# Rotational Velocity in the Milky Way

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## Abstract

In this report we present a model for the rotation of stars in the Milky Way around its center. Firstly (in a simpler model), we assume that the stars, as well as the LSR (Local Standard of Rest), move in circular orbits around the origin of the GC (Galactic Center) frame with a constant velocity $V_{\mathrm{rot}}$ tangential to the orbit, and that the Sun is moving with respect to the LSR with a velocity $U_\odot$ in the x-direction and $V_\odot$ in the y-direction in the GC frame. Then, we present a more complex model, in which for each star, a random component $v'_{\mathrm{rand}} \sim N(0, \sigma)$ is added to its velocity.
In both cases we aim to model the velocity of stars in the radial direction in the Sun frame of reference. Using Bayesian inference on the data provided by GAIA DR2 [**GAIADR2**] we find the distributions of parameters of the two models, namely $\theta_{\mathbf{1}} = (V_{\mathrm{rot}}, U_\odot, V_\odot)$ and $\theta_{\mathbf{2}} = (V_{\mathrm{rot}}, U_\odot, V_\odot, \sigma)$. For each star on the galactic plane we consider its measured longitude, parallax, and radial velocity, with respect to the Sun frame of reference. We also consider the provided uncertainties of the measurements of parallax and radial velocity.
We used MCMC (Monte Carlo Markov Chain) implementation as emcee package in python [**EMCEE**] to estimate the posterior of the two models.
In the first model, we estimate the parameters and their 95% confidence interval to be: $V_{\mathrm{rot}} = 211.45^{+0.04}_{-0.04}$ $\mathrm{km\,s^{-1}}$; $U_\odot = 11.638^{+0.005}_{-0.005}$ $\mathrm{km\,s^{-1}}$; $U_\odot = 21.604^{+0.005}_{-0.005}$ $\mathrm{km\,s^{-1}}$.
With the second model instead, we got: $V_{\mathrm{rot}} = 204^{+2}_{-2}$ $\mathrm{km\,s^{-1}}$; $U_\odot = 11.7^{+0.3}_{-0.3}$ $\mathrm{km\,s^{-1}}$; $U_\odot = 21.7^{+0.3}_{-0.3}$ $\mathrm{km\,s^{-1}}$; $\sigma = 30.6^{+0.2}_{-0.2}$ $\mathrm{km\,s^{-1}}$.

## 1 Data

In this work we use data collected by GAIA DR2 [**GAIADR2**]. From the vast dataset of stars analyzed by GAIA, we select only those for which radial velocities $v_{\mathrm{rad}}$ were measured relative to the Sun using the Doppler effect. To manage the dataset size efficiently, we then apply a random selection to significantly reduce the quantity of data (imposing the random index of data to be less than 100000000).

For each selected star, we extract key parameters from GAIA DR2, including parallax $p$ and its associated error $\sigma_{\mathrm{p}}$, radial velocity $v_{\mathrm{rad}}$ with its measurement uncertainty $\sigma_{\mathrm{v}}$, and galactic coordinates, i.e. latitude $b$ and longitude $l$.

To focus our analysis on stars located approximately on the galactic plane, we impose a selection criterion of $|b| < 5°$. Additionally, to ensure the reliability of the data, we retain only stars with a relative parallax error smaller than 20% and a radial velocity error below 5 $\mathrm{km\,s^{-1}}$. Following these criteria, 75659 stars were selected. The distribution of their radial velocity with respect to the Sun is plotted in the upper panel of figure 1 as a function of their longitude. Only values in the interval [-200, 200] $\mathrm{km\,s^{-1}}$ were reported for better visualization. However, values up to about 500 $\mathrm{km\,s^{-1}}$ were observed. These are likely outliers, and a more sophisticated data selection procedure may improve the quality of the following analysis.

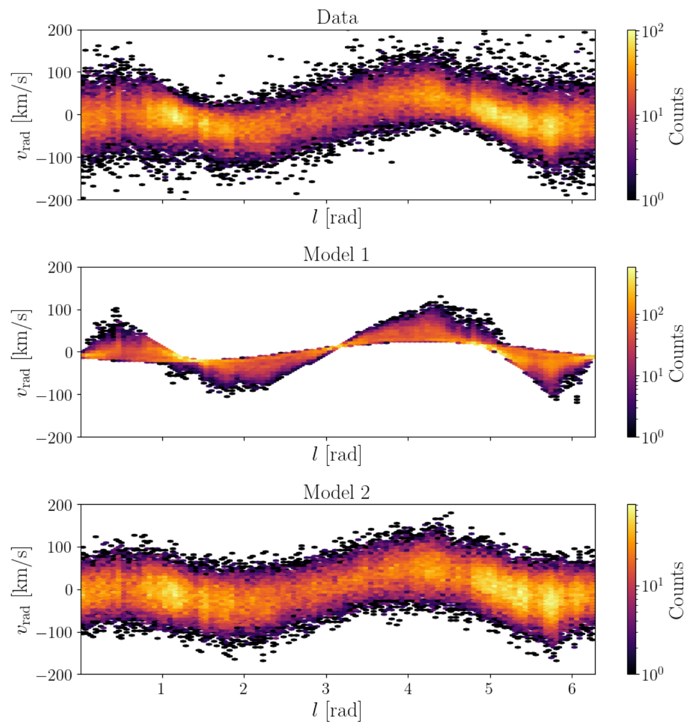


Figure 1: Upper: radial velocities in the interval $[-200,\ 200]$ $\mathrm{km\,s^{-1}}$ of the data. Middle: predictions of the radial velocities with the first model. Lower: simulation of the distribution of the radial velocities with the second model.

# 2 Model

In this section we present the physical and the statistical models we developed for the study of the Galactic kinematics of the Milky Way. Our physical assumptions, presented in the paragraph 2.1, lead to a prediction for the radial motion of stars with respect to the Sun as a function of their longitude and parallax. The prediction, for each star, can then be compared to the direct measurement. In the paragraph 2.2 we formally present our statistical assumptions on the data and the parameters of the two models with their priors and likelihoods.

## 2.1 Physical Model

In our model, we only consider the motion on the galactic plane ($b \approx 0$). We assume that each star moves with a circular orbit around the Galactic Center GC with the same speed $V_{\mathrm{rot}}$. Fixing the frame of reference as in figure 2, a generic star s moves with velocity $\boldsymbol{v}_{\mathrm{s}}$ given by:

$$\boldsymbol{v}_{\mathrm{s}} = V_{\mathrm{rot}} \begin{pmatrix} \sin(\varphi) \\ -\cos(\varphi) \end{pmatrix} \tag{1}$$

where $\varphi$ is the angle from the x-axis of the star with respect to the GC. In the following, primate vectors are in the frame of reference of the Sun, whereas unprimed ones are in the frame of reference of the GC. Angles are supposed to be expressed as radians.

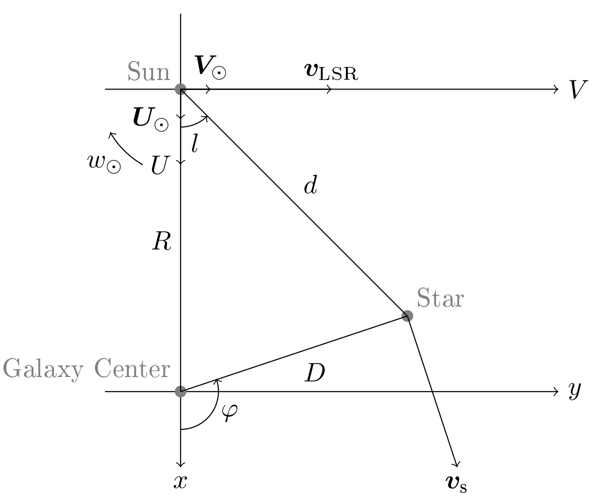


Figure 2: Frames of reference. The angular frequency $w_\odot$ only contributes to the tangential component of the velocities of the stars in the Sun's frame of reference, and is therefore omitted in equation 3.

In this frame of reference, the Sun has $\varphi = \pi$, and we assume it to be at a fixed distance $R = 8300\,\mathrm{pc}$ from the GC [**GalacticKinematics**]. In this model, the drift velocity of the Local Standard of Rest LSR is also given by equation 1. In addition, the Sun has its own peculiar motion with respect to the LSR, with components, $U_\odot$ along the $x$-axis, and $V_\odot$ along the $y$ one. Therefore, the total velocity $\boldsymbol{v}_\odot$ of the Sun is:

$$\boldsymbol{v}_\odot = \begin{pmatrix} 0 \\ V_{\mathrm{rot}} \end{pmatrix} + \begin{pmatrix} U_\odot \\ V_\odot \end{pmatrix} \tag{2}$$

The velocity $\boldsymbol{v}_s'$ of a star s in the frame of reference of the Sun is given by the equation:

$$\boldsymbol{v}_s' = \boldsymbol{v}_s - \boldsymbol{v}_\odot \tag{3}$$

We can now project the velocity $\boldsymbol{v}_s'$ onto radial direction with respect to the Sun $\hat{e}_r'$, obtaining the radial component $v_{\mathrm{rad}'}$ of the velocity of a star with longitude $l$ in the Sun's frame of reference:

$$\begin{aligned} v_{\mathrm{rad}'} &= \boldsymbol{v}_s' \cdot \hat{e}_r' = \\ &= V_{\mathrm{rot}} \Big[ \sin\varphi \cos l - (1 + \cos\varphi) \sin l \Big] - U_\odot \cos l - V_\odot \sin l \end{aligned} \tag{4}$$

Eq.4 must be adapted to the actual data provided by GAIA, which means expressing $sin\varphi$ and $cos\varphi$ in terms of $l$ and the parallax $p$, expressed in milliarcseconds (mas). First of all, the distance $d$ in parsec of a star from the Sun can be computed as:

$$d[\mathrm{pc}] = \frac{1000}{p[\mathrm{mas}]} \tag{5}$$

Then, by geometric considerations we can evaluate the distance $D$ of a star from the GC, $\sin\varphi$ and $\cos\varphi$ as a function of $R$, $d$ and $l$ (see figure 2), obtaining:

$$\begin{aligned} D =& \sqrt{d^2 + R^2 - 2dR\cos l} \\ \sin\varphi =& \pm\sqrt{1 - \cos^2\varphi} = \frac{d \sin l}{D} \\ \cos\varphi =& \frac{d\cos l - R}{D} \end{aligned} \tag{6}$$

By substituting eq.5-6 into eq.4, we get an expression for the prediction of the model for the radial component of the velocity of star $i$ $\hat{v}^{(1)}_{\mathrm{rad},i}$ as a function of the measurements of its longitude $l_i$ and parallax $p_i$:

$$\begin{aligned} \hat{v}^{(1)}_{\mathrm{rad},i}(l_i, p_i) =& V_{\mathrm{rot}} \sin l_i \left( \frac{R}{\sqrt{(\frac{1000}{p_i})^2 + R^2 - 2(\frac{1000}{p_i}) R \cos l_i}} - 1 \right) - \\ & - U_\odot \cos l_i - V_\odot \sin l_i \end{aligned} \tag{7}$$

In a second, more sophisticated model, we also consider the random motion of all stars. Neglecting the difference of mass between the stars, we assume that the components of their velocities along the radial direction with respect to the Sun $v_{\mathrm{rand,i}}$ are random variables extracted from the same gaussian distribution, centered in 0 with standard deviation $\sigma$, which has the dimensions of a velocity ($v_{\mathrm{rand,i}} \sim N(0, \sigma)$). The prediction of this model is therefore:

$$\begin{aligned} \hat{v}^{(2)}_{\mathrm{rad},i}(l_i, p_i) =& V_{\mathrm{rot}} \sin l_i \left( \frac{R}{\sqrt{(\frac{1000}{p_i})^2 + R^2 - 2(\frac{1000}{p_i}) R \cos l_i}} - 1 \right) - \\ & - U_\odot \cos l_i - V_\odot \sin l_i + v_{\mathrm{rand,i}} \end{aligned} \tag{8}$$

## 2.2 Statistical Model

In the following we describe the Bayesian inference we have made on the parameters of the two models. We used Monte Carlo Markov Chains (MCMC) provided by the emcee package in python [**EMCEE**] to get an approximation of the posteriors of our models. We provided emcee with the $\mathbb{P}^*(\theta|\mathcal{D})$ - the numerator of Bayes' theorem (posterior) to sample from:

$$\mathbb{P}^*(\theta|\mathcal{D}) = \mathcal{L}(\mathcal{D}|\theta)\mathbb{P}(\theta) \tag{9}$$

where $\mathcal{L}(\mathcal{D}|\theta)$ is the likelihood and $\mathbb{P}(\theta)$ is the prior. In practice, the logarithms of these quantities were used to achieve numerical stability.

In both models, we assumed each measurement and each parameter to be independent of all the others. Under this assumption, the likelihood and the prior are a product of individual terms, so the logarithms can be written as sums of terms. Then, we assume each measured value $m_{measured}$ to be the sum of its true value $m_{true}$, and a random error $\epsilon$ coming from a normal distribution centered at 0 with standard deviation $\sigma_m$, given by the statistical uncertainty provided by GAIA database ($\epsilon \sim N(0, \sigma_\mathrm{m})$). So we have: $m_{measured} = m_{true} + \epsilon$.

For the first model we have a set of three parameters $\boldsymbol{\theta_1} = (V_\mathrm{rot}, U_\odot, V_\odot)$. Neglecting the uncertainties associated to the parallax measurements, the difference between the measure of the radial velocity $v_{\mathrm{rad},i}$ and its prediction $\hat{v}^{(1)}_{\mathrm{rad},i}$ is a random variable extracted from a normal distribution centered in 0, with standard deviation only given by the statistical uncertainty on the measurements of the radial velocity $\sigma_{\mathrm{v},i}$. The log-likelihood of this model is therefore given by the sum of independent terms as:

$$\log \mathcal{L}^{(1)}(\mathcal{D}|\theta_1) = -\frac{1}{2}\sum_i [\log(2\pi\sigma^2_{\mathrm{v},i}) + \frac{(v_{\mathrm{rad},i} - \hat{v}^{(1)}_{\mathrm{rad},i})^2}{\sigma^2_{\mathrm{v},i}}] \tag{10}$$

Then, we chose a flat prior for $V_\mathrm{rot} \in [0, 500\,\mathrm{km\,s^{-1}}]$, in order to consider typical values of the rotational motion of stars in spiral barred (Sb) galaxies which are found in the range [144, 330] $\mathrm{km\,s^{-1}}$[**Schneider2015**]. For $U_\odot$ and $V_\odot$ we chose a gaussian prior centered in 0, $\log \mathbb{P}^{(1)}(U_\odot) + \log \mathbb{P}^{(1)}(U_\odot) \sim -\frac{U^2_\odot + V^2_\odot}{v^2_\mathrm{gal}}$, assuming the peculiar motion of the Sun to be analogous to a stochastic thermal motion. As a value for $v_\mathrm{gal}$, we chose 200 $\mathrm{km\,s^{-1}}$ since it is the typical scale of stars' velocities in a Sb galaxy.

In our second model, there are four parameters: $\boldsymbol{\theta_2} = (V_\mathrm{rad}, U_\odot, V_\odot, \sigma)$. Accounting also for the errors on the parallax measurements, the model prediction $\hat{v}^{(2)}_{\mathrm{rad},i}$ (see equation 8) is a random variable extracted from a normal distribution centered in 0 with variance given by the sum of the variance of the random component $\sigma^2$, and the contribution $\left(\frac{\partial \hat{v}^{(2)}_{\mathrm{rad},i}}{\partial \mathrm{p}_i}\right)^2 \sigma^2_{\mathrm{p}_i}$ originating from the error on the parallax measurement, where the derivative can be computed analytically from equation 8. Then, the difference between the measured value of the radial velocity $v_{\mathrm{rad},i}$ and the model prediction $\hat{v}^{(2)}_{\mathrm{rad},i}$ is a random variable extracted from a normal distribution centered in 0 with total variance given by the sum of the variances of the error on the radial velocity measurements, and that on the model prediction, which results in:

$$\sigma^2_{\mathrm{tot},i} = \sigma^2_{\mathrm{v},i} + \left(\frac{\partial \hat{v}^{(2)}_{\mathrm{rad},i}}{\partial \mathrm{p}_i}\right)^2 \sigma^2_{\mathrm{p}_i} + \sigma^2 \tag{11}$$

Under these assumptions, the log-likelihood of this model is:

$$\log \mathcal{L}^{(2)}(\mathcal{D}|\theta_2) = -\frac{1}{2}\sum_i \left\{ \log[2\pi(\sigma^2_i + \sigma^2)] + \frac{(v_{\mathrm{rad},i} - \hat{v}^{(2)}_{\mathrm{rad},i})^2}{\sigma^2_{\mathrm{tot},i}} \right\} \tag{12}$$

We keep for the first three parameters the same priors decided in the first model, and we assume the fourth parameter $\sigma$ to be uncorrelated to the others. We then chose for it the non-informative prior of the standard deviation of a gaussian likelihood, $\log \mathbb{P}(\sigma) = -\log(\sigma)$ [**mackay2003**], since, in our model, it has a similar role.

# 3 Conclusions

The median values of the posterior probability distributions of the two models, and their 95% confidence intervals, are reported in table 1. It can be observed that the estimates for $U_\odot$ and $V_\odot$ of the two models are compatible with one another, whereas the 95% confidence interval of the two estimates of $V_\mathrm{rot}$ do not overlap. Both estimates of $U_\odot$ are in good agreement with that found in literature [**LocalKinematics**], whereas both estimates of $V_\odot$ are far from being compatible with those in the same [**LocalKinematics**]. The results for $V_\mathrm{rot}$ were also compared to those obtained in literature [**GalacticKinematics**]. Our estimates result to be lower, but both models yield a non-null overlap of their respective 95% confidence interval with that of the value found in the literature. The posterior distributions we obtained for the parameters are reported in Appendix A. Finally, the middle and lower panels of fig.1 present the predictions of the two models, made with the median values obtained for the parameters, using the longitude and parallax of the sample of stars. In the second model, we simulated the contribution of $v_{\mathrm{rand},i}$ by adding a random number, sampled from a Gaussian centered in 0, with standard deviation given by the median value found for $\sigma$. Visually, it is clear that the second model reproduces the data better than the first one, and that the random motion of stars is crucial to model the radial velocities of stars correctly.

| Parameter | Model 1 | Model 2 | Literature |
|---|---|---|---|
| $V_\mathrm{rot}$ [$\mathrm{km\,s^{-1}}$] | $211.45^{+0.04}_{-0.04}$ | $204^{+2}_{-2}$ | $225 \pm 10$ [**GalacticKinem** |
| $U_\odot$ [$\mathrm{km\,s^{-1}}$] | $11.638^{+0.004}_{-0.005}$ | $11.7^{+0.3}_{-0.3}$ | $11.1^{+0.7}_{-0.8}$ [**LocalKinema** |
| $V_\odot$ [$\mathrm{km\,s^{-1}}$] | $21.604^{+0.005}_{-0.005}$ | $21.7^{+0.3}_{-0.3}$ | $12.2^{+0.5}_{-0.5}$ [**LocalKinema** |
| $\sigma$ [$\mathrm{km\,s^{-1}}$] | — | $30.6^{+0.2}_{-0.2}$ | — |

Table 1: Estimated parameters and their 95% confidence intervals for the two models. The value found in the literature for $V_\mathrm{rot}$ is reported with its total uncertainty (one standard deviation), obtained by summing in quadrature the statistical and systematic uncertainties reported in the original article [**GalacticKinematics**].

# A Posterior distributions

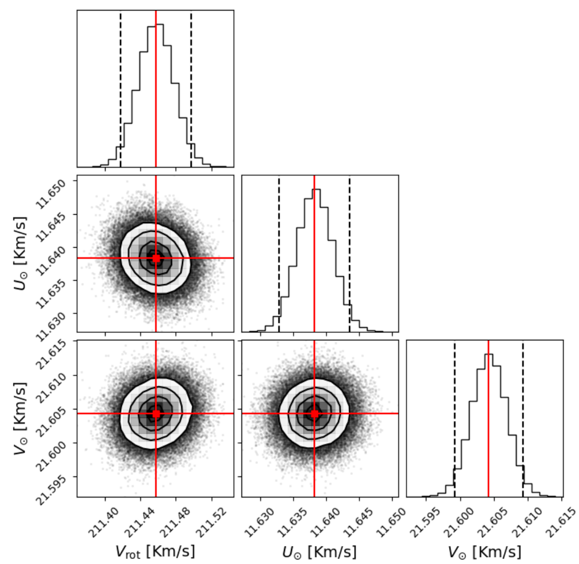


Figure 3: Posterior distribution of the parameters for the first model.

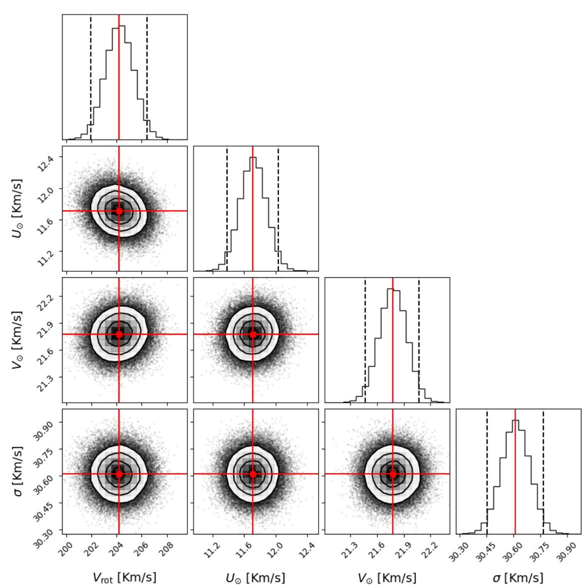


Figure 4: Posterior distribution of the parameters for the second model.