Rotational Velocity in the Milky Way

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1 Introduction

In this excercise we aim to model the rotation of stars within the Milky Way around its center. We introduce an inertial frame of reference centered at the center of the galaxy, with xy-plane coinciding with the galactic plane and x-axis pointing away from the Sun. We will denote it as CG frame. We are going to assume that the stars in the CG frame move in circular orbits around its origin with a constant velocity V_{rot}. We are going to assume that also LSR (Local Standard of Rest) is moving with a velocity V_{rot} in the y-direction in the CG frame. We also introduce a frame of reference centered at the Sun, with the x-axis pointing towards the center of the galaxy, and the z-axis pointing towards the North Galactic Pole. We will denote it as Sun frame. The Sun frame is rotating with an angular velocity w around the z-axis due to the rotation of the galaxy with respect to the CG frame. We are going to assume that Sun frame is moving with a velocity U_0 in the x-direction and V_0 in the y-direction in the CG frame with respect to the LSR.

Our goal is to model the velocity of stars in the radial direction in the Sun frame of reference, and find V_rot, U_0 and V_0 using Baysian Inference on the data provided by the GAIA mission. The data contains the parallax and the radial motion as well as the longitude of stars with respect to the Sun frame of reference.

2 Model

2.1 Introduction

2.2 Coordinate change

GAIA provides measurements of the radial velocity relative to the Sun's frame of reference. In our model, the Sun's frame moves around the center of the galaxy with a drift velocity (that of the LSR) plus a random vector. In the following, primate vectors are in the frame of reference of the Sun, whereas unprimed ones are in the frame of reference of the center of the galaxy. Angles are supposed to be expressed as radians. Calling $\boldsymbol{v}_0$ the total velocity of the Sun relative to the center of the galaxy, we have the following relation:

$$\boldsymbol{v}_0 = \boldsymbol{v}_{LSR} + \boldsymbol{v}_{rand} \tag{1}$$

We can fix the frames of reference in the center of the Galaxy and on the Sun as in fig.1. In the picture, all the velocities are represented in the frame of reference fixed at the center of the galaxy. In our model, in this frame, all the stars (and the LSR frame) move around the center with velocity V_{rot}, therefore, the velocity for a star s at angle φ from the x-axis is:

$$\begin{aligned} \boldsymbol{v}_s &= V_{rot}(-\hat{e}_\varphi) \\ \hat{e}_\varphi &= \begin{pmatrix} -\sin(\varphi) \\ \cos(\varphi) \end{pmatrix} \end{aligned} \tag{2}$$

In particular, we fix $\varphi = \pi$ for the Sun. Therefore, the velocity of the Sun, in the rest frame of the Galaxy is given by the equation:

$$\boldsymbol{v}_0 = \begin{pmatrix} 0 \\ V_{rot} \end{pmatrix} + \begin{pmatrix} U_0 \\ V_0 \end{pmatrix} \tag{3}$$

The frame of reference of the sun is moving with velocity $\boldsymbol{v}_0$ given by eq.1, and its axis are rotating with an angular velocity $\boldsymbol{w}_{sun} = -w_{sun}\hat{e}_z$. Therefore, the velocity $\boldsymbol{v}_s'$ of a star s at distance d from the Sun is given by the equation

$$\boldsymbol{v}_s' = \boldsymbol{v}_s - \boldsymbol{v}_0 - \boldsymbol{w}_{sun} \times \hat{e}_r' d = \boldsymbol{v}_s - \boldsymbol{v}_0 + w_{sun}\hat{e}_l' d \tag{4}$$

The radial component of the velocity of a star with longitude l in the sun frame of reference is finally given by:

$$\begin{aligned} \hat{e}_r' &= \begin{pmatrix} \cos(l) \\ \sin(l) \end{pmatrix} \\ v_s^{\text{rad}'} &= \boldsymbol{v}_s' \cdot \hat{e}_r' = \\ &= V_{rot}\Big[\sin\varphi\cos l - (1+\cos\varphi)\sin l\Big] - U_0\cos l - V_0 \sin l \end{aligned} \tag{5}$$

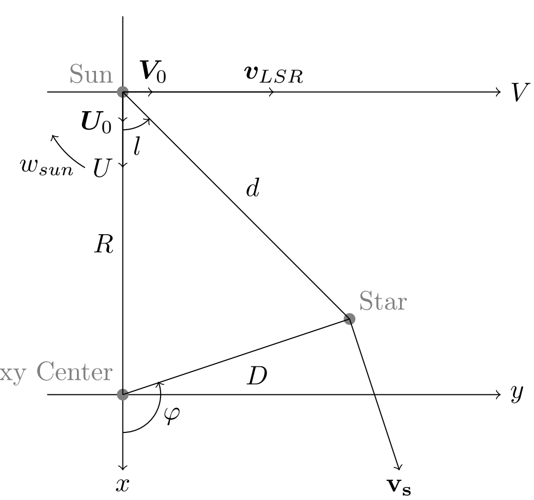

Figure 1: Frames of reference.

Eq.5 must be adapted to the actual data provided by GAIA, which means expressing $sin\varphi$ and $cos\varphi$ in terms of l and the parallax p, expressed in arcoseconds. First of all, the distance in parsec can be computed as:

$$d[pc] = \frac{1000}{p[arcsec]} \tag{6}$$

Then, by applying the cosine theorem two times for R, d, D, l, φ (fig.1), $cos\varphi$ can be written as:

$$\cos\varphi = \frac{d\cos l - R}{\sqrt{d^2 + R^2 - 2dR\cos l}} \tag{7}$$

and, therefore,

$$\sin\varphi = \pm\sqrt{1 - \cos^2\varphi} = \frac{d\sin l}{\sqrt{d^2 + R^2 - 2dR\cos l}} \tag{8}$$

By substituting eq.6-**??** into eq.5, we get an expression for the prediction of the model for the radial component of the velocity of star i $v_{rad}^{mod}(l_i, p_i)$ as a function of the measurements of its longitude and parallax l_i, p_i.

2.3 Statistical Model

2.4 Uncertainties

GAIA measurements are affected by statistical uncertainties on the evaluations of the parallax and the radial velocity. We assume the measurements to be random variables sampled from a gaussian distributrion centered at the true value of the respective quantity, with standard deviation given by the error reported by GAIA ($v_{rad} \sim N(\tilde{v}_{rad})$). Assuming the pysical model to be exact, and the measurements to be independant, the difference between the direct measure of the radial velocity, and the corresponding value given by the model by eq.5 is a random variable with variance that can be computed through propagation of errors as described in the following.

equation

3 Data Analysis

4 Conclusions