

Lab experience

Global warming

LABORATORY EXPERIENCE 3

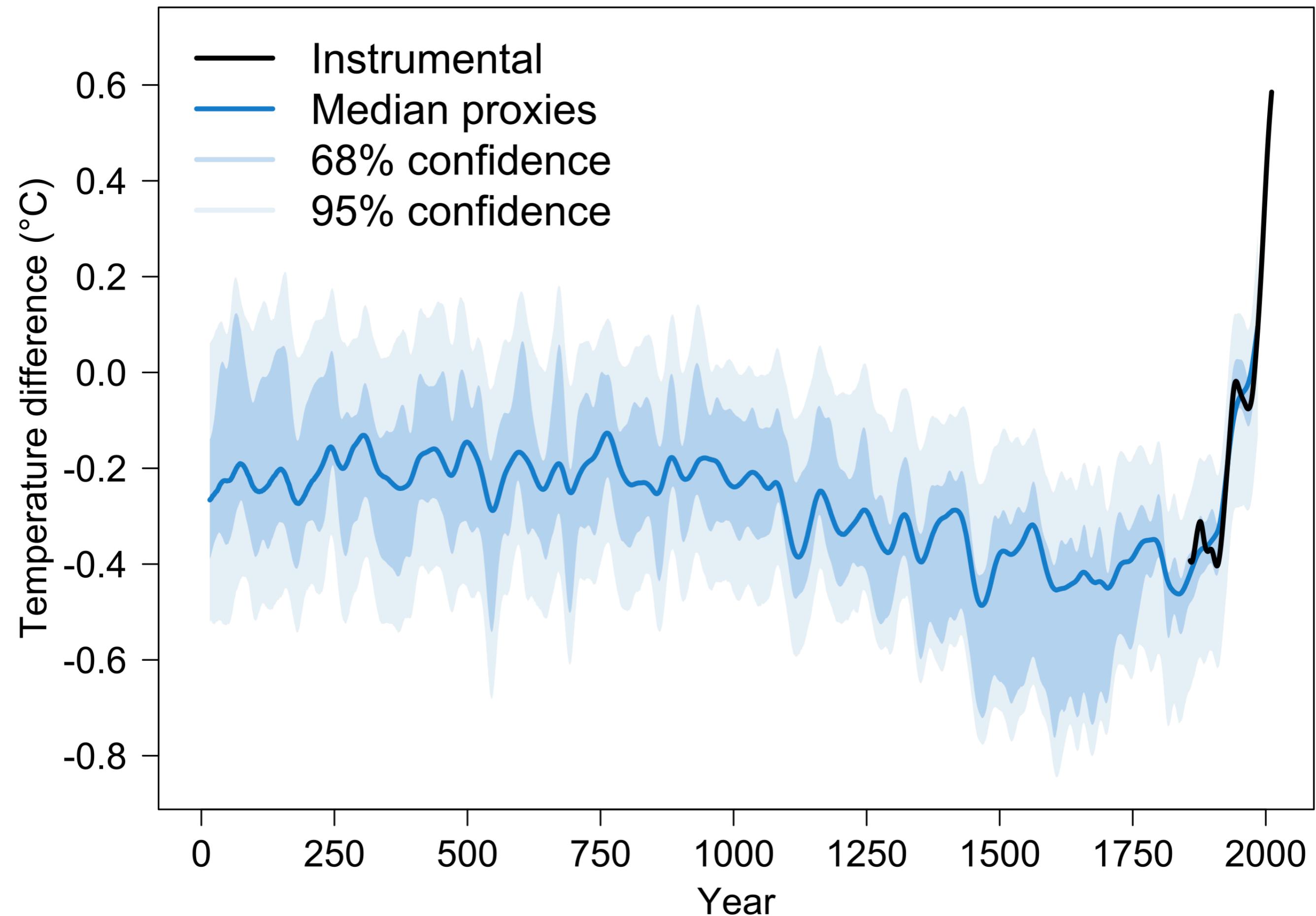
GLOBAL WARMING



Mean global temperature record

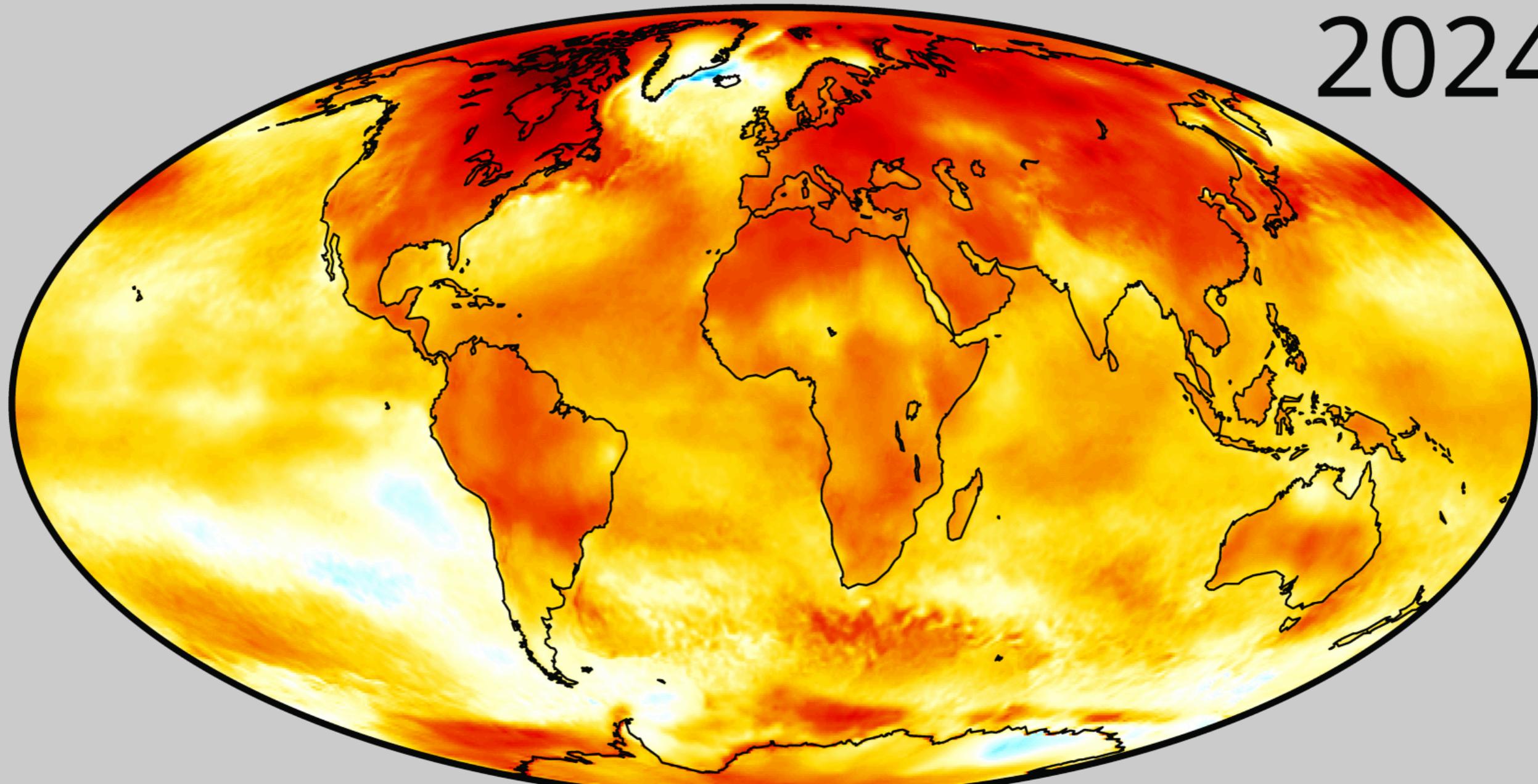
- Since 1850, we have global historical **record measurements** of the mean Earth temperature
- For older dates, we need to resort to **temperature proxies**:
 - **Tree rings** widths (dendroclimatology) for the last ~1000 years
 - **Ice cores** can identify temperature changes from the abundance of oxygen isotopes (last 800 kyr)
 - Fossils, sediments, and geologic studies for even older periods

Temperature last two millennia



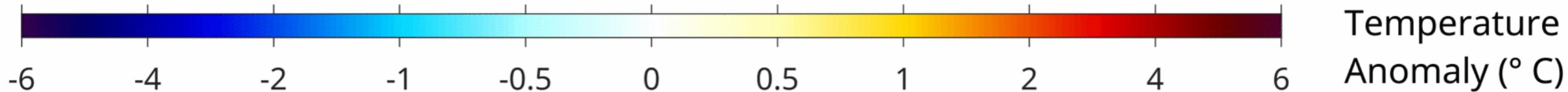
Global Warming and local changes

2024



Relative to 1951-1980 Averages

www.BerkeleyEarth.org



Temperature
Anomaly ($^{\circ}$ C)

Thermal equilibrium

- Earth's temperature is determined from an **equilibrium condition**: heat received from the Sun is equal to heat dispersed into space
- In the simplest model, both quantities can be computed using **Stefan-Boltzmann's law**
- In the computation, one needs to take into account the **albedo**, that is the amount of light reflected into space (for example because of the clouds).

Exercise. Suppose the Earth has an albedo of $A = 0.3$ (that is, 30% of the Sun light is reflected back into space). What is its equilibrium temperature?

Solution

$$P_{\text{in}} = \sigma T_{\odot}^4 (1 - A) \frac{R_{\odot}^2}{a^2} \pi R_{\oplus}^2 = \sigma T_{\oplus}^4 \cdot 4\pi R_{\oplus}^2 = P_{\text{out}}$$

and therefore

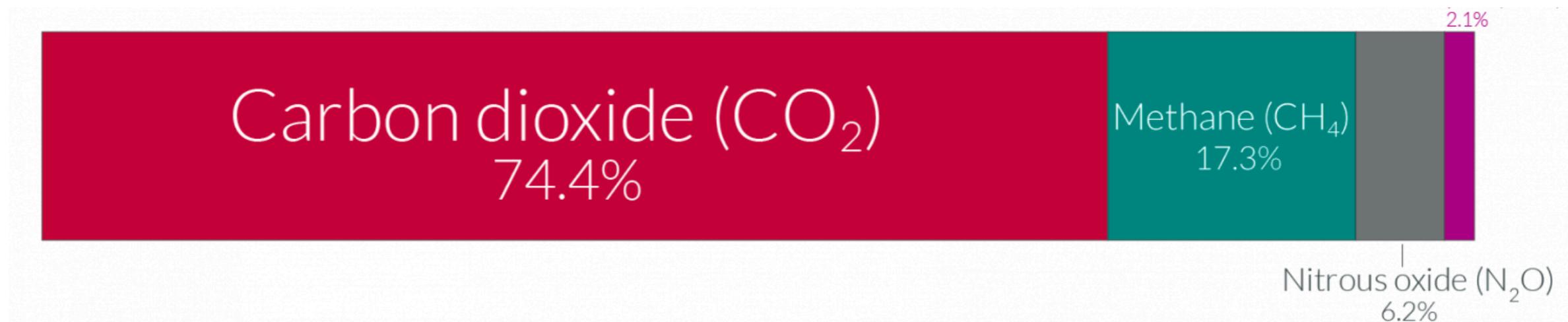
$$T_{\oplus} = T_{\odot} \sqrt[4]{1 - A} \sqrt{\theta/2} \simeq 255 \text{ K}$$

where $\theta \equiv R_{\odot}/a = 0.00465$ is the angular radius of the Sun expressed in radians.

Observation. This result is, somewhat surprisingly, about 33 K less than the average global temperature on Earth. Can you explain this difference?

Greenhouse effect!

- This temperature difference is mostly due to the **greenhouse effect**
- Some gases (CO_2 , CH_4 e N_2O) in the atmosphere are transparent in the visible radiation, but are **opaque in the infrared (IR)**



- The Sun radiation P_{in} (mostly in the visible) arrives mostly undisturbed; however, the emitted radiation P_{out} (in the IR) is mostly reflected back to the Earth
- Although the whole problem is very complex, we can describe the main characteristic with a simple model...

Troposphere and stratosphere

- The atmosphere is made of various layers. For our purposes, we just need to consider two of them:

Troposphere: occupies the first 11 km and has a lapse rate of $6.5 \text{ }^{\circ}\text{C} / \text{km}$; it is relatively dense and opaque in the IR

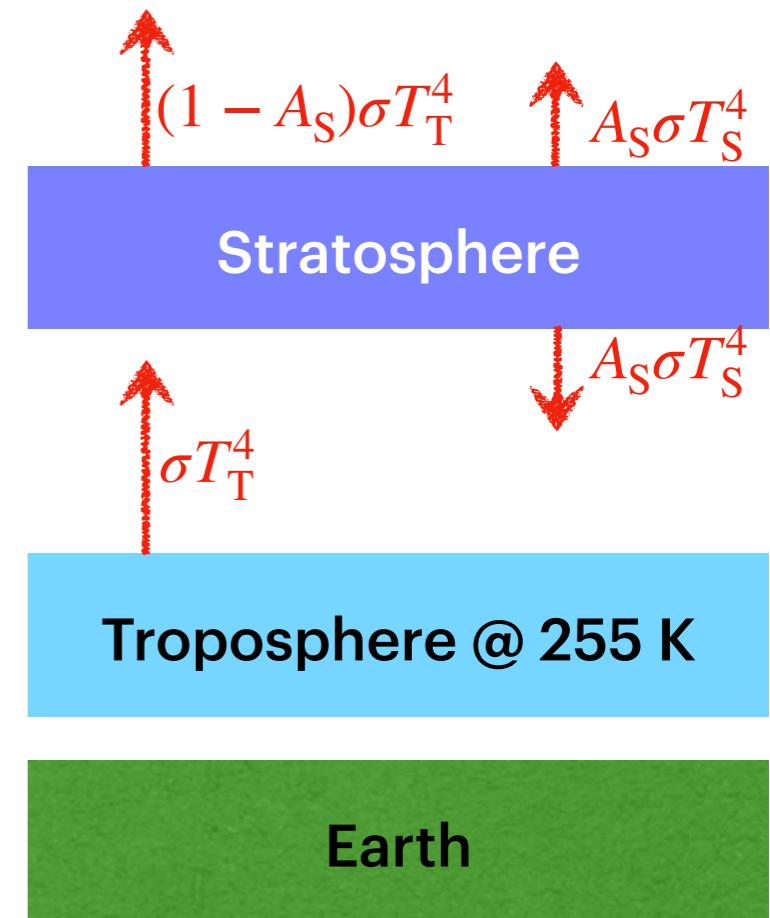
Stratosphere: reaches 50–60 km and is a far less dense layer; it is separate by the troposphere by the **tropopause**

- The stratosphere, not very dense and essentially without greenhouse gases, is transparent to the IR: for this reason its **temperature is constant**

Two-layer model

- The top of the troposphere has an effective temperature $T_T = 255 \text{ K}$, i.e. the temperature we computed earlier
- Assuming an albedo A_S for the stratosphere, its temperature T_S can be obtained from the energy balance:

$$A_S \sigma T_T^4 = 2A_S \sigma T_S^4 \implies T_S = 214 \text{ K}$$

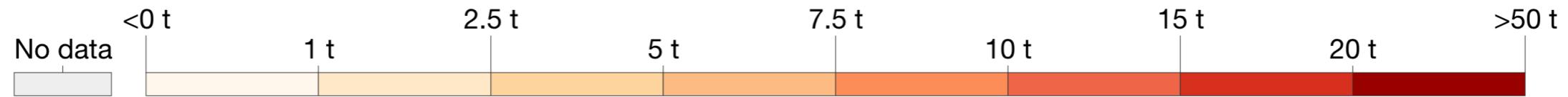
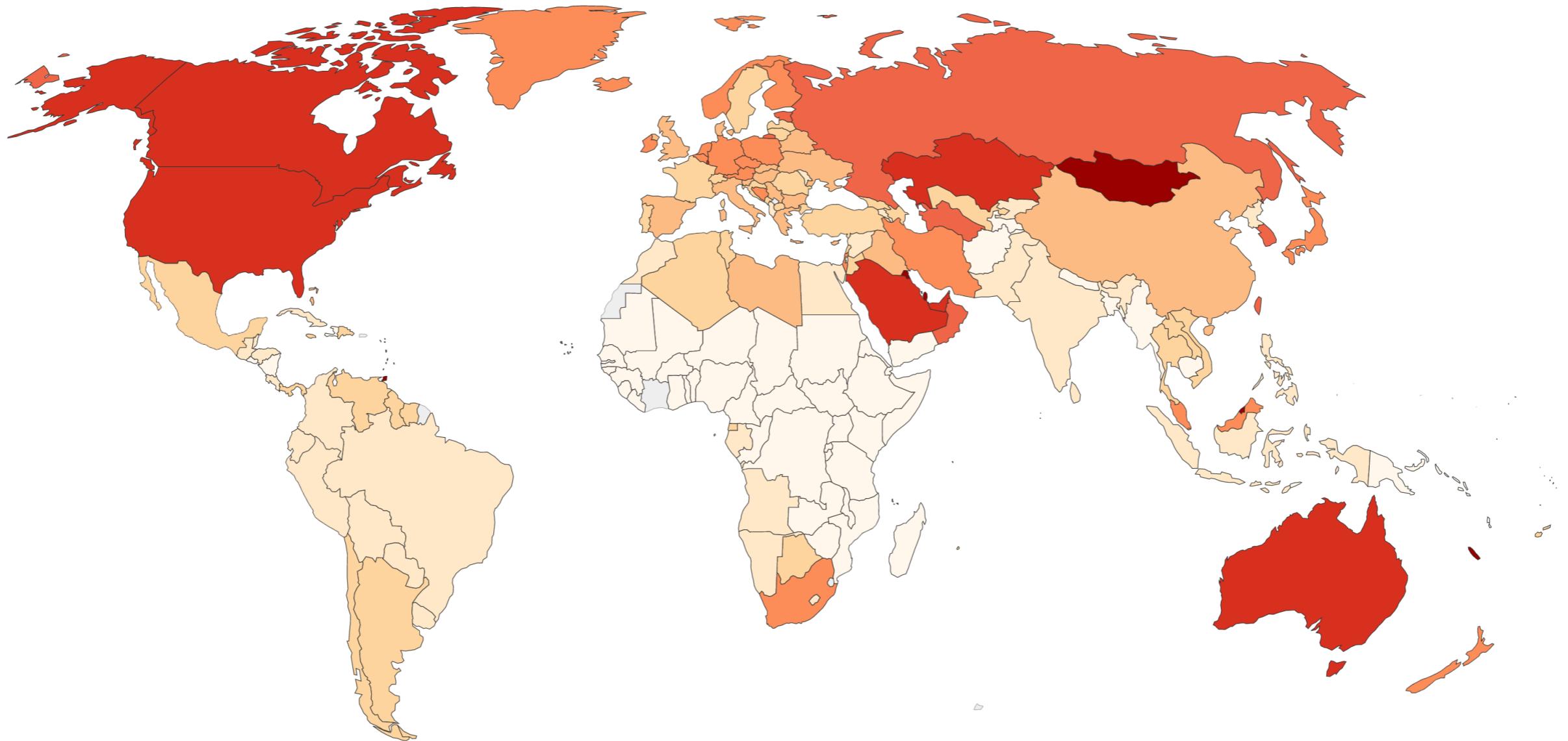


- The Earth surface temperature is computed using the lapse rate and the height off the troposphere:

$$T_\oplus = T_S + 6.5 \times 11 = 286 \text{ K}$$

Per capita CO₂ emissions, 2019

Carbon dioxide (CO₂) emissions from the burning of fossil fuels for energy and cement production. Land use change is not included.



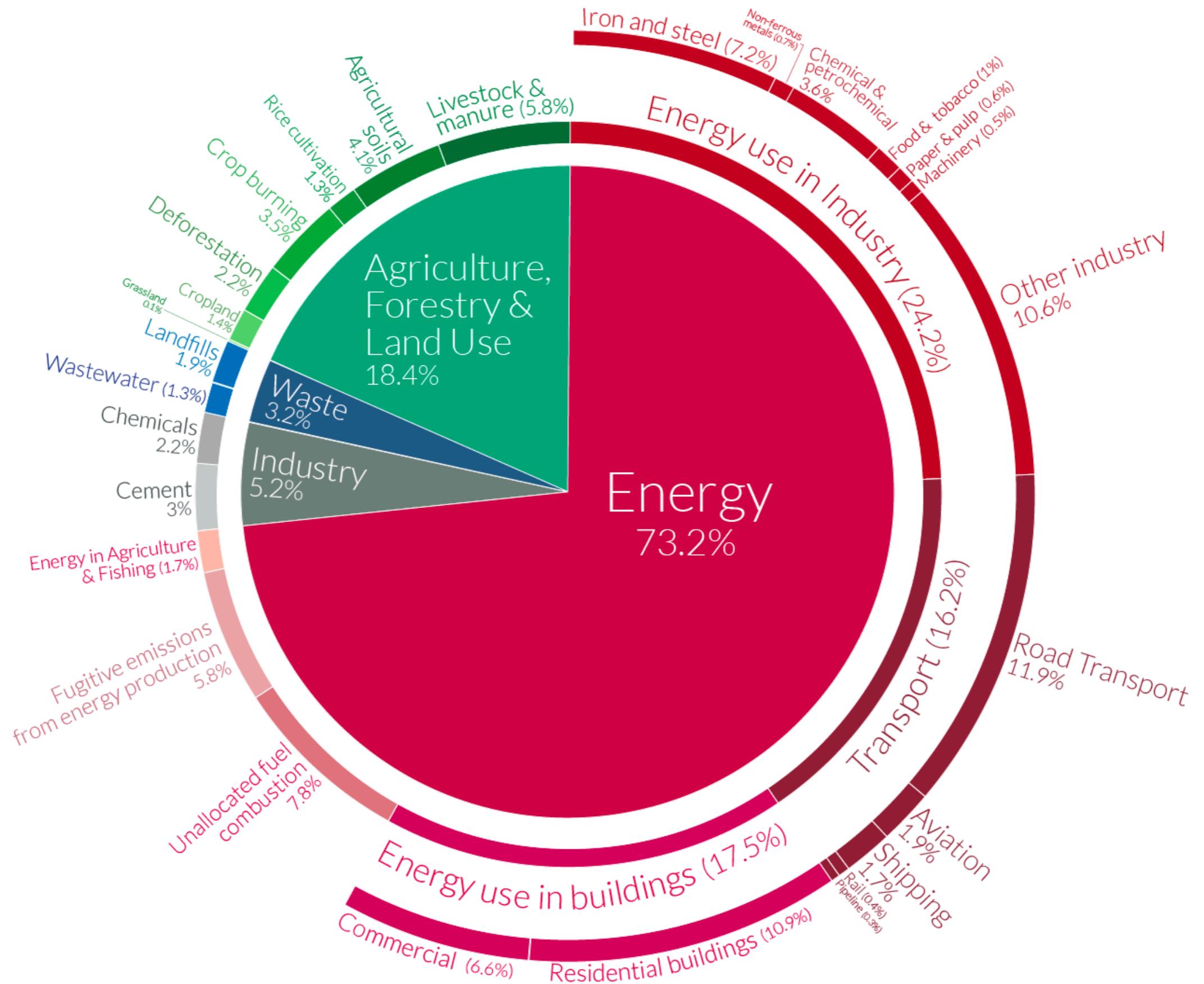
Source: Our World in Data based on the Global Carbon Project; Gapminder & UN

Note: CO₂ emissions are measured on a production basis, meaning they do not correct for emissions embedded in traded goods.

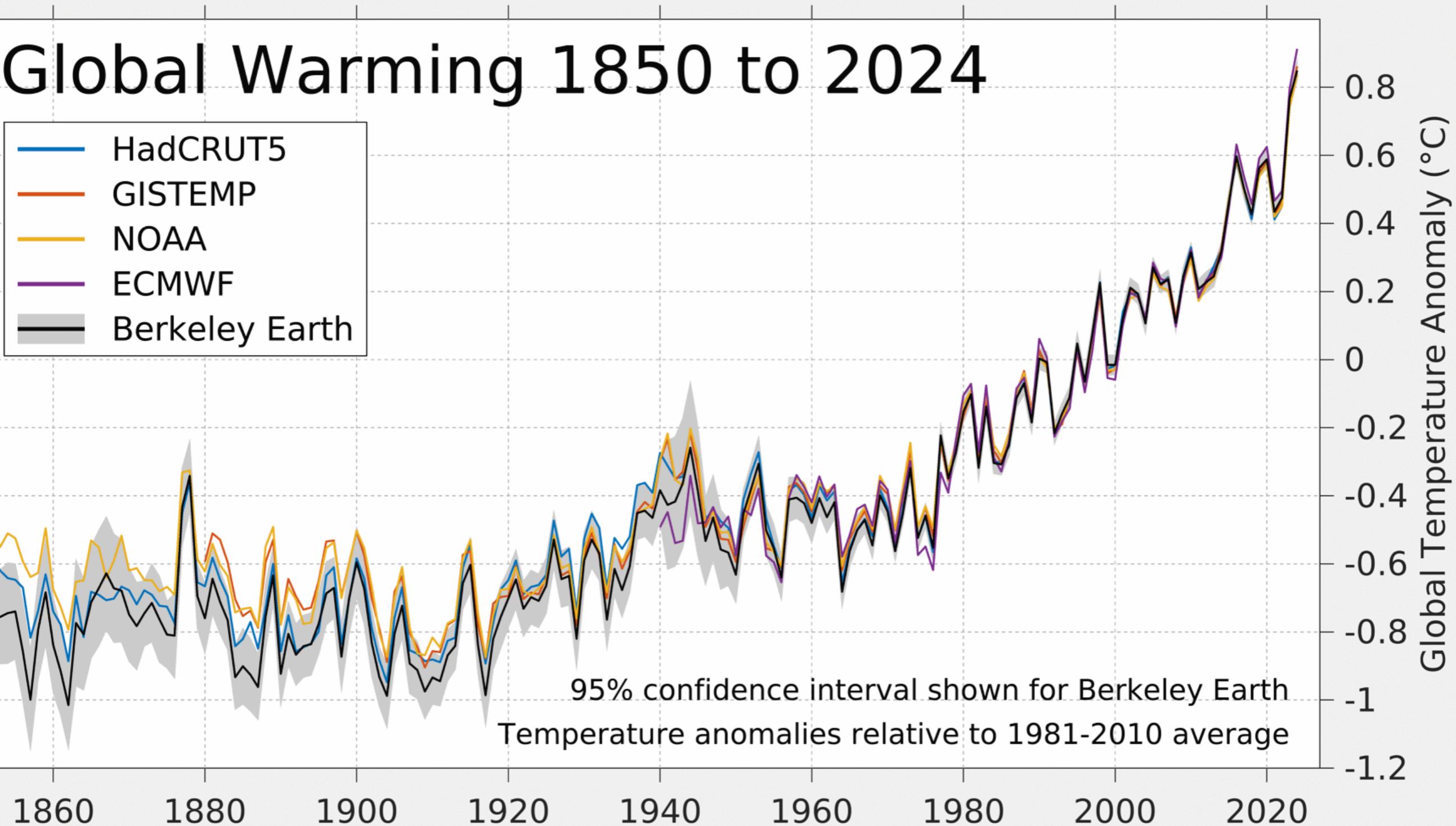
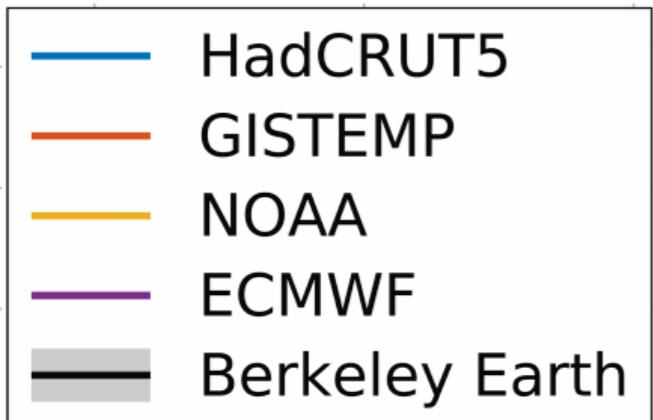
OurWorldInData.org/co2-and-other-greenhouse-gas-emissions/ • CC BY

Global greenhouse gas emissions by sector

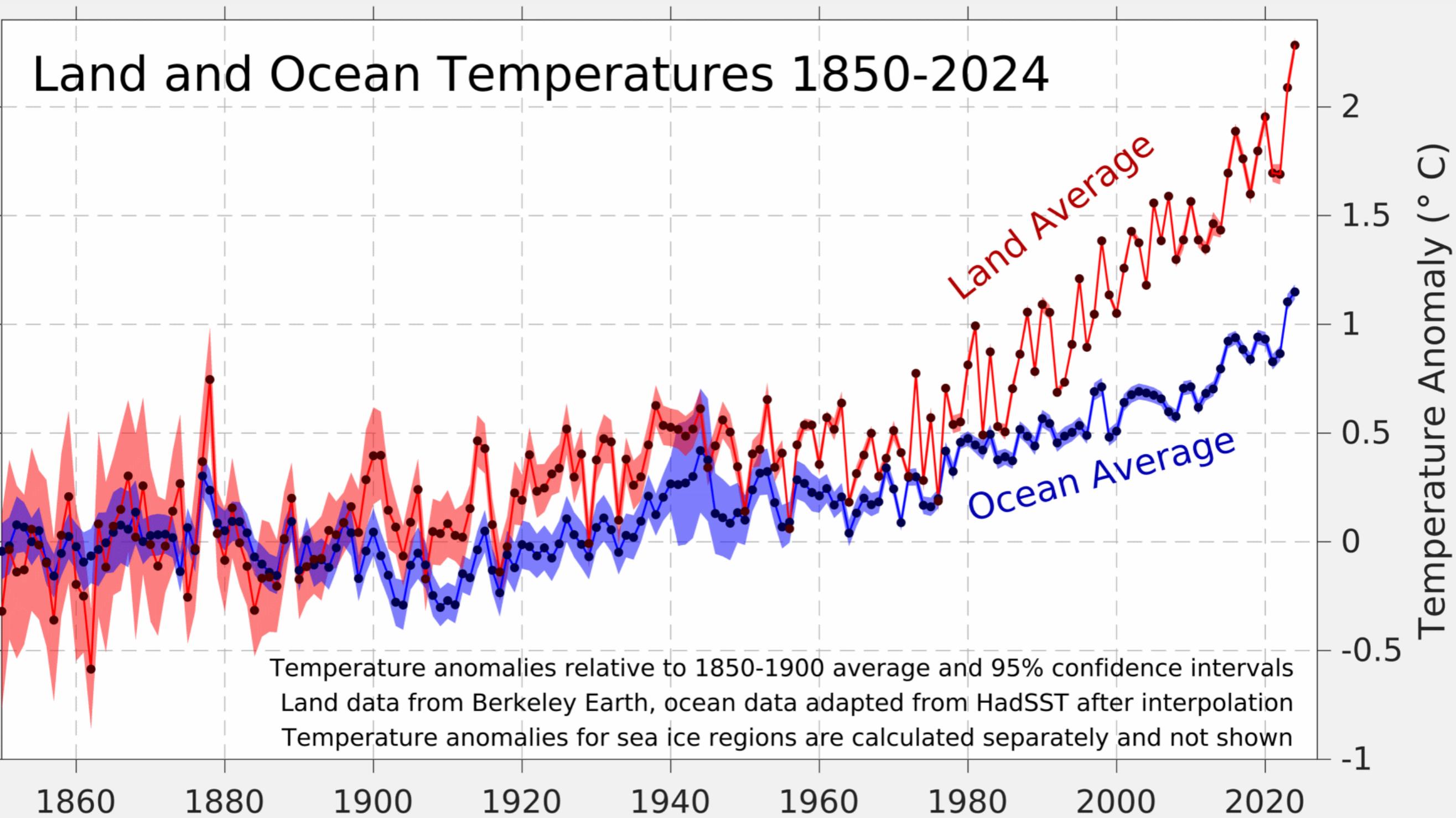
This is shown for the year 2016 – global greenhouse gas emissions were 49.4 billion tonnes CO₂eq.



Global Warming 1850 to 2024



Land and Ocean Temperatures 1850-2024



Task

- Model the **global mean temperature historical record** (1850-present) with a number of models
 - Perform **Bayesian inference** on the models' parameters and **model inference** using the Bayesian evidence
 - Use at least the following models: constant temperature, linear growth, exponential growth, gaussian process with or without trend
- Data: **Berkeley Earth** “Land + Ocean (1850 – Recent)” data
<http://berkeleyearth.org/data/>
- Inference library: **Ultranest**
<https://johannesbuchner.github.io/UltraNest/index.html>
- Gaussian Process library: **George**
<https://george.readthedocs.io/en/latest/>

Gaussian Processes

- **Informal definition:** a probability distribution over possible functions that fit a set of points
- A Gaussian Process (GP) is a sort of **interpolation**: given a set of evaluations $y_i = f(x_i)$ of a real-valued function, estimate the function at some other points $\{x_i^*\}$.
- However, a GP also provides **uncertainties** on the interpolated values
- It also naturally implements the idea of **smoothness** of the interpolated function through the covariance (**kernel**)

Gaussian Process for our model

- GPs can be used to interpolate the mean global temperature record...
 - Depending on the model, we can then obtain an estimate of the future trend
 - A GPs interpolation also provides the evidence of the interpolation!
 - Useful to assess the merit of this specific model
 - GPs do not need to assume a zero-mean: we can interpolate also a function with a trend
 - For example, in case of linear trend, one can infer at the same time the two parameters for the linear function (slope and intercept) together with the GPs kernel parameter

References on GPs

THE BOOK

Ramussen & Williams, "Gaussian Processes for Machine Learning", MIT Press, 2006

<http://www.gaussianprocess.org/gpml/chapters/RW.pdf>

A nice tutorial

<https://www.youtube.com/watch?v=92-98SYOdIY>