

CZECH TECHNICAL UNIVERSITY IN PRAGUE

FACULTY OF ELECTRICAL ENGINEERING  
DEPARTMENT OF CYBERNETICS  
MULTI-ROBOT SYSTEMS



# Development of a Safe Flocking Algorithm for UAVs Using 3D Lidar and Collaborative Multi-Robot Coordination

Bachelor's Thesis

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Study programme: Electrical Engineering and Information Technology  
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Bachelor's thesis title in English:

**Development of a Safe Flocking Algorithm for UAVs Using 3D Lidar and Collaborative Multi-Robot Coordination**

Bachelor's thesis title in Czech:

**Vývoj bezpečného algoritmu pro koordinaci UAV pomocí 3D lidaru a koordinace více robotů**

Guidelines:

- (1) Develop a safe flocking algorithm for UAVs in C++ within the MRS system framework. The algorithm must ensure collision-free and efficient movement of UAV.
- (2) Compare your solution to some of the most promising algorithms in the literature within the MRS simulator, in particular [1],[2],[3].
- (3) The novel contributions of the thesis will include
  - i) Extend existing multi-robot algorithm from 2d to 3d.
  - ii) Use 3d lidar to localize, sense, process, and react to environments such as a forest.
  - iii) Research on possible enhancements of the algorithm for example using neural networks or learning-based techniques.
- (4) Conduct an experimental campaign to validate the theoretical findings. Include different real-world scenarios, such as navigating through forests or crowded spaces.

Bibliography / sources:

- [1] Boldrer, M., Serra-Gomez, A., Lyons, L., Alonso-Mora, J., & Ferranti, L. (2024). Rule-Based Lloyd Algorithm for Multi-Robot Motion Planning and Control with Safety and Convergence Guarantees. arXiv preprint arXiv:2310.19511v2 (2023).
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- [3] Ahmad, A., Licea, D. B., Silano, G., Bába, T., & Saska, M. (2022). PACNav: a collective navigation approach for UAV swarms deprived of communication and external localization. Bioinspiration & Biomimetics, 17(6), 066019

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## Abstract

The study of autonomous Unmanned Aerial Vehicles (UAVs) has become a prominent sub-field of mobile robotics.

**Keywords** TODOUnmanned Aerial Vehicles, Automatic Control

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## Abbreviations

**FOV** Field of View

**GNSS** Global Navigation Satellite System

**GPS** Global Positioning System

**IMU** Inertial Measurement Unit

**LiDAR** Light Detection and Ranging

**RTK** Real-time Kinematics

**SLAM** Simultaneous Localization And Mapping

**UAV** Unmanned Aerial Vehicle

**UGV** Unmanned Ground Vehicle

**RBL** Rule-based Lloyd Algorithm

**ToF** Time-of-Flight

**PCL** Point Cloud Library

**GNSS** Global Navigation Satellite System

**3D** three-dimensional

**2D** two-dimensional

**EMI** Electromagnetic Interference

**IMU** Inertial Measurement Unit

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## ■ 1 Introduction

TODO

### ■ 1.1 Related Works

TODO

### ■ 1.2 Problem Statement

### ■ 1.3 Contributions

TODO

### ■ 1.4 Mathematical Notation

TODO

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$\mathcal{N}_i$	set of neighboring agents for the $i$ -th agent
$\mathbf{x}, \boldsymbol{\alpha}$	vector, pseudo-vector, or tuple
$\hat{\mathbf{x}}, \hat{\boldsymbol{\omega}}$	unit vector or unit pseudo-vector
$\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3$	elements of the <i>standard basis</i>
$\mathbf{X}, \Omega$	matrix
$\mathbf{I}$	identity matrix
$x = \mathbf{a}^T \mathbf{b}$	inner product of $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$
$\mathbf{x} = \mathbf{a} \times \mathbf{b}$	cross product of $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$
$\mathbf{x} = \mathbf{a} \circ \mathbf{b}$	element-wise product of $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$
$\mathbf{x}_{(n)} = \mathbf{x}^T \hat{\mathbf{e}}_n$	$n^{\text{th}}$ vector element (row), $\mathbf{x}, \mathbf{e} \in \mathbb{R}^3$
$X_{(a,b)}$	matrix element, (row, column)
$x_d$	$x_d$ is <i>desired</i> , a reference
$\dot{x}, \ddot{x}, \dot{\dot{x}}, \ddot{\dot{x}}$	$1^{\text{st}}, 2^{\text{nd}}, 3^{\text{rd}},$ and $4^{\text{th}}$ time derivative of $x$
$x^{[n]}$	$x$ at the sample $n$
$\mathbf{A}, \mathbf{B}, \mathbf{x}$	LTI system matrix, input matrix and input vector
$SO(3)$	3D special orthogonal group of rotations
$SE(3)$	$SO(3) \times \mathbb{R}^3$ , special Euclidean group

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Table 1.1: Mathematical notation, nomenclature and notable symbols.

## ■ 2 Extension of the Rule-Based Lloyd Algorithm to 3D

### ■ 2.1 Introduction

#### ■ Motivation

Motivation in using Rule-based Lloyd Algorithm (RBL) algorithm offers communication-less approach for guiding multiple agents from point A to point B. It requires an estimate of its position—whether through Global Positioning System (GPS), Real-time Kinematics (RTK) or methods like Simultaneous Localization And Mapping (SLAM)—along with onboard sensors that provide information about the surrounding environment, enabling it to avoid obstacles and move toward the goal. So far, the algorithm has mainly been tested in two-dimensional (2D) scenarios with a fixed altitude, and the results have looked promising. This were the main motivation factors to try and extend it to full three-dimensional (3D) navigation, where altitude and sensor orientation also play a role.

#### ■ Problem Statement

Many real-world robotic applications, particularly involving Unmanned Aerial Vehicle (UAV)s, require navigation within 3D environments containing obstacles. While algorithms like the RBL [1] have proven effective in 2D goal convergence, they are inherently limited when applied directly to 3D space. The core problem arises because the existing RBL algorithm is fundamentally designed for 2D planar navigation, making it unsuitable for applications like UAV operations that require navigating in 3D space. Specifically, there is a need to extend existing 2D algorithms to 3D while intelligently incorporating vertical exploration to ensure safe, efficient, and effective navigation through obstacle-rich 3D environments without significantly compromising performance or requiring excessive computational resources. This work addresses the challenge of adapting the RBL algorithm for 3D space by developing and evaluating specific rules to govern vertical movement, aiming to achieve robust 3D navigation capabilities.

The primary challenge involved the extension of an existing 2D algorithm into a 3D space. To achieve improved results in this higher dimension, new rules controlling vertical exploration were introduced. The subsequent sections of this chapter detail the fundamental principles of the RBL algorithm, with specific attention devoted to its application in both 2D and 3D environments. The distinct rules governing the algorithm's behavior in these two dimensions are described separately to provide clarity. Finally, the chapter concludes in the presentation and evaluation of simulation experiments designed to assess the efficiency of the proposed 3D extension and the impact of the introduced vertical exploration rules.

#### ■ Objectives

- Enhance the convergence speed of agents towards their goals by promoting exploration, with focus on the vertical exploration.
- Demonstrate the algorithm's enhanced applicability, achieved by extending goal placement to the z-axis.

## ■ Chapter Overview

The principles of the RBL algorithm, presented in [1], form the basis of the work in this chapter. These principles are clarified and modified to address the challenges of 3D extension. The chapter details the fundamental principles of RBL in 2D and 3D spaces, describes the specific rules applied to enhance convergence, and concludes with a series of simulation scenarios designed to demonstrate the performance of the proposed solution.

## ■ 2.2 Basic principles of Rule-Based Lloyd Algorithm

### ■ Overview

The algorithm is a communication-less approach designed to navigate agents from point A to point B. The algorithm's ability to determine each agent's position relative to its goal depends on the availability of positioning data. This can be either global positioning data, such as from GPS, or relative positioning data obtained through techniques like SLAM in GNSS-denied environments, combined with sensor inputs that provide information about the agent's surrounding environment. These sensors can include LiDAR, depth cameras, or standard cameras with estimation techniques, allowing the agent to detect and avoid obstacles or other agents. The algorithm enables autonomous navigation without requiring direct communication between agents, making it suitable for scalable and decentralized applications.

### ■ Applications and Limitations in 2D

Modern robotics relies on the capability to navigate from point A to point B. Navigation plays a crucial role in various robotic applications, such as Unmanned Ground Vehicle (UGV)s, which are commonly used in manufacturing and logistics. UGVs typically follow predefined 2D trajectories guided by visual [9], magnetic [12], or LiDAR-based navigation [11]. Additionally, 2D navigation is widely employed in robotic vacuum cleaners, enabling them to systematically cover an area while avoiding obstacles.

An obvious limitation for algorithms in 2D is scalability. As the number of agents in a system increases, the complexity of managing their movements and coordination also grows significantly. Obstacle avoidance in 2D can also be less efficient compared to 3D environments, as agents have fewer options for evading obstacles. In 3D, agents can change their altitude in addition to their horizontal trajectory, giving them more freedom to maneuver around obstacles.

### ■ Key principles

RBL ensures convergence to the goal and provides sufficient conditions for achieving it. The problem involves individual control of  $N$  agents from their initial position  $\mathbf{p}_i(0)$  toward a goal region, represented as circle. This goal region is denoted as  $G(\mathbf{g}_i, r_g)$ , where  $\mathbf{g}_i$  is center and  $r_g$  is radius of goal region. The agent is progressing towards its designated destination,  $\mathbf{d}_i$ . Each agent is known of its current position  $\mathbf{p}_i$ , encumbrance  $\delta_i$ , which determines safe space around agent. Additionally, each agent also knows the positions and encumbrances of its neighboring agents  $\mathcal{N}_i$ , agent  $j \in \mathcal{N}_i$  if  $\|\mathbf{p}_i - \mathbf{p}_j\| \leq \frac{r_{s,i}}{\eta}$ , where  $r_{s,i}$  is sensing radius of the  $i$ -th agent and  $\eta$  is a scaling parameter. For simplicity  $r_{s,i}$  is considered to be same for all agents, therefore  $r_{s,i} = r_s$ .

The core objective of the algorithm is to minimize the coverage cost function, which accounts for the distribution of agents and obstacles over the environment. This function is expressed as:

$$J_{cov}(\mathbf{p}) = \sum_{i=1}^N \int_{\mathcal{V}_i} \|\mathbf{q} - \mathbf{p}_i\|^2 \varphi_i(\mathbf{q}) d\mathbf{q}, \quad (2.1)$$

where  $\mathbf{p}_i$  is the position of agent  $i$ ,  $\mathcal{V}_i$  is the Voronoi cell of the  $i$ -th robot,  $\|\mathbf{q} - \mathbf{p}_i\|^2$  is squared Euclidian distance between point in the mission space  $\mathbf{q} \in \mathcal{Q}$  and agent's position  $p_i$ , and  $\varphi_i(\mathbf{q})$  is the weighting function.

Voronoi cell is defined as:

$$\mathcal{V}_i = \{\mathbf{q} \in \mathcal{Q} \mid \|\mathbf{q} - \mathbf{p}_i\| \leq \|\mathbf{q} - \mathbf{p}_j\|, \forall j \neq i\} \quad (2.2)$$

For visual representation see Fig. 2.1a.

However, this standard definition of Voronoi cells does not take into account the physical space occupied by the agents, or their encumbrances. To address this, a Modified Voronoi cell is introduced, which takes into account the encumbrances of agents. This modified version adjusts the boundaries of each Voronoi cell to account for the encumbrances of neighboring agents. Also to enhance the algorithm performance, the Voronoi cell are scaled using a scaling parameter  $\eta \in [0, 1]$ . The modified Voronoi cell definition is as follows:

$$\tilde{\mathcal{V}}_i = \begin{cases} \{\mathbf{q} \in Q \mid \|\mathbf{q} - \mathbf{p}_i\| \leq \eta \|\mathbf{q} - \mathbf{p}_j\|\}, & \text{if } \Delta_{ij} \leq \frac{\|\mathbf{p}_i - \mathbf{p}_j\|}{2} \\ \{\mathbf{q} \in Q \mid \|\mathbf{q} - \mathbf{p}_i\| \leq \eta \|\mathbf{q} - \tilde{\mathbf{p}}_j\|\}, & \text{otherwise,} \end{cases} \quad (2.3)$$

$\forall j \in \mathcal{N}_i$ , where  $\Delta_{ij} = \delta_i + \delta_j$  and  $\tilde{\mathbf{p}}_j = \mathbf{p}_j + 2(\Delta_{ij} - \frac{\|\mathbf{p}_i - \mathbf{p}_j\|}{2}) \frac{\mathbf{p}_i - \mathbf{p}_j}{\|\mathbf{p}_i - \mathbf{p}_j\|}$ . The parameter  $\eta$  controls the degree of overlap or empty space between cells. Specifically, for the classical Voronoi cell definition ( $\eta = 0.5$ ), each agent needs to detect its neighbors within a distance of  $2r_s$ . In general each agent needs to know its neighbors within distance of  $\frac{r_s}{\eta}$ .

Together with cell  $\mathcal{S}_i$  defined as:

$$\mathcal{S}_i = \{\mathbf{q} \in \mathcal{Q} \mid \|\mathbf{q} - \mathbf{p}_i\| \leq r_{s,i}\} \quad (2.4)$$

the cell  $\mathcal{A}_i$  is obtained as  $\mathcal{A}_i = \tilde{\mathcal{V}}_i \cap \mathcal{S}_i$ .

Convergence to goal region  $G(\mathbf{g}_i, r_g)$  depends on the choice of weighting function that assigns weights to points  $\mathbf{q}$  in the mission space  $\mathcal{Q}$ . The weighting function  $\varphi_i(\mathbf{q})$  is defined as follows:

$$\varphi_i(\mathbf{q}) = \exp\left(-\frac{\|\mathbf{q} - \mathbf{d}_i\|}{\beta_i}\right), \quad (2.5)$$

where  $\beta_i$  is the weighting factor for points  $\mathbf{q}$ , and  $\mathbf{d}_i$  represents the current destination of the agent. The destination is computed as follows:

$$\mathbf{d}_i = \mathbf{p}_i + R(\theta)(\mathbf{g}_i - \mathbf{p}_i) \quad (2.6)$$

where  $R$  is the azimuthal rotation. Rules:

■ **Weighting rule**

$$\dot{\beta}_i(A_i) = \begin{cases} -k & \text{if } \beta_i > \beta_{min} \wedge \|\mathbf{c}_{A_i} - \mathbf{p}_i\| < d_1 \wedge \|\mathbf{c}_{A_i} - \mathbf{c}_{\mathcal{S}_i}\| > d_2 \\ 0 & \text{if } \beta_i \leq \beta_{min} \wedge \|\mathbf{c}_{A_i} - \mathbf{p}_i\| < d_1 \wedge \|\mathbf{c}_{A_i} - \mathbf{c}_{\mathcal{S}_i}\| > d_2 \\ -(\beta_i - \beta_i^D) & \text{otherwise} \end{cases} \quad (2.7)$$

where the first case decreases  $\beta_i$  over time, the second case ensures that  $\beta_i$  does not decrease below its minimum threshold  $\beta_{min}$  (saturation) and the third case provides a general update rule when the previous conditions are not met.

#### ■ Azimuth update rule

$$\dot{\theta}_i = \begin{cases} k & \text{if } \theta < \frac{\pi}{2} \wedge \|\mathbf{c}_{\mathcal{A}_i} - \mathbf{c}_{\mathcal{S}_i}\| > d_4 \wedge \|\mathbf{p}_i - \mathbf{c}_{\mathcal{A}_i}\| > d_3 \\ -k & \text{if } \theta > 0 \wedge \neg(\|\mathbf{c}_{\mathcal{A}_i} - \mathbf{c}_{\mathcal{S}_i}\| > d_4 \wedge \|\mathbf{p}_i - \mathbf{c}_{\mathcal{A}_i}\| > d_3) \\ 0 & \text{otherwise} \end{cases} \quad (2.8)$$

where the first case increases  $\theta_i$  over time, the second case ensures that  $\theta_i$  converges back when the distance constraints are not satisfied, and the third case keeps  $\theta_i$  unchanged.

#### ■ Azimuth reset rule

$$\theta = 0 \quad \text{if } \theta = \frac{\pi}{2} \wedge \|\mathbf{p}_i - \bar{\mathbf{c}}_{\mathcal{A}_i}\| \quad (2.9)$$

where  $\bar{\mathbf{c}}_{\mathcal{A}_i}$  represents the centroid computed from the cell  $\mathcal{A}_i$ , which is weighted using the unrotated destination, meaning  $\mathbf{d}_i = \mathbf{g}_i$ .

Weighted centroid is computed as follows:

$$\mathbf{c}_{\mathcal{A}_i} = \frac{\int_{\mathcal{A}_i} \mathbf{q} \varphi_i(\mathbf{q}) d\mathbf{q}}{\int_{\mathcal{A}_i} \varphi_i(\mathbf{q}) d\mathbf{q}}, \quad (2.10)$$

where  $\mathbf{q}$  represents the point in mission space, and  $\varphi_i(\mathbf{q})$  is a weighting function. The centroids for other relevant cells are computed using a similar approach, but over different sets.

By applying the previously defined rules and computations, the RBL algorithm successfully guides agents movement toward the goal. However, these rules focus solely on rotating the centroid  $\mathcal{A}_i$  in the azimuthal plane. To enhance performance in a fully three-dimensional space, additional rules are required to account for elevation adjustments.

### ■ 2.3 Extension of the RBL algorithm to 3D

#### ■ Motivation for 3D Extension

In many practical applications, agents must operate in three-dimensional spaces, considering not only horizontal movement but also vertical positioning. A 3D extension is necessary to navigate complex environments that feature obstacles in all directions. In 2D, agents are restricted to a flat plane, which simplifies navigation but limits the ability to interact with objects and environments that exist in the third dimension. The ability to utilize vertical space can enhance energy efficiency, as agents can optimize their paths by ascending or descending to avoid obstacles or to find more favorable environmental conditions, such as discovering more open space. The transition from 2D to 3D also opens up possibilities for more advanced movement strategies, such as navigating through multi-level environments or optimizing trajectories by utilizing vertical space. Moreover, the use of 3D models allows for more accurate representations of real-world scenarios, where elevation plays a crucial role in decision-making and task execution.

#### ■ Differences between 2D and 3D

The primary distinction in the 3D extension is that each goal region is now represented as a sphere rather than a circle. Similarly, the sensing cell  $\mathcal{S}_i$  is also modeled as a sphere

instead of a circle, allowing for a more accurate representation of the UAV's perception in three-dimensional space. This change introduces a key modification when defining  $\tilde{V}_i$ : in the 2D case, a line was sufficient to slice the sensing region, whereas in 3D, a plane must be computed to properly segment the spherical sensing cell.

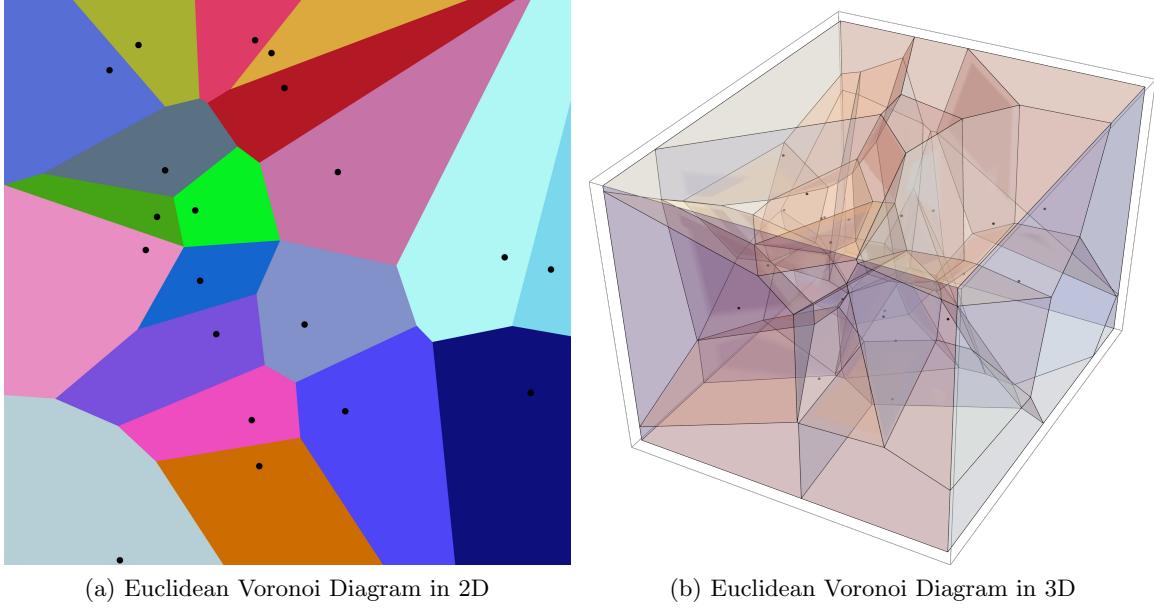


Figure 2.1: (a) an example of 20 Voronoi cells in 2D [5] (b) 25 Voronoi cells in 3D [8]

## ■ Additional Constraints and Modifications

To extend the approach to the 3D case, several adjustments are made. The weighting rule (2.7) remains unchanged. However, in both the azimuth update rule (2.8) and the azimuth reset rule (2.9), only the displacement of centroids projected onto the  $xy$ -plane is taken into account. The modified formulations for the 3D case are as follow:

### ■ Azimuth update rule 3D

$$\dot{\theta}_i = \begin{cases} k & \text{if } \theta < \frac{\pi}{2} \wedge \|\mathbf{c}_{\mathcal{A}_i} - \mathbf{c}_{\mathcal{S}_i}\|_{xy} > d_4 \wedge \|\mathbf{p}_i - \mathbf{c}_{\mathcal{A}_i}\|_{xy} > d_3 \\ -k & \text{if } \theta > 0 \wedge \neg(\|\mathbf{c}_{\mathcal{A}_i} - \mathbf{c}_{\mathcal{S}_i}\|_{xy} > d_4 \wedge \|\mathbf{p}_i - \mathbf{c}_{\mathcal{A}_i}\|_{xy} > d_3) \\ 0 & \text{otherwise} \end{cases} \quad (2.11)$$

### ■ Azimuth reset rule 3D

$$\theta = 0 \quad \text{if } \theta = \frac{\pi}{2} \wedge \|\mathbf{p}_i - \bar{\mathbf{c}}_{\mathcal{A}_i}\|_{xy} \quad (2.12)$$

The notation  $\|\cdot\|_{xy}$  denotes the Euclidean norm computed in the  $xy$ -plane only, defined as:

$$\|\mathbf{p}_i - \mathbf{p}_j\|_{xy} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (2.13)$$

Additionally,  $Z_{clipping}$  is applied to each sensing cell  $\mathcal{S}_i$ , constraining it within the vertical limits defined by  $\min_z$  and  $\max_z$ :

$$\mathcal{S}_i = \{\mathbf{q} \in \mathcal{Q} \mid \|\mathbf{q} - \mathbf{p}_i\| \leq r_{s,i}, \quad \min_z \leq q_z \leq \max_z\} \quad (2.14)$$

where  $\min_z$  and  $\max_z$  define the vertical bounds within which the sensing region  $\mathcal{S}_i$  is restricted. This ensures that the agent cannot exceed these limits, as it follows the computed centroid  $\mathbf{c}_{V_i}$ . By constraining the sensing radius, the agent remains confined within the specified region, preventing it from moving outside the vertical interval  $\min_z$  to  $\max_z$ .

The destination rotation rule  $Z_{rule}$  is introduced to enhance agents avoidance by rotating the computed destination  $\mathbf{d}_i$  by an angle  $\phi$ . For vertical rotation angle  $\phi$ , the following condition is introduced

$$\Omega = \|\mathbf{c}_{\mathcal{A}_i} - \mathbf{c}_{\mathcal{S}_i}\|_z < d_6 \wedge \|\mathbf{p}_i - \mathbf{c}_{\mathcal{A}_i}\|_z \vee (\|\mathbf{p}_i - \mathbf{c}_{\mathcal{S}_i}\|_{xy} - \|\mathbf{p}_i - \mathbf{c}_{\mathcal{A}_i}\|_{xy}) > d_7 \quad (2.15)$$

This condition has two main parts, combined by a logical or. The first part evaluates the vertical displacement of the centroid of the partitioned cell  $\mathbf{c}_{\mathcal{A}_i}$  from the centroid of the sensing cell  $\mathbf{c}_{\mathcal{S}_i}$  and the displacement of the agents position  $\mathbf{p}_i$  from  $\mathbf{c}_{\mathcal{A}_i}$ . The second part considers the difference in the horizontal plane between the agent's position and the centroids  $\mathbf{c}_{\mathcal{A}_i}, \mathbf{c}_{\mathcal{S}_i}$ .

To improve agent distribution in the vertical dimension, a directional influence on vertical exploration is introduced. Given that each agent's position  $\mathbf{p}_i$  and final goal  $\mathbf{g}_i$ , are known, this direct influence can be included into the update rule for  $\phi_i$ .

First, the global heading angle,  $\theta_{goal}$ , towards the goal is calculated:

$$\theta_{goal} = \text{atan2}(g_{i,y} - p_{i,y}, g_{i,x} - p_{i,x}) \quad (2.16)$$

where  $\mathbf{g}_{i,x}, \mathbf{g}_{i,y}$  represent the x and y coordinates of the goal, and  $\mathbf{p}_{i,x}, \mathbf{p}_{i,y}$  represent the x and y coordinates of the agent. The resulting angle is in the range  $[-\pi, \pi]$ . Next, the heading angle is linearly mapped to a direction influence  $D_{influence}$  value in the range [-1, 1]:

$$D_{influence} = \frac{\theta_{goal}}{\pi} \quad (2.17)$$

This mapping ensures that agents traveling directly northward (from South to North) have a directional influence close to one, that will promote upward vertical exploration. Similarly, agents traveling southward have a direction influence close to -1, promoting downward exploration. Agents moving primarily eastward or westward have a directional influence close to 0, resulting in minimal vertical exploration. This strategy encourages a more uniform distribution of agents throughout the 3D space.

To determine the adjustment to the agent's vertical orientation, the directional influence  $D_{influence}$  is combined with the relative vertical displacement of the agent and  $\mathbf{c}_{\mathcal{S}_i}$ . Weighted average is used to combine them:

$$C_{influence} = \frac{w_1 \cdot (\mathbf{c}_{\mathcal{S}_i,z} - \mathbf{p}_{i,z}) + w_2 \cdot D_{influence}}{w_1 + w_2} \quad (2.18)$$

where  $w_1, w_2$  are weighting factors. The resulting combined influence  $C_{influence}$  is then used to update the agent's rotation of its destination  $\mathbf{d}_i$  by  $\phi_i$ .

Based on condition (2.15) and (2.18) update rule for  $\phi_i$  can be constructed as follows:

$$\dot{\phi}_i = \begin{cases} k & \text{if } \phi_i < \frac{\pi}{4} \wedge C_{influence} > 0 \wedge \Omega \\ 0 & \text{if } \phi_i > \frac{\pi}{4} \wedge C_{influence} > 0 \wedge \Omega \\ -k & \text{if } \phi_i > -\frac{\pi}{4} \wedge C_{influence} \leq 0 \wedge \Omega \\ 0 & \text{if } \phi_i < -\frac{\pi}{4} \wedge C_{influence} \leq 0 \wedge \Omega \\ -k & \text{if } \phi_i > 0 \wedge \neg \Omega \\ k & \text{if } \phi_i < 0 \wedge \neg \Omega \\ 0 & \text{otherwise} \end{cases} \quad (2.19)$$

where the first four cases control the rotation of the agent's destination  $\mathbf{d}_i$  and the final three cases handle the smooth convergence of  $\phi_i$  back to 0, when the condition  $\Omega$  is not met.

After the application of the rules for  $\theta_i$  and  $\phi_i$ , the agent's destination is updated as:

$$\mathbf{d}_i = \begin{pmatrix} p_{i,x} + \|\mathbf{p}_i - \mathbf{g}_i\| \cdot \sin(\phi_{\mathbf{g}_i} + \phi_i) \cdot \cos(\theta_{\mathbf{g}_i} - \theta_i) \\ p_{i,y} + \|\mathbf{p}_i - \mathbf{g}_i\| \cdot \sin(\phi_{\mathbf{g}_i} + \phi_i) \cdot \sin(\theta_{\mathbf{g}_i} - \theta_i) \\ p_{i,z} + \|\mathbf{p}_i - \mathbf{g}_i\| \cdot \cos(\phi_{\mathbf{g}_i} + \phi_i) \end{pmatrix} \quad (2.20)$$

where  $\phi_{\mathbf{g}_i} = \arccos(\frac{\mathbf{g}_{i,z} - \mathbf{p}_{i,z}}{\|\mathbf{p}_i - \mathbf{g}_i\|})$  is the polar angle from z-axis to the goal,  $\theta_{\mathbf{g}_i} = \text{atan2}(\mathbf{g}_{i,y} - \mathbf{p}_{i,y}, \mathbf{g}_{i,x} - \mathbf{p}_{i,x})$  is the azimuthal angle in the xy-plane to the goal.

## ■ Algorithm Walkthrough

At each iteration, the proposed algorithm performs a sequence of actions to control agent movement. These actions can be summarized as follows:

### (i) Cell Generation:

For each agent, two cells are generated: a partitioned cell, denoted as ' $\mathcal{A}$ ' and a sensing cell, denoted as ' $\mathcal{S}$ '.

### (ii) Centroid Computation:

- The centroid of cell  $\mathcal{A}$ ,  $c_{\mathcal{A}}$  is computed using a weighted function that considers both the agent's goal position,  $\mathbf{g}_i$ , and its current destination,  $\mathbf{d}_i$ .
- The centroid of cell  $\mathcal{S}$ ,  $c_{\mathcal{S}}$  is computed using a weighted function that considers only the agent's current destination,  $\mathbf{d}_i$ .

### (iii) Rule Application and Destination Update:

A set of rules is applied to:

- Adjust weighting function used in the centroid computations (2.7).
- Rules to update  $\theta_i$  (2.11) and  $\phi_i$  (2.19) are applied.
- Update the agent's destination  $\mathbf{d}_i$  (2.20).

### (iv) Agent movement

Finally, the agent is directed towards a centroid location  $c_{\mathcal{A}}$ .

This process is repeated until all agents reach their goals.

## ■ 2.4 Simulation and Results Analysis

### ■ Computational Implementation Details

The continuous algorithm presented in the previous section can be easily discretized for implementation in a computational environment. Instead of continuous sets, the cells  $\mathcal{A}$  and

$\mathcal{S}$  are approximated using a set of discrete points. Discrete points are partitioned out when definition (2.3) is not met.

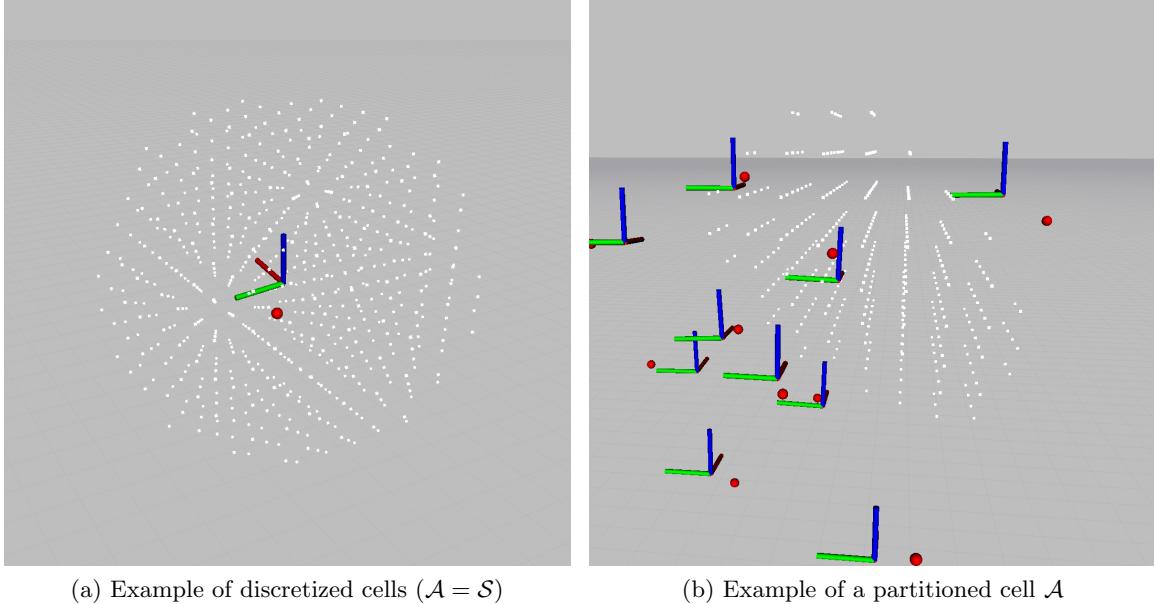


Figure 2.2: Discrete approximation of Voronoi cells, with centroid  $c_A$  shown as a red dot. (a) Example where the partitioned cell  $\mathcal{A}$  and the sensing cell  $\mathcal{S}$  are the same. (b) Example where the partitioned cell  $\mathcal{A}$  is partitioned by other UAVs.

## ■ Simulation Environment

In this simulation, the term 'agent' refers to the UAV, which were simulated using the framework provided by [2]. Each UAV obtains its global ground truth position from the ROS simulator. The positions of neighboring UAVs were estimated using simulated blinking ultraviolet markers, following the method described in [14] and [3]. The following constraints are relevant for each UAV:

Parameter	Value
Maximal horizontal velocity [ $\text{m s}^{-1}$ ]	4.0
Horizontal acceleration [ $\text{m s}^{-2}$ ]	2.0
Maximal ascending velocity [ $\text{m s}^{-1}$ ]	2.0
Vertical ascending acceleration [ $\text{m s}^{-2}$ ]	1.0
Maximal descending velocity [ $\text{m s}^{-1}$ ]	2.0
Vertical descending acceleration [ $\text{m s}^{-2}$ ]	1.0

Table 2.1: Motion constraints of the UAV.

## ■ Simulation Scenarios

To evaluate the safety and performance of the UAVs during interactions, a series of experiments was conducted. Experiments were performed primarily using circular formations,

with UAV counts of  $N = 5$ ,  $10$ , and  $15$ . Each of these circular formation experiments was repeated  $10$  times. Additionally, to extend the analysis to  $3D$  and assess the algorithm's performance, a set of spherical formation experiments was conducted with  $N = 10$ , also repeated  $10$  times. After each UAV was flown from the ground to its designated starting position, the RBL algorithm was initiated.

These formations provide a controlled environment compared to random initial position and goal locations. In both circular and spherical formations, the UAVs were evenly distributed along the circle or the surface of the sphere, with the goal position located on the opposite side. The radius of both circle and the sphere was set to  $5$  meters.

The following metrics were measured across the  $10$  repetitions of each experiment, along with their standard deviations: success rate  $SR$  [%], average trajectory length  $\bar{L}$  [m], average time to reach the goal  $\bar{t}$  [s], average of the maximal time for UAV to reach the goal  $\bar{t}_{\max}$  [s], and average velocity  $\bar{v}$  [m/s].

The specific values of the parameters used in the experiments are summarized in following table 2.2:

Table 2.2: Parameters Used in Experiments

Parameter	Value
Sensing radius ( $r_s$ )	3.5 m
Update rate	10 Hz
Encumbrance	0.5 m
$d_1 = d_3 = d_5$	0.5 m
$d_2 = d_4 = d_6$	1.0 m
$d_7$	0.2
$\beta_i^D$	1.5
$\eta$	0.9
$min_z$	1.0 m
$max_z$	10.0 m

## ■ Experimental Results

### ■ $N = 5$ Circular Formation:

	$SR$ [%]	$\bar{L}$ [m]	$\bar{t}$ [s]	$\bar{t}_{\max}$ [s]	$\bar{v}$ [m/s]
RBL 2D	100.00	$21.06 \pm 0.10$	$25.15 \pm 0.21$	$25.15 \pm 0.19$	$0.83 \pm 0.01$
RBL 3D	100.00	$20.77 \pm 0.29$	$26.04 \pm 0.51$	$26.79 \pm 0.27$	$0.79 \pm 0.02$
RBL 3D <sub>clipped</sub>	100.00	$20.60 \pm 0.24$	$26.73 \pm 0.47$	$27.39 \pm 0.28$	$0.77 \pm 0.02$
RBL 3D <sub>z</sub>	100.00	$20.97 \pm 0.52$	$25.54 \pm 0.97$	$26.72 \pm 0.60$	$0.81 \pm 0.03$

### ■ $N = 10$ Circular Formation:

	$SR$ [%]	$\bar{L}$ [m]	$\bar{t}$ [s]	$\bar{t}_{\max}$ [s]	$\bar{v}$ [m/s]
RBL 2D	100.00	$22.95 \pm 1.64$	$30.79 \pm 2.28$	$34.73 \pm 0.77$	$0.74 \pm 0.07$
RBL 3D	100.00	$22.22 \pm 0.89$	$30.05 \pm 2.61$	$34.39 \pm 4.19$	$0.73 \pm 0.05$
RBL 3D <sub>clipped</sub>	100.00	$22.31 \pm 0.69$	$30.22 \pm 1.83$	$33.22 \pm 0.93$	$0.73 \pm 0.04$
RBL 3D <sub>z</sub>	100.00	$22.38 \pm 0.88$	$28.80 \pm 2.30$	$32.35 \pm 1.25$	$0.77 \pm 0.05$

- **$N = 15$  Circular Formation:**

	$SR$ [%]	$\bar{L}$ [m]	$\bar{t}$ [s]	$\bar{t}_{\max}$ [s]	$\bar{v}$ [m/s]
RBL 2D	100.00	$23.56 \pm 1.75$	$33.89 \pm 2.48$	$38.75 \pm 1.45$	$0.69 \pm 0.06$
RBL 3D	100.00	$22.36 \pm 0.97$	$30.65 \pm 3.07$	$35.77 \pm 3.72$	$0.72 \pm 0.07$
RBL 3D <sub>clipped</sub>	100.00	$22.32 \pm 0.81$	$30.69 \pm 2.61$	$34.50 \pm 0.47$	$0.72 \pm 0.06$
RBL 3D <sub>z</sub>	100.00	$22.64 \pm 0.95$	$29.67 \pm 2.54$	$34.27 \pm 0.88$	$0.76 \pm 0.06$

- **$N = 10$  Spherical Formation:**

	$SR$ [%]	$\bar{L}$ [m]	$\bar{t}$ [s]	$\bar{t}_{\max}$ [s]	$\bar{v}$ [m/s]
RBL 3D	100.00	$14.32 \pm 1.52$	$27.76 \pm 3.06$	$32.48 \pm 2.26$	$0.51 \pm 0.05$
RBL 3D <sub>z</sub>	100.00	$14.06 \pm 0.97$	$27.17 \pm 2.66$	$31.86 \pm 1.24$	$0.55 \pm 0.06$

The figure below visualizes the UAV trajectories, showing the effect of the applied simulation rules.

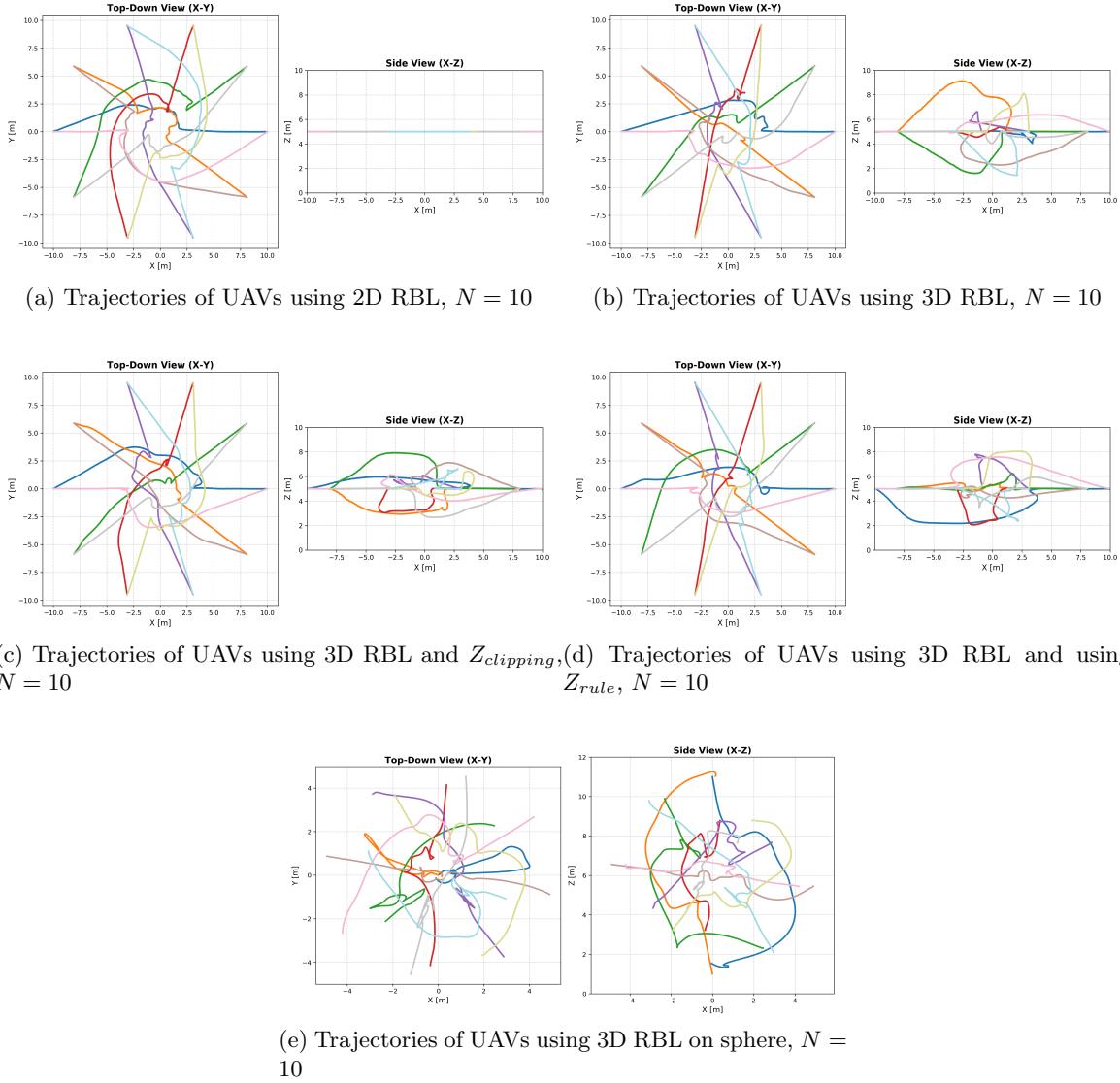


Figure 2.3: Visualization of UAV formations and algorithm behavior for  $N = 10$  (shown with top-down and side views). (a) Trajectories in a 2D circular crossing. (b) Trajectories in a 3D circular crossing. (c) Effect of Z-axis clipping on the circular crossing. (d) Effect of the Z-axis rule on the circular crossing. (e) Trajectories in a 3D spherical crossing.

## 2.5 Comparison with State-of-the-Art Method

As an alternative approach developed within the MRS framework, the 3D RBL algorithm presented here can be compared to other state-of-the-art methods like PACNav. todo cite Both algorithms target communication-less UAV coordination, a feature desirable for scalability. Furthermore, implementations described for both approaches have utilized UVDAR to determine the positions of neighbouring UAVs.

A key requirement for both is some form of self-position estimation. PACNav achieves this in GNSS-denied settings using SLAM techniques. Similarly, RBL can also operate without GNSS provided a robust odometry source like SLAM is available to determine the UAV's own position within a consistent frame.

Where the approaches differ notably is in their coordination strategy. PACNav utilizes a leader-follower dynamic: 'uninformed' UAVs identify and follow 'informed' (or otherwise reliable) leaders based on observed motion characteristics like path persistence and similarity, encouraging group cohesion. In contrast, the RBL algorithm assigns an individual goal to each UAV. Coordination emerges as each agent calculates its path based on its local modified Voronoi cell, moving towards its computed centroid while implicitly accounting for neighbours.

Regarding validation, PACNav has demonstrated performance through real-world experiments, including navigation in a forest environment. Validation of the multi-agent 3D RBL algorithm was conducted through simulation. The integration and experimental validation of 3D LiDAR for environmental perception and single-agent navigation within the RBL framework are addressed in the subsequent chapter.

Another distinct approach in decentralized, communication-less coordination is the purely vision-based model presented by Mezey et al.. TODO cite Unlike RBL, which relies on explicit position information, the Mezey et al. model derives control actions solely from a 1D visual projection field generated by detecting neighbours via onboard camera. It requires no localization data. Coordination emerges from low-level visual attraction-repulsion forces, resulting in patterns like flocking or swarming rather than directed motion towards individual goals. While effectively demonstrated in 2D with UGVs, extending this purely vision-based model to complex 3D environments presents challenges regarding goal achievement. Furthermore, the use of toroidal simulation environments, which mitigate group fragmentation and can facilitate specific emergent patterns like leader-follower dynamics, potentially limits the direct applicability of some observed behaviors to real-world 3D scenarios. The 3D RBL, by utilizing explicit position information, aims for more predictable goal-oriented navigation in 3D, although with different sensing requirements compared to the purely reactive, vision-only model of Mezey et al.

## ■ 2.6 Summary and Key Insights

This chapter detailed the extension of RBL algorithm, originally tested for 2D coordination, to operate in a full three-dimensional space. Key modification included adapting The Voronoi cell partitioning for 3D, utilizing projections for already existing azimuth update rules, and introducing a new elevation rotation angle. This new rule dynamically adjusts the agent's vertical destination based on relative centroid positions and directional influence towards the goal, aiming to improve distribution. Note that for the directional influence to work the UAVs need to have some sort of information about frame of reference between each other at the start of the algorithm. TODO rephrase Vertical movement has also been constrained within defined interval ( $\min_z$ ,  $\max_z$ ).

The proposed 3D RBL extensions were evaluated through simulations within the MRS framework, comparing performance of UAVs crossing circle and sphere. The results indicate that the extension variants successfully enabled multi-agent coordination in three dimensions. Specifically, analysis of the simulation results reveals a trend where the performance of the 3D approach with applied rules compared to the basic 2D appears to increase with scaling number of agents. For instance, based on the average completion times, the 3D variant was calculated to be approximately 1.99% faster than the 2D variant for N=5 agents. This performance scales for larger swarms. For N=15 the 3D approach was calculated to be 4.22% faster than the 2D. This suggests that the extension becomes increasingly beneficial in more crowded scenarios.

While the magnitude of this observed improvement was perhaps less than initially anticipated, it is important to consider the conservative motion constraints for UAVs on the Z-axis (velocity/acceleration), reflecting realistic UAV dynamics but limiting vertical exploration. Furthermore, the parameters used for testing simulation were also conservative, suggesting that finer tuning - particularly of vertical exploration weights and weighting function aggressiveness - could potentially yield better performance. Nonetheless, the trend indicated by these results supports the conclusion that the 3D extension effectiveness crowedness rises. Furthermore, simulations in the spherical formation scenario confirmed the algorithm's capability to manage 3D navigation and coordination, with the Z-rule again showing performance benefits.

In conclusion, the simulations presented in this chapter demonstrate the feasibility of extending the RBL algorithm to 3D. The introduced elevation control rule appears particularly beneficial, enhancing the efficiency and convergence speed of the agents towards their goals in simulated 3D environments. This provides a promising foundation for applying the 3D RBL algorithm to real-world UAV navigation tasks, explored further in subsequent chapters.

## ■ 3 UAV Navigation Using the RBL Algorithm and LiDAR Sensing

### ■ 3.1 Introduction

#### ■ Motivation

UAVs are increasingly being deployed in challenging, unstructured environments like dense forests, moving beyond their traditional use in open areas. This shift is driven by a growing demand for autonomous solutions in sectors like environmental monitoring, forestry management (health assessment [7]), and search and rescue, where ground-based access is difficult. Furthermore, this work aligns with the broader trend of deploying UAVs for increasingly complex tasks, such as detailed infrastructure inspection (e.g., power lines, where autonomous navigation in cluttered, potentially high Electromagnetic Interference (EMI) environments is essential (Fly4Future [15])). Enabling UAVs to reliably navigate point-to-point within these challenging settings is a foundational step towards realizing these advanced applications safely and efficiently.

While the primary objective of this work is to enable successful point-to-point navigation within a forest using the RBL algorithm, a significant secondary benefit emerges from this process. By successfully navigating from point A to point B while simultaneously building a map of the traversed environment, the UAV generates valuable spatial data about the forest structure. This autonomously generated map can then serve a vital purpose: enabling enhanced planning for future missions within the same area. Once a map exists, subsequent UAV operations could potentially transition from purely reactive navigation strategies to more efficient, globally informed path planning algorithms such as TODO find some. Leveraging prior knowledge of obstacle locations could significantly improve the safety and efficiency of future routine tasks.

#### ■ Problem Statement

Operating UAVs effectively within complex, three-dimensional environments like forests presents significant navigational challenges that hinder widespread autonomous deployment. The primary problems addressed in this work stem from:

(i) **Global Navigation Satellite System (GNSS)-Denied Conditions:**

Within forests, the dense canopy and other obstructions frequently block or scatter GNSS signals, leading to unreliable or completely absent reception. This necessitates reliance on onboard sensors and algorithms (like SLAM based on Light Detection and Ranging (LiDAR) and Inertial Measurement Unit (IMU) data) for accurate localization and state estimation.

(ii) **Cluttered and Unpredictable Environments:**

Forests are inherently cluttered with numerous static obstacles (trees, trunks, branches) and potentially dynamic ones (animals, leaves, falling branches). The navigation system must be capable of perceiving these obstacles in real-time and planning safe paths around them without prior knowledge of their exact layout.

Therefore, the core problem is to develop and validate a robust autonomous navigation system that allows a UAV to reliably traverse between specified points in a cluttered, GNSS-denied

forest environment using only onboard sensing and computation.

## ■ Objectives

The primary objectives within this chapter are:

### ■ Integrate the RBL with Onboard Sensing:

Adapt the core of the RBL to utilize real-time sensor data. This includes modification of sensing cell  $\mathcal{S}$ , because used LiDAR doesn't see in all directions. Sensing cell needs to remain convex so the centroid computed from partitioned cell  $\mathcal{A}$  is never computed outside of the set - for safety convergence towards the goal.

### ■ Process Point Cloud Data:

Implement necessary filtration to refine the raw point cloud data acquired from the LiDAR.

### ■ Integrate external packages on the UAV:

Integrate and configure external software packages for simultaneous environmental mapping and robust state estimation, suitable for operation within the GNSS-denied forest environment.

### ■ Experimental Validation:

Conduct experiments to evaluate the performance, robustness, and effectiveness of the complete navigation solution. This validation will be performed through real-world flight tests in an actual forest.

## ■ Chapter Overview

TODO Summary of what this chapter covers

### ■ 3.2 LiDAR-Based Perception and Point Cloud Processing

#### ■ Overview of LiDAR for UAV Navigation

LiDAR is a crucial sensing technology widely used in applications such as SLAM [17], autonomous vehicles [13], UAVs TODOcite, and precision agriculture [4]. It provides high-resolution spatial data about the surrounding environment, making it a valuable tool for perception and navigation in dynamic and complex environments. For UAV applications, LiDAR serves several essential functions:

- **3D Mapping** – Capturing a detailed representation of terrain, structures, and obstacles.
- **Obstacle Detection** – Identifying objects and estimating their position relative to the UAV for collision avoidance.
- **Autonomous Path Planning** – Assisting navigation algorithms by providing spatial information for decision-making.
- **Terrain Following** – Helping the UAV maintain a safe altitude by detecting variations in ground elevation.

LiDAR offers several benefits that make it an attractive choice for UAV-based navigation:

- **High Accuracy** – Provides precise distance measurements, crucial for obstacle avoidance and localization.
- **Environment Agnostic** – Functions effectively in various conditions, including low-light environments and featureless terrain where cameras may fail.

- **Fast Data Acquisition** – Captures thousands to millions of points per second, enabling real-time processing.
  - **Rich Depth Information** – Unlike cameras that provide only 2D images, LiDAR generates accurate depth data, improving spatial awareness and 3D perception.
- Despite its advantages, LiDAR also presents certain challenges and limitations:
- **Computational Complexity** – Processing large point clouds in real-time requires significant computational power, which may be a limitation for UAVs with low processing resources.
  - **Sensor Noise and Artifacts** – External factors such as vibrations and motion of UAV can introduce errors in point cloud data.
  - **Limited Field of View (FoV)** – The placement of the LiDAR sensor on the UAV affects its coverage, requiring strategies to compensate for blind spots.
  - **Environmental Interference** – Performance may degrade in challenging conditions such as fog, rain, or dense vegetation due light deviation.
  - **Power Consumption** – LiDAR sensors can consume a significant amount of power, which reduces the overall flight time of the UAV.
  - **Interference with Other LiDARs** – LiDAR sensors can experience interference when multiple units are used nearby, potentially leading to faulty measurements.

#### ■ Point Cloud Data Acquisition

LiDAR systems determine object distances by emitting laser pulses and measuring the time it takes for the reflected light to return. This process, known as Time-of-Flight (ToF), involves scanning the environment with laser beams directed at varying horizontal and vertical angles. The reflected light, modulated in intensity, phase, or frequency, is captured by a receiver, which uses a lens to focus the signal onto a photodetector. This detector converts the light into an electrical signal via the photoelectric effect [10].

The system calculates distance based on the light's travel time, considering its near-light-speed propagation. To distinguish transmitted from received signals, the laser's wavelength is often adjusted. Subsequent signal processing filters and analyzes the electrical signal, accounting for surface material and environmental variations. The output is a 3D point cloud representing the scanned environment, along with reflected laser energy intensities. All these data points are stored in a ROS message of type *sensor\_msgs :: PointCloud2*

#### ■ Preprocessing Techniques

To efficiently process LiDAR data and reduce computational complexity, the raw point cloud undergoes downsampling and filtering. The point cloud density is reduced using a voxel grid filter. Subsequently, points associated with the UAV's structure are removed based on its known encumbrance.

- **Voxel Grid DownSampling** – The raw LiDAR point cloud often contains a large number of points, which can be computationally expensive to process in real-time. To address this, we apply a voxel grid filter using the Point Cloud Library (PCL) [16]. This method partitions the 3D space into a grid of voxels with a given resolution (*leafSize*) and retains a single representative point per voxel. The filtering process reduces the number of points while preserving the overall structure of the environment.
- **Filtering Points Corresponding to the UAV Structure** – LiDAR sensors mounted on UAVs can capture unwanted points originating from the UAV itself, such as reflections

from its frame or rotor rods. To prevent these points from interfering with navigation, additional filtering step has been applied. Points falling outside a specified distance range (closer than a minimum threshold or farther than a maximum threshold) are filtered out.

The resulting filtered point cloud contains only relevant environmental features while eliminating unnecessary points, improving efficiency for future processes. The point cloud is then fed into Point lio state estimator and bonxai mapping. TODO get access to repo.

## ■ Bonxai Mapping

As will be shown later in this chapter, prior terrain knowledge is beneficial for navigation. For this purpose, environmental mapping was accomplished using a simple package developed by the MRS group, implementing the Bonxai mapping approach. The resulting map is represented as a voxel grid, which is then used by the RBL algorithm for navigation planning.

Initially, an approach involving surface reconstruction from this voxel data was investigated. This involved estimating surface normals from the point cloud combined with Greedy Projection Triangulation (GP3) method from the PCL library to generate polygonal mesh approximating the environment's surface. While this method could produce relatively accurate surface representation, it turned out to be computationally expensive and too slow for real-time execution on the UAV's onboard computer.

Due to these limitations, the surface reconstruction approach was abandoned. Instead a simpler and more efficient method was adopted, which involves directly using the information about the environment stored in voxel grid map. This direct voxel usage approach is detailed in the subsequent section. Consequently, further development of the mesh-based surface reconstruction method is not recommended for this application.

## ■ Voxel-Based Modification of Cell $\mathcal{A}$

Given that the size of the voxels is known, this information can be used to refine the partitioning of cell  $\mathcal{A}$  by treating voxels as obstacles. First, it is necessary to determine which voxels are relevant to the RBL algorithm. This can be achieved by considering the scaling parameter  $\eta$ . A voxel is considered relevant if it lies within radius of  $\frac{r_s}{\eta}$  of the agent. For each discrete point in the set composing cell  $\mathcal{S}$ , the nearest voxel is identified using k-d tree algorithm from the PCL library. Because the found point is the voxel's center, the closest point within that voxel's boundaries is calculated to accurately partition the cell  $\mathcal{A}$  from cell  $\mathcal{S}$ .

Given a point  $\mathbf{p}_S$ , the voxel center  $\mathbf{v}_c$  and the voxel edge length  $\mathbf{e}$ ,  $\mathbf{p}_{closest}$ , is computed as:

$$\mathbf{p}_{closest} = \begin{pmatrix} \text{clamp}(p_S, x, v_{c,x} - \frac{\mathbf{e}}{2}, v_{c,x} + \frac{\mathbf{e}}{2}) \\ \text{clamp}(p_S, y, v_{c,y} - \frac{\mathbf{e}}{2}, v_{c,y} + \frac{\mathbf{e}}{2}) \\ \text{clamp}(p_S, z, v_{c,z} - \frac{\mathbf{e}}{2}, v_{c,z} + \frac{\mathbf{e}}{2}) \end{pmatrix} \quad (3.1)$$

where function  $\text{clamp}(x, a, b)$  constrains the value  $x$  to the range  $[a, b]$ . This constrains  $\mathbf{p}_S$  to the voxel boundaries.

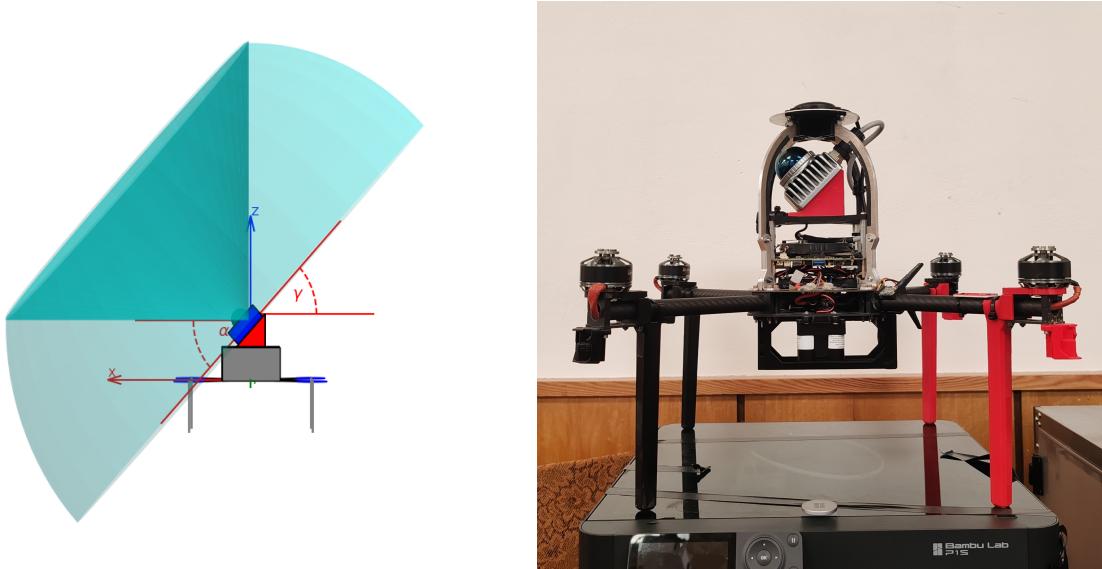
A point is excluded from the set if  $\|\mathbf{p}_i - \mathbf{p}_S\| \leq \eta \cdot \|\mathbf{p}_i - \mathbf{p}_{closest}\|$ .

### ■ 3.3 Implementation and Integration on UAV

#### ■ Challenges in Integration

The algorithm's performance is influenced by the limitations of the LiDAR sensor used for environmental sensing. While the algorithm functions optimally with a full 360° horizontal and 180° vertical Field of View (FOV), which would require multiple sensors, the experiments used a single Livox Mid 360 LiDAR [6]. This LiDAR provides a 360° horizontal FOV and only a 59° vertical FOV, resulting in a sensing blind spot.

To solve this limitation, several modifications were implemented. The LiDAR was mounted at an angle  $\gamma$ , as shown in Figure Fig. 3.1, to enhance ground sensing and increase forward visibility.



(a) UAV model with visualized LiDAR mounting parameters.  
(b) UAV used in the experiment with mounted LiDAR.

Figure 3.1: UAV and LiDAR mounting scheme. Subfigure (a) shows a modeled UAV with visualized LiDAR mounting parameters, including the LiDAR elevation field of view  $\alpha$  and tilt angle  $\gamma$ . Subfigure (b) displays the real UAV used in the experiment with the LiDAR mounted according to the same configuration.

In addition to the LiDAR mounting angle, a software modification is implemented to account for the LiDAR's limited FOV. The algorithm uses map to partition cell  $\mathcal{S}$  into cell  $\mathcal{A}$ , from which the centroid is computed to guide UAV movement. To ensure safety, UAV should only move only within their visible FOV. However, map information from areas outside the current FOV can still be valuable.

Therefore, a constraint is introduced - the UAV maintains its yaw rotation towards the current centroid  $\mathbf{c}_A$  and moves towards it only if the centroid lies within a defined angular range in front of the UAV.

To incorporate the LiDAR's FOV into cell  $\mathcal{S}$ , two planes are defined based on the LiDAR's mounting configuration and vertical FOV. Let  $\mathbf{e}_z$  be unit vector along the z-axis,  $R_{off} \in SO(3)$  the offset rotation, and  $R_{rpy} \in SO(3)$  the roll-pitch-yaw rotation. The normal

vectors,  $\mathbf{n}_1$  and  $\mathbf{n}_2$  defining two planes are given by:

$$\mathbf{n}_1 = R_{rpy} \cdot R_y(\gamma) \cdot \mathbf{e}_z \quad (3.2)$$

$$\mathbf{n}_2 = R_{rpy} \cdot R_y(\gamma - \alpha) \cdot \mathbf{e}_z \quad (3.3)$$

where  $\alpha$  is the LiDAR's FOV and  $\gamma$  is the the mounting angle of the LiDAR.

Cell  $\mathcal{S}$  is then modified by excluding points that lie outside the LiDAR's FOV, defined by these two planes:

$$\mathcal{S}'_i = \{\mathbf{q} \in \mathcal{S}_i \mid \mathbf{n}_1 \cdot (\mathbf{q} - \mathbf{p}_{LiDAR}) \geq 0 \wedge \mathbf{n}_2 \cdot (\mathbf{q} - \mathbf{p}_{LiDAR}) \leq 0\} \quad (3.4)$$

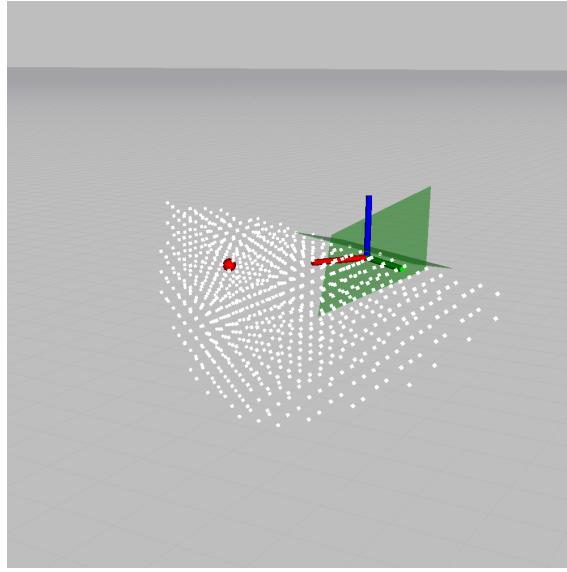
where  $\mathcal{S}'_i$  is the modified cell  $\mathcal{S}$  and  $\mathbf{p}_{LiDAR}$  is the exact position of the LiDAR sensor.

This modification effectively restricts the points considered in the calculation of the centroid to only those within the LiDAR's FOV.

To incorporate map information from the surrounding area outside of the LiDAR's current FOV, the following procedure is used. Firstly, both cells  $\mathcal{S}_i$  and  $\mathcal{S}'_i$  are created. These cells are partitioned into cells  $\mathcal{A}_i$  and  $\mathcal{A}'_i$  using voxels. Cell  $\mathcal{A}'_i$  is partitioned using voxels taht are actively sensed, while cell  $\mathcal{A}_i$  is partitioned using the voxels from the whole surrounding.

If the centroid computed from cell  $\mathcal{A}_i$ ,  $\mathbf{c}_{\mathcal{A}_i}$ , is within the cell  $\mathcal{A}'_i$ , the UAV is commanded to move towards  $\mathbf{c}_{\mathcal{A}_i}$  while simu rotating its yaw towards it. However, if  $\mathbf{c}_{\mathcal{A}_i}$  lies outside cell  $\mathcal{A}'_i$ , it is projected onto the closest point on the boundary of cell  $\mathcal{A}'_i$ . As the UAV only moves if the centroid is within a certain angle in front of it, this projection onto the boundary of  $\mathcal{A}'_i$  ensures that the UAV primarily rotates towards the centroid until the centorid is within an angle in front of the UAV.

This procedure effectively utilizes the information fro mthe map while ensuring that the UAV moves only within its actively sensed FOV.



(a) Visualization of planes defining cell  $\mathcal{S}'_i$

Figure 3.2: RViz visualization of LiDAR field-of-view (FOV) planes. The UAV is represented by the XYZ coordinate axes. The green squares represent the two planes, defined by normal vectors  $\mathbf{n}_1$  and  $\mathbf{n}_2$ , used to slice cell  $\mathcal{S}_i$  and determine the modified cell  $\mathcal{S}'_i$ . The red sphere represents the centroid  $\mathbf{c}_{\mathcal{A}_i}$ .

■ **3.4 Experimental Results**

TODO

■ **Simulation Setup**

virtual environment modeling, tools used

■ **Performance in Simulated Environments**

Success rates, obstacle avoidance, efficiency metrics - same as in the rbl

■ **Real-World Experiments in a Forest Environment**

■ **Comparative Analysis**

Simulation vs. real-world performance

■ **3.5 Conclusion**

TODO Summary of contributions and some directions for future research

## ■ 4 Researching on possible enhancements of the algorithm for example using neural networks or learning-based techniques.

tune the parameters of RBL to suit 3D

I will have to read something first, but what I found is that I could try object detection and classification using models like [PointNet](#), [PointNet++](#) or [VoxelNet](#)

Pointcloud denoising

Segmentation of LiDAR data. [RangeNet++](#), [SalsaNext](#) to differentiate between terrain trees and free space

NN approaches for approximating point clouds with simple convex shapes:

[CvxNet: Learnable Convex Decomposition by Deng et al. \(CVPR 2020\)](#)

[Label-Efficient Learning on Point Clouds using Approximate Convex Decompositions by Gadelha et al. \(Arxiv 2020\)](#)

[Region Segmentation via Deep Learning and Convex Optimization by Sonntag and Morgenshtern \(Arxiv 2019\)](#)

[Point Density-Aware Voxels for LiDAR 3D Object Detection](#)

## ■ 5 Conclusion

Summarize the achieved results. Can be similar as an abstract or an introduction, however, it should be written in past tense.

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## ■ A Appendix A