

## Active Force Control of 3-RRR Planar Parallel Manipulator

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**Abstract**— This paper presents a new and novel method to control a 3-RRR (revolute-revolute-revolute) planar parallel manipulator using an active force control (AFC) strategy. A traditional proportional-integral-derivative (PID) controller was first designed and developed to demonstrate the basic and stable response of the manipulator in performing trajectory tracking tasks. Later, the AFC section was incorporated into the control scheme in cascade form by adding it in series with the PID controller (PID+AFC), its primary aim of which is to improve the overall system dynamic performance particularly when the manipulator is subjected to different loading conditions. Results clearly illustrate the robustness and effectiveness of the proposed AFC-based scheme in rejecting the disturbances compared to the traditional PID controller.

**Keywords**- 3-RRR parallel manipulator; active force control; trajectory tracking

### I. INTRODUCTION

In recent years, new types of robots have been proposed called parallel robots in order to do certain tasks that serial robots are not capable to perform. Although serial robots have advantages of easy dynamic and kinematic models, large workspace but, parallel robots have high stiffness, high speed and large load carrying capacity. The most important advantage of parallel robots compared to serial robot is the possibility of fixing actuators on the base, which can be used for fast and accurate operations.

Unlike serial robots, the literatures on control of parallel manipulators are quite limited [1]. Ghorbel showed that the methods used for the control of serial robots can be applied to parallel manipulators as well [2]. Amirat *et al.* applied a classic proportional-integral-derivative (PID) controller to a six degree-of-freedom (DOF) parallel robot with C3 links related to equestrian gait simulation [3]. Walker worked on adaptive control of manipulators containing closed kinematic loops [4]. He presented a robust independent joint control scheme that is implemented to a high-speed parallel robot. He showed that the common independent joint

control methods such as the PID controller in industrial robot manipulators cannot achieve satisfactory performance because of their inherent low rejection to disturbances and parameter variations. Honegger *et al.* introduced an adaptive control method on a new 6 DOF parallel manipulator intended to be used as a high speed milling machine [5]. They showed that one of the advantages of the parallel robots over the serial counterparts, is the possibility of former to keep the motors fixed to the base. This feature typically makes use of light links which are capable to perform fast movements. Tao *et al.* suggested an adaptive robust posture control on a pneumatic muscle-driven parallel manipulator with actuation redundancy [6].

In this paper, a new feedback control employing an active force control (AFC) strategy in conjunction with a classic PID controller (PID+AFC) was applied to a 3-RRR (revolute-revolute-revolute) three DOF planar parallel manipulator to improve performance of the system in presence of disturbances. The robustness and effectiveness of the AFC strategy as a ‘disturbance rejector’ is presented through simulation using MATLAB/Simulink software package. The results show that the traditional PID controller is very useful and common in general industrial operations and applications in the absence of any known or unknown disturbances, but AFC controller is proven to be more robust and powerful than PID alone in the presence of known and unknown disturbances [7, 8, 9].

### II. KINEMATICS OF 3-RRR PARALLEL MANIPULATOR

#### A. Inverse Kinematics

A three DOF planar parallel manipulator is shown in Fig. 1. The manipulator has nine revolute joints, i.e., three actuated joints fixed to the base and six unactuated joints that form three closed kinematic chains. The triangular plate in the middle of the figure is supposed to be the end-effector. The manipulator is symmetric and the corresponding length of each leg is the same.

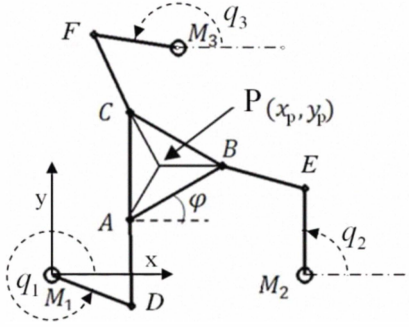


Figure 1. General form of a planar three DOF parallel manipulator [10]

Inverse and direct kinematic analyses of the manipulator have been derived and presented in [10]. In inverse kinematic problem, the active joint angles can be obtained from the end-effector position and orientation. A general solution of the inverse kinematic for leg  $i$ , is shown as follows:

$$\theta_i = \alpha_i = \psi_i \quad i = 1, 2, 3 \quad (1)$$

$$\alpha_i = \text{atan2}(x_{2i}, y_{2i}) \quad (2)$$

$\psi_i$  can be obtained from the following equation:

$$\psi_i = \cos^{-1} \left[ \frac{l_1^2 - l_2^2 + x_{2i}^2 + y_{2i}^2}{2l_1 \sqrt{x_{2i}^2 + y_{2i}^2}} \right] \quad (3)$$

Coordinates  $x_{2i}$  and  $y_{2i}$  are defined as:

$$x_{2i} = x - l_3 \cos \phi_i - x_{oi} \quad (4)$$

$$y_{2i} = y - l_3 \sin \phi_i - y_{oi} \quad (5)$$

Where,

$$\phi_1 = \phi + \frac{\pi}{6}, \phi_2 = \phi + \frac{5\pi}{6}, \phi_3 = \phi - \frac{\pi}{2} \quad (6)$$

The Cartesian positions of the motors are considered as follows:

$$x_{oi} = \{0, 1, \frac{1}{2}\}, y_{oi} = \{0, 0, \frac{\sqrt{3}}{2}\} \quad (7)$$

Solving the inverse kinematics is very useful since the robot tasks are commonly formulated in terms of end-effector's specified position and motion.

#### B. Direct Kinematics

In direct kinematic problem, with given position of joint angles, the position and direction of end effector can be obtained. Gosselin illustrated that the solution of the direct kinematic problem of three DOF planar parallel manipulator

leads to a maximum of six possible results [10]. The following equations can be used to solve the direct kinematics using numerical methods.

$$x_c = x_D + l_2 \cos(\alpha_1 + \psi) + \sqrt{3}l_3 \cos(\alpha_1 + \alpha_2 + \theta) \quad (8)$$

$$y_c = y_D + l_2 \sin(\alpha_1 + \psi) + \sqrt{3}l_3 \sin(\alpha_1 + \alpha_2 + \theta) \quad (9)$$

and

$$\alpha_1 = \text{atan2} \left[ \frac{y_E - y_D}{x_E - x_D} \right], \alpha_2 = \frac{\pi}{3} \quad (10)$$

$$\theta_{1,2} = 2 \tan^{-1} \left[ \frac{B \pm \sqrt{B^2 - AC}}{A} \right] \quad (11)$$

A, B, C are given as follows:

$$A = \frac{-d^2 - 3l_3^2}{2\sqrt{3}l_2l_3} - \frac{d}{l_2} + \left(1 + \frac{d}{\sqrt{3}l_3}\right) \cos \psi \quad (12)$$

$$B = \sin \psi \quad (13)$$

$$C = \frac{-d^2 - 3l_3^2}{2\sqrt{3}l_2l_3} + \frac{d}{l_2} + \left(\frac{d}{\sqrt{3}l_3} - 1\right) \cos \psi \quad (14)$$

Where

$$d = \sqrt{(x_E - x_D)^2 + (y_E - y_D)^2} \quad (15)$$

Therefore the following nonlinear equation can be solved by:

$$(x_c - x_F)^2 + (y_c - y_F)^2 = l_2^2 \quad (16)$$

Fig. 2 shows the equivalent four bar linkage that can be used to solve the direct kinematics problem of the manipulator.

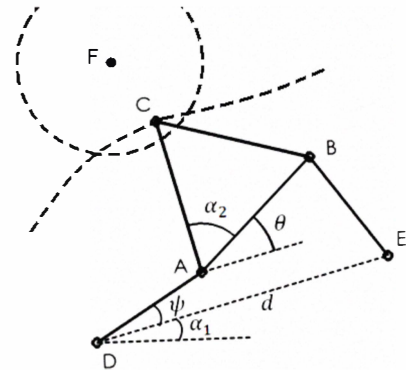


Figure 2. Equivalent four bar linkage [10]

The details of the direct kinematic analysis can be found in [10].

### III. DYNAMICS OF 3-RRR PARALLEL MANIPULATOR

In this paper, Natural Orthogonal Complement (NOC) method is used to solve the dynamics of the 3-RRR robot [11]. From the power equations of all links using natural orthogonal complement of the constraint equations, a set of *Euler-Lagrange* equations can be derived. More detailed description of the NOC method can be found in [12] which is applied to a class of parallel manipulator. To control the manipulator, direct dynamics of the system is modeled and simulated in order to predict the motion of the manipulator, given the driving forces of the system. A general form of the manipulator dynamic is expressed as follows:

$$\mathbf{T}^T \mathbf{M}_{\text{total}} \mathbf{T} \ddot{\mathbf{q}} + (\mathbf{T}^T \mathbf{M}_{\text{total}} \dot{\mathbf{T}} + \mathbf{T}^T \Omega \mathbf{M}_{\text{total}} \mathbf{T}) \dot{\mathbf{q}}^a - \mathbf{T}^T \mathbf{W}^g = \boldsymbol{\tau}^a \quad (17)$$

Where

$$\mathbf{W}^g = \begin{bmatrix} 0 \\ m_1 g \\ 0 \\ m_2 g \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ m_r g \end{bmatrix} \quad (18)$$

$$\mathbf{M}_{\text{total}} = \text{diag}(M_1, M_2, \dots, M_r) \quad (19)$$

$$\Omega = \text{diag}(\Omega_1, \Omega_2, \dots, \Omega_r) \quad (20)$$

$$\mathbf{M}_i = \begin{bmatrix} I_i & \mathbf{0} \\ \mathbf{0} & m_i \times \mathbf{1} \end{bmatrix}, \Omega_i = \begin{bmatrix} \omega_i \times \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (21)$$

The  $\mathbf{0}$  and  $\mathbf{1}$  in Eq. (19) represent the  $(3 \times 3)$  zero matrix and  $(3 \times 3)$  identity matrix, respectively.  $m_i$  and  $\omega_i$  are the mass and angular velocity of each link, respectively and  $\mathbf{T}$  is the NOC matrix. Equation (16) could be simplified by considering the following expressions:

$$\mathbf{M} = \mathbf{M}(q) = \mathbf{T}^T \mathbf{M}_{\text{total}} \mathbf{T}$$

$$\mathbf{C} = \mathbf{C}(q, \dot{q}) = \mathbf{T}^T \mathbf{M}_{\text{total}} \dot{\mathbf{T}} + \mathbf{T}^T \Omega \mathbf{M}_{\text{total}} \mathbf{T} \quad (22)$$

$$\mathbf{G} = \mathbf{G}(q) = -\mathbf{T}^T \mathbf{W}^g$$

Therefore the equation becomes:

$$\mathbf{M} \ddot{\mathbf{q}}^a + \mathbf{C} \dot{\mathbf{q}}^a + \mathbf{G} = \boldsymbol{\tau}^a \quad (23)$$

Where  $q^a, \dot{q}^a, \ddot{q}^a$  are corresponding displacement, velocity and acceleration of actuated joints, respectively.

### IV. PID CONTROL

In proportional-integral-derivative (PID) controller, the signal produced by the controller is proportional to the error sensed by a sensor. Classic PID controller is most commonly used method in industry and do not require the specific knowledge of the robot's model. In PID controller, in order to have a satisfactory controller, the main problem is how to tune the parameters of PID controller to obtain a desired performance.

A general form of a classic PID controller is shown as follows:

$$u_i = K_p e_i + K_i \int e_i dt + K_d \frac{de_i}{dt} \quad (24)$$

Where  $K_p, K_i$  and  $K_d$  are the PID gains related to  $i^{th}$  actuator.

### V. ACTIVE FORCE CONTROL

Active force control (AFC) method was first proposed by Hewit and Burdett in the early eighties [7]. They presented the application of the AFC technique to a robot arm in the presence of known or unknown disturbances. They showed that by using this method, the system remains stable and robust even in presence of noises, parametric changes, uncertainties and disturbances. The effectiveness and robustness of AFC method as a disturbance rejector scheme compared to the traditional control methods such as the PID controller is proven in the literature [8, 9]. AFC method applied to a two link serial robot was demonstrated in [8]. The research involves the application of AFC and its robustness on a robot in presence of different types of disturbances through a specified trajectory. Two different intelligent methods, namely, neural network and iterative learning algorithms were used to estimate the unknown parameters in the AFC loop. From *Newton* second law of motion for rotating bodies, the sum of all torques applied to the system is equal to the product of the mass moment of inertia ( $I$ ) and the angular acceleration ( $\alpha$ ) of the system, i.e.:

$$\sum \tau = I \alpha \quad (25)$$

When the disturbance is considered, (25) becomes:

$$\tau + \tau_d = I(\theta) \alpha \quad (26)$$

Where,

- $\tau$  is the applied torque to the system
- $\tau_d$  is the known and unknown disturbance torque
- $\theta$  and  $\ddot{\theta}$  are the joint angle and angular acceleration of the robot system, respectively

Disturbances have to be estimated and are formulated as follows:

$$\tau_d^* = \tau - IN\ddot{\theta} \quad (27)$$

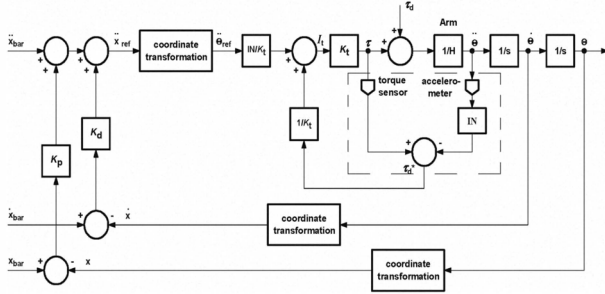


Figure 3. Schematic diagram of an AFC scheme [9]

Where  $IN$  is the estimated inertia matrix that can be obtained by assuming a perfect model, crude approximation or other intelligent methods can be found in literature.  $\tau$  is the measured applied control torque that can be measured using a force or current sensor and  $\ddot{\theta}$  is the measured acceleration that can be obtained using an accelerometer. From (27), it is clear that if the applied torque and angular acceleration are accurately measured and that the estimated inertial parameter is appropriately approximated, it will lead to the rejection of the total torque disturbance in the AFC loop without having to acquire the knowledge about actual magnitude of the disturbance. In the study, the estimated  $IN$  is acquired through a crude approximation method.

## VI. SIMULATION AND RESULTS

To simulate dynamics and control of 3-RRR parallel robot, MATLAB/Simulink software was used. The geometric dimensions and the other properties of the manipulator plus the controllers parameters are as follows:

$$\begin{aligned} l_1 &= l_2 = 0.5 \text{ m}, l_3 = 0.2 \text{ m} \\ m_1 &= m_2 = m_3 = 0.5 \text{ kg}, I_1 = I_2 = I_3 = 0.05 \text{ kgm}^2 \\ m_1 &= m_2 = m_3 = 0.3 \text{ kg}, I_1 = I_2 = I_3 = 0.01 \text{ kgm}^2 \\ m_7 &= 1 \text{ kg}, I_7 = 0.1 \text{ kgm}^2 \\ \text{AFC: } IN &= [0.05 \text{ kgm}^2, 0.05 \text{ kgm}^2, 0.05 \text{ kgm}^2] \\ \text{PID: } K_p &= 30, K_i = 0, K_d = 5 \end{aligned}$$

The block diagram simulation of the proposed scheme is illustrated in Fig. 4 which shows the kinematics, controllers, manipulator dynamics and applied disturbances that are modelled using equations expressed earlier. The PID gains were heuristically tuned after a number of trial runs, taking into account initial ideal situation, i.e., in the absence of disturbances. Two types of disturbances, namely, the constant and harmonic torques of suitable magnitudes and frequency were later introduced into the system. The results obtained for both the PID controller only and PID+AFC controller were presented, analysed and compared. Note also, that, two trajectories were chosen based on triangular and rectangular paths.

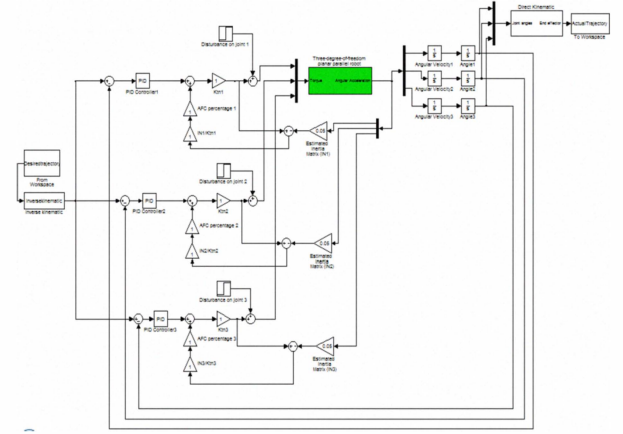


Figure 4. Proposed control of a 3-RRR robot using MATLAB/Simulink

Figs. 5 and 6 show the effect of using PID+AFC control scheme compared to the pure PID controller when three constant disturbance torques (of magnitude 5Nm) were applied to the actuated joints separately and the end-effector designed to follow specified trajectories

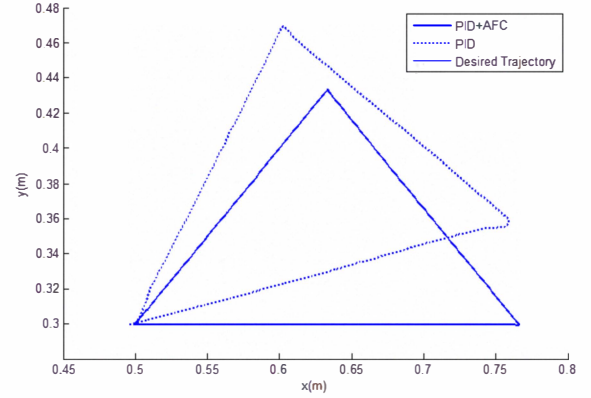


Figure 5. Tracked trajectory using PID and PID+AFC control methods in the presence of constant disturbances

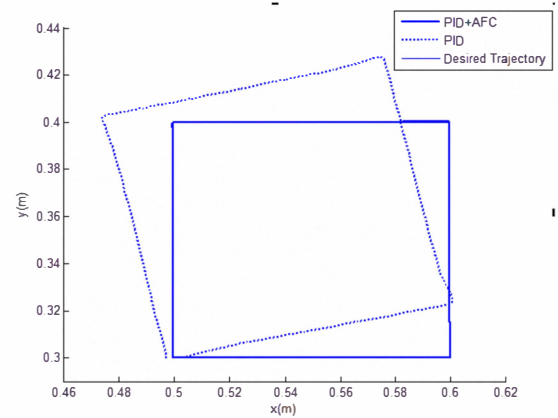


Figure 6. Tracked trajectory using PID and PID+AFC control methods in the presence of constant disturbances

The trajectories generated by the PID controller are very much distorted compared to the PID+AFC scheme in

which the end-effector tends to produce a 'tilt' effect about the lower left tips of both trajectories. On the other hand, AFC results closely resemble those of the desired trajectories, implying that the control method is very robust. The robustness and effectiveness of AFC-based scheme is further demonstrated in Figs. 7 and 8, this time considering the presence of three separate harmonic disturbance torques ( $2 \sin 5t$  Nm) applied to the actuated joints. Again, the PID control produces distorted response for both trajectories, implying that it is not robust in the presence of the disturbances. Thus, the AFC scheme performs much better than the PID counterpart for both types of disturbances introduced to the system.

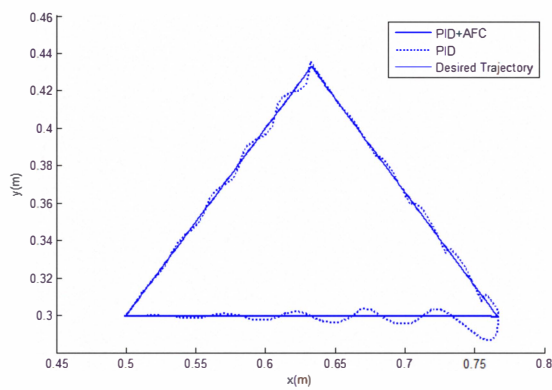


Figure 7. Tracked trajectory using PID and PID+AFC control methods in the presence of harmonic disturbances

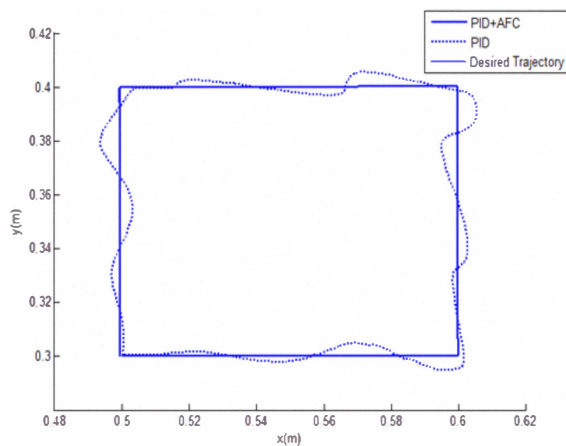


Figure 8. Tracked trajectory using PID and PID+AFC control methods in the presence of harmonic disturbances

## VII. CONCLUSION

In this paper, two control methods, namely the classic PID controller and AFC-based (PID+AFC) scheme were studied to compare their performance in terms of robustness in trajectory tracking control of a 3-RRR planar parallel manipulator in the presence of known disturbances. The PID control, though simple and relatively stable, could not provide satisfactory performance in presence of disturbances and the performance degrades considerably due to inherent system dynamics and friction.

Results clearly show the effectiveness and robustness of the AFC-based controller compared to traditional PID counterpart as a disturbance rejector scheme. In the AFC controller, the estimated inertial parameters required in the AFC loop were directly substituted based on crude approximation method; thus there is no need to compute the matrices on-line and can be easily implemented in real-time. Further research could be carried out to complement the research findings. This may include the computation of the estimated inertia matrices using intelligent methods such as neural networks or fuzzy logic and applying other types of loading and operating conditions.

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