Rotational Velocity in the Milky Way

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Abstract

In this report we present a model for the rotation of stars in the Milky Way around its center. In a fisrt simpler model, we assume that the stars in the GC (Galactic Center) frame move in circular orbits around its origin with a constant velocity V_{rot}, as well as the LSR (Local Standard of Rest), and that the Sun frame is moving with a velocity $U_\odot$ in the x-direction and $V_\odot$ in the y-direction in the GC frame with respect to the LSR. Then, we present a second, more complex model, in which we add, for each star, a random component $v'_{\mathrm{rand}} \sim N(0, \sigma^2)$ to its radial velocity.

In this report, we used the data provided by GAIA DR2 [1], considering, for each star on the galactic plane, its longitude, parallax, and radial velocity, along with the respective uncertainties.

These data are used to make Bayesian inference on the parameters of the two models, namely $\boldsymbol{\theta_1} = (V_{rot}, U_\odot, V_\odot)$ and $\boldsymbol{\theta_2} = (V_{rot}, U_\odot, V_\odot, \sigma)$, using MCMC (Monte Carlo Markov Chain) to estimate the non-normalized posterior of the two models.

1 Datas

In this work we will use data taken from GAIA DR2 [1]. From the vast dataset of stars analyzed by GAIA, we first select only those for which radial velocities, v_{rad}, were measured relative to the Sun using the Doppler effect. To manage the dataset size efficiently, we then apply a random selection to significantly reduce the quantity of data, imposing the random index of data to be less than 100000000.

For each selected star, we extract key parameters from GAIA DR2, including parallax p and its associated error σ_{p}, radial velocity v_{rad} with its measurement uncertainty σ_{v}, and galactic coordinates, i.e. latitude b and longitude l.

To focus our analysis on stars located within the galactic plane, we impose a selection criterion of $|b| < 5°$. Additionally, to ensure the reliability of the data, we retain only stars with a relative parallax error smaller than 20% and a radial velocity error below 5 $\mathrm{km\,s^{-1}}$.

2 Model

2.1 Physical Model

In our model, we only consider the motion on the galactic plane ($b \approx 0$). We assume that each star moves with a circular orbit around the Galactic Center GC with the same speed, V_{rot}. Fixing the frame of reference as in figure **??**, a generic star s has a velocity $\boldsymbol{v}_{\mathrm{s}}$ given by:

$$\boldsymbol{v}_{\mathrm{s}} = V_{\mathrm{rot}} \begin{pmatrix} \sin(\varphi) \\ -\cos(\varphi) \end{pmatrix} \tag{1}$$

where φ is the angle from the x-axis of the star with respect to the GC.

In the following, primate vectors are in the frame of reference of the Sun, whereas unprimed ones are in the frame of reference of the GC. Angles are supposed to be expressed as radians.

In particular, the Sun in this frame of reference has $\varphi = \pi$, and we assume it to be at a fixed distance $R = 8300\,\mathrm{pc}$ with respect to the GC. In this model, the drift velocity of the Local Standard of Rest, LSR, is also given by equation **??**. In addition, for the Sun, we consider also its peculiar motion with respect to the LSR, given by a vector composed by two component, $U_\odot$ along the x-axis, and $V_\odot$ along the y one. In this way its velocity is given by the equation:

$$\boldsymbol{v}_\odot = \begin{pmatrix} 0 \\ V_{rot} \end{pmatrix} + \begin{pmatrix} U_\odot \\ V_\odot \end{pmatrix} \tag{2}$$

Therefore, the velocity $\boldsymbol{v'_s}$ of a star s (in the frame of reference of the Sun) at distance d from the Sun is given by the equation:

$$\boldsymbol{v'_s} = \boldsymbol{v_s} - \boldsymbol{v}_\odot \tag{3}$$

We can now project the velocity $\boldsymbol{v'_s}$ onto radial direction with respect to the Sun $\hat{e}'_r$, given by

$$\hat{e}'_r = \begin{pmatrix} \cos(l) \\ \sin(l) \end{pmatrix} \tag{4}$$

The radial component $v_{\mathrm{rad}'}$ of the velocity of a star with longitude l in the sun frame of reference is finally given by:

$$\begin{aligned} v_{\mathrm{rad}'} &= \boldsymbol{v'_s} \cdot \hat{e}'_r = \\ &= V_{\mathrm{rot}} \Big[\sin\varphi \cos l - (1 + \cos\varphi) \sin l \Big] - U_\odot \cos l - V_\odot \sin l \end{aligned} \tag{5}$$

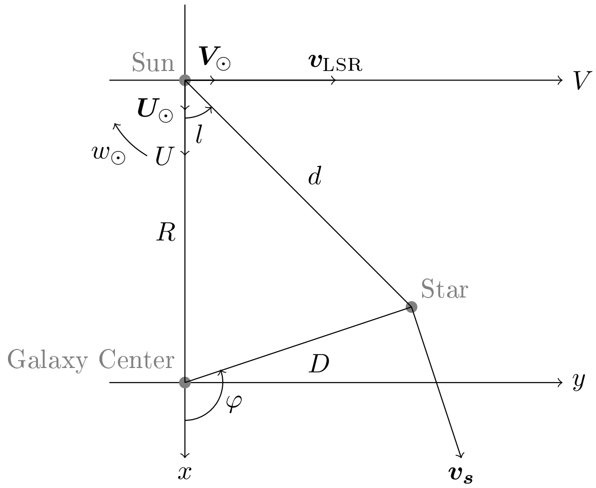

Figure 1: Frames of reference. The angular frequency $w_\odot$ will only contributes to the tangential component of the velocities of the stars with respect to the Sun's frame of reference.

Eq.**??** must be adapted to the actual data provided by GAIA, which means expressing $sin\varphi$ and $cos\varphi$ in terms of l and the parallax p, expressed in milliarcoseconds (mas). First of all, the distance in parsec can be computed as:

$$d[\text{pc}] = \frac{1000}{p[\text{mas}]} \tag{6}$$

Then, by geometric considerations we can evaluate $D, \sin\varphi$ and $\cos\varphi$ as a function of R, d and l (see figure **??**), obtaining:

$$D = \sqrt{d^2 + R^2 - 2dR\cos l} \tag{7}$$

$$\sin\varphi = \pm\sqrt{1-\cos^2\varphi} = \frac{d\sin l}{D} \tag{8}$$

$$\cos\varphi = \frac{d\cos l - R}{D} \tag{9}$$

By substituting eq.**??**-**??** into eq.**??**, we get an expression for the prediction of the model for the radial component of the velocity of star i $\hat{v}^{(1)}_{rad,i}(l_i, p_i)$ as a function of the measurements of its longitude and parallax l_i, p_i:

$$\begin{aligned}\hat{v}^{(1)}_{rad,i}(l_i,p_i) =& V_{\text{rot}}\sin l_i\Big(\frac{R}{\sqrt{(\frac{1000}{p_i})^2 + R^2 - 2(\frac{1000}{p_i})R\cos l_i}} - 1\Big) - \\ & - U_\odot\cos l_i - V_\odot \sin l_i\end{aligned} \tag{10}$$

In a second, more sofisticated model, we add to the model for the radial velocities a random variable $v_{\text{rand,i}}$ distributed with null expected value and variance σ^2, in order to consider also the peculiar motion of each star, obtaining:

$$\begin{aligned}\hat{v}^{(2)}_{rad,i}(l_i,p_i) =& V_{\text{rot}}\sin l_i\Big(\frac{R}{\sqrt{(\frac{1000}{p_i})^2 + R^2 - 2(\frac{1000}{p_i})R\cos l_i}} - 1\Big) - \\ & - U_\odot\cos l_i - V_\odot \sin l_i + v_{\text{rand,i}}\end{aligned} \tag{11}$$

2.2 Statistical Model

In the following we describe the Bayesian inference we have made on the parameters of the two models. We are going to use Monte Carlo Markov Chains MCMC in order to get an approximation for the non-normalized propability distributrions of our parameters $\mathbb{P}^*(\theta|\mathcal{D})$, considering only the numerator of Bayes' theorem (posterior):

$$\mathbb{P}^*(\theta|\mathcal{D}) = \mathbb{P}(\mathcal{D}|\theta)\mathbb{P}(\theta) \tag{12}$$

where $\mathbb{P}(\mathcal{D}|\theta)$ is the likelihood and $\mathbb{P}(\theta)$ is the prior. In practice, we use, both for the likelihood and for the prior, their logarithm.

For the first model we have a set of three parameters, named $\boldsymbol{\theta_1} = (V_{\text{rot}}, U_\odot, V_\odot)$ and for this model we will assume each measurement of the stars' radial velocity to be a random variable sampled from a gaussian distributrion centered at the true value v_{rad}, with standard deviation given by the error reported by GAIA, σ_{v} ($v_{\text{rad}} \sim N(v_{\text{rad}}, \sigma_{\text{v}})$). Moreover, we will also assume the measurements to be mutually indipendent. From this two assumptions follow directly that our likelihood will be a product of gaussians. So the log-likelihood for this model is:

$$\log\mathbb{P}^{(1)}(\mathcal{D}|\theta_1) = -\frac{1}{2}\sum_i[\log(2\pi\sigma^2_{\text{v},i}) + \frac{(v_{rad,i} - \hat{v}^{(1)}_{rad,i})^2}{\sigma^2_{\text{v},i}}] \tag{13}$$

As prior for this model, we firstly assume the three parameters to be indipendent each other, which is pretty realistic since the rotational motion of all the stars shouldn't influence the peculiar motion of the Sun, and its two components on the galactic should be indipendent a priori. For this reason the prior is the product of the priors of each single parameter. Then, we choose a flat prior for $V_{\text{rot}} \in [0, 500\,\text{km}\,\text{s}^{-1}]$, in order to be sure to contain typical star rotational motion in galaxies (**METTERE REFERENZA**). For $U_\odot$ and $V_\odot$, instead, we chose a gaussian prior, $\log\mathbb{P}^{(1)}(U_\odot) \sim (\frac{U_\odot}{v_{\text{gal}}})^2$ (the same for $V_\odot$), since we expect it to be like a thermal motion, perfectly described by a gaussian probability distribution centered in $0\,\text{km}\,\text{s}^{-1}$; we chose for the standard deviation of these priors to be $v_{\text{gal}} = 200\,\text{km}\,\text{s}^{-1}$ since, as previously described, these are the order of magnitude for stars speed in galaxies.

For the second model, the set of parameters is made by four, $\boldsymbol{\theta_2} = (V_{rot}, U_\odot, V_\odot, \sigma)$. In addition to the assumption made in the first simple model we also assume the measurement of the stars' parallax to be a random variable sampled from a gaussian distributrion centered at the true value p, with standard deviation given by the error reported by GAIA, σ_{p} ($p \sim N(p, \sigma_{\text{p}})$), and it is also quite realistic that these measurement are independent of each other, so the log-likelihood for this model is:

$$\log\mathbb{P}^{(2)}(\mathcal{D}|\theta_2) = -\frac{1}{2}\sum_i\left\{\log[2\pi(\sigma_i^2 + \sigma^2)] + \frac{(v_{rad,i} - \hat{v}^{(2)}_{rad,i})^2}{\sigma_i^2 + \sigma^2}\right\} \tag{14}$$

where σ_i^2 is the sum of the variance for the normal distribution on the radial velocitiy measurements and for the one on the parallax propagated through the model (see equation **??**).

We keep for the first three parameters the same priors decided in the first model, while we also assume the fourth parameter σ to be uncorrelated to the others (stars peculiar motion at a first glance shouldn't depend on the Sun's one either on their average rotational motion).

GAIA measurements are affected by statistical uncertainties on the evaluations of the parallax and the radial velocity. We assume the measurements to be random variables sampled from a gaussian distributrion centered at the true value of the respective quantity, with standard deviation given by the error reported by GAIA ($v_{rad} \sim N(v_{\mathrm{rad}}, \sigma_{\mathrm{v}})$). Assuming the pysical model to be exact, and the measurements to be independant, the difference between the direct measure of the radial velocity, and the corresponding value given by the model by eq.**??** is a random variable with variance that can be computed through propagation of errors as described in the following.

3 DataAnalysis

4 Conclusions

References

[1] A. G. A. Brown et al. "Gaia Data Release 2: Summary of the contents and survey properties". In: *Astronomy amp; Astrophysics* 616 (Aug. 2018), A1. ISSN: 1432-0746. DOI: 10.1051/0004-6361/201833051. URL: http://dx.doi.org/10.1051/0004-6361/201833051.