

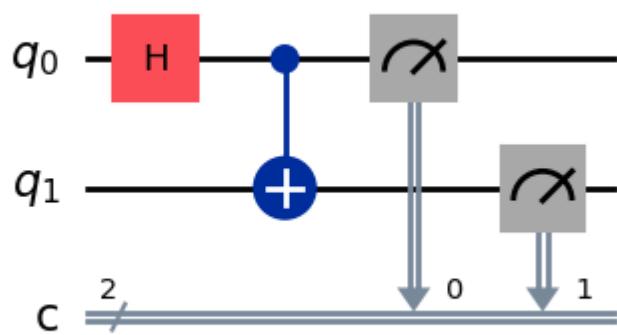
Michał Kałmucki 151944

Quantum Computing Lab 2 Report

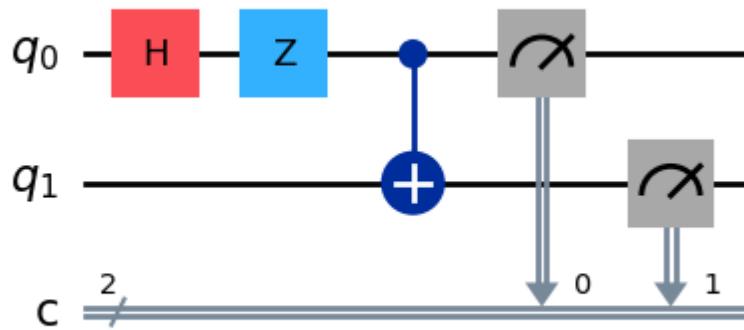
1. Circuit Plots (Tasks 1-7)

Bell State Circuits (Tasks 1-4)

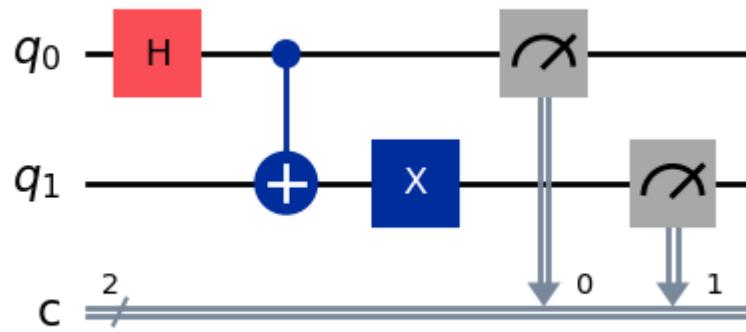
Phi+ Circuit



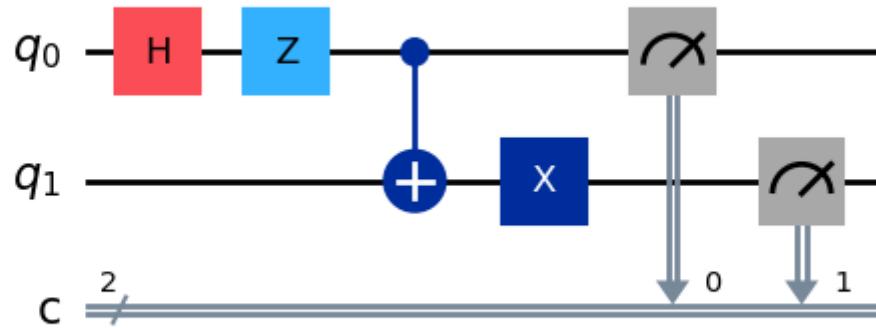
Phi- Circuit



Psi+ Circuit

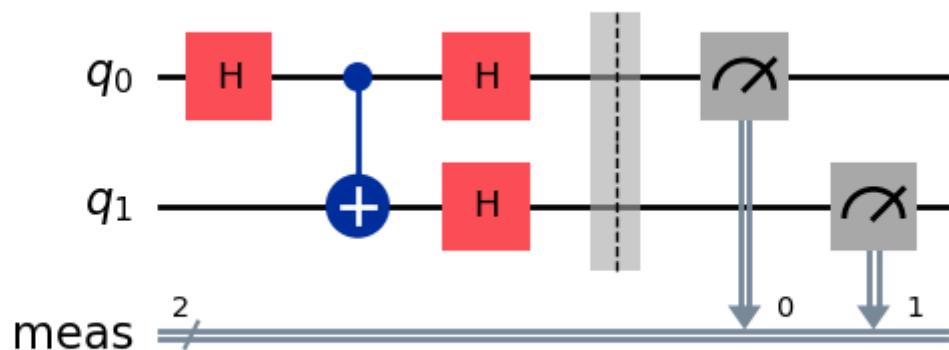


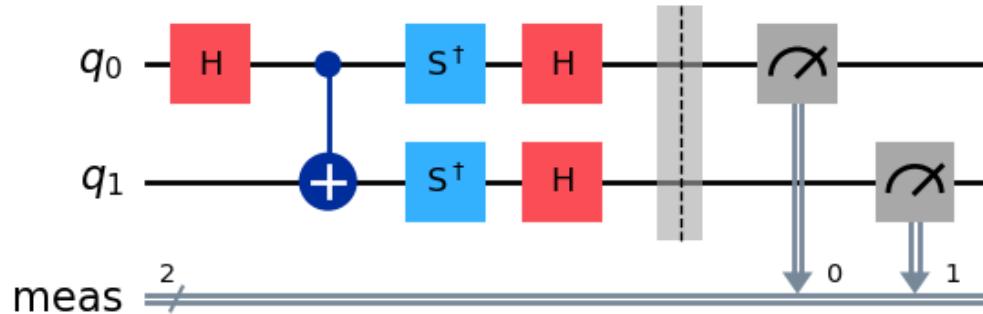
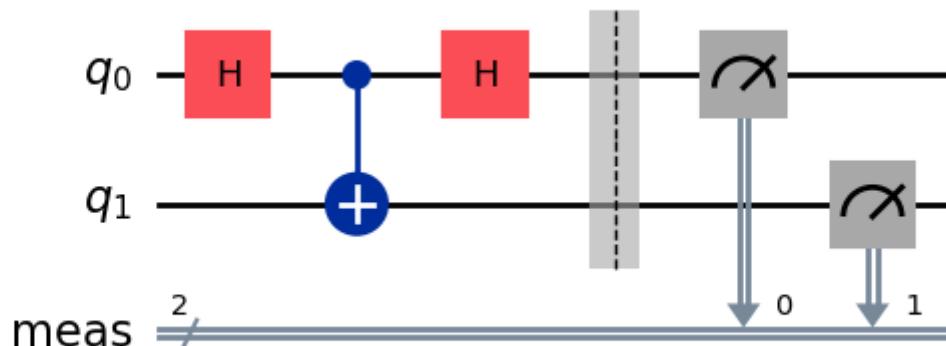
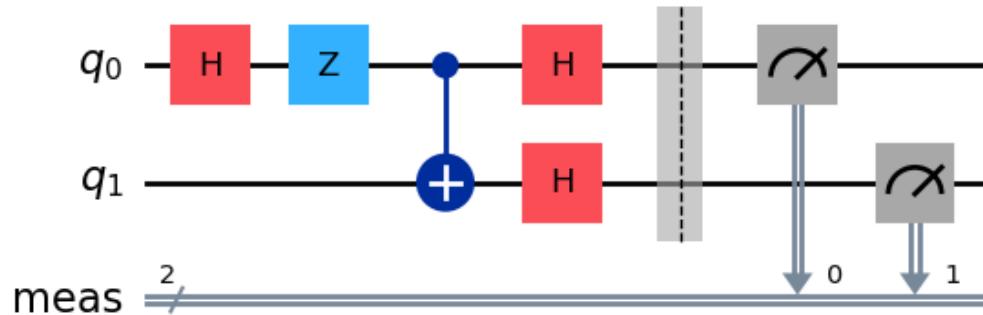
Psi- Circuit

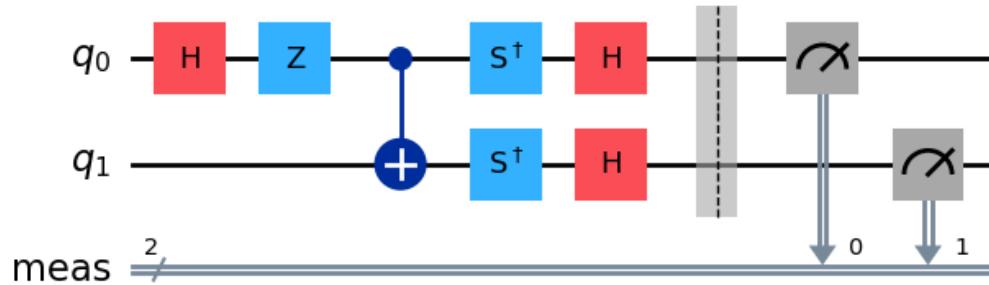


Measurement Circuits (Tasks 5-7)

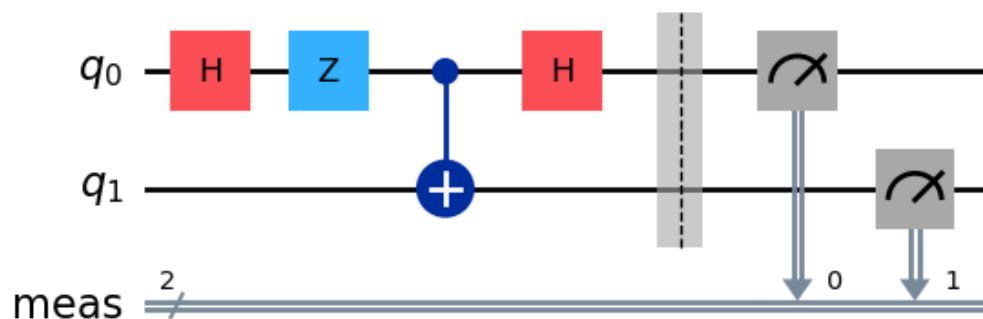
Phi+ XX Measurement



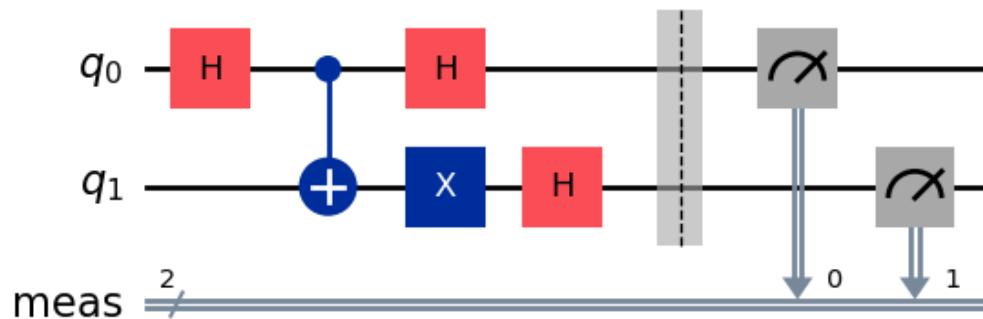
Phi+ YY Measurement**Phi+ XZ Measurement****Phi- XX Measurement****Phi- YY Measurement**



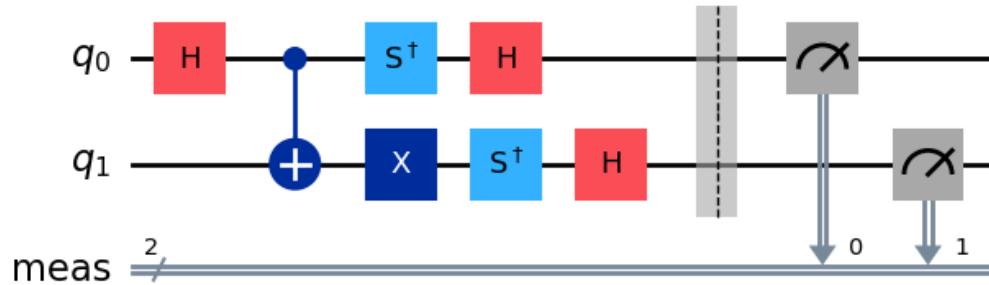
Phi- XZ Measurement



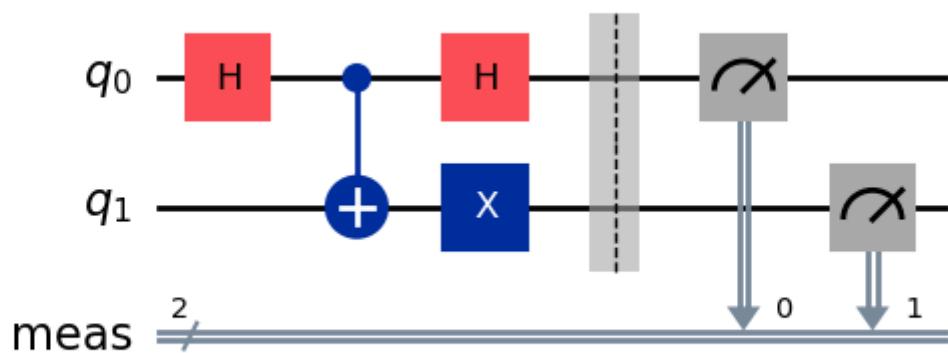
Psi+ XX Measurement



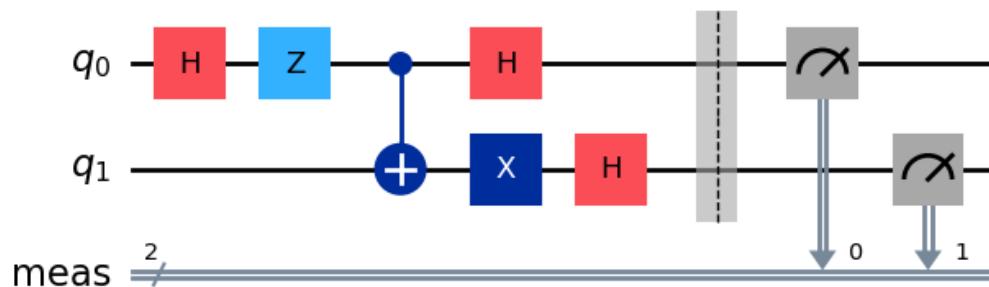
Psi+ YY Measurement



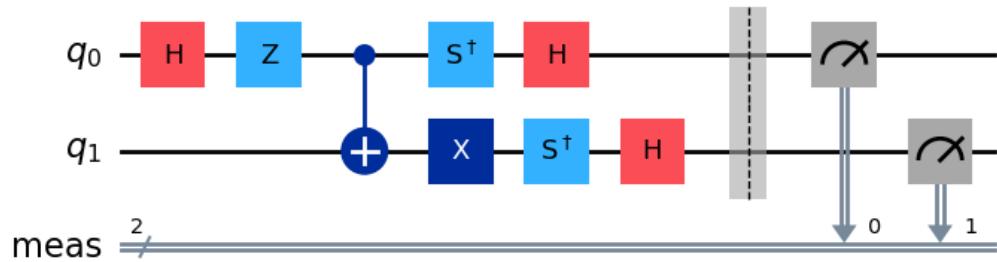
Psi+ XZ Measurement



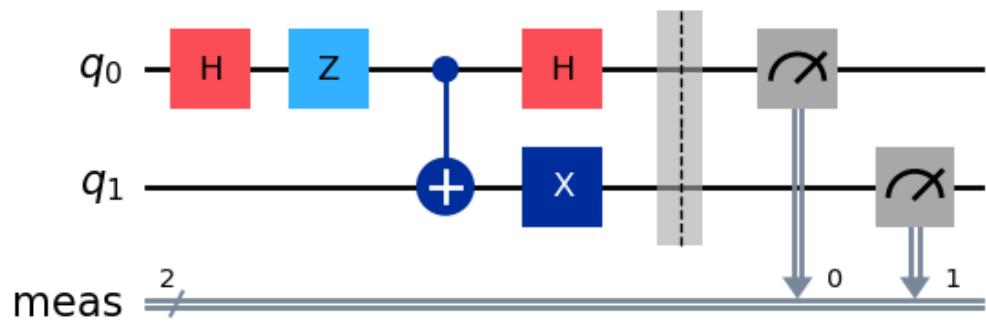
Psi- XX Measurement



Psi- YY Measurement



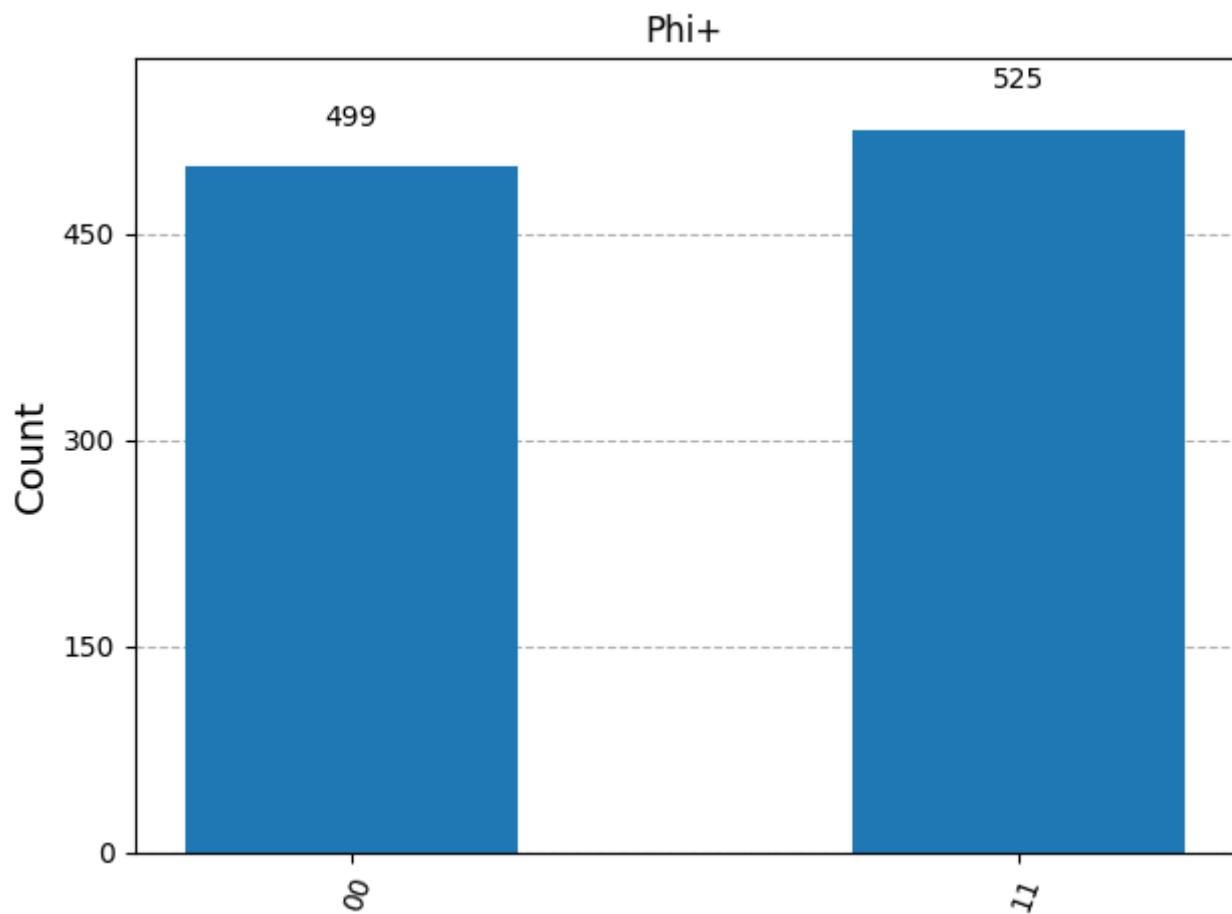
Psi- XZ Measurement



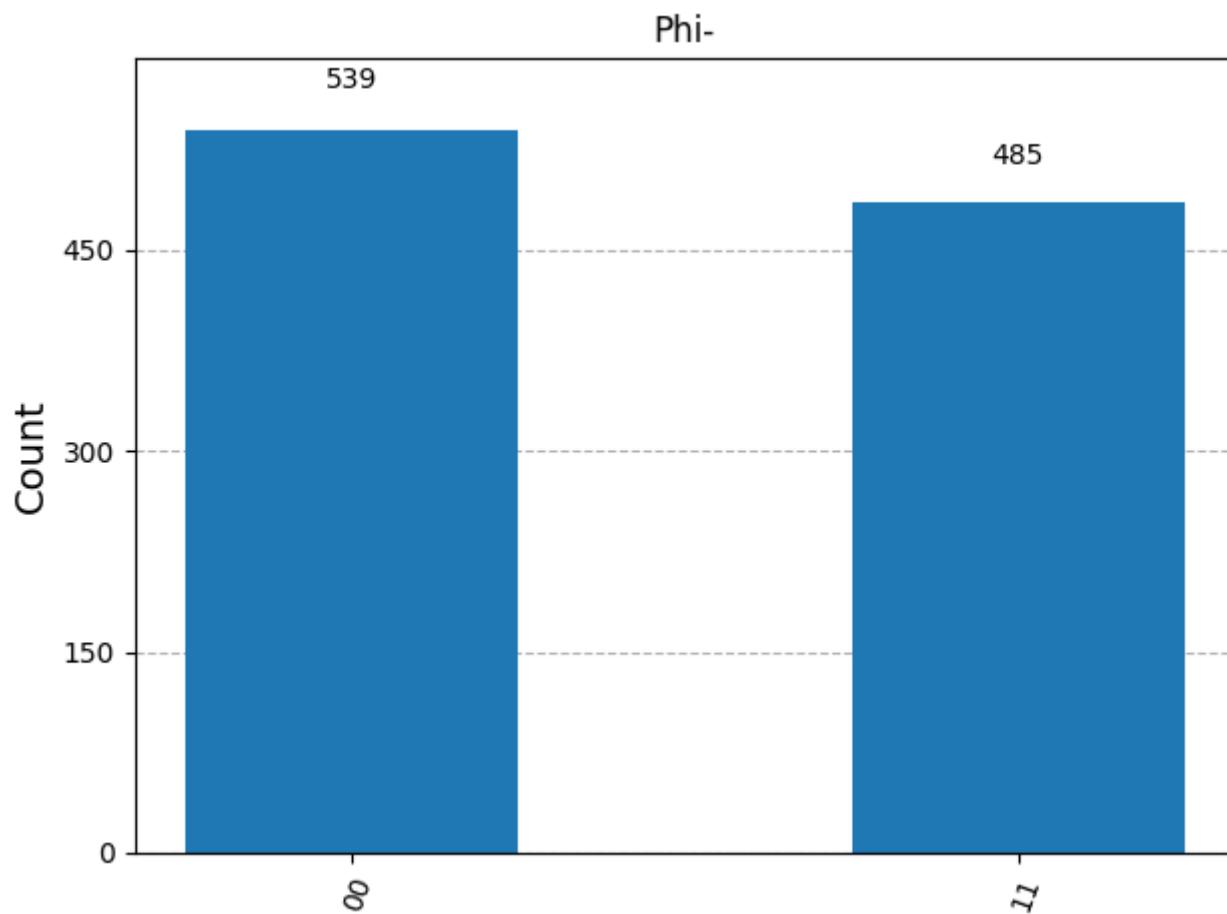
2. Results Plots (Tasks 1-7)

Bell State Results (Tasks 1-4)

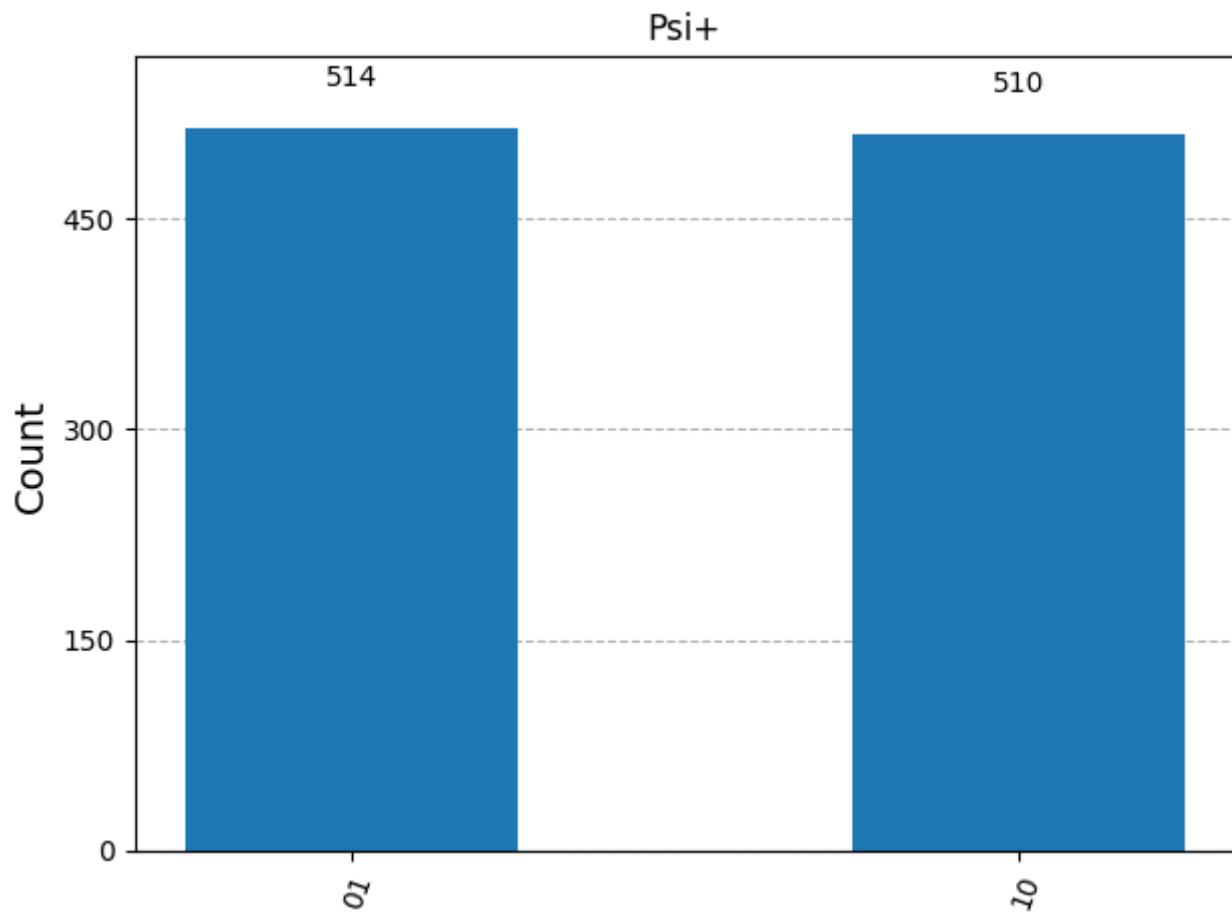
Phi+ Histogram



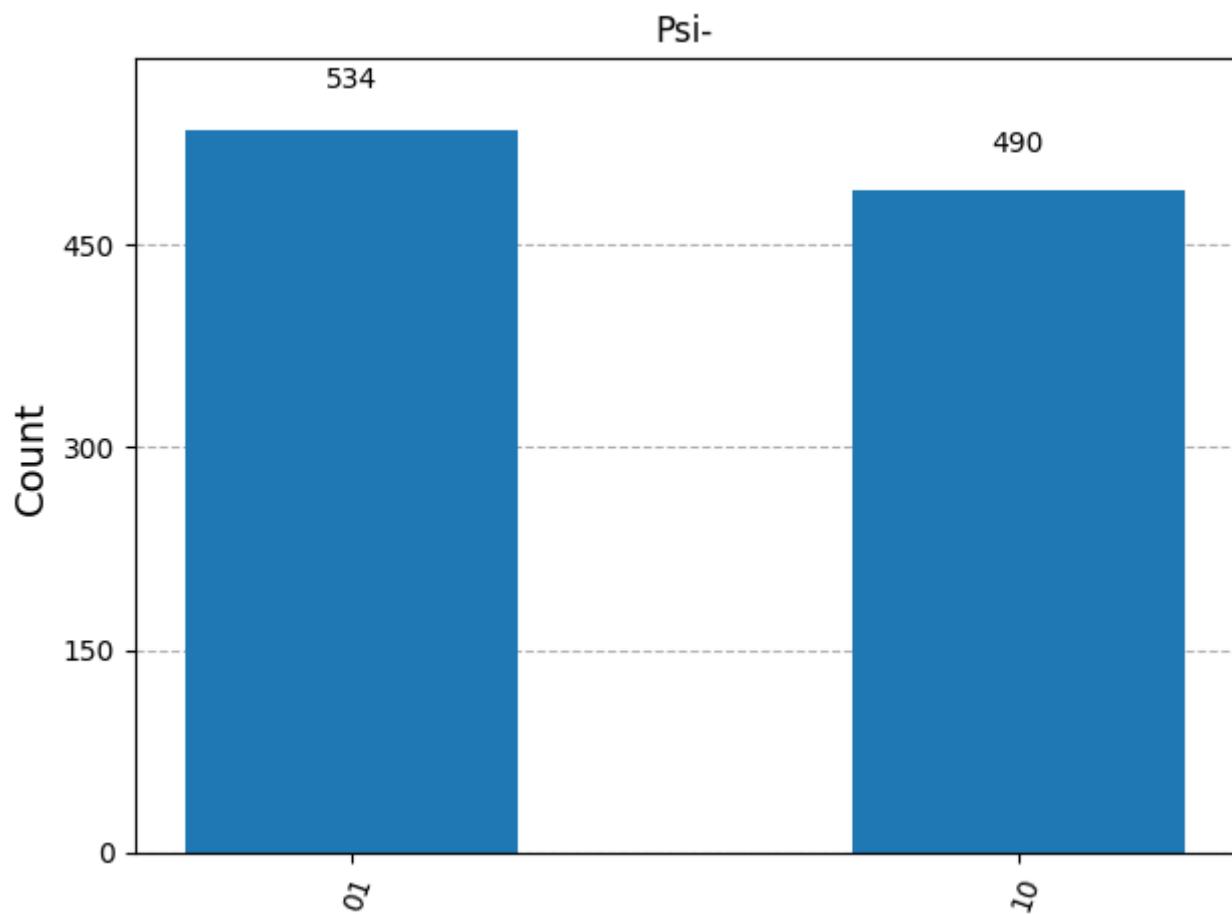
Phi- Histogram



Psi+ Histogram

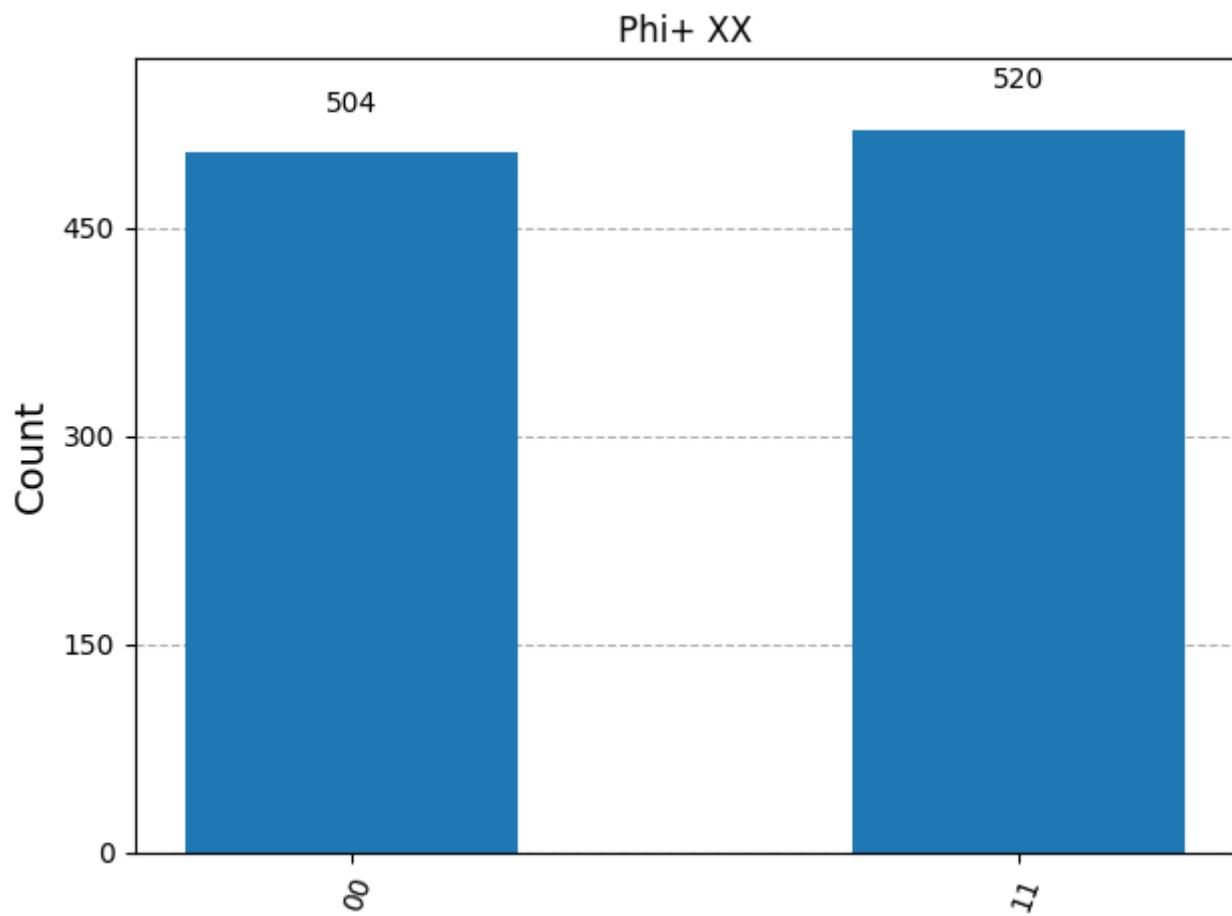


Psi- Histogram

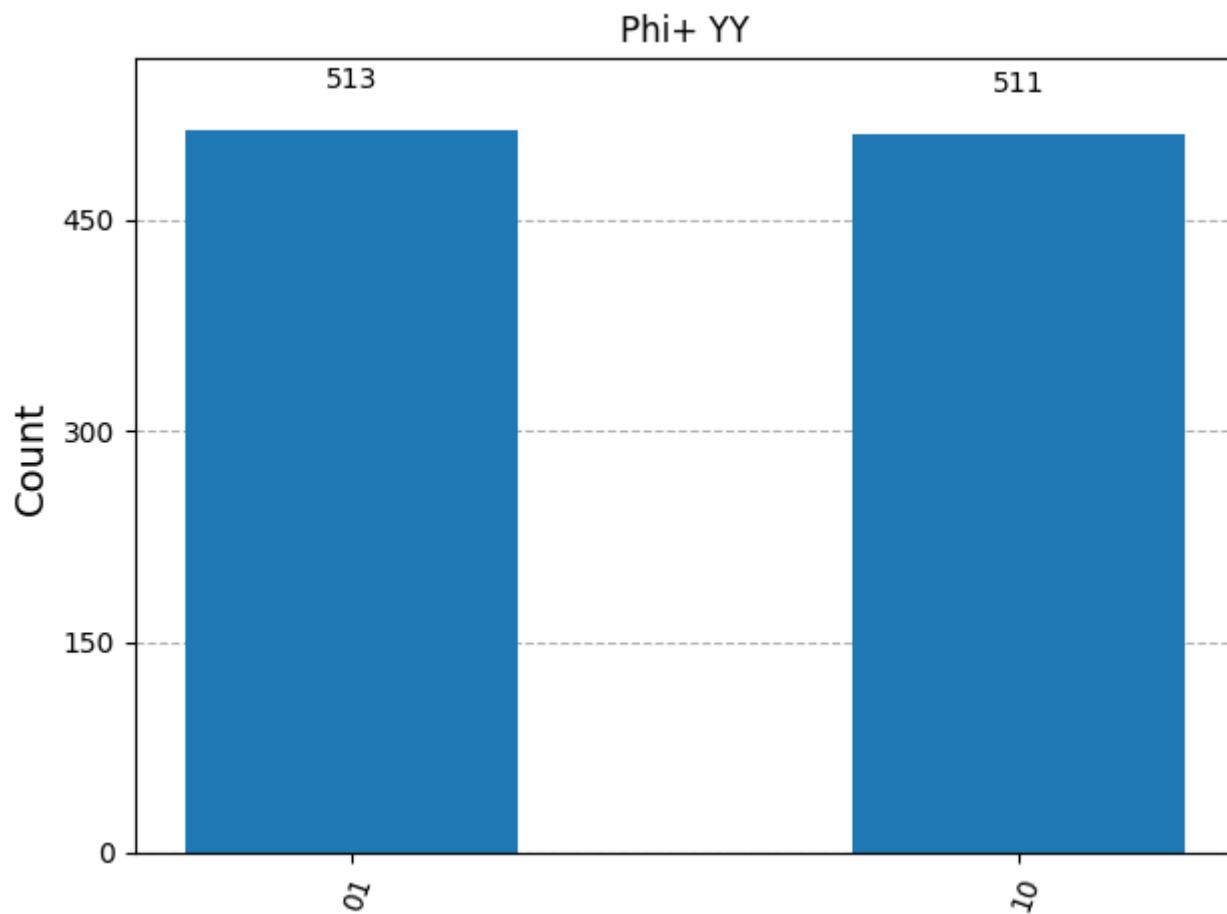


Measurement Results (Tasks 5-7)

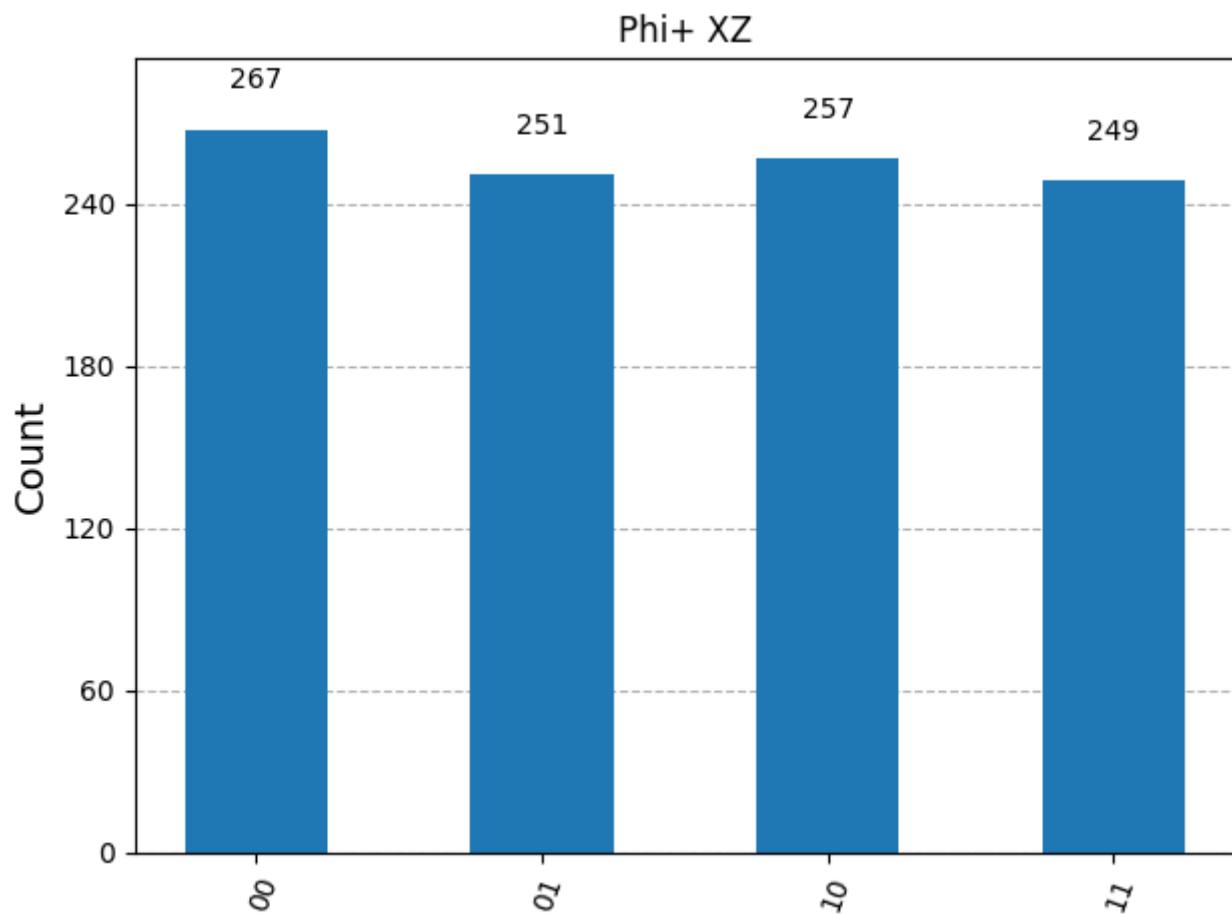
Phi+ XX Histogram



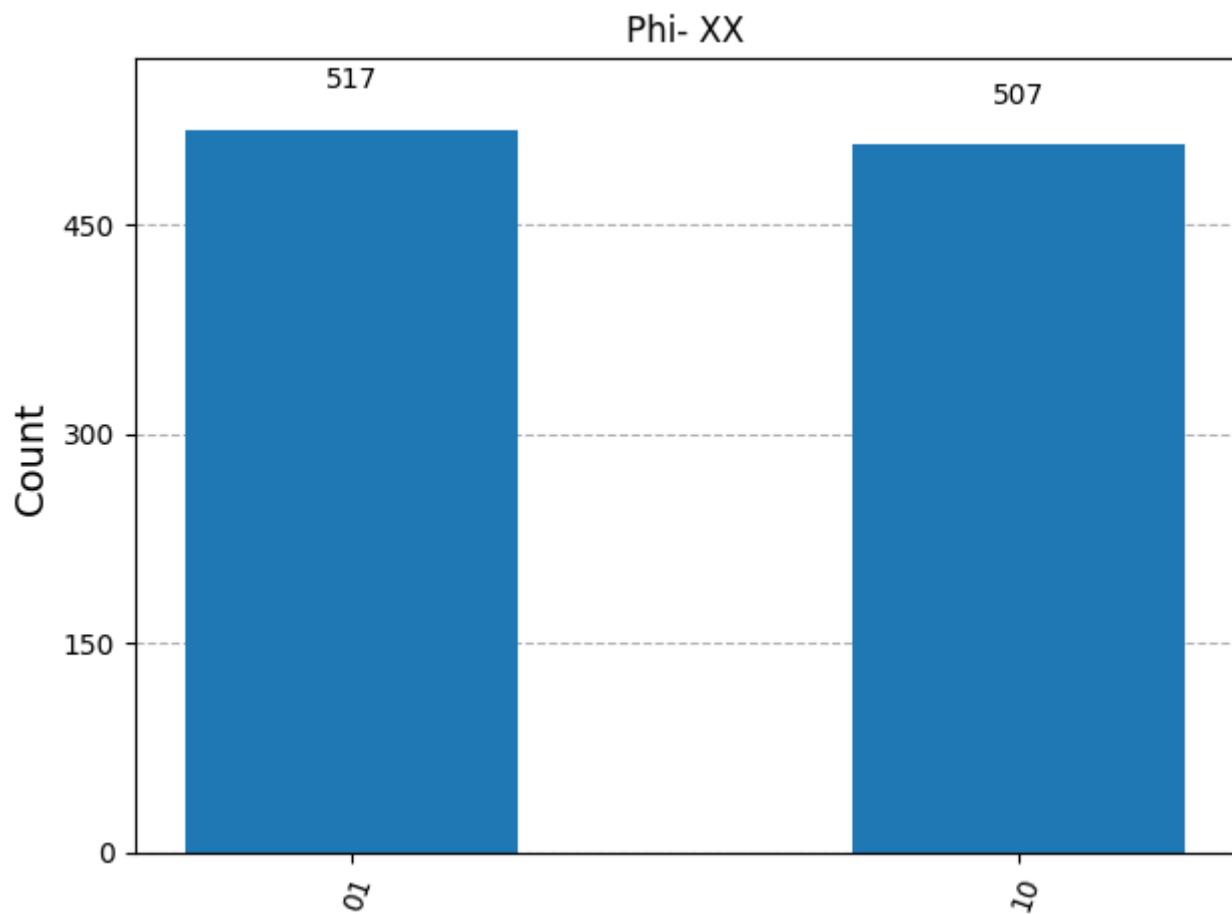
Phi+ YY Histogram



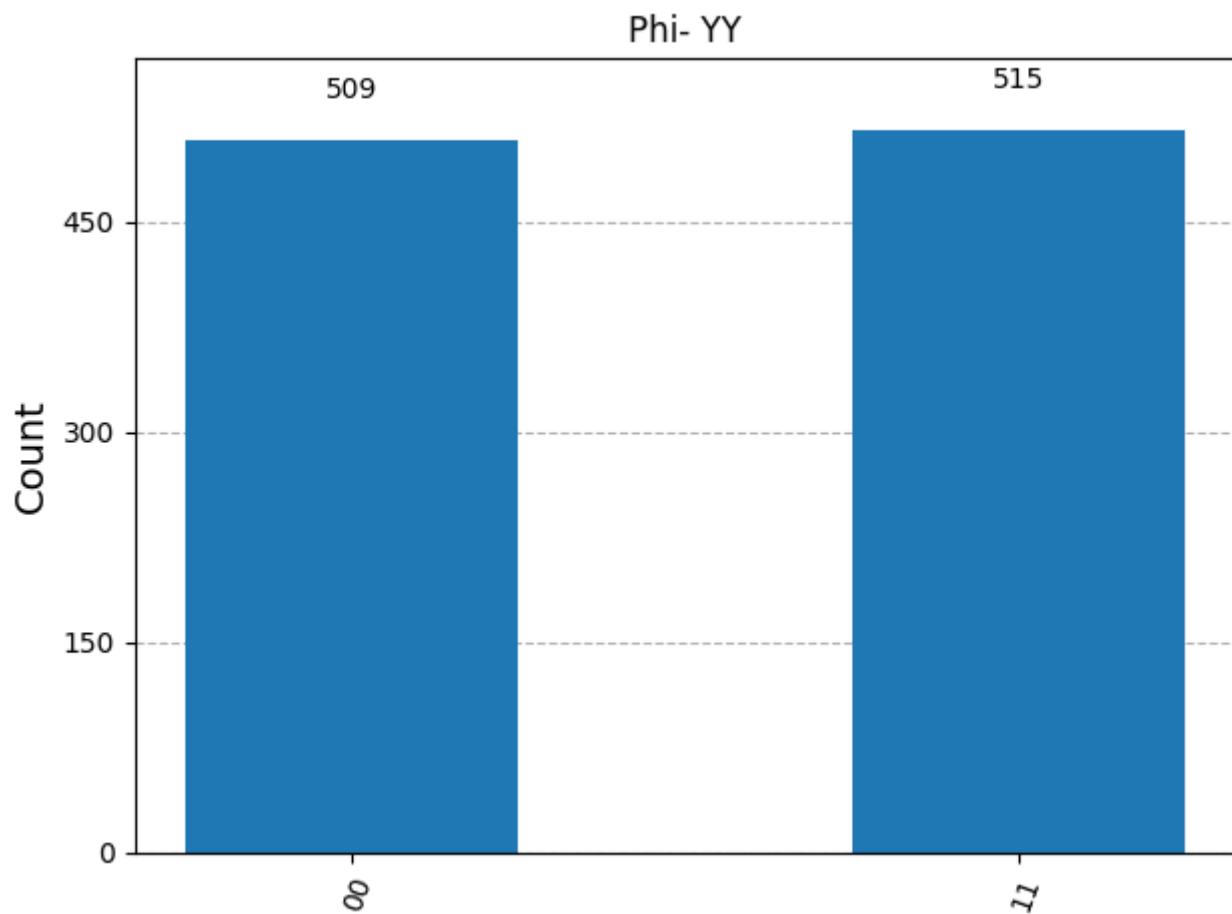
Phi+ XZ Histogram



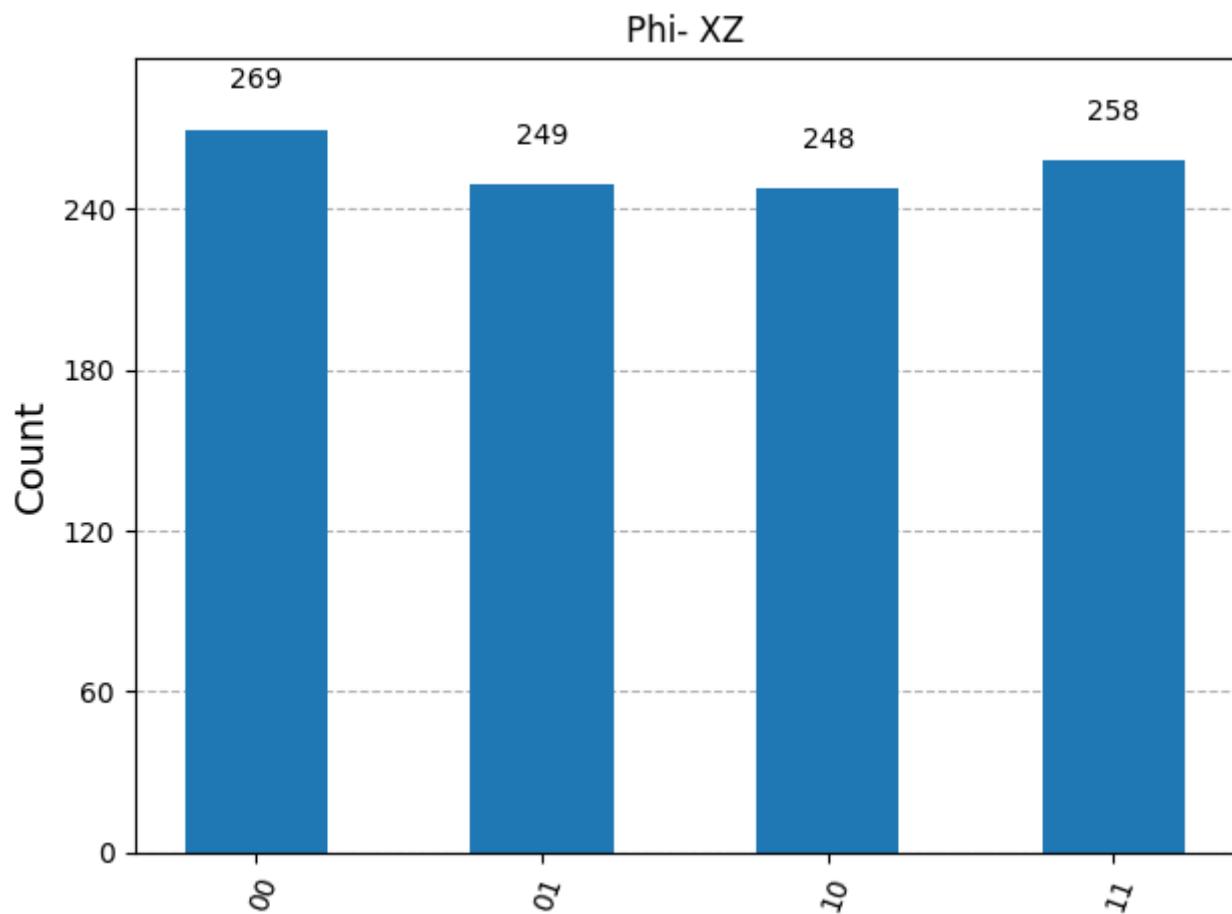
Phi- XX Histogram



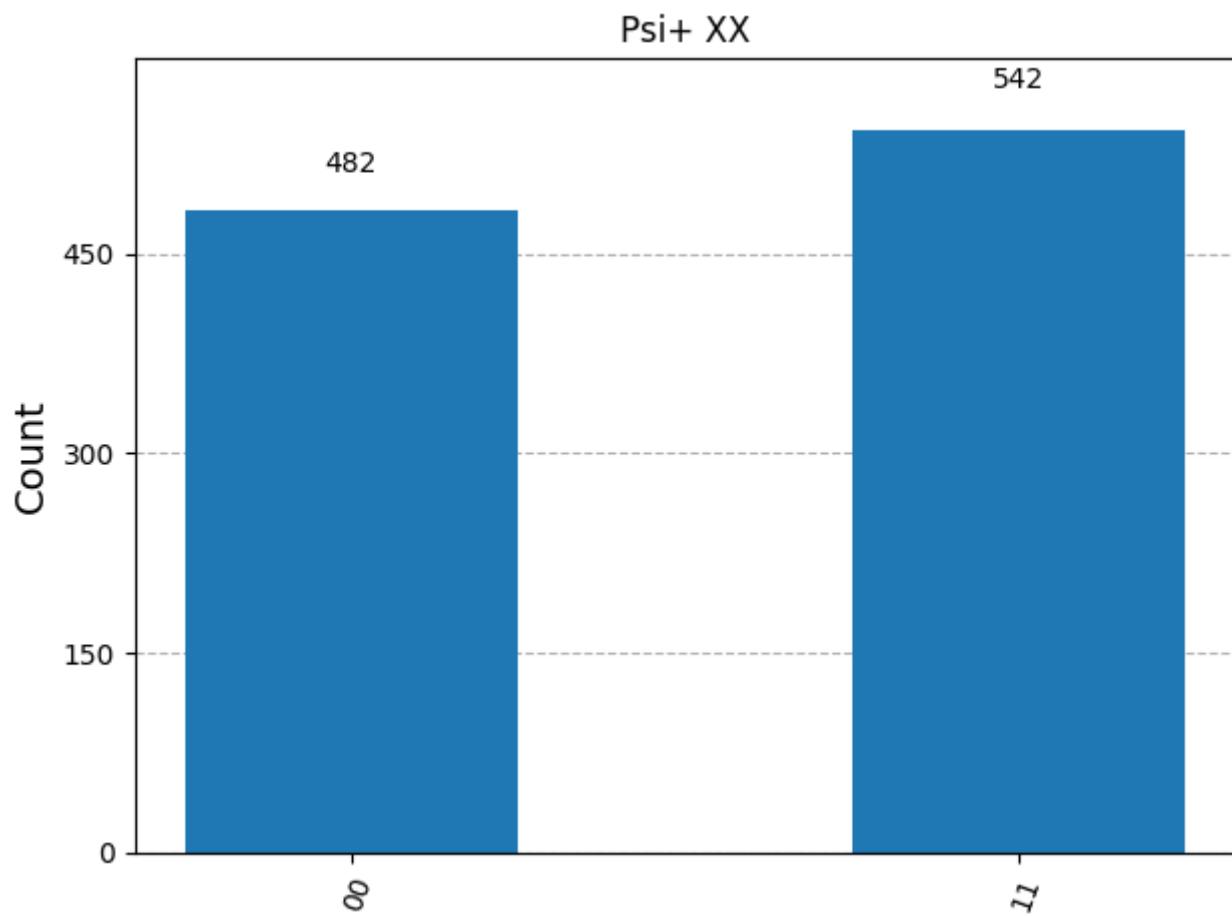
Phi- YY Histogram



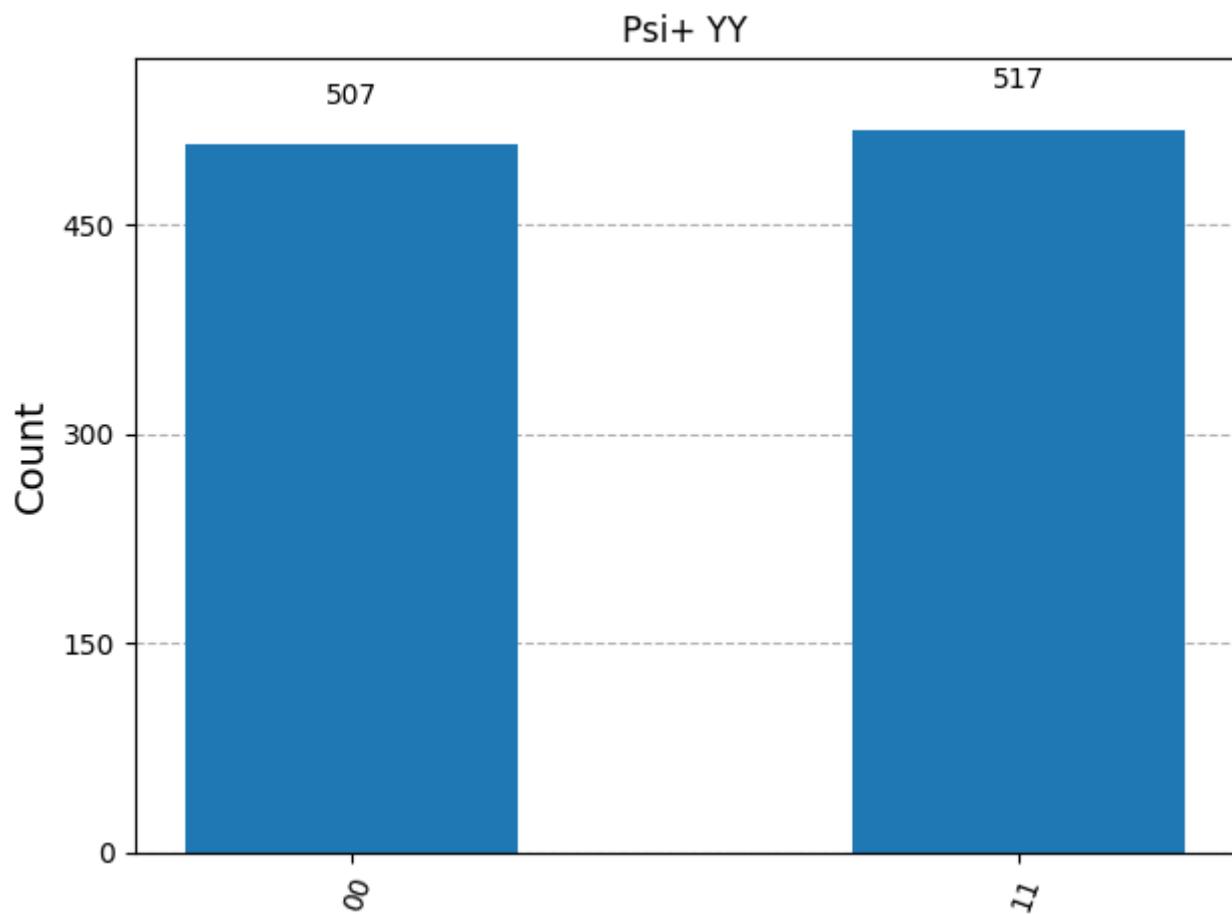
Phi- XZ Histogram

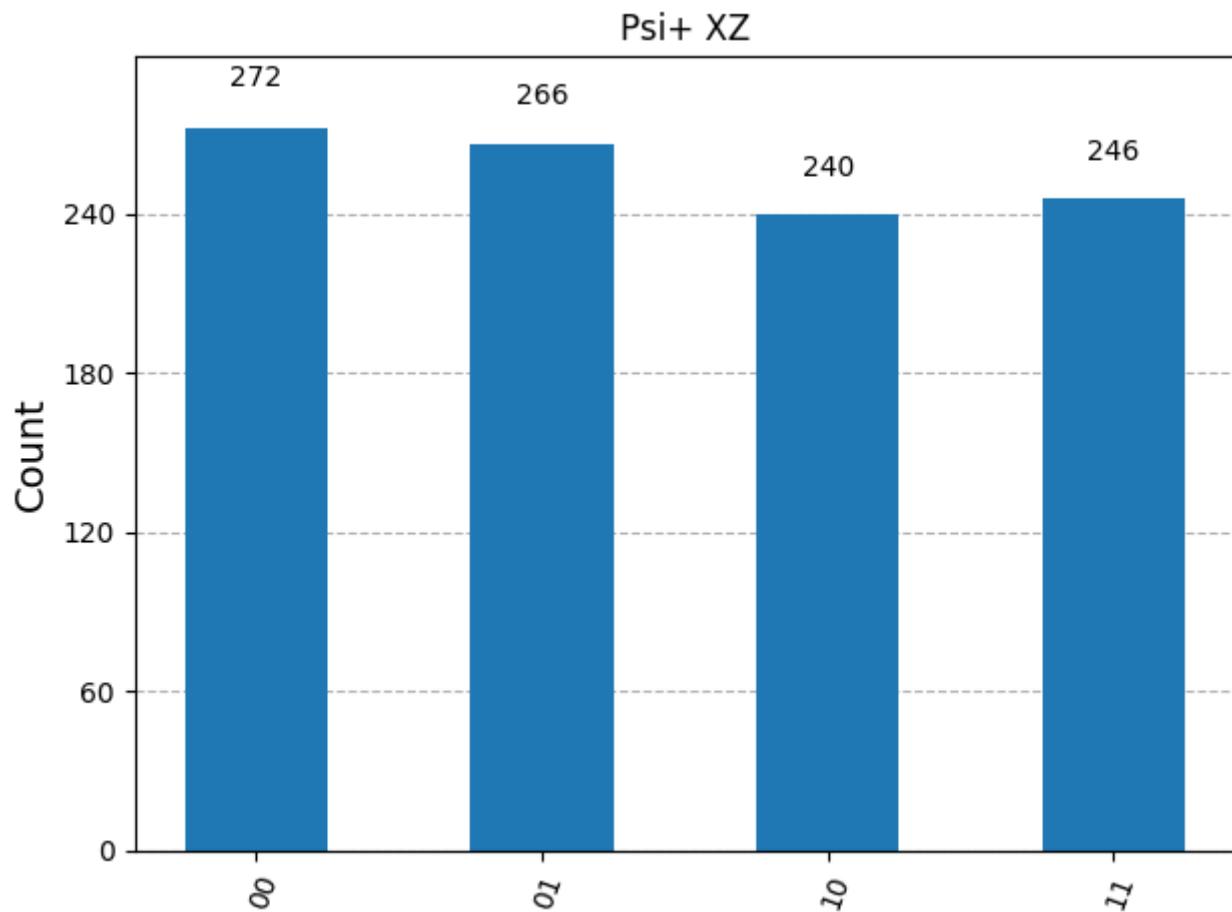


Psi+ XX Histogram

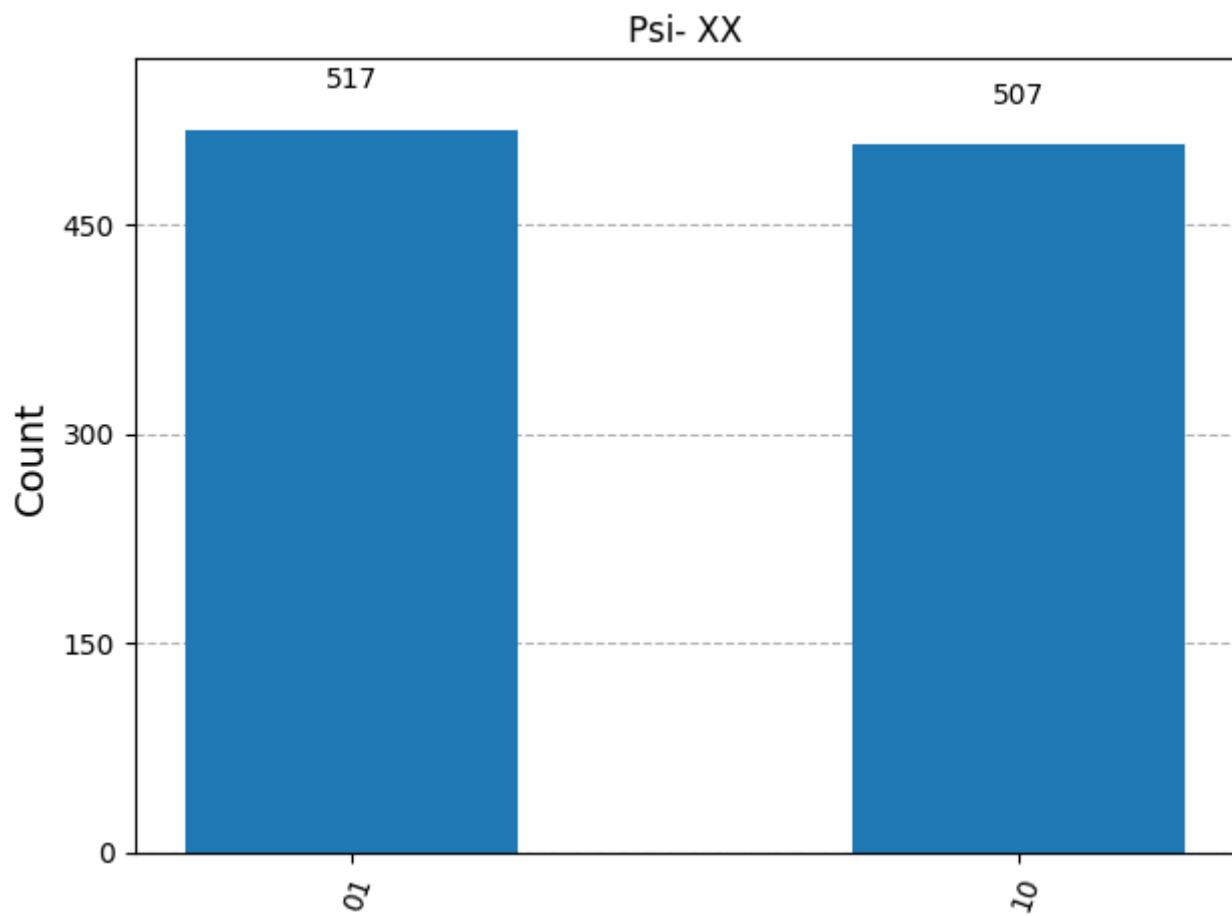


Psi+ YY Histogram

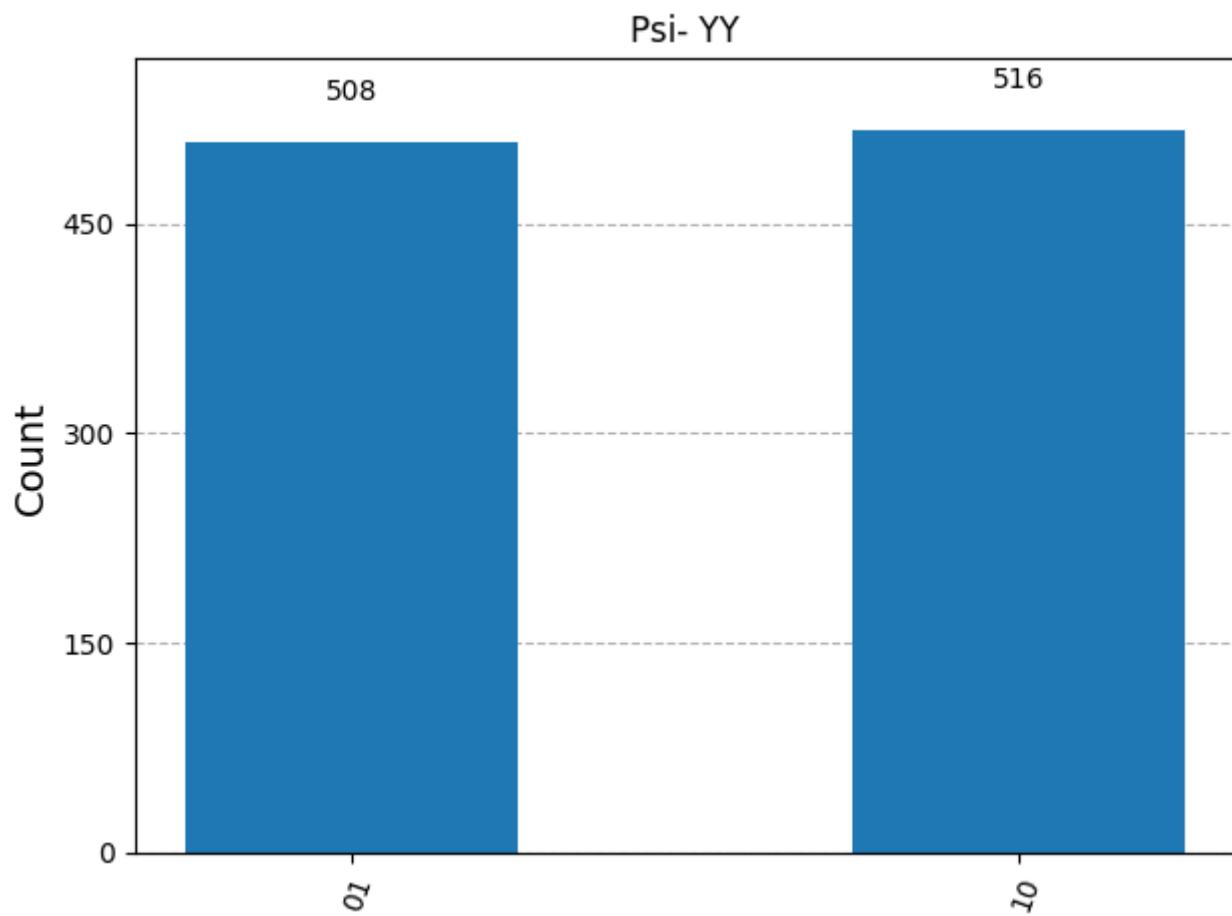
**Psi+ XZ Histogram**



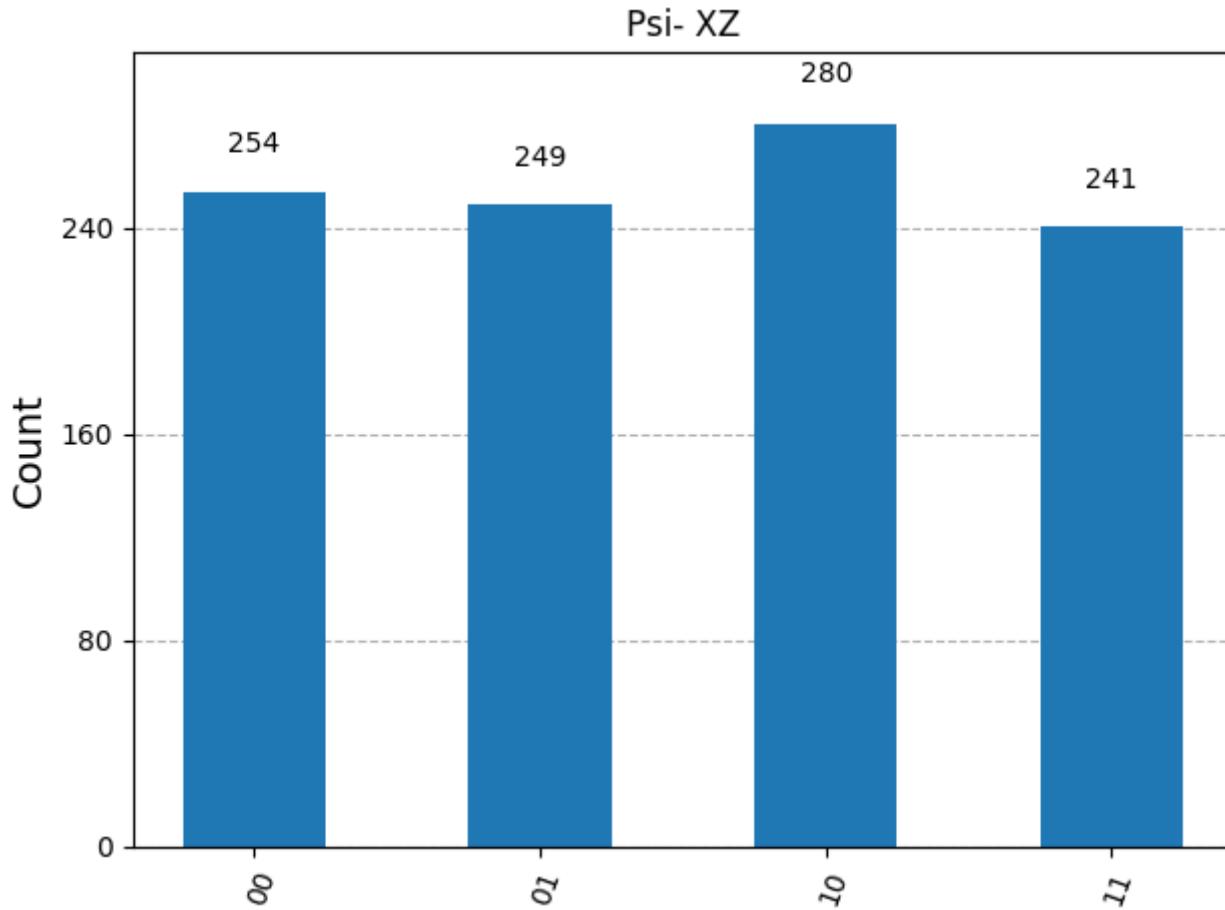
Psi- XX Histogram



Psi- YY Histogram



Psi- XZ Histogram



3. Mathematical Calculations (Tasks 2-4)

Matrix Form (Task 2: Phi-)

Matrices used:

$$\$ \$ H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & -1 \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix} \$ \$$$

$$\$ \$ H \otimes I = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & -1 \end{pmatrix} \$ \$$$

$$\$ \$ Z \otimes I = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix} \$ \$$$

$$\$ \$ CX = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \$ \$$$

Derivation:

Starting from the initial state: $\$ \$ |00\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \$ \$$

Apply $H \otimes I$: $\$ \$ (H \otimes I) |00\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$

Apply $Z \otimes I$: $\$ \$ (Z \otimes I) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$

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\ 0 \ 1 \ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \ 0 \ -1 \ 0 \end{pmatrix} $$
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Apply CNOT (CX): $$ \text{CX} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} $$
```

Final state: $\$ \$ \backslash Phi^- \rangle = \begin{pmatrix} 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \end{pmatrix}$ $\$ \$$

Operator Form (Task 3: Psi+)

Derivation:

Starting from $|00\rangle$.

Apply $H \otimes I$: $\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$

Apply CX: $\text{CX} \cdot \frac{1}{\sqrt{2}} (\lvert 00 \rangle + \lvert 10 \rangle) = \frac{1}{\sqrt{2}} (\lvert 00 \rangle + \lvert 11 \rangle)$

Apply $I \otimes X$: $\$ (I \backslash otimes X) \backslash cdot \backslash frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = \backslash frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)$ \$\$

Final state: $\$ \$ \backslash \text{Psi}^+ \backslash \text{rangle = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \end{pmatrix} \$ \$$

Operator Form (Task 4: Psi-)

Derivation:

Starting from $|00\rangle$.

Apply $H \otimes I$: $\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$

Apply $Z \otimes I$: $\frac{1}{\sqrt{2}} (|00\rangle + |10\rangle) = \frac{1}{\sqrt{2}} (|00\rangle - |10\rangle)$

Apply CX: $\text{CX} \cdot \frac{1}{\sqrt{2}} (\lvert 00 \rangle - \lvert 10 \rangle) = \frac{1}{\sqrt{2}} (\lvert 00 \rangle - \lvert 11 \rangle)$

Apply $I \otimes X$: $\$ (I \backslash otimes X) \backslash cdot \backslash frac{1}{\sqrt{2}} (\lvert 00 \rangle - \lvert 11 \rangle) = \backslash frac{1}{\sqrt{2}} (\lvert 01 \rangle - \lvert 10 \rangle)$ \$\$

Final state: $\$ \$ \Psi^-\rangle = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \end{pmatrix} \$ \$$

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Matrix Form (Task 2: Phi-)

Matrices used:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$H \otimes I = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$Z \otimes I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Derivation:

Starting from the initial state:

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Apply $H \otimes I$:

$$(H \otimes I)|00\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Apply $Z \otimes I$:

$$(Z \otimes I) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

Apply CNOT (CX):

$$CX \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

Final state:

$$|\Phi^-\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Operator Form (Task 3: Psi+)

Derivation:

Starting from $|00\rangle$.

Apply $H \otimes I$:

$$(H \otimes I)|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

Apply CX:

$$CX \cdot \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Apply $I \otimes X$:

$$(I \otimes X) \cdot \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

Final state:

$$|\Psi^+\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

Operator Form (Task 4: Psi-)

Derivation:

Starting from $|00\rangle$.

Apply $H \otimes I$:

$$(H \otimes I)|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

Apply $Z \otimes I$:

$$(Z \otimes I) \cdot \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) = \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)$$

Apply CX:

$$CX \cdot \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle) = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

Apply $I \otimes X$:

$$(I \otimes X) \cdot \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Final state:

$$|\Psi^-\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$