ENGR2741 Prac 1 Report

Pin Jointed Framework: Forces in a Simple Cantilever Truss

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***Abstract*—A simple three member pin-jointed cantilever truss supported by one pin joint and a roller support is loaded at the cantilever point. The internal member forces are calculated theoretically and measured experimentally with strain gauges, and comparisons made between the two. The two members directly supporting the load showed to behave as expected while the member between the two supports showed to experience some unexpected compression. This is possibly due to friction at the roller support or pin joint that was initially considered negligible, or offset external loads. Overall the theory is a good predictor of actual forces.**

***Keywords*—cantilever truss, internal member forces, stress, strain**

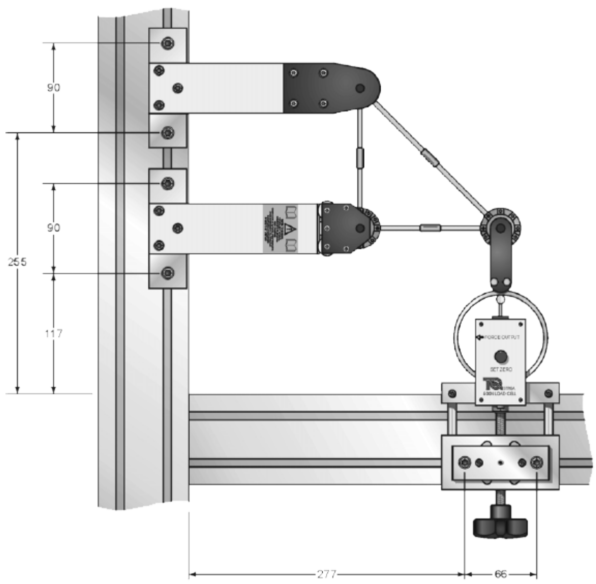

Figure 1: Diagram showing front view of experiment apparatus. Adapted from Holyoak [1].

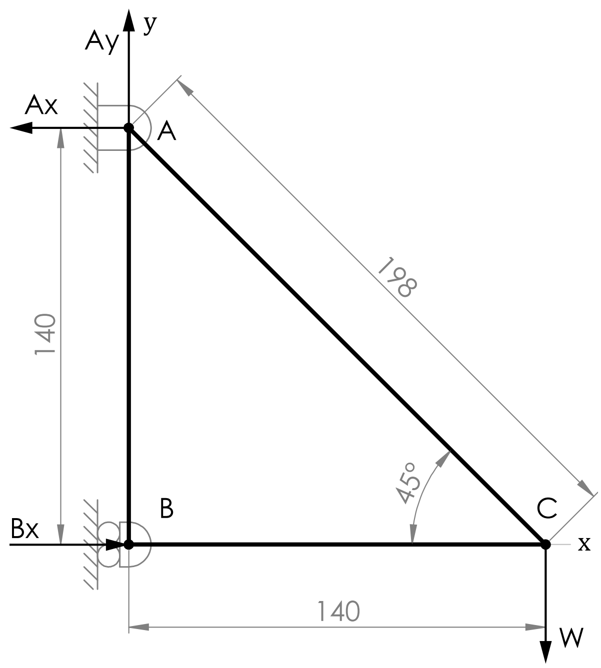

Figure 2: Free body diagram of the truss.

Symbols

F_x	force in x-direction	newton
F_y	force in y-direction	newton
M	moment	newton meter
W	external load	newton
F_{AB}	force in member AB	newton
F_{AC}	force in member AC	newton
F_{BC}	force in member BC	newton
E	Young's modulus	pascal
$\varnothing$	rod diameter	meter
A	cross-section area	meter^2
σ	stress	pascal
ϵ	strain	ratio

1 Introduction

When designing a structure it is important to ensure the forces within the members are properly understood so that the design will meet requirements. By utilising equilibrium analysis the theoretical forces and moments can be calculated. It is also important to test a design experimentally to verify the structure in the real world.

In this report a simple three member pin-jointed cantilever truss, shown in Figure 2 is examined, both theoretically and experimentally.

2 Methods

Three truss members AB, AC, and BC are mounted together with pin joints. The truss is connected to the apparatus frame at a fixed pin joint at A and a roller pin joint at B. Each three truss members have a strain gauge attached to their midpoints. These strain gauges report to a digital interface that is accessed via a computer. At pin joint C of the truss a screw-adjustable mechanism is attached that allows a tension force to be generated between that point and the apparatus frame, simulating a hanging weight, designated W. This load mechanism has a force sensor that reports back to the digital interface.

Using the equations for stress and strain, the forces within the members of the truss can be calculated from the reported strain.

The rod members are stainless steel with Young's modulus of $E = 210\,\mathrm{GPa}$, and of circular cross section with diameter $\varnothing = 6\,\mathrm{mm}$.

A diagram of the experimental setup is shown in Figure 1.

2.1 Assumptions

The materials are assumed to be elastic, not loaded beyond their yield stress, and be homogeneous. The structure is assumed to be in equilibrium, with negligible friction at pin joints and roller supports. The weight of the members is not considered, and they are assumed to be two-force members with purely axial loading.

3 Results

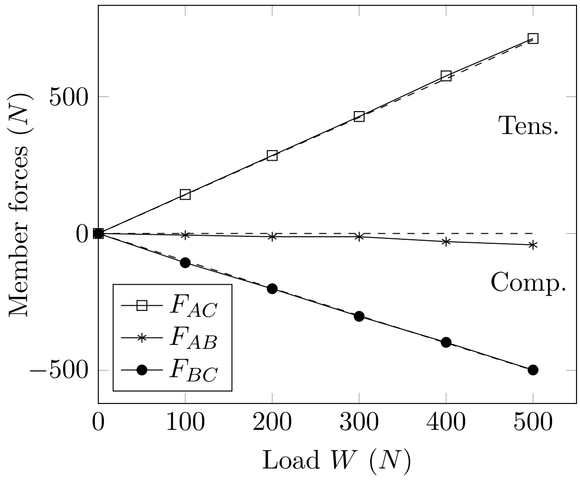

Figure 3: Plot showing internal member forces for the three rods AB, AC, and BC. Solid lines —— show experimental results, while the dashed lines - - - show the theoretical forces. Positive forces indicate tension, while negative forces indicate compression.

4 Analysis

The forces follow a linear trend as expected from the equations governing their behaviour, and there is good agreement between the theoretical and experimental forces in members AC and BC. There is a compression force in member AB that is not accounted for in theory, possibly due to untrue assumptions.

Strain gauge readings are positive for tension and negative in compression [2], and so it can be seen from Figure 3 and Table 2 that in all experimental cases when a load W is applied in the downward direction at point C, member AB is in compression, member AC is in tension, and member BC is in compression.

The type of loading of each member (compression/tension) can also be determined theoretically from the loadings calculated at the joints.

Of note is that member AB is expected to have no loading, however showed to experience a slight compression loading in practise.

Using a linear regression from the experimental data the load in member AB is proportional to the load as:

$$F_{AB} = -7.13 \times 10^{-2} \times W \tag{1}$$

which can be interpreted as possibly frictional influence of joint B or some moment produced at point C. As the friction in the roller support at joint B is dependant on the normal load ($F = \mu N$, N is load in member BC), it would follow the described linear behaviour, however the reason of the direction of the force (compression, rather than tension) remains undetermined.

Assuming the rods do not deform in bending but only in-axis, the force experienced in member AB cannot be due to a moment produced at joint A or B as they are inline with member AB and hence the perpendicular distance there is zero.

This anomaly is not a result of the mass of member AB as this would introduce a tension load and also be constant with different external loading, rather than the proportional relationship experienced.

One possible reason for the unexpected compression could be the external loading being applied at an angle offset from the vertical, but this would also have an affect on the loading experienced in the other members.

4.1 Theoretical Forces

The pin joint at A has x and y support reactions and no moments, the roller joint at B has x reactions and no moments, and the external load W is applied in the vertical direction at C, that is purely in the y direction.

The equations of equilibrium can be used for analysis:

$$\overset{+}{\rightarrow} \sum F_x = 0 \tag{2}$$
$$+\uparrow \sum F_y = 0 \tag{3}$$
$$\circlearrowleft + \sum M = 0 \tag{4}$$

Taking moments about A:

$$\sum M_A = 0 \tag{5}$$
$$(-BC \times W) + (AB \times B_x) = 0 \tag{6}$$
$$B_x = \frac{BC}{AB} W \tag{7}$$

For the case where the truss is set at 45°, AB and BC are equal in length, then $B_x = W$.

$$\sum F_y = 0 \tag{8}$$
$$A_y = W \tag{9}$$
$$\sum F_x = 0 \tag{10}$$
$$B_x = A_x = W \tag{11}$$

By inspecting the external loadings at the pin joints the forces in the members can be shown as:

$$F_{AB} = 0 \tag{12}$$
$$F_{AC} = \sqrt{{A_x}^2 + {A_y}^2} \tag{13}$$
$$= W\sqrt{2} \quad \text{(Tens.)} \tag{14}$$
$$F_{BC} = W \quad \text{(Comp.)} \tag{15}$$

For several forces approximately equivalent to the experimental loads W the theoretical internal forces of each member are listed in Table 1.

4.2 Experimental Analysis

From the reported strain reading the force can be determined by applying stress and stain relationships.

The stress is proportional to the strain by the Young's modulus of the material [2, 3]:

$$\sigma = E\epsilon \tag{16}$$

and the stress is equivalent to the force per unit area [2, 3]:

$$\sigma = \frac{F}{A} \tag{17}$$

and so the force is related to the strain as:

$$F = \epsilon E A \tag{18}$$

The rod members have cross sectional area:

$$A = \pi r^2 \tag{19}$$
$$= \pi \left(3 \times 10^{-3}\right)^2 \tag{20}$$
$$= 28.3 \times 10^{-6}\,\mathrm{m}^2 \tag{21}$$

For example, an external load of $W = 200\,\mathrm{N}$ produced a strain of $48\,\mu\epsilon$ in rod AC, and so the force can be calculated as:

$$F = \epsilon E A \tag{22}$$
$$= \left(48 \times 10^{-6}\right) \times \left(210 \times 10^{9}\right) \times \left(28.3 \times 10^{-6}\right) \tag{23}$$
$$= 285\,\mathrm{N} \tag{24}$$

and comparing this value to the theoretical value of 283 N shows good agreement.

A complete table of experimental results is shown in Table 2.

4.3 Errors

Although unknown, the errors introduced by the strain and force gauges can be considered negligible as the force in members AC and BC show good agreement with the theory, as shown in Figure 3. The only anomaly was the unexpected compression experienced in member AB, which has been discussed.

5 Conclusion

The experimental results showed good agreement with theoretical calculations in two members of the truss while a third showed some unexpected compression. This was possibly caused by untrue assumptions such as friction in the roller support, a generated moment at pin connection C, or non vertical external loads. Overall the stress and strain relationships are shown to describe real wold behaviour accurately and the experiment shows that it is important to consider assumptions carefully.

References

[1] N. Holyoak, *ENGR2741 and ENGR8791: Mechanics & Structures (And GE) Practical 1 Pin Jointed Frameworks: Forces in a Simple Cantilever Truss*, 2020.

[2] J. William D. Callister and D. G. Rethwisch, *Fundamentals of Materials Sscience and Engineering: An Integrated Approach*, 9th ed. John Wiley & Sons, Inc., 2014, pp. 204, 399, 400, ISBN: 978-1-118-32457-8.

[3] R. Hibbeler, *Statics And Mechanics Of Materials*, 5th ed. United Kingdom: Pearson Education Limited, 2019, pp. 172, 174, ISBN: 1-292-17791-8.

Appendix A Data

Table 1: Table of theoretical internal forces in each truss member for several external loadings. The external loads are approximately equivalent to experimental values. All forces in newtons. Positive values indicate tension, and negative compression. Derived in Section 4.1.

Load (N)	F_{AB}	F_{AC}	F_{BC}
0	0	0	0
100	0	141.42	−100
200	0	282.84	−200
300	0	424.26	−300
400	0	565.69	−400
500	0	707.11	−500

Table 2: Table of experimental results showing measured strain and calculated stress and force in each member rod for each applied load W, as derived in Section 4.2.

External	Member AB			Member AC			Member BC		
Load	ϵ	σ	F	ϵ	σ	F	ϵ	σ	F
N	$\mu\epsilon$	MPa	N	$\mu\epsilon$	MPa	N	$\mu\epsilon$	MPa	N
0	0	0	0	0	0	0	0	0	0
100	−1	−0.21	−5.94	24	5.04	142.5	−18	−3.78	−106.88
200	−2	−0.42	−11.88	48	10.08	285.01	−34	−7.14	−201.88
300	−2	−0.42	−11.88	72	15.12	427.51	−51	−10.71	−302.82
400	−5	−1.05	−29.69	97	20.37	575.95	−67	−14.07	−397.82
500	−7	−1.47	−41.56	120	25.2	712.51	−84	−17.64	−498.76