

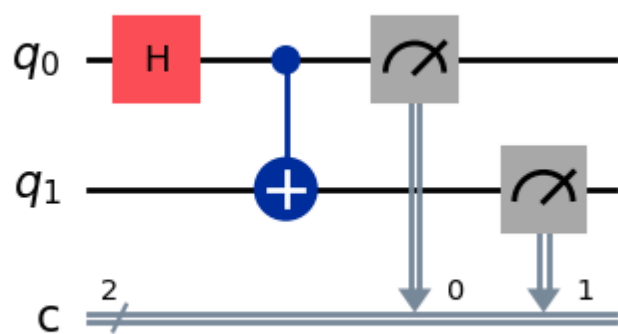
Michał Kałmucki 151944

# Quantum Computing Lab 2 Report

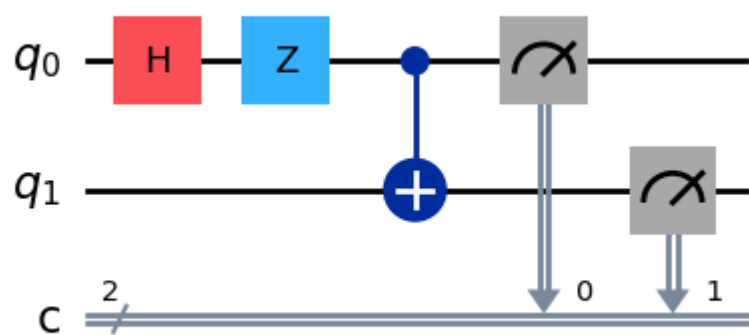
## 1. Circuit Plots (Tasks 1-7)

### Bell State Circuits (Tasks 1-4)

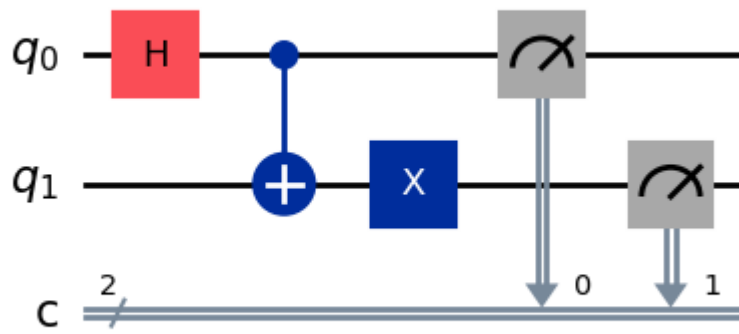
#### Phi+ Circuit



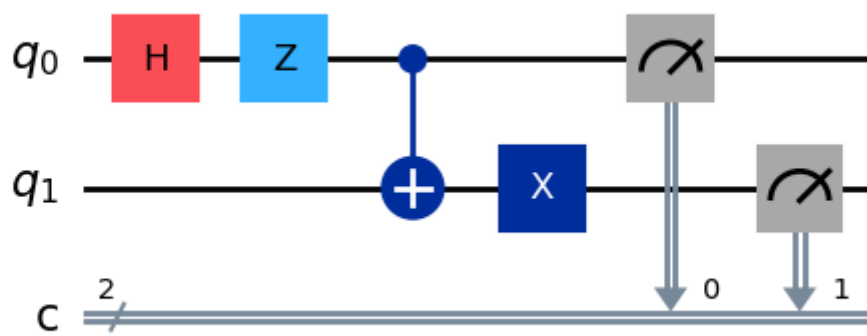
#### Phi- Circuit



#### Psi+ Circuit

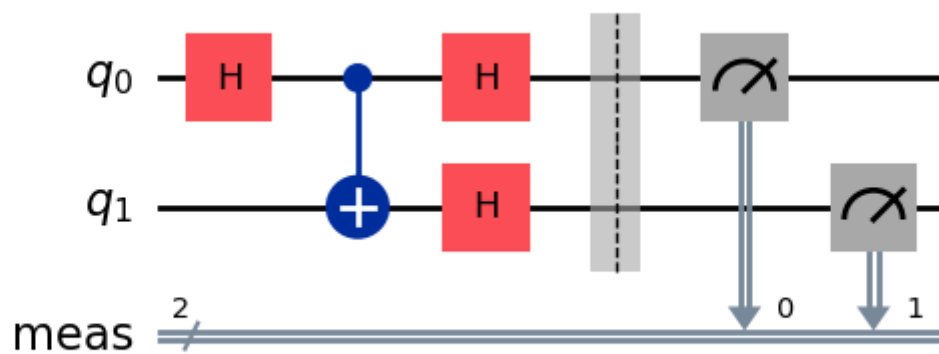


### Psi- Circuit

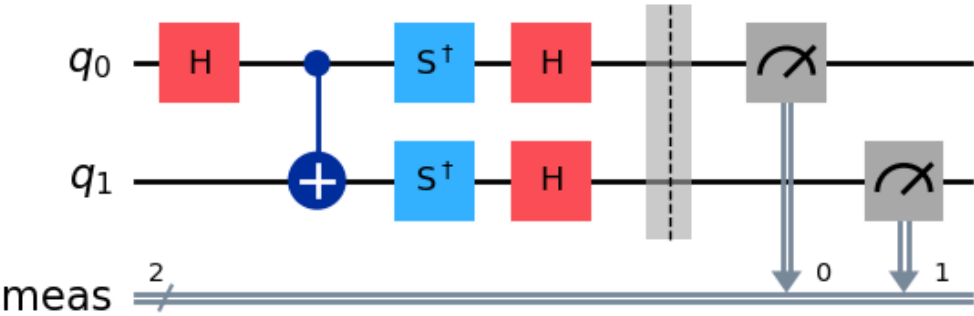


### Measurement Circuits (Tasks 5-7)

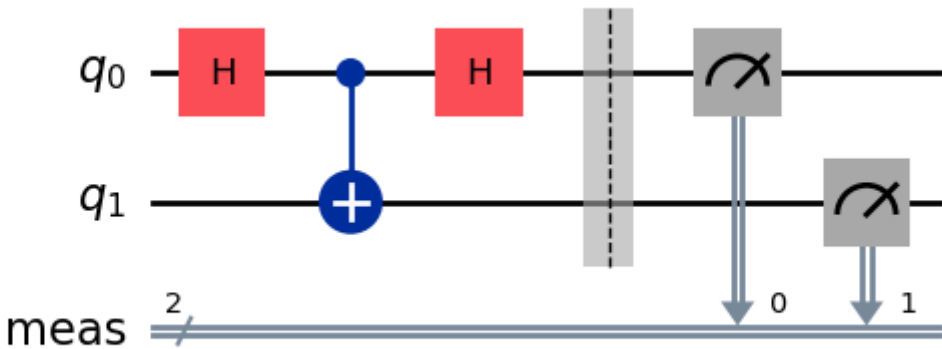
#### Phi+ XX Measurement



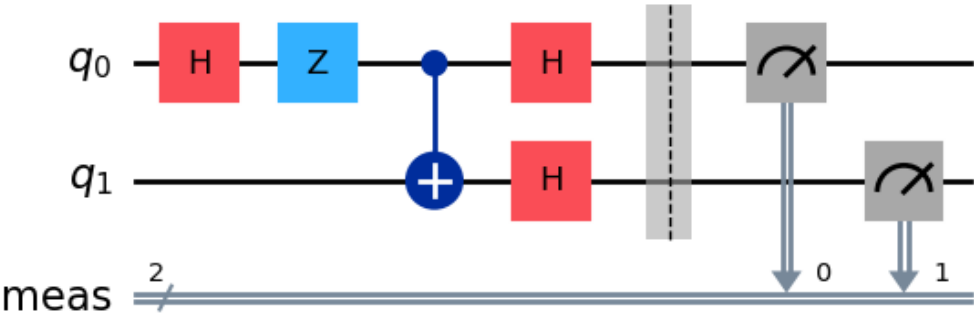
Phi+ YY Measurement



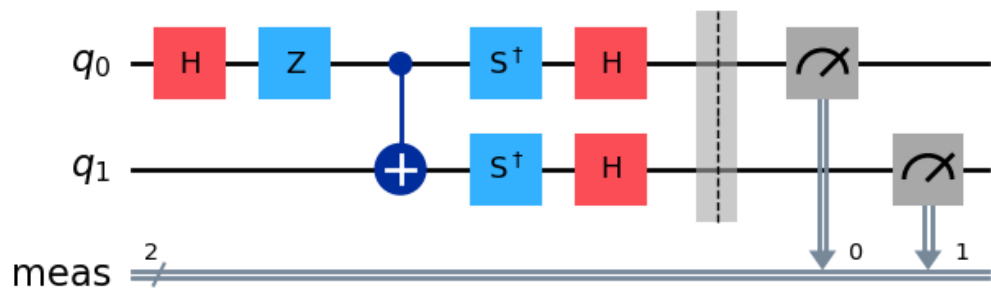
Phi+ XZ Measurement



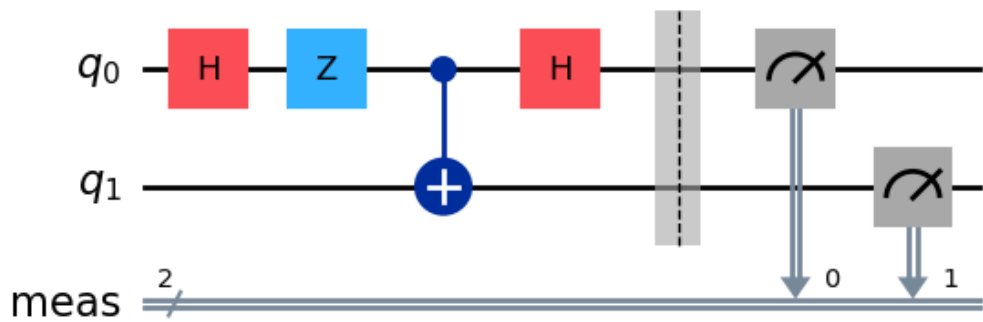
Phi- XX Measurement



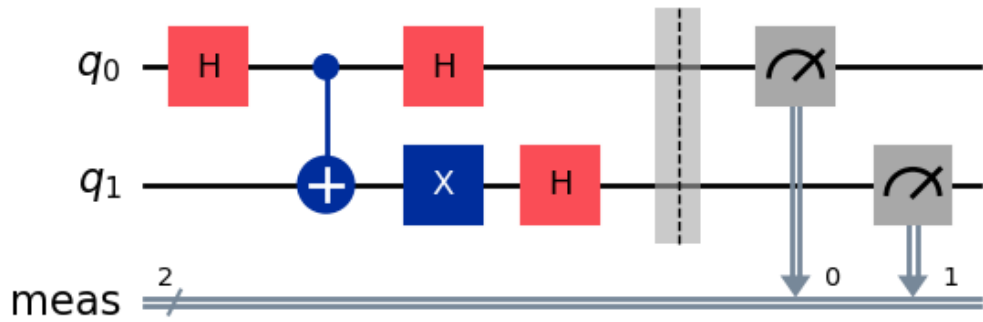
Phi- YY Measurement



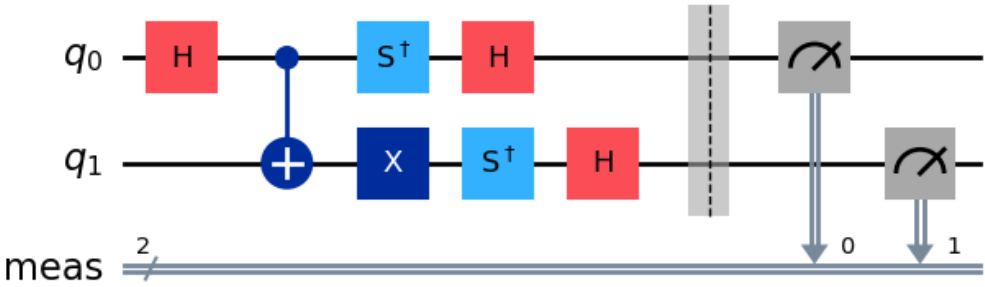
Phi- XZ Measurement



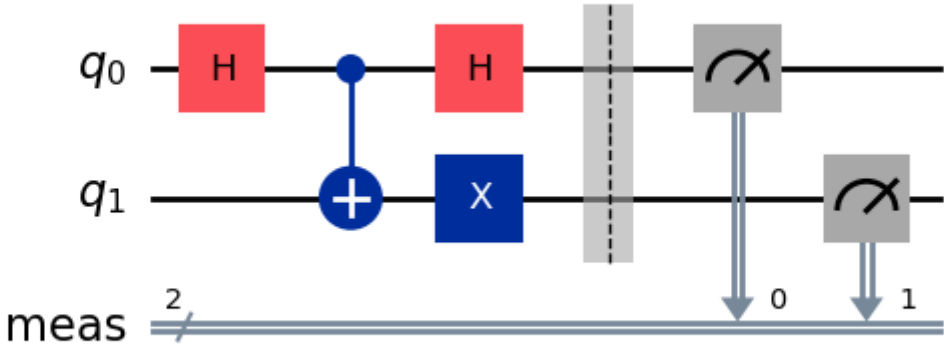
Psi+ XX Measurement



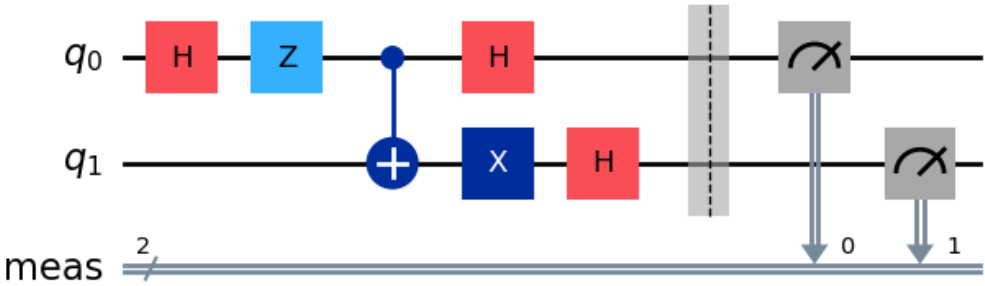
Psi+ YY Measurement



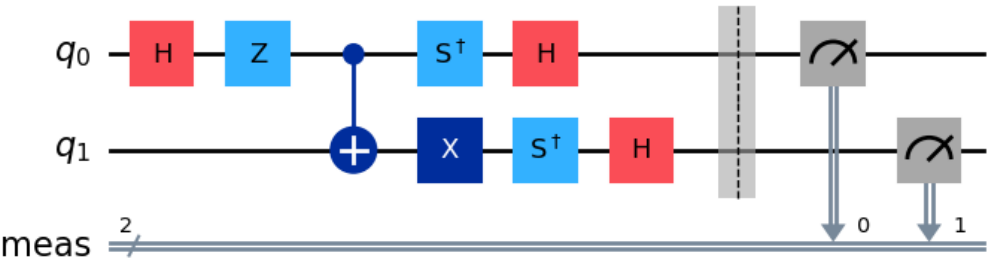
**Psi+ XZ Measurement**



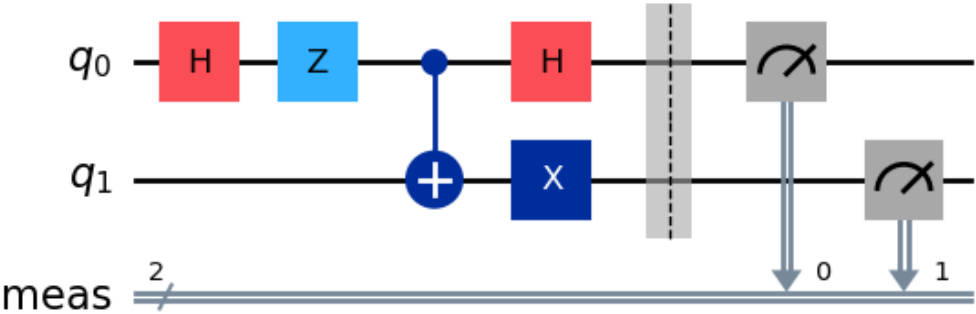
**Psi- XX Measurement**



**Psi- YY Measurement**



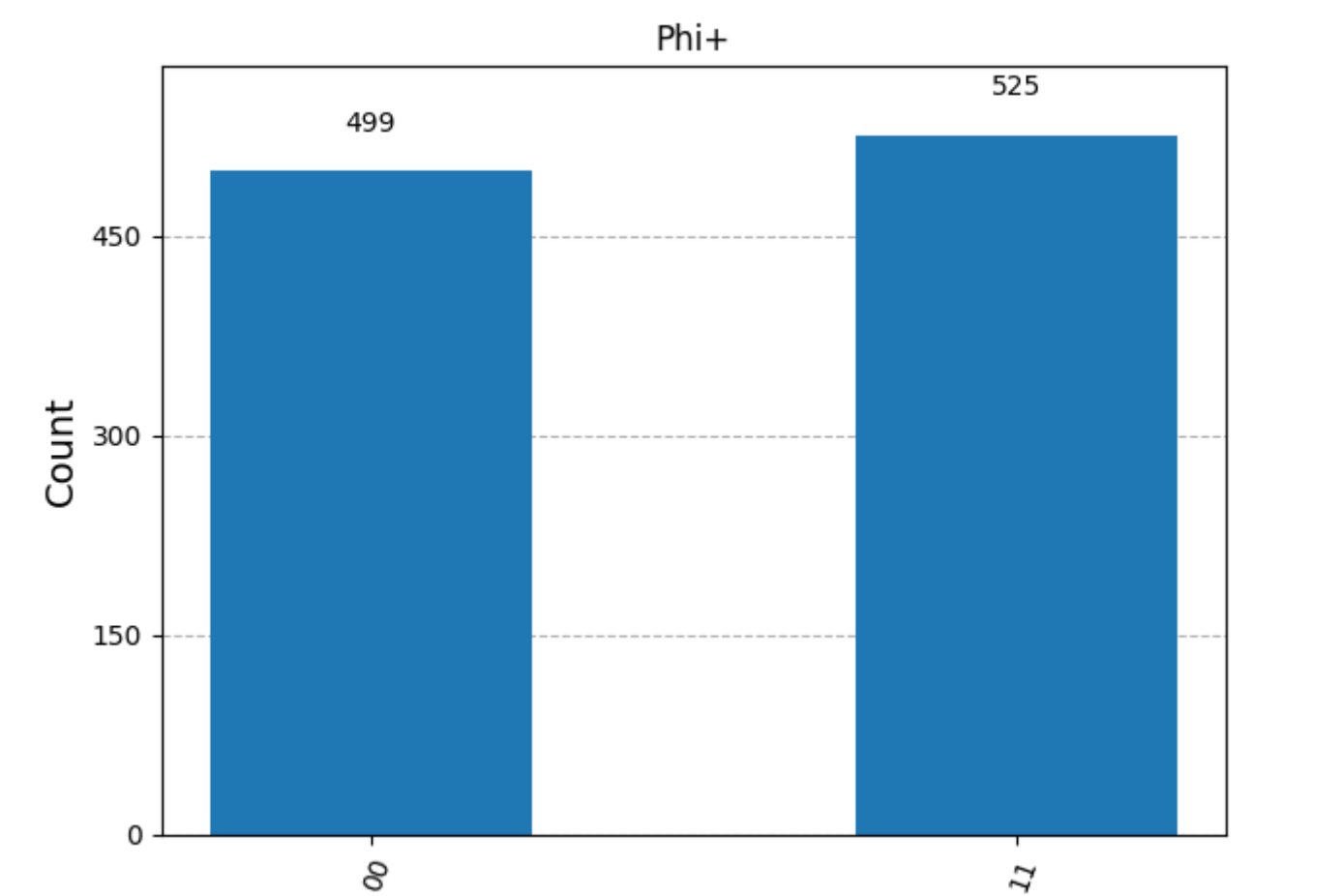
**Psi- XZ Measurement**



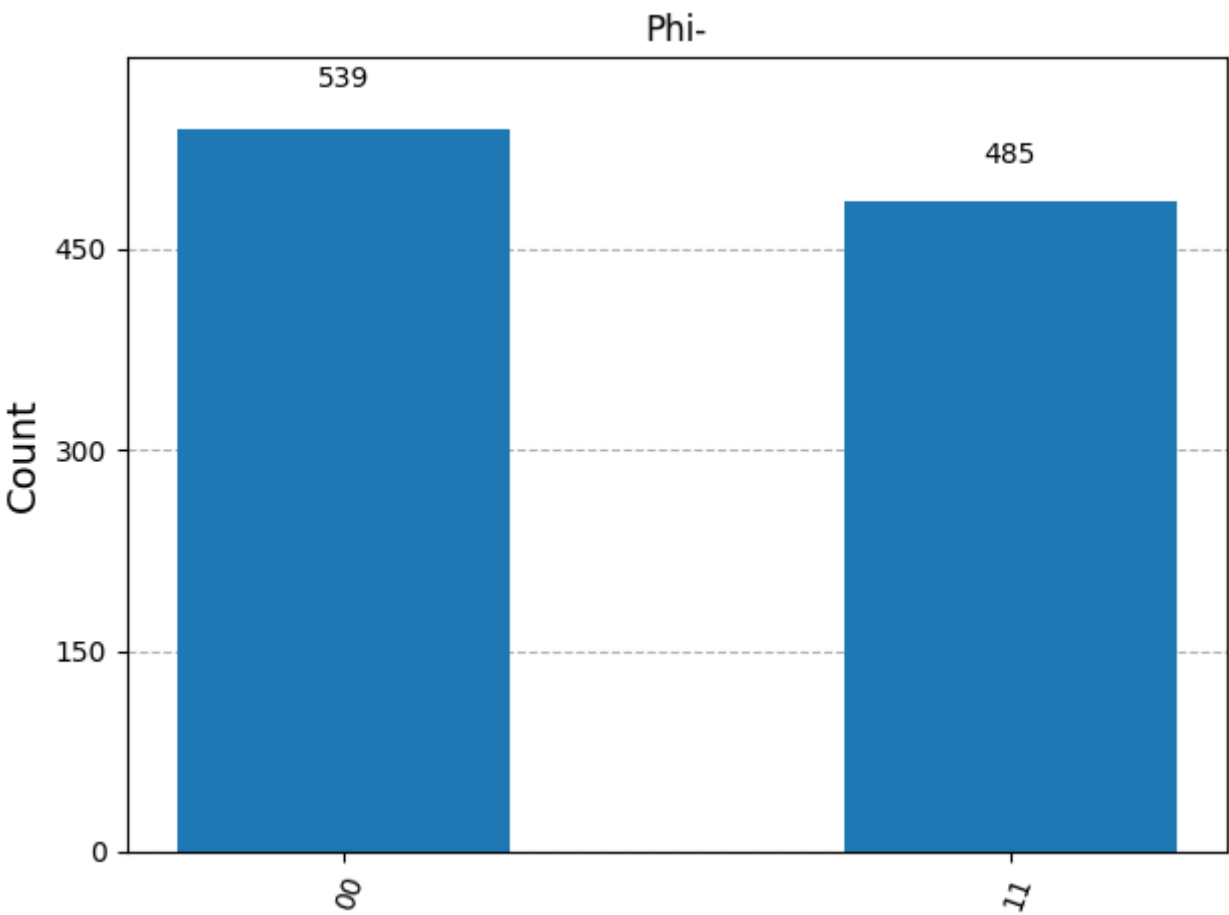
2. Results Plots (Tasks 1-7)

Bell State Results (Tasks 1-4)

**Phi+ Histogram**

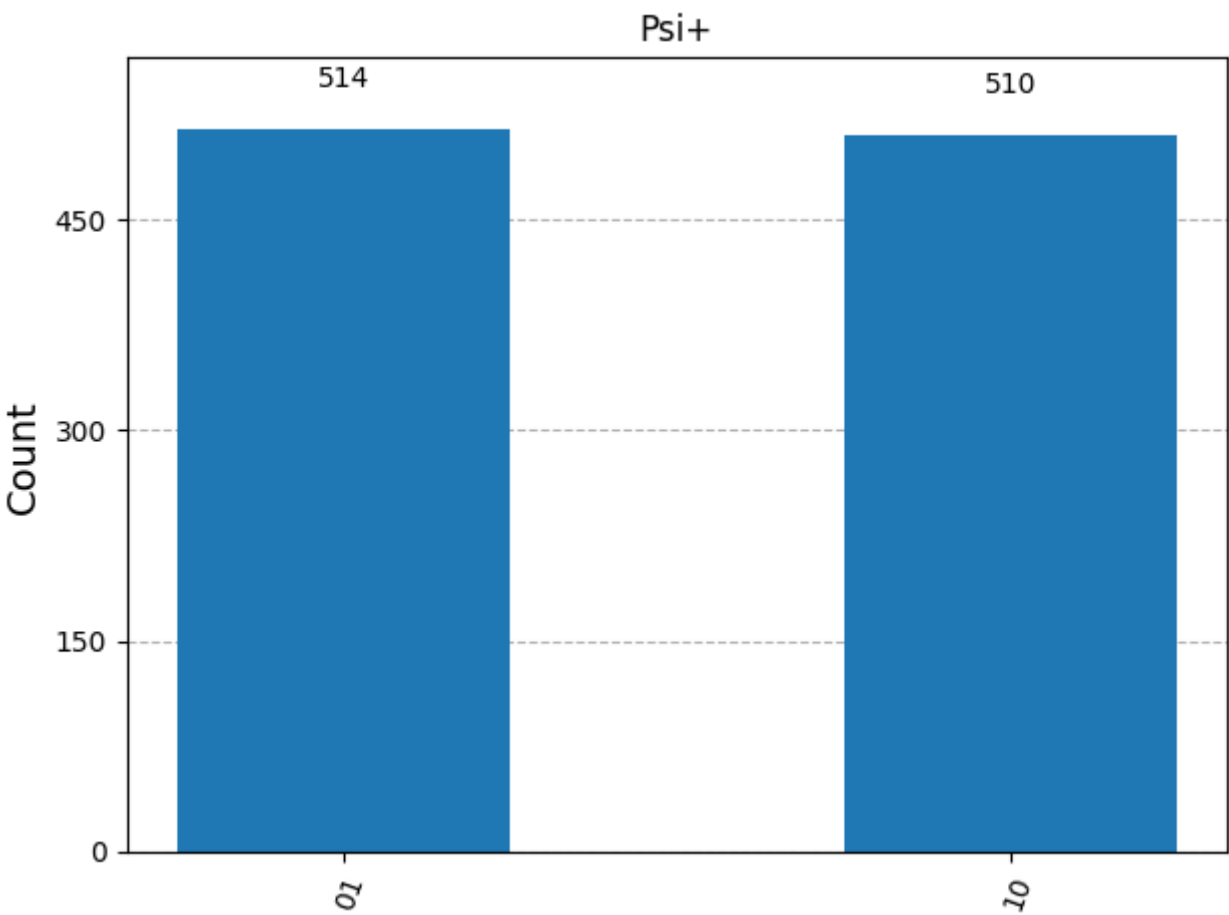


Phi- Histogram

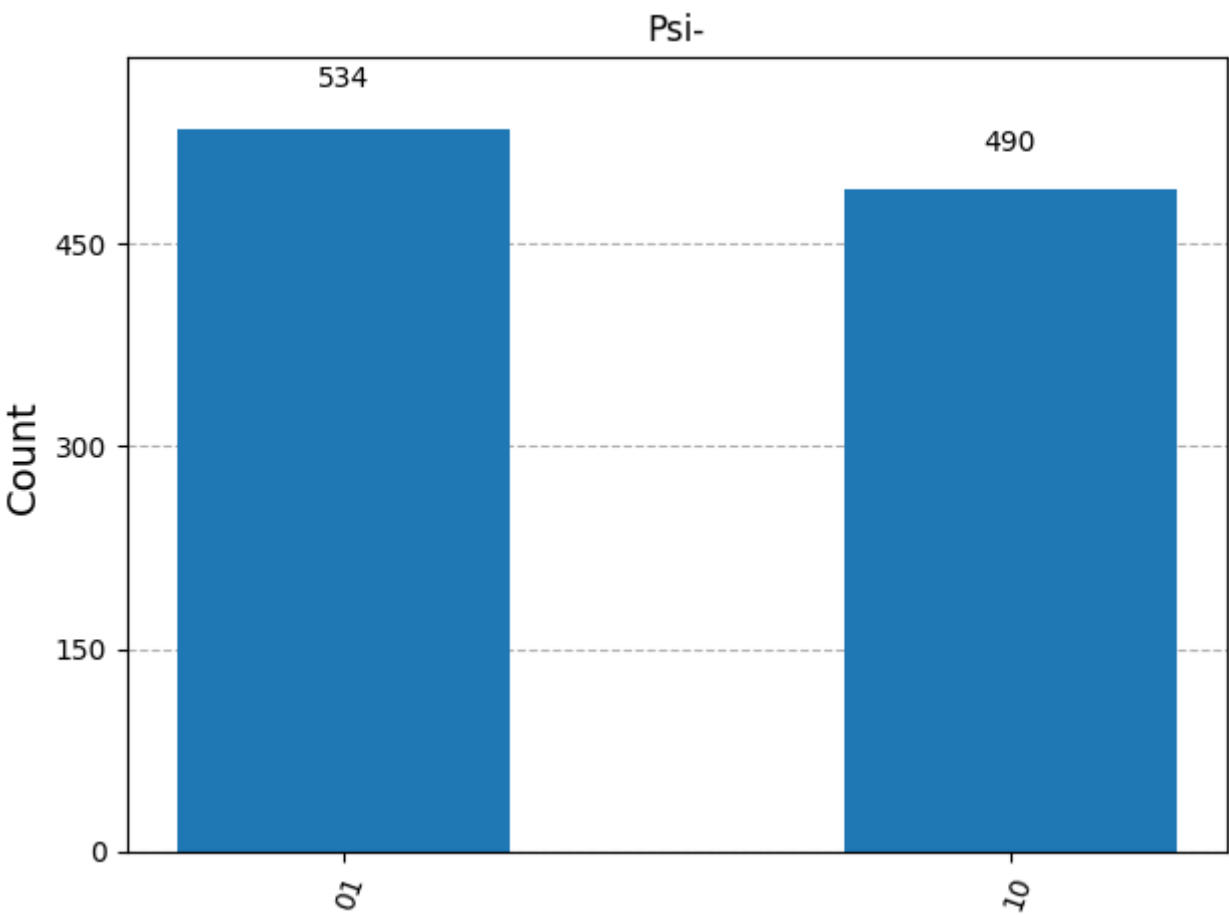


Psi+ Histogram



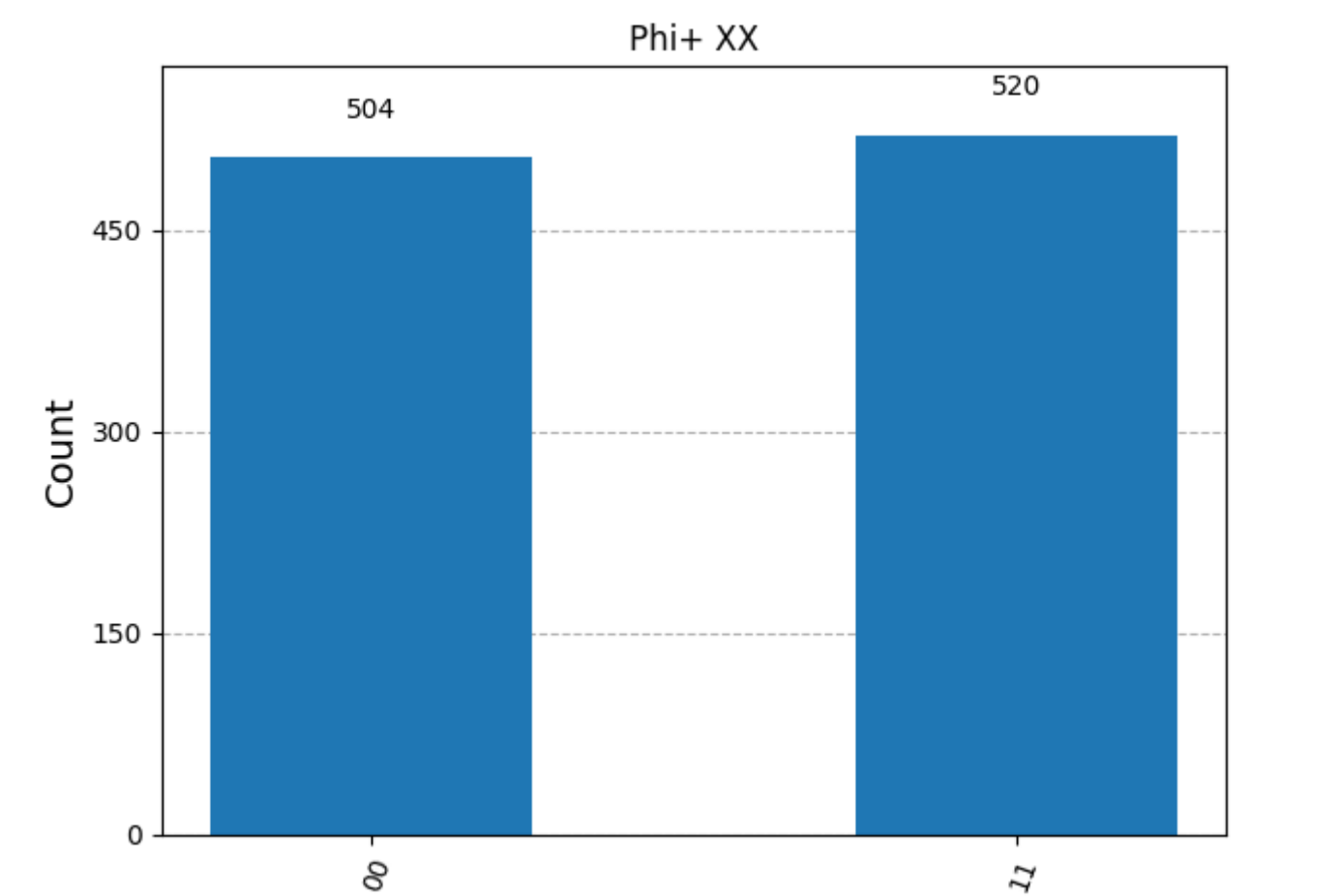


Psi- Histogram

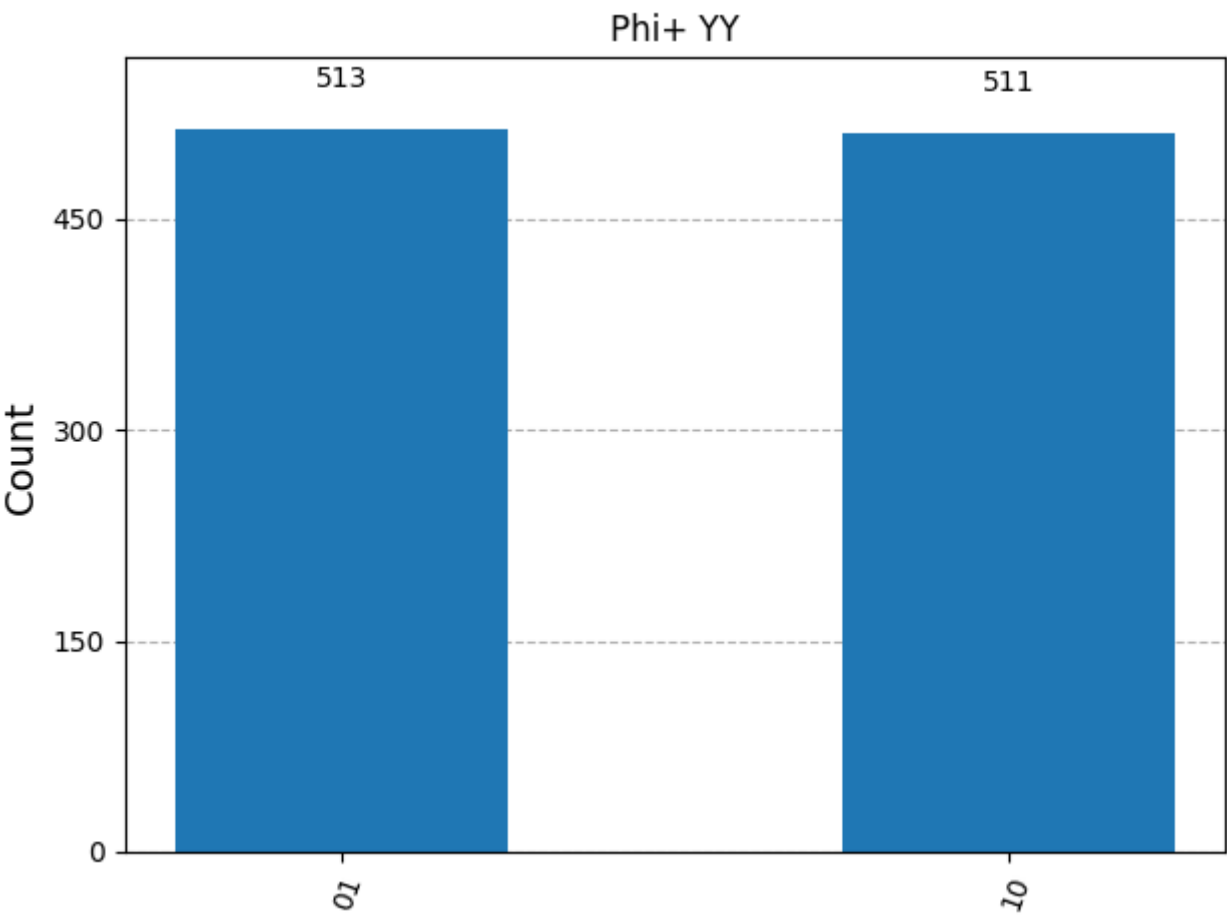


Measurement Results (Tasks 5-7)

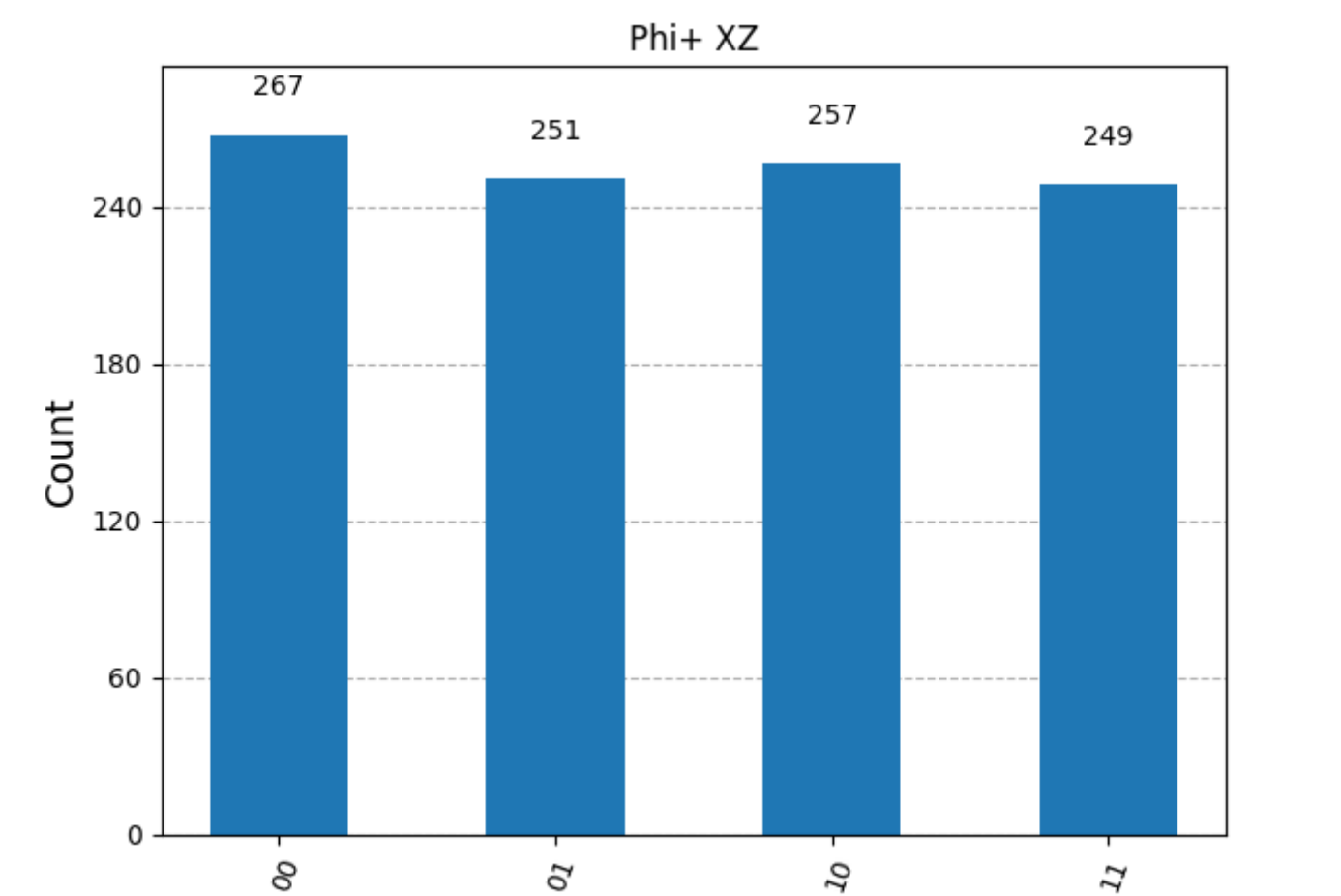
**Phi+ XX Histogram**



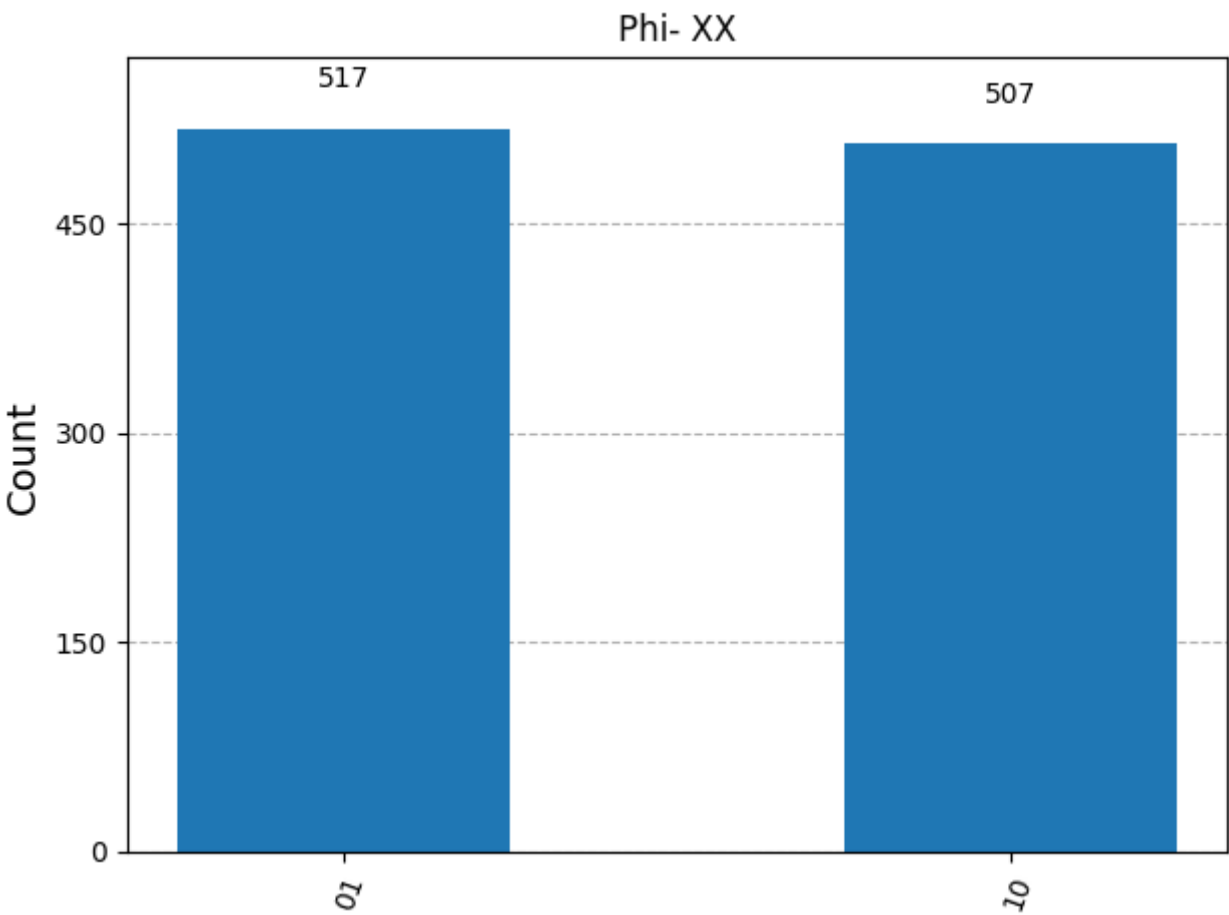
Phi+ YY Histogram



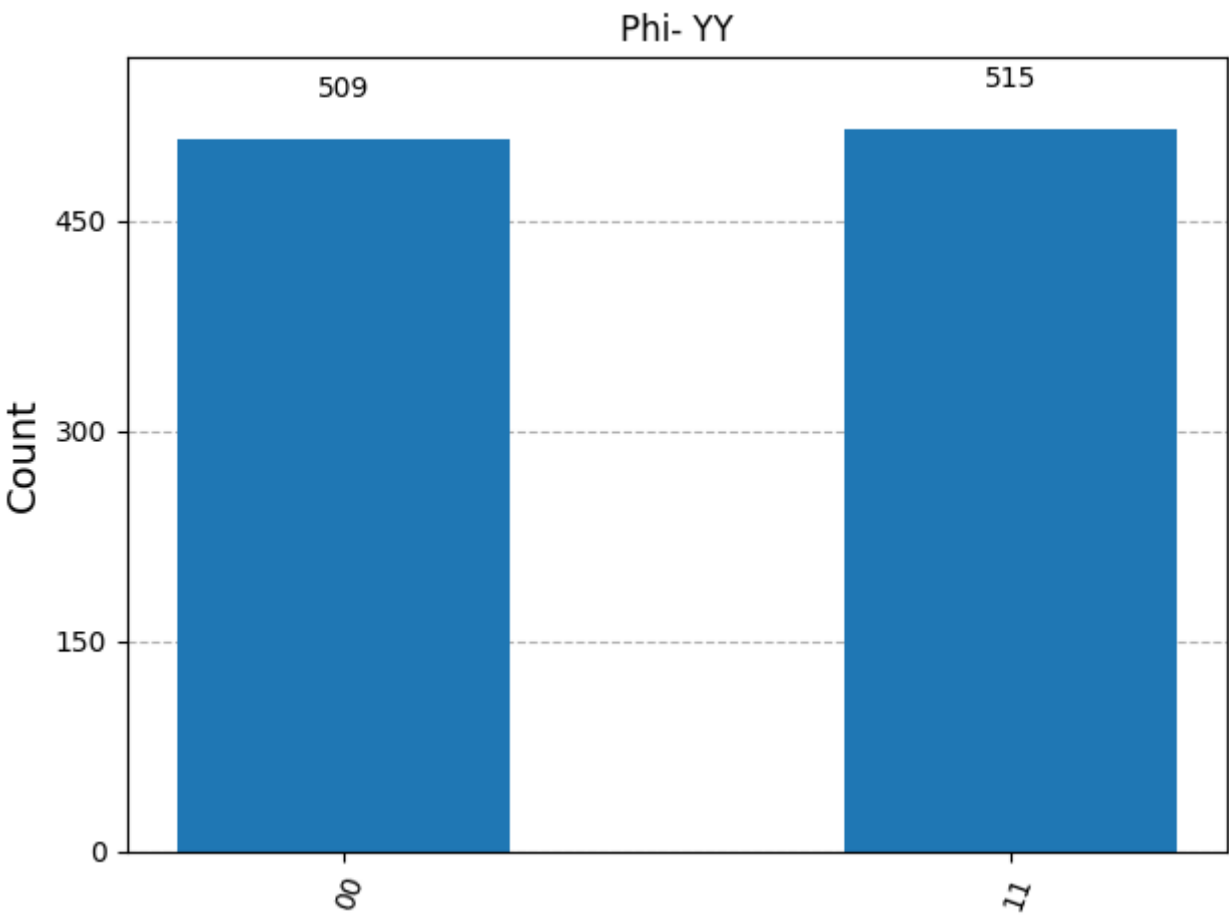
Phi+ XZ Histogram



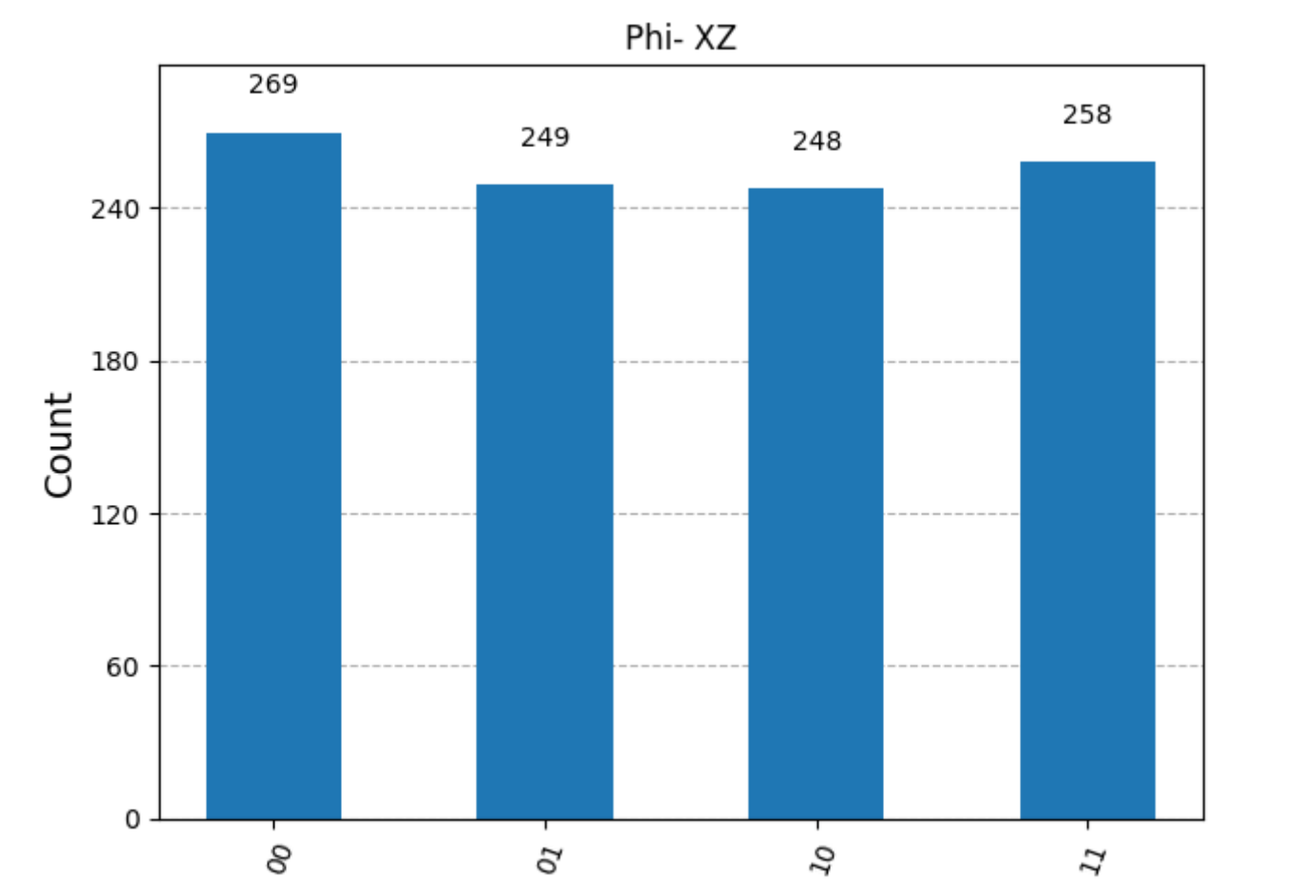
Phi- XX Histogram



Phi- YY Histogram

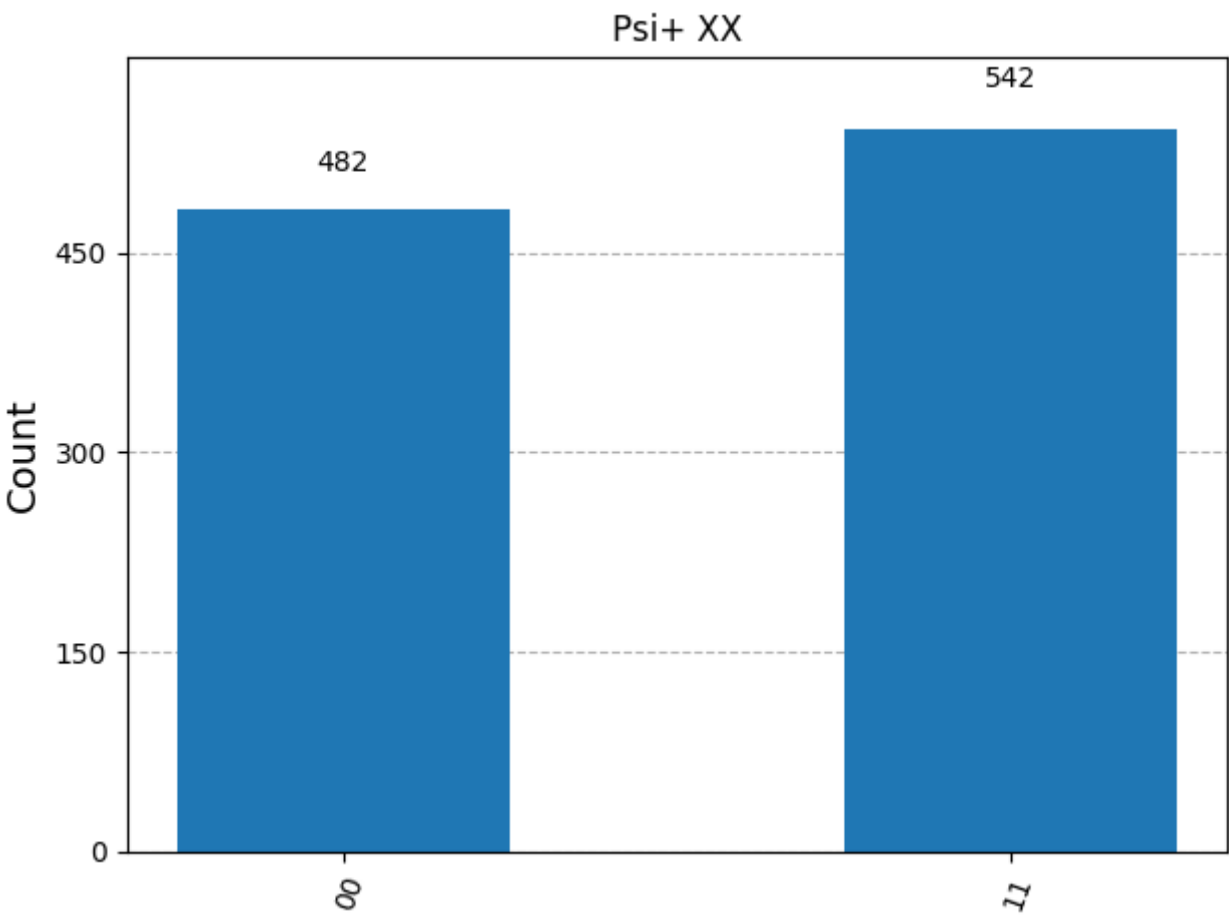


Phi- XZ Histogram

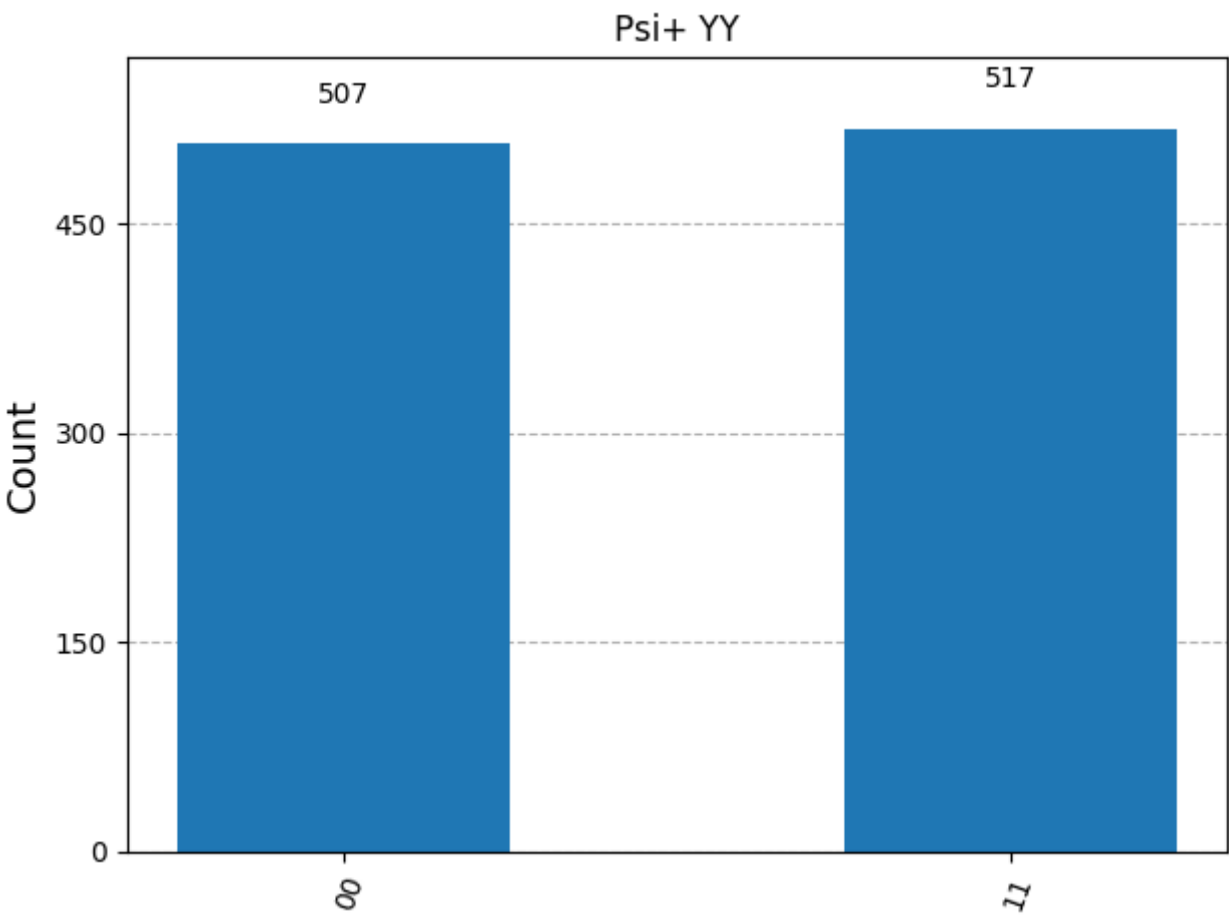


Psi+ XX Histogram

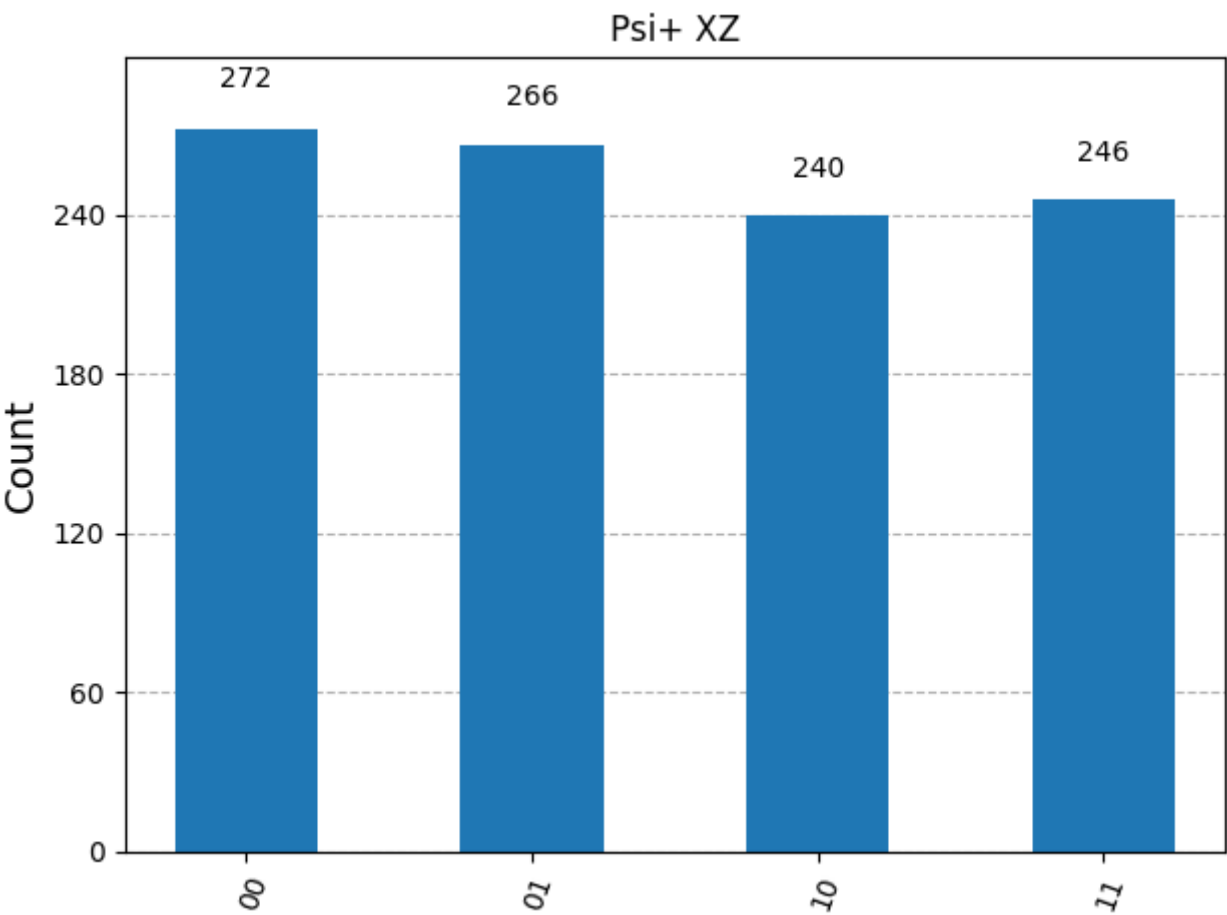




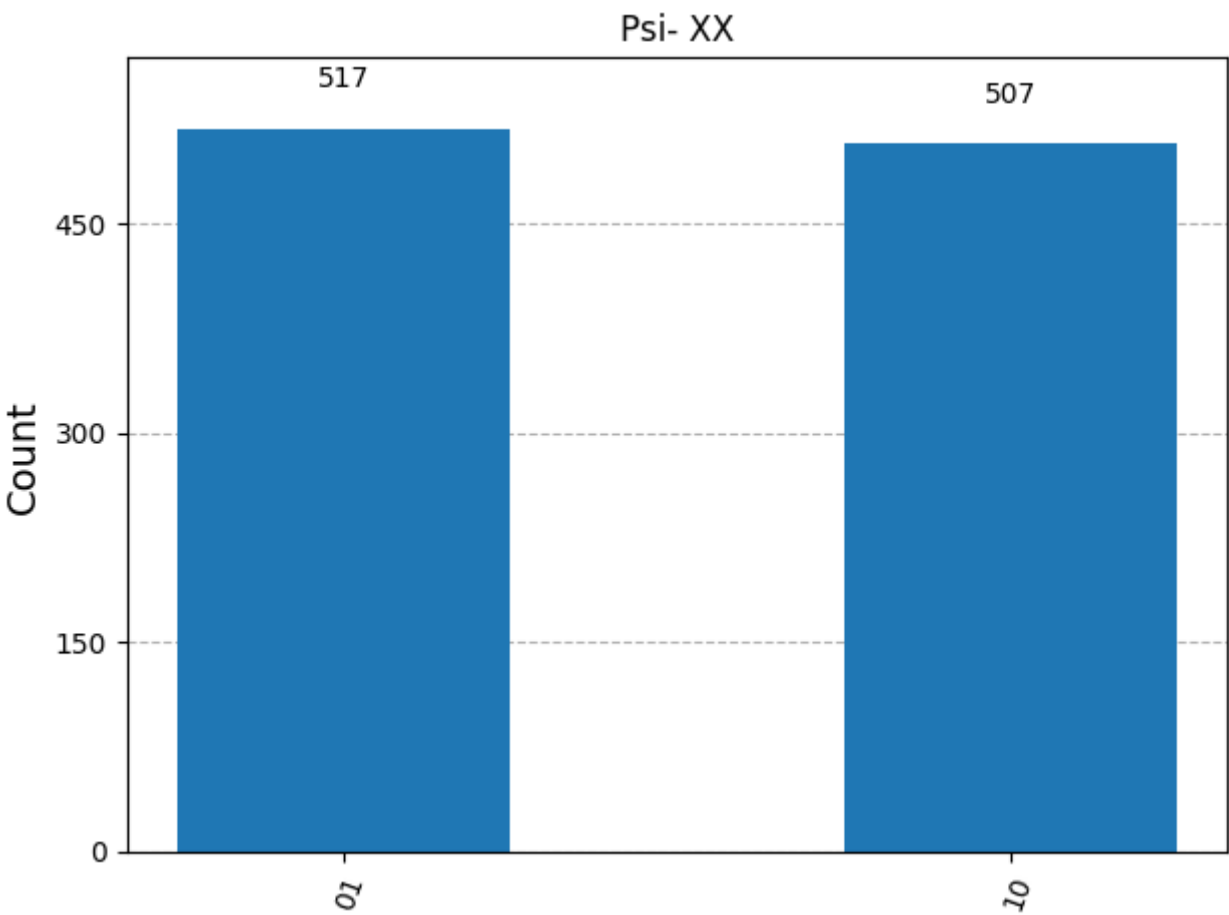
Psi+ YY Histogram



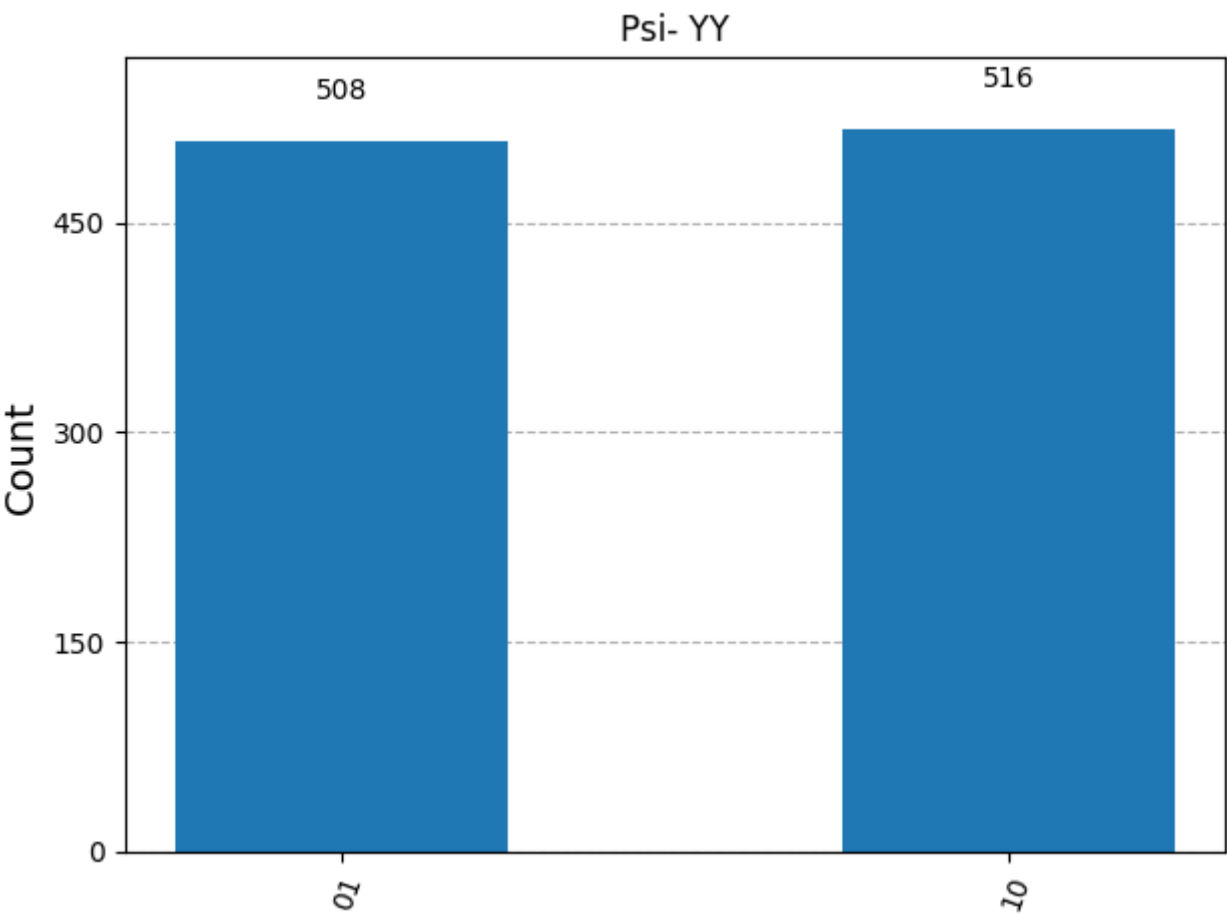
Psi+ XZ Histogram



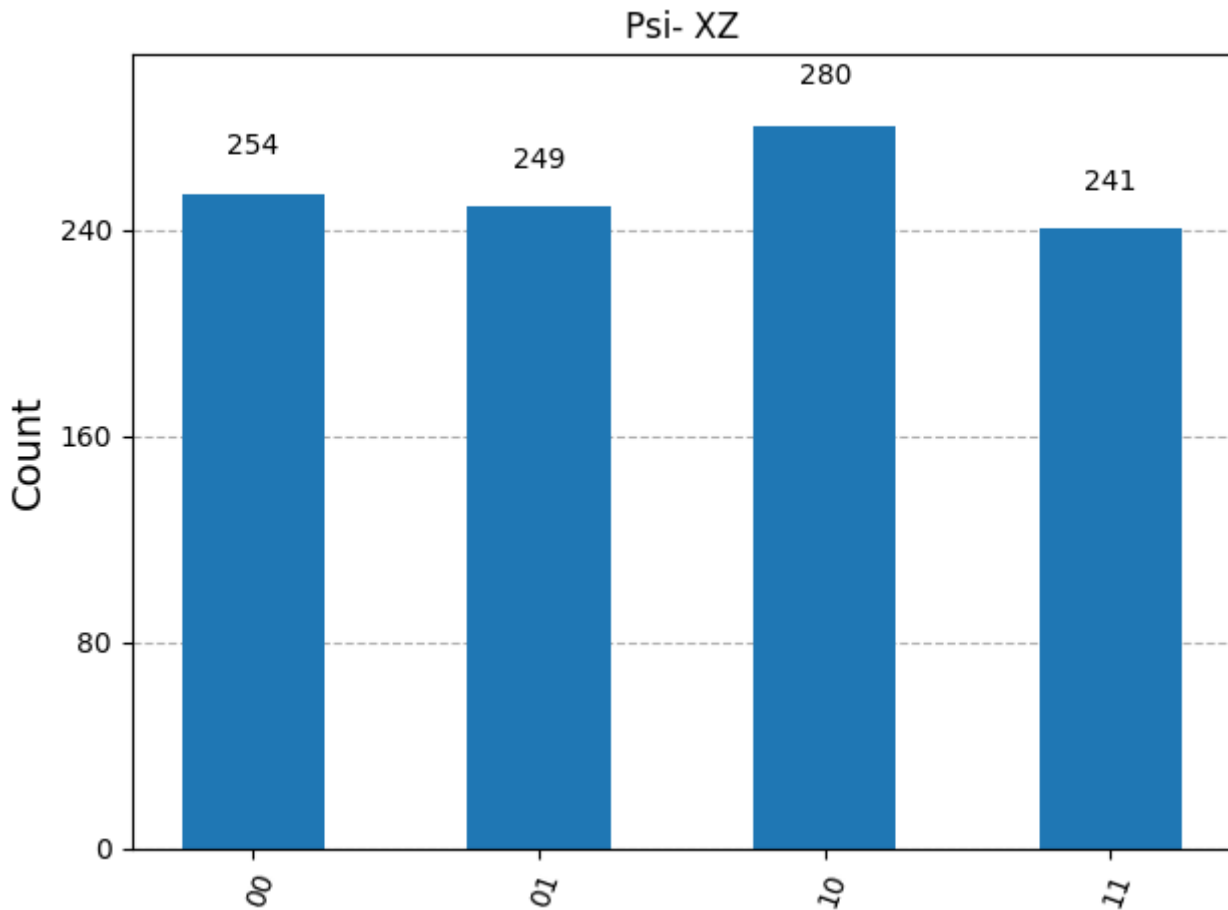
Psi- XX Histogram



Psi- YY Histogram



Psi- XZ Histogram



### 3. Mathematical Calculations (Tasks 2-4)

Matrix Form (Task 2: Phi-)

**Matrices used:**

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$H \otimes I = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$Z \otimes I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

**Derivation:**

Starting from the initial state:  $|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$$\text{Apply } H \otimes I: (H \otimes I) |00\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\text{Apply } Z \otimes I: (Z \otimes I) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Apply CNOT (CX):  $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

Final state:  $|\Phi^-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

### Operator Form (Task 3: Psi+)

#### Derivation:

Starting from  $|00\rangle$ .

Apply  $H \otimes I$ :  $(H \otimes I) |00\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |10\rangle)$

Apply CX:  $\frac{1}{\sqrt{2}} (|00\rangle + |10\rangle) = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$

Apply  $I \otimes X$ :  $\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)$

Final state:  $|\Psi^+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

### Operator Form (Task 4: Psi-)

#### Derivation:

Starting from  $|00\rangle$ .

Apply  $H \otimes I$ :  $(H \otimes I) |00\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |10\rangle)$

Apply  $Z \otimes I$ :  $\frac{1}{\sqrt{2}} (|00\rangle + |10\rangle) = \frac{1}{\sqrt{2}} (|00\rangle - |10\rangle)$

Apply CX:  $\frac{1}{\sqrt{2}} (|00\rangle - |10\rangle) = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle)$

Apply  $I \otimes X$ :  $\frac{1}{\sqrt{2}} (|00\rangle - |11\rangle) = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)$

Final state:  $|\Psi^-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

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## Matrix Form (Task 2: Phi-)

Matrices used:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$H \otimes I = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$Z \otimes I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Derivation:

Starting from the initial state:

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Apply  $H \otimes I$ :

$$(H \otimes I)|00\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Apply  $Z \otimes I$ :

$$(Z \otimes I) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

Apply CNOT (CX):

$$CX \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

Final state:

$$|\Phi^-\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$



### Operator Form (Task 3: Psi+)

#### Derivation:

Starting from  $|00\rangle$ .

Apply  $H \otimes I$ :

$$(H \otimes I)|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

Apply CX:

$$CX \cdot \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Apply  $I \otimes X$ :

$$(I \otimes X) \cdot \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

Final state:

$$|\Psi^+\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

### Operator Form (Task 4: Psi-)

#### Derivation:

Starting from  $|00\rangle$ .

Apply  $H \otimes I$ :

$$(H \otimes I)|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

Apply  $Z \otimes I$ :

$$(Z \otimes I) \cdot \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) = \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)$$

Apply CX:

$$CX \cdot \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle) = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

Apply  $I \otimes X$ :

$$(I \otimes X) \cdot \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Final state:

$$|\Psi^-\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$