ENGR2741 Prac 2 Report

Torsion of Circular Sections

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***Abstract*—Torque applied to a rod will induce a shear stress and shear strain. One method of expressing the amount of deformation is through the angle of twist the rod experiences due to the applied torque. The theoretical equations predict a linear relationship between the applied torque over constant length, or constant torque over varying length and the angle of twist. The experimental results show linear relationships, as predicted.**

***Keywords*—torsion, angle of twist, shear modulus of elasticity**

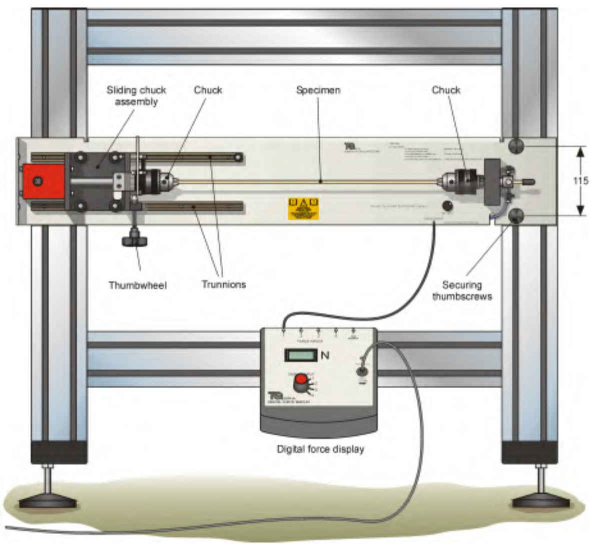

Figure 1: Front view of experiment apparatus. Adapted from Holyoak [1].

Symbols

ϕ	angle of twist	radians
τ	shear stress	pascals
γ	shear strain	unitless
G	shear modulus of elasticity	pascals
T	torque	newton-meters
L	length	meters
J	polar moment of inertia	meter^4
c	radius	meters

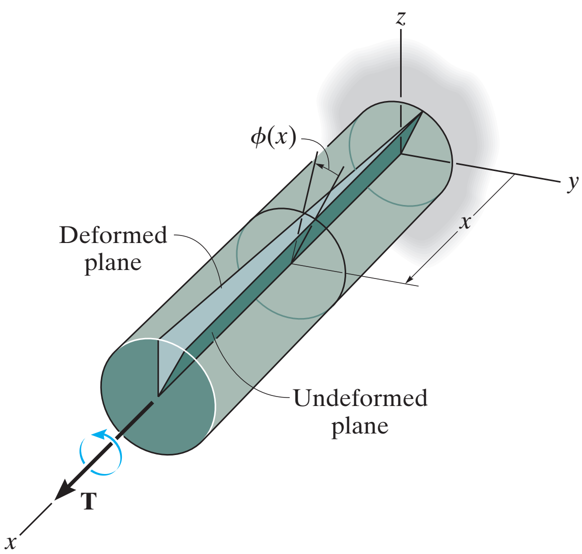

Figure 2: Visualisation of the angle of twist ϕ showing that it is dependent on length x. Adapted from [2].

1 Introduction

If a torque is applied to a material, the internal structure of the material will experience torsion, a shear deformation. Analysing torsion is important in any application where a torque is applied, for example axles, shafts and drills. Similar to axial stress and strain, within a material's elastic region of deformation, shear stress and strain are also related linearly by:

$$\tau = G\gamma \tag{1}$$

where τ is the shear stress, G is the shear modulus of elasticity, and γ is the shear strain.

As a torque is applied a torsional shear stress is developed which induces a torsional shear strain. Torsional shear can be quantified by the angle of twist a material experiences.

$$\phi = \frac{TL}{JG} \tag{2}$$

where ϕ in the angle of twist in radians, T is the torque, L is the length, J is the polar moment of inertia, and G is the shear modulus of elasticity.

The polar moment of inertia for a solid circular cross section is:

$$J = \frac{\pi}{2}c^4 \tag{3}$$

where c is the radius.

This experiment will analyse the relationship between the torsional deformation by the angle of twist of a brass and steel rod of solid circular cross sections and the applied torque and compare experimental results with theoretical equations.

2 Methods

The experiment apparatus consists of two chucks (as commonly seen on drills) that hold the ends of the rods. The length of rod that has torque applied is altered by moving the left chuck axially, while the right chuck is fixed in translation. The distance is confirmed by meter rule.

Torque is applied to the rod by adjusting a thumbscrew that applies torque to the left chuck. An angle scale attached to the left chuck indicates the current amount of rotation the chuck has undergone. The rotation of the right chuck is resisted by a force meter set at a distance of 50 mm from the centre of rotation. The force value is shown as a readout on an electronic display and allows the torque to be calculated by use of $T = Fd$.

The rods are loaded in two case, firstly the steel and brass rods are held at a fixed length $L = 0.5$ m and the torque is adjusted, and secondly the brass rod has a constant torque $T = 0.15$ Nm applied at varying lengths. The results are plotted in Figures 3 and 4.

A diagram of the experiment apparatus is shown in Figure 1.

The steel rod has diameter of $\varnothing = 3.2$ mm and shear modulus of elasticity of $G = 79.6$ GPa.

The brass rod has diameter of $\varnothing = 3.1$ mm and shear modulus of elasticity of $G = 38$ GPa.

2.1 Assumptions

Material is not loaded into the plastic region. The rods are homogeneous in material type, and cross-section area. Torque is applied about rod axis.

3 Results

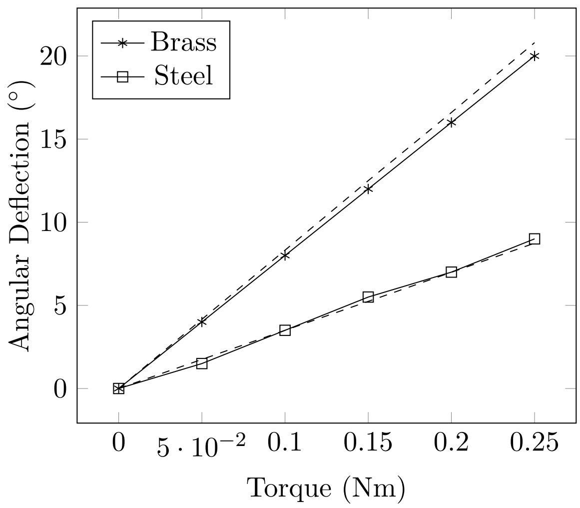

Figure 3: Plot of angular deflection vs applied torque for a steel and brass rod. Torque is applied at a fixed length of $L = 0.5$ m. Solid lines —— show experimental results, while the dashed lines - - - show the theoretical values.

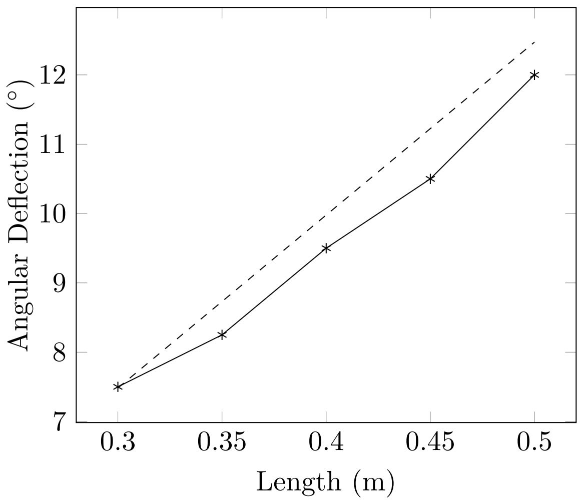

Figure 4: Plot of angular deflection vs torque applied over length for a brass rod. Torque is fixed at $T = 0.15\,\mathrm{Nm}$. Solid lines —— show experimental results, while the dashed lines - - - show the theoretical forces.

4 Analysis

As shown in Figure 3, the angular deformation of the steel and brass rods follow a linear relationship proportional to the applied torque. This is the expected behaviour as shown by Equation 2 when the length, cross sectional area, and shear modulus are kept constant.

The experimental results for the steel rod appear to be affected by a small random error, whereby the brass rod appears to be affected by a systematic error and no random errors. As the torque increases the brass rod deforms less than expected by $\approx 4\%$. The most likely cause of these errors are the rough gradations of the angle meter and potential parallax reading error.

As shown in Figure 4, the angular deformation of the brass rod is roughly linearly proportional to the length over which the torque is applied. This is the expected behaviour referring to Equation 2 whereby torque, cross section area, and shear modulus are kept constant the angle of twist is only a function of length, proportional by a constant.

The experimental results appear to be affected by a systematic error and a random error occurring every two readings. This is again possibly caused by the angle scale having low granularity.

4.1 Theoretical Calculations

The anticipated angle of twist in the steel rod for a load of $3\,\mathrm{N}$ is as follows:

Given:

$$\begin{aligned} T &= 3\,\mathrm{N} \times 0.05\,\mathrm{m} = 0.15\,\mathrm{Nm} \\ L &= 0.5\,\mathrm{m} \\ G &= 79.6 \times 10^{9}\,\mathrm{Pa} \end{aligned} \tag{4}$$

and

$$\begin{aligned} J &= \frac{\pi}{2}\left(1.6 \times 10^{-3}\,\mathrm{m}\right)^{4} \\ &= 1.65 \times 10^{-10}\,\mathrm{m}^{4} \end{aligned} \tag{5}$$

thus

$$\begin{aligned} \phi &= \frac{0.15 \times 0.5}{1.65 \times 10^{-10} \times 79.6 \times 10^{9}} \\ &= 9.15 \times 10^{-2}\,\mathrm{rad} \\ &= 5.24^{\circ} \end{aligned} \tag{6}$$

For brass:

$$\begin{aligned} T &= 3\,\mathrm{N} \times 0.05\,\mathrm{m} = 0.15\,\mathrm{Nm} \\ L &= 0.5\,\mathrm{m} \\ G &= 38 \times 10^{9}\,\mathrm{Pa} \end{aligned} \tag{7}$$

and

$$\begin{aligned} J &= \frac{\pi}{2}\left(1.55 \times 10^{-3}\,\mathrm{m}\right)^{4} \\ &= 1.45 \times 10^{-10}\,\mathrm{m}^{4} \end{aligned} \tag{8}$$

thus

$$\begin{aligned} \phi &= \frac{0.15 \times 0.5}{1.45 \times 10^{-10} \times 38 \times 10^{9}} \\ &= 0.218\,\mathrm{rad} \\ &= 12.5^{\circ} \end{aligned} \tag{9}$$

Comparing these values to the experimental results shows good agreement within the expected margin of error.

5 Conclusion

The angle of twist was shown to be linearly related to applied torque, or torque applied over length, when all other factors were kept constant. This is in agreement with the theoretical equations. There were some experimental errors likely caused by low resolution measuring equipment.

Academic Disclaimer

Sections of this report repeat work previously submitted as *ENGR2741 Prac 2 Report: Torsion of Circular Sections* by Cedrych *et al.* 2019 [3].

References

[1] N. Holyoak, *ENGR2741 and ENGR8791: Mechanics & Structures (and GE) Practical 2 Torsion of Circular Sections*, 2020.

[2] R. Hibbeler, *Statics And Mechanics Of Materials*, 5th ed. United Kingdom: Pearson Education Limited, 2019, p. 473, ISBN: 1-292-17791-8.

[3] M. Cedrych, A. John, E. Neil, and J. Orwa, "ENGR2741 Prac 2 Report: Torsion of Circular Sections," Flinders University, 2019.