# Lab 6

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## Contents



# 1 Introduction

Today we will be starting off using Stan, looking at the kid's test score data set (available in resources for the Gelman Hill textbook).

```
library(tidyverse)
library(rstan)
library(tidybayes)
library(here)
```

The data look like this:

```
kidiq <- read_rds(paste0(here(),"/data/kidiq.RDS"))
kidiq
```

```
## # A tibble: 434 x 4
##    kid_score mom_hs mom_iq mom_age
##        <int>  <dbl>  <dbl>   <int>
##  1        65      1  121.       27
##  2        98      1   89.4      25
##  3        85      1  115.       27
##  4        83      1   99.4      25
##  5       115      1   92.7      27
##  6        98      0  108.       18
```

```
##  7        69      1  139.       20
##  8       106      1  125.       23
##  9       102      1   81.6      24
## 10        95      1   95.1      19
## # ... with 424 more rows
```

As well as the kid's test scores, we have a binary variable indicating whether or not the mother completed high school, the mother's IQ and age.

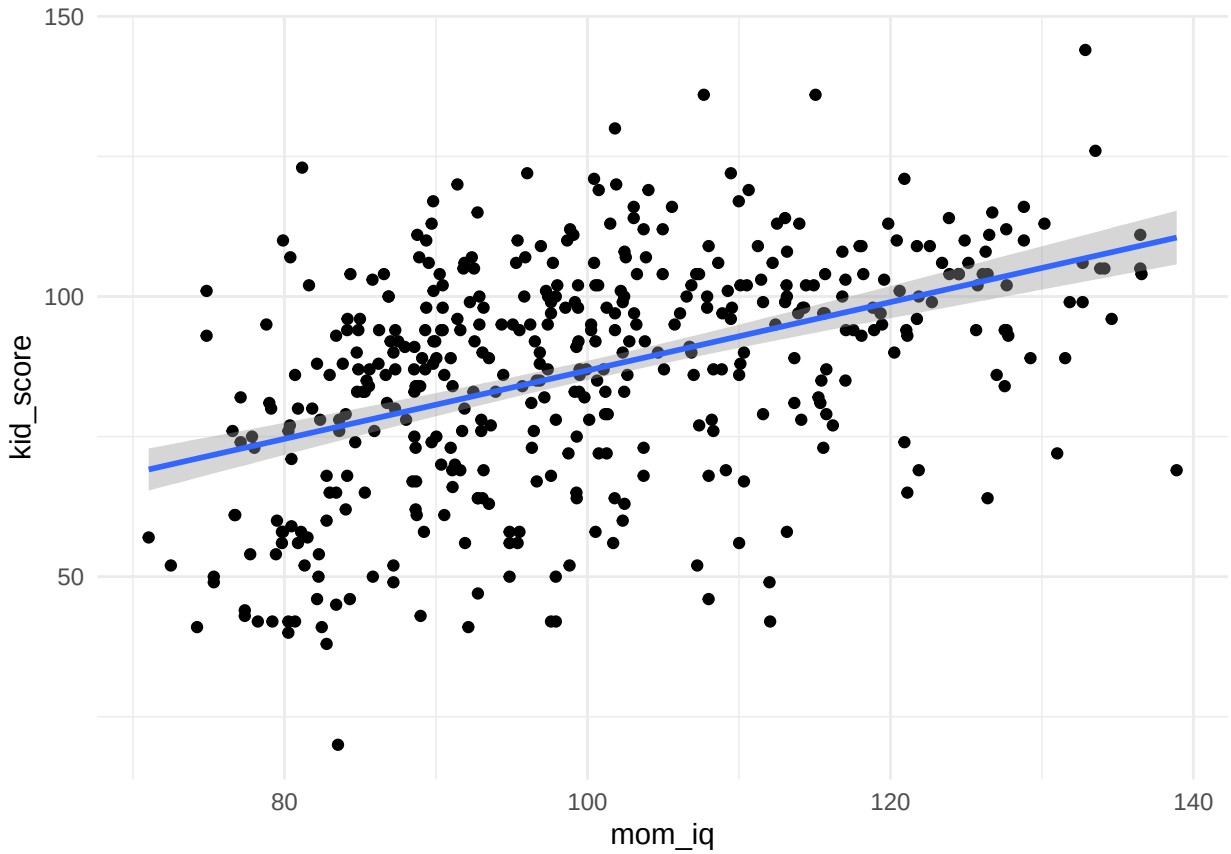
# 2 Descriptives

## 2.1 Question 1

Use plots or tables to show three interesting observations about the data. Remember:

First let's take a look at mom IQ vs kid score:

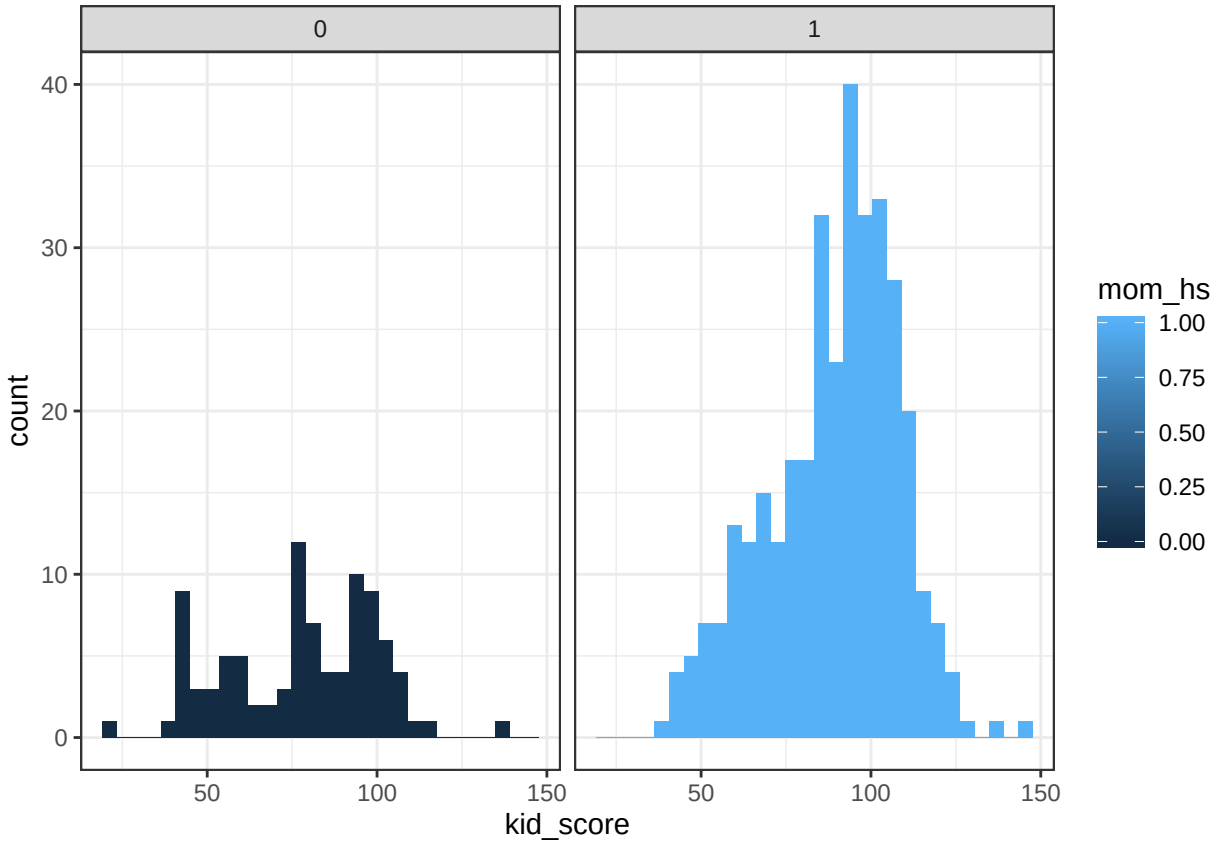
```
kidiq %>%
  ggplot(aes(x = mom_iq, y = kid_score)) +
  geom_point() +
  theme_minimal() +
  geom_smooth(method = "lm")
```




There seems to be a pattern of kid scores increasing with mom's IQ, which is very surprising given all the arguments I've seen from Taleb (yeah I know) about IQ only being a valid measure for low numbers.

Next I wanna take a peek at the distribution of kid scores split by whether their mother completed high school.

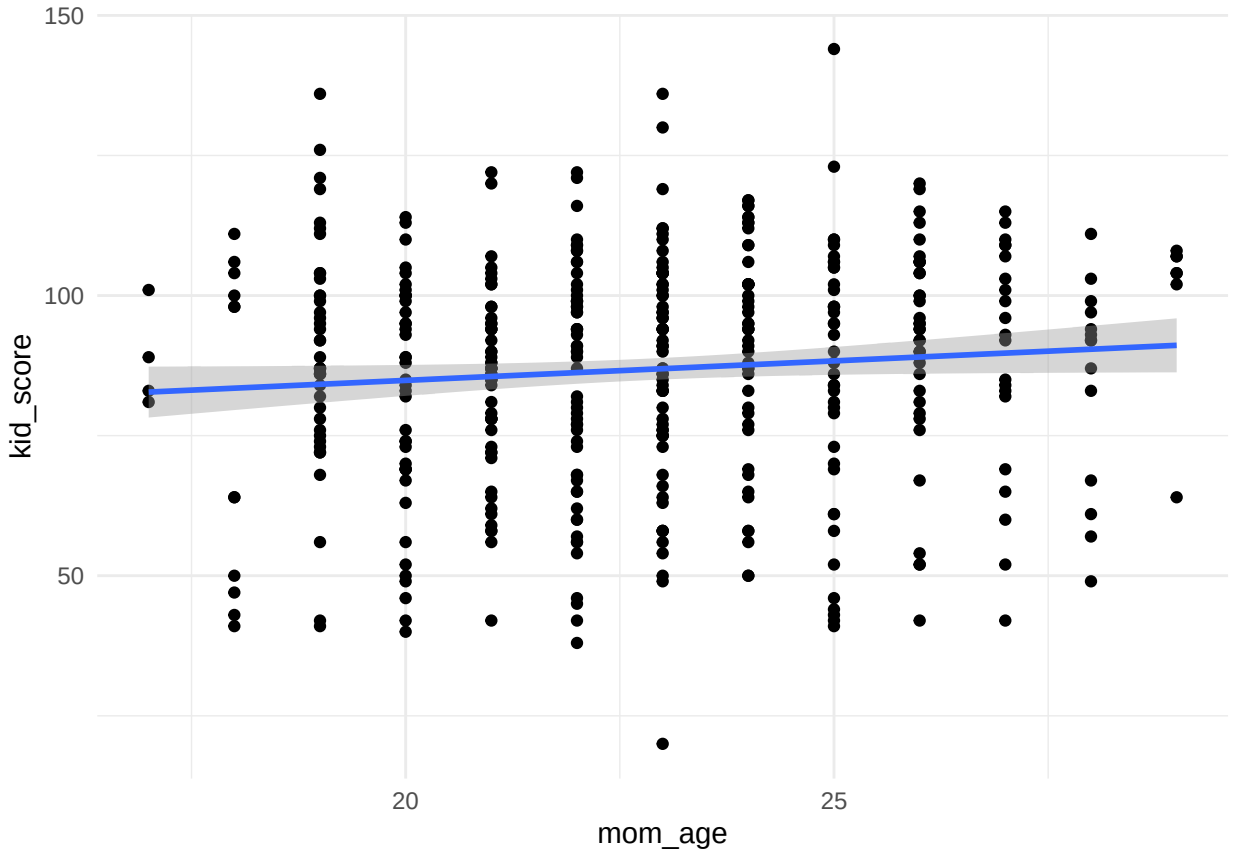
```
kidiq %>%
  ggplot(aes(x = kid_score, facet = mom_hs, fill = mom_hs)) +
  geom_histogram() +
  theme_bw() +
  facet_wrap(mom_hs ~ . )
```




Seems like kids of mothers with no high shool have a much flatter distribution with less of a peak at the high values. This is less surprising as no highschool can be good proxy for lower income / lower resources.

Now for my most interesting plot (because I have no expectations and little prior knowledge), I want to see kid score by mother age.

```
kidiq %>%
  ggplot(aes(x = mom_age, y = kid_score)) +
  geom_point() +
  theme_minimal() +
  geom_smooth(method = "lm")
```




Seems to be not much going on, there is little to no relationship between mom age and kid score. Not surprised but a little disappointed that there is nothing extraordinary for me to make wild claims about possible causes.

## 3 Estimating mean, no covariates

In class we were trying to estimate the mean and standard deviation of the kid's test scores. The `kids2.stan` file contains a Stan model to do this. If you look at it, you will notice the first `data` chunk lists some inputs that we have to define: the outcome variable `y`, number of observations `N`, and the mean and standard deviation of the prior on `mu`. Let's define all these values in a `data` list.

```
y <- kidiq$kid_score
mu0 <- 80
sigma0 <- 100

data <- list(y = y,
             N = length(y),
             mu0 = mu0,
             sigma0 = sigma0)
```

Now we can run the model:

```
fit <- stan(file = paste0(here(), "/code/models/kids2.stan"),
            data = data)
```

```
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 1).
## Chain 1:
```

```
## Chain 1: Gradient evaluation took 2.2e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.22 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.048034 seconds (Warm-up)
## Chain 1:                0.037442 seconds (Sampling)
## Chain 1:                0.085476 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 1e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.1 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.064445 seconds (Warm-up)
## Chain 2:                0.063554 seconds (Sampling)
## Chain 2:                0.127999 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 7e-06 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.07 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
```

```
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.05102 seconds (Warm-up)
## Chain 3:                0.029244 seconds (Sampling)
## Chain 3:                0.080264 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 8e-06 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.08 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.049057 seconds (Warm-up)
## Chain 4:                0.040148 seconds (Sampling)
## Chain 4:                0.089205 seconds (Total)
## Chain 4:
```

Look at the summary

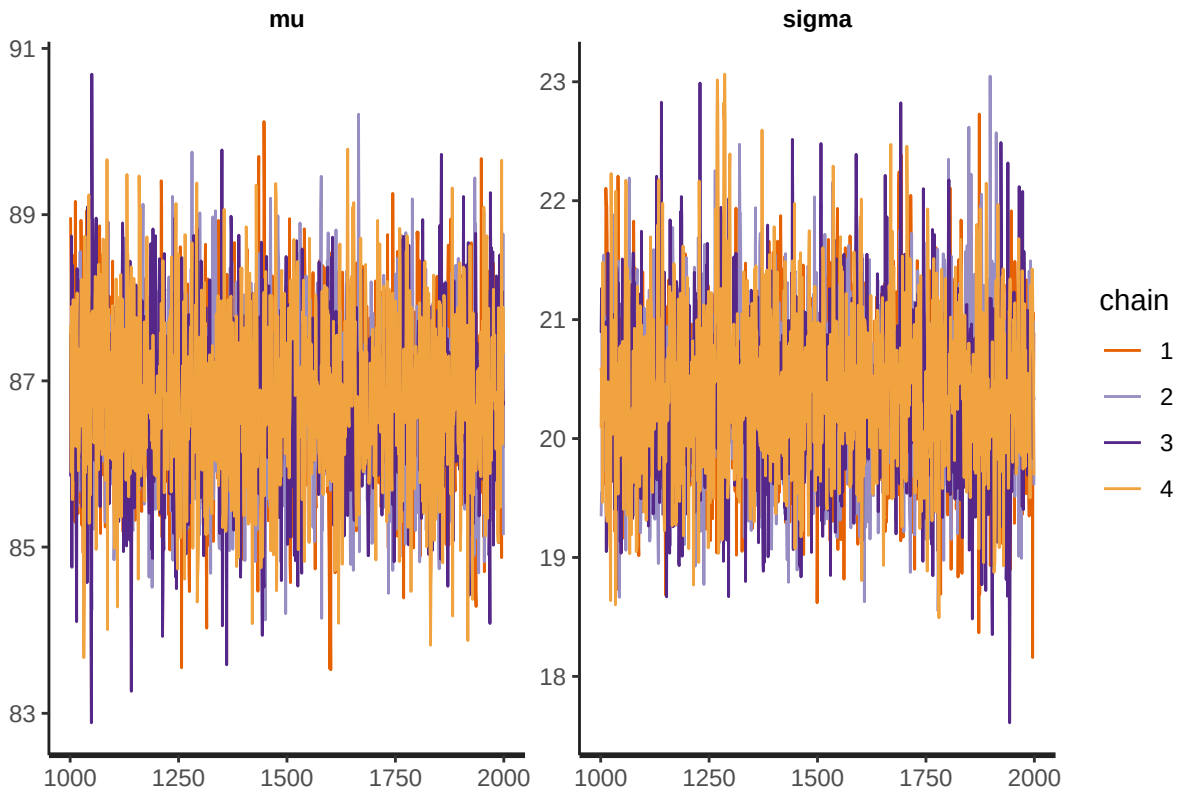
```
fit
```

```
## Inference for Stan model: kids2.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##           mean se_mean   sd     2.5%      25%      50%      75%    97.5% n_eff
## mu       86.78    0.02 0.99    84.85    86.11    86.77    87.47    88.74  2909
## sigma    20.38    0.01 0.70    19.08    19.90    20.36    20.83    21.80  3227
```

```
## lp__  -1525.56    0.03 1.06 -1528.39 -1525.95 -1525.22 -1524.80 -1524.55  1639
##       Rhat
## mu       1
## sigma    1
## lp__     1
##
## Samples were drawn using NUTS(diag_e) at Wed Feb 12 15:59:41 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

Traceplot

```
traceplot(fit)
```




All looks fine.

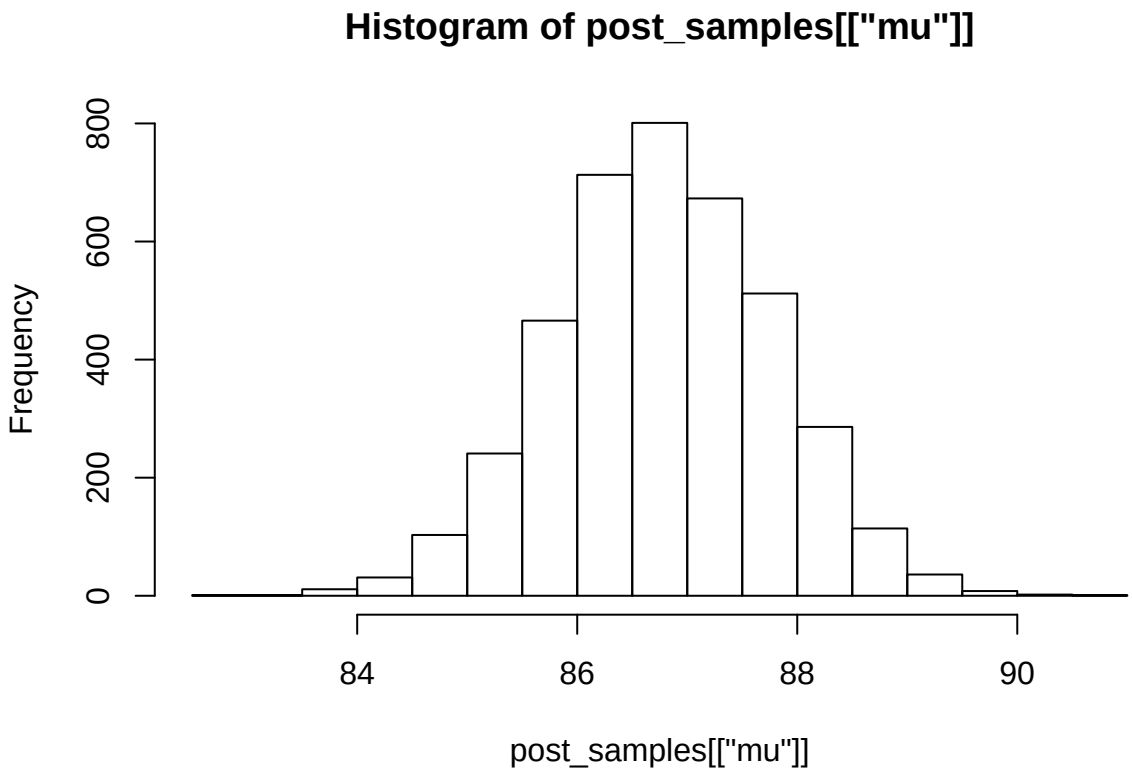
## 3.1 Understanding output

What does the model actually give us? A number of samples from the posteriors. To see this, we can use `extract` to get the samples.

```
post_samples <- extract(fit)
```

This is a list, and in this case, each element of the list has 4000 samples. E.g. quickly plot a histogram of mu

```
hist(post_samples[["mu"]])
```




### 3.2 Plot estimates

There are a bunch of packages, built-in functions that let you plot the estimates from the model, and I encourage you to explore these options (particularly in `bayesplot`, which we will most likely be using later on). I like using the `tidybayes` package, which allows us to easily get the posterior samples in a tidy format (e.g. using gather draws to get in long format). Once we have that, it's easy to just pipe and do ggplots as usual. `tidybayes` also has a bunch of fun visualizations, see more info here: https://mjskay.github.io/tidybayes/articles/tidybayes.html#introduction

Get the posterior samples for mu and sigma in long format:

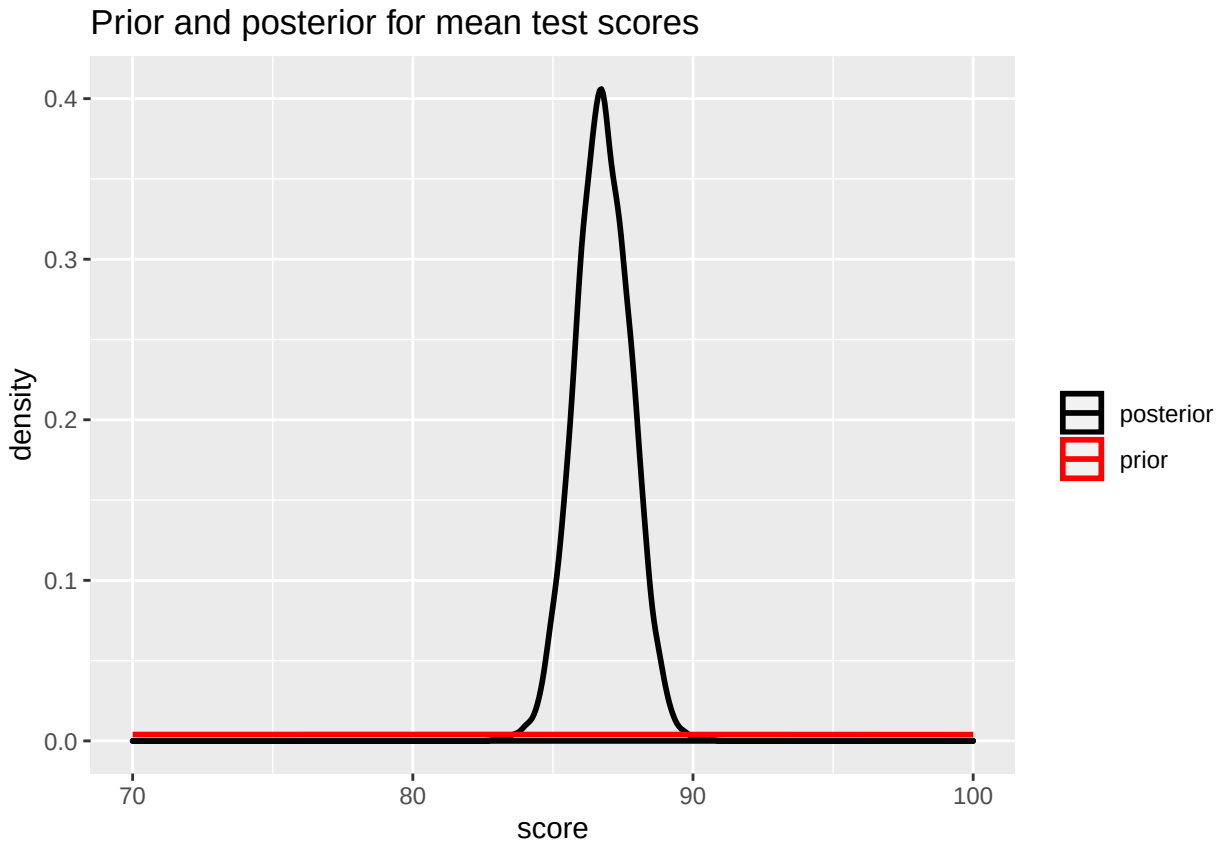
```
dsamples <- fit %>%
  gather_draws(mu, sigma)
dsamples
```

```
## # A tibble: 8,000 x 5
## # Groups:   .variable [2]
##    .chain .iteration .draw .variable .value
##     <int>      <int> <int> <chr>      <dbl>
##  1      1          1     1 mu          89.0
##  2      1          2     2 mu          88.3
##  3      1          3     3 mu          86.6
##  4      1          4     4 mu          87.4
##  5      1          5     5 mu          86.0
##  6      1          6     6 mu          85.8
##  7      1          7     7 mu          85.6
##  8      1          8     8 mu          85.3
##  9      1          9     9 mu          88.5
```

```
## 10      1         10    10 mu          86.3
## # ... with 7,990 more rows
```

Let's plot the density of the posterior samples for mu and add in the prior distribution

```
dsamples %>%
  filter(.variable == "mu") %>%
  ggplot(aes(.value, color = "posterior")) + geom_density(size = 1) +
  xlim(c(70, 100)) +
  stat_function(fun = dnorm,
        args = list(mean = mu0,
                    sd = sigma0),
        aes(colour = 'prior'), size = 1) +
  scale_color_manual(name = "", values = c("prior" = "red", "posterior" = "black")) +
  ggtitle("Prior and posterior for mean test scores") +
  xlab("score")
```




### 3.3 Question 2

Change the prior to be much more informative (by changing the standard deviation to be 0.1). Rerun the model. Do the estimates change? Plot the prior and posterior densities.

```
sigma0 = 0.1
data2 <- list(y = y,
              N = length(y),
              mu0 = mu0,
              sigma0 = sigma0)

fit2 <- stan(file = paste0(here(), "/code/models/kids2.stan"),
```

```
            data = data2)
```

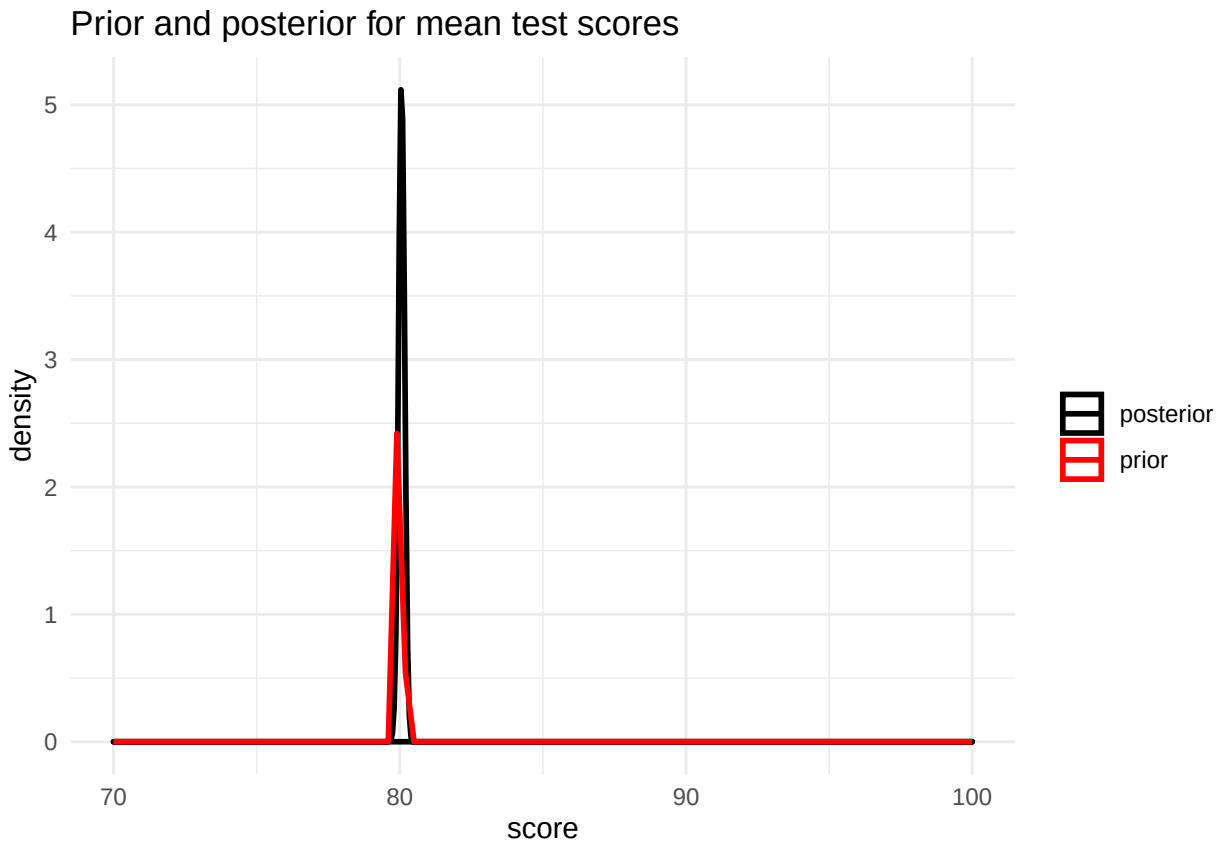
```
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 8e-06 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.08 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.041113 seconds (Warm-up)
## Chain 1:                0.044315 seconds (Sampling)
## Chain 1:                0.085428 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 1.7e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.17 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.031615 seconds (Warm-up)
## Chain 2:                0.047917 seconds (Sampling)
## Chain 2:                0.079532 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 3).
```

```
## Chain 3:
## Chain 3: Gradient evaluation took 1.1e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.11 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.038554 seconds (Warm-up)
## Chain 3:                0.063943 seconds (Sampling)
## Chain 3:                0.102497 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 1.3e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.13 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.040095 seconds (Warm-up)
## Chain 4:                0.033137 seconds (Sampling)
## Chain 4:                0.073232 seconds (Total)
## Chain 4:
```

```
dsamples2 <- fit2 %>%
  gather_draws(mu, sigma)

dsamples2 %>%
  filter(.variable == "mu") %>%
```

```r
ggplot(aes(.value, color = "posterior")) + geom_density(size = 1) +
xlim(c(70, 100)) +
stat_function(fun = dnorm,
      args = list(mean = mu0,
                  sd = sigma0),
      aes(colour = 'prior'), size = 1) +
scale_color_manual(name = "", values = c("prior" = "red", "posterior" = "black")) +
ggtitle("Prior and posterior for mean test scores") +
xlab("score") +
theme_minimal()
```




# 4 Adding covariates

Now let's see how kid's test scores are related to mother's education. We want to run the simple linear regression

$$Score = \alpha + \beta X$$

where $X = 1$ if the mother finished high school and zero otherwise.

`kid3.stan` has the stan model to do this. Notice now we have some inputs related to the design matrix $X$ and the number of covariates (in this case, it's just 1).

Let's get the data we need and run the model.

```r
X <- as.matrix(kidiq$mom_hs, ncol = 1)
K <- 1
```

```
data <- list(y = y, N = length(y),
             X = X, K = K)
fit3 <- stan(file = paste0(here(), "/code/models/kids3.stan"),
            data = data,
            iter = 1000)
```

```
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 9.9e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.99 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.239053 seconds (Warm-up)
## Chain 1:                0.155154 seconds (Sampling)
## Chain 1:                0.394207 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 3.2e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.32 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.257276 seconds (Warm-up)
```

```
## Chain 2:                0.146169 seconds (Sampling)
## Chain 2:                0.403445 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 3.1e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.31 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.225201 seconds (Warm-up)
## Chain 3:                0.131769 seconds (Sampling)
## Chain 3:                0.35697 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 3.1e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.31 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.303361 seconds (Warm-up)
## Chain 4:                0.138705 seconds (Sampling)
## Chain 4:                0.442066 seconds (Total)
## Chain 4:
```

## 4.1 Question 3

Confirm that the estimates of the intercept and slope are comparable to results from `lm()`

```
linmod <- lm(data = kidiq, formula = kid_score ~ mom_hs)
summary(linmod)
```

```
##
## Call:
## lm(formula = kid_score ~ mom_hs, data = kidiq)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -57.55 -13.32   2.68  14.68  58.45
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   77.548      2.059  37.670  < 2e-16 ***
## mom_hs        11.771      2.322   5.069 5.96e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 19.85 on 432 degrees of freedom
## Multiple R-squared:  0.05613,    Adjusted R-squared:  0.05394
## F-statistic: 25.69 on 1 and 432 DF,  p-value: 5.957e-07
```

```
fit3
```

```
## Inference for Stan model: kids3.
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
##             mean se_mean   sd     2.5%      25%      50%      75%    97.5%
## alpha      78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## beta[1]    11.17    0.07 2.23     6.68     9.70    11.23    12.66    15.49
## sigma      19.83    0.02 0.69    18.57    19.36    19.80    20.24    21.32
## mu[1]      89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[2]      89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[3]      89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[4]      89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[5]      89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[6]      78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[7]      89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[8]      89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[9]      89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[10]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[11]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[12]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[13]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[14]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[15]     78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[16]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[17]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[18]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[19]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[20]     78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
```

```
## mu[21]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[22]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[23]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[24]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[25]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[26]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[27]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[28]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[29]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[30]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[31]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[32]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[33]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[34]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[35]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[36]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[37]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[38]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[39]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[40]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[41]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[42]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[43]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[44]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[45]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[46]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[47]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[48]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[49]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[50]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[51]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[52]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[53]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[54]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[55]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[56]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[57]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[58]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[59]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[60]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[61]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[62]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[63]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[64]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[65]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[66]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[67]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[68]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[69]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[70]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[71]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[72]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[73]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[74]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
```

```
## mu[75]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[76]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[77]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[78]     78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[79]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[80]     78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[81]     78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[82]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[83]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[84]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[85]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[86]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[87]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[88]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[89]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[90]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[91]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[92]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[93]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[94]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[95]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[96]     78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[97]     89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[98]     78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[99]     78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[100]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[101]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[102]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[103]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[104]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[105]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[106]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[107]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[108]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[109]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[110]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[111]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[112]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[113]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[114]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[115]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[116]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[117]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[118]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[119]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[120]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[121]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[122]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[123]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[124]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[125]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[126]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[127]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[128]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
```

```
## mu[129]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[130]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[131]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[132]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[133]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[134]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[135]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[136]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[137]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[138]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[139]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[140]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[141]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[142]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[143]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[144]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[145]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[146]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[147]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[148]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[149]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[150]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[151]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[152]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[153]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[154]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[155]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[156]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[157]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[158]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[159]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[160]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[161]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[162]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[163]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[164]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[165]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[166]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[167]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[168]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[169]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[170]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[171]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[172]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[173]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[174]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[175]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[176]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[177]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[178]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[179]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[180]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[181]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[182]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
```

```
## mu[183]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[184]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[185]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[186]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[187]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[188]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[189]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[190]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[191]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[192]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[193]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[194]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[195]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[196]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[197]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[198]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[199]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[200]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[201]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[202]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[203]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[204]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[205]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[206]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[207]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[208]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[209]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[210]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[211]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
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## mu[213]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[214]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[215]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[216]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[217]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[218]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[219]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[220]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[221]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[222]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[223]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[224]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[225]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[226]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[227]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[228]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[229]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[230]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[231]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[232]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[233]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[234]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[235]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[236]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
```

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## mu[237]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[238]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[239]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[240]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[241]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[242]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[243]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[244]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[245]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[246]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[247]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[248]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[249]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[250]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[251]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[252]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[253]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[254]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[255]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[256]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
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## mu[258]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
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## mu[260]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[261]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[262]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[263]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[264]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[265]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[266]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[267]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[268]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[269]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[270]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[271]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[272]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[273]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
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## mu[275]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
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## mu[279]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[280]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[281]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[282]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[283]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[284]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[285]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
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## mu[287]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
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## mu[289]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[290]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
```

```
## mu[291]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[292]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[293]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[294]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[295]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
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## mu[297]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[298]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[299]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
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## mu[301]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[302]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[303]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[304]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[305]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[306]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[307]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[308]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
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## mu[312]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[313]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[314]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[315]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[316]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[317]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[318]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[319]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[320]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[321]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[322]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[323]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[324]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[325]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[326]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[327]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[328]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[329]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[330]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[331]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[332]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[333]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
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## mu[335]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[336]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[337]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[338]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[339]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[340]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[341]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[342]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[343]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[344]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
```

```
## mu[345]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[346]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[347]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[348]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[349]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[350]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[351]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[352]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[353]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[354]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[355]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[356]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[357]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[358]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[359]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[360]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[361]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[362]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[363]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[364]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[365]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[366]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[367]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[368]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[369]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[370]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[371]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[372]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[373]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[374]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[375]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[376]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[377]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[378]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[379]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[380]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[381]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[382]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[383]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[384]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[385]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[386]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[387]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[388]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[389]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[390]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[391]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[392]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[393]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[394]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[395]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[396]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[397]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[398]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
```

```
## mu[399]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[400]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[401]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[402]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[403]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[404]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[405]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[406]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[407]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[408]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[409]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[410]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[411]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[412]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[413]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[414]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[415]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[416]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[417]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[418]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[419]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[420]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[421]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[422]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[423]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[424]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[425]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[426]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[427]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[428]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[429]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[430]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[431]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[432]    78.06    0.07 2.00    74.28    76.72    78.03    79.34    82.17
## mu[433]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## mu[434]    89.23    0.02 1.10    87.06    88.50    89.22    89.96    91.37
## lp__    -1514.11    0.05 1.28 -1517.29 -1514.74 -1513.76 -1513.17 -1512.69
##         n_eff Rhat
## alpha     938 1.00
## beta[1]   922 1.00
## sigma    1174 1.00
## mu[1]    2600 1.00
## mu[2]    2600 1.00
## mu[3]    2600 1.00
## mu[4]    2600 1.00
## mu[5]    2600 1.00
## mu[6]     938 1.00
## mu[7]    2600 1.00
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## mu[9]    2600 1.00
## mu[10]   2600 1.00
## mu[11]   2600 1.00
## mu[12]   2600 1.00
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```

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## mu[14]   2600 1.00
## mu[15]    938 1.00
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## mu[28]   2600 1.00
## mu[29]   2600 1.00
## mu[30]   2600 1.00
## mu[31]   2600 1.00
## mu[32]   2600 1.00
## mu[33]   2600 1.00
## mu[34]    938 1.00
## mu[35]   2600 1.00
## mu[36]   2600 1.00
## mu[37]   2600 1.00
## mu[38]   2600 1.00
## mu[39]   2600 1.00
## mu[40]   2600 1.00
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## mu[45]    938 1.00
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## mu[67]   2600 1.00
```

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## mu[68]   2600 1.00
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## mu[78]    938 1.00
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## mu[82]   2600 1.00
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## mu[87]   2600 1.00
## mu[88]   2600 1.00
## mu[89]   2600 1.00
## mu[90]   2600 1.00
## mu[91]   2600 1.00
## mu[92]   2600 1.00
## mu[93]   2600 1.00
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## mu[176]  2600 1.00
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## mu[230]   938 1.00
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## mu[284]   938 1.00
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## mu[338]  2600 1.00
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## mu[341]   938 1.00
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## mu[392]  2600 1.00
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## mu[410]   938 1.00
## mu[411]  2600 1.00
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## mu[421]   938 1.00
## mu[422]   938 1.00
## mu[423]   938 1.00
## mu[424]  2600 1.00
## mu[425]  2600 1.00
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## mu[428]   938 1.00
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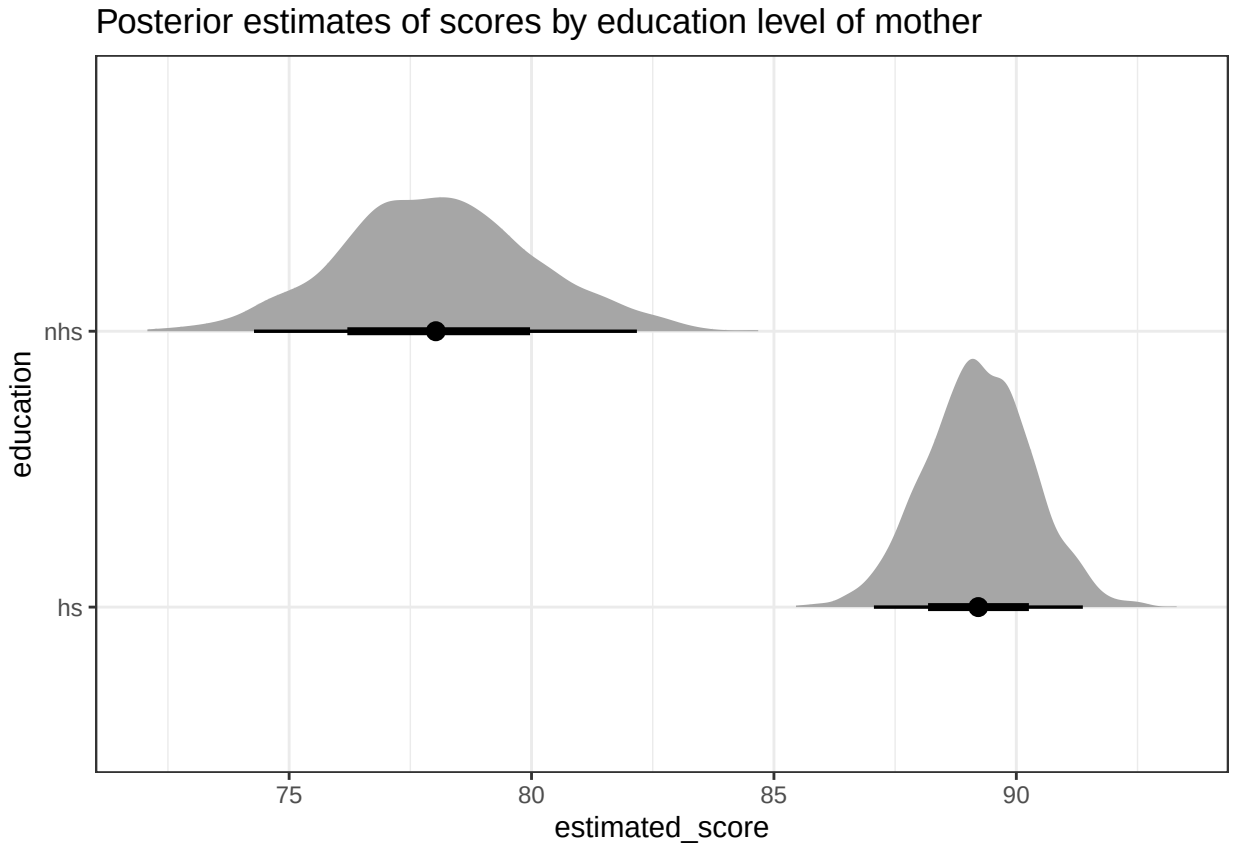
## mu[430]   938 1.00
## mu[431]  2600 1.00
## mu[432]   938 1.00
## mu[433]  2600 1.00
## mu[434]  2600 1.00
## lp__      771 1.01
##
## Samples were drawn using NUTS(diag_e) at Wed Feb 12 16:01:22 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

So the results are 78 and 11 for STAN and 77.5 and 11 for LM. No objections there.

## 4.2 Plotting results

It might be nice to plot the posterior samples of the estimates for the non-high-school and high-school mothered kids. Here's some code that does this: notice the `beta[condition]` syntax. Also notice I'm using `spread_draws`, because it's easier to calculate the estimated effects in wide format

```
fit3 %>%
  spread_draws(alpha, beta[condition], sigma) %>%
     mutate(nhs = alpha, # no high school is just the intercept
            hs = alpha + beta) %>%
  pivot_longer(nhs:hs, names_to = "education", values_to = "estimated_score") %>%
  ggplot(aes(y = education, x = estimated_score)) +
  stat_halfeyeh() +
  theme_bw() +
  ggtitle("Posterior estimates of scores by education level of mother")
```




## 4.3 Question 4

Add in mother's IQ as a covariate and rerun the model. You will probably want to mean center the covariate before putting it into the model. Interpret the coefficient on the (centered) mum's IQ.

```
kidiq <- kidiq %>% mutate(mom_iq_centered = scale(mom_iq, scale = FALSE))

X <- as.matrix(kidiq$mom_hs, ncol = 1)
X <- cbind(X, as.matrix(kidiq$mom_iq_centered, ncol = 1))
K <- 2

data <- list(y = y, N = length(y),
```

```
             X = X, K = K)
fit4 <- stan(file = paste0(here(), "/code/models/kids4.stan"),
             data = data,
             iter = 1000)
```

```
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 3.3e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.33 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.327612 seconds (Warm-up)
## Chain 1:                0.197781 seconds (Sampling)
## Chain 1:                0.525393 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 3.7e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.37 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.268 seconds (Warm-up)
## Chain 2:                0.172797 seconds (Sampling)
## Chain 2:                0.440797 seconds (Total)
```

```
## Chain 2:
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 3.6e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.36 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.266388 seconds (Warm-up)
## Chain 3:                0.173903 seconds (Sampling)
## Chain 3:                0.440291 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 3.5e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.35 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.329432 seconds (Warm-up)
## Chain 4:                0.163779 seconds (Sampling)
## Chain 4:                0.493211 seconds (Total)
## Chain 4:
```

```
fit4
```

```
## Inference for Stan model: kids3.
```

```
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
##             mean se_mean   sd     2.5%      25%      50%      75%    97.5%
## alpha      82.34    0.06 1.92    78.65    81.04    82.40    83.60    86.05
## beta[1]     5.71    0.06 2.16     1.57     4.27     5.75     7.20     9.91
## beta[2]     0.57    0.00 0.06     0.46     0.53     0.57     0.61     0.68
## sigma      18.13    0.02 0.61    16.92    17.72    18.13    18.55    19.34
## mu[1]     100.06    0.03 1.49    97.14    99.04   100.05   101.08   102.91
## mu[2]      82.01    0.03 1.22    79.68    81.14    82.04    82.88    84.37
## mu[3]      96.83    0.03 1.26    94.40    95.95    96.82    97.70    99.26
## mu[4]      87.74    0.02 0.99    85.78    87.07    87.76    88.43    89.65
## mu[5]      83.93    0.03 1.12    81.77    83.15    83.97    84.71    86.06
## mu[6]      86.83    0.06 2.06    82.82    85.42    86.86    88.19    90.90
## mu[7]     110.16    0.06 2.35   105.41   108.64   110.17   111.71   114.82
## mu[8]     102.34    0.04 1.67    99.06   101.23   102.36   103.49   105.55
## mu[9]      77.61    0.04 1.52    74.73    76.52    77.65    78.69    80.48
## mu[10]     85.26    0.03 1.06    83.18    84.52    85.28    86.00    87.31
## mu[11]     81.56    0.03 1.25    79.19    80.69    81.58    82.46    83.98
## mu[12]     85.14    0.03 1.06    83.04    84.40    85.16    85.88    87.19
## mu[13]     81.78    0.03 1.23    79.42    80.91    81.81    82.67    84.17
## mu[14]     96.08    0.03 1.21    93.78    95.24    96.06    96.92    98.39
## mu[15]     82.65    0.06 1.92    78.95    81.35    82.69    83.89    86.33
## mu[16]     99.66    0.03 1.46    96.82    98.65    99.64   100.66   102.43
## mu[17]     96.25    0.03 1.22    93.92    95.40    96.24    97.10    98.60
## mu[18]     94.64    0.03 1.14    92.47    93.88    94.62    95.44    96.85
## mu[19]    107.29    0.05 2.09   103.06   105.91   107.30   108.69   111.41
## mu[20]     80.79    0.06 1.89    77.14    79.50    80.84    82.04    84.45
## mu[21]     93.79    0.02 1.10    91.68    93.05    93.76    94.57    95.89
## mu[22]    103.24    0.04 1.74    99.82   102.09   103.25   104.45   106.61
## mu[23]     86.87    0.02 1.01    84.88    86.17    86.88    87.55    88.83
## mu[24]     88.01    0.02 0.99    86.05    87.35    88.03    88.70    89.91
## mu[25]     80.98    0.06 1.89    77.30    79.68    81.02    82.23    84.64
## mu[26]    100.41    0.04 1.51    97.45    99.38   100.41   101.46   103.32
## mu[27]     87.34    0.02 1.00    85.37    86.67    87.36    88.02    89.27
## mu[28]     86.87    0.02 1.01    84.88    86.18    86.89    87.55    88.83
## mu[29]     76.89    0.04 1.58    73.87    75.77    76.95    78.02    79.90
## mu[30]     96.19    0.03 1.22    93.87    95.34    96.17    97.03    98.52
## mu[31]     93.25    0.02 1.07    91.18    92.51    93.22    94.01    95.32
## mu[32]     89.09    0.02 0.99    87.13    88.43    89.08    89.77    90.97
## mu[33]     98.26    0.03 1.36    95.60    97.32    98.24    99.21   100.86
## mu[34]     87.25    0.06 2.08    83.21    85.83    87.29    88.64    91.38
## mu[35]     86.08    0.02 1.03    84.05    85.35    86.10    86.80    88.09
## mu[36]     84.01    0.03 1.11    81.85    83.22    84.04    84.78    86.13
## mu[37]     85.73    0.03 1.04    83.68    85.00    85.75    86.45    87.75
## mu[38]     92.04    0.02 1.03    90.02    91.34    92.01    92.78    94.01
## mu[39]     80.78    0.03 1.30    78.34    79.85    80.80    81.70    83.27
## mu[40]     82.01    0.03 1.22    79.68    81.14    82.04    82.88    84.37
## mu[41]     89.49    0.02 0.99    87.54    88.83    89.49    90.18    91.39
## mu[42]    105.20    0.05 1.91   101.40   103.94   105.21   106.50   108.94
## mu[43]     78.63    0.04 1.45    75.88    77.60    78.66    79.65    81.36
## mu[44]    102.69    0.04 1.70    99.37   101.56   102.70   103.86   105.95
## mu[45]     74.28    0.05 1.92    70.59    72.99    74.31    75.58    78.05
## mu[46]    103.13    0.04 1.73    99.74   101.98   103.14   104.32   106.46
```

```
## mu[47]     76.24    0.04 1.63    73.11    75.08    76.30    77.41    79.36
## mu[48]     95.54    0.03 1.18    93.29    94.73    95.51    96.36    97.84
## mu[49]     93.93    0.03 1.10    91.81    93.18    93.90    94.70    96.05
## mu[50]     87.72    0.02 0.99    85.75    87.04    87.74    88.40    89.63
## mu[51]     89.43    0.02 0.99    87.48    88.77    89.43    90.12    91.33
## mu[52]    102.21    0.04 1.66    98.95   101.10   102.22   103.34   105.39
## mu[53]     85.14    0.03 1.06    83.04    84.40    85.16    85.88    87.19
## mu[54]     83.72    0.03 1.13    81.54    82.92    83.77    84.50    85.86
## mu[55]     89.09    0.02 0.99    87.13    88.43    89.08    89.77    90.97
## mu[56]     70.88    0.05 2.02    67.03    69.50    70.93    72.26    74.83
## mu[57]     85.91    0.03 1.04    83.87    85.18    85.94    86.63    87.92
## mu[58]     72.32    0.05 1.97    68.57    70.94    72.37    73.65    76.16
## mu[59]     69.03    0.06 2.10    65.02    67.60    69.10    70.47    73.15
## mu[60]     93.75    0.02 1.09    91.64    93.00    93.72    94.52    95.84
## mu[61]     94.64    0.03 1.14    92.47    93.88    94.62    95.44    96.85
## mu[62]     95.82    0.03 1.20    93.55    94.99    95.79    96.66    98.14
## mu[63]     83.00    0.06 1.93    79.33    81.70    83.03    84.25    86.74
## mu[64]     88.82    0.02 0.99    86.85    88.15    88.81    89.50    90.71
## mu[65]     89.02    0.02 0.99    87.06    88.36    89.01    89.71    90.90
## mu[66]     87.39    0.02 1.00    85.41    86.71    87.40    88.06    89.31
## mu[67]     84.36    0.03 1.10    82.22    83.59    84.40    85.13    86.46
## mu[68]     80.84    0.03 1.29    78.41    79.91    80.85    81.75    83.32
## mu[69]     80.85    0.03 1.29    78.42    79.92    80.87    81.77    83.33
## mu[70]     80.22    0.03 1.33    77.70    79.27    80.24    81.17    82.75
## mu[71]     82.28    0.03 1.20    79.97    81.43    82.31    83.13    84.60
## mu[72]     94.77    0.07 2.50    89.76    93.07    94.79    96.47    99.75
## mu[73]     98.00    0.08 2.73    92.54    96.19    98.01    99.87   103.36
## mu[74]     93.73    0.02 1.09    91.63    92.99    93.70    94.50    95.83
## mu[75]     82.63    0.03 1.18    80.36    81.78    82.66    83.46    84.91
## mu[76]     81.75    0.03 1.23    79.38    80.88    81.77    82.63    84.14
## mu[77]     95.47    0.03 1.18    93.23    94.67    95.45    96.29    97.77
## mu[78]     69.85    0.06 2.06    65.88    68.45    69.91    71.27    73.90
## mu[79]     85.44    0.03 1.05    83.38    84.71    85.46    86.18    87.48
## mu[80]     73.03    0.05 1.95    69.30    71.69    73.08    74.33    76.81
## mu[81]     74.53    0.05 1.91    70.84    73.24    74.55    75.83    78.30
## mu[82]     85.44    0.03 1.05    83.38    84.71    85.46    86.18    87.48
## mu[83]     94.45    0.03 1.13    92.30    93.69    94.42    95.24    96.63
## mu[84]    103.41    0.04 1.76    99.93   102.24   103.42   104.62   106.81
## mu[85]     82.35    0.03 1.20    80.04    81.49    82.38    83.19    84.66
## mu[86]     87.64    0.02 1.00    85.66    86.97    87.66    88.33    89.55
## mu[87]     77.35    0.04 1.54    74.42    76.25    77.40    78.45    80.27
## mu[88]     94.03    0.03 1.11    91.91    93.28    94.01    94.81    96.17
## mu[89]    108.79    0.05 2.23   104.31   107.34   108.80   110.27   113.21
## mu[90]    101.62    0.04 1.61    98.48   100.53   101.62   102.72   104.69
## mu[91]    107.72    0.05 2.13   103.42   106.33   107.73   109.14   111.91
## mu[92]     83.00    0.03 1.16    80.77    82.17    83.03    83.80    85.23
## mu[93]     89.91    0.02 0.99    87.97    89.24    89.91    90.61    91.83
## mu[94]    102.99    0.04 1.72    99.64   101.85   103.00   104.19   106.30
## mu[95]    103.71    0.04 1.78   100.18   102.52   103.72   104.94   107.19
## mu[96]     78.07    0.05 1.87    74.42    76.78    78.10    79.34    81.72
## mu[97]     79.42    0.04 1.39    76.75    78.42    79.45    80.40    82.06
## mu[98]     82.65    0.06 1.92    78.95    81.35    82.69    83.89    86.33
## mu[99]     86.89    0.06 2.06    82.87    85.48    86.92    88.25    90.96
## mu[100]    70.30    0.05 2.04    66.41    68.90    70.36    71.71    74.28
```

```
## mu[101]   100.06    0.03 1.49    97.14    99.04   100.05   101.08   102.91
## mu[102]    89.79    0.02 0.99    87.86    89.12    89.79    90.48    91.73
## mu[103]    88.30    0.02 0.99    86.34    87.64    88.32    88.99    90.19
## mu[104]    81.55    0.03 1.25    79.18    80.67    81.57    82.44    83.96
## mu[105]    79.71    0.04 1.37    77.09    78.72    79.73    80.68    82.32
## mu[106]    99.11    0.03 1.42    96.34    98.12    99.10   100.09   101.79
## mu[107]    82.69    0.03 1.18    80.43    81.85    82.72    83.52    84.96
## mu[108]    85.97    0.03 1.03    83.93    85.24    86.00    86.69    87.98
## mu[109]   103.77    0.04 1.79   100.23   102.58   103.77   105.00   107.25
## mu[110]    99.34    0.03 1.43    96.53    98.34    99.33   100.32   102.06
## mu[111]    89.17    0.07 2.17    84.99    87.68    89.20    90.64    93.52
## mu[112]    83.80    0.03 1.12    81.63    83.01    83.85    84.58    85.94
## mu[113]    85.80    0.03 1.04    83.75    85.07    85.82    86.52    87.82
## mu[114]    83.18    0.03 1.15    80.96    82.36    83.21    83.97    85.39
## mu[115]    89.81    0.02 0.99    87.87    89.14    89.80    90.50    91.74
## mu[116]    88.19    0.02 0.99    86.23    87.52    88.21    88.88    90.08
## mu[117]    82.27    0.03 1.20    79.95    81.42    82.30    83.12    84.59
## mu[118]    96.61    0.03 1.25    94.21    95.75    96.60    97.47    99.01
## mu[119]    80.44    0.05 1.89    76.83    79.16    80.48    81.70    84.07
## mu[120]    91.23    0.02 1.01    89.24    90.55    91.20    91.94    93.19
## mu[121]    86.76    0.02 1.01    84.77    86.07    86.79    87.45    88.72
## mu[122]   106.15    0.05 1.99   102.16   104.84   106.16   107.50   110.06
## mu[123]    89.09    0.02 0.99    87.13    88.43    89.08    89.77    90.97
## mu[124]    84.36    0.03 1.10    82.22    83.59    84.40    85.13    86.46
## mu[125]    90.34    0.02 0.99    88.40    89.68    90.34    91.05    92.27
## mu[126]    90.16    0.02 0.99    88.21    89.50    90.16    90.87    92.09
## mu[127]    86.53    0.06 2.05    82.56    85.13    86.56    87.88    90.58
## mu[128]    83.82    0.03 1.12    81.65    83.03    83.87    84.60    85.96
## mu[129]    84.90    0.03 1.07    82.78    84.16    84.94    85.65    86.97
## mu[130]    84.16    0.03 1.10    82.01    83.38    84.20    84.93    86.27
## mu[131]    78.63    0.04 1.45    75.88    77.60    78.66    79.65    81.36
## mu[132]    71.60    0.05 2.03    67.66    70.18    71.61    73.04    75.55
## mu[133]   103.78    0.04 1.79   100.24   102.59   103.79   105.01   107.27
## mu[134]    96.97    0.03 1.27    94.51    96.09    96.95    97.85    99.42
## mu[135]    77.22    0.05 1.88    73.57    75.93    77.24    78.51    80.90
## mu[136]   106.73    0.05 2.04   102.60   105.39   106.74   108.11   110.74
## mu[137]    72.43    0.05 1.95    68.67    71.07    72.46    73.80    76.20
## mu[138]    88.46    0.02 0.99    86.49    87.80    88.46    89.15    90.35
## mu[139]   106.63    0.05 2.03   102.52   105.29   106.64   108.01   110.62
## mu[140]    87.23    0.02 1.00    85.25    86.55    87.24    87.91    89.17
## mu[141]    96.96    0.03 1.27    94.50    96.07    96.94    97.83    99.41
## mu[142]    97.75    0.03 1.32    95.16    96.82    97.73    98.68   100.29
## mu[143]   108.79    0.05 2.23   104.31   107.34   108.80   110.27   113.21
## mu[144]    91.78    0.02 1.02    89.76    91.08    91.74    92.51    93.73
## mu[145]   107.45    0.05 2.11   103.19   106.07   107.45   108.85   111.59
## mu[146]    90.70    0.02 1.00    88.75    90.03    90.69    91.42    92.63
## mu[147]    87.70    0.02 1.00    85.73    87.03    87.72    88.38    89.61
## mu[148]    88.30    0.02 0.99    86.34    87.64    88.32    88.99    90.19
## mu[149]    82.10    0.03 1.21    79.78    81.25    82.13    82.97    84.45
## mu[150]    99.95    0.03 1.48    97.05    98.93    99.93   100.97   102.77
## mu[151]    95.52    0.03 1.18    93.28    94.72    95.50    96.35    97.82
## mu[152]   103.07    0.04 1.73    99.70   101.92   103.07   104.26   106.39
## mu[153]   100.98    0.04 1.56    97.95    99.92   100.98   102.04   103.98
## mu[154]    96.79    0.03 1.26    94.36    95.92    96.78    97.66    99.21
```

```
## mu[155]    83.29    0.03 1.15    81.07    82.48    83.32    84.07    85.47
## mu[156]    81.61    0.03 1.24    79.24    80.74    81.63    82.50    84.02
## mu[157]    92.14    0.02 1.03    90.11    91.43    92.11    92.87    94.10
## mu[158]    90.88    0.02 1.00    88.93    90.21    90.86    91.60    92.82
## mu[159]    90.25    0.02 0.99    88.30    89.58    90.25    90.95    92.18
## mu[160]    93.79    0.02 1.10    91.68    93.05    93.76    94.57    95.89
## mu[161]    75.78    0.05 1.89    72.17    74.52    75.80    77.08    79.44
## mu[162]    92.02    0.07 2.33    87.52    90.40    92.03    93.57    96.64
## mu[163]   102.64    0.04 1.69    99.33   101.51   102.64   103.80   105.88
## mu[164]   100.90    0.04 1.55    97.88    99.84   100.90   101.96   103.89
## mu[165]   100.02    0.03 1.48    97.11    99.00   100.01   101.04   102.87
## mu[166]    84.16    0.03 1.10    82.01    83.38    84.20    84.93    86.27
## mu[167]    87.70    0.02 1.00    85.73    87.03    87.72    88.38    89.61
## mu[168]   101.63    0.04 1.61    98.49   100.55   101.64   102.74   104.71
## mu[169]    89.75    0.07 2.20    85.57    88.23    89.78    91.26    94.16
## mu[170]    97.02    0.03 1.27    94.55    96.13    97.00    97.90    99.48
## mu[171]    96.54    0.03 1.24    94.15    95.68    96.53    97.40    98.94
## mu[172]    84.43    0.03 1.09    82.29    83.67    84.47    85.19    86.53
## mu[173]   100.41    0.04 1.51    97.45    99.38   100.41   101.46   103.32
## mu[174]    82.57    0.03 1.19    80.29    81.73    82.60    83.41    84.86
## mu[175]    72.92    0.05 1.95    69.18    71.56    72.96    74.22    76.72
## mu[176]    87.76    0.02 0.99    85.80    87.09    87.78    88.45    89.67
## mu[177]    95.96    0.03 1.21    93.68    95.12    95.94    96.81    98.28
## mu[178]    86.69    0.02 1.01    84.70    85.99    86.71    87.38    88.66
## mu[179]    82.27    0.03 1.20    79.95    81.42    82.30    83.12    84.59
## mu[180]    99.19    0.03 1.42    96.40    98.19    99.18   100.17   101.88
## mu[181]    80.86    0.06 1.89    77.19    79.56    80.90    82.11    84.53
## mu[182]    79.46    0.04 1.39    76.81    78.47    79.49    80.45    82.10
## mu[183]    89.38    0.02 0.99    87.44    88.71    89.37    90.07    91.27
## mu[184]    87.64    0.02 1.00    85.66    86.97    87.66    88.33    89.55
## mu[185]    81.82    0.06 1.91    78.12    80.51    81.85    83.09    85.53
## mu[186]    86.31    0.02 1.02    84.29    85.60    86.33    87.02    88.31
## mu[187]    92.78    0.02 1.05    90.73    92.06    92.75    93.53    94.80
## mu[188]    88.36    0.02 0.99    86.40    87.70    88.37    89.05    90.25
## mu[189]    87.76    0.02 0.99    85.80    87.09    87.78    88.45    89.67
## mu[190]    96.00    0.03 1.21    93.72    95.17    95.98    96.85    98.32
## mu[191]    78.23    0.05 1.87    74.60    76.95    78.27    79.50    81.90
## mu[192]    75.54    0.05 1.90    71.91    74.27    75.56    76.84    79.22
## mu[193]    85.82    0.06 2.02    81.93    84.42    85.85    87.14    89.79
## mu[194]    84.18    0.06 1.96    80.35    82.83    84.21    85.47    87.96
## mu[195]    87.66    0.02 1.00    85.68    86.98    87.67    88.34    89.57
## mu[196]    86.17    0.06 2.03    82.24    84.78    86.20    87.51    90.20
## mu[197]    87.59    0.02 1.00    85.60    86.91    87.60    88.27    89.50
## mu[198]    89.45    0.02 0.99    87.50    88.78    89.44    90.14    91.34
## mu[199]    84.00    0.03 1.11    81.84    83.22    84.04    84.78    86.13
## mu[200]    97.62    0.03 1.31    95.06    96.69    97.59    98.54   100.13
## mu[201]    79.77    0.04 1.36    77.17    78.79    79.80    80.74    82.38
## mu[202]    91.19    0.07 2.28    86.80    89.61    91.20    92.71    95.72
## mu[203]    95.54    0.03 1.18    93.29    94.73    95.52    96.37    97.84
## mu[204]    91.95    0.02 1.03    89.94    91.26    91.92    92.69    93.92
## mu[205]    85.69    0.03 1.04    83.64    84.96    85.71    86.42    87.71
## mu[206]    69.34    0.06 2.09    65.35    67.92    69.40    70.77    73.42
## mu[207]    81.97    0.03 1.22    79.64    81.10    82.00    82.84    84.34
## mu[208]    95.11    0.03 1.16    92.89    94.33    95.08    95.92    97.39
```

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## mu[209]    94.10    0.03 1.11    91.98    93.35    94.07    94.88    96.24
## mu[210]    93.48    0.02 1.08    91.40    92.74    93.44    94.24    95.56
## mu[211]    86.83    0.06 2.06    82.82    85.42    86.86    88.19    90.90
## mu[212]    90.88    0.02 1.00    88.93    90.21    90.86    91.60    92.82
## mu[213]    86.71    0.06 2.06    82.72    85.30    86.73    88.07    90.77
## mu[214]    93.44    0.02 1.08    91.36    92.70    93.40    94.20    95.52
## mu[215]   107.11    0.05 2.08   102.91   105.74   107.12   108.50   111.20
## mu[216]   104.43    0.04 1.84   100.78   103.21   104.44   105.70   108.02
## mu[217]    88.78    0.02 0.99    86.81    88.11    88.77    89.47    90.67
## mu[218]    80.85    0.03 1.29    78.42    79.92    80.87    81.77    83.33
## mu[219]    98.34    0.03 1.36    95.66    97.38    98.31    99.29   100.93
## mu[220]    99.57    0.03 1.45    96.73    98.56    99.55   100.56   102.32
## mu[221]    82.57    0.03 1.19    80.29    81.73    82.60    83.41    84.85
## mu[222]    76.64    0.04 1.60    73.58    75.50    76.69    77.79    79.69
## mu[223]   108.84    0.05 2.23   104.35   107.38   108.84   110.32   113.27
## mu[224]    89.43    0.02 0.99    87.48    88.77    89.43    90.12    91.33
## mu[225]   106.63    0.05 2.03   102.52   105.29   106.64   108.01   110.62
## mu[226]    85.95    0.03 1.04    83.91    85.22    85.98    86.67    87.96
## mu[227]    98.40    0.03 1.37    95.72    97.45    98.38    99.36   101.00
## mu[228]    98.34    0.03 1.36    95.66    97.38    98.31    99.29   100.93
## mu[229]    88.91    0.02 0.99    86.95    88.24    88.90    89.60    90.79
## mu[230]    79.12    0.05 1.87    75.50    77.84    79.17    80.37    82.79
## mu[231]    79.35    0.04 1.39    76.67    78.35    79.38    80.34    82.00
## mu[232]    80.54    0.03 1.31    78.08    79.60    80.55    81.47    83.03
## mu[233]   104.68    0.04 1.86   100.99   103.45   104.68   105.96   108.30
## mu[234]   103.84    0.04 1.79   100.30   102.65   103.85   105.08   107.34
## mu[235]    91.95    0.02 1.03    89.94    91.26    91.92    92.69    93.92
## mu[236]    91.33    0.02 1.01    89.34    90.64    91.30    92.04    93.28
## mu[237]    97.73    0.03 1.32    95.15    96.80    97.71    98.66   100.27
## mu[238]    99.05    0.03 1.41    96.29    98.06    99.04   100.03   101.73
## mu[239]    89.63    0.02 0.99    87.69    88.96    89.62    90.32    91.54
## mu[240]    87.01    0.06 2.07    82.97    85.59    87.04    88.37    91.09
## mu[241]   103.07    0.04 1.73    99.70   101.92   103.07   104.26   106.39
## mu[242]    89.14    0.02 0.99    87.18    88.48    89.12    89.82    91.02
## mu[243]    81.86    0.06 1.91    78.17    80.56    81.90    83.13    85.57
## mu[244]    83.40    0.03 1.14    81.19    82.60    83.44    84.19    85.58
## mu[245]    79.04    0.04 1.42    76.33    78.04    79.07    80.05    81.73
## mu[246]    79.17    0.04 1.41    76.47    78.17    79.20    80.16    81.84
## mu[247]    76.19    0.04 1.64    73.06    75.03    76.25    77.37    79.32
## mu[248]    83.18    0.03 1.15    80.96    82.36    83.21    83.97    85.39
## mu[249]    82.63    0.03 1.18    80.36    81.78    82.66    83.46    84.91
## mu[250]    76.12    0.04 1.64    72.98    74.96    76.19    77.30    79.27
## mu[251]    93.32    0.02 1.07    91.24    92.58    93.28    94.09    95.40
## mu[252]    74.85    0.05 1.75    71.48    73.62    74.91    76.09    78.17
## mu[253]    86.69    0.02 1.01    84.70    85.99    86.71    87.38    88.66
## mu[254]   104.43    0.04 1.84   100.78   103.21   104.44   105.70   108.02
## mu[255]    90.10    0.07 2.22    85.87    88.58    90.12    91.61    94.54
## mu[256]    90.42    0.07 2.24    86.14    88.87    90.43    91.93    94.88
## mu[257]    95.17    0.03 1.16    92.94    94.38    95.14    95.97    97.45
## mu[258]    68.33    0.06 2.13    64.21    66.89    68.41    69.80    72.48
## mu[259]    71.84    0.05 1.99    68.07    70.47    71.88    73.18    75.72
## mu[260]    89.56    0.02 0.99    87.61    88.89    89.55    90.25    91.46
## mu[261]    88.73    0.02 0.99    86.77    88.06    88.72    89.42    90.62
## mu[262]    95.00    0.03 1.15    92.79    94.22    94.97    95.81    97.26
```

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## mu[263]    95.64    0.08 2.56    90.57    93.91    95.65    97.39   100.71
## mu[264]    80.42    0.03 1.32    77.95    79.48    80.45    81.36    82.92
## mu[265]    88.12    0.02 0.99    86.16    87.45    88.14    88.81    90.01
## mu[266]    89.38    0.02 0.99    87.44    88.71    89.37    90.07    91.27
## mu[267]    81.21    0.03 1.27    78.81    80.30    81.23    82.11    83.66
## mu[268]    71.12    0.05 2.01    67.29    69.75    71.18    72.49    75.04
## mu[269]    71.12    0.05 2.01    67.29    69.75    71.18    72.49    75.04
## mu[270]    82.99    0.03 1.16    80.76    82.15    83.02    83.79    85.22
## mu[271]    79.86    0.04 1.36    77.27    78.89    79.88    80.82    82.46
## mu[272]    95.54    0.03 1.18    93.29    94.73    95.51    96.36    97.84
## mu[273]    94.91    0.03 1.15    92.70    94.14    94.88    95.71    97.16
## mu[274]    88.78    0.02 0.99    86.81    88.11    88.77    89.47    90.67
## mu[275]    72.02    0.05 1.98    68.26    70.64    72.06    73.36    75.89
## mu[276]    71.60    0.05 2.00    67.79    70.23    71.64    72.94    75.49
## mu[277]    77.45    0.04 1.54    74.54    76.35    77.49    78.53    80.35
## mu[278]    99.77    0.03 1.46    96.91    98.76    99.76   100.77   102.55
## mu[279]    89.45    0.02 0.99    87.50    88.78    89.44    90.14    91.34
## mu[280]    82.93    0.03 1.17    80.70    82.09    82.96    83.74    85.16
## mu[281]    90.16    0.02 0.99    88.21    89.50    90.16    90.87    92.09
## mu[282]    89.38    0.02 0.99    87.44    88.71    89.37    90.07    91.27
## mu[283]   100.48    0.04 1.52    97.51    99.44   100.47   101.53   103.40
## mu[284]    75.90    0.05 1.89    72.29    74.64    75.91    77.19    79.54
## mu[285]    76.68    0.05 1.88    73.02    75.41    76.70    77.97    80.35
## mu[286]    72.99    0.05 1.95    69.25    71.63    73.03    74.29    76.78
## mu[287]    83.46    0.03 1.14    81.26    82.66    83.51    84.25    85.64
## mu[288]    80.78    0.03 1.30    78.34    79.85    80.80    81.70    83.27
## mu[289]    71.24    0.05 2.01    67.41    69.86    71.30    72.60    75.16
## mu[290]    73.43    0.05 1.94    69.72    72.11    73.49    74.72    77.19
## mu[291]    69.69    0.06 2.07    65.73    68.29    69.75    71.13    73.75
## mu[292]    72.27    0.05 1.97    68.52    70.89    72.32    73.60    76.12
## mu[293]    75.20    0.05 1.72    71.90    73.98    75.26    76.43    78.45
## mu[294]    82.21    0.03 1.21    79.89    81.35    82.24    83.07    84.54
## mu[295]    70.92    0.05 2.02    67.07    69.54    70.98    72.30    74.86
## mu[296]    77.91    0.04 1.50    75.08    76.85    77.95    78.97    80.73
## mu[297]    71.13    0.05 2.01    67.29    69.75    71.18    72.49    75.05
## mu[298]    69.49    0.06 2.08    65.51    68.09    69.55    70.92    73.56
## mu[299]    72.56    0.05 1.96    68.84    71.19    72.60    73.89    76.39
## mu[300]    81.92    0.03 1.22    79.58    81.05    81.95    82.80    84.29
## mu[301]    81.86    0.03 1.23    79.50    80.99    81.88    82.73    84.24
## mu[302]    76.08    0.05 1.89    72.49    74.82    76.10    77.38    79.76
## mu[303]    98.81    0.03 1.39    96.07    97.83    98.79    99.79   101.46
## mu[304]    78.88    0.04 1.43    76.15    77.86    78.91    79.89    81.58
## mu[305]    78.99    0.04 1.42    76.27    77.97    79.02    80.00    81.68
## mu[306]    90.93    0.02 1.00    88.96    90.26    90.91    91.65    92.87
## mu[307]    92.60    0.02 1.05    90.56    91.88    92.57    93.35    94.60
## mu[308]    86.57    0.02 1.02    84.58    85.87    86.59    87.26    88.54
## mu[309]    90.21    0.02 0.99    88.26    89.54    90.21    90.92    92.14
## mu[310]    80.98    0.06 1.89    77.30    79.68    81.02    82.23    84.64
## mu[311]    81.61    0.03 1.24    79.24    80.74    81.63    82.50    84.02
## mu[312]    92.17    0.02 1.03    90.14    91.47    92.14    92.91    94.14
## mu[313]    82.28    0.03 1.20    79.97    81.43    82.31    83.13    84.60
## mu[314]    93.12    0.02 1.07    91.06    92.39    93.09    93.88    95.18
## mu[315]    88.42    0.02 0.99    86.45    87.76    88.43    89.11    90.31
## mu[316]    98.76    0.03 1.39    96.03    97.78    98.74    99.74   101.40
```

```
## mu[317]    85.51    0.03 1.05    83.46    84.78    85.53    86.25    87.54
## mu[318]    79.40    0.04 1.39    76.74    78.41    79.43    80.39    82.05
## mu[319]    88.48    0.02 0.99    86.52    87.82    88.48    89.17    90.37
## mu[320]    80.67    0.03 1.30    78.22    79.74    80.69    81.60    83.16
## mu[321]    69.76    0.06 2.07    65.79    68.36    69.82    71.19    73.81
## mu[322]    79.70    0.04 1.37    77.08    78.72    79.73    80.68    82.32
## mu[323]    83.95    0.03 1.11    81.78    83.16    83.99    84.72    86.07
## mu[324]    73.77    0.05 1.84    70.20    72.48    73.80    75.07    77.31
## mu[325]    96.72    0.03 1.25    94.29    95.86    96.71    97.59    99.13
## mu[326]    92.25    0.02 1.04    90.22    91.53    92.22    92.99    94.21
## mu[327]    89.31    0.02 0.99    87.36    88.64    89.30    90.00    91.20
## mu[328]    86.87    0.02 1.01    84.88    86.17    86.88    87.55    88.83
## mu[329]    86.27    0.02 1.03    84.25    85.55    86.29    86.97    88.28
## mu[330]    94.57    0.03 1.13    92.41    93.80    94.55    95.36    96.77
## mu[331]    93.44    0.02 1.08    91.36    92.70    93.40    94.20    95.52
## mu[332]    77.97    0.04 1.50    75.15    76.91    78.01    79.02    80.77
## mu[333]    83.95    0.03 1.11    81.78    83.16    83.99    84.72    86.07
## mu[334]    96.90    0.03 1.26    94.45    96.02    96.89    97.77    99.34
## mu[335]    73.28    0.05 1.94    69.55    71.94    73.33    74.57    77.04
## mu[336]    97.26    0.03 1.29    94.76    96.36    97.25    98.16    99.74
## mu[337]    87.66    0.02 1.00    85.68    86.98    87.67    88.34    89.57
## mu[338]    80.78    0.03 1.30    78.34    79.85    80.80    81.70    83.27
## mu[339]    74.82    0.05 1.75    71.45    73.60    74.88    76.07    78.15
## mu[340]    77.09    0.04 1.57    74.11    75.98    77.13    78.20    80.05
## mu[341]    72.38    0.05 1.97    68.64    71.00    72.43    73.71    76.22
## mu[342]    79.63    0.04 1.37    77.00    78.65    79.66    80.61    82.26
## mu[343]    84.12    0.03 1.11    81.96    83.34    84.16    84.89    86.24
## mu[344]    81.56    0.03 1.25    79.19    80.69    81.58    82.46    83.98
## mu[345]    89.79    0.02 0.99    87.86    89.12    89.79    90.48    91.73
## mu[346]    74.89    0.05 1.91    71.20    73.60    74.91    76.19    78.63
## mu[347]    81.16    0.06 1.89    77.49    79.86    81.21    82.41    84.83
## mu[348]    99.95    0.03 1.48    97.05    98.93    99.93   100.97   102.77
## mu[349]    83.36    0.03 1.14    81.14    82.55    83.39    84.14    85.54
## mu[350]    80.96    0.03 1.28    78.55    80.04    80.98    81.87    83.44
## mu[351]    82.63    0.03 1.18    80.36    81.78    82.66    83.46    84.91
## mu[352]    79.06    0.04 1.42    76.35    78.05    79.09    80.06    81.74
## mu[353]    86.15    0.02 1.03    84.13    85.43    86.18    86.86    88.16
## mu[354]    85.61    0.03 1.05    83.57    84.89    85.64    86.35    87.64
## mu[355]    81.67    0.03 1.24    79.31    80.81    81.69    82.56    84.07
## mu[356]    77.88    0.05 1.87    74.23    76.59    77.91    79.16    81.52
## mu[357]    92.60    0.02 1.05    90.57    91.89    92.57    93.35    94.60
## mu[358]    76.91    0.04 1.58    73.89    75.79    76.96    78.04    79.91
## mu[359]    82.23    0.06 1.91    78.52    80.94    82.28    83.48    85.94
## mu[360]    82.39    0.03 1.20    80.09    81.54    82.42    83.23    84.70
## mu[361]    84.61    0.03 1.08    82.48    83.85    84.64    85.37    86.70
## mu[362]    90.16    0.02 0.99    88.21    89.50    90.16    90.87    92.09
## mu[363]    75.05    0.05 1.73    71.72    73.82    75.11    76.28    78.33
## mu[364]    98.03    0.03 1.34    95.39    97.10    98.01    98.97   100.61
## mu[365]    76.60    0.04 1.60    73.53    75.46    76.65    77.75    79.65
## mu[366]    78.75    0.04 1.44    76.00    77.72    78.78    79.76    81.46
## mu[367]    81.65    0.03 1.24    79.28    80.78    81.67    82.54    84.06
## mu[368]    78.27    0.04 1.47    75.48    77.22    78.31    79.32    81.04
## mu[369]    80.60    0.03 1.31    78.15    79.66    80.61    81.53    83.09
## mu[370]    81.65    0.03 1.24    79.28    80.78    81.67    82.54    84.06
```

```
## mu[371]    83.65    0.03 1.13    81.45    82.84    83.69    84.42    85.79
## mu[372]    82.69    0.03 1.18    80.43    81.85    82.72    83.52    84.96
## mu[373]    79.04    0.04 1.42    76.33    78.04    79.07    80.05    81.73
## mu[374]    84.12    0.03 1.11    81.96    83.34    84.16    84.89    86.24
## mu[375]    74.29    0.05 1.92    70.60    73.01    74.33    75.59    78.06
## mu[376]    87.41    0.02 1.00    85.43    86.73    87.42    88.08    89.33
## mu[377]    85.51    0.03 1.05    83.46    84.78    85.53    86.25    87.54
## mu[378]    88.66    0.02 0.99    86.70    87.99    88.66    89.35    90.56
## mu[379]    80.06    0.04 1.34    77.51    79.11    80.08    81.01    82.62
## mu[380]    78.27    0.04 1.47    75.48    77.22    78.31    79.32    81.04
## mu[381]    87.30    0.02 1.00    85.32    86.62    87.31    87.98    89.23
## mu[382]    72.20    0.05 1.98    68.44    70.82    72.24    73.54    76.05
## mu[383]    74.17    0.05 1.92    70.48    72.88    74.20    75.47    77.93
## mu[384]    70.66    0.05 2.03    66.79    69.26    70.71    72.05    74.59
## mu[385]    68.33    0.06 2.13    64.21    66.89    68.41    69.80    72.48
## mu[386]    77.20    0.04 1.56    74.24    76.09    77.24    78.30    80.14
## mu[387]    97.02    0.03 1.27    94.55    96.13    97.00    97.90    99.48
## mu[388]    86.44    0.02 1.02    84.43    85.73    86.46    87.14    88.42
## mu[389]    71.48    0.05 2.00    67.67    70.11    71.52    72.83    75.38
## mu[390]    76.41    0.04 1.62    73.31    75.26    76.47    77.57    79.50
## mu[391]    86.93    0.02 1.01    84.94    86.23    86.94    87.61    88.88
## mu[392]    93.09    0.02 1.07    91.03    92.37    93.06    93.86    95.16
## mu[393]    71.37    0.05 2.00    67.55    70.00    71.42    72.73    75.28
## mu[394]    88.48    0.02 0.99    86.52    87.82    88.48    89.17    90.37
## mu[395]    83.00    0.03 1.16    80.77    82.17    83.03    83.80    85.23
## mu[396]    78.39    0.04 1.47    75.61    77.35    78.42    79.42    81.13
## mu[397]    82.21    0.03 1.21    79.89    81.35    82.24    83.07    84.54
## mu[398]   105.68    0.05 1.95   101.78   104.40   105.69   107.01   109.53
## mu[399]   101.99    0.04 1.64    98.77   100.89   102.00   103.11   105.12
## mu[400]    83.29    0.03 1.15    81.07    82.48    83.32    84.08    85.47
## mu[401]    78.36    0.05 1.87    74.72    77.07    78.39    79.63    82.02
## mu[402]    77.73    0.05 1.87    74.10    76.44    77.77    79.02    81.39
## mu[403]    75.92    0.05 1.89    72.31    74.65    75.93    77.21    79.56
## mu[404]   102.88    0.04 1.71    99.54   101.75   102.89   104.07   106.16
## mu[405]    73.81    0.05 1.93    70.11    72.50    73.86    75.11    77.58
## mu[406]    72.67    0.05 1.96    68.95    71.31    72.72    74.00    76.50
## mu[407]    89.08    0.02 0.99    87.12    88.42    89.07    89.76    90.96
## mu[408]    88.19    0.02 0.99    86.23    87.52    88.21    88.88    90.08
## mu[409]    93.75    0.02 1.09    91.64    93.00    93.72    94.52    95.84
## mu[410]    75.85    0.05 1.89    72.24    74.59    75.87    77.15    79.50
## mu[411]    91.89    0.02 1.03    89.87    91.19    91.85    92.62    93.85
## mu[412]    85.38    0.03 1.05    83.32    84.66    85.40    86.13    87.42
## mu[413]    92.78    0.02 1.05    90.73    92.06    92.75    93.53    94.80
## mu[414]    93.93    0.03 1.10    91.81    93.18    93.90    94.70    96.05
## mu[415]    97.62    0.03 1.31    95.06    96.69    97.59    98.54   100.14
## mu[416]    74.78    0.05 1.91    71.10    73.49    74.81    76.08    78.54
## mu[417]    86.27    0.02 1.03    84.25    85.55    86.29    86.97    88.28
## mu[418]    76.80    0.05 1.88    73.15    75.54    76.82    78.10    80.48
## mu[419]    81.97    0.03 1.22    79.64    81.10    82.00    82.84    84.34
## mu[420]    67.70    0.06 2.16    63.51    66.22    67.77    69.18    71.90
## mu[421]    77.73    0.05 1.87    74.10    76.44    77.77    79.02    81.39
## mu[422]    80.33    0.05 1.88    76.72    79.06    80.38    81.59    83.96
## mu[423]    80.87    0.06 1.89    77.20    79.57    80.92    82.12    84.54
## mu[424]   105.97    0.05 1.98   102.02   104.68   105.99   107.32   109.87
```

```
## mu[425]    75.69    0.04 1.68    72.47    74.51    75.75    76.89    78.88
## mu[426]   103.78    0.04 1.79   100.25   102.59   103.79   105.02   107.27
## mu[427]   101.99    0.04 1.64    98.77   100.89   102.00   103.11   105.12
## mu[428]    71.24    0.05 2.01    67.41    69.86    71.30    72.60    75.16
## mu[429]    68.06    0.06 2.15    63.92    66.59    68.13    69.53    72.23
## mu[430]    73.75    0.05 1.93    70.05    72.44    73.80    75.05    77.52
## mu[431]    84.07    0.03 1.11    81.92    83.30    84.11    84.85    86.19
## mu[432]    79.42    0.05 1.88    75.80    78.13    79.46    80.68    83.10
## mu[433]    86.27    0.02 1.03    84.25    85.55    86.29    86.97    88.28
## mu[434]    83.09    0.03 1.16    80.87    82.26    83.12    83.89    85.31
## lp__    -1474.08    0.05 1.34 -1477.44 -1474.77 -1473.79 -1473.03 -1472.38
##         n_eff Rhat
## alpha    1150    1
## beta[1]  1109    1
## beta[2]  1459    1
## sigma    1493    1
## mu[1]    1826    1
## mu[2]    1526    1
## mu[3]    1884    1
## mu[4]    1799    1
## mu[5]    1599    1
## mu[6]    1116    1
## mu[7]    1675    1
## mu[8]    1783    1
## mu[9]    1431    1
## mu[10]   1662    1
## mu[11]   1512    1
## mu[12]   1656    1
## mu[13]   1519    1
## mu[14]   1893    1
## mu[15]   1146    1
## mu[16]   1834    1
## mu[17]   1891    1
## mu[18]   1905    1
## mu[19]   1707    1
## mu[20]   1170    1
## mu[21]   1923    1
## mu[22]   1767    1
## mu[23]   1751    1
## mu[24]   1814    1
## mu[25]   1167    1
## mu[26]   1819    1
## mu[27]   1777    1
## mu[28]   1751    1
## mu[29]   1422    1
## mu[30]   1892    1
## mu[31]   1932    1
## mu[32]   1867    1
## mu[33]   1860    1
## mu[34]   1114    1
## mu[35]   1707    1
## mu[36]   1602    1
## mu[37]   1687    1
## mu[38]   1939    1
```

```
## mu[39]   1491    1
## mu[40]   1526    1
## mu[41]   1884    1
## mu[42]   1736    1
## mu[43]   1446    1
## mu[44]   1777    1
## mu[45]   1299    1
## mu[46]   1769    1
## mu[47]   1415    1
## mu[48]   1898    1
## mu[49]   1921    1
## mu[50]   1798    1
## mu[51]   1882    1
## mu[52]   1786    1
## mu[53]   1656    1
## mu[54]   1590    1
## mu[55]   1867    1
## mu[56]   1377    1
## mu[57]   1697    1
## mu[58]   1344    1
## mu[59]   1416    1
## mu[60]   1924    1
## mu[61]   1905    1
## mu[62]   1896    1
## mu[63]   1142    1
## mu[64]   1855    1
## mu[65]   1864    1
## mu[66]   1780    1
## mu[67]   1618    1
## mu[68]   1492    1
## mu[69]   1492    1
## mu[70]   1477    1
## mu[71]   1535    1
## mu[72]   1119    1
## mu[73]   1131    1
## mu[74]   1924    1
## mu[75]   1547    1
## mu[76]   1518    1
## mu[77]   1899    1
## mu[78]   1399    1
## mu[79]   1672    1
## mu[80]   1328    1
## mu[81]   1293    1
## mu[82]   1672    1
## mu[83]   1909    1
## mu[84]   1765    1
## mu[85]   1537    1
## mu[86]   1794    1
## mu[87]   1428    1
## mu[88]   1919    1
## mu[89]   1689    1
## mu[90]   1797    1
## mu[91]   1702    1
## mu[92]   1560    1
```

```
## mu[93]   1900    1
## mu[94]   1772    1
## mu[95]   1760    1
## mu[96]   1217    1
## mu[97]   1460    1
## mu[98]   1146    1
## mu[99]   1115    1
## mu[100]  1390    1
## mu[101]  1826    1
## mu[102]  1896    1
## mu[103]  1829    1
## mu[104]  1512    1
## mu[105]  1466    1
## mu[106]  1845    1
## mu[107]  1549    1
## mu[108]  1701    1
## mu[109]  1759    1
## mu[110]  1840    1
## mu[111]  1110    1
## mu[112]  1593    1
## mu[113]  1691    1
## mu[114]  1567    1
## mu[115]  1896    1
## mu[116]  1824    1
## mu[117]  1534    1
## mu[118]  1887    1
## mu[119]  1175    1
## mu[120]  1932    1
## mu[121]  1745    1
## mu[122]  1723    1
## mu[123]  1867    1
## mu[124]  1618    1
## mu[125]  1914    1
## mu[126]  1908    1
## mu[127]  1117    1
## mu[128]  1594    1
## mu[129]  1644    1
## mu[130]  1609    1
## mu[131]  1446    1
## mu[132]  1385    1
## mu[133]  1759    1
## mu[134]  1882    1
## mu[135]  1234    1
## mu[136]  1715    1
## mu[137]  1389    1
## mu[138]  1837    1
## mu[139]  1716    1
## mu[140]  1771    1
## mu[141]  1882    1
## mu[142]  1869    1
## mu[143]  1689    1
## mu[144]  1938    1
## mu[145]  1705    1
## mu[146]  1923    1
```

```
## mu[147]  1797    1
## mu[148]  1829    1
## mu[149]  1529    1
## mu[150]  1828    1
## mu[151]  1898    1
## mu[152]  1771    1
## mu[153]  1809    1
## mu[154]  1885    1
## mu[155]  1572    1
## mu[156]  1514    1
## mu[157]  1939    1
## mu[158]  1926    1
## mu[159]  1911    1
## mu[160]  1923    1
## mu[161]  1265    1
## mu[162]  1112    1
## mu[163]  1778    1
## mu[164]  1810    1
## mu[165]  1827    1
## mu[166]  1609    1
## mu[167]  1797    1
## mu[168]  1796    1
## mu[169]  1110    1
## mu[170]  1881    1
## mu[171]  1888    1
## mu[172]  1622    1
## mu[173]  1819    1
## mu[174]  1545    1
## mu[175]  1331    1
## mu[176]  1801    1
## mu[177]  1895    1
## mu[178]  1741    1
## mu[179]  1534    1
## mu[180]  1843    1
## mu[181]  1169    1
## mu[182]  1461    1
## mu[183]  1880    1
## mu[184]  1794    1
## mu[185]  1156    1
## mu[186]  1719    1
## mu[187]  1937    1
## mu[188]  1832    1
## mu[189]  1801    1
## mu[190]  1894    1
## mu[191]  1213    1
## mu[192]  1270    1
## mu[193]  1120    1
## mu[194]  1131    1
## mu[195]  1795    1
## mu[196]  1118    1
## mu[197]  1791    1
## mu[198]  1883    1
## mu[199]  1602    1
## mu[200]  1872    1
```

```
## mu[201]  1468    1
## mu[202]  1111    1
## mu[203]  1898    1
## mu[204]  1939    1
## mu[205]  1685    1
## mu[206]  1410    1
## mu[207]  1525    1
## mu[208]  1901    1
## mu[209]  1917    1
## mu[210]  1929    1
## mu[211]  1116    1
## mu[212]  1926    1
## mu[213]  1116    1
## mu[214]  1929    1
## mu[215]  1710    1
## mu[216]  1748    1
## mu[217]  1853    1
## mu[218]  1492    1
## mu[219]  1859    1
## mu[220]  1836    1
## mu[221]  1545    1
## mu[222]  1419    1
## mu[223]  1689    1
## mu[224]  1882    1
## mu[225]  1716    1
## mu[226]  1699    1
## mu[227]  1858    1
## mu[228]  1859    1
## mu[229]  1859    1
## mu[230]  1197    1
## mu[231]  1459    1
## mu[232]  1485    1
## mu[233]  1744    1
## mu[234]  1757    1
## mu[235]  1939    1
## mu[236]  1934    1
## mu[237]  1870    1
## mu[238]  1846    1
## mu[239]  1890    1
## mu[240]  1115    1
## mu[241]  1771    1
## mu[242]  1869    1
## mu[243]  1155    1
## mu[244]  1576    1
## mu[245]  1453    1
## mu[246]  1456    1
## mu[247]  1415    1
## mu[248]  1567    1
## mu[249]  1547    1
## mu[250]  1414    1
## mu[251]  1931    1
## mu[252]  1403    1
## mu[253]  1741    1
## mu[254]  1748    1
```

```
## mu[255]  1110    1
## mu[256]  1110    1
## mu[257]  1900    1
## mu[258]  1430    1
## mu[259]  1355    1
## mu[260]  1887    1
## mu[261]  1851    1
## mu[262]  1901    1
## mu[263]  1122    1
## mu[264]  1482    1
## mu[265]  1820    1
## mu[266]  1880    1
## mu[267]  1502    1
## mu[268]  1372    1
## mu[269]  1372    1
## mu[270]  1560    1
## mu[271]  1469    1
## mu[272]  1898    1
## mu[273]  1901    1
## mu[274]  1853    1
## mu[275]  1351    1
## mu[276]  1361    1
## mu[277]  1429    1
## mu[278]  1832    1
## mu[279]  1883    1
## mu[280]  1558    1
## mu[281]  1908    1
## mu[282]  1880    1
## mu[283]  1818    1
## mu[284]  1262    1
## mu[285]  1245    1
## mu[286]  1329    1
## mu[287]  1579    1
## mu[288]  1491    1
## mu[289]  1369    1
## mu[290]  1319    1
## mu[291]  1403    1
## mu[292]  1346    1
## mu[293]  1406    1
## mu[294]  1532    1
## mu[295]  1376    1
## mu[296]  1435    1
## mu[297]  1372    1
## mu[298]  1407    1
## mu[299]  1339    1
## mu[300]  1523    1
## mu[301]  1521    1
## mu[302]  1258    1
## mu[303]  1850    1
## mu[304]  1451    1
## mu[305]  1452    1
## mu[306]  1927    1
## mu[307]  1938    1
## mu[308]  1734    1
```

```
## mu[309]  1910    1
## mu[310]  1167    1
## mu[311]  1514    1
## mu[312]  1939    1
## mu[313]  1535    1
## mu[314]  1934    1
## mu[315]  1835    1
## mu[316]  1851    1
## mu[317]  1675    1
## mu[318]  1460    1
## mu[319]  1838    1
## mu[320]  1488    1
## mu[321]  1401    1
## mu[322]  1466    1
## mu[323]  1599    1
## mu[324]  1396    1
## mu[325]  1886    1
## mu[326]  1939    1
## mu[327]  1877    1
## mu[328]  1751    1
## mu[329]  1717    1
## mu[330]  1907    1
## mu[331]  1929    1
## mu[332]  1436    1
## mu[333]  1599    1
## mu[334]  1883    1
## mu[335]  1322    1
## mu[336]  1878    1
## mu[337]  1795    1
## mu[338]  1491    1
## mu[339]  1403    1
## mu[340]  1425    1
## mu[341]  1343    1
## mu[342]  1465    1
## mu[343]  1607    1
## mu[344]  1512    1
## mu[345]  1896    1
## mu[346]  1285    1
## mu[347]  1165    1
## mu[348]  1828    1
## mu[349]  1574    1
## mu[350]  1495    1
## mu[351]  1547    1
## mu[352]  1454    1
## mu[353]  1711    1
## mu[354]  1681    1
## mu[355]  1515    1
## mu[356]  1220    1
## mu[357]  1938    1
## mu[358]  1423    1
## mu[359]  1151    1
## mu[360]  1538    1
## mu[361]  1630    1
## mu[362]  1908    1
```

```
## mu[363]  1405    1
## mu[364]  1865    1
## mu[365]  1419    1
## mu[366]  1448    1
## mu[367]  1515    1
## mu[368]  1441    1
## mu[369]  1486    1
## mu[370]  1515    1
## mu[371]  1586    1
## mu[372]  1549    1
## mu[373]  1453    1
## mu[374]  1607    1
## mu[375]  1299    1
## mu[376]  1781    1
## mu[377]  1675    1
## mu[378]  1847    1
## mu[379]  1474    1
## mu[380]  1441    1
## mu[381]  1775    1
## mu[382]  1347    1
## mu[383]  1301    1
## mu[384]  1382    1
## mu[385]  1430    1
## mu[386]  1426    1
## mu[387]  1881    1
## mu[388]  1727    1
## mu[389]  1363    1
## mu[390]  1417    1
## mu[391]  1754    1
## mu[392]  1934    1
## mu[393]  1366    1
## mu[394]  1838    1
## mu[395]  1560    1
## mu[396]  1442    1
## mu[397]  1532    1
## mu[398]  1729    1
## mu[399]  1790    1
## mu[400]  1572    1
## mu[401]  1211    1
## mu[402]  1223    1
## mu[403]  1262    1
## mu[404]  1774    1
## mu[405]  1310    1
## mu[406]  1336    1
## mu[407]  1867    1
## mu[408]  1824    1
## mu[409]  1924    1
## mu[410]  1263    1
## mu[411]  1939    1
## mu[412]  1669    1
## mu[413]  1937    1
## mu[414]  1921    1
## mu[415]  1872    1
## mu[416]  1287    1
```

```
## mu[417]  1717    1
## mu[418]  1242    1
## mu[419]  1525    1
## mu[420]  1442    1
## mu[421]  1223    1
## mu[422]  1177    1
## mu[423]  1169    1
## mu[424]  1725    1
## mu[425]  1410    1
## mu[426]  1758    1
## mu[427]  1790    1
## mu[428]  1369    1
## mu[429]  1436    1
## mu[430]  1311    1
## mu[431]  1605    1
## mu[432]  1192    1
## mu[433]  1717    1
## mu[434]  1564    1
## lp__      822    1
##
## Samples were drawn using NUTS(diag_e) at Wed Feb 12 16:01:31 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

Interpretation:

Since the coefficient estimate was about 0.57 we can say that for every IQ point higher than average the estimated kid score increases by 0.57

## 4.4 Question 5

Confirm the results from Stan agree with `lm()`

```
linmod2 <- lm(data = kidiq, formula = kid_score ~ mom_hs + mom_iq)
summary(linmod2)
```

```
##
## Call:
## lm(formula = kid_score ~ mom_hs + mom_iq, data = kidiq)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -52.873 -12.663   2.404  11.356  49.545
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 25.73154    5.87521   4.380 1.49e-05 ***
## mom_hs       5.95012    2.21181   2.690  0.00742 **
## mom_iq       0.56391    0.06057   9.309  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.14 on 431 degrees of freedom
## Multiple R-squared:  0.2141, Adjusted R-squared:  0.2105
## F-statistic: 58.72 on 2 and 431 DF,  p-value: < 2.2e-16
```

It seems to agree very well: 0.56 vs 0.57 for LM vs STAN for the IQ 5.95 vs 5.64 for LM vs STAN for the Age and finall 25.7 vs 82.39 for LM vs STAN which does not agree at all.

### 4.5 Question 6

Plot the posterior estimates of scores by education of mother for mothers who have an IQ of 110.

```
fit4 %>%
  spread_draws(alpha, beta[condition], sigma) %>%
  pivot_wider(names_from = condition, names_prefix = "beta", values_from = beta) %>%
     mutate(nhs = alpha + beta2 * 10, # no high school is just the intercept
            hs = alpha + beta1 + beta2 * 10) %>%
  pivot_longer(nhs:hs, names_to = "education", values_to = "estimated_score") %>%
  ggplot(aes(y = education, x = estimated_score)) +
  stat_halfeyeh() +
  theme_bw() +
  ggtitle("Posterior estimates of scores by education level of mother for mothers with IQ 110")
```