Tutorial 5 7.11

1. How to simulate a Turing machine using nondeterministic counter automata with 0-tests?

   * WLOG assume that we have only 1s and 0s on the tape (we can encode any alphabet using a two-character alphabet)
   * simulate a binary stack using two counter automata with 0-tests
   * interpret machine's tape as two stacks and simulate it using four automata
   * details: https://www.cs.usfca.edu/~galles/cs411/lecture/lecture14.pdf
   * another idea: use one counter to store tape's content, second one to remember where machine's head is and some helper counters to support tape operations

2. For every $m > 0$ create a Petri net of size $O(n+m)$ that is bounded by $F_m(n)$ and not bounded by any smaller number, where

i) $F_1(n) = 2n$

ii) $F_{m+1}(n) = F_m^n(1) = \overbrace{F_m(F_m(\ldots(F_m(1))\ldots))}^{n \text{ times}}$

* we can construct a VASS instead of a Petri net (since the two models are equivalent and we defined how to simulate one with another during the previous tutorial; however, here it will be easier to create a Petri net based on the VASS itself)
* for a given n-dimensional VASS (Q, T) with initial configuration (p, u) we define its size as

$$|Q| + |T| + \sum_{i=1}^{n} u(i) + \sum_{t \in T} \sum_{i=1}^{n} |t(i)|$$

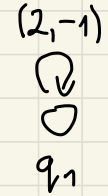
* how does a closed-form expression for $F_m(n)$ look like?

$F_2(n) = F_1(F_1(\dots(F_1(1))\dots)) = 2^n$

$F_3(n) = F_2(F_2(\dots(F_2(1))\dots)) = 2^{2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}}$ (n twos)

and so on

* first, compute $F_1(1)$

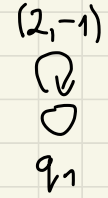


initial config.

$(q_1, (0,1))$

interpretation: first coordinate stores the result of the current computation, second coordinate is the argument to F_1

thus, for $F_1(n)$ we have

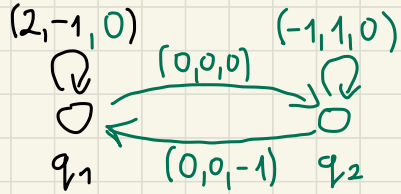


initial config.

$(q_1, (0,n))$

* now, we can build a VASS for $F_2(n)$ using the construction above

* VASS for $F_2(n) = F_1(F_1(\dots(F_1(1))\dots))$



initial config.

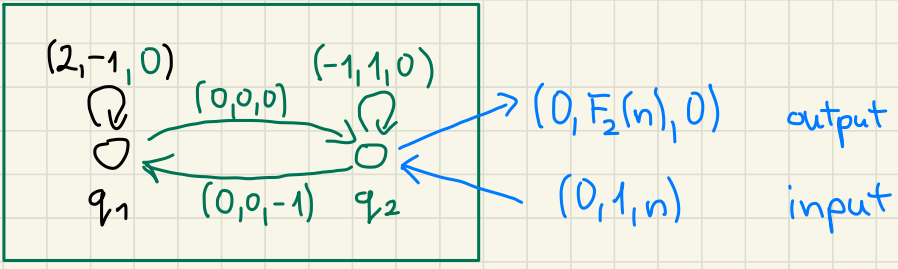
$(q_2, (0,1,n))$

interpretation: two first coordinates serve the same purpose as previously, the third one can be interpreted as the argument for F_2 – the number of times we have to compose F_1 with itself

the initial configuration says that we apply the n-th composition of F_1 to 1

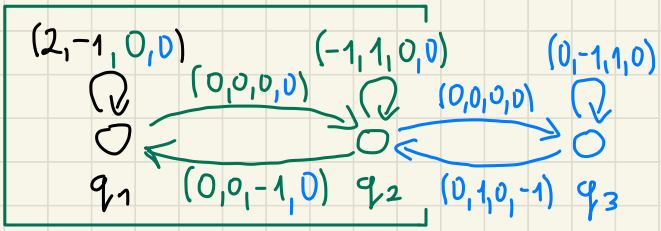
loop in state q_2 updates the parameter for F_1 to the current computation result

when we reach deadlock, the second coordinate stores the final result, hence, we can interpret the construction above as a blackbox computing $F_2(n)$



we can use this observation to construct a VASS computing $F_3(n)$

* "induction" step – VASS for $F_3(n)$



initial configuration: $(q_3, (0,0,1,n))$

it says that we have to apply the n-th composition of F_2 to 1

first we go from q_3 to q_2: we subtract 1 from the fourth coordinate, which can be interpreted as the start of computation of the most inner call of F_2 in $F_3(n) = F_2(F_2(\dots(F_2(1))\dots))$

moreover, we add 1 to the second coordinate to obtain $0,1,1$ as the prefix of the current configuration – this starts the computation of $F_2(1)$ in the green box

the computation ends in state q_2 with configuration prefix equal to $0, F_2(1), 0$

next, we go through q_3 and approach green box again with a new request – to compute F_2 with the argument $F_2(1)$ (as the conf. prefix is $0, 1, F_2(1)$) ...

* it is easy to see how to continue the construction for $F_4(n)$, $F_5(n)$, ...

3. Prove that in the pessimistic case the maximum size of the coverability tree is not smaller than $Ack(n)$, where n is the size of the net.

* we can assume $Ack(n) \approx F_n(n)$

* formally:

$$Ack(m,n) = \begin{cases} n+1 & m=0 \\ Ack(m-1,1) & m>0,\ n=0 \\ Ack(m-1, Ack(m,n-1)) & m>0,\ n>0 \end{cases}$$

$m \backslash n$	0	1	2	3	4	n
0	1	2	3	4	5	$n+1$
1	2	3	4	5	6	$n+2 = 2+(n+3)-3$
2	3	5	7	9	11	$2n+3 = 2\cdot(n+3)-3$
3	5	13	29	61	125	$2^{(n+3)}-3$
4	$13 = 2^{2^2}-3$	$65533 = 2^{2^{2^2}}-3$	$2^{65536}-3 = 2^{2^{2^{2^2}}}-3$	$2^{2^{65536}}-3 = 2^{2^{2^{2^{2^2}}}}-3$	$2^{2^{2^{65536}}}-3 = 2^{2^{2^{2^{2^{2^2}}}}}-3$	$\underbrace{2^{2^{\cdot^{\cdot^{\cdot^2}}}}}_{n+3}-3$

* thus the proof follows the construction presented for the previous problem

4. Improve Lipton's construction presented during the lecture to avoid the exponential blowup of the instruction number.

* consider $Dec_{m+1}(x)$ and think about how to improve it (during tutorials), then analyze the whole construction once again (homework)

$Dec_{m+1}(x)$: ← we would like to have one universal Dec_i for all possible calls

loop

| a_m ++, $\hat{a}_m$ --

| loop

| | b_m ++, $\hat{b}_m$ --

| | x --, $\hat{x}$ ++ ← if there is one gadget for Dec_i, how does it know what to increase?

| $Dec_m(b_m)$ ← how does it know where to return?

$Dec_m(a_m)$

reverse the side-effect on a_m, b_m and ensure that

this part is repeated $2^{2^m} \cdot 2^{2^m} = 2^{2^{m+1}}$ times

$Dec_{m+1}(x)$:

loop

| $a_m{+}{+}$, $\hat{a}_m{-}{-}$
| loop
| | $b_m{+}{+}$, $\hat{b}_m{-}{-}$
| | $x{-}{-}$, $\hat{x}{+}{+}$
| $Dec_m(b_m)$

$Dec_m(a_m)$

new variables with dashes representing the flags / semaphores for previously defined variables

$\bar{x}{-}{-}$
$x{-}{-}$, $\hat{x}{+}{+}$
$\bar{x}{+}{+}$
or
$\bar{y}{-}{-}$
$y{-}{-}$, $\hat{y}{+}{+}$
$\bar{y}{+}{+}$
or
...

← used only when $\bar{x} = 1$

← used only when $\bar{y} = 1$

$c_i{+}{+}$, $\overline{b_m{+}{+}}$, $Dec_m(b_m)$, $\overline{b_m{-}{-}}$, $c_i{-}{-}$

$c_i{+}{+}$ can be interpreted as a call statement and $\overline{b_m{+}{+}}$ as a parameter for the called function (it informs that some action have to be performed on b_m)

$c_i{-}{-}$ asserts that the execution goes back to the right place

* it remains to work on the details
* we update the other functions in a similar way