

IMPORTS

EXTENDS

*Naturals*,  
*Integers*,  
*Sequences*,  
*TLC*,  
*FiniteSets*

CONSTANTS

*Shoppers*, Set of customers  $\sim RM$  (Resource Manager)  
*Products* Set of products (available / unavailable)

ASSUME

$Cardinality(Shoppers) > 0 \quad \wedge$   
 $Cardinality(Products) \geq 0$

\*\*\*\* Supporting methods \*\*\*\*

$IsInjective(f) \triangleq \forall a, b \in \text{DOMAIN } f : f[a] = f[b] \Rightarrow a = b$

$SetToSeq(S) \triangleq \text{CHOOSE } f \in [1 \dots Cardinality(S) \rightarrow S] : IsInjective(f)$

\*\*\*\*\*

$Customers \triangleq [Shoppers \rightarrow 1 \dots 5]$  The set of all arrays indexed by the elements of *Shoppers* indexed with values from integers

$Commodities \triangleq [Products \rightarrow 1 \dots 2]$  Index of the products represents individual prices

VARIABLES *shopperState*, *availableProducts*, *customers*, *commodities*, *boughtProducts*, *commodity*, *availableProducts*

$vars \triangleq \langle shopperState, availableProducts, customers, commodities, boughtProducts, commodity, availableProducts \rangle$

$State \triangleq (Shoppers)$

Type Control Invariants

$TCTypeOK \triangleq shopperState \in [Shoppers \rightarrow \{\text{"idle"}, \text{"browsing"}, \text{"selecting"}, \text{"ordering"}, \text{"shipped"}, \text{"served"}\}]$

Global variables

$TCInint \triangleq \wedge shopperState = [shopper \in Shoppers \mapsto \text{"browsing"}]$   
 $\wedge customers \in Customers$   
 $\wedge commodities \in Commodities$   
 $\wedge bought = \{\}$

Variables for the states of actual shopping activities

$\wedge availableProductsSet \in [Shoppers \rightarrow \text{SUBSET DOMAIN commodities}]$   
 $\wedge availableProducts = [shopper \in Shoppers \mapsto SetToSeq(availableProductsSet[shopper])]$

$$\begin{aligned}
& \wedge \text{availableProductsIndex} = [\text{shopper} \in \text{Shoppers} \mapsto 1] \\
& \wedge \text{boughtProducts} = [\text{shopper} \in \text{Shoppers} \mapsto \{\}] \\
& \wedge \text{commodity} = [\text{shopper} \in \text{State} \mapsto 0]
\end{aligned}$$

$$\begin{aligned}
& \text{Shopper is on the web page and selects its product(s)} \\
\text{ProductsBrowsing}(\text{shopper}) & \triangleq \wedge \text{shopperState}[\text{shopper}] = \text{"browsing"} \\
& \wedge \text{shopperState}' = [\text{shopperState} \text{ EXCEPT } ![\text{shopper}] = \text{"browsing"}] \text{ All other sho} \\
& \wedge \text{UNCHANGED } \langle \text{customers}, \text{commodities}, \text{boughtProducts}, \text{availableProducts}, \text{a}
\end{aligned}$$

$$\begin{aligned}
& \text{Shopper tries to select the product (add to basket)} \\
\text{ProductSelection}(\text{shopper}) & \triangleq \wedge \text{shopperState}[\text{shopper}] = \text{"selecting"} \\
& \wedge \text{IF } \text{availableProductsIndex}[\text{shopper}] \leq \text{Len}(\text{availableProducts}[\text{shopper}]) \\
& \quad \text{THEN } \wedge \text{commodity}' = [\text{commodity} \text{ EXCEPT } ![\text{shopper}] = \text{availableProducts}[\text{shopper}]] \\
& \quad \wedge \text{IF } \text{commodity}[\text{shopper}] \in \text{bought} \\
& \quad \quad \text{THEN } \wedge \text{FALSE} \\
& \quad \quad \text{ELSE } \wedge \text{shopperState}' = [\text{shopperState} \text{ EXCEPT } ![\text{shopper}] = \text{"ordering"}] \\
& \quad \text{ELSE } \wedge \text{shopperState}' = [\text{shopperState} \text{ EXCEPT } ![\text{shopper}] = \text{"served"}] \\
& \quad \wedge \text{UNCHANGED } \text{commodity} \\
& \wedge \text{UNCHANGED } \langle \text{customers}, \text{commodities}, \text{boughtProducts}, \text{availableProducts}, \text{a}
\end{aligned}$$

$$\begin{aligned}
& \text{Shopper proceeds to the basket finalize the order} \\
\text{ProductOrdering}(\text{shopper}) & \triangleq \wedge \text{shopperState}[\text{shopper}] = \text{"ordering"} \\
& \wedge \text{IF } \text{commodities}[\text{commodities}[\text{shopper}]] \leq \text{customers}[\text{shopper}] \\
& \quad \text{THEN } \wedge \text{customers}' = [\text{customers} \text{ EXCEPT } ![\text{shopper}] = \text{customers}[\text{shopper}]] \\
& \quad \wedge \text{boughtProducts}' = [\text{boughtProducts} \text{ EXCEPT } ![\text{shopper}] = \text{boughtProducts}[\text{shopper}]] \\
& \quad \wedge \text{bought}' = (\text{bought} \cup \text{commodities}[\text{shopper}]) \\
& \quad \text{ELSE } \wedge \text{TRUE} \\
& \quad \wedge \text{UNCHANGED } \langle \text{customers}, \text{bought}, \text{boughtProducts} \rangle \\
& \wedge \text{shopperState}' = [\text{shopperState} \text{ EXCEPT } ![\text{shopper}] = \text{"selecting"}] \\
& \wedge \text{UNCHANGED } \langle \text{commodities}, \text{availableProducts}, \text{availableProductsIndex}, \text{availableProducts} \rangle
\end{aligned}$$

$$\begin{aligned}
& \text{Shopper ordered product and specifies shippment} \\
\text{ProductShipping}(\text{shopper}) & \triangleq \wedge \text{shopperState}[\text{shopper}] = \text{"shipping"} \\
& \wedge \text{shopperState}' = [\text{shopperState} \text{ EXCEPT } ![\text{shopper}] = \text{"served"}] \\
& \wedge \text{UNCHANGED } \langle \text{availableProducts}, \text{customers}, \text{commodities}, \text{boughtProducts}, \text{commodities} \rangle
\end{aligned}$$

$$\begin{aligned}
& \text{Shopper does not do anything at all – TODO: Integrate this state} \\
\text{NonActive}(\text{shopper}) & \triangleq \wedge \text{shopperState}[\text{shopper}] = \text{"idle"} \\
& \wedge (\text{shopperState}' = [\text{shopperState} \text{ EXCEPT } ![\text{shopper}] = \text{"served"}] \vee \text{shopperState}' = [\text{shopperState} \text{ EXCEPT } ![\text{shopper}] = \text{"idle"}]) \\
& \wedge \text{UNCHANGED } \langle \text{availableProducts}, \text{customers}, \text{commodities}, \text{boughtProducts}, \text{commodities} \rangle
\end{aligned}$$

$$\begin{aligned}
& \text{State Automata formula of the Online Store} \\
\text{OnlineShopping}(\text{shopper}) & \triangleq \wedge \text{ProductsBrowsing}(\text{shopper}) \\
& \vee \text{ProductSelection}(\text{shopper})
\end{aligned}$$

$$\begin{aligned} & \vee \textit{ProductOrdering}(\textit{shopper}) \\ & \vee \textit{ProductShipping}(\textit{shopper}) \end{aligned}$$

#### TERMINATION ASSERTIONS

$$\begin{aligned} \textit{Terminating} \triangleq & \quad \wedge (\forall \textit{shopper} \in \textit{State} : \textit{shopperState}[\textit{shopper}] = \text{"served"}) \\ & \wedge \text{UNCHANGED } \textit{vars} \end{aligned}$$

$$\begin{aligned} \textit{Next} \triangleq & \quad (\exists \textit{shopper} \in \textit{Shoppers} : \textit{OnlineShopping}(\textit{shopper})) \\ & \vee \textit{Terminating} \end{aligned}$$

$$\textit{Spec} \triangleq \textit{TCInint} \wedge \Box[\textit{Next}]_{\textit{vars}}$$

$$\textit{Termination} \triangleq \Diamond(\forall \textit{shopper} \in \textit{State} : \textit{shopperState}[\textit{shopper}] = \text{"served"} \vee \textit{shopperState}[\textit{shopper}] = \text{"browsing"})$$

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