecue grill has two racks, one for the food and one for the charcoal. The food rack is a great circle of the grill and has a radius of 11 inches. The charcoal rack is 5 inches below the food rack. Find the difference in the areas of the two racks.



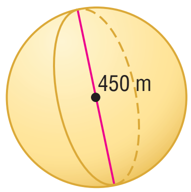
Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

15.

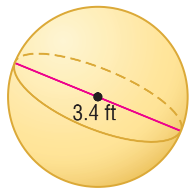


16.

17.



18.



19. hemisphere: The circumference of a great circle is 40.8 inches.

20. sphere: The circumference of a great circle is 30.2 feet.

21. sphere: The area of a great circle is 814.3 square meters.

22. hemisphere: The area of a great circle is 227.0 square kilometers.

23. ARCHITECTURE The Reunion Tower is a distinguishing landmark in the Dallas, Texas, skyline. The geodesic dome is about 118 feet in diameter. Determine the surface area of the dome, assuming that it is a sphere.

24. IGLOOS An igloo is made of hard-packed snow blocks. The blocks are arranged in a spiral that is increasingly smaller near the top to form a hemisphere. Find the surface area of the living area if the diameter is 13 feet.

Real-World Link

The diameter of Earth is 7899.83 miles from the North Pole to the South Pole and 7926.41 miles from opposite points at the equator.

EARTH For Exercises 25–27, use the information at the left.

25. Approximate the surface area of Earth using each measure.

26. If the atmosphere of Earth extends to about 100 miles above the surface, find the surface area of the atmosphere surrounding Earth. Use the mean of the two diameters.

27. About 75% of Earth's surface is covered by water. Find the surface area of water on Earth, using the mean of the two diameters.

Real-World Career

Physicist

About 29% of physicists work for the government. Physicists can also work for universities or companies in technology or medical fields.

Source: www.bls.gov

Math Online
For more information, go to geometryonline.com.

Determine whether each statement is *true* or *false*. If false, give a counterexample.

28. The radii of a sphere are congruent to the radius of its great circle.

29. In a sphere, two different great circles intersect in only one point.

30. Two spheres with congruent radii can intersect in a circle.

31. A sphere's longest chord will pass through the center of the circle.

32. Two spheres can intersect in one point.

CHANGING DIMENSIONS **Find the indicated unit ratio of two spheres with the given information.**

33. Surface area: The radius of one is twice the radius of the second sphere.

34. Radii: The surface area of one is one half the surface area of the other.

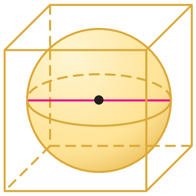
35. Surface area: The radius of one is three times the radius of the other.

ASTRONOMY **For Exercises 36 and 37, use the following information.**
In 2002, NASA's Chandra X-Ray Observatory found two unusual neutron stars. These two stars are smaller than previously found neutron stars, but they have the mass of a larger neutron star, causing astronomers to think this star may not be made of neutrons, but a different form of matter.

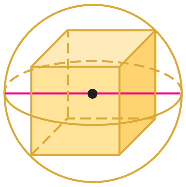
36. Neutron stars have diameters from 12 to 20 miles in size. Find the range of the surface areas.

37. One of the new stars has a diameter of 7 miles. Find the surface area of this star.

38. A sphere is inscribed in a cube. Describe how the radius of the sphere is related to the dimensions of the cube.



39. A sphere is circumscribed about a cube. Find the length of the radius of the sphere in terms of the dimensions of the cube.



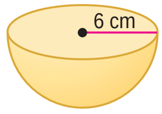
EXTRA PRACTICE
See pages 825, 839.
Math Online
Self-Check Quiz at geometryonline.com

H.O.T. Problems

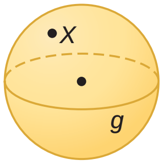
40. **OPEN ENDED** Draw a sphere with a chord $\overline{AB}$. Draw a tangent parallel to $\overline{AB}$.

41. **FIND THE ERROR** Loesha and Tim are finding the surface area of a hemisphere with a radius of 6 centimeters. Who is correct? Explain.

Loesha	Tim
$T = \frac{1}{2}(4\pi r^2)$	$T = \frac{1}{2}(4\pi r^2) + \pi r^2$
$= 2\pi(6^2)$	$= 2\pi(6^2) + \pi(6^2)$
$= 72\pi$	$= 72\pi + 36\pi$
	$= 108\pi$



42. **CHALLENGE** In spherical geometry, a plane is the surface of a sphere and a line is a great circle. How many lines exist that contain point X and do not intersect line g?



43. **Writing in Math** Describe how to find the surface area of a sphere. Include another example of a sport that uses spheres.

Fermilab

STANDARDIZED TEST PRACTICE

44. A rectangular solid that is 4 inches long, 5 inches high, and 7 inches wide is inscribed in a sphere. What is the radius of this sphere?

A $\frac{3\sqrt{10}}{2}$ in.

B $\sqrt{41}$ in.

C $\sqrt{65}$ in.

D $3\sqrt{10}$ in.

45. REVIEW Between 2000 and 2004, North Carolina experienced a 6.1% population increase. If x represents the population before 2000, which expression represents the population of North Carolina at the end of 2004?

F $x + 2000(0.061)$

G $x + x(0.061)$

H $x + 2005(0.061)$

J $x + 6.1x$

Spiral Review

Find the surface area of each cone. Round to the nearest tenth. (Lesson 12-5)

46. $h = 13$ inches, $\ell = 19$ inches

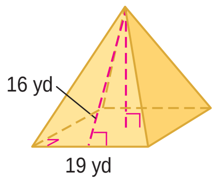
47. $r = 7$ meters, $h = 10$ meters

48. $r = 4.2$ cm, $\ell = 15.1$ cm

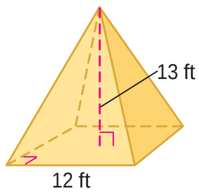
49. $d = 11.2$ ft, $h = 7.4$ ft

Find the surface area of each regular pyramid. Round to the nearest tenth if necessary. (Lesson 12-4)

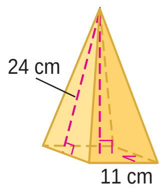
50.



51.



52.



53. CRAFTS Find the area of fabric needed to cover one side of a circular placemat with a diameter of 11 inches. Allow an additional 3 inches around the placemat. Round to the nearest tenth. (Lesson 11-3)

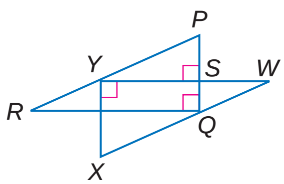
Write an equation for each circle. (Lesson 10-8)

54. a circle with center at $(-2, 7)$ and a radius with endpoint at $(3, 2)$

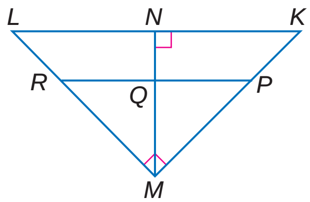
55. a diameter with endpoints at $(6, -8)$ and $(2, 5)$

Use the given information to find each measure. (Lesson 7-3)

56. If $\overline{PR} \parallel \overline{WX}$, $WX = 10$, $XY = 6$, $WY = 8$, $RY = 5$, and $PS = 3$, find PY, SY, and PQ.



57. If $\overline{PR} \parallel \overline{KL}$, $KN = 9$, $LN = 16$, $PM = 2(KP)$, find KP, KM, MR, ML, MN, and PR.



Find the distance between each pair of points. (Lesson 1-3)

58. $A(-1, -8)$, $B(3, 4)$

59. $C(0, 1)$, $D(-2, 9)$

60. $E(-3, -12)$, $F(5, 4)$

61. $G(4, -10)$, $H(9, -25)$

62. $J\left(1, \frac{1}{4}\right)$, $K\left(-3, -\frac{7}{4}\right)$

63. $L\left(-5, \frac{8}{5}\right)$, $M\left(5, -\frac{2}{5}\right)$

EXTEND 12-6

Geometry Lab
Locus and Spheres

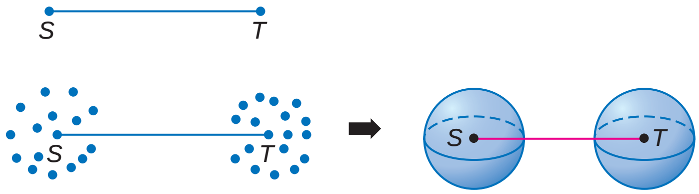
Spheres are defined in terms of a locus of points in space. The definition of a sphere is the set of all points that are a given distance from a given point.

Concepts in Motion
Animation geometryonline.com

ACTIVITY 1

Find the locus of points a given distance from the endpoints of a segment.

- Draw a given line segment with endpoints *S* and *T*.
- Create a set of points that are equidistant from *S* and a set of points that are equidistant from *T*.



Analyze the Results

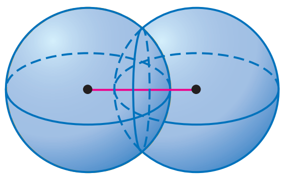
1. Draw a figure and describe the locus of points in space that are 5 units from each endpoint of a given segment that is 25 units long.
2. Are the two figures congruent?
3. What is the radius and diameter of each figure?
4. Find the distance between the two figures.

ACTIVITY 2

Investigate spheres that intersect.

Find the locus of all points that are equidistant from the centers of two intersecting spheres with the same radius.

- Draw a line segment.
- Draw congruent overlapping spheres, with the centers at the endpoints of the given line segment.



Concepts in Motion
Interactive Lab geometryonline.com

Analyze the Results

5. What is the shape of the intersection of the spheres?
6. Can this be described as a locus of points in space or on a plane? Explain.
7. Describe the intersection as a locus.
8. **MINING** What is the locus of points that describes how particles will disperse in an explosion at ground level if the expected distance a particle could travel is 300 feet?

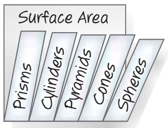
Study Guide and Review

Download Vocabulary Review from geometryonline.com

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Three-Dimensional Figures (Lesson 12-1)

- A solid can be determined from its orthographic drawing.
- Solids can be classified by bases, faces, edges, and vertices.

Surface Areas of Prisms (Lesson 12-2)

- The lateral faces of a prism are the faces that are not bases of the prism.
- The lateral surface area of a right prism is the product of the perimeter of a base of the prism and the height of the prism.

Surface Areas of Cylinders (Lesson 12-3)

- The lateral surface area of a cylinder is 2π multiplied by the product of the radius of a base of the cylinder and the height of the cylinder.
- The surface area of a cylinder is the lateral surface area plus the area of both circular bases.

Surface Areas of Pyramids (Lesson 12-4)

- The slant height ℓ of a regular pyramid is the length of an altitude of a lateral face.
- The lateral area of a pyramid is $\frac{1}{2}P\ell$, where ℓ is the slant height of the pyramid and P is the perimeter of the base of the pyramid.

Surface Areas of Cones (Lesson 12-5)

- A cone is a solid with a circular base and a single vertex.
- The lateral area of a right cone is $\pi r\ell$, where ℓ is the slant height of the cone and r is the radius of the circular base.

Surface Areas of Spheres (Lesson 12-6)

- The set of all points in space a given distance from one point is a sphere.
- The surface area of a sphere is $4\pi r^2$, where r is the radius of the sphere.

Key Vocabulary

axis (p. 693)
circular cone (p. 706)
corner view (p. 680)
cross section (p. 681)
great circle (p. 711)
hemisphere (p. 712)
lateral area (p. 686)
lateral edges (p. 686)
lateral faces (p. 686)
oblique cone (p. 706)
perspective view (p. 680)
reflection symmetry (p. 684)
regular pyramid (p. 699)
right cone (p. 706)
right cylinder (p. 693)
right prism (p. 686)
slant height (p. 699)

Vocabulary Check

State whether each sentence is *true* or *false*. If false, replace the underlined term to make a true sentence.

1. In a cylinder, the axis is the segment with endpoints that are the centers of the bases.
2. A perspective view is the view of a three-dimensional figure from the corner.
3. For a given sphere, the intersection of the sphere and a plane that contains the center of the sphere is called a hemisphere.
4. A circular cone is one of the two congruent parts into which a great circle separates a sphere.
5. For prisms, pyramids, cylinders, and cones, the lateral area is the area of the figure, not including the bases.
6. A pyramid with bases that are isosceles triangles is called an oblique pyramid.
7. A right cone is a cone with an axis that is also an altitude.
8. The height of each lateral edge is called the slant height ℓ of the pyramid.

Vocabulary Review at geometryonline.com

CHAPTER 12

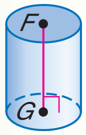
Study Guide and Review

Lesson-by-Lesson Review

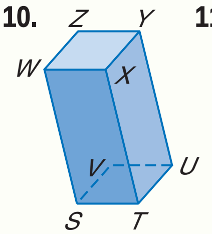
12-1 Representations of Three-Dimensional Figures (pp. 680–685)

Identify each solid. Name the bases, faces, edges, and vertices.

9.



10.



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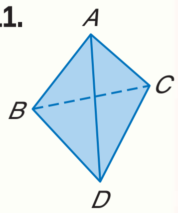
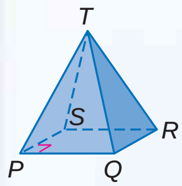


12. **GEMOLOGY** A well-cut diamond enhances the natural beauty of the stone. These cuts are called *facets.* What shapes are seen in the emerald-cut diamond shown?

Example 1 **Identify the solid. Name the bases, faces, edges, and vertices.**



The base is a rectangle, and all of the lateral faces intersect at point T, so this solid is a rectangular pyramid.

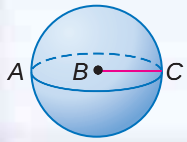
Base: $\square PQRS$

Faces: $\triangle TPQ$, $\triangle TQR$, $\triangle TRS$, $\triangle TSP$

Edges: $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$, $\overline{PS}$, $\overline{PT}$, $\overline{QT}$, $\overline{RT}$, $\overline{ST}$

Vertices: P, Q, R, S, T

Example 2 **Identify the solid. Name the bases, faces, edges, and vertices.**

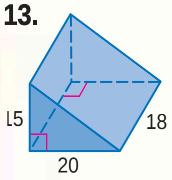


This solid has no bases, faces, or edges. It is a sphere.

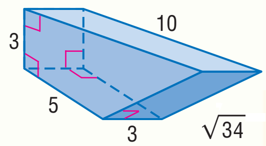
12-2 Surface Areas of Prisms (pp. 686–691)

Find the lateral area of each prism.

13.



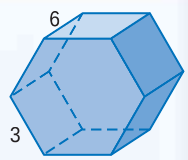
14.



15. **GIFT WRAPPING** Kim is wrapping a board game as a birthday gift for her nephew. The board game is 20 inches long, 11 inches wide and 4 inches high. Find the surface area to be wrapped.

Example 3 **Find the lateral area of the regular hexagonal prism.**

The bases are regular hexagons. So the perimeter of one base is 6(3) or 18. Substitute this value into the formula.



$L = Ph$	Lateral area of a prism
$= (18)(6)$	$P = 18, h = 6$
$= 108$	Multiply.

The lateral area is 108 square units.

V. Fleming/Photo Researchers

Mixed Problem Solving
For mixed problem-solving practice, see page 839.

12-3 Surface Areas of Cylinders (pp. 693–697)

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

16. $d = 4$ in., $h = 12$ in.

17. $r = 6$ ft, $h = 8$ ft

18. $r = 4$ mm, $h = 58$ mm

19. $d = 4$ km, $h = 8$ km

20. **CANS** Soft drinks are sold in aluminum cans that measure 6 inches in height and 2 inches in diameter. How many square inches of aluminum are needed to make a soft drink can?

Example 4 **Find the surface area of a cylinder with a radius of 38 centimeters and a height of 123 centimeters.**

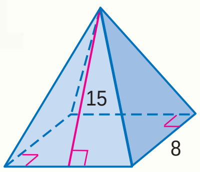
$T = 2\pi rh + 2\pi r^2$	Surface area of a cylinder
$= 2\pi(38)(123) + 2\pi(38)^2$	$r = 38, h = 123$
$\approx 38{,}440.5$	Use a calculator.

The surface area of the cylinder is approximately 38,440.5 square centimeters.

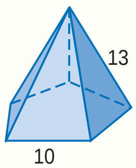
12-4 Surface Areas of Pyramids (pp. 699–705)

Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

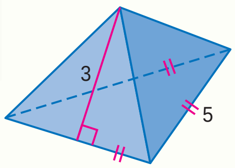
21.



22.



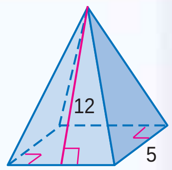
23.



24. **HOTELS** The Luxor Hotel in Las Vegas is a black glass pyramid. The base is a square with edges 646 feet long. The hotel is 350 feet tall. Find the area of the glass.

25. **MAYAN RUINS** The base of a Mayan pyramid is square with edge length 55.3 meters. The average angle of inclination of the faces is 53.3°. Find the surface area of the pyramid. Round to the nearest tenth.

Example 5 **Find the surface area of the regular pyramid.**



The perimeter of the base is 4(5) or 20 units, and the area of the base is 5^2 or 25 square units. Substitute these values into the formula for the surface area of a pyramid.

$T = \frac{1}{2}P\ell + B$	Surface area of a regular pyramid
$= \frac{1}{2}(20)(12) + 25$	$P = 20, \ell = 12, B = 25$
$= 145$	Simplify.

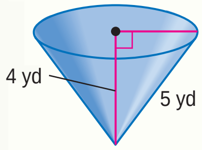
The surface area is 145 square units.

Study Guide and Review

12-5 Surface Areas of Cones (pp. 706–710)

Find the surface area of each cone. Round to the nearest tenth.

26.



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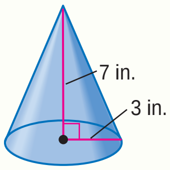
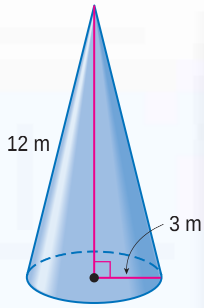


28. TOWERS In 1921, Italian immigrant Simon Rodia bought a home in Los Angeles, California, and began building conical towers in his backyard. The structures are made of steel mesh and cement mortar. Suppose the height of one tower is 55 feet and the diameter of the base is 8.5 feet, find the lateral area of the tower.

Example 6 **Find the surface area of the cone.**



Substitute the known values into the formula for the surface area of a right cylinder.

$T = \pi r\ell + \pi r^2$ Surface area of a cone

$= \pi(3)(12) + \pi(3)^2$ $r = 3, \ell = 12$

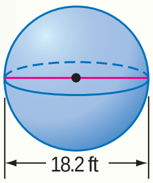
≈ 141.4 Use a calculator.

The surface area is approximately 141.4 square meters.

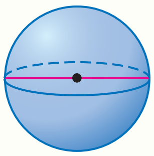
12-6 Surface Areas of Spheres (pp. 711–717)

Find the surface area of each sphere or hemisphere. Round to the nearest tenth if necessary.

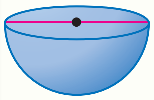
29.



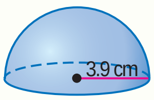
30. Area of great circle = 218 in^2



31. Area of great circle = 121 mm^2

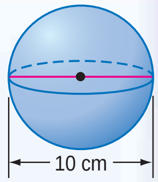


32.



33. CAMPING Jen has a tent in the shape of a hemisphere. The canvas that makes up the floor is a great circle of the tent and has a radius 5 feet. Jen needs to buy a rain tarp for her tent. Find the lateral area to be covered by the rain tarp.

Example 7 **Find the surface area of a sphere with a diameter of 10 centimeters.**



$T = 4\pi r^2$ Surface area of a sphere

$= 4\pi(5)^2$ $r = 5$

≈ 314.2 Use a calculator.

The surface area is approximately 314.2 square centimeters.

Example 8 **Find the surface area of a hemisphere with radius 6.3 inches.**

To find the surface area of a hemisphere, add the area of the great circle to half of the surface area of the sphere.

surface area $= \frac{1}{2}(4\pi r^2) + \pi r^2$ Surface area of a hemisphere

$= \frac{1}{2}[4\pi(6.3)^2] + \pi(6.3)^2$ $r = 6.3$

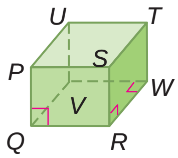
≈ 374.1 Use a calculator.

The surface area is approximately 374.1 square inches.

CHAPTER 12 Practice Test

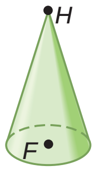
Identify each solid. Name the bases, faces, edges, and vertices.

1.



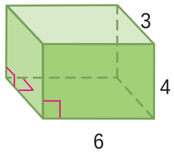
2.

3.

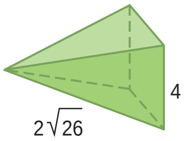


Find the surface area for each solid. Round to the nearest tenth.

4.



5.



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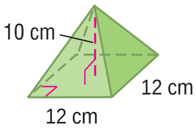
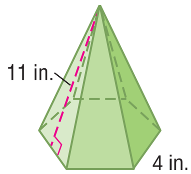
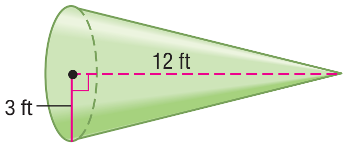


7.

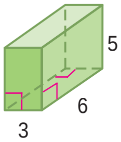


8.



Find the lateral area of each prism.

9.



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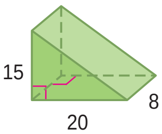
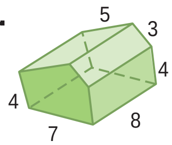


11.



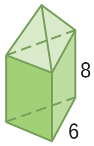
Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

12. $r = 8$ ft, $h = 22$ ft

13. $r = 3$ mm, $h = 2$ mm

14. $r = 78$ m, $h = 100$ m

The figure shown is a composite solid of a tetrahedron and a triangular prism. Find each measure in the solid. Round to the nearest tenth if necessary.



15. height

16. lateral area

17. surface area

Find the surface area of each cone. Round to the nearest tenth.

18. $h = 24$, $r = 7$

19. $h = 3$ m, $\ell = 4$ m

20. $r = 7$, $\ell = 12$

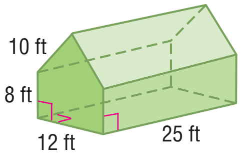
Find the surface area of each sphere. Round to the nearest tenth if necessary.

21. $r = 15$ in.

22. $d = 14$ m

23. The area of a great circle of the sphere is 116 square feet.

24. GARDENING The surface of a greenhouse is covered with plastic or glass. Find the amount of plastic needed to cover the greenhouse shown.



25. MULTIPLE CHOICE A cube has a surface area of 150 square centimeters. What is the length of each edge?

A 25 cm

B 15 cm

C 12.5 cm

D 5 cm

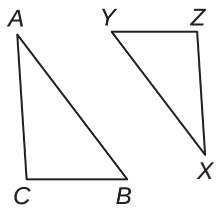
CHAPTER 12

Standardized Test Practice

Cumulative, Chapters 1–12

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. In the triangles below, $\angle C \cong \angle Z$.



Which of the following would be sufficient to prove that $\triangle ABC \sim \triangle XYZ$?

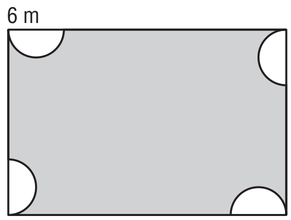
A $\frac{AB}{BC} = \frac{XY}{YZ}$

B $\frac{AC}{BC} = \frac{XY}{YZ}$

C $\frac{AC}{BC} = \frac{XZ}{YZ}$

D $\frac{AB}{AC} = \frac{XY}{XZ}$

2. The rectangle shown below has a length of 30 meters and a width of 20 meters.



If four congruent half circles are removed from the rectangle as shown, what will be the area of the remaining figure?

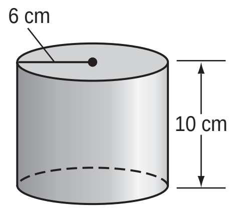
F $(100 - 24\pi)\ m^2$

G $(600 - 36\pi)\ m^2$

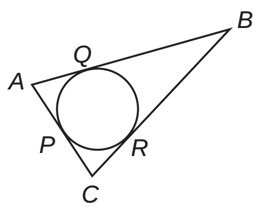
H $(600 - 18\pi)\ m^2$

J $(600 - 12\pi)\ m^2$

3. **GRIDDABLE** What is the lateral area of the can of soup shown below to the nearest tenth of a centimeter?



4. Triangle *ABC* is circumscribed about, with points of tangency at *P*, *Q*, and *R*.



If $AB = 13$ and $BC = 11$, and $BR = 9$, what is the perimeter of $\triangle ABC$?

A 28

B 30

C 33

D 36

TEST-TAKING TIP

Question 4 Read the question carefully to check that you answered the question asked. In Question 4, you are asked to find the perimeter of $\triangle ABC$, not the length of $\overline{AC}$.

5. A regular hexagon is inscribed in a circle with a diameter of 12 centimeters. What is the area of the hexagon?

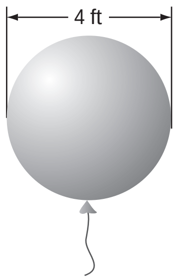
F $54\sqrt{3}\ cm^2$

G $90\ cm^2$

H $72\sqrt{3}\ cm^2$

J $144\ cm^2$

6. What is the surface area of the spherical weather balloon shown below?



A 8π ft^2

B 16π ft^2

C 32π ft^2

D 64π ft^2

7. **ALGEBRA** The height of a triangle is 3 meters less than half its base. The area of the triangle is 54 square meters. Find the length of the base.

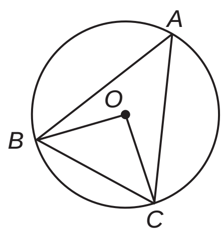
F 12 m

G 18 m

H 24 m

J 27 m

8. If O is the center of the circle and $m\angle BAC = 46°$, what is $m\angle BOC$?



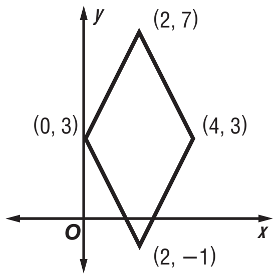
A 23°

B 46°

C 92°

D 111°

9. The figure graphed below is a rhombus. What is the area, in square units, of the rhombus?



F 16

G 20

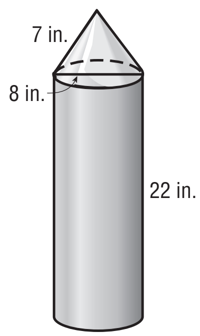
H 32

J 64

Pre-AP

Record your answers on a sheet of paper. Show your work.

10. Aliya is constructing a model of a rocket. She uses a right cylinder for the base and a right cone for the top as shown.

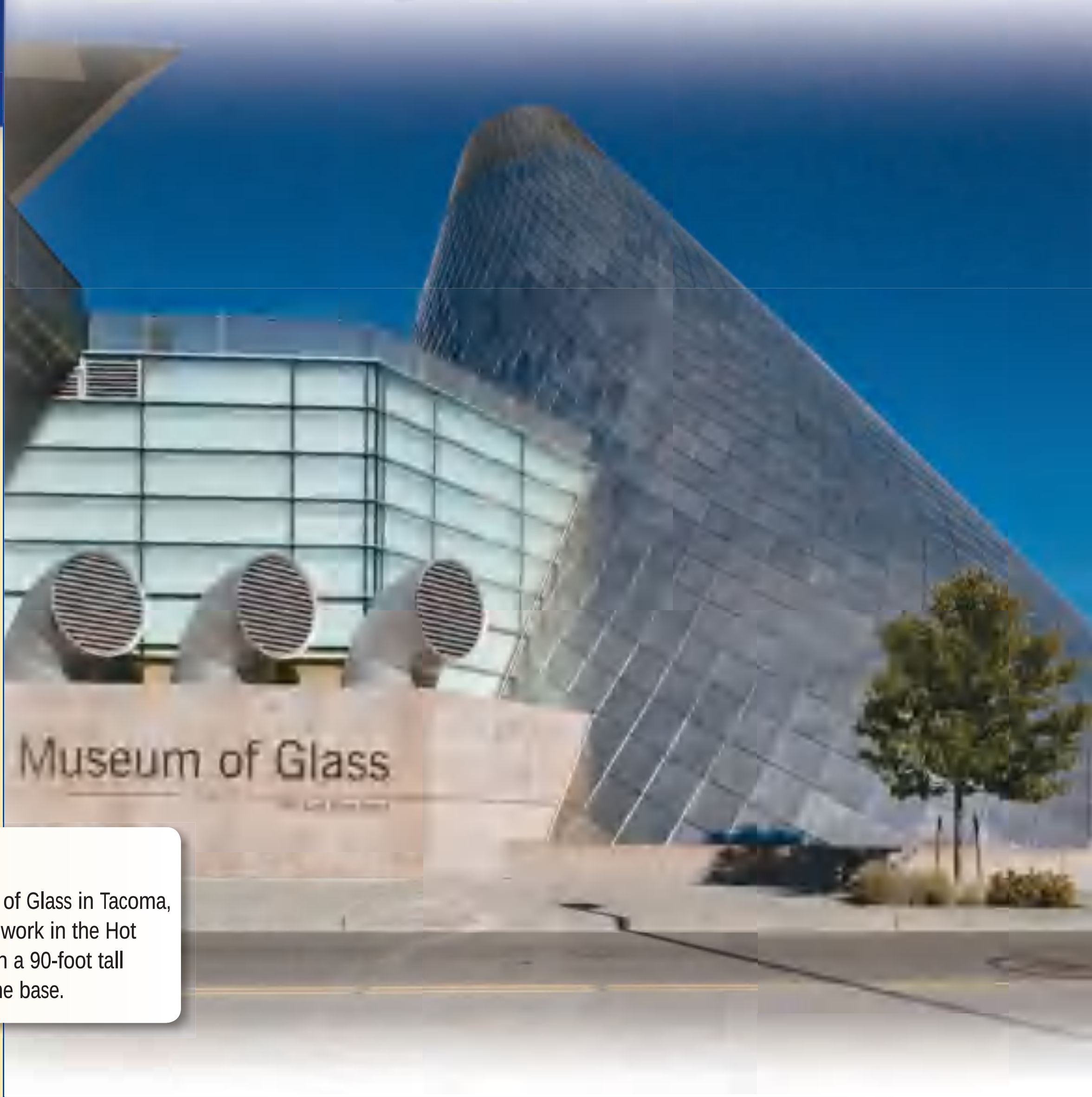


Bottles of model paint sell for \$1.49 each. If one bottle of model paint covers 3 square feet, how much will it cost to paint the outer surface of the rocket, including its bottom, with one color of paint?

NEED EXTRA HELP?										
If You Missed Question...	1	2	3	4	5	6	7	8	9	10
Go to Lesson or Page...	7-3	11-4	12-3	10-5	11-3	12-6	796	10-2	11-2	12-5

CHAPTER 13

Extending Volume



BIG Ideas

- Find volumes of prisms, cylinders, pyramids, cones, and spheres.
- Identify congruent and similar solids.
- Graph solids in space and use the Distance and Midpoint Formulas in space.

Key Vocabulary

similar solids (p. 750)
congruent solids (p. 751)
ordered triple (p. 758)

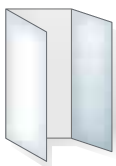
Real-World Link

Architecture Visitors to the Museum of Glass in Tacoma, Washington, can watch glass artists at work in the Hot Shop Amphitheater, which is housed in a 90-foot tall angled cone, 100 feet in diameter at the base.

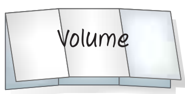
FOLDABLES

Volume Make this Foldable to help you organize your notes. Begin with one sheet of 11" × 17" paper.

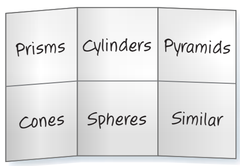
1 **Fold** in thirds.



2 **Fold** in half lengthwise. Label as shown.



3 **Unfold** book. Draw lines along the folds and label as shown.



Art on File/CORBIS

GET READY for Chapter 13

Diagnose Readiness **You have two options for checking Prerequisite Skills.**

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Find the value of the variable in each equation. (Prerequisite Skill)

1. $a^2 + 12^2 = 13^2$
2. $(4\sqrt{3})^2 + b^2 = 8^2$
3. $b^2 + 3b^2 = 192$
4. $256 + 7^2 = c^2$
5. Write and solve an equation for *The square of x is equal to the square of the product of 2 and x, minus 18.*

Simplify. (Prerequisite Skill)

6. $(5b)^2$
7. $\left(\frac{n}{4}\right)^2$
8. $\left(\frac{3x}{4y}\right)^2$
9. $\left(\frac{4y}{7}\right)^2$

Write and simplify an algebraic expression for each verbal expression. (Prerequisite Skill)

10. the square of the product of a and 3
11. the square of the quotient of $4z$ and 12

***W* is the midpoint of $\overline{AB}$. For each pair of points, find the coordinates of the third point.** (Lesson 1-3)

12. $A(0, -1)$, $B(-5, 4)$
13. $A(1, -1)$, $W(10, 10)$
14. **MAPS** Hallie lives halfway between Raleigh, North Carolina, and Indianapolis, Indiana. If the latitude and longitude of Raleigh is (35.5°, 78.4°) and that of Indianapolis is (39.5°, 86.1°), find the latitude and longitude of where she lives. (Lesson 1-3)

QUICK Review

EXAMPLE 1

Find the value of the variable in $4^2 + b^2 = (4\sqrt{2})^2$.

$4^2 + b^2 = (4\sqrt{2})^2$	Original equation
$16 + b^2 = 32$	Evaluate the exponents.
$b^2 = 16$	Subtract 16 from each side.
$b = 4$	Take the square root of each side.

EXAMPLE 2

Simplify $\left(\frac{2x}{3y}\right)^2$.

$\left(\frac{2x}{3y}\right)^2 = \frac{(2x)^2}{(3y)^2}$	Power of a Quotient Property
$= \frac{2^2x^2}{3^2y^2}$	Power of a Product Property
$= \frac{4x^2}{9y^2}$	Simplify.

EXAMPLE 3

***M* is the midpoint of $\overline{RS}$. If $S(1, 2)$ and $M(3, 4)$, find the coordinates of $R(x, y)$.**

By the Midpoint formula, the coordinates of M are $\left(\frac{x+1}{2}, \frac{y+2}{2}\right)$. Write two equations to solve for x and y.

$\frac{x+1}{2} = 3$	$\frac{y+2}{2} = 4$
$x + 1 = 6$	$y + 2 = 8$
$x = 5$	$y = 6$

The coordinates of R are (5, 6).

13-1 Volumes of Prisms and Cylinders

Main Ideas

- Find volumes of prisms.
- Find volumes of cylinders.

GET READY for the Lesson

Creators of comics occasionally use mathematics.

SHOE

In the comic above, the teacher is getting ready to teach a geometry lesson on volume. Shoe seems to be confused about the mathematical meaning of volume.

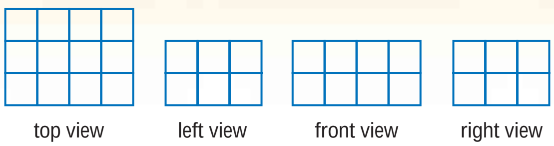
Volumes of Prisms You can create a rectangular prism from different views of the figure to verify the formula for its volume.

GEOMETRY LAB

Volume of a Rectangular Prism

MODEL

Use cubes to make a model of the solid with the given orthographic drawing.



ANALYZE

1. How many cubes make up the prism?
2. Find the product of the length, width, and height of the prism.
3. Compare the number of cubes to the product of the length, width, and height.
4. Repeat the activity with a prism of different dimensions.
5. **Make a conjecture** about the formula for the volume of a right rectangular prism.

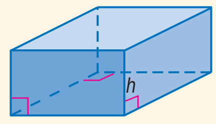
Study Tip

Lookback

To review identifying the base(s) of three dimensional figures, see Lesson 1-7.

KEY CONCEPT — *Volume of a Prism*

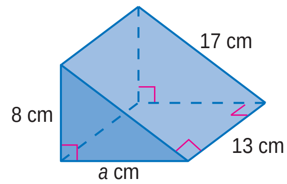
If a prism has a volume of V cubic units, a height of h units, and each base has an area of B square units, then $V = Bh$.



Area of base = B

EXAMPLE Volume of a Triangular Prism

1 Find the volume of the triangular prism.



The bases of the prism are congruent right triangles. Use the Pythagorean Theorem to find the leg of one base of the prism.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$a^2 + 8^2 = 17^2$	$b = 8$, $c = 17$
$a^2 + 64 = 289$	Multiply.
$a^2 = 225$	Subtract 64 from each side.
$a = 15$	Take the square root of each side.

Next, find the volume of the prism.

$V = Bh$	Volume of a prism
$= \frac{1}{2}(8)(15)(13)$	$B = \frac{1}{2}(8)(15)$, $h = 13$
$= 780$	Simplify.

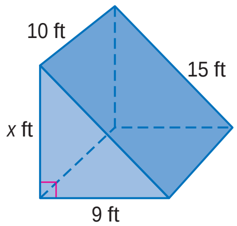
The volume of the prism is 780 cubic centimeters.

Study Tip

Volume and Area

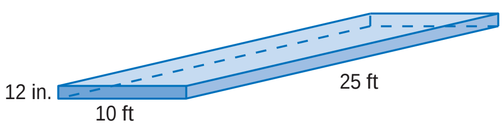
Area is two-dimensional, so it is measured in square units. Volume is three-dimensional, so it is measured in cubic units.

1. Find the volume of the triangular prism.



2 **SNOW** **The weight of wet snow in pounds is 0.003993 pounds/cubic inch times the volume of snow in cubic inches. How many pounds of wet snow would a person shovel in a rectangular driveway 25 feet by 10 feet after 12 inches of snow have fallen?**

First, make a drawing.
Then convert feet to inches.
25 feet = 25 × 12 or 300 inches
10 feet = 10 × 12 or 120 inches



(continued on the next page)

To find the pounds of wet snow shoveled, first find the volume of snow on the driveway.

$V = Bh$ — Volume of a prism

$= 300(120)(12)$ — $B = 300(120)$, $h = 12$

$= 432{,}000$ — The volume is 432,000 cubic inches.

Now multiply the volume by 0.003993 pounds/cubic inch.

$0.003993(432{,}000) \approx 1725$ — Simplify.

A person shoveling 12 inches of snow on a rectangular driveway 25 feet by 10 feet would shovel approximately 1725 pounds of snow.

CHECK Your Progress

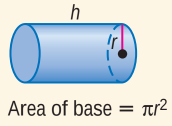
2. SWIMMING POOL A rectangular lap pool measures 80 feet long by 20 feet. If it needs to be filled to four feet deep and each cubic foot holds 7.5 gallons, how many gallons will it take to fill the lap pool?

Personal Tutor at geometryonline.com

Volumes of Cylinders Like the volume of a prism, the volume of a cylinder is the product of the area of the base and the height.

KEY CONCEPT — ***Volume of a Cylinder***

If a cylinder has a volume of V cubic units, a height of h units, and the bases have radii of r units, then $V = Bh$ or $V = \pi r^2 h$.



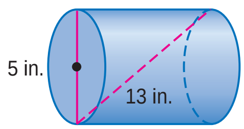
Study Tip

Square Roots

When you take the square root of each side of $h^2 = 144$, the result is actually ± 12. Since this is a measure, the negative value is not reasonable.

EXAMPLE Volume of a Cylinder

3 Find the volume of the cylinder.



The diameter of the base, the diagonal, and the lateral edge of the cylinder form a right triangle. Use the Pythagorean Theorem to find the height.

$a^2 + b^2 = c^2$ — Pythagorean Theorem

$h^2 + 5^2 = 13^2$ — $a = h$, $b = 5$, and $c = 13$

$h^2 + 25 = 169$ — Multiply.

$h^2 = 144$ — Subtract 25 from each side.

$h = 12$ — Take the square root of each side.

Now find the volume.

$V = \pi r^2 h$ — Volume of a cylinder

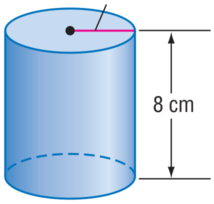
$= \pi(2.5^2)(12)$ — $r = 2.5$ and $h = 12$

≈ 235.6 — Use a calculator.

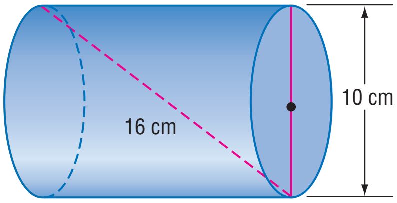
The volume is approximately 235.6 cubic inches.

Find the volume of each cylinder.

3A.



3B.



Review Vocabulary

oblique solids: a solid that is not right (Lesson 12-2)

Thus far, we have only studied the volumes of right solids. Do the formulas for volume apply to oblique solids as well as right solids?

Study the two stacks of quarters. The stack on the left represents a right cylinder, and the stack on the right represents an oblique cylinder. Since each stack has the same number of coins, with each coin the same size and shape, the two cylinders must have the same volume. Cavalieri, an Italian mathematician of the seventeenth century, was credited with making this observation first.

Study Tip

Cavalieri's Principle

This principle applies to the volumes of all solids.

KEY CONCEPT *Cavalieri's Principle*

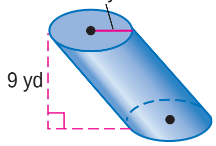
If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

If a cylinder has a base with an area of B square units and a height of h units, then its volume is Bh cubic units, whether it is right or oblique.

EXAMPLE Volume of an Oblique Solid

4 **Find the volume of the oblique cylinder.**

To find the volume, use the formula for a right cylinder.



$V = \pi r^2 h$ Volume of a cylinder

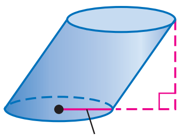
$= \pi(4^2)(9)$ $r = 4, h = 9$

≈ 452.4 Use a calculator.

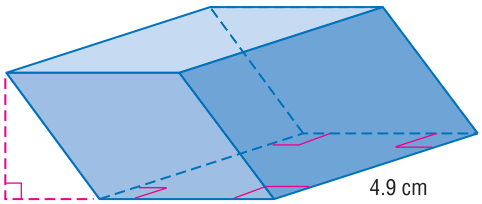
The volume is approximately 452.4 cubic yards.

Find the volume of each oblique solid.

4A.



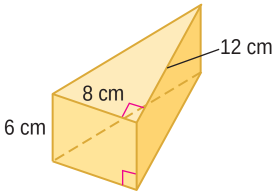
4B.



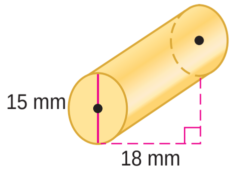
CHECK Your Understanding

Find the volume of each prism or cylinder. Round to the nearest tenth if necessary.

Examples 1, 3, 4
(pp. 729–731)

1.


2.


3.


Example 2
(p. 729)

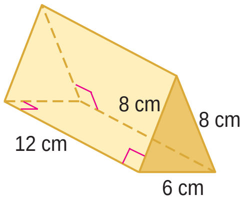
4. **DIGITAL CAMERA** The world's most powerful digital camera is located in New Mexico at the Apache Point Observatory. It is surrounded by a rectangular prism made of aluminum that protects the camera from wind and unwanted light. If the prism is 12 feet long, 12 feet wide, and 14 feet high, find its volume to the nearest cubic foot.

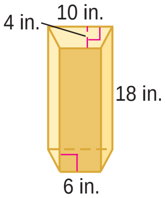
Exercises

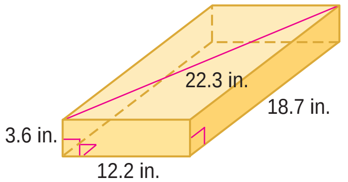
HOMEWORK HELP

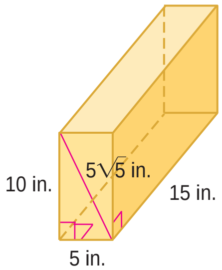
For Exercises	See Examples
5, 6	1
7, 8, 15	2
9, 10, 14	3
11–13	4

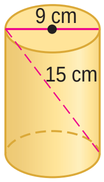
Find the volume of each prism or cylinder. Round to the nearest tenth if necessary.

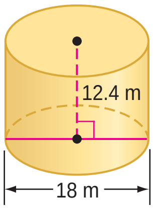
5.


6.


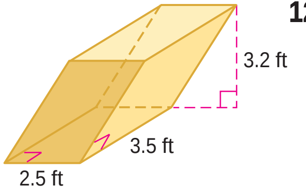
7.


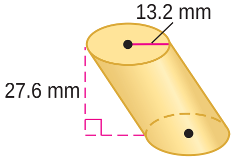
8.


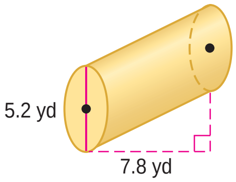
9.


10.


Find the volume of each oblique prism or cylinder. Round to the nearest tenth if necessary.

11.


12.


13.


14. The volume of a cylinder is 615.8 cubic meters, and the height is 4 meters. Find the length of the diameter of the cylinder.

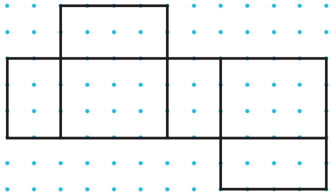
15. The volume of a rectangular prism is 1152 cubic inches, and the area of the base is 64 square inches. Find the length of the lateral edge of the prism.

Study Tip

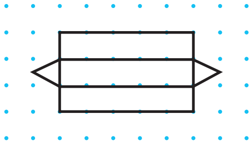
Look Back
To review **nets,** see Extend Lesson 1-7.

Find the volume of the solid formed by each net.

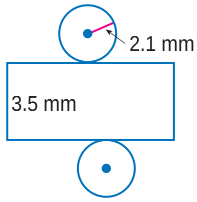
16.



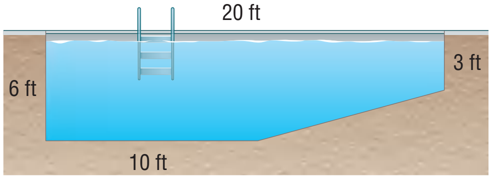
17.



18.



19. SWIMMING POOLS The base of a rectangular swimming pool is sloped so one end of the pool is 6 feet deep and the other end is 3 feet deep. If the length of the pool is 20 feet and the width is 15 feet, find the volume of water it takes to fill the pool.



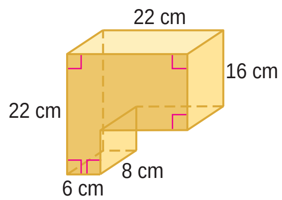
Real-World Link
The rules for some sandcastle competitions include not adding or removing sand from your designated plot and using only materials natural to the beach in the finished sculpture.

Source: cannon-beach.net

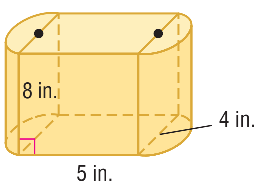
20. SANDCASTLES In a sandcastle competition, contestants are only allowed to use water, shovels, and 10 cubic yards of sand. In order to transport the correct amount of sand, they want to create cylinders that are 6 feet tall to hold enough sand for one contestant. What should the diameter of the cylinders be?

COMPOSITE SOLIDS **Find the volume of each solid to the nearest tenth.**

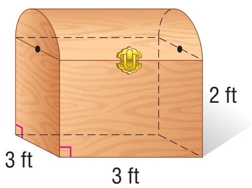
21.



22.

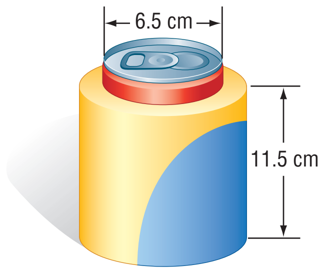


23. BUILDING Manny is building a blanket chest for his sister. His design is a composite of a square prism and half of a cylinder. What is the volume of the hope chest?



24. AQUARIUM The New England Aquarium in Boston, Massachusetts, has one of the world's largest cylindrical tanks. The Giant Ocean Tank holds approximately 200,000 gallons and is 23 feet deep. If it takes about $7\frac{1}{2}$ gallons of water to fill a cubic foot, what is the radius of the Giant Ocean Tank?

25. MANUFACTURING A can 12 centimeters tall fits into a rubberized cylindrical holder that is 11.5 centimeters tall, including 1 centimeter, which is the thickness of the base of the holder. The thickness of the rim of the holder is 1 centimeter. What is the volume of the rubberized material that makes up the holder?



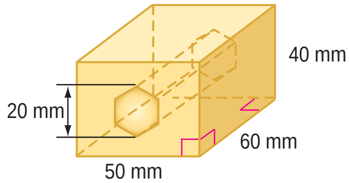
A&E The Graphics Complex

26. **CHANGING DIMENSIONS** A soy milk company is planning a promotion in which the volume of soy milk in each container will be increased by 25%. The company wants the base of the containers to stay the same. What will be the height of the new containers?

27. **PATIOS** Mrs. Blackwell is planning to remove an old patio and install a new rectangular concrete patio 20 feet long, 12 feet wide, and 4 inches thick. One contractor bid $2225 for the project. A second contractor bid $500 per cubic yard for the new patio and $700 for removal of the old patio. Which is the less expensive option? Explain.

28. **MEASUREMENT** Find a real prism or cylinder. Measure its dimensions and find its volume.

ENGINEERING For Exercises 29 and 30, use the following information.
Machinists make parts for intricate pieces of equipment. Suppose a part has a regular hexagonal hole drilled in a brass block.



29. Find the volume of the resulting part.

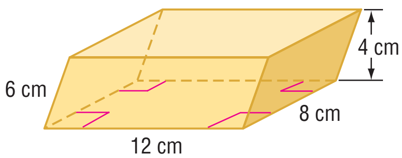
30. The *density* of a substance is its mass per unit volume. At room temperature, the density of brass is 8.0 grams per cubic centimeter. What is the mass of this block of brass?

EXTRA PRACTICE
See pages 825, 840.
Math Online
Self-Check Quiz at geometryonline.com

H.O.T. Problems

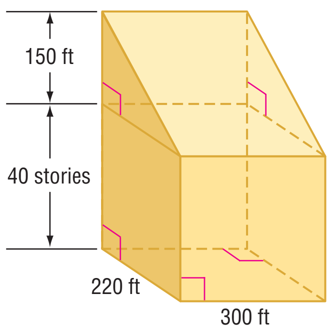
31. **OPEN ENDED** Draw a prism that has a volume of 50 cubic centimeters.

32. **FIND THE ERROR** Che and Julia are trying to find the volume of the oblique prism. Who is correct? Explain your reasoning.



Che	Julia
$V = Bh$	$V = Bh$
$= (12)(8)(6)$	$= (12)(8)(4)$
$= 576 \text{ cm}^3$	$= 384 \text{ cm}^3$

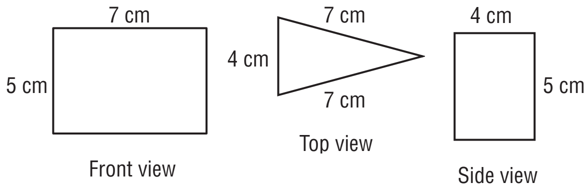
33. **CHALLENGE** A 40-story building is a rectangular prism with a length of 300 feet and a width of 220 feet. On top of the rectangular prism is a triangular prism the base of which has a height of 150 feet and a base of 220 feet. If each story is 11 feet, find the volume of the building.



34. **Writing in Math** Refer to the information on page 728 to explain how mathematics is sometimes used in comics. Differentiate between the mathematical meaning of volume and Shoe's meaning of volume.

STANDARDIZED TEST PRACTICE

35. What is the volume of a 3-dimensional object with the dimensions shown in the three views below?



A 59.1 cm^3

B 67.1 cm^3

C 68.4 cm^3

D 103.4 cm^3

36. REVIEW Every person on a basketball team gets one paper cup for water during each practice. The table shows the number of paper cups remaining *c* after each practice *p*.

Number of Practices	Cups Left
1	262
2	244
3	226
4	208
5	190

Which function can be used to describe this relationship?

F $c = 262 - 12p$ **H** $c = 262 - 18p$

G $c = 280 - 12p$ **J** $c = 280 - 18p$

Find the surface area of each sphere. Round to the nearest tenth. (Lesson 12-6)

37. $d = 12$ ft   **38.** $r = 41$ cm   **39.** $r = 8.5$ in.   **40.** $d = 18$ m

Find the surface area of each cone. Round to the nearest tenth. (Lesson 12-5)

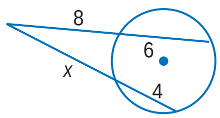
41. slant height = 11 m, radius = 6 m

42. diameter = 16 cm, slant height = 13.5 cm

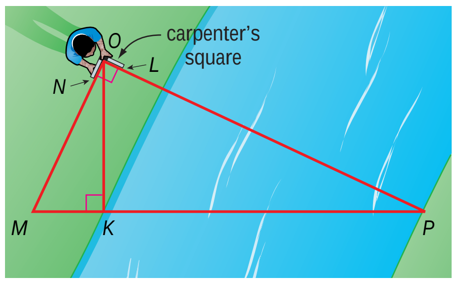
43. radius = 5 in., height = 12 in.

44. diameter = 14 in., height = 24 in.

45. Find *x* to the nearest tenth. (Lesson 10-7)



46. SURVEYING Mr. Glover uses a carpenter's square to find the distance across a stream. The carpenter's square models right angle *NOL*. He puts the square on top of a pole that is high enough to sight along $\overline{OL}$ to point *P* across the river. Then he sights along $\overline{ON}$ to point *M*. If *MK* is 1.5 feet and *OK* = 4.5 feet, find the distance *KP* across the stream. (Lesson 7-3)



GET READY for the Next Lesson

PREREQUISITE SKILL Find the area of each polygon with given side length *s*. Round to the nearest hundredth. (Lesson 11-3)

47. equilateral triangle, $s = 7$ in.

48. regular hexagon, $s = 12$ cm

49. regular pentagon, $s = 6$ m

50. regular octagon, $s = 50$ ft

EXTEND 13-1

Spreadsheet Lab

Changing Dimensions

Changing the dimensions of a prism affects the surface area and the volume of the prism. You can investigate the changes by using a spreadsheet.

- Create a spreadsheet by entering the length of the rectangular prism in column B, the width in column C, and the height in column D.
- In cell E2, enter the formula for the total surface area. Enter the formula for the volume of the prism in cell F2.
- Copy the formulas in cells E2 and F2 to the other cells in columns E and F.

Prisms.xls

◇	A	B	C	D	E	F
1	Prism	*l*	*w*	*h*	Surface Area	Volume
2	1	1	2	3	22	6
3	2					
4	3					
5	4					
6	5					
7						

Sheet 1 | Sheet 2 | Sheet 3

Use your spreadsheet to find the surface areas and volumes of prisms with the dimensions given in the table below.

Set 1			
Prism	Length	Width	Height
A	1	2	3
B	1	2	6
C	1	2	9

Set 2			
Prism	Length	Width	Height
D	2	3	4
E	2	6	8
F	2	9	12

Set 3			
Prism	Length	Width	Height
G	4	5	6
H	8	10	12
I	12	15	18

Exercises **For Exercises 1–3, compare the following in sets 1, 2, and 3.**

1. dimensions of prisms **2.** surface areas of prisms **3.** volumes of prisms

4. Write a statement about the change in the surface area and volume of a prism when one or more dimensions are changed.

For Exercises 5 and 6, use the following information.
Candles are made by pouring wax in cylindrical molds.

5. If a candle is twice as tall as another candle but has the same radius, does the larger candle require twice as much wax? Explain.

6. A candle is half as tall and has half the radius of another candle. Does the smaller candle require half the wax? Explain.

13-2 Volumes of Pyramids and Cones

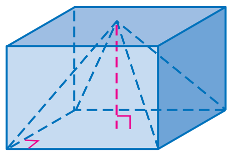
Main Ideas

- Find volumes of pyramids.
- Find volumes of cones.

GET READY for the Lesson

The Transamerica Pyramid is the tallest skyscraper in San Francisco. The 48-story building is a square pyramid. The building was designed to allow more light to reach the street.

Volumes of Pyramids The pyramid and the prism at the right share a base and have the same height. As you can see, the volume of the pyramid is less than the volume of the prism.

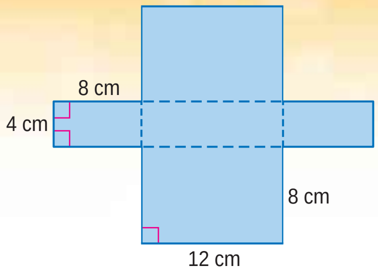


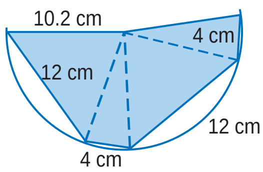
GEOMETRY LAB

Investigating the Volume of a Pyramid

ACTIVITY

- Measure each side to draw each net on card stock.
- Cut out the nets. Fold on the dashed lines.
- Tape the edges together to form models of the solids with one face removed.
- Estimate how much greater the volume of the prism is than the volume of the pyramid.
- Fill the pyramid with rice. Then pour this rice into the prism. Repeat until the prism is filled.



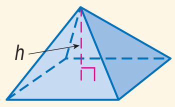


ANALYZE THE RESULTS

1. How many pyramids of rice did it take to fill the prism?
2. Compare the areas of the bases of the prism and pyramid.
3. Compare the heights of the prism and the pyramid.
4. Make a conjecture about the formula for the volume of a pyramid.

KEY CONCEPT — *Volume of a Pyramid*

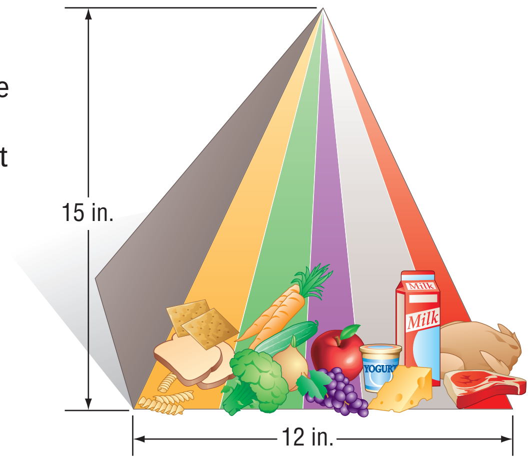
If a pyramid has a volume of V cubic units, a height of h units, and a base with an area of B square units, then $V = \frac{1}{3}Bh$.



Area of base $= B$

EXAMPLE 1 Volume of a Pyramid

NUTRITION Rebeca is making a plaster model of the Food Guide Pyramid for a class presentation. The model is a square pyramid with a base edge of 12 inches and a height of 15 inches. Find the volume of plaster needed to make the model.

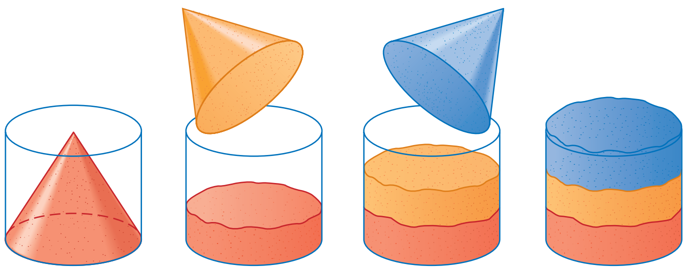


$V = \frac{1}{3}Bh$	Volume of a pyramid
$= \frac{1}{3}s^2h$	$B = s^2$
$= \frac{1}{3}(12^2)(15)$	$s = 12$, $h = 15$
$= 720$	Multiply.

Rebeca needs 720 cubic inches of plaster to make the model.

1. The Pyramid Arena in Memphis, Tennessee, is the third largest pyramid in the world. It is approximately 350 feet tall, and its base is 600 feet wide. Find the volume of this pyramid.

Volumes of Cones The derivation of the formula for the volume of a cone is similar to that of a pyramid. If the areas of the bases of a cone and a cylinder are the same and if the heights are equal, then the volume of the cylinder is three times as much as the volume of the cone.

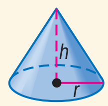


Study Tip

Volume Formulas Notice how the formulas for the volumes of a cone and pyramid are the same. However, the area of the base of a cone is πr^2.

KEY CONCEPT — *Volume of a Cone*

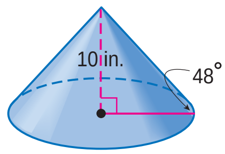
If a right circular cone has a volume of V cubic units, a height of h units, and the base has a radius of r units, then $V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2h$.



Area of base $= \pi r^2$

EXAMPLE Volume of a Cone

2 **Find the volume of the cone.**



Use trigonometry to find the radius of the base.

$\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$ — Definition of tangent

$\tan 48° = \dfrac{10}{r}$ — $A = 48°$, opposite $= 10$, and adjacent $= r$

$r = \dfrac{10}{\tan 48°}$ — Solve for r.

$r \approx 9.0$ — Use a calculator.

Now find the volume.

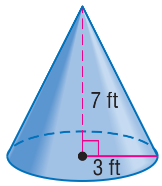
$V = \frac{1}{3}\pi r^2 h$ — Volume of a cone

$\approx \frac{1}{3}\pi(9^2)(10)$ — $r \approx 9$, $h = 10$

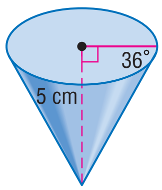
≈ 848.23 — Use a calculator.

Find the volume of each cone.

2A.



2B.



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Study Tip

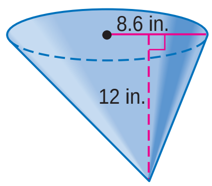
Common Misconceptions

The formula for the surface area of a cone only applies to right cones. However, a right cone and an oblique cone with the same radius and height have the same volume but different surface areas.

Recall that Cavalieri's Principle applies to all solids. So, the formula for the volume of an oblique cone is the same as that of a right cone.

EXAMPLE Volume of an Oblique Cone

3 **Find the volume of the oblique cone.**



$V = \frac{1}{3}\pi r^2 h$ — Volume of a cone

$= \frac{1}{3}\pi(8.6^2)(12)$ — $r = 8.6$, $h = 12$

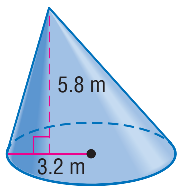
≈ 929.4 — Use a calculator.

The volume of the oblique cone is approximately 929.4 cubic inches.

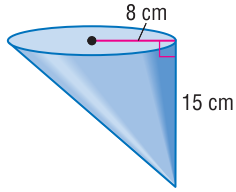
Find the volume of each oblique cone.

Interactive Lab
geometryonline.com

3A.



3B.

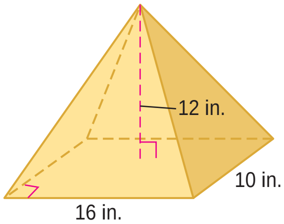


CHECK Your Understanding

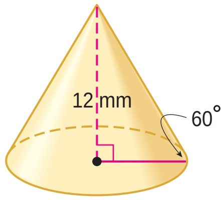
Find the volume of each pyramid or cone. Round to the nearest tenth if necessary.

Example 1, 2, 3
(pp. 738, 739)

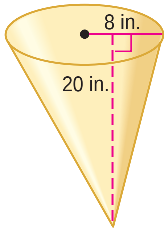
1.



2.



3.



Example 2
(p. 739)

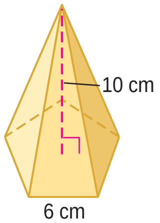
4. MUSEUMS The skydome of the National Corvette Museum in Bowling Green, Kentucky, is a conical building. If the height is 100 feet and the area of the base is about 15,400 square feet, find the volume of air that the heating and cooling systems would have to accommodate. Round to the nearest tenth.

Exercises

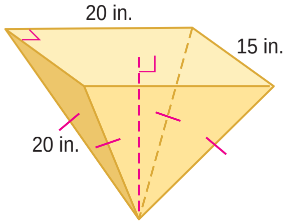
HOMEWORK HELP	
For Exercises	See Examples
5–7	1
8–10	2
11–13	3

Find the volume of each pyramid or cone. Round to the nearest tenth if necessary.

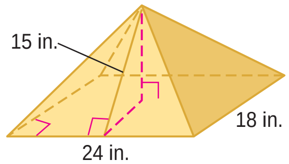
5.



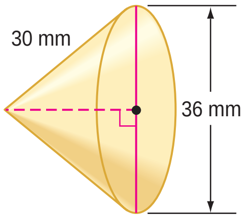
6.



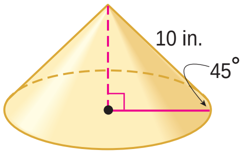
7.



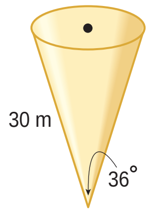
8.



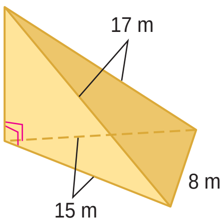
9.



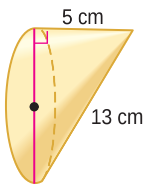
10.



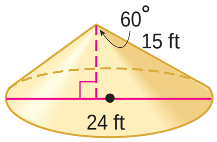
11.



12.

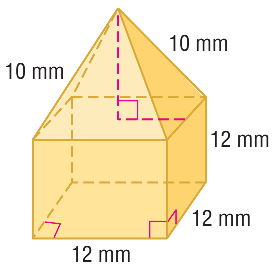


13.

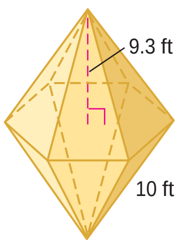


COMPOSITE SOLIDS **Find the volume of each solid. Round to the nearest tenth.**

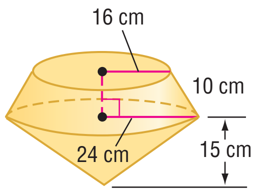
14.



15.



16.

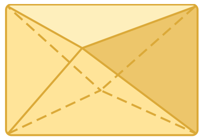


Real-World Link

There are three major types of volcanoes: cinder cone, shield dome, and composite. Shield dome volcanoes are formed almost exclusively from molten lava. Composite volcanoes are formed from layers of molten lava and hardened chunks of lava.

Source: pubs.usgs.gov

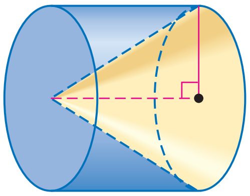
17. SCIENCE Marta is making a model of an alum crystal for science class. The crystal shape is a composite of two congruent rectangular pyramids. If the base of each pyramid is to be 1 inch by $1\frac{1}{2}$ inches and the total height of the model is 4 inches, determine the volume of clay needed to make the model.



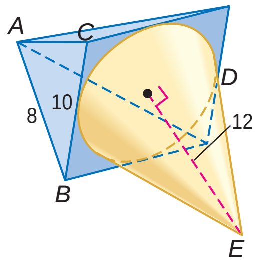
18. VOLCANOES Mount Rainier, which is an active, composite volcano in Washington, is 4392 meters tall and is about 18 kilometers across at its base. Assume that Mount Rainier can be modeled by a cone. Find the volume in cubic kilometers of rock it would take to fill Mt. Rainier.

19. HISTORY The Great Pyramid of Khufu is a square pyramid. The lengths of the sides of the base are 755 feet. The original height was 481 feet. The current height is 449 feet. What volume of material has been lost?

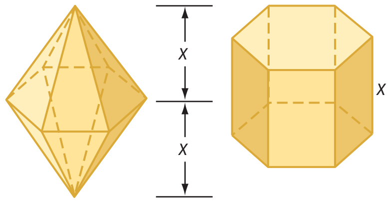
20. PROBABILITY What is the probability of choosing a point inside the cylinder, but not inside the cone that has the same base and height as the cylinder?



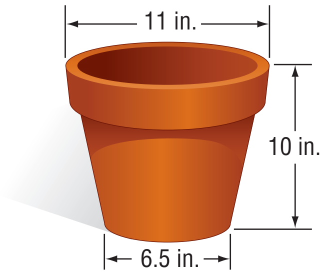
21. A pyramid with a square base is next to a circular cone as shown at the right. The circular base is inscribed in the square. Isosceles $\triangle ABC$ is perpendicular to the base of the pyramid. $\overline{DE}$ is a slant height of the cone. Find the volume of the figure.



22. Write a ratio comparing the volumes of the solids shown. Explain your answer.



23. GARDENING Potting soil is sold in 3 cubic feet bags. What volume of soil will fill one planter? How many planters could be filled with one bag of potting soil? (*Hint*: Draw the cone that contains the frustum.)



24. MEASUREMENT Find a real-world object in the shape of a pyramid or cone. Measure the object and find its volume.

EXTRA PRACTICE

See pages 826, 840.

Math Online

Self-Check Quiz at geometryonline.com

H.O.T. Problems

25. REASONING Describe the effect on the volumes of a cone and a pyramid if the dimensions are doubled.

26. REASONING Explain how the volume of a pyramid is related to that of a prism with the same height and a base congruent to that of the pyramid.

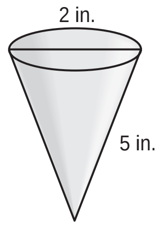
27. OPEN ENDED Draw and label two cones with different dimensions, but with the same volume.

28. **CHALLENGE** Find the volume of a regular tetrahedron with one side measuring 12 inches.

29. **Writing in Math** Refer to the information on page 737 to explain why an architect would use geometry. Discuss how a pyramidal building allows more light onto the street than a rectangular prism building.

STANDARDIZED TEST PRACTICE

30. An ice cream cone is shown below.



What is the volume of this ice cream cone?

A $\frac{2\pi}{3}\sqrt{6}$ in^3

B $\frac{5\pi}{3}$ in^3

C $\frac{\pi}{3}\sqrt{21}$ in^3

D $\frac{4\pi}{3}\sqrt{21}$ in^3

31. **REVIEW** The table shows the results of a number cube being rolled.

Outcome	Frequency
1	2
2	8
3	5
4	3
5	6
6	6

Based on these results, what is the experimental probability of rolling a 5?

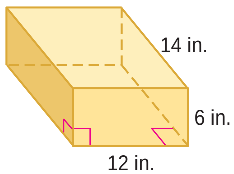
F $\frac{1}{6}$

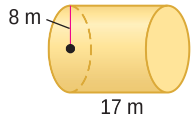
G $\frac{1}{5}$

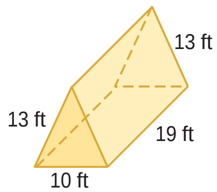
H 0.5

J 0.6

Find the volume of each prism or cylinder. Round to the nearest tenth. (Lesson 13-1)

32.


33.


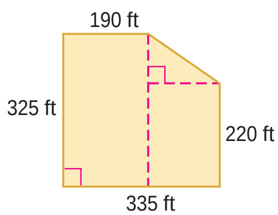
34.


Find the surface area of each sphere. Round to the nearest tenth if necessary. (Lesson 12-7)

35. The circumference of a great circle is 86 centimeters.

36. The area of a great circle is 64.5 square yards.

37. **BASEBALL** A baseball diamond has the shape of a rectangle with a corner cut out as shown at the right. What is the total area of the baseball field? (Lesson 11-4)



GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression. Round to the nearest hundredth.** (Page 780)

38. $4\pi r^2,\ r = 3.4$

39. $\frac{4}{3}\pi r^3,\ r = 7$

40. $4\pi r^2,\ r = 12$

13-3 Volumes of Spheres

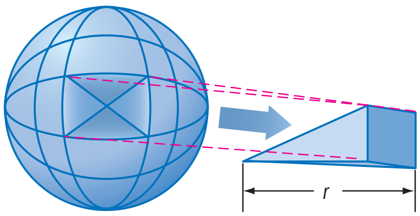
Main Ideas

- Find volumes of spheres.
- Solve problems involving volumes of spheres.

Eratosthenes was an ancient Greek mathematician who estimated the circumference of Earth. He assumed that Earth was a sphere and estimated that the circumference was about 40,000 kilometers. From the circumference, the radius of Earth can be calculated. Then the volume of Earth can be determined.

Volumes of Spheres You can relate finding a formula for the volume of a sphere to finding the volume of a right pyramid and the surface area of a sphere.

Suppose the space inside a sphere is separated into infinitely many near-pyramids, all with vertices located at the center of the sphere. Observe that the height of these pyramids is equal to the radius r of the sphere. The sum of the areas of all the pyramid bases equals the surface area of the sphere.



Each pyramid has a volume of $\frac{1}{3}Bh$, where B is the area of its base and h is its height. The volume of the sphere is equal to the sum of the volumes of all of the small pyramids.

$$V = \frac{1}{3}B_1h_1 + \frac{1}{3}B_2h_2 + \frac{1}{3}B_3h_3 + \ldots + \frac{1}{3}B_nh_n$$ Sum of the volumes of all of the pyramids

$$= \frac{1}{3}B_1r + \frac{1}{3}B_2r + \frac{1}{3}B_3r + \ldots + \frac{1}{3}B_nr$$ Replace h with r.

$$= \frac{1}{3}r(B_1 + B_2 + B_3 + \ldots + B_n)$$ Distributive Property

$$= \frac{1}{3}r(4\pi r^2)$$ Replace $B_1 + B_2 + B_3 + \ldots + B_n$ with $4\pi r^2$.

$$= \frac{4}{3}\pi r^3$$ Simplify.

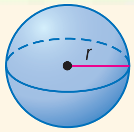
Study Tip

Look Back

Recall that the surface area of a sphere, $4\pi r^2$, is equal to $B_1 + B_2 + B_3 + \ldots + B_n$. To review **surface area of a sphere,** see Lesson 12-7.

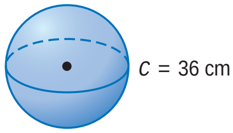
KEY CONCEPT — Volume of a Sphere

If a sphere has a volume of V cubic units and a radius of r units, then $V = \frac{4}{3}\pi r^3$.



EXAMPLE 1 Volumes of Spheres

Find the volume of the sphere. Round to the nearest tenth.



First find the radius of the sphere.

$C = 2\pi r$	Circumference of a circle
$36 = 2\pi r$	$C = 36$
$\frac{18}{\pi} = r$	Solve for r.

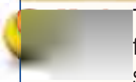
Now find the volume.

$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$= \frac{4}{3}\pi\left(\frac{18}{\pi}\right)^3$	$r = \frac{18}{\pi}$
$\approx 787.9 \text{ cm}^3$	Use a calculator.

Find the volume of a sphere with the given measure. Round to the nearest tenth.

1A. radius: 24 in. **1B.** diameter: 15 cm

Cross-Curricular Project

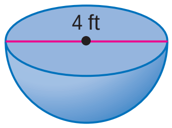


The formulas for the surface area and volume of a sphere can help you find the surface area and volume of the hemisphere. Visit geometryonline.com to continue work on your project.

The volume of a hemisphere is one-half the volume of the related sphere.

EXAMPLE 2 Volume of a Hemisphere

Find the volume of the hemisphere. Round to the nearest tenth.



$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$	Volume of a hemisphere
$= \frac{2}{3}\pi(2^3)$	$r = 2$
$\approx 16.8 \text{ ft}^3$	Use a calculator.

CHECK Your Progress

2. Find the volume of a hemisphere in which the radius of the sphere is 3.4 yards.

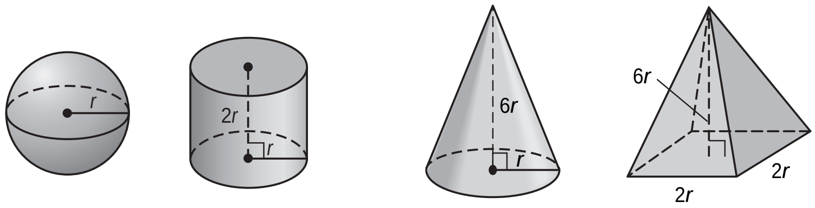
Solve Problems Involving Volumes of Spheres Often spherical objects are contained in other solids. A comparison of the volumes is necessary to know if one object can be contained in the other.

Test-Taking Tip

Estimation
You can estimate the volume of each solid. This can help you eliminate one or two answer choices.

STANDARDIZED TEST EXAMPLE **Volume Comparison**

3 Compare the volumes of the sphere, cylinder, cone, and pyramid. Determine which volume is the greatest.



A sphere
B cylinder
C cone
D pyramid

Read the Test Item

You are asked to compare the volumes of four solids.

Solve the Test Item

Volume of the sphere: $\frac{4}{3}\pi r^3$

Volume of the cylinder: $\pi r^2 h = \pi r^2(2r)$ $h = 2r$
$= 2\pi r^3$ Simplify.

Volume of the cone: $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(6r)$ $h = 6r$
$= 2\pi r^3$ Simplify.

Volume of the pyramid: $\frac{1}{3}Bh = \frac{1}{3}(4r^2)(6r)$ $B = 4r^2, h = 6r$
$= 8r^3$ Simplify.

Compare the volumes. Since 8 is greater than $\frac{4}{3}\pi$ and 2π, $8r^3$ is greater than $\frac{4}{3}\pi r^3$ and $2\pi r^3$. The volume of the pyramid is the greatest. D is the correct answer.

3. Compare the volumes of a sphere with radius 8 inches, a square pyramid with a base that is 15 inches long and height of 20 inches, a cone with height of 20 inches and a base with radius 7.5 inches, and a cylinder with a height of 16 inches and a base with radius 8 inches. Determine which volume is the greatest.
F sphere **G** square pyramid **H** cone **J** cylinder

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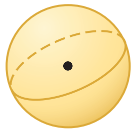
CHECK Your Understanding

Example 1 (p. 744)

Find the volume of each sphere. Round to the nearest tenth.

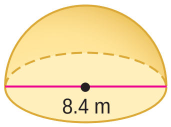


1. The radius is 13 inches.
2. $C = 18$ cm
3. The diameter is 12.5 centimeters.

Example 2 (p. 744)

Find the volume of each hemisphere. Round to the nearest tenth.

4.



5. The radius is 5.2 miles.

Example 3 (p. 745)

6. **STANDARDIZED TEST PRACTICE** Compare the volumes of a sphere with a radius of 5 inches, a cone with a height of 20 inches and a base with a diameter of 10 inches, a rectangular prism with length 5 inches, width 8 inches, and height 10 inches, and a square pyramid with a base of length 10 inches and a height of 18 inches. Determine which volume is the least.

A cone **B** prism **C** pyramid **D** sphere

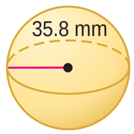
Exercises

HOMEWORK HELP	
For Exercises	See Examples
7–18	1–2
19–22	3

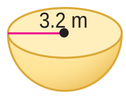
Find the volume of each sphere or hemisphere. Round to the nearest tenth.

7. sphere: radius is 7.62 m
8. sphere: diameter is 33 in.
9. sphere: diameter is 18.4 ft
10. sphere: radius is $\frac{\sqrt{3}}{2}$ cm
11. hemisphere: radius is 8.4 yd
12. hemisphere: diameter is 21.8 cm
13. $C = 24$ in.

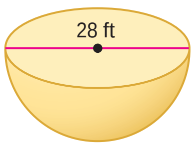
14.



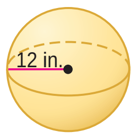
15.



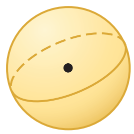
16.



17.



18. $C = 48$ cm



19. **ASTRONOMY** The diameter of the Moon is 3476 kilometers. Find its volume.

20. **SPORTS** A golf ball has a diameter of 4.3 centimeters, and a tennis ball has a diameter of 6.9 centimeters. How much greater is the volume of the tennis ball?

21. **TENNIS** Find the volume of the empty space in a cylindrical tube of three tennis balls. The diameter of each ball is about 2.5 inches. The cylinder is 2.5 inches in diameter and is 7.5 inches tall.

22. **MEASUREMENT** Measure the circumference of a real-world sphere. Find the volume of the sphere.

23. **SNOW CONES** Suppose a paper cone is 4 inches tall and the diameter of the base is 3 inches. A spherical scoop of shaved ice with a diameter of 3 inches rests on the top of the cone. If all the ice melts into the cone, will the cone overflow? Explain. If not, what percent of the cone will be filled?

24. **PROBABILITY** Find the probability of choosing a point at random inside a sphere that has a radius of 6 centimeters and is inscribed in a cylinder.

25. **BALLOONS** A spherical helium-filled balloon with a diameter of 30 centimeters can lift an object of about 14 grams. Find the size of a balloon that could lift a person who weighs 65 kilograms.

26. **BALLOONS** Troy inflates a spherical balloon to a circumference of about 14 inches. He then adds more air to the balloon until the circumference is about 18 inches. What volume of air was added to the balloon?

Real-World Career

Agricultural Engineer

Agricultural engineers use engineering techniques to develop equipment and methods that assist farmers in efficient agricultural production.

Source: www.bls.gov

Math Online

For more information, visit geometryonline.com

Find the volume of each sphere or hemisphere. Round to the nearest tenth.

27. The surface area of a sphere is 784π square inches.

28. A hemisphere has a surface area of 18.75π square meters.

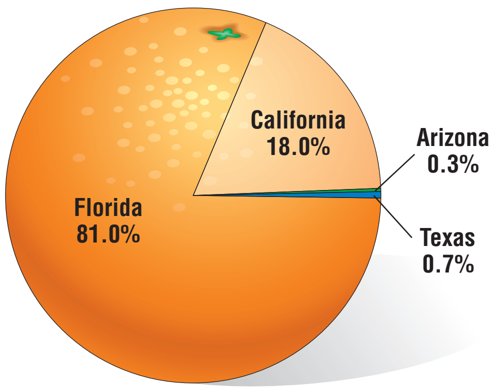
29. **ARCHITECTURE** The Pantheon in Rome is able to contain a perfect sphere. The building is a cylinder 142 feet in diameter with a hemispherical domed roof. The total height is 142 feet. Find the volume of the interior of the Pantheon.

FOOD **For Exercises 30 and 31, use the following information.**
Suppose the orange in the graphic is a sphere with a radius of 4 centimeters. Round to the nearest tenth.

30. What is the volume of the portion of the sphere that represents orange production in California?

31. What is the surface area of the portion of the sphere that represents orange production in Florida?

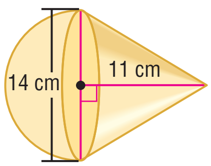
Orange Production



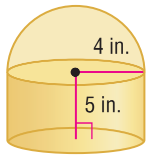
Source: USDA

COMPOSITE SOLIDS **Find the volume of each solid. Round to the nearest tenth.**

32.



33.



EXTRA PRACTICE

See pages 826, 840.

Math Online

Self-Check Quiz at geometryonline.com

34. **VITAMINS** A vitamin capsule consists of a cylinder with a hemisphere on each end. The capsule is 12 millimeters long and 4 millimeters thick. What is the volume of the capsule?

H.O.T. Problems

35. **CHALLENGE** Find the volume of a sphere that is circumscribed about a cube with a volume of 216 cubic inches.

36. FIND THE ERROR Winona and Kenji found the volume of a sphere with a radius of 12 centimeters. Who is correct? Explain your reasoning.

Winona	Kenji
$V = \frac{4}{3}\pi(12)^3$	$V = \frac{4}{3}$à$(12)^3$
$= 4\pi(4)^3$	$= \frac{4}{3}$à(1728)
$= 256\pi$ cm^3	$= 2304$à cm^3

37. Which One Doesn't Belong? Identify the solid that does not belong with the other three. Explain your reasoning.

sphere with radius r	cylinder with radius r, height h	square prism with $B = r^2$, $h =$ àr	cone with radius r, height $3h$

38. Writing in Math Describe how you can find the volume of Earth. Include the radius and volume of Earth from this estimate. Then use the Internet or other source to find current calculations for the volume of Earth.

STANDARDIZED TEST PRACTICE

39. If the radius of a sphere is increased from 3 units to 5 units, what percent would the volume of the smaller sphere be of the volume of the larger sphere?

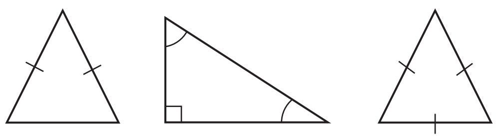
A 21.6%

B 40%

C 60%

D 463%

40. REVIEW Which statement about the triangles is true?



F All the triangles are scalene.

G All the triangles are equiangular.

H All the triangles are equilateral.

J All the triangles are isosceles.

Spiral Review

Find the volume of each cone. Round to the nearest tenth. (Lesson 13-2)

41. height = 9.5 meters, radius = 6 meters

42. height = 7 meters, diameter = 15 meters

43. REFRIGERATORS A refrigerator has a volume of 25.9 cubic feet. If the interior height is 5.0 feet and the width is 2.4 feet, find the depth. (Lesson 13-1)

Write an equation for each circle. (Lesson 10-8)

44. center at $(2, -1)$, $r = 8$

45. diameter with endpoints at $(5, -4)$ and $(-1, 6)$

GET READY for the Next Lesson

PREREQUISITE SKILL Simplify. (Page 780)

46. $(2a)^2$

47. $(3x)^3$

48. $\left(\frac{5a}{b}\right)^2$

49. $\left(\frac{2k}{5}\right)^3$

CHAPTER 13

Mid-Chapter Quiz

Lessons 13-1 through 13-3

1. The volume of a cylinder is 706.9 cubic feet, and its height is 9 feet. Find the diameter of the cylinder. (Lesson 13-1)

2. The volume of a right rectangular prism is 2912 cubic centimeters, and the area of each base is 91 square centimeters. Find the length of the lateral edge of the prism. (Lesson 13-1)

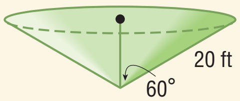
3. **FOOD** A canister of oatmeal is 10 inches tall with a diameter of 4 inches. Find the maximum volume of oatmeal that the canister can hold to the nearest tenth. (Lesson 13-1)

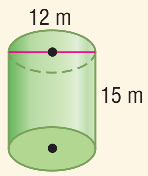
4. **MULTIPLE CHOICE** A rectangular swimming pool has a volume of 16,320 cubic feet, a depth of 8 feet, and a length of 85 feet. What is the width of the swimming pool? (Lesson 13-1)

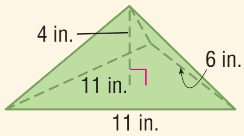
   A 24 ft **C** 192 ft

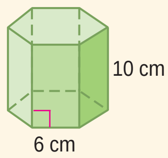
   B 48 ft **D** 2040 ft

Find the volume of each solid. Round to the nearest tenth. (Lessons 13-1 and 13-2)

5.


6.


7.


8.


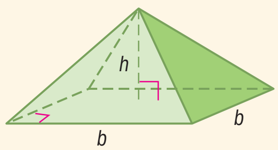
9. **MULTIPLE CHOICE** Which of the following is the volume of the square pyramid if $b = 2h$? (Lesson 13-2)

   F $\frac{h^3}{3}$

   G $\frac{4h^3}{3}$

   H $4h^3$

   J $\frac{8h^3 - 4h}{3}$



VOLCANOES For Exercises 10–13, use the following information. (Lesson 13-2)

The slope of a volcano is the angle made by the side of the cone and a horizontal line. Find the volume of material in each volcano, assuming that it is a solid cone.

Volcano	Characteristics
Mauna Loa	4170 m tall, 103 km across at base
Mount Fuji	3776 m tall, slope of 9°
Paricutin	410 m tall, 33° slope
Vesuvius	22.3 km across at base, 1220 m tall

10. Mauna Loa
11. Mount Fuji
12. Paricutin
13. Vesuvius

Find the volume of each sphere. Round to the nearest tenth. (Lesson 13-3)

14. radius = 25.3 ft
15. diameter = 36.8 cm

Find the volume of each hemisphere. Round to the nearest tenth.

16. radius = 47.9 in.
17. diameter = 10.2 m

FOOD For Exercises 18 and 19, use the following information. (Lesson 13-2)

Suppose a sugar cone is 10 centimeters deep and has a diameter of 4 centimeters. A spherical scoop of ice cream with a diameter of 4 centimeters rests on top of the cone.

18. If all the ice cream melts into the cone, will the cone overflow? Explain.

19. If the cone does not overflow, what percent of the cone will be filled?

20. The drama class is using a large foam sphere as a boulder in their next production. The circumference of the foam boulder is 21 feet. The backstage area where the drama class stores all their props is 10 feet × 12 feet × 8 feet. Will the foam boulder fit backstage? Explain. (Lessons 13-3)

13-4 Congruent and Similar Solids

Main Ideas

- Identify congruent or similar solids.
- State the properties of similar solids.

New Vocabulary

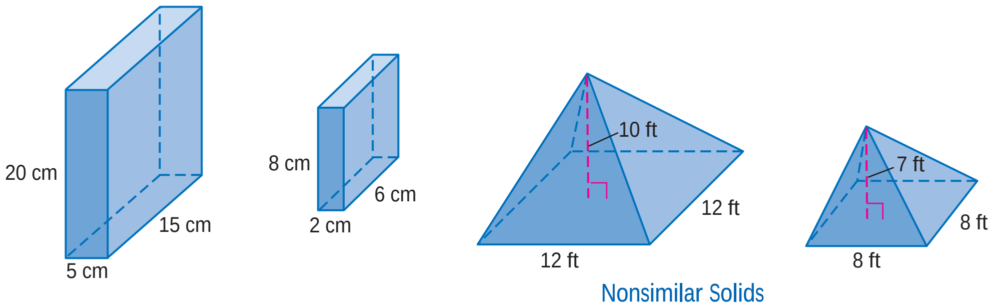
similar solids
congruent solids

GET READY for the Lesson

People collect miniatures of race cars, farm equipment, and monuments such as the Statue of Liberty. The scale factors commonly used for miniatures include 1:16, 1:24, 1:32, and 1:64. One of the smallest miniatures has a scale factor of 1:1000. If a car is 108 inches long, then a 1:24 scale model would be 108 ÷ 24 or 4.5 inches long.

Congruent or Similar Solids **Similar solids** are solids that have exactly the same shape but not necessarily the same size. You can determine if two solids are similar by comparing the ratios of corresponding linear measurements.

In two similar polyhedra, all of the corresponding faces are similar, and all of the corresponding edges are proportional.
All spheres are similar, just as all circles are similar.



Similar Solids

Nonsimilar Solids

Review Vocabulary

Scale Factor: the ratio of the lengths of two corresponding sides of two similar polygons or two similar solids (Lesson 7-2)

In the similar solids above, $\frac{8}{20} = \frac{2}{5} = \frac{6}{15}$. In the nonsimilar solids, $\frac{10}{7} \neq \frac{12}{8} = \frac{12}{8}$. Recall that the ratio of the measures is called the *scale factor*. If the ratio of corresponding measurements of two solids is 1:1, then the solids are congruent. For two solids to be congruent, all of the following conditions must be met. **Congruent solids** are exactly the same shape and exactly the same size. They are a special case of similar solids. They have a scale factor of 1.

KEY CONCEPT — Similar and Congruent Solids

Two solids are similar if the ratios of corresponding linear measures are proportional. Two solids are congruent if:

- the corresponding angles are congruent,
- the corresponding edges are congruent,
- the corresponding faces are congruent, and
- the volumes are equal.

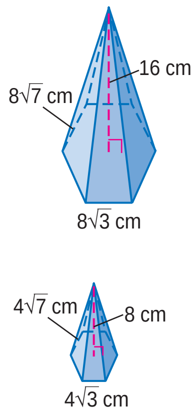
Review Vocabulary

Regular Polygonal Pyramid: a pyramid with a base that is a regular polygon (Lesson 1-7)

EXAMPLE Similar and Congruent Solids

1 **Determine whether each pair of solids is *similar, congruent,* or *neither*.**

a.



Find the ratios between the corresponding parts of the regular hexagonal pyramids.

$\frac{\text{base edge of larger pyramid}}{\text{base edge of smaller pyramid}} = \frac{8\sqrt{3}}{4\sqrt{3}}$ Substitution

$= 2$ Simplify.

$\frac{\text{height of larger pyramid}}{\text{height of smaller pyramid}} = \frac{16}{8}$ Substitution

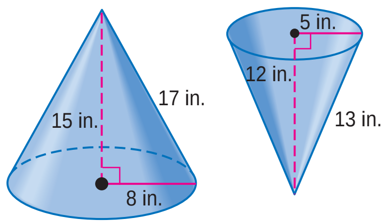
$= 2$ Simplify.

$\frac{\text{lateral edge of larger pyramid}}{\text{lateral edge of smaller pyramid}} = \frac{8\sqrt{7}}{4\sqrt{7}}$ Substitution

$= 2$ Simplify.

The ratios of the measures are equal, so we can conclude that the pyramids are similar. Since the scale factor is not 1, the solids are not congruent.

b.



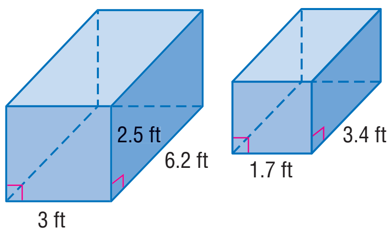
Compare the ratios between the corresponding parts of the cones.

$\frac{\text{radius of larger cone}}{\text{radius of smaller cone}} = \frac{8}{5}$ Substitution

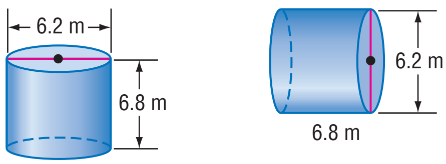
$\frac{\text{height of larger cone}}{\text{height of smaller cone}} = \frac{15}{12}$ Substitution

Since the ratios are not the same, there is no need to find the ratio of the slant heights. The cones are not similar.

1A.



1B.



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Properties of Similar Solids You can investigate the relationships between similar solids using spreadsheets.

SPREADSHEET INVESTIGATION

Explore Similar Solids

COLLECT THE DATA

Step 1 In Column A, enter the labels as shown. Columns B, C, D, E, and F will be used for five similar prisms.

Step 2 Enter the formula for the surface area of the prism in cell B4. Copy the formula into the other cells in row 4.

Step 3 Write a similar formula to find the volume of the prism. Copy the formula in the cells in row 5.

Step 4 Enter the formula B1*C6 in cell C1, enter B2*C6 in cell C2 and enter B3*C6 in cell C3. These formulas find the dimensions of prism C based on the dimensions of prism B and the scale factor you enter. Copy the formulas into columns D, E, and F.

Step 5 Type the formula C4/B4 in cell C7, type D4/B4 in cell D7, and so on. This formula will find the ratio of the surface area of prism B to the surface areas of each of the other prisms.

Step 6 Write a formula for the ratio of the volume of prism C to the volume of prism B. Enter the formula in cell C8. Enter similar formulas in the cells in row 8.

Step 7 Use Columns D, E, and F to find the surface areas, volumes, and ratios for prisms with scale factors of 3, 4, and 5.

Similar Solids.xls

◇	A	B	C
1	length	1	2
2	width	4	8
3	height	6	12
4	surface area	68	272
5	volume	24	192
6	scale factor		2
7	ratios of surface area		4
8	ratios of volume		8

Sheet 1 Sheet 2 Sheet 3

ANALYZE

1. Compare the ratios in cells 6, 7, and 8 of columns C, D, and E. What do you observe?

2. **Make a conjecture** about the ratio of the surface areas of two solids if the scale factor is a:b. Change the scale factors in the spreadsheet to verify your conjecture.

3. What is the ratio of the volumes of two solids if the scale factor is a:b?

Study Tip

Standardized Tests

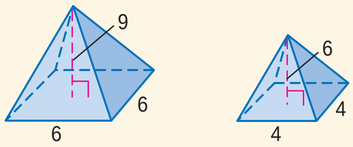
Problems using Theorem 13.1 are frequently on standardized tests. Be sure to note that the solids must be similar for the theorem to apply.

The Spreadsheet Investigation suggests the following theorem.

THEOREM 13.1

If two solids are similar with a scale factor of $a:b$, then the surface areas have a ratio of $a^2:b^2$, and the volumes have a ratio of $a^3:b^3$.

Example:



Scale factor 3:2

Ratio of surface areas $3^2:2^2$ or 9:4

Ratio of volumes $3^3:2^3$ or 27:8

EXAMPLE Mirror Balls

2 **ENTERTAINMENT** **Mirror balls are spheres that are covered with reflective tiles. One ball has a diameter of 4 inches, and another has a diameter of 20 inches.**

a. Find the scale factor of the two spheres.

Write the ratio of the corresponding measures of the spheres.

$$\frac{\text{diameter of the smaller sphere}}{\text{diameter of the larger sphere}} = \frac{4}{20} \quad \text{Substitution}$$

$$= \frac{1}{5} \quad \text{Simplify.}$$

The scale factor is 1:5.

b. Find the ratio of the surface areas of the two spheres.

If the scale factor is $a:b$, then the ratio of the surface areas is $a^2:b^2$.

$$\frac{\text{surface area of the smaller sphere}}{\text{surface area of the larger sphere}} = \frac{a^2}{b^2} \quad \text{Theorem 13.1}$$

$$= \frac{1^2}{5^2} \quad a = 1 \text{ and } b = 5$$

$$= \frac{1}{25} \quad \text{Simplify.}$$

The ratio of the surface areas is 1:25.

c. Find the ratio of the volumes of the two spheres.

If the scale factor is $a:b$, then the ratio of the volumes is $a^3:b^3$.

$$\frac{\text{volume of the smaller sphere}}{\text{volume of the larger sphere}} = \frac{a^3}{b^3} \quad \text{Theorem 13.1}$$

$$= \frac{1^3}{5^3} \quad a = 1 \text{ and } b = 5$$

$$= \frac{1}{125} \quad \text{Simplify.}$$

The ratio of the volumes is 1:125.

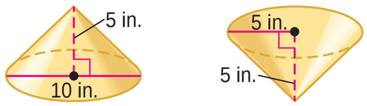
2. SPORTS A regulation volleyball has a circumference of about 66 centimeters. A smaller ball sold with a children's net has a circumference of about 52 centimeters. Find the scale factor and the ratios of the surface areas and volumes of the two spheres.

CHECK Your Understanding

Example 1 **(p. 751)**

Determine whether each pair of solids is *similar, congruent,* or *neither*.

1.



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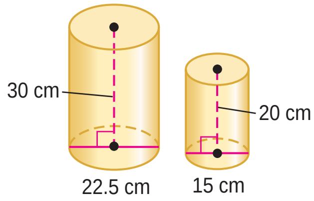


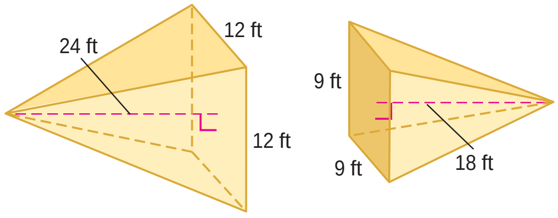
Example 2 **(p. 753)**

For Exercises 3–5, refer to the pyramids on the right.

3. Find the scale factor of the two pyramids.

4. Find the ratio of the surface areas of the two pyramids.

5. Find the ratio of the volumes of the two pyramids.



LAWN ORNAMENTS **For Exercises 6–8, use the following information.** There are two gazing balls in a garden. One has a diameter of 6 inches, and the other has a diameter of 18 inches.

6. Find the scale factor of the two gazing balls.

7. Determine the ratio of the surface areas of the two spheres.

8. What is the ratio of the volumes of the gazing balls?

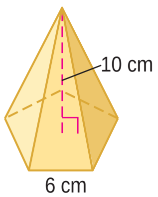
Exercises

HOMEWORK HELP

For Exercises	See Examples
9–14	1
15–21	2

Determine whether each pair of solids is *similar, congruent,* or *neither*.

9.



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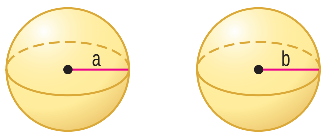

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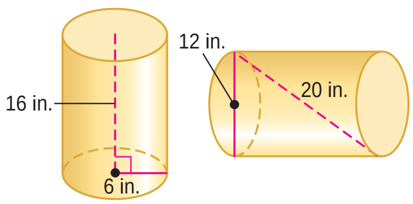

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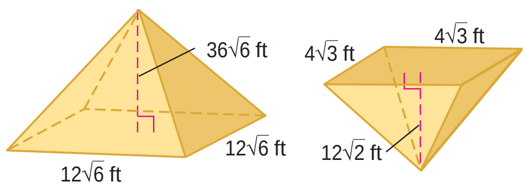

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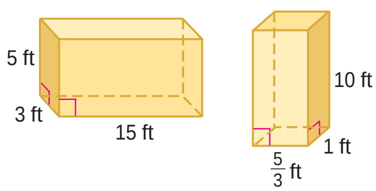

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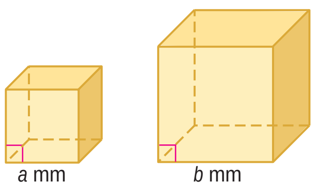


Real-World Link

The National Collegiate Athletic Association (NCAA) states that the maximum circumference of a basketball for men is 30 inches. The maximum circumference of a women's basketball is 29 inches.

Source: www.ncaa.org

BASKETBALL **For Exercises 15–17, use the information at the left.** Find the indicated ratio of the smaller ball to the larger ball.

15. scale factor **16.** ratio of surface areas **17.** ratio of the volumes

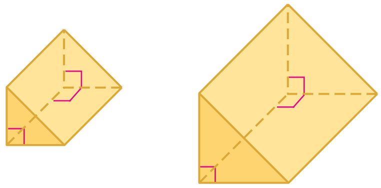
For Exercises 18–21, refer to the two similar right prisms with a scale factor of 1:2.5.

18. Find the ratio of the perimeters of the bases.

19. What is the ratio of the surface areas?

20. What is the ratio of the volumes?

21. Suppose the volume of the smaller prism is 48 cubic inches. Find the volume of the larger prism.



22. **ARCHITECTURE** To encourage recycling, the people of Rome, Italy, built a model of Basilica di San Pietro from empty beverage cans. The model was built to a 1:5 scale. The model was a rectangular prism that measured 26 meters high, 49 meters wide, and 93 meters long. Find the dimensions of the actual Basilica di San Pietro.

23. **SCULPTURE** The sculpture shown at the right is a scale model of a cornet in Texas. If the sculpture is 26 feet long and a standard cornet is 14 inches long, what is the scale factor of the sculpture to a standard cornet?

Determine whether each statement is *sometimes, always,* or *never* true. Justify your answer.

24. Two spheres are similar.

25. Congruent solids have equal surface areas.

26. Similar solids have equal volumes.

27. A pyramid is similar to a cone.

28. Cones and cylinders with the same height and base are similar.

29. Nonsimilar solids have different surface areas.

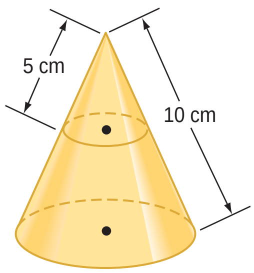
30. The diameters of two similar cones are in the ratio 5 to 6. If the volume of the smaller cone is 125π cubic centimeters and the diameter of the larger cone is 12 centimeters, what is the height of the larger cone?

31. **FESTIVALS** The world's largest circular pumpkin pie was made for the Circleville Pumpkin Show in Circleville, Ohio. The diameter was 5 feet. If the record pie is similar to an 8-inch pie that weighs $1\frac{1}{4}$ pounds, estimate the weight of the record pie. Explain your solution.

For Exercises 32 and 33, use the following information. When a cone is cut by a plane parallel to its base, a cone similar to the original is formed.

32. What is the ratio of the volume of the frustum to that of the original cone? to the smaller cone?

33. What is the ratio of the lateral area of the frustum to that of the original cone? to the smaller cone?



EXTRA PRACTICE

See pages 826, 840.

Math Online

Self-Check Quiz at geometryonline.com

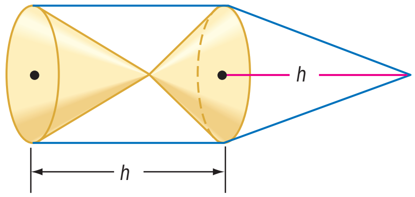
34. **MEASUREMENT** Measure two real-world objects that appear to be similar. Are they similar? Explain.

H.O.T. Problems

35. **OPEN ENDED** Draw and label the dimensions of a pair of cones that are similar and a pair of cones that are neither similar nor congruent.

36. **REASONING** Explain the relationship between the surface areas of similar solids and volumes of similar solids.

CHALLENGE For Exercises 37 and 38, refer to the figure.



37. Is it possible for the two cones inside the cylinder to be congruent? Explain.

38. Is the volume of the cone on the right equal to, greater than, or less than the sum of the volume of the cones inside the cylinder? Explain.

39. **Writing in Math** Describe how the geometry of similar solids can be applied to miniature collectibles. Include the scale factors that are commonly used. If a miniature is 4.5 inches with a scale factor of 1:24, then how long is the object?

STANDARDIZED TEST PRACTICE

40. The ratio of the surface areas of two similar solids is 4:9. What is the ratio between the volumes of the two solids?

A 64 to 729 **C** 8 to 27

B 16 to 81 **D** 2 to 3

41. Which statement is true about the volume of a rectangular prism?

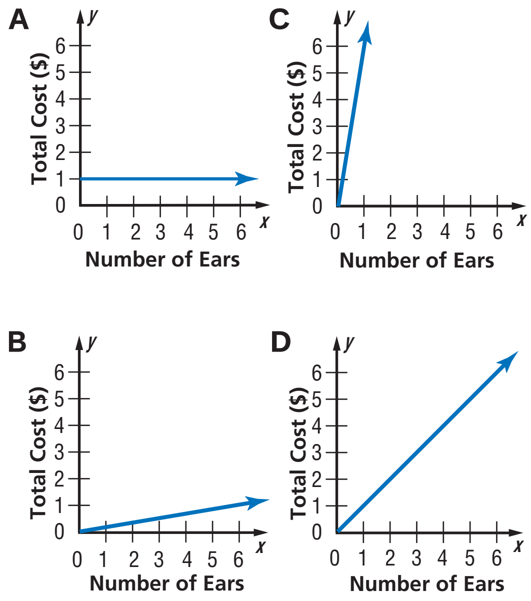
F The volume of the prism depends on the sum of the length, width, and height of the prism.

G The volume of the prism depends only on the product of the length and the width of the prism.

H The volume of the prism depends only on the sum of the width and the height.

J The volume of the prism depends only on the product of the length, width, and height of the prism.

42. **REVIEW** Corn is on sale at the price of 6 ears for $1. Which graph shows the relationship between the number of ears of corn purchased and the total cost?



Find the volume of each sphere. Round to the nearest tenth. (Lesson 13-3)

43. diameter = 8 feet

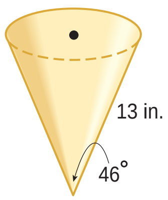
44. radius = 9.5 meters

45. radius = 15.1 centimeters

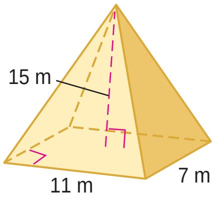
46. diameter = 23 inches

Find the volume of each pyramid or cone. Round to the nearest tenth. (Lesson 13-2)

47.



48.



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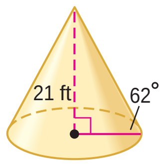
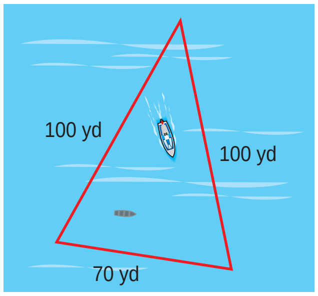


Find the radius of the base of each cylinder. Round to the nearest tenth. (Lesson 12-4)

50. The surface area is 430 square centimeters, and the height is 7.4 centimeters.

51. The surface area is 224.7 square yards, and the height is 10 yards.

NAVIGATION **For Exercises 52–54, use the following information.** As part of a scuba diving exercise, a 12-foot by 3-foot rectangular-shaped rowboat was sunk in a quarry. A boat takes a scuba diver to a random spot in the enclosed section of the quarry and anchors there so that the diver can search for the rowboat. (Lesson 11-5)



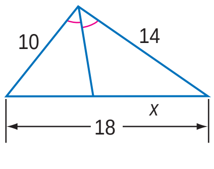
52. What is the approximate area of the enclosed section of the quarry?

53. What is the area of the rowboat?

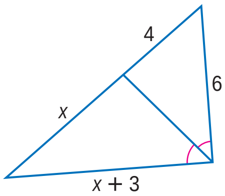
54. What is the probability that the boat will anchor over the sunken rowboat?

Find *x*. (Lesson 7-5)

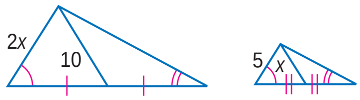
55.



56.



57.



Find the range for the measure of the third side of a triangle given the measures of two sides. (Lesson 5-4)

58. 7 and 12

59. 14 and 23

60. 22 and 34

61. 15 and 18

62. 30 and 30

63. 64 and 88

GET READY for the Next Lesson

PREREQUISITE SKILL **Determine whether the ordered pair is on the graph of the given equation. Write *yes* or *no*.** (Lesson 1-1)

64. $y = 3x + 5$, $(4, 17)$

65. $y = -4x + 1$, $(-2, 9)$

66. $y = 7x - 4$, $(-1, 3)$

13-5 Coordinates in Space

Main Ideas

- Graph solids in space.
- Use the Distance and Midpoint Formulas for points in space.

New Vocabulary

ordered triple

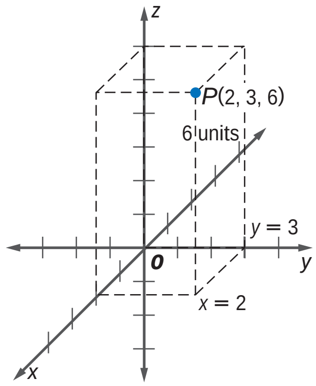
GET READY for the Lesson

The initial step in computer animation is creating a three-dimensional image. A *mesh* is created first. This is an outline that shows the size and shape of the image. Then the image is *rendered*, adding color and texture. The image is animated using software. Coordinates are used to describe the location of each point in the image.

Graph Solids in Space

To describe the location of a point on the coordinate plane, we use an ordered pair of two coordinates. In space, each point requires three numbers, or coordinates, to describe its location because space has three dimensions. In space, the *x*-, *y*-, and *z*-axes are perpendicular to each other.

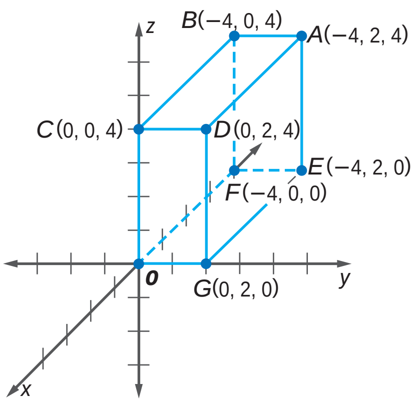
A point in space is represented by an **ordered triple** of real numbers (x, y, z). In the figure at the right, the ordered triple (2, 3, 6) locates point *P*. Notice that a rectangular prism is used to show perspective.



EXAMPLE Graph a Rectangular Solid

1 **Graph a rectangular solid that has $A(-4, 2, 4)$ and the origin as vertices. Label the coordinates of each vertex.**

- Plot the *x*-coordinate first. Draw a segment from the origin 4 units in the negative direction.
- To plot the *y*-coordinate, draw a segment 2 units in the positive direction.
- Next, to plot the *z*-coordinate, draw a segment 4 units long in the positive direction.
- Label the coordinate *A*.
- Draw the rectangular prism and label each vertex.



Study Tip

Drawing in Three Dimensions

Use the properties of a rectangluar prism to correctly locate the *z*-coordinate. *A* is the vertex farthest from the origin.

Graph a rectangular solid that contains the given point and the origin as vertices. Label the coordinates of each vertex.

1A. $W(3, 3, 4)$ **1B.** $L(4, -5, 2)$

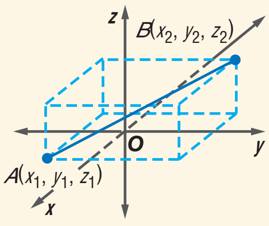
Personal Tutor at geometryonline.com

Distance and Midpoint Formula Recall that the Distance Formula is derived from the Pythagorean Theorem. The Pythagorean Theorem can also be used to find the formula for the distance between two points in space.

KEY CONCEPT *Distance Formula in Space*

Given two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in space, the distance between A and B is given by the following equation.

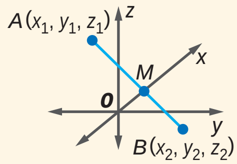
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



The Midpoint Formula can also be extended to points in space.

KEY CONCEPT *Midpoint Formula in Space*

Given two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in space, the midpoint of $\overline{AB}$ is at $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$.



EXAMPLE Distance and Midpoint Formulas in Space

2 **a. Determine the distance between $T(6, 0, 0)$ and $Q(-2, 4, 2)$.**

$$TQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad \text{Distance Formula in Space}$$

$$= \sqrt{[6 - (-2)]^2 + (0 - 4)^2 + (0 - 2)^2} \quad \text{Substitution}$$

$$= \sqrt{84} \text{ or } 2\sqrt{21} \quad \text{Simplify.}$$

b. Determine the coordinates of the midpoint M of $\overline{TQ}$.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) \quad \text{Midpoint Formula in Space}$$

$$= \left(\frac{6 - 2}{2}, \frac{0 + 4}{2}, \frac{0 + 2}{2}\right) \text{ or } (2, 2, 1) \quad \text{Substitute and simplify.}$$

2. Determine the distance between $J(2, 4, 9)$ and $K(-3, -5, 10)$. Then determine the coordinates of the midpoint M of $\overline{JK}$.

Extra Examples at geometryonline.com

Review Vocabulary

translation: a transformation that moves all points of a figure the same distance in the same direction (Lesson 9-2)

EXAMPLE 3 Translating a Solid

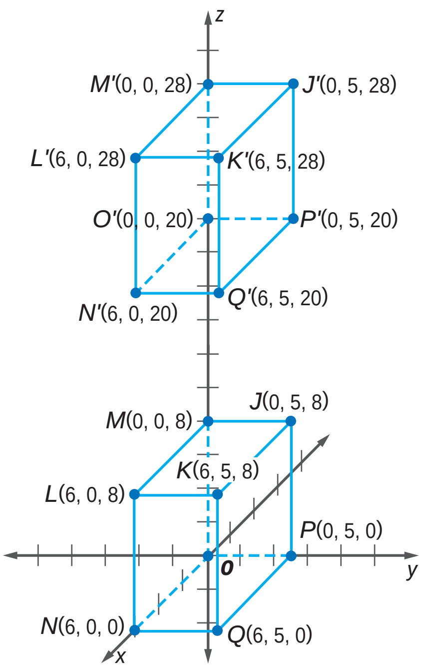
ELEVATORS Suppose an elevator is 5 feet wide, 6 feet deep, and 8 feet tall. Position the elevator on the ground floor at the origin of a three dimensional space. If the distance between the floors of a warehouse is 10 feet, write the coordinates of the vertices of the elevator after it goes up to the third floor.

Explore Since the elevator is a rectangular prism, use positive values for x, y, and z. Write the coordinates of each corner. The points on the elevator will rise 10 feet for each floor. When the elevator ascends to the third floor, it will have traveled 20 feet.

Plan Use the translation $(x, y, z) \rightarrow (x, y, z + 20)$ to find the coordinates of each vertex of the rectangular prism that represents the elevator.

Solve

Coordinates of the vertices, (x, y, z) Preimage	Translated coordinates, $(x, y, z + 20)$ Image
$J(0, 5, 8)$	$J'(0, 5, 28)$
$K(6, 5, 8)$	$K'(6, 5, 28)$
$L(6, 0, 8)$	$L'(6, 0, 28)$
$M(0, 0, 8)$	$M'(0, 0, 28)$
$N(6, 0, 0)$	$N'(6, 0, 20)$
$O(0, 0, 0)$	$O'(0, 0, 20)$
$P(0, 5, 0)$	$P'(0, 5, 20)$
$Q(6, 5, 0)$	$Q'(6, 5, 20)$



Check Check that the distance between corresponding vertices is 20 feet.

CHECK Your Progress

3A. Suppose a trolley is 12 feet long, 10 feet tall, and 8 feet wide. Position the trolley on the ground floor such that the far back corner is at the origin of a three-dimensional space. If the trolley moves 15 feet along the x-axis, write the coordinates of the vertices. Assume that the trolley is a rectangular prism.

3B. Use the Distance Formula in Space to verify the distance the trolley traveled.

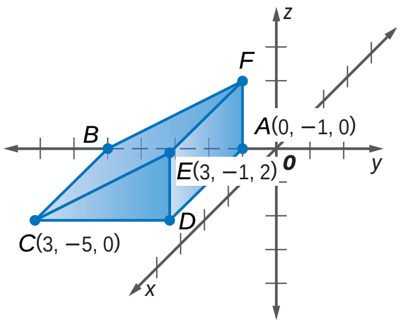
3C. Use the Midpoint Formula in Space to find the coordinates of the center of the trolley.

Review Vocabulary

reflection: a transformation representing a flip of a figure in a point, line, or plane (Lesson 9-1)

EXAMPLE Reflections in Space

4 **Reflect the right triangular prism in the *xz*-plane. Graph the image under the reflection.**



Use the grid to find a corresponding point for each vertex so that the vertices of the image will be equidistant from the *xz*-plane.

$A(0, -1, 0) \rightarrow A'(0, 1, 0)$

$B(3, -1, 2) \rightarrow B'(3, 1, 2)$

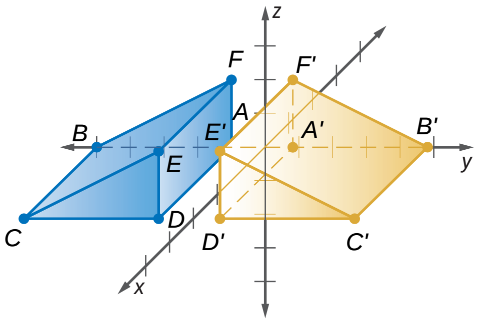
$C(3, -5, 0) \rightarrow C'(3, 5, 0)$

$D(3, -1, 0) \rightarrow D'(3, 1, 0)$

$E(3, -1, 2) \rightarrow E'(3, 1, 2)$

$F(0, -1, 2) \rightarrow F'(0, 1, 2)$

Plot the coordinates of the vertices of the image. The *x*- and *z*-coordinates remain the same, the *y*-coordinates are opposites. That is, $(a, b, c) \rightarrow (a, -b, c)$.



4. The vertices of a triangular prism are $A(0, 0, 2)$, $B(0, 2, 2)$, $C(4, 2, 2)$, $D(4, 0, 2)$, $E(4, 1, 5)$, and $F(0, 1, 5)$. Reflect the prism in the *xy*-plane. Graph the prism and its image under the reflection.

CHECK Your Understanding

Example 1 (p. 758)

Graph a rectangular solid that contains the given point and the origin as vertices. Label the coordinates of each vertex.

1. $A(2, 1, 5)$
2. $P(-1, 4, 2)$

Example 2 (p. 759)

Determine the distance between each pair of points. Then determine the coordinates of the midpoint *M* of the segment joining the pair of points.

3. $D(0, 0, 0)$ and $E(1, 5, 7)$
4. $G(-3, -4, 6)$ and $H(5, -3, -5)$

Example 3 (p. 760)

5. **COMPUTER ANIMATION** An animator has drawn a box that rests against a wall. The coordinates of the vertices are (5, 0, 0), (5, 2, 0), (3, 2, 0), (3, 0, 0), (5, 0, 2), (5, 2, 2), (3, 2, 2), (3, 0, 2). Describe the translation that would move this box two inches to the right and 5 inches forward. Write the coordinates of the vertices after the translation.

Example 4 (p. 761)

6. Reflect the right triangular prism with coordinates $G(2, -5, 4)$, $H(0, -5, 4)$, $J(0, -2, 0)$, $K(2, -2, 0)$, $L(2, -5, 0)$, and $M(0, -5, 0)$ in the *xz*-plane. Graph the image under the reflection.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
7–12	1
13–20	2
21–24	3
25–28	4

Graph the rectangular solid that contains the given point and the origin as vertices. Label the coordinates of each vertex.

7. $C(-2, 2, 2)$

8. $R(3, -4, 1)$

9. $P(4, 6, -3)$

10. $G(4, 1, -3)$

11. $K(-2, -4, -4)$

12. $W(-1, -3, -6)$

Determine the distance between each pair of points. Then determine the coordinates of the midpoint *M* of the segment joining the pair of points.

13. $K(2, 2, 0)$ and $L(-2, -2, 0)$

14. $P(-2, -5, 8)$ and $Q(3, -2, -1)$

15. $A(4, 7, 9)$ and $B(-3, 8, -8)$

16. $W(-12, 8, 10)$ and $Z(-4, 1, -2)$

17. $F\left(\frac{3}{5}, 0, \frac{4}{5}\right)$ and $G(0, 3, 0)$

18. $G(1, -1, 6)$ and $H\left(\frac{1}{5}, -\frac{2}{5}, 2\right)$

19. $B(\sqrt{3}, 2, 2\sqrt{2})$ and $C(-2\sqrt{3}, 4, 4\sqrt{2})$

20. $S(6\sqrt{3}, 4, 4\sqrt{2})$ and $T(4\sqrt{3}, 5, \sqrt{2})$

Consider a rectangular prism with the given coordinates. Find the coordinates of the vertices of the prism after the translation.

21. $P(-2, -3, 3)$, $Q(-2, 0, 3)$, $R(0, 0, 3)$, $S(0, -3, 3)$, $T(-2, 0, 0)$, $U(-2, -3, 0)$, $V(0, -3, 0)$, and $W(0, 0, 0)$; $(x, y, z) \rightarrow (x + 2, y + 5, z - 5)$

22. $A(2, 0, 1)$, $B(2, 0, 0)$, $C(2, 1, 0)$, $D(2, 1, 1)$, $E(0, 0, 1)$, $F(0, 1, 1)$, $G(0, 1, 0)$ and $H(0, 0, 0)$; $(x, y, z) \rightarrow (x - 2, y + 1, z - 1)$

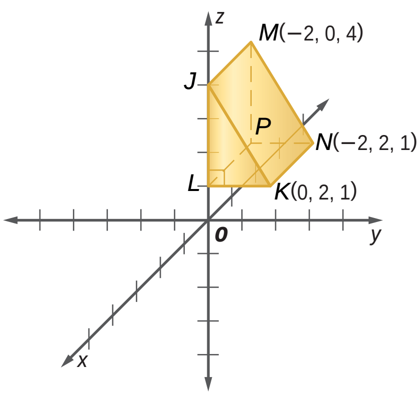
STORAGE For Exercises 23 and 24, use the following information.
A crane is used to move a crate from the loading dock to a place in a warehouse. To allow the storage company to locate and identify the crates, they assign ordered triples to the corners using positive *x*, *y*, and *z* values.

23. A crate is 12 feet deep, 8 feet wide, and 8 feet high. If the crate is stored 16 feet up and 48 feet back in the warehouse, find the ordered triples of the vertices describing the new location. Use the translation $(x, y, z) \rightarrow (x - 48, y, z + 16)$.

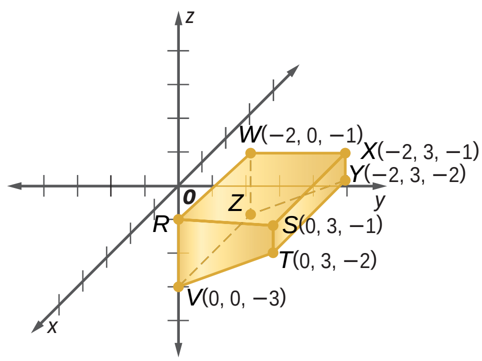
24. Another crate is a cube 4 feet on each side. If the crate is stored 15 feet back, 11 feet to the right, and 25 feet up, find the ordered triples of the vertices describing the new location. Use the translation $(x, y, z) \rightarrow (x - 15, y + 11, z + 25)$.

Reflect each prism in the *xy*-plane. Graph the image under the reflection.

25.

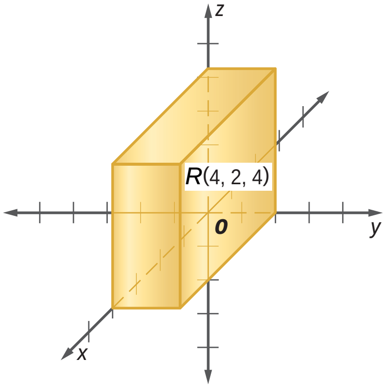


26.

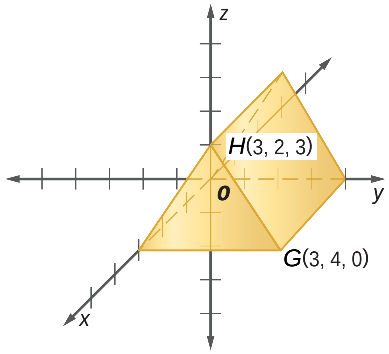


Find the volume of each prism.

27.



28.



Consider a cube with coordinates *A*(3, 3, 3), *B*(3, 0, 3), *C*(0, 0, 3), *D*(0, 3, 3), *E*(3, 3, 0), *F*(3, 0, 0), *G*(0, 0, 0), and *H*(0, 3, 0). Find the coordinates of the image under each transformation. Graph the preimage and the image.

29. Use the translation equation $(x, y, z) \rightarrow (x + 1, y + 2, z - 2)$.

30. Use the translation equation $(x, y, z) \rightarrow (x - 2, y - 3, z + 2)$.

31. Dilate the cube by a factor of 2. What is the volume of the image?

32. Dilate the cube by a factor of $\frac{1}{3}$. What is the ratio of the volumes for these two cubes?

33. If $M(5, 1, 2)$ is the midpoint of segment $\overline{AB}$ and point A has coordinates $(2, 4, 7)$, then what are the coordinates of point B?

34. The center of a sphere is at $(4, -2, 6)$, and the endpoint of a diameter is at $(8, 10, -2)$. What are the coordinates of the other endpoint of the diameter?

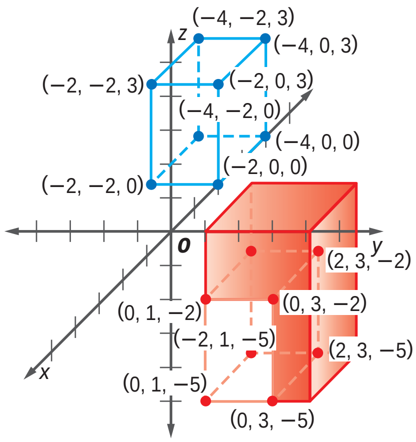
35. Find the center and the radius of a sphere if the diameter has endpoints at $(-12, 10, 12)$ and $(14, -8, 2)$.

36. A rotation in three-space is a composition of reflections in intersecting planes. A triangular prism has vertices $A(0, -3, 2)$, $B(-2, -3, 2)$, $C(-2, 0, 1)$, $D(0, 0, 1)$, $E(0, -3, 1)$, and $F(-2, -3, 1)$. Graph the original image and the reflected image in the *xz*-plane. Then graph the reflection in the *xy*-plane. Describe the rotation.

EXTRA PRACTICE
See pages 827, 840.

Math Online
Self-Check Quiz at
geometryonline.com

37. GAMES The object of a video game is to move a rectangular prism around to fit with other solids. The prism at the right has moved to combine with the red L-shaped solid. Write the translation that moved the prism to the new location.



H.O.T. Problems

38. REASONING Compare and contrast the number of regions on the coordinate plane and in three-dimensional coordinate space.

39. OPEN ENDED Draw and label the vertices of a rectangular prism that has a volume of 24 cubic units.

40. REASONING Find a counterexample for the following statement. *Every rectangular prism will be congruent to its image from any type of transformation.*

41. **CHALLENGE** A sphere with its center at (2, 4, 6) and a radius of 4 units is inscribed in a cube. Graph the cube and determine the coordinates of the vertices.

42. **Writing in Math** Explain why a computer animator would use three-dimensional graphing instead of two-dimensional graphing.

STANDARDIZED TEST PRACTICE

43. The center of a sphere is at $(4, -5, 3)$, and the endpoint of a diameter is at $(5, -4, -2)$. What are the coordinates of the other endpoint of the diameter?

A $(-1, -1, 5)$

B $\left(-\frac{1}{2}, -\frac{1}{2}, \frac{5}{2}\right)$

C $(3, -6, 8)$

D $(13, -14, 4)$

44. **REVIEW** Which expression is equivalent to $\dfrac{48x^3y^{-2}z}{6xy^{-2}z^2}$?

F $\dfrac{8x^2}{z}$

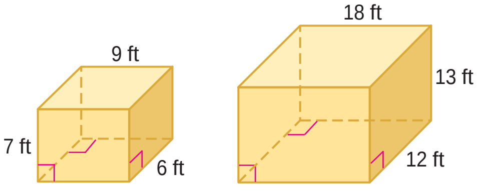
G $\dfrac{8x^2}{y^4z}$

H $\dfrac{8x^2}{yz}$

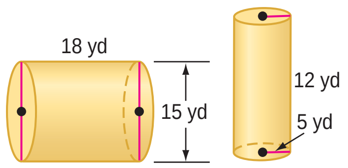
J $\dfrac{8x^{-2}y^4}{z^3}$

Determine whether each pair of solids is *similar*, *congruent*, or *neither*. (Lesson 13-4)

45.



46.



Find the volume of a sphere given the radius or diameter. Round to the nearest tenth. (Lesson 13-3)

47. $r = 10$ cm

48. $d = 13$ yd

49. $r = 17.2$ m

50. $d = 29$ ft

51. **FOOD** An oatmeal container is a cardboard cylinder 10 inches tall with a plastic lid that has a diameter of $2\frac{1}{2}$ inches. What is the lateral area of the container? (Lesson 12-3)

Cross-Curricular Project

Geometry and Architecture

Memorials Help Us Pay Tribute It's time to complete your project. Use the information and data you have gathered about designing your memorial. Add some additional data or pictures to your portfolio or Web page. Be sure to include your scale drawings and calculations in the presentation.

Math Online **Cross-Curricular Project at** geometryonline.com

CHAPTER 13

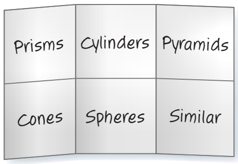
Study Guide and Review

Download Vocabulary Review from geometryonline.com

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Volumes of Prisms and Cylinders (Lesson 13-1)

- The volumes of prisms and cylinders are given by the formula $V = Bh$.

Volumes of Pyramids and Cones (Lesson 13-2)

- The volume of a pyramid is given by the formula $V = \frac{1}{3}Bh$.
- The volume of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$.

Volumes of Spheres (Lesson 13-3)

- The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

Congruent and Similar Solids (Lesson 13-4)

- Similar solids have the same shape, but not necessarily the same size.
- Congruent solids are similar solids with a scale factor of 1.

Coordinates in Space (Lesson 13-5)

- The Distance Formula in Space is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
- Given $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the midpoint of $\overline{AB}$ is at $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$.

Key Vocabulary

congruent solids (p. 751)
ordered triple (p. 758)
similar solids (p. 750)

Vocabulary Check

Complete each sentence with the correct term.

1. You can use $V = \frac{1}{3}Bh$ to find the volume of a (prism, pyramid).
2. (Similar, Congruent) solids always have the same volume.
3. Every point in space can be represented by (an ordered triple, an ordered pair).
4. $V = \pi r^2 h$ is the formula for the volume of a (sphere, cylinder).
5. In (similar, congruent) solids, if $a \neq b$ and $a{:}b$ is the ratio of the lengths of corresponding edges, then $a^3{:}b^3$ is the ratio of the volumes.
6. The formula $V = Bh$ is used to find the volume of a (prism, pyramid).
7. To find the length of an edge of a pyramid, you can use (the Distance Formula in Space, Cavalieri's Principle).
8. You can use $V = \frac{4}{3}\pi r^3$ to find the volume of a (cylinder, sphere).
9. To find the volume of an oblique pyramid, you can use (the Distance Formula in Space, Cavalieri's Principle).
10. The formula $V = \frac{1}{3}Bh$ is used to find the volume of a (cylinder, cone).

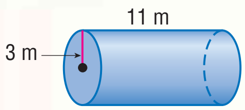
Vocabulary Review at geometryonline.com

Lesson-by-Lesson Review

13–1 Volumes of Prisms and Cylinders (pp. 728–735)

Find the volume of each prism or cylinder. Round to the nearest tenth if necessary.

11.



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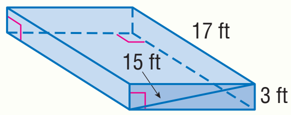
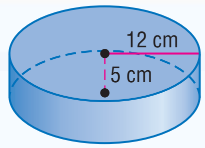


13. FOOD Soda is sold in aluminum cans that measure 6 inches high and 2 inches in diameter. How many cubic inches of soda are contained in a full can?

Example 1 **Find the volume of the cylinder.**



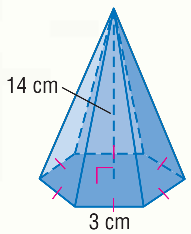
V	$= \pi r^2 h$	Volume of a cylinder
	$= \pi(12^2)(5)$	$r = 12$ and $h = 5$
	≈ 2261.9	Use a calculator.

The volume is approximately 2261.9 cubic centimeters.

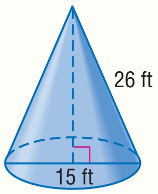
13–2 Volumes of Pyramids and Cones (pp. 737–742)

Find the volume of each pyramid or cone. Round to the nearest tenth.

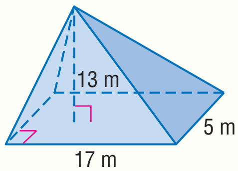
14.



15.

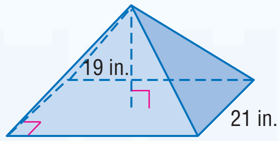


16.



17. RESTAURANT A restaurant is built in the shape of a pyramid. If the height is 63 feet, and the area of the base is 15,625 square feet, find the volume of air that the heating and cooling system would have to accommodate.

Example 2 **Find the volume of the square pyramid.**



V	$= \frac{1}{3}Bh$	Volume of a pyramid
	$= \frac{1}{3}(21^2)(19)$	$B = 21^2$ and $h = 19$
	$= 2793$	Simplify.

The volume of the pyramid is 2793 cubic inches.

Mixed Problem Solving
For mixed problem-solving practice, see page 840.

13–3 Volumes of Spheres (pp. 743–748)

Find the volume of each sphere. Round to the nearest tenth.

18. The radius of the sphere is 2 feet.

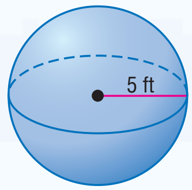
19. The circumference of the sphere is 65 millimeters.

20. The surface area of the sphere is 126 square centimeters.

21. The area of a great circle of the sphere is 25π square units.

22. **ASTRONOMY** The diameter of Mercury is about 3000 miles and the diameter of Earth is about 7900 miles. Find the difference between the volumes of the two planets.

Example 3 **Find the volume of the sphere.**



$V = \frac{4}{3}\pi r^3$ Volume of a sphere

$= \frac{4}{3}\pi(5^3)$ $r = 5$

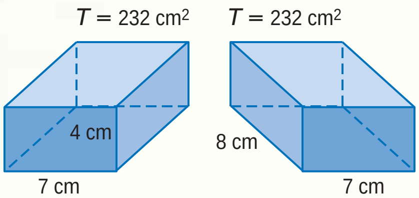
≈ 523.6 Use a calculator.

The volume of the sphere is about 523.6 cubic feet.

13–4 Congruent and Similar Solids (pp. 750–757)

Determine whether the two solids are *congruent, similar,* or *neither.*

23.



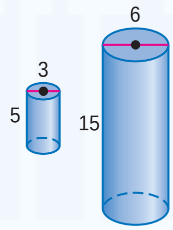
TOURISM **For Exercises 24 and 25, use the following information.**

Dale Ungerer, a farmer in Hawkeye, Iowa, constructed a gigantic ear of corn to attract tourists to his farm. The ear of corn is 32 feet long and has a circumference of 12 feet. Each "kernel" is a one-gallon milk jug with a volume of 231 cubic inches.

24. If a real ear of corn is 10 inches long, what is the scale factor between the gigantic ear of corn and the similar real ear of corn?

25. Estimate the volume of a kernel of the real ear of corn.

Example 4 **Determine whether the two cylinders are *congruent, similar,* or *neither.***



$\frac{\text{diameter of larger cylinder}}{\text{diameter of smaller cylinder}} = \frac{6}{3}$ Substitution

$= 2$ Simplify.

$\frac{\text{height of larger cylinder}}{\text{height of smaller cylinder}} = \frac{15}{5}$ Substitution

$= 3$ Simplify.

The ratios of the measures are not equal, so the cylinders are not similar.

13–5 Coordinates in Space (pp. 758–764)

Determine the distance between each pair of points. Then determine the coordinates of the midpoint M of the segment joining the pair of points.

26. $A(-5, -8, -2)$ and $B(3, -8, 4)$

27. $C(-9, 2, 4)$ and $D(-9, 9, 7)$

28. $E(-4, 5, 5)$ and the origin

29. $F(5\sqrt{2}, 3\sqrt{7}, 6)$ and $G(-2\sqrt{2}, 3\sqrt{7}, -12)$

30. **RECREATION** Two hot-air balloons take off from the same site. One hot-air balloon is 12 miles west and 12 miles south of the takeoff point and 0.4 mile above the ground. The other balloon is 4 miles west and 10 miles south of the takeoff site and 0.3 mile above the ground. Find the distance between the two balloons to the nearest tenth of a mile.

Example 5 **Consider $\triangle ABC$ with vertices $A(13, 7, 10)$, $B(17, 18, 6)$, and $C(15, 10, 10)$. Find the length of the median from A to $\overline{BC}$ of ABC.**

Use the Midpoint Formula.

$$M = \left(\frac{17+15}{2}, \frac{18+10}{2}, \frac{6+10}{2}\right)$$
$$= (16, 14, 8)$$

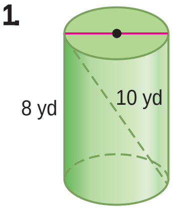
$\overline{AM}$ is the desired median, so AM is the length of the median. Use the Distance Formula.

$$AM = \sqrt{(16-13)^2 + (14-7)^2 + (8-10)^2}$$
$$= \sqrt{62}$$

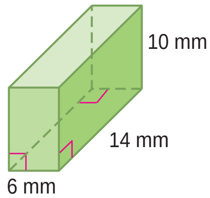
CHAPTER 13 Practice Test

Find the volume of each solid. Round to the nearest tenth if necessary.

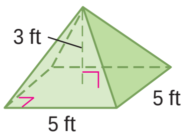
1.



2.



3.



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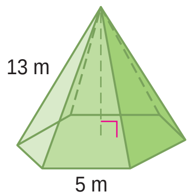
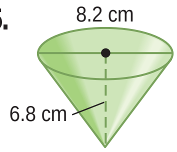
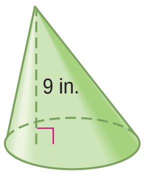


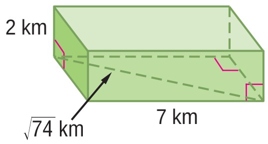
5.



6. $C = 22\pi$



7.



Find the volume of each sphere. Round to the nearest tenth.

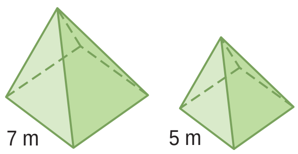
8. The radius has a length of 3 cm.

9. The circumference of the sphere is 34 ft.

10. The surface area of the sphere is 184 in^2.

11. The area of a great circle is 157 mm^2.

The two square pyramids are similar.



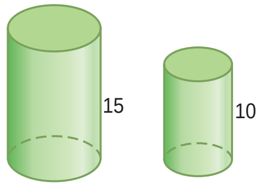
12. Find the scale factor of the pyramids.

13. What is the ratio of the surface areas?

14. What is the ratio of the volumes?

15. SPORTS The diving pool at the Georgia Tech Aquatic Center was used for the springboard and platform diving competitions of the 1996 Olympic Games. The pool is 78 feet long and 17 feet deep, and it is 110.3 feet from one corner on the surface of the pool to the opposite corner on the surface. If it takes about 7.5 gallons of water to fill one cubic foot of space, approximately how many gallons of water are needed to fill the diving pool?

The two cylinders are similar.



16. Find the ratio of the radii of the bases of the cylinders.

17. What is the ratio of the surface areas?

18. What is the ratio of the volumes?

Determine the distance between each pair of points in space. Then determine the coordinates of the midpoint M of the segment joining the pair of points.

19. the origin and $(0, -3, 5)$

20. the origin and $(-1, 10, -5)$

21. the origin and $(9, 5, -7)$

22. $(-2, 2, 2)$ and $(-3, -5, -4)$

23. $(9, 3, 4)$ and $(-9, -7, 6)$

24. $(8, -6, 1)$ and $(-3, 5, 10)$

25. MULTIPLE CHOICE A rectangular prism has a volume of 360 cubic feet. If the prism has a length of 15 feet and a height of 2 feet, what is the width?

A 30 ft
B 24 ft
C 12 ft
D 7.5 ft

Standardized Test Practice

Cumulative, Chapters 1–13

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Neil is having an in-ground pool installed in his back yard. According to the original plan, the pool would be 10 feet wide, 20 feet long, and 8 feet deep. Neil decides to keep the same depth but to make the pool 5 feet wider and 5 feet longer. How much additional water is required to fill this pool?

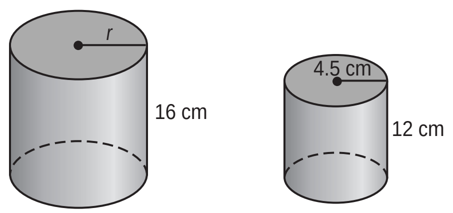
A 3000 ft^3

B 1600 ft^3

C 1400 ft^3

D 375 ft^3

2. If the two cylinders are similar, what is the volume of the larger cylinder to the nearest tenth of a cubic centimeter?



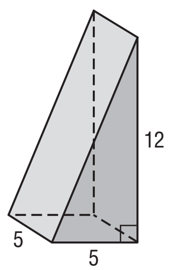
F 729.6 cm^3

G 1016.9 cm^3

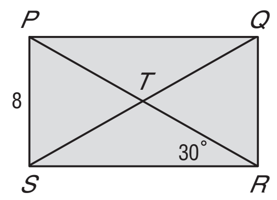
H 1808.6 cm^3

J 2121.5 cm^3

3. **GRIDDABLE** Find the surface area of the prism below, in square units.



4. If *PQRS* is a rectangle, what is its area in square units?



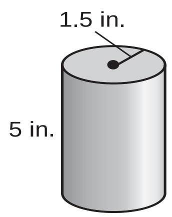
A $4\sqrt{3}$

B $8\sqrt{3}$

C $64\sqrt{3}$

D 128

5. Sara is coordinating the Student Council's charity drive and wants to place 20 cans for donations around the school. She is using old soup cans and making new labels out of construction paper to go around the sides of the cans. Use the diagram of the can below to calculate how much construction paper she will need to the nearest tenth.



F 942.0 in^2

G 706.5 in^2

H 47.1 in^2

J 17.3 in^2

6. The center of a sphere has coordinates (3, 1, 4). A point on the surface of the sphere has coordinates (9, –2, –2). What is the measure of the radius of the sphere?

A 7

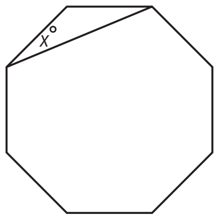
B $\sqrt{61}$

C $\sqrt{73}$

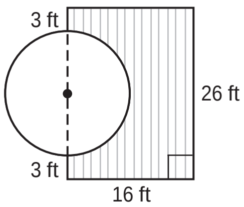
D 9

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 841–856.

7. GRIDDABLE The figure is a regular octagon. Find x.



8. Mr. Jiliana built a wooden deck around half of his circular swimming pool. He needs to know the area of the deck so he can buy cans of stain. Which is closest to the area of the deck?



F 102 ft^2
G 259 ft^2
H 388 ft^2
J 402 ft^2

TEST-TAKING TIP

Question 8 Sometimes more than one step is required to find the answer. In this question, you need to determine which dimensions are important. Then calculate the area of both the rectangle and the circle to solve the problem.

9. What is the equation of the circle with center (−2, 3) and diameter of 4 units?

A $(x + 2)^2 + (y - 3)^2 = 4$
B $(x - 2)^2 + (y + 3)^2 = 4$
C $(x + 2)^2 + (y - 3)^2 = 16$
D $(x - 2)^2 + (y + 3)^2 = 16$

10. ALGEBRA The equation of line m is $5x + 2y = 20$. The equation of line n is $2x + 5y = 50$. Which statement about the two lines is true?

F Lines m and n are parallel.
G Lines m and n are perpendicular.
H Lines m and n have the same x-intercept.
J Lines m and n have the same y-intercept.

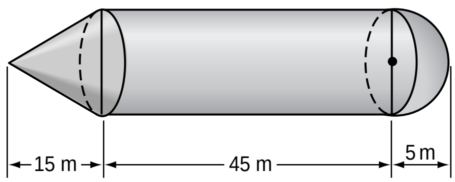
11. The circumference of a regulation soccer ball is 25 inches. What is the volume of the soccer ball to the nearest cubic inch?

A 94 in^3
B 264 in^3
C 333 in^3
D 8177 in^3

Pre-AP

Record your answers on a sheet of paper. Show your work.

12. College engineering students designed an enlarged external fuel tank for a space shuttle as part of an assignment.



a. What is the volume of the entire fuel tank?
b. Would replacing the cone with another hemisphere increase or decrease the volume of the tank?
c. If the students were to double the radius and length of the tank, how would the volume be affected?
d. What would be the affect of halving both the radius and length? Tripling both the radius and length?

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson...	13-1	13-4	12-3	6-4	12-4	13-5	6-1	11-5	10-8	3-4	12-6	13-3

Student Handbook

Built-In Workbooks

Reference

Eclipse Studios

How to Use the Student Handbook

The Student Handbook is the additional skill and reference material found at the end of the text. This handbook can help you answer these questions.

What If I Forget What I Learned Last Year?

Use the **Prerequisite Skills** section to refresh your memory about things you have learned in other math classes. Here is a list of the topics covered in your book.

1. Graphing Ordered Pairs
2. Changing Units of Measure Within Systems
3. Operations with Integers
4. Evaluating Algebraic Expressions
5. Solving Linear Equations
6. Solving Inequalities in One Variable
7. Graphing Using Intercepts and Slope
8. Writing Linear Functions
9. Solving Systems of Linear Equations
10. Square Roots and Simplifying Radicals
11. Multiplying Polynomials
12. Dividing Polynomials
13. Factoring to Solve Equations
14. Operations with Matrices

What If I Need More Practice?

You, or your teacher, may decide that working through some additional problems would be helpful. The **Extra Practice** section provides these problems for each lesson so you have ample opportunity to practice new skills.

What If I Have Trouble with Word Problems?

The **Mixed Problem Solving and Proof** portion of the book provides additional word problems and proofs that use the skills presented in each lesson. These problems give you real-world situations where the math can be applied.

What If I Need Help on Taking Tests?

The **Preparing for Standardized Tests** section gives you tips and practice on how to answer different types of questions that appear on tests.

What If I Need to Find a Postulate or Theorem?

The **Postulates, Theorems, and Corollaries** section lists all of the postulates, theorems, and corollaries in the text along with the page where it appears.

What If I Forget a Vocabulary Word?

The **English-Spanish Glossary** provides a list of important or difficult words used throughout the textbook. It provides a definition in English and Spanish as well as the page number(s) where the word can be found.

What If I Need to Check a Homework Answer?

The answers to odd-numbered problems are all included in **Selected Answers.** Check your answers to make sure you understand how to solve all of the assigned problems.

What If I Need to Find Something Quickly?

The **Index** alphabetically lists the subjects covered throughout the entire textbook and the pages on which each subject can be found.

What If I Forget a Formula?

Inside the back cover of your math book is a list of **Formulas and Symbols** that are used in the book.

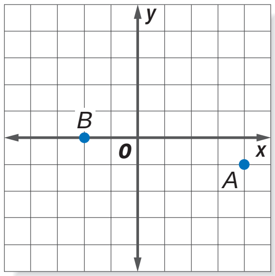
Prerequisite Skills

1 Graphing Ordered Pairs

Points in the coordinate plane are named by **ordered pairs** of the form (x, y). The first number, or **x-coordinate**, corresponds to a number on the x-axis. The second number, or **y-coordinate**, corresponds to a number on the y-axis.

EXAMPLE

1 Write the ordered pair for each point.



a. ***A***

The x-coordinate is 4.

The y-coordinate is -1.

The ordered pair is $(4, -1)$.

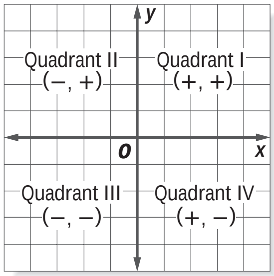
b. ***B***

The x-coordinate is -2.

The point lies on the x-axis, so its y-coordinate is 0.

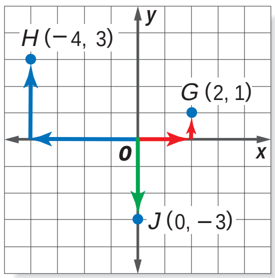
The ordered pair is $(-2, 0)$.

The x-axis and y-axis separate the coordinate plane into four regions, called **quadrants**. The point at which the axes intersect is called the **origin**. The axes and points on the axes are not located in any of the quadrants.



EXAMPLE

2 Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.



a. ***G*(2, 1)**

Start at the origin. Move 2 units right, since the x-coordinate is 2. Then move 1 unit up, since the y-coordinate is 1. Draw a dot, and label it G. Point $G(2, 1)$ is in Quadrant I.

b. ***H*(−4, 3)**

Start at the origin. Move 4 units left, since the x-coordinate is -4. Then move 3 units up, since the y-coordinate is 3. Draw a dot, and label it H. Point $H(-4, 3)$ is in Quadrant II.

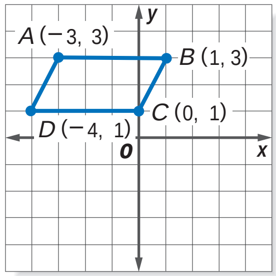
c. ***J*(0, −3)**

Start at the origin. Since the x-coordinate is 0, the point lies on the y-axis. Move 3 units down, since the y-coordinate is -3. Draw a dot, and label it J. Because it is on one of the axes, point $J(0, -3)$ is not in any quadrant.

EXAMPLE

3 Graph a polygon with vertices $A(-3, 3)$, $B(1, 3)$, $C(0, 1)$, and $D(-4, 1)$.

Graph the ordered pairs on a coordinate plane. Connect each pair of consecutive points. The polygon is a parallelogram.



EXAMPLE

4 Graph four points that satisfy the equation $y = 4 - x$.

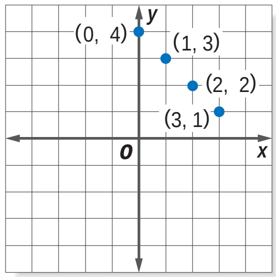
Make a table.

Choose four values for x.

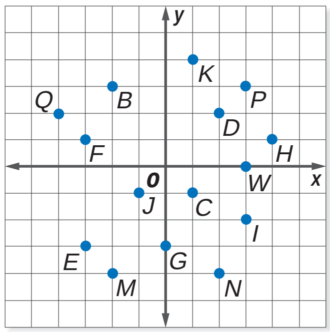
Evaluate each value of x for $4 - x$.

x	$4 - x$	y	(x, y)
0	$4 - 0$	4	(0, 4)
1	$4 - 1$	3	(1, 3)
2	$4 - 2$	2	(2, 2)
3	$4 - 3$	1	(3, 1)

Plot the points.



Exercises **Write the ordered pair for each point shown at the right.**



1. B **2.** C **3.** D

4. E **5.** F **6.** G

7. H **8.** I **9.** J

10. K **11.** W **12.** M

13. N **14.** P **15.** Q

Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.

16. $M(-1, 3)$ **17.** $S(2, 0)$ **18.** $R(-3, -2)$ **19.** $P(1, -4)$

20. $B(5, -1)$ **21.** $D(3, 4)$ **22.** $T(2, 5)$ **23.** $L(-4, -3)$

Graph the following geometric figures.

24. a square with vertices $W(-3, 3)$, $X(-3, -1)$, $Y(1, 3)$, and $Z(1, -1)$

25. a polygon with vertices $J(4, 2)$, $K(1, -1)$, $L(-2, 2)$, and $M(1, 5)$

26. a triangle with vertices $F(2, 4)$, $G(-3, 2)$, and $H(-1, -3)$

Graph four points that satisfy each equation.

27. $y = 2x$ **28.** $y = 1 + x$ **29.** $y = 3x - 1$ **30.** $y = 2 - x$

2 Changing Units of Measure within Systems

Metric Units of Length
1 kilometer (km) = 1000 meters (m)
1 m = 100 centimeters (cm)
1 cm = 10 millimeters (mm)

Customary Units of Length
1 foot (ft) = 12 inches (in.)
1 yard (yd) = 3 ft
1 mile (mi) = 5280 ft

- To convert from larger units to smaller units, multiply.
- To convert from smaller units to larger units, divide.

EXAMPLE 1

State which metric unit you would use to measure the length of your pen.

Since a pen has a small length, the *centimeter* is the appropriate unit of measure.

EXAMPLE 2

Complete each sentence.

a. 4.2 km = __?__ m

There are 1000 meters in a kilometer.
4.2 km × 1000 = 4200 m

b. 39 ft = __?__ yd

There are 3 feet in a yard.
39 ft ÷ 3 = 13 yd

EXAMPLE 3

Complete each sentence.

a. 17 mm = __?__ m

There are 100 centimeters in a meter. First change *millimeters* to *centimeters.*

17 mm = __?__ cm — smaller unit → larger unit
17 mm ÷ 10 = 1.7 cm — Since 10 mm = 1 cm, divide by 10.

Then change *centimeters* to *meters.*

1.7 cm = __?__ m — smaller unit → larger unit
1.7 cm ÷ 100 = 0.017 m — Since 100 cm = 1 m, divide by 100.

b. 6600 yd = __?__ mi

There are 5280 feet in one mile. First change *yards* to *feet.*

6600 yd = __?__ ft — larger unit → smaller unit
6600 yd × 3 = 19,800 ft — Since 3 ft = 1 yd, multiply by 3.

Then change *feet* to *miles.*

19,800 ft = __?__ mi — smaller unit → larger unit
19,800 ft ÷ 5280 = $3\frac{3}{4}$ or 3.75 mi — Since 5280 ft = 1 mi, divide by 5280.

Metric Units of Capacity
1 liter (L) = 1000 milliliters (mL)

Customary Units of Capacity	
1 cup (c) = 8 fluid ounces (fl oz)	1 quart (qt) = 2 pt
1 pint (pt) = 2 c	1 gallon (gal) = 4 qt

EXAMPLE

4 Complete each sentence.

a. 3.7 L = __?__ mL

There are 1000 milliliters in a liter.

3.7 L × 1000 = 3700 mL

b. 16 qt = __?__ gal

There are 4 quarts in a gallon.

16 qt ÷ 4 = 4 gal

c. 7 pt = __?__ fl oz

There are 8 fluid ounces in a cup.

First change *pints* to *cups*.

7 pt = __?__ c

7 pt × 2 = 14 c

Then change *cups* to *fluid ounces*.

14 c = __?__ fl oz

14 c × 8 = 112 fl oz

d. 4 gal = __?__ pt

There are 4 quarts in a gallon.

First change *gallons* to *quarts*.

4 gal = __?__ qt

4 gal × 4 = 16 qt

Then change *quarts* to *pints*.

16 qt = __?__ pt

16 qt × 2 = 32 pt

The mass of an object is the amount of matter that it contains.

Metric Units of Mass
1 kilogram (kg) = 1000 grams (g)
1 g = 1000 milligrams (mg)

Customary Units of Weight
1 pound (lb) = 16 ounces (oz)
1 ton (T) = 2000 lb

EXAMPLE

5 Complete each sentence.

a. 5.47 kg = __?__ mg

There are 1000 milligrams in a gram.

Change *kilograms* to *grams*.

5.47 kg = __?__ g

5.47 kg × 1000 = 5470 g

Then change *grams* to *milligrams*.

5470 g = __?__ mg

5470 g × 1000 = 5,470,000 mg

b. 5 T = __?__ oz

There are 16 ounces in a pound.

Change *tons* to *pounds*.

5 T = __?__ lb

5 T × 2000 = 10,000 lb

Then change *pounds* to *ounces*.

10,000 lb = __?__ oz

10,000 lb × 16 = 160,000 oz

Exercises **State which metric unit you would probably use to measure each item.**

1. radius of a tennis ball
2. length of a notebook
3. mass of a textbook
4. mass of a beach ball
5. liquid in a cup
6. water in a bathtub

Complete each sentence.

7. 120 in. = __?__ ft
8. 18 ft = __?__ yd
9. 10 km = __?__ m
10. 210 mm = __?__ cm
11. 180 mm = __?__ m
12. 3100 m = __?__ km
13. 90 in. = __?__ yd
14. 5280 yd = __?__ mi
15. 8 yd = __?__ ft
16. 0.62 km = __?__ m
17. 370 mL = __?__ L
18. 12 L = __?__ mL
19. 32 fl oz = __?__ c
20. 5 qt = __?__ c
21. 10 pt = __?__ qt
22. 48 c = __?__ gal
23. 4 gal = __?__ qt
24. 36 mg = __?__ g
25. 13 lb = __?__ oz
26. 130 g = __?__ kg
27. 9.05 kg = __?__ g

3 Operations with Integers

The absolute value of any number *n* is its distance from zero on a number line and is written as $|n|$. Since distance cannot be less than zero, the absolute value of a number is always greater than or equal to zero.

EXAMPLE 1 **Evaluate each expression.**

a. $|3|$

$|3| = 3$ Definition of absolute value

b. $|-7|$

$|-7| = 7$ Definition of absolute value

c. $|-4 + 2|$

$|-4 + 2| = |-2|$ $\quad -4 + 2 = -2$

$= 2$ Simplify.

To add integers with the same sign, add their absolute values. Give the result the same sign as the integers. To add integers with different signs, subtract their absolute values. Give the result the same sign as the integer with the greater absolute value.

EXAMPLE 2 **Find each sum.**

a. $-3 + (-5)$ Both numbers are negative, so the sum is negative.

$-3 + (-5) = -8$ Add $|-3|$ and $|-5|$.

b. $-4 + 2$ The sum is negative because $|-4| > |2|$.

$-4 + 2 = -2$ Subtract $|2|$ from $|-4|$.

c. $6 + (-3)$ The sum is positive because $|6| > |-3|$.

$6 + (-3) = 3$ Subtract $|-3|$ from $|6|$.

To subtract an integer, add its additive inverse.

EXAMPLE 3 **Find each difference.**

a. $4 - 7$

$4 - 7 = 4 + (-7)$ To subtract 7, add -7.

$= -3$

b. $2 - (-4)$

$2 - (-4) = 2 + 4$ To subtract -4, add 4.

$= 6$

The product of two integers with different signs is negative. The product of two integers with the same sign is positive. Similarly, the quotient of two integers with different signs is negative, and the quotient of two integers with the same sign is positive.

EXAMPLE

4 **Find each product or quotient.**

a. $\mathbf{4(-7)}$ — The factors have different signs.
$4(-7) = -28$ — The product is negative.

b. $\mathbf{-64 \div (-8)}$ — The dividend and divisor have the same sign.
$-64 \div (-8) = 8$ — The quotient is positive.

c. $\mathbf{-9(-6)}$ — The factors have the same sign.
$-9(-6) = 54$ — The product is positive.

d. $\mathbf{-55 \div 5}$ — The dividend and divisor have different signs.
$-55 \div 5 = -11$ — The quotient is negative.

e. $\mathbf{\frac{24}{-3}}$ — The dividend and divisor have different signs.
$\frac{24}{-3} = -8$ — The quotient is negative.

To evaluate expressions with absolute value, evaluate the absolute values first and then perform the operation.

EXAMPLE

5 **Evaluate each expression.**

a. $\mathbf{|-3| - |5|}$
$|-3| - |5| = 3 - 5$ — $|-3| = 3, |5| = 5$
$= -2$ — Simplify.

b. $\mathbf{|-5| + |-2|}$
$|-5| + |-2| = 5 + 2$ — $|-5| = 5, |-2| = 2$
$= 7$ — Simplify.

Exercises **Evaluate each absolute value.**

1. $|-3|$ **2.** $|4|$ **3.** $|0|$ **4.** $|-5|$

Find each sum or difference.

5. $-4 - 5$ **6.** $3 + 4$ **7.** $9 - 5$ **8.** $-2 - 5$
9. $3 - 5$ **10.** $-6 + 11$ **11.** $-4 + (-4)$ **12.** $5 - 9$
13. $-4 - (-2)$ **14.** $3 - (-3)$ **15.** $3 + (-4)$ **16.** $-3 - (-9)$

Evaluate each expression.

17. $|-4| - |6|$ **18.** $|-7| + |-1|$ **19.** $|1| + |-2|$ **20.** $|2| - |-5|$
21. $|-5 + 2|$ **22.** $|6 + 4|$ **23.** $|3 - 7|$ **24.** $|-3 - 3|$

Find each product or quotient.

25. $-36 \div 9$ **26.** $-3(-7)$ **27.** $6(-4)$ **28.** $-25 \div 5$
29. $-6(-3)$ **30.** $7(-8)$ **31.** $-40 \div (-5)$ **32.** $11(3)$
33. $44 \div (-4)$ **34.** $-63 \div (-7)$ **35.** $6(5)$ **36.** $-7(12)$
37. $-10(4)$ **38.** $80 \div (-16)$ **39.** $72 \div 9$ **40.** $39 \div 3$

Prerequisite Skills

4 Evaluating Algebraic Expressions

An expression is an algebraic expression if it contains sums and/or products of variables and numbers. To evaluate an algebraic expression, replace the variable or variables with known values, and then use the order of operations.

Order of Operations	
Step 1	Evaluate expressions inside grouping symbols.
Step 2	Evaluate all powers.
Step 3	Do all multiplications and/or divisions from left to right.
Step 4	Do all additions and/or subtractions from left to right.

EXAMPLE 1

Evaluate each expression.

a. $x - 5 + y$ **if** $x = 15$ **and** $y = -7$

$x - 5 + y = 15 - 5 + (-7)$ Substitute.

$= 10 + (-7)$ Subtract.

$= 3$ Add.

b. $6ab^2$ **if** $a = -3$ **and** $b = 3$

$6ab^2 = 6(-3)(3)^2$ Substitute.

$= 6(-3)(9)$ $3^2 = 9$

$= (-18)(9)$ Multiply.

$= -162$ Multiply.

EXAMPLE 2

Evaluate if $m = -2$, $n = -4$, **and** $p = 5$.

a. $\dfrac{2m + n}{p - 3}$

$\dfrac{2m + n}{p - 3} = \dfrac{2(-2) + (-4)}{5 - 3}$ Substitute.

$= \dfrac{-4 - 4}{5 - 3}$ Multiply.

$= \dfrac{-8}{2}$ or -4 Subtract.

b. $-3(m^2 + 2n)$

$-3(m^2 + 2n) = -3[(-2)^2 + 2(-4)]$

$= -3[4 + (-8)]$

$= -3(-4)$ or 12

EXAMPLE 3

Evaluate $3|a - b| + 2|c - 5|$ **if** $a = -2$, $b = -4$, **and** $c = 3$.

$3|a - b| + 2|c - 5| = 3|-2 - (-4)| + 2|3 - 5|$ Substitute for a, b, and c.

$= 3|2| + 2|-2|$ Simplify.

$= 3(2) + 2(2)$ Find absolute values.

$= 10$ Simplify.

Exercises

Evaluate each expression if $a = 2$, $b = -3$, $c = -1$, **and** $d = 4$.

1. $2a + c$
2. $\dfrac{bd}{2c}$
3. $\dfrac{2d - a}{b}$
4. $3d - c$
5. $\dfrac{3b}{5a + c}$
6. $5bc$
7. $2cd + 3ab$
8. $\dfrac{c - 2d}{a}$

Evaluate each expression if $x = 2$, $y = -3$, **and** $z = 1$.

9. $24 + |x - 4|$
10. $13 + |8 + y|$
11. $|5 - z| + 11$
12. $|2y - 15| + 7$

5 Solving Linear Equations

If the same number is added to or subtracted from each side of an equation, the resulting equation is true.

EXAMPLE

1 Solve each equation.

a. $x - 7 = 16$

$x - 7 = 16$	Original equation
$x - 7 + 7 = 16 + 7$	Add 7 to each side.
$x = 23$	Simplify.

b. $m + 12 = -5$

$m + 12 = -5$	Original equation
$m + 12 + (-12) = -5 + (-12)$	Add -12 to each side.
$m = -17$	Simplify.

c. $k + 31 = 10$

$k + 31 = 10$	Original equation
$k + 31 - 31 = 10 - 31$	Subtract 31 from each side.
$k = -21$	Simplify.

If each side of an equation is multiplied or divided by the same number, the resulting equation is true.

EXAMPLE

2 Solve each equation.

a. $4d = 36$

$4d = 36$	Original equation
$\frac{4d}{4} = \frac{36}{4}$	Divide each side by 4.
$x = 9$	Simplify.

b. $-\frac{t}{8} = -7$

$-\frac{t}{8} = -7$	Original equation
$-8\left(-\frac{t}{8}\right) = -8(-7)$	Multiply each side by -8.
$t = 56$	Simplify.

c. $\frac{3}{5}x = -8$

$\frac{3}{5}x = -8$	Original equation
$\frac{5}{3}\left(\frac{3}{5}\right)x = \frac{5}{3}(-8)$	Multiply each side by $\frac{5}{3}$.
$x = -\frac{40}{3}$	Simplify.

To solve equations with more than one operation, often called *multi-step equations*, undo operations by working backward.

EXAMPLE

3 **Solve each equation.**

a. $8q - 15 = 49$

$8q - 15 = 49$ Original equation

$8q = 64$ Add 15 to each side.

$q = 8$ Divide each side by 8.

b. $12y + 8 = 6y - 5$

$12y + 8 = 6y - 5$ Original equation

$12y = 6y - 13$ Subtract 8 from each side.

$6y = -13$ Subtract $6y$ from each side.

$y = -\frac{13}{6}$ Divide each side by 6.

When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

EXAMPLE

4 **Solve $3(x - 5) = 13$.**

$3(x - 5) = 13$ Original equation

$3x - 15 = 13$ Distributive Property

$3x = 28$ Add 15 to each side.

$x = \frac{28}{3}$ Divide each side by 3.

Exercises Solve each equation.

1. $r + 11 = 3$
2. $n + 7 = 13$
3. $d - 7 = 8$
4. $\frac{8}{5}a = -6$
5. $-\frac{p}{12} = 6$
6. $\frac{x}{4} = 8$
7. $\frac{12}{5}f = -18$
8. $\frac{y}{7} = -11$
9. $\frac{6}{7}y = 3$
10. $c - 14 = -11$
11. $t - 14 = -29$
12. $p - 21 = 52$
13. $b + 2 = -5$
14. $q + 10 = 22$
15. $-12q = 84$
16. $5s = 30$
17. $5c - 7 = 8c - 4$
18. $2\ell + 6 = 6\ell - 10$
19. $\frac{m}{10} + 15 = 21$
20. $-\frac{m}{8} + 7 = 5$
21. $8t + 1 = 3t - 19$
22. $9n + 4 = 5n + 18$
23. $5c - 24 = -4$
24. $3n + 7 = 28$
25. $-2y + 17 = -13$
26. $-\frac{t}{13} - 2 = 3$
27. $\frac{2}{9}x - 4 = \frac{2}{3}$
28. $9 - 4g = -15$
29. $-4 - p = -2$
30. $21 - b = 11$
31. $-2(n + 7) = 15$
32. $5(m - 1) = -25$
33. $-8a - 11 = 37$
34. $\frac{7}{4}q - 2 = -5$
35. $2(5 - n) = 8$
36. $-3(d - 7) = 6$

6 Solving Inequalities in One Variable

Statements with greater than (>), less than (<), greater than or equal to (≥), or less than or equal to (≤) are inequalities.

If any number is added or subtracted to each side of an inequality, the resulting inequality is true.

EXAMPLE 1 **Solve each inequality.**

a. $x - 17 > 12$

$x - 17 > 12$ Original inequality

$x - 17 + 17 > 12 + 17$ Add 17 to each side.

$x > 29$ Simplify.

The solution set is $\{x \mid x > 29\}$.

b. $y + 11 \leq 5$

$y + 11 \leq 5$ Original inequality

$y + 11 - 11 \leq 5 - 11$ Subtract 11 from each side.

$y \leq -6$ Simplify.

The solution set is $\{y \mid y \leq -6\}$.

If each side of an inequality is multiplied or divided by a positive number, the resulting inequality is true.

EXAMPLE 2 **Solve each inequality.**

a. $\frac{t}{6} \geq 11$

$\frac{t}{6} \geq 11$ Original inequality

$(6)\frac{t}{6} \geq (6)11$ Multiply each side by 6.

$t \geq 66$ Simplify.

The solution set is $\{t \mid t \geq 66\}$.

b. $8p < 72$

$8p < 72$ Original inequality

$\frac{8p}{8} < \frac{72}{8}$ Divide each side by 8.

$p < 9$ Simplify.

The solution set is $\{p \mid p < 9\}$.

If each side of an inequality is multiplied or divided by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is true.

EXAMPLE 3 **Solve each inequality.**

a. $-5c > 30$

$-5c > 30$ Original inequality

$\frac{-5c}{-5} < \frac{30}{-5}$ Divide each side by -5. Change $>$ to $<$.

$c < -6$ Simplify.

The solution set is $\{c \mid c < -6\}$.

b. $-\frac{d}{13} \leq -4$

$-\frac{d}{13} \leq -4$	Original inequality
$(-13)\left(\frac{-d}{13}\right) \geq (-13)(-4)$	Multiply each side by -13. Change $\leq$ to $\geq$.
$d \geq 52$	Simplify.

The solution set is $\{d \mid d \geq 52\}$.

Inequalities involving more than one operation can be solved by undoing the operations in the same way you would solve an equation with more than one operation.

EXAMPLE

4 Solve each inequality.

a. $-6a + 13 < -7$

$-6a + 13 < -7$	Original inequality
$-6a + 13 - 13 < -7 - 13$	Subtract 13 from each side.
$-6a < -20$	Simplify.
$\frac{-6a}{-6} > \frac{-20}{-6}$	Divide each side by -6. Change $<$ to $>$.
$a > \frac{10}{3}$	Simplify.

The solution set is $\left\{a \mid a > \frac{10}{3}\right\}$.

b. $4z + 7 \geq 8z - 1$

$4z + 7 \geq 8z - 1$	Original inequality
$4z + 7 - 7 \geq 8z - 1 - 7$	Subtract 7 from each side.
$4z \geq 8z - 8$	Simplify.
$4z - 8z \geq 8z - 8 - 8z$	Subtract $8z$ from each side.
$-4z \geq -8$	Simplify.
$\frac{-4z}{-4} \leq \frac{-8}{-4}$	Divide each side by -4. Change $\geq$ to $\leq$.
$z \leq 2$	Simplify.

The solution set is $\{z \mid z \leq 2\}$.

Exercises **Solve each inequality.**

1. $x - 7 < 6$
2. $4c + 23 \leq -13$
3. $-\frac{p}{5} \geq 14$
4. $-\frac{a}{8} < 5$
5. $\frac{t}{6} > -7$
6. $\frac{a}{11} \leq 8$
7. $d + 8 \leq 12$
8. $m + 14 > 10$
9. $2z - 9 < 7z + 1$
10. $6t - 10 \geq 4t$
11. $3z + 8 < 2$
12. $a + 7 \geq -5$
13. $m - 21 < 8$
14. $x - 6 \geq 3$
15. $-3b \leq 48$
16. $4y < 20$
17. $12k \geq -36$
18. $-4h > 36$
19. $\frac{2}{5}b - 6 \leq -2$
20. $\frac{8}{3}t + 1 > -5$
21. $7q + 3 \geq -4q + 25$
22. $-3n - 8 > 2n + 7$
23. $-3w + 1 \leq 8$
24. $-\frac{4}{5}k - 17 > 11$

7 Graphing Using Intercepts and Slope

The x-coordinate of the point at which a line crosses the x-axis is called the ***x*-intercept**. The y-coordinate of the point at which a line crosses the y-axis is called the ***y*-intercept**. Since two points determine a line, one method of graphing a linear equation is to find these intercepts.

EXAMPLE

1 Determine the x-intercept and y-intercept of $4x - 3y = 12$. Then graph the equation.

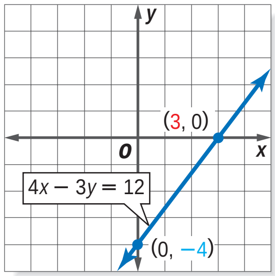
To find the x-intercept, let $y = 0$.

$4x - 3y = 12$	Original equation
$4x - 3(0) = 12$	Replace y with 0.
$4x = 12$	Simplify.
$x = 3$	Divide each side by 4.

To find the y-intercept, let $x = 0$.

$4x - 3y = 12$	Original equation
$4(0) - 3y = 12$	Replace x with 0.
$-3y = 12$	Divide each side by -3.
$y = -4$	Simplify.

Put a point on the x-axis at 3 and a point on the y-axis at -4. Draw the line through the two points.



A linear equation of the form $y = mx + b$ is in *slope-intercept* form, where m is the slope and b is the y-intercept.

EXAMPLE

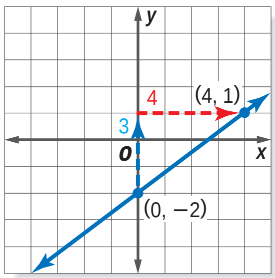
2 Graph $y = \frac{3}{4}x - 2$.

Step 1 The y-intercept is -2. So, plot a point at $(0, -2)$.

Step 2 The slope is $\frac{3}{4}$. $\frac{\text{rise}}{\text{run}}$

From $(0, -2)$, move up 3 units and right 4 units. Plot a point.

Step 3 Draw a line connecting the points.



Exercises

Graph each equation using both intercepts.

1. $-2x + 3y = 6$
2. $2x + 5y = 10$
3. $3x - y = 3$

Graph each equation using the slope and y-intercept.

4. $y = -x + 2$
5. $y = x - 2$
6. $y = x + 1$

Graph each equation using either method.

7. $y = \frac{2}{3}x - 3$
8. $y = \frac{1}{2}x - 1$
9. $y = 2x - 2$
10. $-6x + y = 2$
11. $2y - x = -2$
12. $3x + 4y = -12$

Prerequisite Skills

8 Writing Linear Equations

The linear equation $y = mx + b$ is written in **slope-intercept form**, where m is the slope and b is the y-intercept. The equation of a vertical line cannot be written in slope-intercept form. The equation of a horizontal line can be written in slope-intercept form as $y = b$.

The linear equation $y - y_1 = m(x - x_1)$ is written in **point-slope form**, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.

EXAMPLE

1 Write an equation of the line in slope-intercept form for each situation.

a. the line with slope $\frac{3}{4}$ and y-intercept -8

$y = mx + b$	Slope-intercept form
$y = \frac{3}{4}x + (-8)$	$m = \frac{3}{4}$ and $b = -8$
$y = \frac{3}{4}x - 8$	Simplify.

b. the line through (2, 5) with slope 1

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 5 = 1(x - 2)$	$(x_1, y_1) = (2, 5)$ and $m = 1$
$y = x + 3$	Add 5 to each side.

c. the line through (−4, 4) and (2, 1)

Step 1 Find the slope of the line.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$= \frac{1 - 4}{2 - (-4)}$	$(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (-4, 4)$
$= \frac{-3}{6}$ or $-\frac{1}{2}$	Simplify.

Step 2 Use the point-slope form.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 1 = -\frac{1}{2}(x - 2)$	$(x_1, y_1) = (2, 1)$ and $m = -\frac{1}{2}$
$y - 1 = -\frac{1}{2}x + 1$	Distributive Property
$y = -\frac{1}{2}x + 2$	Add 1 to each side.

d. the line through (−2, −1) with slope 0

A line with $m = 0$ can be written as $y = b$.
This is a horizontal line.

$y = b$	Slope-intercept form
$y = -1$	$b = -1$

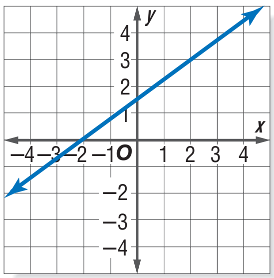
Exercises **Write the slope-intercept form of an equation for the line with the given slope and y-intercept.**

1. $m = -3$, y-intercept: 5
2. $m = 1$, y-intercept: -2
3. $m = 0$, y-intercept: -4
4. $m = -2$, y-intercept: 0
5. $m = \frac{1}{2}$, y-intercept: 1
6. $m = -\frac{3}{5}$, y-intercept: -6
7. $m = -\frac{2}{3}$, y-intercept: 0
8. $m = \frac{1}{4}$, y-intercept: 7

Write the slope-intercept form of an equation for the line with the given slope through the given point.

9. $m = -3$, $(-1, 4)$
10. $m = 5$, $(7, 2)$
11. $m = 6$, $(9, 1)$
12. $m = -7$, $(-1, -5)$
13. $m = \frac{2}{3}$, $(-3, 6)$
14. $m = \frac{4}{5}$, $(-2, -2)$
15. $m = -\frac{1}{6}$, $(2, 9)$
16. $m = -\frac{5}{2}$, $\left(\frac{1}{2}, -4\right)$

Write the slope-intercept form of an equation for each line.

17.


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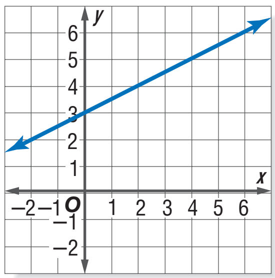

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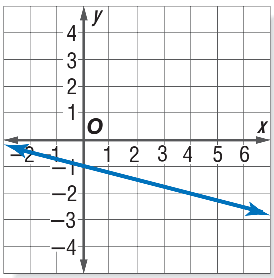

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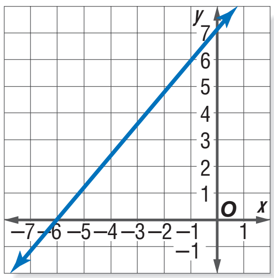

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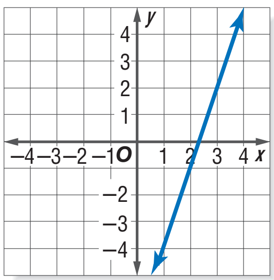

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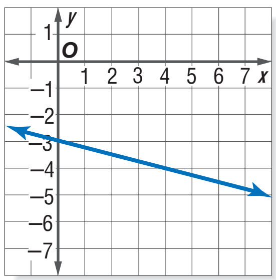


Write the slope-intercept form of an equation for the line that passes through the given points.

23. $(6, 9)$, $(-2, 9)$
24. $(2, 1)$, $(3, 8)$
25. $(0, 1)$, $(3, -8)$
26. $(4, -2)$, $(12, 2)$
27. $(16, -7)$, $(4, 2)$
28. $\left(\frac{1}{2}, 1\right)$, $(2, 10)$
29. $(-2, -5)$, $(-1, -1)$
30. $(5, 1)$, $(10, -3)$

9 Solving Systems of Linear Equations

Two or more equations that have common variables are called a **system of equations**. The solution of a system of equations in two variables is an ordered pair of numbers that satisfies both equations. A system of two linear equations can have zero, one, or an infinite number of solutions. There are three methods by which systems of equations can be solved: graphing, elimination, and substitution.

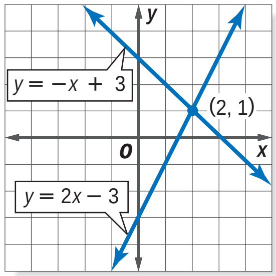
EXAMPLE 1

Solve each system of equations by graphing. Then determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.

a. $y = -x + 3$

$y = 2x - 3$

The graphs appear to intersect at (2, 1). Check this estimate by replacing x with 2 and y with 1 in each equation.



Check

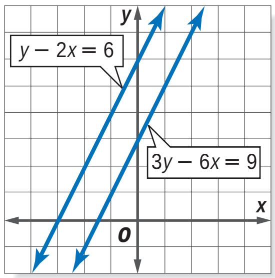
$y = -x + 3$	$y = 2x - 3$
$1 \stackrel{?}{=} -2 + 3$	$1 \stackrel{?}{=} 2(2) - 3$
$1 = 1$ ✓	$1 = 1$ ✓

The system has one solution at (2, 1).

b. $y - 2x = 6$

$3y - 6x = 9$

The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions of this system of equations. Notice that the lines have the same slope but different y-intercepts. Equations with the same slope *and* the same y-intercepts have an infinite number of solutions.



It is difficult to determine the solution of a system when the two graphs intersect at noninteger values. There are algebraic methods by which an exact solution can be found. One such method is **substitution**.

EXAMPLE 2

Use substitution to solve the system of equations.

$y = -4x$

$2y + 3x = 8$

Since $y = -4x$, substitute $-4x$ for y in the second equation.

$2y + 3x = 8$	Second equation
$2(-4x) + 3x = 3$	$y = -4x$
$-8x + 3x = 8$	Simplify.
$-5x = 8$	Combine like terms.
$\frac{-5x}{-5} = \frac{8}{-5}$	Divide each side by -5.
$x = -\frac{8}{5}$	Simplify.

Use $y = -4x$ to find the value of y.

$y = -4x$	First equation
$= -4\left(-\frac{8}{5}\right)$	$x = -\frac{8}{5}$
$= \frac{32}{5}$	Simplify.

The solution is $\left(-\frac{8}{5}, \frac{32}{5}\right)$.

Sometimes adding or subtracting two equations together will eliminate one variable. Using this step to solve a system of equations is called **elimination**.

EXAMPLE

3 Use elimination to solve the system of equations.

$\mathbf{3x + 5y = 7}$

$\mathbf{4x + 2y = 0}$

Either x or y can be eliminated. In this example, we will eliminate x.

$3x + 5y = 7$ Multiply by 4. $\quad 12x + 20y = 28$

$4x + 2y = 0$ Multiply by -3. $\quad + (-12x) - 6y = 0$

$14y = 28$ Add the equations.

$\frac{14y}{14} = \frac{28}{14}$ Divide each side by 14.

$y = 2$ Simplify.

Now substitute 2 for y in either equation to find the value of x.

$4x + 2y = 0$ Second equation

$4x + 2(2) = 0$ $y = 2$

$4x + 4 = 0$ Simplify.

$4x + 4 - 4 = 0 - 4$ Subtract 4 from each side.

$4x = -4$ Simplify.

$\frac{4x}{4} = \frac{-4}{4}$ Divide each side by 4.

$x = -1$ Simplify.

The solution is $(-1, 2)$.

Exercises Solve by graphing.

1. $y = -x + 2$
$y = -\frac{1}{2}x + 1$

2. $y = 3x - 3$
$y = x + 1$

3. $y - 2x = 1$
$2y - 4x = 1$

Solve by substitution.

4. $-5x + 3y = 12$
$x + 2y = 8$

5. $x - 4y = 22$
$2x + 5y = -21$

6. $y + 5x = -3$
$3y - 2x = 8$

Solve by elimination.

7. $-3x + y = 7$
$3x + 2y = 2$

8. $3x + 4y = -1$
$-9x - 4y = 13$

9. $-4x + 5y = -11$
$2x + 3y = 11$

Name an appropriate method to solve each system of equations. Then solve the system.

10. $4x - y = 11$
$2x - 3y = 3$

11. $4x + 6y = 3$
$-10x - 15y = -4$

12. $3x - 2y = 6$
$5x - 5y = 5$

13. $3y + x = 3$
$-2y + 5x = 15$

14. $4x - 7y = 8$
$-2x + 5y = -1$

15. $x + 3y = 6$
$4x - 2y = -32$

10 Square Roots and Simplifying Radicals

A radical expression is an expression that contains a square root. The expression is in simplest form when the following three conditions have been met.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

The **Product Property** states that for two numbers a and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

EXAMPLE

1 Simplify.

a. $\sqrt{45}$

$\sqrt{45} = \sqrt{3 \cdot 3 \cdot 5}$ Prime factorization of 45

$= \sqrt{3^2} \cdot \sqrt{5}$ Product Property of Square Roots

$= 3\sqrt{5}$ Simplify.

b. $\sqrt{6} \cdot \sqrt{15}$

$\sqrt{6} \cdot \sqrt{15} = \sqrt{6 \cdot 15}$ Product Property

$= \sqrt{3 \cdot 2 \cdot 3 \cdot 5}$ Prime factorization

$= \sqrt{3^2} \cdot \sqrt{10}$ Product Property

$= 3\sqrt{10}$ Simplify.

For radical expressions in which the exponent of the variable inside the radical is *even* and the resulting simplified exponent is *odd,* you must use absolute value to ensure nonnegative results.

EXAMPLE

2 $\sqrt{20x^3y^5z^6}$

$\sqrt{20x^3y^5z^6} = \sqrt{2^2 \cdot 5 \cdot x^3 \cdot y^5 \cdot z^6}$ Prime factorization

$= \sqrt{2^2} \cdot \sqrt{5} \cdot \sqrt{x^3} \cdot \sqrt{y^5} \cdot \sqrt{z^6}$ Product Property

$= 2 \cdot \sqrt{5} \cdot x \cdot \sqrt{x} \cdot y^2 \cdot \sqrt{y} \cdot |z^3|$ Simplify.

$= 2xy^2|z^3|\sqrt{5xy}$ Simplify.

The **Quotient Property** states that for any numbers a and b, where $a \geq 0$ and $b \geq 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

EXAMPLE

3 Simplify $\sqrt{\frac{25}{16}}$**.**

$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}}$ Quotient Property

$= \frac{5}{4}$ Simplify.

Rationalizing the denominator of a radical expression is a method used to eliminate radicals from the denominator of a fraction. To rationalize the denominator, multiply the expression by a fraction equivalent to 1 such that the resulting denominator is a perfect square.

EXAMPLE

4 Simplify.

a. $\frac{2}{\sqrt{3}}$

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \quad \text{Multiply by } \frac{\sqrt{3}}{\sqrt{3}}.$$

$$= \frac{2\sqrt{3}}{3} \quad \text{Simplify.}$$

b. $\frac{\sqrt{13y}}{\sqrt{18}}$

$$\frac{\sqrt{13y}}{\sqrt{18}} = \frac{\sqrt{13y}}{\sqrt{2 \cdot 3 \cdot 3}} \quad \text{Prime factorization}$$

$$= \frac{\sqrt{13y}}{3\sqrt{2}} \quad \text{Product Property}$$

$$= \frac{\sqrt{13y}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \quad \text{Multiply by } \frac{\sqrt{2}}{\sqrt{2}}.$$

$$= \frac{\sqrt{26y}}{6} \quad \text{Product Property}$$

Sometimes, conjugates are used to simplify radical expressions. Conjugates are binomials of the form $p\sqrt{q} + r\sqrt{s}$ and $p\sqrt{q} - r\sqrt{s}$.

EXAMPLE

5 Simplify $\frac{3}{5 - \sqrt{2}}$.

$$\frac{3}{5 - \sqrt{2}} = \frac{3}{5 - \sqrt{2}} \cdot \frac{5 + \sqrt{2}}{5 + \sqrt{2}} \quad \frac{5 + \sqrt{2}}{5 + \sqrt{2}} = 1$$

$$= \frac{3(5 + \sqrt{2})}{5^2 - (\sqrt{2})^2} \quad (a - b)(a + b) = a^2 - b^2$$

$$= \frac{15 + 3\sqrt{2}}{25 - 2} \quad \text{Multiply. } (\sqrt{2})^2 = 2$$

$$= \frac{15 + 3\sqrt{2}}{23} \quad \text{Simplify.}$$

Exercises Simplify.

1. $\sqrt{32}$
2. $\sqrt{75}$
3. $\sqrt{50} \cdot \sqrt{10}$
4. $\sqrt{12} \cdot \sqrt{20}$
5. $\sqrt{6} \cdot \sqrt{6}$
6. $\sqrt{16} \cdot \sqrt{25}$
7. $\sqrt{98x^3y^6}$
8. $\sqrt{56a^2b^4c^5}$
9. $\sqrt{\frac{81}{49}}$
10. $\sqrt{\frac{121}{16}}$
11. $\sqrt{\frac{63}{8}}$
12. $\sqrt{\frac{288}{147}}$
13. $\frac{\sqrt{10p^3}}{\sqrt{27}}$
14. $\frac{\sqrt{108}}{\sqrt{2q^6}}$
15. $\frac{4}{5 - 2\sqrt{3}}$
16. $\frac{7\sqrt{3}}{5 - 2\sqrt{6}}$
17. $\frac{3}{\sqrt{48}}$
18. $\frac{\sqrt{24}}{\sqrt{125}}$
19. $\frac{3\sqrt{5}}{2 - \sqrt{2}}$
20. $\frac{3}{-2 + \sqrt{13}}$

11 Multiplying Polynomials

The **Product of Powers** rule states that for any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

EXAMPLE 1 **Simplify each expression.**

a. $(4p^5)(p^4)$

$$(4p^5)(p^4) = (4)(1)(p^5 \cdot p^4)$$
$$= (4)(1)(p^{5+4})$$
$$= 4p^9$$

b. $(3yz^5)(-9y^2z^2)$

$$(3yz^5)(-9y^2z^2) = (3)(-9)(y \cdot y^2)(z^5 \cdot z^2)$$
$$= -27(y^{1+2})(z^{5+2})$$
$$= -27y^3z^7$$

The Distributive Property can be used to multiply a monomial by a polynomial.

EXAMPLE 2 **Simplify $3x^3(-4x^2 + x - 5)$.**

$3x^3(-4x^2 + x - 5) = 3x^3(-4x^2) + 3x^3(x) - 3x^3(5)$ Distributive Property

$= -12x^5 + 3x^4 - 15x^3$ Multiply.

To find the power of a power, multiply the exponents. This is called the **Power of a Power** rule.

EXAMPLE 3 **Simplify each expression.**

a. $(-3x^2y^4)^3$

$$(-3x^2y^4)^3 = (-3)^3(x^2)^3(y^4)^3$$
$$= -27x^6y^{12}$$

b. $(xy)^3(-2x^4)^2$

$$(xy)^3(-2x^4)^2 = x^3y^3(-2)^2(x^4)^2$$
$$= x^3y^3(4)x^8$$
$$= 4x^3 \cdot x^8 \cdot y^3$$
$$= 4x^{11}y^3$$

To multiply two binomials, find the sum of the products of

F the *First* terms,
O the *Outer* terms,
I the *Inner* terms, and
L the *Last* terms.

EXAMPLE 4 **Find $(2x - 3)(x + 1)$.**

$(2x - 3)(x + 1) = \underbrace{(2x)(x)}_{F} + \underbrace{(2x)(1)}_{O} + \underbrace{(-3)(x)}_{I} + \underbrace{(-3)(1)}_{L}$ FOIL method

$= 2x^2 + 2x - 3x - 3$ Multiply.

$= 2x^2 - x - 3$ Combine like terms.

The Distributive Property can be used to multiply any two polynomials.

EXAMPLE 5 **Find $(3x - 2)(2x^2 + 7x - 4)$.**

$$(3x - 2)(2x^2 + 7x - 4) = 3x(2x^2 + 7x - 4) - 2(2x^2 + 7x - 4) \quad \text{Distributive Property}$$
$$= 6x^3 + 21x^2 - 12x - 4x^2 - 14x + 8 \quad \text{Distributive Property}$$
$$= 6x^3 + 17x^2 - 26x + 8 \quad \text{Combine like terms.}$$

Three special products are $(a + b)^2 = a^2 + 2ab + b^2$,
$(a - b)^2 = a^2 - 2ab + b^2$, and
$(a + b)(a - b) = a^2 - b^2$.

EXAMPLE 6 **Find each product.**

a. $(2x - z)^2$

$$(a - b)^2 = a^2 - 2ab + b^2 \quad \text{Square of a difference}$$
$$(2x - z)^2 = (2x)^2 - 2(2x)(z) + (z)^2 \quad a = 2x \text{ and } b = z$$
$$= 4x^2 - 4xz + z^2 \quad \text{Simplify.}$$

b. $(3x + 7)(3x - 7)$

$$(a + b)(a - b) = a^2 - b^2 \quad \text{Product of sum and difference}$$
$$(3x + 7)(3x - 7) = (3x)^2 - (7)^2 \quad a = 3x \text{ and } b = 7$$
$$= 9x^2 - 49 \quad \text{Simplify.}$$

Exercises Find each product.

1. $(3q^2)(q^5)$
2. $(5m)(4m^3)$
3. $\left(\frac{9}{2}c\right)(8c^5)$
4. $(n^6)(10n^2)$
5. $(fg^8)(15f^2g)$
6. $(6j^4k^4)(j^2k)$
7. $(2ab^3)(4a^2b^2)$
8. $\left(\frac{8}{5}x^3y\right)\left(4x^3y^2\right)$
9. $-2q^2(q^2 + 3)$
10. $5p(p - 18)$
11. $15c(-3c^2 + 2c + 5)$
12. $8x(-4x^2 - x + 11)$
13. $4m^2(-2m^2 + 7m - 5)$
14. $8y^2(5y^3 - 2y + 1)$
15. $\left(\frac{3}{2}m^3n^2\right)^2$
16. $(-2c^3d^2)^2$
17. $(-5wx^5)^3$
18. $(6a^5b)^3$
19. $(k^2\ell)^3(13k^2)^2$
20. $(-5w^3x^2)^2(2w^5)^2$
21. $(-7y^3z^2)(4y^2)^4$
22. $\left(\frac{1}{2}p^2q^2\right)^2\left(4pq^3\right)^3$
23. $(m - 1)(m - 4)$
24. $(s - 7)(s - 2)$
25. $(x - 3)(x + 4)$
26. $(a + 3)(a - 6)$
27. $(5d + 3)(d - 4)$
28. $(q + 2)(3q + 5)$
29. $(2q + 3)(5q + 2)$
30. $(2a - 3)(2a - 5)$
31. $(d + 1)(d - 1)$
32. $(4a - 3)(4a + 3)$
33. $(s - 5)^2$
34. $(3f - g)^2$
35. $(2r - 5)^2$
36. $\left(t + \frac{8}{3}\right)^2$
37. $(x + 4)(x^2 - 5x - 2)$
38. $(x - 2)(x^2 + 3x - 7)$
39. $(3b - 2)(3b^2 + b + 1)$
40. $(2j + 7)(j^2 - 2j + 4)$

12 Dividing Polynomials

The **Quotient of Powers** rule states that for any nonzero number a and all integers m and n, $\frac{a^m}{a^n} = a^{m-n}$.

To find the power of a quotient, find the power of the numerator and the power of the denominator.

EXAMPLE

1 Simplify.

a. $\frac{x^5y^8}{-xy^3}$

$$\frac{x^5y^8}{-xy^3} = \left(\frac{x^5}{-x}\right)\left(\frac{y^8}{y^3}\right)$$ Group powers that have the same base.

$$= -(x^{5-1})(y^{8-3})$$ Quotient of powers

$$= -x^4y^5$$ Simplify.

b. $\frac{w^{-2}x^4}{2w^{-5}}$

$$\frac{w^{-2}x^4}{2w^{-5}} = \frac{1}{2}\left(\frac{w^{-2}}{w^{-5}}\right)x^4$$ Group powers that have the same base.

$$= \frac{1}{2}(w^{-2-(-5)})x^4$$ Quotient of powers

$$= \frac{1}{2}w^3x^4$$ Simplify.

You can divide a polynomial by a monomial by separating the terms of the numerator.

EXAMPLE

2 Simplify $\frac{15x^3 - 3x^2 + 12x}{3x}$.

$$\frac{15x^3 - 3x^2 + 12x}{3x} = \frac{15x^3}{3x} - \frac{3x^2}{3x} + \frac{12x}{3x}$$ Divide each term by $3x$.

$$= 5x^2 - x + 4$$ Simplify.

Division can sometimes be performed using factoring.

EXAMPLE

3 Find $(n^2 - 8n - 9) \div (n - 9)$.

$$(n^2 - 8n - 9) \div (n - 9) = \frac{n^2 - 8n - 9}{(n - 9)}$$ Write as a rational expression.

$$= \frac{(n - 9)(n + 1)}{(n - 9)}$$ Factor the numerator.

$$= \frac{\cancel{(n - 9)}(n + 1)}{\cancel{(n - 9)}}$$ Divide by the GCF.

$$= n + 1$$ Simplify.

When you cannot factor, you can use a long division process similar to the one you use in arithmetic.

EXAMPLE

4 **Find $(n^3 - 4n^2 - 9) \div (n - 3)$.**

In this case, there is no n term, so you must rename the dividend using 0 as the coefficient of the missing term.

$(n^3 - 4n^2 + 9) \div (n - 3) = (n^3 - 4n^2 + 0n + 9) \div (n - 3)$

Divide the first term of the dividend, n^3, by the first term of the divisor, n.

$$
\begin{array}{rll}
 & n^2 - n - 3 & \\
n - 3 \,\big)\!\! & \overline{n^3 - 4n^2 + 0n + 12} & \\
 & \underline{(-)n^3 - 3n^2} & \text{Multiply } n^2 \text{ and } n - 3. \\
 & -n^2 + 0n & \text{Subtract and bring down } 0n. \\
 & \underline{(-)-n^2 + 3n} & \text{Multiply } -n \text{ and } n - 3. \\
 & -3n + 12 & \text{Subtract and bring down 12.} \\
 & \underline{(-)-3n + 9} & \text{Multiply } -3 \text{ and } n - 3. \\
 & 3 & \text{Subtract.}
\end{array}
$$

Therefore, $(n^3 - 4n^2 + 9) \div (n - 3) = n^2 - n - 3 + \frac{3}{n - 3}$. Since the quotient has a nonzero remainder, $n - 3$ is not a factor of $n^3 - 4n^2 + 9$.

Exercises Find each quotient.

1. $\frac{a^2c^2}{2a}$
2. $\frac{5q^5r^3}{q^2r^2}$
3. $\frac{b^2d^5}{8b^{-2}d^3}$
4. $\frac{5p^{-3}x}{2p^{-7}}$
5. $\frac{3r^{-3}s^2t^4}{2r^2st^{-3}}$
6. $\frac{3x^3y^{-1}z^5}{xyz^2}$
7. $\left(\frac{w^4}{6}\right)^3$
8. $\left(\frac{-3q^2}{5}\right)^3$
9. $\left(\frac{-2y^2}{7}\right)^2$
10. $\left(\frac{5m^2}{3}\right)^4$
11. $\frac{4z^2 - 16z - 36}{4z}$
12. $(5d^2 + 8d - 20) \div 10d$
13. $(p^3 - 12p^2 + 3p + 8) \div 4p$
14. $(b^3 + 4b^2 + 10) \div 2b$
15. $\frac{a^3 - 6a^2 + 4a - 3}{a^2}$
16. $\frac{8x^2y - 10xy^2 + 6x^3}{2x^2}$
17. $\frac{s^2 - 2s - 8}{s - 4}$
18. $(r^2 + 9r + 20) \div (r + 5)$
19. $(t^2 - 7t + 12) \div (t - 3)$
20. $(c^2 + 3c - 54) \div (c + 9)$
21. $(2q^2 - 9q - 5) \div (q - 5)$
22. $\frac{3z^2 - 2z - 5}{z + 1}$
23. $\frac{(m^3 + 3m^2 - 5m + 1)}{m - 1}$
24. $(d^3 - 2d^2 + 4d + 24) \div (d + 2)$
25. $(2j^3 + 5j + 26) \div (j + 2)$
26. $\frac{2x^3 + 3x^2 - 176}{x - 4}$
27. $(x^2 + 6x - 3) \div (x + 4)$
28. $\frac{h^3 + 2h^2 - 6h + 1}{h - 2}$

13 Factoring to Solve Equations

Some polynomials can be factored using the Distributive Property.

EXAMPLE

1 Factor $5t^2 + 15t$.

Find the greatest common factor (GCF) of $5t^2$ and $15t$.

$5t^2 = 5 \cdot t \cdot t$, $15t = 3 \cdot 5 \cdot t$ — GCF: $5 \cdot t$ or $5t$

$5t^2 + 15t = 5t(t) + 5t(3)$ — Rewrite each term using the GCF.

$= 5t(t + 3)$ — Distributive Property

To factor polynomials of the form $x^2 + bx + c$, find two integers m and n so that $mn = c$ and $m + n = b$. Then write $x^2 + bx + c$ using the pattern $(x + m)(x + n)$.

To factor polynomials of the form $ax^2 + bx + c$, find two integers m and n with a product equal to ac and with a sum equal to b. Write $ax^2 + bx + c$ using the pattern $ax^2 + mx + nx + c$. Then factor by grouping.

EXAMPLE

2 Factor each polynomial.

a. $x^2 - 8x + 15$

b is negative and *c* is positive.

In this equation, b is -8 and c is 15. This means that $m + n$ is negative and mn is positive. So m and n must both be negative.

Factors of 15	Sum of Factors
−1, −15	−16
−3, −5	−8

$x^2 - 8x + 15 = (x + m)(x + n)$

$= (x - 3)(x - 5)$

The correct factors are −3 and −5.

Write the pattern; $m = -3$ and $n = -5$

b. $5x^2 - 19x - 4$

b is negative and *c* is negative.

In this equation, a is 5, b is -19, and c is -4. Find two numbers with a product of -20 and with a sum of -19.

Factors of −20	Sum of Factors
−2, 10	8
2, −10	−8
−1, 20	19
1, −20	−19

$5x^2 - 19x - 4 = 5x^2 + mx + nx - 4$

$= 5x^2 + x + (-20)x - 4$

$= (5x^2 + x) - (20x + 4)$ — Factor the GCF from each group.

$= x(5x + 1) - 4(5x + 1)$

$= (x - 4)(5x + 1)$ — Distributive Property

Here are some special products.

Perfect Square Trinomials

$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$

Difference of Squares

$a^2 - b^2 = (a + b)(a - b)$

Prerequisite Skills

EXAMPLE

3 **Factor each polynomial.**

a. $9x^2 + 6x + 1$

The first and last terms are perfect squares, and the middle term is equal to $2(3x)(1)$.

$$\begin{aligned} 9x^2 + 6x + 1 &= (3x)^2 + 2(3x)(1) + 1^2 && \text{Write as } a^2 + 2ab + b^2. \\ &= (3x + 1)^2 && \text{Factor using the pattern.} \end{aligned}$$

b. $x^2 - 9 = 0$

This is a difference of squares.

$$\begin{aligned} x^2 - 9 &= x^2 - (3)^2 && \text{Write in the form } a^2 - b^2. \\ &= (x - 3)(x + 3) && \text{Factor the difference of squares.} \end{aligned}$$

The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$. Since 0 times any number is equal to zero, this implies that we can use factoring to solve equations.

EXAMPLE

4 **Solve $x^2 - 5x + 4 = 0$ by factoring.**

$$\begin{aligned} x^2 - 5x + 4 &= 0 && \text{Original equation} \\ (x - 1)(x - 4) &= 0 && \text{Factor the polynomial.} \\ x - 1 = 0 \quad \text{or} \quad x - 4 &= 0 && \text{Zero Product Property} \\ x = 1 \qquad\qquad x &= 4 \end{aligned}$$

Exercises **Factor each polynomial.**

1. $u^2 - 12u$	**2.** $w^2 + 4w$	**3.** $7j^2 - 28j$
4. $2g^2 + 24g$	**5.** $6x^2 + 2x$	**6.** $5t^2 - 30t$
7. $z^2 + 10z + 21$	**8.** $n^2 + 8n + 15$	**9.** $h^2 + 8h + 12$
10. $x^2 + 14x + 48$	**11.** $m^2 + 6m - 7$	**12.** $b^2 + 2b - 24$
13. $q^2 - 9q + 18$	**14.** $p^2 - 5p + 6$	**15.** $a^2 - 3a - 4$
16. $k^2 - 4k - 32$	**17.** $n^2 - 7n - 44$	**18.** $y^2 - 3y - 88$
19. $3z^2 + 4z - 4$	**20.** $2y^2 + 9y - 5$	**21.** $5x^2 + 7x + 2$
22. $3s^2 + 11s - 4$	**23.** $6r^2 - 5r + 1$	**24.** $8a^2 + 15a - 2$
25. $w^2 - \frac{9}{4}$	**26.** $c^2 - 64$	**27.** $r^2 + 14r + 49$
28. $b^2 + 18b + 81$	**29.** $j^2 - 12j + 36$	**30.** $4t^2 - 25$

Solve each equation by factoring.

31. $10r^2 - 35r = 0$	**32.** $3x^2 + 15x = 0$	**33.** $k^2 + 13k + 36 = 0$
34. $w^2 - 8w + 12 = 0$	**35.** $c^2 - 5c - 14 = 0$	**36.** $z^2 - z - 42 = 0$
37. $2y^2 - 5y - 12 = 0$	**38.** $3b^2 - 4b - 15 = 0$	**39.** $t^2 + 12t + 36 = 0$
40. $u^2 + 5u + \frac{25}{4} = 0$	**41.** $q^2 - 8q + 16 = 0$	**42.** $a^2 - 6a + 9 = 0$

Prerequisite Skills

14 Operations with Matrices

A **matrix** is a rectangular arrangement of numbers in rows and columns. Each entry in a matrix is called an **element**. A matrix is usually described by its **dimensions**, or the number of **rows** and **columns**, with the number of rows stated first. For example, matrix A has dimensions 3×2 and matrix B has dimensions 2×4.

$$\text{matrix } A = \begin{bmatrix} 6 & -2 \\ 0 & 5 \\ -4 & 10 \end{bmatrix} \qquad \text{matrix } B = \begin{bmatrix} 7 & -1 & -2 & 0 \\ 3 & 6 & -5 & 2 \end{bmatrix}$$

If two matrices have the same dimensions, you can add or subtract them. To do this, add or subtract corresponding elements of the two matrices.

EXAMPLE

1 **If** $A = \begin{bmatrix} 12 & 7 & -3 \\ 0 & -1 & -6 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 0 & 5 \\ 2 & 7 & -7 \end{bmatrix}$, **and** $C = \begin{bmatrix} 9 & 1 & -5 \\ 0 & -1 & 15 \end{bmatrix}$, **find the sum and difference.**

a. $A + B$

$$A + B = \begin{bmatrix} 12 & 7 & -3 \\ 0 & -1 & -6 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 5 \\ 2 & 7 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 12 + (-3) & 7 + 0 & -3 + 5 \\ 0 + 2 & -1 + 7 & -6 + (-7) \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 7 & 2 \\ 2 & 6 & -13 \end{bmatrix}$$

b. $B - C$

$$B - C = \begin{bmatrix} -3 & 0 & 5 \\ 2 & 7 & -7 \end{bmatrix} - \begin{bmatrix} 9 & 1 & -5 \\ 0 & -1 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} -3 - 9 & 0 - 1 & 5 - (-5) \\ 2 - 0 & 7 - (-1) & -7 - 15 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & -1 & 10 \\ 2 & 8 & -22 \end{bmatrix}$$

You can multiply any matrix by a constant called a *scalar*. This is called **scalar multiplication**. To perform scalar multiplication, multiply each element by the scalar.

EXAMPLE

2 **If** $D = \begin{bmatrix} -4 & 6 & -1 \\ 0 & 7 & 2 \\ -3 & -8 & -4 \end{bmatrix}$, **find** $2D$.

$$2D = 2\begin{bmatrix} -4 & 6 & -1 \\ 0 & 7 & 2 \\ -3 & -8 & -4 \end{bmatrix} \quad \text{Substitution}$$

$$= \begin{bmatrix} 2(-4) & 2(6) & 2(-1) \\ 2(0) & 2(7) & 2(2) \\ 2(-3) & 2(-8) & 2(-4) \end{bmatrix} \text{ or } \begin{bmatrix} -8 & 12 & -2 \\ 0 & 14 & 4 \\ -6 & -16 & -8 \end{bmatrix} \quad \text{Simplify.}$$

You can multiply two matrices if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix. The product of two matrices is found by multiplying columns and rows. The entry in the first row and first column of AB, the resulting product, is found by multiplying corresponding elements in the first row of A and the first column of B and then adding.

EXAMPLE

3 **Find EF if $E = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix}$ and $F = \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix}$.**

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix}$$

Multiply the numbers in the first row of E by the numbers in the first column of F and add the products.

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 3(-1) + (-2)(6) & \\ & \end{bmatrix}$$

Multiply the numbers in the first row of E by the numbers in the second column of F and add the products.

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 3(-1) + (-2)(6) & 3(5) + (-2)(-3) \\ & \end{bmatrix}$$

Multiply the numbers in the second row of E by the numbers in the first column of F and add the products.

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 3(-1) + (-2)(6) & 3(5) + (-2)(-3) \\ 0(-1) + 6(6) & \end{bmatrix}$$

Multiply the numbers in the second row of E by the numbers in the second column of F and add the products.

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 3(-1) + (-2)(6) & 3(5) + (-2)(-3) \\ 0(-1) + 6(6) & 0(5) + 6(-3) \end{bmatrix}$$

Simplify the matrix.

$$\begin{bmatrix} 3(-1) + (-2)(6) & 3(5) + (-2)(-3) \\ 0(-1) + 6(6) & 0(5) + 6(-3) \end{bmatrix} = \begin{bmatrix} -15 & 21 \\ 36 & -18 \end{bmatrix}$$

Exercises

If $A = \begin{bmatrix} 10 & -9 \\ 4 & -3 \\ -1 & 11 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -3 \\ 2 & 8 \\ 7 & 6 \end{bmatrix}$, and $C = \begin{bmatrix} 8 & 0 \\ -2 & 2 \\ -10 & 6 \end{bmatrix}$, find each sum, difference, or product.

1. $A + B$ **2.** $B + C$ **3.** $A - C$ **4.** $C - B$

5. $3A$ **6.** $5B$ **7.** $-4C$ **8.** $\frac{1}{2}C$

9. $2A + C$ **10.** $A - 5C$ **11.** $\frac{1}{2}C + B$ **12.** $3A - 3B$

If $X = \begin{bmatrix} 2 & -8 \\ 10 & 4 \end{bmatrix}$, $Y = \begin{bmatrix} -1 & 0 \\ 6 & -5 \end{bmatrix}$, and $Z = \begin{bmatrix} 4 & -8 \\ -7 & 0 \end{bmatrix}$, find each sum, difference, or product.

13. $X + Z$ **14.** $Y + Z$ **15.** $X - Y$ **16.** $3Y$

17. $-6X$ **18.** $\frac{1}{2}X + Z$ **19.** $5Z - 2Y$ **20.** XY

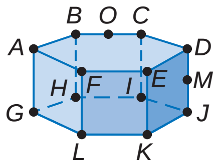
21. YZ **22.** XZ **23.** $\frac{1}{2}(XZ)$ **24.** $XY + 2Z$

Extra Practice

Lesson 1-1

Pages 7–11

For Exercises 1–7, refer to the figure.



1. How many planes are shown in the figure?
2. Name three collinear points.
3. Name all planes that contain point G.
4. Name the intersection of plane ABD and plane DJK.
5. Name two planes that do not intersect.
6. Name a plane that contains $\overleftrightarrow{FK}$ and $\overleftrightarrow{EL}$.
7. Is the intersection of plane ACD and plane EDJ a point or a line? Explain.

Draw and label a figure for each relationship.

8. Line a intersects planes A, B, and C at three distinct points.
9. Planes X and Z intersect in line m. Line b intersects the two planes in two distinct points.

Lesson 1-2

Pages 13–20

Find the precision for each measurement. Explain its meaning.

1. 42 in. .
2. 86 mm
3. 251 cm
4. 33.5 in.
5. $5\frac{1}{4}$ ft
6. 89 m

Find the value of the variable and BC if B is between A and C.

7. $AB = 4x$, $BC = 5x$; $AB = 16$
8. $AB = 17$, $BC = 3m$, $AC = 32$
9. $AB = 9a$, $BC = 12a$, $AC = 42$
10. $AB = 25$, $BC = 3b$, $AC = 7b + 13$
11. $AB = 5n + 5$, $BC = 2n$; $AC = 54$
12. $AB = 6c - 8$, $BC = 3c + 1$, $AC = 65$

Lesson 1-3

Pages 21–29

Use the Pythagorean Theorem to find the distance between each pair of points.

1. $A(0, 0)$, $B(-3, 4)$
2. $C(-1, 2)$, $N(5, 10)$
3. $X(-6, -2)$, $Z(6, 3)$
4. $M(-5, -8)$, $O(3, 7)$
5. $T(-10, 2)$, $R(6, -10)$
6. $F(5, -6)$, $N(-5, 6)$

Use the Distance Formula to find the distance between each pair of points.

7. $D(0, 0)$, $M(8, -7)$
8. $X(-1, 1)$, $Y(1, -1)$
9. $Z(-4, 0)$, $A(-3, 7)$
10. $K(6, 6)$, $D(-3, -3)$
11. $T(-1, 3)$, $N(0, 2)$
12. $S(7, 2)$, $E(-6, 7)$

Find the coordinates of the midpoint of a segment having the given endpoints.

13. $A(0, 0)$, $D(-2, -8)$
14. $D(-4, -3)$, $E(2, 2)$
15. $K(-4, -5)$, $M(5, 4)$
16. $R(-10, 5)$, $S(8, 4)$
17. $B(2.8, -3.4)$, $Z(1.2, 5.6)$
18. $D(-6.2, 7)$, $K(3.4, -4.8)$

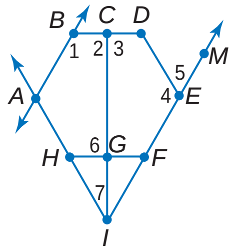
Find the coordinates of the missing endpoint given that B is the midpoint of $\overline{AC}$.

19. $C(0, 0)$, $B(5, -6)$
20. $C(-7, -4)$, $B(3, 5)$
21. $C(8, -4)$, $B(-10, 2)$
22. $C(6, 8)$, $B(-3, 5)$
23. $C(6, -8)$, $B(3, -4)$
24. $C(-2, -4)$, $B(0, 5)$

Lesson 1-4

Pages 31–38

For Exercises 1–14, use the figure at the right.



Name the vertex of each angle.

1. $\angle 1$
2. $\angle 4$
3. $\angle 6$
4. $\angle 7$

Name the sides of each angle.

5. $\angle AIE$
6. $\angle 4$
7. $\angle 6$
8. $\angle AHF$

Write another name for each angle.

9. $\angle 3$
10. $\angle DEF$
11. $\angle 2$

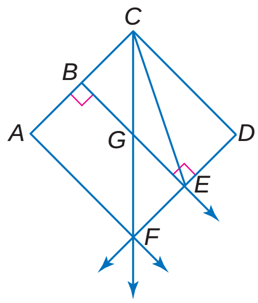
Measure each angle and classify it as *right*, *acute*, or *obtuse*.

12. $\angle ABC$
13. $\angle CGF$
14. $\angle HIF$

Lesson 1-5

Pages 40–47

For Exercises 1–7, use the figure at the right and a protractor.



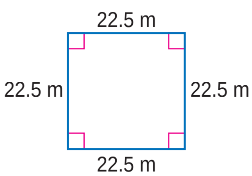
1. Name two acute vertical angles.
2. Name two obtuse vertical angles.
3. Name a pair of complementary adjacent angles.
4. Name a pair of supplementary adjacent angles.
5. Name a pair of congruent supplementary adjacent angles.
6. If $m\angle BGC = 4x + 5$ and $m\angle FGE = 6x - 15$, find $m\angle BGF$.
7. If $m\angle BCG = 5a + 5$, $m\angle GCE = 3a - 4$, and $m\angle ECD = 4a - 7$, find the value of a so that $\overline{AC} \perp \overline{CD}$.
8. The measure of $\angle A$ is nine less than the measure of $\angle B$. If $\angle A$ and $\angle B$ form a linear pair, what are their measures?
9. The measure of an angle's complement is 17 more than the measure of the angle.

Lesson 1-6

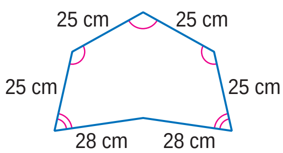
Pages 49–57

Name each polygon by its number of sides. Classify it as *convex* or *concave* and *regular* or *irregular.*

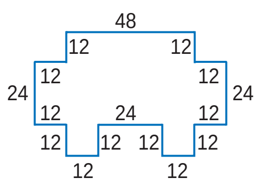
1.



2.



3.



COORDINATE GEOMETRY **Find the perimeter and area of each figure with the given vertices.**

4. triangle with vertices $X(3, 3)$, $Y(-2, 1)$, and $Z(3, 1)$
5. triangle with vertices $A(1, 6)$, $B(1, 2)$, and $C(3, 2)$
6. rectangle with vertices $J(-3, 5)$, $K(2, 5)$, $L(2, -3)$, and $M(-3, -3)$
7. parallelogram with vertices $P(-3, 3)$, $Q(4, 3)$, $R(3, -1)$, $S(-4, -1)$

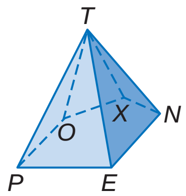
Extra Practice

Lesson 1-7

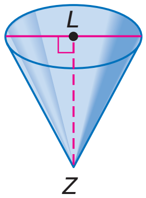
Pages 60–66

Identify each solid. Name the bases, faces, edges, and vertices.

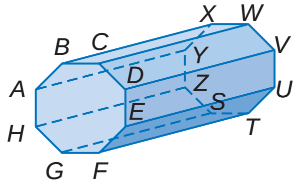
1.



2.

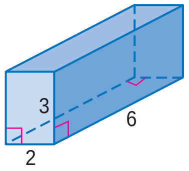


3.

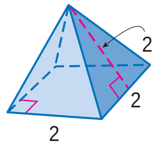


For each solid, draw a net and find the surface area. Round to the nearest tenth if necessary.

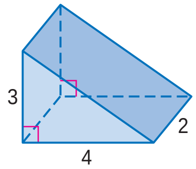
4.



5.



6.



Lesson 2-1

Pages 78–82

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

1. Lines j and k are parallel.
2. $A(-1, -7)$, $B(4, -7)$, $C(4, -3)$, $D(-1, -3)$
3. $\overline{AB}$ bisects $\overline{CD}$ at K.
4. $\overrightarrow{SR}$ is an angle bisector of $\angle TSU$.

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

5. **Given:** EFG is an equilateral triangle.
   Conjecture: $EF = FG$
6. **Given:** r is a rational number.
   Conjecture: r is a whole number.
7. **Given:** n is a whole number.
   Conjecture: n is a rational number.
8. **Given:** $\angle 1$ and $\angle 2$ are supplementary angles.
   Conjecture: $\angle 1$ and $\angle 2$ form a linear pair.

Lesson 2-2

Pages 83–90

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

p: $(-3)^2 = 9$ **q: A robin is a fish.** **r: An acute angle measures less than 90°.**

1. p and q
2. p or q
3. p and r
4. p or r
5. $\sim p$ or q
6. p or $\sim r$
7. $q \wedge r$
8. $(p \wedge q) \vee r$
9. $\sim p \vee \sim r$

Copy and complete each truth table.

10.

p	q	$\sim q$	$p \vee \sim q$
T			
T			
F			
F			

11.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T			
T	F			
F	T			
F	F			

Lesson 2-3

Pages 91–97

Identify the hypothesis and conclusion of each statement.

1. If no sides of a triangle are equal, then it is a scalene triangle.
2. If it rains today, you will be wearing your raincoat.
3. If $6 - x = 11$, then $x = -5$.

Write each statement in if-then form.

4. The sum of the measures of two supplementary angles is 180.
5. A triangle with two congruent sides is an isosceles triangle.
6. Two lines that do not intersect are parallel lines.

Write the converse, inverse, and contrapositive of each conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

7. All triangles are polygons.
8. If two angles are congruent angles, then they have the same measure.
9. If three points lie on the same line, then they are collinear.

Lesson 2-4

Pages 99–104

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it. If not, write *no conclusion.*

1. (1) If it rains, then the field will be muddy.
   (2) If the field is muddy, then the game will be cancelled.
2. (1) If you read a book, then you enjoy reading.
   (2) If you are in the 10th grade, then you passed the 9th grade.

Determine if statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid.*

3. (1) If it snows outside, you will wear your winter coat.
   (2) It is snowing outside.
   (3) You will wear your winter coat.
4. (1) Two complementary angles are both acute angles.
   (2) $\angle 1$ and $\angle 2$ are acute angles.
   (3) $\angle 1$ and $\angle 2$ are complementary.

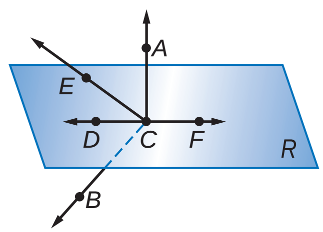
Lesson 2-5

Pages 106–109

Determine whether each statement is *always, sometimes,* or *never* true. Explain.

1. $\overleftrightarrow{RS}$ is perpendicular to $\overleftrightarrow{PS}$.
2. Three points will lie on one line.
3. Points B and C are in plane K. A line perpendicular to line BC is in plane K.

In the figure at the right, $\overleftrightarrow{EC}$ and $\overleftrightarrow{CD}$ are in plane $\mathcal{R}$, and F is on $\overleftrightarrow{CD}$. State the postulate that can be used to show each statement is true.



4. $\overleftrightarrow{DF}$ lies in plane $\mathcal{R}$.
5. E and C are collinear.
6. D, F, and E are coplanar.
7. E and F are collinear.

Extra Practice

Lesson 2-6

Pages 111–117

State the property that justifies each statement.

1. If $x - 5 = 6$, then $x = 11$.
2. If $a - b = r$, then $r = a - b$.
3. If $AB = CD$ and $CD = EF$, then $AB = EF$.
4. Copy and complete the following proof.

Given: $\frac{5x - 1}{8} = 3$

Prove: $x = 5$

Proof:

Statements	Reasons
a. ?	**a.** Given
b. ?	**b.** Multiplication Prop.
c. $5x - 1 = 24$	**c.** ?
d. $5x = 25$	**d.** ?
e. ?	**e.** Division Property

Lesson 2-7

Pages 118–123

Justify each statement with a property of equality or a property of congruence.

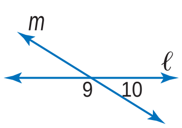
1. If $CD = OP$, then $CD + GH = OP + GH$.
2. If $\overline{MN} \cong \overline{PQ}$, then $\overline{PQ} \cong \overline{MN}$.
3. If $\overline{TU} \cong \overline{JK}$ and $\overline{JK} \cong \overline{DF}$, then $\overline{TU} \cong \overline{DF}$.
4. If $AB = 10$ and $CD = 10$, then $AB = CD$.
5. $\overline{XB} \cong \overline{XB}$
6. If $GH = RS$, then $GH - VW = RS - VW$.
7. If $EF = XY$, then $EF + KL = XY + KL$.
8. If $\overline{JK} \cong \overline{XY}$ and $\overline{XY} \cong \overline{LM}$, then $\overline{JK} \cong \overline{LM}$.

Lesson 2-8

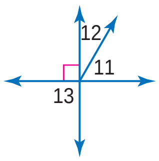
Pages 124–131

Find the measure of each numbered angle.

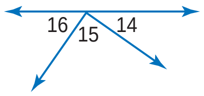
1. $m\angle 9 = 141 + x$
   $m\angle 10 = 25 + x$



2. $m\angle 11 = x + 40$
   $m\angle 12 = x + 10$
   $m\angle 13 = 3x + 30$



3. $m\angle 14 = x + 25$
   $m\angle 15 = 4x + 50$
   $m\angle 16 = x + 45$



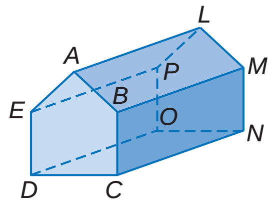
Determine whether each statement is *always, sometimes*, or *never* true.

4. Two angles that are complementary are congruent.
5. Two angles that form a linear pair are complementary.
6. Two congruent angles are supplementary.
7. Perpendicular lines form four right angles.
8. Two right angles are supplementary.
9. Two lines intersect to form four right angles.

Lesson 3-1

Pages 142–147

For Exercises 1–3, refer to the figure at the right.

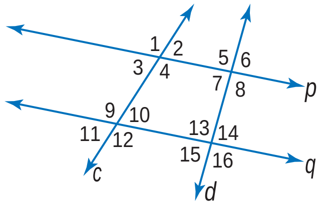


1. Name all segments parallel to $\overline{AE}$.

2. Name all planes intersecting plane *BCN*.

3. Name all segments skew to $\overline{DC}$.

Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.



4. $\angle 2$ and $\angle 5$

5. $\angle 9$ and $\angle 13$

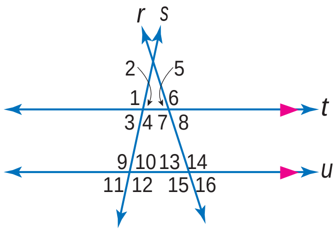
6. $\angle 12$ and $\angle 13$

7. $\angle 3$ and $\angle 6$

Lesson 3-2

Pages 149–154

In the figure, $m\angle 5 = 72$ and $m\angle 9 = 102$. Find the measure of each angle.



1. $m\angle 1$

2. $m\angle 13$

3. $m\angle 4$

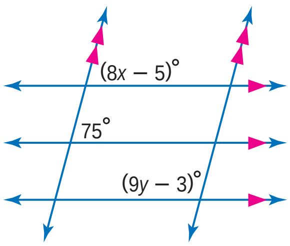
4. $m\angle 10$

5. $m\angle 7$

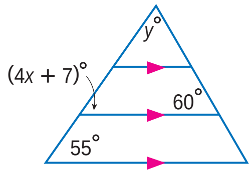
6. $m\angle 16$

Find *x* and *y* in each figure.

7.



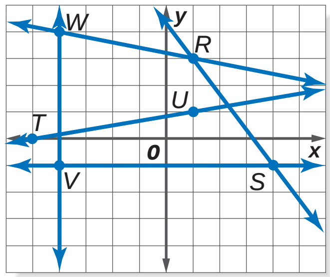
8.



Lesson 3-3

Pages 156–163

Find the slope of each line.



1. $\overleftrightarrow{RS}$

2. $\overleftrightarrow{TU}$

3. $\overleftrightarrow{WV}$

4. $\overleftrightarrow{WR}$

5. a line parallel to $\overleftrightarrow{TU}$

6. a line perpendicular to $\overleftrightarrow{WR}$

7. a line perpendicular to $\overleftrightarrow{WV}$

Determine whether $\overleftrightarrow{RS}$ and $\overleftrightarrow{TU}$ are *parallel, perpendicular*, or *neither.*

8. $R(3, 5)$, $S(5, 6)$, $T(-2, 0)$, $U(4, 3)$

9. $R(5, 11)$, $S(2, 2)$, $T(-1, 0)$, $U(2, 1)$

10. $R(-1, 4)$, $S(-3, 7)$, $T(5, -1)$, $U(8, 1)$

11. $R(-2, 5)$, $S(-4, 1)$, $T(3, 3)$, $U(1, 5)$

Extra Practice

Lesson 3-4

Pages 165–170

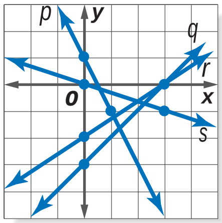
Write an equation in slope-intercept form of the line having the given slope and *y*-intercept.

1. $m = 1$, y-intercept: -5
2. $m = -\frac{1}{2}$, y-intercept: $\frac{1}{2}$
3. $m = 3$, $b = -\frac{1}{4}$

Write an equation in point-slope form of the line having the given slope thatcontains the given point.

4. $m = 3$, $(-2, 4)$
5. $m = -4$, $(0, 3)$
6. $m = \frac{2}{3}$, $(5, -7)$

Write an equation in slope-intercept form for each line.

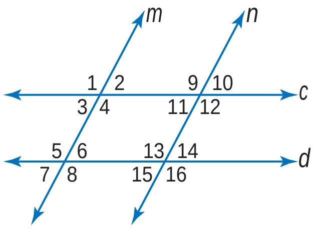


7. p
8. q
9. r
10. s
11. parallel to line q, contains $(2, -5)$
12. perpendicular to line p, contains $(0, 0)$

Lesson 3-5

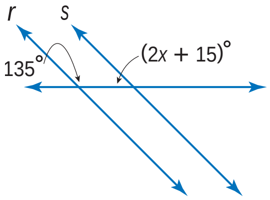
Pages 172–179

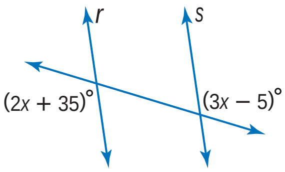
Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

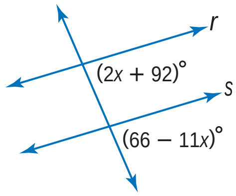


1. $\angle 9 \cong \angle 16$
2. $\angle 10 \cong \angle 16$
3. $\angle 12 \cong \angle 13$
4. $m\angle 12 + m\angle 14 = 180$

Find *x* so that r || s.

5. 

6. 

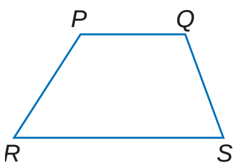
7. 

Lesson 3-6

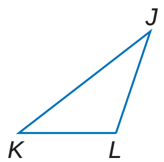
Pages 181–187

Copy each figure. Draw the segment that represents the distance indicated.

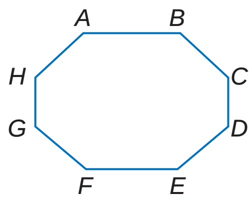
1. P to $\overleftrightarrow{RS}$



2. J to $\overleftrightarrow{KL}$



3. B to $\overleftrightarrow{FE}$



Find the distance between each pair of parallel lines with the given equations.

4. $y = \frac{2}{3}x - 2$
   $y = \frac{2}{3}x + \frac{1}{2}$
5. $y = 2x + 4$
   $y - 2x = -5$
6. $x + 4y = -6$
   $x + 4y = 4$

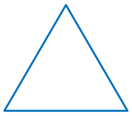
COORDINATE GEOMETRY Construct a line perpendicular to ℓ through *P*. Then find the distance from *P* to ℓ.

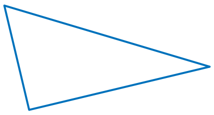
7. Line ℓ contains points $(0, 4)$ and $(-4, 0)$. Point P has coordinates $(2, -1)$.
8. Line ℓ contains points $(3, -2)$ and $(0, 2)$. Point P has coordinates $(-2.5, 3)$.

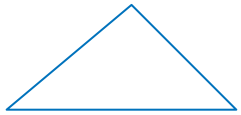
Lesson 4-1

Pages 202–208

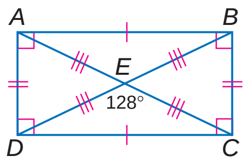
Use a protractor to classify each triangle as *acute, equiangular, obtuse*, or *right*.

1. 

2. 

3. 

Identify the indicated type of triangles in the figure.



4. right

5. obtuse

6. acute

7. isosceles

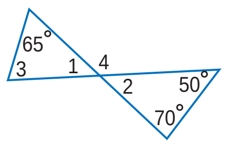
8. Find a and the measure of each side of equilateral triangle MNO if $MN = 5a$, $NO = 4a + 6$, and $MO = 7a - 12$.

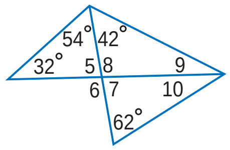
9. Triangle TAC is an isosceles triangle with $\overline{TA} \cong \overline{AC}$. Find b, TA, AC, and TC if $TA = 3b + 1$, $AC = 4b - 11$, and $TC = 6b - 2$.

Lesson 4-2

Pages 210–216

Find each measure.





1. $m\angle 1$

2. $m\angle 2$

3. $m\angle 3$

4. $m\angle 4$

5. $m\angle 5$

6. $m\angle 6$

7. $m\angle 7$

8. $m\angle 8$

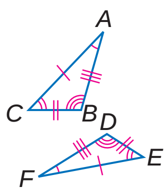
9. $m\angle 9$

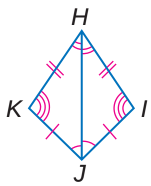
10. $m\angle 10$

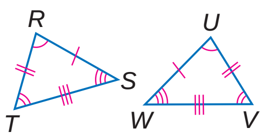
Lesson 4-3

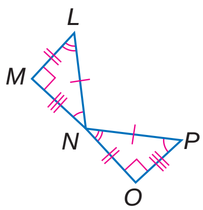
Pages 217–223

Identify the congruent triangles in each figure.

1. 

2. 

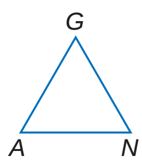
3. 

4. 

5. Write a two-column proof.

Given: $\triangle ANG \cong \triangle NGA$
$\triangle NGA \cong \triangle GAN$

Prove: $\triangle AGN$ is equilateral and equiangular.



Lesson 4-4

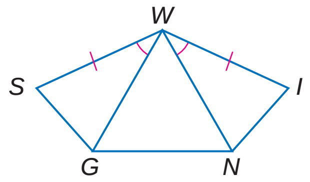
Pages 225–232

Determine whether $\triangle RST \cong \triangle JKL$ given the coordinates of the vertices. Explain.

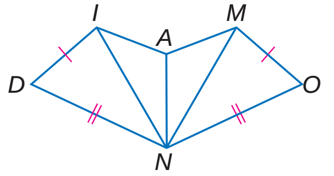
1. $R(-6, 2)$, $S(-4, 4)$, $T(-2, 2)$, $J(6, -2)$, $K(4, -4)$, $L(2, -2)$
2. $R(-6, 3)$, $S(-4, 7)$, $T(-2, 3)$, $J(2, 3)$, $K(5, 7)$, $L(6, 3)$

Write a two-column proof.

3. **Given:** $\triangle GWN$ is equilateral.
$\overline{WS} \cong \overline{WI}$
$\angle SWG \cong \angle IWN$
Prove: $\triangle SWG \cong \triangle IWN$



4. **Given:** $\triangle ANM \cong \triangle ANI$
$\overline{DI} \cong \overline{OM}$
$\overline{ND} \cong \overline{NO}$
Prove: $\triangle DIN \cong \triangle OMN$

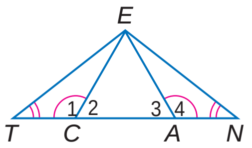


Lesson 4-5

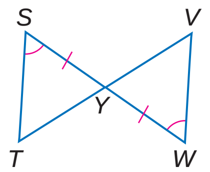
Pages 234–241

Write a paragraph proof.

1. **Given:** $\triangle TEN$ is isosceles with base $\overline{TN}$.
$\angle 1 \cong \angle 4$, $\angle T \cong \angle N$
Prove: $\triangle TEC \cong \triangle NEA$

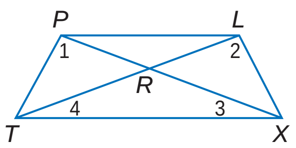


2. **Given:** $\angle S \cong \angle W$
$\overline{SY} \cong \overline{YW}$
Prove: $\overline{ST} \cong \overline{WV}$

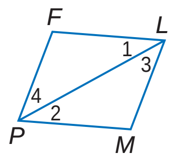


Write a flow proof.

3. **Given:** $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$
Prove: $\overline{PT} \cong \overline{LX}$



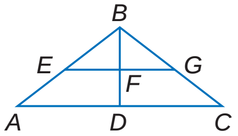
4. **Given:** $\overline{FP} \parallel \overline{ML}$, $\overline{FL} \parallel \overline{MP}$
Prove: $\overline{MP} \cong \overline{FL}$



Lesson 4-6

Pages 244–250

Refer to the figure for Exercises 1–6.



1. If $\overline{AD} \cong \overline{BD}$, name two congruent angles.
2. If $\overline{BF} \cong \overline{FG}$, name two congruent angles.
3. If $\overline{BE} \cong \overline{BG}$, name two congruent angles.
4. If $\angle FBE \cong \angle FEB$, name two congruent segments.
5. If $\angle BCA \cong \angle BAC$, name two congruent segments.
6. If $\angle DBC \cong \angle BCD$, name two congruent segments.

Lesson 4-7

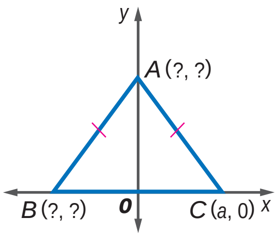
Pages 251–255

Position and label each triangle on the coordinate plane.

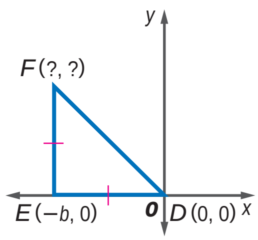
1. isosceles $\triangle ABC$ with base $\overline{BC}$ that is r units long
2. equilateral $\triangle XYZ$ with sides $4b$ units long
3. isosceles right $\triangle RST$ with hypotenuse $\overline{ST}$ and legs $(3 + a)$ units long
4. equilateral $\triangle CDE$ with base $\overline{DE}$ $\frac{1}{4}b$ units long

Name the missing coordinates of each triangle.

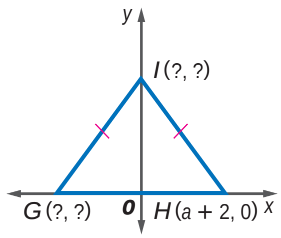
5.



6.



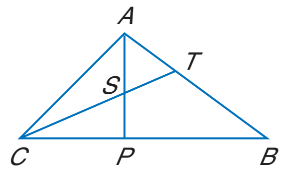
7.

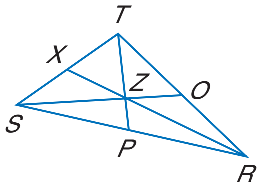


Lesson 5-1

Pages 269–278

For Exercises 1–4, refer to the figures at the right.





1. Suppose $CP = 7x - 1$ and $PB = 6x + 3$. If S is the circumcenter of $\triangle ABC$, find x and CP.
2. Suppose $m\angle ACT = 15a - 8$ and $m\angle ACB = 74$. If S is the incenter of $\triangle ABC$, find a and $m\angle ACT$.
3. Suppose $TO = 7b + 5$, $OR = 13b - 10$, and $TR = 18b$. If Z is the centroid of $\triangle TRS$, find b and TR.
4. Suppose $XR = 19n - 14$ and $ZR = 10n + 4$. If Z is the centroid of $\triangle TRS$, find n and ZR.

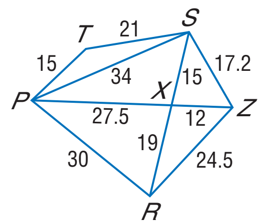
State whether each sentence is *always, sometimes,* or *never* true.

5. The circumcenter and incenter of a triangle are the same point.
6. The three altitudes of a triangle intersect at a point inside the triangle.
7. In an equilateral triangle, the circumcenter, incenter, and centroid are the same point.
8. The incenter is inside of a triangle.

Lesson 5-2

Pages 280–287

Determine the relationship between the measures of the given angles.

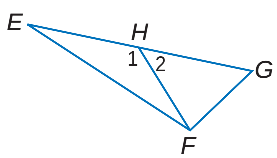


1. $\angle TPS$, $\angle TSP$
2. $\angle PRZ$, $\angle ZPR$
3. $\angle SPZ$, $\angle SZP$
4. $\angle SPR$, $\angle SRP$

Write a two-column proof.

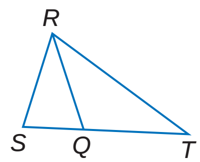
5. **Given:** $FH > FG$

   Prove: $m\angle 1 > m\angle 2$



6. **Given:** $\overline{RQ}$ bisects $\angle SRT$.

   Prove: $m\angle SQR > m\angle SRQ$



Lesson 5-3

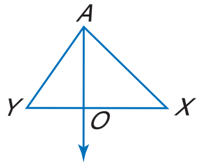
Pages 288–293

Write the assumption you would make to start an indirect proof of each statement.

1. $\angle ABC \cong \angle XYZ$
2. $\overrightarrow{RS}$ bisects $\angle ARC$.
3. An angle bisector of an equilateral triangle is also a median.

Write an indirect proof.

4. **Given:** $\angle AOY \cong \angle AOX$; $\overline{XO} \not\cong \overline{YO}$
   Prove: $\overrightarrow{AO}$ is not the angle bisector of $\angle XAY$.



5. **Given:** $\triangle RUN$
   Prove: There can be no more than one right angle in $\triangle RUN$.

Lesson 5-4

Pages 296–301

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*.

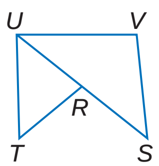
1. 2, 2, 6
2. 2, 3, 4
3. 6, 8, 10
4. 1, 1, 2
5. 15, 20, 30
6. 1, 3, 5
7. 2.5, 3.5, 6.5
8. 0.3, 0.4, 0.5

Find the range for the measure of the third side of a triangle given the measures of two sides.

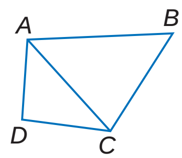
9. 6 and 10
10. 2 and 5
11. 20 and 12
12. 8 and 8
13. 18 and 36
14. 32 and 34
15. 2 and 29
16. 80 and 25

Write a two-column proof.

17. **Given:** $RS = RT$
    Prove: $UV + VS > UT$



18. **Given:** quadrilateral $ABCD$
    Prove: $AD + CD + AB > BC$

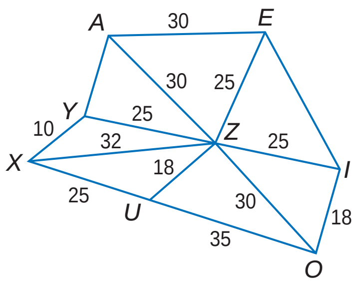


Lesson 5-5

Pages 302–309

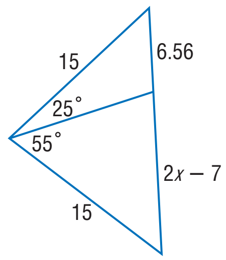
Write an inequality relating the given pair of angle or segment measures.

1. XZ, OZ
2. $m\angle ZIO$, $m\angle ZUX$
3. $m\angle AEZ$, $m\angle AZE$
4. IO, AE
5. $m\angle AZE$, $m\angle IZO$

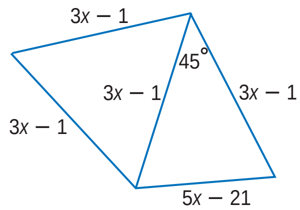


Write an inequality to describe the possible values of *x*.

6.



7.



Lesson 6-1

Pages 318–323

Find the sum of the measures of the interior angles of each convex polygon.

1. 25-gon
2. 30-gon
3. 22-gon
4. 17-gon
5. $5a$-gon
6. b-gon

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

7. 156
8. 168
9. 162

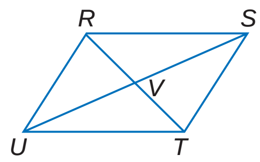
Find the measures of an interior angle and an exterior angle given the number of sides of a regular polygon. Round to the nearest tenth if necessary.

10. 15
11. 13
12. 42

Lesson 6-2

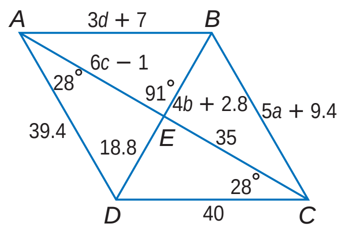
Pages 325–331

Complete each statement about ▱*RSTU*. Justify your answer.



1. $\angle SRU \cong$ __?__
2. $\angle UTS$ is supplementary to __?__.
3. $\overline{RU} \parallel$ __?__
4. $\overline{RU} \cong$ __?__
5. $\triangle RST \cong$ __?__
6. $\overline{SV} \cong$ __?__

ALGEBRA **Use ▱*ABCD* to find each measure or value.**

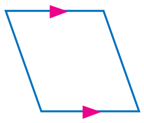


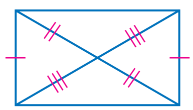
7. $m\angle BAE =$ __?__
8. $m\angle BCE =$ __?__
9. $m\angle BEC =$ __?__
10. $m\angle CED =$ __?__
11. $m\angle ABE =$ __?__
12. $m\angle EBC =$ __?__
13. $a =$ __?__
14. $b =$ __?__
15. $c =$ __?__
16. $d =$ __?__

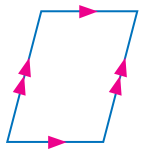
Lesson 6-3

Pages 333–339

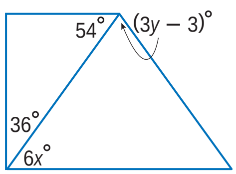
Determine whether each quadrilateral is a parallelogram. Justify your answer.

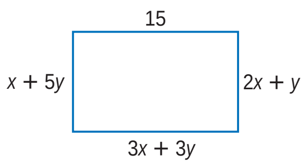
1. 

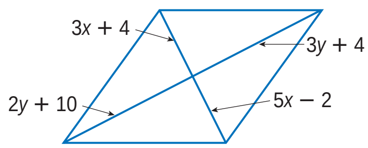
2. 

3. 

ALGEBRA **Find *x* and *y* so that each quadrilateral is a parallelogram.**

4. 

5. 

6. 

Determine whether the figure with the given vertices is a parallelogram. Use the method indicated.

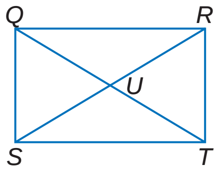
7. $L(-3, 2)$, $M(5, 2)$, $N(3, -6)$, $O(-5, -6)$; Slope Formula
8. $W(-5, 6)$, $X(2, 5)$, $Y(-3, -4)$, $Z(-8, -2)$; Distance Formula
9. $Q(-5, 4)$, $R(0, 6)$, $S(3, -1)$, $T(-2, -3)$; Midpoint Formula
10. $G(-5, 0)$, $H(-13, 5)$, $I(-10, 9)$, $J(-2, 4)$; Distance and Slope Formulas

Extra Practice

Lesson 6-4

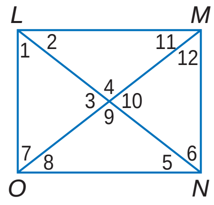
Pages 340–346

ALGEBRA Quadrilateral *QRST* is a rectangle.



1. If $QU = 2x + 3$ and $UT = 4x - 9$, find SU.
2. If $RU = 3x - 6$ and $UT = x + 9$, find RS.
3. If $QS = 3x + 40$ and $RT = 16 - 3x$, find QS.
4. If $m\angle STQ = 5x + 3$ and $m\angle RTQ = 3 - x$, find x.
5. If $m\angle SRQ = x^2 + 6$ and $m\angle RST = 36 - x$, find $m\angle SRT$.

Quadrilateral *LMNO* is a rectangle. Find each measure if $m\angle 5 = 38$.

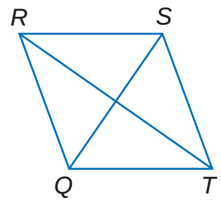


6. $m\angle 1$
7. $m\angle 2$
8. $m\angle 3$
9. $m\angle 4$
10. $m\angle 6$
11. $m\angle 7$
12. $m\angle 8$
13. $m\angle 9$
14. $m\angle 10$
15. $m\angle 11$
16. $m\angle 12$
17. $m\angle OLM$

Lesson 6-5

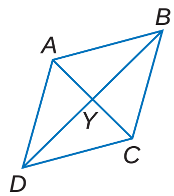
Pages 348–354

In rhombus *QRST*, $m\angle QRS = m\angle TSR - 40$ and $TS = 15$.



1. Find $m\angle TSQ$.
2. Find $m\angle QRS$.
3. Find $m\angle SRT$.
4. Find QR.

ALGEBRA Use rhombus *ABCD* with $AY = 6$, $DY = 3r + 3$, and $BY = \frac{10r - 4}{2}$.



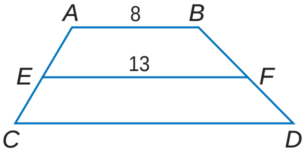
5. Find $m\angle ACB$.
6. Find $m\angle ABD$.
7. Find BY.
8. Find AC.

Lesson 6-6

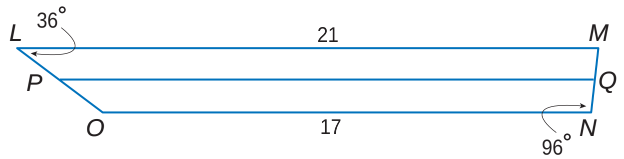
Pages 356–362

COORDINATE GEOMETRY For each quadrilateral with the given vertices,
a. verify that the quadrilateral is a trapezoid, and
b. determine whether the figure is an isosceles trapezoid.

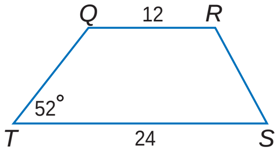
1. $A(0, 9)$, $B(3, 4)$, $C(-5, 4)$, $D(-2, 9)$
2. $Q(1, 4)$, $R(4, 6)$, $S(10, 7)$, $T(1, 1)$
3. $L(1, 2)$, $M(4, -1)$, $N(3, -5)$, $O(-3, 1)$
4. $W(1, -2)$, $X(3, -1)$, $Y(7, -2)$, $Z(1, -5)$
5. For trapezoid *ABDC*, *E* and *F* are midpoints of the legs. Find *CD*.



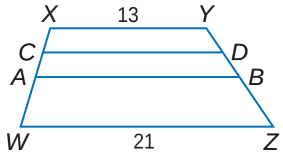
6. For trapezoid *LMNO*, *P* and *Q* are midpoints of the legs. Find *PQ*, $m\angle M$, and $m\angle O$.



7. For isosceles trapezoid *QRST*, find the length of the median, $m\angle S$, and $m\angle R$.



8. For trapezoid *XYZW*, *A* and *B* are midpoints of the legs. For trapezoid *XYBA*, *C* and *D* are midpoints of the legs. Find *CD*.

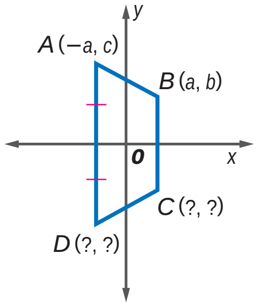


Lesson 6-7

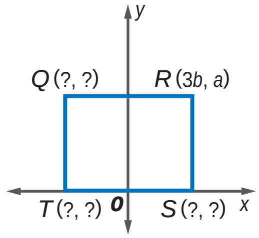
Pages 363–368

Name the missing coordinates for each quadrilateral.

1. isosceles trapezoid *ABCD*



2. rectangle *QRST*



Position and label each figure on the coordinate plane. Then write a coordinate proof for each of the following.

3. The diagonals of a square are congruent.
4. Quadrilateral *EFGH* with vertices $E(0, 0)$, $F\left(a\sqrt{2}, a\sqrt{2}\right)$, $G\left(2a + a\sqrt{2}, a\sqrt{2}\right)$, and $H(2a, 0)$ is a rhombus.

Lesson 7-1

Pages 380–386

1. **ARCHITECTURE** The ratio of the height of a model of a house to the actual house is 1:63. If the width of the model is 16 inches, find the width of the actual house in feet.
2. **CONSTRUCTION** A 64-inch long board is divided into lengths in the ratio 2:3. What are the two lengths into which the board is divided?

Solve each proportion.

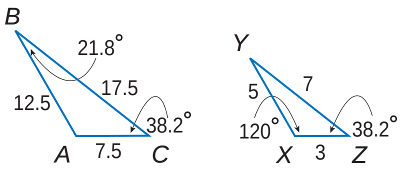
3. $\frac{x+4}{26} = -\frac{1}{3}$
4. $\frac{3x+1}{14} = \frac{5}{7}$
5. $\frac{x-3}{4} = \frac{x+1}{5}$
6. $\frac{2x+2}{2x-1} = \frac{1}{3}$
7. Find the measures of the sides of a triangle if the ratio of the measures of three sides of a triangle is 9:6:5, and its perimeter is 100 inches.
8. Find the measures of the angles in a triangle if the ratio of the measures of the three angles is 13:16:21.

Lesson 7-2

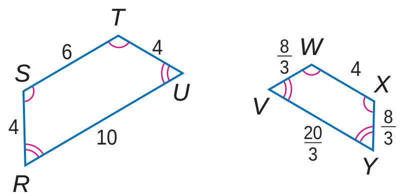
Pages 388–396

Determine whether each pair of figures is similar. Justify your answer.

1.



2.



For Exercises 3 and 4, use $\triangle RST$ with vertices $R(3, 6)$, $S(1, 2)$, and $T(3, -1)$. Explain.

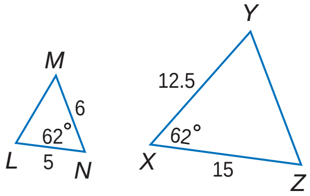
3. If the coordinates of each vertex are decreased by 3, describe the new figure. Is it similar to $\triangle RST$?
4. If the coordinates of each vertex are multiplied by 0.5, describe the new figure. Is it similar to $\triangle RST$?

Lesson 7-3

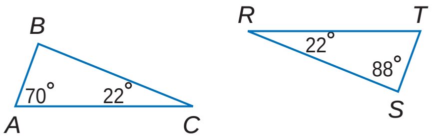
Pages 397–403

Determine whether each pair of triangles is similar. Justify your answer.

1.

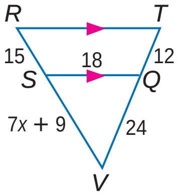


2.

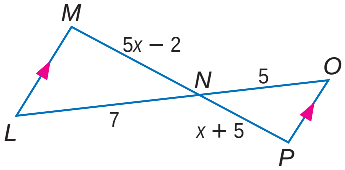


ALGEBRA **Identify the similar triangles. Find *x* and the measures of the indicated sides.**

3. *RT* and *SV*



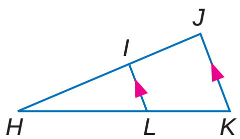
4. *PN* and *MN*



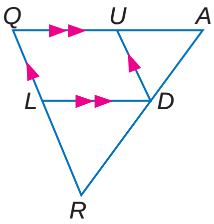
Lesson 7-4

Pages 405–414

1. If $HI = 28$, $LH = 21$, and $LK = 8$, find *IJ*.



2. Find *x*, *AD*, *DR*, and *QR* if $AU = 15$, $QU = 25$, $AD = 3x + 6$, $DR = 8x - 2$, and $UD = 15$.

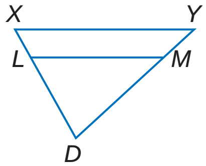


Find *x* so that $\overline{XY} \parallel \overline{LM}$.

3. $XL = 3$, $YM = 5$, $LD = 9$, $MD = x + 3$

4. $YM = 3$, $LD = 3x + 1$, $XL = 4$, $MD = x + 7$

5. $MD = 5x - 6$, $YM = 3$, $LD = 5x + 1$, $XL = 5$

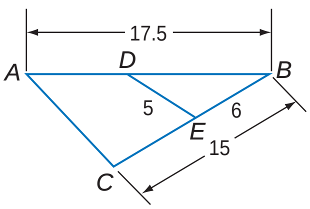


Lesson 7-5

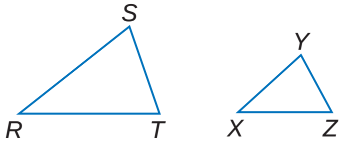
Pages 415–422

Find the perimeter of each triangle.

1. $\triangle ABC$ if $\triangle ABC \sim \triangle DBE$, $AB = 17.5$, $BC = 15$, $BE = 6$, and $DE = 5$



2. $\triangle RST$ if $\triangle RST \sim \triangle XYZ$, $RT = 12$, $XZ = 8$, and the perimeter of $\triangle XYZ = 22$



3. $\triangle LMN$ if $\triangle LMN \sim \triangle NXY$, $NX = 14$, $YX = 11$, $YN = 9$, and $LN = 27$

4. $\triangle GHI$ if $\triangle ABC \sim \triangle GHI$, $AB = 6$, $GH = 10$, and the perimeter of $\triangle ABC = 25$

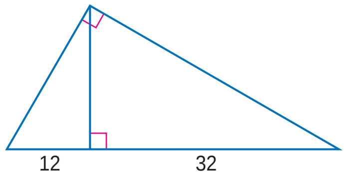
Lesson 8-1

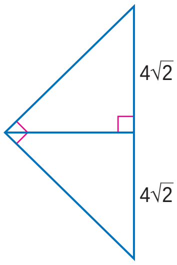
Pages 432–438

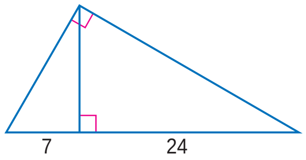
Find the geometric mean between each pair of numbers.

1. 8 and 12
2. 15 and 20
3. 1 and 2
4. 4 and 16
5. $3\sqrt{2}$ and $6\sqrt{2}$
6. $\frac{1}{2}$ and 10
7. $\frac{3}{8}$ and $\frac{1}{2}$
8. $\frac{\sqrt{2}}{2}$ and $\frac{3\sqrt{2}}{2}$
9. $\frac{1}{10}$ and $\frac{7}{10}$

Find the measure of the altitude drawn to the hypotenuse.

10.


11.


12.


Lesson 8-2

Pages 440–446

Determine whether $\triangle DEF$ is a right triangle for the given vertices. Explain.

1. $D(0, 1)$, $E(3, 2)$, $F(2, 3)$
2. $D(-2, 2)$, $E(3, -1)$, $F(-4, -3)$
3. $D(2, -1)$, $E(-2, -4)$, $F(-4, -1)$
4. $D(1, 2)$, $E(5, -2)$, $F(-2, -1)$

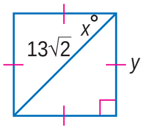
Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

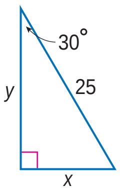
5. 1, 1, 2
6. 21, 28, 35
7. 3, 5, 7
8. 2, 5, 7
9. 24, 45, 51
10. $\frac{1}{3}, \frac{5}{3}, \frac{\sqrt{26}}{3}$
11. $\frac{6}{11}, \frac{8}{11}, \frac{10}{11}$
12. $\frac{1}{2}, \frac{1}{2}, 1$
13. $\frac{\sqrt{6}}{3}, \frac{\sqrt{10}}{5}, \frac{\sqrt{240}}{15}$

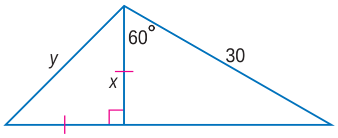
Lesson 8-3

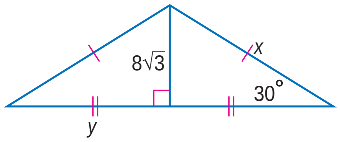
Pages 448–454

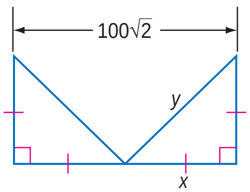
Find *x* and *y*.

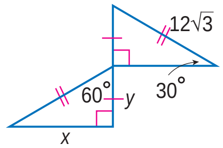
1.


2.


3.


4.


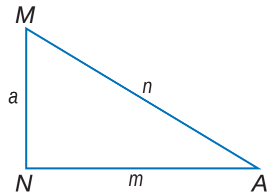
5.


6.


Lesson 8-4

Pages 456–462

Use $\triangle MNA$ with right angle N to find sin M, cos M, tan M, sin A, cos A, and tan A. Express each ratio as a fraction and as a decimal to the nearest hundredth.

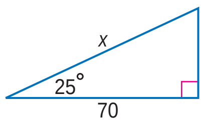


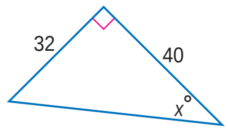
1. $m = 21, a = 28, n = 35$
2. $m = \sqrt{2}, a = \sqrt{3}, n = \sqrt{5}$
3. $m = \frac{\sqrt{2}}{2}, a = \frac{\sqrt{2}}{2}, n = 1$
4. $m = 3\sqrt{5}, a = 5\sqrt{3}, n = 2\sqrt{30}$

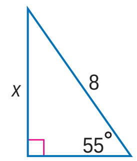
Find the measure of each angle to the nearest tenth of a degree.

5. $\cos A = 0.6293$
6. $\sin B = 0.5664$
7. $\tan C = 0.2665$
8. $\sin D = 0.9352$
9. $\tan M = 0.0808$
10. $\cos R = 0.1097$

Find x. Round to the nearest tenth.

11. 

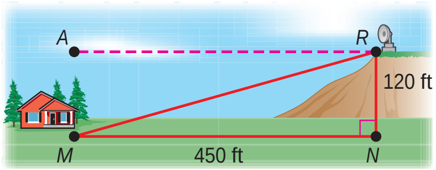
12. 

13. 

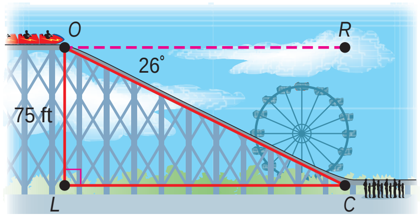
Lesson 8-5

Pages 464–470

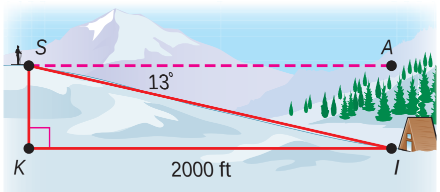
1. **COMMUNICATIONS** A house is located below a hill that has a satellite dish. If $MN = 450$ feet and $RN = 120$ feet, what is the measure of the angle of elevation to the top of the hill?



2. **AMUSEMENT PARKS** Mandy is at the top of the Mighty Screamer roller coaster. Her friend Bryn is at the bottom of the coaster waiting for the next ride. If the angle of depression from Mandy to Bryn is 26° and OL is 75 feet, what is the distance from L to C?



3. **SKIING** Mitchell is at the top of the Bridger Peak ski run. His brother Scott is looking up from the ski lodge at I. If the angle of elevation from Scott to Mitchell is 13° and the distance from K to I is 2000 ft, what is the length of the ski run SI?



Lesson 8-6

Pages 471–477

Find each measure using the given measures from $\triangle ANG$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $m\angle N = 32$, $m\angle A = 47$, and $n = 15$, find a.
2. If $a = 10.5$, $m\angle N = 26$, $m\angle A = 75$, find n.
3. If $n = 18.6$, $a = 20.5$, $m\angle A = 65$, find $m\angle N$.
4. If $a = 57.8$, $n = 43.2$, $m\angle A = 33$, find $m\angle N$.

Solve each $\triangle AKX$ described below. Round measures to the nearest tenth.

5. $m\angle X = 62, a = 28.5, m\angle K = 33$
6. $k = 3.6, x = 3.7, m\angle X = 55$
7. $m\angle K = 35, m\angle A = 65, x = 50$
8. $m\angle A = 122, m\angle X = 15, a = 33.2$

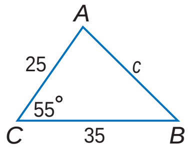
Lesson 8-7

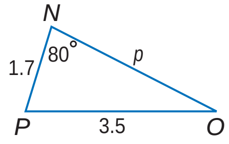
Pages 479–485

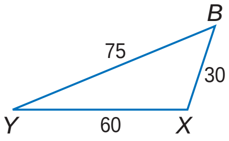
In △*CDE*, given the lengths of the sides, find the measure of the stated angle to the nearest tenth.

1. $c = 100$, $d = 125$, $e = 150$; $m\angle E$
2. $c = 5$, $d = 6$, $e = 9$; $m\angle C$
3. $c = 1.2$, $d = 3.5$, $e = 4$; $m\angle D$
4. $c = 42.5$, $d = 50$, $e = 81.3$; $m\angle E$

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

5. 

6. 

7. 

Lesson 9-1

Pages 497–503

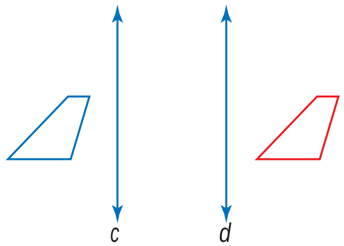
COORDINATE GEOMETRY Graph each figure and its image under the given reflection.

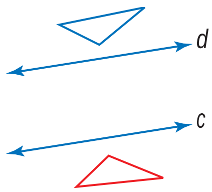
1. △*ABN* with vertices $A(2, 2)$, $B(3, -2)$, and $N(-3, -1)$ in the x-axis
2. rectangle *BARN* with vertices $B(3, 3)$, $A(3, -4)$, $R(-1, -4)$, and $N(-1, 3)$ in the line $y = x$
3. trapezoid *ZOID* with vertices $Z(2, 3)$, $O(2, -4)$, $I(-3, -3)$, and $D(-3, 1)$ in the origin
4. △*PQR* with vertices $P(-2, 1)$, $Q(2, -2)$, and $R(-3, -4)$ in the y-axis
5. square *BDFH* with vertices $B(-4, 4)$, $D(-1, 4)$, $F(-1, 1)$, and $H(-4, 1)$ in the origin
6. quadrilateral *QUAD* with vertices $Q(1, 3)$, $U(3, 1)$, $A(-1, 0)$, and $D(-3, 4)$ in the line $y = -1$
7. △*CAB* with vertices $C(0, 4)$, $A(1, -3)$, and $B(-4, 0)$ in the line $x = -2$

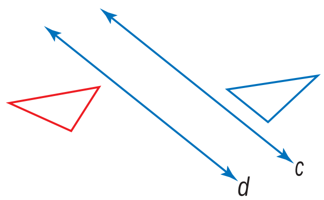
Lesson 9-2

Pages 504–509

In each figure, $c \parallel d$. Determine whether the red figure is a translation image of the blue figure. Write *yes* or *no*. Explain your answer.

1. 

2. 

3. 

COORDINATE GEOMETRY Graph each figure and its image under the given translation.

4. $\overline{LM}$ with endpoints $L(2, 3)$ and $M(-4, 1)$ under the translation $(x, y) \rightarrow (x + 2, y + 1)$
5. △*DEF* with vertices $D(1, 2)$, $E(-2, 1)$, and $F(-3, -1)$ under the translation $(x, y) \rightarrow (x - 1, y - 3)$
6. quadrilateral *WXYZ* with vertices $W(1, 1)$, $X(-2, 3)$, $Y(-3, -2)$, and $Z(2, -2)$ under the translation $(x, y) \rightarrow (x + 1, y - 1)$
7. pentagon *ABCDE* with vertices $A(1, 3)$, $B(-1, 1)$, $C(-1, -2)$, $D(3, -2)$, and $E(3, 1)$ under the translation $(x, y) \rightarrow (x - 2, y + 3)$
8. △*RST* with vertices $R(-4, 3)$, $S(-2, -3)$, and $T(2, -1)$ under the translation $(x, y) \rightarrow (x + 3, y - 2)$

Extra Practice

Lesson 9-3

Pages 510–517

COORDINATE GEOMETRY Draw the rotation image of each figure 90° in the given direction about the center point and label the vertices.

1. $\triangle KLM$ with vertices $K(4, 2)$, $L(1, 3)$, and $M(2, 1)$ counterclockwise about the point $P(1, -1)$
2. $\triangle FGH$ with vertices $F(-3, -3)$, $G(2, -4)$, and $H(-1, -1)$ clockwise about the point $P(0, 0)$

COORDINATE GEOMETRY Draw the rotation image of each triangle by reflecting the triangle in the given lines. State the coordinates of the rotation image and the angle of rotation.

3. $\triangle HIJ$ with vertices $H(2, 2)$, $I(-2, 1)$, and $J(-1, -2)$, reflected in the x-axis and then in the y-axis
4. $\triangle NOP$ with vertices $N(3, 1)$, $O(5, -3)$, and $P(2, -3)$, reflected in the y-axis and then in the line $y = x$
5. $\triangle QUA$ with vertices $Q(0, 4)$, $U(-3, 2)$, and $A(1, 1)$, reflected in the x-axis and then in the line $y = x$
6. $\triangle AEO$ with vertices $A(-5, 3)$, $E(-4, 1)$, and $O(-1, 2)$, reflected in the line $y = -x$ and then in the y-axis

Lesson 9-4

Pages 519–524

Determine whether a semi-regular tessellation can be created from each set of figures. Assume each figure has a side length of 1 unit.

1. regular hexagons and squares
2. squares and regular pentagons
3. regular hexagons and regular octagons

Determine whether each statement is *always*, *sometimes*, or *never* true.

4. Any right isosceles triangle forms a uniform tessellation.
5. A semi-regular tessellation is uniform.
6. A polygon that is not regular can tessellate the plane.
7. If the measure of one interior angle of a regular polygon is greater than 120, it cannot tessellate the plane.

Lesson 9-5

Pages 525–532

Find the measure of the dilation image $\overline{O'M'}$ or the preimage of $\overline{OM}$ using the given scale factor.

1. $OM = 1, r = -2$
2. $OM = 3, r = \frac{1}{3}$
3. $O'M' = \frac{3}{4}, r = 3$
4. $OM = \frac{7}{8}, r = -\frac{5}{7}$
5. $O'M' = 4, r = -\frac{2}{3}$
6. $O'M' = 4.5, r = -1.5$

COORDINATE GEOMETRY Find the image of each polygon, given the vertices, after a dilation centered at the origin with scale factor of 3. Then graph a dilation centered at the origin with a scale factor of $\frac{1}{3}$.

7. $T(1, 1)$, $R(-1, 2)$, $I(-2, 0)$
8. $E(2, 1)$, $I(3, -3)$, $O(-1, -2)$
9. $A(0, -1)$, $B(-1, 1)$, $C(0, 2)$, $D(1, 1)$
10. $B(1, 0)$, $D(2, 0)$, $F(3, -2)$, $H(0, -2)$

Lesson 9-6

Pages 534–541

Find the magnitude and direction of $\overrightarrow{XY}$ for the given coordinates.

1. $X(1, 1)$, $Y(-2, 3)$
2. $X(-1, -1)$, $Y(2, 2)$
3. $X(-5, 4)$, $Y(-2, -3)$
4. $X(2, 1)$, $Y(-4, -4)$
5. $X(-2, -1)$, $Y(2, -2)$
6. $X(3, -1)$, $Y(-3, 1)$

Graph the image of each figure under a translation by the given vector.

7. $\triangle HIJ$ with vertices $H(2, 3)$, $I(-4, 2)$, $J(-1, 1)$; $\vec{\mathbf{a}} = \langle 1, 3 \rangle$
8. quadrilateral $RSTW$ with vertices $R(4, 0)$, $S(0, 1)$, $T(-2, -2)$, $W(3, -1)$; $\vec{\mathbf{x}} = \langle -3, 4 \rangle$
9. pentagon $AEIOU$ with vertices $A(-1, 3)$, $E(2, 3)$, $I(2, 0)$, $O(-1, -2)$, $U(-3, 0)$; $\vec{\mathbf{b}} = \langle -2, -1 \rangle$

Find the magnitude and direction of each resultant for the given vectors.

10. $\vec{\mathbf{c}} = \langle 2, 3 \rangle$, $\vec{\mathbf{d}} = \langle 3, 4 \rangle$
11. $\vec{\mathbf{a}} = \langle 1, 3 \rangle$, $\vec{\mathbf{b}} = \langle -4, 3 \rangle$
12. $\vec{\mathbf{x}} = \langle 1, 2 \rangle$, $\vec{\mathbf{y}} = \langle 4, -6 \rangle$
13. $\vec{\mathbf{s}} = \langle 2, 5 \rangle$, $\vec{\mathbf{t}} = \langle -6, -8 \rangle$
14. $\vec{\mathbf{m}} = \langle 2, -3 \rangle$, $\vec{\mathbf{n}} = \langle -2, 3 \rangle$
15. $\vec{\mathbf{u}} = \langle -7, 2 \rangle$, $\vec{\mathbf{v}} = \langle 4, 1 \rangle$

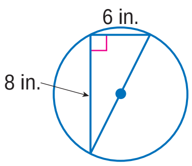
Lesson 10-1

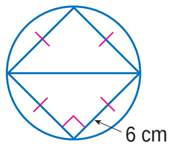
Pages 554–561

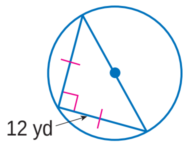
The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

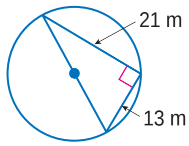
1. $r = 18$ in., $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$
2. $d = 34.2$ ft, $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$
3. $C = 12\pi$ m, $r = \underline{\ ?\ }$, $d = \underline{\ ?\ }$
4. $C = 84.8$ mi, $r = \underline{\ ?\ }$, $d = \underline{\ ?\ }$
5. $d = 8.7$ cm, $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$
6. $r = 3b$ in., $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

Find the exact circumference of each circle.

7. 

8. 

9. 

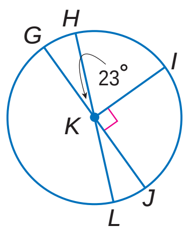
10. 

Lesson 10-2

Pages 563–569

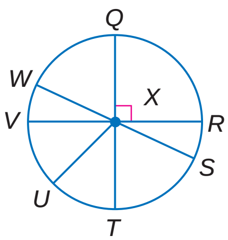
Find each measure.

1. $m\angle GKI$
2. $m\angle LKJ$
3. $m\angle LKI$
4. $m\angle LKG$
5. $m\angle HKI$
6. $m\angle HKJ$



In $\odot X$, $\overline{WS}$, $\overline{VR}$, and $\overline{QT}$ are diameters, $m\angle WXV = 25$ and $m\angle VXU = 45$. Find each measure.

7. $m\widehat{QR}$
8. $m\widehat{QW}$
9. $m\widehat{TU}$
10. $m\widehat{WRV}$
11. $m\widehat{SV}$
12. $m\widehat{TRW}$

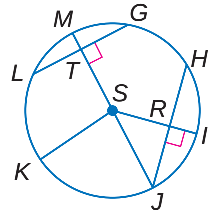


Extra Practice

Lesson 10-3

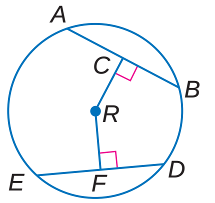
Pages 570–577

In ⊙*S*, *HJ* = 22, *LG* = 18, $m\widehat{IJ} = 35$, and $m\widehat{LM} = 30$. Find each measure.



1. *HR*
2. *RJ*
3. *LT*
4. *TG*
5. $m\widehat{HJ}$
6. $m\widehat{LG}$
7. $m\widehat{MG}$
8. $m\widehat{HI}$

In ⊙*R*, *CR* = *RF*, and *ED* = 30. Find each measure.



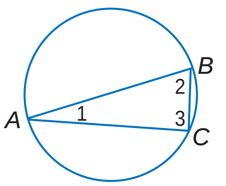
9. *AB*
10. *EF*
11. *DF*
12. *BC*

Lesson 10-4

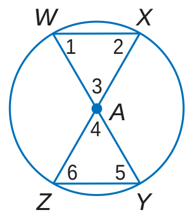
Pages 578–586

Find the measure of each numbered angle for each figure.

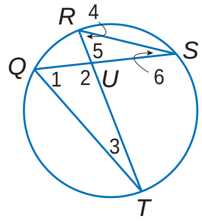
1. $m\widehat{AB} = 176$, and $m\widehat{BC} = 42$



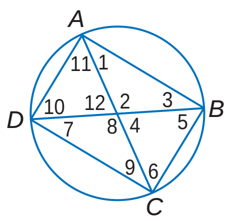
2. $\overline{WX} \cong \overline{ZY}$, and $m\widehat{ZW} = 120$



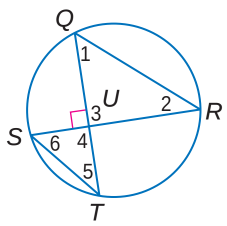
3. $m\widehat{QR} = 40$, and $m\widehat{TS} = 110$



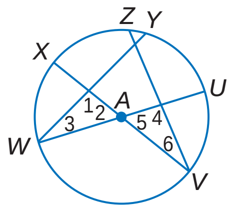
4. ▭*ABCD* is a rectangle, and $m\widehat{BC} = 70$.



5. $m\widehat{TR} = 100$, and $\overline{SR} \perp \overline{QT}$



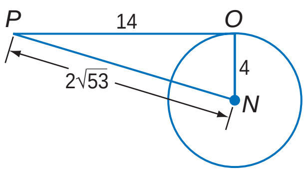
6. $m\widehat{UY} = m\widehat{XZ} = 56$ and $m\widehat{UV} = m\widehat{XW} = 56$

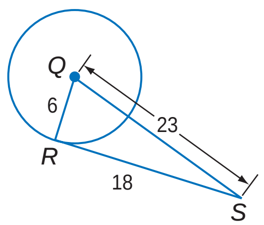


Lesson 10-5

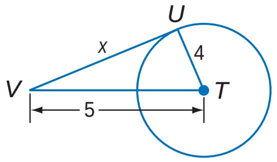
Pages 588–596

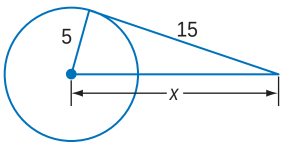
Determine whether each segment is tangent to the given circle.

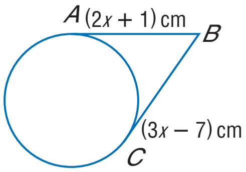
1. 

2. 

Find *x*. Assume that segments that appear to be tangent are tangent.

3. 

4. 

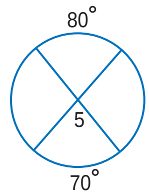
5. 

Lesson 10-6

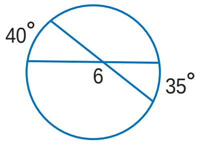
Pages 599–606

Find each measure.

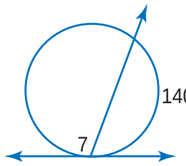
1. $m\angle 5$



2. $m\angle 6$

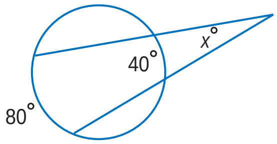


3. $m\angle 7$

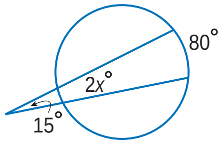


Find *x*. Assume that any segment that appears to be tangent is tangent.

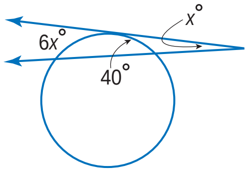
4.



5.



6.

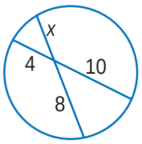


Lesson 10-7

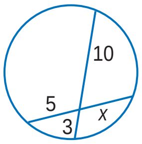
Pages 607–613

Find *x*. Assume that segments that appear to be tangent are tangent.

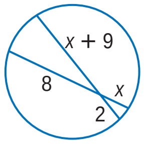
1.



2.

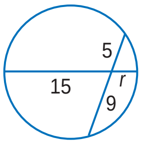


3.

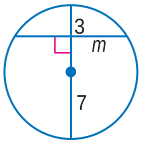


Find each variable to the nearest tenth.

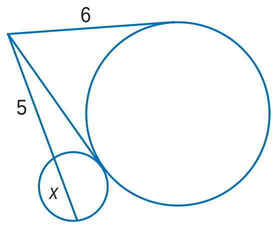
4.



5.



6.



Lesson 10-8

Pages 614–619

Write an equation for each circle.

1. center at $(1, -2)$, $r = 2$
2. center at origin, $r = 4$
3. center at $(-3, -4)$, $r = \sqrt{11}$
4. center at $(3, -1)$, $d = 6$
5. center at $(6, 12)$, $r = 7$
6. center at $(4, 0)$, $d = 8$
7. center at $(6, -6)$, $d = 22$
8. center at $(-5, 1)$, $d = 2$

Graph each equation.

9. $x^2 + y^2 = 25$
10. $x^2 + y^2 - 3 = 1$
11. $(x - 3)^2 + (y + 1)^2 = 9$
12. $(x - 1)^2 + (y - 4)^2 = 1$

13. Find the radius of a circle whose equation is $(x + 3)^2 + (y - 1)^2 = r^2$ and contains $(-2, 1)$.

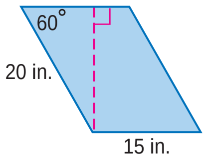
14. Find the radius of a circle whose equation is $(x - 4)^2 + (y - 3)^2 = r^2$ and contains $(8, 3)$.

Extra Practice

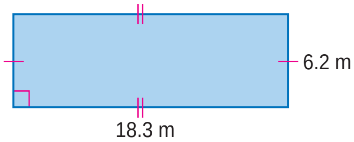
Lesson 11-1

Pages 630–636

Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

1. 

2. 

3. 

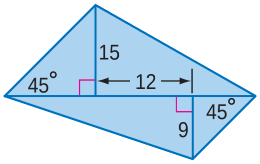
COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.**

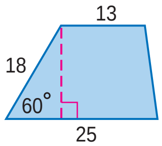
4. $Q(-3, 3)$, $R(-1, 3)$, $S(-1, 1)$, $T(-3, 1)$
5. $A(-7, -6)$, $B(-2, -6)$, $C(-2, -3)$, $D(-7, -3)$
6. $L(5, 3)$, $M(8, 3)$, $N(9, 7)$, $O(6, 7)$
7. $W(-1, -2)$, $X(-1, 1)$, $Y(2, 1)$, $Z(2, -2)$

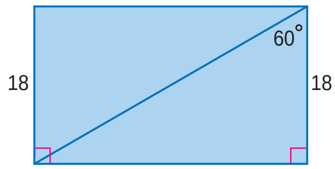
Lesson 11-2

Pages 638–647

Find the area of each quadrilateral.

1. 

2. 

3. 

COORDINATE GEOMETRY **Find the area of trapezoid *ABCD* given the coordinates of the vertices.**

4. $A(1, 1)$, $B(2, 3)$, $C(4, 3)$, $D(7, 1)$
5. $A(-2, 2)$, $B(2, 2)$, $C(7, -3)$, $D(-4, -3)$
6. $A(1, -1)$, $B(4, -1)$, $C(8, 5)$, $D(1, 5)$
7. $A(-2, 2)$, $B(4, 2)$, $C(3, -2)$, $D(1, -2)$

COORDINATE GEOMETRY **Find the area of rhombus *LMNO* given the coordinates of the vertices.**

8. $L(-3, 0)$, $M(1, -2)$, $N(-3, -4)$, $O(-7, -2)$
9. $L(-3, -2)$, $M(-4, 2)$, $N(-3, 6)$, $O(-2, 2)$
10. $L(-1, -4)$, $M(3, 4)$, $N(-1, 12)$, $O(-5, 4)$
11. $L(-2, -2)$, $M(4, 4)$, $N(10, -2)$, $O(4, -8)$

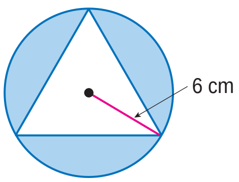
Lesson 11-3

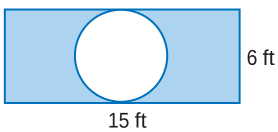
Pages 649–656

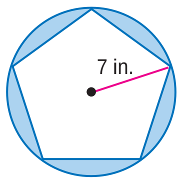
Find the area of each regular polygon. Round to the nearest tenth.

1. a square with perimeter 54 feet
2. a triangle with side length 9 inches
3. an octagon with side length 6 feet
4. a decagon with apothem length of 22 centimeters

Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.

5. 

6. 

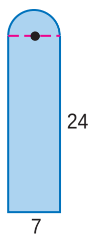
7. 

Lesson 11-4

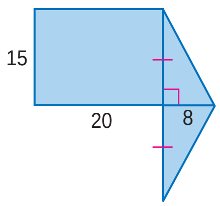
Pages 658–663

Find the area of each figure. Round to the nearest tenth if necessary.

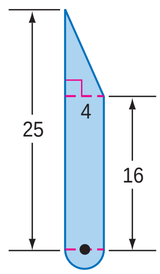
1.



2.



3.



COORDINATE GEOMETRY **The vertices of a composite figure are given. Find the area of each figure.**

4. $R(0, 5)$, $S(3, 3)$, $T(3, 0)$

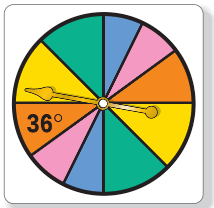
5. $A(-5, -3)$, $B(-3, 0)$, $C(2, -1)$, $D(2, -3)$

Lesson 11-5

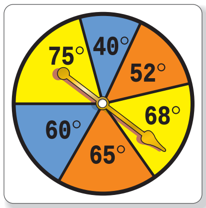
Pages 665–671

Find the total area of the indicated sectors. Then find the probability of spinning the color indicated if the diameter of each spinner is 20 inches.

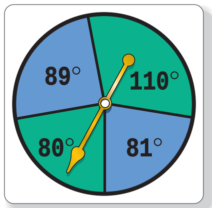
1. orange



2. blue

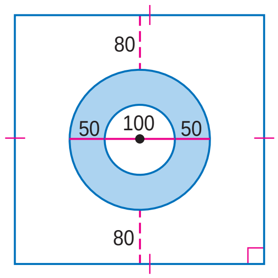


3. green

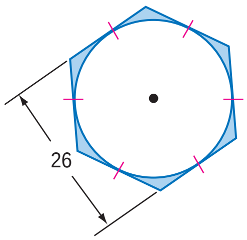


Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region.

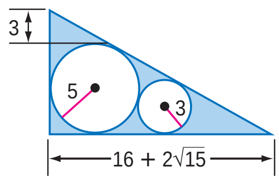
4.



5.



6.

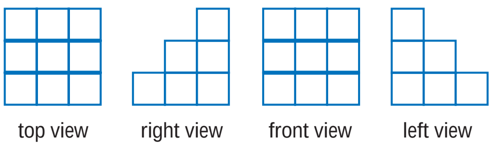


Lesson 12-1

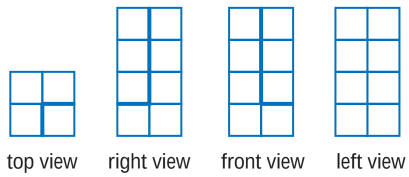
Pages 680–685

Draw the back view and corner view of a figure given each orthographic drawing.

1.



2.



Sketch each solid using isometric dot paper.

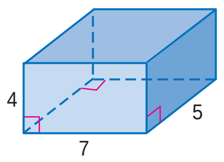
3. rectangular prism 2 units high, 3 units long, and 2 units wide

4. triangular prism 3 units high with bases that are right triangles with legs 3 units and 4 units long

Lesson 12-2

Pages 686–691

Find the lateral area and the surface area of each prism. Round to the nearest tenth if necessary.

1.


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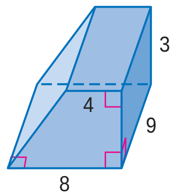
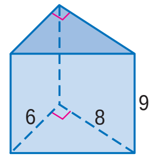
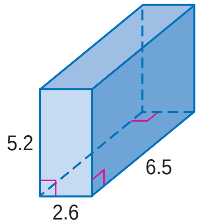
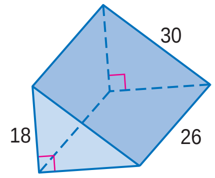
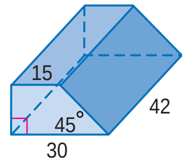


3.


4.


5.


6.


7. The surface area of a right triangular prism is 228 square inches. The base is a right triangle with legs measuring 6 inches and 8 inches. Find the height of the prism.

Lesson 12-3

Pages 693–697

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

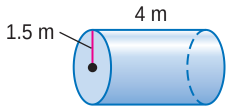
1. $r = 2$ ft, $h = 3.5$ ft

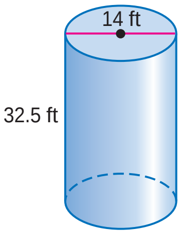
2. $d = 15$ in., $h = 20$ in.

3. $r = 3.7$ m, $h = 6.2$ m

4. $d = 19$ mm, $h = 32$ mm

Find the surface area of each cylinder. Round to the nearest tenth.

5.


6.


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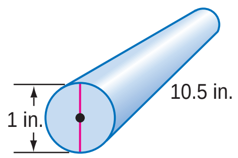

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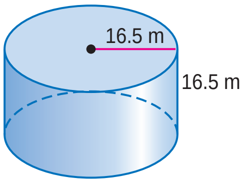
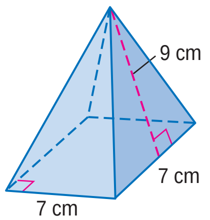
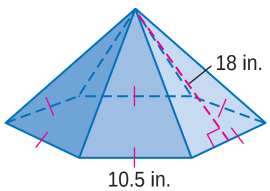


Lesson 12-4

Pages 699–705

Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

1.


2.


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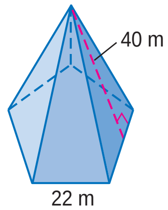

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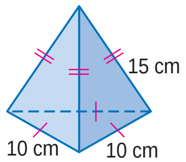
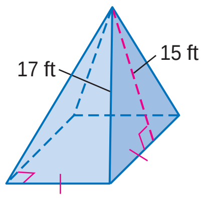
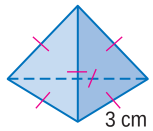


5.


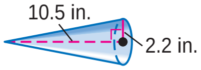
6.


Lesson 12-5

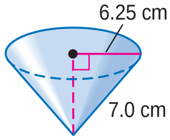
Pages 706–710

Find the surface area of each cone. Round to the nearest tenth.

1.



2.



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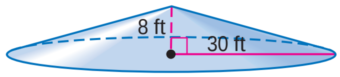


4. Find the surface area of a cone if the height is 28 inches and the slant height is 40 inches.

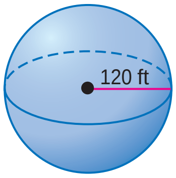
5. Find the surface area of a cone if the height is 7.5 centimeters and the radius is 2.5 centimeters.

Lesson 12-6

Pages 711–717

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

1.



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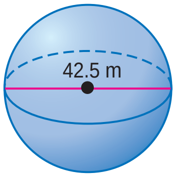

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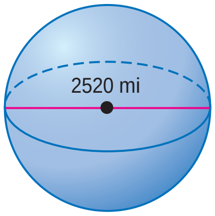

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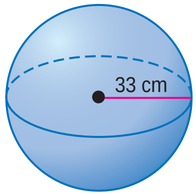


5. a hemisphere with the circumference of a great circle 14.1 cm

6. a sphere with the circumference of a great circle 50.3 in.

7. a sphere with the area of a great circle 98.5 m^2

8. a hemisphere with the circumference of a great circle 3.1 in.

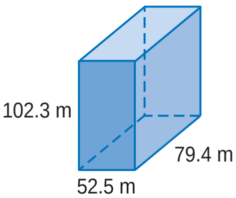
9. a hemisphere with the area of a great circle 31,415.9 ft^2

Lesson 13-1

Pages 728–735

Find the volume of each prism or cylinder. Round to the nearest tenth if necessary.

1.



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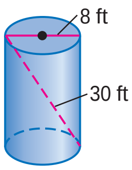

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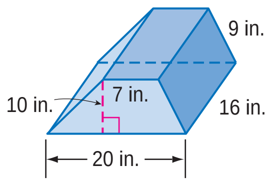
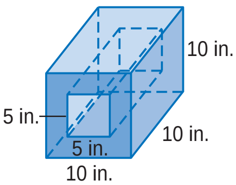


COMPOSITE SOLIDS **Find the volume of each solid to the nearest tenth.**

4.



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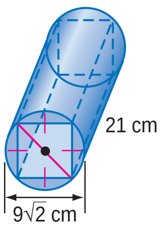

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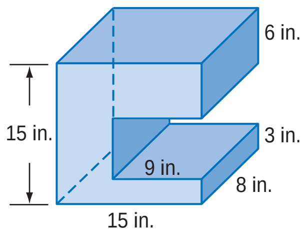
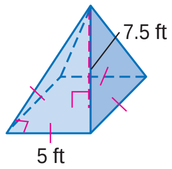


Lesson 13-2

Pages 737–742

Find the volume of each pyramid or cone. Round to the nearest tenth if necessary.

1.


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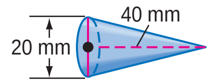

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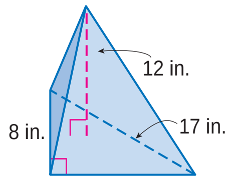

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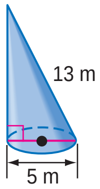

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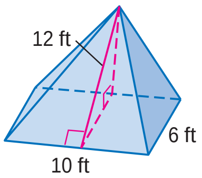

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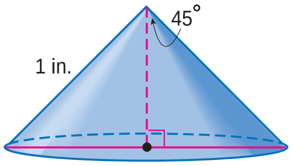
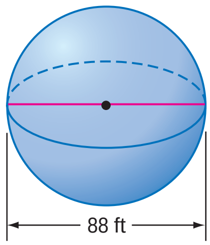
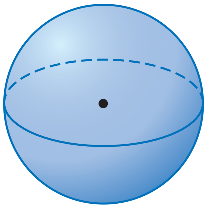


Lesson 13-3

Pages 743–748

Find the volume of each sphere or hemisphere. Round to the nearest tenth.

1.


2. $C = 4$ m


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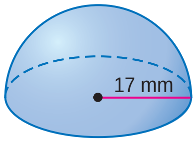


4. The diameter of the sphere is 3 cm.

5. The radius of the hemisphere is $7\sqrt{2}$ m.

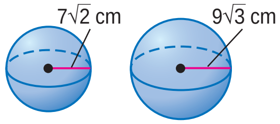
6. The diameter of the hemisphere is 90 ft.

7. The radius of the sphere is 0.5 in.

Lesson 13-4

Pages 750–757

Determine whether each pair of solids are *similar*, *congruent*, or *neither*.

1.


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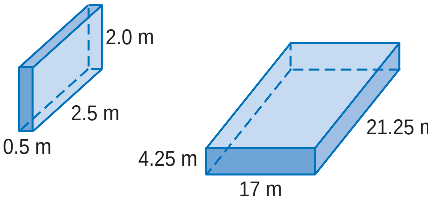

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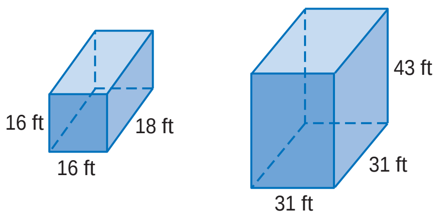

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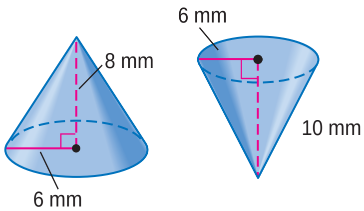
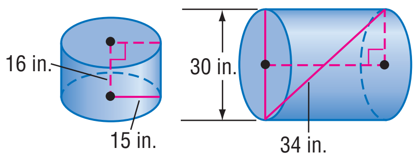
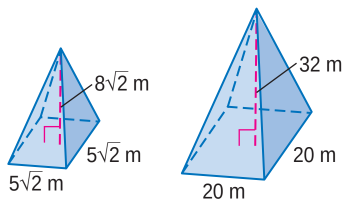


5.


6.


Lesson 13-5

Pages 758–764

Graph the rectangular solid that contains the given point and the origin as vertices. Label the coordinates of each vertex.

1. $A(3, -3, -3)$
2. $E(-1, 2, -3)$
3. $I(3, -1, 2)$
4. $Z(2, -1, 3)$
5. $Q(-4, -2, -4)$
6. $Y(-3, 1, -4)$

Determine the distance between each pair of points. Then determine the coordinates of the midpoint *M* of the segment joining the pair of points.

7. $A(-3, 3, 1)$ and $B(3, -3, -1)$
8. $O(2, -1, -3)$ and $P(-2, 4, -4)$
9. $D(0, -5, -3)$ and $E(0, 5, 3)$
10. $J(-1, 3, 5)$ and $K(3, -5, -3)$
11. $A(2, 1, 6)$ and $Z(-4, -5, -3)$
12. $S(-8, 3, -5)$ and $T(6, -1, 2)$

Mixed Problem Solving and Proof

Chapter 1 Tools of Geometry

(pages 4–75)

ARCHITECTURE For Exercises 1–4, use the following information.
The Burj Al Arab in Dubai, United Arab Emirates, is one of the world's tallest hotels. (Lesson 1-1)

1. Trace the outline of the building on your paper.
2. Label three different planes suggested by the outline.
3. Highlight three lines in your drawing that, when extended, do not intersect.
4. Label three points on your sketch. Determine if they are coplanar and collinear.

SKYSCRAPERS For Exercises 5–7, use the following information. (Lesson 1-2)

Tallest Buildings in San Antonio, TX	
Name	**Height (ft)**
Tower of the Americas	622
Marriot Rivercenter	546
Weston Centre	444
Tower Life	404

Source: www.skyscrapers.com

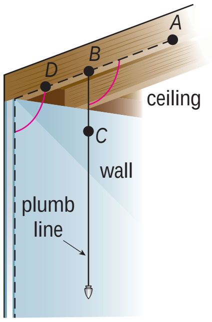
5. What is the precision for the measures of the heights of the buildings?
6. What does the precision mean for the measure of the Tower of the Americas?
7. What is the difference in height between Weston Centre and Tower Life?

8. **TRANSPORTATION** Mile markers are used to name the exits on Interstate 70 in Kansas. The exit for Hays is 3 miles farther than halfway between Exits 128 and 184. What is the exit number for the Hays exit? (Lesson 1-3)

9. **ENTERTAINMENT** The Ferris wheel at the Navy Pier in Chicago has forty gondolas. What is the measure of an angle with a vertex that is the center of the wheel and with sides that are two consecutive spokes on the wheel? Assume that the gondolas are equally spaced. (Lesson 1-4)

CONSTRUCTION For Exercises 10 and 11, use the following information.
A framer is installing a cathedral ceiling in a newly built home. A protractor and a plumb bob are used to check the angle at the joint between the ceiling and wall. The wall is vertical, so the angle between the vertical plumb line and the ceiling is the same as the angle between the wall and the ceiling. (Lesson 1-5)

10. How are $\angle ABC$ and $\angle CBD$ related?
11. If $m\angle ABC = 110$, what is $m\angle CBD$?



STRUCTURES For Exercises 12 and 13, use the following information.

The picture shows the Hongkong and Shanghai Bank located in Hong Kong, China. (Lesson 1-6)

12. Name five different polygons suggested by the picture.
13. Classify each polygon you identified as *convex* or *concave* and *regular* or *irregular*.

14. **HOCKEY** A regulation hockey puck is a cylinder made of vulcanized rubber 1 inch thick and 3 inches in diameter. Find the surface area and volume of a hockey puck. (Lessons 1-6 and 1-7)

(l)Walter Bibikow/Stock Boston, (r)Serge Attal/TimePix

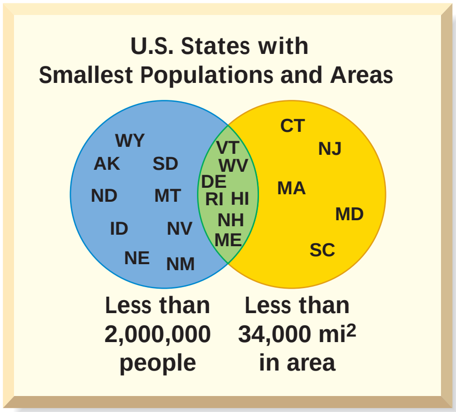
POPULATION **For Exercises 1–2, use the table showing the population density for various states in 1960, 1980, and 2000. The figures represent the number of people per square mile.** (Lesson 2-1)

State	1960	1980	2000
CA	100.4	151.4	217.2
CT	520.6	637.8	702.9
DE	225.2	307.6	401.0
HI	98.5	150.1	188.6
MI	137.7	162.6	175.0

Source: U.S. Census Bureau

1. Find a counterexample for the following statement. *The population density for each state in the table increased by at least 30 during each 20-year period.*
2. Write two conjectures for the year 2010.

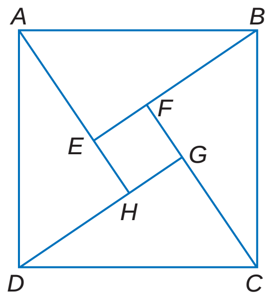
STATES **For Exercises 3 and 4, refer to the Venn diagram.** (Lesson 2-2)



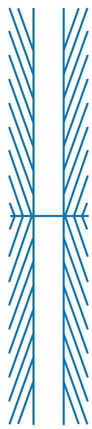
Source: *The World Almanac*

3. How many states have less than 2,000,000 people?
4. How many states have less than 2,000,000 people and are less than 34,000 square miles in area?
5. **QUOTATIONS** Write the quote in if-then form. "A champion is afraid of losing." *(Billy Jean King)* (Lesson 2–3)
6. **AIRLINE SAFETY** Airports in the United States post a sign stating *If any unknown person attempts to give you any items including luggage to transport on your flight, do not accept it and notify airline personnel immediately*. Write a valid conclusion to the hypothesis *If a person Candace does not know attempts to give her an item to take on her flight*, . . . (Lesson 2-4)
7. **PROOF** Write a paragraph proof to show that $\overline{AB} \cong \overline{CD}$ if B is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{BD}$. (Lesson 2–5)

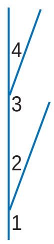
8. **PROOF** Write a two-column proof.
   If $8 + x = -2(x - 3)$, then $x = -\frac{2}{3}$.
   (Lesson 2-6)
9. **PROOF** Write a two-column proof.
   (Lesson 2-7)
   Given: $ABCD$ has 4 congruent sides.
   $DH = BF = AE$; $EH = FE$
   Prove: $AB + BE + AE = AD + AH + DH$



ILLUSIONS **This drawing was created by German psychologist Wilhelm Wundt.** (Lesson 2-8)



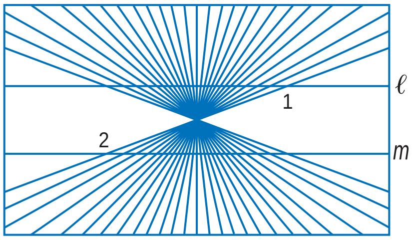
10. Describe the relationship between each pair of vertical lines.
11. A close-up of the angular lines is shown below. If $\angle 4 \cong \angle 2$, write a two-column proof to show that $\angle 3 \cong \angle 1$.



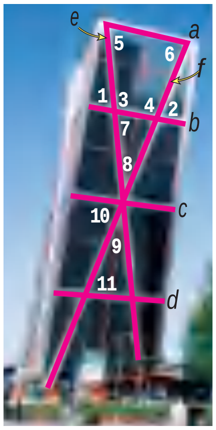
Chapter 3 Parallel and Perpendicular Lines

(pages 140–197)

1. **OPTICAL ILLUSIONS** Lines ℓ and m are parallel, but appear to be bowed due to the transversals drawn through ℓ and m. Make a conjecture about the relationship between $\angle 1$ and $\angle 2$. (Lesson 3-1)



ARCHITECTURE For Exercises 2–10, use the following information.
The picture shows one of two towers of the Puerta de Europa in Madrid, Spain. Lines a, b, c, and d are parallel. The lines are cut by transversals e and f. If $m\angle 1 = m\angle 2 = 75$, find the measure of each angle. (Lesson 3-2)



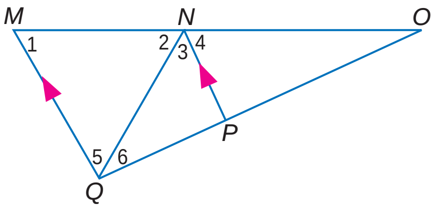
2. $\angle 3$
3. $\angle 4$
4. $\angle 5$
5. $\angle 6$
6. $\angle 7$
7. $\angle 8$
8. $\angle 9$
9. $\angle 10$
10. $\angle 11$

11. **PROOF** Write a two-column proof. (Lesson 3-2)

Given: $\overline{MQ} \parallel \overline{NP}$

$\angle 4 \cong \angle 3$

Prove: $\angle 1 \cong \angle 5$



12. **EDUCATION** Between 1995 and 2000, the average cost for tuition and fees for American universities increased by an average rate of $84.20 per year. In 2000, the average cost was $2600. If costs increase at the same rate, what will the total average cost be in 2010? (Lesson 3-3)

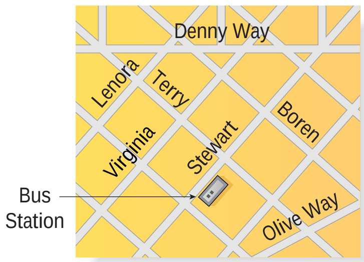
RECREATION For Exercises 13 and 14, use the following information.
The Three Forks community swimming pool holds 74,800 gallons of water. At the end of the summer, the pool is drained and winterized. (Lesson 3-4)

13. If the pool drains at the rate of 1200 gallons per hour, write an equation to describe the number of gallons left after x hours.

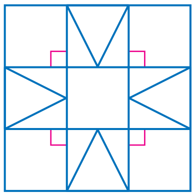
14. How many hours will it take to drain the pool?

15. **CONSTRUCTION** Martin makes two cuts at an angle of 120° with the edge of the wood through points D and P. Explain why these cuts will be parallel. (Lesson 3-5)

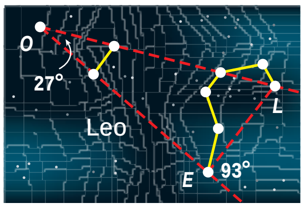
16. **SEATTLE** Describe a segment that represents the shortest distance from the bus station to Denny Way. Can you walk the route indicated by your segment? Explain. (Lesson 3-6)



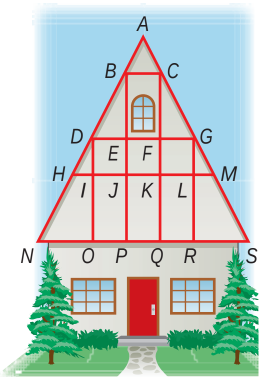
QUILTING For Exercises 1 and 2, trace the quilt pattern square. (Lesson 4-1)



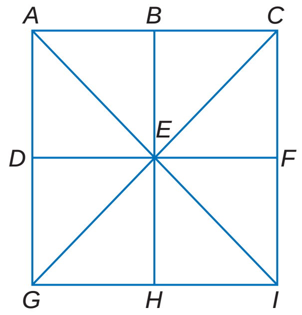
1. Shade all right triangles red. Do they appear to be scalene or isosceles? Explain.
2. Shade all acute triangles blue. Do they appear to be scalene, isosceles, or equilateral? Explain.
3. **ASTRONOMY** Leo is a constellation that represents a lion. Three of the brighter stars in the constellation form $\triangle LEO$. If the angles have measures as shown in the figure, find $m\angle OLE$. (Lesson 4-2)



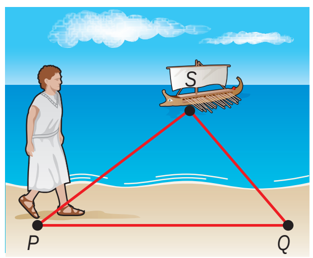
4. **ARCHITECTURE** The diagram shows an A-frame house with various points labeled. Assume that segments and angles that appear to be congruent in the diagram are congruent. Indicate which triangles are congruent. (Lesson 4-3)



RECREATION For Exercises 5–7, use the following information.
Tapatan is a game played in the Philippines on a square board, like the one shown at the top right. Players take turns placing each of their three pieces on a different point of intersection. After all the pieces have been played, the players take turns moving a piece along a line to another intersection. A piece cannot jump over another piece. The player who gets all of his or her pieces in a straight line wins. Point E bisects all four line segments that pass through it. (Lesson 4-4)



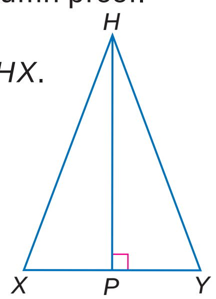
5. Is $\triangle GHE \cong \triangle CBE$? Explain.
6. Is $\triangle AEG \cong \triangle IEG$? Explain.
7. Write a flow proof to show that $\triangle ACI \cong \triangle CAG$.
8. **HISTORY** It is said that Thales determined the distance from the shore to the Greek ships by sighting the angle to the ship from a point P on the shore, walking to point Q, and then sighting the angle to the ship from Q. He then reproduced the angles on the other side of $\overline{PQ}$ and continued these lines until they intersected. Is this method valid? Explain. (Lesson 4-5)



9. **PROOF** Write a two-column proof. (Lesson 4-6)

   Given: $\overline{PH}$ bisects $\angle YHX$.
   $\overline{PH} \perp \overline{YX}$

   Prove: $\triangle YHX$ is an isosceles triangle.

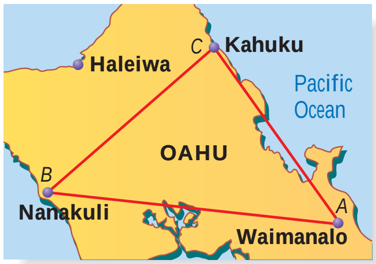


10. **PROOF** $\triangle ABC$ is a right isosceles triangle with hypotenuse $\overline{AB}$. M is the midpoint of $\overline{AB}$. Write a coordinate proof to show that $\overline{CM}$ is perpendicular to $\overline{AB}$. (Lesson 4-7)

CONSTRUCTION For Exercises 1–4, draw a large, acute scalene triangle. Use a compass and straightedge to make the required constructions. (Lesson 5-1)

1. Find the circumcenter. Label it *C*.
2. Find the centroid of the triangle. Label it *D*.
3. Find the orthocenter. Label it *O*.
4. Find the incenter of the triangle. Label it *I*.

RECREATION For Exercises 5–7, use the following information.
Kailey plans to fly over the route marked on the map of Oahu in Hawaii. (Lesson 5-2)



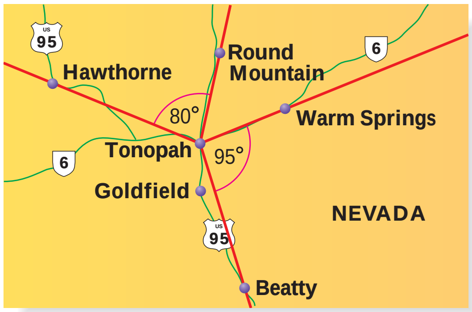
5. The measure of angle *A* is two degrees more than the measure of angle *B*. The measure of angle *C* is fourteen degrees less than twice the measure of angle *B*. What are the measures of the three angles?
6. Write the lengths of the legs of Kailey's trip in order from least to greatest.
7. The length of the entire trip is about 68 miles. The middle leg is 11 miles greater than one-half the length of the shortest leg. The longest leg is 12 miles greater than three-fourths of the shortest leg. What are the lengths of the legs of the trip?

8. **LAW** A man is accused of committing a crime. If the man is telling the truth when he says, "I work every Tuesday from 3:00 P.M. to 11:00 P.M.," what fact about the crime could be used to prove by indirect reasoning that the man was innocent? (Lesson 5-3)

TRAVEL For Exercises 9 and 10, use the following information.
The total air distance to fly from Bozeman, Montana, to Salt Lake City, Utah, to Boise, Idaho, is just over 634 miles.

9. Write an indirect proof to show that at least one of the legs of the trip is longer than 317 miles. (Lesson 5-3)
10. The air distance from Bozeman to Salt Lake City is 341 miles and the distance from Salt Lake to Boise is 294 miles. Find the range for the distance from Bozeman to Boise. (Lesson 5-4)

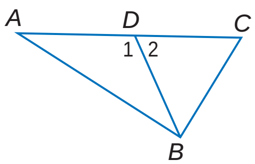
11. **GEOGRAPHY** The map shows a portion of Nevada. The distance from Tonopah to Round Mountain is the same as the distance from Tonopah to Warm Springs. The distance from Tonopah to Hawthorne is the same as the distance from Tonopah to Beatty. Use the angle measures to determine which distance is greater, Round Mountain to Hawthorne or Warm Springs to Beatty. (Lesson 5-5)



12. **PROOF** Write a two-column proof. (Lesson 5-5)

Given: $\overline{DB}$ is a median of $\triangle ABC$.
$m\angle 1 > m\angle 2$

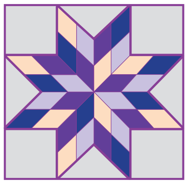
Prove: $m\angle C > m\angle A$



ENGINEERING For Exercises 1 and 2, use the following information.
The London Eye in London, England, is the world's largest observation wheel. The ride has 32 enclosed capsules for riders. (Lesson 6-1)

1. Suppose each capsule is connected with a straight piece of metal forming a 32-gon. Find the sum of the measures of the interior angles.
2. What is the measure of one interior angle of the 32-gon?

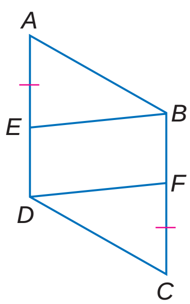
3. **QUILTING** The quilt square shown is called the Lone Star pattern. Describe two ways that the quilter could ensure that the pieces will fit properly. (Lesson 6-2)



4. **PROOF** Write a two-column proof. (Lesson 6-3)

   Given: $\square ABCD$, $\overline{AE} \cong \overline{CF}$

   Prove: Quadrilateral *EBFD* is a parallelogram.

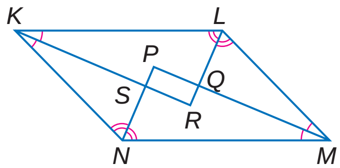


5. **MUSIC** Why will the keyboard stand shown always remain parallel to the floor? (Lesson 6-3)

6. **PROOF** Write a paragraph proof. (Lesson 6-4)

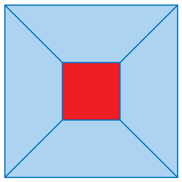
   Given: $\square KLMN$

   Prove: *PQRS* is a rectangle.



7. **CONSTRUCTION** Mr. Redwing is building a sandbox. He placed stakes at what he believes will be the four vertices of a square with a distance of 5 feet between each stake. How can he be sure that the sandbox will be a square? (Lesson 6-5)

DESIGN For Exercises 8 and 9, use the square floor tile design shown. (Lesson 6-6)



8. Explain how you know that the trapezoids in the design are isosceles.
9. The perimeter of the floor tile is 48 inches, and the perimeter of the interior red square is 16 inches. Find the perimeter of one trapezoid.

10. **PROOF** Position a quadrilateral on the coordinate plane with vertices $Q(-a, 0)$, $R(a, 0)$, $S(b, c)$, and $T(-b, c)$. Prove that the quadrilateral is an isosceles trapezoid. (Lesson 6-7)

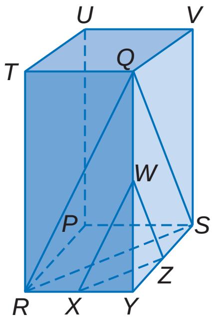
1. **TOYS** In 2000, $34,554,900,000 was spent on toys in the U.S. The U.S. population in 2000 was 281,421,906, with 21.4% of the population 14 years and under. If all of the toys purchased in 2000 were for children 14 years and under, what was the average amount spent per child? (Lesson 7-1)

QUILTING For Exercises 2–4, use the following information.
Felicia found a pattern for a quilt square. The pattern measures three-quarters of an inch on a side. Felicia wants to make a quilt that is 77 inches by 110 inches when finished. (Lesson 7-2)

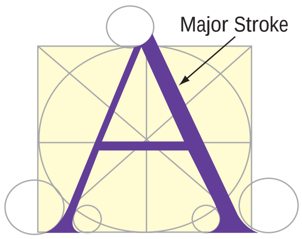
2. If Felicia wants to use only whole quilt squares, what is the greatest side length she can use for each square?
3. How many quilt squares will she need?
4. By what scale factor will she need to increase the pattern for the quilt square?

PROOF For Exercises 5 and 6, write a paragraph proof. (Lesson 7-3)

5. **Given:** $\triangle WYX \sim \triangle QYR$, $\triangle ZYX \sim \triangle SYR$
   Prove: $\triangle WYZ \sim \triangle QYS$
6. **Given:** $\overline{WX} \parallel \overline{QR}$, $\overline{ZX} \parallel \overline{SR}$
   Prove: $\overline{WZ} \parallel \overline{QS}$



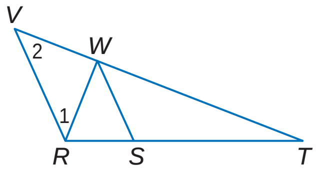
HISTORY For Exercises 7 and 8, use the following information.
In the fifteenth century, mathematicians and artists tried to construct the perfect letter. Damiano da Moile used a square as a frame to design the letter "A" as shown in the diagram. The thickness of the major stroke of the letter was to be $\frac{1}{12}$ of the height of the letter. (Lesson 7-4)



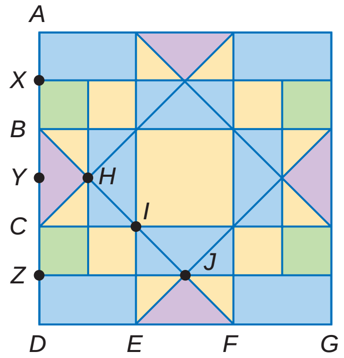
7. Explain why the bar through the middle of the A is half the length of the space between the outside bottom corners of the sides of the letter.
8. If the letter were 3 centimeters tall, how wide would the major stroke be?
9. **PROOF** Write a two-column proof. (Lesson 7-5)

   Given: $\overline{WS}$ bisects $\angle RWT$. $\angle 1 \cong \angle 2$.

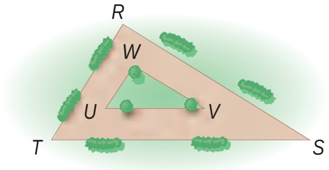
   Prove: $\frac{VW}{WT} = \frac{RS}{TS}$



ART For Exercises 10 and 11, use the diagram of a square mosaic tile. $AB = BC = CD = \frac{1}{3}AD$ and $DE = EF = FG = \frac{1}{3}DG$. (Lesson 7-5)



10. What is the ratio of the perimeter of $\triangle BDF$ to the perimeter of $\triangle BCI$? Explain.
11. Find two triangles such that the ratio of their perimeters is 2:3. Explain.
12. **TRACK** On the triangular track below, $\triangle RST \sim \triangle WVU$. If UV = 500 feet, VW = 400 feet, UW = 300 feet, and ST = 1000 feet, find the perimeter of $\triangle RST$. (Lesson 7-5)



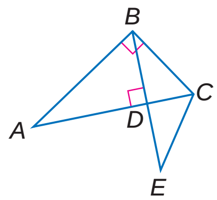
Chapter 8 Right Triangles and Trigonometry

(pages 430–493)

1. **PROOF** Write a two-column proof. (Lesson 8-1)

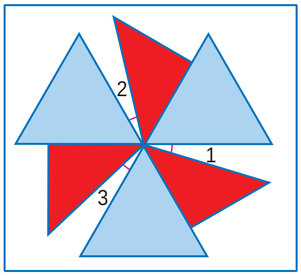
   Given: D is the midpoint of $\overline{BE}$; $\overline{BD}$ is an altitude of right triangle $\triangle ABC$.

   Prove: $\frac{AD}{DE} = \frac{DE}{DC}$



2. **CONSTRUCTION** Carlotta drew a diagram of a right triangular brace with side measures of 2.7 centimeters, 3.0 centimeters, and 5.3 centimeters. Is the diagram correct? Explain. (Lesson 8-2)

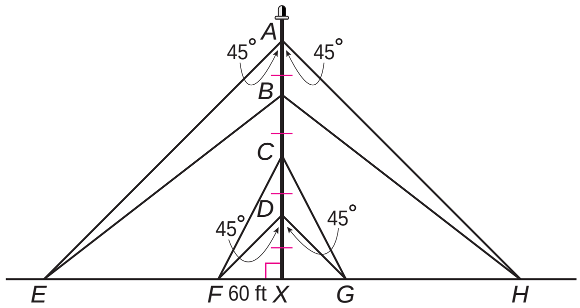
DESIGN For Exercises 3 and 4, use the following information.
In the pinwheel, the blue triangles are congruent equilateral triangles, each with an altitude of 4 inches. The red triangles are congruent isosceles right triangles. The hypotenuse of a red triangle is congruent to a side of a blue triangle. (Lesson 8-3)



3. If angles 1, 2, and 3 are congruent, find the measure of each angle.

4. Find the perimeter of the pinwheel. Round to the nearest inch.

COMMUNICATION For Exercises 5–8, use the following information. (Lesson 8-4)
The diagram shows a radio tower secured by four pairs of guy wires that are equally spaced apart with $DX = 60$ feet. Round to the nearest tenth if necessary.



5. Name the isosceles triangles above.

6. Find $m\angle BEX$ and $m\angle CFX$.

7. Find AE, EB, CF, and DF.

8. Find the total amount of wire used.

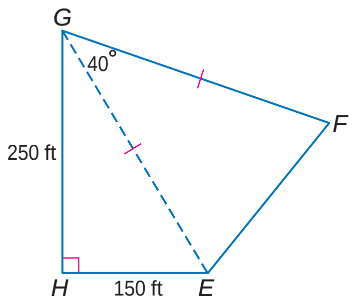
9. **METEOROLOGY** A searchlight is 6500 feet from a weather station. If the angle of elevation to the spot of light on the clouds above the station is 47°, how high is the cloud ceiling? (Lesson 8-5)

GARDENING For Exercises 10 and 11, use the information below.
A flower bed at Magic City Rose Garden is in the shape of an obtuse scalene triangle with the shortest side measuring 7.5 feet. Another side measures 14 feet and the measure of the opposite angle is 103°. (Lesson 8-6)

10. Find the measures of the other angles of the triangle to the nearest degree.

11. Find the perimeter of the garden. Round to the nearest tenth.

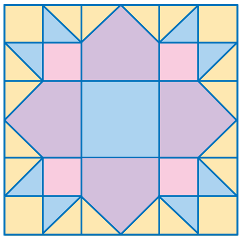
12. **HOUSING** Mr. and Mrs. Abbott bought a lot at the end of a cul-de-sac. They want to build a fence on three sides of the lot, excluding $\overline{HE}$. To the nearest foot, how much fencing will they need to buy? (Lesson 8-7)



Chapter 9 Transformations

(pages 494–549)

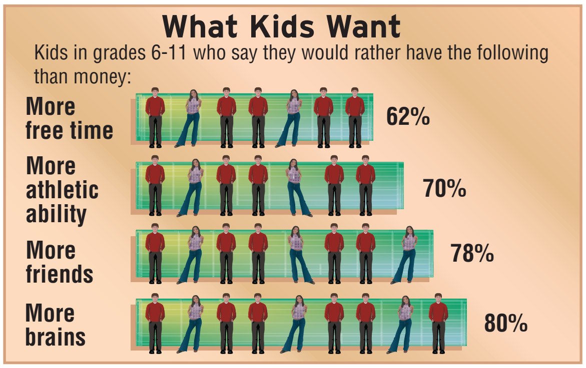
QUILTING For Exercises 1 and 2, use the diagram of a quilt square. (Lesson 9-1)



1. How many lines of symmetry are there for the entire quilt square?
2. Consider different sections of the quilt square. Describe at least three different lines of reflection and the figures reflected in those lines.

3. **ENVIRONMENT** A cloud of dense gas and dust pours out of Surtsey, a volcanic island off the south coast of Iceland. If the cloud blows 40 miles north and then 30 miles east, make a sketch to show the translation of the smoke particles. Then find the distance of the shortest path that would take the particles to the same position. (Lesson 9-2)

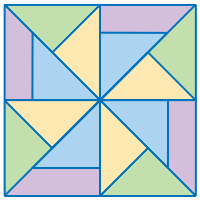
STUDENTS For Exercises 4–6, refer to the graphic below. Each bar of the graph is made up of boy-girl-boy units. (Lesson 9-2)



Source: Yankelovich Partners for Lutheran Brotherhood

4. Which categories show a boy-girl-boy unit that is translated within the bar?
5. Which categories show a boy-girl-boy unit that is reflected within the bar?
6. Does each person shown represent the same percent? Explain.

ART For Exercises 7–10, use the mosaic tile.



7. Identify the order and magnitude of rotation that takes a yellow triangle to a blue triangle. (Lesson 9-3)
8. Identify the order and magnitude of rotation that takes a blue triangle to a yellow triangle. (Lesson 9-3)
9. Identify the magnitude of rotation that takes a trapezoid to a consecutive trapezoid. (Lesson 9-3)
10. Can the mosaic tile tessellate the plane? Explain. (Lesson 9-4)

11. **CRAFTS** Eduardo found a pattern for cross-stitch on the Internet. The pattern measures 2 inches by 3 inches. He would like to enlarge the piece to 4 inches by 6 inches. The copy machine available to him enlarges 150% or less by increments of whole number percents. Find two whole number percents by which he can consecutively enlarge the piece and get as close to the desired dimensions as possible without exceeding them. (Lesson 9-5)

AVIATION For Exercises 12 and 13, use the following information.
A small aircraft flies due south at an average speed of 190 miles per hour. The wind is blowing due west at 30 miles per hour. (Lesson 9-6)

12. Draw a diagram using vectors to represent this situation.
13. Find the resultant velocity and direction of the plane.

1. **CYCLING** A bicycle tire travels about 50.27 inches during one rotation. What is the diameter of the tire? (Lesson 10-1)

SPACE For Exercises 2–4, use the following information. (Lesson 10-2)
The table shows the results of a survey of school children who were asked what the most important reason is to explore Mars.

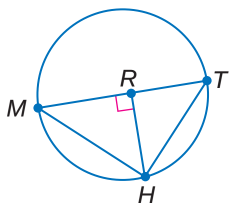
Reason to Visit Mars	Number of Students
Learn about life beyond Earth	910
Learn more about Earth	234
Seek potential for human inhabitance	624
Use as a base for further exploration	364
Increase human knowledge	468

Source: *USA TODAY*

2. If you were to construct a circle graph of these data, how many degrees would be allotted to each category?
3. Describe the type of arc for each category.
4. Construct a circle graph for these data.
5. **CRAFTS** Yvonne uses wooden spheres to make paperweights to sell at craft shows. She cuts off a flat surface for each base. If the original sphere has a radius of 4 centimeters and the diameter of the flat surface is 6 centimeters, what is the height of the paperweight? (Lesson 10-3)
6. **PROOF** Write a two-column proof. (Lesson 10-4)

Given: $\widehat{MHT}$ is a semicircle. $\overline{RH} \perp \overline{TM}$.

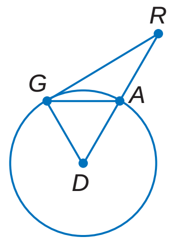
Prove: $\frac{TR}{RH} = \frac{TH}{HM}$



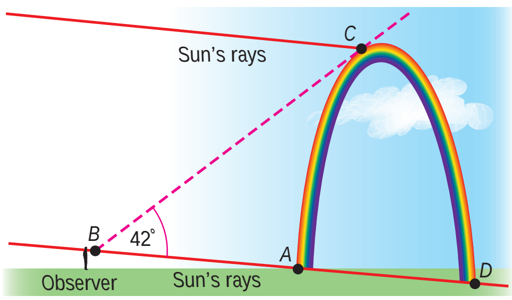
7. **PROOF** Write a paragraph proof. (Lesson 10-5)

Given: $\overline{GR}$ is tangent to $\odot D$ at G.
$\overline{AG} \cong \overline{DG}$

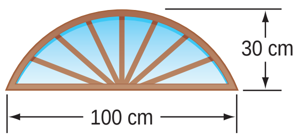
Prove: $\overline{AG}$ bisects $\overline{RD}$.



8. **METEOROLOGY** A rainbow is really a full circle with its center at a point in the sky directly opposite the Sun. The position of a rainbow varies according to the viewer's position, but its angular size, $\angle ABC$, is always 42°. If $m\widehat{CD} = 160$, find the measure of the visible part of the rainbow, $m\widehat{AC}$. (Lesson 10-6)



9. **CONSTRUCTION** An arch over an entrance is 100 centimeters wide and 30 centimeters high. Find the radius of the circle that contains the arch. (Lesson 10-7)

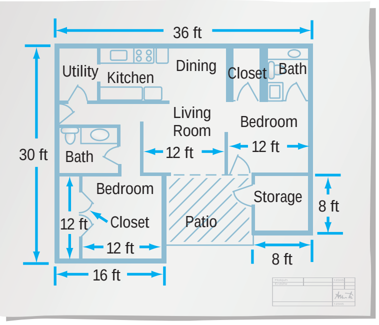


10. **SPACE** Objects that have been left behind in Earth's orbit from space missions are called "space junk." These objects are a hazard to current space missions and satellites. Eighty percent of space junk orbits Earth at a distance of 1,200 miles from the surface of Earth, which has a diameter of 7,926 miles. Write an equation to model the orbit of 80% of space junk with Earth's center at the origin. (Lesson 10-8)

Chapter 11 Areas of Polygons and Circles

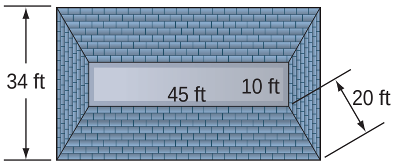
(pages 628–677)

REMODELING For Exercises 1–3, use the following information.
The diagram shows the floor plan of the home that the Summers are buying. They want to replace the patio with a larger sunroom to increase their living space by one-third. (Lesson 11-1)



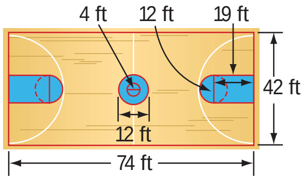
1. Excluding the patio and storage area, how many square feet of living area are in the current house?
2. What area should be added to the house to increase the living area by one-third?
3. The Summers want to connect the bedroom and storage area with the sunroom. What will be the dimensions of the sunroom?

HOME REPAIR For Exercises 4 and 5, use the following information.
Scott needs to replace the shingles on the roof of his house. The roof is composed of two large isosceles trapezoids, two smaller isosceles trapezoids, and a rectangle. Each trapezoid has the same height. (Lesson 11-2)

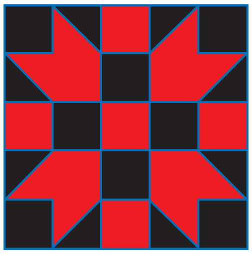


4. Find the height of the trapezoids.
5. Find the area of the roof covered by shingles.

6. **SPORTS** The Moore High School basketball team wants to paint their basketball court as shown. They want the center circle and the free throw areas painted blue. What is the area of the court that they will paint blue? (Lesson 11-3)

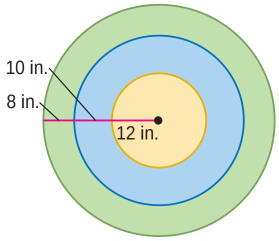


MUSEUMS For Exercises 7–9, use the following information.
The Hyalite Hills Museum plans to install the square mosaic pattern shown below in the entry hall. It is 10 feet on each side with each small black or red square tile measuring 2 feet on each side. (Lesson 11-4)



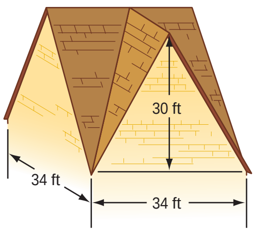
7. Find the area of black tiles.
8. Find the area of red tiles.
9. Which is greater, the total perimeter of the red tiles or the total perimeter of the black tiles? Explain.

10. **GAMES** If the dart lands on the target, find the probability that it lands in the blue region. (Lesson 11-5)



1. **ARCHITECTURE** Sketch an orthographic drawing of the Eiffel Tower. (Lesson 12-1)

2. **CONSTRUCTION** The roof shown below is a hip-and-valley style. Use the dimensions given to find the area of the roof that would need to be shingled. (Lesson 12-2)



3. **MANUFACTURING** Many baking pans are given a special nonstick coating. A rectangular cake pan is 9 inches by 13 inches by 2 inches deep. What is the area of the inside of the pan that needs to be coated? (Lesson 12-2)

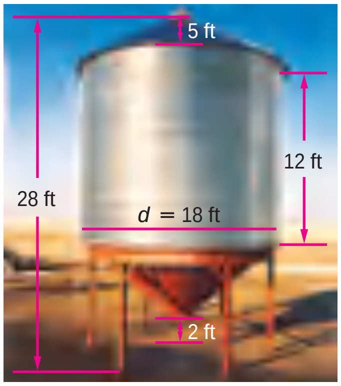
4. **COMMUNICATIONS** Coaxial cable is used to transmit long-distance telephone calls, cable television programming, and other communications. A typical coaxial cable contains 22 copper tubes and has a diameter of 3 inches. What is the lateral area of a coaxial cable that is 500 feet long? (Lesson 12-3)

COLLECTIONS For Exercises 5 and 6, use the following information.
Soledad collects unique salt-and-pepper shakers. She inherited a pair of tetrahedral shakers from her mother. (Lesson 12-4)

5. Each edge of a shaker measures 3 centimeters. Make a sketch of one shaker.

6. Find the total surface area of one shaker.

7. **FARMING** The picture below shows a combination hopper cone and bin used by farmers to store grain after harvest. The cone at the bottom of the bin allows the grain to be emptied more easily. Use the dimensions shown in the diagram to find the entire surface area of the bin with a conical top and bottom. Write the exact answer and the answer rounded to the nearest square foot. (Lesson 12-5)



GEOGRAPHY For Exercises 8–10, use the following information.
Joaquin is buying Dennis a globe for his birthday. The globe has a diameter of 16 inches. (Lesson 12-6)

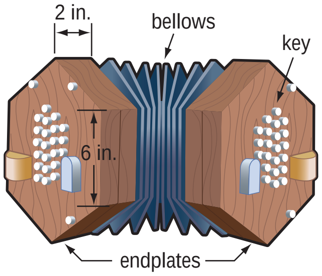
8. What is the surface area of the globe?

9. If the diameter of Earth is 7926 miles, find the surface area of Earth.

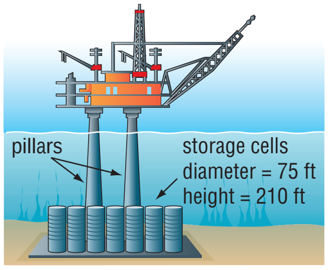
10. The continent of Africa occupies about 11,700,000 square miles. How many square inches will be used to represent Africa on the globe?

1. **METEOROLOGY** The TIROS weather satellites were a series of weather satellites, the first being launched on April 1, 1960. These satellites carried television and infrared cameras and were covered by solar cells. If the cylinder-shaped body of a TIROS had a diameter of 42 inches and a height of 19 inches, what was the volume available for carrying instruments and cameras? Round to the nearest tenth. (Lesson 13-1)

2. **MUSIC** To play a concertina, you push and pull the end plates and press the keys. The air pressure causes vibrations of the metal reeds that make the notes. When fully expanded, the concertina is 36 inches from end to end. If the concertina is compressed, it is 7 inches from end to end. Find the volume of air in the instrument when it is fully expanded and when it is compressed. (*Hint*: Each endplate is a regular hexagonal prism and contains no air.) (Lesson 13-1)

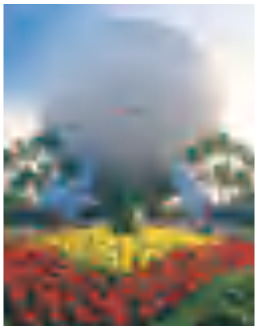


3. **ENGINEERING** The base of an oil drilling platform is made up of 24 concrete cylindrical cells. Twenty of the cells are used for oil storage. The pillars that support the platform deck rest on the four other cells. Find the total volume of the storage cells. (Lesson 13-1)



4. **HOME BUSINESS** Jodi has a home-based business selling homemade candies. She is designing a pyramid-shaped box for the candy. The base is a square measuring 14.5 centimeters on a side. The slant height of the pyramid is 16 centimeters. Find the volume of the box. Round to the nearest cubic centimeter. (Lesson 13-2)

ENTERTAINMENT For Exercises 5–8, use the following information.
Some people think that the Spaceship Earth geosphere at Epcot® in Disney World resembles a golf ball. The building is a sphere measuring 165 feet in diameter. A typical golf ball has a diameter of approximately 1.5 inches.



5. Find the volume of Spaceship Earth. Round to the nearest cubic foot. (Lesson 13-3)

6. Find the volume of a golf ball. Round to the nearest tenth. (Lesson 13-3)

7. What is the scale factor that compares Spaceship Earth to a golf ball? (Lesson 13-4)

8. What is the ratio of the volume of Spaceship Earth to the volume of a golf ball? (Lesson 13-4)

ASTRONOMY For Exercises 9 and 10, use the following information.
A museum has set aside a children's room containing objects suspended from the ceiling to resemble planets and stars. Suppose an imaginary coordinate system is placed in the room with the center of the room at (0, 0, 0). Three particular stars are located at $S(-10, 5, 3)$, $T(3, -8, -1)$, and $R(-7, -4, -2)$, where the coordinates represent the distance in feet from the center of the room. (Lesson 13-5)

9. Find the distance between each pair of stars.

10. Which star is farthest from the center of the room?

F. Stuart Westmorland/Photo Researchers

Mixed Problem Solving and Proof

Preparing for Standardized Tests

Becoming a Better Test-Taker

At some time in your life, you will have to take a standardized test. Sometimes this test may determine if you go on to the next grade or course, or even if you will graduate from high school. This section of your textbook is dedicated to making you a better test-taker.

TYPES OF TEST QUESTIONS In the following pages, you will see examples of four types of questions commonly seen on standardized tests. A description of each type of question is shown in the table below.

Type of Question	Description	See Pages
multiple choice	Four or five possible answer choices are given from which you choose the best answer.	842–843
gridded response	You solve the problem. Then you enter the answer in a special grid and color in the corresponding circles.	844–847
short response	You solve the problem, showing your work and/or explaining your reasoning.	848–851
extended response	You solve a multi-part problem, showing your work and/or explaining your reasoning.	852–856

PRACTICE After being introduced to each type of question, you can practice that type of question. Each set of practice questions is divided into five sections that represent the categories most commonly assessed on standardized tests.

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

USING A CALCULATOR On some tests, you are permitted to use a calculator. You should check with your teacher to determine if calculator use is permitted on the test you will be taking, and if so, what type of calculator can be used.

TEST-TAKING TIPS In addition to Test-Taking Tips like the one shown at the right, here are some additional thoughts that might help you.

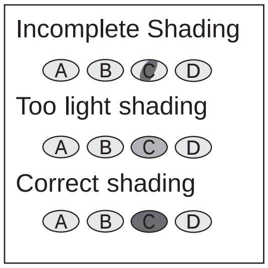
- Get a good night's rest before the test. Cramming the night before does not improve your results.
- Budget your time when taking a test. Don't dwell on problems that you cannot solve. Just make sure to leave that question blank on your answer sheet.
- Watch for key words like NOT and EXCEPT. Also look for order words like LEAST, GREATEST, FIRST, and LAST.

TEST-TAKING TIP

If you are allowed to use a calculator, make sure you are familiar with how it works so that you won't waste time trying to figure out the calculator when taking the test.

Multiple-Choice Questions

Multiple-choice questions are the most common type of question on standardized tests. These questions are sometimes called *selected-response questions.* You are asked to choose the best answer from four or five possible answers.



To record a multiple-choice answer, you may be asked to shade in a bubble that is a circle or an oval, or to just write the letter of your choice. Always make sure that your shading is dark enough and completely covers the bubble.

Sometime a question does not provide you with a figure that represents the problem. Drawing a diagram may help you to solve the problem. Once you draw the diagram, you may be able to eliminate some of the possibilities by using your knowledge of mathematics. Another answer choice might be that the correct answer is not given.

EXAMPLE

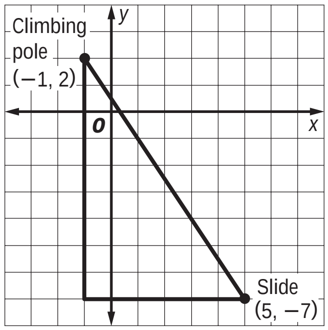
1 **A coordinate plane is superimposed on a map of a playground. Each side of each square represents 1 meter. The slide is located at (5, –7), and the climbing pole is located at (–1, 2). What is the distance between the slide and the pole?**

STRATEGY

Diagrams
Draw a diagram of the playground.

A $\sqrt{15}$ **m** **B 6 m** **C 9 m** **D** $9\sqrt{13}$ **m** **E none of these**

Draw a diagram of the playground on a coordinate plane. Notice that the difference in the *x*-coordinates is 6 meters and the difference in the *y*-coordinates is 9 meters.



Since the two points are two vertices of a right triangle, the distance between the two points must be greater than either of these values. So we can eliminate Choices B and C.

Use the Distance Formula or the Pythagorean Theorem to find the distance between the slide and the climbing pole. Let's use the Pythagorean Theorem.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$6^2 + 9^2 = c^2$	Substitution
$36 + 81 = c^2$	Simplify.
$117 = c^2$	Add.
$3\sqrt{13} = c$	Take the square root of each side and simplify.

So, the distance between the slide and pole is $3\sqrt{13}$ meters. Since this is not listed as choice A, B, C, or D, the answer is Choice E.

If you are short on time, you can test each answer choice to find the correct answer. Sometimes you can make an educated guess about which answer choice to try first.

Multiple Choice Practice

Choose the best answer.

Number and Operations

1. Carmen designed a rectangular banner that was 5 feet by 8 feet for a local business. The owner of the business asked her to make a larger banner measuring 10 feet by 20 feet. What was the percent increase in size from the first banner to the second banner?

A 4% **C** 80%

B 20% **D** 400%

2. A roller coaster casts a shadow 57 yards long. Next to the roller coaster is a 35-foot tree with a shadow that is 20 feet long at the same time of day. What is the height of the roller coaster to the nearest whole foot?

F 98 ft **H** 299 ft

G 100 ft **J** 388 ft

Algebra

3. At Speedy Car Rental, it costs \$32 per day to rent a car and then \$0.08 per mile. If y is the total cost of renting the car and x is the number of miles, which equation describes the relation between x and y?

A $y = 32x + 0.08$ **C** $y = 0.08x + 32$

B $y = 32x - 0.08$ **D** $y = 0.08x - 32$

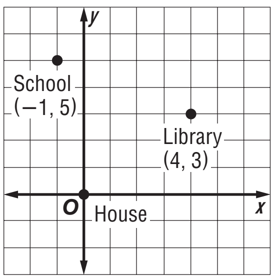
4. Eric plotted his house, school, and the library on a coordinate plane. Each side of each square represents one mile. What is the distance from his house to the library?

F $\sqrt{24}$ mi

G 5 mi

H $\sqrt{26}$ mi

J $\sqrt{29}$ mi



TEST-TAKING TIP

Questions 2, 5 and 7
The units of measure given in the question may not be the same as those given in the answer choices. Check that your solution is in the proper unit.

Geometry

5. The grounds outside of the Custer County Museum contain a garden shaped like a right triangle. One leg of the triangle measures 8 feet, and the area of the garden is 18 square feet. What is the length of the other leg?

A 2.25 in. **C** 13.5 in. **E** 54 in.

B 4.5 in. **D** 27 in.

6. The circumference of a circle is equal to the perimeter of a regular hexagon with sides that measure 22 inches. What is the length of the radius of the circle to the nearest inch? Use 3.14 for π.

F 7 in. **H** 21 in. **K** 28 in.

G 14 in. **J** 24 in.

Measurement

7. Eduardo is planning to install carpeting in a rectangular room that measures 12 feet 6 inches by 18 feet. How many square yards of carpet does he need for the project?

A 25 yd^2 **C** 225 yd^2

B 50 yd^2 **D** 300 yd^2

8. Marva is comparing two containers. One is a cylinder with diameter 14 centimeters and height 30 centimeters. The other is a cone with radius 15 centimeters and height 14 centimeters. What is the ratio of the volume of the cylinder to the volume of the cone?

F 3 to 1 **H** 7 to 5

G 2 to 1 **J** 7 to 10

Data Analysis and Probability

9. Refer to the table. Which statement is true about this set of data?

Country	Spending per Person
Japan	\$8622
United States	\$8098
Switzerland	\$6827
Norway	\$6563
Germany	\$5841
Denmark	\$5778

A The median is less than the mean.

B The mean is less than the median.

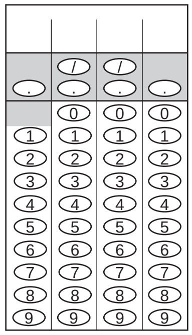
C The range is 2844.

D A and C are true.

E B and C are true.

Preparing for Standardized Tests

Gridded-Response Questions



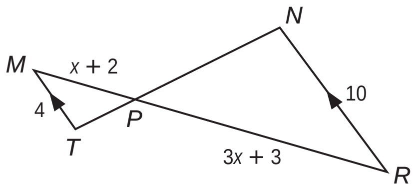
Gridded-response questions are another type of question on standardized tests. These questions are sometimes called *student-produced response griddable*, or *grid-in*, because you must create the answer yourself, not just choose from four or five possible answers.

For gridded response, you must mark your answer on a grid printed on an answer sheet. The grid contains a row of four or five boxes at the top, two rows of ovals or circles with decimal and fraction symbols, and four or five columns of ovals, numbered 0–9. Since there is no negative symbol on the grid, answers are never negative. An example of a grid from an answer sheet is shown at the right.

How do you correctly fill in the grid?

EXAMPLE

1 **In the diagram, $\triangle MPT \sim \triangle RPN$. Find PR.**



What do you need to find?

You need to find the value of x so that you can substitute it into the expression $3x + 3$ to find PR. Since the triangles are similar, write a proportion to solve for x.

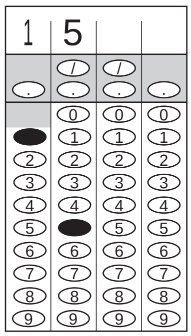
$\frac{MT}{RN} = \frac{PM}{PR}$	Definition of similar polygons
$\frac{4}{10} = \frac{x + 2}{3x + 3}$	Substitution
$4(3x + 3) = 10(x + 2)$	Cross products
$12x + 12 = 10x + 20$	Distributive Property
$2x = 8$	Subtract 12 and $10x$ from each side.
$x = 4$	Divide each side by 2.

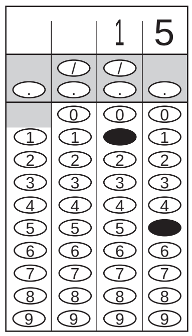
Now find PR.

$PR = 3x + 3$

$= 3(4) + 3$ or 15

How do you fill in the grid for the answer?





- Write your answer in the answer boxes.
- Write only one digit or symbol in each answer box.
- Do not write any digits or symbols outside the answer boxes.
- You may write your answer with the first digit in the left answer box, or with the last digit in the right answer box. You may leave blank any boxes you do not need on the right or the left side of your answer.
- Fill in only one bubble for every answer box that you have written in. Be sure not to fill in a bubble under a blank answer box.

Many gridded-response questions result in an answer that is a fraction or a decimal. These values can also be filled in on the grid.

How do you grid decimals and fractions?

EXAMPLE

2 **A triangle has a base of length 1 inch and a height of 1 inch. What is the area of the triangle in square inches?**

Use the formula $A = \frac{1}{2}bh$ to find the area of the triangle.

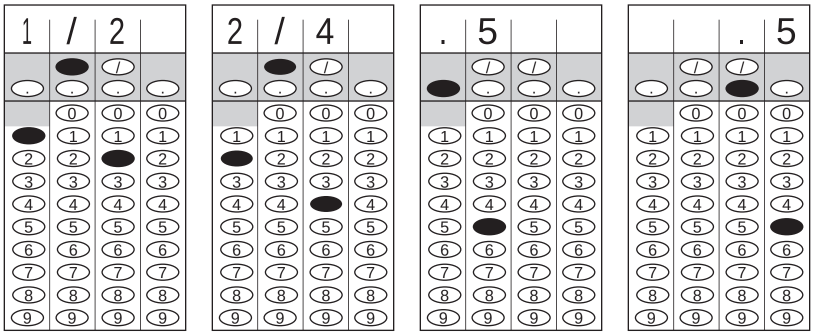
$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(1)(1)$ Substitution

$= \frac{1}{2}$ or 0.5 Simplify.

How do you grid the answer?

You can either grid the fraction or the decimal. Be sure to write the decimal point or fraction bar in the answer box. The following are acceptable answer responses.



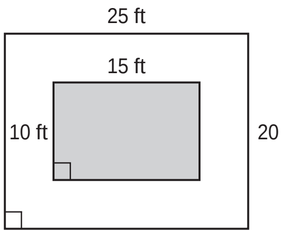
Do not leave a blank answer box in the middle of an answer.

Sometimes an answer is an improper fraction. Never change the improper fraction to a mixed number. Instead, grid either the improper fraction or the equivalent decimal.

How do you grid mixed numbers?

EXAMPLE

3 **The shaded region of the rectangular garden will contain roses. What is the ratio of the area of the garden to the area of the shaded region?**



STRATEGY

Formulas
If you are unsure of a formula, check the reference sheet.

First, find the area of the garden.

$A = \ell w$

$= 25(20)$ or 500

Then find the area of the shaded region.

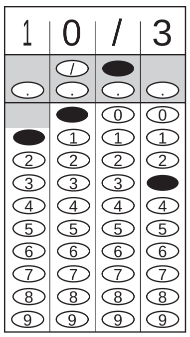
$A = \ell w$

$= 15(10)$ or 150

Write the ratio of the areas as a fraction.

$\frac{\text{area of garden}}{\text{area of shaded region}} = \frac{500}{150}$ or $\frac{10}{3}$

Leave the answer as the improper fraction $\frac{10}{3}$, as there is no way to correctly grid $3\frac{1}{3}$.



Gridded-Response Practice

Solve each problem. Then copy and complete a grid.

Number and Operations

1. A large rectangular meeting room is being planned for a community center. Before building the center, the planning board decides to increase the area of the original room by 40%. When the room is finally built, budget cuts force the second plan to be reduced in area by 25%. What is the ratio of the area of the room that is built to the area of the original room?

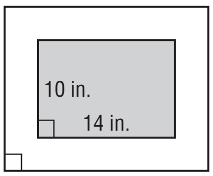
2. Greenville has a spherical tank for the city's water supply. Due to increasing population, they plan to build another spherical water tank with a radius twice that of the current tank. How many times as great will the volume of the new tank be as the volume of the current tank?

3. In Earth's history, the Precambrian period was about 4600 million years ago. If this number of years is written in scientific notation, what is the exponent for the power of 10?

4. A virus is a type of microorganism so small it must be viewed with an electron microscope. The largest shape of virus has a length of about 0.0003 millimeter. To the nearest whole number, how many viruses would fit end to end on the head of a pin measuring 1 millimeter?

Algebra

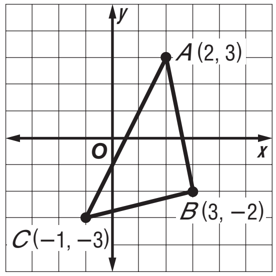
5. Kaia has a painting that measures 10 inches by 14 inches. She wants to make her own frame that has an equal width on all sides. She wants the total area of the painting and frame to be 285 square inches. What will be the width of the frame in inches?



TEST-TAKING TIP

Question 1
Remember that you have to grid the decimal point or fraction bar in your answer. If your answer does not fit on the grid, convert to a fraction or decimal. If your answer still cannot be gridded, then check your computations.

6. The diagram shows a triangle graphed on a coordinate plane. If $\overline{AB}$ is extended, what is the value of the y-intercept?



7. Tyree networks computers in homes and offices. In many cases, he needs to connect each computer to every other computer with a wire. The table shows the number of wires he needs to connect various numbers of computers. Use the table to determine how many wires are needed to connect 20 computers.

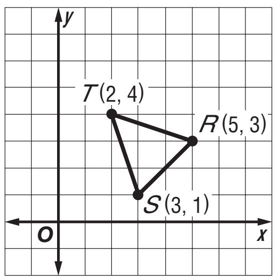
Computers	Wires	Computers	Wires
1	0	5	10
2	1	6	15
3	3	7	21
4	6	8	28

8. A line perpendicular to $9x - 10y = -10$ passes through $(-1, 4)$. Find the x-intercept of the line.

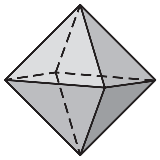
9. Find the positive solution of $6x^2 - 7x = 5$.

Geometry

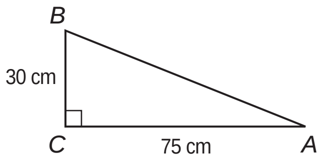
10. The diagram shows $\triangle RST$ on the coordinate plane. The triangle is first rotated 90° counterclockwise about the origin and then reflected in the y-axis. What is the x-coordinate of the image of T after the two transformations?



11. An octahedron is a solid with eight faces that are all equilateral triangles. How many edges does the octahedron have?

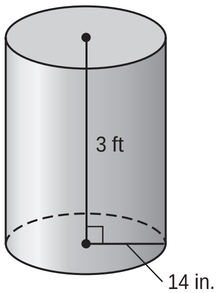


12. Find the measure of $\angle A$ to the nearest tenth of a degree.

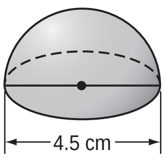


Measurement

13. The Pep Club plans to decorate some large garbage barrels for Spirit Week. They will cover only the sides of the barrels with decorated paper. How many square feet of paper will they need to cover 8 barrels like the one in the diagram? Use 3.14 for π. Round to the nearest square foot.



14. Kara makes decorative paperweights. One of her favorites is a hemisphere with a diameter of 4.5 centimeters. What is the surface area of the hemisphere including the bottom on which it rests? Use 3.14 for π. Round to the nearest tenth of a square centimeter.



15. The record for the fastest land speed of a car traveling for one mile is approximately 763 miles per hour. The car was powered by two jet engines. What was the speed of the car in feet per second? Round to the nearest whole number.

16. On average, a B-777 aircraft uses 5335 gallons of fuel on a 2.5-hour flight. At this rate, how much fuel will be needed for a 45-minute flight? Round to the nearest gallon.

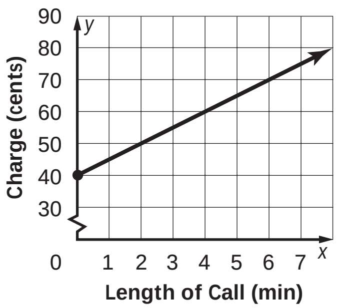
Data Analysis and Probability

17. The table shows the heights of the tallest buildings in Kansas City, Missouri. To the nearest tenth, what is the positive difference between the median and the mean of the data?

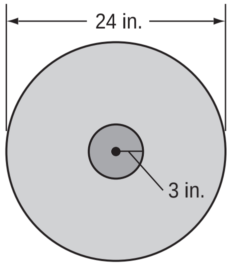
Name	Height (m)
One Kansas City Place	193
Town Pavilion	180
Hyatt Regency	154
Power and Light Building	147
City Hall	135
1201 Walnut	130

Source: skyscrapers.com

18. A long-distance telephone service charges 40 cents per call and 5 cents per minute. If a function model is written for the graph, what is the rate of change of the function?



19. In a dart game, the dart must land within the innermost circle on the dartboard to win a prize. If a dart hits the board, what is the probability, as a percent, that it will hit the innermost circle?



Short-Response Questions

Short-response questions require you to provide a solution to the problem, as well as any method, explanation, and / or justification you used to arrive at the solution. These are sometimes called *constructed-response, open-response, open-ended, free-response,* or *student-produced questions.* The following is a sample rubric, or scoring guide, for scoring short-response questions.

Credit	Score	Criteria
Full	2	Full credit: The answer is correct and a full explanation is provided that shows each step in arriving at the final answer.
Partial	1	Partial credit: There are two different ways to receive partial credit. • The answer is correct, but the explanation provided is incomplete or incorrect. • The answer is incorrect, but the explanation and method of solving the problem is correct.
None	0	No credit: Either an answer is not provided or the answer does not make sense.

On some stanardized tests, no credit is given for a correct answer if your work is not shown.

EXAMPLE

1 Mr. Solberg wants to buy all the lawn fertilizer he will need for this season. His front yard is a rectangle measuring 55 feet by 32 feet. His back yard is a rectangle measuring 75 feet by 54 feet. Two sizes of fertilizer are available—one that covers 5000 square feet and another covering 15,000 square feet. He needs to apply the fertilizer four times during the season. How many bags of each size should he buy to have the least amount of waste?

Full Credit Solution

STRATEGY

Estimation
Use estimation to check your solution.

Find the area of each part of the lawn and multiply by 4 since the fertilizer is to be applied 4 times. Each portion of the lawn is a rectangle, so A = lw.

$$4[(55 \times 32) + (75 \times 54)] = 23{,}240 \text{ ft}^2$$

If Mr. Solberg buys 2 bags that cover 15,000 ft^2, he will have too much fertilizer. If he buys 1 large bag, he will still need to cover 23,240 – 15,000 or 8240 ft^2.

The steps, calculations and reasoning are clearly stated.

Find how many small bags it takes to cover 8240 ft^2.

$$8240 \div 5000 = 1.648$$

Since he cannot buy a fraction of a bag, he will need to buy 2 of the bags that cover 5000 ft^2 each.

The solution of the problem is clearly stated.

Mr. Solberg needs to buy 1 bag that covers 15,000 square feet and 2 bags that cover 5000 square feet each.

Partial Credit Solution

In this sample solution, the answer is correct. However, there is no justification for any of the calculations.

There is not an explanation of how 23,240 was obtained.

23,240

$23{,}240 - 15{,}000 = 8240$

$8240 \div 5000 = 1.648$

Mr. Solberg needs to buy 1 large bag and 2 small bags.

Partial Credit Solution

In this sample solution, the answer is incorrect. However, after the first statement all of the calculations and reasoning are correct.

The first step of multiplying the area by 4 was left out.

First find the total number of square feet of lawn.

Find the area of each part of the yard.

$(55 \times 32) + (75 \times 54) = 5810 \text{ ft}^2$

The area of the lawn is greater than 5000 ft^2, which is the amount covered by the smaller bag, but buying the bag that covers 15,000 ft^2 would result in too much waste.

$5810 \div 5000 = 1.162$

Therefore, Mr. Solberg will need to buy 2 of the smaller bags of fertilizer.

No Credit Solution

In this sample solution, the response is incorrect and incomplete.

The wrong operations are used, so the answer is incorrect. Also, there are no units of measure given with any of the calculations.

$55 + 75 = 130$

$32 + 54 = 86$

$130 \times 86 \times 4 = 44{,}720$

$44{,}720 \div 15{,}000 = 2.98$

Mr. Solberg will need 3 bags of fertilizer.

Short-Response Practice

Solve each problem. Show all your work.

Number and Operations

1. In 2000, approximately \$191 billion in merchandise was sold by a popular retail chain store in the United States. The population at that time was 281,421,906. Estimate the average amount of merchandise bought from this store by each person in the U.S.

2. At a theme park, three educational movies run continuously all day long. At 9 a.m., the three shows begin. One runs for 15 minutes, the second for 18 minutes, and the third for 25 minutes. At what time will the movies all begin at the same time again?

3. Ming found a sweater on sale for 20% off the original price. However, the store was offering a special promotion, where all sale items were discounted an additional 60%. What was the total percent discount for the sweater?

4. The serial number of a DVD player consists of three letters of the alphabet followed by five digits. The first two letters can be any letter, but the third letter cannot be O. The first digit cannot be zero. How many serial numbers are possible with this system?

Algebra

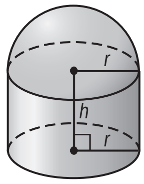
5. Solve and graph $2x - 9 \leq 5x + 4$.

6. Vance rents rafts for trips on the Jefferson River. You have to reserve the raft and provide a \$15 deposit in advance. Then the charge is \$7.50 per hour. Write an equation that can be used to find the charge for any amount of time, where y is the total charge in dollars and x is the amount of time in hours.

TEST-TAKING TIP

Question 4
Be sure to completely and carefully read the problem before beginning any calculations. If you read too quickly, you may miss a key piece of information.

7. Hector is working on the design for the container shown below that consists of a cylinder with a hemisphere on top. He has written the expression $\pi r^2 + 2\pi rh + 2\pi r^2$ to represent the surface area of any size container of this shape. Explain the meaning of each term of the expression.

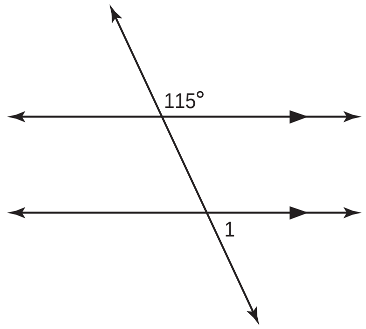


8. Find all solutions of the equation $6x^2 + 13x = 5$.

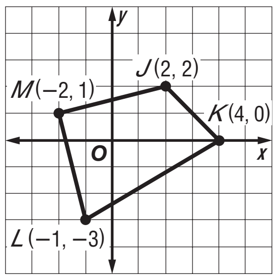
9. In 1999, there were 2,192,070 farms in the U.S., while in 2001, there were 2,157,780 farms. Let x represent years since 1999 and y represent the total number of farms in the U.S. Suppose the number of farms continues to decrease at the same rate as from 1999 to 2001. Write an equation that models the number of farms for any year after 1999.

Geometry

10. Refer to the diagram. What is the measure of $\angle 1$?

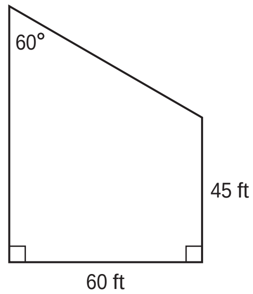


11. Quadrilateral $JKLM$ is to be reflected in the line $y = x$. What are the coordinates of the vertices of the image?



12. Write an equation in standard form for a circle that has a diameter with endpoints at $(-3, 2)$ and $(4, -5)$.

13. In the Columbia Village subdivision, an unusually shaped lot, shown below, will be used for a small park. Find the exact perimeter of the lot.

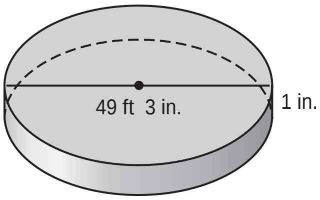


Measurement

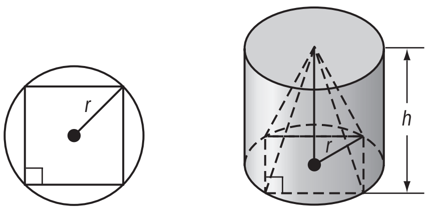
14. The Astronomical Unit (AU) is the distance from Earth to the Sun. It is usually rounded to 93,000,000 miles. The star Alpha Centauri is 25,556,250 million miles from Earth. What is this distance in AU?

15. Linesse handpaints unique designs on shirts and sells them. It takes her about 4.5 hours to create a design. At this rate, how many shirts can she design if she works 22 days per month for an average of 6.5 hours per day?

16. The world's largest pancake was made in England in 1994. To the nearest cubic foot, what was the volume of the pancake?



17. Find the ratio of the volume of the cylinder to the volume of the pyramid.



Top view Front view

Data Analysis and Probability

18. The table shows the winning times for the Olympic men's 1000-meter speed skating event. Make a scatter plot of the data and describe the pattern in the data. Times are rounded to the nearest second.

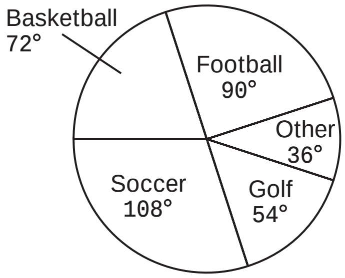
Men's 1000-m Speed Skating Event

Year	Country	Time (s)
1976	U.S.	79
1980	U.S.	75
1984	Canada	76
1988	USSR	73
1992	Germany	75
1994	U.S.	72
1998	Netherlands	71
2002	Netherlands	67
2006	U.S.	69

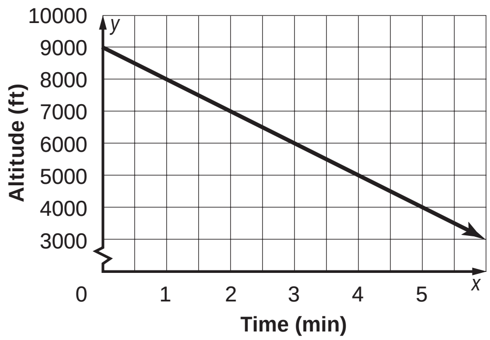
Source: *The World Almanac*

19. Bradley surveyed 70 people about their favorite spectator sport. If a person is chosen at random from the people surveyed, what is the probability that the person's favorite spectator sport is basketball?

Favorite Spectator Sport



20. The graph shows the altitude of a small airplane. Write a function to model the graph. Explain what the model means in terms of the altitude of the airplane.



Preparing for Standardized Tests

Extended-Response Questions

Extended-response questions are often called *open-ended* or *constructed-response questions.* Most extended-response questions have multiple parts. You must answer all parts correctly to receive full credit.

Extended-response questions are similar to short-response questions in that you must show all of your work in solving the problem. A rubric is used to determine whether you receive full, partial, or no credit. The following is a sample rubric for scoring extended-response questions.

Credit	Score	Criteria
Full	4	Full credit: A correct solution is given that is supported by well-developed, accurate explanations.
Partial	3, 2, 1	Partial credit: A generally correct solution is given that may contain minor flaws in reasoning or computation or an incomplete solution. The more correct the solution, the greater the score.
None	0	No credit: An incorrect solution is given indicating no mathematical understanding of the concept, or no solution is given.

On some standardized tests, no credit is given for a correct answer if your work is not shown.

Make sure that when the problem says to *Show your work,* show every part of your solution including figures, sketches of graphing calculator screens, or reasoning behind computations.

EXAMPLE

1 Polygon *WXYZ* with vertices *W*(-3, 2), *X*(4, 4), *Y*(3, -1), and *Z*(-2, -3) is a figure represented on a coordinate plane to be used in the graphics for a video game. Various transformations will be performed on the polygon to use for the game.

a. Graph *WXYZ* and its image *W'X'Y'Z'* under a reflection in the *y*-axis. Be sure to label all of the vertices.

b. Describe how the coordinates of the vertices of *WXYZ* relate to the coordinates of the vertices of *W'X'Y'Z'*.

c. Another transformation is performed on *WXYZ*. This time, the vertices of the image *W'X'Y'Z'* are *W'*(2, -3), *X'*(4, 4), *Y'*(-1, 3), and *Z'*(-3, -2). Graph *WXYZ* and its image under this transformation. What transformation produced *W'X'Y'Z'*?

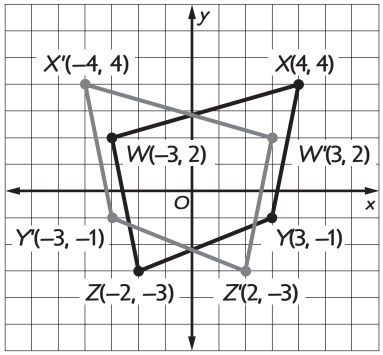
STRATEGY

Make a List Write notes about what to include in your answer for each part of the question.

Full Credit Solution

Part a A complete graph includes labels for the axes and origin and labels for the vertices, including letter names and coordinates.

- The vertices of the polygon should be correctly graphed and labeled.
- The vertices of the image should be located such that the transformation shows a reflection in the *y*-axis.
- The vertices of the polygons should be connected correctly. Optionally, the polygon and its image could be graphed in two contrasting colors.

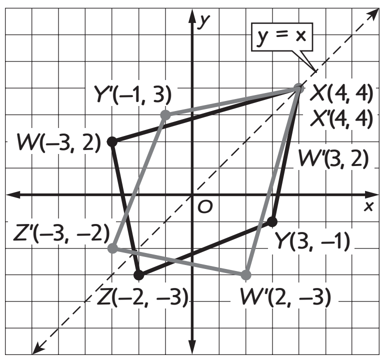


Part b

The coordinates of W and W' are (- 3, 2) and (3, 2). The x-coordinates are the opposite of each other and the y-coordinates are the same. For any point (a, b), the coordinates of the reflection in the y-axis are (- a, b).

Part c

For full credit, the graph in Part C must also be accurate, which is true for this graph.

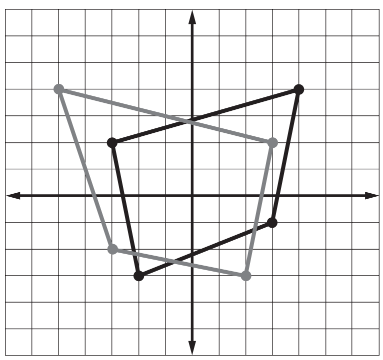


The coordinates of Z and Z' have been switched. In other words, for any point (a, b), the coordinates of the reflection in the y-axis are (b, a). Since X and X' are the same point, the polygon has been reflected in the line y = x.

Partial Credit Solution

Part a This sample graph includes no labels for the axes and for the vertices of the polygon and its image. Two of the image points have been incorrectly graphed.

More credit would have been given if all of the points were reflected correctly. The images of *X* and *Y* are not correct.



(continued on the next page)

Part b Partial credit is given because the reasoning is correct, but the reasoning was based on the incorrect graph in Part a.

> For two of the points, W and Z, the y-coordinates are the same and the x-coordinates are opposites. But, for points X and Y, there is no clear relationship.

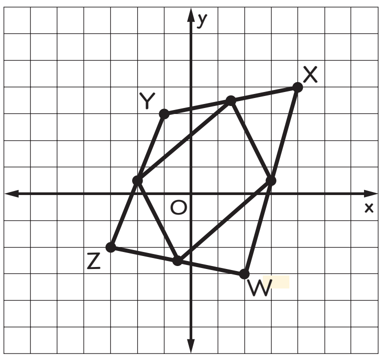
Part c Full credit is given for Part c. The graph supplied by the student was identical to the graph shown for the full credit solution for Part c. The explanation below is correct, but slightly different from the previous answer for Part c.

> I noticed that point X and point X' were the same. I also guessed that this was a reflection, but not in either axis. I played around with my ruler until I found a line that was the line of reflection. The transformation from WXYZ to W'X'Y'Z' was a reflection in the line y = x.

This sample answer might have received a score of 2 or 1, depending on the judgment of the scorer. Had the student graphed all points correctly and gotten Part b correct, the score would probably have been a 3.

No Credit Solution

Part a The sample answer below includes no labels on the axes or the coordinates of the vertices of the polygon. The polygon *WXYZ* has three vertices graphed incorrectly. The polygon that was graphed is not reflected correctly either.



Part b

> I don't see any way that the coordinates relate.

Part c

> It is a reduction because it gets smaller.

In this sample answer, the student does not understand how to graph points on a coordinate plane and also does not understand the reflection of figures in an axis or other line.

Extended-Response Practice

Solve each problem. Show all your work.

Number and Operations

1. Refer to the table.

Population		
City	1990	2000
Phoenix, AZ	983,403	1,321,045
Austin, TX	465,622	656,562
Charlotte, NC	395,934	540,828
Mesa, AZ	288,091	396,375
Las Vegas, NV	258,295	478,434

Source: census.gov

a. For which city was the increase in population the greatest? What was the increase?

b. For which city was the percent of increase in population the greatest? What was the percent increase?

c. Suppose that the population increase of a city was 30%. If the population in 2000 was 346,668, find the population in 1990.

2. Molecules are the smallest units of a particular substance that still have the same properties as that substance. The diameter of a molecule is measured in angstroms (Å). Express each value in scientific notation.

a. An angstrom is exactly 10^{-8} centimeter. A centimeter is approximately equal to 0.3937 inch. What is the approximate measure of an angstrom in inches?

b. How many angstroms are in one inch?

c. If a molecule has a diameter of 2 angstroms, how many of these molecules placed side by side would fit on an eraser measuring $\frac{1}{4}$ inch?

Algebra

3. The Marshalls are building a rectangular in-ground pool in their backyard. The pool will be 24 feet by 29 feet. They want to build a deck of equal width all around the pool. The area of the pool and deck will be 1800 square feet.

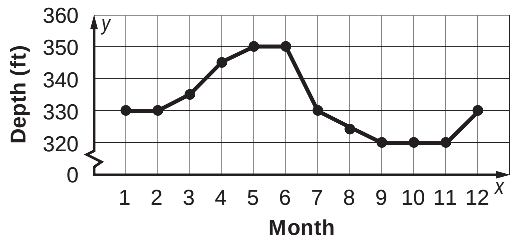
a. Draw and label a diagram.

b. Write an equation that can be used to find the width of the deck.

c. Find the width of the deck.

4. The depth of a reservoir was measured on the first day of each month. (Jan. = 1, Feb. = 2, and so on.)

Depth of the Reservoir



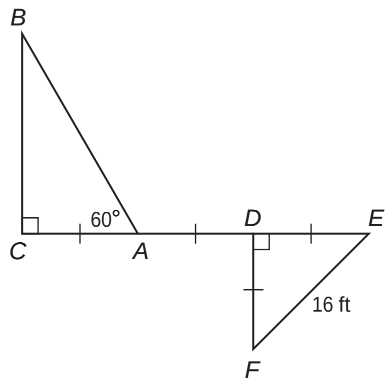
a. What is the slope of the line joining the points with x-coordinates 6 and 7? What does the slope represent?

b. Write an equation for the segment of the graph from 5 to 6. What is the slope of the line and what does this represent?

c. What was the lowest depth of the reservoir? When was this depth first measured and recorded?

Geometry

5. The Silver City Marching Band is planning to create this formation with the members.



a. Find the missing side measures of $\triangle EDF$. Explain.

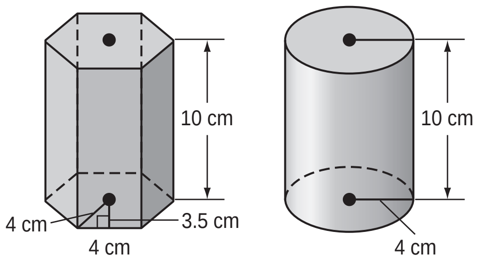
b. Find the missing side measures of $\triangle ABC$. Explain.

c. Find the total distance of the path: A to B to C to A to D to E to F to D.

d. The director wants to place one person at each point A, B, C, D, E, and F. He then wants to place other band members approximately one foot apart on all segments of the formation. How many people should he place on each segment of the formation? How many total people will he need?

Measurement

6. Two containers have been designed. One is a hexagonal prism, and the other is a cylinder.



a. What is the volume of the hexagonal prism?

b. What is the volume of the cylinder?

c. What is the percent of increase in volume from the prism to the cylinder?

7. Kabrena is working on a project about the solar system. The table shows the maximum distances from Earth to the other planets in millions of miles.

Distance from Earth to Other Planets			
Planet	Distance	Planet	Distance
Mercury	138	Saturn	1031
Venus	162	Uranus	1962
Mars	249	Neptune	2913
Jupiter	602		

Source: *The World Almanac*

a. The maximum speed of the Apollo moon missions spacecraft was about 25,000 miles per hour. Make a table showing the time it would take a spacecraft traveling at this speed to reach each of the four closest planets.

b. Describe how to use scientific notation to calculate the time it takes to reach any planet.

c. Which planet would it take approximately 13.3 years to reach? Explain.

TEST-TAKING TIP

Question 6
While preparing to take a standardized test, familiarize yourself with the formulas for surface area and volume of common three-dimensional figures. Some tests provide a list of formulas for use during testing.

Data Analysis and Probability

8. The table shows the average monthly temperatures in Barrow, Alaska. The months are given numerical values from 1-12. (Jan. = 1, Feb. = 2, and so on.)

Average Monthly Temperature			
Month	°F	Month	°F
1	–14	7	40
2	–16	8	39
3	–14	9	31
4	–1	10	15
5	20	11	–1
6	35	12	–11

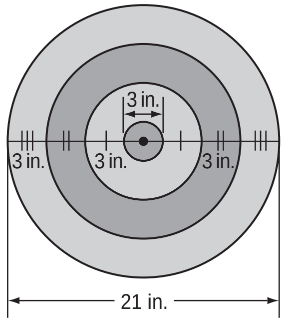
a. Make a scatter plot of the data. Let x be the numerical value assigned to the month and y be the temperature.

b. Describe any trends shown in the graph.

c. Find the mean of the temperature data.

d. Describe any relationship between the mean of the data and the scatter plot.

9. A dart game is played using the board shown. The inner circle is pink, the next ring is blue, the next red, and the largest ring is green. A dart must land on the board during each round of play.



a. What is the probability that a dart landing on the board hits the pink circle?

b. What is the probability that the first dart thrown lands in the blue ring and the second dart lands in the green ring?

c. Suppose players throw a dart twice. For which outcome of two darts would you award the most expensive prize? Explain your reasoning.

Postulates, Theorems, and Corollaries

Chapter 2 Reasoning and Proof

Postulate 2.1 Through any two points, there is exactly one line. (p. 105)

Postulate 2.2 Through any three points not on the same line, there is exactly one plane. (p. 105)

Postulate 2.3 A line contains at least two points. (p. 106)

Postulate 2.4 A plane contains at least three points not on the same line. (p. 106)

Postulate 2.5 If two points lie in a plane, then the entire line containing those points lies in that plane. (p. 106)

Postulate 2.6 If two lines intersect, then their intersection is exactly one point. (p. 106)

Postulate 2.7 If two planes intersect, then their intersection is a line. (p. 106)

Theorem 2.1 **Midpoint Theorem** If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$. (p. 107)

Postulate 2.8 **Ruler Postulate** The points on any line or line segment can be paired with real numbers so that, given any two points A and B on a line, A corresponds to zero, and B corresponds to a positive real number. (p. 118)

Postulate 2.9 **Segment Addition Postulate** If A, B, and C are collinear and B is between A and C, then $AB + BC = AC$. If $AB + BC = AC$, then B is between A and C. (p. 119)

Theorem 2.2 **Segment Congruence** Congruence of segments is reflexive, symmetric, and transitive. (p. 119)

Postulate 2.10 **Protractor Postulate** Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r. (p. 124)

Postulate 2.11 **Angle Addition Postulate** If R is in the interior of $\angle PQS$, then $m\angle RQS + m\angle PQR = m\angle PQS$. If $m\angle PQR + m\angle RQS = m\angle PQS$, then R is in the interior of $\angle PQS$. (p. 124)

Theorem 2.3 **Supplement Theorem** If two angles form a linear pair, then they are supplementary angles. (p. 125)

Theorem 2.4 **Complement Theorem** If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles. (p. 125)

Theorem 2.5 Congruence of angles is reflexive, symmetric, and transitive. (p. 126)

Theorem 2.6 Angles supplementary to the same angle or to congruent angles are congruent. (p. 126) Abbreviation: ∡ suppl. to same ∠ or ≅ ∡ are ≅.

Theorem 2.7 Angles complementary to the same angle or to congruent angles are congruent. (p. 126) Abbreviation: ∡ compl. to same ∠ or ≅ ∡ are ≅.

Theorem 2.8 **Vertical Angle Theorem** If two angles are vertical angles, then they are congruent. (p. 127)

Theorem 2.9 Perpendicular lines intersect to form four right angles. (p. 128)

Theorem 2.10 All right angles are congruent. (p. 128)

Theorem 2.11 Perpendicular lines form congruent adjacent angles. (p. 128)

Theorem 2.12 If two angles are congruent and supplementary, then each angle is a right angle. (p. 128)

Theorem 2.13 If two congruent angles form a linear pair, then they are right angles. (p. 128)

Chapter 3 Parallel and Perpendicular Lines

Postulate 3.1 **Corresponding Angles Postulate** If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent. (p. 149)

Theorem 3.1 **Alternate Interior Angles Theorem** If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent. (p. 150)

Theorem 3.2 **Consecutive Interior Angles Theorem** If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary. (p. 150)

Theorem 3.3 **Alternate Exterior Angles Theorem** If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent. (p. 150)

Theorem 3.4 **Perpendicular Transversal Theorem** In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other. (p. 150)

Postulate 3.2 Two nonvertical lines have the same slope if and only if they are parallel. (p. 158)

Postulate 3.3 Two nonvertical lines are perpendicular if and only if the product of their slopes is -1. (p. 158)

Postulate 3.4 If two lines in a plane are cut by a transversal so that corresponding angles are congruent, then the lines are parallel. (p. 172) Abbreviation: If corr. $\angle$s are $\cong$, lines are $\parallel$.

Postulate 3.5 **Parallel Postulate** If given a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line. (p. 173)

Theorem 3.5 If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel. (p. 173)
Abbreviation: If alt. ext. $\angle$s are $\cong$, then lines are $\parallel$.

Theorem 3.6 If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel. (p. 173)
Abbreviation: If cons. int. $\angle$s are suppl., then lines are $\parallel$.

Theorem 3.7 If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel. (p. 173)
Abbreviation: If alt. int. $\angle$s are $\cong$, then lines are $\parallel$.

Theorem 3.8 In a plane, if two lines are perpendicular to the same line, then they are parallel. (p. 173) Abbreviation: If 2 lines are $\perp$ to the same line, then lines are $\parallel$.

Theorem 3.9 In a plane, if two lines are equidistant from a third line, then the two lines are parallel to each other. (p. 183)

Chapter 4 Congruent Triangles

Theorem 4.1 **Angle Sum Theorem** The sum of the measures of the angles of a triangle is 180. (p. 210)

Theorem 4.2 **Third Angle Theorem** If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent. (p. 211)

Theorem 4.3 **Exterior Angle Theorem** The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles. (p. 212)

Corollary 4.1 The acute angles of a right triangle are complementary. (p. 213)

Corollary 4.2 There can be at most one right or obtuse angle in a triangle. (p. 213)

Theorem 4.4 **Properties of Triangle Congruence** Congruence of triangles is reflexive, symmetric, and transitive. (p. 218)

Postulate 4.1 **Side-Side-Side Congruence (SSS)** If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent. (p. 226)

Postulate 4.2 **Side-Angle-Side Congruence (SAS)** If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. (p. 227)

Postulate 4.3 **Angle-Side-Angle Congruence (ASA)** If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. (p. 235)

Theorem 4.5 **Angle-Angle-Side Congruence (AAS)** If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent. (p. 236)

Theorem 4.6 **Leg-Leg Congruence (LL)** If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent. (p. 243)

Theorem 4.7 **Hypotenuse-Angle Congruence (HA)** If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the two triangles are congruent. (p. 243)

Theorem 4.8 **Leg-Angle Congruence (LA)** If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent. (p. 243)

Postulate 4.4 **Hypotenuse-Leg Congruence (HL)** If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent. (p. 243)

Theorem 4.9 **Isosceles Triangle Theorem** If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (p. 245)

Theorem 4.10 If two angles of a triangle are congruent, then the sides opposite those angles are congruent. (p. 246) Abbreviation: Conv. of Isos. △Th.

Corollary 4.3 A triangle is equilateral if and only if it is equiangular. (p. 247)

Corollary 4.4 Each angle of an equilateral triangle measures 60°. (p. 247)

Chapter 5 Relationships in Triangles

Theorem 5.1 Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment. (p. 269)

Theorem 5.2 Any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment. (p. 269)

Theorem 5.3 **Circumcenter Theorem** The circumcenter of a triangle is equidistant from the vertices of the triangle. (p. 270)

Theorem 5.4 Any point on the angle bisector is equidistant from the sides of the angle. (p. 271)

Theorem 5.5 Any point equidistant from the sides of an angle lies on the angle bisector. (p. 271)

Theorem 5.6 **Incenter Theorem** The incenter of a triangle is equidistant from each side of the triangle. (p. 271)

Theorem 5.7 **Centroid Theorem** The centroid of a triangle is located two-thirds of the distance from a vertex to the midpoint of the side opposite the vertex on a median. (p. 271)

Theorem 5.8 **Exterior Angle Inequality Theorem** If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its corresponding remote interior angles. (p. 281)

Theorem 5.9 If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side. (p. 282)

Theorem 5.10 If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle. (p. 283)

Theorem 5.11 **Triangle Inequality Theorem** The sum of the lengths of any two sides of a triangle is greater than the length of the third side. (p. 296)

Theorem 5.12 The perpendicular segment from a point to a line is the shortest segment from the point to the line. (p. 298)

Corollary 5.1 The perpendicular segment from a point to a plane is the shortest segment from the point to the plane. (p. 298)

Theorem 5.13 **SAS Inequality/Hinge Theorem** Two sides of a triangle are congruent to two sides of another triangle. If the included angle in the first triangle has a greater measure than the included angle in the second triangle, then the third side of the first triangle is longer than the third side of the second triangle. (p. 302)

Theorem 5.14 **SSS Inequality** If two sides of a triangle are congruent to two sides of another triangle and the third side in one triangle is longer than the third side in the other, then the angle between the pair of congruent sides in the first triangle is greater than the corresponding angle in the second triangle. (p. 304)

Chapter 6 Quadrilaterals

Theorem 6.1 **Interior Angle Sum Theorem** If a convex polygon has n sides and S is the sum of the measures of its interior angles, then $S = 180(n - 2)$. (p. 318)

Theorem 6.2 **Exterior Angle Sum Theorem** If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360. (p. 320)

Theorem 6.3 Opposite sides of a parallelogram are congruent. (p. 326)
Abbreviation: Opp. sides of ▱ are $\cong$.

Theorem 6.4 Opposite angles of a parallelogram are congruent. (p. 326)
Abbreviation: Opp. $\angle$s of ▱ are $\cong$.

Theorem 6.5 Consecutive angles in a parallelogram are supplementary. (p. 326)
Abbreviation: Cons. $\angle$s in ▱ are suppl.

Theorem 6.6 If a parallelogram has one right angle, it has four right angles. (p. 326)
Abbreviation: If ▱ has 1 rt. $\angle$, it has 4 rt. $\angle$s.

Theorem 6.7 The diagonals of a parallelogram bisect each other. (p. 327)
Abbreviation: Diag. of ▱ bisect each other.

Theorem 6.8 Each diagonal of a parallelogram separates the parallelogram into two congruent triangles. (p. 328) Abbreviation: Diag. separates ▱ into 2 $\cong$ $\triangle$s.

Theorem 6.9 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (p. 334)
Abbreviation: If both pairs of opp. sides are $\cong$, then quad. is ▱.

Theorem 6.10 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (p. 334)
Abbreviation: If both pairs of opp. $\angle$s are $\cong$, then quad. is ▱.

Theorem 6.11 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (p. 334) Abbreviation: If diag. bisect each other, then quad. is ▱.

Theorem 6.12 If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram. (p. 334)
Abbreviation: If one pair of opp. sides is $\parallel$ and $\cong$, then the quad. is a ▱.

Theorem 6.13 If a parallelogram is a rectangle, then the diagonals are congruent. (p. 340)
Abbreviation: If ▱ is rectangle, diag. are $\cong$.

Theorem 6.14 If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. (p. 342) Abbreviation: If diagonals of ▱ are $\cong$, ▱ is a rectangle.

Theorem 6.15 The diagonals of a rhombus are perpendicular. (p. 348)

Theorem 6.16 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (p. 348)

Theorem 6.17 Each diagonal of a rhombus bisects a pair of opposite angles. (p. 348)

Theorem 6.18 Each pair of base angles of an isosceles trapezoid are congruent. (p. 356)

Theorem 6.19 The diagonals of an isosceles trapezoid are congruent. (p. 356)

Theorem 6.20 The median of a trapezoid is parallel to the bases, and its measure is one-half the sum of the measures of the bases. (p. 359)

Chapter 7 Proportions and Similarity

Postulate 7.1 **Angle-Angle (AA) Similarity** If the two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar. (p. 397)

Theorem 7.1 **Side-Side-Side (SSS) Similarity** If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar. (p. 398)

Theorem 7.2 **Side-Angle-Side (SAS) Similarity** If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar. (p. 398)

Theorem 7.3 Similarity of triangles is reflexive, symmetric, and transitive. (p. 399)

Theorem 7.4 **Triangle Proportionality Theorem** If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional lengths. (p. 405)

Theorem 7.5 **Converse of the Triangle Proportionality Theorem** If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side. (p. 406)

Theorem 7.6 **Triangle Midsegment Theorem** A midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side. (p. 407)

Corollary 7.1 If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally. (p. 408)

Corollary 7.2 If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal. (p. 408)

Theorem 7.7 **Proportional Perimeters Theorem** If two triangles are similar, then the perimeters are proportional to the measures of corresponding sides. (p. 415)

Theorem 7.8 If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides. (p. 416)
Abbreviation: ~ △s have corr. altitudes proportional to the corr. sides.

Theorem 7.9 If two triangles are similar, then the measures of the corresponding angle bisectors of the triangles are proportional to the measures of the corresponding sides. (p. 416) Abbreviation: ~ △s have corr. ∠ bisectors proportional to the corr. sides.

Theorem 7.10 If two triangles are similar, then the measures of the corresponding medians are proportional to the measures of the corresponding sides. (p. 416)
Abbreviation: ~ △s have corr. medians proportional to the corr. sides.

Theorem 7.11 **Angle Bisector Theorem** An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides. (p. 418)

Chapter 8 Right Triangles and Trigonometry

Theorem 8.1 If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and to each other. (p. 433)

Theorem 8.2 The measure of an altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse. (p. 433)

Theorem 8.3 If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the measure of a leg of the triangle is the geometric mean between the measures of the hypotenuse and the segment of the hypotenuse adjacent to that leg. (p. 434)

Theorem 8.4 **Pythagorean Theorem** In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse. (p. 440)

Theorem 8.5 **Converse of the Pythagorean Theorem** If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle. (p. 442)

Theorem 8.6 In a 45°-45°-90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg. (p. 448)

Theorem 8.7 In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg. (p. 450)

Theorem 8.8 **Law of Sines** In any $\triangle ABC$ with a, b, and c representing the measures of the sides opposite the angles with measures A, B, C, respectively, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. (p. 471)

Theorem 8.9 **Law of Cosines** In any $\triangle ABC$ with a, b, and c representing the measures of the sides opposite the angles with measures A, B, C, respectively, then the following equations are true. (p. 479)

$a^2 = b^2 + c^2 - 2bc \cos A$ $\quad b^2 = a^2 + c^2 - 2ac \cos B$ $\quad c^2 = a^2 + b^2 - 2ab \cos C$

Chapter 9 Transformations

Theorem 9.1 In a given rotation, if A is the preimage, A' is the image, and P is the center of rotation, then the measure of the angle of rotation $\angle APA'$ is twice the measure of the acute or right angle formed by the intersecting lines of reflection. (p. 512)

Corollary 9.1 Reflecting an image successively in two perpendicular lines results in a 180° rotation. (p. 512)

Theorem 9.2 If a dilation with center C and a scale factor of r transforms A to E and B to D, then $ED = |r|(AB)$. (p. 526)

Theorem 9.3 If $P(x, y)$ is the preimage of a dilation centered at the origin with a scale factor r, then the image is $P'(rx, ry)$. (p. 527)

Chapter 10 Circles

Theorem 10.1 In the same or in congruent circles, two arcs are congruent if and only if their corresponding central angles are congruent. (p. 563)

Postulate 10.1 **Arc Addition Postulate** The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. (p. 563)

Theorem 10.2 In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. (p. 569)

Abbreviations: In $\odot$, 2 minor arcs are $\cong$, *iff* corr. chords are $\cong$.
In $\odot$, 2 chords are $\cong$, *iff* corr. minor arcs are $\cong$.

Theorem 10.3 In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc. (p. 571)

Theorem 10.4 In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center. (p. 572)

Theorem 10.5 **Inscribed Angle Theorem** If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc (or the measure of the intercepted arc is twice the measure of the inscribed angle). (p. 577)

Theorem 10.6 If two inscribed angles of a circle (or congruent circles) intercept congruent arcs or the same arc, then the angles are congruent. (p. 579)
Abbreviations: Inscribed ∡s of same arc are ≅.
Inscribed ∡s of ≅ arcs are ≅.

Theorem 10.7 If an inscribed angle intercepts a semicircle, the angle is a right angle. (p. 580)

Theorem 10.8 If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary. (p. 581)

Theorem 10.9 If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency. (p. 588)

Theorem 10.10 If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle. (p. 589)

Theorem 10.11 If two segments from the same exterior point are tangent to a circle, then they are congruent. (p. 590)

Theorem 10.12 If two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle. (p. 598)

Theorem 10.13 If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc. (p. 599)

Theorem 10.14 If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs. (p. 600)

Theorem 10.15 If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal. (p. 606)

Theorem 10.16 If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment. (p. 607)

Theorem 10.17 If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment. (p. 608)

Chapter 11 Area of Polygons and Circles

Postulate 11.1 Congruent figures have equal areas. (p. 642)

Postulate 11.2 The area of a region is the sum of the areas of all of its nonoverlapping parts. (p. 658)

Chapter 13 Volume

Theorem 13.1 If two solids are similar with a scale factor of a:b, then the surface areas have a ratio of a^2:b^2, and the volumes have a ratio of a^3:b^3. (p. 752)

Glossary/Glosario

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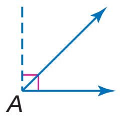
Cómo usar el glosario en español:

1. Busca el término en inglés que desees encontrar.
2. El término en español, junto con la definición, se encuentran en la columna de la derecha.

English | Español

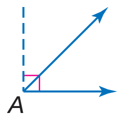
A

acute angle (p. 32) An angle with a degree measure less than 90.



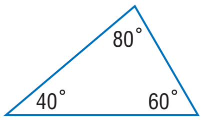
$0 < m\angle A < 90$

ángulo agudo Ángulo cuya medida en grados es menos de 90.



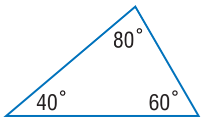
$0 < m\angle A < 90$

acute triangle (p. 202) A triangle in which all of the angles are acute angles.



three acute angles

triángulo acutángulo Triángulo cuyos ángulos son todos agudos.

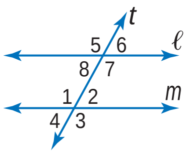


tres ángulos agudos

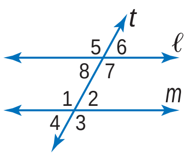
adjacent angles (p. 40) Two angles that lie in the same plane, have a common vertex and a common side, but no common interior points.

ángulos adyacentes Dos ángulos que yacen sobre el mismo plano, tienen el mismo vértice y un lado en común, pero ningún punto interior.

alternate exterior angles (p. 144) In the figure, transversal t intersects lines ℓ and m. $\angle 5$ and $\angle 3$, and $\angle 6$ and $\angle 4$ are alternate exterior angles.



ángulos alternos externos En la figura, la transversal t interseca las rectas ℓ y m. $\angle 5$ y $\angle 3$, y $\angle 6$ y $\angle 4$ son ángulos alternos externos.



alternate interior angles (p. 144) In the figure above, transversal t intersects lines ℓ and m. $\angle 1$ and $\angle 7$, and $\angle 2$ and $\angle 8$ are alternate interior angles.

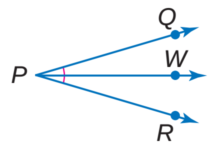
ángulos alternos internos En la figura anterior, la transversal t interseca las rectas ℓ y m. $\angle 1$ y $\angle 7$, y $\angle 2$ y $\angle 8$ son ángulos alternos internos.

altitude **1.** (p. 272) In a triangle, a segment from a vertex of the triangle to the line containing the opposite side and perpendicular to that side. **2.** (p. 693) In a prism or cylinder, a segment perpendicular to the bases with an endpoint in each plane. **3.** (p. 699) In a pyramid or cone, the segment that has the vertex as one endpoint and is perpendicular to the base. **4.** (p. 706) In a parallelogram, any segment perpendicular to the bases, with endpoints on each base.

ambiguous case of the Law of Sines (p. 478) Given the measures of two sides and a nonincluded angle, there exist two possible triangles.

angle (p. 31) The intersection of two noncollinear rays at a common endpoint. The rays are called *sides* and the common endpoint is called the *vertex*.

angle bisector (p. 35) A ray that divides an angle into two congruent angles.



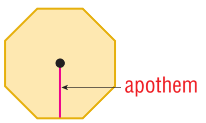
$\overrightarrow{PW}$ is the bisector of $\angle P$.

angle of depression (p. 465) The angle between the line of sight and the horizontal when an observer looks downward.

angle of elevation (p. 464) The angle between the line of sight and the horizontal when an observer looks upward.

angle of rotation (p. 510) The angle through which a preimage is rotated to form the image.

apothem (p. 649) A segment that is drawn from the center of a regular polygon perpendicular to a side of the polygon.



arc (p. 563) A part of a circle that is defined by two endpoints.

area (p. 51) The number of square units needed to cover a surface.

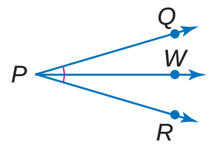
axiom (p. 105) A statement that is accepted as true.

altura **1.** En un triángulo, segmento trazado desde el vértice de un triángulo hasta el lado opuesto y que es perpendicular a dicho lado. **2.** El segmento perpendicular a las bases de prismas y cilindros que tiene un extremo en cada plano. **3.** El segmento que tiene un extremo en el vértice de pirámides y conos y que es perpendicular a la base. **4.** En un paralelogramo, todo segmento de recta perpendicular a las bases y cuyos extremos se hallan en las bases.

caso ambiguo de la ley de los senos Dadas las medidas de dos lados y de un ángulo no incluido, existen dos triángulos posibles.

ángulo La intersección de dos semirrectas no colineales en un punto común. Las semirrectas se llaman *lados* y el punto común se llama *vértice*.

bisectriz de un ángulo Semirrecta que divide un ángulo en dos ángulos congruentes.



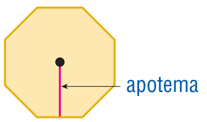
$\overrightarrow{PW}$ es la bisectriz del $\angle P$.

ángulo de depresión Ángulo formado por la horizontal y la línea de visión de un observador que mira hacia abajo.

ángulo de elevación Ángulo formado por la horizontal y la línea de visión de un observador que mira hacia arriba.

ángulo de rotación El ángulo a través del cual se rota una preimagen para formar la imagen.

apotema Segmento perpendicular trazado desde el centro de un polígono regular hasta uno de sus lados.



arco Parte de un círculo definida por los dos extremos de una recta.

área El número de unidades cuadradas que se requieren para cubrir una superficie.

axioma Enunciado que se acepta como verdadero.

axis **1.** (p. 693) In a cylinder, the segment with endpoints that are the centers of the bases. **2.** (p. 711) In a cone, the segment with endpoints that are the vertex and the center of the base.

B

base angle of an isosceles triangle (p. 203) See *isosceles triangle*.

between (p. 15) For any two points A and B on a line, there is another point C between A and B if and only if A, B, and C are collinear and $AC + CB = AB$.

betweenness of points (p. 15) See *between*.

biconditional (p. 98) The conjunction of a conditional statement and its converse.

C

center of circle (p. 554) The central point where radii form a locus of points called a circle.

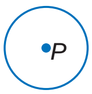
center of rotation (p. 510) A fixed point around which shapes move in a circular motion to a new position.

central angle (p. 562) An angle that intersects a circle in two points and has its vertex at the center of the circle.

centroid (p. 271) The point of concurrency of the medians of a triangle.

chord **1.** (p. 554) For a given circle, a segment with endpoints that are on the circle. **2.** (p. 711) For a given sphere, a segment with endpoints that are on the sphere.

circle (p. 554) The locus of all points in a plane equidistant from a given point called the *center* of the circle.



P is the center of the circle.

circumcenter (p. 270) The point of concurrency of the perpendicular bisectors of a triangle.

circumference (pp. 51, 555) The distance around a circle.

eje **1.** El segmento en un cilindro cuyos extremos forman el centro de las bases. **2.** El segmento en un cono cuyos extremos forman el vértice y el centro de la base.

ángulo de la base de un triángulo isósceles Ver *triángulo isósceles*.

ubicado entre Para cualquier par de puntos A y B de una recta, existe un punto C ubicado entre A y B si y sólo si A, B y C son colineales y $AC + CB = AB$.

intermediación de puntos Ver *ubicado entre*.

bicondicional La conjunción entre un enunciado condicional y su recíproco.

centro de un círculo Punto central a partir del cual los radios forman un lugar geométrico de puntos llamado círculo.

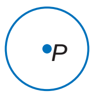
centro de rotación Punto fijo alrededor del cual gira una figura hasta alcanzar una posición determinada.

ángulo central Ángulo que interseca un círculo en dos puntos y cuyo vértice se localiza en el centro del círculo.

centroide Punto de intersección de las medianas de un triángulo.

cuerda **1.** Segmento cuyos extremos están en un círculo. **2.** Segmento cuyos extremos están en una esfera.

círculo Lugar geométrico formado por el conjunto de puntos en un plano, equidistantes de un punto dado llamado *centro*.

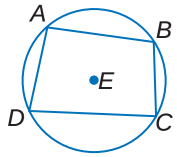


P es el centro del círculo.

circuncentro Punto de intersección de las mediatrices de un triángulo.

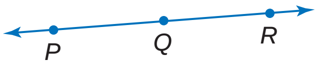
circunferencia Distancia alrededor de un círculo.

circumscribed (p. 571) A circle is circumscribed about a polygon if the circle contains all the vertices of the polygon.



⊙*E* is circumscribed about quadrilateral *ABCD*.

collinear (p. 6) Points that lie on the same line.



P, *Q*, and *R* are collinear.

complementary angles (p. 42) Two angles with measures that have a sum of 90.

component form (p. 533) A vector expressed as an ordered pair, ⟨change in *x*, change in *y*⟩.

composite figure (p. 658) A figure that cannot be separated into regions that are basic figures.

composition of reflections (p. 505) Successive reflections in parallel lines.

compound statement (p. 83) A statement formed by joining two or more statements.

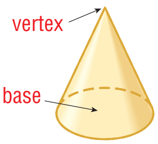
concave polygon (p. 49) A polygon for which there is a line containing a side of the polygon that also contains a point in the interior of the polygon.

conclusion (p. 91) In a conditional statement, the statement that immediately follows the word *then*.

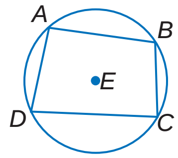
concurrent lines (p. 270) Three or more lines that intersect at a common point.

conditional statement (p. 91) A statement that can be written in *if-then* form.

cone (p. 61) A solid with a circular base, a vertex not contained in the same plane as the base, and a lateral surface area composed of all points in the segments connecting the vertex to the edge of the base.

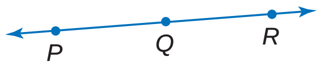


circunscrito Un polígono está circunscrito a un círculo si todos sus vértices están contenidos en el círculo.



⊙*E* está circunscrito al cuadrilátero *ABCD*.

colineal Puntos que yacen en la misma recta.



P, Q y *R* son colineales.

ángulos complementarios Dos ángulos cuya suma es igual a 90 grados.

componente Vector representado en forma de par ordenado, ⟨cambio en *x*, cambio en *y*⟩.

figura compuesta Figura que no se puede separar en regiones que tengan la forma de figuras básicas.

composición de reflexiones Reflexiones sucesivas en rectas paralelas.

enunciado compuesto Enunciado formado por la unión de dos o más enunciados.

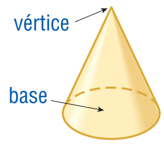
polígono cóncavo Polígono para el cual existe una recta que contiene un lado del polígono y un punto interior del polígono.

conclusión Parte del enunciado condicional que está escrita después de la palabra *entonces*.

rectas concurrentes Tres o más rectas que se intersecan en un punto común.

enunciado condicional Enunciado escrito en la forma *si-entonces*.

cono Sólido de base circular cuyo vértice no se localiza en el mismo plano que la base y cuya superficie lateral está formada por todos los segmentos que unen el vértice con los límites de la base.



congruence transformations (p. 219) A mapping for which a geometric figure and its image are congruent.

transformación de congruencia Transformación en un plano en la que la figura geométrica y su imagen son congruentes.

congruent (p. 15) Having the same measure.

congruente Que miden lo mismo.

congruent solids (p. 751) Two solids are congruent if all of the following conditions are met.

1. The corresponding angles are congruent.
2. Corresponding edges are congruent.
3. Corresponding faces are congruent.
4. The volumes are congruent.

sólidos congruentes Dos sólidos son congruentes si cumplen todas las siguientes condiciones:

1. Los ángulos correspondientes son congruentes.
2. Las aristas correspondientes son congruentes.
3. Las caras correspondientes son congruentes.
4. Los volúmenes son congruentes.

congruent triangles (p. 217) Triangles that have their corresponding parts congruent.

triángulos congruentes Triángulos cuyas partes correspondientes son congruentes.

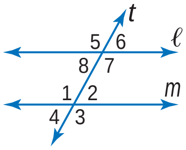
conjecture (p. 78) An educated guess based on known information.

conjetura Juicio basado en información conocida.

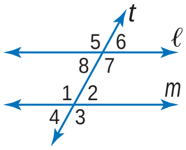
conjunction (p. 84) A compound statement formed by joining two or more statements with the word *and*.

conjunción Enunciado compuesto que se obtiene al unir dos o más enunciados con la palabra *y*.

consecutive interior angles (p. 144) In the figure, transversal t intersects lines ℓ and m. There are two pairs of consecutive interior angles: $\angle 8$ and $\angle 1$, and $\angle 7$ and $\angle 2$.



ángulos internos consecutivos En la figura, la transversal t interseca las rectas ℓ y m. La figura presenta dos pares de ángulos consecutivos internos: $\angle 8$ y $\angle 1$, y $\angle 7$ y $\angle 2$.



construction (p. 16) A method of creating geometric figures without the benefit of measuring tools. Generally, only a pencil, straightedge, and compass are used.

construcción Método para dibujar figuras geométricas sin el uso de instrumentos de medición. En general, sólo requiere de un lápiz, una regla sin escala y un compás.

contrapositive (p. 93) The statement formed by negating both the hypothesis and conclusion of the converse of a conditional statement.

antítesis Enunciado formado por la negación de la hipótesis y la conclusión del recíproco de un enunciado condicional dado.

converse (p. 93) The statement formed by exchanging the hypothesis and conclusion of a conditional statement.

recíproco Enunciado que se obtiene al intercambiar la hipótesis y la conclusión de un enunciado condicional dado.

convex polygon (p. 49) A polygon for which there is no line that contains both a side of the polygon and a point in the interior of the polygon.

polígono convexo Polígono para el cual no existe recta alguna que contenga un lado del polígono y un punto en el interior del polígono.

coordinate proof (p. 251) A proof that uses figures in the coordinate plane and algebra to prove geometric concepts.

prueba de coordenadas Demostración que usa álgebra y figuras en el plano de coordenadas para demostrar conceptos geométricos.

coplanar (p. 6) Points that lie in the same plane.

coplanar Puntos que yacen en un mismo plano.

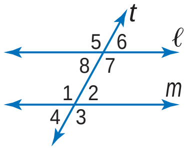
corner view (p. 680) The view from a corner of a three-dimensional figure, also called the *perspective view*.

vista de esquina Vista de una figura tridimensional desde una esquina. También se conoce como *vista de perspectiva*.

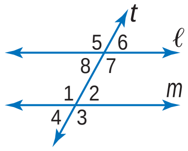
corollary (p. 213) A statement that can be easily proved using a theorem is called a corollary of that theorem.

corolario La afirmación que puede demostrarse fácilmente mediante un teorema se conoce como corolario de dicho teorema.

corresponding angles (p. 144) In the figure, transversal t intersects lines ℓ and m. There are four pairs of corresponding angles: $\angle 5$ and $\angle 1$, $\angle 8$ and $\angle 4$, $\angle 6$ and $\angle 2$, and $\angle 7$ and $\angle 3$.



ángulos correspondientes En la figura, la transversal t interseca las rectas ℓ y m. La figura muestra cuatro pares de ángulos correspondientes: $\angle 5$ y $\angle 1$, $\angle 8$ y $\angle 4$, $\angle 6$ y $\angle 2$, y $\angle 7$ y $\angle 3$.



cosine (p. 456) For an acute angle of a right triangle, the ratio of the measure of the leg adjacent to the acute angle to the measure of the hypotenuse.

coseno Para un ángulo agudo de un triángulo rectángulo, la razón entre la medida del cateto adyacente al ángulo agudo y la medida de la hipotenusa de un triángulo rectángulo.

counterexample (p. 79) An example used to show that a given statement is not always true.

contraejemplo Ejemplo que se usa para demostrar que un enunciado dado no siempre es verdadero.

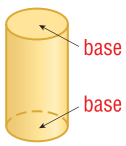
cross products (p. 381) In the proportion $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, the cross products are ad and bc. The proportion is true if and only if the cross products are equal.

productos cruzados En la proporción, $\frac{a}{b} = \frac{c}{d}$, donde $b \neq 0$ y $d \neq 0$, los productos cruzados son ad y bc. La proporción es verdadera si y sólo si los productos cruzados son iguales.

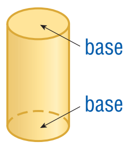
cross section (p. 681) The intersection of a solid and a plane.

sección transversal Intersección de un sólido con un plano.

cylinder (p. 61) A figure with bases that are formed by congruent circles in parallel planes.



cilindro Figura cuyas bases son círculos congruentes localizados en planos paralelos.



D

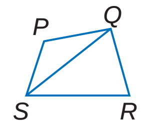
deductive argument (p. 111) A proof formed by a group of algebraic steps used to solve a problem.

argumento deductivo Demostración que consta del conjunto de pasos algebraicos que se usan para resolver un problema.

deductive reasoning (p. 99) A system of reasoning that uses facts, rules, definitions, or properties to reach logical conclusions.

degree (p. 31) A unit of measure used in measuring angles and arcs. An arc of a circle with a measure of 1° is $\frac{1}{360}$ of the entire circle.

diagonal (p. 318) In a polygon, a segment that connects nonconsecutive vertices of the polygon.



$\overline{SQ}$ is a diagonal.

diameter **1.** (p. 554) In a circle, a chord that passes through the center of the circle. **2.** (p. 711) In a sphere, a segment that contains the center of the sphere, and has endpoints that are on the sphere.

dilation (p. 525) A transformation determined by a center point C and a scale factor k. When $k > 0$, the image P' of P is the point on $\overrightarrow{CP}$ such that $CP' = |k| \cdot CP$. When $k < 0$, the image P' of P is the point on the ray opposite $\overrightarrow{CP}$ such that $CP' = k \cdot CP$.

direct isometry (p. 516) An isometry in which the image of a figure is found by moving the figure intact within the plane.

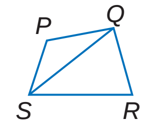
direction (p. 533) The measure of the angle that a vector forms with the positive x-axis or any other horizontal line.

disjunction (p. 84) A compound statement formed by joining two or more statements with the word *or*.

razonamiento deductivo Sistema de razonamiento que emplea hechos, reglas, definiciones y propiedades para obtener conclusiones lógicas.

grado Unidad de medida que se usa para medir ángulos y arcos. El arco de un círculo que mide 1° equivale a $\frac{1}{360}$ del círculo completo.

diagonal Recta que une vértices no consecutivos de un polígono.



$\overline{SQ}$ es una diagonal.

diámetro **1.** Cuerda que pasa por el centro de un círculo. **2.** Segmento que incluye el centro de una esfera y cuyos extremos se localizan en la esfera.

dilatación Transformación determinada por un punto central C y un factor de escala k. Cuando $k > 0$, la imagen P' de P es el punto en $\overrightarrow{CP}$ tal que $CP' = |k| \cdot CP$. Cuando $k < 0$, la imagen P' de P es el punto en la semirrecta opuesta $\overrightarrow{CP}$ tal que $CP' = k \cdot CP$.

isometría directa Isometría en la cual se obtiene la imagen de una figura, al mover la figura intacta junto con su plano.

dirección Medida del ángulo que forma un vector con el eje positivo x o con cualquier otra recta horizontal.

disyunción Enunciado compuesto que se forma al unir dos o más enunciados con la palabra *o*.

E

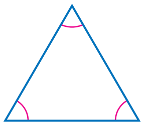
edge of a polygon (p. 60) A line segment where the faces of a polygon intersect.

equal vectors (p. 534) Vectors that have the same magnitude and direction.

arista de un polígono Segmento de recta en el que se intersecan las caras de un polígono.

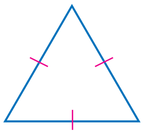
vectores iguales Vectores que poseen la misma magnitud y dirección.

equiangular triangle (p. 202) A triangle with all angles congruent.

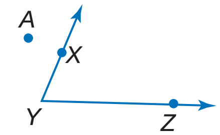


equidistant (p. 183) The distance between two lines measured along a perpendicular line is always the same.

equilateral triangle (p. 203) A triangle with all sides congruent.

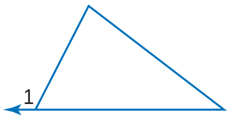


exterior (p. 31) A point is in the exterior of an angle if it is neither on the angle nor in the interior of the angle.



A is in the exterior of $\angle XYZ$.

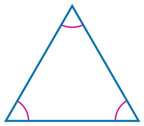
exterior angle (p. 211) An angle formed by one side of a triangle and the extension of another side.



$\angle 1$ is an exterior angle.

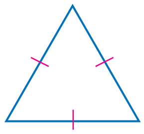
extremes (p. 381) In $\frac{a}{b} = \frac{c}{d}$, the numbers *a* and *d*.

triángulo equiangular Triángulo cuyos ángulos son congruentes entre sí.

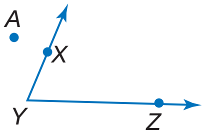


equidistante Distancia entre dos rectas que permanece siempre constante, medida a lo largo de su perpendicular.

triángulo equilátero Triángulo cuyos lados son congruentes entre sí.

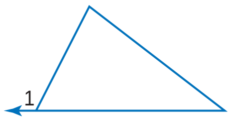


exterior Un punto yace en el exterior de un ángulo si no se localiza ni en el ángulo ni en el interior del ángulo.



A está en el exterior del $\angle XYZ$.

ángulo externo Ángulo formado por un lado de un triángulo y la extensión de otro de sus lados.



$\angle 1$ es un ángulo externo.

extremos Los números *a* y *d* en $\frac{a}{b} = \frac{c}{d}$.

F

face of a polygon (p. 60) A flat surface of a polygon.

flow proof (p. 212) A proof that organizes statements in logical order, starting with the given statements. Each statement is written in a box with the reason verifying the statement written below the box. Arrows are used to indicate the order of the statements.

formal proof (p. 112) A two-column proof containing statements and reasons.

cara de un polígono Superficie plana de un polígono.

demostración de flujo Demostración en que se ordenan los enunciados en orden lógico, empezando con los enunciados dados. Cada enunciado se escribe en una casilla y debajo de cada casilla se escribe el argumento que verifica el enunciado. El orden de los enunciados se indica mediante flechas.

prueba formal Prueba en dos columnas que contiene enunciados y razonamientos.

fractal (p. 423) A figure generated by repeating a special sequence of steps infinitely often. Fractals often exhibit self-similarity.

fractal Figura que se obtiene mediante la repetición infinita de una sucesión particular de pasos. Los fractales a menudo exhiben autosemejanza.

frustum (p. 704) The part of a solid that remains after the top portion has been cut by a plane parallel to the base.

tronco La parte de un sólido que queda después de cortar la parte superior por un plano paralelo a la base.

G

geometric mean (p. 432) For any positive numbers a and b, the positive number x such that $\frac{a}{x} = \frac{x}{b}$.

media geométrica Para todo número positivo a y b, existe un número positivo x tal que $\frac{a}{x} = \frac{x}{b}$.

geometric probability (p. 665) Using the principles of length and area to find the probability of an event.

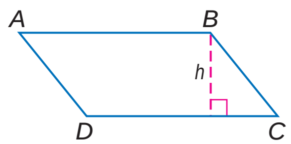
probabilidad geométrica El uso de los principios de longitud y área para calcular la probabilidad de un evento.

great circle (p. 711) For a given sphere, the intersection of the sphere and a plane that contains the center of the sphere.

círculo máximo La intersección entre una esfera dada y un plano que contiene el centro de la esfera.

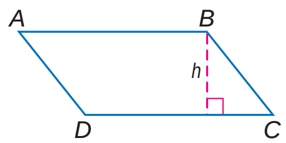
H

height of a parallelogram (p. 630) The length of an altitude of a parallelogram.



h is the height of parallelogram *ABCD.*

altura de un paralelogramo La longitud de la altura de un paralelogramo.



h es la altura del paralelogramo *ABCD.*

hemisphere (p. 712) One of the two congruent parts into which a great circle separates a sphere.

hemisferio Cada una de las dos partes congruentes en que un círculo máximo divide una esfera.

hypothesis (p. 91) In a conditional statement, the statement that immediately follows the word *if*.

hipótesis El enunciado escrito a continuación de la palabra *si* en un enunciado condicional.

I

if-then statement (p. 91) A compound statement of the form "if *A*, then *B*," where *A* and *B* are statements.

enunciado si-entonces Enunciado compuesto de la forma "si *A*, entonces *B*," donde *A* y *B* son enunciados.

incenter (p. 271) The point of concurrency of the angle bisectors of a triangle.

incentro Punto de intersección de las bisectrices interiores de un triángulo.

included angle (p. 227) In a triangle, the angle formed by two sides is the included angle for those two sides.

ángulo incluido En un triángulo, el ángulo formado por dos lados cualesquiera del triángulo es el ángulo incluido de esos dos lados.

included side (p. 234) The side of a triangle that is a side of each of two angles.

indirect isometry (p. 516) An isometry that cannot be performed by maintaining the orientation of the points, as in a direct isometry.

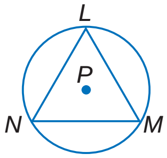
indirect proof (p. 288) In an indirect proof, one assumes that the statement to be proved is false. One then uses logical reasoning to deduce that a statement contradicts a postulate, theorem, or one of the assumptions. Once a contradiction is obtained, one concludes that the statement assumed false must in fact be true.

indirect reasoning (p. 288) Reasoning that assumes that the conclusion is false and then shows that this assumption leads to a contradiction of the hypothesis or some other accepted fact, like a postulate, theorem, or corollary. Then, since the assumption has been proved false, the conclusion must be true.

inductive reasoning (p. 78) Reasoning that uses a number of specific examples to arrive at a plausible generalization or prediction. Conclusions arrived at by inductive reasoning lack the logical certainty of those arrived at by deductive reasoning.

informal proof (p. 106) A paragraph proof.

inscribed (p. 570) A polygon is inscribed in a circle if each of its vertices lie on the circle.



$\triangle LMN$ is inscribed in $\odot P$.

intercepted (p. 577) An angle intercepts an arc if and only if each of the following conditions are met.

1. The endpoints of the arc lie on the angle.
2. All points of the arc except the endpoints are in the interior of the circle.
3. Each side of the angle contains an endpoint of the arc.

lado incluido El lado de un triángulo que es común a de sus dos ángulos.

isometría indirecta Tipo de isometría que no se puede obtener manteniendo la orientación de los puntos, como ocurre durante la isometría directa.

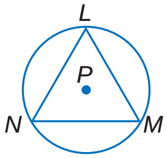
demostración indirecta En una demostración indirecta, se asume que el enunciado por demostrar es falso. Después, se deduce lógicamente que existe un enunciado que contradice un postulado, un teorema o una de las conjeturas. Una vez hallada una contradicción, se concluye que el enunciado que se suponía falso debe ser, en realidad, verdadero.

razonamiento indirecto Razonamiento en que primero se asume que la conclusión es falsa y, después, se demuestra que esto contradice la hipótesis o un hecho aceptado como un postulado, un teorema o un corolario. Finalmente, dado que se ha demostrado que la conjetura es falsa, entonces la conclusión debe ser verdadera.

razonamiento inductivo Razonamiento que usa varios ejemplos específicos para lograr una generalización o una predicción creíble. Las conclusiones obtenidas mediante el razonamiento inductivo carecen de la certidumbre lógica de aquellas obtenidas mediante el razonamiento deductivo.

prueba informal Prueba en forma de párrafo.

inscrito Un polígono está inscrito en un círculo si todos sus vértices yacen en el círculo.

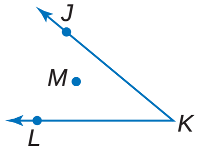


$\triangle LMN$ está inscrito en $\odot P$.

intersecado Un ángulo interseca un arco si y sólo si se cumplen todas las siguientes condiciones.

1. Los extremos del arco yacen en el ángulo.
2. Todos los puntos del arco, exceptuando sus extremos, yacen en el interior del círculo.
3. Cada lado del ángulo contiene un extremo del arco.

interior (p. 31) A point is in the interior of an angle if it does not lie on the angle itself and it lies on a segment with endpoints that are on the sides of the angle.

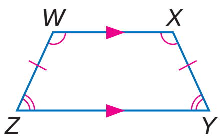


M is in the interior of ∠*JKL*.

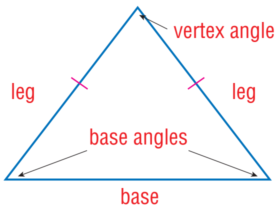
inverse (p. 93) The statement formed by negating both the hypothesis and conclusion of a conditional statement.

isometry (p. 497) A mapping for which the original figure and its image are congruent.

isosceles trapezoid (p. 356) A trapezoid in which the legs are congruent, both pairs of base angles are congruent, and the diagonals are congruent.

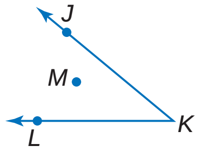


isosceles triangle (p. 203) A triangle with at least two sides congruent. The congruent sides are called *legs*. The angles opposite the legs are *base angles*. The angle formed by the two legs is the *vertex angle*. The side opposite the vertex angle is the *base*.



iteration (p. 423) A process of repeating the same procedure over and over again.

interior Un punto se localiza en el interior de un ángulo, si no yace en el ángulo mismo y si está en un segmento cuyos extremos yacen en los lados del ángulo.

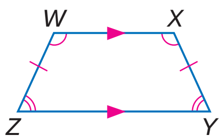


M está en el interior del ∠*JKL*.

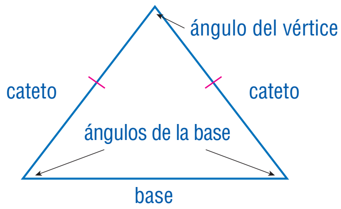
inversa Enunciado que se obtiene al negar la hipótesis y la conclusión de un enunciado condicional.

isometría Transformación en que la figura original y su imagen son congruentes.

trapecio isósceles Trapecio cuyos catetos son congruentes, ambos pares de ángulos son congruentes y las diagonales son congruentes.



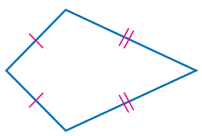
triángulo isósceles Triángulo que tiene por lo menos dos lados congruentes. Los lados congruentes se llaman *catetos*. Los ángulos opuestos a los catetos son los *ángulos de la base*. El ángulo formado por los dos catetos es el *ángulo del vértice*. Los lados opuestos al ángulo del vértice forman la *base*.



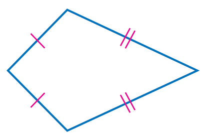
iteración Proceso de repetir el mismo procedimiento una y otra vez.

K

kite (p. 355) A quadrilateral with exactly two distinct pairs of adjacent congruent sides.



cometa Cuadrilátero que tiene exactamente dospares de lados congruentes adyacentes distintivos.



lateral area (p. 686) For prisms, pyramids, cylinders, and cones, the area of the figure, not including the bases.

área lateral En prismas, pirámides, cilindros y conos, es el área de la figura, sin incluir el área de las bases.

lateral edges **1.** (p. 686) In a prism, the intersection of two adjacent lateral faces. **2.** (p. 686) In a pyramid, lateral edges are the edges of the lateral faces that join the vertex to vertices of the base.

aristas laterales **1.** En un prisma, la intersección de dos caras laterales adyacentes. **2.** En una pirámide, las aristas de las caras laterales que unen el vértice de la pirámide con los vértices de la base.

lateral faces **1.** (p. 686) In a prism, the faces that are not bases. **2.** (p. 699) In a pyramid, faces that intersect at the vertex.

caras laterales **1.** En un prisma, las caras que no forman las bases. **2.** En una pirámide, las caras que se intersecan en el vértice.

Law of Cosines (p. 479) Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite the angles with measures A, B, and C respectively. Then the following equations are true.

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Ley de los cosenos Sea $\triangle ABC$ cualquier triángulo donde a, b y c son las medidas de los lados opuestos a los ángulos que miden A, B y C respectivamente. Entonces las siguientes ecuaciones son ciertas.

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Law of Detachment (p. 99) If $p \rightarrow q$ is a true conditional and p is true, then q is also true.

Ley de indiferencia Si $p \rightarrow q$ es un enunciado condicional verdadero y p es verdadero, entonces q es verdadero también.

Law of Sines (p. 471) Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite the angles with measures A, B, and C respectively. Then, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Ley de los senos Sea $\triangle ABC$ cualquier triángulo donde a, b y c representan las medidas de los lados opuestos a los ángulos A, B y C respectivamente. Entonces, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Law of Syllogism (p. 100) If $p \rightarrow q$ and $q \rightarrow r$ are true conditionals, then $p \rightarrow r$ is also true.

Ley del silogismo Si $p \rightarrow q$ y $q \rightarrow r$ son enunciados condicionales verdaderos, entonces $p \rightarrow r$ también es verdadero.

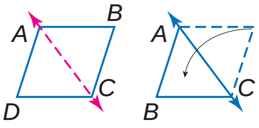
line (p. 6) A basic undefined term of geometry. A line is made up of points and has no thickness or width. In a figure, a line is shown with an arrowhead at each end. Lines are usually named by lowercase script letters or by writing capital letters for two points on the line, with a double arrow over the pair of letters.

recta Término primitivo en geometría. Una recta está formada por puntos y carece de grosor o ancho. En una figura, una recta se representa con una flecha en cada extremo. Por lo general, se designan con letras minúsculas o con las dos letras mayúsculas de dos puntos sobre la línea. Se escribe una flecha doble sobre el par de letras mayúsculas.

line of reflection (p. 497) A line through a figure that separates the figure into two mirror images.

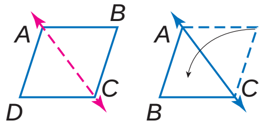
línea de reflexión Línea que divide una figura en dos imágenes especulares.

line of symmetry (p. 500) A line that can be drawn through a plane figure so that the figure on one side is the reflection image of the figure on the opposite side.



$\overleftrightarrow{AC}$ is a line of symmetry.

eje de simetría Recta que se traza a través de una figura plana, de modo que un lado de la figura es la imagen reflejada del lado opuesto.

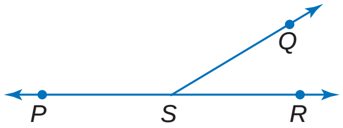


$\overleftrightarrow{AC}$ es un eje de simetría.

line segment (p. 13) A measurable part of a line that consists of two points, called endpoints, and all of the points between them.

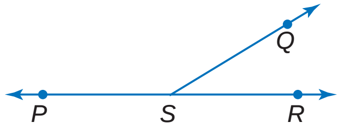
segmento de recta Sección medible de una recta. Consta de dos puntos, llamados extremos, y todos los puntos localizados entre ellos.

linear pair (p. 40) A pair of adjacent angles whose non-common sides are opposite rays.



$\angle PSQ$ and $\angle QSR$ are a linear pair.

par lineal Par de ángulos adyacentes cuyos lados no comunes forman semirrectas opuestas.



$\angle PSQ$ y $\angle QSR$ forman un par lineal.

locus (p. 11) The set of points that satisfy a given condition.

lugar geométrico Conjunto de puntos que satisfacen una condición dada.

logically equivalent (p. 93) Statements that have the same truth values.

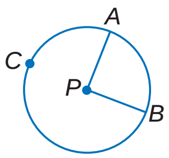
equivalente lógico Enunciados que poseen el mismo valor de verdad.

M

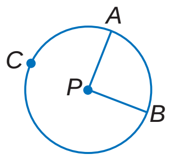
magnitude (p. 533) The length of a vector.

magnitud La longitud de un vector.

major arc (p. 563) An arc with a measure greater than 180.
$\overset{\frown}{ACB}$ is a major arc.



arco mayor Arco que mide más de 180.
$\overset{\frown}{ACB}$ es un arco mayor.



means (p. 381) In $\frac{a}{b} = \frac{c}{d}$, the numbers b and c.

medios Los números b y c en la proporción $\frac{a}{b} = \frac{c}{d}$.

median **1.** (p. 271) In a triangle, a line segment with endpoints that are a vertex of a triangle and the midpoint of the side opposite the vertex. **2.** (p. 358) In a trapezoid, the segment that joins the midpoints of the legs.

mediana **1.** Segmento de recta de un triángulo cuyos extremos son un vértice del triángulo y el punto medio del lado opuesto a dicho vértice. **2.** Segmento que une los puntos medios de los catetos de un trapecio.

midpoint (p. 22) The point on a segment exactly halfway between the endpoints of the segment.

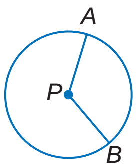
punto medio El punto en un segmento que yace exactamente entre los extremos del segmento.

midsegment (p. 406) A segment with endpoints that are the midpoints of two sides of a triangle.

segmento medio Segmento cuyos extremos son los puntos medios de dos lados de un triángulo.

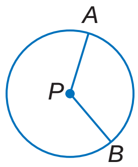
minor arc (p. 563) An arc with a measure less than 180.

$\overset{\frown}{AB}$ is a minor arc.



arco menor Arco que mide menos de 180.

$\overset{\frown}{AB}$ es un arco menor.



N

negation (p. 83) If a statement is represented by *p*, then *not p* is the negation of the statement.

negación Si *p* representa un enunciado, entonces *no p* representa la negación del enunciado.

net (p. 67) A two-dimensional figure that when folded forms the surfaces of a three-dimensional object.

red Figura bidimensional que al ser plegada forma las superficies de un objeto tridimensional.

***n*-gon** (p. 50) A polygon with *n* sides.

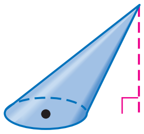
enágono Polígono con *n* lados.

non-Euclidean geometry (p. 188) The study of geometrical systems that are not in accordance with the Parallel Postulate of Euclidean geometry.

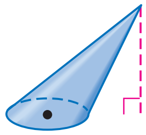
geometría no euclidiana El estudio de sistemas geométricos que no satisfacen el Postulado de las Paralelas de la geometría euclidiana.

O

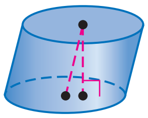
oblique cone (p. 706) A cone that is not a right cone.



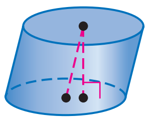
cono oblicuo Cono que no es un cono recto.



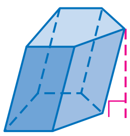
oblique cylinder (p. 692) A cylinder that is not a right cylinder.



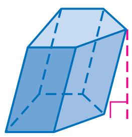
cilindro oblicuo Cilindro que no es un cilindro recto.



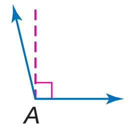
oblique prism (p. 692) A prism in which the lateral edges are not perpendicular to the bases.



prisma oblicuo Prisma cuyas aristas laterales no son perpendiculares a las bases.

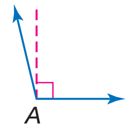


obtuse angle (p. 32) An angle with degree measure greater than 90 and less than 180.



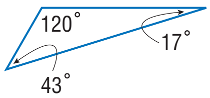
$90 < m\angle A < 180$

ángulo obtuso Ángulo que mide más de 90 y menos de 180.



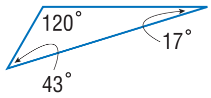
$90 < m\angle A < 180$

obtuse triangle (p. 202) A triangle with an obtuse angle.



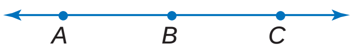
one obtuse angle

triángulo obtusángulo Triángulo que tiene un ángulo obtuso.

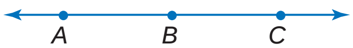


un ángulo obtuso

opposite rays (p. 31) Two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ such that B is between A and C.



semirrectas opuestas Dos semirrectas $\overrightarrow{BA}$ y $\overrightarrow{BC}$ tales que B se localiza entre A y C.



ordered triple (p. 758) Three numbers given in a specific order used to locate points in space.

triple ordenado Tres números dados en un orden específico que sirven para ubicar puntos en el espacio.

orthocenter (p. 272) The point of concurrency of the altitudes of a triangle.

ortocentro Punto de intersección de las alturas de un triángulo.

orthographic drawing (p. 67) The two-dimensional top view, left view, front view, and right view of a three-dimensional object.

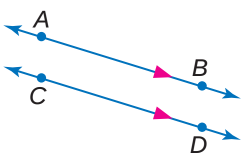
vista ortogonal Vista bidimensional desde arriba, desde la izquierda, desde el frente o desde la derecha de un cuerpo tridimensional.

P

paragraph proof (p. 106) An informal proof written in the form of a paragraph that explains why a conjecture for a given situation is true.

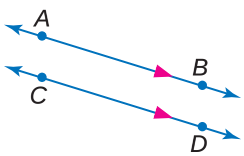
demostración de párrafo Demostración informal escrita en forma de párrafo que explica por qué una conjetura acerca de una situación dada es verdadera.

parallel lines (p. 142) Coplanar lines that do not intersect.



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

rectas paralelas Rectas coplanares que no se intersecan.



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

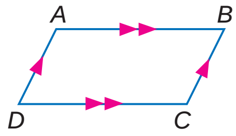
parallel planes (p. 142) Planes that do not intersect.

planos paralelos Planos que no se intersecan.

parallel vectors (p. 534) Vectors that have the same or opposite direction.

vectores paralelos Vectores que tienen la misma dirección o la dirección opuesta.

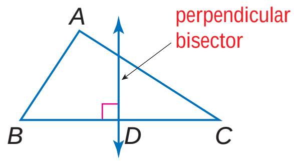
parallelogram (p. 327) A quadrilateral with parallel opposite sides. Any side of a parallelogram may be called a *base*.



$\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$

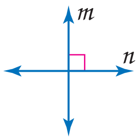
perimeter (p. 51) The sum of the lengths of the sides of a polygon.

perpendicular bisector (p. 267) In a triangle, a line, segment, or ray that passes through the midpoint of a side and is perpendicular to that side.



D is the midpoint of $\overline{BC}$.

perpendicular lines (p. 43) Lines that form right angles.



line $m \perp$ line n

perspective view (p. 680) The view of a three-dimensional figure from the corner.

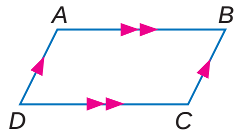
pi (π) (p. 557) An irrational number represented by the ratio of the circumference of a circle to the diameter of the circle.

plane (p. 6) A basic undefined term of geometry. A plane is a flat surface made up of points that has no depth and extends indefinitely in all directions. In a figure, a plane is often represented by a shaded, slanted four-sided figure. Planes are usually named by a capital script letter or by three noncollinear points on the plane.

plane Euclidean geometry (p. 188) Geometry based on Euclid's axioms dealing with a system of points, lines, and planes.

Platonic solids (p. 61) The five regular polyhedra: tetrahedron, hexahedron, octahedron, dodecahedron, or icosahedron.

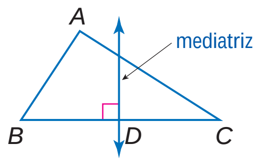
paralelogramo Cuadrilátero cuyos lados opuestos son paralelos entre sí. Cualquier lado del paralel-ogramo puede ser la *base*.



$\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$

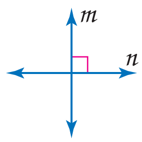
perímetro La suma de la longitud de los lados de un polígono.

mediatriz Recta, segmento o semirrecta que atraviesa el punto medio del lado de un triángulo y que es perpendicular a dicho lado.



D es el punto medio de $\overline{BC}$.

rectas perpendiculares Rectas que forman ángulos rectos.



recta $m \perp$ recta n

vista de perspectiva Vista de una figura tridimensional desde una de sus esquinas.

pi (π) Número irracional representado por la razón entre la circunferencia de un círculo y su diámetro.

plano Término primitivo en geometría. Es una superficie formada por puntos y sin profundidad que se extiende indefinidamente en todas direcciones. Los planos a menudo se representan con un cuadrilátero inclinado y sombreado. Los planos en general se designan con una letra mayúscula o con tres puntos no colineales del plano.

geometría del plano euclidiano Geometría basada en los axiomas de Euclides, los que integran un sistema de puntos, rectas y planos.

sólidos platónicos Cualquiera de los siguientes cinco poliedros regulares: tetraedro, hexaedro, octaedro, dodecaedro e icosaedro.

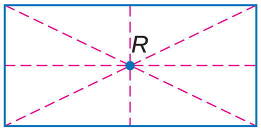
point (p. 6) A basic undefined term of geometry. A point is a location. In a figure, points are represented by a dot. Points are named by capital letters.

punto Término primitivo en geometría. Un punto representa un lugar o localización. En una figura, se representa con una marca puntual. Los puntos se designan con letras mayúsculas.

point of concurrency (p. 270) The point of intersection of concurrent lines.

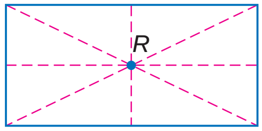
punto de concurrencia Punto de intersección de rectas concurrentes.

point of symmetry (p. 500) The common point of reflection for all points of a figure.



R is a point of symmetry.

punto de simetría El punto común de reflexión de todos los puntos de una figura.



R es un punto de simetría.

point of tangency (p. 587) For a line that intersects a circle in only one point, the point at which they intersect.

punto de tangencia Punto de intersección de una recta que interseca un círculo en un solo punto, el punto en donde se intersecan.

point-slope form (p. 166) An equation of the form $y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of any point on the line and m is the slope of the line.

forma punto-pendiente Ecuación de la forma $y - y_1 = m(x - x_1)$, donde (x_1, y_1) representan las coordenadas de un punto cualquiera sobre la recta y m representa la pendiente de la recta.

polygon (p. 49) A closed figure formed by a finite number of coplanar segments called *sides* such that the following conditions are met.

1. The sides that have a common endpoint are noncollinear.
2. Each side intersects exactly two other sides, but only at their endpoints, called the *vertices*.

polígono Figura cerrada formada por un número finito de segmentos coplanares llamados *lados*, y que satisface las siguientes condiciones:

1. Los lados que tienen un extremo común son no colineales.
2. Cada lado interseca exactamente dos lados, pero sólo en sus extremos, formando los *vértices*.

polyhedrons (p. 60) Closed three-dimensional figures made up of flat polygonal regions. The flat regions formed by the polygons and their interiors are called *faces*. Pairs of faces intersect in segments called *edges*. Points where three or more edges intersect are called *vertices*.

poliedro Figura tridimensional cerrada formada por regiones poligonales planas. Las regiones planas definidas por un polígono y sus interiores se llaman *caras*. Cada intersección entre dos caras se llama *arista*. Los puntos donde se intersecan tres o más aristas se llaman *vértices*.

postulate (p. 105) A statement that describes a fundamental relationship between the basic terms of geometry. Postulates are accepted as true without proof.

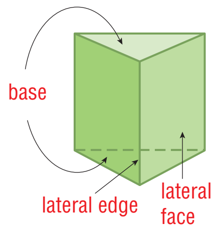
postulado Enunciado que describe una relación fundamental entre los términos primitivos de geometría. Los postulados se aceptan como verdaderos sin necesidad de demostración.

precision (p. 14) The precision of any measurement depends on the smallest unit available on the measuring tool.

precisión La precisión de una medida depende de la unidad de medida más pequeña del instrumento de medición.

prism (p. 60) A solid with the following characteristics.

1. Two faces, called *bases*, are formed by congruent polygons that lie in parallel planes.
2. The faces that are not bases, called *lateral faces*, are formed by parallelograms.
3. The intersections of two adjacent lateral faces are called *lateral edges* and are parallel segments.



triangular prism

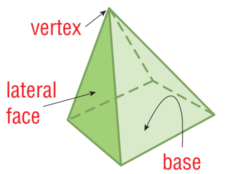
proof (p. 106) A logical argument in which each statement you make is supported by a statement that is accepted as true.

proof by contradiction (p. 288) An indirect proof in which one assumes that the statement to be proved is false. One then uses logical reasoning to deduce a statement that contradicts a postulate, theorem, or one of the assumptions. Once a contradiction is obtained, one concludes that the statement assumed false must in fact be true.

proportion (p. 381) An equation of the form $\frac{a}{b} = \frac{c}{d}$ that states that two ratios are equal.

pyramid (p. 60) A solid with the following characteristics.

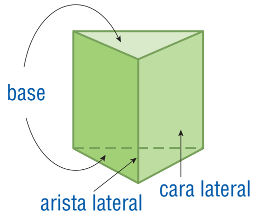
1. All of the faces, except one face, intersect at a point called the *vertex*.
2. The face that does not contain the vertex is called the *base* and is a polygonal region.
3. The faces meeting at the vertex are called *lateral faces* and are triangular regions.



rectangular pyramid

prisma Sólido que posee las siguientes características:

1. Tiene dos caras llamadas *bases*, formadas por polígonos congruentes que yacen en planos paralelos.
2. Las caras que no son las bases, llamadas *caras laterales*, son formadas por paralelogramos.
3. Las intersecciones de dos aristas laterales adyacentes se llaman *aristas laterales* y son segmentos paralelos.



prisma triangular

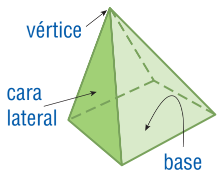
demostración Argumento lógico en que cada enunciado está basado en un enunciado que se acepta como verdadero.

demostración por contradicción Demostración indirecta en que se asume que el enunciado que se va a demostrar es falso. Después, se razona lógicamente para deducir un enunciado que contradiga un postulado, un teorema o una de las conjeturas. Una vez que se obtiene una contradicción, se concluye que el enunciado que se supuso falso es, en realidad, verdadero.

proporción Ecuación de la forma $\frac{a}{b} = \frac{c}{d}$ que establece que dos razones son iguales.

pirámide Sólido con las siguientes características:

1. Todas, excepto una de las caras, se intersecan en un punto llamado *vértice*.
2. La cara que no contiene el vértice se llama *base* y es una región poligonal.
3. Las caras que se encuentran en los vértices se llaman *caras laterales* y son regiones triangulares.



pirámide rectangular

Pythagorean triple (p. 443) A group of three whole numbers that satisfies the equation $a^2 + b^2 = c^2$, where c is the greatest number.

R

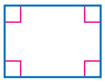
radius **1.** (p. 554) In a circle, any segment with endpoints that are the center of the circle and a point on the circle. **2.** (p. 711) In a sphere, any segment with endpoints that are the center and a point on the sphere.

rate of change (p. 157) Describes how a quantity is changing over time.

ratio (p. 380) A comparison of two quantities using division.

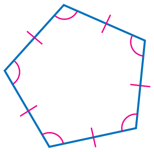
ray (p. 31) $\overrightarrow{PQ}$ is a ray if it is the set of points consisting of $\overline{PQ}$ and all points S for which Q is between P and S.

rectangle (p. 340) A quadrilateral with four right angles.



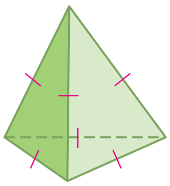
reflection (p. 597) A transformation representing a flip of the figure over a point, line, or plane.

regular polygon (p. 50) A convex polygon in which all of the sides are congruent and all of the angles are congruent.



regular pentagon

regular polyhedron (p. 60) A polyhedron in which all of the faces are regular congruent polygons.



regular prism (p. 60) A right prism with bases that are regular polygons.

triplete de Pitágoras Grupo de tres números enteros que satisfacen la ecuación $a^2 + b^2 = c^2$, donde c es el número más grande.

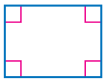
radio **1.** Cualquier segmento cuyos extremos están en el centro de un círculo y en un punto cualquiera del mismo. **2.** Cualquier segmento cuyos extremos forman el centro y en punto de una esfera.

tasa de cambio Describe cómo cambia una cantidad a través del tiempo.

razón Comparación de dos cantidades mediante división.

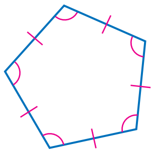
semirrecta $\overrightarrow{PQ}$ es una semirrecta si consta del conjunto de puntos formado por $\overline{PQ}$ y todos los puntos S para los que Q se localiza entre P y S.

rectángulo Cuadrilátero que tiene cuatro ángulos rectos.



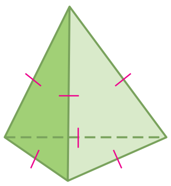
reflexión Transformación que se obtiene cuando se “voltea” una imagen sobre un punto, una línea o un plano.

polígono regular Polígono convexo en el que todos los lados y todos los ángulos son congruentes entre sí.



pentágono regular

poliedro regular Poliedro cuyas caras son polígonos regulares congruentes.



prisma regular Prisma recto cuyas bases son polígonos regulares.

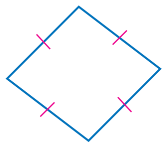
regular pyramid (p. 699) A pyramid with a base that is a regular polygon.

regular tessellation (p. 520) A tessellation formed by only one type of regular polygon.

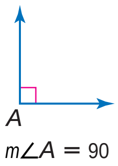
remote interior angles (p. 211) The angles of a triangle that are not adjacent to a given exterior angle.

resultant (p. 535) The sum of two vectors.

rhombus (p. 348) A quadrilateral with all four sides congruent.



right angle (p. 32) An angle with a degree measure of 90.

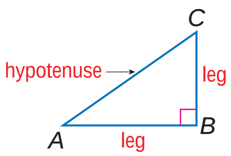


right cone (p. 706) A cone with an axis that is also an altitude.

right cylinder (p. 692) A cylinder with an axis that is also an altitude.

right prism (p. 692) A prism with lateral edges that are also altitudes.

right triangle (p. 202) A triangle with a right angle. The side opposite the right angle is called the *hypotenuse*. The other two sides are called *legs*.



rotation (p. 510) A transformation that turns every point of a preimage through a specified angle and direction about a fixed point, called the *center of rotation*.

rotational symmetry (p. 573) If a figure can be rotated less than 360° about a point so that the image and the preimage are indistinguishable, the figure has rotational symmetry.

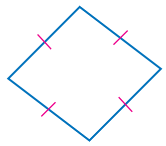
pirámide regular Pirámide cuya base es un polígono regular.

teselado regular Teselado formado por un solo tipo de polígono regular.

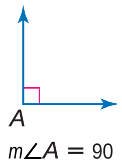
ángulos internos no adyacentes Ángulos de un triángulo que no son adyacentes a un ángulo exterior dado.

resultante La suma de dos vectores.

rombo Cuadrilátero cuyos cuatro lados son congruentes.



ángulo recto Ángulo cuya medida en grados es 90.

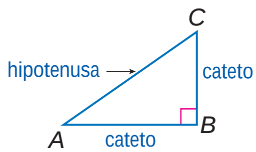


cono recto Cono cuyo eje es también su altura.

cilindro recto Cilindro cuyo eje es también su altura.

prisma recto Prisma cuyas aristas laterales también son su altura.

triángulo rectángulo Triángulo con un ángulo recto. El lado opuesto al ángulo recto se conoce como *hipotenusa*. Los otros dos lados se llaman catetos.



rotación Transformación en que se hace girar cada punto de la preimagen a través de un ángulo y una dirección determinadas alrededor de un punto, conocido como *centro de rotación*.

simetría de rotación Si se puede rotar una imagen menos de 360° alrededor de un punto y la imagen y la preimagen son idénticas, entonces la figura presenta simetría de rotación.

scalar (p. 536) A constant multiplied by a vector.

escalar Una constante multiplicada por un vector.

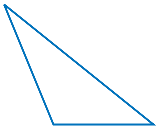
scalar multiplication (p. 536) Multiplication of a vector by a scalar.

multiplicación escalar Multiplicación de un vector por una escalar.

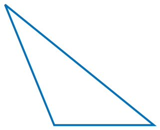
scale factor (p. 389) The ratio of the lengths of two corresponding sides of two similar polygons or two similar solids.

factor de escala La razón entre las longitudes de dos lados correspondientes de dos polígonos o sólidos semejantes.

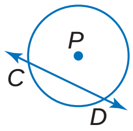
scalene triangle (p. 203) A triangle with no two sides congruent.



triángulo escaleno Triángulo cuyos lados no son congruentes.

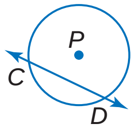


secant (p. 598) Any line that intersects a circle in exactly two points.



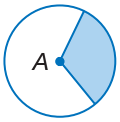
$\overleftrightarrow{CD}$ is a secant of $\odot P$.

secante Cualquier recta que interseca un círculo exactamente en dos puntos.



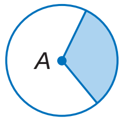
$\overleftrightarrow{CD}$ es una secante de $\odot P$.

sector of a circle (p. 666) A region of a circle bounded by a central angle and its intercepted arc.



The shaded region is a sector of $\odot A$.

sector de un círculo Región de un círculo que está limitada por un ángulo central y el arco que interseca.



La región sombreada es un sector de $\odot A$.

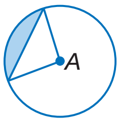
segment (p. 13) See *line segment*.

segmento Ver *segmento de recta*.

segment bisector (p. 25) A segment, line, or plane that intersects a segment at its midpoint.

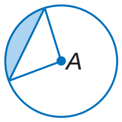
bisectriz de segmento Segmento, recta o plano que interseca un segmento en su punto medio.

segment of a circle (p. 667) The region of a circle bounded by an arc and a chord.



The shaded region is a segment of $\odot A$.

segmento de un círculo Región de un círculo limitada por un arco y una cuerda.



La región sombreada un segmento de $\odot A$.

self-similar (p. 423) If any parts of a fractal image are replicas of the entire image, the image is self-similar.

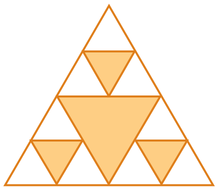
autosemejante Si cualquier parte de una imagen fractal es una réplica de la imagen completa, entonces la imagen es autosemejante.

semicircle (p. 563) An arc that measures 180.

semi-regular tessellation (p. 520) A uniform tessellation formed using two or more regular polygons.

sides of the angle (p. 31) The rays of an angle.

Sierpinski Triangle (p. 423) A self-similar fractal described by Waclaw Sierpinski. The figure was named for him.



similar solids (p. 750) Solids that have exactly the same shape, but not necessarily the same size.

similarity transformation (p. 526) When a figure and its transformation image are similar.

sine (p. 456) For an acute angle of a right triangle, the ratio of the measure of the leg opposite the acute angle to the measure of the hypotenuse.

skew lines (p. 143) Lines that do not intersect and are not coplanar.

slant height (p. 699) The height of the lateral side of a pyramid or cone.

slope (p. 156) For a (nonvertical) line containing two points (x_1, y_1) and (x_2, y_2), the number m given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ where $x_2 \neq x_1$.

slope-intercept form (p. 165) A linear equation of the form $y = mx + b$. The graph of such an equation has slope m and y-intercept b.

solving a triangle (p. 472) Finding the measures of all of the angles and sides of a triangle.

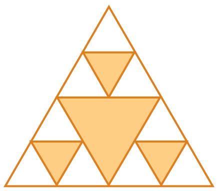
space (p. 8) A boundless three-dimensional set of all points.

semicírculo Arco que mide 180.

teselado semirregular Teselado uniforme compuesto por dos o más polígonos regulares.

lados de un ángulo Los rayos de un ángulo.

triángulo de Sierpinski Fractal descubierto por el matemático Waclaw Sierpinski. La figura se nombró en su honor.



sólidos semejantes Sólidos que tienen exactamente la misma forma, pero no necesariamente el mismo tamaño.

transformación de semejanza Aquélla en que la figura y su imagen transformada son semejantes.

seno Es la razón entre la medida del cateto opuesto al ángulo agudo y la medida de la hipotenusa de un triángulo rectángulo.

rectas alabeadas Rectas que no se intersecan y que no son coplanares.

altura oblicua La altura de la cara lateral de una pirámide o un cono.

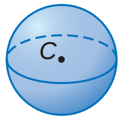
pendiente Para una recta (no vertical) que contiene dos puntos (x_1, y_1) y (x_2, y_2), el número m dado por la fórmula $m = \frac{y_2 - y_1}{x_2 - x_1}$ donde $x_2 \neq x_1$.

forma pendiente-intersección Ecuación lineal de la forma $y = mx + b$. En la gráfica de tal ecuación, la pendiente es m y la intersección y es b.

resolver un triángulo Calcular las medidas de todos los ángulos y todos los lados de un triángulo.

espacio Conjunto tridimensional no acotado de todos los puntos.

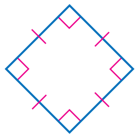
sphere (p. 61) In space, the set of all points that are a given distance from a given point, called the *center*.



C is the center of the sphere.

spherical geometry (p. 188) The branch of geometry that deals with a system of points, great circles (lines), and spheres (planes).

square (p. 349) A quadrilateral with four right angles and four congruent sides.



standard position (p. 533) When the initial point of a vector is at the origin.

statement (p. 83) Any sentence that is either true or false, but not both.

supplementary angles (p. 42) Two angles with measures that have a sum of 180.

surface area (p. 62) The sum of the areas of all faces and side surfaces of a three-dimensional figure.

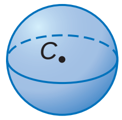
T

tangent **1.** (p. 456) For an acute angle of a right triangle, the ratio of the measure of the leg opposite the acute angle to the measure of the leg adjacent to the acute angle. **2.** (p. 587) A line in the plane of a circle that intersects the circle in exactly one point. The point of intersection is called the *point of tangency*. **3.** (p. 711) A line that intersects a sphere in exactly one point.

tessellation (p. 519) A pattern that covers a plane by transforming the same figure or set of figures so that there are no overlapping or empty spaces.

theorem (p. 106) A statement or conjecture that can be proven true by undefined terms, definitions, and postulates.

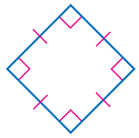
esfera El conjunto de todos los puntos en el espacio que se encuentran a cierta distancia de un punto dado llamado *centro*.



C es el centro de la esfera.

geometría esférica Rama de la geometría que estudia los sistemas de puntos, círculos máximos (rectas) y esferas (planos).

cuadrado Cuadrilátero con cuatro ángulos rectos y cuatro lados congruentes.



posición estándar Ocurre cuando la posición inicial de un vector es el origen.

enunciado Una oración que puede ser falsa o verdadera, pero no ambas.

ángulos suplementarios Dos ángulos cuya suma es igual a 180.

área de superficie La suma de las áreas de todas las caras y superficies laterales de una figura tridimensional.

tangente **1.** La razón entre la medida del cateto opuesto al ángulo agudo y la medida del cateto adyacente al ángulo agudo de un triángulo rectángulo. **2.** La recta situada en el mismo plano de un círculo y que interseca dicho círculo en un sólo punto. El punto de intersección se conoce como *punto de tangencia*. **3.** Recta que interseca una esfera en un sólo punto.

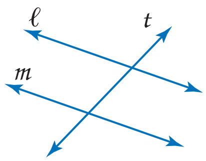
teselado Patrón que cubre un plano y que se obtiene transformando la misma figura o conjunto de figuras, sin que haya traslapes ni espacios vacíos.

teorema Enunciado o conjetura que se puede demostrar como verdadera mediante el uso de términos primitivos, definiciones y postulados.

transformation (p. 496) In a plane, a mapping for which each point has exactly one image point and each image point has exactly one preimage point.

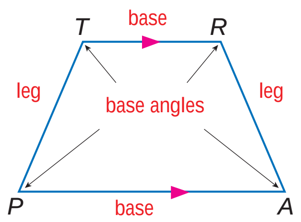
translation (p. 504) A transformation that moves all points of a figure the same distance in the same direction.

transversal (p. 143) A line that intersects two or more lines in a plane at different points.



Line t is a transversal.

trapezoid (p. 356) A quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called *bases*. The nonparallel sides are called *legs*. The pairs of angles with their vertices at the endpoints of the same base are called *base angles*.



trigonometric ratio (p. 456) A ratio of the lengths of sides of a right triangle.

trigonometry (p. 456) The study of the properties of triangles and trigonometric functions and their applications.

truth table (p. 86) A table used as a convenient method for organizing the truth values of statements.

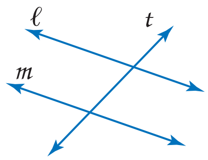
truth value (p. 83) The truth or falsity of a statement.

two-column proof (p. 112) A formal proof that contains statements and reasons organized in two columns. Each step is called a *statement*, and the properties that justify each step are called *reasons*.

transformación La relación en el plano en que cada punto tiene un único punto imagen y cada punto imagen tiene un único punto preimagen.

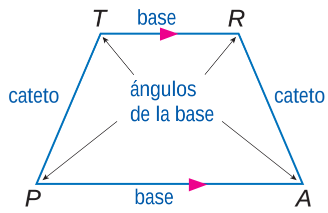
traslación Transformación en que todos los puntos de una figura se trasladan la misma distancia, en la misma dirección.

transversal Recta que interseca en diferentes puntos dos o más rectas en el mismo plano.



La recta t es una transversal.

trapecio Cuadrilátero con un sólo par de lados paralelos. Los lados paralelos del trapecio se llaman *bases*. Los lados no paralelos se llaman *catetos*. Los ángulos cuyos vértices se encuentran en los extremos de la misma base se llaman *ángulos de la base*.



razón trigonométrica Razón de las longitudes de los lados de un triángulo rectángulo.

trigonometría Estudio de las propiedades de los triángulos y de las funciones trigonométricas y sus aplicaciones.

tabla verdadera Tabla que se utiliza para organizar de una manera conveniente los valores de verdad de los enunciados.

valor verdadero La condición de un enunciado de ser verdadero o falso.

demostración a dos columnas Aquélla que contiene enunciados y razones organizadas en dos columnas. Cada paso se llama *enunciado* y las propiedades que lo justifican son las *razones*.

U

undefined terms (p. 6) Words, usually readily understood, that are not formally explained by means of more basic words and concepts. The basic undefined terms of geometry are point, line, and plane.

términos primitivos Palabras que por lo general se entienden fácilmente y que no se explican formalmente mediante palabras o conceptos más básicos. Los términos básicos primitivos de la geometría son el punto, la recta y el plano.

uniform tessellations (p. 520) Tessellations containing the same arrangement of shapes and angles at each vertex.

teselado uniforme Teselados que contienen el mismo patrón de formas y ángulos en cada vértice.

V

vector (p. 533) A directed segment representing a quantity that has both magnitude, or length, and direction.

vector Segmento dirigido que representa una cantidad que posee tanto magnitud, o longitud, como dirección.

vertex angle of an isosceles triangle (p. 244) See *isosceles triangle*.

ángulo vértice de un triángulo isosceles Ver *triángulo isósceles*.

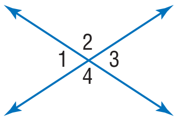
vertex of an angle (p. 31) The common endpoint of an angle.

vértice de un ángulo Extremo común de los lados de un ángulo.

vertex of a polyhedron (p. 60) The intersection of three edges of a polyhedron.

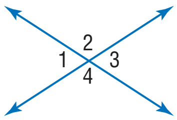
vértice de un poliedro Intersección de las aristas de un poliedro.

vertical angles (p. 40) Two nonadjacent angles formed by two intersecting lines.



$\angle 1$ and $\angle 3$ are vertical angles.
$\angle 2$ and $\angle 4$ are vertical angles.

ángulos opuestos por el vértice Dos ángulos no adyacentes formados por dos rectas que se intersecan.



$\angle 1$ y $\angle 3$ son ángulos opuestos por el vértice.
$\angle 2$ y $\angle 4$ son ángulos opuestos por el vértice.

volume (p. 62) A measure of the amount of space enclosed by a three-dimensional figure.

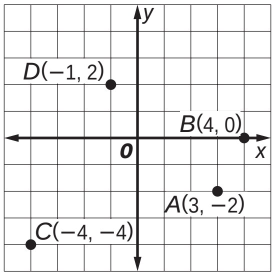
volumen La medida de la cantidad de espacio dentro de una figura tridimensional.

Selected Answers

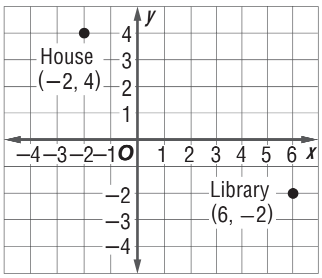
Chapter 1 Tools of Geometry

Page 5 *Chapter 1* *Get Ready*

1–4.



5.

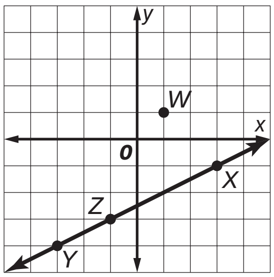


7. $7\frac{7}{16}$
9. $2\frac{1}{16}$
11. 5
13. 1444
15. 25

Pages 9–11 *Lesson 1-1*

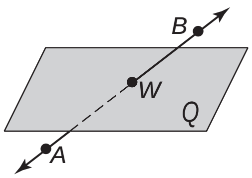
1. Sample answer: line *p* **3.** line

5.

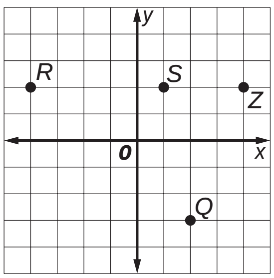


7. 6 **9.** No; *A*, *C*, and *J* lie in plane *ABC*, but *D* does not. **11.** *F* **13.** *W* **15.** Yes; it intersects both *m* and *n* when all three lines are extended. **17.** lines
19. plane **21.** intersecting lines

23.

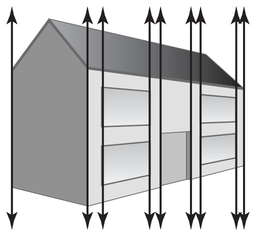


25.

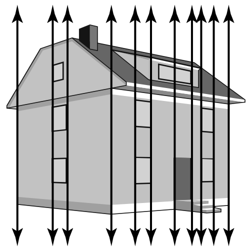


27. 5 **29.** *E*, *F*, *C* **31.** (C, 5) **33.** Cameron

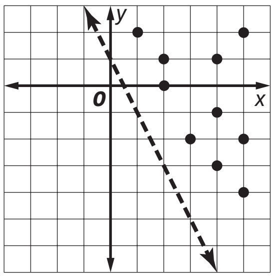
35.



39. Sample answer:



41. vertical
43. part of the coordinate plane above the line $y = -2x + 1$

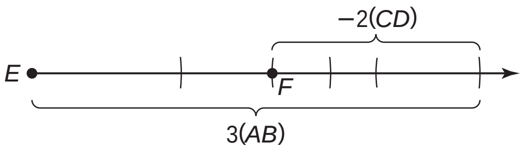


45. Micha; the points must be noncollinear to determine a plane. **47.** Sample answer: Chairs wobble because all four legs do not touch the floor at the same time. The ends of the legs represent points. If all points lie in the same plane, the chair will not wobble. Because it only takes three points to determine a plane, a chair with three legs will never wobble. **49.** H
51. > **53.** > **55.** <

Pages 17–20 *Lesson 1-2*

1. 1.3 cm or 13mm **3.** $1\frac{3}{4}$ in. **5.** 0.5 m; 14 m could be 13.5 to 14.5 m. **7.** 3.7 cm **9.** $x = 3$; $LM = 9$ **11.** $\overline{BC} \cong \overline{CD}$, $\overline{BE} \cong \overline{ED}$, $\overline{BA} \cong \overline{DA}$ **13.** 3.8 cm or 38 mm **15.** $1\frac{1}{4}$ in.
17. 0.5 mm; 21.5 to 22.5 mm **19.** 0.5 cm; 307.5 to 308.5 cm
21. $\frac{1}{8}$ ft; $3\frac{1}{8}$ to $3\frac{3}{8}$ ft **23.** $1\frac{1}{4}$ in. **25.** 2.8 cm **27.** $1\frac{1}{4}$ in.
29. $x = 11$; $ST = 22$ **31.** $x = 2$; $ST = 4$ **33.** $y = 2$; $ST = 3$
35. no **37.** yes **39.** yes **41.** 12 cm **43.** 12.5 cm; Each measurement is accurate within 0.5 cm, so the least perimeter is 2.5 cm + 4.5 cm + 5.5 cm. **45.** 50,000 visitors **47.** No; the number of visitors to Washington state parks could be as low as 46.35 million or as high as 46.45 million. The visitors to Illinois state parks could be as low as 44.45 million or as high as 44.55 million visitors. The difference in visitors could be as high as 2.0 million. **49.** 1.7% **51.** 0.08%

53.

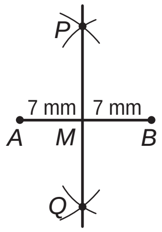


55. Sample answers: rectangle, square, equilateral triangle **57.** 5 **59.** Units of measure are used to differentiate between size and distance, as well as for precision. An advantage is that the standard of measure for a cubit is always available. A disadvantage is that a cubit would vary in length depending on whose arm was measured. **61.** J **63.** Sample answer: planes *ABC* and *BCD* **65.** 5 **67.** line **69.** 22 **71.** $\frac{1}{2}$

Pages 25–29 ***Lesson 1-3***

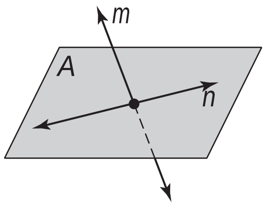
1. 8 **3.** 10 **5.** -6 **7.** $(-2.5, 4)$ **9.** $(3, 5)$ **11.** 2 **13.** 3 **15.** 11 **17.** 10 **19.** 13 **21.** $\sqrt{89} \approx 9.4$ **23.** 15 **25.** $\sqrt{90} \approx 9.5$ **27.** $\sqrt{61} \approx 7.8$ **29.** -3 **31.** 2.5 **33.** 1 **35.** $\left(-\frac{1}{2}, \frac{1}{2}\right)$ **37.** $(10, 3)$ **39.** $(-10, -3)$ **41.** $(5.6, 2.85)$ **43.** $R(2, 7)$ **45.** $T\left(\frac{8}{3}, 11\right)$ **47.** Norris Corner, MA **49.** Collinsville, CT **51.** Sample answer: =SQRT((A2 - C2)^2+(B2 - D2)^2) **53.** 212.0 **55.** 420.3

57. Sample answer:

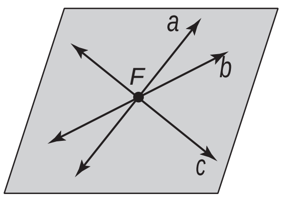


59. $(-1, -3)$
61. C
63. D
65. 5.5 cm

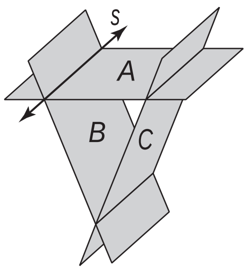
67.



69.



71.

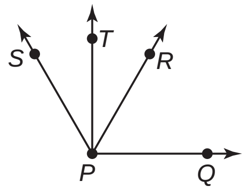


73. 19.5 **75.** 5 **77.** 6.25

Pages 35–38 ***Lesson 1-4***

1. *C* **3.** $\angle CDB$, $\angle 1$ **5.** 45°, acute **7.** 47 **9.** *E* **11.** *A* **13.** $\overrightarrow{DA}$, $\overrightarrow{DB}$ **15.** $\overrightarrow{ED}$, $\overrightarrow{EG}$ **17.** $\angle ABC$, $\angle CBA$ **19.** $\angle 2$, $\angle DBA$, $\angle EBA$, $\angle ABE$, $\angle FBA$, $\angle ABF$ **21.** *D*, *H* **23.** Sample answer: $\angle 4$, $\angle 3$ **25.** 90°, right **27.** 30°, acute **29.** 150°, obtuse **31.** 30 **33.** 6 **35.** 35 **37.** 60, 30, 90, 60, 120, 60

39. Sample answer:



$m\angle QPR = 60$; $m\angle QPT = 90$; $m\angle QPS = 120$ **41.** 1, 3, 6, 10, 15 **43.** 21, 45 **45.** You can only compare the measures of the angles. The arcs indicate both measures are the same regardless of the length of the rays. **47.** D **49.** 5; (3.5, 5) **51.** $\sqrt{164} \approx 12.8$; (1, 3) **53.** 11.4 mm **55.** 5 **57.** *G* or *L* **59.** 75 **61.** 12 **63.** 12

Pages 45–47 ***Lesson 1-5***

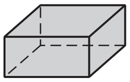
1. Sample answer: $\angle ABF$, $\angle CBD$ **3.** $m\angle 1 = 120$, $m\angle 2 = 60$, $m\angle 3 = 120$ **5.** 148 **7.** Yes; they share a common side and vertex, so they are adjacent. Since $\overline{PR}$ falls between $\overline{PQ}$ and $\overline{PS}$, $m\angle QPR < 90$, so the two angles cannot be complementary or supplementary. **9.** $\angle PHG$, $\angle THQ$ **11.** $\angle TPH$, $\angle QRG$ **13.** $\angle HTP$ **15.** 15 **17.** 112, 68 **19.** 143, 37 **21.** 4 **23.** Yes; the symbol denotes that $\angle DAB$ is a right angle. **25.** Yes, they are vertical angles. **27.** Yes; the sum of their measures is $m\angle ADC$, which is 90°. **29.** always **31.** sometimes **33.** 53, 37 **35.** $\ell \perp \overleftrightarrow{AB}$, $m \perp \overleftrightarrow{AB}$, $n \perp \overleftrightarrow{AB}$ **37.** Sample answer: When two angles form a linear pair, then their noncommon sides form a straight angle, which measures 180. When the sum of the measures of two angles is 180, then the angles are supplementary. **39.** Because $\angle WUT$ and $\angle TUV$ are supplementary, let $m\angle WUT = x$ and $m\angle TUV = 180 - x$. A bisector creates measures that are half of the original angle, so $m\angle YUT = \frac{1}{2}m\angle WUT$ or $\frac{x}{2}$ and $m\angle TUZ = \frac{1}{2}m\angle TUV$ or $\frac{180 - x}{2}$. Then $m\angle YUZ = m\angle YUT + m\angle TUZ$ or $\frac{x}{2} + \frac{180 - x}{2}$. This sum simplifies to $\frac{180}{2}$ or 90. Because $m\angle YUZ = 90$, $\overline{YU} \perp \overline{UZ}$. **41.** D **43.** The angle at which the dogs must turn to get the scent of the article they wish to find is an acute angle. **45.** 8 **47.** $\sqrt{173} \approx 13.2$ **49.** $\sqrt{20} \approx 4.5$ **51.** $n = 3$, $QR = 20$ **53.** 24 **55.** 40

Pages 54–57 ***Lesson 1-6***

1. pentagon; concave; irregular **3.** 44 ft; 121 ft^2 **5.** C **7.** quadrilateral; convex; irregular **9.** pentagon **11.** dodecagon **13.** ≈25.1 in.; ≈50.3 in^2 **15.** ≈ 36.4 cm; ≈ 105.7 cm^2 **17.** 42.2 km; 108.6 km^2 **19.** 7.85 ft **21.** $P = 11 + \sqrt{61}$ or about 18.8 units; $A = 15$ units^2 **23.** $P = 2\sqrt{32} + 2\sqrt{18}$ or about 19.8 units; $A = 24$ units^2 **25.** 13 ft **27.** It doubles to 26 ft; It quadruples to 40 ft^2. **29.** 12.25 yd^2 **31.** 24 in. **33.** 10π or ≈31.4 units **35.** 24 in. **37.** All are 15 cm. **39.** 13 units, 13 units, 5 units **41.** 21 km **43.** The perimeter is doubled to 42 km. **45.** ≈ 36.9 **47.** ≈ 110.7 **49.** Divide the perimeter by 10. **51.** It is a square with side length of 3 units. **53.** Circle; the other shapes are polygons. **55.** D **57.** always **59.** 58 **61.** 68 **63.** 164 **65.** 673.5

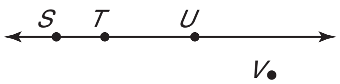
Pages 63–66 ***Lesson 1-7***

1. hexagonal pyramid; base: *ABCDEF*, faces: *ABCDEF*, $\triangle AGF$, $\triangle FGE$, $\triangle EGD$, $\triangle DGC$, $\triangle CGB$, $\triangle BGA$; edges: $\overline{AF}$, $\overline{FE}$, $\overline{ED}$, $\overline{DC}$, $\overline{CB}$, $\overline{BA}$, $\overline{AG}$, $\overline{FG}$, $\overline{EG}$, $\overline{DG}$, $\overline{CG}$, and $\overline{BC}$; vertices: *A*, *B*, *C*, *D*, *E*, *F*, and *G* **3.** 66 units^2 **5.** $\approx$ 27.2 in^3 **7.** rectangular pyramid; base: $\square DEFG$; faces: $\square DEFG$, $\triangle DHG$, $\triangle GHF$, $\triangle FHE$, $\triangle DHE$; edges: $\overline{DG}$, $\overline{GF}$, $\overline{FE}$, $\overline{ED}$, $\overline{DH}$, $\overline{EH}$, $\overline{FH}$, $\overline{GH}$; vertices: *D*, *E*, *F*, *G*, and *H* **9.** cylinder; bases: circles *S* and *T* **11.** cone; base: circle *B*; vertex: *A* **13.** 121.5 units^2; 91.125 units^3 **15.** 800 units^2; 1280 units^3 **17.** 2217.1 in^3 **19.** 27.5 ft^3 **21.** 9 cm **23.** The volume of the original prism is 4752 cubic centimeters. The volume of the new prism is 38,016 cubic centimeters. The volume increased by a factor of 8 when each dimension was doubled. **25.** Yes, there is a pattern. The number of sides of the base of a prism is 2 less than the number of faces in the polyhedron. The number of sides of the base of a pyramid is 1 less than the number of faces.

29. Sample answer: 

31. cone **33.** Sample answer: Classifying the pyramids of Egypt as square or rectangular pyramids allows you to find the surface area or volume of the structures. **35.** F **37.** $P = 39.8$ in., $A = 65.94$ in^2 **39.** $P = 24.6$ ft, $A = 37.1$ ft^2 **41.** 37°, 143° **43.** 7; 14 **45.** 5; 30 **47.** 5; 3 **49.** Charlotte

Pages 68–72 ***Chapter 1*** ***Study Guide and Review***

1. true **3.** false, is congruent to **5.** false; line segment **7.** Sample answer: 

9. $x = 6$; $PB = 18$ **11.** $k = -5$; $PB = 1$ **13.** not enough information **15.** $\sqrt{20} \approx 4.5$ **17.** $\sqrt{17} \approx 4.1$ **19.** (3, -5) **21.** (1, -1) **23.** $\overrightarrow{EF}$ and $\overrightarrow{EH}$ **25.** $\angle J$, $\angle H$, and $\angle K$ are right; $\angle F$ and $\angle G$ are obtuse. **27.** $\angle YWT$ **29.** 70° left turn **31.** not a polygon **33.** pentagonal prism; bases: pentagons *RBCJH* and *KYDEG*; faces: pentagon *RBCJH*, pentagon *KYDEG*, quadrilaterals *GHJE*, *EJCD*, *YBCD*, *KRBY*, *KRGH*; edges: $\overline{KR}$, $\overline{RH}$, $\overline{GH}$, $\overline{GK}$, $\overline{KY}$, $\overline{YB}$, $\overline{RB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DY}$, $\overline{CJ}$, $\overline{EJ}$, $\overline{ED}$, $\overline{JH}$, $\overline{EG}$; vertices: *K*, *R*, *B*, *H*, *G*, *Y*, *J*, *C*, *E*, *D* **35.** 238 ft^2 **37.** $\approx$ 410.7 cm^3 **39.** Company B

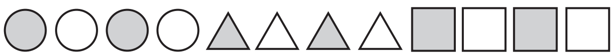
Chapter 2 Reasoning and Proof

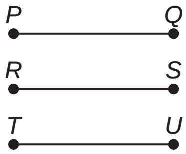
Page 77 ***Chapter 2*** ***Get Ready***

1. 10 **3.** 0 **5.** 50 **7.** $3 + n^2$ **9.** 21 **11.** -9 **13.** $-\frac{18}{5}$ **15.** $3x = 24$; \$8 **17.** Sample answer: $\angle AGC$ and $\angle CGE$ **19.** 6

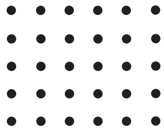
Pages 80–82 ***Lesson 2-1***

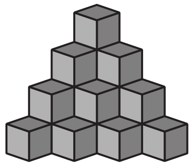
1.

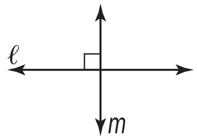


3. $PQ = TU$ 

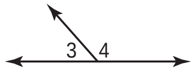
5. In California 31% of the total anglers are youth.

7.  **9.** 32 **11.** $\frac{11}{3}$ **13.** 162

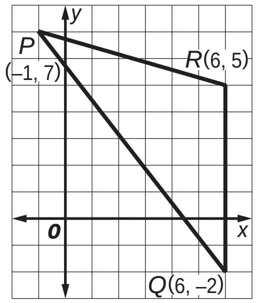
15. 

17. 

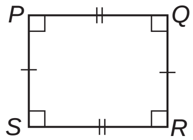
Lines ℓ and *m* form four right angles.

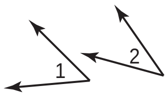
19. 

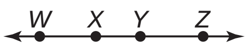
$\angle 3$ and $\angle 4$ are supplementary.

21. 

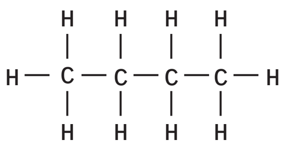
$\triangle PQR$ is a scalene triangle.

23. Sample answer: 

$PQ = SR$, $QR = PS$ **25.** false 

27. false 

29. true

31. Sample answer: Snow will not stick on a roof with a steep angle.
33. Butane will have 4 carbon atoms and 10 hydrogen atoms. 

35. C_nH_{2n+2} **37.** Sample answer: When it is cloudy, it rains. Counterexample: It is often cloudy and it does not rain. **39.** Sample answer: In the ancient Orient, math teachers presented several similar problems to their students. The students then used inductive reasoning to determine a rule. Current teaching methods vary. **41.** G **43.** hexagon, convex, not regular **45.** heptagon, concave, not regular **47.** 13, 14

Pages 87–90 ***Lesson 2-2***

1. False; 9 + 5 = 14 and February has 30 days. **3.** False; February has 30 days and a square has four sides. **5.** True; February has 30 days or a square has four sides. **7.** 14 **9.** 3

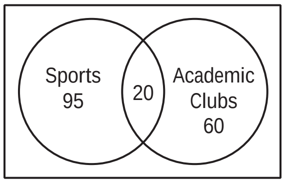
11.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

13. False; $\sqrt{-64} = 8$ and a triangle has three sides. **15.** False; $\sqrt{-64} = 8$ and $0 \geq 0$. **17.** True; a triangle has three sides or $0 < 0$. **19.** False; $\sqrt{-64} = 8$ and an obtuse angle measures greater than 90° and less than 180°. **21.** False; $0 < 0$ or $\sqrt{-64} = 8$ **23.** True; $\sqrt{-64} \neq 8$ and a triangle has three sides, or an obtuse angle measures greater than 90° and less than 180°.

25. 42 **27.** 25 **29.**

Level of Participation Among 310 Students



31. 135

33.

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

35.

p	q	p or q
T	T	T
T	F	T
F	T	T
F	F	F

37.

p	q	p and q
T	T	T
T	F	F
F	T	F
F	F	F

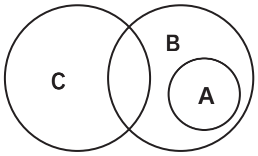
39.

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

41.

p	q	r	$\sim q$	$\sim r$	$\sim q \vee \sim r$	$p \wedge (\sim q \vee \sim r)$
T	T	T	F	F	F	F
T	T	F	F	T	T	T
T	F	T	T	F	T	T
T	F	F	T	T	T	T
F	T	T	F	F	F	F
F	T	F	F	T	T	F
F	F	T	T	F	T	F
F	F	F	T	T	T	F

43. Sample answer: A client visited England and France. **45.** true **47.** false **49.** Sample answer: A square has five right angles and the Postal Service does not deliver mail on Sundays. **51.**



53. Sample answer: Logic can be used to eliminate choices on a multiple choice problem. A conjunction is the intersection of two statements. It is true when both statements are true. A disjunction is the union of two statements. It is true if either statement is true or if both statements are true. **55.** H **57.** 81 **59.** 1 **61.** 405 **63.** 33.1 **65.** 30.4 **67.** 145°, obtuse **69.** 90°, right **71.** 14 **73.** -10

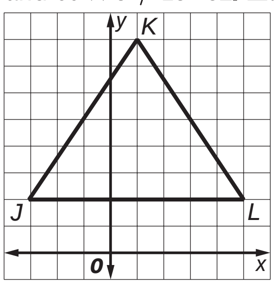
Pages 94–97 ***Lesson 2-3***

1. H: it rains on Monday; C: I will stay home **3.** If a pitcher is a 32-ounce pitcher, then it holds a quart of liquid. **5.** If you are in Colorado, then aspen trees cover high areas of the mountains. If you are in Florida, then cypress trees rise from the swamps. If you are in Vermont, then maple trees are prevalent. **7.** true **9.** Converse: If plants grow, then they have water; true. Inverse: If plants do not have water, then they will not grow; true. Contrapositive: If plants do not grow, then they do not have water. False; they may have been killed by overwatering.

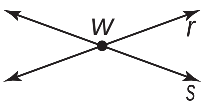
11. H: you are a teenager; C: you are at least 13 years old **13.** H: $2x + 6 = 10$, C: $x = 2$ **15.** H: there is no struggle; C: there is no progress **17.** H: a quadrilateral has four congruent sides; C: it is a square **19.** If you buy a 1-year gym membership, then you get a free water bottle. **21.** If I think, then I am. **23.** If two angles are vertical, then they are congruent. **25.** If you listen to jazz music, then you will hear trumpet or saxophone. If you listen to rock music, then you will hear guitar and drums. If you listen to hip-hop music, then you will hear the bass. **27.** true **29.** true **31.** true **33.** true **35.** false **37.** Converse: If you live in Texas, then you live in Dallas. False; you could live in Austin. Inverse: If you don't live in Dallas, then you don't live in Texas. False; you could live in Austin. Contrapositive: If you don't live in Texas, then you don't live in Dallas; true. **39.** Converse: If the sum of two angles is 90, then they are complementary; true. Inverse: If two angles are not complementary, then their sum is not 90; true. Contrapositive: If the sum of two angles is not 90, then they are not complementary; true. **41.** Converse: If an angle has a measure of 90, then it is a right angle; true. Inverse: If an angle is not a right angle, then its measure is not 90; true. Contrapositive: If an angle does not have a measure of 90, then it is not a right angle; true. **43.** Sample answer: In Alaska, if it is summer, then there are more hours of daylight than darkness. In Alaska, if it is winter, then there are more hours of darkness than daylight. **45.** Sample answer: If you eat your peas, then you will have dessert. **47.** Sample answer: If you live in Chicago then you live in Texas. Yes; If you live in Chicago then you do not live in Texas. **49.** A
51. Sample answer:

Trial	A	B	C
1	4	9	5
2	6	3	4
3	4	8	5
4	8	2	5
5	7	4	5

53. If A < 2 or B $\neq$ 3, then C = 5. **55.** False; a hexagon has five sides or $60 \times 3 = 18$. **57.** True; a hexagon doesn't have five sides or $60 \times 3 = 18$. **59.** False; George Washington was not the first president of the United States and $60 \times 3 \neq 18$. **61.** $\triangle JKL$ has two sides congruent.



63. 14 cm **65.** It doubles to 28 cm. **67.** $\sqrt{20}$ or 4.5 **69.** $\sqrt{29}$ or 5.4 **71.**



73. Subtract 4 from each side. **75.** Divide each side by 8.

Pages 101–104 **Lesson 2-4**

1. valid **3.** no conclusion **5.** valid; Law of Syllogism **7.** $14.35 **9.** Invalid; 10 + 12 = 22. **11.** Valid; since 11 and 23 are odd, the Law of Detachment indicates that their sum is even. **13.** Valid; *A*, *B*, and *C* are noncollinear, and by definition three noncollinear points determine a plane. **15.** Invalid; the hypothesis is false as there are only two points. **17.** no conclusion **19.** If *X* is the midpoint of $\overline{YZ}$, then $\overline{YX} \cong \overline{XZ}$ **21.** yes; Law of Syllogism **23.** invalid **25.** yes; Law of Detachment **27.** then he could hear the grating noise of the fish canneries. **29.** Sample answer: a: If it is rainy, the game will be cancelled. b: It is rainy. c: The game will be cancelled. **31.** Lakeisha; if you are dizzy, that does not necessarily mean that you are seasick and thus have an upset stomach. **33.** Sample answer: Doctors and nurses use charts to assist in determining medications and their doses for patients. Doctors need to note a patient's symptoms to determine which medication to prescribe, then determine how much to prescribe based on weight, age, severity of the illness, and so on. Doctors use what is known to be true about diseases and when symptoms appear, then deduce that the patient has a particular illness. **35.** J **37.** They are fast and reliable. **39.**

q	r	$q \wedge r$
T	T	T
T	F	F
F	T	F
F	F	F

41.

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

43. $\angle HDG$ **45.** Sample answer: $\angle JHE$ and $\angle DHE$ **47.** Yes, slashes on the segments indicate that they are congruent. **49.** Sample answer: $\angle 1$ and $\angle 2$ are complementary, $m\angle 1 + m\angle 2 = 90$.

Pages 107–109 **Lesson 2-5**

1. 6 **3.** 15 ribbons **5.** definition of collinear **7.** Since *P* is the midpoint of $\overline{QR}$ and $\overline{ST}$, $PQ = PR = \frac{1}{2}QR$ and $PS = PT = \frac{1}{2}ST$ by the definition of midpoint. We are given $\overline{QR} \cong \overline{ST}$ so $QR = ST$ by the definition of congruent segments. By the Multiplication Property, $\frac{1}{2}QR = \frac{1}{2}ST$. So, by substitution, $PQ = PT$. **9.** 10 **11.** Sometimes; the three points cannot be on the same line. **13.** Never; the intersection of a line and a plane can be a point, but the intersection of two planes is a line.

15. Given: C is the midpoint of $\overline{AB}$.
B is the midpoint of $\overline{CD}$.

Prove: $\overline{AC} \cong \overline{BD}$

Proof:
We are given that C is the midpoint of $\overline{AB}$, and B is the midpoint of $\overline{CD}$. By the definition of midpoint $\overline{AC} \cong \overline{CB}$ and $\overline{CB} \cong \overline{BD}$. Using the definition of congruent segments, $AC = CB$, and $CB = BD$. $AC = BD$ by the Transitive Property. Thus, $\overline{AC} \cong \overline{BD}$ by the definition of congruent segments.

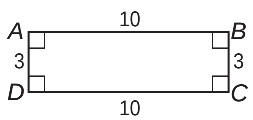
17. Postulate 2.1; through any two points, there is exactly one line. **19.** Postulate 2.2; through any three points not on the same line, there is exactly one plane. **21.** Sample answer: Lawyers make final arguments, which is a speech that uses deductive reasoning, in court cases. **23.** Deductive reasoning is used to support claims that are made in a proof. Postulates, theorems, algebraic properties, and definitions can be used as reasons in a proof. **25.** Conjecture; postulates, theorems, and axioms are used as reasons in proofs. A conjecture cannot be used in a proof unless it is proven true. **27.** Sample answer: The forms and structures of different types of writing are accepted as valid, such as the structure of a poem. The Declaration of Independence, “We hold these truths to be self-evident, . . .” **29.** F **31.** Converse: If you have a computer, then you have access to the Internet at your house. False; you can have a computer and not have access to the Internet. Inverse: If you do not have access to the Internet at your house, then you do not have a computer. False; it is possible to not have access to the Internet and still have a computer. Contrapositive: If you do not have a computer, then you do not have access to the Internet at your house. False; you could have Internet access through your television or wireless phone. **33.** 19 **35.** -24

Pages 114–117 ***Lesson 2-6***

1. Multiplication Prop. **3.** Addition Prop. **5.** B

7. Given: Rectangle $ABCD$, $AD = 3$, $AB = 10$

Prove: $AC = BD$



Proof:

Statements (Reasons)

1. Rectangle $ABCD$, $AD = 3$, $AB = 10$ (Given)
2. Draw segments AC and DB. (Two points determine a line.)
3. $\triangle ABC$ and $\triangle BCD$ are right triangles. (Def. of rt. $\triangle$)
4. $AC = \sqrt{3^2 + 10^2}$, $DB = \sqrt{3^2 + 10^2}$ (Pyth. Th.)
5. $AC = BD$ (Substitution)

9. Subt. Prop. **11.** Substitution **13.** Reflexive Prop. **15.** Substitution **17.** Transitive Prop.

19a. $2x - 7 = \frac{1}{3}x - 2$

19b. $3(2x - 7) = 3\left(\frac{1}{3}x - 2\right)$

19c. Dist. Prop.

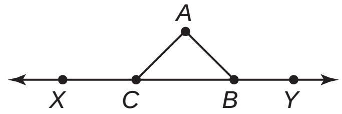
19d. $5x - 21 = -6$

19e. Add. Prop.

19f. $x = 3$

21. Given: $m\angle ACB = m\angle ABC$

Prove: $m\angle XCA = m\angle YBA$



Proof:

Statements(Reasons)

1. $m\angle ACB = m\angle ABC$ (Given)
2. $m\angle XCA + m\angle ACB = 180$; $m\angle YBA + m\angle ABC = 180$ (Def. of suppl. $\angle$s)
3. $m\angle XCA + m\angle ACB = m\angle YBA + m\angle ABC$ (Substitution)
4. $m\angle XCA + m\angle ACB = m\angle YBA + m\angle ACB$ (Substitution)
5. $m\angle XCA = m\angle YBA$ (Subt. Prop.)

23. Given: $5 - \frac{2}{3}z = 1$

Prove: $z = 6$

Proof:

Statements (Reasons)

1. $5 - \frac{2}{3}z = 1$ (Given)
2. $3\left(5 - \frac{2}{3}z\right) = 3(1)$ (Mult. Prop.)
3. $15 - 2z = 3$ (Dist. Prop.)
4. $15 - 2z - 15 = 3 - 15$ (Subt. Prop.)
5. $-2z = -12$ (Substitution)
6. $\frac{-2z}{-2} = \frac{-12}{-2}$ (Div. Prop.)
7. $z = 6$ (Substitution)

25. Given: $-2y + \frac{3}{2} = 8$

Prove: $y = -\frac{13}{4}$

Proof:

Statements (Reasons)

1. $-2y + \frac{3}{2} = 8$ (Given)
2. $2\left(-2y + \frac{3}{2}\right) = 2(8)$ (Mult. Prop.)
3. $-4y + 3 = 16$ (Dist. Prop.)
4. $-4y = 13$ (Subt. Prop.)
5. $y = -\frac{13}{4}$ (Div. Prop.)

27. Proof:

Statements (Reasons)

1. $PV = nRT$ (Given)
2. $\frac{PV}{nR} = \frac{nRT}{nR}$ (Div. Prop.)
3. $\frac{PV}{nR} = T$ (Div. Prop.)

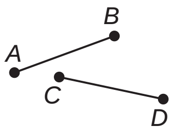
29. Sample answer: If $x = 2$ and $x + y = 6$, then $2 + y = 6$. **31.** Sample answers are: Michael has a symmetric relationship of first cousin with Chris, Kevin, Diane, Dierdre, and Steven. Diane, Dierdre, and Steven have a symmetric and transitive relationship of sibling. Any direct line from bottom to top has a transitive descendent relationship.

33. B **35.** 6 **37.** Invalid; $27 \div 6 = 4.5$, which is not an integer. **39.** If you can have patience, then you can have what you will. **41.** If you respect yourself, then others will respect you. **43.** 11 **45.** 47

Pages 121–123 ***Lesson 2-7***

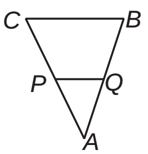
1a. $\overline{PQ} \cong \overline{RS}$, $\overline{QS} \cong \overline{ST}$
1b. Def. of $\cong$ segments
1c. Seg. Add. Post.
1d. $PS = RS + ST$
1e. $PS = RT$
1f. Def. of $\cong$ segments
3a. Given
3b. Def. of $\cong$ segments
3c. $WA = AY$, $ZA = AX$
3d. Seg. Add. Post.
3e. Substitution
3f. Substitution
3g. Substitution
3h. $WA = ZA$
3i. Def. of $\cong$ segments

5. Given: $\overline{AB} \cong \overline{CD}$
Prove: $\overline{CD} \cong \overline{AB}$



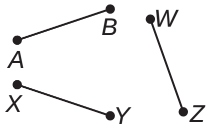
Proof:
Statements (Reasons)
1. $\overline{AB} \cong \overline{CD}$ (Given)
2. $AB = CD$ (Def. of $\cong$ segs.)
3. $CD = AB$ (Symmetric Prop.)
4. $\overline{CD} \cong \overline{AB}$ (Def. of $\cong$ segs.)

7. Given: $\overline{AB} \cong \overline{AC}$ and $\overline{PC} \cong \overline{QB}$
Prove: $\overline{AP} \cong \overline{AQ}$



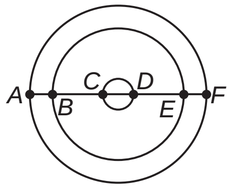
Proof:
Statements (Reasons)
1. $\overline{AB} \cong \overline{AC}$ and $\overline{PC} \cong \overline{QB}$ (Given)
2. $AB = AC$, $PC = QB$ (Def. of $\cong$ segs.)
3. $AB = AQ + QB$, $AC = AP + PC$ (Seg. Add. Post.)
4. $AQ + QB = AP + PC$ (Substitution)
5. $AQ + QB = AP + QB$ (Substitution)
6. $QB = QB$ (Reflexive Prop.)
7. $AQ = AP$ (Subt. Prop.)
8. $AP = AQ$ (Symmetric Prop.)
9. $\overline{AP} \cong \overline{AQ}$ (Def. of $\cong$ segs.)

9. Given: $\overline{XY} \cong \overline{WZ}$ and $\overline{WZ} \cong \overline{AB}$
Prove: $\overline{XY} \cong \overline{AB}$



Proof:
Statements (Reasons)
1. $\overline{XY} \cong \overline{WZ}$ and $\overline{WZ} \cong \overline{AB}$ (Given)
2. $XY = WZ$ and $WZ = AB$ (Def. of $\cong$ segs.)
3. $XY = AB$ (Transitive Prop.)
4. $\overline{XY} \cong \overline{AB}$ (Def. of $\cong$ segs.)

11. Given: $\overline{AB} \cong \overline{EF}$ and $\overline{BC} \cong \overline{DE}$
Prove: $\overline{AC} \cong \overline{DF}$



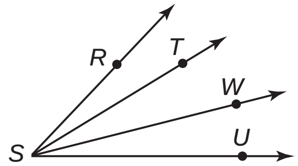
Proof:
Statements (Reasons)
1. $\overline{AB} \cong \overline{EF}$ and $\overline{BC} \cong \overline{DE}$ (Given)
2. $AB = EF$ and $BC = DE$ (Def. of $\cong$ segs.)
3. $AB + BC = DE + EF$ (Add. Prop.)
4. $AC = AB + BC$, $DF = DE + EF$ (Seg. Add. Post.)
5. $AC = DF$ (Substitution)
6. $\overline{AC} \cong \overline{DF}$ (Def. of $\cong$ segs.)

13. Sample answer: The distance from Cleveland to Chicago is the same as the distance from Cleveland to Chicago. **15.** Sample answer: You can use segment addition to find the total distance between two destinations by adding the distances of various points in between. A passenger can add the distance from San Diego to Phoenix and the distance from Phoenix to Dallas to find the distance from San Diego to Dallas. The Segment Addition Postulate can be useful if you are traveling in a straight line. **17.** F **19.** Substitution **21.** Add. Prop. **23.** never **25.** always **27.** 30 **29.** 25

Pages 129–131 ***Lesson 2-8***

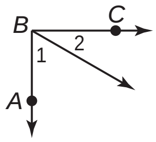
1. $m\angle 6 = 43$, $m\angle 7 = 90$
3a. Given
3b. Def. of suppl. $\measuredangle$s
3c. Substitution
3d. Def. of $\cong$ $\measuredangle$s
3e. Subt. Prop.
3f. Def. of $\cong$ $\measuredangle$s
5. 26 **7.** $m\angle 5 = 61$, $m\angle 7 = 29$, $m\angle 8 = 61$
9. $m\angle 11 = 124$, $m\angle 12 = 56$ **11.** $m\angle 15 = 58$, $m\angle 16 = 58$
13. $m\angle 13 = 112$, $m\angle 14 = 112$

15. Given: $m\angle RSW = m\angle TSU$
Prove: $m\angle RST = m\angle WSU$



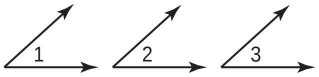
Proof:
Statements (Reasons)
1. $m\angle RSW = m\angle TSU$ (Given)
2. $m\angle RSW = m\angle RST + m\angle TSW$, $m\angle TSU = m\angle TSW + m\angle WSU$ (Angle Addition Postulate)
3. $m\angle RST + m\angle TSW = m\angle TSW + m\angle WSU$ (Substitution)
4. $m\angle TSW = m\angle TSW$ (Reflexive Prop.)
5. $m\angle RST = m\angle WSU$ (Subt. Prop.)

17. Given: $\angle ABC$ is a right angle.
Prove: $\angle 1$ and $\angle 2$ are complementary angles.



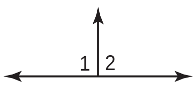
Proof:
Statements (Reasons)
1. $\angle ABC$ is a right angle. (Given)
2. $m\angle ABC = 90$ (Def. of rt. $\angle$s)
3. $m\angle ABC = m\angle 1 + m\angle 2$ ($\angle$ Add. Post.)
4. $90 = m\angle 1 + m\angle 2$ (Subst.)
5. $\angle 1$ and $\angle 2$ are comp. angles. (Def. of comp. $\angle$s)

19. Given: $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$
Prove: $\angle 1 \cong \angle 3$



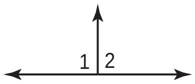
Proof:
Statements (Reasons)
1. $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$ (Given)
2. $m\angle 1 = m\angle 2$, $m\angle 2 = m\angle 3$ (Def. of $\cong$ angles)
3. $m\angle 1 = m\angle 3$ (Trans. Prop.)
4. $\angle 1 \cong \angle 3$ (Def. of $\cong$ angles)

21. Given: $\angle 1$ and $\angle 2$ are rt. $\angle$s.
Prove: $\angle 1 \cong \angle 2$



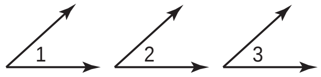
Proof:
Statements (Reasons)
1. $\angle 1$ and $\angle 2$ are rt. $\angle$s. (Given)
2. $m\angle 1 = 90$, $m\angle 2 = 90$ (Def. of rt. $\angle$s)
3. $m\angle 1 = m\angle 2$ (Substitution)
4. $\angle 1 \cong \angle 2$ (Def. of $\cong$ $\angle$s)

23. Given: $\angle 1 \cong \angle 2$, $\angle 1$ and $\angle 2$ are supplementary.
Prove: $\angle 1$ and $\angle 2$ are rt. $\angle$s.



Proof:
Statements (Reasons)
1. $\angle 1 \cong \angle 2$, $\angle 1$ and $\angle 2$ are supplementary. (Given)
2. $m\angle 1 + m\angle 2 = 180$ (Def. of supplementary $\angle$s)
3. $m\angle 1 = m\angle 2$ (Def. of $\cong$ $\angle$s)
4. $m\angle 1 + m\angle 1 = 180$ (Substitution)
5. $2(m\angle 1) = 180$ (Add. Prop.)
6. $m\angle 1 = 90$ (Div. Prop.)
7. $m\angle 2 = 90$ (Substitution)
8. $\angle 1$ and $\angle 2$ are rt. $\angle$s. (Def. of rt. $\angle$s)

25. 152 **27.** Sample answer: If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

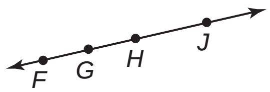


29. Sometimes; if the two angles are formed by the intersection of two lines then the nonadjacent angles are vertical. **31.** $m\angle 1 + m\angle 4 = 90$;

$$\begin{aligned} m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 &= 180 \\ m\angle 1 + m\angle 1 + m\angle 4 + m\angle 4 &= 180 \\ 2(m\angle 1) + 2(m\angle 4) &= 180 \\ 2(m\angle 1 + m\angle 4) &= 180 \\ m\angle 1 + m\angle 4 &= 90 \end{aligned}$$

33. A

35. Given: G is between F and H.
H is between G and J.
Prove: $FG + GJ = FH + HJ$



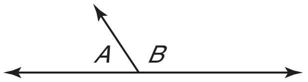
Proof:
Statements (Reasons)
1. G is between F and H; H is between F and J. (Given)
2. $FG + GJ = FJ$, $FH + HJ = FJ$ (Seg. Add. Post.)
3. $FJ = FH + HJ$ (Sym. Prop.)
4. $FG + GJ = FH + HJ$ (Transitive Prop.)

37. Given: $AC = BD$
Prove: $AB = CD$

Proof:
Statements (Reasons)
1. $AC = BD$ (Given)
2. $AB + BC = AC$, $BC + CD = BD$ (Seg. Add. Post.)
3. $BC = BC$ (Reflexive Prop.)
4. $AB + BC = BC + CD$ (Substitution)
5. $AB = CD$ (Subt. Prop.)

Pages 132–136 ***Chapter 2*** ***Study Guide and Review***

1. false, postulates (or axioms) **3.** true **5.** true **7.** true **9.** false; converse **11.** $m\angle A + m\angle B = 180$



13. Sample answer: There was an accident jamming traffic ahead of her on the freeway. **15.** $-1 \leq 0$ or the sum of the measures of two supplementary angles is not 180°; true because $\sim p$ is true. It does not matter that $\sim r$ is false. **17.** Conditional: If the month is March, then it has 31 days. Converse: If the month has 31 days, then it is March. False; January has 31 days. Inverse: If it is not March, then it does not have 31 days. False; January is not March and January has 31 days. Contrapositive: If the month does not have 31 days, then it is not March; true. **19.** true **21.** false **23.** yes, Law of Detachment **25.** Never true; the intersection of two different lines is a point. **27.** sometimes true; only if no more than two of the four points are collinear **29.** There are 15 different possibilities for hanging Marlene's hammock. **31.** Division Property **33.** Transitive Property

35. Given: $x - 1 = \frac{x - 10}{-2}$

Prove: $x = 4$

Proof:

Statements (Reasons)

1. $x - 1 = \frac{x - 10}{-2}$ (Given)

2. $-2(x - 1) = -2\left(\frac{x - 10}{-2}\right)$ (Mult. Prop.)

3. $-2x + 2 = x - 10$ (Dist. Prop.)

4. $-2x + 2 - 2 = x - 10 - 2$ (Subt. Prop.)

5. $-2x = x - 12$ (Substitution)

6. $-2x - x = x - 12 - x$ (Subt. Prop.)

7. $-3x = -12$ (Substitution)

8. $\frac{-3x}{-3} = \frac{-12}{-3}$ (Div. Prop.)

9. $x = 4$ (Substitution)

37. Given: $MN = PQ$, $PQ = RS$

Prove: $MN = RS$

Proof:

Statements (Reasons)

1. $MN = PQ$, $PQ = RS$ (Given)

2. $MN = RS$ (Transitive Prop.)

39. Given: $BC = EC$, $CA = CD$

Prove: $BA = DE$

Proof:

Statements (Reasons)

1. $BC = EC$, $CA = CD$ (Given)

2. $BC + CA = EC + CA$ (Add. Prop.)

3. $BC + CA = EC + CD$ (Substitution)

4. $BC + CA = BA$, $EC + CD = DE$ (Seg. Add. Post.)

5. $BA = DE$ (Substitution)

41. Given: $AB = CB$

M is the midpoint of $\overline{AB}$.

N is the midpoint of $\overline{CD}$.

Prove: $AM = CN$

Proof:

Statements (Reasons)

1. $AB = CB$; M is the midpoint of $\overline{AB}$; N is the midpoint of $\overline{CD}$. (Given)

2. $AM = MB$, $CN = ND$ (Midpoint Th.)

3. $AM + MB = AB$, $CN + NB = CB$ (Seg. Add. Post.)

4. $AM + MB = CN + NB$ (Substitution)

5. $AM + AM = CN + CN$ (Substitution)

6. $2(AM) = 2(CN)$ (Substitution)

7. $AM = CN$ (Div. Prop.)

43. 23

45. Given: $\angle 1$ and $\angle 2$ form a linear pair.

$m\angle 2 = 2(m\angle 1)$

Prove: $m\angle 1 = 60$

Proof:

Statements (Reasons)

1. $\angle 1$ and $\angle 2$ form a linear pair. (Given)

2. $\angle 1$ and $\angle 2$ are supplementary. (Supplement Th.)

3. $m\angle 1 + m\angle 2 = 180$ (Def. of suppl. $\angle$s)

4. $m\angle 2 = 2(m\angle 1)$ (Given)

5. $m\angle 1 + 2(m\angle 1) = 180$ (Substitution)

6. $3(m\angle 1) = 180$ (Substitution)

7. $\frac{3(m\angle 1)}{3} = \frac{180}{3}$ (Division Prop.)

8. $m\angle 1 = 60$ (Substitution)

Chapter 3 Parallel and Perpendicular Lines

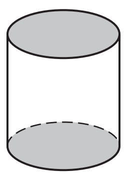
Page 141 ***Chapter 3*** ***Get Ready***

1. $\overleftrightarrow{PQ}$ **3.** $\overleftrightarrow{ST}$ **5.** $\angle 4, \angle 6, \angle 8$ **7.** $\angle 1, \angle 5, \angle 7$ **9.** $2x + 2 = 19$; $x = 8.50$

Pages 144–147 ***Lesson 3-1***

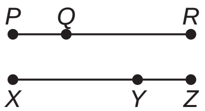
1. ABC, JKL, ABK, CDM **3.** $\overline{BK}$, $\overline{CL}$, $\overline{JK}$, $\overline{LM}$, $\overline{BL}$, $\overline{KM}$ **5.** p and q, p and t, q and t **7.** p and q, p and r, q and r **9.** corr. **11.** alt. ext. **13.** ABC, ABQ, PQR, CDS, APU, DET **15.** $\overline{AP}$, $\overline{BQ}$, $\overline{CR}$, $\overline{FU}$, $\overline{PU}$, $\overline{QR}$, $\overline{RS}$, $\overline{TU}$ **17.** $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{QR}$, $\overline{RS}$, $\overline{ST}$, $\overline{TU}$ **19.** $\overline{CR}$, $\overline{BQ}$, $\overline{AP}$, $\overline{FW}$, $\overline{ET}$ **21.** a and c, a and r, r and c **23.** a and b, a and c, b and c **25.** alt. ext. **27.** corr. **29.** alt. int. **31.** cons. int. **33.** corr. **35.** alt.ext. **37.** The pillars form parallel lines. **39.** One of the west pillars and the base on the east side form skew lines. **41.** p; cons. int. **43.** q; alt. int. **45.** $\overline{CG}$, $\overline{DH}$, $\overline{EI}$ **47.** No; plane ADE will intersect all the planes if they are extended.

49. Sample answer: The bottom and top of a cylinder are contained in parallel planes.



51. infinite number **53.** Sample answer: Parallel lines and planes are used in architecture to make structures that will be stable. Answers should include the following. Opposite walls should form parallel planes; the floor may be parallel to the ceiling. The plane that forms a stairway will not be parallel to some of the walls. **55.** F

57. Given: $\overline{PQ} \cong \overline{ZY}$, $\overline{QR} \cong \overline{XY}$
Prove: $\overline{PR} \cong \overline{XZ}$



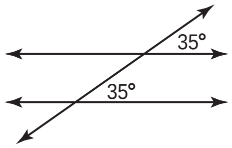
Proof:
Since $\overline{PQ} \cong \overline{ZY}$ and $\overline{QR} \cong \overline{XY}$, $PQ = ZY$ and $QR = XY$ by the definition of congruent segments. By the Addition Property, $PQ + QR = ZY + XY$. Using the Segment Addition Postulate, $PR = PQ + QR$ and $XZ = XY + YZ$. By substitution, $PR = XZ$. Because the measures are equal, $\overline{PR} \cong \overline{XZ}$ by the definition of congruent segments.

59. $m\angle EFG$ is less than 90; Detachment. **61.** 90, 90

Pages 152–154 ***Lesson 3-2***

1. 110 **3.** 70 **5.** $x = 13$, $y = 6$ **7.** 137 **9.** 137 **11.** 43 **13.** 50 **15.** 110 **17.** 110 **19.** $x = 31$, $y = 45$ **21.** $m\angle 1 = 107$ **23a.** Given **23b.** Corr. ⦣ Post. **23c.** Vertical ⦣ Th. **23d.** Transitive Prop. **25.** 115; The angles are consecutive interior angles and are supplementary. **27.** Sometimes; if the transversal is perpendicular to the parallel lines, then $\angle 1$ and $\angle 2$ are right angles and are congruent.

29. Sample answer:

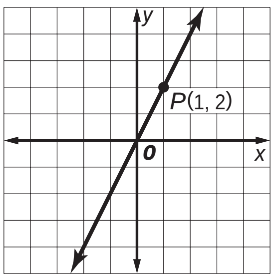


31. $\angle 2$ and $\angle 6$ are consecutive interior angles for the same transversal, which makes them supplementary because $\overline{WX} \parallel \overline{YZ}$. $\angle 4$ and $\angle 6$ are not necessarily supplementary because $\overline{WZ}$ may not be parallel to $\overline{XY}$. **33.** A **35.** $\overline{FG}$ **37.** CDH **39.** 53 **41.** H: you eat a balanced diet; C: it will keep you healthy **43.** $\frac{3}{2}$ **45.** $-\frac{8}{3}$

Pages 160–163 ***Lesson 3-3***

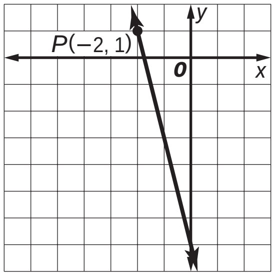
1. $-\frac{1}{2}$ **3.** $-\frac{2}{25}$ or $\frac{2}{25}$ **5.** 1505 m

7.

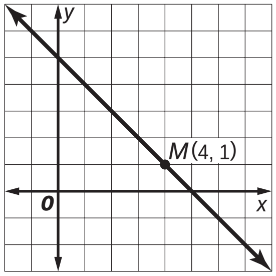


9. −3 **11.** 6 **13.** 6 **15.** undefine **17.** $\frac{1}{7}$ **19.** −5 **21.** 2,816,000 **23.** perpendicular **25.** neither **27.** parallel

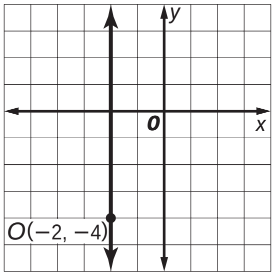
29.



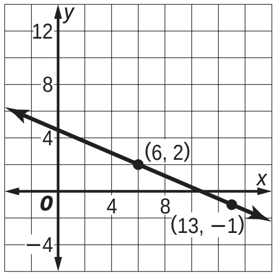
31.



33.



35. 13



37. Sample answer: 0.24 **39.** 2016 **41.** 470,896 **43.** Curtis; Lori added the coordinates instead of finding the difference. **45.** Skew; skew refers to the orientation of a line with regard to another line or plane. The other three terms refer to the slope of a line. **47.** Sample answer: Slope is used when driving through hills to determine how fast to go. Drivers should be notified of the grade so that they can adjust their speed accordingly. A positive slope indicates that the driver must speed up, while a negative slope indicates that the driver should slow down. An escalator must be at a steep enough slope to be efficient, but also must be gradual enough to ensure comfort. **49.** G **51.** 131 **53.** 49 **55.** 49 **57.** ℓ; alt. ext. **59.** p; alt. int. **61.** m; alt. int.

63. Given: $AC = DF$, $AB = DE$
Prove: $BC = EF$
Proof:
Statements (Reasons)
1. $AC = DF$, $AB = DE$ (Given)
2. $AC = AB + BC$; $DF = DE + EF$ (Segment Addition Postulate)
3. $AB + BC = DE + EF$ (Substitution Property)
4. $BC = EF$ (Subtraction Property)

65. 26.69 **67.** If you do not observe Daylight Savings Time, then you live in Hawaii or Arizona; true.

69.

p	q	~q	p or ~q
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

71.

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
T	F	T	T	T

73. $XZ + ZY = XY$ 

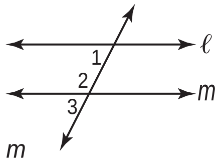
75. $y = -2x + 7$ **77.** $y = \frac{5}{2}x + 2$

Pages 168–170 ***Lesson 3-4***

1. $y = \frac{1}{2}x + 4$ **3.** $y = -\frac{3}{5}x - 2$ **5.** $y - 5 = 3(x - 7)$
7. $y = 2x + 5$ **9.** $y = 2x - 4$ **11.** $y = 10$, $y = 0.79x + 5$
13. $y = 3x - 4$ **15.** $y = \frac{5}{8}x - 6$ **17.** $y = -x - 3$
19. $y - 1 = 2(x - 3)$ **21.** $y + 5 = -\frac{4}{5}(x + 12)$
23. $y - 17.12 = 0.48(x - 5)$ **25.** $y = -3x - 2$
27. $y = 2x - 4$ **29.** $y = -x + 5$ **31.** $y = -\frac{1}{8}x$
33. $y = -750x + 10{,}800$ **35.** $y = -x + 10$
37. $y = -\frac{3}{5}x + 3$ **39.** $y = -\frac{1}{5}x - 4$ **41.** no slope-intercept form, $x = -6$ **43.** Sample answer: $y = 2x - 3$, $y = -x - 6$ **45.** Sample answer: In the equation of a line, the b value indicates the fixed rate, while the mx value indicates charges based on usage. We can find where the equations intersect to see where the cost of the plans would be equal. **47.** F **49.** 58 **51.** 75 **53.** 73 **55.** $\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 8$ **57.** $\angle 1$ and $\angle 7$, $\angle 4$ and $\angle 6$

Pages 175–179 ***Lesson 3-5***

1. $\ell \parallel m$; corr. $\angle$s **3.** $p \parallel q$; cons. int. $\angle$s **5.** 9
7. The slope of $\overleftrightarrow{CD}$ is $\frac{1}{8}$, and the slope of $\overleftrightarrow{AB}$ is $\frac{1}{7}$. The slopes are not equal, so the lines are not parallel.
9. $\overleftrightarrow{AE} \parallel \overleftrightarrow{BF}$; $\cong$ corr. $\angle$s **11.** $\overleftrightarrow{AC} \parallel \overleftrightarrow{EG}$; suppl. consec. int. $\angle$s **13.** 15 **15.** -8 **17.** 21.6 **19.** No; the slopes are not the same.
21. Given: $\angle 1$ and $\angle 2$ are supplementary.



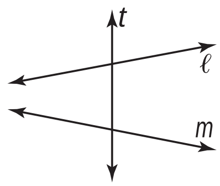
Prove: $\ell \parallel m$
Proof:
Statements (Reasons)
1. $\angle 1$ and $\angle 2$ are supplementary. (Given)
2. $\angle 2$ and $\angle 3$ form a linear pair. (Def. of linear pair)
3. $\angle 2$ and $\angle 3$ are supplementary. (Supplement Th.)
4. $\angle 1 \cong \angle 3$ ($\angle$s suppl. to same $\angle$ are $\cong$.)
5. $\ell \parallel m$ (If corr. $\angle$s are $\cong$, then lines are $\parallel$.)

23. Given: $\angle 2 \cong \angle 1$
$\angle 1 \cong \angle 3$
Prove: $\overline{ST} \parallel \overline{UV}$
Proof:
Statements (Reasons)
1. $\angle 2 \cong \angle 1$, $\angle 1 \cong \angle 3$ (Given)
2. $\angle 2 \cong \angle 3$ (Trans. Prop.)
3. $\overline{ST} \parallel \overline{UV}$ (If alt. int. $\angle$s are $\cong$, lines are $\parallel$.)

25. Given: $\angle RSP \cong \angle PQR$
$\angle QRS$ and $\angle PQR$ are supplementary.
Prove: $\overline{PS} \parallel \overline{QR}$
Proof:
Statements (Reasons)
1. $\angle RSP \cong \angle PQR$, $\angle QRS$ and $\angle PQR$ are suppl. (Given)
2. $m\angle RSP = m\angle PQR$ (Def. of $\cong$ $\angle$s)
3. $m\angle QRS + m\angle PQR = 180$ (Definition of suppl. $\angle$s)
4. $m\angle QRS + m\angle RSP = 180$ (Substitution)
5. $\angle QRS$ and $\angle RSP$ are suppl. (Def. of suppl. $\angle$s)
6. $\overline{PS} \parallel \overline{QR}$ (If cons. int. $\angle$s are suppl., lines are $\parallel$.)

29. The 10-yard lines will be parallel because they are all perpendicular to the sideline and two or more lines perpendicular to the same line are parallel. **31.** Yes; when two pieces are put together, they form a 90° angle. Two lines that are perpendicular to the same line are parallel.

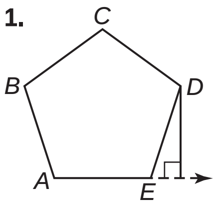
33. Sample answer:



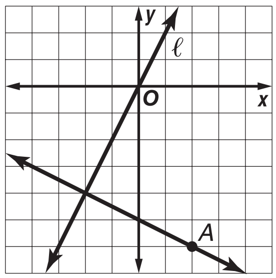
35. Consecutive angles are supplementary; opposite angles are congruent; the sum of the measures of the angles is 360. **37.** B **39.** $y = 0.3x - 6$
41. $y = -\frac{1}{2}x + \frac{19}{2}$ **43.** $-\frac{5}{4}$ **45.** 1 **47.** undefined
49. complementary angles **51.** $\sqrt{85} \approx 9.22$

Pages 185–187 ***Lesson 3-6***

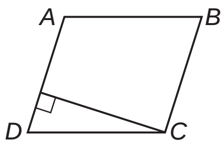
1.



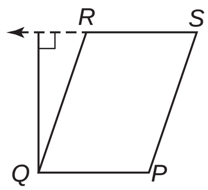
3. $\sqrt{20} \approx 4.47$



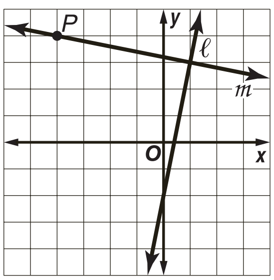
5.



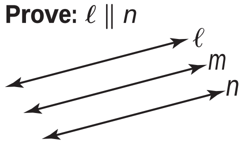
7.



9. $\sqrt{26}$

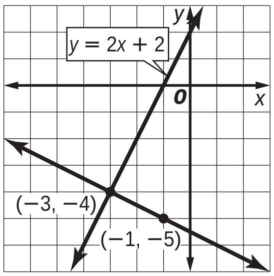


11. 6 **13.** $1.5\sqrt{10}$ **15.** 19 **17.** $\sqrt{9.8}$
19. Given: ℓ is equidistant to m.
n is equidistant to m.
Prove: $\ell \parallel n$



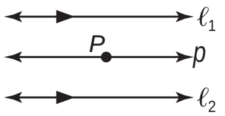
Paragraph proof: We are given that ℓ is equidistant to m, and n is equidistant to m. By definition of equidistant, ℓ is parallel to m, and n is parallel to m. By definition of parallel lines, slope of ℓ = slope of m, and slope of n = slope of m. By substitution, slope of ℓ = slope of n. Then, by definition of parallel lines, $\ell \parallel n$.

21. $\sqrt{5}$



23. The plumb line will be vertical and will be perpendicular to the floor. The shortest distance from a point to the floor will be along the plumb line.

27.



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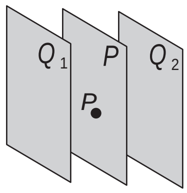
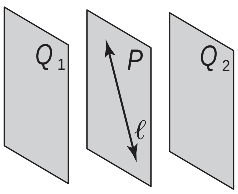


31.

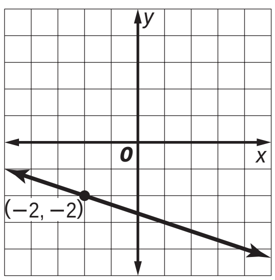


33. Sample answer: We want new shelves to be parallel so they will line up. Answers should include the following. After making several points, a slope can be calculated, which should be the same slope as the original brace. Building walls requires parallel lines.
35. G **37.** $\overleftrightarrow{DA} \parallel \overleftrightarrow{EF}$; corr. $\angle$s **39.** $y = \frac{1}{2}x + 3$
41. $y = \frac{2}{3}x - 2$ **43.** $y = \frac{2}{3}x + \frac{11}{3}$

Pages 191–194 Chapter 3 Study Guide and Review

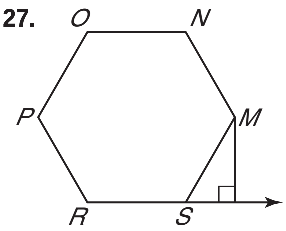
1. alternate **3.** parallel **5.** alternate exterior
7. consecutive **9.** alternate interior **11.** alternate exterior **13.** The lines formed by the paths of these two eagles are skew. **15.** 115

17.



19. $y + 5 = 2(x - 1)$ **21.** $y - 1 = 23(x - 9)$ or $y + 7 = \frac{2}{3}(x + 3)$ **23.** $y = -32x + 8500$ **25.** $\overleftrightarrow{AL}$ and $\overleftrightarrow{BJ}$, cons. int. $\angle$s suppl.

27.



29. ≈ 138 mi

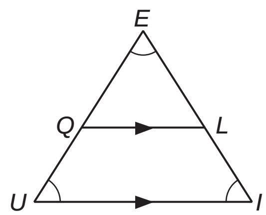
Chapter 4 Congruent Triangles

Page 23 Chapter 201 Get Ready

1. -6.5 **3.** $2\frac{3}{4}$ **5.** $4g + 5 = 6$; $g = 0.25$ **7.** $\angle 4$, $\angle 16$, $\angle 11$, $\angle 14$, $\angle 5$, $\angle 1$, $\angle 7$, $\angle 10$ **9.** 14.6

Pages 205–208 Lesson 4-1

1. obtuse **3.** $\triangle ABC$ **5.** $x = 11$, $LN = 29$, and $MN = 29$ **7.** $TW = \sqrt{125}$, $WZ = \sqrt{74}$, $TZ = \sqrt{61}$; scalene **9.** right **11.** acute **13.** $\triangle MJK$, $\triangle KLM$, $\triangle JKN$, $\triangle LMN$ **15.** $x = 4$, $JM = 3$, $MN = 3$, $JN = 2$
17. $AB = \sqrt{29}$, $BC = 4$, $AC = \sqrt{29}$; isosceles
19. $AB = 10$, $BC = 11$, $AC = \sqrt{221}$; scalene
21. 8 scalene triangles (green), 8 isosceles triangles in the middle (blue), 4 isosceles triangles around the middle (yellow) and 4 isosceles at the corners of the square (purple) **23.** $\triangle AGB$, $\triangle AGC$, $\triangle DGB$, $\triangle DGC$ **25.** $\triangle AGB$, $\triangle AGC$, $\triangle DGB$, $\triangle DGC$ **27.** equilateral, equiangular **29.** $x = 6$, $GH = 13$, $GJ = 13$, $HJ = 5$
31. $x = 8$, $QR = 14$, $RS = 14$, $QS = 14$ **33.** Scalene; it is 130 miles from Charlotte to Raleigh, 92 miles from Raleigh to Winston-Salem, and 70 miles from Winston-Salem to Charlotte.
35. Given: $\triangle EUI$ is equiangular. $\overline{QL} \parallel \overline{UI}$.
Prove: $\triangle EQL$ is equiangular.

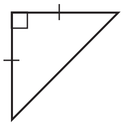


Proof:
Statements (Reasons)
1. $\triangle EUI$ is equiangular. $\overline{QL} \parallel \overline{UI}$ (Given)
2. $\angle E \cong \angle EUI \cong \angle EIU$ (Def. of equiangular $\triangle$)
3. $\angle EUI \cong \angle EQL$, $\angle EIU \cong \angle ELQ$ (Corr. $\angle$s Post.)
4. $\angle E \cong \angle E$ (Reflexive Property)
5. $\angle E \cong \angle EQL \cong \angle ELQ$ (Transitive Property)
6. $\triangle EQL$ is equiangular. (Def. of equiangular $\triangle$)

Selected Answers

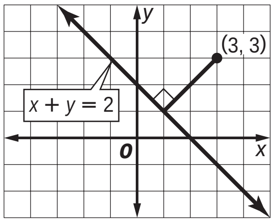
37. $TS = \sqrt{(-7-(-4))^2 + (8-14)^2} = \sqrt{9+36} = \sqrt{45}$
$SR = \sqrt{(-10-(-7))^2 + (2-8)^2} = \sqrt{9+36} = \sqrt{45}$
S is the midpoint of $\overline{RT}$.
$UT = \sqrt{(-0-(-4))^2 + (8-14)^2} = \sqrt{16+36} = \sqrt{52}$
$VU = \sqrt{(4-(0))^2 + (2-8)^2} = \sqrt{16+36} = \sqrt{52}$
U is the midpoint of $\overline{TV}$.

39. Sample answer:



41. Never; right triangles have one right angle and acute triangles have all acute angles. **43.** Sample answer: Triangles are used in construction as structural support. Triangles can be classified by sides and angles. If the measure of each angle is less than 90, the triangle is acute. If the measure of one angle is greater than 90, the triangle is obtuse. If one angle equals 90°, the triangle is right. If each angle has the same measure, the triangle is equiangular. If no two sides are congruent, the triangle is scalene. If at least two sides are congruent, it is isosceles. If all of the sides are congruent, the triangle is equilateral. Isosceles triangles seem to be used more often in architecture and construction. **45.** H

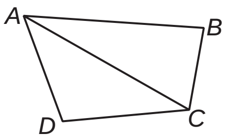
47. $\sqrt{8}$



49. 15
51. 44 **53.** ∠1 and ∠4, ∠1 and ∠10, ∠5 and ∠2, ∠5 and ∠8, ∠9 and ∠6, ∠9 and ∠12 **55.** ∠1, ∠4, and ∠10

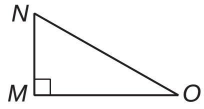
Pages 214–216 Lesson 4-2

1. 43 **3.** 55 **5.** 147 **7.** 25 **9.** 93 **11.** 65, 65 **13.** 64 **15.** 116 **17.** 32 **19.** 37 **21.** 50 **23.** 54 **25.** 137 **27.** 53 **29.** 153
31. Given: $ABCD$ is a quadrilateral.

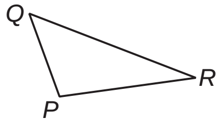


Prove: $m\angle DAC + m\angle B + m\angle BCD + m\angle D = 360$
Proof:
Statements (Reasons)
1. $ABCD$ is a quadrilateral. (Given)
2. $m\angle 2 + m\angle 3 + m\angle B = 180$;
$m\angle 1 + m\angle 4 + m\angle D = 180$ (∠ Sum Theorem)
3. $m\angle 2 + m\angle 3 + m\angle B + m\angle 1 + m\angle 4 + m\angle D = 360$ (Addition Prop.)
4. $m\angle DAB = m\angle 1 + m\angle 2$; $m\angle BCD = m\angle 3 + m\angle 4$ (∠ Addition)
5. $m\angle DAB + m\angle B + m\angle BCD + m\angle D = 360$ (Substitution)

33. Given: $\triangle MNO$
$\angle M$ is a right angle.
Prove: There can be at most one right angle in a triangle.

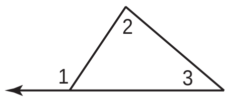


Proof: In $\angle MNO$, M is a right angle.
$m\angle M + m\angle N + m\angle O = 180$. $m\angle M = 90$, so $m\angle N + m\angle O = 90$. If N were a right angle, then $m\angle O = 0$. But that is impossible, so there cannot be two right angles in a triangle.
Given: $\triangle PQR$
$\angle P$ is obtuse.
Prove: There can be at most one obtuse angle in a triangle.



Proof: In $\triangle PQR$, $\angle P$ is obtuse. So $m\angle P > 90$.
$m\angle P + m\angle Q + m\angle R = 180$. It must be that $m\angle Q + m\angle R < 90$. So, $\angle Q$ and $\angle R$ must be acute.

35. Sample answer: ∠2 and ∠3 are the remote interior angles of exterior ∠1.

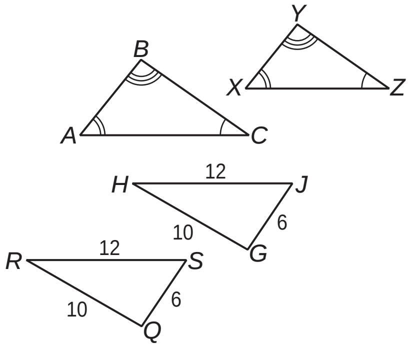


37. Najee; the sum of the measures of the remote interior angles is equal to the measure of the corresponding exterior angle. **39.** A **41.** $\triangle AED$ **43.** $\triangle BEC$ **45.** $2\sqrt{5}$ units **47.** Reflexive **49.** Symmetric

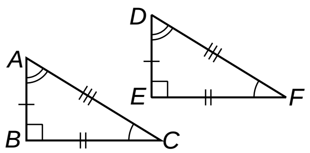
Pages 220–223 Lesson 4-3

1. $\triangle AFC \cong \triangle DFB$; $\angle A \cong \angle D$; $\angle C \cong \angle B$; $\angle BFD \cong \angle CFA$; $\overline{AC} \cong \overline{DB}$; $\overline{AF} \cong \overline{DF}$; $\overline{FC} \cong \overline{FB}$
3. $\triangle BNC \cong EOF \cong \triangle HPJ \cong \triangle KML$ **5.** $QR = 5$, $Q'R' = 5$, $RT = 3$, $R'T' = 3$, $QT = \sqrt{34}$, and $Q'T' = \sqrt{34}$; use a protractor to confirm that the corresponding angles are congruent; flip.
7. $\triangle RSV \cong \triangle TSV$; $\angle R \cong \angle T$; $\angle RSV \cong \angle TSV$; $\angle RVS \cong \angle TVS$; $RS \cong TS$; $SV \cong SV$; $RV \cong TV$
9. $\triangle EFH \cong \triangle GHF$; $\angle E \cong \angle G$; $\angle EFH \cong \angle GHF$; $\angle EHF \cong \angle GFH$; $\overline{EF} \cong \overline{GH}$; $\overline{EH} \cong \overline{GF}$; $\overline{FH} \cong \overline{FH}$
11. Flip; $MN = 8$, $M'N' = 8$, $NP = 2$, $N'P' = 2$, $MP = \sqrt{68}$, and $M'P' = \sqrt{68}$. Use a protractor to confirm that the corresponding angles are congruent.
13. Turn; $JK = \sqrt{40}$, $J'K' = \sqrt{40}$, $KL = \sqrt{29}$, $K'L' = \sqrt{29}$, $JL = \sqrt{17}$, and $J'L' = \sqrt{17}$. Use a protractor to confirm that the corresponding angles are congruent.
15. $\angle C \cong \angle R$; $\angle D \cong \angle S$, $\angle G \cong \angle W$, $\overline{CD} \cong \overline{RS}$, $\overline{DG} \cong \overline{SW}$, $\overline{CG} \cong \overline{RW}$ **17.** $\angle A \cong \angle H$, $\angle D \cong \angle K$; $\angle G \cong \angle L$, $\overline{AD} \cong \overline{HK}$, $\overline{DG} \cong \overline{KL}$, $\overline{AG} \cong \overline{HL}$
19. We need to know if all of the vertex angles are congruent, if all of the base angles are congruent, and if the sides are congruent. **21.** △s 1 - 4, △s 5 - 12, △s 13 - 20 **23.** False; $\angle A \cong X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$, but corresponding sides are not congruent.

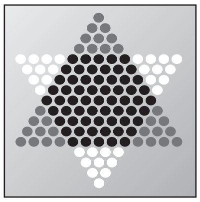
25.



27.



29. $\angle G \cong \angle K$, $\angle H \cong L$, $\angle J \cong P$, $\overline{GH} \cong \overline{KL}$, $\overline{HJ} \cong \overline{LP}$, $\overline{GJ} \cong \overline{KP}$ **31a.** Given **31b.** Given **31c.** Congruence of segments is reflexive. **31d.** Given **31e.** Def. of ⊥ lines **31f.** Given **31g.** Def. of ⊥ lines **31h.** All right ∡ are ≅ **31i.** Given **31j.** Alt. Int. ∡ are ≅ **31k.** Given **31l.** Alt. Int. ∡ are ≅ **31m.** Def. of ≅ △s
33. Sample answer:

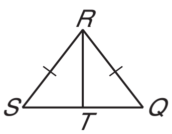


Chinese Checkers Board; each triangle shown is congruent. The length of each side is 4.

35. Sample answer: Triangles are used in bridge design for structure and support. Triangles spread weight and stress evenly throughout the bridge. **37.** J **39.** 75
41. 75 **43.** $x = 5$; $HK = 12$, $HT = 12$, $KT = 12$
45. $\sqrt{32}$ **47.** $\sqrt{34}$

Pages 229–232 ***Lesson 4-4***

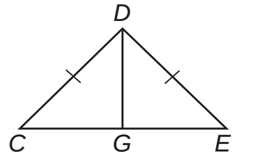
1. Given: T is the midpoint of $\overline{SQ}$; $\overline{SR} \cong \overline{QR}$.



Prove: $\triangle SRT \cong \triangle QRT$
Proof:
Statements (Reasons)
1. T is the midpoint of $\overline{SQ}$. (Given)
2. $\overline{ST} \cong \overline{TQ}$ (Def. of midpoint)
3. $\overline{SR} \cong \overline{QR}$ (Given)
4. $\overline{RT} \cong \overline{RT}$ (Reflexive Prop.)
5. $\triangle SRT \cong \triangle QRT$ (SSS)

3. $EG = \sqrt{10}$, $FG = \sqrt{26}$, $EF = \sqrt{68}$, $MP = \sqrt{2}$, $NP = \sqrt{26}$, and $MN = \sqrt{20}$. The corresponding sides are not congruent; the triangles are not congruent.
5. SAS

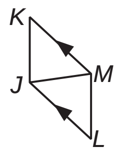
7. Given: $\triangle CDE$ is an isosceles triangle.
G is the midpoint of $\overline{CE}$.
Prove: $\triangle CDG \cong \triangle EDG$
Proof:



Statements (Reasons)
1. $\triangle CDE$ is an isosceles triangle; G is the midpoint of $\overline{CE}$. (Given)
2. $\overline{CD} \cong \overline{DE}$ (Def. of isos. △)
3. $\overline{CG} \cong \overline{GE}$ (Midpoint Th.)
4. $\overline{DG} \cong \overline{DG}$ (Reflexive Prop.)
5. $\triangle CDG \cong \triangle EDG$ (SSS)

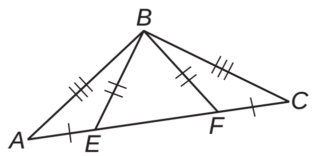
9. $JK = \sqrt{18}$, $KL = \sqrt{17}$, $JL = \sqrt{17}$, $FG = \sqrt{18}$, $GH = \sqrt{17}$, and $FH = \sqrt{17}$. Each pair of corresponding sides has the same measure so they are congruent. $\triangle JKL \cong \triangle FGH$ by SSS. **11.** $JK = \sqrt{50}$, $KL = \sqrt{13}$, $JL = 5$, $FG = \sqrt{8}$, $GH = \sqrt{13}$, and $FH = 5$. The corresponding sides are not congruent so $\triangle JKL$ is not congruent to $\triangle FGH$.

13. Given: $\overline{KM} \parallel \overline{JL}$, $\overline{KM} \cong \overline{LJ}$
Prove: $\triangle JKM \cong \triangle MLJ$
Proof:



Statements (Reasons)
1. $\overline{KM} \parallel \overline{JL}$, $\overline{KM} \cong \overline{LJ}$ (Given)
2. $\angle KMJ \cong \angle LJM$ (Alt. Int. ∡ Th.)
3. $\overline{JM} \cong \overline{JM}$ (Reflexive Prop.)
4. $\triangle JKM \cong \triangle MLJ$ (SAS)

15. SSS or SAS **17.** not possible
19. Given: $\overline{AE} \cong \overline{CF}$; $\overline{AB} \cong \overline{CB}$; $\overline{BE} \cong \overline{BF}$
Prove: $\triangle AFB \cong \triangle CEB$

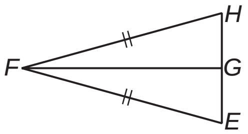


Proof:

$\overline{AE} \cong \overline{CF}$ — **Given**
↓
$AE = CF$ — Def. of ≅ seg.
↓
$AE + EF = EF + CF$ — Addition Prop. ← $EF = EF$ — Reflexive Prop.

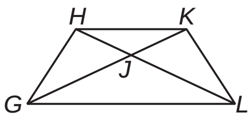
$AF = CE$ — Substitution ← $AE + EF = AF$, $EF + CF = CE$ — Seg. Addition Post.
↓
$\overline{AF} \cong \overline{CE}$ — Def. of ≅ seg.
↓
$\Delta AFB \cong \Delta CEB$ — SSS ← $\overline{AB} \cong \overline{CB}$, $\overline{BE} \cong \overline{BF}$ — **Given**

21. Given: $\overline{EF} \cong \overline{HF}$
G is the midpoint of $\overline{EH}$.
Prove: $\triangle EFG \cong \triangle HFG$



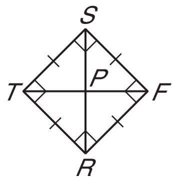
Proof:
Statements (Reasons)
1. $\overline{EF} \cong \overline{HF}$; *G* is the midpoint of *EH*. (Given)
2. $\overline{EG} \cong \overline{HG}$ (Def. of midpoint)
3. $\overline{FG} \cong \overline{FG}$ (Reflexive Prop.)
4. $\triangle EFG \cong \triangle HFG$ (SSS)

23. Given: $\triangle GHJ \cong \triangle LKJ$
Prove: $\triangle GHL \cong \triangle LKG$



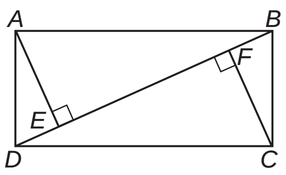
Proof:
Statements (Reasons)
1. $\triangle GHJ \cong \triangle LKJ$ (Given)
2. $\overline{HJ} \cong \overline{KJ}$, $\overline{GJ} \cong \overline{LJ}$, $\overline{GH} \cong \overline{LK}$ (CPCTC)
3. $HJ = KJ$, $GJ = LJ$ (Def. of $\cong$ segments)
4. $HJ + LJ = KJ + JG$ (Addition Prop.)
5. $KJ + GJ = KG$; $HJ + LJ = HL$ (Segment Addition)
6. $\overline{KG} = \overline{HL}$ (Substitution)
7. $\overline{KG} \cong \overline{HL}$ (Def. of $\cong$ segments)
8. $\overline{GL} \cong \overline{LG}$ (Reflexive Prop.)
9. $\triangle GHL \cong \triangle LKG$ (SSS)

25. Given: $\overline{TS} \cong \overline{SF} \cong \overline{FH} \cong \overline{HT}$ $\angle TSF$, $\angle SFH$, $\angle FHT$, and $\angle HTS$ are right angles.
Prove: $\angle SRT \cong \angle SRF$



Proof:
Statements (Reasons)
1. $\overline{TS} \cong \overline{SF} \cong \overline{FR} \cong \overline{RT}$ (Given)
2. $\angle TSF$, $\angle SFR$, $\angle FRT$, and $\angle RTS$ are rt $\angle$s (Given)
3. $\angle STR \cong \angle SFR$ (All rt $\angle$s are $\cong$.)
4. $\triangle STR \cong \triangle SFR$ (SAS)
5. $\angle SRT \cong \angle SRF$ (CPCTC)

29. Given: $\overline{DE} \cong \overline{BF}$, $\overline{AE} \cong \overline{CF}$, $\overline{AE} \perp \overline{DB}$, $\overline{CF} \perp \overline{BD}$
Prove: $\triangle ABD \cong \triangle CDB$

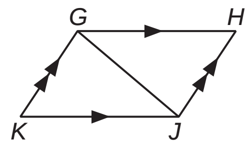


Plan: First use SAS to show that $\triangle ADE \cong \triangle CBF$. Next use CPCTC and Reflexive Property for segments to show $\triangle ABD \cong \triangle CDB$.
Proof:
Statements (Reasons)
1. $\overline{DE} \cong \overline{BF}$, $\overline{AE} \cong \overline{CF}$ (Given)
2. $\overline{AE} \perp \overline{DB}$, $\overline{CF} \perp \overline{BD}$ (Given)
3. $\angle AED$ and $\angle CFB$ are right angles. ($\perp$ lines form right $\angle$s.)
4. $\angle AED \cong \angle CFB$ (All right angles are $\cong$.)
5. $\triangle ADE \cong \triangle CBF$ (SAS)
6. $\overline{AD} \cong \overline{CB}$ (CPCTC)
7. $\overline{DB} \cong \overline{DB}$ (Reflexive Prop.)
8. $\angle CBD \cong \angle ADB$ (CPCTC)
9. $\triangle ABD \cong \triangle CDB$ (SAS)

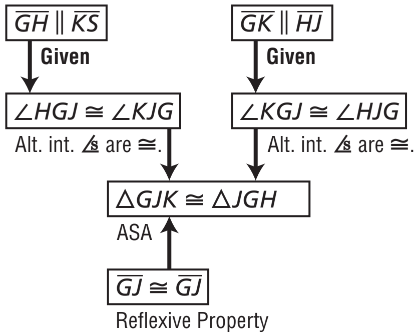
31. C **33.** $\triangle ACB \cong \triangle DCE$ **35.** $\triangle LMP \cong \triangle NPM$ **37.** 102 **39.** 22 **41.** 34 **43.** There is a steeper rate of change from the first quarter to the second. **45.** $\angle CBD$ **47.** $\overline{CD}$ **49.** $\angle DXE$

Pages 238–241 ***Lesson 4-5***

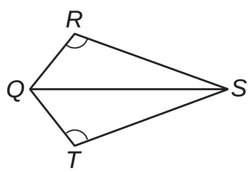
1. Given: $\overline{GH} \parallel \overline{KJ}$, $\overline{GK} \parallel \overline{HJ}$
Prove: $\triangle GJK \cong \triangle JGH$



Proof:

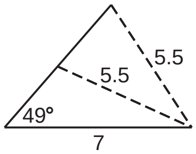


3. Given: $\overline{QS}$ bisects $\angle RST$; $\angle R \cong \angle T$
Prove: $\triangle QRS \cong \triangle QTS$
Proof:

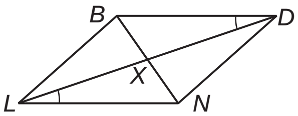


We are given that $\angle R \cong \angle T$. We are also given that $\overline{QS}$ bisects $\angle RST$, so by definition of angle bisector, $\angle RSQ \cong \angle TSQ$. By the Reflexive Property, $\overline{QS} \cong \overline{QS}$. $\triangle QRS \cong \triangle QTS$ by AAS.

5. This cannot be determined. The information given cannot be used with any of the triangle congruence postulates, theorems or the definition of congruent triangles. By construction, two different triangles can be shown with the given information. Therefore, it cannot be determined if $\triangle SRT \cong \triangle MKL$.

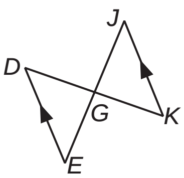


7. Given: $\overline{DL}$ bisects $\overline{BN}$.
$\angle XLN \cong \angle XDB$
Prove: $\overline{LN} \cong \overline{DB}$
Proof:



Since $\overline{DL}$ bisects $\overline{BN}$, $\overline{BX} \cong \overline{XN}$. $\angle XLN \cong \angle XDB$. $\angle LXN \cong \angle DXB$ because vertical angles are congruent. $\triangle LXN \cong \triangle DXB$ by AAS. $\overline{LN} \cong \overline{DB}$ by CPCTC.

9. Given: $\overline{DE} \parallel \overline{JK}$
$\overline{DK}$ bisects $\overline{JE}$.
Prove: $\triangle EGD \cong \triangle JGK$



Proof:

$\overline{DE} \parallel \overline{JK}$ (Given) → $\angle E \cong \angle J$ (Alt. int. ∠s are ≅.)

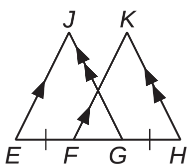
$\overline{DK}$ bisects $\overline{JE}$. (Given) → $\overline{EG} \cong \overline{JG}$ (Def. of seg. bisector)

$\angle DGE \cong \angle KGJ$ (Vert. ∠s are ≅.)

$\angle E \cong \angle J$, $\overline{EG} \cong \overline{JG}$, $\angle DGE \cong \angle KGJ$ → $\triangle EGD \cong \triangle JGK$ (ASA)

11. Since F is the midpoint of $\overline{DG}$, $\overline{DF} \cong \overline{FG}$. F is also the midpoint of $\overline{CH}$, so $\overline{CF} \cong \overline{FH}$. Since $\overline{DG} \cong \overline{CH}$, $\overline{DF} \cong \overline{CF}$ and $\overline{FG} \cong \overline{FH}$. $\angle CFD \cong \angle HFG$ because vertical angles are congruent. $\triangle CFD \cong \triangle HFG$ by SAS.

13. Given: $\overline{EJ} \parallel \overline{FK}$, $\overline{JG} \parallel \overline{KH}$, $\overline{EF} \cong \overline{GH}$
Prove: $\triangle EJG \cong \triangle FKH$
Proof:



$\overline{EF} \cong \overline{GH}$ (Given) → $EF = GH$ (Def. of ≅ seg.)

$EF = GH$, $FG = FG$ (Reflexive Prop.) → $EF + FG = FG + GH$ (Addition Prop.)

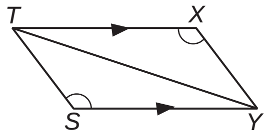
$EF + FG = EG$, $FG + GH = FH$ (Seg. Addition Post.) → $EG = FH$ (Substitution)

$EG = FH$ → $\overline{EG} \cong \overline{FH}$ (Def. of ≅ seg.)

$\overline{EJ} \parallel \overline{FK}$, $\overline{JG} \parallel \overline{KH}$ (Given) → $\angle JEG \cong \angle KFH$, $\angle JGE \cong \angle KHF$ (Corr. ∠s are ≅.)

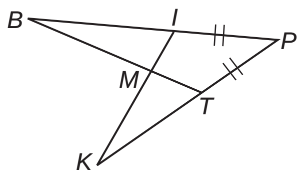
$\overline{EG} \cong \overline{FH}$, $\angle JEG \cong \angle KFH$, $\angle JGE \cong \angle KHF$ → $\triangle EJG \cong \triangle FKH$ (ASA)

15. Given: $\overline{TX} \parallel \overline{SY}$
$\angle TXY \cong \angle YST$
Prove: $\triangle TSY \cong \triangle YXT$
Proof:



Since $\overline{TX} \parallel \overline{SY}$, $\angle YTX \cong \angle TYS$ by the Alternate Interior Angle Theorem. $\overline{TY} \cong \overline{YT}$ by the Reflexive Property. Given $\angle TXY \cong \angle YST$, $\triangle TSY \cong \triangle YXT$ by AAS.

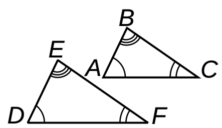
17. Given: $\triangle BMI \cong \triangle KMT$
$\overline{IP} \cong \overline{TP}$
Prove: $\triangle IPK \cong \triangle TPB$



Proof:
Statements (Reasons)
1. $\triangle BMI \cong \triangle KMT$ (Given)
2. $\angle B \cong \angle K$ (CPCTC)
3. $\overline{IP} \cong \overline{TP}$ (Given)
4. $\angle P \cong \angle P$ (Reflexive Prop.)
5. $\triangle IPK \cong \triangle TPB$ (AAS)

19. It is given that $\overline{JM} \cong \overline{LM}$ and $\angle NJM \cong \angle NLM$. By the Reflexive Property, $\overline{NM} \cong \overline{NM}$. It cannot be determined whether $\triangle JNM \cong \triangle LNM$. The information given does not lead to a unique triangle.

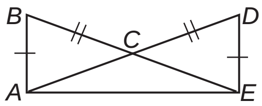
21. $\triangle VMN$, ASA or AAS **23.** AAA is not a congruence postulate or theorem because two triangles can have corresponding congruent angles without corresponding congruent sides $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$. However, $\overline{DF}$ is not congruent to $\overline{AC}$.



25. Since Aiko is perpendicular to the ground, two right angles are formed and right angles are congruent. The angles of sight are the same and her height is the same for each triangle. The triangles are congruent by ASA. By CPCTC, the distances are the same. The method is valid. **27.** B

29. Given: $\overline{BA} \cong \overline{DE}$, $\overline{DA} \cong \overline{BE}$

Prove: $\triangle BEA \cong \triangle DAE$



Proof:

$\overline{DA} \cong \overline{BE}$ — Given

$\overline{BA} \cong \overline{DE}$ — Given

$\overline{AE} \cong \overline{AE}$ — Reflexive Prop.

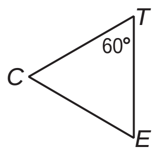
$\rightarrow$ $\triangle BEA \cong \triangle DAE$ — SSS

31. Turn; $RS = \sqrt{2}$, $R'S' = \sqrt{2}$, $ST = 1$, $S'T' = 1$, $R'T = 1$, $R'T' = 1$. Use a protractor to confirm that the corresponding angles are congruent. **33.** If people are happy, then they rarely correct their faults. **35.** isosceles **37.** isosceles

Pages 248–250 ***Lesson 4-6***

1. Given: $\triangle CTE$ is isosceles with vertex $\angle C$. $m\angle T = 60$

Prove: $\triangle CTE$ is equilateral.



Proof:

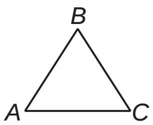
Statements (Reasons)

1. $\triangle CTE$ is isosceles with vertex $\angle C$. (Given)
2. $\overline{CT} \cong \overline{CE}$ (Def. of isosceles $\triangle$)
3. $\angle E \cong \angle T$ (Isosceles $\triangle$ Th.)
4. $m\angle E = m\angle T$ ($\cong$ $\triangle$s have the same measure.)
5. $m\angle T = 60$ (Given)
6. $m\angle E = 60$ (Substitution)
7. $m\angle C = 60$ (Sum of $\angle$s in a $\triangle$ is 180.)
8. $\triangle CTE$ is equiangular. (Def. of equiangular $\triangle$)
9. $\triangle CTE$ is equilateral. (Equiangular $\triangle$s are equilateral.)

3. $\angle ADH \cong \angle AHD$ **5.** $\angle LTR \cong \angle LRT$
7. $\angle LSQ \cong \angle LQS$ **9.** $\overline{LS} \cong \overline{LR}$
11. Case I

Given: $\triangle ABC$ is an equilateral triangle.

Prove: $\triangle ABC$ is an equiangular triangle.

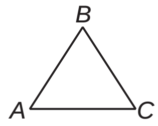


Proof:
1. $\triangle ABC$ is an equilateral triangle. (Given)
2. $\overline{AB} \cong \overline{AC} \cong \overline{BC}$ (Def. equilateral $\triangle$)
3. $\angle A \cong \angle B \cong \angle C$ (Isosceles $\triangle$ Th.)
4. $\triangle ABC$ is an equiangular triangle. (Def. equiangular)

Case II

Given: $\triangle ABC$ is an equiangular triangle.

Prove: $\triangle ABC$ is an equilateral triangle.



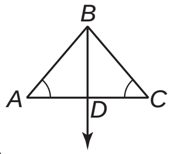
Proof:

Statements (Reasons)

1. $\triangle ABC$ is an equiangular triangle. (Given)
2. $\angle A \cong \angle B \cong \angle C$ (Def. of equiangular $\triangle$)
3. $\overline{AB} \cong \overline{AC} \cong \overline{BC}$ (If 2 $\angle$s of a $\triangle$ are $\cong$ then the sides opp. those $\angle$s are $\cong$.)
4. $\triangle ABC$ is an equilateral triangle. (Def. of equiangular)

13. Given: $\triangle ABC$
$\angle A \cong \angle C$

Prove: $AB \cong CB$



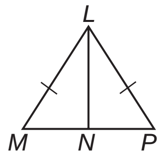
Proof:

Statements (Reasons)

1. Let $\overrightarrow{BD}$ bisect $\angle ABC$. (Protractor Post.)
2. $\angle ABD \cong \angle CBD$ (Def. of bisector)
3. $\angle A \cong \angle C$ (Given)
4. $BD \cong BD$ (Reflexive Prop.)
5. $\triangle ABD \cong \triangle CBD$ (AAS)
6. $\overline{AB} \cong \overline{CB}$ (CPCTC)

15. 10 **17.** 140 **19.** 106 **21.** 30.5 **23.** 124 **25.** 68
27. 111
29. Given: $\triangle MLP$ is isosceles.
N is the midpoint of $\overline{MP}$.

Prove: $\overline{LN} \perp \overline{MP}$



Proof:

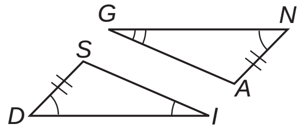
Statements (Reasons)

1. $\triangle MLP$ is isosceles. (Given)
2. $\overline{ML} \cong \overline{LP}$ (Definition of isosceles $\triangle$)
3. $\angle M \cong \angle P$ (Isosceles $\triangle$ Th.)
4. N is the midpoint of $\overline{MP}$. (Given)
5. $\overline{MN} \cong \overline{NP}$ (Def. of midpoint)
6. $\triangle MNL \cong \triangle PNL$ (SAS)
7. $\angle LNM \cong \angle LNP$ (CPCTC)
8. $m\angle LNM = m\angle LNP$ (Def. of $\cong$ $\angle$s)
9. $\angle LNM$ and $\angle LNP$ are a linear pair. (Def. of linear pair)
10. $m\angle LNM + m\angle LNP = 180$ (Sum of measure of linear pair of $\angle$s $= 180$)
11. $2m\angle LNM = 180$ (Substitution)
12. $m\angle LNM = 90$ (Division)
13. $\angle LNM$ is a right angle. (Def. of rt. $\angle$)
14. $\overline{LN} \perp \overline{MP}$ (Def. of $\perp$)

31. 18 **33.** 30 **35.** $m\angle 1 = 18$, $m\angle 2 = 17$, $m\angle 3 = 26$, $m\angle 4 = 17$, $m\angle 5 = 18$ **37.** A

39. Given: $\angle N \cong \angle D$, $\angle G \cong \angle I$, $\overline{AN} \cong \overline{SD}$

Prove: $\triangle ANG \cong \triangle SDI$



Proof:

We are given $\angle N \cong \angle D$ and $\angle G \cong \angle I$. We are also given $\overline{AN} \cong \overline{SD}$. $\triangle ANG \cong \triangle SDI$ by AAS.

41. $QR = \sqrt{17}$, $RS = \sqrt{20}$, $QS = \sqrt{13}$, $EG = \sqrt{17}$, $GH = \sqrt{20}$, and $EH = \sqrt{13}$. Each pair of corresponding sides have the same measure so they are congruent. $\triangle QRS \cong \triangle EGH$ by SSS. **43.** $(-6, -3)$ to $(6, 3)$

45.

p	$\sim p$	q	$\sim q$	$\sim p$ or $\sim q$
T	F	T	F	F
T	F	F	T	T
F	T	T	F	T
F	T	F	T	T

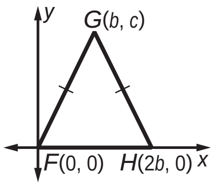
47.

y	$\sim y$	z	$\sim y$ or z
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T

49. $(-1, -3)$

Pages 253–255 **Lesson 4-7**

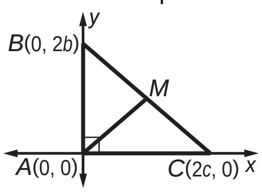
1. Sample answer:



3. $P(0, b)$

5. Given: $\triangle ABC$ is a right triangle with hypotenuse $\overline{BC}$. M is the midpoint of $\overline{BC}$.

Prove: M is equidistant from the vertices.



Proof:

The coordinates of M, the midpoint of $\overline{BC}$, will be $\left(\frac{2c}{2}, \frac{2b}{2}\right) = (c, b)$.

The distance from M to each of the vertices can be found using the Distance Formula.

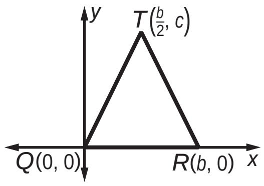
$MB = \sqrt{(c-0)^2 + (b-2b)^2} = \sqrt{c^2 + b^2}$

$MC = \sqrt{(c-2c)^2 + (b-0)^2} = \sqrt{c^2 + b^2}$

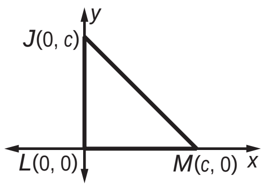
$MA = \sqrt{(c-0)^2 + (b-0)^2} = \sqrt{c^2 + b^2}$

Thus, $MB = MC = MA$, and M is equidistant from the vertices.

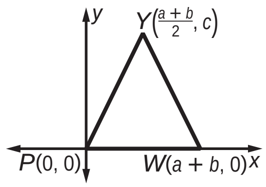
7. Sample answer:



9. Sample answer:



11. Sample answer:

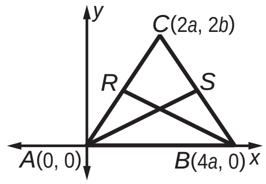


13. $R(a, b)$ **15.** $N(0, 2c)$ **17.** $B(-a, 0)$, $E(0, b)$

19. Given: isosceles $\triangle ABC$ with $\overline{AC} \cong \overline{BC}$

R and S are midpoints of legs $\overline{AC}$ and $\overline{BC}$.

Prove: $\overline{AS} \cong \overline{BR}$



Proof:

The coordinates of S are $\left(\frac{2a+4a}{2}, \frac{2b+0}{2}\right)$ or $(3a, b)$.

The coordinates of R are $\left(\frac{2a+0}{2}, \frac{2b+0}{2}\right)$ or (a, b).

$AS = \sqrt{(3a-0)^2 + (b-0)^2} = \sqrt{(3a)^2 + (b)^2}$ or $\sqrt{9a^2 + b^2}$

$BR = \sqrt{(4a-a)^2 + (0-b)^2} = \sqrt{(3a)^2 + (b)^2}$ or $\sqrt{9a^2 + b^2}$

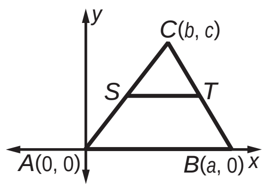
Since $AS = BR$, $\overline{AS} \cong \overline{BR}$.

21. Given: $\triangle ABC$

S is the midpoint of $\overline{AC}$.

T is the midpoint of $\overline{BC}$.

Prove: $\overline{ST} \parallel \overline{AB}$



Proof:

Midpoint S is $\left(\frac{b+0}{2}, \frac{c+0}{2}\right)$ or $\left(\frac{b}{2}, \frac{c}{2}\right)$.

Midpoint T is $\left(\frac{a+b}{2}, \frac{0+c}{2}\right)$ or $\left(\frac{a+b}{2}, \frac{c}{2}\right)$.

Slope of $\overline{ST} = \dfrac{\frac{c}{2} - \frac{c}{2}}{\frac{a+b}{2} - \frac{b}{2}} = \dfrac{0}{\frac{a}{2}}$ or 0.

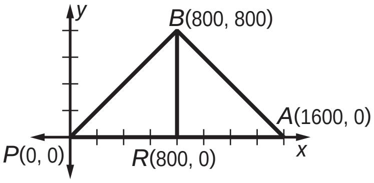
Slope of $\overline{AB} = \dfrac{0-0}{a-0} = \dfrac{0}{a}$ or 0.

$\overline{ST}$ and $\overline{AB}$ have the same slope so $\overline{ST} \parallel \overline{AB}$.

23. Given: $\triangle BPR$

$PR = 800$, $BR = 800$

Prove: $\triangle BPR$ is an isosceles right triangle.



Proof:

Since PR and BR have the same measure, $\overline{PR} \cong \overline{BR}$.

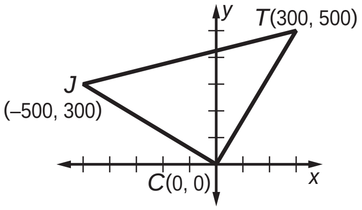
The slope of $PR = \frac{0 - 0}{800 - 0}$ or 0.

The slope of $BR = \frac{800 - 0}{800 - 800}$, which is undefined.

$PR \perp BR$, so $m\angle PRB = 90$. $\triangle BPR$ is an isosceles right triangle.

25. Given: $\triangle JCT$

Prove: $\triangle JCT$ is a right triangle.



Proof:

The slope of $\overline{JC} = \frac{300 - 0}{500 - 0}$ or $-\frac{3}{5}$.

The slope of $\overline{TC} = \frac{500 - 0}{300 - 0}$ or $\frac{5}{3}$.

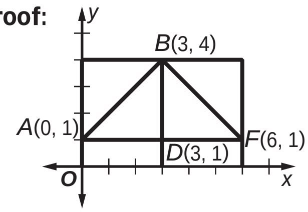
The slope of $\overline{TC}$ is the negative reciprocal of the slope of $\overline{JC}$. $\overline{JC} \cong \overline{TC}$. So $\triangle JCT$ is a right triangle.

27. Given: $\triangle ABD$, $\triangle FBD$

$AF = 6$, $BD = 3$

Prove: $\triangle ABD \cong \triangle FBD$

Proof:



$\overline{BD} \cong \overline{BD}$ by the Reflexive Property.

$AD = \sqrt{(3 - 0)^2 + (1 - 1)^2} = \sqrt{9 + 0}$ or 3

$FD = \sqrt{(6 - 3)^2 + (1 - 1)^2} = \sqrt{9 + 0}$ or 3

Since $AD = FD$, $\overline{AD} \cong \overline{FD}$.

$AB = \sqrt{(3 - 0)^2 + (4 - 1)^2} = \sqrt{9 + 9}$ or $3\sqrt{2}$

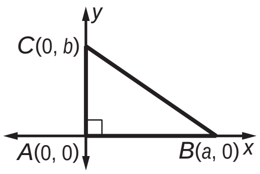
$FB = \sqrt{(6 - 3)^2 + (1 - 4)^2} = \sqrt{9 + 9}$ or $3\sqrt{2}$

Since $AB = FB$, $\overline{AB} \cong \overline{FB}$.

$\triangle ABD \cong \triangle FBD$ by SSS.

29. $(2a, 0)$ or $(0, 2b)$

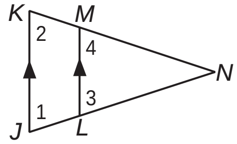
31.



Placing the legs of the triangle on the x- and y-axis and one point on the origin gives you one concrete point (0, 0) and two points with only one variable $(a, 0)$ and $(0, b)$ to work with. This also gives you a concrete reference point to work from as opposed to three ambiguous points. **33.** Placing the figures on the coordinate plane is useful in proofs. We can use coordinate geometry to prove theorems and verify properties. Some different types of proofs are flow proofs, two-column proofs, paragraph proofs, and informal proof. The Isosceles Triangle Theorem can be proved using coordinate proof. **35.** J

37. Given: isosceles triangle JKN with vertex $\angle N$, $\overline{JK} \parallel \overline{LM}$

Prove: $\triangle NML$ is isosceles.



Proof:

__Statements (Reasons)__

1. isosceles triangle JKN with vertex $\angle N$ (Given)
2. $\overline{NJ} \cong \overline{NK}$ (Def. of isosceles triangle)
3. $\angle 2 \cong \angle 1$ (Isosceles Triangle Theorem)
4. $\overline{JK} \parallel \overline{LM}$ (Given)
5. $\angle 1 \cong \angle 3$, $\angle 4 \cong \angle 2$ (Corr. ∠s Post.)
6. $\angle 2 \cong \angle 3$, $\angle 4 \cong \angle 1$ (Congruence of ∠s is transitive.)
7. $\angle 4 \cong \angle 3$ (Congruence of ∠s is transitive.)
8. $\overline{LN} \cong \overline{MN}$ (If 2 ∠s of a △ are ≅, then the sides opp. those ∠s are ≅)
9. $\triangle NML$ is an isosceles triangle. (Def. of isosceles △)

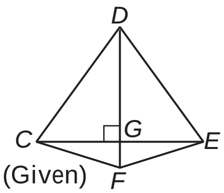
39. $m = 42t + 450$; \$1164

Pages 256–260 ***Chapter 4*** ***Study Guide and Review***

1. obtuse triangle **3.** right triangle **5.** coordinate proof **7.** congruent triangles **9.** right **11.** Sufjan's to Carol's 7.2 miles, Carol's to Steven's 3.56, Sufjan's to Steven's 8.01; scalene. **13.** 25 **15.** 54° **17.** $\angle N \cong \angle RKE$, $\angle C \cong \angle E$, $\angle CKN \cong \angle R$, $\overline{NC} \cong \overline{KE}$, $\overline{CK} \cong \overline{ER}$, $\overline{KN} \cong \overline{RK}$ **19.** $MN = 4$, $NP = 3$, $MP = 5$, $QR = 3$, $RS = 4$, and $QS = 5$. Each pair of corresponding sides does not have the same measure. Therefore, $\triangle MNP$ is not congruent to $\triangle QRS$. $\triangle MNP$ is congruent to $\triangle SRQ$. **21.** No. They form an isosceles triangle.

23. Given: $\overline{DF}$ bisects $\angle CDE$; $\overline{CE} \perp \overline{DF}$.

Prove: $\triangle DGC \cong \triangle DGE$

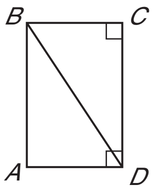


Proof:

__Statements (Reasons)__

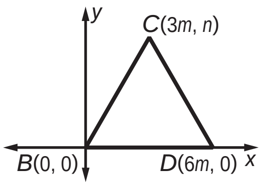
1. $\overline{DF}$ bisects $\angle CDE$; $\overline{CE} \perp \overline{DF}$. (Given)
2. $\overline{DG} \cong \overline{DG}$ (Reflexive Prop.)
3. $\angle CDF \cong \angle EDF$ (Def. of ∠ bisector)
4. $\angle DGC$ is a rt. ∠; $\angle DGE$ is a rt. ∠ (Def. of ⊥ segments)
5. $\angle DGC \cong \angle DGE$ (All rt. ∠s are ≅.)
6. $\triangle DGC \cong \triangle DGE$ (ASA)

25. Given: $\overline{BC} \parallel \overline{AD}$
Prove: $\triangle ABD \cong \triangle CDB$



Proof:
Statements (Reasons)
1. $\overline{BC} \parallel \overline{AD}$ (Given)
2. $\angle CBD \cong \angle ADB$, $\angle CDB \cong \angle ABD$ (Alt. int. ∠s are ≅.)
3. $\overline{BD} \cong \overline{BD}$ (Reflexive prop.)
4. $\triangle ABD \cong \triangle CDB$ (ASA)

27. 30 **29.** See students' work; the triangles in each set appear to be acute
31. Sample answer:

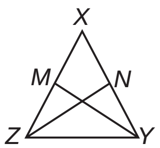


33. $DS \approx 471.7$ m, $DC \approx 471.7$; $\triangle DSC$ is an isosceles triangle.

Chapter 5 Number Patterns and Functions

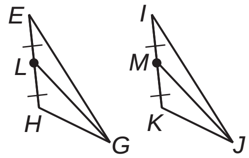
Page 265 ***Chapter 5*** ***Get Ready***
1. (-4, 5) **3.** (-5, 4.5) **5.** 68 **7.** 40 **9.** 26 **11.** 14

Pages 274–278 ***Lesson 5-1***
1. Given: $\overline{XY} \cong \overline{XZ}$, $\overline{YM}$ and $\overline{ZN}$ are medians.
Prove: $\overline{YM} \cong \overline{ZN}$



Proof:
Statements (Reasons)
1. $\overline{XY} \cong \overline{XZ}$, $\overline{YM}$ and $\overline{ZN}$ are medians. (Given)
2. M is the midpoint of $\overline{XZ}$. N is the midpoint of XY. (Def. of median)
3. $XY = XZ$ (Def. of ≅)
4. $\overline{XM} \cong \overline{MZ}$, $\overline{XN} \cong \overline{NY}$ (Midpoint Theorem)
5. $XM = MZ$, $XN = NY$ (Def. of ≅)
6. $XM + MZ = XZ$, $XN + NY = XY$ (Segment Addition Postulate)
7. $XM + MZ = XN + NY$ (Substitution)
8. $MZ + MZ = NY + NY$ (Substitution)
9. $2MZ = 2NY$ (Addition Property)
10. $MZ = NY$ (Division Property)
11. $\overline{MZ} \cong \overline{NY}$ (Def. of ≅)
12. $\angle XZY \cong \angle XYZ$ (Isosceles Triangle Theorem)
13. $\overline{YZ} \cong \overline{ZY}$ (Reflexive Property)
14. $\triangle MYZ \cong \triangle NZY$ (SAS)
15. $\overline{YM} \cong \overline{ZN}$ (CPCTC)

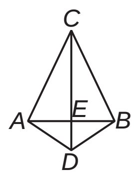
3. $\left(-\frac{17}{38}, -\frac{7}{38}\right)$
5. Given: $\overline{GL}$ is a median of $\triangle EGH$. $\overline{JM}$ is a median of $\triangle IJK$. $\triangle EGH \cong \triangle IJK$.
Prove: $\overline{GL} \cong \overline{JM}$



Proof:
Statements (Reasons)
1. $\overline{GL}$ is a median of $\triangle EGH$, $\overline{JM}$ is a median of $\triangle IJK$, and $\triangle EGH \cong \triangle IJK$. (Given)
2. $\overline{GH} \cong \overline{JK}$, $\angle GHL \cong \angle JKM$, $\overline{EH} \cong \overline{IK}$ (CPCTC)
3. $EH = IK$ (Def. of ≅)
4. $\overline{EL} \cong \overline{LH}$, $\overline{IM} \cong \overline{MK}$ (Def. of median)
5. $EL = LH$, $IM = MK$ (Def of ≅)
6. $EL + LH = EH$, $IM + MK = IK$ (Segment Addition Postulate)
7. $EL + LH = IM + MK$ (Substitution)
8. $LH + LH = MK + MK$ (Substitution)
9. $2LH = 2MK$ (Addition Prop.)
10. $LH = MK$ (Division Prop.)
11. $\overline{LH} \cong \overline{MK}$ (Def of ≅)
12. $\triangle GHL \cong \triangle JKM$ (SAS)
13. $\overline{GL} \cong \overline{JM}$ (CPCTC)

7. 15; no; because $m\angle MSQ = 106$ **9.** 24 **11.** $r = 6$, $q = 7$, $m\angle HWP = 17$ **13.** $m\angle PRZ = 35$ **15.** 12
17. $\left(-\frac{4}{5}, 4\frac{4}{5}\right)$ **19.** (0, 7) **21.** $-\frac{4}{3}$

23. Given: $\overline{CA} \cong \overline{CB}$, $\overline{AD} \cong \overline{BD}$
Prove: C and D are on the perpendicular bisector of $\overline{AB}$.

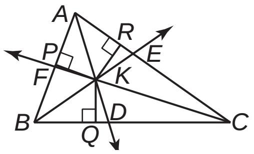


Proof:
Statements (Reasons)
1. $\overline{CA} \cong \overline{CB}$, $\overline{AD} \cong \overline{BD}$ (Given)
2. $\overline{CD} \cong \overline{CD}$ (Congruence of segments is reflexive.)
3. $\triangle ACD \cong \triangle BCD$ (SSS)
4. $\angle ACD \cong \angle BCD$ (CPCTC)
5. $\overline{CE} \cong \overline{CE}$ (Congruence of segments is reflexive.)
6. $\triangle CEA \cong \triangle CEB$ (SAS)
7. $\overline{AE} \cong \overline{BE}$ (CPCTC)
8. E is the midpoint of AB. (Def. of midpoint)
9. $\angle CEA \cong \angle CEB$ (CPCTC)
10. $\angle CEA$ and $\angle CEB$ form a linear pair. (Def. of linear pair)
11. $\angle CEA$ and $\angle CEB$ are supplementary. (Supplement Theorem)
12. $m\angle CEA + m\angle CEB = 180$ (Def. of supplementary)
13. $m\angle CEA + m\angle CEA = 180$ (Substitution Prop.)
14. $2m\angle CEA = 180$ (Substitution Prop.)
15. $m\angle CEA = 90$ (Division Prop.)

Selected Answers

16. $\angle CEA$ and $\angle CEB$ are rt. $\angle$. (Def. of rt. $\angle$)
17. $\overline{CD} \perp \overline{AB}$ (Def. of $\perp$)
18. $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$. (Def. of $\perp$ bisector)
19. C and D are on the perpendicular bisector of $\overline{AB}$. (Def. of point on a line)

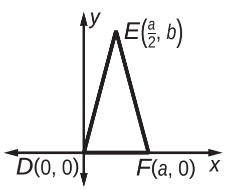
25. Given: $\triangle ABC$, angle bisectors $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$, $\overline{KP} \perp \overline{AB}$, $\overline{KQ} \perp \overline{BC}$, $\overline{KR} \perp \overline{AC}$
Prove: $KP = KQ = KR$



Proof:
Statements (Reasons)
1. $\triangle ABC$, angle bisectors $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$, $\overline{KP} \perp \overline{AB}$, $\overline{KQ} \perp \overline{BC}$, $\overline{KR} \perp \overline{AC}$ (Given)
2. $KP = KQ$, $KQ = KR$, $KP = KR$ (Any point on the $\angle$ bisector is equidistant from the sides of the angle.)
3. $KP = KQ = KR$ (Transitive Property)

27. The three entrances to the school form a triangle. The circumcenter of a triangle is equidistant from the vertices of the triangle. The circumcenter is the intersection of the three perpendicular bisectors of the sides of the triangle. **29.** 8 **31.** The centroid has the same coordinates as the means of the vertices' coordinates. **33.** Sometimes; when the triangle is a right triangle the altitudes intersect at the vertex of the right angle. **35.** Sometimes; the perpendicular bisectors intersect in the exterior of a triangle when the triangle is obtuse. **37.** Sample answer: An altitude and angle bisector of a triangle are the same segment in an equilateral triangle. **39.** Sample answer: Circumcenter; the circumcenter of a triangle is the point equidistant from each vertex. The orthocenter is the point of concurrency formed by the intersection of the three altitudes of a triangle. Altitude could also be correct. Each of the other terms describe a point, an altitude describes a segment. **41.** Sample answer: You can balance a triangle on a pencil point by locating the center of gravity of the triangle, which is the centroid. The centroid is the point of intersection of the medians of the triangle. Yes, in an equilateral triangle, the incenter is the same point as the centroid. **43.** F

45. Sample answer:



47. $\overline{MT} \cong \overline{MR}$
49. $\angle 7 \cong \angle 10$ **51.** It is everywhere equidistant.
53. undefined

55.

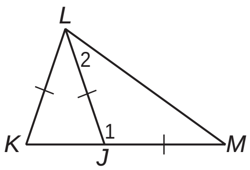
p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

57. true **59.** > **61.** >

Pages 284–287 *Lesson 5-2*

1. $\angle 2$ **3.** $\angle 3$ **5.** $\angle 1$, $\angle 7$ **7.** $m\angle WXY > m\angle XYW$
9. $m\angle WYX < m\angle XWY$ **11.** $\angle 1$ **13.** $\angle 7$
15. $\angle 7$ **17.** $\angle 2$, $\angle 7$, $\angle 8$, $\angle 10$ **19.** $\angle 3$, $\angle 5$
21. $m\angle KAJ < m\angle AJK$ **23.** $m\angle SMJ > m\angle MJS$
25. $m\angle MYJ < m\angle JMY$ **27.** $ZY > YR$
29. $RZ > SR$ **31.** $TY < ZY$

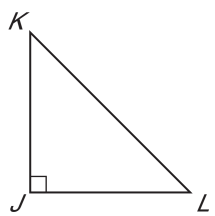
33. Given: $\overline{JM} \cong \overline{JL}$, $\overline{JL} \cong \overline{KL}$
Prove: $m\angle 1 > m\angle 2$



Statements (Reasons)
1. $\overline{JM} \cong \overline{JL}$, $\overline{JL} \cong \overline{KL}$ (Given)
2. $\angle LKJ \cong \angle LJK$ (Isosceles $\triangle$ Theorem)
3. $m\angle LKJ = m\angle LJK$ (Def. of $\cong$)
4. $m\angle 1 > m\angle LKJ$ (Ext. $\angle$ Inequality Theorem)
5. $m\angle 1 > m\angle LJK$ (Substitution)
6. $m\angle LJK > m\angle 2$ (Ext. $\angle$ Inequality Theorem)
7. $m\angle 1 > m\angle 2$ (Trans. Prop. of Inequality)

35. Austin to Chicago, Atlanta to Austin, Chicago to Atlanta **37.** CO, BN, AM **39.** 4; $\overline{PQ}$, $\overline{QR}$, $\overline{PR}$
41. 12; $\overline{PR}$, $\overline{PQ}$, $\overline{QR}$ **43.** -11; $\overline{QR}$, $\overline{PQ}$, $\overline{PR}$

45. Never; if $m\angle J$ is twice $m\angle K$, then $m\angle K = 45$. Therefore, $m\angle L$ is also 45°. Then $JK = JL$. The length of the side opposite $\angle J$ is the hypotenuse and must be greater in length than the sum of the measures of the legs. If the hypotenuse is exactly twice the length of the side opposite $\angle K$, then the triangle could not exist.



47. Grace; she placed the shorter side with the smaller angle and the longer side with the larger angle. **49.** Sample answer: The largest corner is opposite the longest side. The Exterior Angle Inequality Theorem lets you determine the comparison of the angle measures. The largest angle is the angle opposite the side that is 51 feet long. **51.** J **53.** $(15, -6)$
55. Yes; $\frac{1}{3}(-3) = -1$, and F is the midpoint of $\overline{BD}$.

57. $\angle T \cong \angle X$, $\angle U \cong \angle Y$, $\angle V \cong \angle Z$, $\overline{TU} \cong \overline{XY}$, $\overline{UV} \cong \overline{YZ}$, $\overline{TV} \cong \overline{XZ}$ **59.** $\angle B \cong \angle D$, $\angle C \cong \angle G$, $\angle F \cong \angle H$, $\overline{BC} \cong \overline{DG}$, $\overline{CF} \cong \overline{GH}$, $\overline{BF} \cong \overline{DH}$ **61.** true **63.** true

Pages 291–293 ***Lesson 5-3***

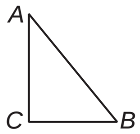
1. $x \geq 5$ **3.** The lines are not parallel.

5. Given: n is odd.
Prove: n^2 is odd.
Proof:
Step 1: Assume n^2 is even.
Step 2: n is odd, so n can be expressed as $2a + 1$.

n^2	$= (2a + 1)^2$	Substitution
	$= (2a + 1)(2a + 1)$	Multiply.
	$= 4a^2 + 4a + 1$	Simplify.
	$= 2(2a^2 + 2a) + 1$	Distributive Property

Step 3: $2(2a^2 + 2a) + 1$ is an odd number. This contradicts the assumption, so the assumption must be false. Thus n^2 is odd.

7. Given: $\triangle ABC$ is a right triangle; $\angle C$ is a right angle.
Prove: $AB > BC$ and $AB > AC$

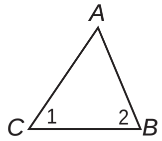


Proof:
Step 1: Assume that the hypotenuse of a right triangle is not the longest side. That is, $AB < BC$ or $AB < AC$.
Step 2: If $AB < BC$, then $m\angle C < m\angle A$. Since $m\angle C = 90$, $m\angle A > 90$. So, $m\angle C + m\angle A > 180$. By the same reasoning, $m\angle C + m\angle B > 180$.
Step 3: Both relationships contradict the fact that the sum of the measures of the angles of a triangle equals 180. Therefore, the hypotenuse must be the longest side of a right triangle.

9. $x \ngtr 4$ or $x \nleq 4$ **11.** A median of an isosceles triangle is not an altitude. **13.** The angle bisector of the vertex angle of an isosceles triangle is not an altitude of the triangle.

15. Given: n^2 is even.
Prove: n^2 is divisible by 4.
Proof:
Step 1: Assume n^2 is not divisible by 4. In other words, 4 is not a factor of n^2.
Step 2: If the square of a number is even, then the number is also even. So, if n^2 is even, n must be even. Let $n = 2a$.
$n = 2a$
$n^2 = (2a)^2$ or $4a^2$
Step 3: 4 is a factor of n^2, which contradicts the assumption.

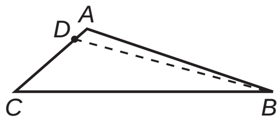
17. Given: $\overline{AB} \ncong \overline{AC}$
Prove: $\angle 1 \ncong \angle 2$



Proof:
Step 1: Assume that $\angle 1 \cong \angle 2$.
Step 2: If $\angle 1 \cong \angle 2$, then the sides opposite the angles are congruent. Thus $\overline{AB} \cong \overline{AC}$.
Step 3: The conclusion contradicts the given information. Thus $\angle 1 \cong \angle 2$ is false. Therefore, $\angle 1 \ncong \angle 2$

19. Given: $m\angle 2 \neq m\angle 1$
Prove: $\ell \nparallel m$
Proof:
Step 1: Assume that $\ell \parallel m$.
Step 2: If $\ell \parallel m$, then $\angle 1 \cong \angle 2$ because they are corresponding angles. Thus, $m\angle 1 = m\angle 2$.
Step 3: This contradicts the given fact that $m\angle 1 \neq m\angle 2$. Thus the assumption $\ell \parallel m$ is false. Therefore, $\ell \nparallel m$.

21. Given: $m\angle A > m\angle ABC$
Prove: $BC > AC$



Proof:
Assume $BC \ngtr AC$. By the Comparison Property, $BC = AC$ or $BC < AC$.
Case 1: If $BC = AC$, then $\angle ABC \cong \angle A$ by the Isosceles Triangle Theorem. (If two sides of a triangle are congruent, then the angles opposite those sides are congruent.) But, $\angle ABC \cong \angle A$ contradicts the given statement that $m\angle A > m\angle ABC$. So, $BC \neq AC$.
Case 2: If $BC < AC$, then there must be a point D between A and C so that $\overline{DC} \cong \overline{BC}$. Draw the auxiliary segment $\overline{BD}$. Since $DC = BC$, by the Isosceles Triangle Theorem $\angle BDC \cong \angle DBC$. Now $\angle BDC$ is an exterior angle of $\triangle BAD$ and by the Exterior Angles Inequality Theorem (the measure of an exterior angle of a triangle is greater than the measure of either corresponding remote interior angle) $m\angle BDC > m\angle A$. By the Angle Addition Postulate, $m\angle ABC = m\angle ABD + m\angle DBC$. Then by the definition of inequality, $m\angle ABC > m\angle DBC$. By Substitution and the Transitive Property of Inequality, $m\angle ABC > m\angle A$. But this contradicts the given statement that $m\angle A > m\angle ABC$. In both cases, a contradiction was found, and hence our assumption must have been false. Therefore, $BC > AC$.

23. A majority is greater than half or 50%.

Proof:

Step 1: Assume that the percent of college-bound seniors influenced by their parents is less than 50%.

Step 2: By examining the graph, you can see that 54% of college-bound seniors were influenced by their parents.

Step 3: Since 54% > 50%, the assumption is false. Therefore, a majority of college-bound seniors were most influenced by their parents in choosing a college.

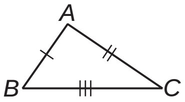
25. teachers and friends; 6% + 5% = 11%, 11% > 8%

27. The door on the left. If the sign on the door on the right were true, then both signs would be true. But one sign is false, so the sign on the door on the right must be false.

29. Sample answer: $\triangle ABC$ is scalene.

Given: $\triangle ABC$; $AB \neq BC$; $BC \neq AC$; $AB \neq AC$

Prove: $\triangle ABC$ is scalene.



Proof:

Assume $\triangle ABC$ is not scalene.

Case 1: $\triangle ABC$ is isosceles.
If $\triangle ABC$ is isosceles, then $AB = BC$, $BC = AC$, or $AB = AC$. This contradicts the given information, so $\triangle ABC$ is not isosceles.

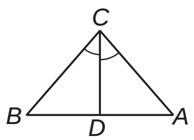
Case 2: $\triangle ABC$ is equilateral.
In order for a triangle to be equilateral, it must also be isosceles, and Case 1 proved that $\triangle ABC$ is not isosceles. Thus, $\triangle ABC$ is not equilateral. Therefore, $\triangle ABC$ is scalene.

31. Sample answer: Sherlock Holmes would disprove all possibilities except the actual solution to a mystery. Indirect proof is used in medical diagnosis, trials, and scientific research.

33. H **35.** $\angle$P

37. Given: $\overline{CD}$ is an angle bisector. $\overline{CD}$ is an altitude.

Prove: $\triangle ABC$ is isosceles.

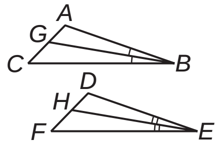


Proof:

<u>**Statements (Reasons)**</u>

1. $\overline{CD}$ is an angle bisector. $\overline{CD}$ is an altitude. (Given)
2. $\angle ACD \cong \angle BCD$ (Def. of $\angle$ bisector)
3. $\overline{CD} \perp \overline{AB}$ (Def. of altitude)
4. $\angle CDA$ and $\angle CDB$ are rt. $\angle$s. ($\perp$ lines form 4 rt. $\angle$s.)
5. $\angle CDA \cong \angle CDB$ (All rt. $\angle$s are $\cong$.)
6. $\overline{CD} \cong \overline{CD}$ (Reflexive Prop.)
7. $\triangle ACD \cong \triangle BCD$ (ASA)
8. $\overline{AC} \cong \overline{BC}$ (CPCTC)
9. $\triangle ABC$ is isosceles. (Def. of isosceles $\triangle$)

39. Given: $\triangle ABC \cong \triangle DEF$; $\overline{BG}$ is an angle bisector of $\angle ABC$. $\overline{EH}$ is an angle bisector of $\angle DEF$.



Prove: $\overline{BG} \cong \overline{EH}$

Proof:

<u>**Statements (Reasons)**</u>

1. $\triangle ABC \cong \triangle DEF$ (Given)
2. $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$, $\angle ABC \cong \angle DEF$ (CPCTC)
3. $\overline{BG}$ is an angle bisector of $\angle ABC$. $\overline{EH}$ is an angle bisector of $\angle DEF$. (Given)
4. $\angle ABG \cong \angle GBC$, $\angle DEH \cong \angle HEF$ (Def. of $\angle$ bisector)
5. $m\angle ABC = m\angle DEF$ (Def. of $\cong$ $\angle$s)
6. $m\angle ABG = m\angle GBC$, $m\angle DEH = m\angle HEF$ (Def. of $\cong$ $\angle$s)
7. $m\angle ABC = m\angle ABG + m\angle GBC$, $m\angle DEF = m\angle DEH + m\angle HEF$ (Angle Addition Property)
8. $m\angle ABC = m\angle ABG + m\angle ABG$, $m\angle DEF = m\angle DEH + m\angle DEH$ (Substitution)
9. $m\angle ABG + m\angle ABG = m\angle DEH + m\angle DEH$ (Substitution)
10. $2m\angle ABG = 2m\angle DEH$ (Substitution)
11. $m\angle ABG = m\angle DEH$ (Division)
12. $\angle ABG \cong \angle DEH$ (Def. of $\cong$ $\angle$s)
13. $\triangle ABG \cong \triangle DEH$ (ASA)
14. $\overline{BG} \cong \overline{EH}$ (CPCTC)

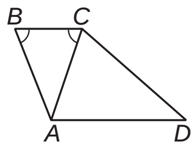
41. true **43.** true

Pages 299–301 ***Lesson 5-4***

1. yes; 3 + 4 > 5 **3.** no; 30.1 + 0.8 $\not>$ 31 **5.** B **7.** no; 1 + 2 $\not>$ 3 **9.** yes; 8 + 8 > 15 **11.** yes; 18 + 21 > 32 **13.** $6 < n < 16$ **15.** $5 < n < 25$ **17.** $26 < n < 68$

19. Given: $\angle B \cong \angle ACB$

Prove: $AD + AB > CD$



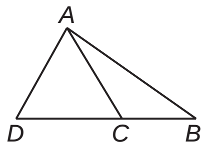
Proof:

<u>**Statements (Reasons)**</u>

1. $\angle B \cong \angle ACB$ (Given)
2. $\overline{AB} \cong \overline{AC}$ (If two $\angle$s are $\cong$, the sides opposite the two $\angle$s are $\cong$.)
3. $AB = AC$ (Def. of $\cong$ segments)
4. $AD + AC > CD$ (Triangle Inequality)
5. $AD + AB > CD$ (Substitution)

21. Given: $\triangle ABC$

Prove: $AC + BC > AB$



Proof:

Statements (Reasons)

1. Construct $\overline{CD}$ so that C is between B and D and $\overline{CD} \cong \overline{AC}$. (Ruler Postulate)
2. $CD = AC$ (Definition of $\cong$)
3. $\angle CAD \cong \angle ADC$ (Isosceles Triangle Theorem)
4. $m\angle CAD = m\angle ADC$ (Definition of $\cong$ angles)
5. $m\angle BAC + m\angle CAD = m\angle BAD$ ($\angle$ Addition Post.)
6. $m\angle BAC + m\angle ADC = m\angle BAD$ (Substitution)
7. $m\angle ADC < m\angle BAD$ (Definition of inequality)
8. $AB < BD$ (If an angle of a triangle is greater than a second angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.)
9. $BD = BC + CD$ (Segment Addition Postulate)
10. $AB < BC + CD$ (Substitution)
11. $AB < BC + AC$ (Substitution (Steps 2, 10))

23. yes; $AB + BC > AC$, $AB + AC > BC$, $AC + BC > AB$ **25.** no; $XY + YZ = XZ$ **27.** 4 **29.** m is either 15 ft or 16 ft; n is 14 ft, 15 ft, or 16 ft. The possible triangles that can be made from sides with those measures are (2 ft, 15 ft, 14 ft), (2 ft, 15 ft, 15 ft), (2 ft, 15 ft, 16 ft), (2 ft, 16 ft, 16 ft). **31.** Sample answer: If the lines are not horizontal, then the segment connecting their y-intercepts is not perpendicular to either line. Since distance is measured along a perpendicular segment, this segment cannot be used. **33.** Jameson; $5 + 10 > 13$ but $5 + 8 \not> 13$. **35.** Sample answer: You can use the Triangle Inequality Theorem to verify the shortest route between two locations. A straight route might not always be available. **37.** G

39. Use $r = \frac{d}{t}$, $t = 3$, and $d = 175$.

Proof:

Step 1: Assume that Maddie's average speed was greater than or equal to 60 miles per hour, $r \geq 60$.

Step 2: **Case 1** | **Case 2**

Case 1	**Case 2**
$R = 60$	$r > 60$
$60 \stackrel{?}{=} \frac{175}{3}$	$\frac{175}{3} \stackrel{?}{>} 60$
$60 \neq 58.3$	$58.3 \not> 60$

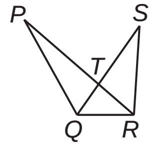
Step 3: The conclusions are false, so the assumption must be false. Therefore, Maddie's average speed was less than 60 miles per hour.

41. $\overline{PQ}$, $\overline{QR}$, $\overline{PR}$ **43.** Label the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ as E, F, and G respectively. Then the coordinates of E, F, and G are $\left(\frac{a}{2}, 0\right)$, $\left(\frac{a+b}{2}, \frac{c}{2}\right)$, and $\left(\frac{b}{2}, \frac{c}{2}\right)$ respectively. The slope of $\overline{AF} = \frac{c}{a+b}$, and the slope of $\overline{AD} = \frac{c}{a+b}$, so D is on $\overline{AF}$. The slope of $\overline{BG} = \frac{c}{b-2a}$ and the slope of $\overline{BD} = \frac{c}{b-2a}$, so D is on $\overline{BG}$. The slope of $\overline{CE} = \frac{2c}{2b-a}$ and the slope of $\overline{CD} = \frac{2c}{2b-a}$, so D is on $\overline{CE}$. Since D is on AF, BG, and CE, it is the intersection point of the three lines.

45. $JK = \sqrt{125}$, $KL = \sqrt{221}$, $JL = \sqrt{226}$, $PQ = \sqrt{125}$, $QR = \sqrt{221}$, and $PR = \sqrt{226}$. The corresponding sides have the same measure and are congruent. $\triangle JKL \cong \triangle PQR$ by SSS. **47.** $x < 12$ **49.** $x < 43.25$

Pages 306–309 *Lesson 5-5*

1. Given: $\overline{PQ} \cong \overline{SQ}$
Prove: $PR > SR$
Proof:

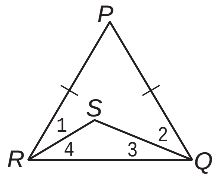


Statements (Reasons)

1. $\overline{PQ} \cong \overline{SQ}$ (Given)
2. $\overline{QR} \cong \overline{QR}$ (Reflexive Property)
3. $m\angle PQR = m\angle PQS + m\angle SQR$ ($\angle$ Addition Post.)
4. $m\angle PQR > m\angle SQR$ (Def. of inequality)
5. $PR > SR$ (SAS Inequality)

3. $AB < CD$ **5.** Sample answer: The pliers are an example of the SAS inequality. As force is applied to the handles, the angle between them decreases causing the distance between them to decrease. As the distance between the ends of the pliers decreases, more force is applied to a smaller area.

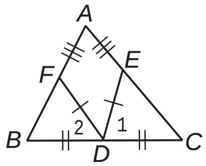
7. Given: $\overline{PR} \cong \overline{PQ}$, $SQ > SR$
Prove: $m\angle 1 < m\angle 2$



Statements (Reasons)

1. $\overline{PR} \cong \overline{PQ}$ (Given)
2. $\angle PRQ \cong \angle PQR$ (If two sides of $\triangle$ are $\cong$, the angles opposite the sides are $\cong$.)
3. $m\angle PRQ = m\angle 1 + m\angle 4$, $m\angle PQR = m\angle 2 + m\angle 3$ (Angle Add. Post.)
4. $m\angle PRQ = m\angle PQR$ (Def. of $\cong$ angles)
5. $m\angle 1 + m\angle 4 = m\angle 2 + m\angle 3$ (Substitution)
6. $SQ > SR$ (Given)
7. $m\angle 4 > m\angle 3$ (If one side of a $\triangle$ is longer than another side, then the $\angle$ opposite the longer side is greater than the $\angle$ opposite the shorter side.)
8. $m\angle 4 = m\angle 3 + x$ (Def. of inequality)
9. $m\angle 1 + m\angle 4 - m\angle 4 = m\angle 2 + m\angle 3 - (m\angle 3 + x)$ (Subtraction Prop.)
10. $m\angle 1 = m\angle 2 - x$ (Substitution)
11. $m\angle 1 + x = m\angle 2$ (Addition Prop.)
12. $m\angle 1 < m\angle 2$ (Def. of inequality)

9. Given: $\overline{ED} \cong \overline{DF}$; $m\angle 1 > m\angle 2$; D is the midpoint of $\overline{CB}$; $\overline{AE} \cong \overline{AF}$.
Prove: $AC > AB$
Proof:

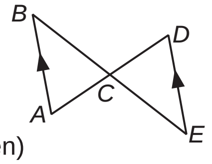


Statements (Reasons)

1. $\overline{ED} \cong \overline{DF}$; D is the midpoint of $\overline{CB}$. (Given)
2. $CD = BD$ (Def. of midpoint)
3. $\overline{CD} \cong \overline{BD}$ (Def. of $\cong$ segments)
4. $m\angle 1 > m\angle 2$ (Given)
5. $EC > FB$ (SAS Inequality)
6. $\overline{AE} \cong \overline{AF}$ (Given)
7. $AE = AF$ (Def. of $\cong$ segments)
8. $AE + EC > AE + FB$ (Add. Prop. of Inequality)
9. $AE + EC > AF + FB$ (Substitution Prop. of Inequality)
10. $AE + EC = AC$, $AF + FB = AB$ (Segment Add. Post.)
11. $AC > AB$ (Substitution)

11. $m\angle BDC < m\angle FDB$ **13.** $AD > DC$
15. $m\angle AOD > m\angle AOB$ **17.** By the SAS Inequality Theorem, if the tree started to lean, one of the angles of the triangle formed by the tree, the ground, and the stake would change, and the side opposite that angle would change as well. However, with the stake in the ground and fixed to the tree, none of the sides of the triangle can change length. Thus, none of the angles can change. This ensures that the tree will stay straight. **19.** $4 < x < 10$ **21.** $x > 7$ **23.** $7 < x < 20$
25. The height of the fulcrum and the length of the lever remain constant. The included angle increases as force is applied to the end of the lever. As this angle increases, the distance from the base of the fulcrum to the Earth increases. **27.** The SSS Inequality Theorem compares the angle between two sides of a triangle for which the two sides are congruent and the third side is different. The SSS Postulate states that two triangles that have three sides congruent are congruent.
29. Sample answer: As the operator lifts the pendulum, the angle increases. When the operator lowers the pendulum, the angle decreases. The distance between the ends of the arms increases as the angle between the arms increases, and decreases as the angle decreases.
31. H **33.** yes; $16 + 6 > 19$ **35.** $\overline{AD}$ is not a median of $\triangle ABC$.

37. Given: $\overline{AD}$ bisects $\overline{BE}$; $\overline{AB} \parallel \overline{DE}$.
Prove: $\triangle ABC \cong \triangle DEC$
Proof:



Statements (Reasons)
1. $\overline{AD}$ bisects $\overline{BE}$; $\overline{AB} \parallel \overline{DE}$. (Given)
2. $\overline{BC} \cong \overline{EC}$ (Def. of seg. Bisector)
3. $\angle B \cong \angle E$ (Alt. int. ∠s Thm.)
4. $\angle BCA \cong \angle ECD$ (Vert. ∠s are ≅.)
5. $\triangle ABC \cong \triangle DEC$ (ASA)

39. $EF = 5$, $FG = \sqrt{50}$, $EG = 5$; isosceles
41. $EF = \sqrt{145}$, $FG = \sqrt{544}$, $EG = 35$; scalene
43. $y - 3 = 2(x - 4)$ **45.** $y + 9 = 11(x + 4)$

Pages 310–312 Chapter 5 Study Guide and Review

1. incenter **3.** Triangle Inequality Theorem **5.** angle bisector **7.** orthocenter **9.** $m\angle ACQ = 55$
11. The median could represent the zipper.
13. $DQ < DR$ **15.** $SR > SQ$ **17.** Assume that Gabriel completed at most 20 passes in each of the five games in which he played. If we let p_1, p_2, p_3, p_4, and p_5 be the number of passes Gabriel completed in games 1, 2, 3, 4, and 5, respectively, then $p_1 + p_2 + p_3 + p_4 + p_5 =$ the total number of passes Gabriel completed $= 101$. Because we have assumed that he completed at most 20 passes in each of the five games, $p_1 \leq 20$ and $p_2 \leq 20$ and $p_3 \leq 20$ and $p_4 \leq 20$ and $p_5 \leq 20$. Then, by a property of inequalities, $p_1 + p_2 + p_3 + p_4 + p_5 \leq 20 + 20 + 20 + 20 + 20$ or 100 passes. But this says that Gabriel completed at most 100 passes this season, which contradicts the information we were given, that he completed 101 passes. So our assumption must be false. Thus, Gabriel completed more than 20 passes in at least one game this season. **19.** yes, $5 + 16 > 20$
21. no, $19 + 19 \not> 41$ **23.** The pole is the same length as itself, the rope is the same length as itself, and the 30° angle is smaller than the 50° angle. By the SAS Inequality, the distance from Wesley to the pole is less than the distance from Nadia to the pole. So, Wesley is standing closer to the pole.

Chapter 6 Quadrilaterals

Page 317 Chapter 6 Get Ready

1. 45 **3.** $-\frac{7}{5}, \frac{5}{7}$; perpendicular **5.** Yes, the slopes of $\overline{AB}$ and $\overline{DC}$ are each $\frac{1}{2}$. The slopes of $\overline{AD}$ and $\overline{BC}$ are each -2. Since the product of the slopes is -1, $\overline{AB} \perp \overline{BC}$ and $\overline{AD} \perp \overline{DC}$.

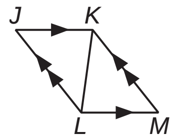
Pages 321–323 Lesson 6-1

1. 540 **3.** 4 **5.** 60, 120 **7.** 5400 **9.** 3060 **11.** $360(2y - 1)$
13. 1080 **15.** 9 **17.** 18 **19.** $m\angle M = 30$, $m\angle P = 120$, $m\angle Q = 60$, $m\angle R = 150$ **21.** $m\angle M = 60$, $m\angle N = 120$, $m\angle P = 60$, $m\angle Q = 120$ **23.** 36, 144 **25.** 40, 140
27. 147.3, 32.7 **29.** 150, 30 **31.** 108, 72 **33.** 105, 110, 120, 130, 135, 140, 160, 170, 180, 190 **35.** A concave polygon has at least one obtuse angle, which means the sum will be different from the formula.

37.
$$\frac{180(n-2)}{n} = \frac{180n - 360}{n} = \frac{180n}{n} - \frac{360}{n} = 180 - \frac{360}{n}$$

39. B **41.** C **43.** $\frac{7}{3} < x < 6$ **45.** no; $5 + 9 \not> 17$
47. yes; $3.5 + 7.2 > 8.4$ **49.** no; $2.2 + 12 \not> 14.3$

51. Given: $\overline{JL} \parallel \overline{KM}$, $\overline{JK} \parallel \overline{LM}$
Prove: $\triangle JKL \cong \triangle MLK$
Proof:

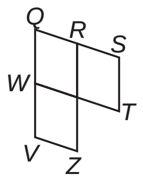


Statements (Reasons)
1. $\overline{JL} \parallel \overline{KM}$, $\overline{JK} \parallel \overline{LM}$ (Given)
2. $\angle MKL \cong \angle JLK$, $\angle JKL \cong \angle MLK$ (Alt. int. ∠s are ≅.)
3. $\overline{KL} \cong \overline{KL}$ (Reflexive Property)
4. $\triangle JKL \cong \triangle MLK$ (ASA)

53. $\angle 1$ and $\angle 4$, $\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$

Pages 328–330 Lesson 6-2

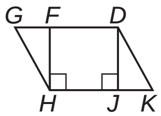
1. Given: ▱$VZRQ$ and ▱$WQST$
Prove: $\angle Z \cong \angle T$
Proof:



Statements (Reasons)
1. ▱$VZRQ$ and ▱$WQST$ (Given)
2. $\angle Z \cong \angle Q$, $\angle Q \cong \angle T$ (Opp. ∠s of a ▱ are ≅.)
3. $\angle Z \cong \angle T$ (Transitive Prop.)

3. $\overline{VQ}$; diag. of ▱ bisect each other. **5.** $\angle STQ$ and $\angle SRQ$; consec. ∠s in ▱ are suppl. **7.** 80 **9.** 30 **11.** 21

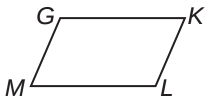
13. Given: ▱$DGHK$, $\overline{FH} \perp \overline{GD}$, $\overline{DJ} \perp \overline{HK}$
Prove: $\triangle DJK \cong \triangle HFG$



Proof:
Statements (Reasons)
1. ▱$DGHK$, $\overline{FH} \perp \overline{GD}$, $\overline{DJ} \perp \overline{HK}$ (Given)
2. $\angle G \cong \angle K$ (Opp. ∠s of ▱ ≅.)
3. $\overline{GH} \cong \overline{DK}$ (Opp. sides of ▱ ≅.)
4. $\angle HFG$ and $\angle DJK$ are rt. ∠s. (⊥ lines form four rt. ∠s.)
5. $\triangle HFG$ and $\triangle DJK$ are rt. △s. (Def. of rt. △s)
6. $\triangle HFG \cong \triangle DJK$ (HA)

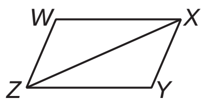
15. $\angle BCD$; opp. ∠s in a ▱ are ≅ **17.** $\overline{DC}$; opp. sides of a ▱ are ∥ **19.** $\triangle CDB$; each diag. of a ▱ separates it into 2 ≅ △s. **21.** 71 **23.** 38 **25.** 97 **27.** 8 **29.** 3.5 **31.** $EQ = 5$, $QG = 5$, $HQ = \sqrt{13}$, $QF = \sqrt{13}$ **33.** Slope of $\overline{EH}$ is undefined, slope of $\overline{EH} = -\frac{1}{3}$; no, the slopes of the sides are not negative reciprocals of each other.

35. Given: ▱$GKLM$
Prove: $\angle G$ and $\angle K$ are supplementary.
$\angle K$ and $\angle L$ are supplementary.
$\angle L$ and $\angle M$ are supplementary.
$\angle M$ and $\angle G$ are supplementary.



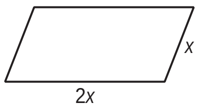
Proof:
Statements (Reasons)
1. ▱$GKLM$ (Given)
2. $\overline{GK} \parallel \overline{ML}$, $\overline{GM} \parallel \overline{KL}$ (Opp. sides of ▱ are ∥.)
3. $\angle G$ and $\angle K$ are supplementary, $\angle K$ and $\angle L$ are supplementary, $\angle L$ and $\angle M$ are supplementary, and $\angle M$ and $\angle G$ are supplementary. (Cons. int. ∠s are suppl.)

37. Given: ▱$WXYZ$
Prove: $\triangle WXZ \cong \triangle YZX$



Proof:
Statements (Reasons)
1. ▱$WXYZ$ (Given)
2. $\overline{WX} \cong \overline{ZY}$, $\overline{WZ} \cong \overline{XY}$ (Opp. sides of ▱ are ≅.)
3. $\angle ZWX \cong \angle XYZ$ (Opp. ∠s of ▱ are ≅.)
4. $\triangle WXZ \cong \triangle YZX$ (SAS)

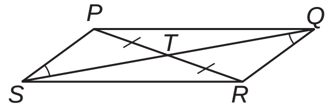
39. $a = 6$, $b = 5$, $DB = 32$
41. Sample answer:



43. Opposite sides are congruent; opposite angles are congruent; consecutive angles are supplementary; and if there is one right angle, there are four right angles. Diagonals bisect each other; each diagonal forms two congruent triangles. **45.** J **47.** 3600 **49.** 6120 **51.** $DG > GH$ **53.** $y = 10x + 82.5$, where $x =$ number of hours worked **55.** diagonal, -14

Pages 337–339 *Lesson 6-3*

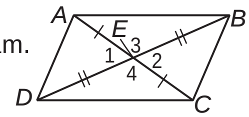
1. Given: $\overline{PT} \cong \overline{TR}$, $\angle TSP \cong \angle TQR$
Prove: $PQRS$ is a parallelogram.



Proof:
Statements (Reasons)
1. $\overline{PT} \cong \overline{TR}$, $\angle TSP \cong \angle TQR$ (Given)
2. $\angle PTS \cong \angle RTQ$ (Vertical ∠s are ≅.)
3. $\triangle PTS \cong \triangle RTQ$ (AAS)
4. $\overline{PS} \cong \overline{QR}$ (CPCTC)
5. $\overline{PS} \parallel \overline{QR}$ (If alt. int. ∠s are ≅, lines are ∥.)
6. $PQRS$ is a parallelogram. (If one pair of opp. sides is ∥ and ≅, then the quad. is a ▱.)

3. No; one pair of opp. sides are not parallel and congruent. **5.** $x = 13$, $y = 4$ **7.** yes **9.** Yes; each pair of opposite angles is congruent. **11.** Yes; opposite angles are congruent. **13.** Yes; one pair of opposite sides is parallel and congruent. **15.** Sample answer: If one pair of opposite sides are congruent and parallel, the quadrilateral is a parallelogram. **17.** Sample answer: If both pairs of opposite sides are parallel and congruent, then the watchbox is a parallelogram.

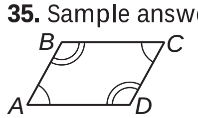
19. Given: $\overline{AE} \cong \overline{EC}$, $\overline{DE} \cong \overline{EB}$
Prove: $ABCD$ is a parallelogram.



Proof:
Statements (Reasons)
1. $\overline{AE} \cong \overline{EC}$, $\overline{DE} \cong \overline{EB}$ (Given)
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$ (Vertical ∠s are ≅.)
3. $\triangle ABE \cong \triangle CDE$, $\triangle ADE \cong \triangle CBE$ (SAS)
4. $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$ (CPCTC)
5. $ABCD$ is a parallelogram. (If both pairs of opp. sides are ≅, then quad is a ▱.)

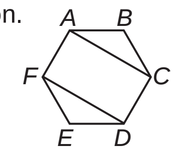
21. $x = 1$, $y = 9$ **23.** $x = 4$, $y = 4$ **25.** $x = 8$, $y = 1\frac{1}{3}$ **27.** no **29.** no **31.** Sample answer: Move Q to (0, 2), S to (1, 2), T to (2, −3), or W to (−8, 0). **33.** (2, −2), (−4, 0), or (0, 4)

35. Sample answer:



Opposite angles are congruent.

37. Given: $ABCDEF$ is a regular hexagon.
Prove: $FDCA$ is a parallelogram.



Proof:
Statements (Reasons)
1. $ABCDEF$ is a regular hexagon. (Given)
2. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\angle E \cong \angle B$, $\overline{FA} \cong \overline{CD}$ (Def. regular hexagon)
3. $\triangle ABC \cong \triangle DEF$ (SAS)
4. $\overline{AC} \cong \overline{DF}$ (CPCTC)
5. $FDCA$ is a ▱. (If both pairs of opp. sides are ≅, then the quad. is a ▱.)

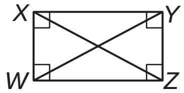
39. B **41.** 12 **43.** 14 units **45.** 8 **47.** 30

49. Given: $P + W > 2$ (P is time spent in the pool, W is time spent lifting weights)
Prove: $P > 1$ or $W > 1$
Proof:
Step 1: Assume $P \leq 1$ and $W \leq 1$.
Step 2: $P + W \leq 2$
Step 3: This contradicts the given statement. Therefore he did at least one of these activities for more than an hour. **51.** 5, $-\frac{3}{2}$; not $\perp$

Pages 344–346 ***Lesson 6-4***

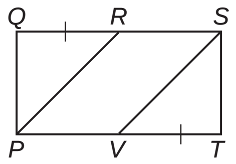
1. 18 **3.** 5 or -2 **5.** Make sure that the angles measure 90 or that the diagonals are congruent. **7.** 84 **9.** 10 or 26 **11.** 22 or 37 **13.** 60 **15.** 30 **17.** 60 **19.** 30 **21.** 120 **23.** about 42 in. **25.** No; consec. sides are not $\perp$. **27.** $WY = \sqrt{130}$; $XZ = \sqrt{130}$ **29.** No; the midpoints of the diagonals are not the same, so the diagonals do not bisect each other. **31.** It is a parallelogram with all sides congruent.

33. Given: $WXYZ$ is a rectangle with diagonals $\overline{WY}$ and $\overline{ZX}$.
Prove: $\overline{WY} \cong \overline{ZX}$



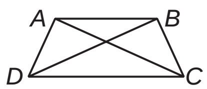
Proof:
1. $WXYZ$ is a rectangle with diagonals $\overline{WY}$ and $\overline{XZ}$. (Given)
2. $\overline{WX} \cong \overline{ZY}$ (Opp. sides of ▱ are $\cong$.)
3. $\overline{WZ} \cong \overline{WZ}$ (Reflexive Property)
4. $\angle XWZ$ and $\angle YZW$ are right angles. (Def. of rectangle)
5. $\angle XWZ \cong \angle YZW$ (All right ∡ are $\cong$.)
6. $\triangle XWZ \cong \triangle YZW$ (SAS)
7. $\overline{WY} \cong \overline{ZX}$ (CPCTC)

35. Given: $PQST$ is a rectangle. $\overline{QR} \cong \overline{VT}$
Prove: $\overline{PR} \cong \overline{VS}$



Proof:
Statements (Reasons)
1. $PQST$ is a rectangle. $\overline{QR} \cong \overline{VT}$ (Given)
2. $PQST$ is a parallelogram. (Def. of rectangle)
3. $\overline{TS} \cong \overline{PQ}$ (Opp. sides of ▱ are $\cong$.)
4. $\angle T$ and $\angle Q$ are rt. ∡. (Definition of rectangle)
5. $\angle T \cong \angle Q$ (All rt. ∡ are $\cong$.)
6. $\triangle RPQ \cong \triangle VST$ (SAS)
7. $\overline{PR} \cong \overline{VS}$ (CPCTC)

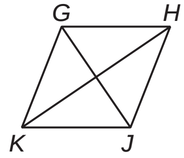
37. No; there are no parallel lines in spherical geometry. **39.** No; the sides are not parallel.
41. Sample answer: $\overline{AC} \cong \overline{BD}$ but $ABCD$ is not a rectangle.



43. McKenna; Consuelo's definition is correct if one pair of opposite sides is parallel and congruent.
45. If consecutive sides are perpendicular or diagonals are congruent, then the parallelogram is a rectangle. **47.** J **49.** 97 **51.** 11 **53.** 5 **55.** 29

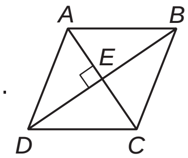
Pages 351–354 ***Lesson 6-5***

1. Given: $\triangle KGH$, $\triangle HJK$, $\triangle GHJ$, and $\triangle JKG$ are isosceles.
Prove: $GHJK$ is a rhombus.



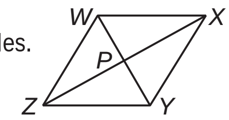
Proof:
Statements (Reasons)
1. $\triangle KGH$, $\triangle HJK$, $\triangle GHJ$, and $\triangle JKG$ are isosceles. (Given)
2. $\overline{KG} \cong \overline{GH}$, $\overline{HJ} \cong \overline{KJ}$, $\overline{GH} \cong \overline{HJ}$, $\overline{KG} \cong \overline{KJ}$ (Def. of isosceles △)
3. $\overline{KG} \cong \overline{HJ}$, $\overline{GH} \cong \overline{KJ}$ (Transitive Property)
4. $\overline{KG} \cong \overline{GH}$, $\overline{HJ} \cong \overline{KJ}$ (Substitution)
5. $GHJK$ is a rhombus. (Def. of rhombus)

3. 5 **5.** 96.8 **7.** None; the diagonals are not congruent or perpendicular



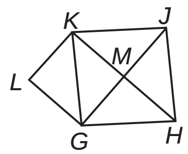
9. Given: $ABCD$ is a parallelogram. $\overline{AC} \perp \overline{BD}$
Prove: $ABCD$ is a rhombus.
Proof: We are given that $ABCD$ is a parallelogram. The diagonals of a parallelogram bisect each other, so $\overline{AE} \cong \overline{EC}$, $\overline{BE} \cong \overline{BE}$, because congruence of segments is reflexive. We are also given that $\overline{AC} \perp \overline{BD}$. Thus, $\angle AEB$ and $\angle BEC$ are right angles by the definition of perpendicular lines. Then $\angle AEB \cong \angle BEC$ because all right angles are congruent. Therefore, $\triangle AEB \cong \triangle CEB$ by SAS. $\overline{AB} \cong \overline{CB}$ by CPCTC. Opposite sides of parallelograms are congruent, so $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$. Then since congruence of segments is transitive, $\overline{AB} \cong \overline{CD} \cong \overline{BC} \cong \overline{AD}$. All four sides of $ABCD$ are congruent, so $ABCD$ is a rhombus by definition.

11. Given: $\triangle WZY \cong \triangle WXY$
$\triangle WZY$ and $\triangle XYZ$ are isosceles.
Prove: $WXYZ$ is a rhombus.



Proof:
Statements (Reasons)
1. $\triangle WZY \cong \triangle WXY$, $\triangle WZY$ and $\triangle XYZ$ are isosceles. (Given)
2. $\overline{WZ} \cong \overline{WX}$, $\overline{ZY} \cong \overline{XY}$ (CPCTC)
3. $\overline{WZ} \cong \overline{ZY}$, $\overline{WX} \cong \overline{XY}$ (Def. of isosceles △)
4. $\overline{WZ} \cong \overline{WX}$, $\overline{ZY} \cong \overline{XY}$ (Substitution Property)
5. $WXYZ$ is a rhombus. (Def. of rhombus)

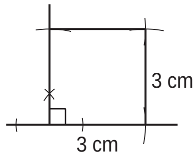
13. Given: $\triangle LGK \cong \triangle MJK$
$GHJK$ is a parallelogram.
Prove: $GHJK$ is a rhombus.



Proof:
Statements (Reasons)
1. $\triangle LGK \cong \triangle MJK$; $GHJK$ is a parallelogram. (Given)
2. $\overline{KG} \cong \overline{KJ}$ (CPCTC)
3. $\overline{KJ} \cong \overline{GH}$, $\overline{KG} \cong \overline{JH}$ (Opp. sides of ▱ are $\cong$.)
4. $\overline{KG} \cong \overline{JH}$, $\overline{GH} \cong \overline{JK}$ (Substitution Property)
5. $GHJK$ is a rhombus. (Def. of rhombus)

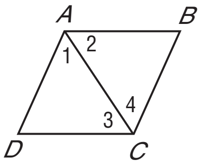
15. 37 **17.** 8 **19.** Rhombus; the diagonals are perpendicular. **21.** Square, rectangle, rhombus; all sides are congruent and perpendicular. **23.** No; it is about 11,662.9 mm.

25. Sample answer:



27. Each of the brown quadrilaterals is a rhombus. Each side is the same length, and the interior angles are not right angles. **29.** 40 cm **31.** always **33.** never **35.** True; squares have all the properties of rectangles.

37. If a diagonal of a parallelogram bisects an angle of a parallelogram, then the parallelogram is a rhombus.
Given: *ABCD* is a parallelogram; diagonal $\overline{AC}$ bisects $\angle DAB$ and $\angle BCD$
Prove: ▱*ABCD* is a rhombus

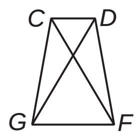


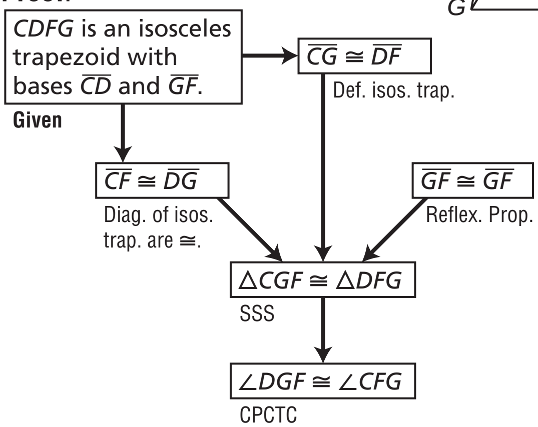
Proof: It is given that *ABCD* is a parallelogram. Since opposite sides of a parallelogram are parallel, $\overline{AB} \parallel \overline{DC}$. By definition, $\angle 2$ and $\angle 3$ are alternate interior angles of parallel lines $\overline{AB}$ and $\overline{DC}$. Since alternate interior angles of parallel lines are congruent, $\angle 2 \cong \angle 3$. Congruence of angles is symmetric, therefore $\angle 3 \cong \angle 2$. It is given that $\overline{AC}$ bisects $\angle DAB$ and $\angle BCD$, so $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ by definition. By the transitive property, $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. The sides opposite congruent in a triangle are congruent, therefore $\overline{AD} \cong \overline{DC}$ and $\overline{AB} \cong \overline{BC}$. So, by definition, *ABCD* is a rhombus.

39. Sample answer: Rhombi and squares both have all four sides congruent, but the diagonals of a square are congruent. A square has four right angles and rhombi have each pair of opposite angles congruent, but not all angles are necessarily congruent. Since the angles of a rhombus are not all congruent, riding over the same road would not be smooth. **41.** F **43.** 140 **45.** yes **47.** 278 mi < *d* < 1160 mi **49.** 2.4

Pages 359–362 ***Lesson 6-6***

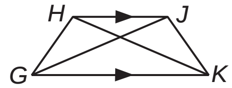
1. Given: *CDFG* is an isosceles trapezoid with bases $\overline{CD}$ and $\overline{GF}$.
Prove: $\angle DGF \cong \angle CFG$
Proof:



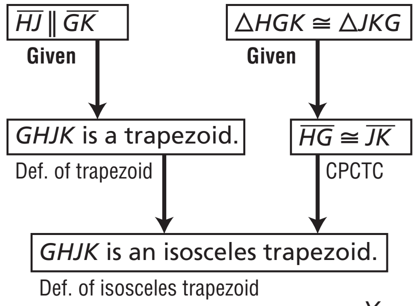


3. $\overline{RS} \parallel \overline{QT}$ and $\overline{QR} \neq \overline{ST}$; *QRST* is a trapezoid. **5.** 4

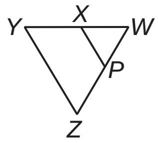
7. Given: $\overline{HJ} \parallel \overline{GK}$, $\overline{HG} \nparallel \overline{JK}$, $\triangle HGK \cong \triangle JKG$
Prove: *GHJK* is an isosceles trapezoid.

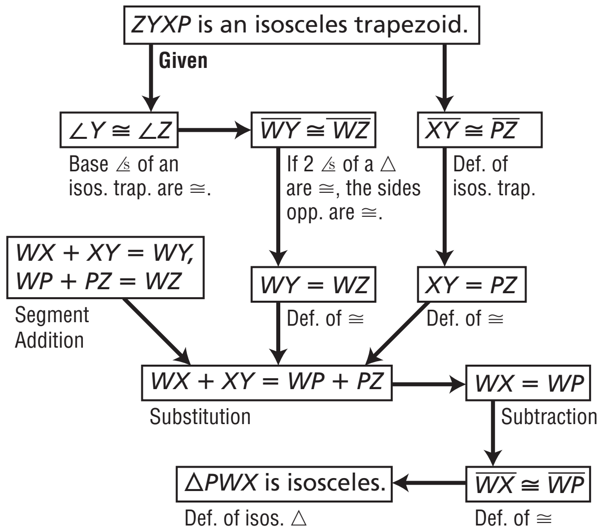


Proof:

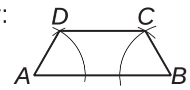


9. Given: *ZYXP* is an isosceles trapezoid.
Prove: △*PWX* is an isosceles.

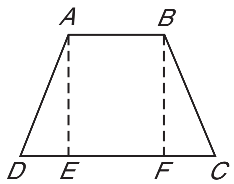




11. The flag of Brunei contains one white parallelogram and one black parallelogram. The flag of Norway contains two red squares, two red rectangles, eight white rectangles, and two blue rectangles. The flag of the Bahamas contains two blue trapezoids.
13a. $\overline{AD} \parallel \overline{BC}$, $\overline{CD} \parallel \overline{AB}$; **13b.** not isosceles, $AB = \sqrt{17}$ and $CD = 5$ **15a.** $\overline{DC} \parallel \overline{FE}$, $\overline{DC} \nparallel \overline{FC}$; **15b.** isosceles, $DE = \sqrt{50}$, $CF = \sqrt{50}$ **17.** c **19.** 14, 110, 110 **21.** 62
23. Sample answer:



25. parallelogram, opp. sides ∥, no rt. ∠, no cons. sides ≅ **27.** $\overline{QR} \parallel \overline{PS}$, not isosceles, $PQ \neq RS$ **29.** 7.5
31. If the base angles of a trapezoid are congruent, then the trapezoid is isosceles.
Given: *ABCD* is a trapezoid; $\angle D \cong \angle C$
Prove: trapezoid *ABCD* is isosceles



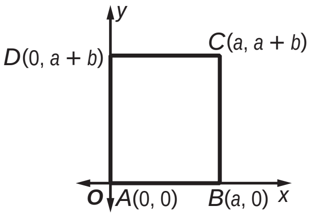
Proof: Draw auxiliary segments so that $\overline{AE} \perp \overline{DC}$ and $\overline{BF} \perp \overline{DC}$. Perpendicular lines form right angles, so $\angle AED$ and $\angle BFC$ are right angles. $\triangle ADE$ and $\triangle BFC$ are right triangles by definition. Since $\overline{AB} \parallel \overline{DC}$ and parallel lines are everywhere equidistant, $\overline{AE} \cong \overline{BF}$. It is given that $\angle D \cong \angle C$. Therefore, $\triangle ADE \cong \triangle BFC$ by LA. $\overline{AD} \cong \overline{BC}$ by CPCTC. Therefore, by definition, trapezoid $ABCD$ is isosceles.

33. Trapezoids have exactly one pair of opposite sides parallel. This is the minimum requirement to prove that a quadrilateral is a trapezoid. **35.** F **37.** 10

39. 70 **41.** $(-2, -1)$ $\left(-2, -\frac{3}{2}\right)$ **43.** about 3.3 million per year **45.** 0

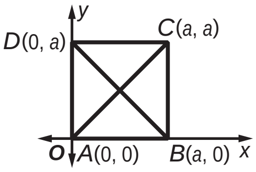
Pages 366–368 ***Lesson 6-7***

1.



3. (c, b)

5. Given: $ABCD$ is a square.
Prove: $\overline{AC} \perp \overline{DB}$

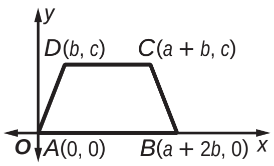


Proof:
Slope of $\overline{DB} = \frac{0 - a}{a - 0}$ or –1

Slope of $\overline{AC} = \frac{0 - a}{0 - a}$ or 1

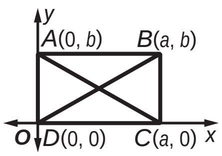
The slope of $\overline{AC}$ is the negative reciprocal of the slope of $\overline{DB}$, so they are perpendicular.

7.



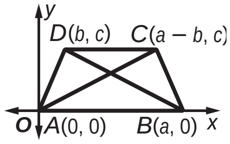
9. $B(-b, c)$ **11.** $G(a, 0)$, $E(-b, c)$ **13.** $T(-2a, c)$, $W(-2a, -c)$

15. Given: $ABCD$ is a rectangle.
Prove: $\overline{AC} \cong \overline{DB}$



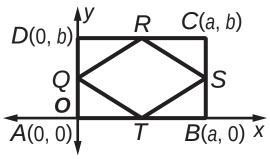
Proof:
Use the Distance Formula to find $AC = \sqrt{a^2 + b^2}$ and $BD = \sqrt{a^2 + b^2}$. $\overline{AC}$ and $\overline{BD}$ have the same length, so they are congruent.

17. Given: isosceles trapezoid $ABCD$ with $\overline{AD} \cong \overline{BC}$
Prove: $\overline{BD} \cong \overline{AC}$



Proof:
$DB = \sqrt{(a - b)^2 + (0 - c)^2} = \sqrt{(a - b)^2 + c^2}$
$AC = \sqrt{((a - b) - 0)^2 + (c - 0)^2} = \sqrt{(a - b)^2 + c^2}$
$BD = AC$ and $\overline{BD} \cong \overline{AC}$

19. Given: $ABCD$ is a rectangle. Q, R, S, and T are midpoints of their respective sides.
Prove: $QRST$ is a rhombus.



Proof:
Midpoint Q is $\left(\frac{0 + 0}{2}, \frac{b + 0}{2}\right)$ or $\left(0, \frac{b}{2}\right)$.

Midpoint R is $\frac{a + 0}{2}, \frac{b + b}{2}$ or $\left(\frac{a}{2}, \frac{2b}{2}\right)$ or $\left(\frac{a}{2}, b\right)$.

Midpoint S is $\left(\frac{a + a}{2}, \frac{b + 0}{2}\right)$ or $\left(\frac{2a}{2}, \frac{b}{2}\right)$ or $\left(a, \frac{b}{2}\right)$.

Midpoint T is $\left(\frac{a + 0}{2}, \frac{0 + 0}{2}\right)$ or $\left(\frac{a}{2}, 0\right)$.

$QR = \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(b - \frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

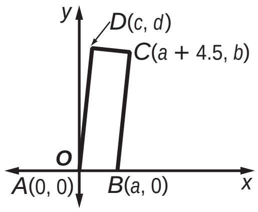
$RS = \sqrt{\left(a - \frac{a}{2}\right)^2 + \left(\frac{b}{2} - b\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2}$ or $\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$ST = \sqrt{\left(a - \frac{a}{2}\right)^2 + \left(\frac{b}{2} - 0\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$QT = \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(0 - \frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2}$ or $\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

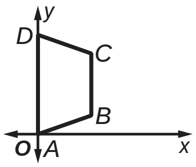
$QR = RS = ST = QT$
$\overline{QR} \cong \overline{RS} \cong \overline{ST} \cong \overline{QT}$
$QRST$ is a rhombus.

21.



23. From the information given, we can approximate the height from the ground to the top level of the tower.

25.

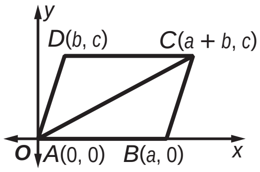


27. Sample answer: The coordinate plane is used in coordinate proofs. The Distance Formula, Midpoint Formula, and Slope Formula are used to prove theorems. Place the figure so one of the vertices is at the origin. Place at least one side of the figure on the positive *x*-axis. Keep the figure in the first quadrant if possible and use coordinates that will simplify calculations. **29.** H **31.** 55 **33.** 160 **35.** 6 **37.** - 12% per year **39.** 2003; In 2002 91% of companies recruit using the Web. Each year 15.5% more companies use the Web to recruit employees, so by 2003 100% of companies will recruit using the Web.

Pages 369–372 ***Chapter 6*** ***Study Guide and Review***

1. true **3.** false; rectangle **5.** false; trapezoid **7.** true **9.** true **11.** octagon **13.** 6.86 **15.** 9 **17.** no **19.** yes **21.** 52 **23.** Yes; the diagonals are congruent and perpendicular (the Pythagorean Theorem proves that the four triangles formed by the diagonals are right triangles). **25.** Sample answer: Measure the lengths of the legs $\overline{AD}$ and $\overline{BC}$ to verify that they are congruent.

27. Given: *ABCD* is a parallelogram.
Prove: $\triangle ABC \cong \triangle CDA$



$AB = \sqrt{(a - 0)^2 + (0 - 0)^2}$
$= \sqrt{a^2 - 0^2}$ or a
$DC = \sqrt{((a + b) - b)^2 + (c - c)^2}$
$= \sqrt{a^2 + 0^2}$ or a
$AD = \sqrt{(b - 0)^2 + (c - 0)^2}$
$= \sqrt{b^2 + c^2}$
$BC = \sqrt{((a + b) - a)^2 + (c - 0)^2}$
$= \sqrt{b^2 + c^2}$

$\overline{AB}$ and $\overline{DC}$ have the same measure, so $\overline{AB} \cong \overline{DC}$. $\overline{AD}$ and $\overline{BC}$ have the same measure, so $\overline{AD} \cong \overline{BC}$. $\overline{AC} \cong \overline{AC}$ by the Reflective Property. Therefore, $\triangle ABC \cong \triangle CDA$ by SSS.

Chapter 7 Proportions and Similarity

Page 379 ***Chapter 7*** ***Getting Ready***

1. 15 **3.** 10 **5.** 37.5 mi **7.** $-\frac{6}{5}$ **9.** yes; ≅ alt. ext. ∠s **11.** no

Pages 383–386 ***Lesson 7-1***

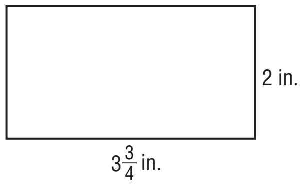
1. 5.9 : 1 **3.** 45, 63, 72 **5.** 2.1275 **7.** 160 **9.** 1:2 **11.** $\frac{1}{12}$ **13.** 18 in., 24 in., 30 in. **15.** 3 cm, 2 cm, 1.2 cm **17.** 43.2, 64.8, 72 **19.** 3.2 **21.** 5 **23.** 3 **25.** - 1, $\frac{-2}{3}$, 1 **27.** 18 ft, 24 ft **29.** about 75 in. **31.** Movie B; 1 : 1 **33.** ≈ 4.7 gal **35.** 3.2 in. by 4 in. **37.** Yes, the ratio of sides is 1.617; 22.83 ft. **39.** See students' work. **41.** Cross multiply and divide by 28. **43.** isosceles triangle **45.** rectangle or parallelogram **47.** D **49.** *B*(3*a*, 0) **51.** 12 **53.** always **55.** never **57.** 75 **59.** 105 **61.** 105 **63.** 20.0 **65.** 1.3

Pages 392–396 ***Lesson 7-2***

1. Yes; because $\angle P \cong \angle Q \cong \angle R \cong \angle G \cong \angle H \cong \angle I$ and $\frac{PQ}{HQ} = \frac{QR}{HI} = \frac{RP}{IG} = \frac{3}{7}$. **3.** $\frac{1}{100}$ **5.** polygon *ABCD* ~ polygon *EFGH*; 23; 28; 20; 32; $\frac{1}{2}$ **7.** 2 h 20 min **9.** $\triangle XYW \sim \triangle XWZ$; $\angle 6 \cong \angle 5$ because if two angles of one triangle are congruent to two angles of a second triangle, then the third angles are congruent. The ratio of the corresponding sides is 1. **11.** $\triangle BCD \sim \triangle PNM$; $\angle B \cong \angle P$, $\angle D \cong \angle M$, and $\angle C \cong \angle N$ because if two angles of one triangle are congruent to two angles of a second triangle, then the third angles are congruent.

13. 9:1 **15.** $\triangle ABC \sim \triangle EDC$; $\frac{3}{5}$; $AC = 7\frac{3}{5}$; $CE = 11\frac{2}{5}$; $\frac{2}{3}$ **17.** $\angle RST \cong \angle EGF$; 7.5, $GF = 7.5$; $EG = 15.525$; $\frac{4}{3}$ **19.** 30; 70 **21.** 3 hrs 32 min **23.** 15 cm, 10 cm

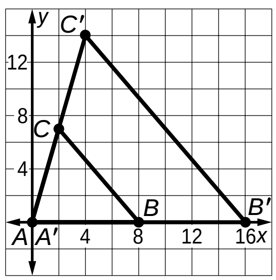
25.



27. 2:1; The two ratios are the same. **29.** 7.5 **31.** 108

33. 73.2 **35.** $\frac{8}{5}$ **37.** always **39.** sometimes **41.** always

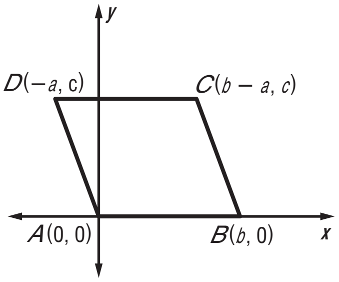
43.



45. $\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$ **47.** The sides are proportional and the angles are congruent, so the triangles are similar. **49.** A rectangle with consecutive sides of 4 in. and 12 in. would not have sides proportional to a rectangle with consecutive sides of 6 in. and 8 in. because $\frac{4}{6} \neq \frac{12}{8}$. **51.** 4:1 **53.** $\frac{a}{3a} = \frac{b}{3b} = \frac{c}{3c} = \frac{(a + b + c)}{(3a + b + c)} = \frac{1}{3}$ **55.** The further the figures are from the center of the circle, the smaller they are. The decreasing size gives a sense of perspective. Each of the black figures are similar and each of the white figures are similar.

57. A **59.** 5.2 **61.** $-\frac{10}{9}$ **63.** $-\frac{4}{11}$

65.



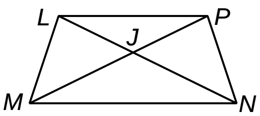
67. 93 **69.** 115 **71.** 118 **73.** 62 **75.** 62

Pages 400–403 ***Lesson 7-3***

1. No; corresponding sides are not proportional. **3.** B **7.** yes; $\triangle QRS \sim \triangle TVU$ by SSS Similarity **9.** yes; $\triangle AEC \sim \triangle BDC$ by AA Similarity **11.** 478 ft **13.** It is difficult to measure shadows within a city. **15.** $\triangle ABE \sim \triangle ACD$; $x = \frac{8}{5}$; $AB = 3\frac{3}{5}$; $AC = 9\frac{3}{5}$ **17.** $\triangle ABC \sim \triangle ARS$; $x = 8$; 15; 8 **19.** $\frac{3}{2}$

21. Given: $\overline{LP} \parallel \overline{MN}$

Prove: $\frac{LJ}{JN} = \frac{PJ}{JM}$



Proof:

Statements (Reasons)

1. $\overline{LP} \parallel \overline{MN}$ (Given)

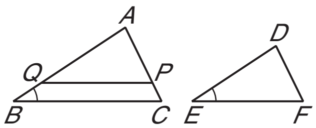
2. $\angle PLN \cong \angle LNM$, $\angle LPM \cong \angle PMN$ (Alt. Int. $\angle$ Thm.)

3. $\angle LPJ \sim \angle NMJ$ (AA Similarity)

4. $\frac{LJ}{JN} = \frac{PJ}{JM}$ (Corr. sides of $\sim \triangle$s are proportional.)

23. Given: $\angle B \cong \angle E$, $\overline{QP} \parallel \overline{BC}$; $\overline{QP} \cong \overline{EF}$, $\frac{AB}{DE} = \frac{BC}{EF}$

Prove: $\triangle ABC \sim \triangle DEF$



Proof:

Statements (Reasons)

1. $\angle B \cong \angle E$, $\overline{QP} \parallel \overline{BC}$; $\frac{AB}{DE} = \frac{BC}{EF}$ (Given)

2. $\angle APQ \cong \angle C$, $\angle AQP \cong \angle B$ (Corr. $\sim \measuredangle$ Post.)

3. $\angle AQP \cong \angle E$ (Transitive Prop.)

4. $\triangle ABC \sim \triangle AQP$ (AA Similarity)

5. $\frac{AB}{AQ} = \frac{BC}{QP}$ (Def. of $\sim \triangle$s)

6. $AB \cdot QP = AQ \cdot BC$; $AB \cdot EF = DE \cdot BC$ (Cross products)

7. $QP = EF$ (Def. of $\cong$ segments)

8. $AB \cdot EF = AQ \cdot BC$ (Substitution)

9. $AQ \cdot BC = DE \cdot BC$ (Substitution)

10. $AQ = DE$ (Div. Prop.)

11. $\overline{AQ} \cong \overline{DE}$ (Def. of $\cong$ segments)

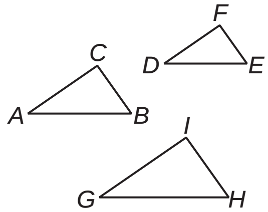
12. $\triangle AQP \cong \triangle DEF$ (SAS)

13. $\angle APQ \cong \angle F$ (CPCTC)

14. $\angle C \cong \angle F$ (Transitive Prop.)

15. $\triangle ABC \sim \triangle DEF$ (AA Similarity)

25.



Given: $\triangle ABC$

Prove: $\triangle ABC \sim \triangle ABC$

Proof:

Statements (Reasons)

1. $\triangle ABC$ (Given)

2. $\angle A \cong \angle A$, $\angle B \cong \angle B$ (Refl. Prop.)

3. $\triangle ABC \sim \triangle ABC$ (AA Similarity)

Symmetric Property of Similarity

Given: $\triangle ABC \sim \triangle DEF$

Prove: $\triangle DEF \sim \triangle ABC$

Proof:

Statements (Reasons)

1. $\triangle ABC \sim \triangle DEF$ (Given)

2. $\angle A \cong \angle D$, $\angle B \cong \angle E$ (Def. of $\sim$ polygons)

3. $\angle D \cong \angle A$, $\angle E \cong \angle B$ (Symmetric Prop.)

4. $\triangle DEF \sim \triangle ABC$ (AA Similarity)

Transitive Property of Similarity

Given: $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle GHI$

Prove: $\triangle ABC \sim \triangle GHI$

Proof:

Statements (Reasons)

1. $\triangle ABC \sim \triangle DEF$, $\triangle DEF \sim \triangle GHI$ (Given)

2. $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle D \cong \angle G$, $\angle E \cong \angle H$ (Def. of $\sim$ polygons)

3. $\angle A \cong \angle G$, $\angle B \cong \angle H$ (Trans. Prop.)

4. $\triangle ABC \sim \triangle GHI$ (AA Similarity)

27. Yes; suppose $\triangle RST$ has angles that measure 46°, 54°, and 80°, $\triangle ABC$ has angles that measure 39°, 63°, and 78°, and $\triangle EFG$ has angles that measure 39°, 63°, and 78°. So $\triangle ABC$ is not similar to $\triangle RST$ and $\triangle RST$ is not similar to $\triangle EFG$, but $\triangle ABC$ is similar to $\triangle EFG$. **29.** Alicia; while both have corresponding sides in a ratio, Alicia has them in proper order with the numerators from the same triangle. **31.** C **33.** D **35.** 5 **37.** 15 **39.** yes; Law of Detachment

41. $\frac{5\sqrt{3}}{9}$ **43.** $\frac{2\sqrt{6}}{9}$

Pages 410–414 ***Lesson 7-4***

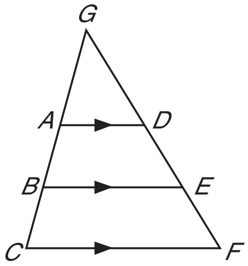
1. 7.5 **3.** 10 **5.** Yes; $\frac{MN}{NP} = \frac{MR}{RQ} = \frac{9}{16}$, so $\overline{RN} \parallel \overline{QP}$. **7.** (4, 3); (−3, 3) **9.** $DE = 7$ and $BC = 14$; $DE = \frac{1}{2}BC$ **11.** $x = 2$; $y = 5$ **13.** 6 **15.** 9 **17.** $x = 6$, $ED = 9$ **19.** $BC = 10$, $FE = 13\frac{1}{3}$, $CD = 9$, $DE = 15$ **21.** no; $\frac{PQ}{QR} \neq \frac{PT}{TS}$ **23.** yes; $\frac{PQ}{QR} = \frac{PT}{TS}$ **25.** The slopes of $\overline{TS}$ and $\overline{WM}$ are both −1, so $\overline{WM} \parallel \overline{TS}$; $\overline{WM}$ is not a midsegment because W and M are not midpoints of their respective sides. **27.** $DE = \frac{\sqrt{90}}{2}$ and $AB = \sqrt{90}$

29. 25 ft **31**. 18.75 ft **33**. $x = 1$, $y = \frac{3}{2}$ **35**. 10 **37**. (3, 8) or (4, 4)

39. $\triangle ABC \sim \triangle ADE$ SAS Similarity

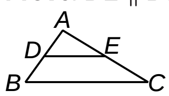
$\frac{AD}{AB} = \frac{DE}{BC}$	$\triangle$Prop. Th.
$\frac{40}{100} = \frac{DE}{BC}$	Substitution
$\frac{2}{5} = \frac{DE}{BC}$	Simplify.
$\frac{2}{5}BC = DE$	Multiply.

41. Given: $\overline{AD} \parallel \overline{BE} \parallel \overline{CF}$
$\overline{AB} \cong \overline{BC}$
Prove: $\overline{DE} \cong \overline{EF}$



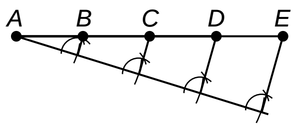
Proof:
From Corollary 7.1, $\frac{AB}{BC} = \frac{DE}{EF}$. Since $\overline{AB} \cong \overline{BC}$, $AB = BC$ by definition of congruence. Therefore, $\frac{AB}{BC} = 1$. By substitution, $1 = \frac{DE}{EF}$. Thus, $DE = EF$. By definition of congruence, $\overline{DE} \cong \overline{EF}$.

43. Given: D is the midpoint of $\overline{AB}$.
E is the midpointof $\overline{AC}$.
Prove: $\overline{DE} \parallel \overline{BC}$; $DE = \frac{1}{2}BC$



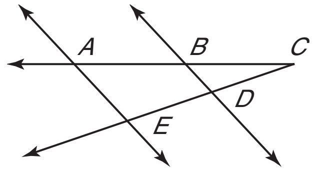
Proof:
Statements (Reasons)
1. D is the midpoint of $\overline{AB}$; E is the midpoint of $\overline{AC}$. (Given)
2. $\overline{AD} \cong \overline{DB}$, $\overline{AE} \cong \overline{EC}$ (Midpoint Theorem)
3. $AD = DB$, $AE = EC$ (Def. of $\cong$ segments)
4. $AB = AD + DB$, $AC = AE +$ EC (Segment Addition Postulate)
5. $AB = AD + AD$, $AC = AE + AE$ (Substitution)
6. $AB = 2AD$, $AC = 2AE$ (Substitution)
7. $\frac{AB}{AD} = 2$, $\frac{AC}{AE} = 2$ (Div. Prop.)
8. $\frac{AB}{AD} = \frac{AC}{AE}$ (Transitive Prop.)
9. $\angle A \cong \angle A$ (Reflexive Prop.)
10. $\triangle ADE \sim \triangle ABC$ (SAS Similarity)
11. $\angle ADE \cong \angle ABC$ (Def. of $\sim$ polygons)
12. $\overline{DE} \parallel \overline{BC}$ (If corr. $\measuredangle$ are $\cong$, the lines are parallel.)
13. $\frac{BC}{DE} = \frac{AB}{AD}$ (Def. of $\sim$ polygons)
14. $\frac{BC}{DE} = 2$ (Substitution Prop.)
15. $2DE = BC$ (Mult. Prop.)
16. $DE = \frac{1}{2}BC$ (Division Prop.)

45.



47. Sample answer: If a line intersects two sides of a triangle and separates sides into corresponding segments of proportional lengths, then it is parallel to the third side.

49. Given: $AB = 4$, $BC = 4$, $ED = DC$
Prove: $\overline{BD} \parallel \overline{AE}$



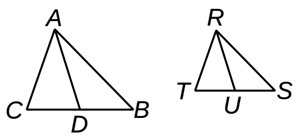
Proof:
Statement (Reason's)
1. $AB = 4$, $BC = 4$ (Given)
2. $4 = BC$ (Symmetric Prop.)
3. $AB = BC$ (Transitive Prop.)
4. $AB + BC = AC$ (Segment Addition Postulate)
5. $BC + BC = AC$ (Substitution)
6. $2BC = AC$ (Substitution)
7. $AC = 2BC$ (Symmetric Prop.)
8. $\frac{AC}{BC} = 2$ (Div. Prop.)
9. $ED = DC$ (Given)
10. $ED + DC = EC$ (Segment Addition Postulate)
11. $DC + DC = EC$ (Substitution)
12. $2DC = EC$ (Substitution)
13. $2 = \frac{EC}{DC}$ (Div. Prop.)
14. $\frac{AC}{BC} = \frac{EC}{DC}$ (Transitive Prop.)
15. $\angle C \cong \angle C$ (Reflexive Prop.)
16. $\triangle ACE \sim \triangle BCD$ (SAS Similarity)
17. $\angle CAE \cong \angle CBD$ (Def. of $\sim$ polygons)
18. $\overline{BD} \parallel \overline{AE}$ (If corr. $\measuredangle$ are $\cong$, lines are $\parallel$.)

51. $\overline{EF} \parallel \overline{GH}$, $\overline{FG} \parallel \overline{EH}$, $\overline{EF} \cong \overline{GH}$, $\overline{FG} \cong \overline{EH}$ **53**. Sample answer: A city planner would need to know that the shortest distance between two parallel lines is the perpendicular distance. **55**. J **57**. yes; SSS **59**. $x = 10$, $y = 18$ **61**. 20, 10, 40, 25 **63**. $\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$

Pages 419–422 ***Lesson 7-5***

1. 10.8

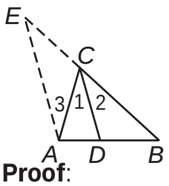
3. **Given:** $\triangle ABC \sim \triangle RST$
$\overline{AD}$ is a median of $\triangle ABC$.
$\overline{RU}$ is a median of $\triangle RST$.
Prove: $\frac{AD}{RU} = \frac{AB}{RS}$



Proof:
Statements (Reasons)

1. $\triangle ABC \sim \triangle RST$; $\overline{AD}$ is a median of $\triangle ABC$; $\overline{RU}$ is a median of $\triangle RST$. (Given)
2. $CD = DB$; $TU = US$ (Def. of median)
3. $\frac{AB}{RS} = \frac{CB}{TS}$ (Def. of $\sim$ polygons)
4. $CB = CD + DB$; $TS = TU + US$ (Seg. Add. Post.)
5. $\frac{AB}{RS} = \frac{CD + DB}{TU + US}$ (Substitution)
6. $\frac{AB}{RS} = \frac{DB + DB}{US + US}$ or (Substitution)
7. $\frac{AB}{RS} = \frac{DB}{US}$ (Substitution)
8. $\angle B \cong \angle S$ (Def. of $\sim$ polygons)
9. $\triangle ABD \sim \triangle RSU$ (SAS Similarity)
10. $\frac{AD}{RU} = \frac{AB}{RS}$ (Def. of $\sim$ polygons)

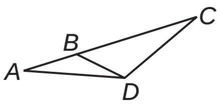
5. 15 **7.** 550 cm or 5.5 m **9.** 63 **11.** 20.25 **13.** 78
15. Given: $\overline{CD}$ bisects $\angle ACB$.
By construction $\overline{AE} \parallel \overline{CD}$.
Prove: $\frac{AD}{DB} = \frac{AC}{BC}$



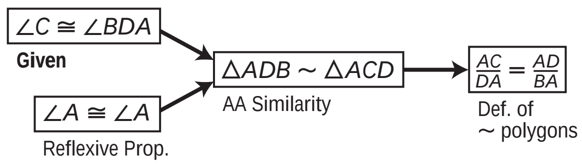
Proof:
Statements (Reasons)

1. $\overline{CD}$ bisects $\angle ACB$; By construction, $\overline{AE} \parallel \overline{CD}$. (Given)
2. $\frac{AD}{DB} = \frac{EC}{BC}$ (Triangle Proportionality Theorem)
3. $\angle 1 \cong \angle 2$ (Definition of Angle Bisector)
4. $\angle 3 \cong \angle 1$ (Alternate Interior Angle Theorem)
5. $\angle 2 \cong \angle E$ (Corresponding Angle Postulate)
6. $\angle 3 \cong \angle E$ (Transitive Prop.)
7. $\overline{EC} \cong \overline{AC}$ (Isosceles $\triangle$ Th.)
8. $EC = AC$ (Def. of congruent segments)
9. $\frac{AD}{DB} = \frac{AC}{BC}$ (Substitution)

17. Given: $\angle C \cong \angle BDA$
Prove: $\frac{AC}{DA} = \frac{AD}{BA}$



Proof:



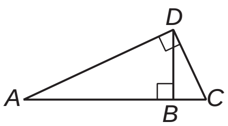
19. 8.5 **21.** 2 **23.** Yes; the perimeters are in the same ratio as the sides, $\frac{300}{600}$ or $\frac{1}{2}$. **25.** 6 **27.** 36 **29.** $\triangle ABC \sim \triangle MNQ$ and $\overline{AD}$ and $\overline{MR}$ are altitudes, angle bisectors, or medians. **31.** Sample answer: 6, 8, 10 and 9, 12, 15 **33.** A **35.** B **37.** not enough information to determine **39.** $\triangle VZW \sim \triangle XYW$; $x = 6$, $VW = 12$, $WX = 10$ **41.** $y = 4x + 5$

Pages 424–426 ***Chapter 7*** ***Study Guide and Review***
1. false; similar **3.** false; proportional **5.** false; ratio **7.** false; one-half **9.** $x = \frac{3}{5}$ **11.** The average weight for an adult who is 71 in. tall would be approximately 26.87 lb. Based on this result, length, and weight do not form a proportion as children grow. **13.** Yes, these are rectangles, so all angles are congruent. Additionally, all sides are in a 3:2 ratio. **15.** The flagpole is 40.6 ft tall. **17.** Yes; $\frac{MT}{SM} = \frac{TN}{TR}$ **19.** The length from your lens to your retina is 1 in.

Chapter 8 Right Triangles and Trigonometry

Page 431 ***Chapter 8*** ***Get Ready***
1. 16 **3.** $d = 24$, $f = 12$ **5.** 42 in. **7.** 10 **9.** 30.41 **11.** 22.5º, 67.5º, 90º

Pages 435–438 ***Lesson 8-1***
1. 6 **3.** $4\sqrt{3} \approx 6.9$ **5.** $2\sqrt{3} \approx 3.5$ **7.** 33.8 ft **9.** $x = 6$, $y = 4\sqrt{3}$ **11.** $10\sqrt{6} \approx 24.5$ **13.** 14 **15.** $\frac{12}{5}$ **17.** $\frac{\sqrt{65}}{7} \approx 1.2$ **19.** 12 **21.** $\sqrt{147} \approx 12.1$ **23.** 5 **25.** about 19.6 mi **27.** $x = \frac{50}{3}$; $y = 10$; $z = \frac{40}{3}$ **29.** $x = 2\sqrt{21} \approx 9.2$; $y = 21$; $z = 25$ **31.** $x = 4\sqrt{6} \approx 9.8$; $y = 4\sqrt{2} \approx 5.7$; $z = 4\sqrt{3} \approx 6.9$ **33.** $4\sqrt{3} \approx 6.9$ **35.** always **37.** sometimes
39. Given: $\angle ADC$ is a right angle. $\overline{DB}$ is an altitude of $\triangle ADC$.
Prove: $\frac{AB}{DB} = \frac{DB}{CB}$

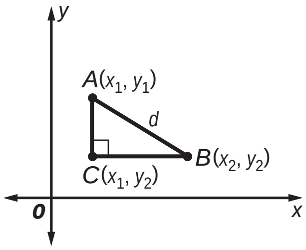


Proof: It is given that $\angle ADC$ is a right angle and $\overline{DB}$ is an altitude of $\triangle ADC$. $\triangle ADC$ is a right triangle by the definition of a right triangle. Therefore, $\triangle ADB \sim \triangle DCB$, because if the altitude is drawn from the vertex of the right angle to the hypotenuse of a right triangle, then the two triangles formed are similar to the given triangle and to each other. So $\frac{AB}{DB} = \frac{DB}{CB}$ by definition of similar polygons.
41. Sample answer: The golden ratio occurs when the geometric mean is approximately 1.68. **43.** Sample answer: 2 and 72, 4 and 36 **45.** Ian; his proportion shows that the altitude is the geometric mean of the two segments of the hypotenuse. **47.** Sample answer: The geometric mean can be used to help determine the optimum viewing distance. If you are too far from a painting, you may not be able to see fine details. If you are too close, you may not be able to see the entire painting. **49.** H **51.** $8\frac{8}{9}$, $11\frac{1}{9}$ **53.** ≈ 3.4 **55.** $\angle 5$, $\angle 7$ **57.** $\angle 2$, $\angle 7$, $\angle 8$ **59.** $y = 2x + 2$ **61.** $y = 4x - 8$ **63.** 13 ft

Pages 444–446 ***Lesson 8-2***
1. 42.5 **3.** $\frac{3}{7}$ **5.** yes; $JK = \sqrt{17}$, $KL = \sqrt{17}$, $JL = \sqrt{34}$, $(\sqrt{17})^2 + (\sqrt{17})^2 = (\sqrt{34})^2$ **7.** no, no **9.** $\sqrt{15} \approx 3.9$

11. $4\sqrt{74} \approx 34.4$ **13.** $4\sqrt{29} \approx 21.5$ **15.** yes; $QR = 6$, $RS = 10$, $QS = 8$; $6^2 + 8^2 = 10^2$ **17.** no; $QR = \sqrt{61}$, $RS = \sqrt{148}$, $QS = \sqrt{113}$; $(\sqrt{61})^2 + (\sqrt{113})^2 \neq (\sqrt{148})^2$ **19.** yes, yes **21.** no, no **23.** yes, no **25.** no, no **27.** These three lengths form a right triangle. The three numbers are a Pythagorean triple: $29^2 = 21^2 + 20^2$.
29. Given: $\triangle ABC$ with right angle at C, $AB = d$

Prove: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

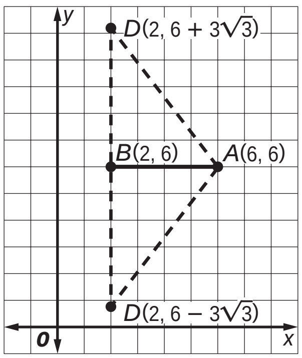


Proof:
Statements (Reasons)
1. $\triangle ABC$ with right angle at C, $AB = d$ (Given)
2. $(CB)^2 + (AC)^2 = (AB)^2$ (Pythagorean Theorem)
3. $|x_2 - x_1| = CB$; $|y_2 - y_1| = AC$ (Distance on a number line)
4. $|x_2 - x_1|^2 + |y_2 - y_1|^2 = d^2$ (Substitution)
5. $(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$ (Substitution)
6. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$ (Take the square root of each side.)
7. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (Reflexive Property)

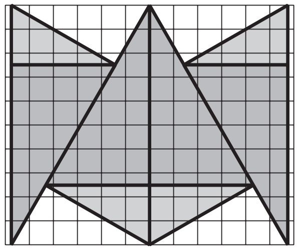
31. $\sqrt{31} \approx 5.6$ **33.** ≈ 10.8 degrees **35.** 270
37. Maria; Colin does not have the longest side as the value of c. **39.** False; counterexample: a right triangle with legs measuring 3 in. and 4 in. has a hypotenuse of 5 in. and an area of $0.5 \cdot 3 \cdot 4$ or 6 in^2. A right triangle with legs measuring 2 in. and $\sqrt{21}$ in. also has a hypotenuse of 5 in., but its area is $0.5 \cdot 2 \cdot \sqrt{21}$ or $\sqrt{21}$ in^2, which is not equivalent to 6 in^2. **41.** Sample answer: The road, the tower that is perpendicular to the road, and the cables form the right triangles. Right triangles are formed by the bridge, the towers, and the cables. The cable is the hypotenuse in each triangle. **43.** 100 **45.** 6 **47.** $\sqrt{77} \approx 8.8$ **49.** 31.8 ft
51. $\frac{7\sqrt{3}}{3}$ **53.** $\sqrt{7}$ **55.** $12\sqrt{2}$

Pages 451–454 ***Lesson 8-3***

1. $90\sqrt{2}$ or about 127.28 ft **3.** $x = 8\sqrt{3}$; $y = 16$ **5.** $a = 6\sqrt{3}$; $c = 12\sqrt{3}$ **7.**

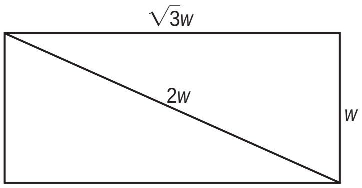


9. $x = \frac{17\sqrt{2}}{2}$; $y = 45$ **11.** $x = 9$; $y = 9\sqrt{3}$ **13.** $x = 5.5$; $y = 5.5\sqrt{3}$ **15.** $a = 14\sqrt{3}$; $CE = 21$; $y = 21\sqrt{3}$; $b = 42$
17. $7.5\sqrt{3}$ cm ≈ 12.99 cm **19.** $14.8\sqrt{3}$ m ≈ 25.63 m
21. $8\sqrt{2} \approx 11.31$ **23.** $a = 3\sqrt{3}$, $b = 9$, $c = 3\sqrt{3}$, $d = 9$
25. (1, 2), (7, 2) **27.** $(8, -5 + 2\sqrt{3})$ **29.** 30° angle
31.



33. $x = 4$; $y = 4\sqrt{3}$; $z = 2\sqrt{6}$

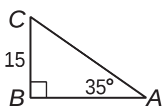
35. 2.25
37. Sample answer: The length of the rectangle is $\sqrt{3}$ times the width; $\ell = \sqrt{3}w$.



39. Sample answer: When a square is cut on the diagonal, two triangles are formed, each with angles measuring 45°-45°-90°. The design uses 45°-45°-90° triangles so the quilt blocks will be square. If 30°-60°-90° triangles were used instead of the 45°-45°-90° triangles, the block would be a rectangle.
41. G **43.** yes, yes **45.** yes, yes **47.** no, no
49. $4\sqrt{6} \approx 9.8$; $4\sqrt{2} \approx 5.7$; $4\sqrt{3} \approx 6.9$
51. $m\angle ALK < m\angle ALN$ **53.** $m\angle OLK > m\angle NLO$
55. Taipa's model will be $\frac{1}{65}$ the size of the real falls.
57. 1.26 **59.** 65 **61.** 7.2 **63.** 500

Pages 460–462 ***Lesson 8-4***

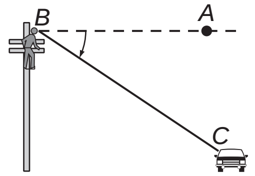
1. $\frac{14}{50} = 0.28$; $\frac{48}{50} = 0.96$; $\frac{14}{48} \approx 0.29$; $\frac{48}{50} = 0.96$; $\frac{14}{50} = 0.28$; $\frac{48}{14} \approx 3.43$ **3.** 0.8387 **5.** 0.8387 **7.** 1.0000
9. 2997 ft **11.** $m\angle B \approx 39.1°$ **13.** $m\angle B \approx 26.6°$
15. $\frac{\sqrt{3}}{3} \approx 0.58$; $\frac{\sqrt{6}}{3} \approx 0.82$; $\frac{\sqrt{2}}{2} \approx 0.71$; $\frac{\sqrt{6}}{3} \approx 0.82$; $\frac{\sqrt{3}}{3} \approx 0.58$; $\sqrt{2} \approx 1.41$ **17.** $\frac{2}{3} \approx 0.67$; $\frac{\sqrt{5}}{5} \approx 0.45$; $\frac{2\sqrt{5}}{5} \approx 0.89$; $\frac{\sqrt{5}}{3} \approx 0.75$; $\frac{2}{3} \approx 0.67$; $\frac{\sqrt{5}}{2} \approx 1.12$ **19.** 0.9260
21. 0.9974 **23.** 0.9239 **25.** 94.1 ft **27.** about 54.5
29. $\frac{\sqrt{26}}{26} \approx 0.1961$ **31.** $\frac{5\sqrt{26}}{26} \approx 0.9806$ **33.** $\frac{\sqrt{26}}{26} \approx 0.1961$
35. $\frac{\sqrt{26}}{26} \approx 0.1961$ **37.** $\frac{5}{1} = 5.0000$ **39.** 75.6 **41.** 27.2
43. 23.2 **45.** 44.9 **47.** 29.1 **49.** 39.8 **51.** 5.18 ft **53.** $x = 17.1$; $y = 23.4$ **55.** Sample answer: $m\angle B = 90$, $m\angle C = 55$, $b \approx 26.2$, $c \approx 21.4$



57. The tan is the ratio of the measure of the opposite side divided by the measure of the adjacent side for a given angle in a right triangle. The $\tan^{-1}$ is the measure of the angle with a certain tangent ratio.
59. D **61.** C **63.** $a = \sqrt{3}$, $c = 2\sqrt{3}$ **65.** yes, yes
67. no, no **69.** 117 **71.** 150 **73.** 63

Pages 466–470 ***Lesson 8-5***

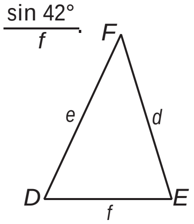
1. about 2.2° **3.** C **5.** 1935.1 ft **7.** about 4° **9.** about 40.2° **11.** about 35.6° **13.** about 5.3° **15.** 100 ft, 300 ft **17.** about 348 m **19.** about 8.3 in. **21.** no **23.** Sample answer: $\angle ABC$



25. about 3.0 mi **27.** B **29.** 47.8 **31.** 65.3 **33.** $6\sqrt{2}$; $6\sqrt{2}$ **35.** $10\sqrt{3}$, 10 **37.** 31.2 cm **39.** true **41.** true **43.** 5 **45.** 34

Pages 475–477 ***Lesson 8-6***

1. 4.6 **3.** 23 **5.** about 237.8 ft **7.** $m\angle Q \approx 43$, $m\angle R \approx 17$, $r = 9.5$ **9.** $m\angle P \approx 37$, $p = 11.1$; $m\angle R \approx 32$ **11.** $m\angle R = 80$, $p = 13.1$, $q = 17.6$ **13.** 2.7 **15.** 29 **17.** 27.7 **19.** $m\angle W = 68$, $w \approx 7.3$, $x \approx 5.1$ **21.** $m\angle Y = 103$, $w \approx 12.6$, $x \approx 6.8$ **23.** $m\angle X = 27$, $x \approx 6.3$, $y \approx 12.5$ **25.** $m\angle Y \approx 17.6$, $m\angle X \approx 55.4$, $x \approx 25.8$ **27.** ≈ 12.72 in. **29.** about 168.8 cm **31.** about 1194 ft **33.** about 246 ft **35.** Sample answer: Let $m\angle D = 65$, $m\angle E = 73$, and $d = 15$. Then $\frac{\sin 65°}{15}$ is the fixed ratio or scale factor for the Law of Sines extended proportion. The length of e is found by using $\frac{\sin 65°}{15} = \frac{\sin 73°}{e}$. The $m\angle F$ is found by evaluating $180 - (m\angle D + m\angle E)$. In this problem $m\angle F = 42$. The length of f is found by using $\frac{\sin 65°}{15} = \frac{\sin 42°}{f}$.



37. Sample answer: The interior framework of the statue is composed of iron rods arranged in a triangular pattern. This gives the statue its strength and balance. **39.** J **41.** about 5.97 ft **43.** $\frac{20}{29} \approx 0.69$; $\frac{21}{29} \approx 0.72$; $\frac{20}{21} \approx 0.95$; $\frac{21}{29} \approx 0.72$; $\frac{20}{29} \approx 0.69$; $\frac{21}{20} = 1.05$ **45.** $\frac{\sqrt{2}}{2} \approx 0.71$; $\frac{\sqrt{2}}{2} \approx 0.71$; 1; $\frac{\sqrt{2}}{2} \approx 0.71$; $\frac{\sqrt{2}}{2} \approx 0.71$; 1 **47.** $\frac{61}{72}$ **49.** $\frac{103}{128}$ **51.** $-\frac{47}{70}$

Pages 482–485 ***Lesson 8-7***

1. 4.1 **3.** 90 **5.** $m\angle X \approx 20$; $m\angle Y \approx 44$; $m\angle Z \approx 116$ **7.** 6.7 ft **9.** $v \approx 18.2$ **11.** $t \approx 14.7$ **13.** 12 **15.** 27 **17.** $p \approx 6.9$; $m\angle M \approx 79$; $m\angle Q \approx 63$ **19.** $m\angle B = 61$; $b \approx 5.4$; $a \approx 4.1$ **21.** $m\angle A \approx 30$; $m\angle B \approx 40$; $m\angle C \approx 110$ **23.** 100; 57 **25.** $m\angle M \approx 18.6$; $m\angle N \approx 138.4$; $n \approx 91.8$ **27.** $m\angle L \approx 101.9$; $m\angle M \approx 36.3$; $m\angle N \approx 41.8$ **29.** $m \approx 6.0$; $m\angle L \approx 22.2$; $m\angle N \approx 130.8$ **31.** $m\angle L \approx 53.6$; $m\angle M \approx 59.6$; $m\angle N \approx 66.8$ **33.** 561.2 units **35.** Adam; 52.3° **37a.** Pythagorean Theorem **37b.** Substitution **37c.** Pythagorean Theorem **37d.** Substitution **37e.** Def. of cosine **37f.** Cross products **37g.** Substitution **37h.** Commutative Property

39. If two angles and one side are given, then the Law of Cosines cannot be used. **41.** Law of Cosines; the other three terms each apply to right triangles only. The Law of Cosines can be used to solve a triangle that is not a right triangle. **43.** C **45.** A **47.** 33 **49.** yes **51.** no **53.** (1.3, 6.7)

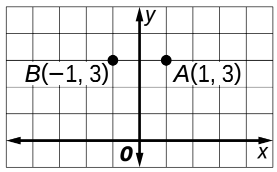
Pages 486–490 ***Chapter 8*** ***Study Guide and Review***

1. true **3.** false, a right triangle **5.** true **7.** false, depression **9.** 8 **11.** $10\sqrt{7} \approx 26.5$ **13.** $4\sqrt{7} \approx 10.6$ **15.** 25 **17.** Yes; the farmer created two right triangles each with a hypotenuse of 625 feet. $625^2 = 175^2 + 600^2$. **19.** $x = 12$, $y = 6\sqrt{3}$ **21.** $a = \frac{28\sqrt{3}}{3}$, $z = \frac{14\sqrt{3}}{3}$, $b = \frac{28}{3}$, $y = \frac{14}{3}$ **23.** $\sin F = \frac{3}{5} = 0.60$, $\cos F = \frac{4}{5} = 0.80$, $\tan F = \frac{3}{4} = 0.75$, $\sin G = \frac{4}{5} = 0.80$, $\cos G = \frac{3}{5} = 0.60$, $\tan G = \frac{4}{3} \approx 1.33$ **25.** $\sin F = \frac{9}{41} \approx 0.22$, $\cos F = \frac{40}{41} \approx 0.98$, $\tan F = \frac{9}{40} \approx 0.23$, $\sin G = \frac{40}{41} \approx 0.98$, $\cos G = \frac{9}{41} \approx 0.22$, $\tan G = \frac{40}{9} \approx 4.44$ **27.** 2.3° **29.** 2.9° **31.** 21.3 **33.** ≈ 77.3 ft **35.** 22.4 **37.** $m\angle B \approx 38$, $m\angle A \approx 89$, $a \approx 8.4$

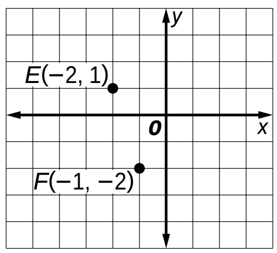
Chapter 9 Transformations

Page 495 ***Chapter 9*** ***Get Ready***

1.



3.



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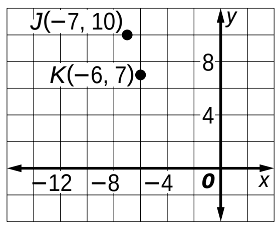
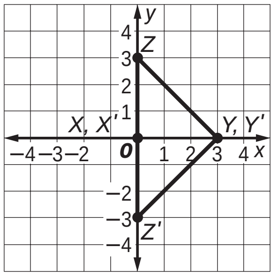
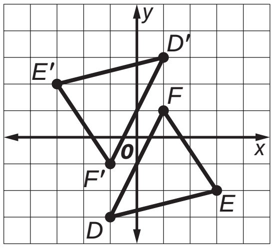


7. (7, F) **9.** 36.9 **11.** 41.8 **13.** 41.4 **15.** 9.2 ft

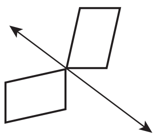
Pages 501–503 ***Lesson 9-1***

1.

3.


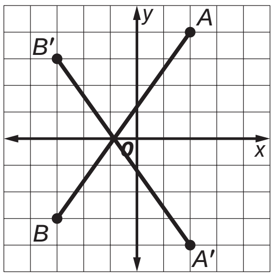
5.


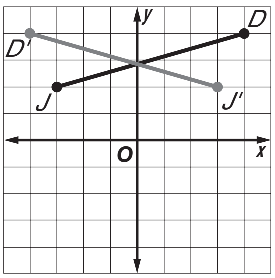
7. Tanya should aim her serve at a point reflected underground a distance equal to the height of the point on her opponent she wants the tennis ball to reach.

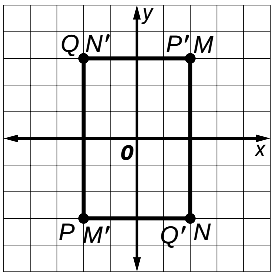
9. 3; no **11.**


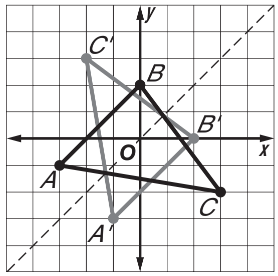
13. $\overline{YZ}$ **15.** T **17.** $\triangle VYX$

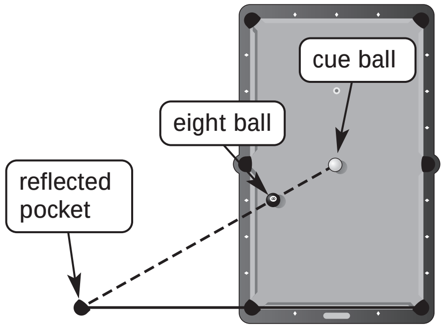
19. $\angle UVZ$ **21.** 1; no **23.** 8; yes

25.


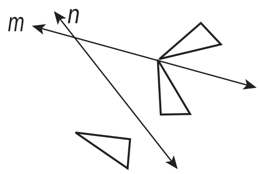
27.


29.


31.


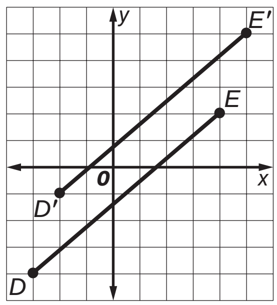
33.


35. $A(4, 7)$, $B(10, -3)$, and $C(-6, -8)$

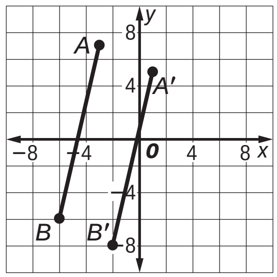
37.

same shape, but turned or rotated **39.** vertical line of symmetry

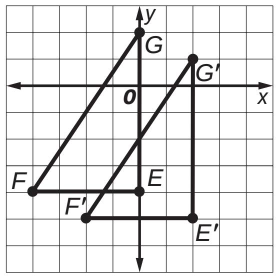
41. vertical, horizontal, diagonal lines of symmetry; point of symmetry at the center **43.** Sample answer: The centroid of an equilateral triangle is not a point of symmetry. **45.** Reflections of the surrounding vistas can be seen in bodies of water. Three examples of line of symmetry in nature are the water's edge in a lake, the line through the middle of a pin oak leaf, and the lines of a four leaf clover. Each point above the water has a corresponding point in the image in the lake. The distance of a point above the water appears the same as the distance of the image below the water.
47. G **49.** 92.1 **51.** 72.8 ft **53.** $\sqrt{10}$ **55.** $\sqrt{29}$

Pages 506–509 ***Lesson 9-2***

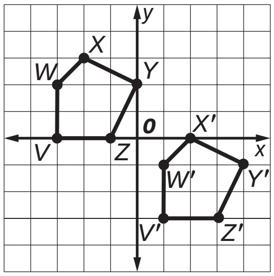
1.


3. $1 \rightarrow 2 = (x, y + 3)$, $3 \rightarrow 4 = (x + 4, y)$
5. No; quadrilateral *WXYZ* is oriented differently than quadrilateral *NPQR*.

7.


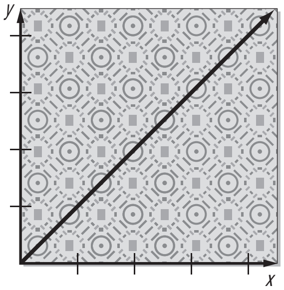
9.


11.



13. Yes; it has the same orientation as the blue figure.
15. Yes; it is a translation followed by another translation.
17. No; it is a reflection followed by a translation.

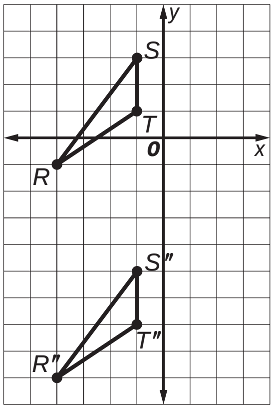
19. down 6 squares **21.** yes; sample answer



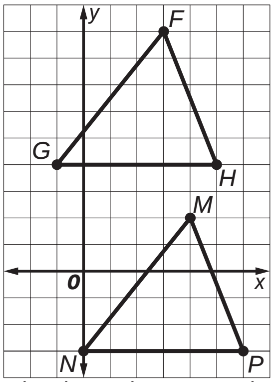
The reflection of the bottom half of the image in the line $y = x$ is congruent to the top half.

23. $B'(5, 0)$

25.

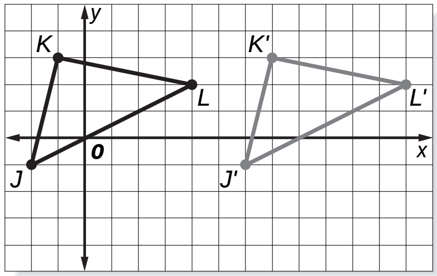


27. $H(5, 4)$, $N(0, -3)$; $(x, y) \rightarrow (x + 1, y - 7)$



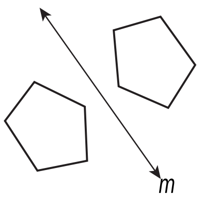
$(x, y) \longrightarrow (x + 1, y - 7)$

29.

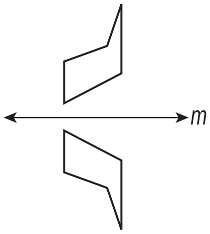


31. 48 in. right **33.** 72 in. right, 48 in. down
35. The properties that are preserved include betweenness of points, collinearity, and angle and distance measure. Since translations are composites of two reflections, all translations are isometries. Thus, all properties preserved by reflections are preserved by translations. **37.** Sample answer: $y = -1$ and $y = -6$

39. D **41.**

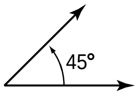


43.

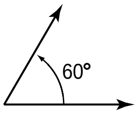


45. Sines; $m\angle C \approx 69.9$, $m\angle A \approx 53.1$, $a \approx 11.9$ **47.** 23 ft
49. $B(a, b)$, $D(d, 0)$ **51.** You did not fill out an application.
53. The two lines are not parallel. **55.** 5 **57.** $3\sqrt{2}$

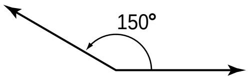
59.



61.

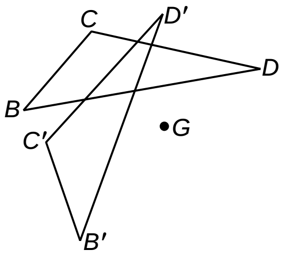


63.

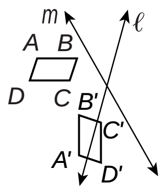


Pages 513–517 ***Lesson 9-3***

1.

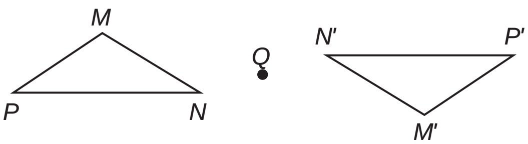


3.

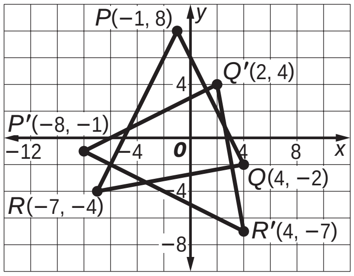


5. 72°

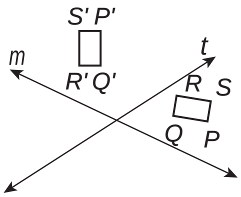
7.



9.

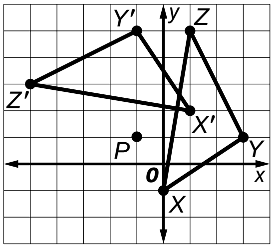


11.



13. order 5 and magnitude 72°; order 4 and magnitude 90°; order 3 and magnitude 120°

15.



17. $T''(0, -4)$, $U''(-2, -3)$, and $V''(-1, -2)$; 180° **19.** Yes; it is a proper successive reflection with respect to the two intersecting lines. **21.** yes **23.** no **25.** Angles of rotation with measures of 90 or 180 would be easier on a coordinate plane because of the grids used in graphing.

27.

Transformation	reflection	translation	rotation
angle measure	yes	yes	yes
betweenness of points	yes	yes	yes
orientation	no	yes	yes
collinearity	yes	yes	yes
distance measure	yes	yes	yes

29. direct **31.** H, I, N, O, S, X, Z **33. Given:** Point A' is the image of point A after a reflection in $\overleftrightarrow{BP}$. A'' is the image of point A' after a reflection in $\overleftrightarrow{PC}$. Points A, B, and A' are collinear. Points A', C', and A'' are collinear. $\angle BPC$ is a right angle.

Prove: $m\angle APA'' = 180$

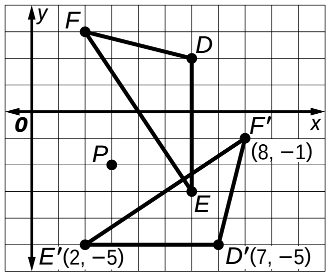
Proof: Since point A' is the image of point A after a reflection in $\overleftrightarrow{BP}$, A'' is the image of point A' after a reflection in $\overleftrightarrow{PC}$, points A, B, and A' are collinear, and points A', C', and A'' are collinear, then $m\angle APA'' = 2(m\angle BPC)$ by Postulate 9.1. Since $\angle BPC$ is a right angle, $m\angle BPC = 90$. By substitution, $m\angle APA'' = 2(90)$ or 180.

35. a point on the line of reflection **37.** no invariant points **39.** D **41.** Yes; it is one reflection after another with respect to the two parallel lines. **43.** Yes; it is one reflection after another with respect to the two parallel lines. **45.** C **47.** $\angle AGF$
49. $-1(1) = 1$ and $-1(1) = -1$ **51.** square **53.** $AB = 7$, $BC = 10$, $AC = 9$ **55.** $\overline{TR}$; diagonals bisect each other
57. $\angle QRS$; opp. $\angle$s $\cong$. **59.** 2 **61.** (0, 8), (1, 5), (2, 2)
63. (0, 6), (1, 3), (2, 0)

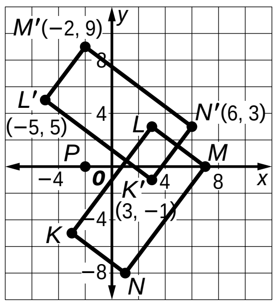
Pages 522–524 *Lesson 9-4*

1. no; measure of interior angle = 144 **3.** no **5.** yes, uniform **7.** Each "postage stamp" is a square that has been tessellated and 90 is a factor of 360. It is a regular tessellation since only one polygon is used **9.** yes; uniform **11.** no **13.** yes; measure of interior angle = 120 **15.** no; measure of interior angle = 150 **17.** no; measure of interior angle = 170 **19.** yes **21.** no **23.** yes; uniform **25.** yes; uniform, semi-regular **27.** always **29.** never **31.** Semi-regular tessellations contain two or more regular polygons, but uniform tessellations can be any combination of shapes. **33.** The figure appears to be a trapezoid, which is not a regular polygon. Thus, the tessellation cannot be regular. **35.** Sample answer: Tessellations can be used in art to create abstract art. The equilateral triangles are arranged to form hexagons, which are arranged adjacent to one another. Sample answers: kites, trapezoids, isosceles triangles. **37.** H

39.



41.

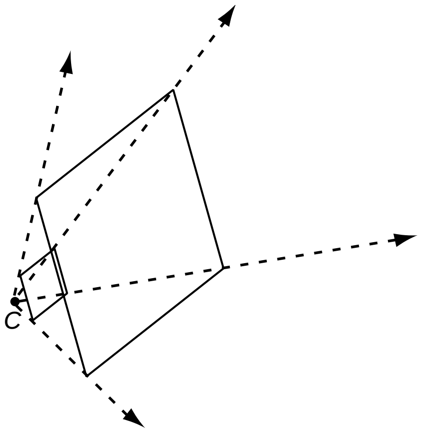


43. $\frac{2}{3}$
45. 15

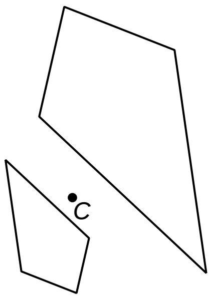
Pages 529–532 **Lesson 9-5**

1. $A'B' = 12$

3.

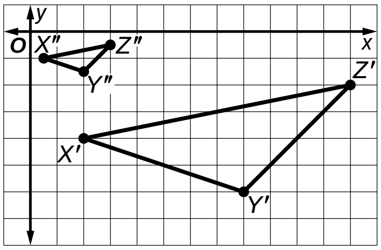


5.

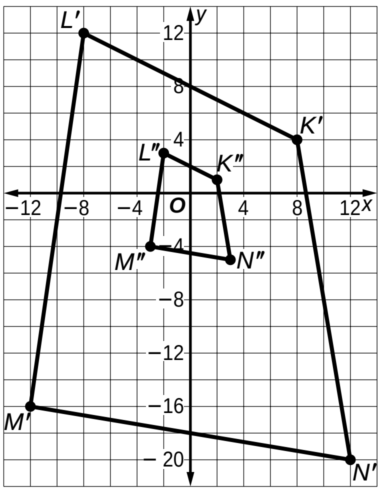


7. $r = 2$; enlargement **9.** $5\frac{1}{3}$ in.
11. $S'T' = \frac{3}{5}$ **13.** $ST = 4$
15. $S'T' = 0.9$

17.



19.



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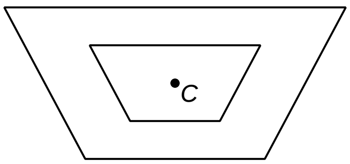
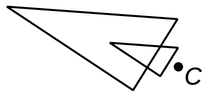
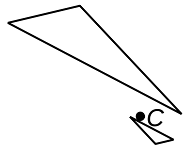


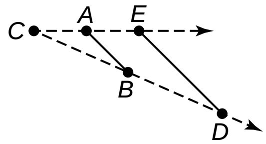
23.



25.

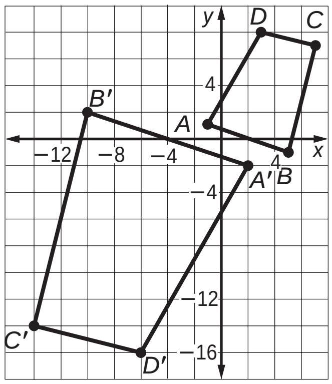


27. $\frac{1}{2}$; reduction **29.** $\frac{1}{3}$; reduction **31.** -2; enlargement
33. It is $\frac{9}{16}$ of the original. **35.** 60%
37. Given: dilation with center C and scale factor k
Prove: $ED = r(AB)$



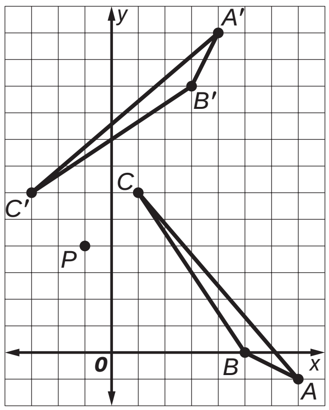
Proof: $CE = r(CA)$ and $CD = r(CB)$ by the definition of a dilation. $\frac{CE}{CA} = r$ and $\frac{CD}{CB} = r$. So, $\frac{CE}{CA} = \frac{CD}{CB}$ by substitution. $\angle ACB \cong \angle ECD$, since congruence of angles is reflexive. Therefore, by SAS Similarity, $\triangle ACB$ is similar to $\triangle ECD$. The corresponding sides of similar triangles are proportional, so $\frac{ED}{AB} = \frac{CE}{CA}$. We know that $\frac{CE}{CA} = r$, so $\frac{ED}{AB} = r$ by substitution. Therefore, $ED = r(AB)$ by the Multiplication Property of Equality.
39. 960 pixels by 720 pixels **41.** $\frac{5}{4}$

43.



45. Always; dilations always preserve angle measure.
47. A cut and paste produces an image congruent to the original. Answers should include the following. Congruent figures are similar, so cutting and pasting is a similarity transformation. If you scale both horizontally and vertically by the same factor, you are creating a dilation. **49.** H **51.** no **53.** yes

55.

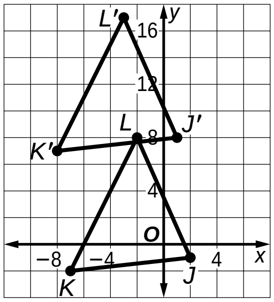


57. Yes; if it is a rectangle the diagonals are congruent.
59. 76.0

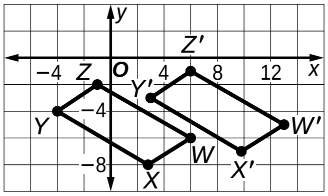
Pages 538–541 **Lesson 9-6**

1. ⟨5, 6⟩ **3.** $\sqrt{41} \approx 6.4$, 218.7°

5.



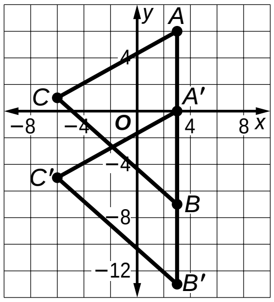
7.



9. $2\sqrt{13} \approx 7.2$, 56.3° **11.** ⟨2, 6⟩ **13.** ⟨−7, −4⟩ **15.** ⟨−3, 5⟩ **17.** $6\sqrt{2} \approx 8.5$, 135°

19. $4\sqrt{10} \approx 12.6$, 198.4° **21.** $2\sqrt{122} \approx 22.1$, 275.2°

23.



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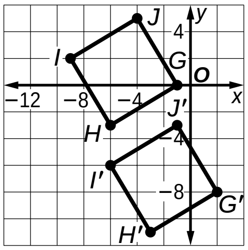

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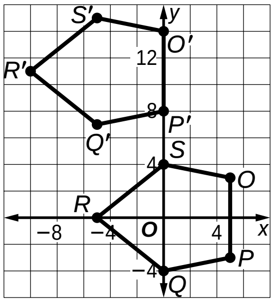

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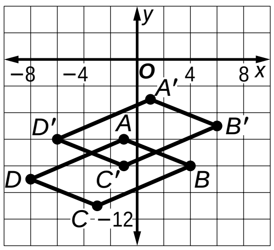

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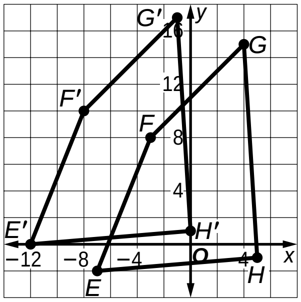
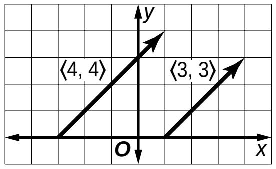


33. about 44.8 mi; about 38.7° south of due east **35.** 5, 0° **37.** $2\sqrt{5} \approx 4.5$, 296.6° **39.** $7\sqrt{5} \approx 15.7$, 26.6° **41.** 25, 73.7° **43.** $5\sqrt{41} \approx 32.0$, 218.6° **45.** 13, 67.4° **47.** 5, 306.9°

49. $2\sqrt{5} \approx 4.5$, 26.6° **51.** ⟨−350, 450⟩ mph **53.** 52.1° north of due west **55.** Sample answer: ⟨7, 7⟩

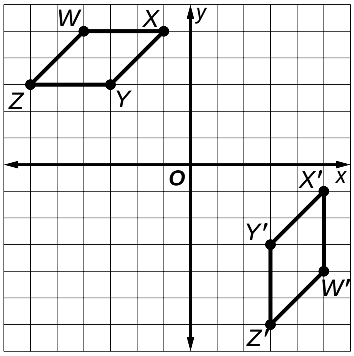


57. Sample answer: ⟨1, 3⟩, ⟨−2, 1⟩, ⟨1, −4⟩ **59.** C **61.** $A'B' = 16$ **63.** $AB = 5$ **65.** yes; uniform; semi-regular **67.** true **69.** False **71.** 25.4 cm, 54.4 cm

Pages 543–546 **Chapter 9** **Study Guide and Review**

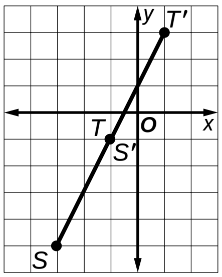
1. false, center **3.** false, component form **5.** false, center of rotation

7.



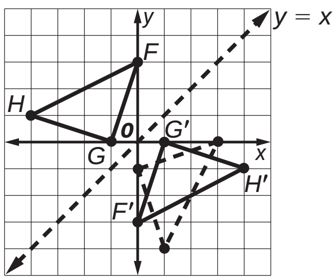
9. 144

11.



13. left 5 seats and back 3 seats

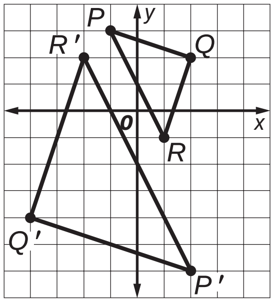
15.



17. 200°

19. yes; not uniform **21.** yes; uniform
23. Yes; the measure of an interior angle is 60 which is a factor of 360. **25.** $C'D' = 24$ **27.** $CD = 4$ **29.** $C'D' = 10$

31.



33. $\frac{1}{48}$ **35.** $\langle -8, 4\rangle$
37. ≈ 29.8, $\approx 346.4°$

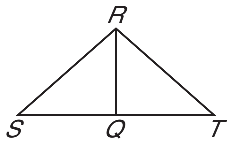
Chapter 10 Circles

Page 553 ***Chapter 10*** ***Get Ready***

1. 162 **3.** 2.4 **5.** $r = \frac{C}{2\pi}$ **7.** 8 **9.** 5.7, -1.7 **11.** 4.4, -3.4

Pages 557–561 ***Lesson 10-1***

1. $\odot E$ **3.** $\overline{AB}$, $\overline{AC}$, or $\overline{BD}$ **5.** 6 mm **7.** 5 **9.** 20 **11.** 753.76 ft, 376.88 ft **13.** $\odot F$ **15.** $\overline{BE}$ or $\overline{CD}$ **17.** $\overline{FA}$ **19.** $\overline{RT}$, $\overline{RU}$, $\overline{RV}$, $\overline{RW}$, $\overline{RX}$, or $\overline{RZ}$ **21.** $\overline{TX}$ or $\overline{WZ}$ **23.** 4 ft **25.** 60 cm **27.** 18 in. **29.** 2 **31.** 12 **33.** 3 **35.** 14 mm, 43.98 mm **37.** 26 mi, 13 mi **39.** $6\frac{1}{4}$ yd, 39.27 yd **41.** a, $6.28a$ **43.** 34π m **45.** $10\pi\sqrt{2}$ in. **47.** 10 **49.** 10 **51.** 5 **53.** 1; This description is the definition of a radius. **55.** 37.68 ft **57.** 5026.5 ft **59.** 3141.6 ft; 3769.9 ft **61.** Radius would be between 1.27 and 1.91 cm. **63.** $\frac{C}{r}$ **65.** Sample answer: about 251.3 feet. The distance the animal travels is approximated by the circumference of the circle. The diameter for the circle on which the animal is located becomes $80 - 2$ or 78. The circumference for this circle is 78π. Multiply by 22 to get a total distance of $22(78\pi)$ or about 5,391 feet. This is a little over a mile. **67.** G **69.** 9.8; 66° **71.** 44.7; 27° **73.** 24
75. Given: $\overrightarrow{RQ}$ bisects $\angle SRT$.
Prove: $m\angle SQR > m\angle QRS$
Proof:



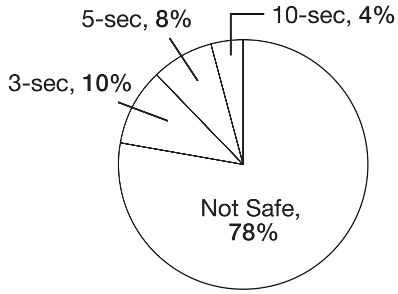
Statements (Reasons)
1. $\overrightarrow{RQ}$ bisects $\angle SRT$. (Given)
2. $\angle QRS \cong \angle QRT$ (Def. of $\angle$ bisector)
3. $m\angle QRS = m\angle QRT$ (Def. of $\cong$ $\angle$s)
4. $m\angle SQR = m\angle T + m\angle QRT$ (Exterior Angle Theorem)
5. $m\angle SQR > m\angle QRT$ (Def. of Inequality)
6. $m\angle SQR > m\angle QRS$ (Substitution)

77. 0.075 **79.** 2014 **81.** 18 **83.** 22.5 **85.** 120

Pages 567–569 ***Lesson 10-2***

1. 120 **3.** 43 **5.** 42 **7.** 222 **9.** quick service, 25°; reasonable prices, 76°; don't know, 14°; atmosphere, 36°; great food, 209° **11.** 120 **13.** 90 **15.** 150 **17.** 115 **19.** 65 **21.** 90 **23.** 90 **25.** 180 **27.** 225 **29.** 78% = 281°; 10% = 36°; 8% = 29°; 4% = 14°

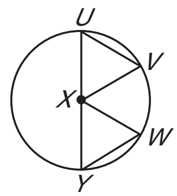
31. **Do you eat food dropped on the floor?**



33. $24\pi \approx 75.40$ units **35.** $4\pi \approx 12.57$ units **37.** 76 **39.** 52 **41.** 256 **43.** 308 **45.** sometimes **47.** always **49.** 56.5 ft **51.** Sample answer: Concentric circles have the same center, but different radius measures; congruent circles usually have different centers but the same radius measure. **53.** No; the radii are not equal, so the proportional part of the circumferences would not be the same. Thus, the arcs would not be congruent. **55.** Sample answer: The hands of the clock form central angles. The hands form right, acute, and obtuse angles. Some times when the angles formed by the minute and hour hand are congruent are at 1:00 and 11:00, 2:00 and 10:00, 3:00 and 9:00, 4:00 and 8:00, and 5:00 and 7:00. They also form congruent angles at many other times of the day, such as 3:05 and 8:55.
57. H **59.** 6.5; 40.84 **61.** 24.00; 12.00 **63.** $8\frac{2}{11}$
65. 9 units **67.** 42 **69.** 100 **71.** 36

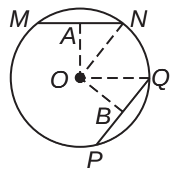
Pages 574–577 ***Lesson 10-3***

1. Proof: Because all radii are congruent, $\overline{XU} \cong \overline{XV} \cong \overline{XW} \cong \overline{XY}$. You are given that $\overline{UV} \cong \overline{WY}$, so $\triangle UVX \cong \triangle WYX$, by SSS. Thus, $\angle UXV \cong \angle WXY$ by CPCTC. Since the central angles have the same measure, their intercepted arcs have the same measure and are therefore, congruent. Thus, $\widehat{UV} \cong \widehat{WY}$.



3. 30 **5.** $5\sqrt{3}$ **7.** $10\sqrt{5} \approx 22.36$ **9.** $m\widehat{AB} = m\widehat{BC} = m\widehat{CD} = m\widehat{DE} = m\widehat{EF} = m\widehat{FG} = m\widehat{GH} = m\widehat{HA} = 45$ **11.** $m\widehat{NP} = m\widehat{RQ} = 120$; $m\widehat{NR} = m\widehat{PQ} = 60$ **13.** 15 **15.** 15 **17.** 80 **19.** 40 **21.** 30 **23.** 15 **25.** 16 **27.** 6 **29.** 8 **31.** 10 **33.** $\sqrt{2} \approx 1.41$

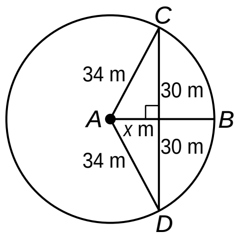
35. Given: $\odot O$, $\overline{MN} \cong \overline{PQ}$
$\overline{ON}$ and $\overline{OQ}$ are radii.
$\overline{OA} \perp \overline{MN}$; $\overline{OB} \perp \overline{PQ}$
Prove: $\overline{OA} \cong \overline{OB}$
Proof:



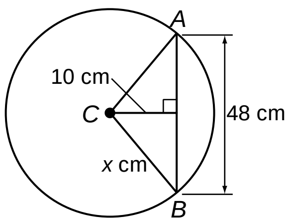
Statements (Reasons)
1. $\odot O$, $\overline{MN} \cong \overline{PQ}$, $\overline{ON}$ and $\overline{OQ}$ are radii, $\overline{OA} \perp \overline{MN}$, $\overline{OB} \perp \overline{PQ}$ (Given)
2. $\overline{OA}$ bisects $\overline{MN}$; $\overline{OB}$ bisects $\overline{PQ}$. ($\overline{OA}$ and $\overline{OB}$ are contained in radii. A radius $\perp$ to a chord bisects the chord.)
3. $AN = \frac{1}{2}MN$; $BQ = \frac{1}{2}PQ$ (Def. of bisector)
4. $MN = PQ$ (Def. of $\cong$ segments)

5. $\frac{1}{2}MN = \frac{1}{2}PQ$ (Mult. Prop.)
6. $AN = BQ$ (Substitution)
7. $\overline{AN} \cong \overline{BQ}$ (Def. of $\cong$ segments)
8. $\overline{ON} \cong \overline{OQ}$ (All radii of a circle are $\cong$.)
9. $\triangle AON \cong \triangle BOQ$ (HL)
10. $\overline{OA} \cong \overline{OB}$ (CPCTC)

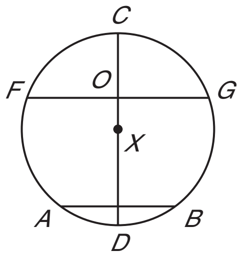
37. 16 m



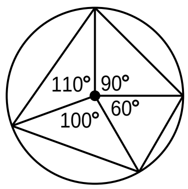
39. 26 cm



43. Given: In $\odot X$, X is on $\overline{CD}$ and $\overline{FG}$ bisects $\overline{CD}$ at O.
Prove: Point O is point X



Proof: Since point X is on $\overline{CD}$ and C and D are on $\odot X$, $\overline{CD}$ is a diameter of $\odot X$. Since $\overline{FG}$ bisects $\overline{CD}$ at O, O is the midpoint of $\overline{CD}$. Since the midpoint of a diameter is the center of a circle, O is the center of the circle $\odot X$. Therefore point O is point X.
45. $m\widehat{AB} = m\widehat{CD}$
47. Sample answer:



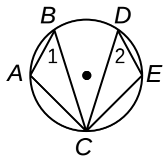
None of the sides are congruent. Architectural drawings for a building.

49. Let r be the radius of $\odot P$. Draw radii to points D and E to create triangles. The length DE is $r\sqrt{3}$ and $AB = 2r$; $r\sqrt{3} \neq \frac{1}{2}(2r)$. **51.** C **53.** 138 **55.** 222 **57.** 14 **59.** 20 **61.** 15

Pages 583–586 ***Lesson 10-4***

1. $m\angle 1 = 30$, $m\angle 2 = 60$, $m\angle 3 = 60$, $m\angle 4 = 30$, $m\angle 5 = 30$, $m\angle 6 = 60$, $m\angle 7 = 60$, $m\angle 8 = 30$ **3.** 1 **5.** 152, 70 **7.** $m\angle 1 = m\angle 2 = 30$, $m\angle 3 = 25$

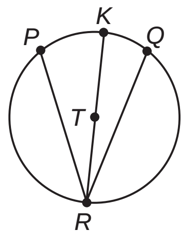
9. Given: $\widehat{AB} \cong \widehat{DE}$, $\widehat{AC} \cong \widehat{CE}$
Prove: $\triangle ABC \cong \triangle EDC$



Proof:
Statements (Reasons)
1. $\widehat{AB} \cong \widehat{DE}$, $\widehat{AC} \cong \widehat{CE}$ (Given)
2. $m\widehat{AB} = m\widehat{DE}$, $m\widehat{AC} = m\widehat{CE}$ (Def. of $\cong$ arcs)
3. $\frac{1}{2}m\widehat{AB} = \frac{1}{2}m\widehat{DE}$, $\frac{1}{2}m\widehat{AC} = \frac{1}{2}m\widehat{CE}$ (Mult. Prop.)
4. $m\angle ACB = \frac{1}{2}m\widehat{AB}$, $m\angle ECD = \frac{1}{2}m\widehat{DE}$, $m\angle 1 = \frac{1}{2}m\widehat{AC}$, $m\angle 2 = \frac{1}{2}m\widehat{CE}$ (Inscribed $\angle$ Theorem)
5. $m\angle ACB = m\angle ECD$, $m\angle 1 = m\angle 2$ (Substitution)
6. $\angle ACB \cong \angle ECD$, $\angle 1 \cong \angle 2$ (Def. of $\cong$ ⦞)
7. $\overline{AB} \cong \overline{DE}$ ($\cong$ arcs have $\cong$ chords.)
8. $\triangle ABC \cong \triangle EDC$ (AAS)

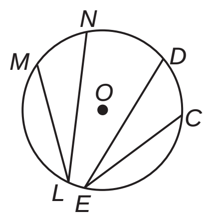
11. $\frac{8}{9}$ **13.** 1 **15.** $m\angle 1 = m\angle 2 = 30$ **17.** $m\angle 1 = 51$, $m\angle 2 = 90$, $m\angle 3 = 39$ **19.** 45, 30, 120 **21.** $m\angle B = 120$, $m\angle C = 120$, $m\angle D = 60$ **23.** Sample answer: diameter of circle; diagonal and angle bisector of $\angle DEG$ and $\angle DFG$ **25.** 72 **27.** 144 **29.** 162 **31.** 9

33. Given: T lies inside $\angle PRQ$. $\overline{RK}$ is a diameter of $\odot T$.
Prove: $m\angle PRQ = \frac{1}{2}m\widehat{PKQ}$



Proof:
Statements (Reasons)
1. $m\angle PRQ = m\angle PRK + m\angle KRQ$ ($\angle$ Addition Th.)
2. $m\widehat{PKQ} = m\widehat{PK} + m\widehat{KQ}$ (Arc Addition Theorem)
3. $\frac{1}{2}m\widehat{PKQ} = \frac{1}{2}m\widehat{PK} + \frac{1}{2}m\widehat{KQ}$ (Multiplication Prop.)
4. $m\angle PRK = \frac{1}{2}m\widehat{PK}$, $m\angle KRQ = \frac{1}{2}m\widehat{KQ}$ (The measure of an inscribed $\angle$ whose side is a diameter is half the measure of the intercepted arc (Case 1).)
5. $\frac{1}{2}m\widehat{PKQ} = m\angle PRK + m\angle KRQ$ (Subst. (Steps 3, 4))
6. $\frac{1}{2}m\widehat{PKQ} = m\angle PRQ$ (Substitution (Steps 5,1))

35. Given: inscribed $\angle MLN$ and $\angle CED$; $\widehat{CD} \cong \widehat{MN}$
Prove: $\angle CED \cong \angle MLN$



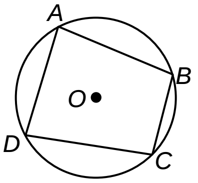
Proof:
Statements (Reasons)
1. $\angle MLN$ and $\angle CED$ are inscribed; $\widehat{CD} \cong \widehat{MN}$ (Given)
2. $m\angle MLN = \frac{1}{2}m\widehat{MN}$; $m\angle CED = \frac{1}{2}m\widehat{CD}$ (Measure of an inscribed $\angle$ = half measure of intercepted arc.)
3. $m\widehat{CD} = m\widehat{MN}$ (Def. of $\cong$ arcs)
4. $\frac{1}{2}m\widehat{CD} = \frac{1}{2}m\widehat{MN}$ (Mult. Prop.)

5. $m\angle CED = m\angle MLN$ (Substitution)

6. $\angle CED \cong \angle MLN$ (Def. of $\cong$ $\angle$s)

37. Given: quadrilateral $ABCD$ inscribed in $\odot O$

Prove: $\angle A$ and $\angle C$ are supplementary. $\angle B$ and $\angle D$ are supplementary.



Proof: By arc addition and the definitions of arc measure and the sum of central angles, $m\widehat{DCB} + m\widehat{DAB} = 360$. Since $m\angle C = \frac{1}{2}m\widehat{DAB}$ and $m\angle A = \frac{1}{2}m\widehat{DCB}$, $m\angle C + m\angle A = \frac{1}{2}(m\widehat{DCB} + m\widehat{DAB})$, but $m\widehat{DCB} + m\widehat{DAB} = 360$, so $m\angle C + m\angle A = \frac{1}{2}(360)$ or 180. This makes $\angle C$ and $\angle A$ supplementary. Because the sum of the measures of the interior angles of the quadrilateral is 360, $m\angle A + m\angle C + m\angle B + m\angle D = 360$. But $m\angle A + m\angle C = 180$, so $m\angle B + m\angle D = 180$, making them supplementary also.

39. Isosceles right triangle; sides are congruent radii making it isosceles and $\angle AOC$ is a central angle for an arc of 90°, making it a right angle. **41.** Square; each angle intercepts a semicircle, making them 90° angles. Each side is a chord of congruent arcs, so the chords are congruent. **45.** Sample answer: The socket is similar to an inscribed polygon because the vertices of the hexagon can be placed on a circle that is concentric with the outer circle of the socket. An inscribed polygon is one in which all of its vertices are points on a circle. The side of the regular hexagon inscribed in a circle $\frac{3}{4}$ inch wide is $\frac{3}{8}$ inch.

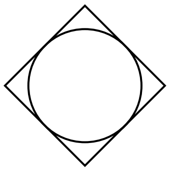
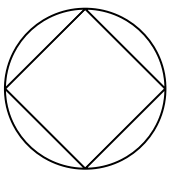
47. H **49.** $\sqrt{135} \approx 11.62$ **51.** 4π units **53.** 10.75 m **55.** sometimes **57.** no **59.** yes

Pages 593–596 ***Lesson 10-5***

1. 12 **3.** $x = 250$; $y = 275$; 1550 units. **5.** 8 **7.** $\sqrt{193}$ **9.** 30 **11.** yes **13.** no **15.** 16 **17.** 3 **19.** 60 units **21.** 24 units **25.** Proof: Assume ℓ is not tangent to circle A. Since ℓ intersects circle A at B, it must intersect the circle in another place. Call this point C. Then $AB = AC$. But if $\overline{AB} \perp \ell$, then $\overline{AB}$ must be the shortest segment from A to ℓ. If $AB = AC$, then $\overline{AC}$ is the shortest segment from A to ℓ. Since B and C are two different points on ℓ, this is a contradiction. Therefore, ℓ is tangent to circle A.

27. 99 mm **29.** $\overline{AD}$ and $\overline{BC}$ **31.** $\overline{AE}$ and $\overline{BF}$ or $\overline{AC}$ and $\overline{BD}$

33. Sample answer:

polygon circumscribed about a circle:	polygon inscribed in a circle:
	

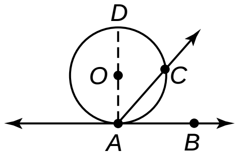
35. If the lines are tangent at the endpoints of a diameter, they are parallel and thus, not intersecting. **37.** D **39.** 45, 45 **41.** 4 **43.** 5

Pages 602–606 ***Lesson 10-6***

1. 138 **3.** 20 **5.** 235 **7.** 60 **9.** 110 **11.** 90 **13.** 50 **15.** 30 **17.** 8 **19.** 130 **21.** 4 **23.** 25 **25.** 10 **27.** 23 **29.** 94 **31.** 141 **33.** 44 **35.** 118 **37.** about 103 ft **39.** 126

41a. Given: $\overleftrightarrow{AB}$ is a tangent to $\odot O$. $\overrightarrow{AC}$ is a secant to $\odot O$. $\angle CAB$ is acute.

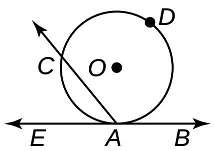
Prove: $m\angle CAB = \frac{1}{2}m\widehat{CA}$



Proof: $\angle DAB$ is a right $\angle$ with measure 90, and $\widehat{DCA}$ is a semicircle with measure 180, since if a line is tangent to a $\odot$, it is $\perp$ to the radius at the point of tangency. Since $\angle CAB$ is acute, C is in the interior of $\angle DAB$, so by the Angle and Arc Addition Postulates, $m\angle DAB = m\angle DAC + m\angle CAB$ and $m\widehat{DCA} = m\widehat{DC} + m\widehat{CA}$. By substitution, $90 = m\angle DAC + m\angle CAB$ and $180 = m\widehat{DC} + m\widehat{CA}$. So, $90 = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{CA}$ by Multiplication Prop., and $m\angle DAC + m\angle CAB = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{CA}$ by substitution. $m\angle DAC = \frac{1}{2}m\widehat{DC}$ since $\angle DAC$ is inscribed, so substitution yields $\frac{1}{2}m\widehat{DC} + m\angle CAB = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{CA}$. By Subtraction Prop., $m\angle CAB = \frac{1}{2}m\widehat{CA}$.

41b. Given: $\overleftrightarrow{AB}$ is a tangent to $\odot O$. $\overrightarrow{AC}$ is a secant to $\odot O$. $\angle CAB$ is obtuse.

Prove: $m\angle CAB = \frac{1}{2}m\widehat{CDA}$



Proof: $\angle CAB$ and $\angle CAE$ form a linear pair, so $m\angle CAB + m\angle CAE = 180$. Since $\angle CAB$ is obtuse, $\angle CAE$ is acute and Case 1 applies, so $m\angle CAE = \frac{1}{2}m\widehat{CA}$. $m\widehat{CA} + m\widehat{CDA} = 360$, so $\frac{1}{2}m\widehat{CA} + \frac{1}{2}m\widehat{CDA} = 180$ by Multiplication Prop., and $m\angle CAE + \frac{1}{2}m\widehat{CDA} = 180$ by substitution. By the Transitive Prop., $m\angle CAB + m\angle CAE = m\angle CAE + \frac{1}{2}m\widehat{CDA}$, so by the Subtraction Prop., $m\angle CAB = \frac{1}{2}m\widehat{CDA}$.

43. 4.6 cm **45.** Sample answer: Each raindrop refracts light from the sun and sends the beam to Earth. The raindrop is actually spherical, but the angle of the light is an inscribed angle from the bent rays. $\angle C$ is an inscribed angle and $\angle F$ is a secant-secant angle. The measure of $\angle F$ can be calculated by finding the positive difference between $m\widehat{BD}$ and the measure of the small intercepted arc containing point C. **47.** G

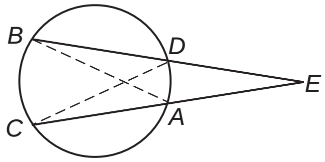
49. 4 **51.** 57 **53.** $\frac{1}{12}$ **55.** 4, -10 **57.** 3, 5

Pages 610–613 ***Lesson 10-7***

1. 2 **3.** 28.05 **5.** 10 **7.** 4.7 **9.** 113.3 cm **11.** 5 **13.** 6 **15.** 8 **17.** 2 **19.** 4 **21.** 11 **23.** 14.3

25. Given: $\overline{EC}$ and $\overline{EB}$ are secant segments.

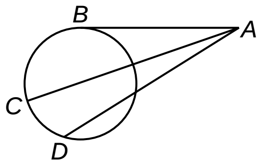
Prove: $EA \cdot EC = ED \cdot EB$



Proof: $\overline{EC}$ and $\overline{EB}$ are secant segments. By the Reflexive Prop., $\angle DEC \cong \angle AEB$. Inscribed $\measuredangle$s that intercept the same arc are $\cong$, so $\angle ECD \cong \angle EBA$. By AA Similarity, $\triangle ABE \sim \triangle DCE$. By the Def. of $\sim$ $\triangle$s, $\frac{EA}{ED} = \frac{EB}{EC}$. The cross products of this proportion are equal, so $EA \cdot EC = ED \cdot EB$.

27. Sample answer: The product equation for secant segments equates the product of exterior segment measure and the whole segment measure for each secant. In the case of secant-tangent, the product involving the tangent segment becomes (measure of tangent segment)^2 because the exterior segment and the whole segment are the same segment.

29.



This could represent rays of light traveling through a magnifying glass and being sent to one point.

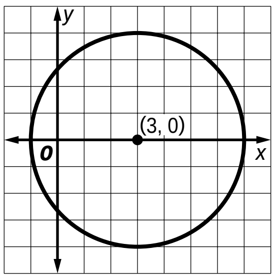
31. Sample answer: The product of the parts on one intersecting chord equals the product of the parts of the other chord. Answers should include the following: $\overline{AF}$, $\overline{FD}$, $\overline{EF}$, $\overline{FB}$, and $AF \cdot FD = EF \cdot FB$

33. H **35.** 157.5 **37.** 7 **39.** 36 **41.** 13 **43.** $\sqrt{40}$

Pages 617–619 ***Lesson 10-8***

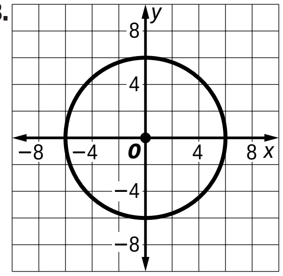
1. $x^2 + y^2 = 1600$ **3.** $x^2 + y^2 = 7$ **5.** $(x + 11)^2 + (y - 2)^2 = 32$

7.



9. $x^2 + y^2 = 9$ **11.** $(x - 1)^2 + (y + 4)^2 = 17$ **13.** $(x - 5)^2 + (y - 10)^2 = 49$ **15.** $(x + 8)^2 + (y - 8)^2 = 64$ **17.** $(x + 3)^2 + (y - 6)^2 = 9$ **19.** $(x + 2)^2 + (y - 1)^2 = 10$ **21.** $(x - 7)^2 + (y - 8)^2 = 25$

23.



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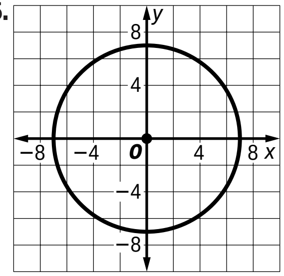

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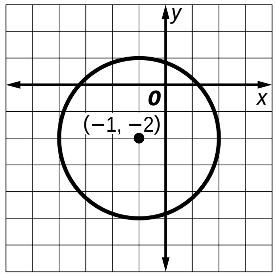

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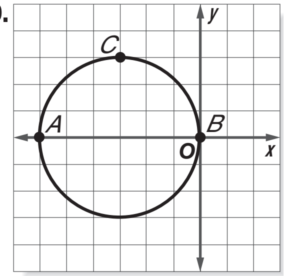
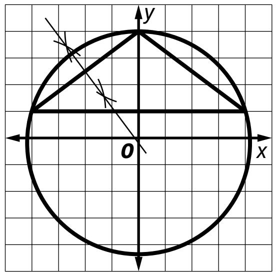


$(x + 3)^2 + y^2 = 9$

31. 2 **33.** $x^2 + y^2 = 810{,}000$ **35.** 3000 ft **37.** (2, -4); $r = 6$ **39.** $x^2 + y^2 = 3{,}705{,}625$

41.

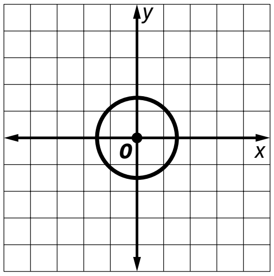


43. A circle is the locus of all points in a plane (coordinate plane) a given distance (the radius) from a given point (the center). The equation of a circle is written from knowing the location of the given point and the radius. **45.** B **47.** 24 **49.** 18 **51.** 59 **53.** 20 **55.** (3, 2), (-4, -1), (0, -4)

Pages 620–624 ***Chapter 10*** ***Study Guide and Review***

1. tangent **3.** central angle **5.** secant **7.** diameter **9.** major arc **11.** circle **13.** 7.5 in.; 47.12 in. **15.** 22 mm; 69.12 mm **17.** 60 **19.** 117 **21.** 30 **23.** 30 **25.** 150 **27.** $\frac{5}{2}\pi$ in. $\approx$ 7.9 in. **29.** 10 **31.** 90 **33.** 90 **35.** 48 **37.** $m\angle 1 = m\angle 3 = 39$, $m\angle 2 = 51$ **39.** $m\angle 2 = 57$, $m\angle 3 = m\angle 1 = 33$ **41.** 9 **43.** 200 ft **45.** 37 **47.** 17.1 **49.** 4 **51.** $(x + 4)^2 + (y - 8)^2 = 9$

53.



55. $(x - 3)^2 + (y - 3)^2 = 18$ **57.** $(x - 7)^2 + (y - 3)^2 = 225$

Chapter 11 Areas of Polygons and Circles

Page 629 ***Chapter 11*** ***Get Ready***

1. 10 **3.** 4.6 **5.** 18 **7.** 10.5 ft **9.** 24 **11.** 88 **13.** 55 **15.** 11

Pages 634–636 ***Lesson 11-1***

1. 30 ft; 43.3 ft^2 **3.** 12.8 m; 10.2 m^2 **5.** parallelogram, $8 + 4\sqrt{10}$ or about 20.65 units, 24 units^2 **7.** 80 in.; 259.8 in^2 **9.** 21.6 cm; 29.2 cm^2 **11.** 44 m; 103.9 m^2 **13.** ≈13.9 ft **15.** 45.7 mm^2 **17.** 108.5 ft^2 **19.** parallelogram, $16 + 2\sqrt{53}$ or about 30.56 units, 56 units^2 **21.** square, $4\sqrt{13}$ or about 14.42 units, 13 units^2 **23.** 77 units^2 **25.** Yes; the dimensions are 32 in. by 18 in. **27.** 38 m, 80 m^2 **29.** The perimeter is 19 m, half of 38 m. The area is 20 m^2. The new perimeter is half of the original. The new area is one half squared or one fourth the area of the original parallelogram. **31.** $A = \left(\frac{1}{2}b\right)\left(\frac{1}{2}h\right)$ or $\frac{1}{4}bh$ The original formula is $A = bh$. When each dimension is divided in half, the area is one-fourth the area of the original parallelogram. $P = 2\left(\frac{1}{2}b\right) + 2\left(\frac{1}{2}h\right)$ or $b + h$. The original formula is $P = 2b + 2h$. When each dimension is divided in half, the perimeter is one-half the perimeter of the original parallelogram. **33.** 5 in., 7 in. **35.** C **37.** (5, 2), $r = 7$ **39.** 32 **41.** 21 **43.** 7 **45.** 15

Pages 643–647 ***Lesson 11-2***

1. 960 m^2 **3.** 240 yd^2 **5.** 16 units^2 **7.** 11 in. **9.** $\frac{8}{39}$ in., 3 in., $6\frac{4}{13}$ in^2 **11.** 99 in^2 **13.** 96.5 yd^2 **15.** 408 cm^2 **17.** 55 units^2 **19.** 70 units^2 **21.** 20 units^2 **23.** 16 units^2 **25.** 26.8 ft **27.** 22.6 m **29.** 20 cm **31.** 13,326.7 ft^2 **33.** about 8.7 ft **35.** 120 in^2 **37.** ≈10.8 in^2 **39.** 21 ft^2 **41.** A rhombus is made up of two congruent triangles. Using d_1 and d_2 instead of b and h, its area in reference to $A = \frac{1}{2}bh$ is $2\left[\frac{1}{2}(d_1)\left(\frac{1}{2}d_2\right)\right]$ or $\frac{1}{2}d_1d_2$. **43.** area = 48, area = 12, perimeter = $8\sqrt{13}$, perimeter = $4\sqrt{13}$; scale factor and ratio of perimeters = $\frac{1}{2}$, ratio of areas = $\left(\frac{1}{2}\right)^2$ **45.** 22.8 units; 11.4 units; The ratio is the same. **47.** The ratio of the areas is the square of the ratio of the perimeters. **49.** The perimeter of new $\triangle DEF$ is 18.2 units, the perimeter of $\triangle DEF$ is 11.4 units. There is no relationship between the perimeters of the triangles. The area of new $\triangle DEF$ is 12 units^2 and that of $\triangle DEF$ is 6 units^2. The area of new $\triangle DEF$ is twice that of $\triangle DEF$. **51.** 6.79 in^2 **53.** 0.92 ft^2 **55.** Sometimes; two rhombi can have different corresponding diagonal lengths and have the same area. **57.** Kiku; she simplified the formula properly by adding the terms in the parentheses before multiplying.**59.** Sample answer: Find the area of a triangle by multiplying the base and the height and dividing by two. Rhombi are composed of two congruent isosceles triangles, and trapezoids are composed of two triangles and a rectangle. **61.** J **63.** 129.9 in^2 **65.** $(x - 1)^2 + (y - 2)^2 = 49$ **67.** $(x + 1.3)^2 + (y - 5.6)^2 = 12.25$ **69.** <123.3, 57.5> **71.** 44.0 **73.** 4.8

Pages 653–656 ***Lesson 11-3***

1. 127.3 yd^2 **3.** about 3.6 yd^2 **5.** 18.5 in^2 **7.** 882 m^2 **9.** 1995.3 in^2 **11.** 482.8 km^2 **13.** 30.4 units^2 **15.** 26.6 units^2 **17.** 4.1 units^2 **19.** 271.2 units^2 **21.** One 16-inch pizza; the area of the 16-inch pizza is greater than the area of two 8-inch pizzas, so you get more pizza for the same price. **23.** $18x^2$ units^2 **25.** 98 units^2 **27.** 907.9 units^2 **29.** 239.0 units^2 **31.** 5.3 cm^2 **33.** Sample answer: Multiply the total area by 37%. **35.** triangles; 629 tiles **37.** 68.7 units^2 **39.** 7.7 units^2 **41.** 58.9 units^2 **43.** 2:3 **45.** ≈ 27.52 cm^2, ≈ 61.94 cm^2; The ratio of the areas is the square of the scale factor. **47.** The ratio of their areas is the square of the scale factor for all similar polygons. In a pair of similar polygons, the apothem and the lengths of the sides are similar by the same scale factor. So the ratio of the area is the square of the scale factor. **49.** The new area is four times that of the original area. The new circumference is twice the original circumference. **51.** 3:4 **53.** D **55.** 260 units^2 **57.** 2829.0 yd^2 **59.** square; 36 units^2 **61.** $F''(-4, 0)$, $G''(-2, -2)$, $H''(-2, 2)$; 90° counterclockwise **63.** 42 **65.** 6 **67.** $4\sqrt{2}$

Pages 660–663 ***Lesson 11-4***

1. 53.4 units^2 **3.** 1247.4 in^2 **5.** 52.6 units^2 **7.** 70.9 units^2 **9.** 4185 units^2 **11.** 154.1 units^2 **13.** 2236.9 in^2 **15.** 23.1 units^2 **17.** 21 units^2 **19.** 33.5 units^2 **21.** 31.1 yd^2 **23.** 462 **25.** Sample answer: Reduce the width of each rectangle. **31.** $\frac{\sqrt{3}}{4}$:1 **33.** B **35.** 42.1 units^2 **37.** 139.1 units^2 **39.** 96 yd^2 **41.** $15 < x < 47$ **43.** $12 < x < 34$ **45.** 0.81 **47.** 0.48

Pages 668–671 ***Lesson 11-5***

1. 0.6 **3.** ≈ 114.2 units^2, ≈ 0.36 **5.** 0.50 **7.** 0.64 **9.** ≈ 58.9 units^2, $0.\overline{3}$ **11.** ≈ 19.6 units^2, 0.11 **13.** ≈ 74.6 units^2, 0.42 **15.** 39.3 units^2, 0.20 **17.** ≈ 0.16 **19.** 0.08 **21.** 0.51 **23.** 0.44 **25.** 0.19 **27.** Rachel; Taimi did not multiply $\frac{62}{360}$ by the area of the circle. **29.** Apothem; the other three terms are related to circles. **31.** No; there is not an equal chance of landing on each color. **33.** C **35.** 1050 units^2 **37.** 110.9 ft^2 **39.** 221.7 in^2

Pages 672–674 **Chapter 11** **Study Guide and Review**

1. composite figure **3.** geometric probability **5.** sector **7.** sector **9.** apothem **11.** square; 49 units^2 **13.** parallelogram; 20 units^2 **15.** 28 in. **17.** 688.2 in^2 **19.** 8 in. **21.** 87.5 units^2 **23.** $0.\overline{3}$ **25.** 0.4

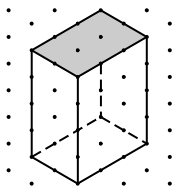
Chapter 12 Extending Surface Area

Page 679 **Chapter 12** **Get Ready**

1. true **3.** cannot be determined **5.** 384 ft^2 **7.** 381.5 in^2 **9.** 7.1 yd^2

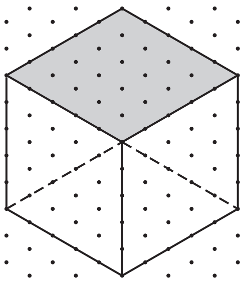
Pages 682–685 **Lesson 12–1**

1.



3. To get round slices of cheese, slice the cheese parallel to the bases. To get rectangular slices, place the cheese on the slicer so the bases are perpendicular to the blade.

5.



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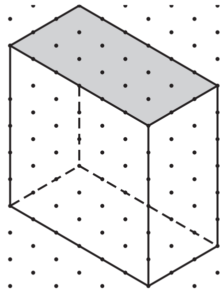

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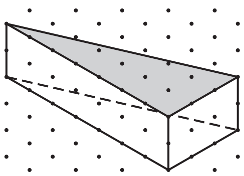
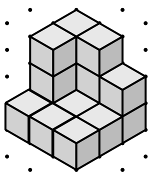
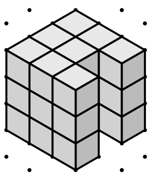


11.



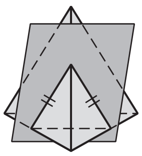
corner view

13.



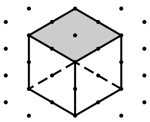
15. triangle **17.** rectangle **19.** square

21. intersecting three faces and edge of base



23. triangles, square or rectangle **25.** hexagon, hexagon **27.** rectangle, rectangle

29.



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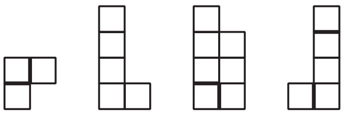


top view left view front view right view

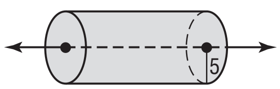
33. 6; there are three planes of symmetry through each vertex, counting each plane only once, there are 6 planes of symmetry. **35.** infinite; Any plane that passes through the center of the sphere is a plane of symmetry. **37.** order 3, magnitude 120° **39.** Sample answer: An orthographic drawing would show the building from each side. This drawing would give the viewer a sense of proportion of the building. **41.** Construction; a construction is a two-dimensional representation of a two-dimensional figure. The other three terms are representations of solids. **43.** D **45.** ≈ 0.242 **47.** ≈ 0.611 **49.** 21 units^2 **51.** $m\angle B \approx 56.3$, $m\angle C \approx 69.7$, $a \approx 6.1$ **53.** $m\angle ADB > m\angle ABD$ **55.** $m\angle BCD < m\angle CDB$ **57.** 224 ft^2 **59.** 13 yd^2

Pages 688–691 **Lesson 12–2**

1. 840 in^2 **3.** 850 m^2 **5.** 1140 ft^2 **7.** 128 units^2 **9.** 162 units^2 **11.** 128 units^2 (square base); 96 units^2 (rectangular base) **13.** 216 in^2 **15.** 352 ft^2 **17.** 114 cm^2 **19.** 522 ft^2 **21.** 454.0 mi^2 **23.** \$80 **25.** 12 in. **27.** The perimeter of the base must be 12 inches. There are three rectangles with integer values for the dimensions that have a perimeter of 12. The dimensions of the base could be 5×1, 4×2, or 3×3. **29.** base of A ≅ base of C; base of A ~ base of B; base of C ~ base of B **31.** A:B = 1:4, B:C = 4:1, A:C = 1:1 **33.** A:B, because the heights of A and B are in the same ratio as perimeters of bases. **35.** 2824.8 cm^2 **37.** 198 cm^2 **39.** C **41.** rectangle **43.** rectangle **45.** 5026.55 cm^2 **47.** 38.48 ft^2

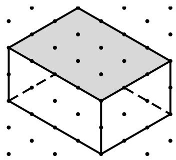
Pages 695–697 **Lesson 12–3**

1. ≈102,353.1 in^2 **3.** 1520.5 m^2 **5.** 10 ft **7.** 371.7 ft^2 **9.** 2463.0 mm^2 **11.** 291.1 yd^2 **13.** 247.3 m^2 **15.** 10 in. **17.** ≈ 11.0 ft **19.** 283.7 in^2 **21.** 42.5 m^2 **23.** ≈ 427.6 in^2 **25.** a cylinder with a radius of 5 units



27. To find the surface area of any solid figure, find the area of the base (or bases) and add to the area of the sides of the figure. The faces and bases of a rectangular prism are rectangles. Since the bases of a cylinder are circles, the “side” of a cylinder is a rectangle. **29.** Jamie; since the cylinder has one base removed, the surface area will be the sum of the lateral area and one base. **31.** Sample answer: To find the lateral area of the full-pipe, multiply the length by the circumference of the base. **33.** H **35.** 300 units^2

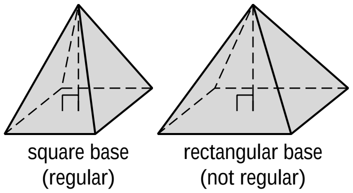
37.



39. 27 **41.** 8 **43.** 170 in^2 **45.** 247 mm^2

Pages 702–705 **Lesson 12–4**

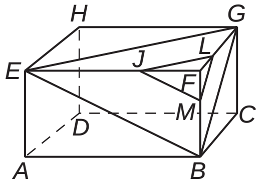
1. 16 in^2 per pyramid, 96 in^2 per star **3.** 86.9 cm^2 **5.** 119 cm^2 **7.** 147.7 ft^2 **9.** 173.2 yd^2 **11.** 326.9 in^2 **13.** 27.7 ft^2 **15.** ≈ 2.3 in. on each side **17.** ≈ 76,452.5 m^2 **19.** 816 ft^2 **21.** The lateral area of the pyramid is 240 ft^2. The lateral area of the cube is 576 ft^2. The base of each triangular face of the pyramid is the same length as the square and the height is less than that of the square. Therefore each square face has a greater area than each triangular face.

23. 

The regular pyramid has a square base and the pyramid that is not regular has a rectangular base.

27. Sample answer: We need to know the dimensions of the base and slant height to find the lateral area and surface area of a pyramid. Sample answer: The roof of a gazebo is often a hexagonal pyramid. **29.** C **31.** 1727.9 cm^2 **33.** 268 in^2 **35.** 90 ft, 433.0 ft^2 **37.** $\approx$ 6.1 mi

39. Given: $\overline{JM} \parallel \overline{EB}$; $\overline{LM} \parallel \overline{GB}$,
Prove: $\overline{JL} \parallel \overline{PG}$

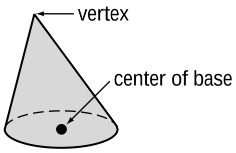


Proof: Since $\overline{JM} \parallel \overline{EB}$ and $\overline{LM} \parallel \overline{GB}$, then $\angle MJF \cong \angle BEF$ and $\angle FML \cong \angle FBG$ because if two parallel lines are cut by a transversal, corresponding angles are congruent. $\angle EFB \cong \angle EFB$ and $\angle BFG \cong \angle BFG$ by the Reflexive Property of Congruence. Then $\triangle EFB \sim \triangle JFM$ and $\triangle FBG \sim \triangle FML$ by AA Similarity. Then $\frac{JF}{EF} = \frac{MF}{BF}, \frac{MF}{BF} = \frac{LF}{GF}$, by the definition of similar triangles. $\frac{JF}{EF} = \frac{LF}{GF}$ by the Transitive Property of Equality, and $\angle EFG \cong \angle EFG$ by the Reflexive Property of Congruence. Thus, $\triangle JFL \sim \triangle EFG$ by SAS Similarity and $\angle FJL \cong \angle FEG$ by the definition of similar triangles. $\overline{JL} \parallel \overline{PG}$ because if two lines are cut by a transversal so that the corresponding angles are congruent, then the lines are parallel. **41.** 21.3 m

Pages 708–710 ***Lesson 12-5***

1. 848.2 cm^2 **3.** 485.4 in^2 **5.** 282.7 cm^2 **7.** 614.3 in^2 **9.** 628.8 m^2 **11.** 679.9 in^2 **13.** 7.9 m **15.** 1613.7 in^2 **17.** No, in order to cover the tepee you would need at least 3160.1 ft^2 of fabric. The canvas that is available is 3000 ft^2. **19.** 8.2 m **21.** 169.6 ft^2 **23.** 101.3133 in^2 **25.** 8.06 in., 101.2849 in^2 **27.** Never; the pyramid could be inscribed in the cone.

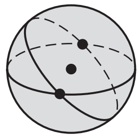
29. Sample answer:



31. As the altitude approaches zero, the slant height of the cone approaches the radius of the base. The lateral area approaches the area of the base. The surface area approaches twice the area of the base. **33.** A **35.** $\approx$ 255,161.7 ft^2 **37.** 2.2 m **39.** 24 **41.** 24 **43.** 90 **45.** 25.1 **47.** 51.5

Pages 714–717 ***Lesson 12-6***

1. 15 **3.** 18 **5.** 150.8 cm^2 **7.** $\approx$ 283.5 in^2 **9.** $\approx$ 8.5 **11.** 8 **13.** 12.8 **15.** 7854.0 in^2 **17.** 636,172.5 m^2 **19.** 397.4 in^2 **21.** 3257.2 m^2 **23.** $\approx$ 43,743.5 ft^2 **25.** pole to pole, 196,058,359.3 mi^2; equator, 197,379,906.2 mi^2 **27.** $\approx$ 147,538,933.4 mi^2 **29.** False; two great circles will intersect at two points.



31. true **33.** 4:1 **35.** 9:1 **37.** $\approx$ 153.9 mi^2 **39.** The radius of the sphere is $\frac{x\sqrt{3}}{2}$, where x is the length of each edge of the cube.

41. Tim; the surface area of a hemisphere is half of the surface area of the sphere plus the area of the great circle. **43.** Sample answer: The surface area of a sphere is four times the area of the great circle of the sphere. Racquetball and basketball are other sports that use balls. **45.** G **47.** 422.4 m^2 **49.** 261.8 ft^2 **51.** 487.6 $units^2$ **53.** 227.0 in^2 **55.** $(x - 4)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{185}{4}$ **57.** $KP = 5$, $KM = 15$, $MR = 13\frac{1}{3}$, $ML = 20$, $MN = 12$, $PR = 16\frac{2}{3}$ **59.** $\sqrt{68} \approx$ 8.25 **61.** $\sqrt{250} \approx 15.81$ **63.** $\sqrt{104} \approx 10.20$

Pages 719–722 ***Chapter 12*** ***Study Guide and Review***

1. true **3.** false, great circle **5.** true **7.** true **9.** cylinder; bases: $\odot F$ and $\odot G$ **11.** Sample answer: triangular prism; base: $\triangle BCD$; faces: $\triangle ABC$, $\triangle ABD$, $\triangle ACD$, and $\triangle BCD$; edges: $\overline{AB}$, $\overline{BC}$, $\overline{AC}$, $\overline{AD}$, $\overline{BD}$, and $\overline{CD}$; vertices: A, B, C, and D **13.** 1080 $units^2$ **15.** 688 in^2 **17.** 527.8 ft^2 **19.** 125.7 km^2 **21.** 304 $units^2$ **23.** 33.3 $units^2$ **25.** 8175.2 m^2 **27.** 100.1 in^2 **29.** 1040.6 ft^2 **31.** 363 mm^2 **33.** 157.1 ft^2 **35.** 880 ft^2 **37.** 235.6 ft^2

Chapter 13 Extending Volume

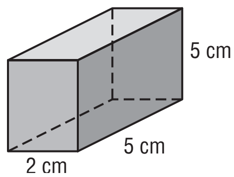
Page 727 ***Chapter 13*** ***Get Ready***

1. ± 5 **3.** $\pm 4\sqrt{3}$ **5.** $x^2 = (2x)^2 - 18$; $x = \sqrt{6}$ **7.** $\frac{n^2}{16}$ **9.** $\frac{16y^2}{49}$ **11.** $\left(\frac{4z}{12}\right)^2 = \frac{z^2}{9}$ **13.** (19, 21)

Pages 732–735 ***Lesson 13-1***

1. 288 cm^3 **3.** 3180.9 mm^3 **5.** 267.0 cm^3 **7.** 821.3 in^3 **9.** 763.4 cm^3 **11.** 28 ft^3 **13.** 165.6 yd^3 **15.** 18 in. **17.** 2.5 $units^3$ **19.** 1575 ft^3 **21.** 3104.0 cm^3 **23.** 28.6 ft^3 **25.** 304.1 cm^3 **27.** There is 2.96 yd^3 of concrete needed. The second contractor is less expensive at \$2181.33. **29.** 104,411.5 mm^3

31. Sample answer:

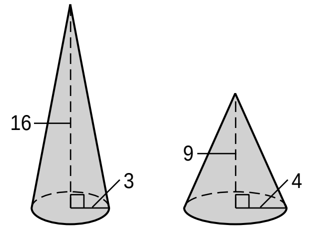


33. 33,990,000 ft^3 **35.** B **37.** 452.4 ft^2 **39.** 907.9 in^2 **41.** 320.4 m^2 **43.** 282.7 in^2 **45.** 8.8 **47.** 21.22 in^2 **49.** 61.94 m^2

Pages 740–742 ***Lesson 13-2***

1. 640 in^3 **3.** 1340.4 in^3 **5.** 206.5 cm^3 **7.** 1728 in^3 **9.** 370.2 in^3 **11.** 154.2 m^3 **13.** 1131.0 ft^3 **15.** 1610.8 ft^3 **17.** 2 in^3 **19.** $\approx$ 6,080,266.7 ft^3 **21.** 522.3 $units^3$ **23.** $\approx$ 614.6 in^3 or about 0.4 ft^3; 8 planters **25.** Each volume is 8 times as large as the original.

27.



$V = \frac{1}{3}\pi(3^2)(16)$

$\quad = 48\pi$ units^3

$V = \frac{1}{3}\pi(4^2)(9)$

$\quad = 48\pi$ units^3

29. Sample answer: Architects use geometry to design buildings that meet the needs of their clients. The silhouette of a pyramid-shaped building is smaller than the silhouette of a rectangular prism with the same height and base. If the light conditions are the same, the shadow cast by the pyramid is smaller than the shadow cast by the rectangular prism. **31.** G **33.** 3418.1 m^3 **35.** 2354.2 cm^2 **37.** 101,262.5 ft^2 **39.** 1436.76

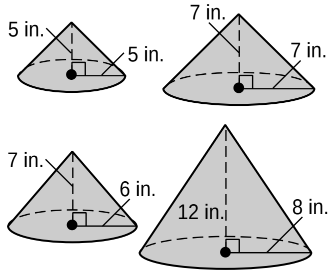
Pages 746–748 ***Lesson 13-3***

1. 9202.8 in^3 **3.** 1022.7 cm^3 **5.** 294.5 mi^3 **7.** 1853.3 m^3 **9.** 3261.8 ft^3 **11.** 1241.4 yd^3 **13.** 233.4 in^3 **15.** 68.6 m^3 **17.** 7238.2 in^3 **19.** $\approx$ 21,990,642,871 km^3 **21.** $\approx$ 12.3 in^3 **23.** The volume of the cone is approximately 9.42 in^3. The volume of ice is approximately 14.1 in^3. If all of the ice melts, the cone will overflow. **25.** The balloon would have to have a diameter of 139,286 cm. **27.** 11,494.0 in^3 **29.** $\approx$ 1,874,017.6 ft^3 **31.** 162.86 cm^2 **33.** 385.4 in^3 **35.** $\approx$ 587.7 in^3 **37.** Sphere; the other three solids have the same volume, πr^3 cubic units. **39.** A **41.** 358.1 m^3 **43.** $\approx$ 2.2 ft **45.** $(x - 2)^2 + (y - 1)^2 = 34$ **47.** $27x^3$ **49.** $\frac{8k^3}{125}$

Pages 754–756 ***Lesson 13-4***

1. congruent **3.** $\frac{4}{3}$ **5.** $\frac{64}{27}$ **7.** 1:64 **9.** neither **11.** congruent **13.** neither **15.** $\frac{29}{30}$ **17.** $\frac{24389}{27000}$ **19.** $\frac{4}{25}$ **21.** 750 in^3 **23.** about 22.3 in.:1 in. **25.** Always; congruent solids have equal dimensions. **27.** Never; different solids cannot be similar. **29.** Sometimes; solids that are not similar can have the same surface area. **31.** About 527 lb; the ratio of the volumes of the pies is $\frac{8}{3375}$. Weights of similar objects are proportional to the volumes of the objects. **33.** 3:4; 3:1

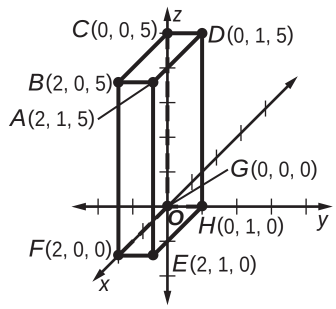
35.



37. Yes; both cones have congruent radii. If the heights are the same measure, the cones are congruent. **39.** Sample answer: Scale factors relate the actual object to the miniatures. The scale factors that are commonly used are 1:24, 1:32, 1:43, and 1:64. The actual object is 108 in. long. **41.** J **43.** 268.1 ft^3 **45.** 14,421.8 cm^3 **47.** 323.3 in^3 **49.** 2741.8 ft^3 **51.** 2.8 yd **53.** 36 ft^2 **55.** 10.5 **57.** 5 **59.** $9 < n < 37$ **61.** $3 < n < 33$ **63.** $24 < n < 152$ **65.** yes

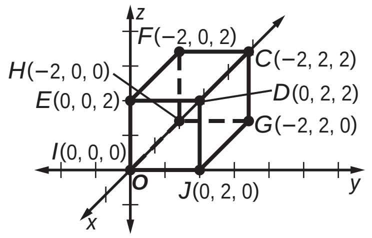
Pages 761–764 ***Lesson 13-5***

1.

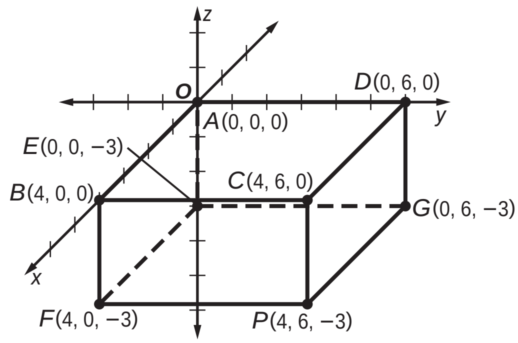


3. $5\sqrt{3} \approx 8.7$; $\left(\frac{1}{2}, \frac{5}{2}, \frac{7}{2}\right)$ **5.** $(x, y, z) \to (x + 5, y + 2, z)$; (10, 2, 0), (10, 4, 0), (8, 4, 0), (8, 2, 0), (10, 2, 2), (10, 4, 2), (8, 4, 2), (8, 2, 2)

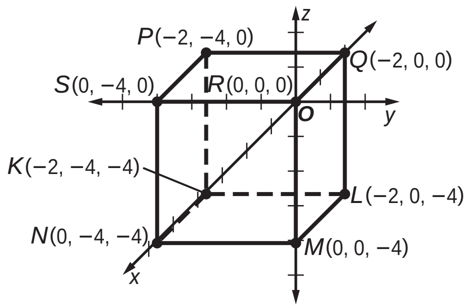
7.



9.



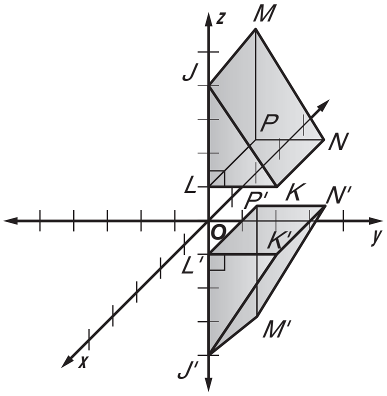
11.



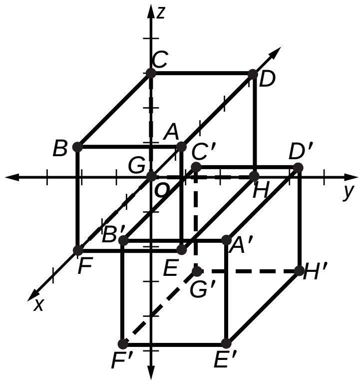
13. $KL = 4\sqrt{2}$; (0, 0, 0) **15.** $AB = \sqrt{339}$; $\left(\frac{1}{2}, \frac{15}{2}, \frac{1}{2}\right)$

17. $FG = \sqrt{10}$; $\left(\frac{3}{10}, \frac{3}{2}, \frac{2}{5}\right)$ **19.** $BC = \sqrt{39}$; $\left(-\frac{\sqrt{3}}{2}, 3, 3\sqrt{2}\right)$

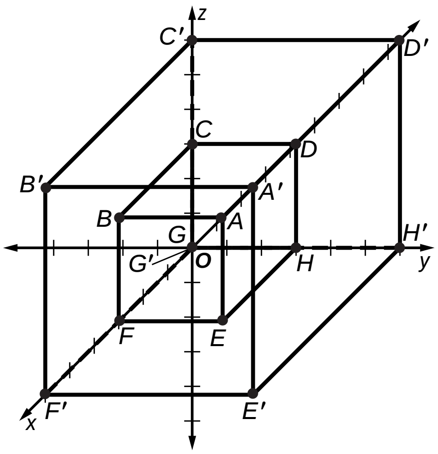
21. $P'(0, 2, -2)$, $Q'(0, 5, -2)$, $R'(2, 5, -2)$, $S'(2, 2, -2)$, $T'(0, 5, -5)$, $U'(0, 2, -5)$, $V'(2, 2, -5)$, and $W'(2, 5, -5)$ **23.** (12, 8, 8), (12, 0, 8), (0, 0, 8), (0, 8, 8), (12, 8, 0), (12, 0, 0), (0, 0, 0), and (0, 8, 0); (−36, 8, 24), (−36, 0, 24), (−48, 0, 24), (−48, 8, 24), (−36, 8, 16), (−36, 0, 16), (−48, 0, 16), and (−48, 8, 16) **25.** $J'(0, 0, -4)$, $K'(0, 2, -1)$, $L'(0, 0, -1)$, $M'(-2, 0, -4)$, $N'(-2, 2, -1)$, $P'(-2, 0, -1)$



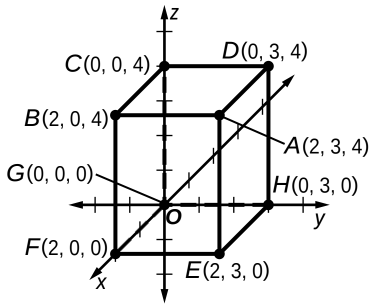
27. 32 units^3 **29.** $A'(4, 5, 1)$, $B'(4, 2, 1)$, $C'(1, 2, 1)$, $D'(1, 5, 1)$, $E'(4, 5, -2)$, $F'(4, 2, -2)$, $G'(1, 2, -2)$, and $H'(1, 5, -2)$;



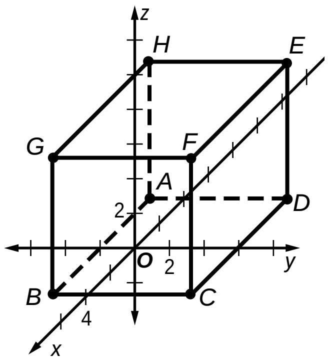
31. $A'(6, 6, 6)$, $B'(6, 0, 6)$, $C'(0, 0, 6)$, $D'(0, 6, 6)$, $E'(6, 6, 0)$, $F'(6, 0, 0)$, $G'(0, 0, 0)$, and $H'(0, 6, 0)$; $V = 216$ cubic units



33. (8, −2, −3) **35.** center (1, 1, 7); radius $= 5\sqrt{11}$
37. $(x, y, z) \rightarrow (x + 2, y + 3, z - 5)$ **39.** Sample answer: Use the point at (2, 3, 4); $A(2, 3, 4)$, $B(2, 0, 4)$, $C(0, 0, 4)$, $D(0, 3, 4)$, $E(2, 3, 0)$, $F(2, 0, 0)$, $G(0, 0, 0)$, and $H(0, 3, 0)$.



41. $A(-2, 0, 2)$, $B(6, 0, 2)$, $C(6, 8, 2)$, $D(-2, 8, 2)$, $E(-2, 8, 10)$, $F(6, 8, 10)$, $G(6, 0, 10)$, and $H(-2, 0, 10)$



43. C **45.** neither **47.** 4188.8 cm^3 **49.** 21,314.4 m^3
51. ≈ 78.5 in^2

Pages 765–768 **Chapter 13** **Study Guide and Review**

1. pyramid **3.** an ordered triple **5.** similar **7.** the Distance Formula in Space **9.** Cavalieri's Principle
11. 311.0 m^3 **13.** 18.8 in^3 **15.** 1466.4 ft^3 **17.** 328,125 ft^3
19. 4637.6 mm^3 **21.** 523.6 units^3 **23.** congruent
25. ≈ 0.004 in^3 **27.** $CD = \sqrt{58}$; (−9, 5.5, 5.5)
29. $FG = \sqrt{422}$; $(1.5\sqrt{2}, 3\sqrt{7}, -3)$

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Index

Index

Index

B

C

D

E

F

G

K

L

M

N

Index

Index

S

Index

T

Index

U

V

Index

W

X

Y

Z

Index